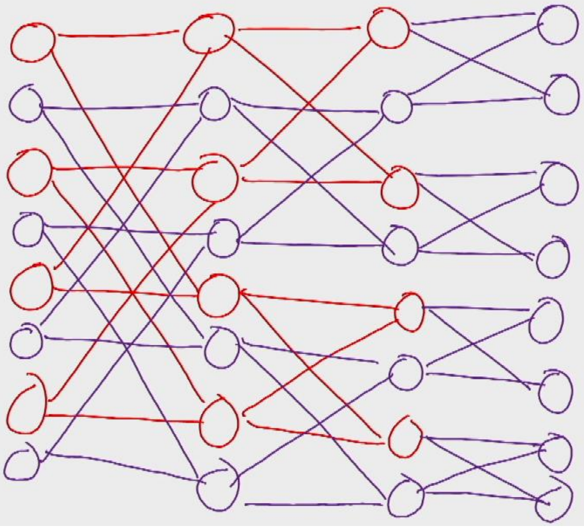
Recursive structure



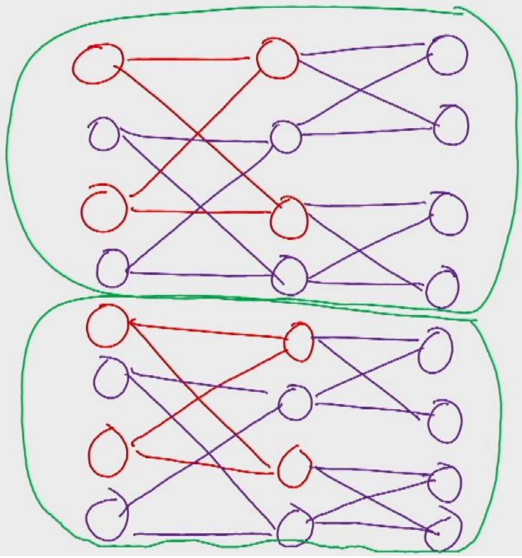
Take an r-D Bfly

Remove the r^{th} col

Recursive structure

2D

2D

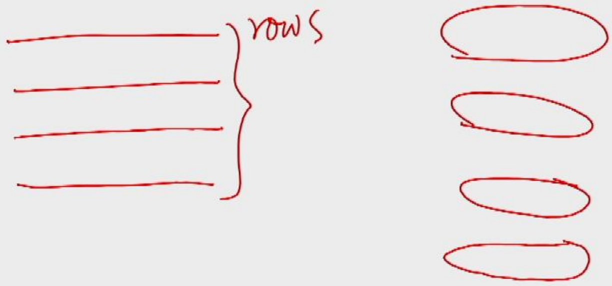


Take an r-D Bfly

Remove the r^{th} col

Wrapped Butterfly

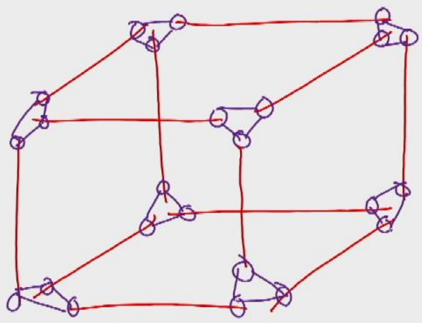
fusing the 0^{th} column & the r^{th} colum



Butterfly with the rows collapsed gives a hypercube

Wrapped Butterfly is Hamiltonian

Cube Connected cycle



3-D hypercube to 3-D CCC

Replace every hypercube node with a cycle of r nodes

the hypercube edges remain

if node w & node w'
differ exactly in the
i^{th} bit
then the i^{th} vertex of cycle w
& i^{th} vertex of cycle w'
are adjacent.

$$\langle w, i \rangle \overset{\text{Hypercube}}{—} \langle w', i \rangle$$

where w & w' differ in i^{th} bit
& only in i^{th} bit

$$\langle w, i \rangle \overset{\text{cycle}}{—} \langle w, i' \rangle$$

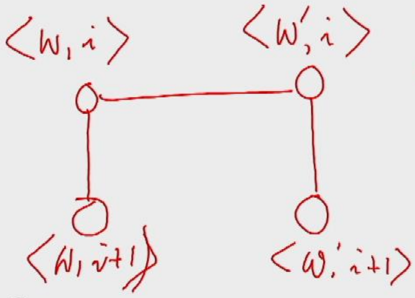
iff $i - i' \equiv 1 \pmod{r}$

each vertex has a degree of 3.

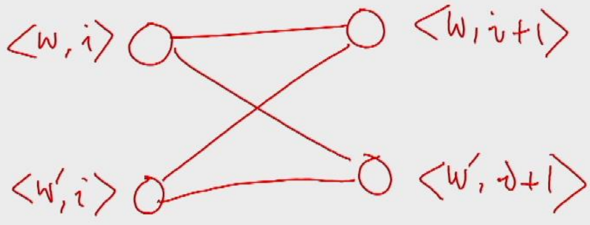
Every r-D hypercube algorithm that runs in T steps can be simulated on a CCC of dimension r in $O(Tr)$ steps.

Close relationship between a CCC and wrapped bfly

CCC



Wrapped bfly



An algorithm that runs in T steps on CCC (resp. wbfly) can be run in $O(T)$ steps on wbfly (resp. CCC)

Butterfly network

0 1 r

$\langle w, 0\rangle$

$\langle w, 0\rangle$

$\langle w', r\rangle$

w & w' agree on the i^{th} bit

take the straight edge between columns i & $i+1$

The diameter of a r-D bfly is $O(r)$

The bisection width of an $\log N$- dimensional bfly is $\theta(N/\log N)$

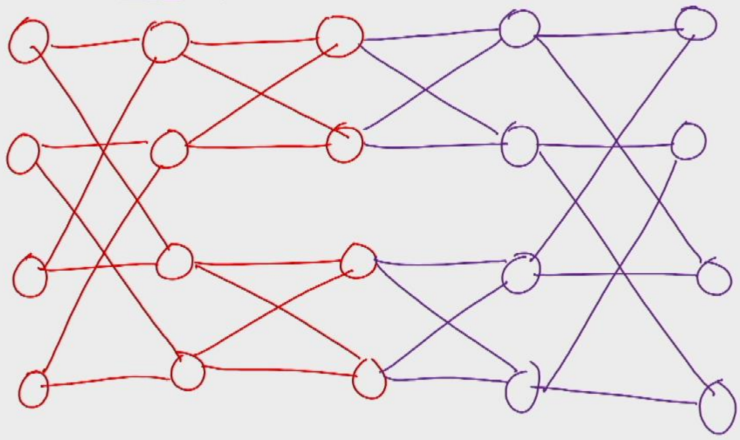
Beneš Network

Paste two butterflies back to back

$2(r+1) - 1$

$2r+1$

2D-Beneš network



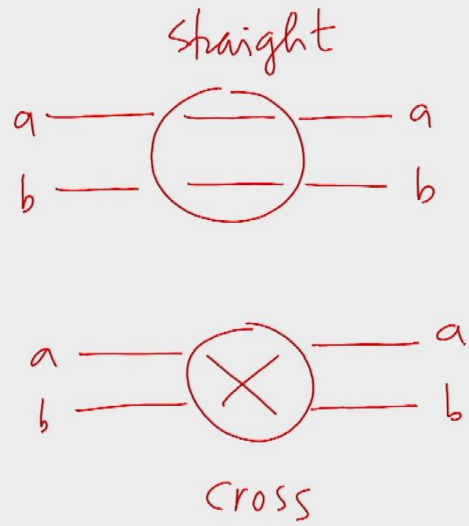
Rearrangeable Network

A network with n inputs & n outputs is a Rearrangeable network if for any permutation π of $\{1, .., n\}$, we can construct edge disjoint paths that connect input i to output $\pi(i)$, for $1 \leq i \leq n$.

A Beneš Network is

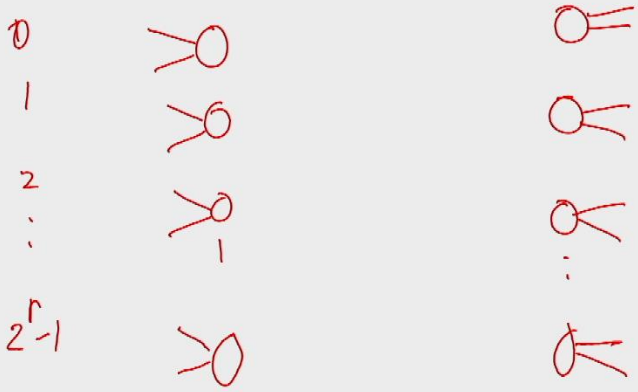
a rearrangeable network

every node is a reconfigurable switch



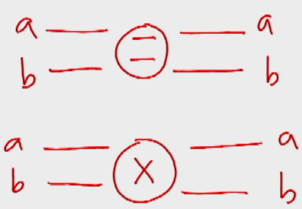
r-D Beneš network

has 2^{r+1} inputs & 2^{r+1} outputs



Proof by induction

Basis $r=0$



a b

b a

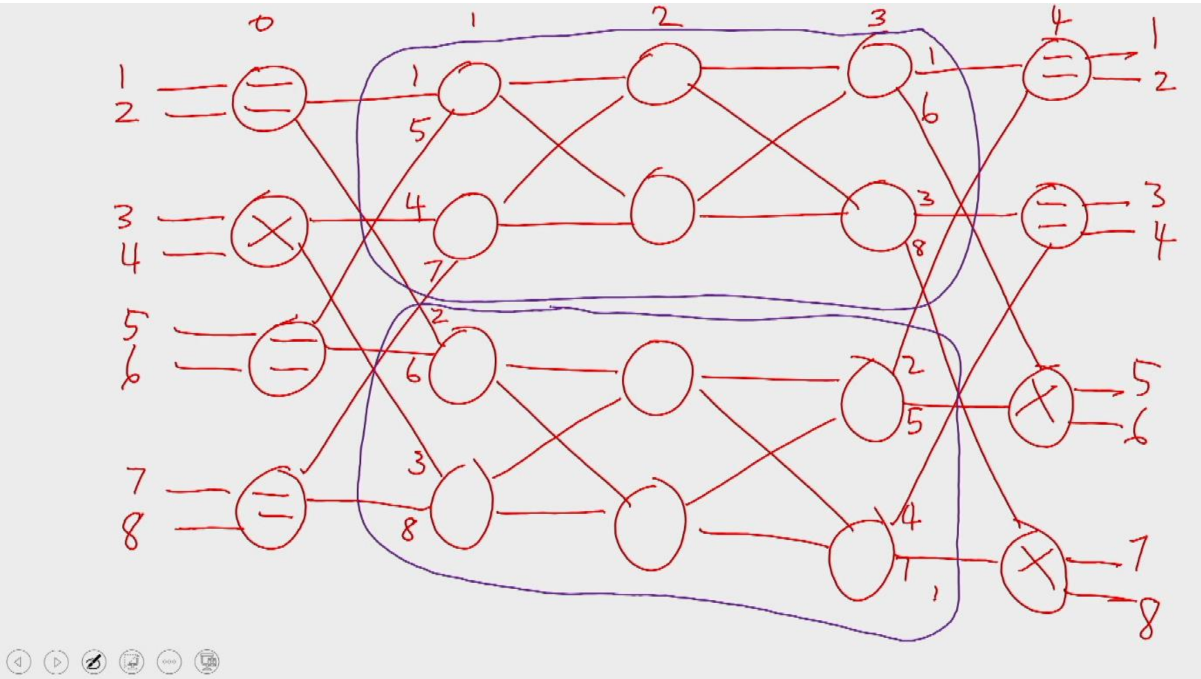
Hypothesis

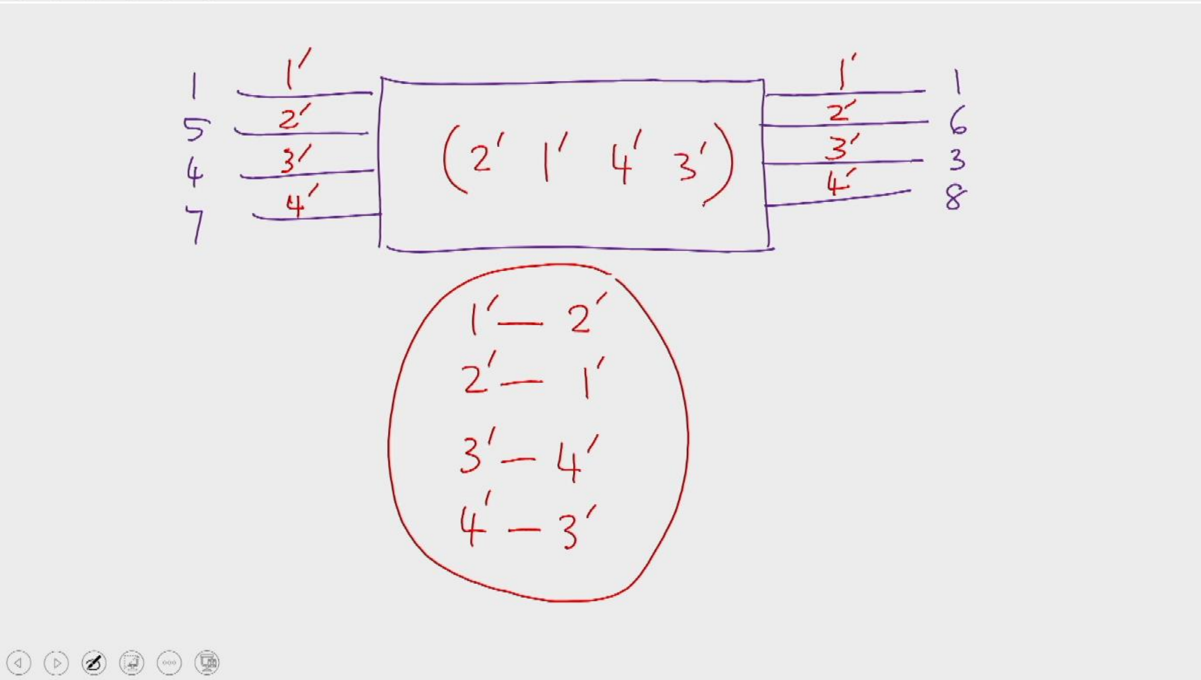
every $(r-1)$ Dimensional Beneš network is rearrangeable

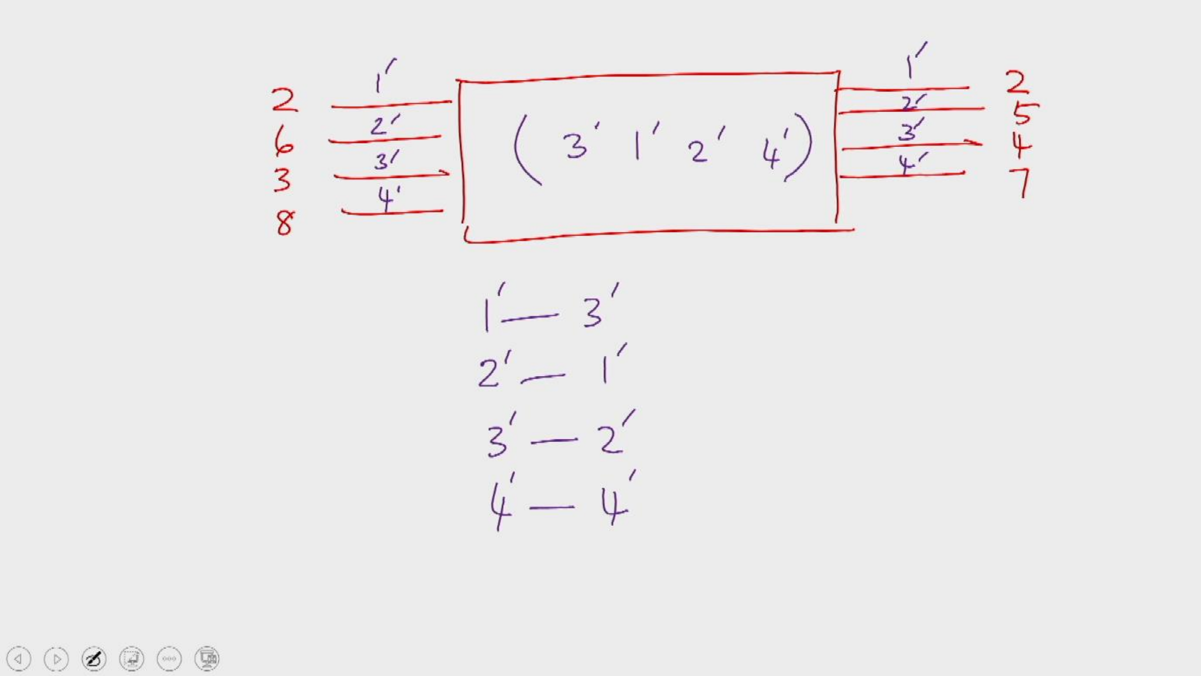
Step

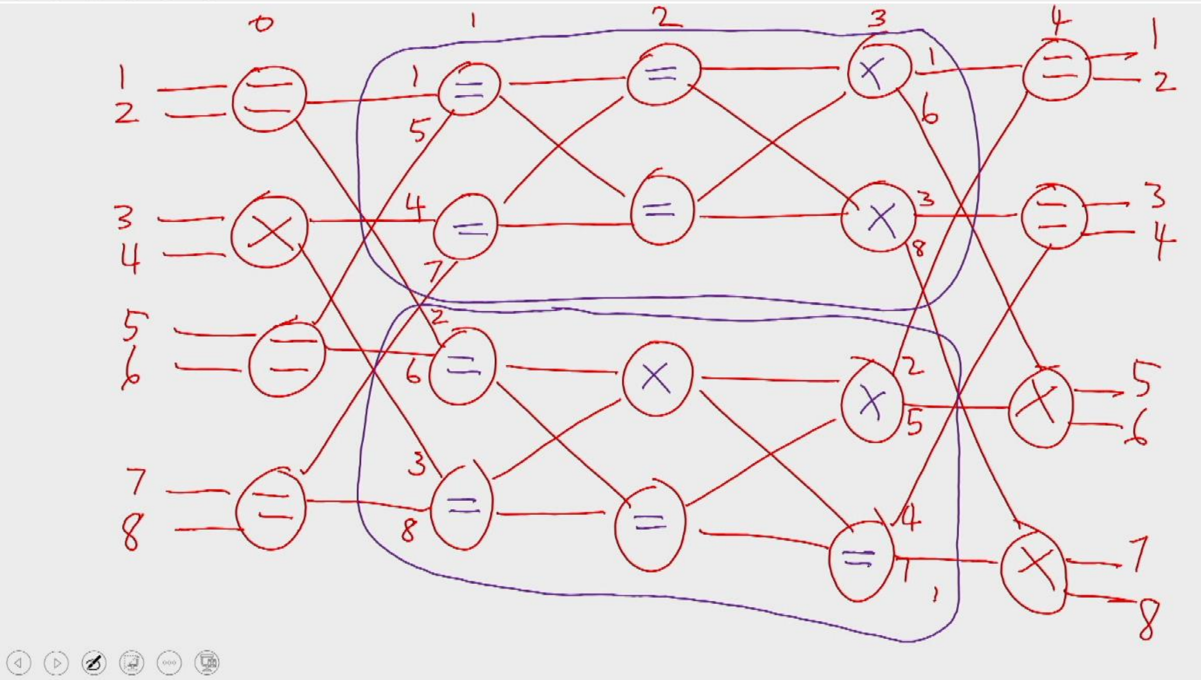
Consider π

i	1	2	3	4	5	6	7	8
$\pi(i)$	6	4	5	8	1	2	3	7


0 1 2 3 4
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8


1 5 4 7
1' 2' 3' 4'
(2' 1' 4' 3')
1' 2' 3' 4'
1 6 3 8
1'— 2'
2'— 1'
3'— 4'
4'— 3'


2 6 3 8
1′ 2′ 3′ 4′
(3′ 1′ 2′ 4′)
1′ 2′ 3′ 4′
2 5 4 7
1′ — 3′
2′ — 1′
3′ — 2′
4′ — 4′


0 1 2 3 4
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8

Start any node i on the input side

Say i is paired with i'

choose a config. for the switch to which i & i' are connected

$=$ or $\times$

we know the half of input i.

Output $\pi(i)$ should be connected to the same half

This gives us the config$\underline{n}$ for the switch $\pi(i)$ is connected

Continue with the pair of $\pi(i)$

$\pi^{-1}(\pi(i)')$

until we get back to the switch to which input is connected.

Why would we loop back?