PH 441 - Assignment 02
Due date: 18/11/2020

November 11, 2020

1 The Sudoku Situation

Bob has been a regular Sudoku solver ever since his school days and has explored all its classical solving algorithms as a sophomore. However, in his final year as an undergraduate, he comes across quantum search algorithms and the speed-ups they provide over the classical ones. Excited, he decides to attempt solving the puzzle using a quantum circuit!

[*One can search up what a Sudoku is and its rules, if not familiar with the classic puzzle! It's not required for this assignment though.*]

Bob starts with a simple objective. He decides to first solve those puzzles of which there are exactly two solutions and picks up the one in Fig. 1a.

Following the rules of the game, he solves the puzzle straightforwardly to obtain the *almost* complete solution in Fig. 1b. And now, his choice of solution boils down to chosing either of the patterns in Fig. 1c, right what he wanted!

To make sure he can solve such problems consisting of just four squares and two unique entries repeating exactly once using a quantum circuit, he creates the following checklist:

1. *Generate a standard state.*

2. Initialize states for the conditions.

3. Initialize an additional qbit as $|-\rangle$.

4. Develop an oracle that checks the entries.

5. *Create a diffuser circuit for the 4-qbit state.*

6. Iterate and measure the result of the ciruit.

For the first step, he uses $4 \to 0$ and $9 \to 1$, such that he can write the superposed state of the solution as,

9		6		7		4		3
			4			2		
	7			2	3		1	
5						1		
	4		2		8		6	
		3						5
	3		7				5	
		7			5			
4		5		1		7		8

(a) An unsolved sudoku puzzle.

9	2	6	5	7	1	4	8	3
3	5	1	4	8	6	2	7	9
8	7	4	9	2	3	5	1	6
5	8	2	3	6	7	1	9	4
1	4	9	2	5	8	3	6	7
7	6	3	1			8	2	5
2	3	8	7			6	5	1
6	1	7	8	3	5	9	4	2
4	9	5	6	1	2	7	3	8

(b) The *almost* complete solution.

9	4
4	9

4	9
9	4

(c) The solutions fitting the blank.

Figure 1: Source: Agnes M. Herzberg and M. Ram Murty, *Sudoku Squares and Chromatic Polynomials*, Notices of the AMS, **54** (6), pp. 708 (2007).

$$S = \frac{1}{4}\sum_{x=0}^{15} |x\rangle \tag{1}$$

where x represents the decimal representation of the 4-qbit state. Thus, his expected solution is either $|1001\rangle$ or $|0110\rangle$.

In the next step, he initializes four qbits to $|0\rangle$ to keep tabs on whether the conditions are met.

The third step is easy!

For the fourth step, he successfully designs an oracle that implements two restrictions — no two elements in the same row or the same column are equal — on the standard qbits (qs). As such, four condition qbits (qc) individually take care of:

- 1st element of 1st row and 2nd element of 1st row ($qs_0 \neq qs_1$).
- 1st element of 1st row and 1st element of 2nd row ($qs_0 \neq qs_2$).
- 2nd element of 1st row and 2nd element of 2nd row ($qs_1 \neq qs_3$).
- 1st element of 2nd row and 2nd element of 2nd row ($qs_2 \neq qs_3$).

The output qbit (qo) takes care of the resultant of the conditions via a multiple-control Toffoli (MCT) gate. After the resultant condition is passed to the output qbit, he resets the conditions back to their initial state for the next round. The circuit representation for Bob's oracle is shown in Fig. 2.

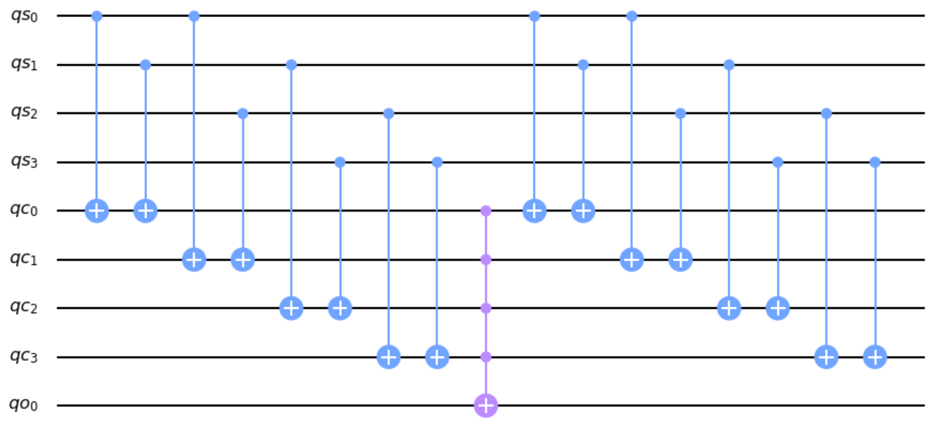

Figure 2: Bob's sudoku oracle. The first 4 lines represent the standard states. The next 4 lines are the conditions. The final line is the output qbit.

2 Now, the Assignment!

However, Bob is a bit forgetful, and time to time, has difficulty recalling some of the basic concepts and their implementations. So, he is stuck at the fifth step and seeks out for your help so that he can proceed with the iteration and measurement in step six! Here's what he expects from you:

1. Find how to get the standard state expressed in Eq. (1) starting with the computational basis and its equivalent circuit diagram. (2 marks)

2. Find the 4-qbit diffusion operator and its circuit representation. (3 marks)