2D-Mesh of trees

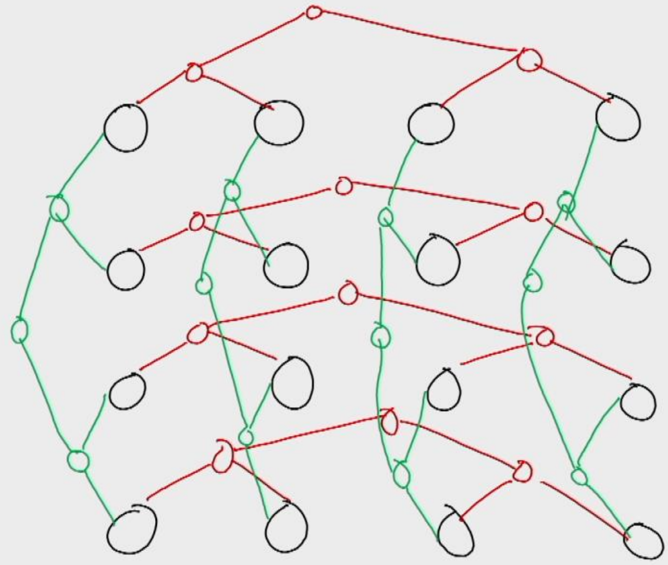
$N \times N$ mesh of processors

leaf processors

binary trees on each row & each column.

$N = 2^k$

4 x 4 Mesh of trees



Leaf processors

N^2 : degree of 2 } N^2

Root processors

2N : degree of 2

other processors

degree 3 } $2N(N-1)$

nodes in an $N \times N$ MoT

$$= N^2 + 2N(N-1)$$

$$= 3N^2 - 2N$$

Sum of degrees $= 2(N^2+2N) + 3(2N^2 - 4N)$

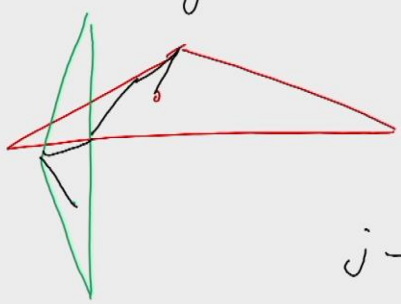
$$= 8N^2 - 8N$$

edges $= 4N^2 - 4N$

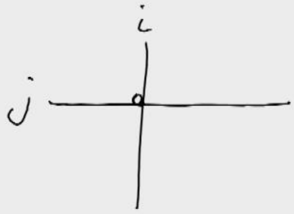
Diameter $\quad 4 \log N$

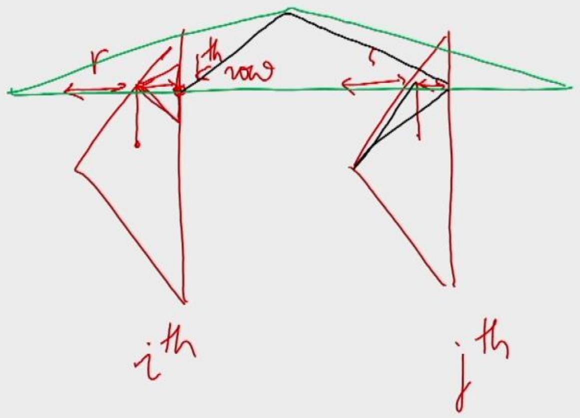
a node in the i^{th} column tree & a node in j^{th} row tree

$\leq 4 \log N$



One common leaf processor





(k, i) (k, j)

$r \geq s$

$$\begin{array}{r} \log N - r \\ 2 \log N \\ \underline{\log N + s} \\ \underline{4 \log N - r + s} \\ \leq 4 \log N \end{array}$$

Bisection Width

is $\Theta(N)$

Routing of packets

N^2 packets

one with each processor

with a unique destination

$\Omega(N)$

N packets

one with each row root

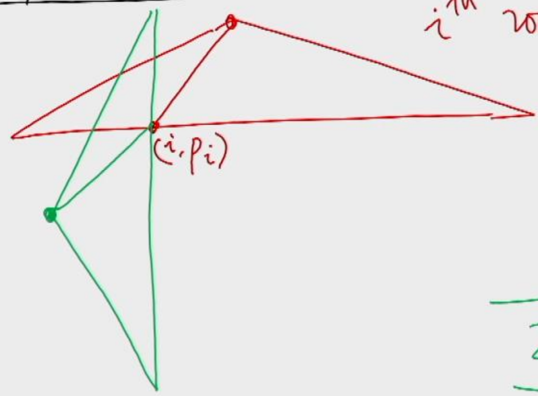
destined to a unique

column root

packet i goes from i^{th} row root

to p_i^{th} column root

i^{th} packet



i^{th} row tree

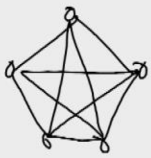
log N steps

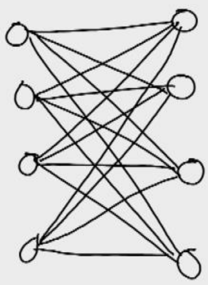
log N steps

2 log N steps

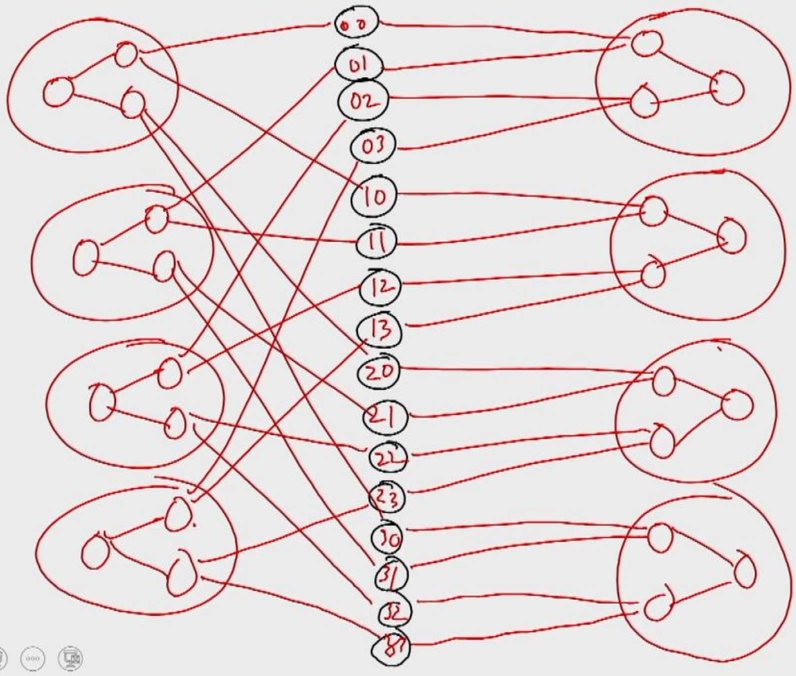
$K_{N,N}$

Complete bipartite graph on 2 vertex sets of size N.





$K_{4,4}$



$$\frac{\begin{array}{c}2N(N-1)\\ +N^2\end{array}}{3N^2-2N}$$

A $K_{N,N}$ can be transformed into a $N \times N$ MoT so that a step on $K_{N,N}$ can simulated on MoT in $2\log N$ steps

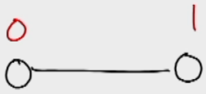
A $K_{N,N}$ algorithm of T steps runs in $\underline{\underline{O(NT)}}$

$O(T \log N)$ steps on $N \times N$ MoTs

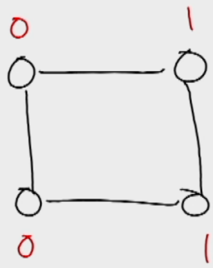
cost of the algo is

$$\underline{O(N^2 T \log N)}$$

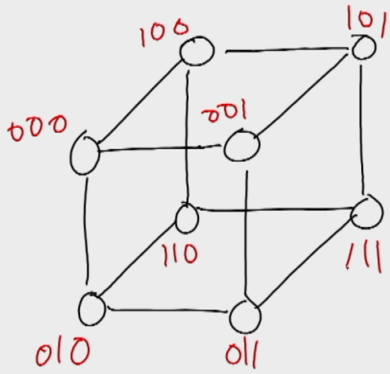
Hypercube

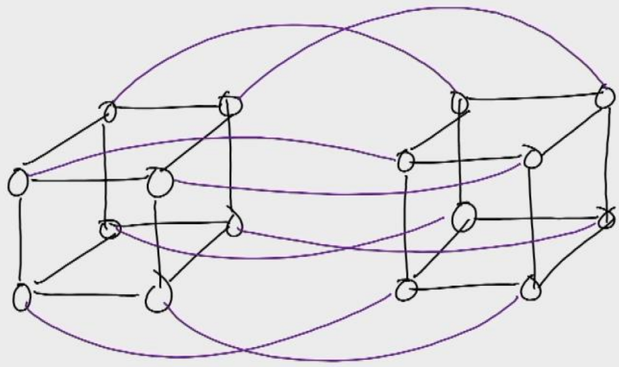


H_1



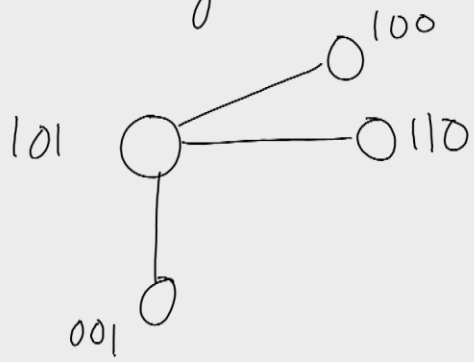
H_2





$\underline{\underline{H_4}}$

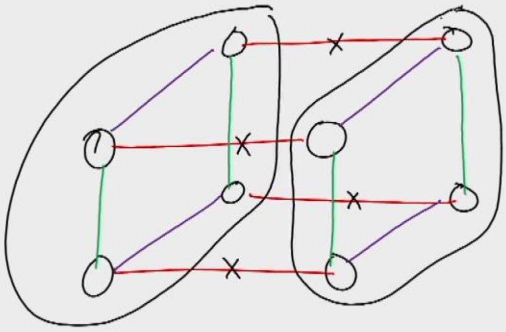
every vertex of H_r has
r neighbours.



Diameter of a Hypercube H_r is r

$$\underbrace{u \longrightarrow u^{1} \longrightarrow u^{12} \longrightarrow u^{123} \longrightarrow - \quad u^{123 \ldots r}}_{\text{distance } r}$$

$H_{\log N}$ has N vertices
diameter, $\log N$



edges of
any one dim.
form a
perfect matching

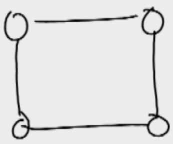
cut the red edges

Bisection Width of a
hypercube is $N/2$

A hypercube is Hamiltonian

A graph $G:(V,E)$ is Hamiltonian
if G has a cycle of
$|V|$ vertices

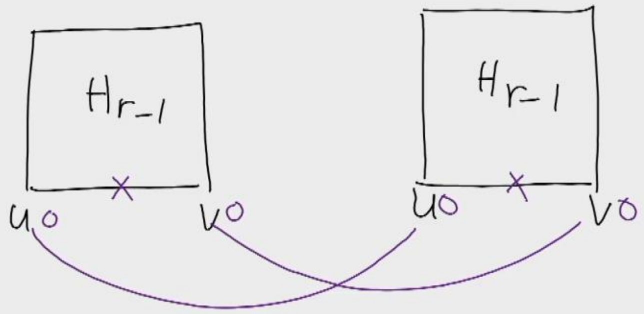
2D : H_2 Basis

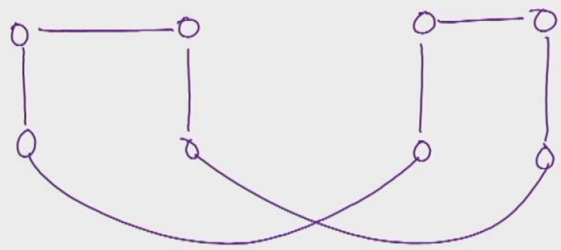


A cycle!

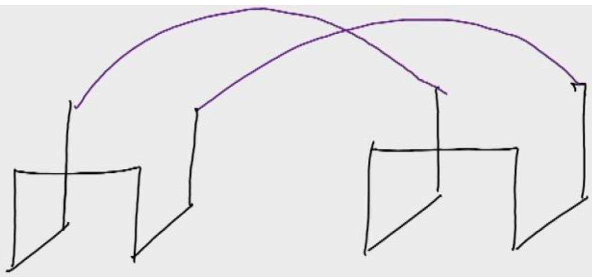
H_r

<u>Hypothesis</u> : H_{r-1} is Hamiltonian





<u>H_3</u>



$\underline{\underline{H_4}}$

H_r is <u>Hamiltonian</u>

Linear Array of size N is a subgraph of $\underline{\underline{H_{\log N}}}$