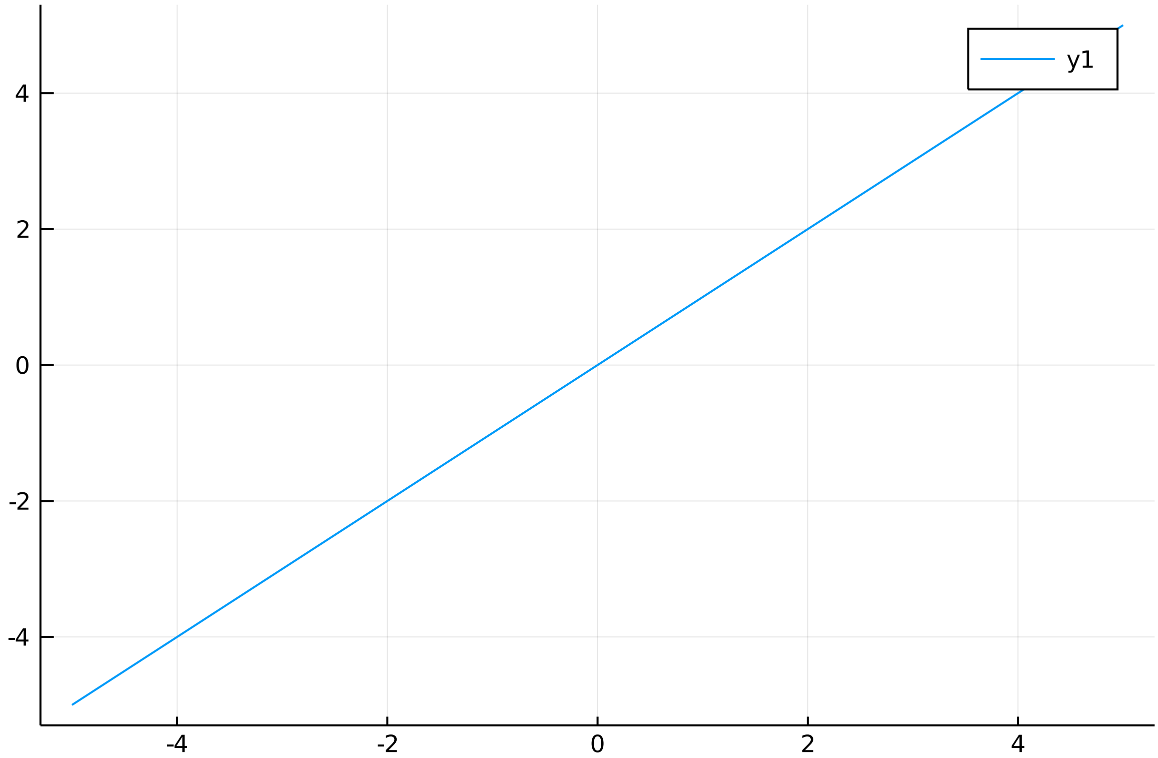
Equation of a straight line



$y = mx + c$

The equation given is the equation for a straight line.

- **m** -> slope
- **c** -> y-intercept
- **x** -> x-coordinate.

Points to be noted

1. Slope is same everywhere on a line
2. Two parellel lines have the same slope
3. Intersecting lines have different slopes

General form of the equation of a straight line

$ax + by + c = 0$

Rearranging to get it into the slope - intercept form gives:

$y = \frac{-c-ax}{b} = \frac{-a}{b}x + \frac{-c}{b}$

Here, m = $\frac{-a}{b}$ & $c = \frac{-c}{b}$

If we generalize the equation to *n* dimensions

$w_1x_1 + w_2x_2 + \dots + w_0 = 0$

$x_1, x_2, \dots$

$w_1, w_2, \dots$

Equation of a plane

$\pi : ax + by + cz + d = 0$

$L : ax + by + c = 0$

π is used to represent plane

We can also write it as a more general notation:

$\pi : w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0$

Equation of a hyperplane

When the number of dimensions is greater than 2, then we call it **hyper-plane** It is represented by: π_d

It's equation is:

$\pi_d : w_1x_1 + w_2x_2 + \dots + w_dx_d + w_0 = 0$

Vectors in Linear Algebra

Below picture is what a vector is. It has a magnitude and a direction.

The magnitude of the above vector is calculated by: $||\vec{u}|| = \sqrt{a^2 + b^2}$. This is a **2-d** vector.

For a **'d'** dimensional vector the formula is generalized to: $||\vec{x}|| = \sqrt{x_1^2 + x_2^2 + \dots + x_d^2}$

The equation of a hyperplane: $\pi_d : w_1x_1 + w_2x_2 + \dots + w_dx_d + w_0 = 0$

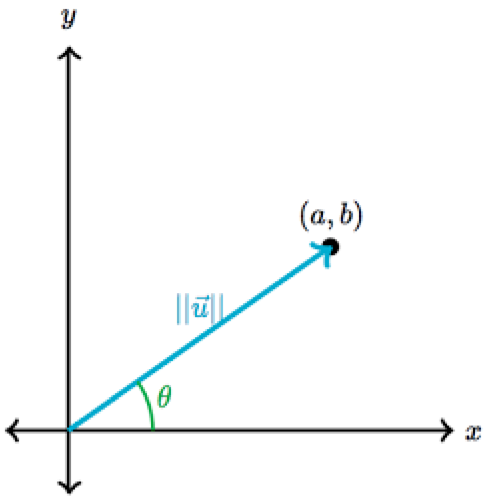
can be rewritten in a matrix form using Linear algebra:

$$[w_1 \dots w_d] \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix} + w_0 = 0$$

$W \in R^d$, which means **W** is a d-dimensional vector and all it's values are real numbers. w_0 is a scalar.

The Vector with all w s is a row vector & the vector with all x s is a column vector In maths, when we say vector we mean a **column vector** by convention

We can write this in a more concise way: $W^T x + w_0 = 0$



Dataset

5 rows × 5 columns

	Id Int64	SepalLengthCm Float64	SepalWidthCm Float64	PetalLengthCm Float64	PetalWidthCm Float64
1	1	5.1	3.5	1.4	0.2
2	2	4.9	3.0	1.4	0.2
3	3	4.7	3.2	1.3	0.2
4	4	4.6	3.1	1.5	0.2
5	5	5.0	3.6	1.4	0.2

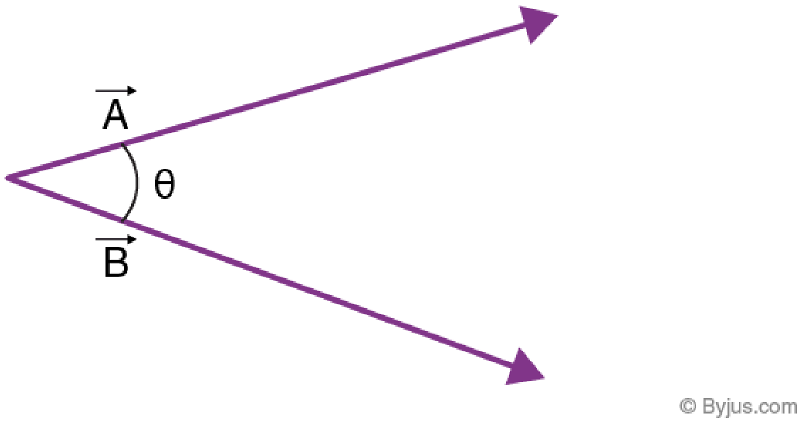
The above is the example of a dataset. This one is the famous Iris dataset.Iris dataset contains 4 features(*SepalLengthCm, SepalWidthCm, PetalLengthCm, PetalWidthCm*) and a target(3 types of flowers)
Each row has 4 columns representing the features and the last column is the target(the thing we want to predict)

We can have a concise mathematical representation of a dataset as:

$$D = \{(x_i, y_i); x_i \in R^d, y_i \in c_1, c_2\}$$

x_i represents one feature vector
y_i represents one target vector

Angle between Vectors



Here we need the notion of a dot product. This is taken from *Wikipedia*

> In mathematics, the dot product or scalar product is an algebraic operation that takes two equal-length sequences of numbers (usually coordinate vectors), and returns a single number. In Euclidean geometry, the dot product of the Cartesian coordinates of two vectors is widely used. It is often called "the" inner product (or rarely projection product) of Euclidean space, even though it is not the only inner product that can be defined on Euclidean space (see Inner product space for more).

Algebraically, the dot product is the sum of the products of the corresponding entries of the two sequences of numbers. Geometrically, it is the product of the Euclidean magnitudes of the two vectors and the cosine of the angle between them.

From the last line of the definition,

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos \theta = \vec{A}^T \vec{B}$$

After rearranging,

$$\theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} \right)$$

If θ is 0 then, vectors are perpendicular

Special cases of equation of a plane

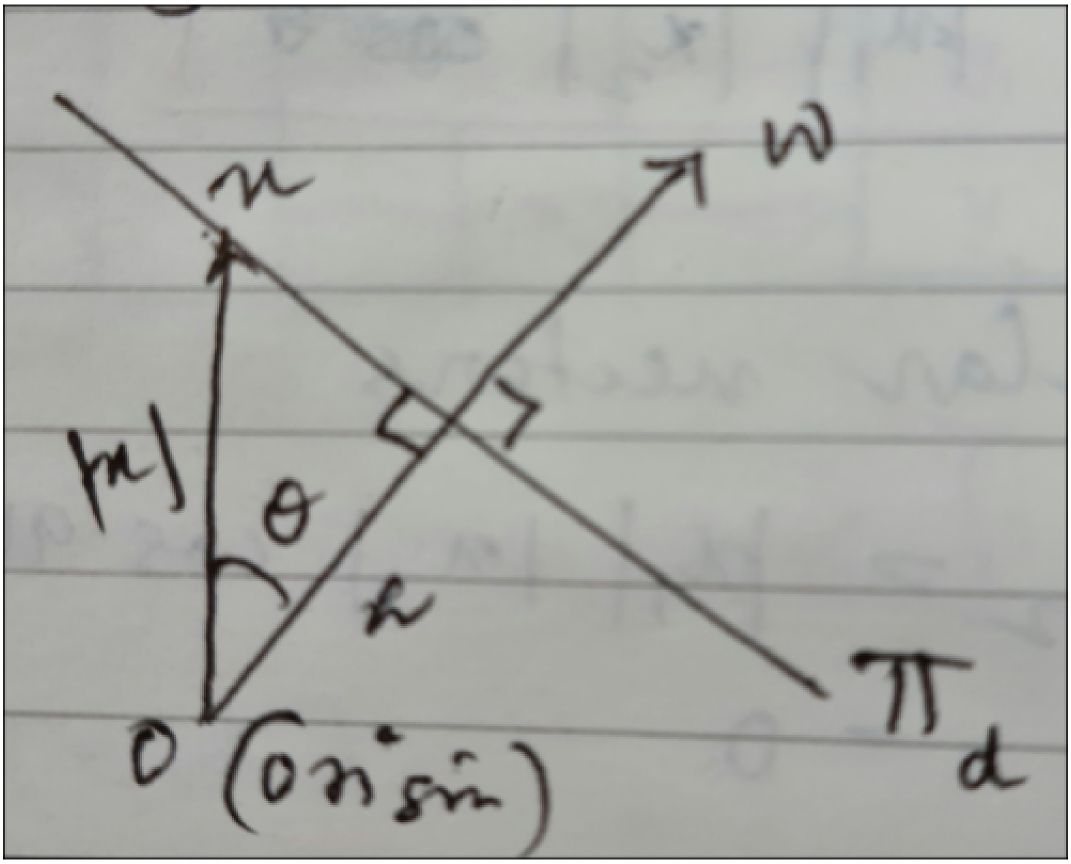
One more concept we need is the concept of a unit vector. A unit vector is a vector whose magnitude is 1. Represented by $\hat{x}$ called the **"hat"** symbol.

1. Passing through origin

Equation recap: $\pi : w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0$

for the plane to pass through origin, $x_1, x_2, x_3 = 0$. Plugging these values in the equation we have: $w_0 = 0$. This is the equation of plane passing through origin

2. Not passing through origin



Equation of plane in vector form:

$w^T x + w_0 = 0$
$=> ||w||||x|| \cos\theta = 0$
$=> ||x|| \cos\theta + w_0 = 0$ (as w is a unit vector) $||x|| \cos\theta = a$ (from the figure)

So, finally $a = -w_0$

As you can see, a is the distance of the plane from the origin. So, what we see is the distance of a plane not passing through origin is $|w_0|$ provided w is a unit vector.

if w is not a unit vector then $a = \frac{|w_0|}{||w||}$

We can prove it like this:
$w \cdot x + w_0 = 0$
$=> ||w||||x|| \cos\theta + w_0 = 0$
$=> ||w||a + w_0 = 0$, as $||x|| \cos\theta = a$ (from the figure)
$=> a = \frac{-w_0}{||w||}$

but ofcourse we can do away with the -ve sign as distance can't be negative. So, $a = \frac{|w_0|}{||w||}$