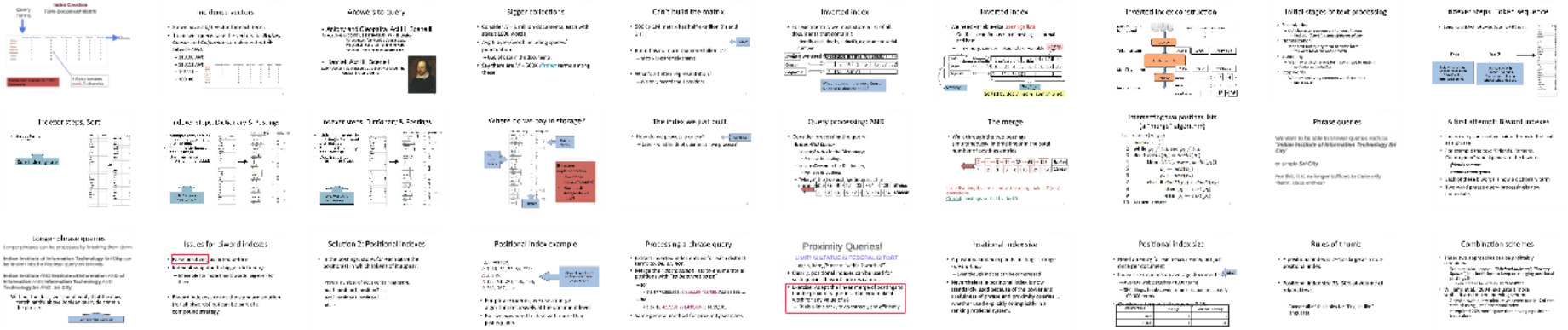
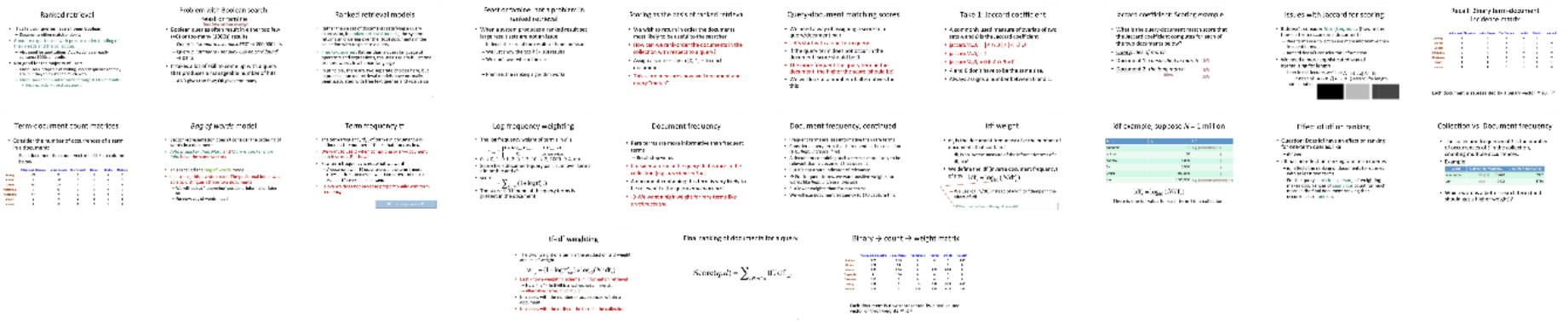
Web Page Indexing: TF-IDF

Problem- Definition

Indexing



Ranked Retrieval



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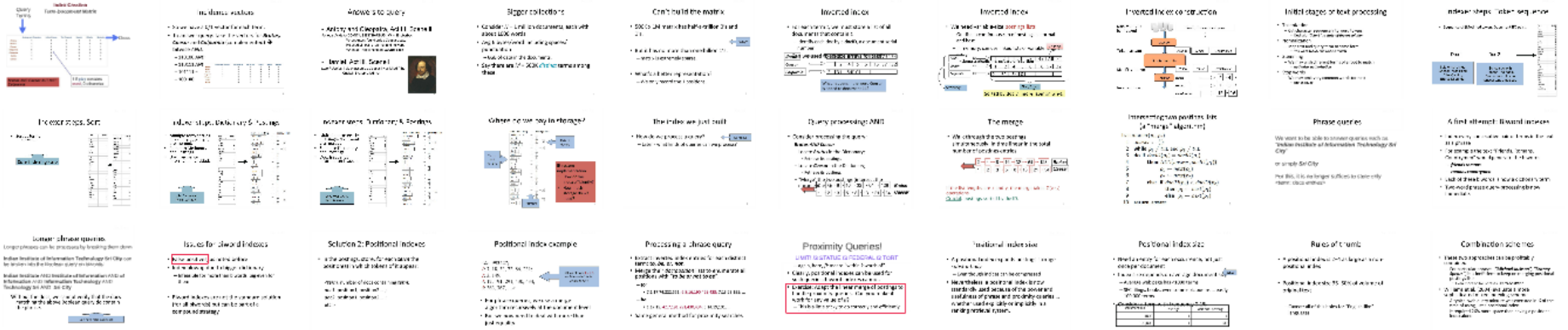

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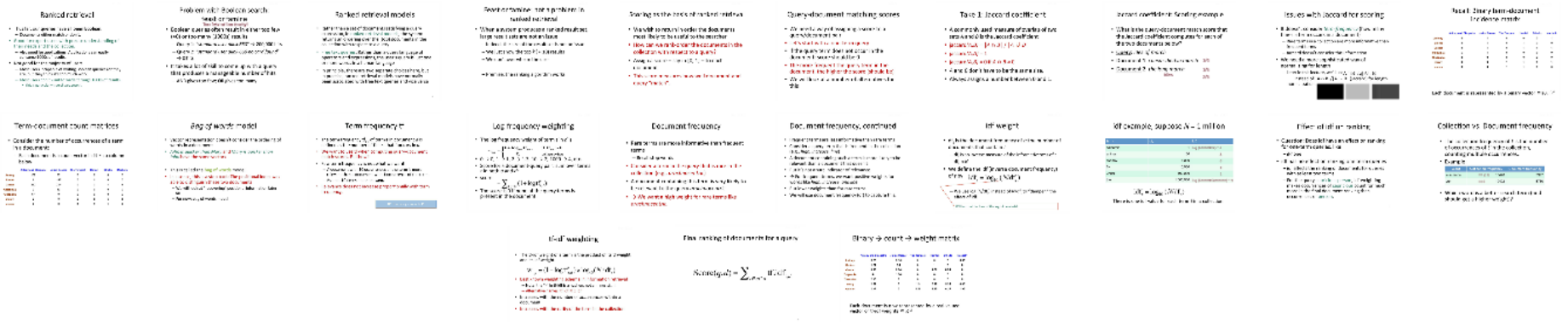

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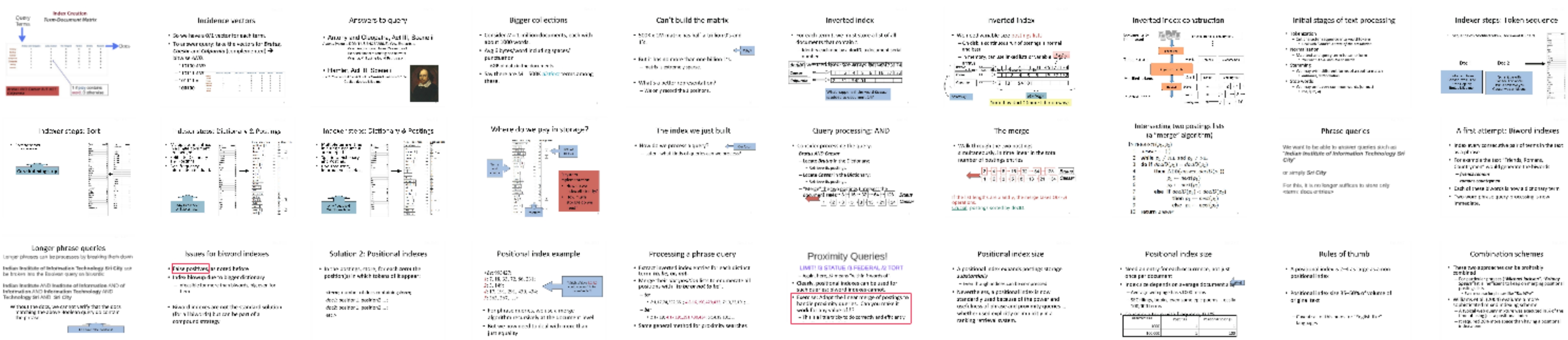


Information Retrieval (IR) - The Problem

Collection: A set of documents Assume it is a static collection for the moment

Goal: Retrieve documents with information that is relevant to the user's information need and helps the user complete a task

Searching is a Solved Problem!

Hash Table: O(1)
Binary Search Tree: O(log(n))
AVL Tree: O(log(n))
.....

Why Do we need a different setup?

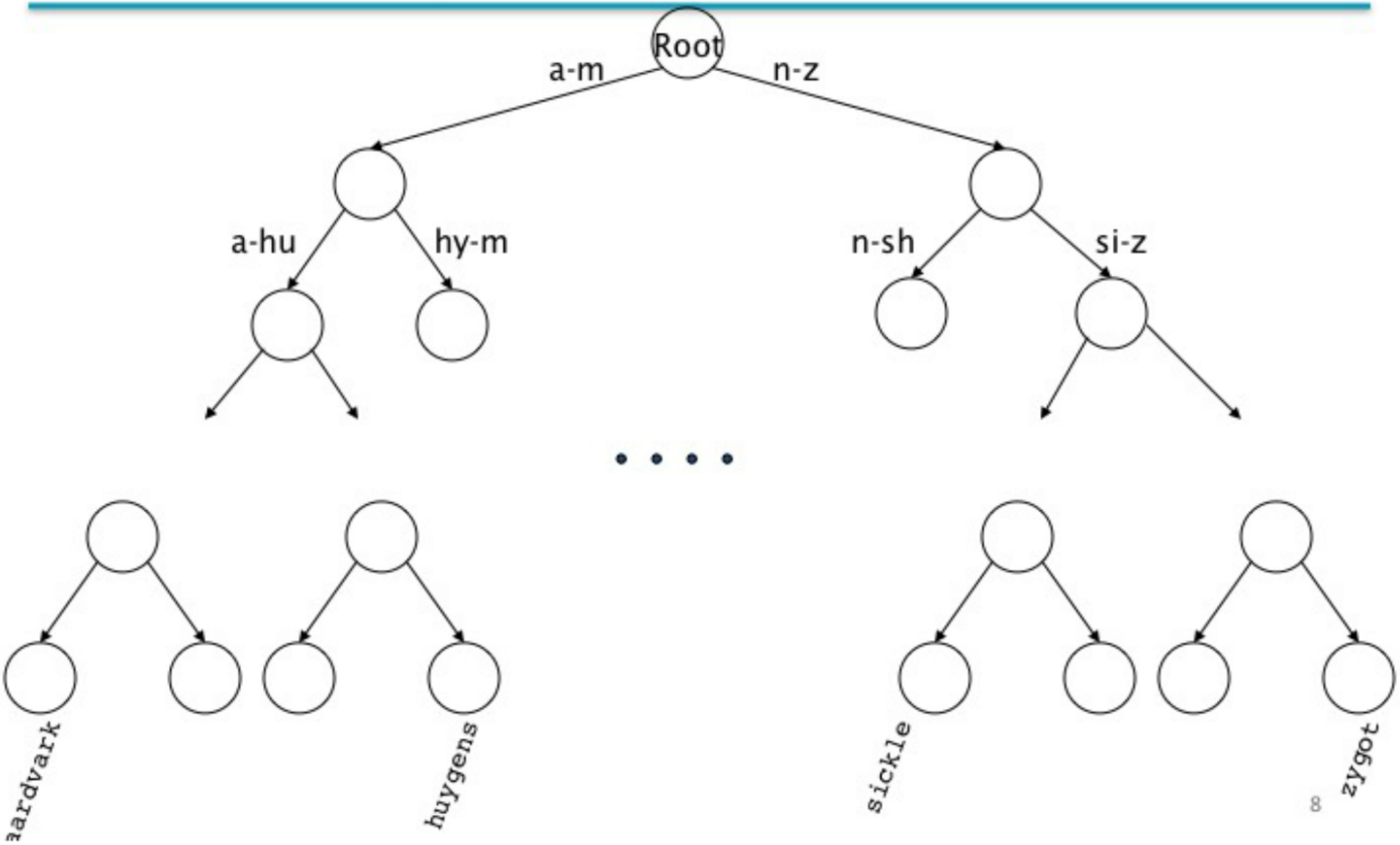
Size of the Data!

- 10 billion web pages
- Average size of webpage = 20KB
- 10 billion * 20KB = 200 TB
- Disk read bandwidth = 50 MB/sec
- Time to read = 4 million seconds = 46+ days
- Say there are M = 500K distinct terms among these.
- Even longer to do something useful with the data

Hashtables

- Each vocabulary term is hashed to an integer
  - (We assume you've seen hashtables before)
- Pros:
  - Lookup is faster than for a tree: O(1)
- Cons:
  - No easy way to find minor variants:
    - judgment/judgement
  - No prefix search [tolerant retrieval]
  - If vocabulary keeps growing, need to occasionally do the expensive operation of rehashing *everything*

Tree: binary tree



Trees

- Simplest: binary tree
- More usual: B-trees
- Trees require a standard ordering of characters and hence strings … but we typically have one
- Pros:
  - Solves the prefix problem (terms starting with *hyp*)
- Cons:
  - Slower: O(log *M*) [and this requires *balanced* tree]
  - Rebalancing binary trees is expensive
    - But B-trees mitigate the rebalancing problem

Real Challenges

Out-of-Vocabulary Words
- 20-25% Named Entities

Multilingual Queries

Wild Card Queries

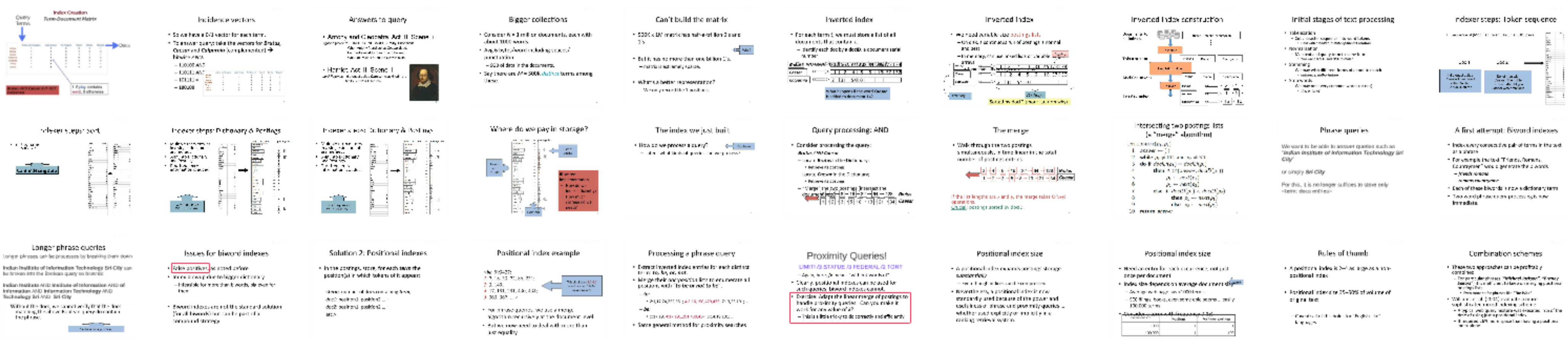
Spell Correction

Phrase Search
- *President of India*

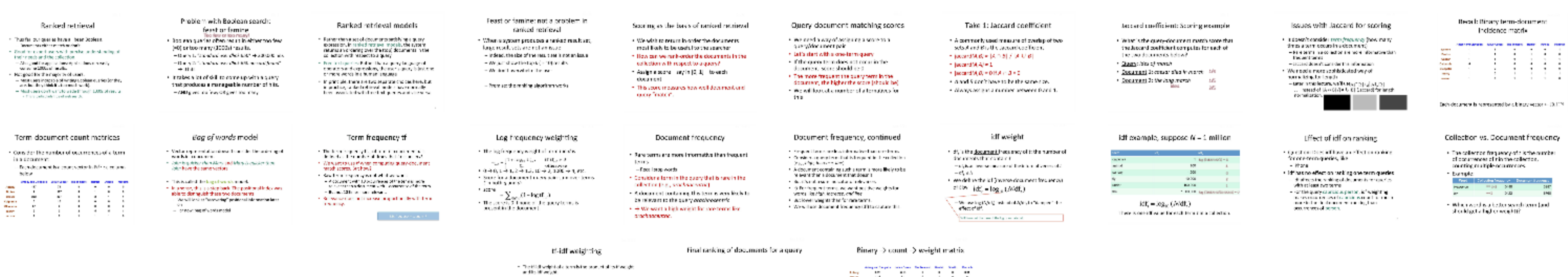
Web Page Indexing: TF-IDF

Problem- Definition

Indexing



Ranked Retrieval



Index Creation

Term-Document Matrix

Query Terms

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Docs

1 if play contains word, 0 otherwise

Brutus** AND **Caesar** BUT NOT **Calpurnia

Incidence vectors

- So we have a 0/1 vector for each term.
- To answer query: take the vectors for ***Brutus, Caesar*** and ***Calpurnia*** (complemented) ➔ bitwise *AND*.
  - 110100 *AND*
  - 110111 *AND*
  - 101111 =
  - **100100**

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Answers to query

- Antony and Cleopatra, Act III, Scene ii

Agrippa [Aside to DOMITIUS ENOBARBUS]: Why, Enobarbus,
When Antony found Julius ***Caesar*** dead,
He cried almost to roaring; and he wept
When at Philippi he found ***Brutus*** slain.

- Hamlet, Act III, Scene ii

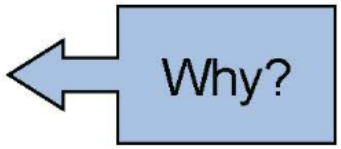
Lord Polonius: I did enact Julius ***Caesar*** I was killed i' the Capitol; ***Brutus*** killed me.

Bigger collections

- Consider N = 1 million documents, each with about 1000 words.
- Avg 6 bytes/word including spaces/ punctuation
  - 6GB of data in the documents.
- Say there are M = 500K *distinct* terms among these.

Can't build the matrix

- 500K x 1M matrix has half-a-trillion 0's and 1's.

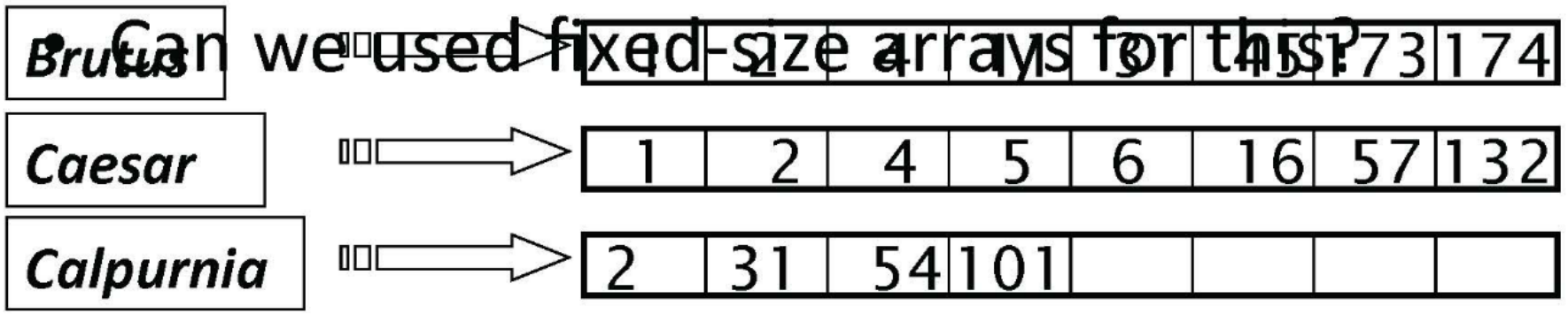


- But it has no more than one billion 1's.
  - matrix is extremely sparse.

- What's a better representation?
  - We only record the 1 positions.

Inverted index

- For each term *t*, we must store a list of all documents that contain *t*.
  - Identify each doc by a **docID**, a document serial number

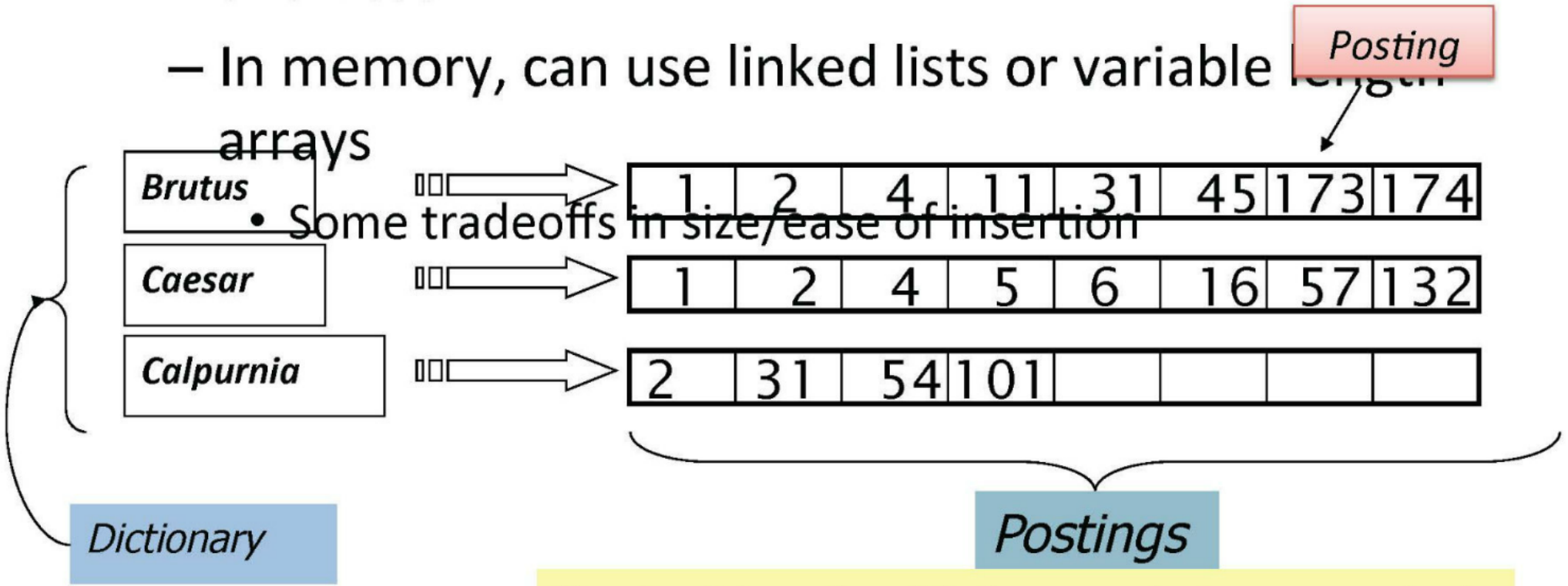


- Can we used fixed-size arrays for this?

What happens if the word ***Caesar*** is added to document 14?

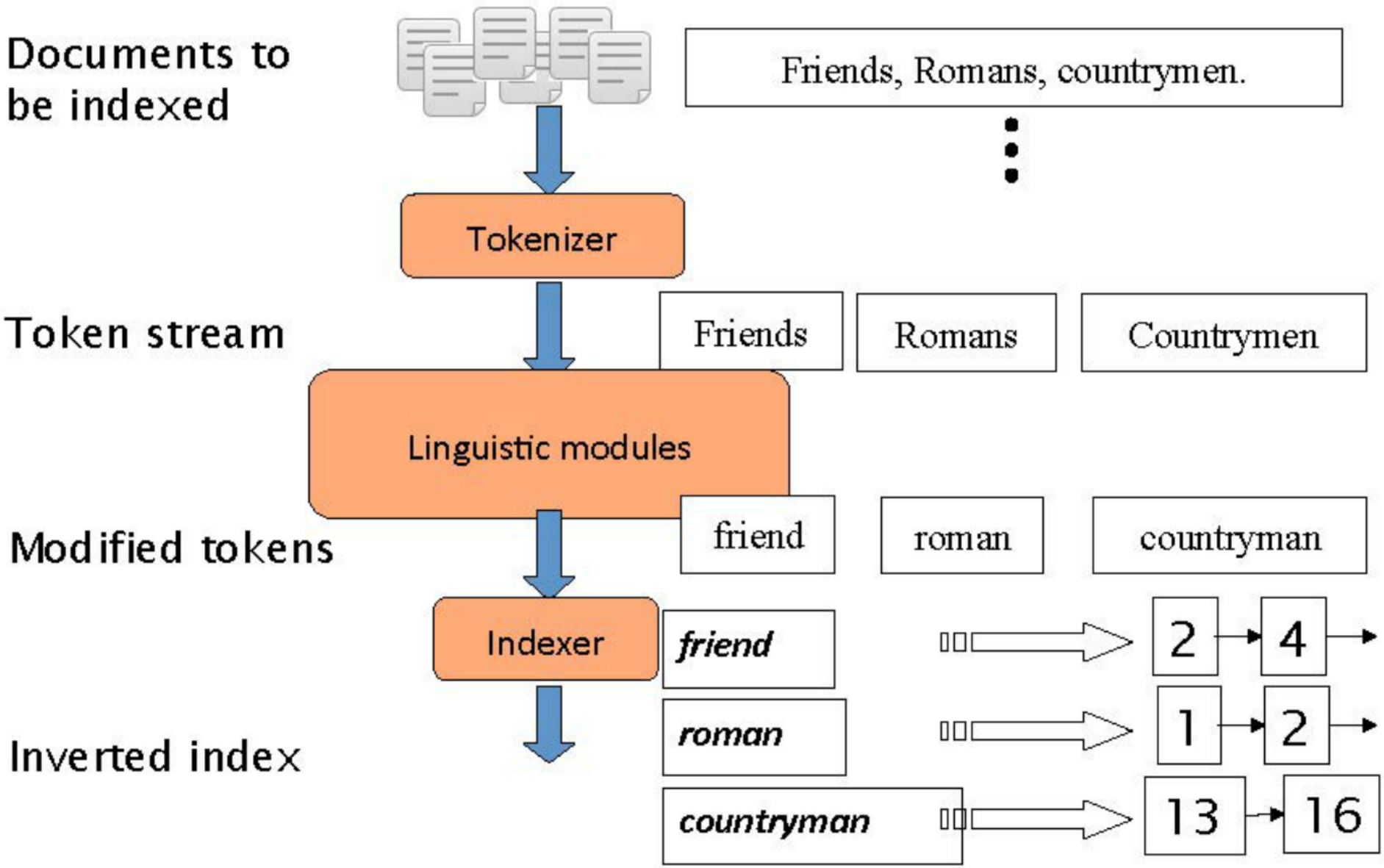
Inverted index

- We need variable-size postings lists
  - On disk, a continuous run of postings is normal and best
  - In memory, can use linked lists or variable length arrays
    - Some tradeoffs in size/ease of insertion



Sorted by docID (more later on why).

Inverted index construction



Initial stages of text processing

- Tokenization
  - Cut character sequence into word tokens
    - Deal with ***"John's"***, ***a state-of-the-art solution***
- Normalization
  - Map text and query term to same form
    - You want ***U.S.A.*** and ***USA*** to match
- Stemming
  - We may wish different forms of a root to match
    - ***authorize***, ***authorization***
- Stop words
  - We may omit very common words (or not)
    - ***the, a, to, of***

Indexer steps: Token sequence

- Sequence of (Modified token, Document ID) pairs.

Doc 1

I did enact Julius Caesar I was killed i' the Capitol; Brutus killed me.

Doc 2

So let it be with Caesar. The noble Brutus hath told you Caesar was ambitious

Term	docID
I	1
did	1
enact	1
julius	1
caesar	1
I	1
was	1
killed	1
i'	1
the	1
capitol	1
brutus	1
killed	1
me	1
so	2
let	2
it	2
be	2
with	2
caesar	2
the	2
noble	2
brutus	2
hath	2
told	2
you	2
caesar	2
was	2
ambitious	2

Indexer steps: Sort

- Sort by terms
  - And then docID

Term	docID
I	1
did	1
enact	1
julius	1
caesar	1
I	1
was	1
killed	1
i'	1
the	1
capitol	1
brutus	1
killed	1
me	1
so	2
let	2
it	2
be	2
with	2
caesar	2
the	2
noble	2
brutus	2
hath	2
told	2
you	2
caesar	2
was	2
ambitious	2

Term	docID
ambitious	2
be	2
brutus	1
brutus	2
capitol	1
caesar	1
caesar	2
caesar	2
did	1
enact	1
hath	1
I	1
I	1
i'	1
it	2
julius	1
killed	1
killed	1
let	2
me	1
noble	2
so	2
the	1
the	2
told	2
you	2
was	1
was	2
with	2

Indexer steps: Dictionary & Postings

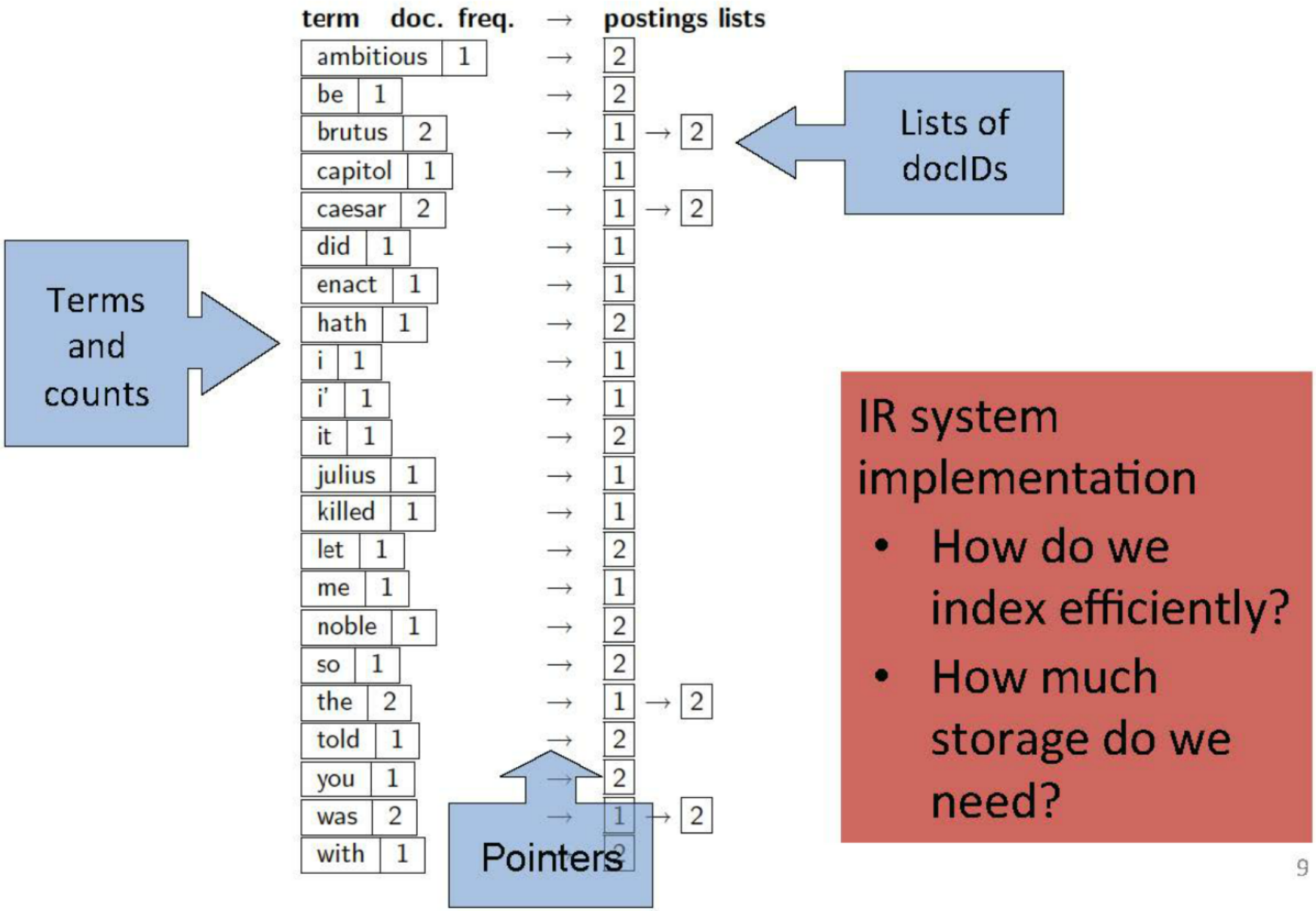
- Multiple term entries in a single document are merged.
- Split into Dictionary and Postings
- Doc. frequency information is added.

Term	docID
ambitious	2
be	2
brutus	1
brutus	2
capitol	1
caesar	1
caesar	2
caesar	2
did	1
enact	1
hath	1
I	1
I	1
i'	1
it	2
julius	1
killed	1
killed	1
let	2
me	1
noble	2
so	2
the	1
the	2
told	2
you	2
was	1
was	2
with	2

term	doc. freq.	→	postings lists
ambitious	1	→	2
be	1	→	2
brutus	2	→	1 → 2
capitol	1	→	1
caesar	2	→	1 → 2
did	1	→	1
enact	1	→	1
hath	1	→	2
i	1	→	1
i'	1	→	1
it	1	→	2
julius	1	→	1
killed	1	→	1
let	1	→	2
me	1	→	1
noble	1	→	2
so	1	→	2
the	2	→	1 → 2
told	1	→	2
you	1	→	2
was	2	→	1 → 2
with	1	→	2

Why frequency?
Will discuss later.

Where do we pay in storage?



IR system implementation

- How do we index efficiently?
- How much storage do we need?

The index we just built

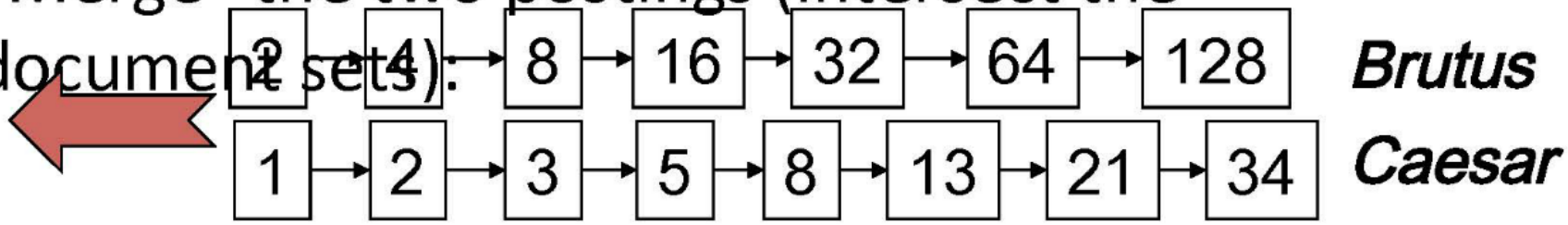
- How do we process a query?
  - Later - what kinds of queries can we process?

Query processing: AND

- Consider processing the query:

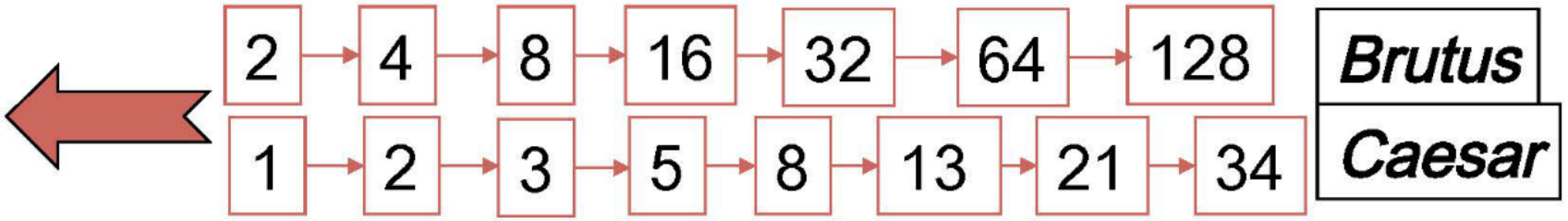
  Brutus *AND* ***Caesar***

  – Locate ***Brutus*** in the Dictionary;
    - Retrieve its postings.
  – Locate ***Caesar*** in the Dictionary;
    - Retrieve its postings.
  – "Merge" the two postings (intersect the document sets):



The merge

- Walk through the two postings simultaneously, in time linear in the total number of postings entries



If the list lengths are *x* and *y*, the merge takes O(*x*+*y*) operations.

Crucial: postings sorted by docID.

Phrase queries

We want to be able to answer queries such as "***Indian Institute of Information Technology Sri City***"

or simply ***Sri City***

For this, it is no longer suffices to store only <term: docs entries>

A first attempt: Biword indexes

- Index every consecutive pair of terms in the text as a phrase
- For example the text "Friends, Romans, Countrymen" would generate the biwords
  - ***friends romans***
  - ***romans countrymen***
- Each of these biwords is now a dictionary term
- Two-word phrase query-processing is now immediate.

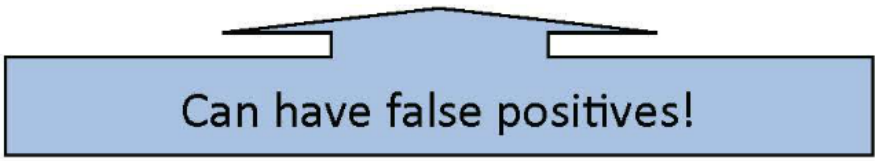
Longer phrase queries

Longer phrases can be processes by breaking them down

Indian Institute of Information Technology Sri City can be broken into the Boolean query on biwords:

Indian Institute AND **Institute of Information** AND **of Information** AND **Information Technology** AND **Technology Sri** AND **Sri City**

Without the docs, we cannot verify that the docs matching the above Boolean query do contain the phrase.



Issues for biword indexes

- False positives, as noted before
- Index blowup due to bigger dictionary
  - Infeasible for more than biwords, big even for them

- Biword indexes are not the standard solution (for all biwords) but can be part of a compound strategy

Solution 2: Positional indexes

- In the postings, store, for each ***term*** the position(s) in which tokens of it appear:

<***term***, number of docs containing ***term***;

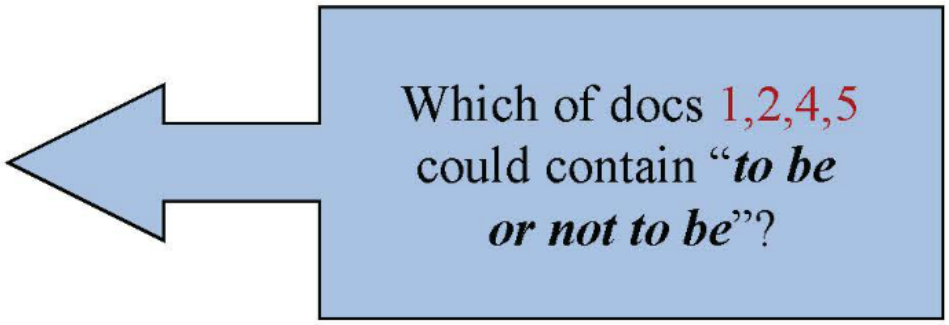
doc1: position1, position2 … ;

doc2: position1, position2 … ;

etc.>

Positional index example

<*be*: 993427;
1: 7, 18, 33, 72, 86, 231;
2: 3, 149;
4: 17, 191, 291, 430, 434;
5: 363, 367, …>



- For phrase queries, we use a merge algorithm recursively at the document level
- But we now need to deal with more than just equality

Processing a phrase query

- Extract inverted index entries for each distinct term: ***to, be, or, not.***
- Merge their *doc:position* lists to enumerate all positions with "***to be or not to be***".
  - ***to****:*
    - *2*:1,17,74,222,551; *4*:8,16,190,429,433; *7*:13,23,191; ...
  - ***be****:*
    - *1*:17,19; *4*:17,191,291,430,434; *5*:14,19,101; ...
- Same general method for proximity searches

Proximity Queries!

LIMIT! /3 STATUE /3 FEDERAL /2 TORT

- Again, here, /*k* means "within *k* words of".

- Clearly, positional indexes can be used for such queries; biword indexes cannot.

- Exercise: Adapt the linear merge of postings to handle proximity queries. Can you make it work for any value of *k*?
  - This is a little tricky to do correctly and efficiently

Positional index size

- A positional index expands postings storage *substantially*
  - Even though indices can be compressed
- Nevertheless, a positional index is now standardly used because of the power and usefulness of phrase and proximity queries … whether used explicitly or implicitly in a ranking retrieval system.

Positional index size

- Need an entry for each occurrence, not just once per document
- Index size depends on average document size
  - Average web page has <1000 terms
  - SEC filings, books, even some epic poems … easily 100,000 terms

Why?

- Consider a term with frequency 0.1%

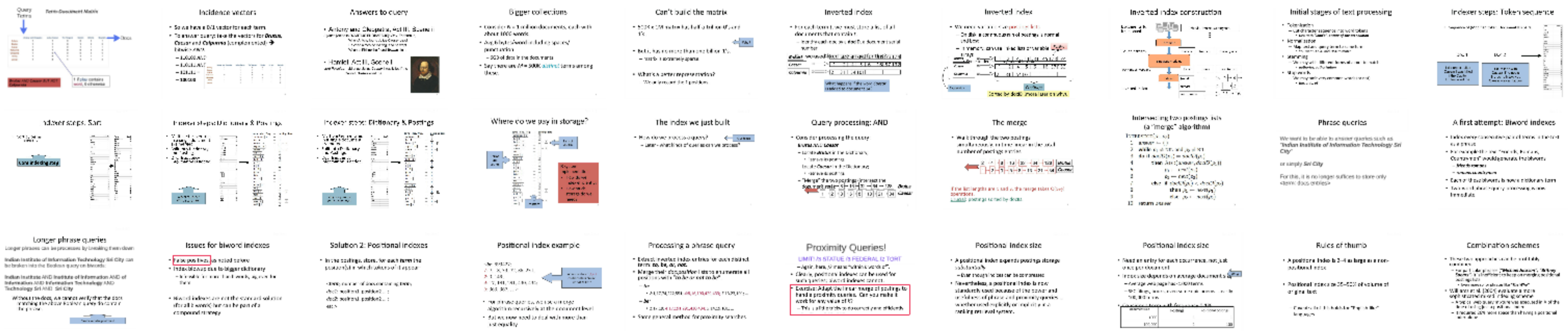
Document size	Postings	Positional postings
1000	1	1
100,000	1	100

Rules of thumb

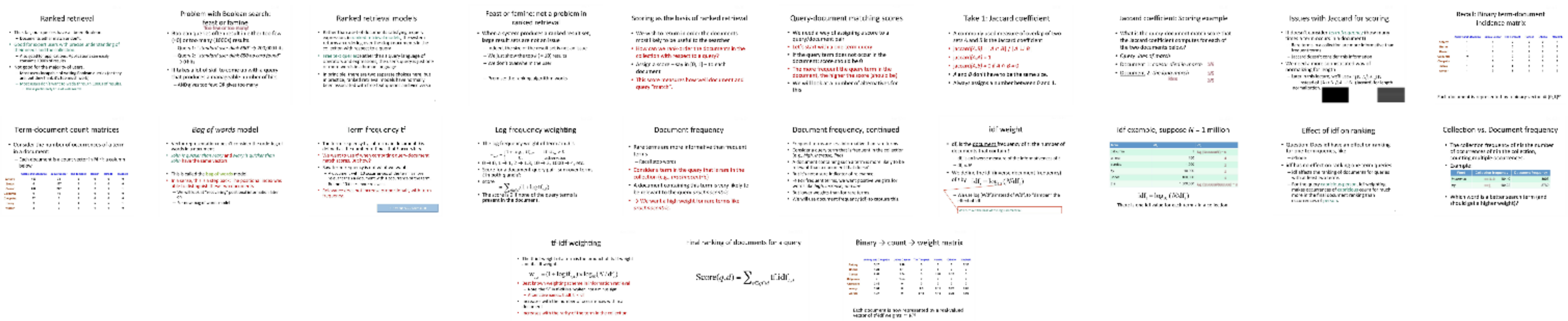
- A positional index is 2–4 as large as a non-positional index
- Positional index size 35–50% of volume of original text
  - Caveat: all of this holds for "English-like" languages

Combination schemes

- These two approaches can be profitably combined
  - For particular phrases (***"Michael Jackson", "Britney Spears"***) it is inefficient to keep on merging positional postings lists
    - Even more so for phrases like ***"The Who"***
- Williams et al. (2004) evaluate a more sophisticated mixed indexing scheme
  - A typical web query mixture was executed in ¼ of the time of using just a positional index
  - It required 26% more space than having a positional index alone



Ranked Retrieval



Ranked retrieval

- Thus far, our queries have all been Boolean.
  - Documents either match or don't.
- Good for expert users with precise understanding of their needs and the collection.
  - Also good for applications: Applications can easily consume 1000s of results.
- Not good for the majority of users.
  - Most users incapable of writing Boolean queries (or they are, but they think it's too much work).
  - Most users don't want to wade through 1000s of results.
    - This is particularly true of web search.

Problem with Boolean search: feast or famine

Too few or too many!

- Boolean queries often result in either too few (≈0) or too many (1000s) results.
  - Query 1: "*standard user dlink 650*" → 200,000 hits
  - Query 2: "*standard user dlink 650 no card found*" → 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
  - AND gives too few; OR gives too many

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in ranked retrieval models, the system returns an ordering over the (top) documents in the collection with respect to a query
- Free text queries: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- In principle, there are two separate choices here, but in practice, ranked retrieval models have normally been associated with free text queries and vice versa

Feast or famine: not a problem in ranked retrieval

- When a system produces a ranked result set, large result sets are not an issue
  - Indeed, the size of the result set is not an issue
  - We just show the top k ($\approx$ 10) results
  - We don't overwhelm the user

  - Premise: the ranking algorithm works

Scoring as the basis of ranked retrieval

- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score – say in [0, 1] – to each document
- This score measures how well document and query "match".

Query-document matching scores

- We need a way of assigning a score to a query/document pair
- Let's start with a one-term query
- If the query term does not occur in the document: score should be 0
- The more frequent the query term in the document, the higher the score (should be)
- We will look at a number of alternatives for this

Take 1: Jaccard coefficient

- A commonly used measure of overlap of two sets A and B is the Jaccard coefficient
- jaccard$(A,B) = |A \cap B| \,/\, |A \cup B|$
- jaccard$(A,A) = 1$
- jaccard$(A,B) = 0$ if $A \cap B = 0$
- A and B don't have to be the same size.
- Always assigns a number between 0 and 1.

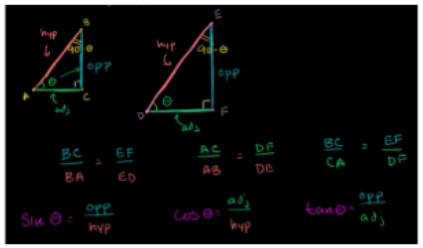
Jaccard coefficient: Scoring example

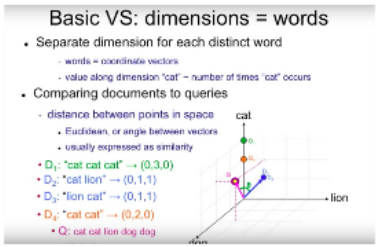
- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?
- Query: *ides of march*
- Document 1: *caesar died in march* 1/6
- Document 2: *the long march* 1/5

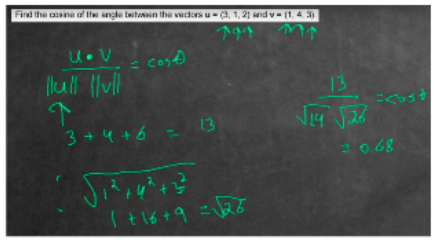
ides 2/5

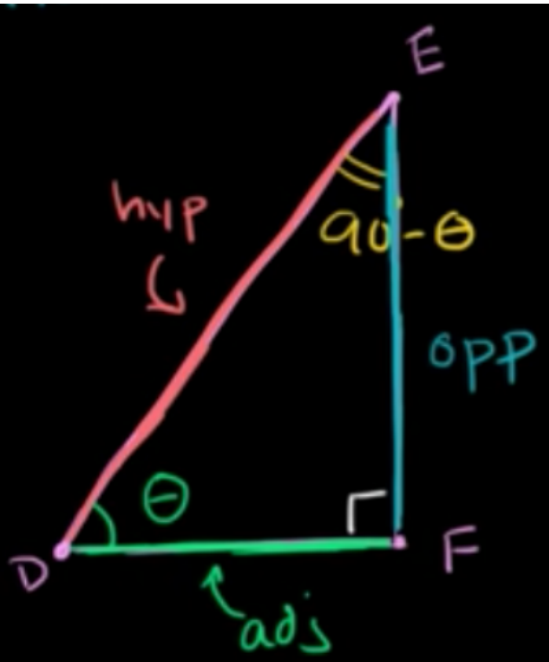
Issues with Jaccard for scoring

- It doesn't consider *term frequency* (how many times a term occurs in a document)
  - Rare terms in a collection are more informative than frequent terms
  - Jaccard doesn't consider this information
- We need a more sophisticated way of normalizing for length
  - Later in this lecture, we'll use $|A \cap B| / \sqrt{|A \cup B|}$ . . . instead of $|A \cap B| / |A \cup B|$ (Jaccard) for length normalization.









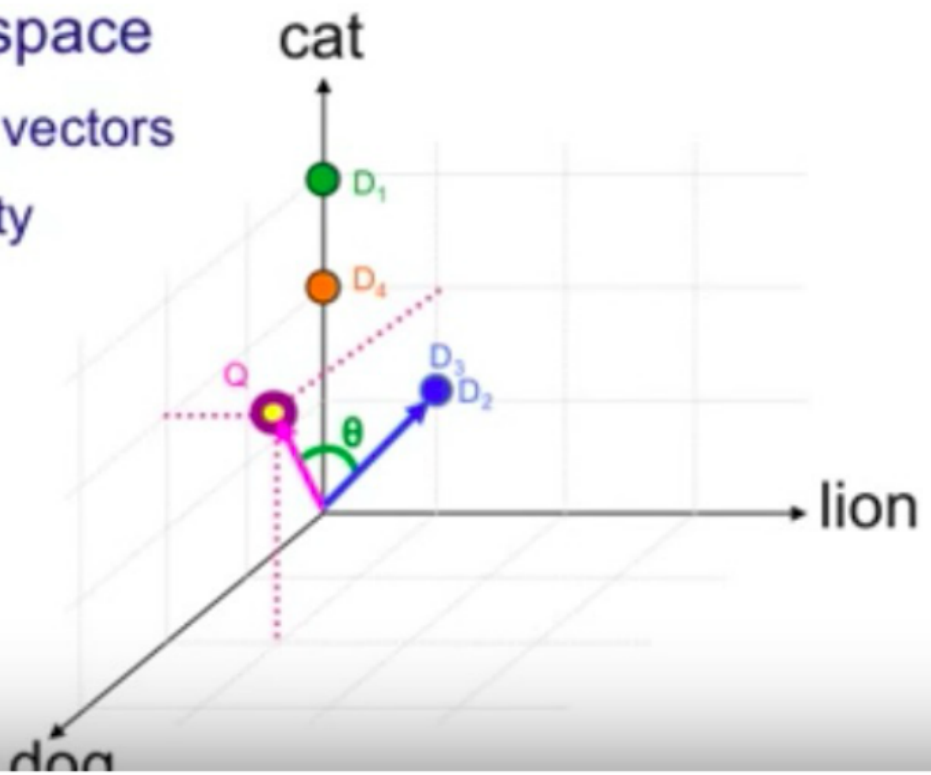
$$\frac{BC}{BA} = \frac{EF}{ED} \qquad \frac{AC}{AB} = \frac{DF}{DE} \qquad \frac{BC}{CA} = \frac{EF}{DF}$$

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

Basic VS: dimensions = words

- Separate dimension for each distinct word
  - words = coordinate vectors
  - value along dimension “cat” ~ number of times “cat” occurs
- Comparing documents to queries
  - distance between points in space
    - Euclidean, or angle between vectors
    - usually expressed as similarity

- D_1: “cat cat cat” → (0,3,0)
- D_2: “cat lion” → (0,1,1)
- D_3: “lion cat” → (0,1,1)
- D_4: “cat cat” → (0,2,0)
  - Q: cat cat lion dog dog



Find the cosine of the angle between the vectors **u** = (3, 1, 2) and **v** = (1, 4, 3).

$$\frac{u \cdot v}{\|u\| \, \|v\|} = \cos\theta$$

$$3 + 4 + 6 = 13$$

$$\sqrt{1^2 + 4^2 + 3^2}$$

$$1 + 16 + 9 = \sqrt{26}$$

$$\frac{13}{\sqrt{14}\sqrt{26}} = \cos\theta$$

$$= 0.68$$

Recall: Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}^{|V|}$

Term-document count matrices

- Consider the number of occurrences of a term in a document:
  - Each document is a count vector in $\mathbb{N}^{|V|}$: a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Bag of words model

- Vector representation doesn't consider the ordering of words in a document
- *John is quicker than Mary* and *Mary is quicker than John* have the same vectors

- This is called the **bag of words** model.
- In a sense, this is a step back: The positional index was able to distinguish these two documents
  - We will look at "recovering" positional information later on
  - For now: bag of words model

Term frequency tf

- The term frequency $tf_{t,d}$ of term *t* in document *d* is defined as the number of times that *t* occurs in *d*.
- We want to use tf when computing query-document match scores. But how?
- Raw term frequency is not what we want:
  - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term.
  - But not 10 times more relevant.
- Relevance does not increase proportionally with term frequency.

NB: frequency = count in IR

Log-frequency weighting

- The log frequency weight of term *t* in *d* is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- 0 → 0, 1 → 1, 2 → 1.3, 10 → 2, 1000 → 4, etc.
- Score for a document-query pair: sum over terms *t* in both *q* and *d*:
- score

$$= \sum_{t \in q \cap d} (1 + \log \mathrm{tf}_{t,d})$$

- The score is 0 if none of the query terms is present in the document.

Document frequency

- Rare terms are more informative than frequent terms
  - Recall stop words
- Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
- A document containing this term is very likely to be relevant to the query *arachnocentric*
- → We want a high weight for rare terms like *arachnocentric*.

Document frequency, continued

- Frequent terms are less informative than rare terms
- Consider a query term that is frequent in the collection (e.g., *high, increase, line*)
- A document containing such a term is more likely to be relevant than a document that doesn't
- But it's not a sure indicator of relevance.
- → For frequent terms, we want positive weights for words like *high, increase, and line*
- But lower weights than for rare terms.
- We will use document frequency (df) to capture this.

idf weight

- df_t is the document frequency of t: the number of documents that contain t
  - df_t is an inverse measure of the informativeness of t
  - $df_t \leq N$
- We define the idf (inverse document frequency) of t by $$idf_t = \log_{10}(N/df_t)$$
  - We use log (N/df_t) instead of N/df_t to "dampen" the effect of idf.

Will turn out the base of the log is immaterial.

idf example, suppose N = 1 million

term	df_t	idf_t
calpurnia	1	log (1000000/1) = 6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	log (1000000/1000000) = 0

$$\mathrm{idf}_t = \log_{10}(N/\mathrm{df}_t)$$

There is one idf value for each term t in a collection.

Effect of idf on ranking

- Question: Does idf have an effect on ranking for one-term queries, like
  - iPhone
- idf has no effect on ranking one term queries
  - idf affects the ranking of documents for queries with at least two terms
  - For the query capricious person, idf weighting makes occurrences of capricious count for much more in the final document ranking than occurrences of person.

Collection vs. Document frequency

- The collection frequency of *t* is the number of occurrences of *t* in the collection, counting multiple occurrences.
- Example:

Word	Collection frequency	Document frequency
insurance	== 2-3 10440	3997
try	==1 10422	8760

- Which word is a better search term (and should get a higher weight)?

tf-idf weighting

- The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$w_{t,d} = (1 + \log \mathrm{tf}_{t,d}) \times \log_{10}(N / \mathrm{df}_t)$$

- Best known weighting scheme in information retrieval
  - Note: the "-" in tf-idf is a hyphen, not a minus sign!
  - Alternative names: tf.idf, tf x idf
- Increases with the number of occurrences within a document
- Increases with the rarity of the term in the collection

Final ranking of documents for a query

$$\text{Score}(q,d) = \sum_{t \in q \cap d} \text{tf.idf}_{t,d}$$

Binary → count → weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathrm{R}^{|V|}$