Module-1 FRICTION

When a rigid body slides over another rigid body, a resisting force is exerted at the surface of contact called force of friction or simply friction.

→ It always acts tangentially to the surface of contact and in a direction, opposite to the tendency of motion or to the direction of motion.

Classification of friction

There are 2 types of friction.

1. Static Friction
2. Dynamic Friction.

Static Friction:

It is the force of friction exerted between two contact surfaces when a body tends to move over another body.

or,

The friction experienced by a body when ~~in motion~~ at rest, is known as static friction.

Dynamic Friction

The friction experienced by a body, when in motion ie called dynamic friction. It is also called ~~Dynamic~~ Kinetic friction.

It is 2 types.

(i) sliding friction

(ii) Rolling friction.

Limiting Friction

The maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, it is known as limiting friction.

Laws of static friction

1. The force of friction always acts in a direction opposite to that in which the body tends to move.
2. The magnitude of force of friction is exactly equal to the force, which tends the body to move.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces.
4. The force of friction i.e. independent of the area of contact between the two forces.
5. The force of friction depends upon the roughness of the surfaces.

Laws of Dynamic or Kinetic friction

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.
2. The Magnitude of the kinetic friction bears a constant ratio to the normal reaction betn the two Surfaces.
3. For moderate Speeds, the force of friction remains constant, But it decreases slightly with the increase of speed.

Co-efficient of friction

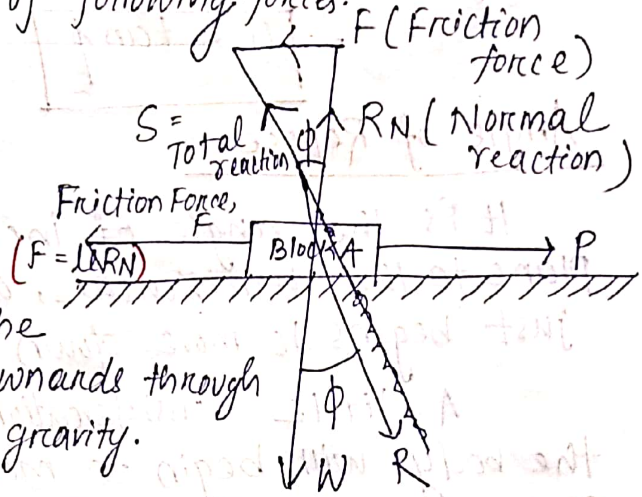
It is defined as the ratio of limiting friction (F) to the normal reaction (R_N), between the bodies.

It is generally denoted by μ.

$$\boxed{\text{Coefficient of friction, } \mu = \frac{F}{R_N}}$$

Total Reaction

The block A resisting on a rough surface is in equilibrium under the action of following forces.



(i) Weight W of the block, acting downwards through its centre of gravity.

(ii) Normal reaction R between the block & Surface.

(iii) Frictional Force $F = \mu R$ where μ is the coefficient of friction.

(iv) Tractive force P.

The forces R & F act at right angle to each other the resultant equals $\sqrt{R^2+F^2}$ and is called the total reaction or resultant reaction.

Angle of friction

It is defined as the angle which the resultant of normal reaction and limiting force of friction makes with the normal reaction.

From the above figure

$R \rightarrow$ Normal reaction

$F \rightarrow$ Limiting force of friction

$$\boxed{S = \sqrt{R^2 + F^2}}$$

= total or resultant reaction.

$$\boxed{\tan\phi = \frac{F}{R}}$$

where ϕ is the angle of friction.

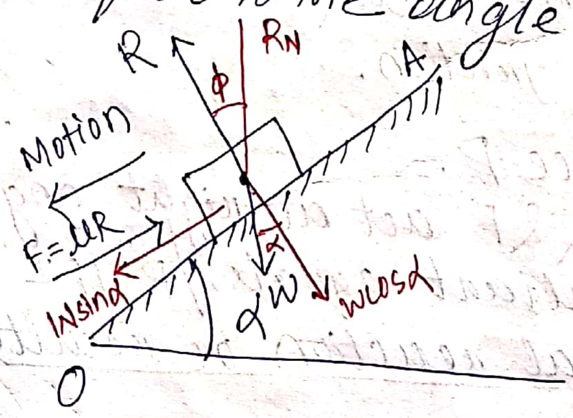
The ratio F/R is also called the co-efficient of friction μ.

$$\therefore \boxed{\mu = \tan\phi}$$

Angle of Repose

It is the angle of inclination (α) of the plane to the horizontal, at which the body just begins to move down the plane.

A little consideration will show that the body will begin to move down the plane, if the angle of inclination (α) of the plane is equal to the angle of friction (ϕ).



Consider a block of weight W resting on an inclined plane OA making an angle α with the horizontal.

 let the angle α be increased gradually till the block is just at the point of sliding. The block i.e then in equilibrium state Under the influence of following set of forces.

(a) Wt W of the block acting vertically downward.

(b) Normal Reaction R acting at right angle to the inclined plane &

(c) Limiting force of friction, $F = \mu R$ acting up the plane

So $\mu R = W\sin\alpha$ —— ①

$R = W\cos\alpha$ —— ②

Equating eqn ① & ② we get,

$$\boxed{\mu = \tan\alpha}$$

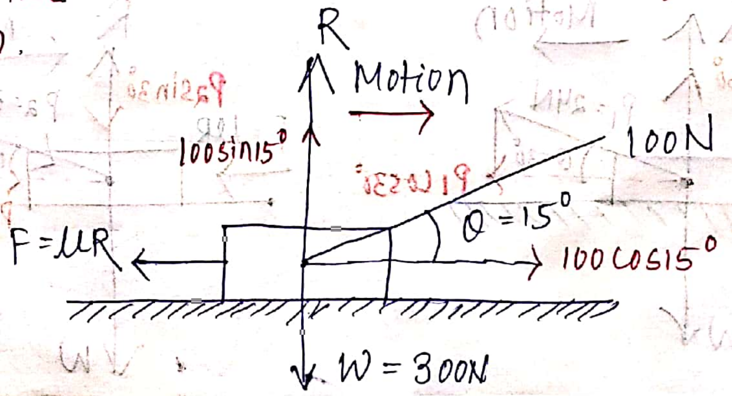
In term of angle of friction ϕ, the coefficient of friction i.e given as $\mu = \tan\phi$

$\tan\phi = \tan\alpha$

$$\boxed{\phi = \alpha}$$

Problems

A body weighing 300N i.e resting on a rough horizontal table. A pull of 100N applied at an angle of 15^0 with the horizontal just causes the body to slide over the table. Make calculations for the normal reaction and the coefficient of friction.



Given

$W = 300N$

$\theta = 15^\circ$

$F = 100N$

$\therefore \Sigma Fx = 0$

$F - 100\cos 15^\circ = 0$

$F = 96.59N$

$\Sigma Fy = 0$

$R + 100\sin 15^\circ - 300 = 0$

$\Rightarrow R = 300 - 100\sin 15^\circ$

$= 300 - 25.88$

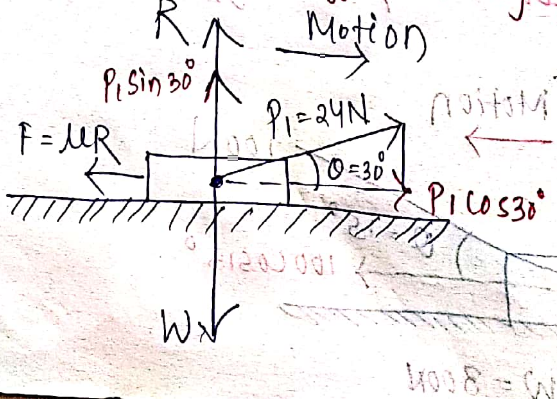
$\boxed{R = 274.12N}$

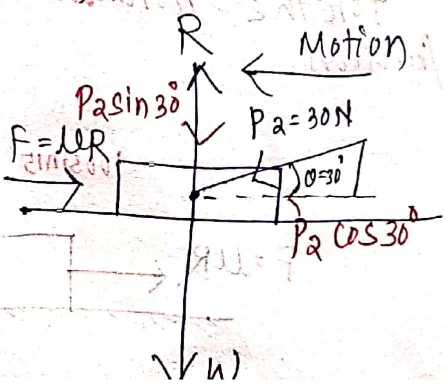
Coefficient of friction $\mu = \frac{F}{R}$

$= \frac{96.59}{274.12}$

$\boxed{\mu = 0.352}$

Q11 A body resting on a rough horizontal plane required a Pull of 24N inclined at 30° to the plane just to move it. It was found that a Push of 30N at 30° to the plane was just enough to cause motion to impend. Make Calculations for the weight of body and the Co-efficient of friction.





Pull force $P_1 = 24$

$\theta = 30^\circ$

Push force $P_2 = 30\,N$

$\theta = 30^\circ$

$\Sigma F_x = 0$, $F = P_1 \cos\theta$ —(1)

$\Sigma F_y = 0$, $R + P_1 \sin\theta = W$ —(2)

$\Rightarrow R = W - P_1 \sin\theta$ —(2)

$\boxed{F = \mu R}$ —(3)

Putting the value of F & R in eq^n (3), we get

$P_1 \cos\theta = \mu(W - P_1 \sin\theta)$

$\Rightarrow P_1 \cos\theta = \mu W - \mu P_1 \sin\theta$

$\Rightarrow P_1 \cos\theta + \mu P_1 \sin\theta = \mu W$

$$\Rightarrow \boxed{P_1 = \frac{\mu W}{\cos\theta + \mu \sin\theta}} \quad \text{—(4)}$$

For Second Fig,

$\Sigma F_x = 0$, $F = P_2 \cos\theta$

$\Sigma F_y = 0$, $R = W + P_2 \sin\theta$

Also $F = \mu R$

$P_2 \cos\theta = \mu(W + P_2 \sin\theta)$

$P_2 \cos\theta - \mu P_2 \sin\theta = \mu W$

$$\boxed{P_2 = \frac{\mu W}{\cos\theta - \mu \sin\theta}} \quad \text{—(5)}$$

From eq^n (4) & (5)

$$\frac{P_1}{P_2} = \frac{\cos\theta - \mu\sin\theta}{\cos\theta + \mu\sin\theta}$$

$$\frac{24}{30} = \frac{\cos 30^\circ - \mu \sin 30^\circ}{\cos 30^\circ + \mu \sin 30^\circ} = \frac{0.866 - 0.5\mu}{0.866 + 0.5\mu}$$

$\boxed{\mu = 0.192}$

From eq^n (4) $P_1 = \dfrac{\mu W}{\cos\theta + \mu\sin\theta}$ $\Rightarrow \boxed{W = 120.25\,N}$

acceleration, whereas the horizontal component ...

20.2. IMPORTANT TERMS

PROJECTILE MOTION (Module 4)

The following terms, which will be frequently used in this chapter, should be clearly understood at this stage :

1. ***Trajectory.*** The path, traced by a projectile in the space, is known as trajectory.
2. ***Velocity of projection.*** The velocity, with which a projectile is projected, is known as the velocity of projection.
3. ***Angle of projection.*** The angle, with the horizontal, at which a projectile is projected, is known as the angle of projection.
4. ***Time of flight.*** The total time taken by a projectile, to reach maximum height and to return back to the ground, is known as the time of flight.
5. ***Range.*** The distance, between the point of projection and the point where the projectile strikes the ground, is known as the *range*. It may be noted that the range of a projectile may be horizontal or inclined.

20.3. MOTION OF A BODY THROWN HORIZONTALLY INTO THE AIR

Consider a body at A thrown horizontally into the *air with a horizontal velocity (v) as shown in Fig. 20.1. A little consideration will show, that this body is subjected to the following two velocities :

1. Horizontal velocity (v), and
2. Vertical velocity due to gravitational acceleration.

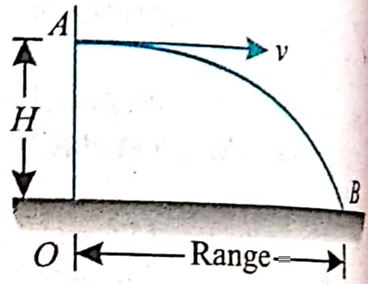


Fig. 20.1.

It is thus obvious, that the body will have some resultant velocity, with which it will travel into the air. We have already discussed in Art 20.1. that the vertical component of this velocity is always subjected to gravitational acceleration, whereas the horizontal component remains constant. Thus the time taken by the body to reach the ground, is calculated from the vertical component of the velocity, whereas the horizontal range is calculated from the horizontal component of the velocity. The velocity, with which the body strikes the ground at B, is the resultant of horizontal and vertical velocities,

Example 20.1. *An aircraft moving horizontally ...*

Example 20.2. *A motor cyclist wants to jump over a ditch as shown in Fig. 20.2.*

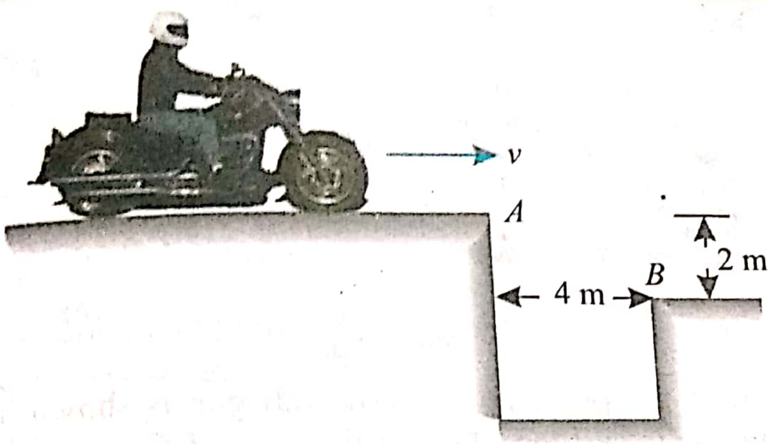


Fig. 20.2.

Find the necessary minimum velocity at A in km. p. hr. of the motor cycle. Also find the inclination and the magnitude of the velocity of the motor cycle just after clearing the ditch.

Solution. Given : Width of ditch $(x) = 4$ m and vertical distance between A and B $(s) = 2$ m.

Minimum velocity of motor cycle at A

Let u = Minimum velocity of motor cycle at A, and

t = Time taken by the motor cycle to clear the ditch.

First of all, let us consider the vertical motion of the motor cycle from A to B due to gravitational acceleration only. In this case, initial velocity of motor cycle $(u) = 0$.

We know that vertical distance between A and B (s),

$$2 = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 9.8\, t^2 = 4.9\, t^2$$

or $$t^2 = \frac{2}{4.9} = 0.41 \qquad \text{or} \qquad t = 0.64 \text{ s}$$

∴ Minimum velocity of the motor cycle at A

$$= \frac{4}{0.64} = 6.25 \text{ m/s} = 22.5 \text{ km.p.h.} \quad \textbf{Ans.}$$

Inclination and magnitude of the velocity of motor cycle just after clearing the ditch (*i.e.* at *B*)

Let θ = Inclination of the velocity with the vertical.

We know that final velocity of the motor cycle in the vertical direction at *B* (*i.e.* after 0.64 second)

$$v = u + gt = 0 + (9.8 \times 0.64) = 6.27 \text{ m/s}$$

$$\therefore \quad \tan\theta = \frac{6.25}{6.27} = 0.9968 \quad \text{or} \quad \theta = 44.9° \quad \textbf{Ans.}$$

and magnitude of the velocity of the motor cycle just after clearing the ditch

$$= \sqrt{(6.25)^2 + (6.27)^2} = 8.85 \text{ m/s} = 31.86 \text{ km.p.h.} \quad \textbf{Ans.}$$

20.4. MOTION OF A PROJECTILE

Consider a particle projected upwards from a point O at an angle α, with the horizontal, with an initial velocity u m/sec as shown in Fig. 20.4.

Now resolving this velocity into its vertical and horizontal components,

$$V = u \sin \alpha \quad \text{and } H = u \cos \alpha$$

We know that the vertical component ($u \sin \alpha$) is subjected to retardation due to gravity. The particle will reach maximum height, when the vertical component becomes zero. After this the particle will come down, due to gravity, and this motion will be subjected to acceleration due to gravity.

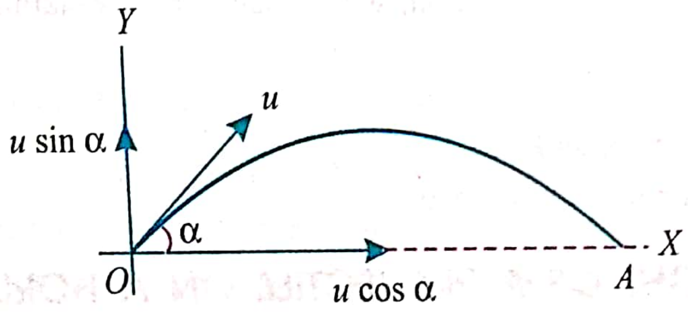


Fig. 20.4. Projectile on a horizontal plane.

The horizontal component ($u \cos \alpha$) will remain constant, since there is no acceleration or retardation (neglecting air resistance). The combined effect of the horizontal and the vertical components will be to move the particle, along some path in the air and then the particle falls on the ground at some point A, other than the point of projection O as shown in Fig. 20.4.

20.5. EQUATION OF THE PATH OF A PROJECTILE

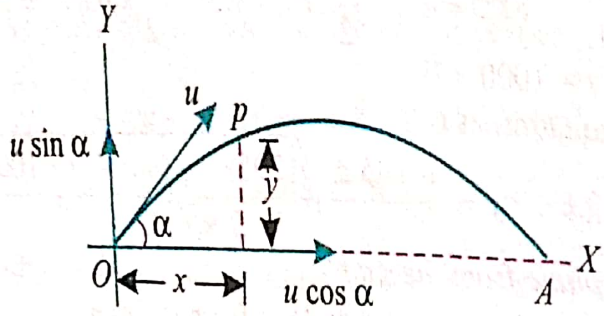


Fig. 20.5. Path of a projectile.

Consider a particle projected from a point O at a certain angle with the horizontal. As already discussed, the particle will move along certain path OPA, in the air, and will fall down at A as shown in Fig. 20.5.

Let u = Velocity of projection, and

α = Angle of projection with the horizontal.

Consider any point P as the position of particle, after t seconds with x and y as co-ordinates as shown in Fig. 20.5. We know that horizontal component of the velocity of projection.

$= u \cos \alpha$

and vertical component $= u \sin \alpha$

$\therefore$
$$y = u \sin \alpha t - \frac{1}{2} g t^2$$

and
$$x = u \cos \alpha \, t \qquad \ldots(i)$$

or
$$t = \frac{x}{u \cos \alpha}$$

Substituting the value of t in equation (i),

$$y = u \sin \alpha \left(\frac{x}{u \cos \alpha} \right) - \frac{1}{2} g \left(\frac{x}{u \cos \alpha} \right)^2$$

$$= x \tan \alpha - \frac{g x^2}{2 u^2 \cos^2 \alpha} \qquad \ldots(ii)$$

Since this is the equation of a parabola, therefore path of a projectile (or the equation of trajectroy) is also a parabola.

Note. It is an important equation, which helps us in obtaining the following relations of a projectile :

1. Time of flight,
2. Horizontal range, and
3. Maximum height of a projectile.

20.6. TIME OF FLIGHT OF A PROJECTILE ON A HORIZONTAL PLANE

It is the time, for which the projectile has remained in the air. We have already discussed in Art. 20.5 that the co-ordinates of a projectile after time t.

$$y = u \sin \alpha t - \frac{1}{2} g t^2$$

We know that when the particle is at A, y is zero. Substituting this value of y in the above equation,

$$0 = u \sin \alpha t - \frac{1}{2} g t^2$$

$$u \sin \alpha t = \frac{1}{2} g t^2$$

or

$$u \sin \alpha = \frac{1}{2} g t \qquad \text{...(Dividing both sides by } t)$$

$$\therefore \quad t = \frac{2u \sin \alpha}{g}$$

Example 20.4. *A projectile is fired upwards at an angle of 30° with a velocity of 40 m/s. Calculate the time taken by the projectile to reach the ground, after the instant of firing.*

Solution. Given : Angle of projection with the horizontal (α) = 30° and velocity of projection (u) = 40 m/s.

We know that time taken by the projectile to reach the ground after the instant of firing,

$$t = \frac{2u \sin \alpha}{g} = \frac{2 \times 40 \sin 30°}{g} = \frac{80 \times 0.5}{9.8} = 4.08 \text{ s} \quad \textbf{Ans.}$$

20.7. HORIZONTAL RANGE OF A PROJECTILE

We have already discussed, that the horizontal distance between the point of projection and the point, where the projectile returns back to the earth is called horizontal range of a projectile. We have also discussed in Arts. 20.4 and 20.6 that the horizontal velocity of a projectile

$$= u \cos \alpha$$

and time of flight,

$$t = \frac{2u \sin \alpha}{g}$$

$\therefore$ Horizontal range

$$= \text{Horizontal velocity} \times \text{Time of flight}$$

$$= u \cos \alpha \times \frac{2u \sin \alpha}{g} = \frac{2u^2 \sin \alpha \cos \alpha}{g}$$

$$R = \frac{u^2 \sin 2\alpha}{g} \qquad \text{...}(\because \sin 2\alpha = 2 \sin \alpha \cos \alpha)$$

Note. For a given velocity of projectile, the range will be maximum when $\sin 2\alpha = 1$. Therefore

$$2\alpha = 90° \quad \text{or} \quad \alpha = 45°$$

or

$$R_{max} = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g} \qquad \text{...}(\because \sin 90° = 1)$$

Example 20.5. *A ball is projected upwards with a velocity of 15 m/s at an angle of 25° with the horizontal. What is the horizontal range of the ball ?*

Solution. Given : Velocity of projection (u) = 15 m/s and angle of projection with the horizontal (α) = 25°

We know that horizontal range of the ball,

$$R = \frac{u^2 \sin 2\alpha}{g} = \frac{(15)^2 \times \sin (2 \times 25°)}{g} = \frac{225 \sin 50°}{g}$$

$$= \frac{225 \times 0.766}{9.8} = 17.6 \text{ m} \quad \textbf{Ans.}$$

20.8. MAXIMUM HEIGHT OF A PROJECTILE ON A HORIZONTAL PLANE

We have already discussed that the vertical component of the initial velocity of a projectile

$$= u \sin \alpha \quad \ldots(i)$$

and vertical component of final velocity

$$= 0 \quad \ldots(ii)$$

$\therefore$ Average velocity of (*i*) and (*ii*),

$$= \frac{u \sin \alpha + 0}{2} = \frac{u \sin \alpha}{2} \quad \ldots(iii)$$

Let H be the maximum height reached by the particle and t be the time taken by the particle to reach maximum height *i.e.*, to attain zero velocity from ($u \sin \alpha$). We have also discussed that time taken by the projectile to reach the maximum height,

$$= \frac{u \sin \alpha}{g}$$

$\therefore$ Maximum height of the projectile,

$$H = \text{Average vertical velocity} \times \text{Time}$$

$$= \frac{u \sin \alpha}{2} \times \frac{u \sin \alpha}{g} = \frac{u^2 \sin^2 \alpha}{2g}$$

Example 20.6. *A bullet is fired with a velocity of 100 m/s at an angle of 45° with the horizontal. How high the bullet will rise ?*

Solution. Given : Velocity of projection (u) = 100 m/s and angle of projection with the horizontal (α) = 45°

We know that maximum height to which the bullet will rise,

$$H = \frac{u^2 \sin^2 \alpha}{2g} = \frac{(100)^2 \times \sin^2 45°}{2 \times 9.8} = \frac{10000 \times (0.707)^2}{19.6} \text{ m}$$

$$= 255.1 \text{ m} \quad \textbf{Ans.}$$

Example 20.7. *If a particle is projected inside a horizontal tunnel which is 5 metres high with a velocity of 60 m/s, find the angle of projection and the greatest possible range.*

Solution. Given : Height of the tunnel (H) = 5 m and velocity of projection (u) = 60 m/s.

Angle of projection

Let $\qquad \alpha$ = Angle of projection.

We know that height of tunnel (H)

$$5 = \frac{u^2 \sin^2 \alpha}{2g} = \frac{(60)^2 \sin^2 \alpha}{2 \times 9.8} = 183.7 \sin^2 \alpha$$

or
$$\sin^2 \alpha = \frac{5}{183.7} = 0.0272$$

$\therefore$
$$\sin \alpha = 0.1650 \quad \text{or} \quad \alpha = 9.5° \quad \textbf{Ans.}$$

Greatest possible range

We know that greatest possible range,

$$R = \frac{u^2 \sin 2\alpha}{g} = \frac{(60)^2 \sin (2 \times 9.5°)}{9.8} = \frac{(60)^2 \sin 19°}{9.8} \text{ m}$$

$$= \frac{3600 \times 0.3256}{9.8} = 119.6 \text{ m} \quad \textbf{Ans.}$$

Example 20.8. *A body is projected at such an angle that the horizontal range is three times the greatest height. Find the angle of projection.*

Solution. Given : Horizontal range $(R) = 3\,H$ (where H is the greatest height). ...(*i*)

Let α = Angle of projection.

We know that horizontal range,

$$R = \frac{u^2 \sin 2\alpha}{g}$$

and the greatest height

$$H = \frac{u^2 \sin^2 \alpha}{2g}$$

Substituting these values of R and H in the given equation (*i*),

$$\frac{u^2 \sin 2\alpha}{g} = 3 \times \frac{u^2 \sin^2 \alpha}{2g}$$

$$\frac{u^2 \times 2 \sin\alpha \cos\alpha}{g} = 3 \times \frac{u^2 \sin^2 \alpha}{2g} \qquad ...(\because\ 2\alpha = 2 \sin\alpha \cos\alpha)$$

or $$2 \cos\alpha = 1.5 \sin\alpha$$

$\therefore$ $$\tan\alpha = \frac{2}{1.5} = 1.333 \quad \text{or} \quad \alpha = 53.1° \quad \textbf{Ans.}$$

Example 20.9. *A particle is thrown with a velocity of 5 m/s at an elevation of 60° to the horizontal. Find the velocity of another particle thrown at an elevation of 45° which will have (a) equal horizontal range, (b) equal maximum height, and (c) equal time of flight.*

Solution. Given : Velocity of projection of first particle $(u_1) = 5$ m/s ; Angle of projection of first particle with the horizontal $(\alpha_1) = 60°$ and angle of projection of second particle with the horizontal $(\alpha_2) = 45°$

Let u_2 = Velocity of projection of the second particle.

(a) Velocity of the second particle for equal horizontal range

We know that horizontal range of a projectile,

$$R = \frac{u^2 \sin 2\alpha}{g}$$

$\therefore$ For equal horizontal range

$$\frac{u_1^2 \sin 2\alpha_1}{g} = \frac{u_2^2 \sin 2\alpha_2}{g}$$

$$(5)^2 \sin (2 \times 60°) = u_2^2 \sin (2 \times 45°)$$

or $$u_2^2 = 25 \times \frac{\sin 120°}{\sin 90°} = 25 \times \frac{0.866}{1.0} = 21.65$$

$\therefore$ $$u_2 = 4.65 \text{ m/s} \quad \textbf{Ans.}$$

(b) Velocity of the second particle for equal maximum height

We know that maximum height of a projectile,

$$H = \frac{u^2 \sin^2 \alpha}{2g}$$

$\therefore$ For equal maximum height

$$\frac{u_1^2 \sin^2 \alpha_1}{2g} = \frac{u_2^2 \sin^2 \alpha_2}{2g}$$

$$(5)^2 \sin^2 60° = u_2^2 \sin^2 45°$$

or $$u_2^2 = 25 \times \frac{\sin^2 60°}{\sin^2 45°} = 25 \times \frac{(0.866)^2}{(0.707)^2} = 37.5$$

$\therefore$ $$u_2 = 6.12 \text{ m/s} \quad \textbf{Ans.}$$

(c) Velocity of the second particle for equal time of flight

We know that time of flight of a projectile

$$t = \frac{2u \sin \alpha}{g}$$

$\therefore$ For equal time of flight

$$\frac{2u_1 \sin \alpha_1}{g} = \frac{2u_2 \sin \alpha_2}{g}$$

$$2 \times 5 \sin 60° = 2u_2 \sin 45°$$

or $$u_2 = 5 \times \frac{\sin 60°}{\sin 45°} = 5 \times \frac{0.866}{0.707} = 6.12 \text{ m/s} \quad \textbf{Ans.}$$