Any interconnection network of a constant vertex degree can be simulated on a wbfly at a low cost

A network of N nodes

$N \leq r \cdot 2^r$ for some r

A wbfly of $r \cdot 2^r$ nodes

① Map the nodes of the network (G) onto the wbfly 1 – 1.

Consider a step of the given G-algorithm

in a step,

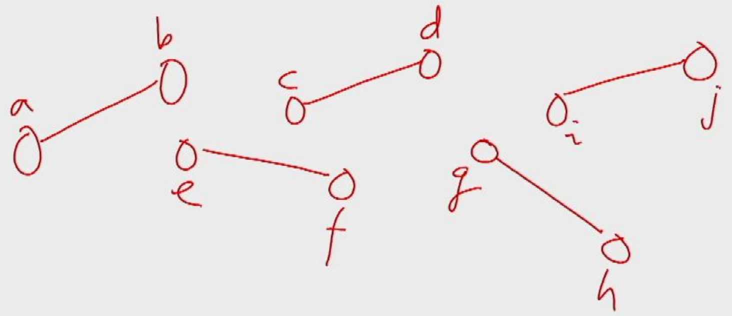
each node of G communicates to all its nbrs

max. V.degree of G : d

d msgs to send & receive for each node

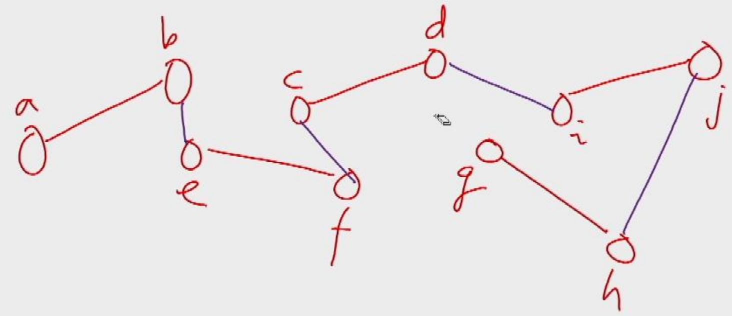
(2) Colour the edges of G
using $(d+1)$ colours
(offline)
Consider each colour class
forms a matching

d - phases
in each phase handle one
colour class



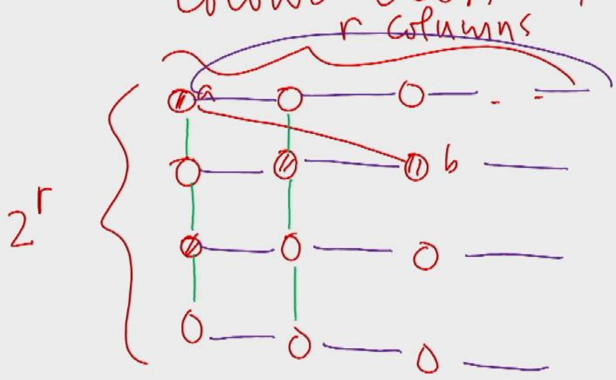
d - phases

in each phase handle one colour class



Consider the 1st phase

colour-class - 1 (matching)



a $2^r \times r$ mesh

Recall the mesh
routing algorithm.
(offline computed)
transmitting along the rows
columns
rows

Step 1 & 3
can be executed as it is
$O(r)$ time

Vertical communication along the columns

2^r

$N = r \cdot 2^r$

$r = \theta(\log N)$

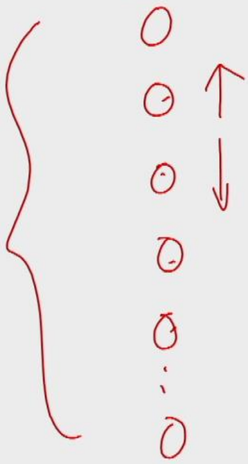
$2^r = \theta(N/\log N)$

$\underline{\underline{O(N/\log N)}}$

Consider the 0^{th} column

0^{th}

2^r nodes



Beneš Network

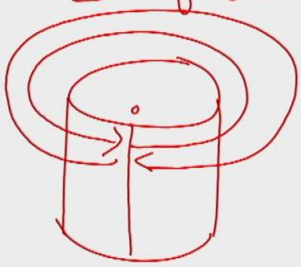
instead of a wbfly

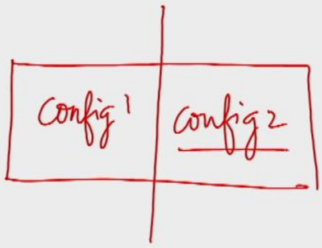
0^{th}

$2r$ steps

Beneš Network

is 2 bflies back to back

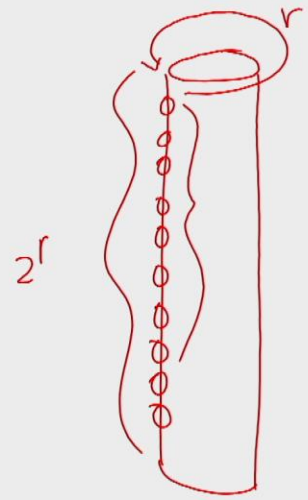




every message of the

0^{th} column is delivered

to the correct destination

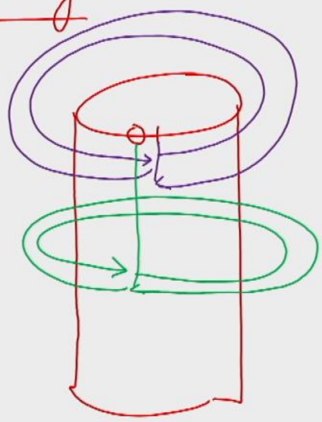


$O(r)$

column 0.

columns 1 to r

Pipelining



Several permutations going on at the same time

$O(r)$ steps

using pipelining

all permutations

Complete

the routing problem

on wbfly

runs in

$r + O(r) + r = O(r)$ steps

N node wbfly

$r = O(\log N)$

d colour classes

$O(d \log N) = O(\log N)$ steps

all messages of the step are delivered

One step on G

can be simulated in

$O(\log N)$ steps in

$Wbfly_{(\log N)}$

A G-algorithm of T steps

$O(T \log N)$ steps on $Wbfly$

Wbfly $\to$ bfly

$\to$ CCC

Shuffle Exchange graph

SEG_r has 2^r nodes

$$\overbrace{00\cdots0}^{r}$$

$$\underbrace{11-1}_{r}$$

Adjacencies of SEG_r

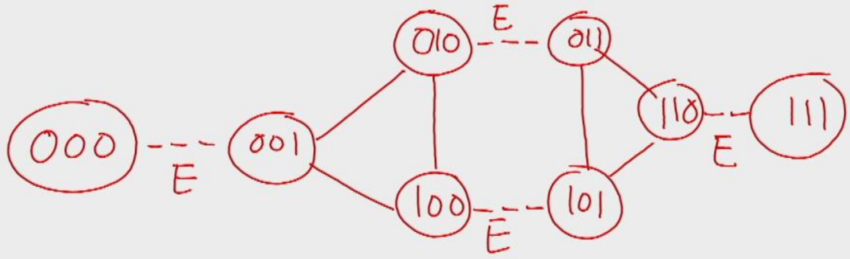
Two kinds of edges

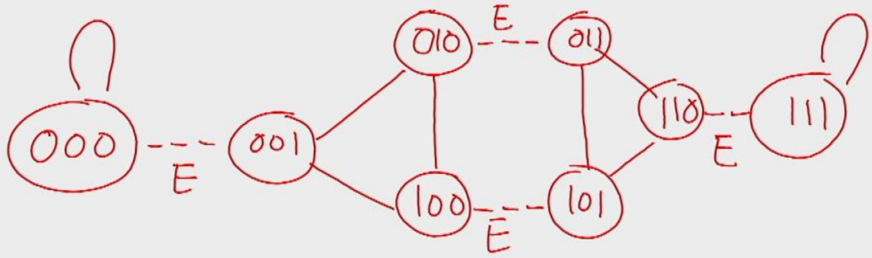
(1) Exchange edges

$$u_1 u_2 \cdots u_r \longrightarrow u_1 u_2 \cdots u_{r-1} \overline{u_r}$$

(2) A shuffle edge

$$u_r u_1 \cdots u_{r-1} \xrightarrow{RS} u_1 \cdots u_r \xrightarrow{LS} u_2 \cdots u_r u_1$$





3-D shuffle exchange graph

Small diameter

$u_1 - u_r$ to $v_1 \cdots v_r$ on SEG

$$u_1 \cdots u_r \xrightarrow{S} u_2 \cdots u_r u_1$$

$$\xrightarrow{E} u_2 - u_r v_1 \xrightarrow{S} u_3 \cdots u_r v_1 u_2$$

$$\xrightarrow{E} u_3 \cdots u_r v_1 v_2 \xrightarrow{S} u_4 \cdots u_r v_1 v_2 u_3$$

$$v_1 - v_r$$

even if we take the E-edge in every step, the path has a length of $2r$

$v_i = \overline{u_i}, \ \forall i$ the $2 \log N$ steps

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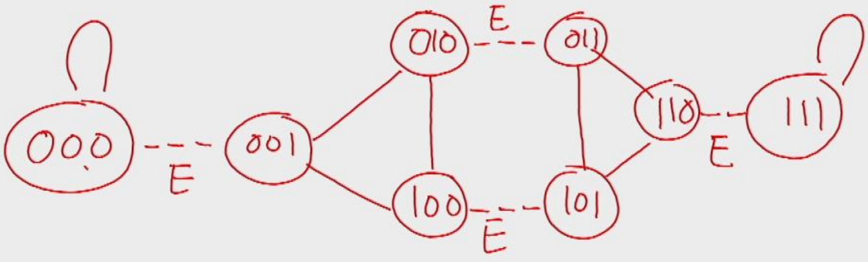

3-D shuffle exchange graph

diameter of SEG_r

is $\underline{2r}$

N nodes SEG

diameter is $\Theta(\log N)$