LAWS OF MOTION MODULE - IV

The entire System of dynamics is based on three laws of motion, which are the fundamental laws & were formulated by "Newton".

Important Terms :-

1. **Mass** :- It is the matter contained in a body.

   Unit :- Kg, Tonne etc.

2. **Weight** :- It is the force, by which the body is attracted towards the centre of the earth.

   Unit :- The units of weight are the same as those of force i.e. N, KN etc.

3. **Momentum** :- It is the quantity of motion Possessed by a body.

   It is expressed as, $\boxed{\text{Momentum} = \text{Mass} \times \text{Velocity}}$

   Unit :- Kg-m/s.

4. **Force** :- It is very important for dynamics and defined as any cause which Produces or tend to Produce, Stops or tends to stop motion.

   Unit :- N, KN etc.

5. **Inertia** :- It is an inherent Property of a body, which offers resistance to the change of its state of rest or Uniform motion.

Rigid Body:- A rigid body is defined as a body on which the distance between two points never changes whatever be the force applied on it.

or the body which doesn't deform under the influence of forces is known as a rigid body.

Newton's Laws of Motion :-

There are three laws of motion by Newton.

First Law :- "Every body continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force."

Second Law:- "The rate of change of momentum is directly proportional to the impressed force and takes place in the same direction, in which the force acts."

Third Law :- To every action, there is always an equal and opposite reaction.

(3)

Newton's first Law of Motion :-

It states "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force."

→ It is also called the law of inertia and consists of the following 2 parts.

* A body at rest continues in the same state unless acted upon by some external force
* A body moving with a uniform velocity continues its state of uniform motion in a straight line, unless it is compelled by some external force to change its state.

Newton's Second Law of Motion :-

It states "The rate of change of momentum is directly proportional to the impressed force and takes place, in the same direction in which the force acts."

→ This law enables us to measure a force, and establishes the fundamental equations of dynamics

→ Consider a body moving in a straight line Let its velocity be changed while moving.

(4) Let m = mass of a body.
u = initial velocity of the body.
v = final velocity of the body.
a = Constant acceleration.
t = time in seconds required to change the velocity from u to v
F = force required to change velocity from u to v in t seconds.

∴ Initial momentum = mu

final momentum = mv

∴ Rate of change of momentum = $\frac{mv - mu}{t}$

$= \frac{m(v-u)}{t} = ma$

$\left[\because \frac{v-u}{t} = a\right]$

According to Newton's Second law of Motion, the rate of change of momentum is directly proportional to the impressed force.

$$F \propto ma = Kma$$

where K = Const of proportionality.

Or $$f = ma = \text{mass} \times \text{acceleration}.$$

Q. A body of mass 10 kg is moving over a smooth surface, whose equation of motion is given by the reation,

$$s = 5t + 2t^2$$

where (s) is in meters and (t) in seconds. find the magnitude of force responsible for the motion.

Sol^n Given, equation of motion $s = 5t + 2t^2$

Differentiating both sides of the above equation w.r.t to 't',

$$\frac{ds}{dt} = \frac{d(5t + 2t^2)}{dt}$$

$$\frac{ds}{dt} = 5 + 4t$$

Again differentiating both sides of the above eqn^n w.r.t to 't',

$$\frac{d^2s}{dt^2} = 4 \Rightarrow \text{acceleration } a = 4 m/sec^2$$

∴ Force responsible for the motion.

$$f = m \times a = 10 \times 4 = 40 N$$

$$\boxed{F = 40 N}$$ Ans

Motion of a Lift :-

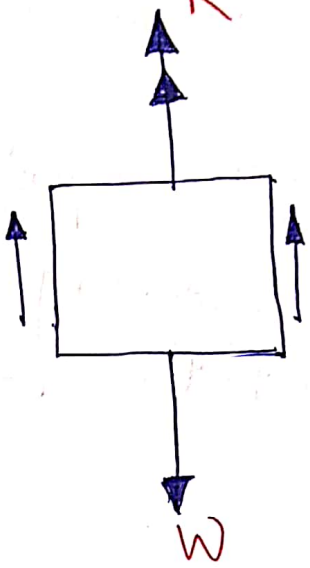
Consider a lift carrying some mass and moving with a uniform acceleration.

Let, m = Mass carried by the lift.
a = Uniform acceleration of the lift
R = Reaction of the lift or tension in the cable, supporting the lift.

There are two cases as follows.

(b) 1. When the lift is moving upwards.
2. When the lift is moving downwards.

1. When the lift is moving upwards :-



We know that downward force due to mass of the Lift = mg

& net upward force on Lift

$$\Rightarrow F = R - mg \quad \text{——} (1)$$

We also know,

thus force = Mass x Acceleration (Lift moving upward)

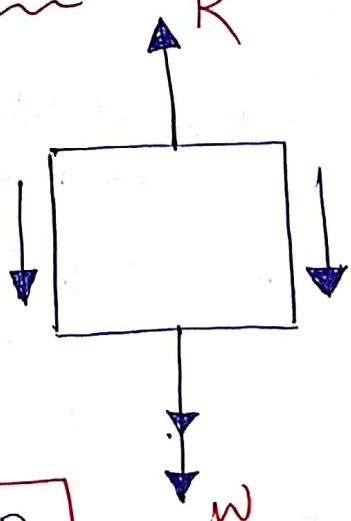
$$f = m \times a \quad \text{——} (2)$$

from eqn (1) & (2), $R - mg = ma$

$$\Rightarrow R = ma + mg$$

$$\boxed{R = m(a+g)}$$

2. When the Lift is moving downwards :-



In this case, the net downward force, which is responsible for the motion of the lift.

$$= mg - R \quad \text{——} (3)$$

From eqn (2) & (3),

$$ma = mg - R$$

$$\Rightarrow R = mg - ma = \boxed{m(g-a) = R}$$

NOTE: In both the cases, we have taken mass carried by the lift only. We have assumed that it includes mass of the lift also, But Sometimes the example contains mass of the lift and mass carried by the lift separately.

In such a case, the mass carried by the lift will exert a pressure on the floor of the lift whereas tension in the cable will be given by the algebraic sum of the masses of the lift and mass carried by the lift.

Mathematically, Pressure exerted by the mass carried by the lift on its floor.

$$= m_2(g+a)$$

& tension in the cable $= (m_1+m_2)(g+a)$

where, m_1 = Mass of the lift.

m_2 = Mass carried by the lift.

Q. In a factory, an elevator is required to carry a body of mass 100 kg. What will be the force exerted by the body on the floor of the lift, when (a) the lift is moving upwards with retardation of $0.8\ m/sec^2$ (b) moving downward with a retardation of $0.8\ m/sec^2$

Given: Mass of the body $(m) = 100\ kg$.

acceleration $a = -0.8\ m/sec^2$ (— due to retardation)

(a) When the lift is moving upwards.

We know force exerted by the body on the floor of the lift $= F_1 = m(g+a) = 100(9.8-0.8) = 900\ N$

(b) When the lift is moving downwards.

(8) We know that force exerted by the body on the floor of the lift

$= f_2 = m(g-a)$

$= 100(9.8+0.8)$

$= 1060N$

D'ALEMBERT'S PRINCIPLE :-

It states that, "If a rigid body is acted upon by a system of forces, that system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the methods of graphic statics.

We know, force acting on a body,

$$P = ma \quad \text{---------- (1)}$$

where m = mass of the body

a = Acceleration of the body.

The eqn (1) may also be written as,

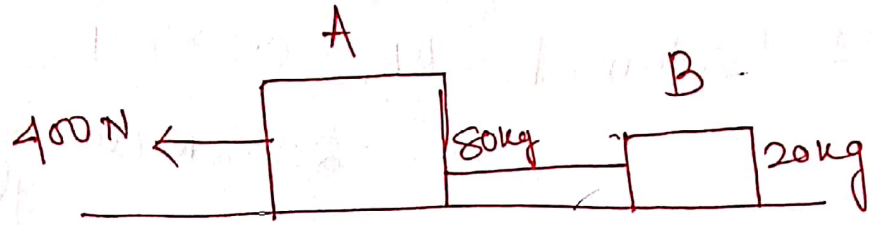
$$P - ma = 0 \quad \text{---------- (2)}$$

NOTE :- Eqn (1) is the equation of Dynamics.

Eqn (2) is the equation of Statics.

* Equation (2) is also known as the equation of dynamic equilibrium under the action of real force 'P'. This Principle is known as D'Alembert's Principle.

Q. Two bodies A & B of mass 80 kg & 20 kg are connected by a thread and move along a rough horizontal plane under the action of a force 400 N applied to the first body of mass 80 kg as shown in fig.



The co-efficient of friction between the sliding surfaces of the bodies and the plane is 0.3. Determine the acceleration of the two bodies and the tension in the thread. Using D'Alembert's Principle.

Given, Mass of the body 'A' $(m_1) = 80$ kg

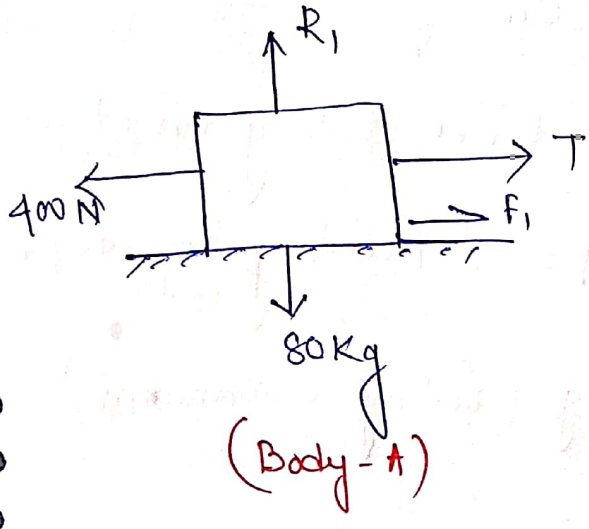
Mass of the body 'B' $(m_2) = 20$ kg.

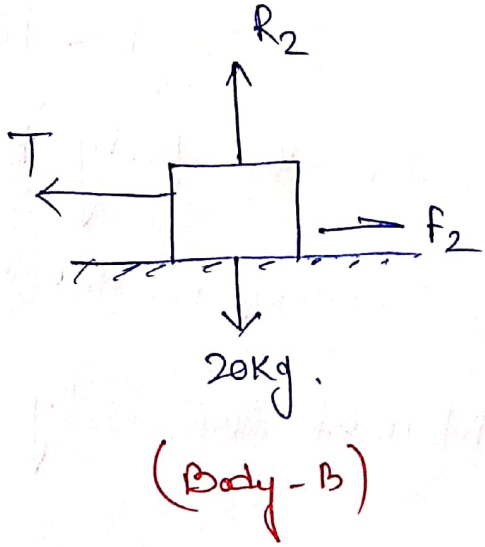
Force applied on 1st body $(P) = 400$ N

Co-efficient of friction $(\mu) = 0.3$.

Let, a = Acceleration of the bodies.

T = Tension in the thread.





(10) Consider the body 'A', the forces acting on it are:

* 400 N force (acting towards left)
* Mass of the body = 80 kg (acting downwards)
* Reaction $R_1 = 80 \times 9.8 = 784$ N (acting upwards)
* Force of friction, $f_1 = \mu R_1 = 0.3 \times 784$

$= 235.2$ N (acting towards right)

* Tension in the thread = T (acting towards right)

∴ Resultant horizontal force,

$P_1 = 400 - T - f_1$

$= 400 - T - 235.2$

$= 164.8 - T$ (acting towards Left)

We know that force causing acceleration to the body 'A' $= m_1 a = 80a$

and according D'Alembert's Principle $(P_1 - m_1 a = 0)$

$= 164.8 - T - 80a = 0$

$\Rightarrow \boxed{T = 164.8 - 80a}$ ——— (1)

Consider the body 'B', the forces acting on it are,

* Tension in the thread = T (acting towards left)
* Mass of the body = 20kg (acting downwards)

(11) * Reaction $R_2 = 20 \times 9.8 = 196\ N$ (acting upwards)

* Force of friction $F_2 = \mu R_2 = 0.3 \times 196$

$\Rightarrow F_2 = 58.8\ N$ (acting towards right)

∴ Resultant horizontal force,

$$P_2 = T - F_2$$

$$P_2 = T - 58.8$$

We know that force causing acceleration to the body 'B' $= m_2 a = 20a$

According to D'Alembert's Principle $(P_2 - m_2 a = 0)$

$$(T - 58.8) - 20a = 0$$

$$\Rightarrow \boxed{T = 58.8 + 20a} \quad \text{——— (2)}$$

Equating two values of T from equation (1) & (2)

$$164.8 - 80a = 58.8 + 20a$$

$$\Rightarrow 100a = 106$$

$$\Rightarrow \boxed{a = 1.06\ m/s^2}$$

Tension in the thread

Substituting the value of a in eqn (2),

$$T = 58.8 + (20 \times 1.06)$$

$$\boxed{T = 80\ N}$$

Newton's Third Law of Motion :-

It states, "To every action, there is always an equal and opposite reaction."

→ By action is meant the force, which a body exerts on another, and the reaction means the equal and opposite force, which the second body exerts on the first.

→ This law, therefore, states that a force always occurs in Pair.

→ Each Pair Consisting of two equal & opposite forces.

Ex When a bullet is fired from a gun, the bullet moves out with a great velocity and the reaction of the bullet, in the opposite direction, gives an unpleasant shock to the man holding the gun.

Recoil of Gun :-

According to Newton's Third law of Motion, when a bullet is fired from a gun, the opposite reaction of the bullet is known as the recoil of Gun.

Let, M = Mass of the Gun
V = Velocity of the Gun with which it recoils.
m = mass of the bullet &
v = Velocity of the bullet after explosion.

(13)

∴ Momentum of the bullet after explosion = mv ——— (1)

and Momentum of the Gun = MV ——— (2)

Equating the equations (1) & (2)

$$MV = mv$$

This relation is Popularly known as Law of Conservation of Momentum.

Q. A machine Gun of mass 25kg fires a bullet of mass 30 gram with a velocity of 250m/s. Find the velocity with which the machine gun will recoil

Given, Mass of the machine gun (M) = 25kg.

Mass of the bullet (m) = 30g = 0.03kg

and velocity of firing (v) = 250 m/s.

Let V = velocity with which the machine gun will recoil.

We know that $MV = mv$

$\Rightarrow 25 \times V = 0.03 \times 250$

$\Rightarrow V = \frac{7.5}{25}$

$\Rightarrow V = 0.3$ m/s. Ans