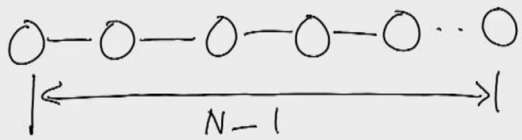
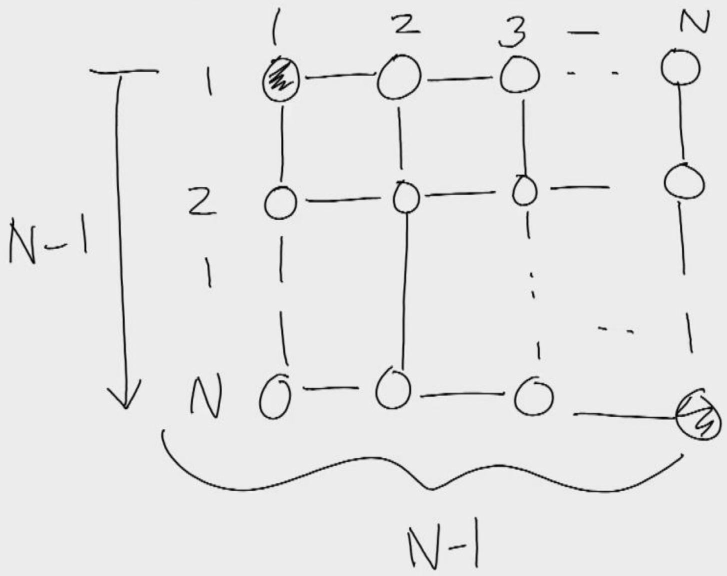
Diameter is the max. distance between any two nodes in the network.



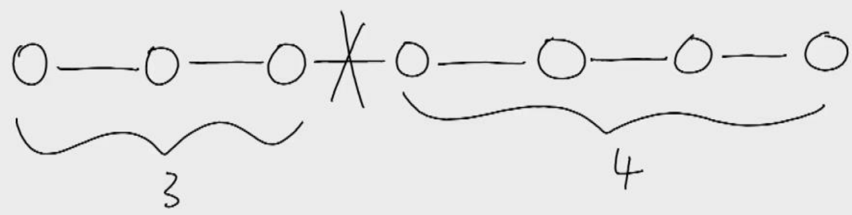


Diameter

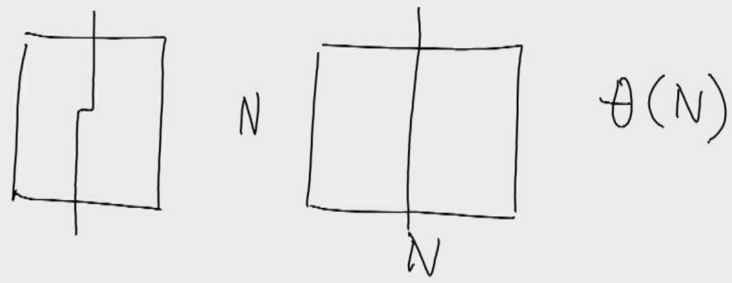
$\underline{2N-2}$

Bisection width

is the minimum no. of edges that must be deleted to bisect the network.

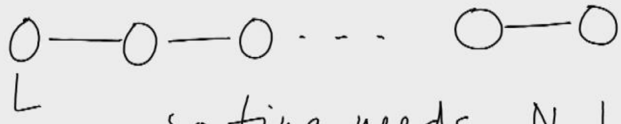


Bisection width = 1



Useful in proving lower bounds

for eg., consider sorting



sorting needs $N-1$ steps

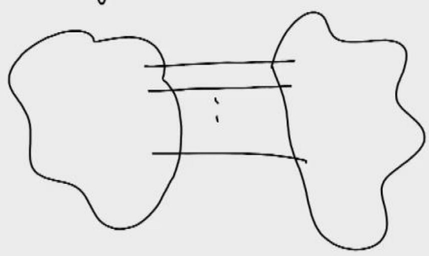
$N \times N$ mesh

diameter is $2N-2$

LB: $2N-2$

Bisection width

Network of ~~da~~ bisection width W.



$N/2$ elements on either side

an edge moves on element / step

$\Rightarrow$ in one step, W elements can move across the bisection

By the time, the i/p is sorted

N elements would have moved

N/W steps would have been spent.

$\Omega(N/W)$ for sorting

Sort on a $\sqrt{N} \times \sqrt{N}$ mesh

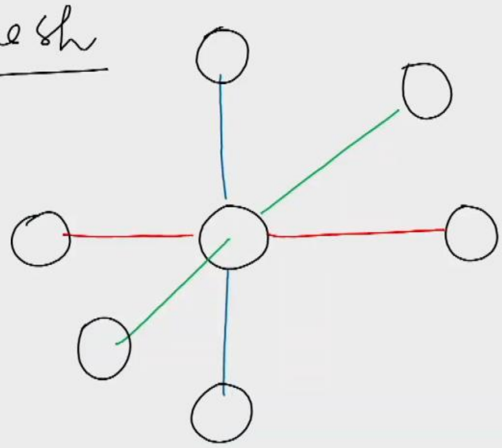
Bisection width $W = \Theta(\sqrt{N})$

traffic $= N$

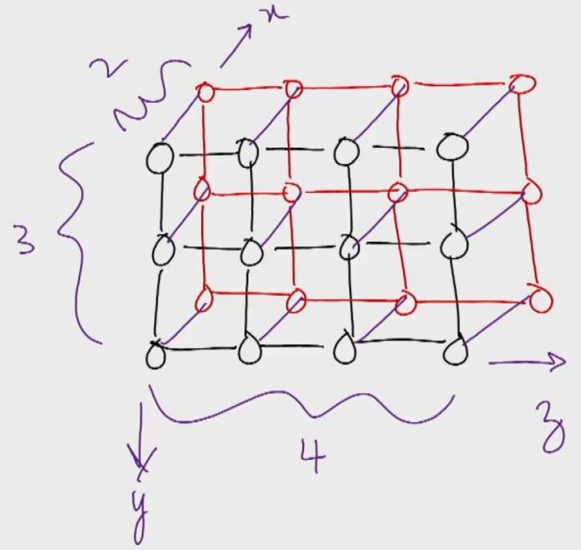
$LB = \Omega(N/\sqrt{N}) = \Omega(\sqrt{N})$

Higher dimensional mesh

3D-mesh



max. degree = 6.



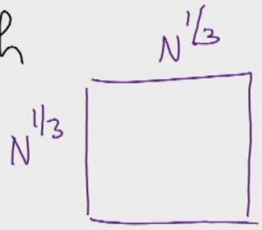
$2 \times 3 \times 4$ mesh

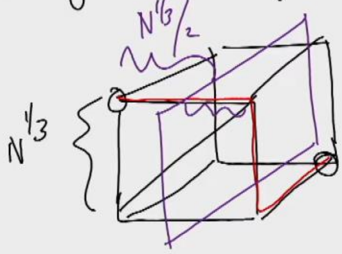
24 processors

$N^{1/3} \times N^{1/3} \times N^{1/3}$ mesh

N-nodes

diameter of this n/w





$3N^{1/3} - 3$

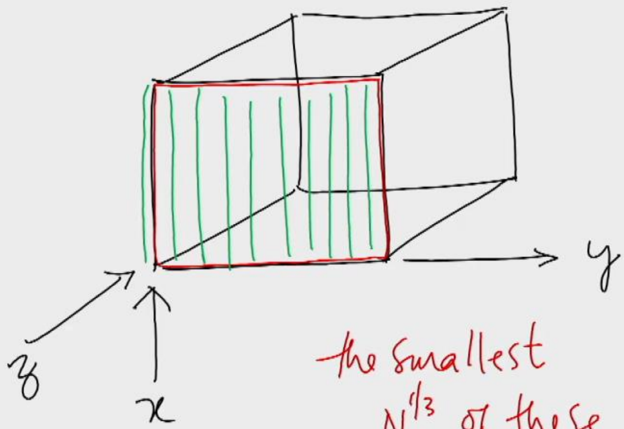
$\Theta(N^{2/3})$

Lower Bound for sorting

$$= \Omega\left(N / N^{2/3}\right) = \Omega(N^{1/3})$$

An algorithm to sort N items in $O(N^{1/3})$ time

<u>0–1 Principle</u>



the smallest $N^{2/3}$ elements will be in the 1st xy-plane

the smallest $N^{1/3}$ of these will be in the first x line

x major order
in the xy-plane

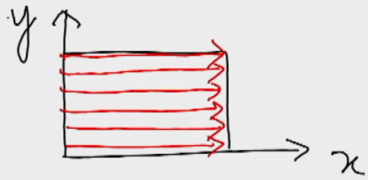
The mesh elements
to be sorted in
zyx-order

$$(i, j, k) \geq (i', j', k')$$

then $k'j'i'$ lexicographically
precedes
kji

zyx-order

xy - plane



yx - order

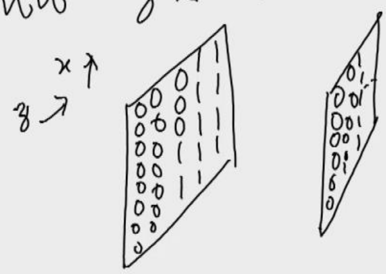
x - major order / row major order

xy - order

y - major order / col major order

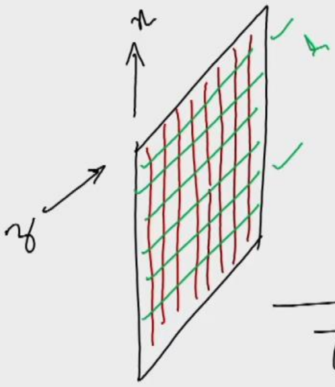
zyx - order sorting

Step 1: Sort each zx - plane into zx - order



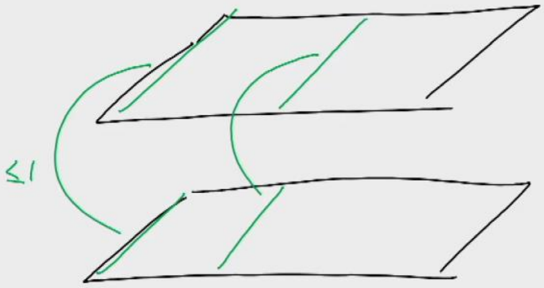
2D - algorithm for the prev. lecture

$O(N^{1/3})$ time



exactly one
x-line is dirty
on each zx-plane

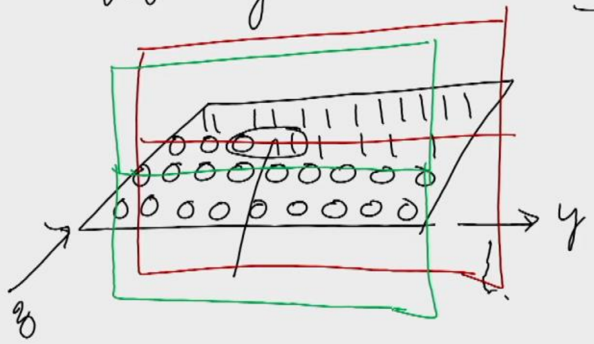
Take 2 z lines from the
same zx-plane
They differ by at most 1
in the # of 1's



2 yz-planes

differ by
$\leq N^{1/3}$ in the
1's.

Step 2: Sort each yz-plane in zy-order



window of $N^{1/3}$ posn

At most 2 dirty xy-planes

which two?

Step 3: Sort each xy-plane in yx-order

alternating the sorting order

odd planes $\longrightarrow$

even planes $\longleftarrow$

0 0 0 0 0 1 1 1 1 1

1 1 1 1 1 1 0 0 0 0

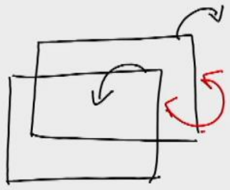
---

0 0 0 0 0 1 0 0 0 0 ×

1 1 1 1 1 1 1 1 1 1 ✓

Step 4

Do ~~4~~ 2 steps of OETS on each z-line



There is at most one dirty xy-plane

Step 5

Sort every xy-plane in yx-order. $\Big\}\ O(N^{1/3})$

Every xy-plane is now clean

The mesh is sorted in zyx-order

$O(N^{1/3})$ time

for $r = O(1)$

N items can be sorted in

$O(N^{1/r})$ time on

a $\underbrace{N^{1/r} \times \dots \times N^{1/r}}_{r-D}$ mesh