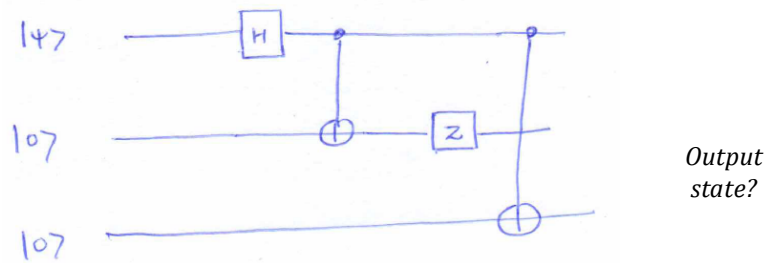


PH 441
End-Sem. Part I
24/11/2020
Marks: 20

1. Find the output state of the following circuit, for:



- (a) $|\psi\rangle = |0\rangle$ and
(b) $|\psi\rangle = |1\rangle$
2. (a) A system is in a statistical mixture of states in which 25% are in the state $|0\rangle$ and the remaining in state $|1\rangle$. If the measurement is made in $\{|+\rangle, |-\rangle\}$ basis, what is the probability of finding the system in the $|+\rangle$ state?
(b) Work out the reduced density matrix corresponding to the first qubit, for following state: $|\psi\rangle = \frac{1}{\sqrt{3}}[|00\rangle + |01\rangle + |10\rangle]$

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1+2=3

3. Consider the following two-qubit state:

$$\frac{1}{\sqrt{11}}(|00\rangle + \sqrt{5}|01\rangle + \sqrt{2}|10\rangle + \sqrt{3}|11\rangle)$$

- (a) If we measure the first qubit and get $|0\rangle$, then find out the state to which the second qubit collapses.
(b) What is the probability that a projective measurement of the first qubit gives $|0\rangle$?

2+2=4

4. What is the fundamental difference between *Super-dense coding* and *quantum teleportation*. Show how the quantum teleportation protocol could be realized using the Bell state: $|\Phi^-\rangle = \frac{1}{\sqrt{2}}[|00\rangle - |11\rangle]$.

1+4=5

5. Consider the following table giving values of a 2 to 1 function $f(x)$ corresponding to three qubit inputs x . There exists a string ξ such that $f(x) = f(y)$ iff $x \oplus \xi = y$.

x	f(x)	x	f(x)
000	101	100	000
001	010	101	110
010	000	110	101
011	110	111	010

Table 1

The string in this case is $|110\rangle$. A Simon algorithm is executed and the second register is measured which gives: $|000\rangle$. If the first register is passed through Hadamard gates, what will be the result of measurement on the first register?

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