Mathematics III (RMA3A001)

Module I

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Lecture - 4

Newton Raphson method

- The Newton Raphson method for finding out the root of the equation $f(x) = 0$ is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$k = 0,\ 1,\ 2,\ 3,\ \ldots.$$

- *First approximation (k=0)*

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

- *Second approximation (k=1)*

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Cont ....

- *Third approximation (k=2)*

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

and so on

- Where x_0 is known as the initial approximation to the root of the equation.
- Since one initial approximations are equal for finding out the root of the equation so it is called a one point formula.
- **NOTE :** The rate of convergence of Newton Raphson method is 2. So it has quadratic rate of convergence.

Example 1

Find the real root of the equation $f(x) = x^4 - x - 10 = 0$ by using Newton Raphson method.

Solution : We have $f(x) = x^4 - x - 10 = 0$, $f'(x) = 4x^3 - 1$

$$f(0) = -10 < 0, \qquad f(1) = -10 < 0, \qquad f(2) = 4 > 0$$

So the root of the equation lies in the interval $(1, 2)$,
Let $x_0 = 1.8$ be the initial approximation to the root of the equation.
We have the Newton Raphson method for finding out the root of the equation $f(x) = 0$ is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$k = 0,\ 1,\ 2,\ 3,\ \ldots.$$

First approximation (k=0)

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1.8 - \frac{f(1.8)}{f'(1.8)} = 1.8583303$$

Second approximation (k=1)

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.8583303 - \frac{f(1.8583303)}{f'(1.8583303)} = 1.8555908$$

Third approximation (k=2)

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.8555908 - \frac{f(1.8555908)}{f'(1.8555908)} = 1.8555845$$

So the root of the equation after three steps by Newton Raphson method is 1.8555845

Example 2

Find the real root of the equation $f(x) = \cos x - xe^x = 0$ by newton Raphson method correct up to three decimal places.

Solution : We have $f(x) = \cos x - xe^x = 0, \qquad f'(x) = -\sin x - xe^x - e^x$

$$f(0) = 1 > 0, \qquad f(1) = -2.177979 < 0$$

So the root of the equation lies in the interval $(0, 1)$
Let $x_0 = 0.5$ be the initial approximation to the root of the equation.
We have the Newton Raphson method for finding out the root of the equation $f(x) = 0$ is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$k = 0,\ 1,\ 2,\ 3,\ \ldots.$$

First approximation (k=0)

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.5180260$$

Second approximation (k=1)

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.5180260 - \frac{f(0.5180260)}{f'(0.5180260)} = 0.517757$$

Third approximation (k=2)

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.517757 - \frac{f(0.517757)}{f'(0.517757)} = 0.517757$$

So the root of the equation by Newton Raphson method correct up to three decimal places is 0.517.

Example 3

Find the cube root of 2 by newton Raphson method correct up to four decimal places.

Solution : Let $x = \sqrt[3]{2}$

$$\implies x^3 = 2$$

$$\implies x^3 - 2 = 0$$

Let $f(x) = x^3 - 2 = 0, \qquad f'(x) = 3x^2$

$$f(0) = -2 < 0, \qquad f(1) = -1 < 0, \qquad f(2) = 6 > 0$$

So the root of the equation lies in the interval $(1, 2)$

Let $x_0 = 1.2$ be the initial approximation to the root of the equation.

We have the Newton Raphson method for finding out the root of the equation $f(x) = 0$ is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$k = 0, 1, 2, 3, \ldots$$

First approximation (k=0)

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1.2 - \frac{f(1.2)}{f'(1.2)} = 1.262962$$

Second approximation (k=1)

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.262962 - \frac{f(1.262962)}{f'(1.262962)} = 1.259928$$

Third approximation (k=2)

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.259928 - \frac{f(1.259928)}{f'(1.259928)} = 1.259921$$

Thus the cube root of 2 correct up to four decimal point by Newton Raphson method is 1.2599.

Any Questions?

Thank You