A hypercube is a Hamiltonian graph.

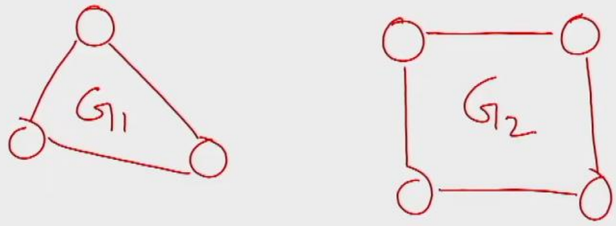
There is a cycle of n nodes in a hypercube of n nodes

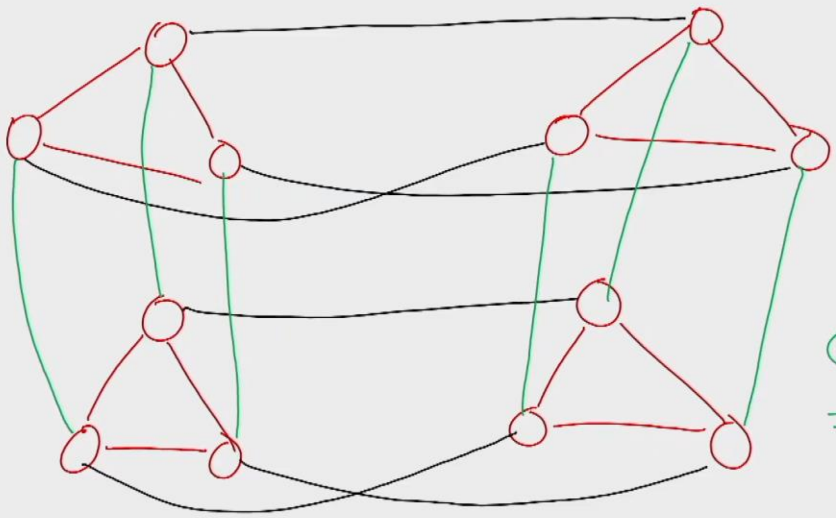
$H_{\log n}$

A linear array can be embedded in a hypercube

What about higher dimensional arrays?

Cross product of graphs





$G_1 \otimes G_2$

G_1, G_2

$G_1 = (V_1, E_1) \quad G_2 = (V_2, E_2)$

$G_1 \otimes G_2 = (V_1 \times V_2, \; E)$

$E = \{ \{(u_1, u_2), (v_1, v_2)\} \mid$

$(u_1, v_1) \in E_1 \; \& \; u_2 = v_2$ or

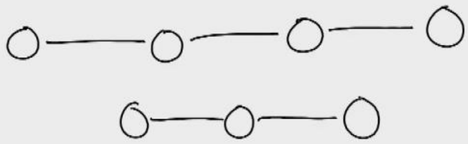
$(u_2, v_2) \in E_2 \; \& \; u_1 = v_1 \}$

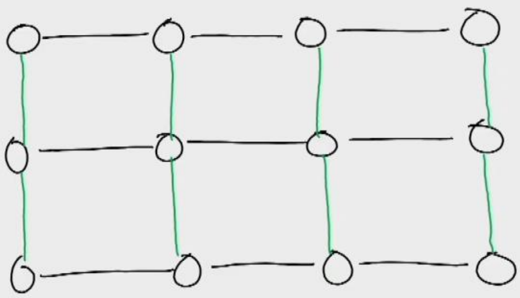
$G_1 \otimes G_2 \otimes G_3$

$= (G_1 \otimes G_2) \otimes G_3$

$= G_1 \otimes (G_2 \otimes G_3)$

A 2D-mesh is a cross product of linear arrays





3 × 4 mesh

$LA_3 \otimes LA_4$

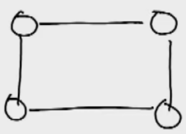
$N_1 \times N_2 \times \cdots \times N_K$

$LA_{N_1} \otimes LA_{N_2} \otimes \cdots \otimes LA_{N_K}$

LAs embed in Hypercubes

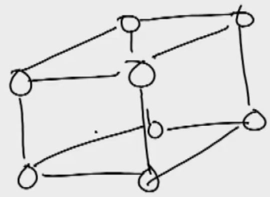
K-D meshes embed in Hypercubes

H_2



2×2 mesh

H_3



$2 \times 2 \times 2$ mesh

H_K is a $\underbrace{2 \times 2 \times \cdots \times 2}_{K\ 2\text{'s}}$ mesh

$$H_K = H_{k_1} \otimes \cdots \otimes H_{k_r}$$

where $k_1 + \cdots + k_r = K$

$$H_K = \underbrace{2 \times 2 \times \cdots 2 \times 2}_{K} \text{ mesh}$$

$$= \underbrace{2 \times 2 \times \cdots 2}_{k_1} \times \underbrace{2 \times \cdots \times 2}_{k_2} \cdots \times \underbrace{2 \times \cdots 2}_{k_r}$$

$$H_K = \underbrace{H_1 \otimes H_1 \otimes \cdots \otimes H_1}_{K \text{ hypercubes } H_1}$$

$$= \underbrace{H_1 \otimes \cdots \otimes H_1}_{k_1} \otimes \underbrace{H_1 \otimes \cdots \otimes H_1}_{k_2} \otimes \cdots \otimes \underbrace{H_1 \otimes \cdots \otimes H_1}_{k_r}$$

$$\underline{k_1 + \cdots + k_r = K}$$

$$H_K = H_{k_1} \otimes H_{k_2} \otimes - \otimes H_{k_r}$$

$k_1 \times k_2 \times \cdots \times k_r$ ✓

r-D mesh is given

$$M = \underline{LA_{k_1} \otimes LA_{k_2} \otimes \cdots \otimes LA_{k_r}}$$

The cross product of

$G_1' - G_k'$ that are subgraphs

of $G_1 \cdots G_k$ is a subgraph

the cross product of $G_1 \cdots G_k$

$$G_1' \otimes - \otimes G_k' \subseteq G_1 \otimes \cdots \otimes G_k$$

A mesh $n_1 \times n_2 \times \dots \times n_r$

is a subgraph of

$$H_{\lceil \log n_1 \rceil} \otimes H_{\lceil \log n_2 \rceil} \otimes \dots \otimes H_{\lceil \log n_r \rceil}$$

$$= H_{\lceil \log n_1 \rceil + \lceil \log n_2 \rceil + \dots + \lceil \log n_r \rceil}$$

3×5 mesh $\longrightarrow 15$

Can be embedded in

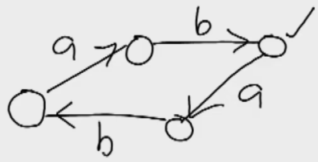
a Hypercube of $\quad H_4 : 16$

dim $\lceil \log 3 \rceil + \lceil \log 5 \rceil$

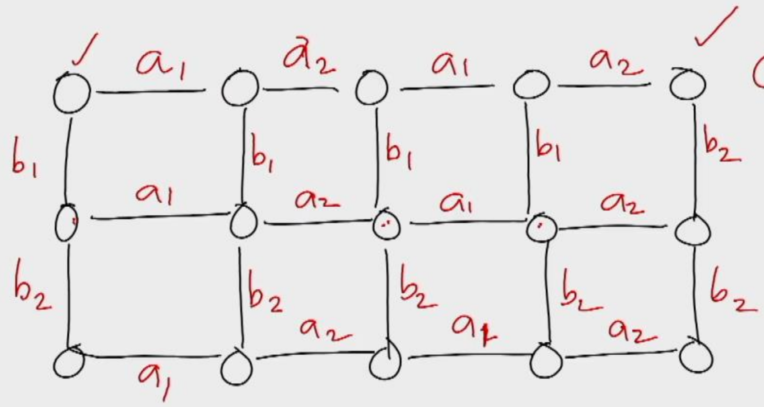
$2 + 3 = \underline{\underline{5}} \longrightarrow 32$

A 3x5 mesh does not embed in ~~H5~~ H_4.

Assume otherwise.



Trace every dimension twice in every cycle



Contradiction

Symmetry in Hypercubes

Permutation π of the dimension of a H_r is given

$\pi : \{1, \dots, r\} \to \{1, \dots, r\}$

A pair of vertices u & u'

Then there exists an automorphism σ of H_r

So that $\sigma(u) = u'$

and the dimensions of the automorphism permutes by π

$\{1, \dots, r\}$

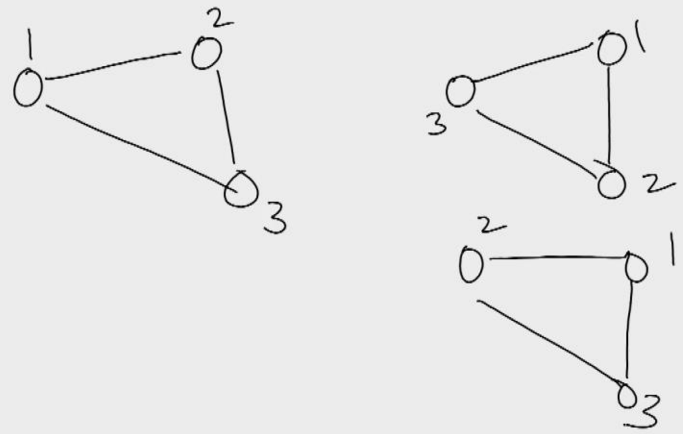
Automorphism of a graph

$G = (V, E)$

is a mapping $\sigma : V \to V$

so that $\{\sigma(u), \sigma(v)\} \in E$

iff $\{u, v\} \in E$



$\pi, \; u, \; u'$

$\sigma(u) = u'$

1 2 3 4 5 6

$$\frac{2}{\pi(2)} \quad \frac{1}{\pi(1)} \quad \frac{3}{\pi(3)}$$

$\sigma(x_1 \, x_2 \ldots x_r)$

$$= x_{\pi(1)} \oplus u_{\pi(1)} \oplus u'_1$$

$$\vdots$$

$$x_{\pi(r)} \oplus u_{\pi(r)} \oplus u'_r$$