Exp: Suppose

Addig first 10 natural nos

S = 0

1	2	3	4	5	6	7	8	9	10
1s	1s	1s	1s	1s	1s	1s	1s	1s	1s

S = 1+2+3+4+5+6+7+8+9+10

= 10s Maximum time (Worst Case)

"O" → Big-Oh

Add()
{
Start.
S = 0
for i = 1 to n
S = S + i
}

5 elements

S = 1+2+3+4+5

= 5s Average time (Average time)

Θ → Big-Theta

S = 1

1s Minimum time (Best Case)

Ω → Big-Omega.

Asymptotic notation.

Why Asymptotic notion:

Running time

Three notations (O, θ, Ω):

Based on mathematical defin.

Two functions

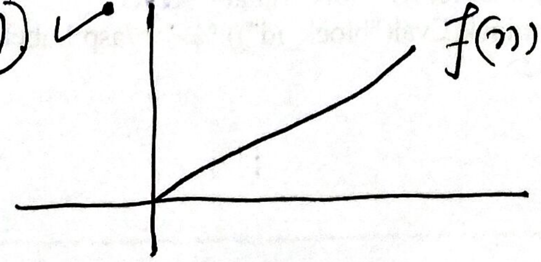
f(n), g(n)

f(n) = n

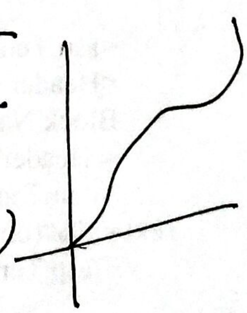
=> f(n) = g(n)

g(n) = n

n	g(n)	f(n)
1	1	1
2	2	2
3	3	3



f(n) = g(n)

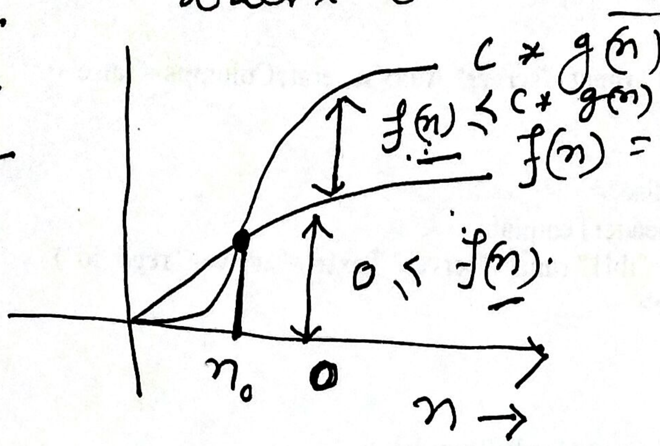


$c = 3 \quad f(n) = n$

$g(n) = c * n$

where 'c' is constant

n	$f(n)$	$g(n)$
1	1	3
2	2	6
3	3	9



$0 \leq f(n) \leq c * g(n)$

There are two consts "c" & n_0

Def^n: $O(g(n)) = \{ f(n)$: there exists two positive constants c & c_0 such that $0 \leq f(n) \leq c * g(n) \; \forall \; n \geq n_0 \}$

Exp: find Big-Oh for the function

$f(n) = 3n + 5$

$\Rightarrow f(n) = 3n + 5n^0$

We take the highest power of n.

$\Rightarrow f(n) = 3n + 5n^1$

$\Rightarrow 3n + 5 \leq 8n$

As per Big-Oh defi^n, we know that

$f(n) \leq c * g(n)$

$c = 8 \quad g(n) = n, \; n_0 = 1$

$\therefore f(n) = O(g(n))$

$= O(n)$

Actual Process

$3n + 5 \leq 8n$

$n = 0$

$5 \leq 0$ ✗

$n = 1$

$8 \leq 8$

$n_0 = 1$

$n = 2$

$11 \leq 16$

$n = 3$

$n \geq n_0$

$$f(n) = \cancel{3n}\ 3n + 5$$

$$\uparrow$$

$$\underline{\underline{g(n)}}$$

$$f(n) = O(g(n))$$

$$= O(n)$$

$$f(n) = 6n^2 + 5n + 2$$

$$f(n) = O(g(n))$$

$$= O(n^2)$$

$$c = ? \quad n_0 = ?$$

- Big-Oh (O) also known as Maximum time taken (Worst case)

Big-Omega: Let $f(n)$, $g(n)$ two functions

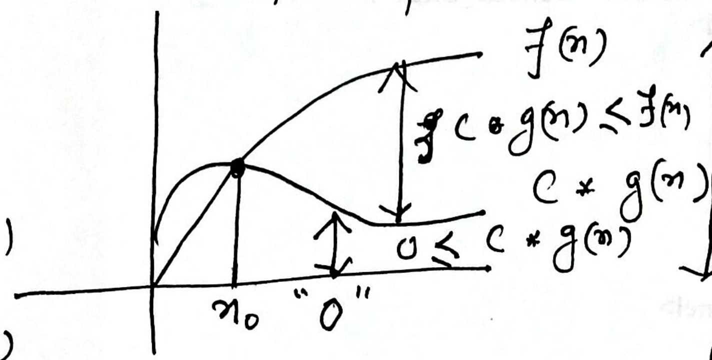
$f(n) = n$

$g(n) = n$

$f(n) = g(n)$

$\overline{c * g(n)}$

$c = -2$



$0 \leq c * g(n) \leq f(n)$

n	$f(n)$	$g(n)$
1	1	-2
2	2	-4
3	3	-8

Defin of Big-Omega (Ω):

$\Omega(g(n)) = \{ f(n) : \exists$ two tve constants c & n_0 such that $0 \leq c * g(n) \leq f(n)\ \forall n \geq n_0 \}$

Find Big-Omega (Ω) for given function $f(n) = 3n + 7$

$$\uparrow$$

$$g(n)$$

$$f(n) = \Omega(g(n)) = \Omega(n)$$

Given $f(n) = 3n + 7$

$$3n \leq \underline{3n + 7}$$

$$\Rightarrow 3n \leq f(n)$$

As per Big-Omega defin

$$c * g(n) \leq f(n)$$

$$\Rightarrow c = 3, \; g(n) = n, \; n_0 = 0$$

$$3n \leq 3n + 7$$

$n = 0$

$0 \leq 7$

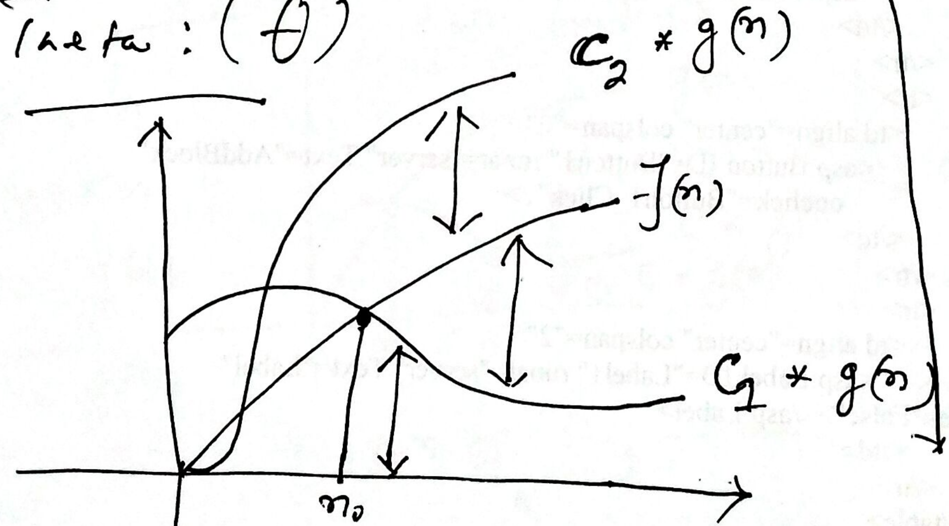
$n = 1$

$3 \leq 10$

$$\therefore f(n) = \Omega(g(n))$$
$$= \Omega(n)$$

$\downarrow$

Big-Theta: (θ)



$$0 \leq c_1 * g(n) \leq f(n) \leq c_2 * g(n)$$

constants c_1, c_2 & n_0

Defin: $\theta(g(n)) = \{ f(n) : \exists$ three constants c_1, c_2 and n_0 such that

$$0 \leq c_1 * g(n) \leq f(n) \leq c_2 * g(n)$$

$\forall \; n \geq n_0 \}$

$f(n) = 3n + 10$

find Big-theta

$f(n) = \theta(g(n))$

$= \theta(n)$

Process:

$f(n) = 3n+10$

$3n \leq 3n+10 \leq 3n+10n^2$

$\Rightarrow 3*n \leq f(n) \leq 13n$

As per Big-theta defin

$\Rightarrow C_1 * g(n) \leq f(n) \leq C_2 * g(n)$

Here $g(n) = n$, $C_1 = 3$, $C_2 = 13$

$n_0 = 1$

$f(n) = \theta(g(n))$

$= \theta(n)$

What is big-Oh of Constant-numeric

$f(n) = 2$
$f(n) = 200$
$f(n) = 439$

$\boxed{f(n) = O(1)}$

$f(n) = 2$

$\Rightarrow f(n) \leq 2 * 1$

As per Big-Oh defin

$f(n) \leq c * g(n)$

$g(n) = 1$, $c = 2$, $n_0 = 1$

$f(n) = O(g(n)) = O(1)$ Constant

$f(n) = 3n^3 + 3n^2 + 9$

Big-Oh

$f(n) = O(n^3)$

Cubic

$f(n) = 3n+7$

$f(n) = O(n)$

linear

$f(n) = 3n^2 + 2n$

$f(n) = O(n^2)$

Square

Big-oh Notation (O) :

Some common asymptotic functions are as follows:

	Exp
a. Constant : 1, ...	$O(1)$
b. logarithmic : $\log n$	$O(\log n)$
c. linear : n	$O(n)$
d. Quadratic : n^2	$O(n^2)$
e. exponential : 2^n	$O(2^n)$
f. factorial : $n!$	$O(n!)$
g. Cubic : n^3	$O(n^3)$