

## Assignment 1: Due Oct 9 (Monday) before Midnight

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PROGRAMMING LANGUAGE USED = R

**Directions:** Respond appropriately to the following questions. Upload your final assignment onto e-learning by the due date. Insert all tables and images, if any, into your word file (or pdf) so that answers are all in one place. Copy and paste your code (no matter what statistical software you are using, including STATA) at the end of the assignment or submit a separate file. Part of your grade (5 points) will be based on the code, and the remaining will be based on your ability to follow directions and *fully* explain econometric models (75 points). This is an individual assignment. You must turn in your own word document (or pdf). Late submissions within 24 hours will receive 50% of the original points, late submissions within 48 hours will receive 25%, and so on.

1. (Lecture 2) (12 points) Use the data in CHARITY [obtained from Franses and Paap (2001)] to answer the following questions:

- i. (2 points) What is the average gift in the sample of 4,268 people (in Dutch guilders)? What percentage of people gave no gift?

Ans : average gift in the sample of 4,268 people = 7.44

```
> mean(charity$gift)
```

```
[1] 7.44447
```

Percentage of people gave on gift = 60.00469%

```
> 100*(sum(charity$gift == 0) / nrow(charity))
```

```
[1] 60.00469
```

- ii. (2 points) What is the average mailing per year? What are the minimum and maximum values?

Ans: Average mailings per year : 2.04955

```
> mean(charity$mailsyear)
```

```
[1] 2.049555
```

Minimum and maximum values = 3.5, 0.25

```
> max(charity$mailsyear)
```

```
[1] 3.5
```

```
> min(charity$mailsyear)
```

```
[1] 0.25
```

- iii. (2 points) Estimate the model

$$gift = \beta_0 + \beta_1 mailsyear + u$$

By OLS and report the results in the usual way (as shown in the lectures), including the sample size and R-squared.

Ans: Linear regression summary

```
lm(formula = gift ~ mailsyear, data = charity)
```

Residuals:

Min	1Q	Median	3Q	Max
-11.287	-7.976	-5.976	2.687	245.999

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	2.0141	0.7395	2.724	0.00648	**
mailsyear	2.6495	0.3431	7.723	1.4e-14	***

---

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	2.0141	0.7395	2.724	0.00648	**
mailsyear	2.6495	0.3431	7.723	1.4e-14	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 14.96 on 4266 degrees of freedom

Multiple R-squared: 0.01379, Adjusted R-squared: 0.01356

F-statistic: 59.65 on 1 and 4266 DF, p-value: 1.404e-14

Sample size =  $df(\text{degrees of freedom}) + 2 = 4266 + 2 = 4268$  R-squared = 0.01379

(4 points) Interpret the slope coefficient. If each mailing costs one guilder, is the charity expected to make a net gain on each mailing? Does this mean the charity makes a net gain on every mailing? Explain.

Ans: The slope coefficient for mailsyear = 2.6495. For each additional mailing sent per year, the charity can expect an increase of approximately 2.6495 Dutch guilders in the average gift received from donors. Yes, the charity makes a net gain of 2.64 on every mailing. Since cost is 1, the net surplus is  $2.649 - 1 = 1.649$ . So, each mailing costs one guilder then the expected profit from each mailing is estimated to be 1.65 guilders.

But some mailings generate less than the mailing cost; other mailings generated much more than the mailing cost.

(2 points) What is the smallest predicted charitable contribution in the sample? Using this simple regression analysis, can you ever predict zero for gift?

Ans: smallest predicted charitable contribution in the sample = 2.0141(intercept)  
+ (2.6495)[slope] \* (min(mailsyear))

```
> 2.0141 + (2.6495) * min(charity$mailsyear)
[1] 2.676475
```

No we can never predict zero for gift. If mailsyear = 0 then the value of gifts will be equal to the intercept which is 2.014. Therefore, with this estimated equation, we can never predict zero for gifts.

2.(Lecture 3) (14 points) The file CEOSAL2 contains data on 177 chief executive officers and can be used to examine the effects of firm performance on CEO salary.

(3 points) Estimate a model relating annual salary to firm sales and market value. Make the model of the constant elasticity variety for both independent variables. Report the results in the usual way.

Ans :

```
> model <- lm(lsalary ~ lsales + lmktval, data = ceosal2)
> summary(model)
```

Call:

```
lm(formula = lsalary ~ lsales + lmktval, data = ceosal2)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-2.28060	-0.31137	-0.01269	0.30645	1.91210

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	4.62092	0.25441	18.163	< 2e-16	***
lsales	0.16213	0.03967	4.087	6.67e-05	***
lmktval	0.10671	0.05012	2.129	0.0347	*

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5103 on 174 degrees of freedom

Multiple R-squared: 0.2991, Adjusted R-squared: 0.2911

F-statistic: 37.13 on 2 and 174 DF, p-value: 3.727e-14

(4 points) Add *profits* to the model from part (i), re-estimate the model and report the results in the usual way. Why can this variable not be included in

logarithmic form? Would you say that these firm performance variables explain most of the variation in CEO salaries?  
Ans :

Variable	Obs	Mean	Std. dev.	Min	Max
profits	177	207.8305	404.4543	-463	2700

We cannot do log transformation to profits, since the minimum value present in the sample for profits column is -463, which strictly indicates that there are negative values. And also speaking generally, the variable "profits" cannot be included in logarithmic form because it is possible for a company to have negative profits. Taking the logarithm of a negative number or zero is undefined.

```
> model <- lm(lsalary ~ lsales + lmktval + profits, data = ceosal2)
> summary(model)
```

Call:

```
lm(formula = lsalary ~ lsales + lmktval + profits, data = ceosal2)
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-2.27002 -0.31026 -0.01027  0.31043  1.91489
```

Coefficients:

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.687e+00   3.797e-01  12.343 < 2e-16 ***
lsales       1.614e-01   3.991e-02   4.043 7.92e-05 ***
lmktval      9.753e-02   6.369e-02   1.531  0.128
profits      3.566e-05   1.520e-04   0.235  0.815
---

```

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

```
Residual standard error: 0.5117 on 173 degrees of freedom
Multiple R-squared:  0.2993,    Adjusted R-squared:  0.2872
F-statistic: 24.64 on 3 and 173 DF,  p-value: 2.53e-13
```

Without adding the profits variable, the adj-r-squared is 0.2911, with adding profits variable the adj-r-squared decreased to 0.2972. These firm performance variables explain most of the variation in CEO salaries. The coefficient profit is very small there, profits are measured in millions so if profits increase by \$1 billion, the predicted salary increases by only 3.6%, with sales and market value held fixed.

Together, these variables explain almost 30% of the sample variation in log (salary). This is certainly not most of the variation

(3 points) Add the variable *ceoten* to the model in part (ii), re-estimate the model and report the results in the usual way. What is the estimated percentage return for another year of CEO tenure, holding other factors fixed?

```
> model <- lm(lsalary ~ lsales + lmktval + profits + ceoten, data = ceosal2)
> summary(model)
```

```
Call:
lm(formula = lsalary ~ lsales + lmktval + profits + ceoten, data = ceosal2)
```

Residuals:

	Min	1Q	Median	3Q	Max		
	-2.48792	-0.29369	0.00827	0.29951	1.85524		
	Source	SS	df	MS	Number of obs	=	177
Model		20.5768102	4	5.14420254	F(4, 172)	=	20.08
Residual		44.0694029	172	.256217459	Prob > F	=	0.0000
					R-squared	=	0.3183
					Adj R-squared	=	0.3024
Total		64.6462131	176	.367308029	Root MSE	=	.50618

lsalary	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
lsales	.1622339	.0394826	4.11	0.000	.0843012	.2401667
lmktval	.1017598	.063033	1.61	0.108	-.022658	.2261775
profits	.0000291	.0001504	0.19	0.847	-.0002677	.0003258
ceoten	.0116847	.005342	2.19	0.030	.0011403	.022229
_cons	4.55778	.3802548	11.99	0.000	3.807213	5.308347

The estimated % return for another year of CEO tenure holding other factors fixed is increased by **1.2%** for predicted salary.

(4 points) Find the sample correlation coefficient between the variables  $\log(\text{mktval})$  and *profits*. Are these variables highly correlated? What does this say about the OLS estimators? [Hint: You can use the stata command **correlate**.]

```
> cor(ceosal2$lmktval, ceosal2$profits, method = "pearson")
[1] 0.7768976
```

The correlation for both  $\log(\text{mktval})$  and *profits* = 0.77, which is highly positively correlated. High correlation (0.77) between  $\log(\text{mktval})$  and *profits* suggests a strong linear relationship between these variables, which can lead to multicollinearity in the above regression model.

3.(Lecture 4) (17 points) Refer to the example used in Lecture 4 to compare the returns to education at junior colleges and four-year colleges. The model after rearrangement is

$$\log(\text{wage}) = \beta_0 + \theta_1 \text{jc} + \beta_2 \text{totcoll} + \beta_3 \text{exper} + u,$$

where *totcoll* is total years of college. Use the data set TWOYEAR, which comes from Kane and Rouse (1995).

i. (5 points) Run the regression above and report the OLS estimates in the usual form, including the standard errors, sample size and R-squared. How do you interpret  $\theta_1$ ? Is it statistically significant?

Ans :

```
call:
lm(formula = lwage ~ jc + totcoll + exper, data = twoyear)

Residuals:
    Min       1Q   Median       3Q      Max
-2.10362 -0.28132  0.00551  0.28518  1.78167

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.4723256   0.0210602   69.910  <2e-16 ***
jc          -0.0101795   0.0069359   -1.468    0.142
totcoll      0.0768762   0.0023087   33.298  <2e-16 ***
exper       0.0049442   0.0001575   31.397  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4301 on 6759 degrees of freedom
Multiple R-squared:  0.2224,    Adjusted R-squared:  0.2221
F-statistic: 644.5 on 3 and 6759 DF,  p-value: < 2.2e-16
```

Null hypothesis =>  $\theta_1 = 0$  Alternative Hypothesis =>  $\theta_1 < 0$

When coming to  $\theta_1$ , the p-value = 0.142 is greater than 0.05, so we cannot reject null hypothesis.  $\theta_1$  is not statistically significant. We accept null hypothesis, i.e ( $\beta_1 - \beta_2$ )

= 0.  $\theta_1 = -0.0101795$ , This is the difference between the coefficients. P-value of  $\theta_1$  is  $0.142 > C_{0.005}$ . The null hypothesis is true.

So, jc is not statistically significant.

ii.)(2 points) The variable *phsrank* is the person's high school percentile. (A higher number is better. Forexample, 90 means you are ranked better than 90 percent of your graduating class.) Find the smallest, largest, and average *phsrank* in the sample.

Ans :

```
> min(twoyear$phsrank)
[1] 0
> max(twoyear$phsrank)
[1] 99
> mean(twoyear$phsrank)
[1] 56.15703
```

Smallest = 0, largest = 99, average = 56.15703

iii.)(4 points) Add *phsrank* to the model and report the OLS estimates in the usual form. Is *phsrank* statistically significant? How much is 10 percentage points of high school rank worth in terms of wage?Ans :

```
Call:
lm(formula = lwage ~ jc + totcoll + exper + phsrank, data = twoyear)

Residuals:
    Min       1Q   Median       3Q      Max
-2.09049 -0.28135  0.00538  0.28543  1.79060

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.4587472  0.0236211  61.756  <2e-16 ***
jc          -0.0093108  0.0069693  -1.336    0.182
totcoll      0.0754756  0.0025588  29.496  <2e-16 ***
exper        0.0049396  0.0001575  31.360  <2e-16 ***
phsrank       0.0003032  0.0002389   1.269    0.204
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4301 on 6758 degrees of freedom
Multiple R-squared:  0.2226,    Adjusted R-squared:  0.2222
F-statistic: 483.8 on 4 and 6758 DF,  p-value: < 2.2e-16
```

The p-value for *phsrank* is 0.204 which is greater than 0.05, null hypothesis cannot be rejected. Therefore, *phsrank* is not statistically significant. . If points of high school rank increase by 10% then there will be a 0.3% increase in wages.

To estimate how much a 10 percentage point increase in high school rank is worth in terms of wage, you can multiply the coefficient by 10 (since we are interested in a 10 percentage point change):

Estimated change in wage =  $0.0003032 * 10$

Estimated change in wage  $\approx 0.003032 * 100 = 0.3\%$  increase in wage

iv.) (3 points) Compare regression results in (i) and (iii), does adding *phsrank* to the model substantively change the conclusions on the returns to two- and four-year colleges? Explain.

Ans: Model (i) (Without *phsrank*):

Source	SS	df	MS	Number of obs	=	6,763
Model	357.752575	3	119.250858	F(3, 6759)	=	644.53
Residual	1250.54352	6,759	.185019014	Prob > F	=	0.0000
				R-squared	=	0.2224
				Adj R-squared	=	0.2221
Total	1608.29609	6,762	.237843255	Root MSE	=	.43014

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
jc	-.0101795	.0069359	-1.47	0.142	-.0237761	.003417
totcoll	.0768762	.0023087	33.30	0.000	.0723504	.0814021
exper	.0049442	.0001575	31.40	0.000	.0046355	.0052529
_cons	1.472326	.0210602	69.91	0.000	1.431041	1.51361

totcoll coefficient estimate: 0.0768762

P-value for totcoll:  $< 2.2e-16$  (highly significant)

Model 2 (With *phsrank*):



Source	SS	df	MS	Number of obs	=	6,763
Model	358.050568	4	89.5126419	F(4, 6758)	=	483.85
Residual	1250.24552	6,758	.185002297	Prob > F	=	0.0000
				R-squared	=	0.2226
				Adj R-squared	=	0.2222
Total	1608.29609	6,762	.237843255	Root MSE	=	.43012

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
jc	-.0093108	.0069693	-1.34	0.182	-.0229728	.0043512
totcoll	.0754756	.0025588	29.50	0.000	.0704595	.0804918
exper	.0049396	.0001575	31.36	0.000	.0046308	.0052483
phsrank	.0003032	.0002389	1.27	0.204	-.0001651	.0007716
cons	1.458747	.0236211	61.76	0.000	1.412442	1.505052

totcoll coefficient estimate: 0.0754756

P-value for totcoll: < 2.2e-16 (highly significant)

In both models, the coefficient estimate for totcoll is highly significant (p-value < 2.2e-16), and the estimated coefficient values are very close. The coefficient for totcoll in Model 2 (with phsrank) is slightly smaller than in Model 1 (without phsrank), but the difference is negligible.

Therefore, adding phsrank to the model does not substantively change the conclusion on the return of two and four-year colleges. T- statistic on jc gets even smaller in absolute value, about 1.33 when phsrank is added. However, the coefficient magnitude is almost equal. Hence, we can say that the base point remains unchanged. This implies that even though the difference is not significant, and the levels are standard, the return to junior college is estimated to be somewhat smaller.

v.)(3 points) The data set contains a variable called *id*. Explain why if you add *id* to the model you expect it to be statistically insignificant. What is the two-sided p-value?

Ans: In most cases, the variable "id" (ID Number) is expected to be statistically insignificant when added to a regression model for the following reasons:

Unique Identifier, No Meaningful Relationship, Low Variability

```

Call:
lm(formula = lwage ~ jc + totcoll + exper + phsrank + id, data = twoyear)

Residuals:
    Min       1Q   Median       3Q      Max
-2.08457 -0.28176  0.00556  0.28680  1.79322

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.452e+00  2.559e-02  56.750  <2e-16 ***
jc           -9.316e-03  6.970e-03  -1.337    0.181
totcoll       7.541e-02  2.561e-03  29.451  <2e-16 ***
exper         4.941e-03  1.575e-04  31.365  <2e-16 ***
phsrank       3.179e-04  2.400e-04   1.325    0.185
id            1.396e-07  2.103e-07   0.664    0.507
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4301 on 6757 degrees of freedom
Multiple R-squared:  0.2227,    Adjusted R-squared:  0.2221
F-statistic: 387.1 on 5 and 6757 DF,  p-value: < 2.2e-16

```

The two-sided p-value = 0.507

4. ) i.)(Lecture 4) (9 points) Use the data set GPA1 to answer this question.

(3 points) Run the regression  $colGPA$  on  $PC$ ,  $hsGPA$ , and  $ACT$  and obtain a 95% confidence interval for  $\beta_{PC}$ . Is the estimated coefficient statistically significant at the 5% level against a two-sided alternative?

Ans :

```
Call:
lm(formula = colGPA ~ PC + hsGPA + ACT, data = gpa1)
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-0.7901 -0.2622 -0.0107  0.2334  0.7570
```

Coefficients:

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.263520   0.333125   3.793 0.000223 ***
PC           0.157309   0.057287   2.746 0.006844 **
hsGPA        0.447242   0.093647   4.776 4.54e-06 ***
ACT          0.008659   0.010534   0.822 0.412513
```

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3325 on 137 degrees of freedom  
Multiple R-squared: 0.2194, Adjusted R-squared: 0.2023  
F-statistic: 12.83 on 3 and 137 DF, p-value: 1.932e-07

95% Confidence Interval for  $\beta_{PC} = 0.157309 \pm (t_{critical} \times 0.057287)$ .  $t_{critical}$  at 95% level is 1.97

colGPA	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
PC	.1573092	.0572875	2.75	0.007	.0440271	.2705913

Lower bound = 0.04402791

Upper bound = 0.2705901

The p-value is  $0.007 < 0.05$ , therefore it is statistically significant at 5% level

ii.) (3 points) discuss the statistical significance of the estimates  $\hat{\beta}_{hsGPA}$  and  $\hat{\beta}_{ACT}$  in part (i). Is *hsGPA* or *ACT* the more important predictor of *colGPA*? Explain.

Ans : Betahsgpa p-value is  $4.54e-06 < 0.05$ , therefore it is statistically significant.

BetaACT p-value is  $0.412 > 0.05$ , it is not statistically significant.

hsGPA is the more important predictor of colGPA than ACT, because of its statistical significance explained above.

iii.) (3 points) Add the two indicators *fathcoll* and *mothcoll* to the regression in part (i). Is either individually significant? Are they jointly statistically significant at the 5% level?

Ans : Individual significance

```
Call:
lm(formula = colGPA ~ PC + hsGPA + ACT + fathcoll + mothcoll,
    data = gpa1)

Residuals:
    Min       1Q   Median       3Q      Max
-0.78149 -0.25726 -0.02121  0.24691  0.74432

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.255554    0.335392   3.744 0.000268 ***
PC           0.151854    0.058716   2.586 0.010762 *
hsGPA        0.450220    0.094280   4.775 4.61e-06 ***
ACT          0.007724    0.010678   0.723 0.470688
fathcoll     0.041800    0.061270   0.682 0.496265
mothcoll    -0.003758    0.060270  -0.062 0.950376
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3344 on 135 degrees of freedom
Multiple R-squared:  0.2222,    Adjusted R-squared:  0.1934
F-statistic: 7.713 on 5 and 135 DF,  p-value: 2.083e-06
```

Both *fathcoll* and *mothcoll* have p-values greater than 0.05, therefore they are not statistically significant at 5% level.

Let's do joint hypothesis testing

```
model1 <- lm(colGPA ~ PC + hsGPA + ACT, data = gpa1)
model2 <- lm(colGPA ~ PC + hsGPA + ACT + fathcoll + mothcoll, data = gpa1)
summary(model2)
anova(model1, model2)
```

```
> anova(model1,model2)
```

Analysis of Variance Table

Model 1: colGPA ~ PC + hsGPA + ACT

Model 2: colGPA ~ PC + hsGPA + ACT + fathcoll + mothcoll

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	137	15.149				
2	135	15.094	2	0.054685	0.2446	0.7834

The estimated p-value is 0.7834, both(fathcoll and mothcoll) jointly are statistically insignificant at 5%level.

5.) (Lecture 5) (10 points) Use the data in WAGE1 for this exercise.i.) (4 points) Estimate the equation  $wage = \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 tenure + u$

and report the OLS estimates in the usual form. Save the residuals and plot a histogram.

[Hint: 1) You can obtain the residuals of each prediction by using the **residuals** command and storing these values in a variable named whatever you'd like, e.g., **predict resid\_wage, residuals**. 2) You can use the **histogram** command to plot a histogram, e.g., **histogram resid\_wage**.]

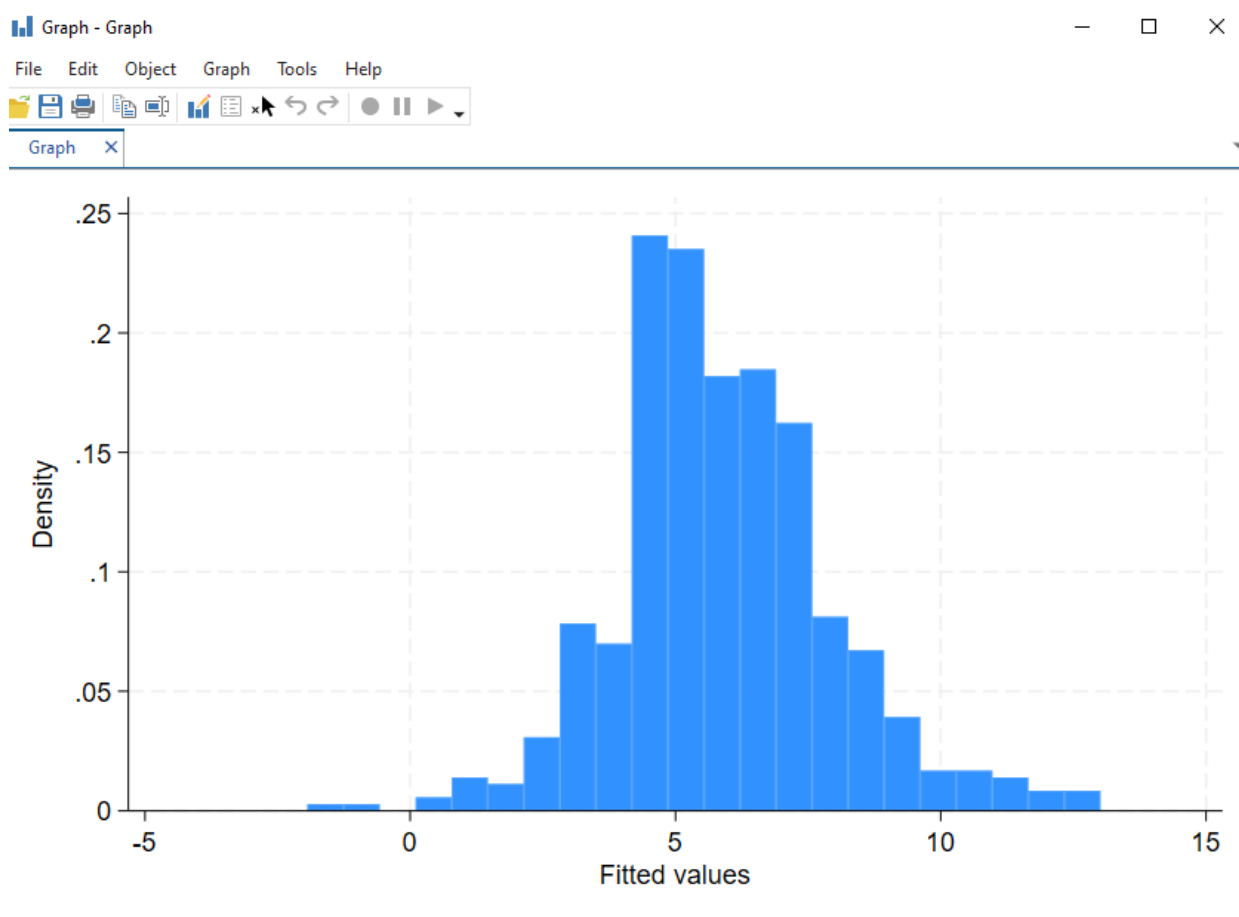
Ans :

```
Call:
lm(formula = wage ~ educ + exper + tenure, data = wage1)

Residuals:
    Min       1Q   Median       3Q      Max
-7.6068 -1.7747 -0.6279  1.1969 14.6536

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.87273    0.72896  -3.941 9.22e-05 ***
educ          0.59897    0.05128  11.679 < 2e-16 ***
exper         0.02234    0.01206   1.853  0.0645 .
tenure        0.16927    0.02164   7.820 2.93e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.084 on 522 degrees of freedom
Multiple R-squared:  0.3064,    Adjusted R-squared:  0.3024
F-statistic: 76.87 on 3 and 522 DF,  p-value: < 2.2e-16
```



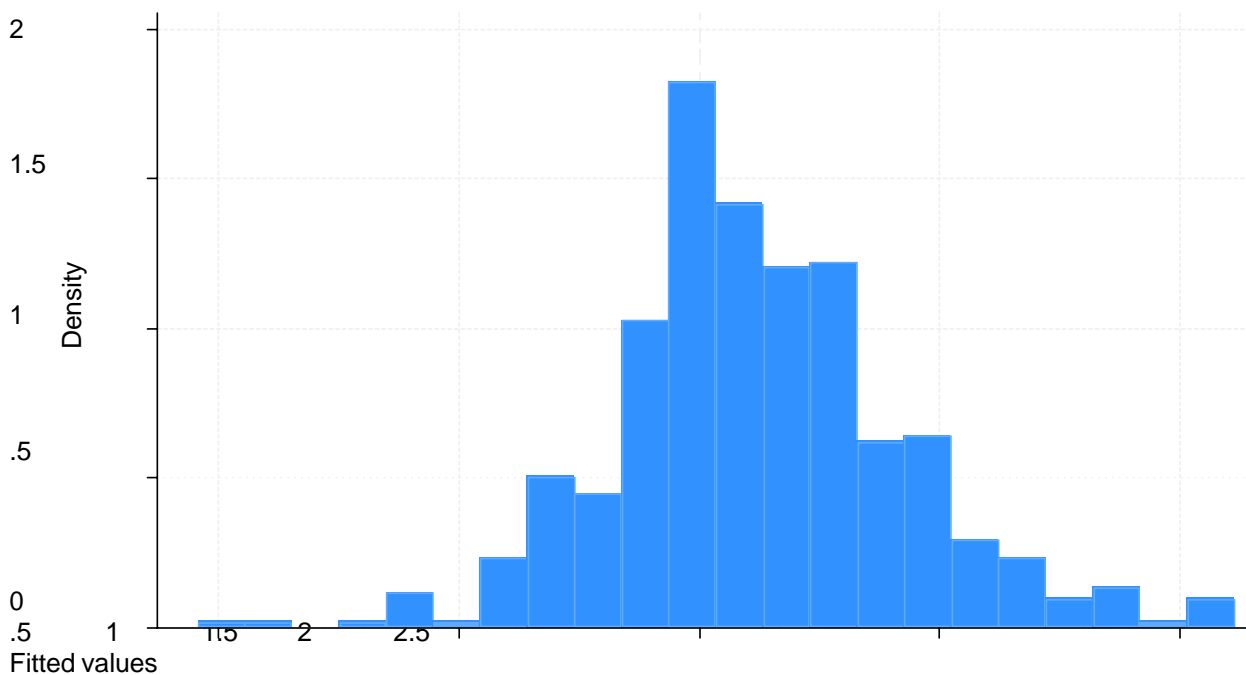
ii.) (4 points) Repeat part (i), but with  $\log(\text{wage})$  as the dependent variable.

```
Call:
lm(formula = lwage ~ educ + exper + tenure, data = wage1)

Residuals:
    Min       1Q   Median       3Q      Max
-2.05802 -0.29645 -0.03265  0.28788  1.42809

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.284360   0.104190   2.729  0.00656 **
educ         0.092029   0.007330  12.555 < 2e-16 ***
exper        0.004121   0.001723   2.391  0.01714 *
tenure       0.022067   0.003094   7.133 3.29e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4409 on 522 degrees of freedom
Multiple R-squared:  0.316,    Adjusted R-squared:  0.3121
F-statistic: 80.39 on 3 and 522 DF,  p-value: < 2.2e-16
```



iii.) (2 points) Would you say that Assumption MLR.6 is closer to being satisfied for the level-level model or the log-level model? Explain.

Ans : The residuals from the log(wage) i.e (ii) regression appear to be more normally distributed. Certainly the histogram in part (ii) fits under its comparable normal density better than in part (i), and the histogram for the wage residuals is notably skewed to the right.

6.) (Lecture 5) (13 points) The model we used in class to explain the standardized outcome on a final exam (*stndfnl*) in terms of percentage of classes attended, prior college grade point average, and ACTscore is  

$$stndfnl = \beta_0 + \beta_1 atndrte + \beta_2 priGPA + \beta_3 ACT + \beta_4 priGPA^2 + \beta_5 ACT^2 + \beta_6 priGPA \cdot atndrte + u.$$

i.) (2 points) Argue that

$$\frac{\Delta stndfnl}{\Delta priGPA} = \beta_2 + 2\beta_4 priGPA + \beta_6 atndrte.$$

ANS :

If we hold everything else constant except priGPA then we get the equation to be

$$\Delta stndfnl = \beta_2 priGPA + \beta_4 priGPA^2 + \beta_6 priGPA \cdot atndrte.$$

Using the calculus assumption  $\beta_4 priGPA^2 = 2\beta_4 priGPA$  and Using partial effect of interaction terms we get:-

$$\frac{\Delta stndfnl}{\Delta priGPA} = \beta_2 + 2\beta_4 priGPA + \beta_6 atndrte$$

ii.) (3 points) Use the equation above to estimate the partial effect of *priGPA* on *stndfnl* when *priGPA* is at its mean value 2.59, and *atndrte* is also at its mean value 82. Interpret your estimate. [Hint: The estimated OLS equation can be found in Lecture 5.]

Ans :

Beta2 = -1.63, Beta4 = 0.296, Beta6 = 0.0056 [Taken from the lecture-5 slides-12,13]

Now substitute these values in the above equation, with mean priGPA value = 2.59, atndrte = 82,

$$-1.63 + 2*(0.296)*(2.59) + (0.0056)*(82) = 0.36428$$

A 10 percentage points increase in priGPA increases stndfnl by 3.6 standard deviations from the mean final exam score



iii.) (4 points) Show that the equation can be re-written as

$$\text{stndfml} = \theta_0 + \beta_1 \text{atndrte} + \theta_2 \text{priGPA} + \beta_3 \text{ACT} + \beta_4 (\text{priGPA} - 2.59)^2 + \beta_5 \text{ACT}^2 + \beta_6 \text{priGPA} \cdot (\text{atndrte} - 82) + u, \text{ where } \theta_2 = \beta_2 + 2\beta_4(2.59) + \beta_6(82). \text{ How do you interpret } \theta_2?$$

Ans:

$$\begin{aligned} \text{stndfml} &= \beta_0 + \beta_1 \text{atndrte} + \beta_2 \text{priGPA} + \beta_3 \text{ACT} + \beta_4 \text{priGPA}^2 + \beta_5 \text{ACT}^2 + \beta_6 \text{priGPA} \cdot \text{atndrte} + u \rightarrow \text{①} \\ \text{Sol:} \quad \text{Substitute } \theta_2 &= \beta_2 + 2\beta_4(2.59) + \beta_6(82) \\ &\text{in the following equation} \\ &= \theta_0 + \beta_1 \text{atndrte} + \theta_2 \text{priGPA} + \beta_3 \text{ACT} \\ &\quad + \beta_4 (\text{priGPA} - 2.59)^2 + \beta_5 \text{ACT}^2 + \beta_6 \cdot \text{priGPA} \cdot (\text{atndrte} - 82) + u \\ &= \theta_0 + \beta_1 \text{atndrte} + (\beta_2 + 2\beta_4(2.59) + \beta_6(82)) \text{priGPA} \\ &\quad + \beta_3 \text{ACT} + \beta_4 \text{priGPA}^2 + 2\beta_4(2.59)^2 \\ &\quad - 2\beta_4 \text{priGPA} \cdot 2.59 + \beta_5 \text{ACT}^2 + \beta_6 \text{priGPA} \cdot \text{atndrte} \\ &\quad - \beta_6 \cdot \text{priGPA} \cdot 82 + u \\ &= \theta_0 + \beta_1 \text{atndrte} + \beta_2 \text{priGPA} + 2\beta_4 \text{priGPA} \cdot (2.59) \\ &\quad + \beta_6(82) \text{priGPA} + \beta_3 \text{ACT} + \beta_4 \text{priGPA}^2 + \beta_4(2.59)^2 \\ &\quad - 2\beta_4 \text{priGPA} \cdot 2.59 + \beta_5 \text{ACT}^2 + \beta_6 \text{priGPA} \cdot \text{atndrte} \\ &\quad - \beta_6 \text{priGPA} \cdot 82 + u \\ &= \theta_0 + \beta_1 \text{atndrte} + \beta_2 \text{priGPA} + \beta_3 \text{ACT} + \beta_4 \text{priGPA}^2 \\ &\quad + \beta_4(2.59)^2 + \beta_5 \text{ACT}^2 + \beta_6 \text{priGPA} \cdot \text{atndrte} + u \\ &\therefore \text{Since } \theta_0 + \beta_4(2.59)^2 \text{ is constant,} \\ &\text{this serves as an Intercept } \beta_0 \\ &\therefore \theta_0 + \beta_4(2.59)^2 = \beta_0 \end{aligned}$$

So, we ~~replace~~ replace  $\beta_0 + \beta_4(2.59)^2$  by  $\beta_0$ ,

our equation then becomes

$$\text{stndfnl} = \beta_0 + \beta_1 \text{atndrte} + \beta_2 \text{priGPA} + \beta_3 \text{ACT} + \beta_4 \text{priGPA}^2 + \beta_5 \text{ACT}^2 + \beta_6 \text{priGPA} \cdot \text{atndrte} + u$$

Interpretation of  $\theta_2$  :  $\theta_2$  can be interpreted as the partial effect of priceGPA on stndfnl if variables atndrte and priGPA<sup>2</sup> take on their mean values.

iv.)(4 points) Following (iii), suppose that, in place of  $\text{priGPA} \cdot (\text{atndrte} - 82)$ , you put  $(\text{priGPA} - 2.59) \cdot (\text{atndrte} - 82)$ . Now how do you interpret the coefficients on atndrte and priGPA?

Ans : Following (iii), if we replace  $\text{priGPA} \cdot (\text{atndrte} - 82)$  by  $(\text{priGPA} - 2.59) \cdot (\text{atndrte} - 82)$ , the equation gets transformed as

$$\text{stndfnl} = \theta_0 + \theta_1 \text{atndrte} + \theta_2 \text{priGPA} + \beta_3 \text{ACT} + \beta_4 (\text{priGPA} - 2.59)^2 + \beta_5 \text{ACT}^2 + \beta_6 \cdot (\text{priGPA} - 2.59) \cdot (\text{atndrte} - 82) + u$$

The coefficient of atndrte =  $\theta_1$

The coefficient of priGPA =  $\theta_2$

Interpretation of atndrte coefficient = The partial effect of atndrte on students final exam performance(stndfnl) at the mean value of priGPA (2.59).

Interpretation of priGPA coefficient = The partial effect of priGPA on students final exam performance(stdndfnl) at the mean value of atndrte (82).

Reference : Lecture-5 slide-11.

#### R-CODE USED

```
#Qno-1 library(wooldridge) data("charity") mean(charity$gift)
100*(sum(charity$gift == 0) / nrow(charity)) mean(charity$mailsyear) max(charity$mailsyear) min(charity$mailsyear)
model <- lm(gift ~ mailsyear, data = charity) summary(model)
2.0141 + (2.6495) * min(charity$mailsyear) #Qno-2
data("ceosal2")
model <- lm(lsalary ~ lsales + lmktval, data = ceosal2) summary(model)
model <- lm(lsalary ~ lsales + lmktval + profits, data = ceosal2) summary(model)
model <- lm(lsalary ~ lsales + lmktval + profits + ceoten, data = ceosal2) summary(model)
percentage_return <- 100 * (exp(1.168e-02) - 1) percentage_return
```

```

cor(ceosal2$lmtval,ceosal2$profits, method = "pearson")#Qno-3
data("twoyear")
model <- lm(lwage ~ jc + totcoll + exper, data = twoyear)summary(model)
min(twoyear$phsrank) max(twoyear$phsrank) mean(twoyear$phsrank)
model <- lm(lwage ~ jc + totcoll + exper + phsrank, data = twoyear)summary(model)
model <- lm(lwage ~ jc + totcoll + exper + phsrank + id, data = twoyear)summary(model)
#Qno-4 data("gpa1")
model <- lm(colGPA ~ PC + hsGPA + ACT, data = gpa1)summary(model)
0.157309 + (1.977431*0.057287)
0.157309 - (1.977431*0.057287)
model1 <- lm(colGPA ~ PC + hsGPA + ACT, data = gpa1)
model2 <- lm(colGPA ~ PC + hsGPA + ACT + fathcoll + mothcoll, data = gpa1)summary(model2)
anova(model1,model2)#Qno-5
data("wage1")
model <- lm(wage ~ educ + exper + tenure,data=wage1)summary(model)
residuals <- resid(model)
hist(residuals, main = "Histogram of Residuals", xlab = "Residuals")model <- lm(lwage ~ educ + exper +
tenure,data=wage1)

```

```

summary(model) residuals <- resid(model)
hist(residuals, main = "Histogram of Residuals", xlab = "Residuals")#Qno-6
-1.63 + (2*(0.296)*(2.59)) + (0.0056*82)
data("attend")
model <- lm(stndfnl ~ atndrte+priGPA+ACT+priGPA^2+ACT^2+priGPA*atndrte,data=attend)summary(model)
mean(attend$atndrte)
attend["priGPA"] = attend["priGPA"] - 2.59attend["priGPA"]
model1 <- lm(stndfnl ~ atndrte+priGPA+ACT+(priGPA^2)+(ACT^2)+(priGPA*atndrte),data=attend)summary(model1)
attend["atndrte"] = attend["atndrte"] - 82
model2 <- lm(stndfnl ~ atndrte+priGPA+ACT+(priGPA^2)+(ACT^2)+(priGPA*atndrte),data=attend)summary(model2)
model2 <- lm(stndfnl ~ atndrte+priGPA+ACT+(priGPA^2)+(ACT^2)+(priGPA*atndrte),data=attend)summary(model2)

```