Mathematics III (RMA3A001)

Module I

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Lecture - 3

Secant method

- The secant method for finding out the root of the equation $f(x) = 0$ is given by

$$x_{k+1} = x_k - \frac{(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})} f(x_k)$$

$$k = 1,\ 2,\ 3,\ \ldots.$$

- *First approximation (k=1)*

$$x_2 = x_1 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} f(x_1)$$

- *Second approximation (k=2)*

$$x_3 = x_2 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} f(x_2)$$

- *Third approximation (k=3)*

$$x_4 = x_3 - \frac{(x_3 - x_2)}{f(x_3) - f(x_2)} f(x_3)$$

and so on

- Where x_0 and x_1 are called the initial approximation to the root of the equation.
- Since two initial approximations are equal for finding out the root of the equation so it is called a two point formula.
- **NOTE :** The rate of convergence of secant method is 1.618.

Example 1

Find the real root of the equation $f(x) = x^3 - 5x + 1 = 0$ correct upto three decimal places by using secant method.

Solution : We have $f(x) = x^3 - 5x + 1 = 0$

$$f(0) = 1 > 0, \qquad f(1) = -3 < 0$$

$$f(0.2) = 0.008 > 0, \qquad f(0.3) = -0.473 < 0$$

So the root of the equation lies in the interval $(0.2, 0.3)$
Let $x_0 = 0.2$ and $x_1 = 0.3$ be the initial approxiamtion to the root of the equation.
We have the secant method for finding out the root of the equation $f(x) = 0$ is given by

$$x_{k+1} = x_k - \frac{(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})} f(x_k)$$

$$k = 1,\ 2,\ 3,\ \ldots.$$

First approximation (k=1)

$$x_2 = x_1 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} f(x_1)$$

$$= 0.3 - \frac{(0.3 - 0.2)}{f(0.3) - f(0.2)} f(0.3) = 0.201663$$

Second approximation (k=2)

$$x_3 = x_2 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} f(x_2)$$

$$= 0.201663 - \frac{(0.201663 - 0.3)}{f(0.201663) - f(0.3)} f(0.201663) = 0.201639$$

So the root of the equation correct upto three decimal places by secant method is 0.201

Example 2

Find the real root of the equation $f(x) = \cos x - xe^x = 0$ by using secant method.

Solution : We have $f(x) = \cos x - xe^x = 0$

$$f(0) = 1 > 0, \qquad f(1) = -2.177979 < 0$$

$$f(0.5) = 0.053221 > 0, \qquad f(0.6) = -0.267935 < 0$$

So the root of the equation lies in the interval $(0.5, 0.6)$
Let $x_0 = 0.5$ and $x_1 = 0.6$ be the initial approximation to the root of the equation.
We have the secant method for finding out the root of the equation $f(x) = 0$ is given by

$$x_{k+1} = x_k - \frac{(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})} f(x_k)$$

$$k = 1, \ 2, \ 3, \ \ldots.$$

First approximation (k=1)

$$x_2 = x_1 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} f(x_1)$$

$$= 0.6 - \frac{(0.6 - 0.5)}{f(0.6) - f(0.5)} f(0.6) = 0.516571$$

Second approximation (k=2)

$$x_3 = x_2 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} f(x_2)$$

$$= 0.516571 - \frac{(0.516571 - 0.6)}{f(0.516571) - f(0.6)} f(0.516571) = 0.0.517678$$

Third approximation (k=3)

$$x_4 = x_3 - \frac{(x_3 - x_2)}{f(x_3) - f(x_2)} f(x_3)$$

$$= 0.517678 - \frac{(0.517678 - 0.516571)}{f(0.517678) - f(0.516571)} f(0.517678) = 0.0.517757$$

So the root of the equation by secant method after three steps is 0.517757

Any Questions?

Thank You