

## ASSIGNMENT-1

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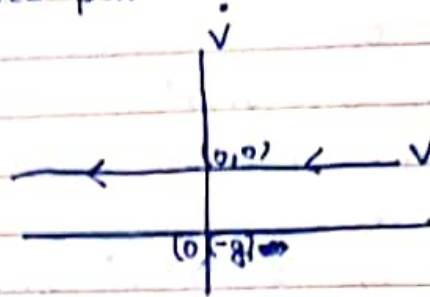
$v^*$ ,  $u^*$  and  $x^*$  denote fixed points

Ans-1-  $\dot{v} = -g = f(v)$

$$f(v^*) = 0$$

$-g = 0$  has no solutions

Since  $g > 0$ , No fixed points.



Ans-2-  $m\dot{v} = -mg - \gamma v$

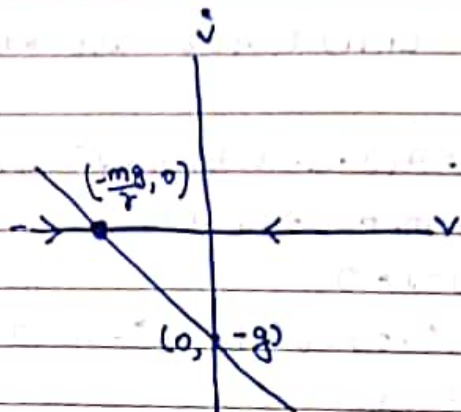
$$\dot{v} = -g - \frac{\gamma}{m}v = f(v)$$

$$f(v^*) = 0$$

$$-g - \frac{\gamma}{m}v^* = 0 \Rightarrow v^* = -\frac{mg}{\gamma}$$

$$f'(v) = -\frac{\gamma}{m}$$

$$f'(v^*) = -\frac{\gamma}{m} < 0$$



Hence stable fixed point

Ans-3-  $m\dot{v} = -mg - cv^2 \operatorname{sgn}(v)$

$$\dot{v} = \begin{cases} -g & v=0 \\ -g - \frac{c}{m}v^2 & v>0 \\ -g + \frac{c}{m}v^2 & v<0 \end{cases} = f(v)$$

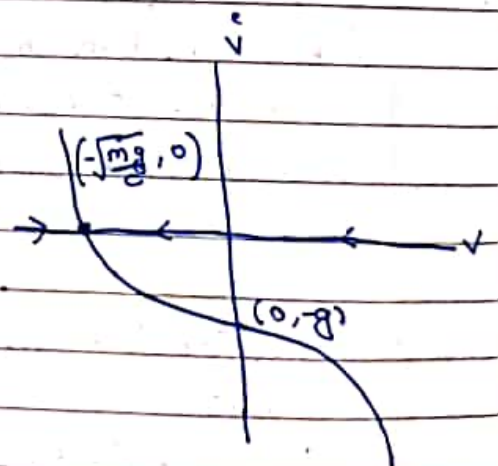
$$f(v^*) = 0$$

$$v^* = 0, f(v^*) = -g \neq 0$$

$$v^* > 0, f(v^*) = -g - \frac{c}{m}v^2 < 0$$

$$v^* < 0, f(v^*) = -g + \frac{c}{m}v^2 = 0$$

$$\Rightarrow v^* = -\sqrt{\frac{mg}{c}}$$



$$f'(v) = \frac{2cv}{m} \text{ for } v < 0 \Rightarrow f'(v^*) = \frac{2v^*c}{m} < 0 \text{ Stable point since } v^* < 0$$

Ans-4-  $\dot{u} = A \sin u = f(u)$

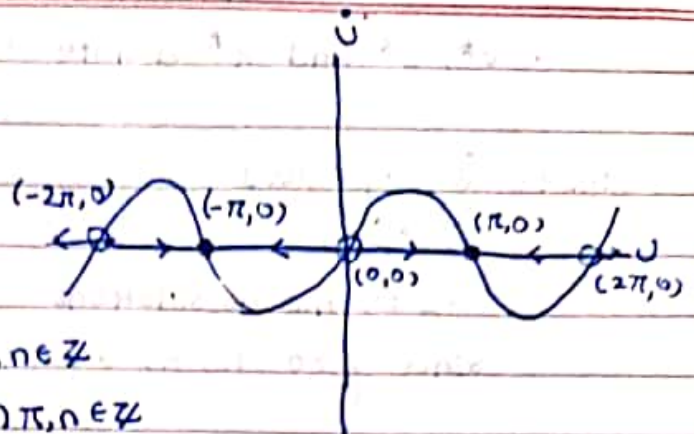
$$f(u^*) = 0$$

$$A \sin u^* = 0 \Rightarrow \sin u^* = 0$$

$$u^* = n\pi \quad \forall n \in \mathbb{Z}$$

$$f'(u) = A \cos u$$

$$f'(u^*) = \begin{cases} A & \forall u^* = 2n\pi, n \in \mathbb{Z} \\ -A & \forall u^* = (2n+1)\pi, n \in \mathbb{Z} \end{cases}$$



$\therefore u^* = 2n\pi, n \in \mathbb{Z}$  are unstable

$u^* = (2n+1)\pi, n \in \mathbb{Z}$  are stable

Ans-5-  $\dot{u} = A(u-a)(u-b)(u-c) = f(u)$

(Assumption:  $0 < a < b < c$ )

$$f(u^*) = 0$$

$$A(u^*-a)(u^*-b)(u^*-c) = 0$$

$$u^* = a, b, c$$

$$f'(u) = A \left( (u-a)(u-b) + (u-a)(u-c) + (u-b)(u-c) \right)$$

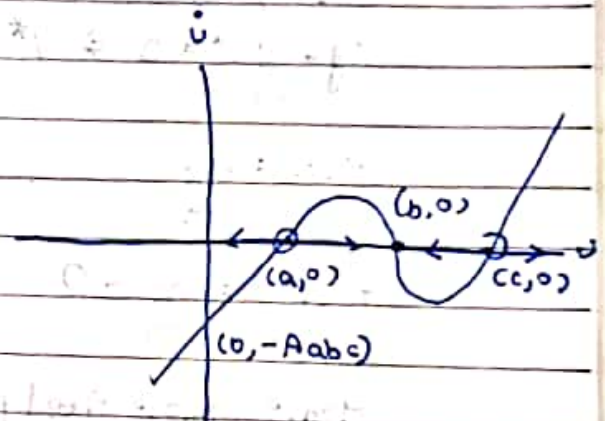
$$f'(a) = A(a-b)(a-c) > 0$$

$$f'(b) = A(b-a)(b-c) < 0$$

$$f'(c) = A(c-a)(c-b) > 0$$

$\therefore a$  and  $c$  are unstable

$b$  is stable



Ans-6-  $\dot{u} = au^2 - bu^3 = f(u)$

$$f(u^*) = 0$$

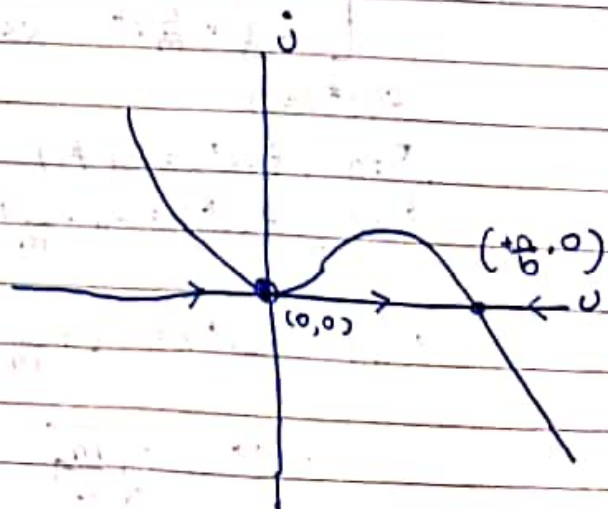
$$u^{*2}(a - bu^*) = 0$$

$$u^* = 0, u^* = \frac{a}{b}$$

$$f'(u) = 2au - 3bu^2$$

$$f''(u) = 2a - 6bu$$

$$f'\left(\frac{a}{b}\right) = \frac{2a^2}{b} - 3\frac{a^2}{b^2} = -\frac{a}{b^2} < 0$$





$$f'(0) = 0, f''(0) = 2a > 0$$

$\therefore$   $\frac{a}{b}$  is stable and 0 is saddle pt. of type I

Ans-7-  $\dot{x} = 4x^2 - 16 = f(x)$

$$f(x^*) = 0$$

$$4x^2 - 16 = 0$$

$$4(x^2 - 2)(x^2 + 2) = 0$$

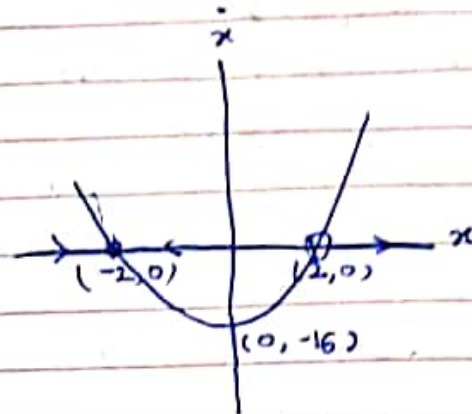
$$x^2 = \pm 2$$

$$f'(x) = 8x$$

$$f'(2) = 16 > 0$$

$$f'(-2) = -16 < 0$$

$\therefore$  2 is unstable and -2 is stable



Ans-8-  $\dot{x} = x - x^3 = f(x)$

$$f(x^*) = 0$$

$$x^* - x^{*3} = 0$$

$$x^*(1 - x^*)(1 + x^*) = 0$$

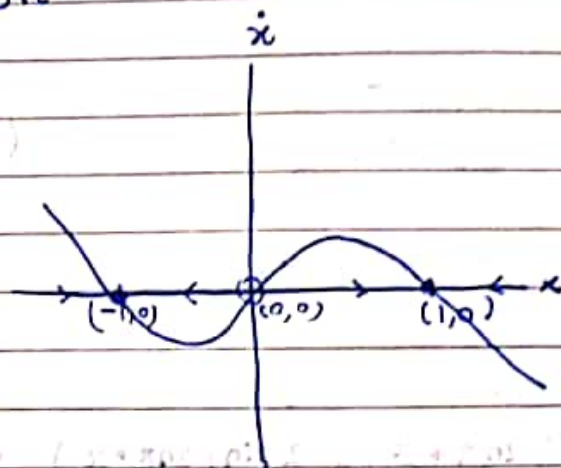
$$x^* = 0, 1, -1$$

$$f'(x) = 1 - 3x^2$$

$$f'(0) = 1 > 0$$

$$f'(1) = f'(-1) = -2 < 0$$

$\therefore$  0 is unstable and 1, -1 are stable



Ans-8-  $\dot{x} = 1 + \frac{1}{2} \cos x = f(x)$

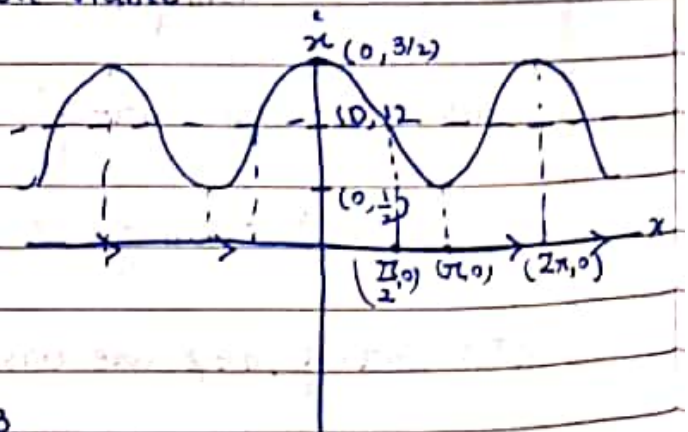
$$f(x^*) = 0$$

$$1 + \frac{1}{2} \cos x^* = 0$$

has no solutions because

$$|\cos x| \leq 1 \Rightarrow 1 + \frac{1}{2} \cos x \geq \frac{1}{2}$$

$$1 + \frac{1}{2} \cos x \geq \frac{1}{2} \quad \therefore \text{No fixed pts.}$$



Ans-10  $\dot{x} = e^{-x} \sin x = f(x)$

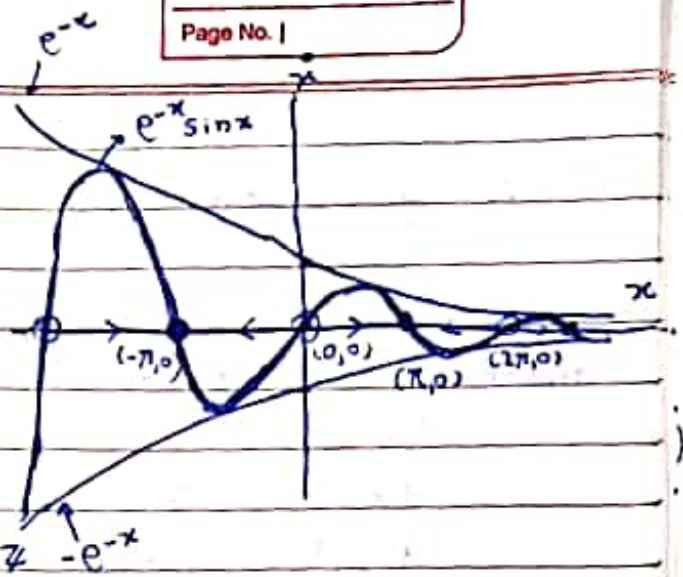
$$f(x^*) = 0$$

$$e^{-x^*} \sin x^* = 0 \Rightarrow \sin x^* = 0$$

$$x^* = n\pi, n \in \mathbb{Z}$$

$$f'(x) = e^{-x} \cos x - e^{-x} \sin x$$

$$f'(x^*) = \begin{cases} e^{-2n\pi} & x^* = 2n\pi, n \in \mathbb{Z} \\ -e^{-(2n+1)\pi} & x^* = (2n+1)\pi, n \in \mathbb{Z} \end{cases}$$



$\therefore x^* = (2n+1)\pi, n \in \mathbb{Z}$  are stable

$x^* = 2n\pi, n \in \mathbb{Z}$  are unstable

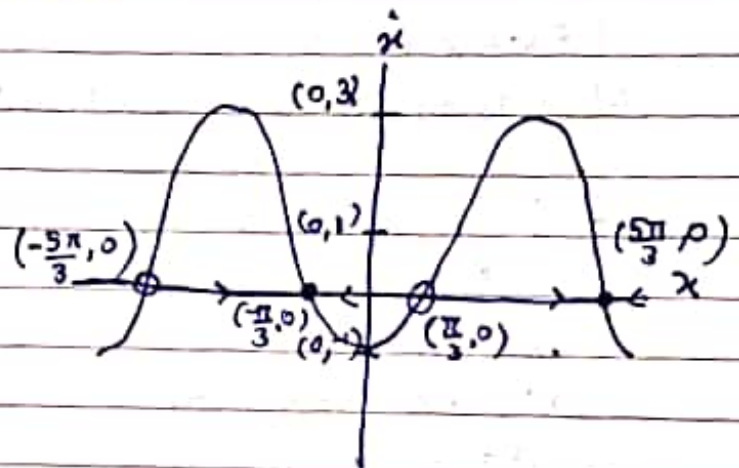
Ans-11  $\dot{x} = 1 - 2\cos x = f(x)$

$$f(x^*) = 0$$

$$1 - 2\cos x^* = 0$$

$$\Rightarrow \cos x^* = \frac{1}{2}$$

$$\Rightarrow x^* = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$



$$f'(x) = 2\sin x$$

$$f'(2n\pi + \frac{\pi}{3}) = 2\sin(2n\pi + \frac{\pi}{3}) \quad n \in \mathbb{Z}$$

$$= 2\sin \frac{\pi}{3} > 0 \quad (\because \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2})$$

$$f'(2n\pi - \frac{\pi}{3}) = 2\sin(2n\pi - \frac{\pi}{3}) \quad n \in \mathbb{Z}$$

$$= -2\sin \frac{\pi}{3} < 0$$

$x^* = 2n\pi + \frac{\pi}{3}, n \in \mathbb{Z}$  are unstable

$x^* = 2n\pi - \frac{\pi}{3}, n \in \mathbb{Z}$  are stable



Ans-12-  $\dot{x} = x(1-x) = f(x)$

$$f(x^*) = 0$$

$$x^*(1-x^*) = 0$$

$$x^* = 0, 1$$

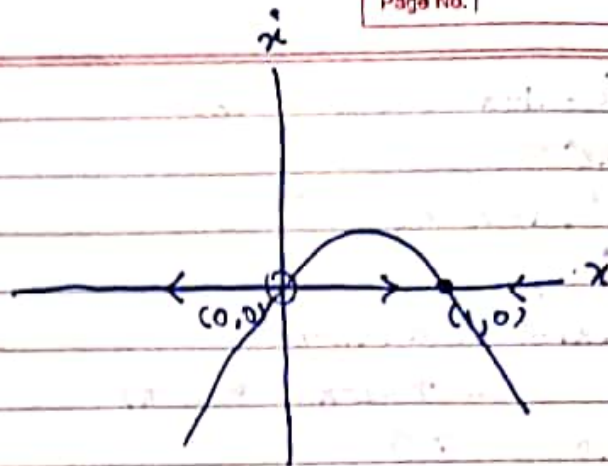
$$f'(x) = 1-2x$$

$$f'(0) = 1 > 0$$

$$f'(1) = -1 < 0$$

$\therefore x^* = 1$  is stable

$x^* = 0$  is unstable



Ans-13-  $\dot{x} = \tan x = f(x)$

$$f(x^*) = 0$$

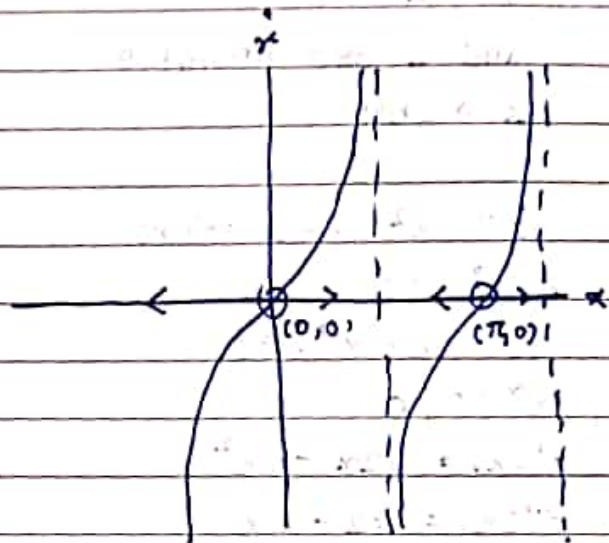
$$\tan x^* = 0$$

$$x^* = n\pi, n \in \mathbb{Z}$$

$$f'(x) = \sec^2 x$$

$$f'(x^*) = \sec^2 n\pi = 1 > 0$$

$\therefore x^* = n\pi, n \in \mathbb{Z}$  are unstable



Ans-14-  $\dot{x} = 1 - e^{-x^2} = f(x)$

$$f(x^*) = 0$$

$$1 - e^{-x^{*2}} = 0$$

$$1 = e^{-x^{*2}} \text{ taking } \log_e \text{ both sides}$$

$$\ln 1 = -x^{*2}$$

$$x^{*2} = 0$$

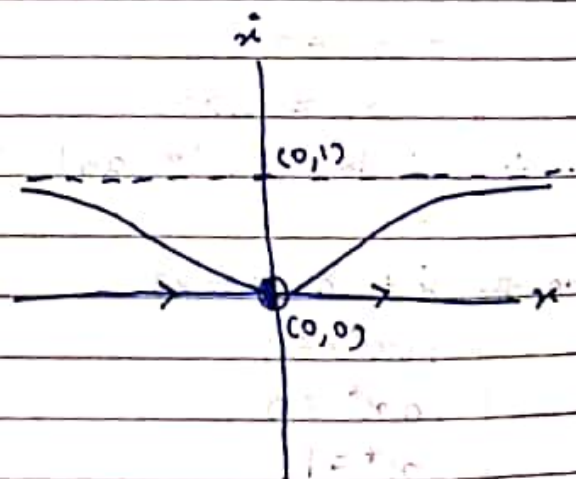
$$x^* = 0$$

$$f'(x) = -e^{-x^2}(-2x) = 2xe^{-x^2}$$

$$f''(x) = 2(e^{-x^2} + e^{-x^2}(-2x)x)$$

$$f'(0) = 0, f''(0) = 2 > 0$$

$\therefore$  Saddle point of type I



Ans-15-  $\dot{x} = (1-x)x(2-x) = f(x)$

$$f(x^*) = 0$$

$$x^*(1-x^*)(2-x^*) = 0$$

$$x^* = 1, 2, 0$$

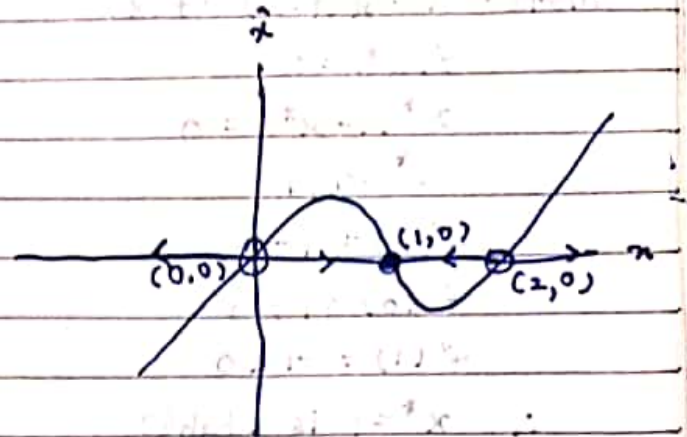
$$f'(x) = (1-x)(2-x) - x(2-x) - x(1-x)$$

$$f'(0) = 2 > 0$$

$$f'(1) = -1 < 0$$

$$f'(2) = 2 > 0$$

$\therefore$  0 and 2 are unstable  
1 is stable



Ans-16-  $\dot{x} = x^2(6-x) = f(x)$

$$f(x^*) = 0$$

$$x^{*2}(6-x^*) = 0$$

$$x^* = 0, 6$$

$$f'(x) = 12x - 3x^2$$

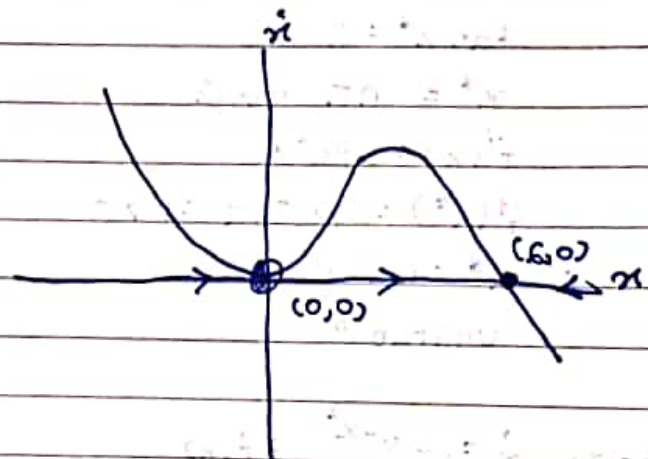
$$f''(x) = 12 - 6x$$

$$f'(6) = 12 \times 6 - 3 \times 36 = 72 - 108 = -36 < 0$$

$$f'(0) = 0$$

$$f''(0) = 12 > 0$$

$\therefore$  6 is stable and 0 is saddle pt. of type I



Ans-17-  $\dot{x} = \ln x = f(x)$

$$f(x^*) = 0$$

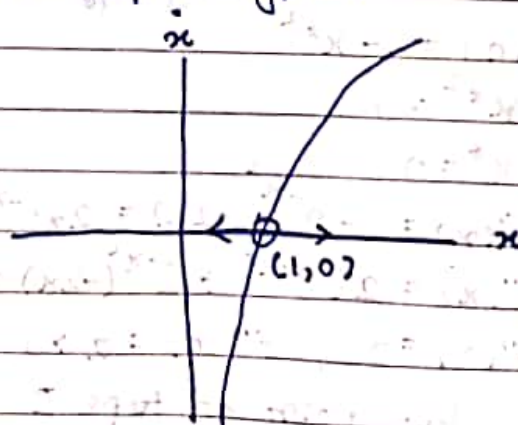
$$\ln x^* = 0$$

$$x^* = 1$$

$$f'(x) = 1/x$$

$$f'(x^*) = f'(1) = 1/1 = 1 > 0$$

$\therefore$   $x^* = 1$  is unstable



Ans-18-  $\dot{x} = 2x = f(x)$

$$f(x^*) = 0$$

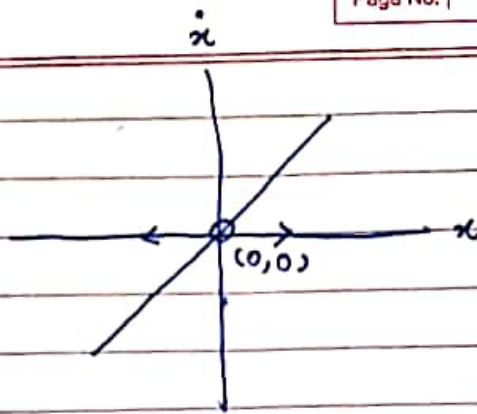
$$2x^* = 0$$

$$x^* = 0$$

$$f'(x) = 2$$

$$f'(x^*) = 2 > 0$$

$\therefore x^* = 0$  is unstable



Ans-19-  $\dot{x} = -x + 1 = f(x)$

$$f(x^*) = 0$$

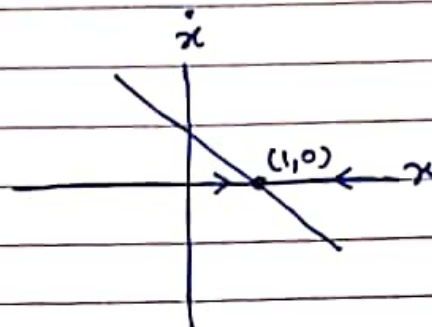
$$-x^* + 1 = 0$$

$$x^* = 1$$

$$f'(x) = -1$$

$$f'(x^*) = -1 < 0$$

$\therefore x^* = 1$  is stable



Ans-20-  $\dot{x} = (x-2)^2 = f(x)$

$$f(x^*) = (x^* - 2)^2 = 0$$

$$x^* = 2$$

$$f'(x) = 2(x-2)$$

$$f'(x) = 2$$

$$f'(2) = 0$$

$$f''(2) = 2 > 0$$

$\therefore x^* = 2$  is a saddle pt.  
of type I

