

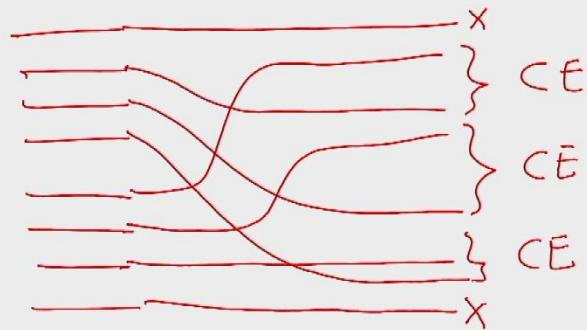
Merging & Sorting

Odd Even Merge Network

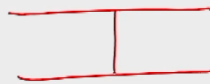
OEM : 2 sorted arrays
of size $n = 2^k$

- odd elements to one side
even elements to the other side
- Merge the odd & even sides
recursively & in parallel

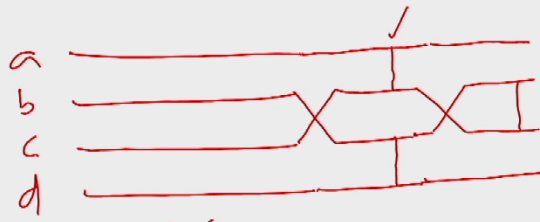
Outputs of the merges
are interleaved



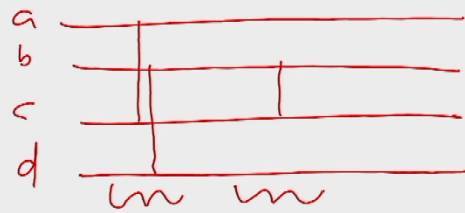
Basis 1×1 OEM



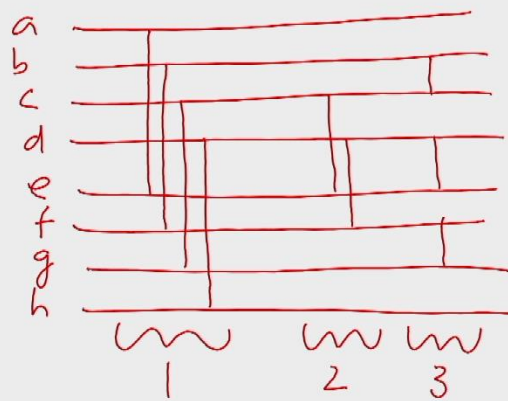
2x2 OEM



Simplify



4x4 OEM



ac eg

bd gh

4 + 2 + 3
= 9 comparisons

3 steps

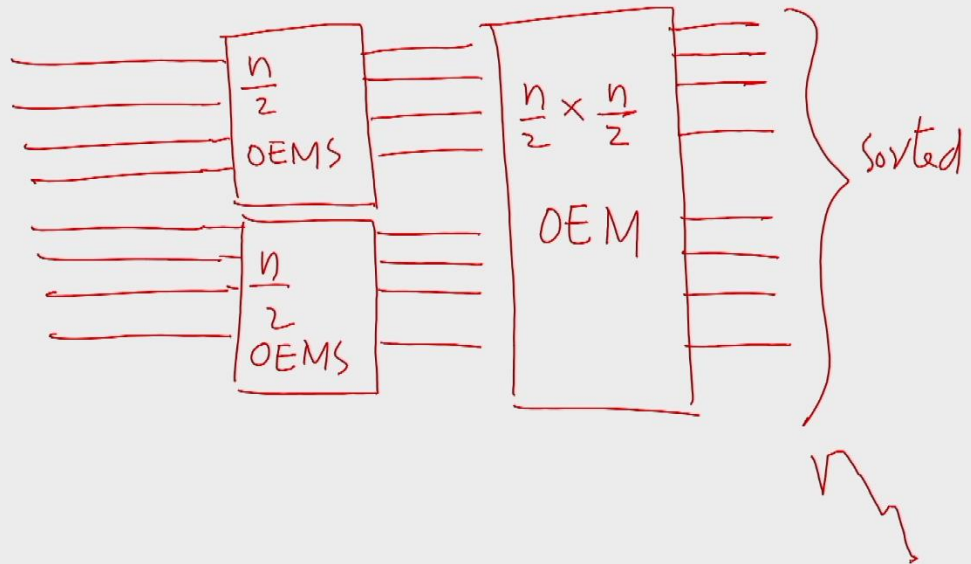
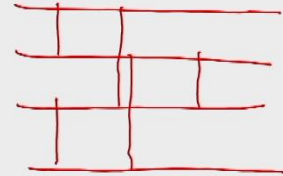
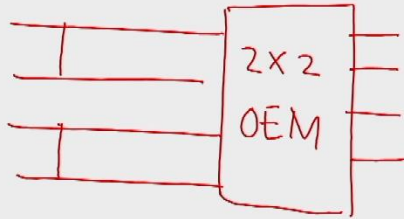


OEM Sorting Network

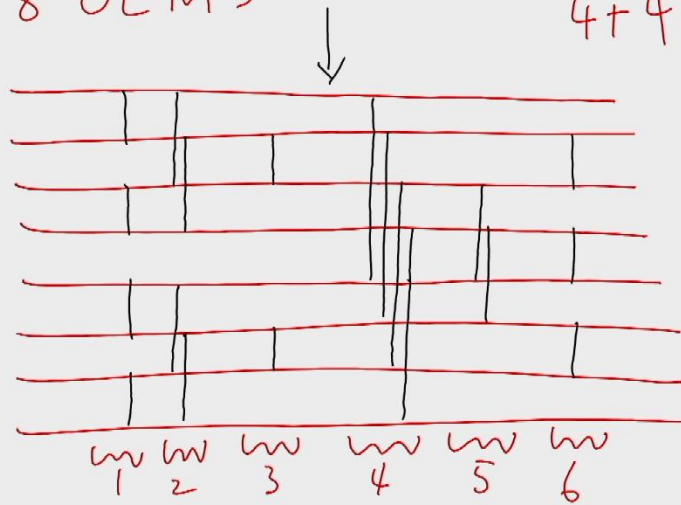
2 Sorter : $\begin{matrix} a \\ b \end{matrix} \begin{array}{|c|} \hline \text{I} \\ \hline \end{array} \begin{matrix} \text{min} \\ \text{max} \end{matrix} \begin{matrix} 2 \text{ OEMS} \\ = 1 \times 1 \text{ OEM} \end{matrix}$

4 Sorter :

4 OEMS



8 OEMS



$$4 + 4 + 2 + 4 + 2 + 3 = 19$$

A Comparator N/w
 Can be simulated on an
 EREW PRAM
 for the same time &
 cost

Correctness

OEM Network correctly merges
two sorted binary sequences

0 0 0 0 0 1 1 1

$0^i \mid 1^{n-i}$, $0^j \mid 1^{n-j}$

odd side

$\lceil i/2 \rceil$ 0's

$\lceil j/2 \rceil$ 0's

$\lceil i/2 \rceil + \lceil j/2 \rceil$

even side

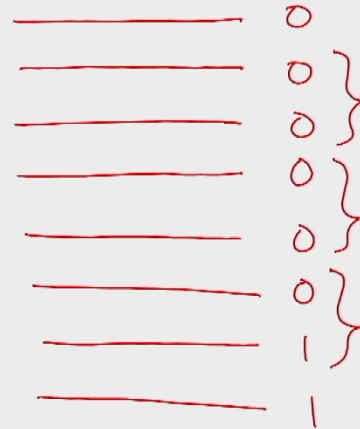
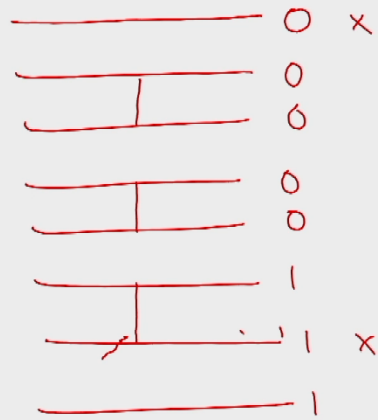
$\lfloor i/2 \rfloor$ 0's

$\lfloor j/2 \rfloor$ 0's

$\lfloor i/2 \rfloor + \lfloor j/2 \rfloor$ 0's

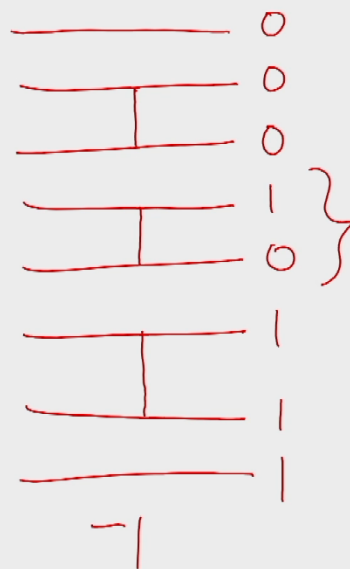
The diff. in no. of 0's : 2, 1, 0

After leaving out — " — : 1, 0, -1



+1

1



+1, 0, -1

OEM N
works correctly
on binary sequences

OEM Sorter

works correctly on
binary sequences.

0-1 Principle Any sorting algo. that correctly
sorts all bin. sequences, correctly
sorts every sequence drawn
from a linearly ordered set

Suppose A is a sorter that works
on all binary sequences

A doesn't work on some sequence
drawn from a LO set.

(\exists a bin. sequence on which A works
incorrectly)

2		1	0	}	(2, 0)
4		2	0		(4, 0)
7		3	0		(7, 1)
1	$\rightarrow A \rightarrow$	5	1		(1, 0)
8		6	1		(8, 1)
3		(4)	0		(3, 0)
5		7	1		(5, 1)
6		8	1		(6, 1)

OEMS works on all
binary sequences
 \rightarrow OEMS works on all
sequences drawn from
a LO set