Time and Cost analyses of OEMS

OEM Network

$$T_M(n,n) = T_M\left(\frac{n}{2}, \frac{n}{2}\right) + 1$$

$$
\begin{aligned}
T_M(n,n) &= T_M\left(\frac{n}{2}, \frac{n}{2}\right) + 1 \\
&= T_M\left(\frac{n}{4}, \frac{n}{4}\right) + 2 \\
&= T_M\left(\frac{n}{8}, \frac{n}{8}\right) + 3 \\
&= T_M\left(\frac{n}{2^k}, \frac{n}{2^k}\right) + k
\end{aligned}
$$

$k = \log_2 n \qquad 2^k = n$

$$\mathrm{I} \quad T_M(n,n) = T_M(1,1) + \log n$$

$$= 1 + \log n$$

Analysis

$$T_M(n,n) = T_M\left(\frac{n}{2}, \frac{n}{2}\right) + 1 \qquad (\text{OEM})$$

$$= \log n + 1$$

$$T_S(n) = T_S\left(\frac{n}{2}\right) + \underbrace{T_M\left(\frac{n}{2}, \frac{n}{2}\right)}_{\log \frac{n}{2} + 1}$$

OEM Sorter

$$T_S(n) = T_S\left(\frac{n}{2}\right) + C_M\left(\frac{n}{2}, \frac{n}{2}\right)$$

$$= T_S\left(\frac{n}{2}\right) + \log n$$

$$= T_S\left(\frac{n}{4}\right) + \log\frac{n}{2} + \log n$$

$$= T_S\left(\frac{n}{8}\right) + \log\frac{n}{4} + \log\frac{n}{2} + \log n$$

$$= T_S\left(\frac{n}{2^k}\right) + \log\frac{n}{2^{k-1}} + \dots + \log n$$

$$\text{I} \quad : T_S\left(\frac{n}{2^k}\right) + k\log n - (0 + 1 + \dots + (k-1))$$

$$k = \log n/2 \ . \ \log n - 1$$

$$T_S(n) = T_S(2) + (\log n - 1)\log n - \frac{(\log n - 1)(\log n - 2)}{2}$$

$$= 1 + \log^2 n - \log n - \frac{\log^2 n - 3\log n + 2}{2}$$

$$= \frac{\log^2 n + \log n}{2} = O(\log^2 n)$$

Cost ?

$$C_M(n, n) = 2\, C_M\left(\frac{n}{2}, \frac{n}{2}\right) + (n-1)$$

$$= 2\left[2\, C_M\left(\frac{n}{4}, \frac{n}{4}\right) + \left(\frac{n}{2} - 1\right)\right] + (n-1)$$

$$= 4\, C_M\left(\frac{n}{4}, \frac{n}{4}\right) + 2n - 2 - 1$$

$$= 4\left[2\, C_M\left(\frac{n}{8}, \frac{n}{8}\right) + \frac{n}{4} - 1\right) + 2n - 3$$

$$= 8\, C_M\left(\frac{n}{8}, \frac{n}{8}\right) + 3n - (1+2+4)$$

$$C_M(n, n) = 2^k\, C_M\left(\frac{n}{2^k}, \frac{n}{2^k}\right) + kn - (\underbrace{2^{k-1} + \dots + 2 + 1})$$

$$\boxed{\begin{array}{c} k = \log_2 n \\ 2^k = n \end{array}}$$

$$= 2^k\, C_M\left(\frac{n}{2^k}, \frac{n}{2^k}\right) + kn - 2^k + 1$$

$$= n\, C_M(1, 1) + n \log n - n + 1$$

$$= \cancel{n} + n \log n - \cancel{n} + 1 = \underline{\underline{n \log n + 1}}$$

$$C_s(n) = 2C_s\left(\frac{n}{2}\right) + C_M\left(\frac{n}{2}, \frac{n}{2}\right)$$

$$= 2C_s\left(\frac{n}{2}\right) + \frac{n}{2}\log\frac{n}{2} + 1$$

$$= 2\left[2C_s\left(\frac{n}{4}\right) + \frac{n}{4}\log\frac{n}{4} + 1\right] + \frac{n}{2}\log\frac{n}{2} + 1$$

$$= 4C_s\left(\frac{n}{4}\right) + \frac{n}{2}\left[\log\frac{n}{4} + \log\frac{n}{2}\right] + (2+1)$$

$$= 4\left[2C_s\left(\frac{n}{8}\right) + \frac{n}{8}\log\frac{n}{8} + 1\right] + \text{——}\,''\,\text{——}$$

$$= 8C_s\left(\frac{n}{8}\right) + \frac{n}{2}\left[\log\frac{n}{8} + \log\frac{n}{4} + \log\frac{n}{2}\right] + (4+2+1)$$

$$\vdots$$

$$= 2^k C_s\left(\frac{n}{2^k}\right) + \frac{n}{2}\left[\log\frac{n}{2^k} + \cdots + \log\frac{n}{2}\right] + 2^k - 1$$

$$= 2^k C_s\left(\frac{n}{2^k}\right) + \frac{kn\log n}{2} - \frac{k(k+1)n}{4} + 2^k - 1$$

$$k = \log\frac{n}{2} \qquad 2^k = \frac{n}{2}$$

$$T_s(n) = \frac{n}{2} C_s(2) + \frac{(L-1)nL}{2} - \frac{(L-1)Ln}{4} + \left(\frac{n}{2} - 1\right)$$

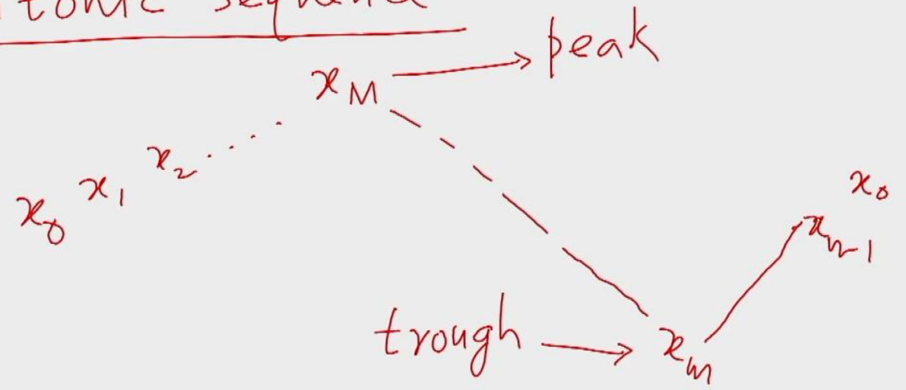
$$= \frac{n}{2} + \frac{L(L-1)n}{4} + \frac{n}{2} - 1$$

$$= \underbrace{\frac{n\log^2 n - n\log n}{4} + n - 1}$$

OEMsort runs in $O(\log^2 n)$ time and $O(n\log^2 n)$ cost on EREW PRAM / comparator network

Bitonic Sorting   2 tones

Bitonic Sequence



Sort the bitonic sequence

6 8 10 12 11 9 3 1 2 4

1 2 3 4 6 8 9 10 11 12

input : $x_0 \ldots x_{n-1}$ $\underline{\underline{x = 2^k}}$

Compare every element with the diametrically opposite element & exchange if necessary, So that the smaller elements occupy positions $0 \ldots n/2-1$

$\checkmark x_0$ x_1 $\ldots$ $x_{n/2-1}$

$\checkmark x_{n/2}$ $x_{n/2+1}$ x_{n-1}

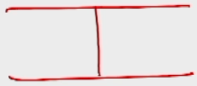
/* the upper seq. & the lower seq. ~~&~~ both bitonic.

every elem of the US < every elem of LS */

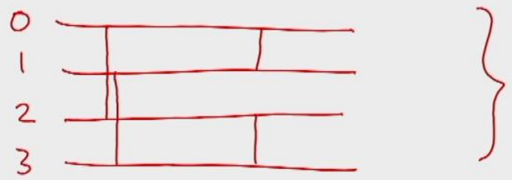
Recurse with the 2 halves

Basis : 2 elements

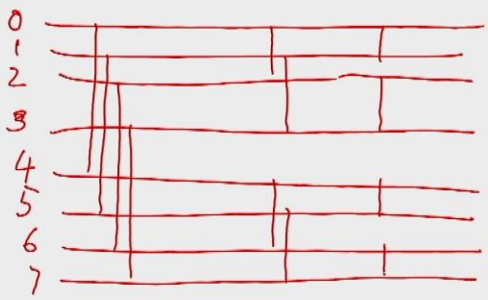
$\langle p, t \rangle$ or $\langle t, p \rangle$

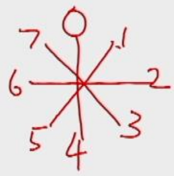


4 BS



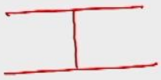
8 BS





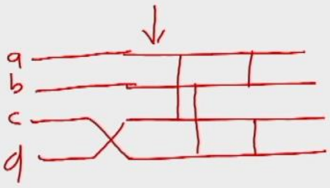
Merge

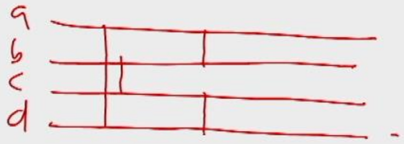
1×1 BSM



2×2 BSM

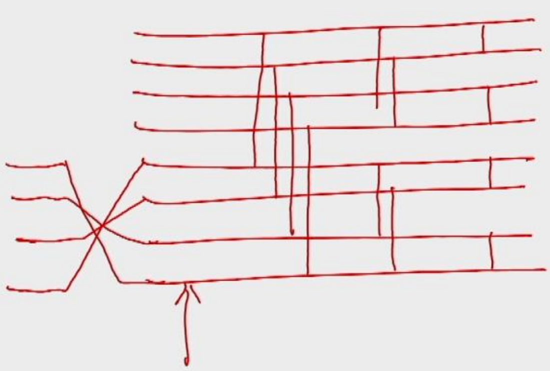
$a<b,\ c<d$

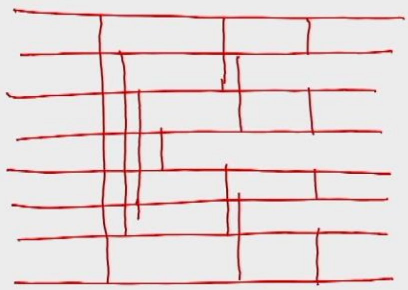




1 2 3 4
6 7 8 9
1 2 3 4 9 8 7 6

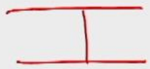
4×4 BSMerger





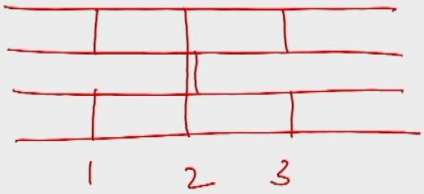
Bitonic-Sorter_Merger-Sorter

2 BSMS



2x2 BSM

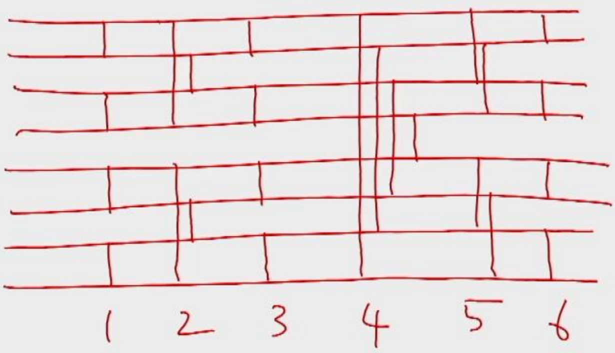
4 BSMS



6 comp.
time 3

8 BSMS

4 x 4 BSM



6 steps

4
4
4
4
4
4

Compare exchange opposite elements $\longrightarrow$

2 bitonic sequences to form. one smaller than the other