

Last Name: _____ First Name: _____ NetID: _____

- 1) (20 pts) For the following forms apply β -reduction and α -conversion where appropriate. Explain each step.

- a) $(\lambda x.x)(\lambda x.x)$
- a) $(\lambda x.x\ x)(\lambda x.\lambda y.x\ x)$
- b) $((\lambda x.(x\ y))(\lambda z.z))$
- c) $(\lambda z.z)(\lambda y.y\ y)(\lambda x.x\ a)$
- d) $(\lambda z.z)(\lambda z.z\ z)(\lambda z.z\ y)$
- e) $(\lambda x.\lambda y.x\ y\ y)(\lambda a.a)\ b$
- f) $(\lambda x.x\ x)(\lambda y.y\ x)\ z$
- g) $(\lambda x.(\lambda y.(x\ y))\ y)\ z$
- h) $((\lambda x.x\ x)(\lambda y.y))(\lambda y.y)$
- i) $((\lambda x.\lambda y.(x\ y))(\lambda y.y))\ w$

- 2) (20 pts) Consider the following BNF grammar for a language with two infix operators @ and \$ and terminals a b c. To receive credit, you must attach a detailed explanation for each of your answers that show the rationale for your answer.

$$S ::= A \mid S @ A$$

$$V ::= S \mid S \$ V$$

$$A ::= a \mid b \mid c$$

- a) What is the associativity of the @ operator: (a) left (b) right (c) neither
 - b) What is the associativity of the \$ operator: (a) left (b) right (c) neither
 - c) Which operator has higher precedence: (a) @ (b) \$ (c) neither
 - d) The grammar is: (a) left recursive (b) right recursive (c) both left and right recursive (d) neither left nor right recursive (c)
 - e) Draw a parse tree for the following string: a @ b @ c \$ a \$ b
- 3) (15 pts) Write programs in Haskell, Python, and Scheme that reverses a simple list of integers using recursion; specifically, in tail recursive form. Do not use higher order functions such as “reverse”, comprehensive lists or functions from a library.
- 4) (5 pts) Prove that the following grammar is ambiguous
- $$\begin{aligned} \langle S \rangle &\rightarrow \langle A \rangle \\ \langle A \rangle &\rightarrow \langle A \rangle + \langle A \rangle \mid \langle \text{id} \rangle \\ \langle \text{id} \rangle &\rightarrow a \mid b \mid c \end{aligned}$$

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5) Given the following grammar:

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle$
 $\quad \mid \langle \text{term} \rangle$

$\langle \text{term} \rangle \rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle$
 $\quad \mid \langle \text{factor} \rangle$

$\langle \text{factor} \rangle \rightarrow (\langle \text{expr} \rangle)$
 $\quad \mid \langle \text{id} \rangle$

a) (10 pts) Rewrite the BNF to give + precedence over * and force + to be right associative.

b) (10 pts) Rewrite the BNF to add the ++ and -- unary operators

6) (20 pts) In your own words what are the key elements of a Functional Programming Language. Compare this with an Imperative Programming Language. What operations are available in Programming Language and how do they differ from each other.