Euler Graph

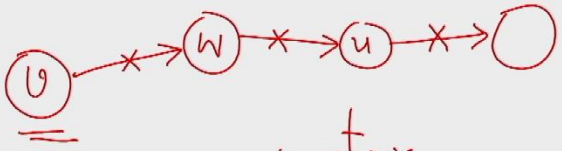
A graph with an Euler circuit

Is a walk of the graph
that starts at a vertex v
visits every edge exactly once
and returns to v

A graph is an Euler Graph
iff
every vertex of it has an
even degree

neighbours of the vertex

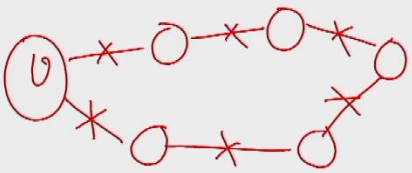
G has an Euler circuit

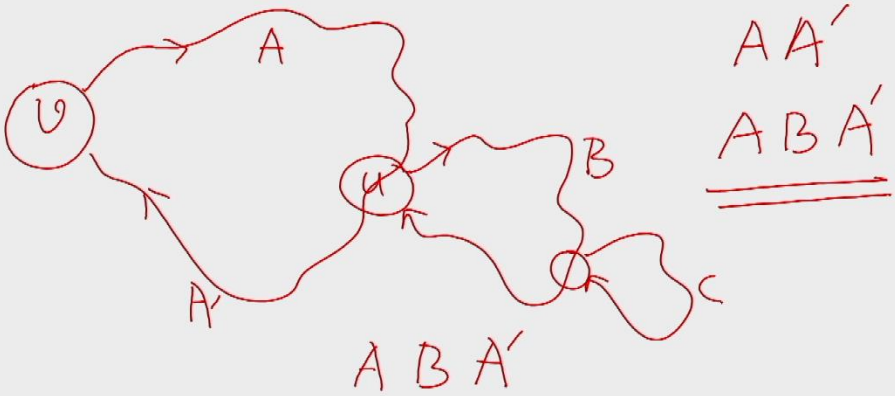


at every vertex,
the incident edges can be
paired off

G=(V,E) has an even degree at
every vertex

arbitrary vertex v. start walking
in the graph.

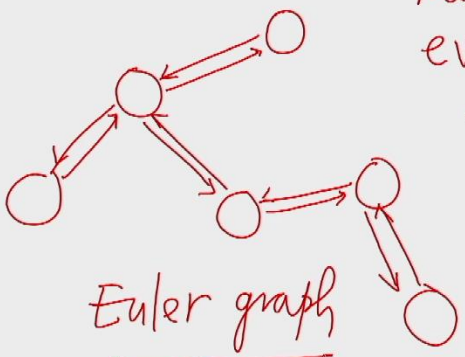




AA'

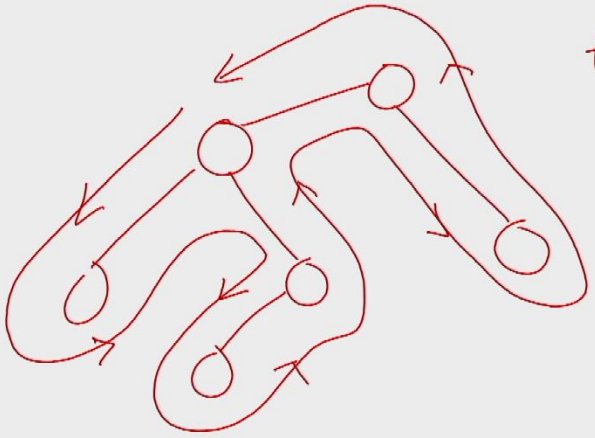
ABA'

Given a tree T (acyclic graph)
undirected now



The degree of every vertex is even.
in-degree = out-degree

Euler graph

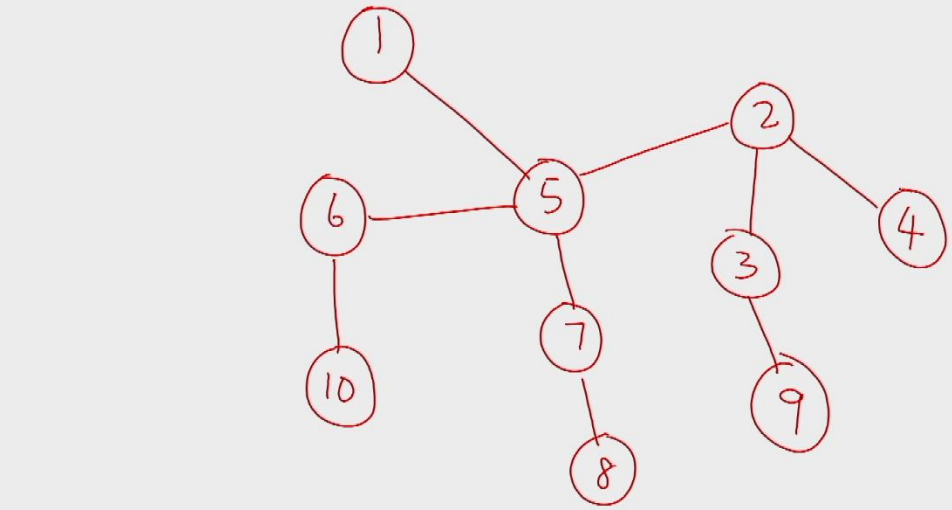


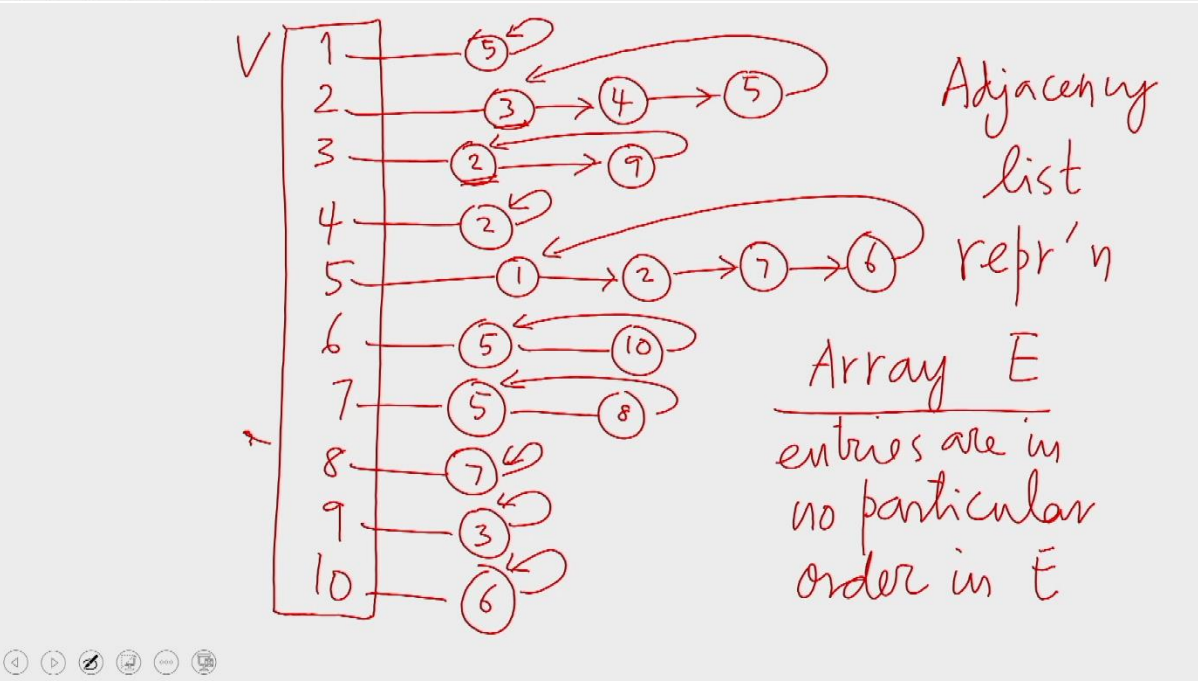
Euler circuit of T

T is given in adjacency list representation

Array of V adjacency lists

$\forall v \in V$, we have a circular linked list of the edges outgoing from v


1
2
5
6
4
3
7
10
9
8


V
1
2
3
4
5
6
7
8
9
10
Adjacency list repr'n
Array E
entries are in no particular order in E

Twin pointers

$\{2,3\}$ $[2,3] \rightleftharpoons [3,2]$

twins

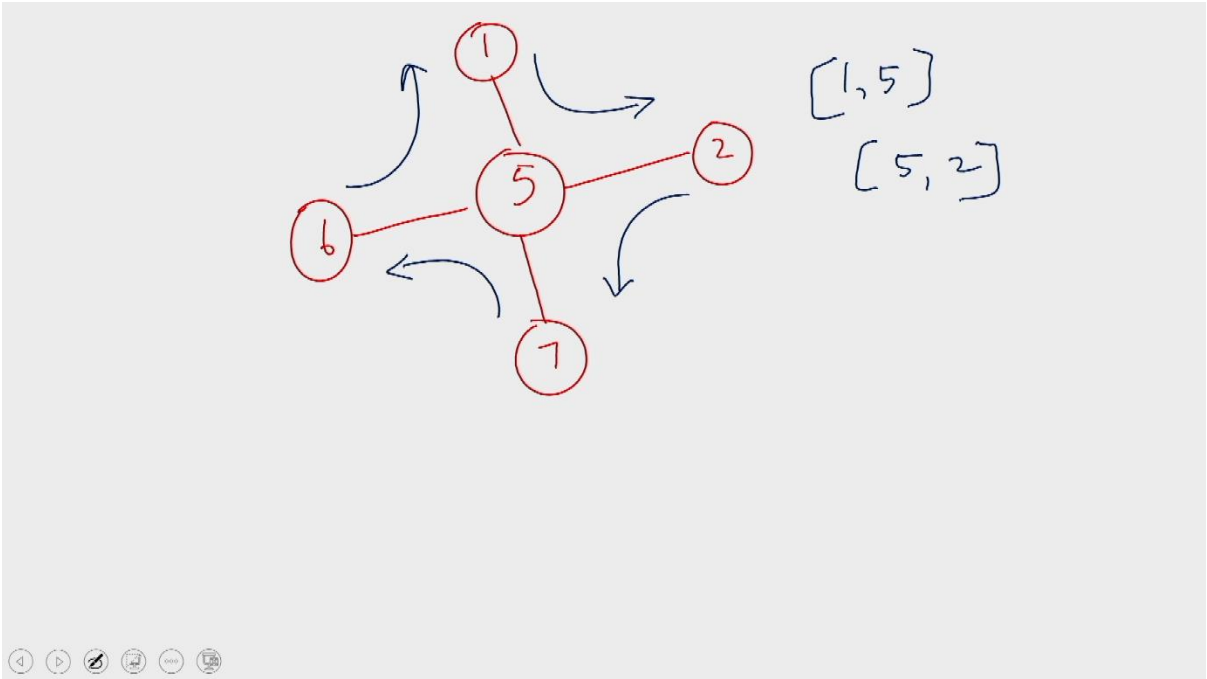
even if they are not available
find them in $O(1)$ time

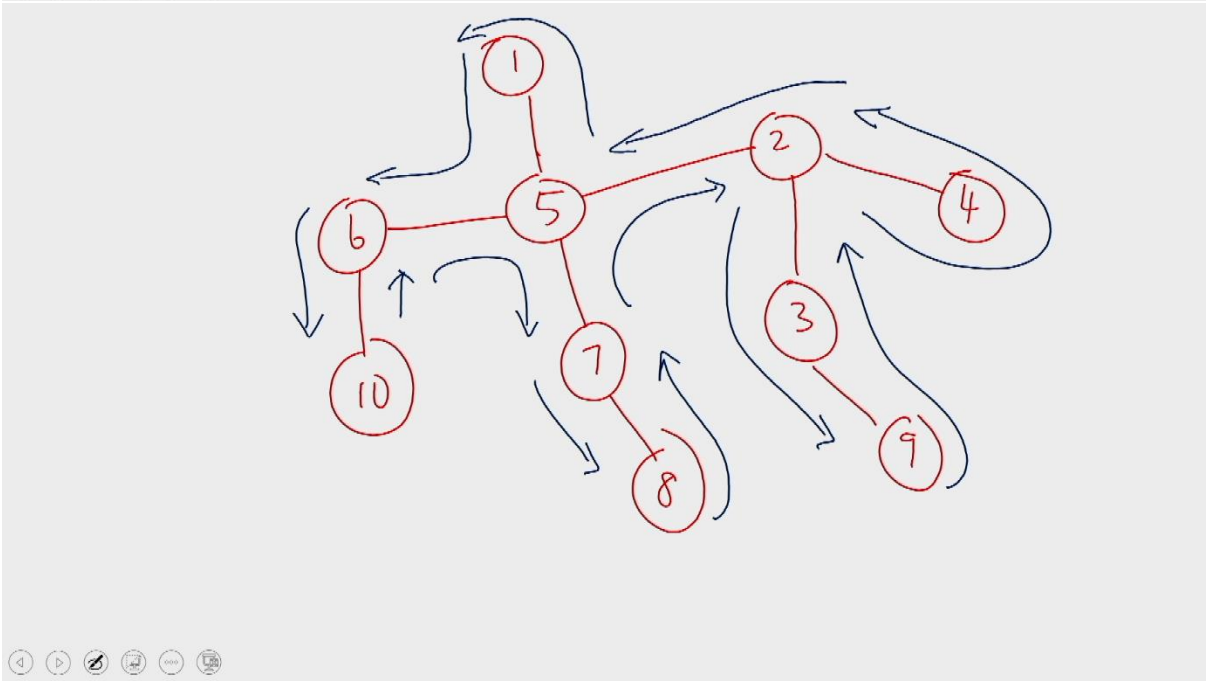
Tree Algorithms

Euler circuit of T

for each adjacency list entry $[i,j]$

Euler_next$[i,j]$
= Adj_next[twin$[i,j]$]


1
2
5
6
7
[1, 5]
[5, 2]


1
2
4
5
6
3
7
10
8
9

$O(1)$ time using $|E|$ processors

T is a tree

$|E| = O(|V|) = O(n)$

n processors $O(1)$ time

EREW PRAM

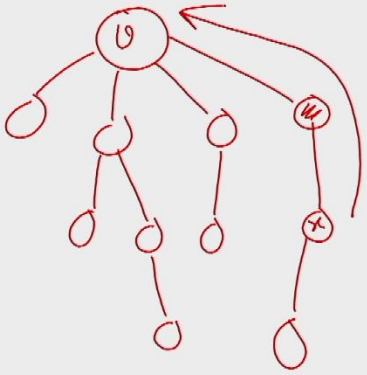
Euler ckt

is a circular linked list

given in an array E

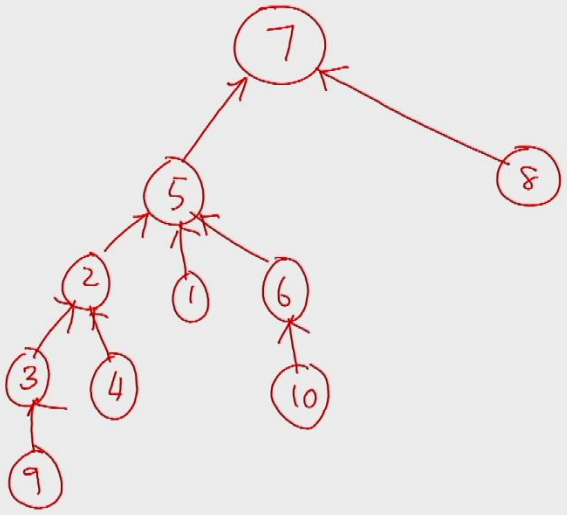
Root the tree at vertex v

tree is hung by vertex v



for each vertex x
find p[x]
parent of x

Root T at vertex 7



find the parent pointers

Take the Euler circuit
break the circuit by deleting
one incoming edge of v
Now the ckt has become a list
Rank the list
Use the ranks ?

15	11	78	17	93	5
56	12	~~87~~		32	6
6	10	13	75	1	24	7
10	6	14	52	2	42	8
6	5	15	23	3	25	9
5	7	16	39	4	51	10

$v = 7$

Compare
$[i, j]$
with $[j, i]$
if rank $[i, j]$
< rank $[j, i]$
$i = p(j)$

$O(\log n)$ time using $\frac{n}{\log n}$ processors

a tree can be rooted

on an EREW PRAM

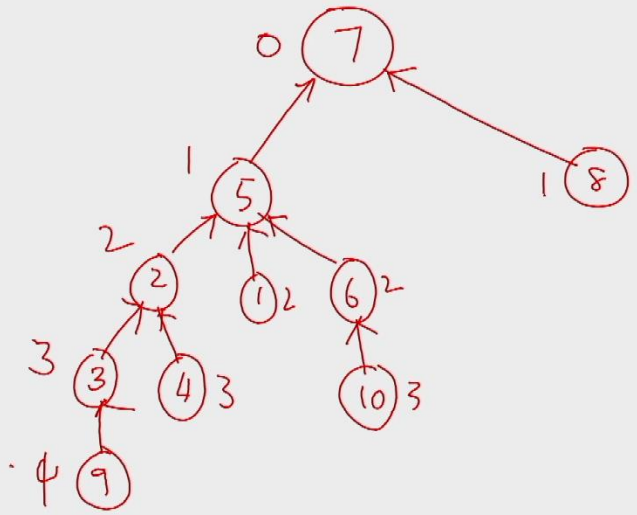
75	1	1
52	1	2
23	1	3
39	1	4
93	-1	3
32	-1	2
24	1	3
42	-1	2

25	-1	1
51	1	2
15	-1	1
56	1	2
6 10	1	3
10 6	-1	2
6 5	-1	1
5 7	-1	0
78	1	1

for parent-child edges : 1

child parent edges : -1

Root T at vertex 7



find the parent pointers

5 – 1
2 – 2
6 – 2

75	1	1
52	1	2
23	1	3
39	1	4
93	−1	3
32	−1	2
24	1	3
42	−1	2

25	−1	1
51	1	2
15	−1	1
56	1	2
6 10	1	3
10 6	−1	2
6 5	−1	1
5 7	−1	0
78	1	1

for parent-child edges : 1
child parent edges : −1

Levels of nodes

$O(\log n)$ time

$n/\log n$ processors

On EREW PRAM