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Unit - 1

Random variables and Discrete probability distribution :

- → Conditional probability
- → Discrete random variables
- → Independent random variables
- → Expectation of discrete random variable.
- → Sums of Independent random variable.
- → Moments
- → Variance of a Sum.
- → Correlation coefficient.
- → Poisson approximation to the binomial distribution.
- → Infinite sequence Bernoulli trials.

a) Trial and event : Let an experiment be repeated under essentially the same conditions and let it result in anyone of the several possible outcomes. Then the experiment is called the trial and the possible outcomes are known as events or cases. For example :

i) Tossing of a coin is a trial and turning of head or tail is an event.
ii) Throwing a dice is a trial and getting 1 to 6 is an event.

b) Exhaustive event : The total number of all possible outcomes in any trial is known as Exhaustive event. For example :

i) In tossing a coin there are two exhaustive event.
ii) In throwing a dice there are six exhaustive event.
iii) In throwing two dice there are thirty six exhaustive event.

c) Mutually exclusive events : Events are said to be mutually exclusive if the happening of anyone of them rules out the happening of all others called mutually exclusive i.e; event can happen symultaneously in the same trial. For example:

i) In tossing a coin the event head and tail are mutually exclusive since if the outcome is head the possibility of getting tail gets ruled out.

ii) In throwing a dice all the six faces number 1, 2, 3, 4, 5, 6 are mutually exclusive. Since any outcomes rules out the possibilties of getting anyother.

d) Equally likely events: Events are said to be equally likely if there is no reson to expect anyone in preference to any other.

e) Independent and dependent event: Two or more events are said to be independent if the happening or non-happening of anyone doesn't depend (or is not affected) by the happening or non-happening of anyother. Otherwise they are said to be dependent.

Probability

If a trial reserves in an exhaustive, mutually exclusive and equally likely cases, 'm' of them are favourable to happening of event E then the probability of happening of E is given by p or

$$P(E) = \frac{\text{Favourable no. of cases}}{\text{exhaustive no. of case (total outcomes)}} = \frac{m}{n}$$

i) Properties of probability:
a) $0 \le p \le 1$
b) if $P(E) = 0$ (impossible event)
c) if $P(E) = 1$ (chance of happening event is 100%)
d) $p + q = 1$ $(q = p - 1)$
e) $0 \le q \le 1$

i) Number of ways happening:
Suppose total number of bulbs are 10 and out of 10, we are choosing 2 bulbs.

Sln. No. of bulbs choosen out of 10 → $^{10}C_2$

$$^{n}C_r = \frac{n!}{(n-r)!\,r!}$$

$$^{10}C_2 = \frac{10!}{2!\,8!}$$

$$\therefore \boxed{^{10}C_2 = 45}$$

Example: If 4 kings cards are there and have to choose 3 out of 4 number of ways.

Soln: No. of cards choosen out of 4 → $^{4}C_3$

$$^{4}C_3 = \frac{4!}{1!\,3!}$$

$$^{4}C_3 = 4!$$

Note 1: If 2 events are simultaneously happening $P(A) \times P(B)$
Note 2: Either A is happening or B happening $P(A) + P(B)$

iii) Conditional probability: Probability of an event B under condition that an event A occurs, this probability is called conditional probability of B, given A and it is denoted by $P(B/A)$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \; ; \; P(A) \neq 0$$

iv) Multiplication rule: If A and B are event in sample space 'S' and $P(A) \neq 0$, $P(B) \neq 0$

Ex:-1 We tossed two dice, 1st dice is 4, then giving this information what is the probability that sum of two dice equals six.

Soln Event A → 1st dice - 4

$m = \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}$; n = total (36)

$\therefore P(A) = m/n$

$\therefore P(A) = 1/6$

$B = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

$\therefore A \cap B = \{4,2\}$

$P(A \cap B) = 1$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B/A) = \frac{6}{36} = \frac{1}{6}$$

Ex-2 Suppose card number 1 to 10 are placed in a hat mixed up and then one of the card is drawn. If we are told that the number of the drawn card is atleast 5.

Soln At least 5 → 5, 6, 7, 8, 9, 10

$P(A) = m/n$

$P(A) = 3/5$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{1/10}{3/5}$$

$$\therefore P(B/A) = \frac{1}{6}$$

Ex-3 If a family has 2 children, what is the conditional probability that both are boys. Given that at least 1 of them is a boy. Assume that the sample space S is given by $\{(b,b),(b,g),(g,b),(g,g)\}$.

Soln $\{(b,b),(b,g),(g,b),(g,g)\}$ - According to the given condition our outcomes are equally likely.

At least 1 boy - $(b,b)\ (b,g)\ (g,b)$

$$P(A) = \frac{3}{4}$$

$$P(A \cap B) = \frac{1}{4}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B/A) = \frac{1}{3}$$

Ex-4 Ten cards numbered 1 to 10 are placed in a box mixed up throughly and then 1 card is drawn randomly. If it is known that the number on the drawn card is more than 3. What is the probability that it is an even number.

Soln $A = 4, 5, 6, 7, 8, 9, 10$

$$P(A) = \frac{7}{10}$$

$$P(A \cap B) = \frac{4}{10}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B/A) = \frac{4}{7}$$

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Ex-5 A dice is thrown twice and the sum of the number appearing is observed to be 6. What is the probability that the number has appeared at least once.

Soln $A = 36$

At least 1 → {(1,5), (2,4), (3,3), (4,2), (5,1)}

$A = 5$

$\therefore P(A) = 5/36$

$\therefore P(A \cap B) = 2/36$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \Rightarrow \frac{2}{5}$$

Ex-6 A dice is thrown 3 times, event A and B are defined as below where event A is already given A→6 on the 1st and 5 on the second, B→4 on the 3rd dice. Find the probability of B given that A has already occured.

Soln $A = \{(6,5,1), (6,5,2), (6,5,3), (6,5,4), (6,5,5), (6,5,6)\}$

Total outcomes $= 6^3 = 216$

$$P(A) = \frac{6}{216}$$

$$P(A \cap B) = \frac{1}{216}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B/A) = \frac{1}{6}$$

Ex-7 A coin is flipped twice assuming that all four coins in the sample space are equally likely what is the conditional probability that both flipped land on head. Given that

i) the first flip land on head, ii) at least on flip land on head.

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Soln i) $A = \{(H,H), (H,T)\}$

$P(A) = \frac{2}{4}$

$P(A \cap B) = \frac{1}{4}$

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$P(B/A) = \frac{1}{2}$

ii) $A = \{(H,H), (H,T), (T,H)\}$

$P(A) = 3/4$

$P(A \cap B) = 1/4$

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$P(B/A) = \frac{1}{3}$

Ex-8 Two dice are thrown, given that the number shown on initial dice is 3. What is the conditional probability that the sum of number on two dice equal 8.

Soln $A = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$

$P(A) = \frac{6}{36}$

$P(A \cap B) = \frac{1}{36}$

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$P(B/A) = \frac{1}{6}$

Ex-9 At DPS school, the probability that a student takes technology and spanish is 0.087. The probability that a student takes technology is 0.68. What is the probability that a student take spanish given that a student is taking technology.

Soln Given- $P(T \cap S) = 0.087$; $P(T) = 0.68$; $P(S/T) = ?$

$$P(S/T) = \frac{P(T \cap S)}{P(T)} = \frac{0.087}{0.68}$$

$P(S/T) = 0.128$

Ex-10 The probability of raining on Sunday is 0.07. If today is sunday then find the probability of rain today.

Soln Given: $P(A \cap B) = 0.07$

Let sunday be S & Rain be R

$P(S \cap R) = 0.07$

$P(S) = \frac{1}{7}$ - - - [∵ Seven days in week]

$P(R/S) = \frac{P(S \cap R)}{P(S)}$

$P(R/S) = \frac{0.07}{1/7}$

$P(R/S) = 0.49$

* Basic questions.

Q-1 A committee of 5 is to be selected from a group of 6 men and 9 women. If the selection is made randomly what is the probability that the committee consists of 3 men and 2 women.

Soln [6 men, 9 women]

i) Combination to get 3 men out of 6 $\rightarrow {}^6C_3 = \frac{6!}{3! \times 3!} \Rightarrow 20$

ii) Combination to get 2 women out of 9 $\rightarrow {}^9C_2 = \frac{9!}{7! \times 2!} \Rightarrow 36$

iii) Total possible outcomes $\rightarrow {}^{15}C_5 = \frac{15!}{5! \times 10!}$

$= 3 \times 7 \times 13 \times 11$

$= 3003$

iv) Required probability $\rightarrow \frac{{}^6C_3 \times {}^9C_2}{{}^{15}C_5} \Rightarrow 0.2397$

Q-2 A box contain 2 white balls and 2 blacks balls and second box contain 3 white balls and 4 black balls. If one ball is drawn at random from each box, what is the probability of that they are of same colour.

Soln Case I $\rightarrow$ Balls drawn from 1st box.

i) Both balls drawn are white $\rightarrow \frac{{}^2C_1}{{}^4C_1} = \frac{1}{2}$

ii) Both balls drawn are black $\rightarrow \frac{{}^2C_1}{{}^4C_1} = \frac{1}{2}$

Case II $\rightarrow$ Balls drawn from 2nd box.

i) Both balls drawn are white $\rightarrow \frac{{}^3C_1}{{}^7C_1} = \frac{3}{7}$

ii) Both balls drawn are black $\rightarrow \frac{{}^4C_1}{{}^7C_1} = \frac{4}{7}$

Probability of 1st time ball drawn $= \frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$

Probability of 2nd time ball drawn $= \frac{1}{2} \times \frac{4}{7} = \frac{4}{14}$

Required probability - Either white balls are drawn or black balls are drawn.

$$= \frac{3}{14} + \frac{4}{14}$$

$$= \frac{\cancel{7}^{1}}{\cancel{14}_{2}} \Rightarrow \frac{1}{2}$$

Ex-11 There are 10 students of which 3 are graduate if a committee of 5 students is formed. What is the probability that they are

i) Only two graduate ; ii) at least 2 graduate ; iii) atmost one.

Sn. 10 students $\rightarrow$ 3G
$\rightarrow$ 7NG

Total outcomes $\rightarrow$ $^{10}C_5 = \frac{10!}{5!\,5!}$

$= 252$

i) Only 2 graduate $= \frac{^3C_2 \times {^7C_3}}{^{10}C_5}$

$= \frac{\frac{3!}{2!} \times \frac{7!}{4!3!}}{252}$

$= 0.4166$

ii) At least 2 graduate $= \frac{^3C_2 \times {^7C_3}}{^{10}C_5} + \frac{^3C_3 \times {^7C_3}}{^{10}C_5}$

$= \frac{\left(\frac{3!}{2!} \times \frac{7!}{4!3!}\right) + \left(1 \times \frac{7!}{5!\,2!}\right)}{252}$

$= 0.5$

iii) Atmost 1 graduate $= \frac{^3C_1 \times {^7C_4}}{^{10}C_5} + \frac{^3C_0 \times {^7C_5}}{^{10}C_5}$

$= \frac{\left(\frac{3!}{2!} \times \frac{7!}{3!\,4!}\right) + \left(1 \times \frac{7!}{2! \times 5!}\right)}{252}$

$= 0.4166 + 0.14$

$= 0.5$

Ex-12 The probability that the person A solves the problem is $\frac{1}{3}$ and B is $\frac{1}{2}$ and for C is $\frac{3}{5}$. If the probability is simultaneously designed to all of them. What is the probability that problem is solved.

Soln. $A \rightarrow \frac{1}{3}$; $B \rightarrow \frac{1}{2}$; $C \rightarrow \frac{3}{5}$ } solve probability.

$\bar{A} = 1 - \frac{1}{3}$; $B = 1 - \frac{1}{2}$; $C = 1 - \frac{3}{5}$

$= \frac{2}{3}$; $= \frac{1}{2}$; $= \frac{2}{5}$ } Not solving probability

Required probability → Problem not solved

$$= \frac{2}{3} \times \frac{1}{2} \times \frac{2}{5}$$

$$= \frac{2}{15}$$

Required probability of solving the problem

$$= 1 - \frac{2}{15}$$

$$= \frac{15-2}{15}$$

$$= \frac{13}{15}$$

Ex-13 Suppose that an urn contain 8 red balls and 4 white balls, we draw 2 balls from the urn one by one without replacement. If we assume that each drawn ball in the urn is equally likely to be choosen. What is the probability that both balls are red.

Soln Required probability → $\frac{{}^{8}C_1}{{}^{12}C_1} \times \frac{{}^{7}C_1}{{}^{11}C_1}$

$$= \frac{8}{12} \times \frac{7}{11}$$

$$= \frac{14}{33}$$

* **Random Variable:** Real value function defined on the sample space are known as random variable. Random variable are denoted by capital letters from the last part of alphabet.
Example: X, Y, Z.

Random variable are of two types:

Random variable

- **Discrete random variable**
  (Countable Integer)
  Example: i) No of students in a class.
  ii) Roll no.
  iii) Total no. of cities in India.
  - Methods
    - Binomial
    - Poisson
- **Continuous random variab**
  (Fraction)
  Example: i) height of stud
  ii) Weight of a person.
  - Methods
    - Exponential
    - Normal distributi

Ex-1 Let X denote the random variable that is defined as the sum of two fair dice then-

Discrete random value: Total outcomes $= 6^2 = 36$

$P(x=2) = P\{(1,1)\} = \frac{1}{36}$

$P(x=3) = P\{(1,2),(2,1)\} = \frac{2}{36}$

$P(x=4) = P\{(1,3),(2,2),(3,1)\} = \frac{3}{36}$

$P(x=5) = p\{(1,4),(2,3),(3,2),(4,1)\} = \frac{4}{36}$

$P(x=6) = p\{(1,5),(2,4),(3,3),(4,2),(5,1)\} = \frac{5}{36}$

$P(x=7) = p\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\} = \frac{6}{36}$

$P(x=8) = p\{(2,6),(3,5),(4,4),(5,3),(6,2)\} = \frac{5}{36}$

$P(x=9) = p\{(3,6),(4,5),(5,4),(6,3)\} = \frac{4}{36}$

$P(x=10) = p\{(4,6),(5,5),(6,4)\} = \frac{3}{36}$

$P(x=11) = p\{(6,5),(5,6)\} = \frac{2}{36}$

$P(x=12) = p\{(6,6)\} = \frac{1}{36}$

PDF or PDT (Probability distribution table or fraction):

X	2	3	4	5	6	7	8	9	10	11	12
P(x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

Submmation of all probability = $\sum_{x=2}^{12} P(x=n)$

$= \frac{1}{36}+\frac{2}{36}+\frac{3}{36}+\frac{4}{36}+\frac{5}{36}+\frac{6}{36}+\frac{5}{36}+\frac{4}{36}+\frac{3}{36}+\frac{2}{36}+\frac{1}{36}$

$= \frac{36}{36} = 1$ //

* Discrete random variable: A Random variable that can take on at most accountable number of possible values is said to be Discrete random variable 'X'.

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We define probability mass function -

$$p(a) = P(x=a)$$

Let a random variable 'X' assume value $x_1, x_2, x_3 \ldots x_n$ with probability $p_1, p_2, p_3 \ldots p_n$ respectively.

where,

$$P(X=x_i) = p_i \geq 0, \forall x_i \quad \text{and} \quad p_1+p_2+p_3+\ldots p_n = 1$$

$$\boxed{\sum_{i=1}^{n} p_i = 1}$$

Then,

$$X : x_1 \quad x_2 \quad x_3 \ldots x_n$$

$$p(x) : p_1 \quad p_2 \quad p_3 \ldots p_n$$

$$E(X) = \mu = \text{mean} = x_1p_1 + x_2p_2 + \ldots x_np_n$$

$$\boxed{\mu = \sum_{i=1}^{n} x_i p_i}$$

$$\text{Variance } V(X) = \sigma^2 = \boxed{\sum_{i=1}^{n} x_i^2 p_i - \mu^2}$$

$$= E(X^2) - [E(X)]^2$$

$$\text{Standard Deviation} = +\sqrt{\text{Variance}}$$

Q-1 Probability distribution of a finite random variable X given by the following table.

X	-2	-1	0	1	2	3
P(x)	0	0.1	K	0.3	2K	0.5

i) Find the value of K.

ii) Find Mean (μ).

iii) Find Variance (σ^2).

iv) $P(-1 < X \leq 2)$

v) $X < 0$

Soln i) $\Sigma p_i = 1$

$0 + 0.1 + K + 0.3 + 2K + 0.5 = 1$

$3K + 0.9 = 1$

$3K = 0.1$

$K = \frac{1}{30} \Rightarrow 0.03$

ii) Mean (μ) $= 0 - 0.1 + 0 + 0.3 + 4K + 1.5$

$= 1.7 + 4 \times 0.03$

$= 1.82$

iii) Variance (σ^2) $= 0 + 0.1 + 0.3 + 8K + 4.5 - (1.82)^2$

$= 4.9 + 8 \times 0.03 - (1.82)^2$

$= 4.9 + 0.24 - 3.3124$

$= 1.8276$

iv) $P(-1 < X \leq 2) = P(0) + P(1) + P(2)$

$= K + 0.3 + 2K$

$= 0.03 + 0.3 + 2 \times 0.03$

$= 0.03 + 0.3 + 0.06$

$= 0.39$

v) $X < 0 = P(-1) + P(-2)$

$= 0.1 + 0$

$= 0.1$

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Q-2 Probability distribution of a finite random variable X given by the following table.

X	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

i) Find the value of K.
ii) Find Mean (μ).
iii) Find Variance (σ^2).
iv) $P(3 < X \le 6)$
v) $X \ge 6$

Soln i) $\Sigma p_i = 1$

$0+K+2K+2K+3K+2K^2+7K^2+K=1$

$9K+10K^2=1$

$10K^2+9K-1=0$

$10K^2+10K-K-1=0$

$10K(K+1)-1(K+1)=0$

$(10K-1)(K+1)=0$

$10K-1=0$ / $K+1=0$

$K=\frac{1}{10}$ / $K=-1$ {probability can't be -ve}

$K=\frac{1}{10} \Rightarrow 0.1$ //

ii) Mean (μ) $= 0+K+4K+6K+12K+5K^2+12K^2+14K^2+7K$

$= 30K+31K^2$

$= 30\times0.1+31\times(0.1)^2$

$= 3+0.31$

$= 3.31$ //

iii) Variance (σ^2) $= 0+K+8K+18K+48K+25K^2+72K^2+343K^2+49K-(3.31)^2$

$= 124K + 440K^2 - 10.9561$

$= 124\times 0.1 + 440\times(0.1)^2 - 10.9561$

$= 12.4 + 4.4 - 10.9561$

$= 5.8439$ //

iv) $P(3<X\leq 6) = P(4)+P(5)+P(6)$

$= 3K + K^2 + 2K^2$

$= 3K + 3K^2$

$= 3\times 0.1 + 3\times(0.1)^2$

$= 0.3 + 0.03$

$= 0.33$ //

v) $X\geq 6 = P(6)+P(7)$

$= 2K^2+7K^2+K$

$= 9K^2+K$

$= 9\times(0.1)^2+0.1$

$= 0.19$ //

Q-3 If Random variable X takes the values 1,2,3,4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$.

i) Find the Probability distribution function of X.

ii) Mean (μ) = ?

iii) Variance (σ^2) = ?

iv) $P(1\leq X<3)$

Soln. i)

X	1	2	3	4
P(X)	P_1	P_2	P_3	P_4

As we know, $p_1+p_2+p_3+p_4=1$

$2p_1=3p_2=p_3=5p_4$

$p_1=\frac{p_3}{2},\ p_2=\frac{p_3}{3},\ p_4=\frac{p_3}{5},\ \cancel{p_3=p_3}$

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$\frac{P_3}{2} + \frac{P_3}{3} + \frac{P_3}{2} + \frac{P_3}{5} = 1$

$P_3 = \frac{30}{61}$

X	1	2	3	4
P(x)	15/61	10/61	30/61	6/61

ii) Mean $(\mu) = \frac{15}{61} + \frac{20}{61} + \frac{90}{61} + \frac{24}{61}$

$= \frac{149}{61} \Rightarrow 2.442$ //

iii) Variance $(\sigma^2) = \frac{15}{61} + \frac{40}{61} + \frac{270}{61} + \frac{96}{61} - (2.442)^2$

$= \frac{421}{61} - 5.963$

$= \frac{421 - 363.743}{61}$

$= 0.9386$ //

iv) $P(1 \leq X < 3) = P(1) + P(2)$

$= \frac{15}{61} + \frac{10}{61}$

$= \frac{25}{61} \Rightarrow 0.4098$ //

Q-4 A coin is tossed twice, a random variable X represent the number of head turning up, find the discrete probility distribution for X. Also find it mean and variance.

Soln.

X	0	1	2
P(x)	1/4	2/4	1/4

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ii) Mean $(\mu) = 0 + \frac{2}{4} + \frac{2}{4}$

$= \frac{\cancel{4}}{\cancel{4}} \Rightarrow 1$

iii) Variance $(\sigma^2) = 0 + \frac{2}{4} + 1 - 1$

$= \frac{\cancel{2}^1}{\cancel{4}_2} \Rightarrow \frac{1}{2}$

Q-5 Let X denote the numbers of hours you study during a randomly selected school day. The probability that X, x has the following form where K is some unknown constant.

$$P(X=x) = \begin{cases} 0.1 & \text{if } x=0 \\ Kx & \text{if } x = 1 \text{ or } 2 \\ K(5-x) & \text{if } x = 3 \text{ or } 4 \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the value of K.

b) i) What is the probability that you studied at least two hours?

ii) Exactly two hours.

iii) Atmost two hours.

Soln

a)

X	0	1	2	3	4
P(X)	0.1	K	2K	2K	K

$6K + 0.1 = 1$

$6K = 1 - 0.1$

$6K = 0.9$

$K = \frac{\cancel{9}3}{\cancel{60}20} \Rightarrow \frac{3}{20}$

b) i) P(studied at least two hours) $= P(2) + P(3) + P(4)$

$= 2K + 2K + K$

$= 5K \Rightarrow 5 \times \frac{3}{20} \Rightarrow \frac{3}{4}$

ii) P(studied exactly two hours) = $P(2)$

$= 2K \Rightarrow 2 \times \frac{3}{20} \Rightarrow \frac{3}{10}$

iii) P(studied atmost two hours) = $P(2) + P(1) + P(0)$

$= 2K + K + 0.1$

$= 3K + 0.1$

$= \frac{9}{20} + 0.1 \Rightarrow 0.55$

Q-6 A Random variable X take the values -3, -2, -1, 0, 1, 2, 3 such that $P\{X=0\} = P\{X<0\}$ & $P(X=-3\} = P\{X=-2\} = P\{X=1\} = P\{X=2\} = P\{X=3\}$. Find the probability distribution.

X	-3	-2	-1	0	1	2	3
P(X)	P_1	P_2	P_3	P_4	P_5	P_6	P_7

* Poisson dist. - Poisson dist. is limiting case of binomial poisson dist. was discovered by S.D Poisson in the year 1837 if the parameter n and p of a binomial dist are known we can find dist. but in situation when n is very large and p is very small then we apply poisson dist. If we assume that if

$n \to \infty$

$p \to 0$

such that $np \to$ finite $\to \lambda$

We get poisson approximation to the binomial dist. then

$$\boxed{P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{r!}}$$ where $r = 0, 1, 2, 3 \dots$

$\boxed{\lambda = np}$

* Mean of the Poisson dist:

$$\mu = \sum_{r=0}^{\infty} r P(r)$$

$$= \sum_{r=0}^{\infty} \frac{\cancel{r}\, e^{-\lambda}\lambda^{r}}{\cancel{r}(r-1)!}$$

$$= e^{-\lambda} \sum_{r=1}^{\infty} \frac{\lambda^{r}}{(r-1)!}$$

$$= e^{-\lambda}\left[\frac{\lambda}{1} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \frac{\lambda^4}{3!} + \frac{\lambda^5}{4!} + \dots\right]$$

$$= e^{-\lambda}\lambda\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} + \dots\right]$$

$$\because \left(e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots\right)$$

$= e^{-\lambda} \lambda e^{\lambda}$

$= e^{-\lambda+\lambda} \lambda$

$= \lambda$

$\boxed{\mu = np = \lambda}$

* Variance $= \sum_{x=0}^{\infty} x^2 p(x) - \lambda^2$

$= \sum x^2 \frac{e^{-\lambda} \lambda^x}{x!} - \lambda^2$

$= e^{-\lambda} \sum_{x=0} \frac{x^2 \lambda^x}{x!} - \lambda^2$

$= e^{-\lambda} \left[\frac{1^2 \lambda^1}{1!} + \frac{2^2 \lambda^2}{2!} + \frac{3^2 \lambda^3}{3!} + \cdots \right] - \lambda^2$

$= \lambda e^{-\lambda} \left[1 + \frac{2^{\cancel{2}} \lambda}{\cancel{2}!} + \frac{3^{\cancel{2}} \lambda^2}{\cancel{3}!} + \frac{4^{\cancel{2}} \lambda^3}{\cancel{4}!} + \cdots \right] - \lambda^2$

$= \lambda e^{-\lambda} \left[1 + 2\lambda + \frac{3\lambda^2}{2!} + \frac{4\lambda^3}{3!} - \cdots \right] - \lambda^2$

$= \lambda e^{-\lambda} \left[1 + (1+1)\lambda + \frac{(1+2)\lambda^2}{2!} + \frac{(1+3)\lambda^3}{3!} + \cdots \right] - \lambda^2$

$= \lambda e^{-\lambda} \left[\left\{ 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots \right\} + \left\{ \lambda + \frac{2\lambda^2}{2!} + \frac{3\lambda^3}{3!} + \cdots \right\} \right] - \lambda^2$

$= \lambda e^{-\lambda} \left[e^{\lambda} + \lambda \left[1 + \frac{\lambda}{1} + \frac{\lambda^2}{2!} + \cdots \right] \right] - \lambda^2$

$= \lambda e^{-\lambda} \left[e^{\lambda} + \lambda e^{\lambda} \right] - \lambda^2$

$= \lambda e^{0} + \lambda^2 e^{0} - \lambda^2$

$= \lambda + \cancel{\lambda^2} - \cancel{\lambda^2}$

$\boxed{\text{Variance} = npq = \lambda}$

1.22

Q1 In the prob. of bad reaction from a certain injection is 0.0002 determine the chance that out of thousand individual more than two will get bad reaction.

Soln $p = 0.0002$

$n = 1000$

$\lambda = n \times p = 1000 \times 0.0002 = 0.2$

$p(r) = \frac{e^{-\lambda} \lambda^{r}}{r!}$

$\frac{\cancel{0.04}}{\cancel{2} \times 1}$

$p(3) + p(4) + p(5) + --- p(1000)$

$= 1 - [p(0) + p(1) + p(2)]$

$= 1 - e^{-\lambda} \left[\frac{(0.2)^0}{0} + \frac{(0.2)^1}{1!} + \frac{(0.2)^2}{2!} \right]$

$= 1 - e^{0.2} [1 + 0.2 + 0.02]$

$= 0.0012$ //

Q-2 A car hire firm has 2 cars which in hire day by day. The number of demand for a car on each day is distributed as Poisson dist. with mean 1.5.
Calculate the propotion of days in which neither car is used and the propotion of days on which some demand is refused

Soln i) $r = 0$

$\frac{e^{-1.5} \times (1.5)^0}{\underline{0}}$

$= 0.2231$ //

ii) $P(3) + P(4) + P(5) + ---$

$= 1 - [P(0) + P(1) + P(2)]$

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$$= 1 - \left[0.2231 + \frac{e^{-1.5} \times (1.5)^1}{1!} + \frac{e^{-1.5} \times (1.5)^2}{2!}\right]$$

$= 0.1913$

Q-3 A manufacturer knows that the condensors he makes contain on an average of 1% defective. He packed them in box of hundred what is the prob. that a box picked at random will contain 4 or more faulty condensors.

Soln $p = 0.01$

$n = 100$

$\lambda = np = 1$

$\frac{12+12+6+2}{12}$

$P(4) + P(5) + P(6) \ ---- \ P(100).$

$= 1 - [P(0) + P(1) + P(2) + P(3)]$

$$= 1 - \left[\frac{e^{-1} 1^0}{0!} + \frac{e^{-1} 1^1}{1!} + \frac{e^{-1} 1^2}{2!} + \frac{e^{-1} 1^3}{3!}\right]$$

$$= 1 - e^{-1}\left[\frac{1^0}{0!} + \frac{1^1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!}\right]$$

$$= 1 - 0.367879\left[1 + 1 + \frac{1}{2} + \frac{1}{6}\right]$$

$$= 1 - 0.3678\left[2.666666\right]$$

$=$

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Q-4 Wireless sets are manufactured with 25 soldiers join on the average 1 joint in 500 is defective. How many sets can be expected to free from defective joints in a conciment of 10000.

Soln $p = \frac{1}{500}$ (for defective join

$= 0.002$

$n = 25$

$\lambda = 25 \times 0.002 \Rightarrow 0.05$

$r = 0$

$\frac{e^{-0.05} \times 1}{0!} = 0.9512$

No. of sets which having no deffective joints = 9512

Most Imp Q The prob. of a poisson variand taking the value 3 and 4 are equal calculate the prob. of variand taking the value 0 and 1

Solve $P(3) = P(4)$

$\frac{e^{-\lambda} \lambda^3}{3!} = \frac{e^{-\lambda} \lambda^4}{4!}$

$\frac{4 \cancel{!3}}{\cancel{!3}} = \lambda$

$\boxed{\lambda = 4}$

$P(0) = ? = \frac{e^{-4} (4)^0}{0!} = e^{-4} = 0.01831$

$P(1) = \frac{e^{-4} (4)^1}{1!} = 4 \times e^{-4} = 0.07326$

Maths

Soln no mistake

$p(x=0) = \frac{1}{e^3}$

$\frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{1}{e^3}$

$\frac{e^{-\lambda}\lambda^{0}}{1} = \frac{1}{e^3}$

$e^{-\lambda} = e^{-3}$

$\boxed{\lambda = 3}$

) $x = 3$

$\frac{e^{-3}3^{3}}{3!} =$

more than 3

$P(4) + P(5) + P(6) + \dots$

$1 - [P(0) + P(1) + P(2) + P(3)]$

$1 - e^{-3}\left[\frac{3^0}{\lfloor 0} + \frac{3^1}{\lfloor 1} + \frac{3^2}{\lfloor 2} + \frac{3^3}{\lfloor 3}\right]$

$= \left[e^{-3}\left[1 + 3 + \frac{9}{2} + \frac{9}{2}\right]\right]$

At most 3 mist.

$P(3) + P(2) + P(1) + P(0)$

If ~~capital~~ x is a poisson variate such that $P(X=2) = 9P(X=4) + 90P(X=6)$ Compute mean and variance.

Soln $\frac{e^{-\lambda}\lambda^2}{2!} = 9\frac{e^{-\lambda}\lambda^4}{4!} + 90\frac{e^{-\lambda}\lambda^6}{6!}$

$\frac{1}{2} = \frac{3\lambda^2}{24} + \frac{90\lambda^4}{6.5.4.3.2.1}$

$1 = \frac{3\lambda^2}{4} + \frac{\lambda^4}{4}$

$\lambda^4 + 3\lambda^2 - 4 = 0$

$\lambda^4 + 4\lambda^2 - \lambda^2 - 4 = 0$

$\lambda^2[\lambda^2+4] - 1[\lambda^2+4] = 0$

$\lambda^2 = -4, \quad \lambda^2 = +1$

$\lambda = \sqrt{-4} \qquad \lambda = +1, -1$

$= \pm 2i$

mean $= +1$

Variance $= +1$

Suppose a number of telephone calls on an operator received from 9:00 to 9:05 with mean of 3. Find the prob. that is the operator will receive no call in that time interval.

ii) In the next three days the operator will receive a total of 1 call in

Most Imp

Soln i) No calls -

$$P(x=0) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \frac{e^{-3}\lambda^{0}}{0!}$$

$$= e^{-3}$$

$$= 0.0497$$

ii) One call in 3 days :

$P(1)\,P(0)\,P(0) + P(0)\,P(1)\,P(0) + P(0)\,P(0)\,P(1)$

$= 3\,[P(0)\,P(0)\,P(1)]$

$$= 3\left[\frac{e^{-3}\,3^{1}}{1!} \times \frac{e^{-3}\,3^{0}}{0!} \times \frac{e^{-3}\,3^{0}}{0!}\right]$$

$\cancel{= 3.\,e^{-3}\,[3\times1\times1]}$

$\cancel{= 9\times e^{-3}}$

$\cancel{= 0.44808}$

$$= 3\left[\frac{0.04978\times3}{1} \times 0.04978\times1 \times 0.04978\times1\right]$$

$= 0.0011102151$

Q Data was collected over a period of 10 years showing number of deaths from horse kicks in each of the 200 army corps. The distribution of death was as follows

No. of deaths	frequency
0	109
1	65
2	22
3	3
4	1
	200

Fit a poisson dist. to the data and calculate the theoritical frequency.

Soln Required poisson dist. $= N\left(\frac{e^{-\lambda}\lambda^r}{r!}\right) = N.P(r)$

$N.(P(r))$	Theo.
$200 \times \frac{e^{-0.61} \times 1}{1}$	108.6701
$200 \times \frac{e^{-0.61} \times (0.61)^1}{1!}$	66.288
$200 \times \frac{e^{-0.61} \times (0.61)^2}{2!}$	20.2180
$200 \times \frac{e^{-0.61} \times (0.61)^3}{3!}$	4.11101
$200 \times \frac{e^{-0.61} \times (0.61)^4}{4!}$	0.62692

H/W

Q1 The number of arrivals of customer during any day follow poisson dist. with a mean of 5 what is the prob. that the total no. of customer on 2 days selected at random is less than 2.

$P(0)P(1) + P(1)P(0) + P(0)P(0)$

0.00673

Q-2 Suppose that a book of 600 pages contain 40 printing mistake. Assume that this error are randomly distributed throughout the book and r. The number of error per page follow poisson dist. What is the prob. that 10 pg. selected at random will be free from error.

Soln $n = 10$ $p = \frac{40}{600}$ $\lambda = np = 0.66666$ $r = 0$ $\frac{e^{-0.666}(0.666)^0}{0!}$

$= 0.51375$

Q Fit a poisson dist. to the following data and calculate the theo freq.

Death	freq.
0	122
1	60
2	15
3	2
4	1
	200

Q A manufacturer knows that the condenser he makes contain on an average 1% of defective. He packs them in boxes of 100. What is the prob. that a box picked at random will contain 4 or more faulty condensers.

Soln $n = 100 \quad p = 0.01$

$\lambda = np = 1$

$P(4 \text{ or more faulty condensers}) = P(4) + P(5) + P(6) \ldots P(10$

$= 1 - [P(0) + P(1) + P(2) + P(3)]$

$$= 1 - \left[\frac{e^{-1}1^0}{0!} + \frac{e^{-1}1^1}{1!} + \frac{e^{-1}1^2}{2!} + \frac{e^{-1}1^3}{3!}\right]$$

$$= 1 - e^{-1}\left[\frac{1^0}{0!} + \frac{1^1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!}\right]$$

$$= 1 - e^{-1}\left[1 + 1 + \frac{1}{2} + \frac{1}{6}\right]$$

$$= 1 - 0.36787\left[\frac{12+12+6+2}{12}\right]$$

$$= 1 - 0.36787\left[2.66666\right]$$

$$= 0.019013$$

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Soln3

Death	freqn.
0	122
1	60
2	15
3	2
4	1
	200

$$\text{Mean } (\lambda) = \frac{\sum f_i x_i}{\sum f_i} = \frac{60+30+6+4}{200} \Rightarrow 0.5$$

$$\text{Required poisson distribution} = N\left(\frac{e^{-\lambda}\lambda^r}{r!}\right) = N.P(r)$$

N.P(r)	theoritical freqn.
$200 \times \frac{e^{-0.5} \times 0.5^0}{0!}$	121.30613
$200 \times \frac{e^{-0.5} \times 0.5^1}{1!}$	60.65306
$200 \times \frac{e^{-0.5} \times 0.5^2}{2!}$	15.16326
$200 \times \frac{e^{-0.5} \times 0.5^3}{3!}$	2.52721
$200 \times \frac{e^{-0.5} \times 0.5^4}{4!}$	0.31590

Soln4 P(customer less than 2 in 2 days) $= P(0)P(1) + P(1)P(0) + P(0)P(0)$

$$= \frac{e^{-5}5^0}{0!} \times \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^1}{1!} \times \frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^0}{0!} \times \frac{e^{-5}5^0}{0!}$$

$$= 0.00022 + 0.00022 + 0.00113$$

$$= 0.00157$$

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* Discrete variat random variable:

* Prob. joint mass fun of (x,y):

If (X,Y) is a two dimensional random variable where $X = x_i$, $Y = y_j$ and $i, j = 1, 2, 3 ----$
then
$$P(X = x_i, Y = y_j) = P_{ij}$$

$P_{ij} \rightarrow$ joint prob. mass fun of (X,Y)

Prop.:

i) $P \geq 0$ ii) $\sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij} = 1$

* Marginal prob. mass function of X :-

$$P(X = x_i) = \sum_{j=1}^{m} P_{ij} = P_{11} + P_{12} + P_{13} ---$$

The collection of $(x_i\ P_{ij})$ is called marginal p.m.f of X.

* Marginal prob. mass function of Y :-

$$P(Y = y_j) = \sum_{i=1}^{n} P_{ij} = P_{11} + P_{21} + P_{31} + ----$$

The pair (y_j, P_{ij}) is called marginal p.m.f of Y.

If X & Y are independent.

$$P_{xy}(x\,y) = P_x(x) \times P_y(y)$$
$$= P_x \cdot P_y$$

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Note: $E(x) = \sum x f(x)$
$E(X) = \sum x\, p_x(x)$
$E(Y) = \sum y\, p_y(y)$

Covariance of X and Y:

Let, (X,Y) are having μ_x, μ_y represent the covariance X & Y
$$Cov(X,Y) = E(x,y) - \mu_x \mu_y$$

Correlation of X and Y

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$$

If X and Y are independent

i) $E(XY) = E(X) \cdot E(Y)$
ii) $Cov(X,Y) = 0$
$\Rightarrow \rho(X,Y) = 0$

Q.1 The joint dist. of two random variable X & Y is as follows

X↓ \ Y→	-4	2	7	Sum	
1	1/8	1/4	1/8	1/2	$\rightarrow f(x_1)$
5	1/4	1/8	1/8	1/2	$\rightarrow f(x_2)$
Sum	3/8	3/8	1/4	1	
	$g(y_1)$	$g(y_2)$	$g(y_3)$		

Q.2 Calculate the following a) $E(X)$, $E(Y)$, b) $E(XY)$, c) σ_x, σ_y
d) $Cov(x,y)$ e) $\rho(x,y)$

$-\frac{4}{8}+\frac{2}{4}+\frac{7}{8}, -\frac{20}{4}$

a) $E(X) = \sum_x p_X(x)$

$= 1 \times \frac{1}{2} + 5 \times \frac{1}{2}$

$= 3$

b) $E(Y) = 1$

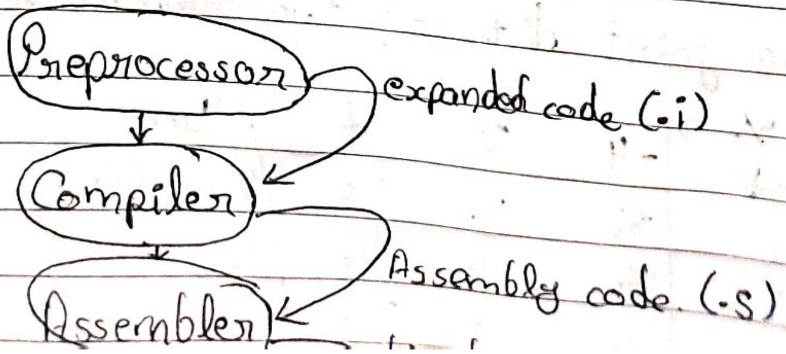
b) $E(XY) = \sum x_i y_j J_{ij}$

$= 1 \times (-4) \times \frac{1}{8} + 1 \times 2 \times \frac{1}{4} + 1 \times 7 \times \frac{1}{8} + 5 \times (-4) \times \frac{1}{4} + 5 \times 2 \times \frac{1}{8}$
$+ 5 \times 7 \times \frac{1}{8}$

---

* **Compiler:** Compiler is used to convert high level language program / source code into machine language / object code.

* **Compilation process:** The process of converting source code into machine code is called compilation process. The compiler checks the source code and if the code is free from errors then it generates the machine code. The compilation process can be divided into four steps.



Maths

c) σ_x, σ_y

$$\sigma_x^2 = E(x^2) - \mu_x^2$$
$$= \sum x^2 p_x(x) - \mu_x^2$$
$$= 1^2 \times \frac{1}{2} + 25 \times \frac{1}{2} - 9$$

$$\sigma_x^2 = 9$$
$$\sigma_x = 3$$

$$\sigma_y^2 = \sum y^2 p_y(y) - \mu_y^2$$
$$= -4^2 \times \frac{3}{8} + 2^2 \times \frac{3}{8} + 7^2 \times \frac{1}{4} - 1$$
$$= \cancel{16}^{\,2} \times \frac{3}{\cancel{8}} + \cancel{4} \times \frac{3}{\cancel{8}_2} + 49 \times \frac{1}{4}$$
$$= 6 + \frac{3}{2} + \frac{49}{4}$$
$$= \frac{48+12+98}{8}$$
$$\sigma_y^2 = \frac{75}{4}$$
$$\sigma_y = \sqrt{75}/2$$

31
~~9~~
18
~~72~~
~~36~~
~~12~~
4
49
×2
98

11
~~33~~
~~72~~
24

d) $$Cov(x,y) = E(xy) - \mu_x \mu_y$$
$$= 3/2 - 3 \times 1$$
$$= \frac{3-6}{2} \Rightarrow -\frac{3}{2}$$

e) $$\rho(x,y) = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{-\cancel{3}/\cancel{2}}{\cancel{3} \times \sqrt{75}/\cancel{2}} \Rightarrow \frac{-1}{\sqrt{75}}$$

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Q.2 The joint prob. mass function of (X,Y) is given by $P(x,y) = K(2x+3y)$ where $X = 0,1,2$. $Y = 1,2,3$
Find i) $K=?$, ii) Marginal p.m.f of X, Y, iii) Conditional p.m.f of x given $Y=1$, iv) Conditional p.m.f of Y given $X=2$, v) Prob. dist. of $X+Y$ independent?

Solu:

$X \downarrow$ \ $Y \rightarrow$	1	2	3	Sum
0	3K	6K	9K	18K
1	5K	8K	11K	24K
2	7K	10K	13K	30K
Sum	15K	24K	33K	72K

$72K = 1 \Rightarrow K = 1/72$

ii) Marginal p.m.f of X

X	0	1	2
P(x)	18K	24K	30K
P(x)	1/4	1/3	5/12

Marginal p.m.f of Y

Y	1	2	3
P(y)	15K	24K	33K
P(y)	5/24	1/3	11/24

iii) Conditional p.m.f of x, when $Y=1$

X	0	1	2
P(x)	3K	5K	7K
P(x)	1/24	5/72	7/72

iv) when $x=2$

y	1	2	3
$P(y)$	$7k$	$10k$	$13k$
$P(y)$	$7/72$	$5/36$	$13/72$

v) Prob. dist. of $(X+Y)$:

$Z = X+Y$

$Z = 1, 2, 3, 2, 3, 4, 3, 4, 5$

Z	1	2	3	4	5
X	0	0 1	0 2 1	1 2	2
	↓	↓ ↓	↓ ↓ ↓	↓ ↓	↓
Y	1	2 1	3 1 2	3 2	3
	$3k$	$6k+5k = 11k$	$9k+8k+7k = 24k$	$11k+10k = 21k$	$13k$

$(X+Y)$ Z	
$P(z)$	

To prove inde.

$P_{(x,y)}(X, Y) = P_x(x)\, P_y(y)$

$P_{(x,y)}(0, 1) = P_x(0)\, P_y(1)$

$3k = 18k \times 15k$

$\frac{1}{24} \neq \frac{1}{4} \times \frac{15}{72}$

X and Y are not Independent.