

Ex1

swap(a,b)

{
temp = a; → 1
a = b; → 1
b = temp; → 1
}

Time

Space

a → 1

b → 1

temp → 1

$f(n) = 3$

$O(1)$ Constant

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$O(1)$ ^{words} Constant

$x = 5 * a + 6 * b \rightarrow 1 \text{ ms}$

$x = 5 * a + 6 * b \rightarrow 1 \text{ ms}$

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$x = 5 * a + 6 * b \rightarrow 1 \text{ ms}$

$f(n) = 4$

Constant

$O(1)$

→ Constant complexity

Big O

$\Theta(1)$
Average
case

$\Omega(1)$
Best
case

Ex

Swap(a,b)

{
temp = a; → 1 ms (stab)
a = b; → 1 ms
b = temp; → 1 ms
}

Time

Space

(memory)

a → 1 unit

b → 1 unit

temp → 1 unit

$f(n) = 3 \text{ units}$

↑
Constant

$O(1)$

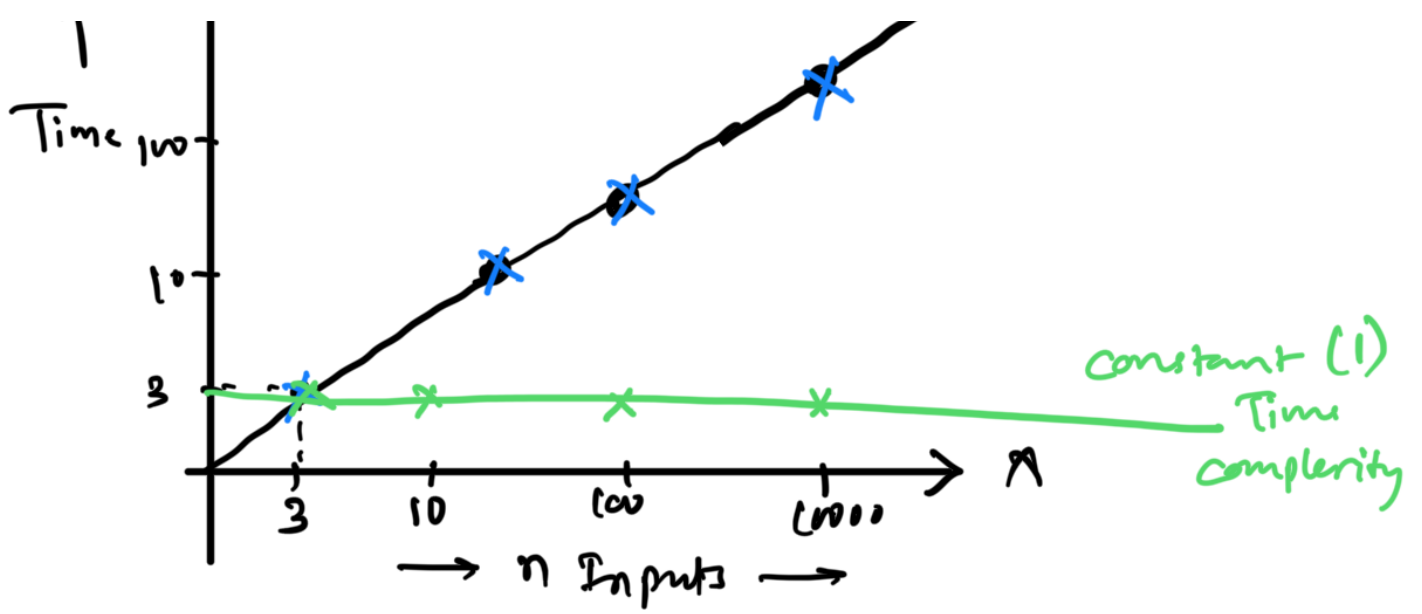
$f(n) = 3 \text{ ms}$

↑
Constant

Constant Time Comp $O(1)$

↑

Linear $O(n)$
Time Complexity



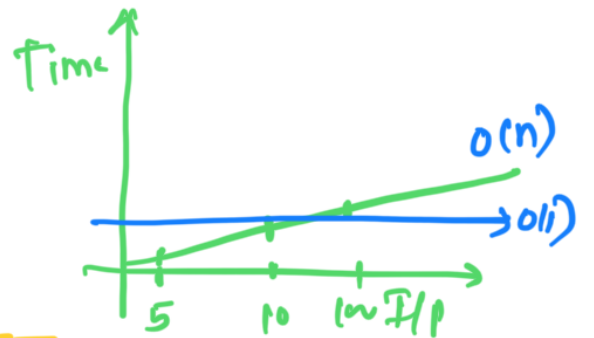
$3 \text{ sec} \leftarrow 3 \text{ inputs} \rightarrow 3 \text{ ms}$
 $3 \text{ sec} \leftarrow 10 \text{ inputs} \rightarrow 10 \text{ ms}$
 $3 \text{ sec} \leftarrow 100 \text{ inputs} \rightarrow 100 \text{ ms}$
 $3 \text{ sec} \leftarrow 10000 \text{ inputs} \rightarrow 10000 \text{ ms}$

Relⁿ betⁿ i/p value and time taken

Constant $O(1)$

Linear $O(n)$

Optimization



Ex:2

$i = 0, 1, 2, 3, 4, 5$ $A = [2, 3, 4, 5, 7]$
 $m = 5$

Algorithm: Sum(A, n)

```

{
    S = 0;
    for (int i = 0; i < n; i++)
    {
        S = S + A[i];
    }
    return S;
}

```

Annotations: $(n+1)$ for the for loop, n for the body, $n+1$ for the return statement.

Calculation: $0 + 2 = 2 + 3 = 5 + 4 = 21 = 7 + 14 = 5 + 9$

Space

$A \rightarrow n$
 $m \rightarrow 1$
 $i \rightarrow 1$
 $S \rightarrow 1$
 $f(n) = n + 3$
 \downarrow
 $O(n)$

$f(n) = 2n + 3$
 \downarrow
 $O(n)$

eg. $f(n) = 2n + 3$

Consider $O(n) + O(1)$

eg. $f(n) = 2n^2 + 3n + 7$

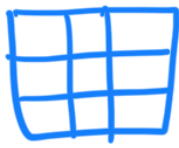
1. Ignore constants
2. Consider Time Complexity

Important Rule

$$= O(n^4) + O(n)$$

$$= O(n^2)$$

$n=3$



3x3

$$A = n \times n = 3 \times 3 = 3^2 \Rightarrow n^2$$

Ex 3 Algorithm

Add (A, B, n)

```

{
  for(i=0; i<n; i++)
  {
    for(j=0; j<n; j++)
    {
      C[i,j] = A[i,j] + B[i,j];
    }
  }
}

```

Time

Space

$A \rightarrow n^2$
 $B \rightarrow n^2$
 $C \rightarrow n^2$
 $n \rightarrow 1$
 $i \rightarrow 1$
 $j \rightarrow 1$

$$f(n) = 3n^2 + 3$$

$$O(n^2)$$

$$f(n) = 2n^2 + 2n + 1$$

$$= O(n^2)$$

square time complexity

$$\begin{aligned}
 f(n) &= n+1 + n(n+1) + n(n) \\
 \text{Calculation} &= n+1 + n^2 + n + n^2 \\
 &= 2n^2 + 2n + 1
 \end{aligned}$$

$A = n \times n$

$n=3$
4



$$4 \times 4 = 16$$

$$n^2$$

$$3 \times 3 = n^2$$

$A = m \times n$

$A = m \times n$

$B = m \times n$

$C = m \times n$



$$\begin{aligned}
 f(n) &= 3mn + 3 \\
 &= 3(n \times n) \\
 &= O(n^2)
 \end{aligned}$$

Ex: 4

mul (A, B, n)

{ for (i=0; i < n; i++) ————— $n+1$

{ for (j=0; j < n; j++) ————— $n \times (n+1)$

{ c[i,j]=0; ————— $n \times n$

for (k=0; k < n; k++) ————— $n \times n \times (n+1)$

{
 c[i,j] = c[i,j] + A[i,k] + B[k,j]; ————— $n \times n \times n$
}

}
}
}

$$\begin{aligned} f(n) &= (n+1) + n(n+1) + n^2 + n^2(n+1) + n^3 \\ &= n+1 + n^2+1 + n^2 + n^3 + n^2 + n^3 \\ &= \cancel{2n^3} + \cancel{3n^2} + \cancel{2n+1} \\ &= O(n^3) \rightarrow \text{Time Complexity} \end{aligned}$$

space

A $\rightarrow n^2$
B $\rightarrow n^2$
C $\rightarrow n^2$
n $\rightarrow 1$
i $\rightarrow 1$
j $\rightarrow 1$
k $\rightarrow 1$

$$f(n) = \cancel{3n^2} + \cancel{4} \\ O(n^2)$$

Remember:

for (i=0; j < n, i++) ————— $n+1$
 {
 stmt ————— n
 }
}

$\rightarrow O(n)$

$$O(n)$$

Ex 2

for ($i=0$; $i > n$; $i--$)
 {
 st;
 }

$\longrightarrow O(n)$

Ex 3

for ($i=0$; $i < n$; $i=i+20$)
 {
 ~~st;~~
 }

$\longrightarrow \frac{n}{2} \quad O(\frac{n}{2})$
 $O(n)$

~~4 6 8 10~~
~~2~~ 10

0 }
 2 }
 4 } (5)
 6 }
 8 }

for ($i=0$; $i < n$; $i=i+2$) $\longrightarrow (n+1)$
 {
 st;
 }

$\longrightarrow \frac{n}{2}$

$$\begin{aligned} f(n) &= n+1 + \frac{n}{2} \\ &= \frac{3n+1}{2} \\ &= O(n) \end{aligned}$$

Ex for ($i=0$; $i < n$; $i=i+20$) $\longrightarrow n+1$
 {
 st;
 }

$\longrightarrow \frac{n}{20}$

$$f(n) = O(n)$$