Indian Institute of Technology Guwahati

Department of Physics

Mid-Sem.: Nonlinear Dynamics and Chaos (PH551)

Open Elective (July., 2021 - Nov., 2021)

Maximum Credits: 30 Saturday, 25th of September, 2021 (09:00Hrs-11:00Hrs IST)

- Answer all questions.
- Use of calculator is allowed.

1. Consider a particle of unit mass is subjected under the influence of an inverted potential $U(x) = -x^2$. Sketch the phase space portrait of the system for the energies E=5, E=0, and E=-5. **[4]**

2. Consider the following system. Find the fixed points, classify them, sketch the neighbouring trajectories, and also try to fill the rest of the phase portrait. **[5+6+5]**

$$(i)\ddot{x} = ax - x^2, \text{for different values of a} \qquad (ii)\dot{r} = r(1-r^2), \dot{\theta} = 1 - \cos\theta. \qquad (iii)\ddot{x} + x\dot{x} + x = 0.$$

3. The growth of cancerous tumors can be modeled using the dynamical equation $\dot{N} = -aNln(bN)$, where $N(t)$ is proportional to the number of cells in the tumor, and $a, b > 0$ are parameters. Sketch the vector field and then plot $N(t)$ for various initial conditions. **[4]**

4. For the followings draw the phase space portraits as μ is varied and also identify the kind of bifurcation. **[3+3]**

(a)$\dot{x} = \mu x - x^2$, $\dot{y} = -y$
(b)$\dot{x} = \mu x + x^3$, $\dot{y} = -y$.