Mathematics III (RMA3A001)

Module I

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Lecture - 1

Solution of non linear equation in one variable

- In the science and engineering the solution or roots x of an equation $f(x) = 0$ occurs in many applications.
- If $f(x)$ is a linear or quadratic equation then we can find the roots of $f(x) = 0$ easily.
- However, many non linear equations, transdental equations can not be solved easily.
- Hence to find out the solution or root of such types of equations we need some numerical successive approximation methods or iterative methods.

Iterative method

- These methods are based on the idea of successive approximations i.e starting with one or more initial approximations to the root or solution we obtain a sequence of approximations or iterates $\{x_k\}$, which is in the limit converges to the root.
- The methods may give only one root at a time.

Definition

A sequence of iterates $\{x_k\}$ is said to be converges to the exact root or solution α if

$$\lim_{k\to\infty} |x_k - \alpha| = 0, \qquad k = 1,\ 2,\ 3,\ \ldots\ldots$$

Non linear / Polynomial / Algebric equations

- An expression of the form

$$f(x) = a_0x^n + a_1x^{n-1} + \ldots\ldots + a_{n-1}x + a_n$$

Where $a_1, a_2, \ldots\ldots a_n$ are constants provided $a_0 \neq 0$ and n is positive integer is called a polynomial in x of degree n. The polynomial equation $f(x) = 0$ is called algebric equation.

Example

$$x^3 - 5x + 1 = 0$$

$$x^4 - 3x^3 + 4x - 3 = 0$$

Transdental equations

- If $f(x)$ is an expression involving trigonometric, logarithmic, exponential functions etc. then it is called transdental equation.

Example

$$\cos x - xe^x = 0$$

$$\log_e x - e^x = 0$$

- A number α is said to be a root or solution of the equation $f(x) = 0$ if $f(\alpha) = 0$.
- One fundamental theorem is used to locate the interval in which the real root of the equation $f(x) = 0$ lies. The theorem is known as intermediate value theorem.
- We have to take that interval or any point on that interval as the initial approximation to the root of the equation in different iterative methods.

Intermediate value theorem

- If $f(x)$ is a continuous function on some interval $[a, b]$ and $f(a)f(b) < 0$ i.e. $f(a)$ and $f(b)$ are of opposite sign then the equation $f(x) = 0$ has at least one real root or an odd number of roots in the interval (a, b).

Example 1

Find the interval in which the real root of the equation lies,

$$f(x) = x^3 - 5x + 1 = 0$$

Solution : We have

$$f(x) = x^3 - 5x + 1 = 0$$

Now

$$f(0) = 1 > 0$$

$$f(1) = -3 < 0$$

So, the root of the equation lies in the interval $(0, 1)$.

'0' point is closer to the root of the equation than the point '1'. Because at 0 point the value of $f(x)$ is 1, where as at 1 point the value of $f(x)$ is -3. Since 1 is closer to 0 rather than -3, thus 0 point is closer to the root.
If we try to find out the closer interval in which the root of $f(x)$ lies, we have to check the points closer to 0.

$$f(0.1) = 0.501 > 0$$

$$f(0.2) = 0.008 > 0$$

$$f(0.3) = -0.473 < 0$$

Thus the closer interval in which the root of the equation lies $(0.2, 0.3)$. Among these two points 0.2 and 0.3, 0.2 is nearer to the root of the equation.

Example 2 :

Find out the interval in which the root the equation

$$f(x) = \cos x - xe^x = 0$$

lies.

Solution : First convert the calcular into radian mode because this equation contains trigonometric function.

We have

$$f(x) = \cos x - xe^x = 0$$

$$f(0) = 1 > 0$$

$$f(1) = -2.1779 < 0$$

So, the root of the equation lies in the interval $(0, 1)$.

Now we have to find out the closer interval in which the root of the equation lies

$$f(0.5) = 0.175 > 0$$

$$f(0.6) = -0.093 < 0$$

So, the root of the equation lies in the interval $(0.5, 0.6)$

Different Numerical methods for finding the root of $f(x) = 0$

There are several numerical methods for finding the root or solution of an equation $f(x) = 0$. Some methods are one point method because one initial approximation is required. Some of them are two point method because two initial approximations are required for finding out the root of the equation.

- Bisection method (Two point method)
- Secant method (Two point method)
- Regular Falasi method (Two point method)
- Newton Raphson method (One point method)
- Fixed point iteration method (One point method)

Any Questions?

Thank You