SCALARS and VECTORS:

1. The scalar(dot) product of two vectors $\vec{A}$ and $\vec{B}$ making an angle θ between them is
   $\vec{A}\,.\vec{B}$ = A B Cos ,where A & B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$ respectively.

Note: $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k}$=1 and $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath}$ =0 ,where $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vectors

1. The vector(cross) product of two vectors $\vec{A}$ and $\vec{B}$ making an angle θ between them is
   $\vec{A}\; x\; \vec{B}$ = AB Sin$\theta\,\hat{n}$ where A & B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$ respectively.

Note: $\hat{\imath}\; x\; \hat{\imath} = \hat{\jmath} x \hat{\jmath} = \hat{k}\; x\; \hat{k} = 0$

$\hat{\imath}\; x\; \hat{\jmath} = \hat{k}$, $\hat{\jmath}\; x\; \hat{\imath} = -\hat{k}$, $\hat{\jmath}\; x\; \hat{k} = \hat{\imath}$, $\hat{k}\; x\; \hat{\jmath} = -\hat{\imath}$ $\hat{k}\; x\; \hat{\imath} = \hat{\jmath}$, $\hat{\imath}\; x\; \hat{k} = -\hat{\jmath}$,

Field is a set of functions of the co-ordinates of a point in space independent of time.

Field can be expressed by a scalar or vector

Scalar Field is the region in space in which a scalar quantity has unique value or magnitude at every point .

Ex: Electric potential, Temperature etc

Vector Field is the region in space in which a vector quantity has unique magnitude and direction at every point.

Ex: Gravitational field intensity , (3D)Electric field intensity, (3D)Magnetic field intensity etc 3D(3dimensional) vector fields are expressed by three components of the vector along
3 co-ordinate axes.

The $(\boldsymbol{Del})$ $\boldsymbol{perator}$:

In mathematics the operator ∇ is used called "Del / Nabla".
When this acts on a scalar quantity, it indicates to differentiate that quantity.
The operator is given by $\nabla = \hat{\imath}\frac{\delta}{\delta x} + \hat{\jmath}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}$ ....(1)
If φ be a scalar function ,then $\nabla\varphi$ states that the ∇ acts/operates on φ.

'∇' can act in three ways namely,

- When ∇ acts on a scalar function φ,then $\boldsymbol{\nabla\varphi}$ **is called Gradient**.
- When ∇ acts on a vector function $\vec{f}$,then *the dot prtoduct* $\boldsymbol{\nabla}.\vec{\boldsymbol{f}}$ **is called Divergence**.
- When ∇ acts on a vector function $\vec{f}$,then *the cross/vector prtoduct* $\boldsymbol{\nabla x}\vec{\boldsymbol{f}}$ **is called Curl.**

Gradient :

Gradient of a continuously differential scalar point function $\emptyset$(x,y,z) is defined as grad $\emptyset = \boldsymbol{\nabla}\emptyset = \hat{\imath}\frac{\partial\emptyset}{\partial x} + \hat{\jmath}\frac{\partial\emptyset}{\partial y} + \hat{k}\frac{\partial\emptyset}{\partial z}$, $\nabla\emptyset$ is a vector of 3 successive components $\frac{\partial\emptyset}{\partial x}, \frac{\partial\emptyset}{\partial y}, \frac{\partial\emptyset}{\partial z}$

Note: 1. If $\emptyset$ is a scalar point function, then $\boldsymbol{\nabla}\emptyset$ **is a vector point function**.

2. $\nabla\emptyset \neq \emptyset$(there is no ' .' or 'x ' between ∇ *and* $\emptyset$

Physical interpretation of Gradient:

The gradient at any point P(x,y,z) of a scalar point Function $\emptyset$(x,y,z) is a vector normal to the level surface of $\emptyset$(x,y,z) and passes through the point P(x,y,z)

The magnitude of the gradient is the rate of change of $\emptyset$(x,y,z) in the direction of the normal to the level surface at the point P(x,y,z).

Ex: The temperature is different at different places in a room. Gradient determines this difference.(Gradient operates on a scalar field temperature T(x,y,z) and gives grad T(x,y,z) as a vector field.

Gradients are important part of life:

- The roof of a house ,Tennis courts, roads, football and cricket grounds are made with a gradient to assist drainage.

Divergence of a Vector point function:

If $\vec{E}$(x,y,z) is a continuously differentiable vector point function, then divergence of $\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k}$ is defined as

div $\vec{E} = \nabla . \vec{E} = (\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}). (E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k})$

div $\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$ $\because\ \hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$ and $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

Thus the Divergence of a vector function is a scalar function.

- Divergence of a vector point function $\vec{E}$ is a scalar quantity. It is a scalar product of operator ∇ with vector $\vec{E}$
- Divergence is an operator, so $\nabla.\vec{E} \neq = \vec{E}.\nabla$
- If $\vec{E}$ is a constant vector, then div $\vec{E} = 0$
- If c is a constant ,then div(c$\vec{E}$) $= c\ div\ \vec{E}$

Physical interpretation of Divergence:

In general divergence of a vector point function representing any physical quantity gives at each point the rate per unit volume at which the physical quantity is moving towards or away from the point.

Curl of a Vector point Function:

If $\vec{H}$(x,y,z) is a continuously differentiable vector point function, then the curl of

$\vec{H} = H_x\hat{\imath} + H_y\hat{\jmath} + H_z\hat{k}$ is defined

as curl $\vec{H} = \nabla \times \vec{H} = (\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}) \times (H_x\hat{\imath} + H_y\hat{\jmath} + H_z\hat{k})$

$$\nabla \times \vec{H} = \hat{\imath} \times \frac{\partial (H_x\hat{\imath} + H_y\hat{\jmath} + H_z\hat{k})}{\partial x} + \hat{\jmath} \times \frac{\partial (H_x\hat{\imath} + H_y\hat{\jmath} + H_z\hat{k})}{\partial y} + \hat{k} \times \frac{\partial (H_x\hat{\imath} + H_y\hat{\jmath} + H_z\hat{k})}{\partial z}$$

$$= \left(\frac{\partial H_y}{\partial x}\hat{k} - \frac{\partial H_z}{\partial x}\hat{\jmath}\right) + \left(\frac{-\partial H_x}{\partial y}\hat{k} + \frac{\partial H_z}{\partial y}\hat{\imath}\right) + \left(\frac{\partial H_x}{\partial z}\hat{\jmath} - \frac{\partial H_y}{\partial z}\hat{\imath}\right)$$

$$= \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\hat{k}$$

$$\text{curl } \vec{H}\ or\ \nabla \times \vec{H} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$\because\ \hat{\imath}x\hat{\imath} = \hat{\jmath}x\hat{\jmath} = \hat{k}x\hat{k} = 0$

$\hat{\imath}x\hat{\jmath} = \hat{k},\ \hat{\jmath}x\hat{\imath} = -\hat{k},$

$\hat{\jmath}x\hat{k} = \hat{\imath},\ \hat{k}x\hat{\jmath} = -\hat{\imath}$

$\hat{k}x\hat{\imath} = \hat{\jmath},\ \hat{\imath}x\hat{k} = -\hat{\jmath},$

Physical interpretation of Curl

The physical significance of a curl operator is that it describes the rotation of the field $\vec{f}$ at a point.

Divergence of a Vector point function:

If $\vec{E}$(x,y,z) is a continuously differentiable vector point function, then divergence of $\vec{E} = E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z$ is defined as

$$\text{div } \vec{E} = \nabla \cdot \vec{E} = \left(\hat{a}_x\frac{\partial}{\partial x} + \hat{a}_y\frac{\partial}{\partial y} + \hat{a}_z\frac{\partial}{\partial z}\right) \cdot \left(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right)$$

$$\text{div } \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$\because \hat{a}_x\cdot\hat{a}_x = \hat{a}_y\cdot\hat{a}_y = \hat{a}_z\cdot\hat{a}_z = 1$ and $\hat{a}_x\cdot\hat{a}_y = \hat{a}_y\cdot\hat{a}_z = \hat{a}_z\cdot\hat{a}_x = 0$

Curl of a Vector point Function:

If $\vec{H}$(x,y,z) is a continuously differentiable vector point function, then the curl of $\vec{H} = H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z$ is defined

as $\text{curl } \vec{H} = \nabla \times \vec{H} = \left(\hat{a}_x\frac{\partial}{\partial x} + \hat{a}_y\frac{\partial}{\partial y} + \hat{a}_z\frac{\partial}{\partial z}\right) \times \left(H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z\right)$

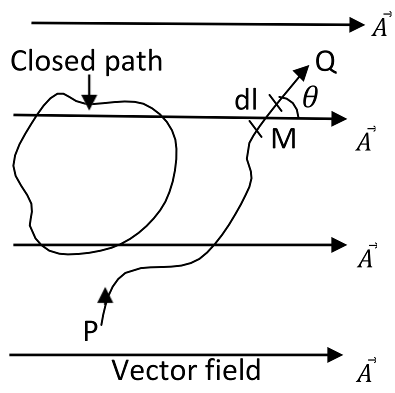
$$\text{curl } \vec{H} \; or \; \nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$\because \hat{a}_x \times \hat{a}_x = \hat{a}_y \times \hat{a}_y = \hat{a}_z \times \hat{a}_z = 0$

$\hat{a}_x \times \hat{a}_y = \hat{a}_z, \hat{a}_y \times \hat{a}_x = -\hat{a}_z, \hat{a}_y \times \hat{a}_z = \hat{a}_x, \hat{a}_z \times \hat{a}_y = -\hat{a}_x, \hat{a}_z \times \hat{a}_x = \hat{a}_y, \hat{a}_x \times \hat{a}_z = -\hat{a}_y$

Different types of integrations viz linear ,surface and volume integrations.

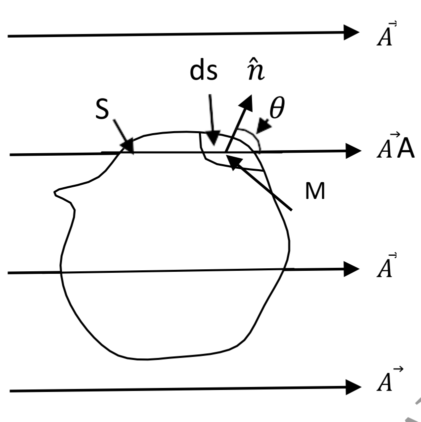
A Line(Path) Integral is an integral of a function along a path or a line.
Line integrals deals only with single variable. Let the curve be a straight line segment. Then the segment has definitely an initial and final point.
Ex: The line integral of a vector field is the amount of work that a force field does on a particle as it moves along a curve.



Consider a vector field $\vec{A}$ and a small element of length 'dl'
Along the path PQ at M and θ is the angle between $\vec{A}$ and dl
Then the line integral along the path PQ is $\int_P^Q \vec{A}.\vec{dl}$

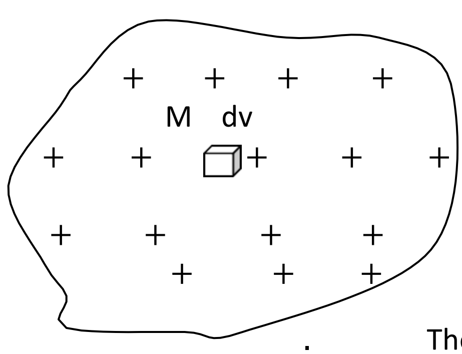
For a closed path the line integral is $\oint \vec{A}.\vec{dl}$
The symbol for closed contour line integral is $\oint$
The line integrals are used to find the potential difference
Two points in an electric field.

Surface Integral is a definite integral taken over a surface.
It is thought of as the double integral analog of a the line integral.
Given a surface,one may integrate over its scalar fields and vector fields.
Surface integrals have applications in Physics particularly with classical theory of electromagnetism.



$Consider\ a\ small\ area\ ds\ on\ the\ surface\ of\ area\ S\ surroundin$
$Point\ M$ in a vector field $\vec{A}$.Let $\hat{n}$ be the unit vector normal to ds.
ds $\hat{n}$ represent the area vector of ds.
Then the surface integral over the entire surface is $\int_S \vec{A}\,\vec{ds}$
The surface integral gives the net outward flux of the vector field through the surface.
The symbol for surface integral is $\int_S$
The surface integral for a closed surface is $\oint_S \vec{A}\,\vec{ds}$
The surface integrals are used to find the net electric flux through the surface in an electric field

Volume Integral refers to an integral over a 3D domain ie; it is a special case of multiple integral.
Volume integrals are important in Physics,because to calculate flux densities(Volume integrals of charge/mass densities gives the charge / mass of in that volume.

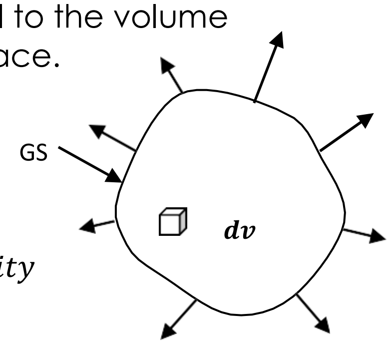


Consider a volume charge V distribution in which the $charges\ are\ distributed$ continuously. Let a small volume dv around a point M inside the charge $distribution$. If ρ be the density of charge at M which is $a\ scalar\ quantity$.
Then the net charge in the volume is given by the $volume\ integral \oint_V \rho\, dv$ or $\int_S \rho\ dv$

. The volume integral symbol is $\oint_V$

Derivation of Gauss divergence theorem .

Gauss Divergence Theorem states that the Surface integral of the normal flux density over any closed surface in an electric field is equal to the volume integral of the divergence of the flux enclosed by the surface.



Mathematically it is given by $\oint_S \vec{D}.\vec{ds} = \oint_V (\nabla.\vec{D})dv$

Consider a Gaussian surface S enclosing a volume V.

Let a charge dQ is enclosed in a small volume dv inside Gaussian surface.

$If\ \rho$ is charge density $inside\ the\ volu, then\ the\ charge\ density$ associated with the volume is given by $\rho = \frac{dQ}{dv}$

ie; dQ = ρ dv

Then the total charge enclosed by the Gaussian surface is given by

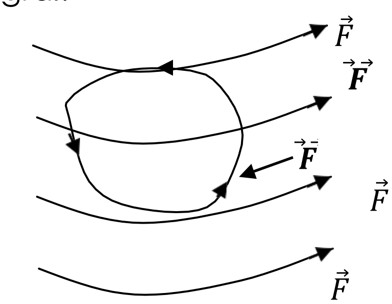
Q = $\oint_v dQ = \oint_v \rho\ dv$ but $= .\vec{D}$

$\therefore$ Q = $\oint_v (\nabla.\vec{D})dv$ .....(1)

According to Gauss' law of electrostatistics,Q = $\oint_S \vec{D}.\vec{ds}$ .....(2)

From equations 1 & 2 ,we get $\oint_S \vec{D}.\vec{ds} = \int_V \nabla.\vec{D}$, This is Gaussian Divergence theorem which relate surface integral with volume integral.

Mention Stoke's theorem:

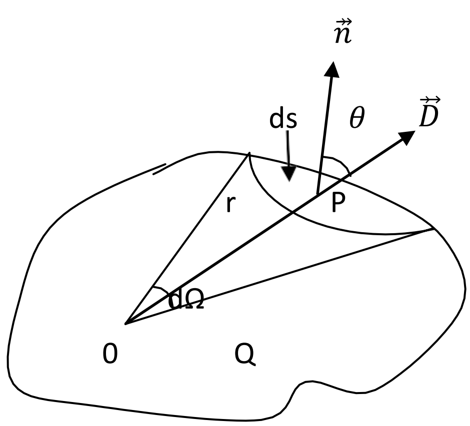


Stoke's theorem states that the surface integral of curl of $\vec{F}$ closed loop throughout a chosen surface is equal to the circulation of $\vec{F}\ around\ theboundary$ of chosen surface.

Mathematically ,$\int_S (\nabla x\vec{F}).\vec{ds} = \oint \vec{F}.\vec{dl}$

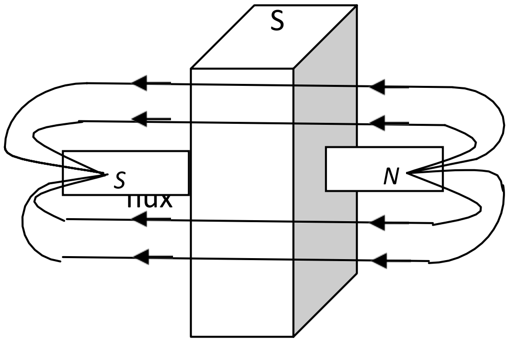
Gauss flux theorem in electrostatics :

Gauss law in electrostatics states that the total electric flux over any closed surface is $\frac{1}{\epsilon_o}$ times the total charge enclosed inside the surface.



Let q be the charge enclosed by a closed surface S. The closed surface is considered to be made up of a large number of elementary areas of each area dS. If $\vec{D}$ be the electric flux at P surrounded by ds and $\vec{n}$ the unit vector normal to ds, then the total flux over the entire surface is given by the surface integral , $\varphi = \oint_S \vec{D}.\vec{ds} = \frac{1}{\varepsilon_o}Q$

Gauss flux theorem in magnetism:



Consider a closed Gaussian surface S. Magnetic lines exist in closed loops. For a closed surface in a magnetic field ,the total inward magnetic flux in to the Gaussian surface is equal to the total outward flux so that the net flux through the Gaussian surface is zero and is given by $\oint_S \vec{B}.\vec{ds} = 0$ where $\vec{B}$ is magnetic flux density.

From Gauss divergent theorem we get

$\oint_S \vec{B}.\vec{ds} = \oint_V (\nabla.\vec{B})\, dv = 0$

Thus $\nabla.\vec{B} = 0$ which is Maxwell's equation.

Ampere's law ,

Ampere's law states that the circulation of magnetic field strength $\vec{H}$ along a closed path is equal to the net current enclosed by the surface,

Mathematically $\oint \vec{H}.\vec{dl} = I \quad \ldots(1)$

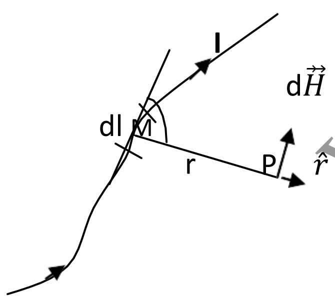
By applying Stoke's law we get $\int_S (\nabla x\vec{H}).\vec{ds} = \oint \vec{H}.\vec{dl} \quad \ldots(2)$

The equation for I could be obtained as I= $\oint_S \vec{J}.\vec{ds} \quad \ldots(3)$

From equation 2&3 we get $\int_S (\nabla x\,\vec{H}).\vec{ds} = \oint_S \vec{J}.\vec{ds}$

Thus we get the Ampere's law $\nabla x\,\vec{H} = \vec{J}$

Biot-Savart law



Consider a small element 'dl' of the conductor carrying current I Let P be the point at a distance r from its centre making an an angle θ with the midpoint Mof the element and $\hat{r}$ the unit vector along MP.

Biot-Savart law states that the magnitude of magnetic field dH at P varies :-

1. directly as the length of the element (dl)
2. directly as the current through the element(I)
3. directly as the sine of the angle(θ) made by r with the element.
   inversely as the square of the distance r(r^2)

Mathematically , dH $\alpha \frac{I\,dl\,sin\theta}{r^2}$

$= (\frac{1}{4\pi})\frac{I\,dl\,sin\theta}{r^2}$ where $\frac{1}{4\pi}$ is proportionality constant.

In vector form , $\vec{dH} = \frac{I\,\vec{dl}x\,\hat{r}}{4\pi r^2}$

$\vec{dH}$ acts at P perpendicular to the plane containing the element and vector $\vec{r}$.

Faraday's laws of EMI.

EMI(Electro Magnetic Induction) is the phenomenon in which an emf and hence current is induced in a loop when the magnetic flux linked with the loop changes.
The emf (electr-motive force) is called the induced emf and the current is called Induced Current.

Faraday's laws of EMI:

First Law states that whenever the magnetic flux linked with a(closed) circuit/conductor changes, an emf is always induced in it.
Second Law states that the magnitude of the emf induced is directly proportional to the rate of change of magnetic flux (linked with the loop)
E or e $= -\frac{d\emptyset}{dt}$,where $\emptyset$ is the magnetic flux linked
For N turns , e $= -N\frac{d\emptyset}{dt}$
($-$ *ve sign indicates that the direction of induced emf is opposite to that* of the current.
(EMI implies that an electric current is induced by a time varying magnetic field)
Faraday's law in integral and differential form:
WKT induced emf , e $= -\frac{d\emptyset}{dt}$ ....(1)
The rate of change of magnetic flux is given by $\frac{d\emptyset}{dt} = \int_S \frac{\partial \vec{B}}{\partial t}.\vec{dS}$ ......(2)
The induced emf in the circuit is given by e $= \oint \vec{E}.\vec{dl}$ ....(3)
From eqns 1,2&3,we get $\oint \vec{E}.\vec{dl} = -\int_S \frac{\partial \vec{B}}{\partial t}.\vec{dS}$
But from Stoke's law , $\oint \vec{E}.\vec{dl} = \int_S (\nabla x\vec{E}).\vec{dS}$
$\therefore \int_S (\nabla x\vec{E}).\vec{dS} = -\int_S \frac{\partial \vec{B}}{\partial t}.\vec{dS}$
Thus , $\nabla x\vec{E} = -\frac{\partial \vec{B}}{\partial t}$, This is Farday's law in differential form ,which is Maxweel's equation

Lenz's Law states that the direction of the induced emf is such that it oppose the change to which it is due. Thus Lenz's law give the direction of induced emf.

Equation of continuity:

Principle of conservation of charge states that the net charge in an isolated system remains constant. ie: The time rate of increase(decrease) of charge with in a closed volume is equal to the net rate of flow of charge into(out of) the volume.This conservation of charge is given by the equation of continuity.

Let d$\vec{S}$ be the small element on the surface 'S' enclosing a volume'V'. If $\vec{J}$ be the current density at a point on the element surface, then the current leaving the volume V bounded by the surface dS is given by

I= $\oint_S \vec{J}.d\vec{S}$ .....(1) If current is not steady, then J is the function of t(x,y,z) and $\oint_S \vec{J}.d\vec{S}$ represent the rate at which charge leave the enclosed volume.

Using divergence theorem,eqn 1 is written as I= $\oint_S \vec{J}.d\vec{S} = \oint_V \nabla.\vec{J}.dv$ ....(2)

As some charge leaves the volume ,the amount of charge decreases within the volume and is given by

$$I = -\frac{dQ}{dt}$$

$$= -\frac{d}{dt}\int_V \rho dv \quad \text{Since } Q = \int_V \rho dv$$

$$= -\int_V \frac{\delta\rho}{\delta t} dv \quad ......(3)$$

From eqn 2&3 ,we get , $\oint_V \nabla.\vec{J}.dv = -\int_V \frac{\delta\rho}{\delta t} dv$

$$\int_V [\nabla.\vec{J}.+\frac{\delta\rho}{\delta t}]\, dv = 0$$

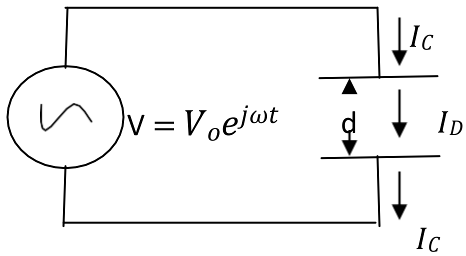
ie ; $\boldsymbol{\nabla.\vec{J}.+\frac{\delta\rho}{\delta t}=0}$ ....(4) This is known as the equation of continuity.

$\boldsymbol{\nabla.\vec{J}} = -\frac{\delta\rho}{\delta t}$ This implies that charge cannot flow away from the given volume without decrease of charge existing within the volume.

Definition of displacement current , $\boldsymbol{I_D}$:

Displacement current is the correction factor in Maxwell's equation that appear in time-varying condition, but doesn't describe any movement of charges though it has an associated magnetic field.

Derivation of an expression for displacement current , I_D:



Consider an AC circuit with a capacitor to which an AC voltage V $= V_o V_o e^{j\omega t}$ is applied. Let A be the area of each plate and d the separation between the plates.

The displacement current in terms of displacement current density is given by $= (\frac{\partial \vec{D}}{\partial t}) \cdot A$ ……..(1)

If D is electric field density ,then $\vec{D} = \epsilon \vec{E}$ …..(2)

Also electric field ,$\vec{E} = \frac{V}{d}$ …..(3)

where the applied voltage , V $= V_o e^{j\omega t}$….(4)

From eqns 2,3 &4 ,we get $\vec{D} = \frac{\epsilon}{d} V_o e^{j\omega t}$ ….(5)

Also from eqns 1 &5 ,we get $I_D = \frac{\partial}{\partial t}(\frac{\epsilon}{d} V_o e^{j\omega t}) \cdot A$

On differentiation we get , $\boldsymbol{I_D = \frac{j\omega\epsilon A}{d} V_o e^{j\omega t}}$

This is the expression for displacement current.

Maxwell's Ampere law.

From Gauss law WKT , $\nabla . \vec{D} = \rho$

Differentiating w.r.t time , we have $\frac{\partial}{\partial t}(\nabla . \vec{D}) = \frac{\partial \rho}{\partial t}$ or $\nabla (\frac{\partial \vec{D}}{\partial t}) = \frac{\partial \rho}{\partial t}$…(1)

but from equation of continuity , $\nabla . \vec{J} = -\frac{\delta \rho}{\delta t}$ …(2)

∴ from eqns 1 ans 2 , we get $\nabla . \vec{J} = -\nabla (\frac{\partial \vec{D}}{\partial t})$ or $\nabla . (\vec{J} + \frac{\partial \vec{D}}{\partial t}) = 0$ …(3)

Thus for time varying case equation (3) to be considered but not $\nabla . \vec{J} = 0$

ie. $\vec{J}$ must be replaced by $\vec{J} + \frac{\partial \vec{D}}{\partial t}$

∴ Ampere circuital law in point form becomes $\boldsymbol{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$, this equation is called Maxwell-Ampere's law.

Maxwell's equations in differential form and in vacuum.

SL.NO	Applicable law	Maxwell's equations in Differential form		
		Medium	Vacuum	For static field
1	Gauss law	$\nabla.\vec{D} = \rho$	$\nabla.\vec{D} = 0$	$\nabla.\vec{D} = \rho$
2	Gauss law for mag. Field	$\nabla.\vec{B} = 0$	$\nabla.\vec{B} = 0$	$\nabla.\vec{B} = 0$
3	Faraday's law	$\nabla x\vec{E} = -\frac{\partial\vec{B}}{\partial t}$	$\nabla x\vec{E} = -\frac{\partial\vec{B}}{\partial t}$	$\nabla x\vec{E} = 0$
4	Ampere's law	$\nabla x\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$	$\nabla x\vec{H} = \frac{\partial\vec{D}}{\partial t}$	$\nabla x\vec{H} = \vec{J}$

Where $\vec{E}$ = Electric field Intensity(V/m)

$\vec{H}$ = Magnetic field intensity(A/m)

$\vec{J}$ = Electric current density(A/m^2)

$\vec{D}$ = Electric flux density (C/m^2)

$\vec{B}$ = Magnetic field density(Wb/m^2 or T)

ρ = Electric charge density (C/m^3)

For free space $\rho = 0, J = 0$ and $\mu_0 = 1, \varepsilon_0 = 1$

EM Waves:

Electromagnetic wave is the wave produce due to mutually perpendicular oscillating Electric fields and Magnetic fields perpendicular to the direction of propagation of the wave.

Derivation of wave equation in terms of electric field using Maxwell's equations.

Consider Maxwell's equations

$\nabla x\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$ and $\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

Substituting for D = εE and B = μH

We get ,$\vec{H} = \vec{J} + \frac{\varepsilon\partial\vec{E}}{\partial t}$ ……….(1)

$$\nabla x\vec{E} = -\frac{\mu\partial\vec{H}}{\partial t} \quad \ldots\ldots\ldots\ldots(2)$$

Taking curl of eqn(2) on both sides ,we get

$$\nabla\ x\ \nabla x\vec{E} = -\frac{\mu\partial(\nabla x\vec{H})}{\partial t} \quad \ldots\ldots(3)$$

From vector analysis $\nabla\ x\ \nabla x\vec{E} = \ (\nabla.\vec{E}\ ) - \nabla^2\vec{E}$

$$= \nabla(\frac{\rho}{\varepsilon}) - \nabla^2\vec{E} \quad \ldots\ldots(4) \because \nabla.\vec{E} = \frac{\rho}{\varepsilon}$$

From eqn 3&4, we get, $\nabla(\frac{\rho}{\varepsilon}) - \nabla^2\vec{E} = -\frac{\mu\partial(\nabla x\vec{H})}{\partial t}$ ……(5)

From eqn 1 & 5,we get $\nabla(\frac{\rho}{\varepsilon}) - \nabla^2\vec{E} = -\frac{\mu\partial(\vec{J}+\frac{\varepsilon\partial\vec{E}}{\partial t})}{\partial t}$

On rearranging, $\boldsymbol{\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = \mu\frac{\partial\vec{J}}{\partial t} + \nabla(\frac{\rho}{\varepsilon})}$ …..(6)

LHS of the above equation is the characteristic form of wave equation and RHS represents sources responsible for the wave field(Charges & currents).

Thus equation (6) represents the wave equation in $\vec{E}$ for a homogeneous isotropic medium with μ and ε

For Free spac , $\rho = 0$ and $J\ = 0$

$$\therefore \quad \boldsymbol{Wave\ equation\ for\ free\ space\ is\quad \nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0,}$$

Mention of plane electromagnetic waves in vacuum along with the equations for E , B and C in terms of μ_o and ε_o and E and B.

Plane waves are waves that vary only in the direction of propagation and their characteristics remain constant across the planes normal to the direction of propagation.

Consider a plane EM wave of wavelength $\boldsymbol{\lambda}$ propagating in vacuum along x-axis.

Let $\vec{E}$ and $\vec{B}_z$ be the electric and magnetic fields associated with the wave of amplitude A along y-axis and z-axis respectively and at any instant 't' they are given by

$$\vec{E}_y = A\cos[\frac{2\pi}{\lambda}(x - Ct)]\ \hat{a}_y \quad \text{and} \quad \vec{B}_z = \frac{1}{c}A\cos[\frac{2\pi}{\lambda}(x - Ct)]\ \hat{a}_z$$

where c is the velocity of electromagnetic wave in vacuum = $3x10^8$m/s.

We can show that C = $\frac{|\vec{E}_y|}{|\vec{B}_z|} = \frac{E_y}{B_z}$

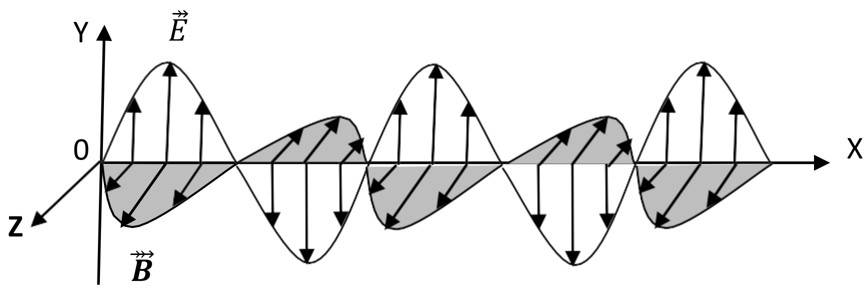
The velocity of the wave is given by v = $\frac{1}{\sqrt{\mu\varepsilon}}$

But for EM wave in vacuum, v = C, $\mu = \mu_0$ and $\varepsilon = \varepsilon_0$

$\therefore$ C = $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Transverse nature of Electromagnetic waves:

Electromagnetic waves are the waves produced by mutually perpendicular oscillating electric and magnetic field vectors, which are perpendicular to the direction of propagation. EM wave is graphically represented as follows.



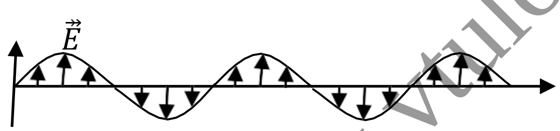
Electromagnetic waves are transverse in nature.

Consider an EM wave travelling along X-axis. If the time varying electric and magnetic fields are along Y and Z-axes respectively, then we have

$\vec{E}_y = A\cos[\frac{2\pi}{\lambda}(x - Ct)]\,\hat{a}_y$ and $\vec{B}_z = \frac{1}{c}A\cos[\frac{2\pi}{\lambda}(x - Ct)]\,\hat{a}_z$

Also the ratio of the amplitudes is given by $\frac{|\vec{E}_y|}{|\vec{B}_z|} = \frac{\boldsymbol{E_y}}{\boldsymbol{B_z}} = \boldsymbol{C}$

Representation of EM waves in polarization:

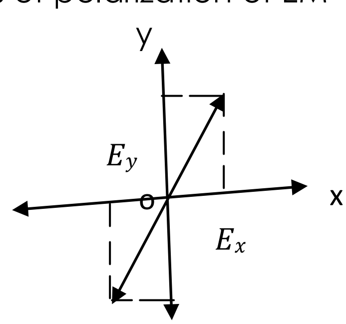


Polarisation of electromagnetic waves:

Electromagnetic wave is said to be polarized if the $\vec{E}$ vector oscillate in a plane along a straight line perpendicular to the direction of propagation of the wave ,as shown in the diagram.

Three types of polarization namely linear , circular and elliptical polarization of $\vec{E}$

Consider the electric field vector of the EM wave makes an angle (θ) with the x-axis.Then the electric vector can be resolved in to two mutually perpendicular vectors $\vec{E}_x$ $and\vec{E}_y$ along x and y-axes respectively. Based on the magnitudes and phase difference between $\vec{E}$ $d\vec{E}_y$ we have three types of polarization of EM waves ,namely:



- **Linear polarisarion** of EM waves
- **Circular polarization** of EM waves
- **Elliptical polarization** of EM waves

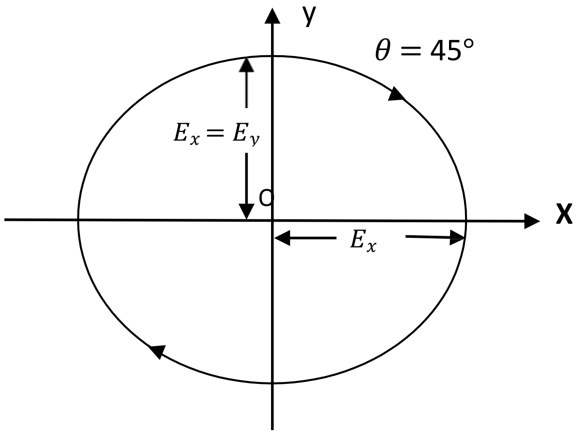
Linear polarisarion of EM waves :

EM wave is said to be linearly polarized if the $\vec{E}$ $d\vec{E}_y$ components must be in phase

and their amplitudes may or may not be equal
If $E_1\ and\ E_2$ be the amplitudes of $E_x\ and\ E_y$ respectively , then for linear polarization phase difference $\delta\ = 0$ and $\frac{E_x}{E_y} = \frac{E_1}{E_2}$

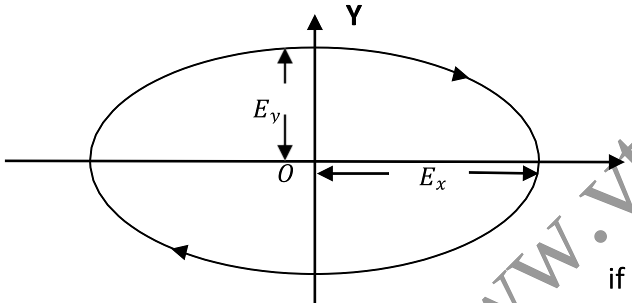
Thus tip of the vector $\vec{E}$ traces straight line.

Circular polarization of EM waves :



EM wave is said to be circularly polarized if the $\vec{E}\ d\vec{E}_y$ components must have same amplitude and a phase difference of 90°.
If $E_1\ and\ E_2$ be the amplitudes of $E_x\ and\ E_y$ respectively , then for circular polarization phase difference $\delta\ = 90°$, $E_1\ =\ E_2\ and\ \frac{E_x}{E_y} = \frac{E_1}{E_2}$
$E_x^2 + E_y^2 = E_1^2$, Thus tip of the vector $\vec{E}$ traces a circle.

Elliptical polarization of EM waves :



EM wave is said to be elliptical polarized if the $\vec{E}\ d\vec{E}_y$ components must have unequal amplitude and a phase difference $\delta \neq 0$. If $_1\ and\ E_2$ be the amplitudes of X $E_x\ and\ E_y$ respectively , then for elliptical polarization phase difference $\delta\ \neq 0, E_x\ \neq E_y$
if $\partial = \frac{\pi}{2}$,then $(\frac{E_x}{E_1})^2\ + (\frac{E_y}{E_2})^2\ = 1$,Thus tip of the vector $\vec{E}$ traces an ellipse.

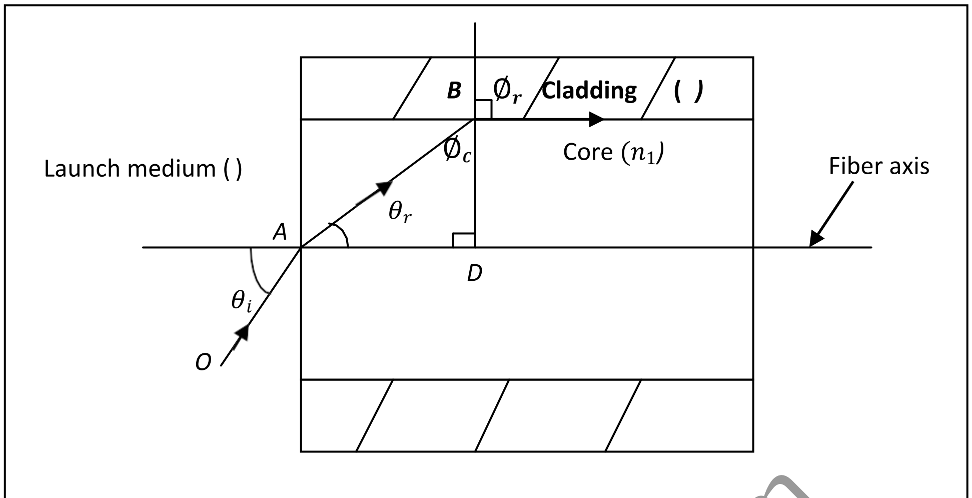
OPTICAL FIBRES

Q:What is an Optical Fiber?

Optical Fiber is a transparent di-electric material (like glass/plastic) which guides/ carry) light along it based on the principle of total reflection of light.

Optical fiber consists of a cylindrical transparent di-electric material of high refractive index called core. It is surrounded by another di-electric transparent material of low refractive index called cladding Cladding in turn is surrounded by cylindrical insulator called Sheath, which gives mechanical strength & protect the fiber from absorption, scattering etc

Q:**What is an acceptance angle of an optical fiber ? Find the expression for acceptance angle or Numerical aperture.**



Acceptance angle is the maximum angle submitted by the ray with the axis of the fibre so that light can be accepted and guided along the fiber.

Let n_1 , n_2 & n_o be the RI of core, cladding and launch medium respectively. Also OA incident ray,AB refracted ray,BC totally reflected ray, θ_i& θ_r be the angles of incidence, refraction at A & $\emptyset$ & $\emptyset_r$ be tha angle of incidence and angle of refraction at B respectively.

By snell's law at A, $n_o \sin\theta_i = n_1 \sin\theta_r$ ⇨ $\sin\theta_i = \frac{n_1}{n_o}\sin\theta_r$ ……(1)

At $\emptyset = \emptyset_C$ Critical angle , $\emptyset_r = \mathbf{90°}$

∴ from Snell's law at B, $n_1 \sin\emptyset_C = n_2 Sin 90°$ or $\sin\emptyset_C = \frac{n_2}{n_1}$……(2)

But, from $\Delta^{le}\ ADB$, =(90°−$\emptyset_C$)⇨ $\sin\theta_r = \sin(90° - \emptyset_C)$=cos $\emptyset_C$ ….(3)

From eqns 1 &3 ,we get $\sin\theta_i = \frac{n_1}{n_o}c(\emptyset_C)$ …….(4)

From eqns 2&3,we get $\sin(\emptyset_C) = \frac{n_2}{n_1}$ and $\cos(\emptyset_C) = \sqrt{(1 - \sin^2\emptyset_C)} = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$ …(5)

From eqns 4 &5,we get $\sin\theta_i = \frac{\sqrt{n_1^2 - n_2^2}}{n_o}$

If $\theta_i = \theta_0$ is the maximum angle of incidence for which total internal reflection takes place,then θ_0 is called the acceptance angle.

$Then,$ $\boldsymbol{\sin\theta_0} = \frac{\sqrt{\boldsymbol{n_1^2 - n_2^2}}}{\boldsymbol{n_o}}$ for air $n_0 = 1$

The condition for propagation is $\boldsymbol{\theta_i} < \boldsymbol{\theta_0}$

ie; Angle of incidence must be less than acceptance angle.

Or

Sine of the incident angle must be less than numerical aperture.

ie; Sin $\theta_i <$ NA

Q: What is meant by numerical aperture(NA) and mention the expression for it.

Numerical aperture is the ability of the optical fiber to accept the light and guide along the fiber and is numerically equal to sine of the acceptance angle.

ie: NA = $sin\,\theta_o = \frac{\sqrt{n_1^2 - n_2^2}}{n_o}$

Q:What is meant by fractional refractive index change(Δ).

The ratio of the difference between refractive indices of core and cladding to that of core is called fractional refractive index change. ie: $\Delta = \frac{(n_1 - n_2)}{n_1}$

Q:What is attenuation and explain types of attenuation in optical fibres.

Loss of power of light signal as it is guided along the fiber is called attenuation.

Attenuation is measured in terms of dB/km.

There are three types of attenuations in the fiber namely:

1. **Absorption losses** are the losses due to impurities & material itself and they are two types namely

a) **Impurity losses** are the losses due to the impurities(Cu, Fe, etc) present in the fiber, which can be minimised by taking care during manufacture of the fiber.

b) **Intrinsic losses** are the losses due to the material itself, these losses decreases with the increase of wavelength.

2. **Scattering losses** are the losses due to imperfections of the fiber called Rayleigh scattering losses which varies inversely as the λ^4.

3. **Radiation losses** are the losses are two types namely:-

a) Microscopic losses are the losses due to non-linearity of the fiber axis ,which can be minimized by providing compressible jacket & taking care during manufacture of the fiber.

a) Macroscopic losses are the losses due to large curvature/bending of the fiber when it is wound over a spool/bent at corners. These losses increase exponentially up to threshold radius and there afterwards losses becomes large.

Expression for attenuation coefficient(α)

Consider an optical fiber of length 'L' and P_i & P_o be the input and output powers respectively.

The according to Lambert's law ,the rate of decrease of intensity $(-\frac{dP}{dL})$ with distance varies directly as Intensity (P)

ie; $-\frac{dP}{dL} \alpha P$,where – sign signifies that intensity decreases with distance.

$= \alpha$ P ,where α is constant called attenuation coefficient.

On rearranging we get, $\frac{dP}{P} = -\alpha dL$

Integrating on both sides ,between limits (P_i , P_o) and (0,L)we have,

$$\int_{P_i}^{P_o} \frac{dP}{P} = -\alpha \int_0^L dL$$

$$[logP]_{P_i}^{P_o} = -\alpha[L]_0^L$$

$$\therefore \log P_o - \log P_i = -\alpha L$$

$$\log\left(\frac{P_o}{P_i}\right) = -\alpha L \quad \text{or} \quad = -\frac{1}{L}\log\left(\frac{P_o}{P_i}\right) \quad \text{Bel/km}$$

$$\therefore \boldsymbol{\alpha = -\frac{10}{L}\log\left(\frac{P_o}{P_i}\right)} \quad \textbf{dB/km}$$

Note: $\frac{P_o}{P_i} = 10^{-\frac{\alpha L}{10}}$.

Q: What is V-Parameter/number ?

V-Parameter/number is the quantity which represent the number of modes of the fibre given by V = $\frac{\pi d}{\lambda}$NA = $\frac{\pi d}{\lambda}\frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

where d = diameter , λ = wavelength,

n_1, n_2 & n_0 RI's of Core, Cladding & medium

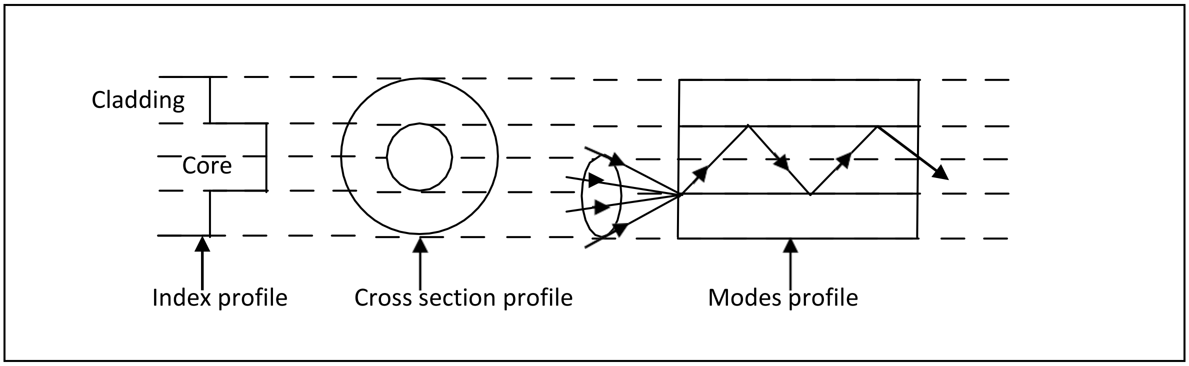
Q:What is meant by modes of propagation.

The paths along which the light is guided in the fiber are called modes of propagation and the number of modes of the fiber is given by N = $\frac{V^2}{2}$

Q: Explain the construction and working of Types of optical Fibers.

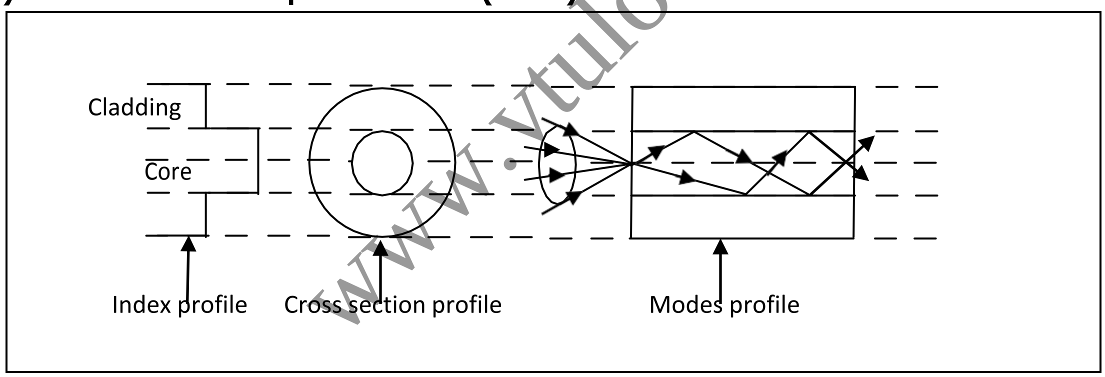
There are three types of optical fibers namely:-

1) Single mode step index fiber(SMF



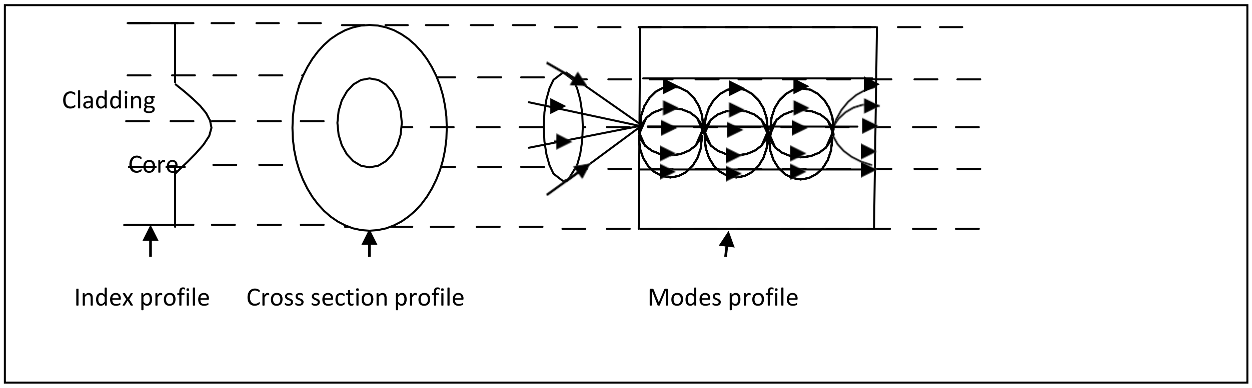
1. SMF core diameter = $8 - 10\mu m$ and cladding diameter = 60-70.
2. The refractive index, cross section and modes profiles are as shown in the diagram.
3. The V-number is < 2.4.
4. Numerical aperture is < 0.12 .
5. Attenuation is in the range 0.25-0.5 dB/km.
6. Information carrying capacity is very large. They are long haul carriers.
7. Laser source is used. Connectors are costly

2) Multimode step index fiber(MMF)



1) MMF has a core diameter $50 - 200\mu m$ of uniform RI and cladding diameter 100-250μm has $uniform\ RI$.
2) The refractive index, cross section and modes profiles are as shown in the diagram.
3) MMF guides light in multi-modes as shown .
4) The V-number is > 2.4.
5) Numerical aperture is $0.2\ to\ 0.3$.
6) Attenuation is in the range 0.5- 4 dB/km.
7) Information carrying capacity is small to medium and short haul carriers.
8) LED source is used & Connectors are cheap.

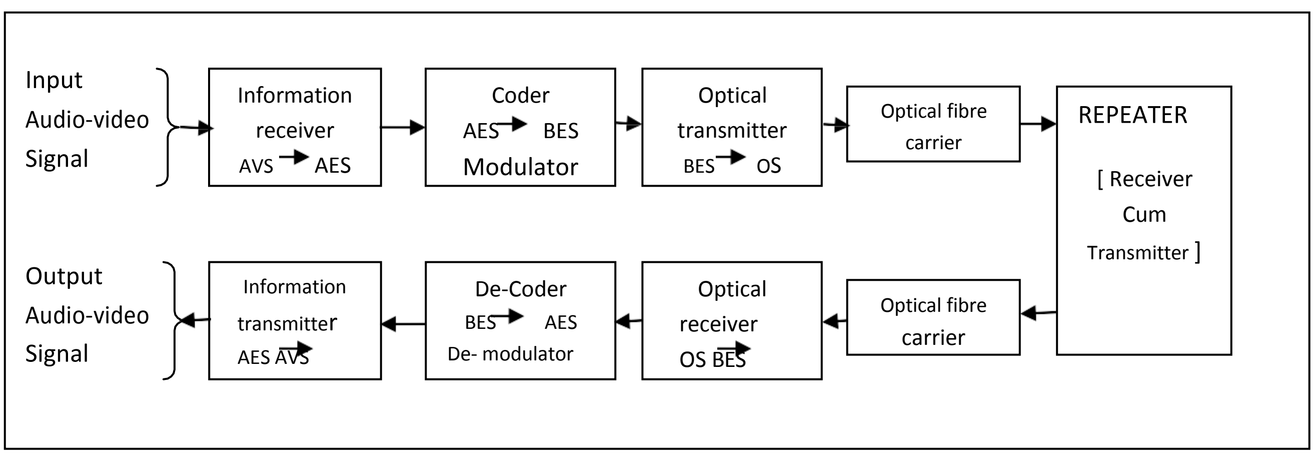
3) Graded index multimode fiber(GRIN)



1. GRIN core diameter = $50 - 200\mu m$ of variable RI and cladding diameter = 100-$250\mu m$
2. The refractive index, cross section and modes profiles are as shown in the diagram
3. The V-number is > 2.4.
4. Numerical aperture is $0.2\ to\ 0.3$.
5. Attenuation is in the range 0.5- 4 dB/km.
6. Information carrying capacity is large and efficient and are short haul carriers.
7. Laser/LED source is used. Connectors are cheap.
8. Easy to splice and interconnect but expensive.

Q:Explain point to point communication system using optical fiber.

The schematic block diagram of point to point communication system using optical fiber is as shown in the diagram.



Note: AVS =audio/video signal. **AES**=analog electrical signal,

BES=binary electrical signal & **OS** = optical signal.

1. **Information receiver-**receives , convert input AVS in to AES & fed to coder.
2. **Coder-** receives, convert AES in to BES and fed in to optical transmitter after modulating it with carrier signal.
3. **Optical transmitter-**receives, convert BES in to OS and fed in to carrier optical fiber.
4. **Carrier optical fiber-**receive OS and guide it along the fiber. Weakened OS is fed in to repeater.
4. **Re-peater(Receiver cum transmitter)-**receives the Weakened OS, restore to original strength and fed back in to carrier optical fiber again, which in turn guide OS and fed in to optical receiver.
6. **Optical receiver-** receive ,convert OS in to BES & fed in to de-coder.
7. **De-coder-**receive, de-modulate & convert BES in to AES & fed in to information transmitter.
8. **Information transmitter-**finally receive, convert AES in to AVS as output

Advantages of Optical Fiber communication system(OFCS):

Merits of point to point Optical Fiber communication system are:

1. **Wide bandwidth**-the bandwidth of OFCS is very large about 10^5 GHz compared to cable communication band width 500MHz.
2. **Electrical isolation**- OFCS cables are electrically isolated as the optical fibers are made of insulating glass and plastics.
3. **No cross talk**- OFCS works on the principle of total internal reflection as there is no leakage of signal.
4. **Economical ,light weight, strong, flexible & small size**- optical fibers used in OFCS are cheap, light weight and small size compared to conventional system.
5. **Immune to electro-magnetic interference**- Electrically neutral light signals in OFCS are unaffected by external electromagnetic waves due to current/magnetic field/lightning/electrical sparks.
6. **High signal security**- In OFCS the signals transmitted are highly secured as the leakage can be easily detectable.
7. **Low attenuation / transmission loss**-In OFCS the transmission loss of the signal is about 0.2dB/km.

Demerits of point to point Optical Fiber communication system:

Optical fibers used in OFCS :

1. are sensitive to temperature changes which leads to loss of signal.
2. may break easily due to bending or accidents.
3. re- connection of broken Optical fibers is skillful work and costly.

@@@end@@@