Project Report: SPY Volatility Forecasting using GARCH

By: Abhishek Patil

1. Project Background:

The project aims to forecast the volatility of the SPY (SPDR S&P 500 ETF Trust) using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. Volatility forecasting plays a crucial role in risk management and investment decision-making. By accurately predicting volatility, investors can assess the level of risk associated with holding the asset and make informed trading decisions.

2. Theory of GARCH Model:

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is a statistical method commonly used in finance for forecasting volatility, which measures the degree of variation or dispersion in financial asset prices. Here's a simplified explanation of how the GARCH model works:

A. Understanding Volatility:

In finance, volatility refers to the degree of variation in the price of a financial asset over time. High volatility indicates large price swings, while low volatility suggests relatively stable prices.

B. Autoregressive Nature:

The GARCH model builds on the autoregressive concept, which means it considers the relationship between a variable and its own past values. In the case of volatility, it examines how past volatility levels affect future volatility.

C. Conditional Heteroskedasticity:

The term "heteroskedasticity" refers to the phenomenon where the variance of a variable changes over time. In financial markets, volatility tends to exhibit this behavior, with periods of high and low volatility. "Conditional" heteroskedasticity means that the variance is dependent on past values of the series.

D. Model Components:

The GARCH model consists of two main components:

- Autoregressive Component (ARCH): This part captures the past squared residuals (errors) of the asset returns, indicating volatility clustering, where periods of high volatility tend to be followed by more high volatility.
- Moving Average Component (GARCH): This component accounts for the lagged conditional variances, representing the persistence of volatility shocks over time.

E. Parameter Estimation:

The GARCH model estimates parameters that describe the behavior of volatility over time. These parameters include the ARCH and GARCH terms, which determine the degree of persistence and volatility clustering in the data.

F. Forecasting:

Once the model parameters are estimated, the GARCH model can be used to forecast future volatility based on past information. By analyzing historical volatility patterns, the model generates predictions about the future variability of asset prices.

3. Steps Involved in the Project:

- Installation and Data Acquisition: Installed the 'arch' package in Python and downloaded SPY data from Yahoo Finance spanning from 2010 to 2024.
- Data Preparation: Calculated daily returns using the daily close price of SPY and plotted the daily returns over the entire time period.
- Model Selection: Conducted Partial Auto Correlation Function (PACF) analysis to determine the lag structure. Selected a GARCH(2,2) model based on the significant decay observed in the PACF plot.
- Model Fitting: Fitted the GARCH(2,2) model to the data and extracted the model summary to understand the estimated parameters.
- Rolling Prediction: Performed a rolling prediction for the last year to forecast the volatility for the next day. Plotted the rolling predictions against the daily returns of SPY.
- Alternative Model: Fitted an ARCH(2,0) model as an alternative to GARCH(2,2) and compared the rolling volatility predictions against the daily returns.
- Future Volatility Prediction: Forecasted the volatility for the next 7 days using the fitted GARCH model.

4. Performance Metrics:

GARCH Model Performance:

The GARCH(2,2) model exhibited a good fit to the data, capturing volatility patterns effectively. The model summary provided insights into the estimated parameters and their significance.

Rolling Volatility Prediction:

The rolling volatility predictions closely tracked the daily returns of SPY over the last year, indicating the model's ability to capture market volatility.

ARCH Model Comparison:

The ARCH(2,0) model also performed well, demonstrating its efficacy in capturing volatility without considering the random factor of lagged returns.

5. Future Applications:

- Volatility Prediction Timeframes: The GARCH model can be utilized to forecast volatility for different timeframes, such as daily, weekly, or monthly, catering to varying investment horizons.
- Stock Volatility Forecasting: Beyond SPY, the GARCH model can be applied to predict the volatility of different stocks, enabling investors to assess risk across various assets.
- Algorithmic Trading Strategies: The predicted volatility can be integrated into algorithmic trading strategies to optimize portfolio management and risk mitigation techniques.

6. Conclusion:

The project successfully demonstrated the application of the GARCH model for forecasting SPY volatility. By leveraging historical data and advanced statistical techniques, the model provided valuable insights into market risk dynamics. Moving forward, the insights gained from this project can be applied to enhance investment strategies and risk management practices in financial markets.

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```
!pip install arch
```

```
Requirement already satisfied: arch in /usr/local/lib/python3.10/dist-packages (6.3.0)
Requirement already satisfied: numpy>=1.19 in /usr/local/lib/python3.10/dist-packages (from arch) (1
Requirement already satisfied: scipy>=1.5 in /usr/local/lib/python3.10/dist-packages (from arch) (1.
Requirement already satisfied: pandas>=1.1 in /usr/local/lib/python3.10/dist-packages (from arch) (1
Requirement already satisfied: statsmodels>=0.12 in /usr/local/lib/python3.10/dist-packages (from ar
Requirement already satisfied: python-dateutil>=2.8.1 in /usr/local/lib/python3.10/dist-packages (fr
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.10/dist-packages (from pandas>
Requirement already satisfied: patsy>=0.5.4 in /usr/local/lib/python3.10/dist-packages (from statsmo
Requirement already satisfied: packaging>=21.3 in /usr/local/lib/python3.10/dist-packages (from stat
Requirement already satisfied: six in /usr/local/lib/python3.10/dist-packages (from patsy>=0.5.4->st
```

Load all the necessary Libraries:

```
import yfinance as yf
from datetime import datetime, timedelta
import pandas as pd
import matplotlib.pyplot as plt
from arch import arch_model
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
import numpy as np
```

SPY Volatility:

```
# Download AAPL data from 2010-01-01 to 2024-01-01
spy_data = yf.download('SPY', start='2010-01-01', end='2024-01-01')

# Print the last few rows of the data
spy_data.tail()
```

```
[*********************100%%**********************]  1 of 1 completed
```

Date	Open	High	Low	Close	Adj Close	Volume
2023-12-22	473.859985	475.380005	471.700012	473.649994	473.649994	67126600
2023-12-26	474.070007	476.579987	473.989990	475.649994	475.649994	55387000
2023-12-27	475.440002	476.660004	474.890015	476.510010	476.510010	68000300
2023-12-28	476.880005	477.549988	476.260010	476.690002	476.690002	77158100
2023-12-29	476.489990	477.029999	473.299988	475.309998	475.309998	122234100

Calculating the Daily returns:

```
#spy_data["Daily Returns"] = (spy_data["Close"].pct_change().dropna()) * 100
```

```
returns = 100 * spy_data.Close.pct_change().dropna()
```

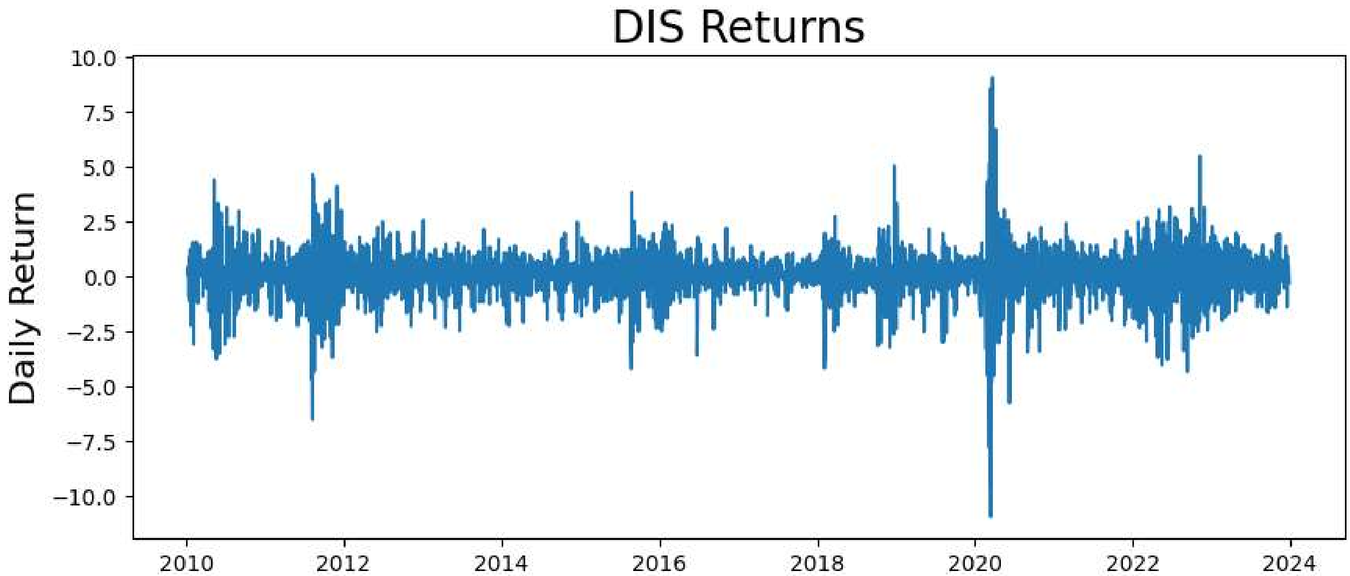
```
spy_data.head()
```

Date	Open	High	Low	Close	Adj Close	Volume
2010-01-04	112.370003	113.389999	111.510002	113.330002	87.129936	118944600
2010-01-05	113.260002	113.680000	112.849998	113.629997	87.360580	111579900
2010-01-06	113.519997	113.989998	113.430000	113.709999	87.422089	116074400
2010-01-07	113.500000	114.330002	113.180000	114.190002	87.791115	131091100
2010-01-08	113.889999	114.620003	113.660004	114.570000	88.083275	126402800

Next steps: Generate code with `spy_data` View recommended plots

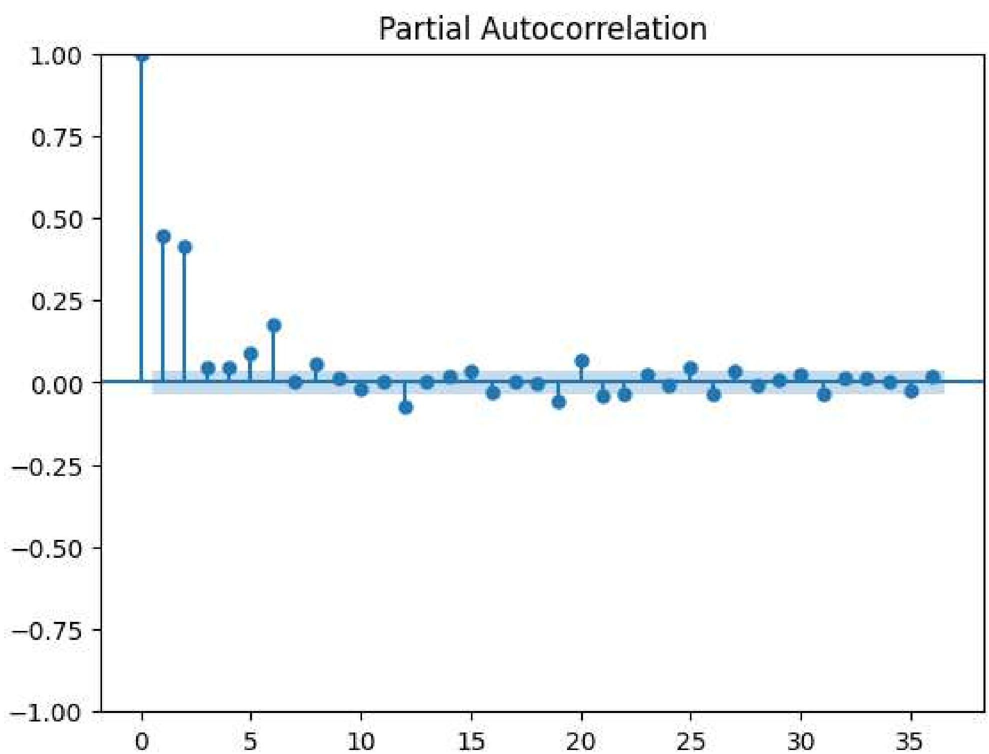
```
# Now lets plot the returns
plt.figure(figsize=(10,4))
plt.plot(returns)
plt.ylabel('Daily Return', fontsize=16)
plt.title('DIS Returns', fontsize=20)
```

```
Text(0.5, 1.0, 'DIS Returns')
```



PACF:

```
plot_pacf(returns**2)
plt.show()
```



Theory of Garch Model:

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4. Model Components: The GARCH model consists of two main components:

A) Autoregressive Component (ARCH): This part captures the past squared residuals (errors) of the asset returns, indicating volatility clustering, where periods of high volatility tend to be followed by more high volatility.

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5. Parameter Estimation: The GARCH model estimates parameters that describe the behavior of volatility over time. These parameters include the ARCH and GARCH terms, which determine the degree of persistence and volatility clustering in the data.

6. Forecasting: Once the model parameters are estimated, the GARCH model can be used to forecast future volatility based on past information. By analyzing historical volatility patterns, the model generates predictions about the future variability of asset prices.

Example:

Suppose we want to forecast the volatility of a stock based on its historical daily returns. We collect data on the stock's returns over the past year and use a GARCH model to analyze the data. The model estimates parameters such as the ARCH and GARCH terms, which describe the past volatility patterns and their persistence. With these parameters, we can then predict the future volatility of the stock, helping investors assess the level of risk associated with holding the asset.

Fit Garch(2,2):

```
model = arch_model(returns, p=2, q=2)
```

```
model_fit = model.fit()
```

```
Iteration:      1,   Func. Count:      8,   Neg. LLF: 33905.99923247029
Iteration:      2,   Func. Count:     20,   Neg. LLF: 16298.83024021532
Iteration:      3,   Func. Count:     32,   Neg. LLF: 6837.065143677712
Iteration:      4,   Func. Count:     41,   Neg. LLF: 6466.456770266234
Iteration:      5,   Func. Count:     50,   Neg. LLF: 4733.958561512231
Iteration:      6,   Func. Count:     58,   Neg. LLF: 4619.797585026682
Iteration:      7,   Func. Count:     67,   Neg. LLF: 4535.103194796102
Iteration:      8,   Func. Count:     75,   Neg. LLF: 4541.062415575578
Iteration:      9,   Func. Count:     83,   Neg. LLF: 4533.6826278861945
Iteration:     10,   Func. Count:     91,   Neg. LLF: 4533.405589891328
Iteration:     11,   Func. Count:     98,   Neg. LLF: 4533.398341300197
Iteration:     12,   Func. Count:    105,   Neg. LLF: 4533.393414410276
Iteration:     13,   Func. Count:    112,   Neg. LLF: 4533.393154417841
Iteration:     14,   Func. Count:    119,   Neg. LLF: 4533.393146706734
Iteration:     15,   Func. Count:    125,   Neg. LLF: 4533.393146706026
Optimization terminated successfully    (Exit mode 0)
            Current function value: 4533.393146706734
            Iterations: 15
            Function evaluations: 125
            Gradient evaluations: 15
```

```
model_fit.summary()
```

Constant Mean - GARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	GARCH	**Log-Likelihood:**	-4533.39
Distribution:	Normal	**AIC:**	9078.79
Method:	Maximum Likelihood	**BIC:**	9115.79
		No. Observations:	3521
Date:	Fri, Mar 01 2024	**Df Residuals:**	3520
Time:	00:23:21	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0801	1.262e-02	6.348	2.187e-10	[5.537e-02, 0.105]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0622	1.541e-02	4.034	5.483e-05	[3.196e-02,9.235e-02]
alpha[1]	0.1350	3.193e-02	4.229	2.345e-05	[7.245e-02, 0.198]
alpha[2]	0.1580	4.291e-02	3.683	2.309e-04	[7.391e-02, 0.242]
beta[1]	0.1596	0.325	0.491	0.623	[-0.477, 0.796]
beta[2]	0.4965	0.271	1.830	6.721e-02	[-3.518e-02, 1.028]

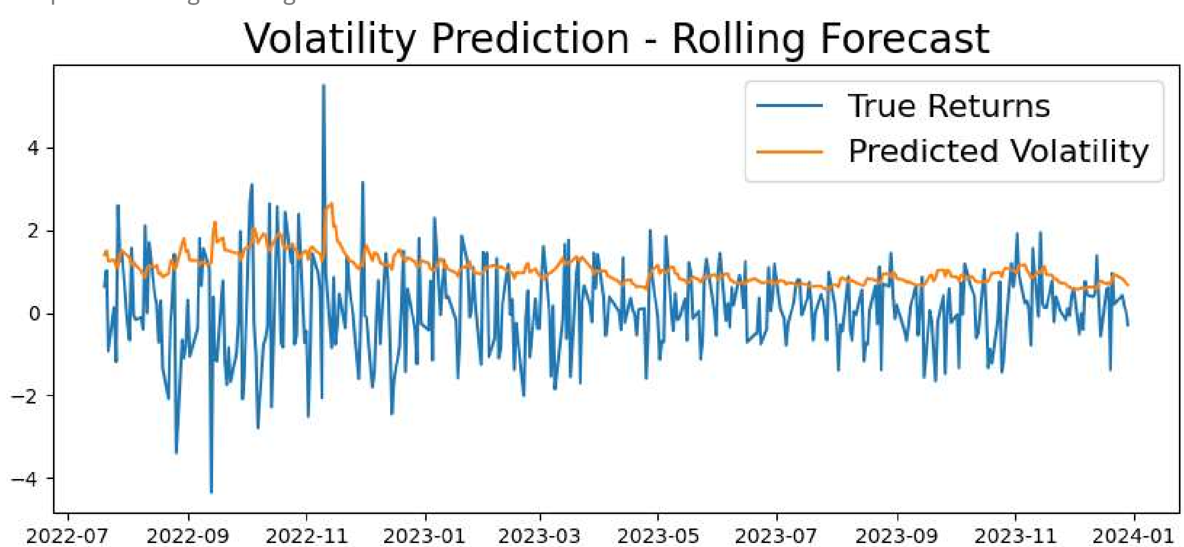
Covariance estimator: robust

```
rolling_predictions = []
test_size = 365

for i in range(test_size):
    train = returns[:-(test_size-i)]
    model = arch_model(train, p=2, q=2)
    model_fit = model.fit(disp='off')
    pred = model_fit.forecast(horizon=1)
    rolling_predictions.append(np.sqrt(pred.variance.values[-1,:][0]))


rolling_predictions = pd.Series(rolling_predictions, index=returns.index[-365:])


print(rolling_predictions)
```

```
Date
2022-07-20    1.398314
2022-07-21    1.501521
2022-07-22    1.246247
2022-07-25    1.281126
2022-07-26    1.116803
                ...
2023-12-22    0.924220
2023-12-26    0.827553
2023-12-27    0.782722
2023-12-28    0.720820
2023-12-29    0.671540
Length: 365, dtype: float64
```

```
plt.figure(figsize=(10,4))
true, = plt.plot(returns[-365:])
preds, = plt.plot(rolling_predictions)
plt.title('Volatility Prediction - Rolling Forecast', fontsize=20)
plt.legend(['True Returns', 'Predicted Volatility'], fontsize=16)
```

```
<matplotlib.legend.Legend at 0x7d3285a0cdc0>
```



Try GARCH(2,0) = ARCH(2)

```
model = arch_model(returns, p=2, q=0)
```

```
model_fit = model.fit()
```

```
Iteration:      1,   Func. Count:      6,   Neg. LLF: 23419.361887263665
Iteration:      2,   Func. Count:     15,   Neg. LLF: 8146.2967403175135
Iteration:      3,   Func. Count:     23,   Neg. LLF: 4622735.770407058
Iteration:      4,   Func. Count:     29,   Neg. LLF: 13696.780549815805
Iteration:      5,   Func. Count:     35,   Neg. LLF: 25454.42639698014
Iteration:      6,   Func. Count:     41,   Neg. LLF: 4826.236170371174
Iteration:      7,   Func. Count:     47,   Neg. LLF: 4757.9353240712635
Iteration:      8,   Func. Count:     52,   Neg. LLF: 4754.289888458895
Iteration:      9,   Func. Count:     57,   Neg. LLF: 4753.699144061156
Iteration:     10,   Func. Count:     62,   Neg. LLF: 4753.584856769439
Iteration:     11,   Func. Count:     67,   Neg. LLF: 4753.572746686398
Iteration:     12,   Func. Count:     72,   Neg. LLF: 4753.570073279083
Iteration:     13,   Func. Count:     77,   Neg. LLF: 4753.570071401244
Iteration:     14,   Func. Count:     81,   Neg. LLF: 4753.570071401204
Optimization terminated successfully    (Exit mode 0)
            Current function value: 4753.570071401244
            Iterations: 14
            Function evaluations: 81
            Gradient evaluations: 14
```

```
model_fit.summary()
```

Constant Mean - ARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	ARCH	**Log-Likelihood:**	-4753.57
Distribution:	Normal	**AIC:**	9515.14
Method:	Maximum Likelihood	**BIC:**	9539.81
		No. Observations:	3521
Date:	Fri, Mar 01 2024	**Df Residuals:**	3520
Time:	00:25:07	**Df Model:**	1

Mean Model

	coef	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
mu	0.0860	1.463e-02	5.882	4.055e-09	[5.737e-02, 0.115]

Volatility Model

	coef	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
omega	0.4543	3.134e-02	14.495	1.300e-47	[0.393, 0.516]
alpha[1]	0.2739	4.373e-02	6.264	3.761e-10	[0.188, 0.360]
alpha[2]	0.3640	4.060e-02	8.965	3.114e-19	[0.284, 0.444]

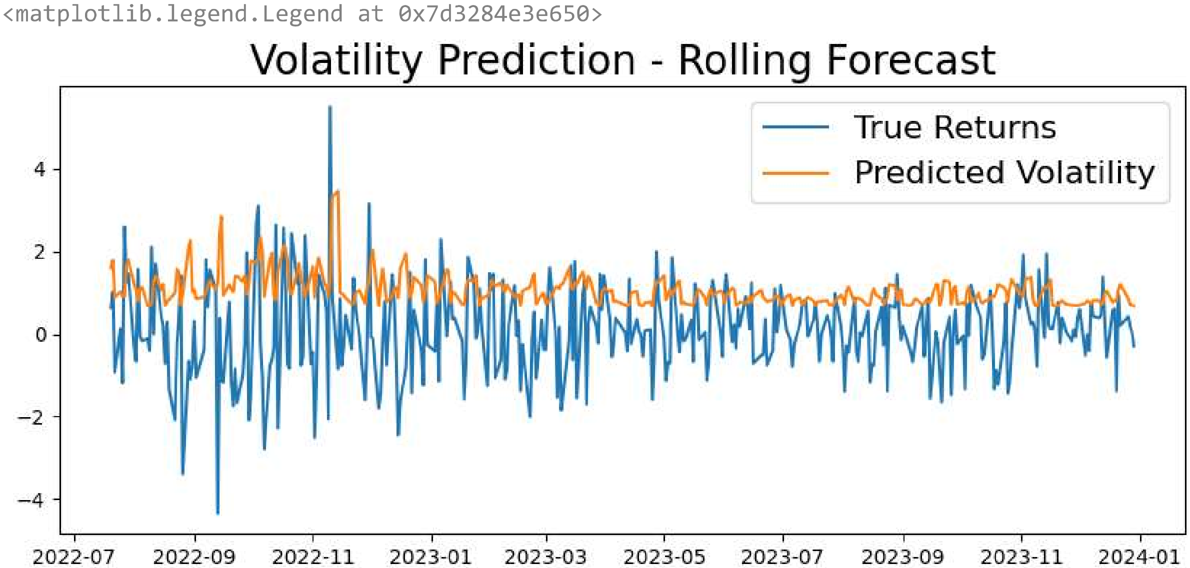
Covariance estimator: robust

```
rolling_predictions = []
test_size = 365

for i in range(test_size):
    train = returns[:-(test_size-i)]
    model = arch_model(train, p=2, q=0)
    model_fit = model.fit(disp='off')
    pred = model_fit.forecast(horizon=1)
    rolling_predictions.append(np.sqrt(pred.variance.values[-1,:][0]))


rolling_predictions = pd.Series(rolling_predictions, index=returns.index[-365:])


print(rolling_predictions)
```

```
Date
2022-07-20    1.598265
2022-07-21    1.783203
2022-07-22    0.882357
2022-07-25    1.019494
2022-07-26    0.911492
                ...
2023-12-22    1.203179
2023-12-26    0.854233
2023-12-27    0.700807
2023-12-28    0.706100
2023-12-29    0.677148
Length: 365, dtype: float64
```

```
plt.figure(figsize=(10,4))
true, = plt.plot(returns[-365:])
preds, = plt.plot(rolling_predictions)
plt.title('Volatility Prediction - Rolling Forecast', fontsize=20)
plt.legend(['True Returns', 'Predicted Volatility'], fontsize=16)
```

```
<matplotlib.legend.Legend at 0x7d3284e3e650>
```



How to use the model:

```
train = returns
model = arch_model(train, p=2, q=2)
model_fit = model.fit(disp='off')
```

```
pred = model_fit.forecast(horizon=7)
future_dates = [returns.index[-1] + timedelta(days=i) for i in range(1,8)]
pred = pd.Series(np.sqrt(pred.variance.values[-1,:]), index=future_dates)
```

```
print(pred)
```

```
2023-12-30    0.640825
2023-12-31    0.654661
2024-01-01    0.676187
2024-01-02    0.690943
2024-01-03    0.708589
2024-01-04    0.722896
2024-01-05    0.738091
dtype: float64
```

```
plt.figure(figsize=(10,4))
plt.plot(pred)
plt.title('Volatility Prediction - Next 7 Days', fontsize=20)
```

```
Text(0.5, 1.0, 'Volatility Prediction - Next 7 Days')
```