# DM Assignment-1

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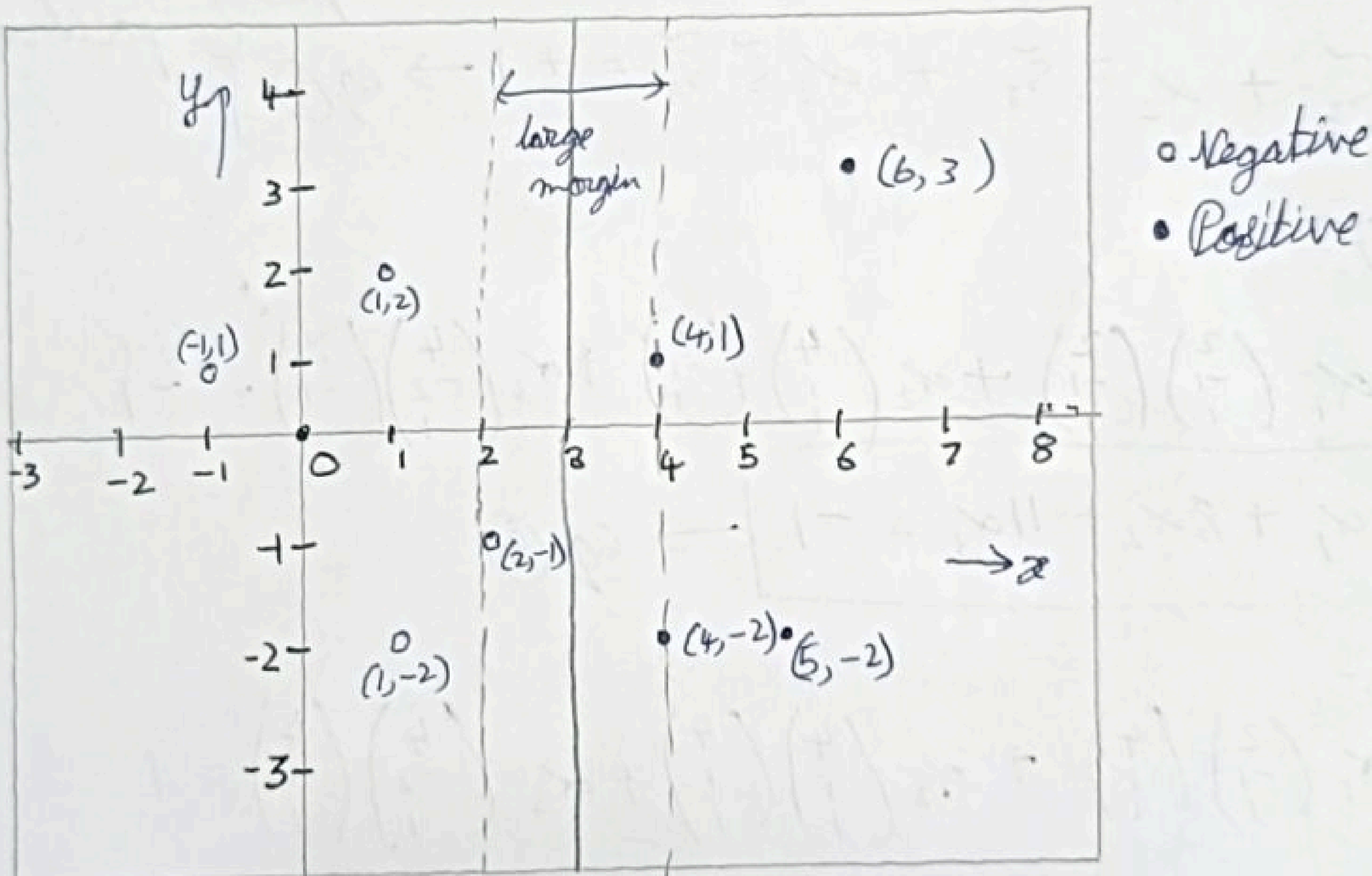
Question:- Find the seperating hyperplane for the following dataset using support vector machine (SVM). $D = \{S_1, S_2\}$ where
$S_1$ is positively labelled data:- (4,1), (4,−2), (5,−2), (6,3)
$S_2$ is negatively labelled data:- (1,2), (2,−1), (1,−2), (−1,1)




→ points (4,1), (4,−2), (5,−2) and (6,3) belongs to class positive
→ points (1,2), (2,−1), (1,−2), (−1,1) belongs to class negative
→ It is observed that (2,−1), (4,1) and (4,−2) are the support vectors.
→ By maximum marginal hyperplane

$$s_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad s_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \quad s_3 = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$

→ Augmented vector can be obtained by adding the bias given as follows:-

$$\tilde{s}_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \quad \tilde{s}_2 = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}, \quad \tilde{s}_3 = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$

→ For, these, a set of three equations can be obtained based on these three support vectors

$$\alpha_1 \tilde{s}_1 \tilde{s}_1 + \alpha_2 \tilde{s}_2 \tilde{s}_1 + \alpha_3 \tilde{s}_3 \tilde{s}_1 = -1 \rightarrow eq\ 1 \rightarrow \text{negative class}$$

$$\left.\begin{aligned} \alpha_1 \tilde{s}_1 \tilde{s}_2 + \alpha_2 \tilde{s}_2 \tilde{s}_2 + \alpha_3 \tilde{s}_3 \tilde{s}_2 = +1 \rightarrow eq\ 2 \\ \alpha_1 \tilde{s}_1 \tilde{s}_3 + \alpha_2 \tilde{s}_2 \tilde{s}_3 + \alpha_3 \tilde{s}_3 \tilde{s}_3 = +1 \rightarrow eq\ 3 \end{aligned}\right\} \text{positive class}$$

Take eq 1:-

$$\alpha_1 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = -1$$

$$\boxed{6\alpha_1 + 8\alpha_2 + 11\alpha_3 = -1} \rightarrow eq\ 4$$

Take eq 2:-

$$\alpha_1 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} = 1$$

$$\boxed{8\alpha_1 + 18\alpha_2 + 15\alpha_3 = 1} \rightarrow eq\ 5$$

Take eq 3:-

$$\alpha_1 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = 1$$

$$\boxed{11\alpha_1 + 15\alpha_2 + 21\alpha_3 = 1} \rightarrow eq\ 6$$

Solving these three equations by Gaussian elimination

$$\left(\begin{array}{ccc|c} 6 & 8 & 11 & -1 \\ 8 & 18 & 15 & 1 \\ 11 & 15 & 21 & 1 \end{array}\right) \xrightarrow{R_2 - \left(\frac{4}{3}\right)R_1 \to R_2} \left(\begin{array}{ccc|c} 6 & 8 & 11 & -1 \\ 0 & \frac{22}{3} & \frac{1}{3} & \frac{7}{3} \\ 11 & 15 & 21 & 1 \end{array}\right)$$

$$\Big\downarrow R_3 - \left(\frac{11}{6}\right)R_1 \to R_3$$

$$\left(\begin{array}{ccc|c} 6 & 8 & 11 & -1 \\ 0 & \frac{22}{3} & \frac{1}{3} & \frac{7}{3} \\ 0 & 0 & \frac{9}{11} & \frac{30}{11} \end{array}\right) \xleftarrow{R_3 - \left(\frac{1}{22}\right)R_2 \to R_3} \left(\begin{array}{ccc|c} 6 & 8 & 11 & -1 \\ 0 & \frac{22}{3} & \frac{1}{3} & \frac{7}{3} \\ 0 & \frac{1}{3} & \frac{5}{6} & \frac{17}{6} \end{array}\right)$$

$$\Downarrow$$

$$\left\{\begin{array}{r} 6\alpha_1 + 8\alpha_2 + 11\alpha_3 = -1 \\ \frac{22}{3}\alpha_2 + \frac{1}{3}\alpha_3 = \frac{7}{3} \\ \frac{9}{11}\alpha_3 = \frac{30}{11} \end{array}\right\}$$

On solving,

we get $\alpha_3 = \frac{30}{9} = \frac{10}{3}$

$$\boxed{\alpha_3 = \frac{10}{3}}$$

$$\frac{22}{3}\alpha_2 + \frac{1}{3}\left(\frac{10}{3}\right) = \frac{7}{3} \Rightarrow \frac{22}{3}\alpha_2 = \frac{7}{3} - \frac{10}{9} = \frac{11}{9}$$

$$\Rightarrow \boxed{\alpha_2 = \frac{1}{6}}$$

$$6\alpha_1 + 8\left(\frac{1}{6}\right) + 11\left(\frac{10}{3}\right) = -1 \Rightarrow \left(\alpha_1 = \frac{-13}{2}\right)$$

We got

$\alpha_1 = -6.5$

$\alpha_2 = 0.167$

$\alpha_3 = 3.33$

The optimal hyperplane is given by

$$\tilde{\omega} = \sum_{i=1}^{3} \alpha_i \times \tilde{s}_i = \alpha_1 \tilde{s}_1 + \alpha_2 \tilde{s}_2 + \alpha_3 \tilde{s}_3$$

$$\tilde{\omega} = -6.5 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + 0.167 \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + 3.33 \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$

$$\tilde{\omega} = \begin{pmatrix} 0.988 \\ 0.007 \\ -3.003 \end{pmatrix}$$

weight vector, $w = \begin{pmatrix} 0.988 \\ 0.007 \end{pmatrix}$ and bias, $b = -3.003$

→ seperating hyperplane can be written as $\boxed{w.x + b = 0}$

$$w_1 x_1 + w_2 x_2 + b = 0$$

$$\rightarrow 0.988 x_1 + 0.007 x_2 - 3.003 = 0$$

This is the seperating hyperplane.