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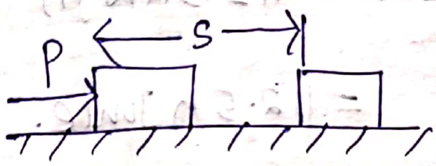
Work, Power & Energy Module-4

Work: Whenever a force acts on a body & the body undergoes some displacement, then work is said to be done.

If a force 'p' acting on a body causes it to move through a distance 's' then workdone by the force.

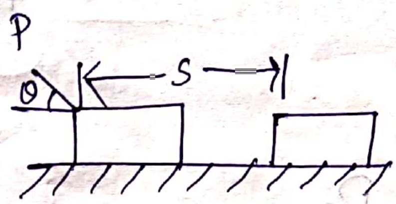
$= \text{force} \times \text{distance}$

$= P \times S$



(Body moving in the direction of force)

$\text{Workdone} = P \times S$



(Body not moving in the direction of force)

$\text{Workdone} = P\cos\theta \times S$

Unit of work

$\text{workdone} = \text{Force} \times \text{distance}$

$\text{Unit} = N \times m$

$1 \text{ joule} = 1 \text{ NM}$

$\boxed{1 \text{ Kilojoule} = 1 \text{ KNM}}$

Q.1 A horse pulling a cart exerts a steady horizontal pull of 300N & walks at the rate of 4.5 kmph. How much work is done by the horse in 5 min?

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Pull = 300N (Force)

$V = 45 Kmph = \frac{4.5 \times 1000}{60}$ m/min

Time = 5 min

Distance (S) = velocity x time

= 75 X 5 = 375 m

Workdone by the horse

W = Force x distance

= 300 X 375

= 112500 Nm = 112.5 KN.m

= 112.5 K Joule.

Power

The Power may be defined as the rate of doing work. It is thus the measure of performance of engines.

Unit

watt (w) = 1 N.m/sec = 1 J/sec

Energy

The energy may be defined as the capacity ~~to~~ to do work. It exists in may form its mechanical, electrical & chemical etc.

Unit

Unit of energy is same as work

$1 Nm = 1$ joule

Mechanical energy

Potential energy

→ It is the energy posseses by a body for doing work by virtue of its position.

Potential energy $= mgh$.

Kinetic energy

→ It is the energy possess by a body for doing work by virtue of its mass & velocity.

$KE = \frac{1}{2} mv^2$.

D' Alemebrt's Principle

It states that if a body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude & direction & the line of action may be foundout by the methods of graphic statics.

Force acting on a body

$P = ma$ ——— (i)

Where P = force

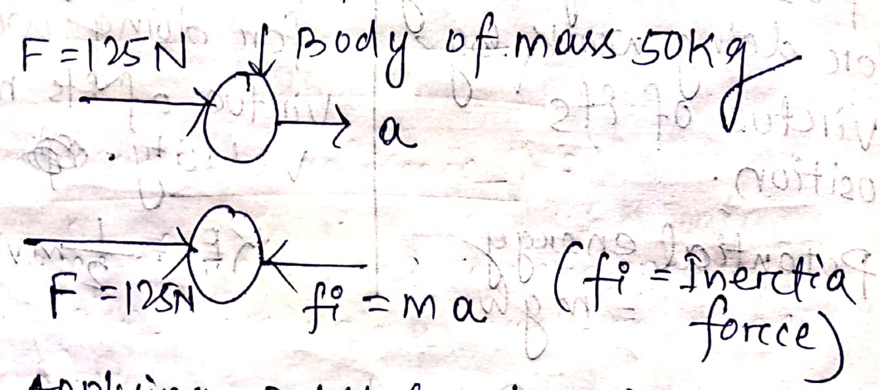
m = mass of the body

a = acceleration of the body

$\boxed{P - ma = 0}$ ——— (ii)

The eqn(i) is the eqn of dynamics whereas eqn (ii) is the eqn of statics. The eqn(ii) is also known as the eqn of dynamic equilibrium under the action of real force P. This is known as D'Almbert's principle.

Q// Determine the accn produced in a body of 50kg mass when it is acted upon by a force of 125N. Use D'Almbert's principle.



By Applying D'Almbert's principle

$$P - ma = 0$$

$$\Rightarrow 125 - 50 \times a = 0$$

$$\Rightarrow 125 = 50a$$

$$\Rightarrow a = 125/50 = 2.5\,m/s^2$$

Impulse of a force

It is defined as the time integral of the force over the interval of time

$$I = \int_{t_1}^{t_2} F\,dt$$

Where, I = Impulse
F = force
t_1 & t_2 = time interval.

The force may vary or remain constant over a period of time. The impulse of a force be visualized as the area under a force vs. time curve.

Momentum

→ It is the quantity of motion possessed by a body.

→ Momentum = Mass x velocity.

→ Unit is kg-m/s.

Concept of inertial & non inertial frame of reference

Inertial frame of reference (Module-3)

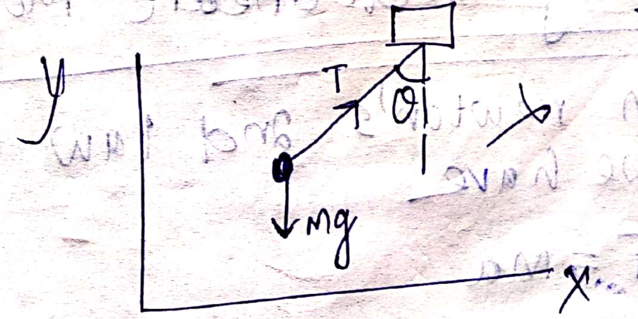
→ A frame of reference either at rest or moving with a uniform velocity (zero acc^n) is known as Inertial frame.

→ All the laws of physics hold good in such a frame.

→ Zero force & Zero acc^n.

$\vec{F} = 0$

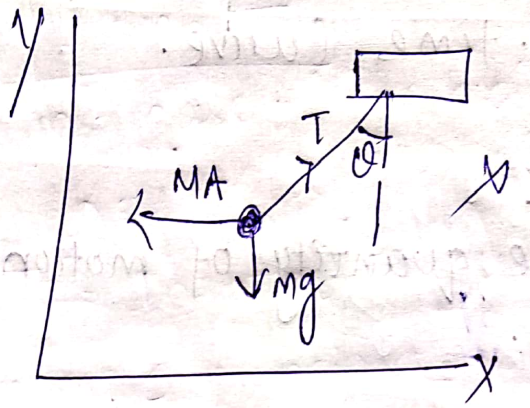
$\vec{a} = 0$.



Inertial frame

Noninertial frame / Accelerated frame

It is a frame of reference which is either having a uniform linear acceleration or is being rotated with uniform speed.



$\vec{F} = 0$

$\vec{a} = 0.$

Inertial frame of reference (IFR)

→ The frame of reference where newton's laws are applicable are known as (IFR)

Ex:- The Earth.

→ When we consider motion w.r.t earth it is known as IFR.

→ The FOR (Frame of reference) at rest or moving at constant velocity w.r.t Inertial frame of reference.

Kinematics of Rectilinear Motion

From newton's 2nd law of motion we have

$F = ma$

$\Rightarrow F - ma = 0$

$F + (-ma) = 0$

$$\boxed{\Sigma F + F_i = 0}$$

where F = external force applied
resultant of forces.
M = mass of the body.
a = acceleration of the body.
F_i = inertial force / D'Almbert's force / reverse affective force.

D'Almbert's principle

The body will be in dynamic equilibrium under the action of external forces and D'Almbert's force (F_i). This is known as D'Almbert's principle.

It state that, "the body will be in dynamic equilibrium if algebric sum of net external forces acting on system/body and inertial force is 0."

$$\boxed{\Sigma F + F_i = 0}$$

~~M-4~~ Module-4

Kinematics of curvilinear Motion

(Rotation of rigid body)

From 2nd law we have,

$T = I\alpha$

$T + (-I\alpha) = 0$

$T + T_i = 0$

T = torque applied / resultant of torque.

I = mass moment of Inertia.

$$\boxed{I = mK^2}$$

m = mass of the body

K = radius of gyration.

T_i = Inertial Torque.

D'Almbert's principle

It states that, "when external torque's act on a system having rotating motion, then the algebric sum of all the torque's acting on the system due to external forces and reverse effective forces including the inertial torques (T_i) taken in opposite direction of angular acc^n is zero.

$$\boxed{T + T_i = 0}$$

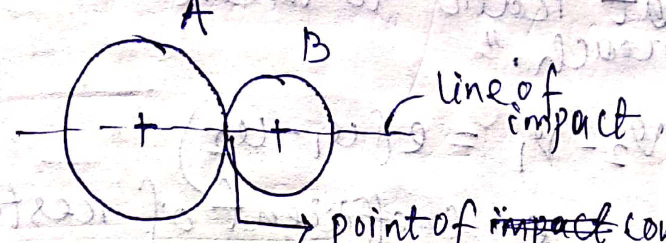
Impact

When two bodies collide with one another, they are called impact.

(I) Direct impact

(II) Indirect impact.

Direct impact



If the two bodies, before impact are moving along the line of impact, the collision is called direct impact.

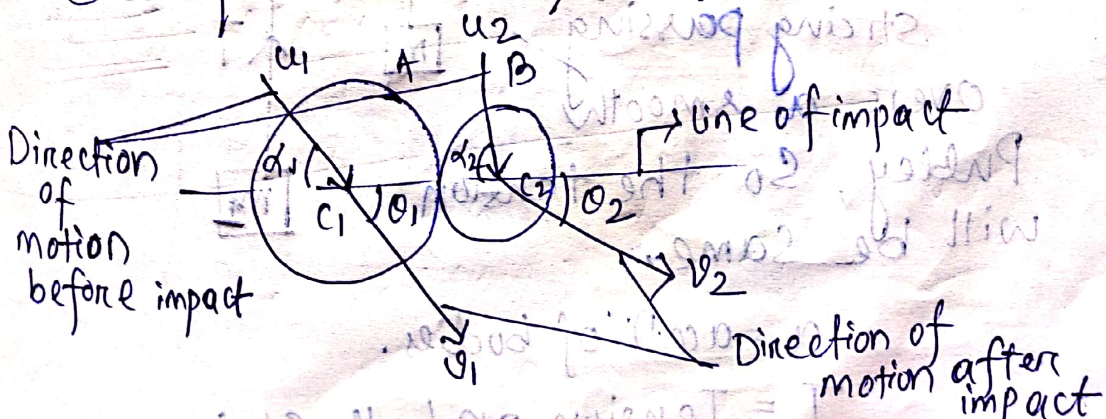
$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2.$$

where m_1, u_1, v_1 = mass, initial and final velocity of the first body

m_2, u_2, v_2 = mass, initial and final velocity of second body.

indirect impact

If the two bodies, before impact are not moving along the line of impact, the collision is called an indirect / oblique impact.



$m_1 u_1 \cos\alpha_1 + m_2 u_2 \cos\alpha_2$
$= m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$

Coefficient of restitution

It states that, "When two moving bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach."

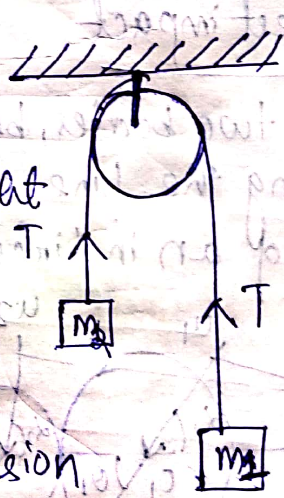
$$v_2 - v_1 = e(u_1 - u_2)$$

e = coefficient of restitution / constant of proportionality.

Motion of Connected Bodies

Case-I

Motion of two bodies connected by a string and passing over a smooth pulley.



string is light.
We shall consider light inextensible string passing over a smooth pulley, so the tension will be same.

a = acc^n of bodies.
T = Tension on both string.

$m_1 > m_2$:

For body 1

Resultant force $= m_1 g - T$ (downward)

$$m_1 g - T = m_1 a$$

$$T = m_1 g - m_1 a = m_1(g-a) \quad \text{— (I)}$$

For body-1

Resultant force $= T - m_1 g$

$$T - m_1 g = m_1 a$$

$$T = m_1(g+a) \quad \text{— (II)}$$

For body-2

Resultant force $= T - m_2 g$ (upward)

$$m_2 a = T - m_2 g$$

$$\Rightarrow T = m_2(g+a) \quad \text{— (II)}$$

In eq^n (I) & (II)

$$m_1(g-a) = m_2(g+a)$$

$$m_1 g - m_1 a = m_2 g + m_2 a$$

$$\Rightarrow m_1 g - m_2 g = m_2 a + m_1 a$$

$$\Rightarrow g(m_1 - m_2) = a(m_1 + m_2)$$

$$\Rightarrow \boxed{a = \frac{g(m_1 - m_2)}{(m_1 + m_2)}}$$

Put the value of a in eq^n (1)

$$T = m_1(g-a)$$

$$\Rightarrow T = m_1 \times \left(g - \frac{g(m_1-m_2)}{m_1+m_2}\right)$$

$$\Rightarrow T = m_1 \times g\left[1 - \frac{m_1-m_2}{m_1+m_2}\right]$$

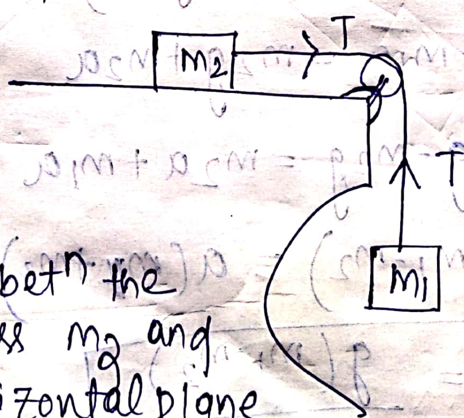
$$\Rightarrow T = m_1 g \times \left(\frac{\cancel{m_1}+m_2-\cancel{m_1}+m_2}{m_1+m_2}\right)$$

$$\Rightarrow T = m_1 g \times \frac{2m_2}{m_1+m_2}$$

$$\Rightarrow \boxed{T = \frac{2m_1m_2g}{m_1+m_2}}$$

Case-II

Motion of two bodies connected by a string, one of which is hanging free and the other laying on a smooth horizontal plane.



friction bet^n the body of mass m_2 and the horizontal plane is neglected.

For body 1

$$m_1g - T = m_1a \quad (1)$$

$$\Rightarrow T = m_1g - m_1a$$

$\Rightarrow T = m_1(g-a) \quad (\downarrow)$

For body-2

$$T = m_2 a \quad \text{—— (ii)}$$

In eqn (i) & (ii)

$$m_1(g-a) = m_2 a$$

$$\Rightarrow \cancel{a = m_1 g}\; m_1 g - m_1 a = m_2 a$$

$$\Rightarrow m_1 g = m_2 a + m_1 a$$

$$\Rightarrow m_1 g = a(m_1+m_2)$$

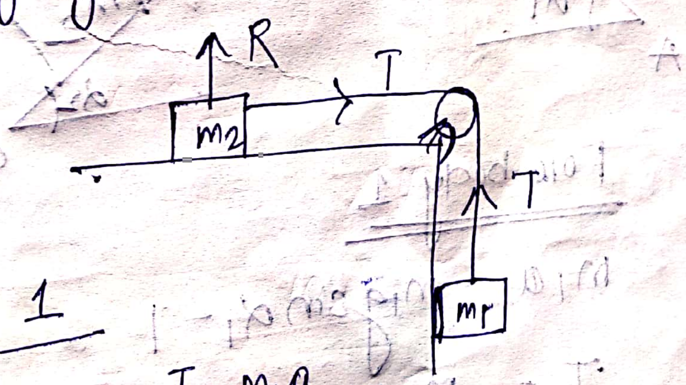
$$\Rightarrow a = \frac{m_1 g}{m_1+m_2}$$

$$T = m_2 \times a = m_2 \times \frac{m_1 g}{m_1+m_2}$$ (Put the value of a in eqn)

$$\boxed{T = \frac{m_1 m_2 g}{m_1+m_2}}$$

Case-III

Motion of two bodies connected by a string, one of which is hanging free and the other lying on a rough horizontal plane



For body 1

$$m_1 g - T = m_1 a$$

$$\Rightarrow T = m_1(g-a) \quad \text{—— (i)}$$

For body 2

$$R = m_2 g$$

$$F = \mu R = \mu m_2 g$$

($T \rightarrow$ Right
$\mu m_2 g / F \rightarrow$ left
friction force produced)

$$m_2 a = T - \mu m_2 g$$

$$T = m_2 a + \mu m_2 g$$

$$T = m_2(a + \mu g) \quad \text{--- (II)}$$

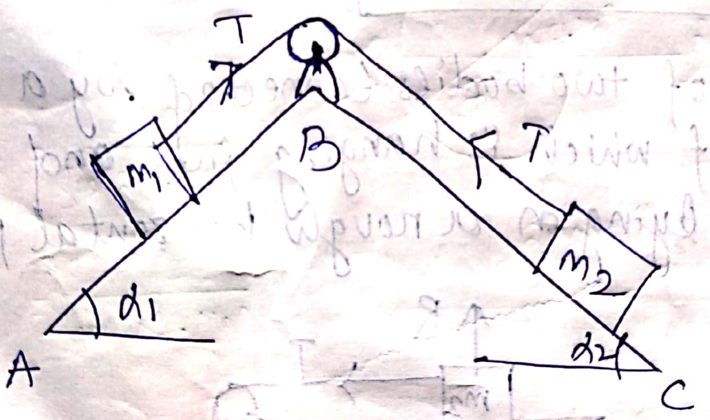
In eq^n (I) & (II)

$$a = \frac{g(m_1 - \mu m_2)}{m_1 + m_2}$$

$$T = \frac{m_1 m_2 g (1 + \mu)}{m_1 + m_2}$$

Case-IV

Motion of two bodies, connected by a String and lying on smooth inclied planes.



For body 1

$$m_1 a = m_1 g \sin\alpha_1 - T$$

$$T = m_1(g\sin\alpha_1 - a) \quad \text{--- (I)}$$

For body 2

$$m_2 a = T - m_2 g \sin\alpha_2$$

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$$T = m_2(a + g\sin\alpha_2) \quad \text{--- (11)}$$

In eqn (1) & (11)

$$a = \frac{g(m_1\sin\alpha_1 - m_2\sin\alpha_2)}{m_1+m_2}$$

$$T = \frac{m_1 m_2 g(\sin\alpha_1 + \sin\alpha_2)}{m_1+m_2}$$