NPTEL MOOC Jan-Apr 2019

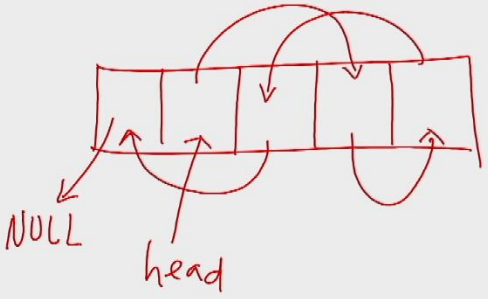
Parallel Algorithms

Lecture 09

Sajith Gopalan
Professor
Dept. of Computer Sc. & Engg.,
IIT Guwahati

Linked Lists

Physical representation

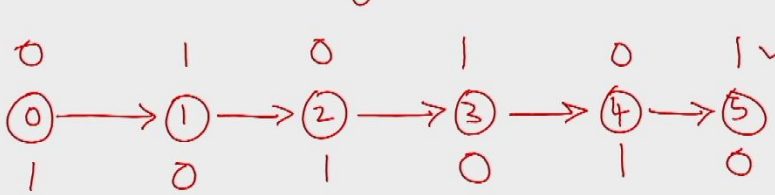


Vertex-colouring of graphs

$G = (V, E)$

a vertex-colouring of G assigns a colour (integer) to each vertex s.t. no 2 adj. vertices get the same colour

Vertex-colouring of a linked list



2-vertex-colourings

lsb of the ranks

$O(\log n)$ time n processors on EREW PRAM.

3-vertex colouring of linked lists in $O(\log^* n)$ time.

$$\log^* n := \min\{ i \mid \log^{(i)} n \leq 2 \}$$

$$2^{65536} \xrightarrow{1} 65536 \xrightarrow{2} 16 \xrightarrow{3} 4 \xrightarrow{4} 2$$

$$\log^*(2^{65536}) = 4$$

$$2^{65536} = 2^{(2^{16})} >> 10^{80}$$

$f(n) = \log^* n$ is a very slow growing function

:

$2^{65536} = 2^{(2^{16})} >> 10^{80}$

$f(n) = \log^* n$ is a very slow growing function

$O(\log^* n)$ time using n processors

on EREW PRAM

$\not\exists$ 3-colouring

Initial Colouring

Give every vertex a unique colour

Colour : physical index

0	1	2	3	4	5

0 to $n-1$: $\lceil \log n \rceil$ bits

iteration

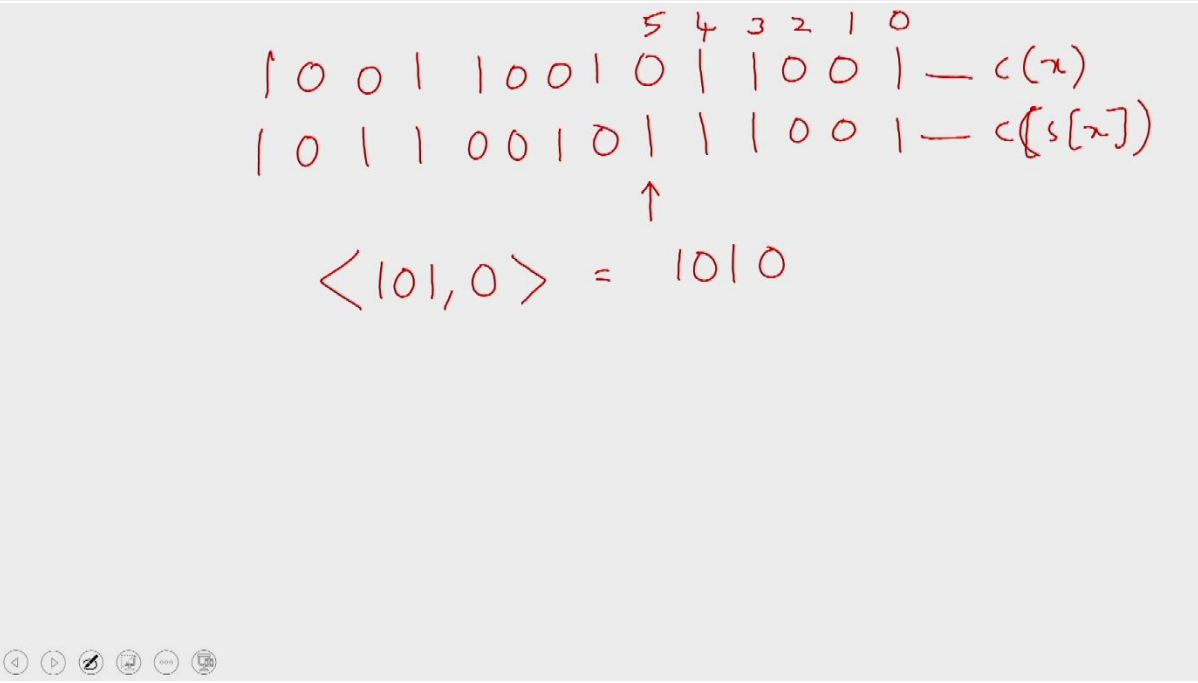
pardo for vertex x in the list

. /* suppose $s[x]$ is the successor
$s[s[x]]$ is the successor's successor */

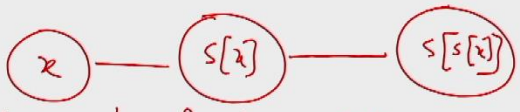
. /* c is the present colour fn */

. Let i be the lsb at which $c[x]$ & $c[s[x]]$ differ

then $c[x] = \langle i, c_i[x] \rangle$



The colouring remains valid after each iteration (if it was valid before)



i is the lsb at which $c(x)$ and $c(s[x])$ differ
j is the lsb ——— $c(s(x))$ $c(s(s(x)))$ differ

$c[x]$ and $c[s[x]]$ are different

$i \neq j$ $\quad \langle i, - \rangle \neq \langle j, - \rangle$

$i = j$ $\quad \langle i, c_i[x] \rangle \quad \langle i, c_i[s[x]] \rangle$

the colouring stays valid

the logically last vertex
let the 1st vertex be its successor

a, b are two colours

find the lsb at which a & b differ

a	1001110010
b	0010100010
c = a ⊕ b	1011010000
d = c − 1	1011001111
c ⊕ d	0000011111

00001111 is the unary representation of '4'

0	0	1
1	1	11
10	2	111
11	3	1111
100	4	11111

n → 1^{n+1}

Table look up

$\log n$ positions

$0 .. \log n$: unary to binary

Dictionary stored in array of size n

00001111 is the unary representation of '4'

0	0	1	1
1	1	11	3
10	2	111	7
11	3	1111	15
100	4	11111	31

n $\quad 1^{n+1}$

.

Masks

3rd posn

7	6	5	4	3	2	1	0
0	0	0	1	0	0	0	
1	0	1	1	0	0	1	
0	0	0	1	0	0	0	

iteration in $O(1)$ time

initial colours

$L_1 = \lceil \log n \rceil$

$0 \text{ to } \lceil \log n \rceil - 1$

$L_2 = \lceil \log \lceil \log n \rceil \rceil + 1$

$= \lceil \log \log n \rceil + 1$

$\leq 2 \lceil \log \log n \rceil$

$0 .. n-1$

$\lceil \log \lceil x \rceil \rceil$

$- \lceil \log x \rceil$

$$L_3 \leq \lceil \log 2\lceil \log\log n \rceil \rceil + 1$$
$$= \lceil \log\log\log n \rceil + 2$$
$$\leq 2\lceil \log\log\log n \rceil$$
$$L_k \leq 2\lceil \log^{(k)} n \rceil$$

$\log^* n$ iterations

$k = \log^* n$

$L_k \leq 2\lceil \log^{(k)} n \rceil \leq 4$

3 2 1 0

00 i
01 i
10 i
11 i

} 3 bits

8 colours

3 bit colours

2 1 0

10 01 00

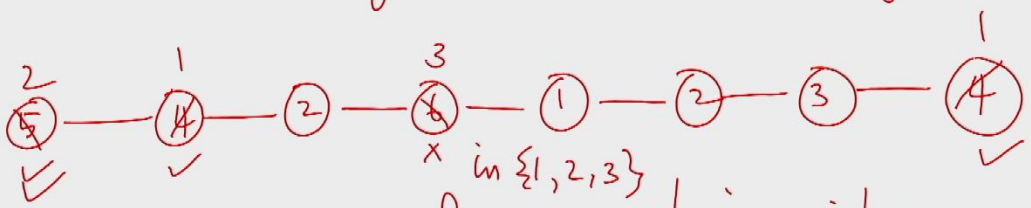
00— }
01— } 3 bit colours
10— } 6 colours

n-colouring

6-colouring

$O(\log^* n)$ steps

6-colouring to a 3-colouring



least colour, not in its neighbourhood

$O(\log^* n)$ time

a linked list of n vertices

is 3-coloured on

EREW PRAM