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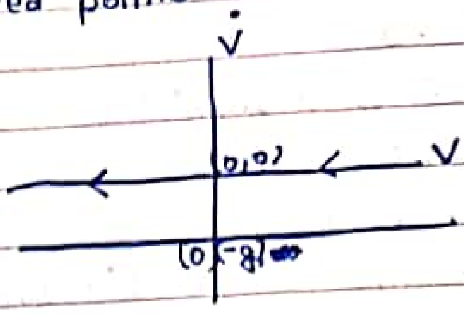
ASSIGMENT-1

- v^*, u^* and x^* denote fixed points



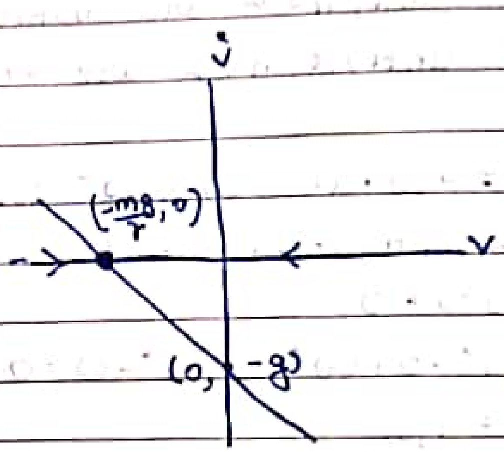
Ans-1- $\dot{v} = -g = f(v)$

$f(v^*) = 0$

$-g = 0$ has no solutions since $g > 0$

Ans-2- $m\dot{v} = -mg - \gamma v$

$\dot{v} = -g - \frac{\gamma}{m} v = f(v)$



$f(v^*) = 0$

$-g - \frac{\gamma}{m} v^* = 0 \Rightarrow v^* = -\frac{mg}{\gamma}$

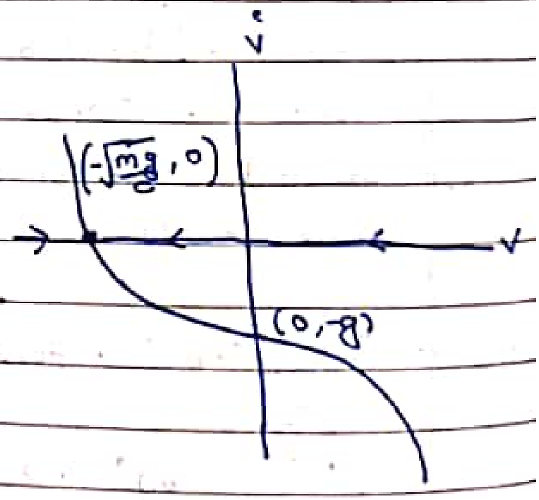
$f'(v) = -\frac{\gamma}{m}$

$f'(v^*) = -\frac{\gamma}{m} < 0$

Hence stable fixed point

Ans-3- $m\dot{v} = -mg - cv^2 \operatorname{sgn}(v)$

$$\dot{v} = \begin{cases} -g & v = 0 \\ -g - \frac{c}{m} v^2 & v > 0 \\ -g + \frac{c}{m} v^2 & v < 0 \end{cases} = f(v)$$



$f(v^*) = 0$

$v^* = 0$, $f(v^*) = -g \neq 0$

$v^* > 0$, $f(v^*) = -g - \frac{c}{m} v^2 < 0$

$v^* < 0$ $f(v^*) = -g + \frac{c}{m} v^2 = 0$

$\Rightarrow v^* = -\sqrt{\frac{mg}{c}}$

$f'(v) = \frac{2cv}{m}$ for $v < 0 \Rightarrow f'(v^*) = \frac{2v^* c}{m} < 0$ since $v^* < 0$ Stable point

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Ans-4 $\dot{u} = A\sin u = f(u)$

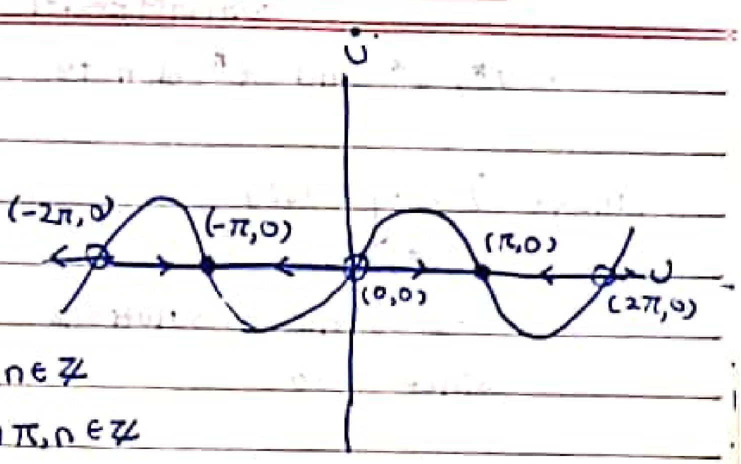
$f(u^*) = 0$

$A\sin u^* = 0 \Rightarrow \sin u^* = 0$

$u^* = n\pi \quad \forall\, n \in \mathbb{Z}$

$f'(u) = A\cos u$

$$f'(u^*) = \begin{cases} A & \forall\ u^* = 2n\pi,\, n \in \mathbb{Z} \\ -A & \forall\ u^* = (2n+1)\pi,\, n \in \mathbb{Z} \end{cases}$$



$\therefore$ $u^* = 2n\pi,\, n \in \mathbb{Z}$ are unstable

$u^* = (2n+1)\pi,\, n \in \mathbb{Z}$ are stable

Ans-5 $\dot{u} = A(u-a)(u-b)(u-c) = f(u)$

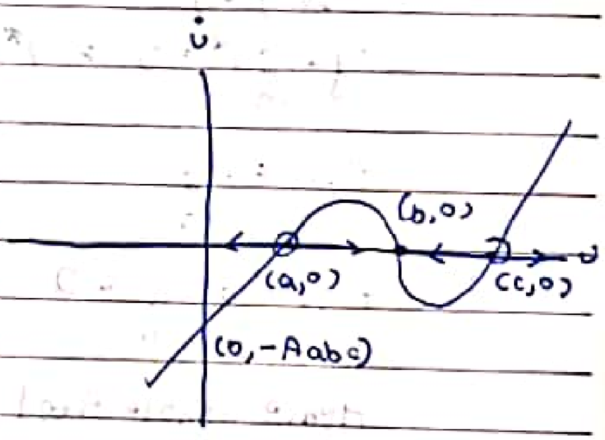
(Assumption: $0 < a < b < c$)

$f(u^*) = 0$

$A(u^* - a)(u^* - b)(u^* - c) = 0$

$u^* = a, b, c$

$$f'(u) = A\Big((u-a)(u-b) + (u-a)(u-c) + (u-b)(u-c)\Big)$$



$f'(a) = A\,(a-b)(a-c) > 0$

$f'(b) = A\,(b-a)(b-c) < 0$

$f'(c) = A\,(c-a)(c-b) > 0$

$\therefore$ a and c are unstable

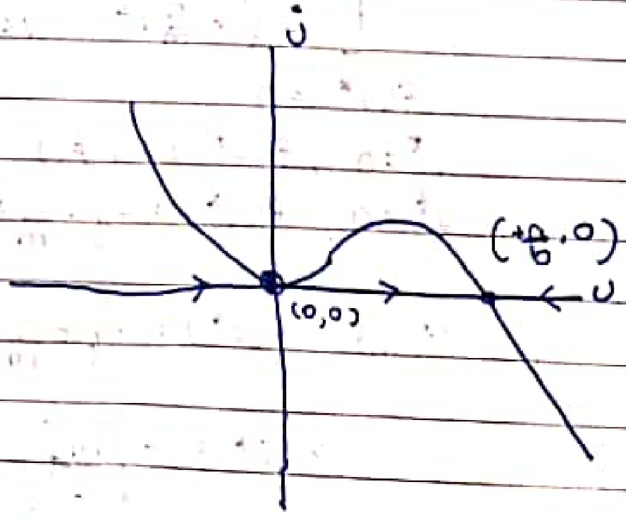
b is stable

Ans-6 $\dot{u} = au^2 - bu^3 = f(u)$

$f(u^*) = 0$

$u^{*2}(a - bu^*) = 0$

$u^* = 0,\ u^* = \dfrac{a}{b}$



$f'(u) = 2au - 3bu^2$

$f''(u) = 2a - 6bu$

$$f'\left(\frac{a}{b}\right) = \frac{2a^2}{b} - \frac{3\cancel{b}a^2}{b^{\cancel{2}}} = \frac{-a}{b^2} < 0$$

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$f'(0) = 0,\ f''(0) = 2a > 0$

$\therefore$ $\frac{a}{b}$ is stable and 0 is saddle pt. of type I

Ans-7- $\dot{x} = 4x^2 - 16 = f(x)$

$f(x^*) = 0$

$4x^{*2} - 16 = 0$

$4(x^* - 2)(x^* + 2) = 0$

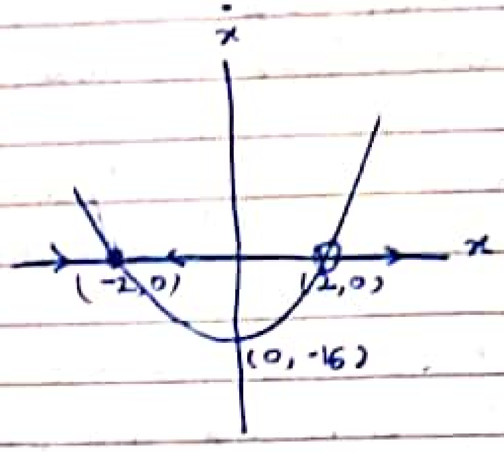
$x^* = \pm 2$

$f'(x) = 8x$

$f'(2) = 16 > 0$

$f'(-2) = -16 < 0$

$\therefore$ 2 is unstable and -2 is stable



Ans-8- $\dot{x} = x - x^3 = f(x)$

$f(x^*) = 0$

$x^* - x^{*3} = 0$

$x^*(1 - x^*)(1 + x^*) = 0$

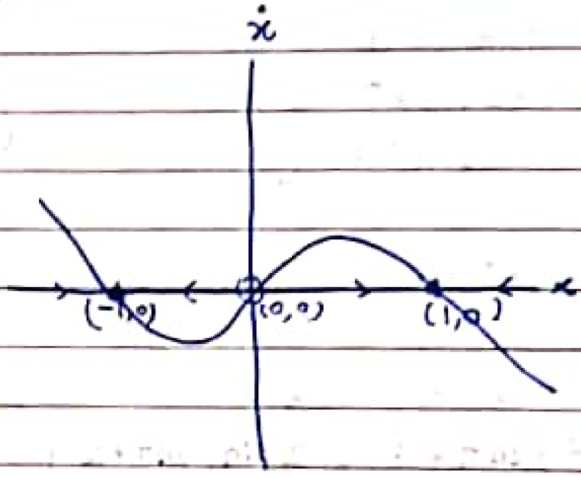
$x^* = 0, 1, -1$

$f'(x) = 1 - 3x^2$

$f'(0) = 1 > 0$

$f'(1) = f'(-1) = -2 < 0$

$\therefore$ 0 is unstable and 1, -1 are stable



Ans-9- $\dot{x} = 1 + \frac{1}{2}\cos x = f(x)$

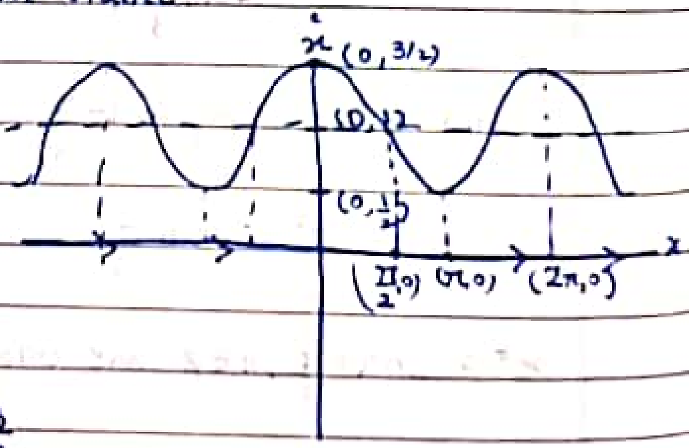
$f(x^*) = 0$

$1 + \frac{1}{2}\cos x^* = 0$

has no solutions because

$|\cos x| \le 1 \Rightarrow 1 + \frac{1}{2}\cos x \le \frac{3}{2}$

$1 + \frac{1}{2}\cos x \ge \frac{1}{2}$



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Ans-10 $\dot{x} = e^{-x}\sin x = f(x)$

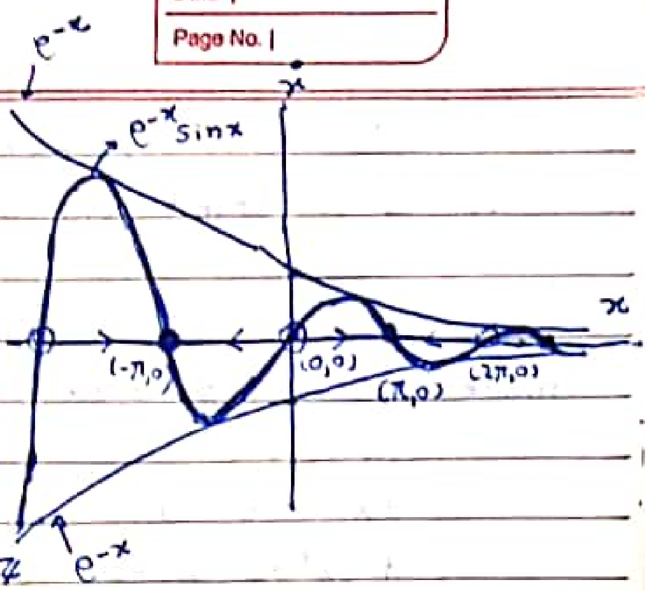
$f(x^*) = 0$

$e^{-x^*}\sin x^* = 0 \Rightarrow \sin x^* = 0$

$x^* = n\pi,\ n \in \mathbb{Z}$

$f'(x) = e^{-x}\cos x - e^{-x}\sin x$

$$f'(x^*) = \begin{cases} e^{-2n\pi} & x^* = 2n\pi,\ n \in \mathbb{Z} \\ e^{-(2n+1)\pi} & x^* = (2n+1)\pi,\ n \in \mathbb{Z} \end{cases}$$



$\therefore$ $x^* = (2n+1)\pi$ $n \in \mathbb{Z}$ are stable

$x^* = 2n\pi,\ n \in \mathbb{Z}$ are unstable

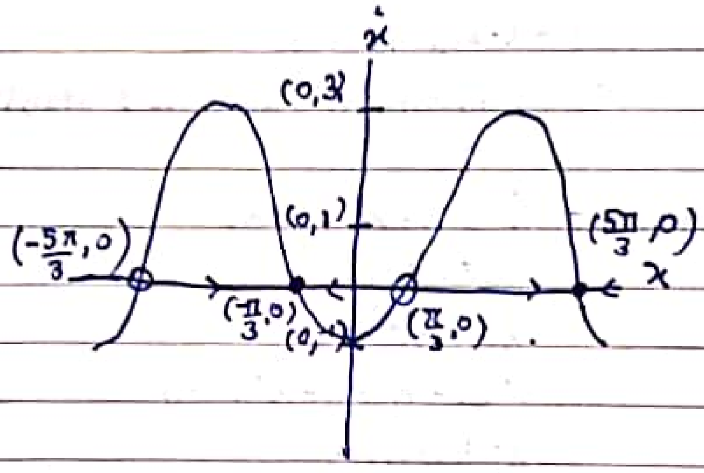
Ans-11 $\dot{x} = 1 - 2\cos x = f(x)$

$f(x^*) = 0$

$1 - 2\cos x^* = 0$

$\Rightarrow \cos x^* = 1/2$

$\Rightarrow x^* = 2n\pi \pm \frac{\pi}{3},\ n \in \mathbb{Z}$



$f'(x) = 2\sin x$

$f'\left(2n\pi + \frac{\pi}{3}\right) = 2\sin\left(2n\pi + \frac{\pi}{3}\right)$ $n \in \mathbb{Z}$

$= 2\sin\frac{\pi}{3} > 0$ $\left(\because \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}\right)$

$f'\left(2n\pi - \frac{\pi}{3}\right) = 2\sin\left(2n\pi - \frac{\pi}{3}\right)$ $n \in \mathbb{Z}$

$= -2\sin\frac{\pi}{3} < 0$

$x^* = 2n\pi + \frac{\pi}{3},\ n \in \mathbb{Z}$ are unstable

$x^* = 2n\pi - \frac{\pi}{3},\ n \in \mathbb{Z}$ are stable

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Ans-12- $\dot{x} = x(1-x) = f(x)$

$f(x^*) = 0$

$x^*(1-x^*) = 0$

$x^* = 0, 1$

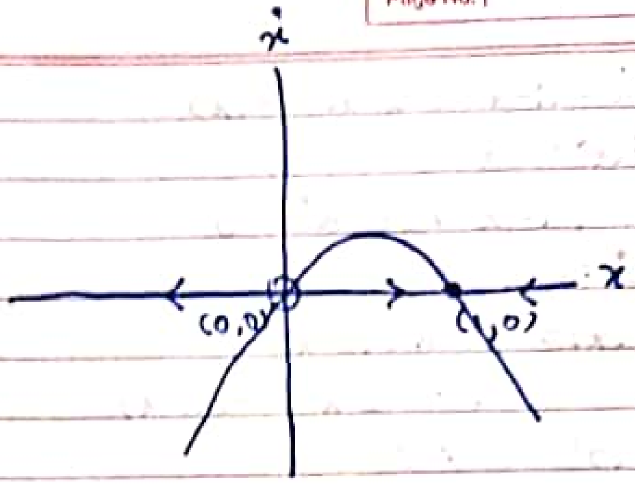
$f'(x) = 1 - 2x$

$f'(0) = 1 > 0$

$f'(1) = -1 < 0$

$\therefore$ $x^* = 1$ is stable

$x^* = 0$ is unstable



Ans-13- $\dot{x} = \tan x = f(x)$

$f(x^*) = 0$

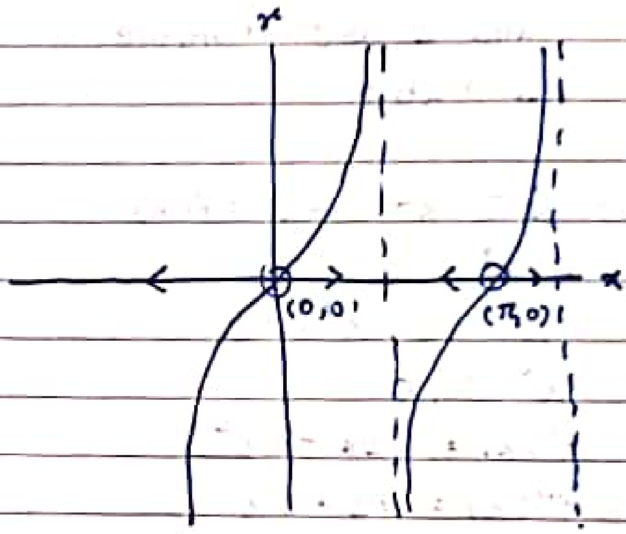
$\tan x^* = 0$

$x^* = n\pi, n \in \mathbb{Z}$

$f'(x) = \sec^2 x$

$f'(x^*) = \sec^2 n\pi = 1 > 0$

$\therefore$ $x^* = n\pi, n \in \mathbb{Z}$ are unstable



Ans-14- $\dot{x} = 1 - e^{-x^2} = f(x)$

$f(x^*) = 0$

$1 - e^{-x^{*2}} = 0$

$1 = e^{-x^{*2}}$ taking $\log_e$ bothsides

$\ln 1 = -x^{*2}$

$x^{*2} = 0$

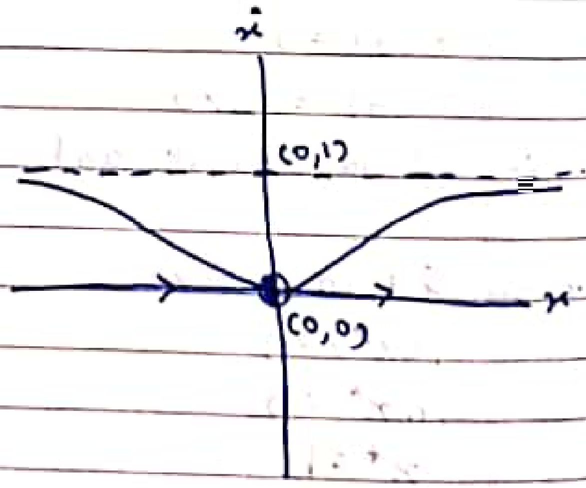
$x^* = 0$

$f'(x) = -e^{-x^2}(-2x) = 2xe^{-x^2}$

$f''(x) = 2(e^{-x^2} + e^{-x^2}(-2x)x)$

$f'(0) = 0$, $f''(0) = 2 > 0$

$\therefore$ Saddle point of type I



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Ans 15 - $\dot{x} = (1-x)x(2-x) = f(x)$

$f(x^*) = 0$

$x^*(1-x^*)(2-x^*) = 0$

$x^* = 1, 2, 0.$

$f'(x) = (1-x)(2-x) - x(2-x) - x(1-x)$

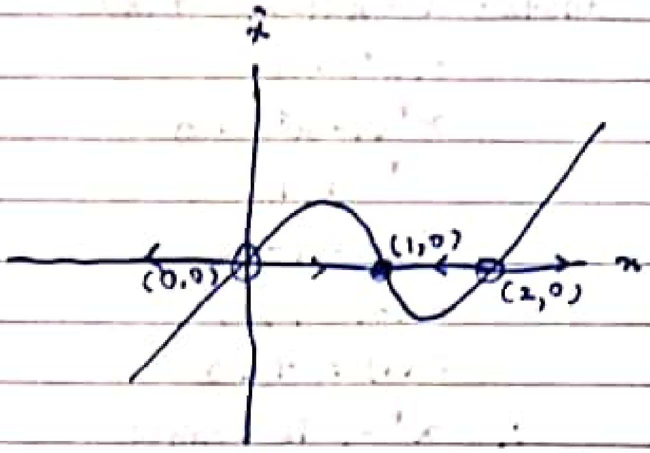
$f'(0) = 2 > 0$

$f'(1) = -1 < 0$

$f'(2) = 2 > 0$

$\therefore$ 0 and 2 are unstable

1 is stable



Ans-16 - $\dot{x} = x^2(6-x) = f(x)$

$f(x^*) = 0$

$x^{*2}(6-x^*) = 0$

$x^* = 0, 6$

$f'(x) = 12x - 3x^2$

$f''(x) = 12 - 6x$

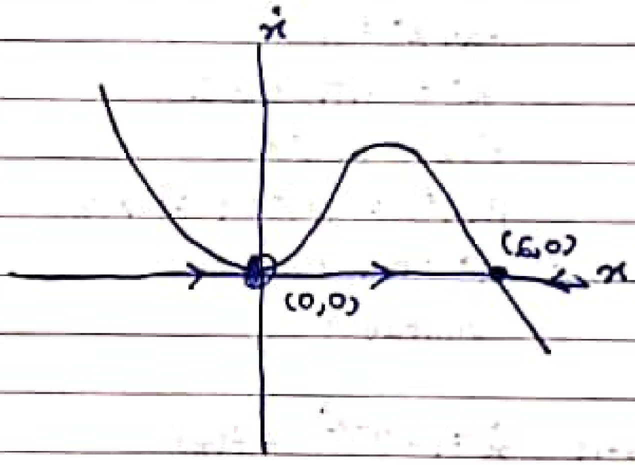
$f'(6) = 12\times6 - 3\times36$

$= 72 - 108 = -36 < 0$

$f'(0) = 0$

$f''(0) = 12 > 0$

$\therefore$ 6 is stable and 0 is saddle pt. of type I



Ans-17 - $\dot{x} = \ln x = f(x)$

$f(x^*) = 0$

$\ln x^* = 0$

$x^* = 1$

$f'(x) = 1/x$

$f'(x^*) = f'(1) = 1/1 = 1 > 0$

$\therefore$ $x^* = 1$ is unstable