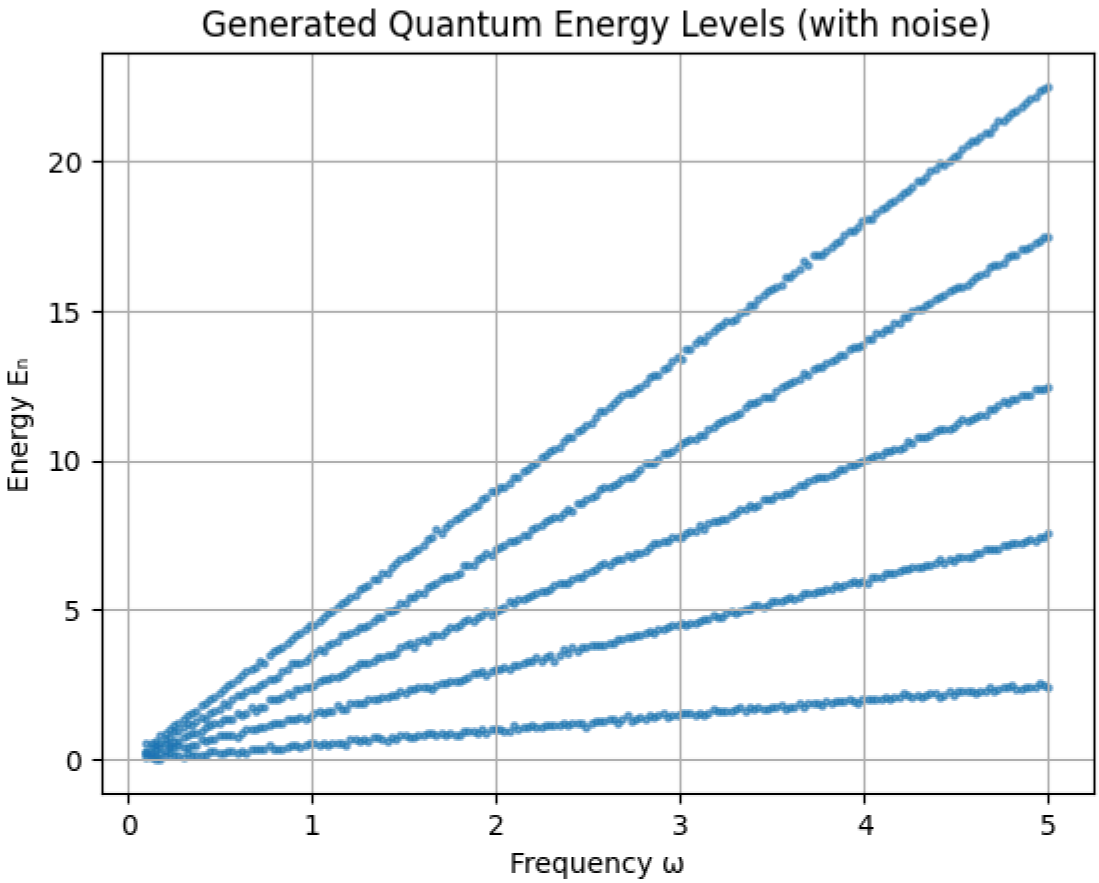
```
import numpy as np
import matplotlib.pyplot as plt

# ℏ = 1, Energy levels: E_n = (n + 0.5) * ω
omegas = np.linspace(0.1, 5.0, 200)  # frequencies
n_levels = 5  # energy levels n = 0 to 4

# Build dataset: For each omega, calculate E0 to E4
X = []
y = []

for omega in omegas:
    for n in range(n_levels):
        X.append([omega, n])  # input: frequency & energy level index
        E_n = (n + 0.5) * omega
        noise = np.random.normal(0, 0.05)  # small noise
        y.append(E_n + noise)

X = np.array(X)
y = np.array(y)

print(f"Input shape: {X.shape}, Output shape: {y.shape}")

# Optional: visualize
plt.scatter(X[:,0], y, s=5, alpha=0.6)
plt.xlabel("Frequency ω")
plt.ylabel("Energy E□")
plt.title("Generated Quantum Energy Levels (with noise)")
plt.grid(True)
plt.show()
```

```
Input shape: (1000, 2), Output shape: (1000,)
```



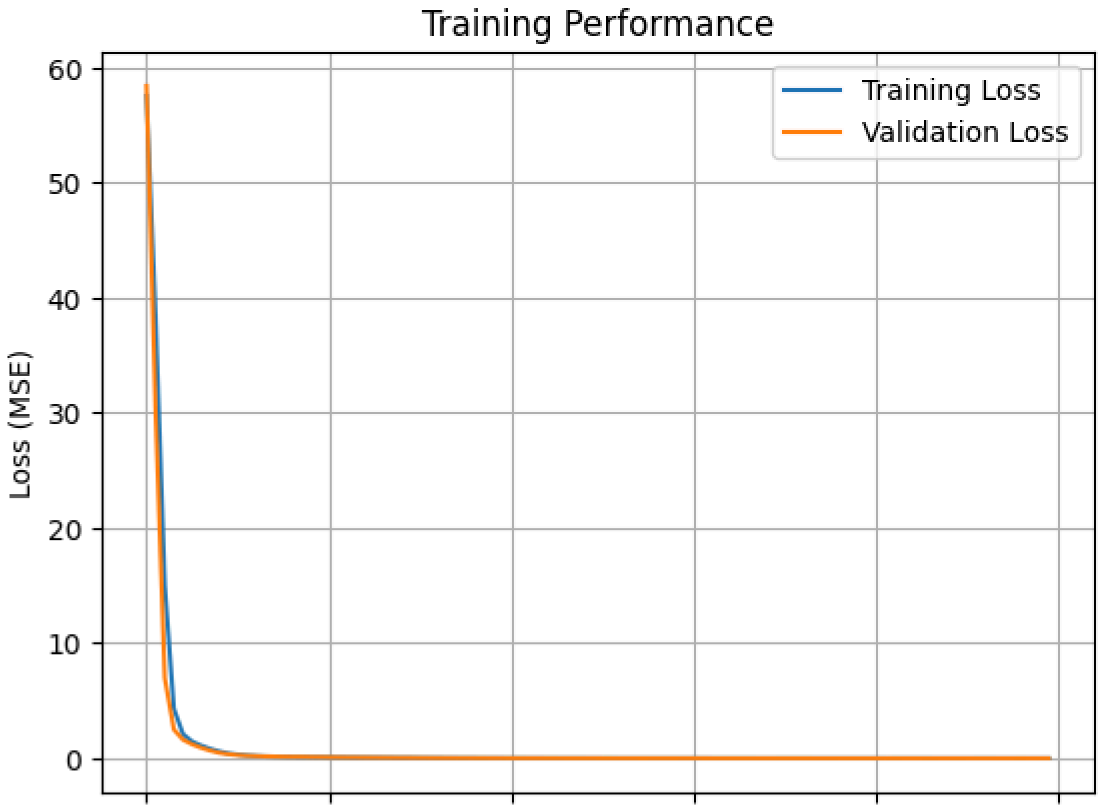
```
import tensorflow as tf
from tensorflow import keras
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
import matplotlib.pyplot as plt

# Scale inputs (omega and n)
scaler = StandardScaler()
X_scaled = scaler.fit_transform(X)

# Split data into training and test sets (80% train, 20% test)
X_train, X_test, y_train, y_test = train_test_split(
    X_scaled, y, test_size=0.2, random_state=42
)

# Build the neural network model
model = keras.Sequential([
    keras.layers.Dense(32, activation='relu', input_shape=(2,)),
    keras.layers.Dense(32, activation='relu'),
    keras.layers.Dense(1)  # Output: predicted energy
])

# Compile the model with optimizer and loss function
model.compile(optimizer='adam', loss='mse', metrics=['mae'])
```

```
# Train the model for 100 epochs
history = model.fit(X_train, y_train, epochs=100, batch_size=16,
                    validation_data=(X_test, y_test), verbose=1)

# Plot training and validation loss curves
plt.plot(history.history['loss'], label='Training Loss')
plt.plot(history.history['val_loss'], label='Validation Loss')
plt.xlabel('Epoch')
plt.ylabel('Loss (MSE)')
plt.title('Training Performance')
plt.legend()
plt.grid(True)
plt.show()
```

```
/usr/local/lib/python3.11/dist-packages/keras/src/layers/core/dense.py:87: UserW
  super().__init__(activity_regularizer=activity_regularizer, **kwargs)
Epoch 1/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 2s 7ms/step - loss: 56.5495 - mae: 5.6821 - val_lo
Epoch 2/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 42.8536 - mae: 4.5953 - val_lo
Epoch 3/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 19.3913 - mae: 3.3286 - val_lo
Epoch 4/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 4.9550 - mae: 1.7785 - val_los
Epoch 5/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 2.5157 - mae: 1.2466 - val_los
Epoch 6/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 1.5066 - mae: 0.9306 - val_los
Epoch 7/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 1.2531 - mae: 0.8507 - val_los
Epoch 8/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.8853 - mae: 0.7041 - val_los
Epoch 9/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.7380 - mae: 0.6573 - val_los
Epoch 10/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.4777 - mae: 0.5091 - val_los
Epoch 11/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.3530 - mae: 0.4357 - val_los
Epoch 12/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.2570 - mae: 0.3694 - val_los
Epoch 13/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.2035 - mae: 0.3385 - val_los
Epoch 14/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.1866 - mae: 0.3167 - val_los
Epoch 15/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.1399 - mae: 0.2784 - val_los
Epoch 16/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.1395 - mae: 0.2733 - val_los
Epoch 17/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.1128 - mae: 0.2483 - val_los
Epoch 18/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.1109 - mae: 0.2378 - val_los
Epoch 19/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.1174 - mae: 0.2476 - val_los
Epoch 20/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.1000 - mae: 0.2224 - val_los
Epoch 21/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0930 - mae: 0.2115 - val_los
Epoch 22/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0860 - mae: 0.2061 - val_los
Epoch 23/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0681 - mae: 0.1839 - val_los
Epoch 24/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0694 - mae: 0.1827 - val_los
Epoch 25/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0653 - mae: 0.1766 - val_los
Epoch 26/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0547 - mae: 0.1633 - val_los
Epoch 27/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0612 - mae: 0.1715 - val_los
```

```
Epoch 28/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0583 - mae: 0.1593 - val_los
Epoch 29/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0489 - mae: 0.1518 - val_los
Epoch 30/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0431 - mae: 0.1433 - val_los
Epoch 31/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0375 - mae: 0.1346 - val_los
Epoch 32/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0332 - mae: 0.1232 - val_los
Epoch 33/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0281 - mae: 0.1188 - val_los
Epoch 34/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0299 - mae: 0.1203 - val_los
Epoch 35/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 5ms/step - loss: 0.0291 - mae: 0.1176 - val_los
Epoch 36/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 6ms/step - loss: 0.0245 - mae: 0.1085 - val_los
Epoch 37/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 1s 5ms/step - loss: 0.0279 - mae: 0.1156 - val_los
Epoch 38/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 6ms/step - loss: 0.0244 - mae: 0.1035 - val_los
Epoch 39/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 1s 5ms/step - loss: 0.0181 - mae: 0.0938 - val_los
Epoch 40/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0163 - mae: 0.0928 - val_los
Epoch 41/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0182 - mae: 0.0949 - val_los
Epoch 42/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0174 - mae: 0.0929 - val_los
Epoch 43/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0157 - mae: 0.0855 - val_los
Epoch 44/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0131 - mae: 0.0815 - val_los
Epoch 45/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0134 - mae: 0.0808 - val_los
Epoch 46/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0126 - mae: 0.0801 - val_los
Epoch 47/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0112 - mae: 0.0774 - val_los
Epoch 48/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0129 - mae: 0.0799 - val_los
Epoch 49/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0099 - mae: 0.0707 - val_los
Epoch 50/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0114 - mae: 0.0757 - val_los
Epoch 51/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0108 - mae: 0.0740 - val_los
Epoch 52/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0089 - mae: 0.0688 - val_los
Epoch 53/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0091 - mae: 0.0682 - val_los
Epoch 54/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0074 - mae: 0.0658 - val_los
Epoch 55/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0076 - mae: 0.0643 - val_los
Epoch 56/100
Epoch 56/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0085 - mae: 0.0685 - val_los
Epoch 57/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0083 - mae: 0.0654 - val_los
Epoch 58/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0088 - mae: 0.0679 - val_los
Epoch 59/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0068 - mae: 0.0610 - val_los
Epoch 60/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0061 - mae: 0.0591 - val_los
Epoch 61/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0075 - mae: 0.0666 - val_los
Epoch 62/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0068 - mae: 0.0617 - val_los
Epoch 63/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0060 - mae: 0.0616 - val_los
Epoch 64/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0055 - mae: 0.0566 - val_los
Epoch 65/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0059 - mae: 0.0581 - val_los
Epoch 66/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0054 - mae: 0.0552 - val_los
Epoch 67/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0047 - mae: 0.0519 - val_los
Epoch 68/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0047 - mae: 0.0532 - val_los
Epoch 69/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0052 - mae: 0.0555 - val_los
Epoch 70/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0056 - mae: 0.0564 - val_los
Epoch 71/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0050 - mae: 0.0555 - val_los
Epoch 72/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0044 - mae: 0.0522 - val_los
Epoch 73/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0041 - mae: 0.0499 - val_los
Epoch 74/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0044 - mae: 0.0529 - val_los
Epoch 75/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0041 - mae: 0.0496 - val_los
Epoch 76/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0043 - mae: 0.0521 - val_los
Epoch 77/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0038 - mae: 0.0486 - val_los
Epoch 78/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0036 - mae: 0.0467 - val_los
Epoch 79/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 5ms/step - loss: 0.0036 - mae: 0.0471 - val_los
Epoch 80/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 5ms/step - loss: 0.0033 - mae: 0.0449 - val_los
Epoch 81/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 5ms/step - loss: 0.0045 - mae: 0.0521 - val_los
Epoch 82/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 5ms/step - loss: 0.0041 - mae: 0.0506 - val_los
Epoch 83/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 5ms/step - loss: 0.0036 - mae: 0.0473 - val_los
Epoch 84/100
```

```
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 7ms/step - loss: 0.0039 - mae: 0.0494 - val_los
Epoch 85/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 5ms/step - loss: 0.0037 - mae: 0.0479 - val_los
Epoch 86/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 5ms/step - loss: 0.0038 - mae: 0.0482 - val_los
Epoch 87/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0035 - mae: 0.0478 - val_los
Epoch 88/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0035 - mae: 0.0467 - val_los
Epoch 89/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0035 - mae: 0.0461 - val_los
Epoch 90/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0033 - mae: 0.0468 - val_los
Epoch 91/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0034 - mae: 0.0457 - val_los
Epoch 92/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0032 - mae: 0.0442 - val_los
Epoch 93/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0039 - mae: 0.0508 - val_los
Epoch 94/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0035 - mae: 0.0476 - val_los
Epoch 95/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0033 - mae: 0.0458 - val_los
Epoch 96/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0031 - mae: 0.0442 - val_los
Epoch 97/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0033 - mae: 0.0469 - val_los
Epoch 98/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0033 - mae: 0.0462 - val_los
Epoch 99/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 4ms/step - loss: 0.0033 - mae: 0.0464 - val_los
Epoch 100/100
50/50 ━━━━━━━━━━━━━━━━━━━━ 0s 3ms/step - loss: 0.0034 - mae: 0.0472 - val_los
```


Training Performance
Training Loss
Validation Loss
Loss (MSE)