

Indian Institute of Technology Guwahati

Department of Physics

End Sem.: Nonlinear Dynamics and Chaos (PH551)

Open Elective (July, 2021 - Nov, 2021)

Maximum Credits: 50 Thursday, 25th of Nov., 2021 (09:00Hrs-12:00Hrs IST)

- Answer all questions.
- Use of calculator is allowed.

1. Consider the map [2+5+3]

$$x_{n+1} = 2x_n, \quad 0 \leq x_n \leq \frac{1}{2}$$
$$= 2 - 2x_n \quad \frac{1}{2} \leq x_n \leq 1$$

- (a) Find all the fixed points, and classify their stability.
 - (b) Show that the map has a period-2 orbit. Is it stable or unstable?
 - (c) Compute the Lyapunov exponent of system. Does system exhibit regular or chaotic nature?
2. Sketch the phase space portrait of the following system [4+4]
- (a) $\dot{r} = r(1 - r^2)(4 - r^2), \dot{\theta} = 2 - r^2$.
 - (b) $\dot{r} = r \sin r, \dot{\theta} = 1$
3. Consider the system [5]
- $$\begin{aligned}\dot{x} &= \nu x + zy \\ \dot{y} &= \nu y + (z - a)x \\ \dot{z} &= 1 - xy,\end{aligned}$$
- where, a, ν are positive parameters.
- (a) Find out whether the system is dissipative?
 - (b) Compute the fixed points of the system.
4. Consider the system $\dot{x} = \mu r - r^3$ and $\dot{\theta} = \omega + br^2$. Find out the kind of bifurcation that will occur as μ is varied. Here ω and b are positive quantity. Is the bifurcation subcritical or supercritical? [6]
5. Consider the map $x_{n+1} = 3x_n - x_n^3$. [3+4]
- (a) Find all the fixed points and classify their stability.
 - (b) Draw cobweb diagrams starting at initial points $x_0 = 1.9$ and $x_0 = 2.1$.
6. Find a conserved quantity for the system $\ddot{x} = a - e^x$ and sketch the phase space portrait for $a < 0$, $a = 0$, and $a > 0$. [6]
7. Consider the quadratic map $x_{n+1} = x_n^2 + c$. [2+3+3]
- (a) Find and classify all the fixed points as a function of c .
 - (b) Find the value of c at which the fixed points bifurcates, and classify those bifurcations.
 - (c) For which values of c is there a stable 2-cycle? When it is superstable?