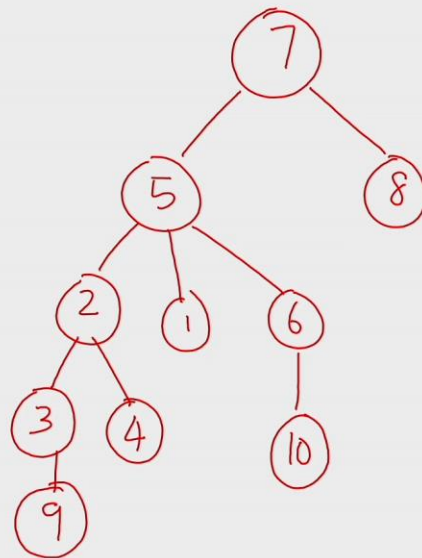


Pre order Traversal

of a rooted tree T is a listing of the vertices of T in which every vertex is listed before its children



Pre order Traversal

7	1
5	6
2	10
3	8
9	
4	



parent-child edges : 1
count up
child-parent edges do not
count : 0



Euler ckt starting at the root
assign weights of
1 to p-c edges
0 to c-p edges
Prefix sums on the Euler ckt



75	1	<u>1</u>	24	1	<u>5</u>	65	0	8
52	1	<u>2</u>	42	0	5	57	0	8
23	1	<u>3</u>	25	0	5	78	1	<u>9</u>
39	1	<u>4</u>	51	1	<u>6</u>	87	0	9
93	0	4	15	0	6			
32	0	4	56	1	<u>7</u>			
			610	1	8			
			106	0	8			
			75	2	3	9	4	1
			0	1	2	3	4	5
						6	7	8
						9		

Preorder Traversal
 can be computed in
 $O(\log n)$ time $\frac{n}{\log n}$ processors
 on an EREW PRAM

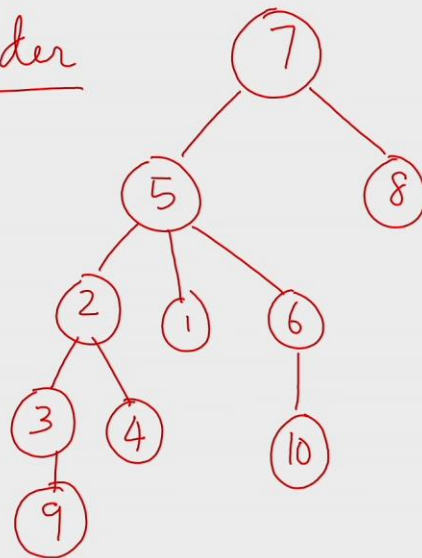
Post Order Traversal

of T, every vertex v is listed after all its children.



Post order

9	6
3	5
4	8
2	7
1	
10	



Pre order Traversal

7	1
5	6
2	10
3	8
9	
4	

Assign weights to the edge

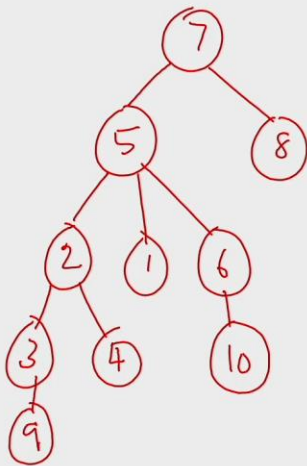
p - c edge : 0

c - p edge : 1

$O(\log n)$ time using $n/\log n$ pr.
EREW PRAM.



Number of descendants of node



a node along with its
children

g. children

g.g. children & so on
from the set of
its descendants



for each parent-child ~~node~~
edge $[i, j]$

$$\text{rank}([j, i]) - \text{rank}([i, j]) \\ = \# \text{ proper descendants of } j$$

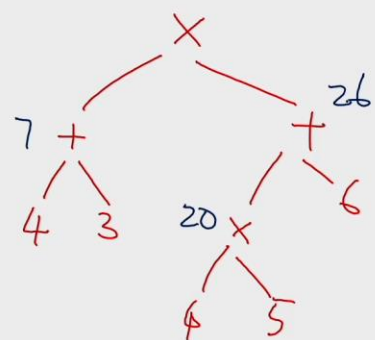
$O(\log n)$ time $n/n \log n$ processors
on EREW PRAM



Arithmetic expression
 $x +$ values are integers

$$(4 + 3) \times ((4 \times 5) + 6)$$

expression tree



An expression tree can be
evaluated bottom up.

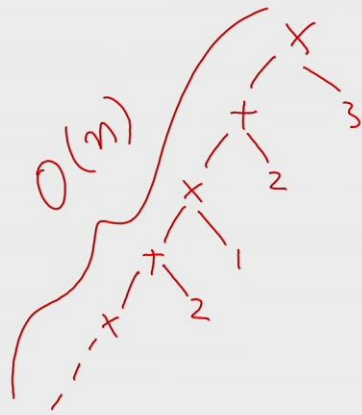
Parallel? Balanced? efficient

Not Balanced:

$O(\text{depth})$.



Depth could be $O(n)$
When n is the size of the tree



$O(n)$ time
will not do

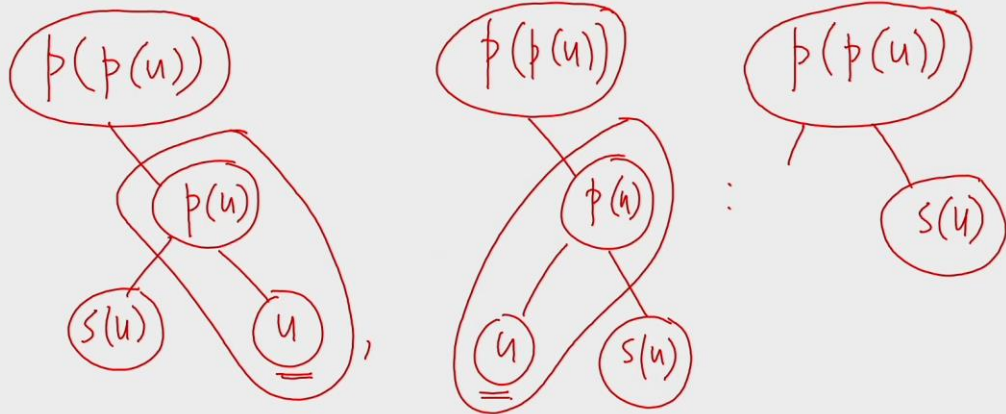


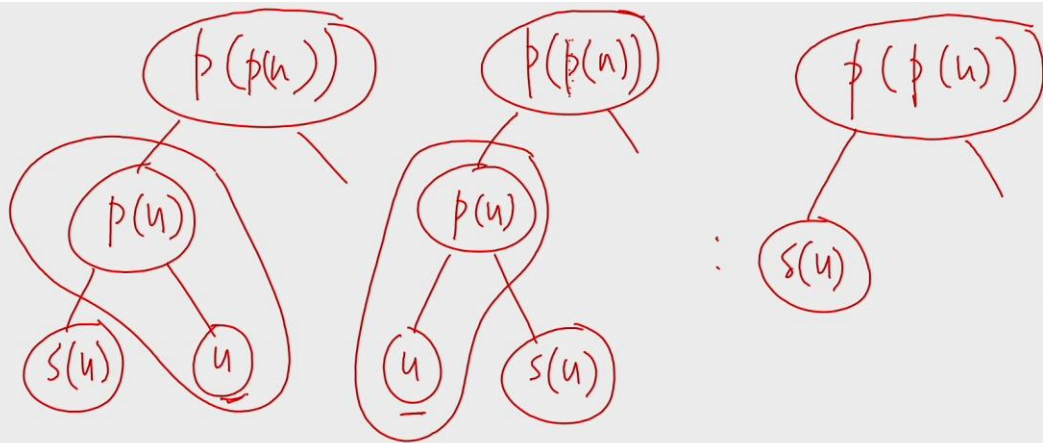
Tree Contraction Technique

every
internal
node
has 2
children


Rake operation

A leaf u s.t. $p(u) \neq \text{root}$
remove u & $p(u)$ and make
the sibling ($s(u)$) a child of
 $p(p(u))$





shrinking of a tree to a single vertex by repeated rake ops and one final reduction of a tree of the form



T s.t. T is a rooted binary tree
each ~~vertex~~ nonleaf has exactly
2 children.
each vertex has $p(\cdot)$
 $s(\cdot)$



1. Get the adjacency list repⁿ of T
2. Find an Euler ckt
3. Break the ckt open by deleting an incoming edge of r
4. Find a preorder traversal



5. Copy the traversal into an array
6. Mark all the leaf nodes
7. Compact the leaf nodes.
Now the leaves are in a L to R order. Let A denote the array of leaves



A_{odd} : the seq. of the odd elements in A

A_{even} : _____ even

a	b	c	d	e	f
	x		x		x



for $\lceil \log(n+1) \rceil$ iterations

- apply rake on all A_{odd} elements that are left children
- apply rake on the remaining elements of A_{odd} .
- $A = A_{\text{even}}$



we have at most $\lfloor m/2 \rfloor$ leaves,
if we began with m
leaves before the iteration

#leaves is 2



$n - 1$

$n/2 - 1$

$n/4 - 1$

\vdots

$O(n) : O(\log n)$

Brent's SP

$O(\log n)$ time

$n/\log n$ processors

