07/08/2017

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* Module - I

- n^{th} derivative of standard function.

1. n^{th} derivative of e^{ax} :-

let $y = e^{ax} \rightarrow (1)$

Diff^n eq^n (1) w.r.t x

$$\frac{dy}{dx} = e^{ax} \cdot a$$

Diff^n again w.r.t x.

$$\frac{d^2y}{dx^2} = a^2 \times e^{ax}$$

$|||^{ly}$ $$\frac{d^3y}{dx^3} = a^3 \times e^{ax}$$

In general

$$\frac{d^ny}{dx^n} = a^n \cdot e^{ax} \quad (\text{or}) \quad D^n(e^{ax}) = a^n e^{ax}$$

Note:- $\cosh x = \frac{e^x + e^{-x}}{2}$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

2. n^{th} derivative of a^{mx}

let $y = a^{mx} \longrightarrow (1)$

Diff^n eq^n (1) w.r.t x.

$$\frac{dy}{dx} = a^{mx} \times \log a \times m = (m \log a)\, a^{mx}$$

Diff^n again w.r.t x

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$$\frac{d^2y}{dx^2} = (m\log a)^2 a^{mx}$$

$|||^{ly}$ $$\frac{d^3y}{dx^3} = (m\log a)^3 a^{mx}$$

In general

$$D^n[a^{mx}] = [m\log a]^n a^{mx}.$$

III n^{th} derivative of $(ax+b)^m$:

let $y = (ax+b)^m \Rightarrow$ ①

Diff^n eq^n ① w.r.t x

$$\frac{dy}{dx} = m(ax+b)^{m-1} \times a = am(ax+b)^{m-1}$$

Diff^n again w.r.t x.

$$\frac{d^2y}{dx^2} = a^2m(m-1)(ax+b)^{m-2}$$

$|||^{ly}$ $$\frac{d^3y}{dx^2} = a^3m(m-1)(m-2)(ax+b)^{m-3}$$

In general

$$= a^n[m(m-1)(m-2)(m-3)\cdots(m-(n-1)](ax+b)^{m-n}$$

** Note:- (01) If $a=1$ and $b=0$

$$D^n(x^m) = m(m-1)(m-2)\cdots(m-(n-1)]x^{m-n} \Rightarrow ②$$

(02) If $m=n$ then eq^n ② becames.

$$D^n(x^n) = \{n(n-1)(n-2)\cdots[\cancel{n-n}+1]\}x^0$$

$$= n(n-1)(n-2)\cdots 1$$

$$= D^n(x^n) = n!$$

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IV n^{th} derivative of $\frac{1}{ax+b}$

Let $y = \left(\frac{1}{ax+b}\right) \rightarrow (1)$

Diff^n eq^n (1) w.r.t x.

$$\frac{dy}{dx} = \frac{-1}{(ax+b)^2} \times a = \frac{(-1)^1 \times a^1 \times 1!}{(ax+b)^2}$$

Diff^n again w.r.t x.

$$\frac{d^2y}{dx^2} = \frac{-a \times (-2) \times a}{(ax+b)^3} = \frac{2a^2}{(ax+b)^3} = \frac{(-1)^2 a^2 \times 2!}{(ax+b)^3}$$

Diff^n again w.r.t to x

$$\frac{d^3y}{dx^3} = \frac{2a^2 \times (-3) \times a}{(ax+b)^4} = \frac{-6a^3}{(ax+b)^4} = \frac{(-1)^3 a^3 \times 3!}{(ax+b)^4}$$

$|||^{ly}$ $$\frac{d^4y}{dx^4} = \frac{24a^4}{(ax+b)^5} = \frac{(-1)^4 a^4 \times 4!}{(ax+b)^5}$$

In general

$$D^n\left[\frac{1}{ax+b}\right] = \frac{(-1)^n a^n n!}{(ax+b)^{n+1}}$$

If $a=1$ and $b=0$

$$D^n\left[1/x\right] = \frac{(-1)^n n!}{x^{n+1}}$$

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V n^{th} derivative of $\log(ax+b)$:-

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Diffⁿ eqⁿ again w.r.t x

$$\frac{d^2y}{dx^2} = \frac{a \times -1}{(ax+b)^2} \times a$$

$$\frac{d^2y}{dx^2} = \frac{-a^2}{(ax+b)^2} = \frac{(-1)^1 a^2 \times 1!}{(ax+b)^2}$$

Diffⁿ w.r.t x.

$$\frac{d^3y}{dx^3} = -a^2 \times \frac{-2}{(ax+b)^3} \times a$$

$$\frac{d^3y}{dx^3} = \frac{2a^3}{(ax+b)^3} = \frac{(-1)^2 a^3 2!}{(ax+b)^3}$$

$$\text{IIIIth} \quad \frac{d^4y}{dx^4} = \frac{-6a^4}{(ax+b)^4} = \frac{(-1)^3 a^4 3!}{(ax+b)^4}$$

$\vdots$

In general

$$D^n[\log(ax+b)] = \frac{(-1)^{n-1}(n-1)!\, a^n}{(ax+b)^n} \,//$$

VI n^{th} derivative of $\sin(ax+b)$.

let $y = \sin(ax+b) \rightarrow (1)$

Diffⁿ eqⁿ w.r.t x:

$$\frac{dy}{dx} = \cos(ax+b) \times a$$

$$\frac{dy}{dx} = a\sin\left(\pi/2 + ax+b\right) \rightarrow (2)$$

Diffⁿ eqⁿ (2) w.r.t x.

$$\frac{d^2y}{dx^2} = a \times \cos\left(\pi/2 + ax+b\right) \times a$$

$$\frac{d^2y}{dx^2} = a^2 \sin\left(\pi/2 + \pi/2 + ax+b\right)$$

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$$\frac{d^2y}{dx^2} = a^2 \sin\left(\frac{2\pi}{2} + ax + b\right)$$

$$\text{III}^{ly} \quad \frac{d^3y}{dx^3} = a^3 \sin\left(\frac{3\pi}{2} + ax + b\right)$$

In general

$$D^n\left[\sin(ax+b)\right] = a^n \sin\left(\frac{n\pi}{2} + ax + b\right)$$

If $a=1$ and $b=0$

$$D^n(\sin x) = \sin\left(\frac{n\pi}{2} + x\right)$$

VII n^{th} derivative of $\cos(ax+b)$

let $y = \cos(ax+b) \rightarrow (1)$

Diff^n eq^n w.r.t x

$$\frac{dy}{dx} = -\sin(ax+b) \times a$$

$$\frac{dy}{dx} = a\cos\left(\pi/2 + ax + b\right) \rightarrow (2)$$

Diff^n eq^n (2) w.r.t x.

$$\frac{d^2y}{dx^2} = -a \times \sin\left(\pi/2 + ax + b\right) \times a$$

$$\frac{d^2y}{dx^2} = a^2 \cos\left(\pi/2 + \pi/2 + ax + b\right)$$

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If $a=1$ and $b=0$

$$D^n(\cos x) = \cos\left(\frac{n\pi}{2} + x\right)$$

VIII ** n^{th} derivative of $e^{ax}\sin(bx+c)$

let $y = e^{ax}\sin(bx+c) \rightarrow ①$

Diff^n eq^n ① w.r.t x $\quad uv' + vu'$

$$\frac{dy}{dx} = e^{ax}\cos(bx+c) \times b + \sin(bx+c)\, e^{ax} \times a$$

$$\frac{dy}{dx} = e^{ax}\{b\cos(bx+c) + a\sin(bx+c)\}$$

put $a = r\cos\theta$ $\quad b = r\sin\theta$

$r = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}(b/a)$

$$\cancel{\frac{dy}{dx} = e^{ax}\{b\cos(bx+c) + a\sin(bx+c)\}}$$

$$\frac{dy}{dx} = e^{ax}\{r\sin\theta\cos(bx+c) + r\cos\theta\sin(bx+c)\}$$

$$\frac{dy}{dx} = re^{ax}\sin(\theta + bx + c)$$

Diff^n again w.r.t x.

$$\frac{d^2y}{dx^2} = r\{e^{ax}\cos(\theta+bx+c) \times b + \sin(\theta+bx+c)\, e^{ax} \times a\}$$

$$= re^{ax}\{r\sin\theta\cos(\theta+bx+c) + r\cos\theta\sin(\theta+bx+c)\}$$

$$= r^2 a e^{ax}\sin(\theta+\theta+bx+c)$$

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$$\frac{d^3y}{dx^3} = r^3 e^{ax} \cos(3\theta + bx + c).$$

In general

$$D^n\left(e^{ax}\cos(bx+c)\right) = r^n e^{ax}\cos(n\theta + bx + c)$$

$$= \left(\sqrt{a^2+b^2}\right)^n e^{ax}\cos\left(n\tan^{-1}(b/a) + bx + c\right)$$

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**

SL.NO	Function	n^{th} derivative
01.	e^{ax}	• $a^n e^{ax}$
02.	a^{mx}	• $(m\log a)^n a^{mx}$
03.	$(ax+b)^m$	• $a^n\left[m(m-1)(m-2)(m-3)\cdots(m-(n-1))\right](ax+b)^{m-n}$
04.	$1/ax+b$	• $\frac{(-1)^n a^n n!}{(ax+b)^{n+1}}$
05.	$\log(ax+b)$	• $\frac{(-1)^n (n-1)!\, a^n}{(ax+b)^n}$
06.	$\sin(ax+b)$	• $a^n \sin\left(n\pi/2 + ax+b\right)$
07.	$\cos(ax+b)$	• $a^n \cos\left(n\pi/2 + ax+b\right)$
08.	$e^{ax}\sin(bx+c)$	• $\left(\sqrt{a^2+b^2}\right)^n e^{ax}\sin\left(n\tan^{-1}(b/a) + bx + c\right)$
09.	$e^{ax}\cos(bx+c)$	• $\left(\sqrt{a^2+b^2}\right)^n e^{ax}\cos\left(n\tan^{-1}(b/a) + bx + c\right)$

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let $y = e^{10x}$

$D^n(e^{ax}) = a^n e^{ax}$

$D^n(e^{10x}) = 10^n e^{10x}$

let $y = e^{3x}$

$D^n(e^{ax}) = a^n e^{ax}$

$D^n(e^{3x}) = 3^n e^{3x}$

02. a^{3x}

let $y = a^{3x}$

$D^n(a^{mx}) = (m \log a)^n a^{mx}$.

$D^n(a^{3x}) = (3 \log a)^n a^{3x}$

03. 3^{2x}

let $y = \cancel{e^{10x}}\ 3^{2x}$

$D^n(a^{mx}) = (m \log a)^n a^{mx}$

$D^n(3^{2x}) = (2 \log 3)^n 3^{2x}$

$y = 3^{5x}$

let $y = 3^{5x}$

$D^n(a^{mx}) = (m \log a)^n a^{mx}$

$D^n(3^{5x}) = 5 \log 3)^n 3^{5x}$.

04. $\frac{1}{3x+2}$

let $u = 1$

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06. $\cos(2x-1)$

let $y = \cos(2x-1)$

$D^n = \cos(ax+b) = a^n\left(\cos\left(n\pi/2 + ax+b\right)\right)$

$D^n = \cos(2x-1) = 2^n\left(\cos\left(n\pi/2 + 2x-1\right)\right)$

07. $\sin bx$.

let $y = \sin bx$

$D^n = \overset{\sin}{\cancel{\cos}}(ax+b) = a^n\left(\sin\left(n\pi/2 + ax+b\right)\right)$

$D^n = \cancel{\cos}(\sin(bx) = b^n\left(\sin\left(n\pi/2 + bx\right)\right)$.

08. $e^{2x}\sin 3x$.

let $y = e^{2x}\sin 3x$.

$D^n = e^{ax}\sin(bx+c) = \left(\sqrt{a^2+b^2}\right)^n e^{ax}\sin\left(n\tan^{-1}(b/a) + bx + c\right)$.

$D^n = e^{2x}\sin 3x = \left(\sqrt{13}\right)^n e^{2x}\sin\left(n\tan^{-1}(3/2) + 3x\right)$

II Find the nth derivative of the following.

01. $\cosh^2 2x$

02. $e^{2x}\cosh 4x$

03. $\sin^3 2x$

04. $\sin^2 4x + \cos^2 x$

05. $\sin 4x \cos 2x$

06. $\cos x \cos 3x \cos 5x$

07. $e^{2x}\cos^3 x$

08. $\log(4x^2-1)$

(9) $\log\left(\sqrt{4x^2+8x+3}\right)$

(10) $\log\{(3x+2)e^{5x+6}\}$

(11) $\dfrac{6x}{(x-1)(x^2-4)}$

(12) $\left(\dfrac{x}{1+3x+2x^2}\right.$

(13) $\dfrac{x^2}{(2x+1)(2x+3)}$ $2x^2+7x+6$

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01. $\cosh^2 2x$

let $y = \cosh^2 2x$

$$\boxed{\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}}$$

$$y = \left(\frac{e^{2x} + e^{-2x}}{2}\right)^2$$

$$= \frac{1}{4}\left\{e^{4x} + e^{-4x} + 2e^{2x} \times e^{-2x}\right\}$$

$$= y = \frac{1}{4}\left\{e^{4x} + e^{-4x} + 2\right\}$$

$$D^n(y) = 1/4\left(D^n(e^{4x}) + D^n(e^{-4x}) + D^n(2)\right)$$

$$D^n(y) = 1/4\left\{4^n e^{4x} - 4^n e^{-4x}\right\}$$

02. $e^{2x}\cosh 4x$

let $y = e^{2x}\cosh 4x$

$$\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

$$y = e^{2x}\left\{\frac{e^{4x} + e^{-4x}}{2}\right\}$$

$$y = \frac{1}{2}e^{2x}\left\{e^{4x} + e^{-4x}\right\}$$

$$y = \frac{1}{2}\left\{e^{6x} + e^{-2x}\right\}$$

$$y = \frac{1}{2}\left\{D^n(e^{6x}) + D^n\{e^{-2x}\}\right\}$$

$$y = \frac{1}{2}\left(6^n e^{6x} + (-2^n)e^{-2x}\right)$$

$$y = 1/2\left(6^n e^{6x} - 2^n e^{-2x}\right)$$

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03. ~~let~~ $\sin^3 2x$.

let $y = \sin^3 2x$

$$\boxed{\sin 3\theta = 3\sin\theta - 4\sin^3\theta}$$

$$4\sin^3\theta = 3\sin\theta - \sin 3\theta.$$

$$\sin^3\theta = \frac{1}{4}\left[3\sin\theta - \sin 3\theta\right]$$

$$\sin^3 2x = \frac{1}{4}\left[3\sin 2x - \sin 6x\right]. \quad (\because \theta = 2x)$$

$$D^n(y) = \frac{1}{4}\left\{3D^n(\sin 2x) - D^n(\sin 6x)\right]$$

$$= \frac{1}{4}\left\{3\times(2^n)\sin\left(\frac{n\pi}{2} + 2x\right) - (6)^n \sin\left(\frac{n\pi}{2} + 6x\right)\right\} //$$

04. $\sin^2 4x + \cos^2 x$

let $y = \sin^2 4x + \cos^2 x \rightarrow (1)$

$$\boxed{\begin{aligned}\cos 2\theta &= 1 - 2\sin^2\theta \\ &= 2\sin^2\theta = 1 - \cos 2\theta\end{aligned}}$$

$$\sin^2\theta = \frac{1}{2}\left[1 - \cos 2\theta\right]$$

$$\sin^2 4x = \frac{1}{2}\left[1 - \cos 8x\right] \quad (\because \theta = 4x).$$

$|||^{ly}$ $\cos^2 x = \frac{1}{2}(1 + \cos 2\theta) \Rightarrow \cos^2 x = \frac{1}{2}(1 + \cos 2x).$

Sub $\sin^2 4x$ and $\cos^2 x$ in (1)

$$y = \frac{1}{2}\left[1 - \cos 8x\right] + \frac{1}{2}\left[1 + \cos 2x\right]$$

$$= \frac{1}{2}\left[1 - \cos 8x + 1 + \cos 2x\right].$$

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$= \frac{1}{2}\{2 - \cos 8x + \cos 2x\}$.

$D^n(y) = 1/2\{D^n(2) - D^n(\cos 8x) + D^n(\cos 2x)\}$

$= \frac{1}{2}\{-8^n \cos\left(\frac{n\pi}{2} + 8x\right) + 2^n \cos\left(\frac{n\pi}{2} + 2x\right)\}$.

05. $\sin 4x \cos 2x$.

let ~~Sin4~~ $y = \sin 4x \cos 2x$

$\boxed{\sin A \cos B = 1/2\{\sin(A+B) + \sin(A-B)\}}$

$\sin 4x \cos 2x = \frac{1}{2}\{\sin(6x) + \sin(2x)\}$.

$D^n(y) = \frac{1}{2}\{D^n(\sin 6x) + D^n(\sin 2x)\}$.

$= \frac{1}{2}\{6^n \sin\left(\frac{n\pi}{2} + 6x\right) + 2^n \sin\left(\frac{n\pi}{2} + 2x\right)\}$.

06. $\cos x \cos 3x \cos 5x$.

let $y = \cos x \cos 3x \cos 5x$.

$\boxed{\cos A \cos B = \frac{1}{2}\{\cos(A+B) + \cos(A-B)\}}$

$y = \frac{1}{2}\{\cos(4x) + \cos(2x)\}\cos 5x$

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let $y = e^{2x} \cdot \cos^3 x$

$\cos^3 x = \frac{1}{4}\{3\cos x + \cos 3x\}$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\cos^3\theta = \frac{\cos 3\theta + 3\cos\theta}{4}$

$e^{2x} \cdot \cos^3 x = \frac{1}{4}\{3e^{2x}\cos x + e^{2x}\cos 3x\}$

$D^n(y) = \frac{1}{4}\{3D^n(e^{2x}\cos x) + D^n(e^{2x}\cos 3x)\}$

$= \frac{1}{4}\{3(\sqrt{5})^n e^{2x}\cos(n\tan^{-1}(1/2)+x) + (\sqrt{13})^n e^{2x}\cos(n\tan^{-1}(3/2)+3x)\}$

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08. $\log(4x^2-1)$

let $y = \log(4x^2-1)$

$y = \log[(2x+1)(2x-1)]$

$y = \log(2x+1) + \log(2x-1)$

$D^n(y) = D^n[\log(2x+1)] + D^n[\log(2x-1)]$

$= \frac{(-1)^{n-1}(n-1)!\,2^n}{(2x+1)^n} + \frac{(-1)^{n-1}(n-1)!\,2^n}{(2x-1)^n}$

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$$= \frac{1}{2}\log(4x^3+8x+3)$$

$$y = \frac{1}{2}\log\left[(2x+1)(2x+3)\right]$$

$$4x^2+8x+3$$
$$= 4x^2+2x+6x+3$$
$$= 2x(2x+1)+3(2x+1)$$
$$= (2x+1)(2x+3).$$

$$y = \frac{1}{2}\left\{\log(2x+1)+\log(2x+3)\right\}$$

$$D^n(y) = \frac{1}{2}\left\{D^n\left[\log(2x+1)\right] + D^n\left[\log(2x+3)\right]\right.$$

$$= \frac{1}{2}\left[\frac{(-1)^{n-1}(n-1)!\,2^n}{(2x+1)^n} + \frac{(-1)^{n-1}(n-1)!\,2^n}{(2x+3)^n}\right]$$

$$(-1)^{n-1}(n-1)!\,2^{n-1}\left\{\frac{1}{(2x+1)^n} + \frac{1}{(2x+3)^n}\right\}.$$

10. $\log\{(3x+2)e^{5x+6}\}$.

let $y = \log\{(3x+2)e^{5x+6}\}$.

$$= \log(3x+2) + \log e^{5x+6}$$

$$y = \log(3x+2) + (5x+6)$$

$$D^n(y) = D^n\left[\log(3x+2)\right] + D^n\left[5x+6\right]$$

$$D^n(y) = \frac{(-1)^{n-1}(n-1)!\,3^n}{(3x+2)^n}, \quad n>1$$

If $n=1$.

$$\frac{dy}{dx} = \frac{3}{(3x+2)} + 5.$$

11. $\dfrac{6x}{(x-1)(x^2-4)}$

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let $y = \frac{6x}{(x-1)(x^2-4)} = \frac{6x}{(x-1)(x-2)(x+2)}$.

Consider.

$$\frac{6x}{(x-1)(x-2)(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x+2)} \Rightarrow (1)$$

$$= \frac{A(x-2)(x+2) + B(x-1)(x+2) + C(x-1)(x-2)}{(x-1)(x-2)(x+2)}.$$

$$6x = A(x-2)(x+2) + B(x-1)(x+2) + C(x-1)(x-2).$$

$x=1$

$6 = -3A$

$= A = -2$ //.

$x = 2$

$12 = 0 + 4B + 0$

$B = 3$.

$x = -2$

$-12 = 0 + 0 + 12$

$C = -1$

sub A, B, C value - (1)

$$y = \frac{-2}{(x-1)} + \frac{3}{(x-2)} - \frac{1}{(x+2)}$$

$$D^n(y) = -2\left[D^n\left(\frac{1}{x-1}\right)\right] + 3\left[D^n\left(\frac{1}{(x-2)}\right)\right] - 1\left[D^n\left(\frac{1}{(x+2)}\right)\right].$$

$$= -2\left[\frac{(-1)^n n!}{(x-1)^{n+1}}\right] + 3\left[\frac{(-1)^n n!}{(x-1)^{n+1}}\right] - 1\left[\frac{(-1)^n n!}{(x-1)^{n+1}}\right] //$$

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$1+3x+2x^2$.

let $y = \dfrac{x}{1+3x+2x^2}$.

$2x^2+3x+1$

$2x^2+2x+x+1$

$2x(x+1)+1(x+1)$

$(x+1)(2x+1)$

$y = \dfrac{x}{(2x+1)(x+1)}$.

Consider.

$$\frac{x}{(2x+1)(x+1)} = \frac{A}{(2x+1)} + \frac{B}{(x+1)} \Rightarrow (1)$$

$$= \frac{A(2x+1)+B(x+1)}{(x+1)(2x+1)}$$

$x = A(2x+1)+B(x+1)$.

put $x=-1$, $-1 = -A = A = 1$

$x = -1/2$, $-1/2 = 0 + 1/2\, B$.

$B = -1$.

Sub value A and B in (1)

$y = \dfrac{1}{\quad} - \dfrac{1}{\quad}$

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Consider.

$$\frac{x}{(2x+1)(2x+3)} = \frac{A}{(2x+1)} + \frac{B}{(2x+3)} \rightarrow (1)$$

$$= \frac{A(2x+3) + B(2x+1)}{(2x+1)(2x+3)}$$

$$x = A(2x+3) + B(2x+1)$$

put $x = \frac{-1}{2}$

$$\frac{-1}{2} = 2A =$$

$$A = \frac{-1}{4}.$$

$x = \frac{-3}{2}.$

$$\frac{+3}{2} \cancel{2B} \cdot \frac{2}{2}. = +2B$$

$$B = \frac{3}{4}.$$

Sub A and B value $\rightarrow (1)$

$$y = \frac{\frac{-1}{4}}{2x+3} + \frac{\frac{3}{4}}{(2x+1)}$$

$$D^n y = \frac{-1}{4}\left[D^n\left(\frac{1}{2x+3}\right)\right] + \frac{3}{4}\left[D^n\left(\frac{1}{2x+1}\right)\right]$$

$$= \frac{-1}{4}\left[\frac{(-1)^n n!\, 2^n}{(2x+3)^{n+1}}\right] + \frac{3}{4}\left[\frac{(-1)^n n!\, 2^n}{(2x+1)^{n+1}}\right] //$$

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Iebnitz's Theorem

If u and v are any two functions of x then the nth derivative of product function uv is defined as

** $D^n(uv)$ or $(uv)_n = uv_n + {}^nC_1 u_1 v_{n-1} + {}^nC_2 u_2 v_{n-2} + {}^nC_3 u_3 v_{n-3} + \dots\dots\dots + u_n v$

** Note (i) $^nC_1 = \frac{n!}{(n-1)! \times 1!} = \frac{n(n-1)!}{(n-1)!}$

$^nC_1 = n$ //

(ii) $^nC_2 = \frac{n!}{(n-2)! \times 2!} = \frac{n(n-1)(n-2)!}{(n-2)! \times 2}$

$^nC_2 = \frac{n(n-1)}{2}$ //

(iii) $^nC_3 = \frac{n(n-1)(n-2)}{6}$ //.

** • Problems

01 If $\tan y = x$ then prove that $(1+x^2)y_{n+2} + 2(n+1)x\, y_{n+1} + n(n+1)y_n = 0$

by putting $n=0$ we get ....

⇒ Given

$\tan y = x$

$y = \tan^{-1} x$

Diff^n w.r.t x

$y_1 = \frac{1}{1+x^2}$

$(1+x^2)\, y_1 = 1$

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Applying the L.T we get.

$$(1+x^2)y_{n+2} + nc_1(2x)y_{n+1} + nc_2(2)y_n + 2xy_{n+1} + nc_1 \times 2y_n = 0$$

$$(1+x^2)y_{n+2} + 2nxy_{n+1} + \frac{n(n-1)}{2} \times 2y_n + 2xy_{n+1} + 2ny_n = 0$$

$$(1+x^2)y_{n+2} + 2xy_{n+1}(n+1) + y_n \times n[(n-1)+2] = 0$$

$$= (1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$$

02. If $\cos^{-1}(y/b) = \log\left(\frac{x}{n}\right)^n$ then prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + 2n$

$\Rightarrow$ Given

$$\cos^{-1}(y/b) = \log\left(\frac{x}{n}\right)^n$$

$$y/b = \cos\left(n\log\left(\frac{x}{n}\right)\right)$$

$$y = b\cos\left(n\log\left(x/n\right)\right) \rightarrow (1)$$

Diff^n eq^n (1) w.r.to x.

$$y_1 = -b\sin\left(n\log\left(\frac{x}{n}\right)\right) \times n \times \frac{1}{x} \times \frac{1}{n}$$

$$xy_1 = -nb\sin\left(n\log\left(\frac{x}{n}\right)\right)$$

Diff^n again w.r.t x.

$$xy_2 + y_1 = -nb\cos\left(n\log\left(\frac{x}{n}\right)\right) n \times \frac{1}{x} \times \frac{1}{n}$$

$$x^2y_2 + xy_1 = -n^2b\cos\left(n\log(x/n)\right) \text{ [by using (1)]}$$

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$x^2y_2 + xy_1 = -n^2y.$

$x^2y_2 + xy_1 + n^2y = 0.$

Using L.T we get.

$x^2y_{n+2} + nc_1 (2x) y_{n+1} + nc_2 (2) y_n + xy_{n+1} + nc_1 (1) y_n + n^2y_n = 0$

$x^2y_{n+2} + 2n\, xy_{n+1} + \frac{n(n-1)}{2} \times 2y_n + xy_{n+1} + ny_n + n^2y_n = 0$

$x^2y_{n+2} + xy_{n+1}(2n+1) + y_n(n^2 - n + n^2 + n) = 0$

$x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y_n = 0$ //.

03. If $x = \tan(\log y)$ the prove that $(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-}$

$\Rightarrow$ Given

$x = \tan(\log y)$

$\tan^{-1}x = \log y$

$y = e^{\tan^{-1}x} \rightarrow (1)$

Diffn w.r.to x.

$y_1 = e^{\tan^{-1}x} \times \frac{1}{(1+x)^2}$

$(1+x^2)y_1 = e^{\tan^{-1}x}.$ (by using eq (1))

$(1+x^2)y_1 = y.$

$(1+x^2)y_1 - y = 0$

by using L.P we get.

$(1+x^2)y_{n+1} + nc_1 (2x) y_n + nc_2 \times 2y_{n-1} - y_n = 0$

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$$(1+x^2)y_{n+1} + 2nxy_n + \frac{n(n-1)}{\cancel{2}} \times \cancel{2}y_{n-1} - y_n = 0$$

$$(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-1} = 0$$

17/08/2016

04. If $\sin^{-1}y = 2\log(x+1)$ then prove that $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$

Given

$\sin^{-1}y = 2\log(x+1)$

$y = \sin[2\log(x+1)] \Rightarrow ①$

Diff^n eq^n ① w.r.t x

$$y_1 = \cos[2\log(x+1)] \times 2 \times \frac{1}{(x+1)}$$

$$(x+1)y_1 = 2\cos(2\log(x+1))$$

Diff^n again w.r.t x.

$$(x+1)y_2 + y_1 = -2\sin(2\log(x+1)) \times 2 \times \frac{1}{(x+1)}$$

$$(x+1)^2 y_2 + (x+1)y_1 = -4\underbrace{\sin[2\log(x+1)]}_{\text{using (i)}}$$

$$(x+1)^2 y_2 + (x+1)y_1 = -4y.$$

$$(x+1)^2 y_2 + (x+1)y_1 + 4y = 0.$$

Using L.T we get.

$$(x+1)^2 y_{n+2} + {}^nC_1\, 2(x+1)y_{n+1} + {}^nC_2\,(2)y_n + (x+1)y_{n+1} + {}^nC_1\,(1)y_n + 4y_n = 0$$

$$(x+1)^2 y_{n+2} + 2n(x+1)y_{n+1} + \frac{n(n-1)}{\cancel{2}} \times \cancel{2}y_n + (x+1)y_{n+1} + ny_n + 4y_n = 0$$

$$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0.$$

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Diff^n eq^n ① w.r.t x.

$$y_1 = e^{m\cos^{-1}x} \cdot \frac{-1}{\sqrt{1-x^2}} m$$

$$= e^{m\cos^{-1}x} \cdot \frac{-m}{\sqrt{1-x^2}}.$$

$$\sqrt{1-x^2}\, y_1 = -m e^{m\cos^{-1}x}.$$

Diff^n again w.r.t x.

$$\sqrt{1-x^2}\, y_2 + y_1 \times \frac{1}{2\sqrt{1-x^2}}(-2x) = -m e^{m\cos^{-1}x} \times m \times \frac{-1}{\sqrt{1-x^2}}$$

$\times^{ly}$ by $\sqrt{1-x^2}$

$$(1-x^2)y_2 - xy_1 = m^2 e^{m\cos^{-1}x} \text{ using ① eq}$$

$$(1-x^2)y_2 - xy_1 = m^2 y$$

$$(1-x^2)y_2 - xy_1 - m^2 y = 0$$

using L.T we get.

$$(1-x^2)y_{n+2} + nc_1(-2x)y_{n+1} + nc_2(-2)y_n - xy_{n+1} - nc_1(1)y_n - m^2 y_n = 0$$

$$(1-x^2)y_{n+2} - 2nxy_{n+1} - \frac{n(n-1)}{2} \times 2y_n - xy_{n+1} - ny_n - m^2 y_n = 0$$

$$(1-x^2)y_{n+2} - xy_{n+2}(2n+1) - y_n(n^2 - n + n + m^2)$$

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Diff^n eq^n w.r.t x.

$$(\sqrt{1+x^2})y_1 + y \times \frac{1}{\cancel{2}\sqrt{1+x^2}} \times \cancel{2}x = \frac{1}{\sqrt{1+x^2}}$$

$\otimes^{ly}$ by $\sqrt{1+x^2}$

$$(1+x^2)y_1 + xy = 1$$

Diff^n again w.r.t x.

$$(1+x^2)y_2 + y_1(2x) + xy_1 + y = 0$$

$$(1+x^2)y_2 + 3xy_1 + y = 0$$

using L.T we get

$$(1+x^2)y_{n+2} + nc_1(2x)y_{n+1} + nc_2(2)y_n + 3x\,y_{n+1} + nc_1(3)y_n + y_n = 0$$

$$(1+x^2)y_{n+2} + 2nxy_{n+1} + \frac{n(n-1)}{\cancel{2}} \times \cancel{2}y_n + 3xy_{n+1} + 3ny_n + y_n = 0$$

$$(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + y_n(n^2-n+3n+1) = 0$$

$$(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n^2+2n+1) = 0$$

$$(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n+1)^2y_n = 0.$$

18/08/2

*08

07. let $y=(x^2-1)^n$ then prove that $(1-x^2)y_{n+2} - 2xy_{n+1} + n(n+1)y_n = 0$

Given

$$y=(x^2-1)^n \rightarrow (1)$$

Diff^n ~~eq^n (1) w.r.t x~~ $\log y = \log[(x^2-1)^n]$.

$$\log y = n\log(x^2-1)$$

Diff^n eq^n w.r.t x.

$$\frac{1}{y}\cdot y_1 = n \times \frac{1}{(x^2-1)} \times 2x$$

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$(x^2-1)y_1 = 2nxy$

$(x^2-1)y_1 - 2nxy = 0$

$\otimes^{ly}$ by -1

$(1-x^2)y + 2nxy = 0$

Diff^n again w.r.t x

$(1-x^2)y_2 + y_1(-2x) + 2nxy_1 + 2ny = 0$

$(1-x^2)y_2 + 2(n-1)xy_1 + 2ny = 0$

Using L.T, we get

$(1-x^2)y_{n+2} + nc_1(-2x)y_{n+1} + nc_2(-2)y_n + 2(n-1)xy_{n+1} + nc_1\, 2(n-1)y_n + 2ny_n = 0$

$(1-x^2)y_{n+2} - 2nxy_{n+1} - \frac{n(n-1)}{2} \times 2y_n + 2(n-1)xy_{n+1} + 2n(n-1)y + 2ny_n = 0$

$(1-x^2)y_{n+2} - 2xy_{n+1}(\cancel{n}-\cancel{n}+1) + y_n(-n^2+n+2n^2-\cancel{2n}+\cancel{2n}) = 0$

$(1-x^2)y_{n+2} - 2xy_{n+1} + n(n+1)y_n = 0$

18/08/2017

Q8. If $x = \sin t$ and $y = \cos mt$ then prove that:

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_1 = 0$

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Diff^n again w.r.t x.

$$(\sqrt{1-x^2})\, y_2 + y_1 \times \frac{1}{2\sqrt{1-x^2}}(-2x) = -m\cos(m\sin^{-1}x) \times m \times \frac{1}{\sqrt{1-x^2}}$$

$\otimes^{ly}$ by $\sqrt{1-x^2}$

$$(1-x^2)\, y_2 - xy_1 = -m^2 \underline{\cos(m\sin^{-1}x)} \rightarrow \text{using (1)}.$$

$$(1-x^2)y_2 - xy_1 = -m^2 y.$$

$$(1-x^2)y_2 - xy_1 + m^2 y = 0$$

using L.P we get.

$$(1-x^2)\, y_{n+2} + nc_1(-2x)\, y_{n+1} + nc_2(-2)\, y_n - xy_{n+1} - nc_1(1)\, y_n + m^2 y_n = 0$$

$$(1-x^2)\, y_{n+2} - 2nxy_{n+1} - \frac{n(n-1)}{\cancel{2}} \times \cancel{2}y_n - xy_{n+1} - ny_n + m^2 y_n = 0$$

$$(1-x^2)\, y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)\, y_n = 0$$

(9) If $y = \sin^{-1}x$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$

Given $y = \sin^{-1}x$

Diff w.r.t x.

$$y_1 = \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\, y_1 = 1$$

diff again w.r.t x.

$$\sqrt{1-x^2}\, y_2 + y_1 \frac{1}{\cancel{2}\sqrt{1-x^2}}(-\cancel{2}x) = 0$$

$\otimes^{ly}$ $\sqrt{1-x^2}$.

$$(1-x^2)y_2 - y_1 x = 0.$$

using L.T we get

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$(1-x^2)y_{n+2} + nc_1(-2x)\,y_{n+1} + nc_2(-2)\,y_n - xy_{n+1} - nc_1(1)\,y_n = 0.$

$(1-x^2)\,y_{n+2} + (-2nxy_{n+1}) - \frac{n(n-1)}{\cancel{2}} \times \cancel{2}y_n - xy_{n+1} - ny_n = 0.$

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$

10. If $y = \log\left(x+\sqrt{1+x^2}\right)$ then prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0.$

Given

$y = \log\left(x+\sqrt{1+x^2}\right).$

Diffn eqn w.r.t x.

$y_1 = \frac{1}{x+\sqrt{1+x^2}}\left\{1 + \frac{1}{\cancel{2}\sqrt{1+x^2}}(\cancel{2}x)\right\}$

$(x+\sqrt{1+x^2})y_1 = 1$

~~Diffn again w.r.t x.~~

$\frac{1}{(x+\sqrt{1+x^2})}\left\{\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right\}$

$y_1 = \frac{1}{\sqrt{1+x^2}}.$

$\sqrt{1+x^2}\,y_1 = 1.$

diffn again.

$(\sqrt{1+x^2})y_2 + \frac{1}{\cancel{2}\sqrt{1+x^2}}(\cancel{2}x)\,y_1 = 1$

$\times^{ly}\sqrt{1+x^2}$ we get

$(1+x^2)\,y_2 + x\,y_1 = 1.$

$(1+x^2)y_{n+2} + nc_1(2x)y_{n+1} + nc_2(2)\,y_n + xy_{n+1} + \cancel{xy_n}\; nc_1(1)y_n$

$(1+x^2)y_{n+2} + 2xy_{n+1} + \frac{n(n-1)}{\cancel{2}}\cancel{2}y_n + xy_{n+1} + y_n.$

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11. If $y = \frac{\log x}{x}$ then prove that

$$\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = \frac{(-1)^n n!}{x^{n+1}}\left\{\log x - 1 - \frac{1}{2} - \frac{1}{3} \cdots\right.$$

Given

$$y = \frac{\log x}{x} = \log x \times \frac{1}{x}$$

$u = \log x$ $\qquad$ $v = \frac{1}{x}$

$u_1 = \frac{1}{x}$ $\qquad$ $v_n = \frac{(-1)^n n!}{x^{n+1}}$

$u_2 = \frac{-1}{x^2}$ $\qquad$ $v_{n-1} = \frac{(-1)^{n-1}(n-1)!}{x^n}$

$u_3 = \frac{2}{x^3}$ $\qquad$ $v_{n-2} = \frac{(-1)^{n-2}(n-2)!}{x^{n-1}}$

$\vdots$ $\qquad$ $v_{n-3} = \frac{(-1)^{n-3}(n-3)!}{x^{n-2}}$

$u_n = \frac{(-1)^{n-1}(n-1)!}{x^n}$

$$D^n(uv) = uv_n + nc_1 u_1 v_{n-1} + nc_2 u_2 v_{n-2} + nc_3 u_3 v_{n-3} + \cdots + u_n v$$

$$D^n\left(\frac{\log x}{x}\right) = \log x \times \frac{(-1)^n n!}{x^{n+1}} + n \times \frac{1}{x} \frac{(-1)^{n-1}(n-1)!}{x^n} + \frac{n(n-1)}{2} \times$$

$$\times \frac{(-1)^{n-2}(n-2)!}{x^{n-1}} + \cdots + \frac{(-1)^{n-1}(n-1)!}{x^n} \times$$

$$= \frac{(-1)^n \cdot n! \log x}{x^{n+1}} - \frac{(-1)^n \times n!}{x^{n+1}} - \frac{(-1)^n n!}{2x^{n+1}} \cdots - \frac{(-1)^n}{x^{n+1}}$$

$$\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = \frac{(-1)^n n!}{x^{n+1}}\left\{\log x - 1 - \frac{1}{2} \cdots \frac{1}{n}\right\}$$ //.

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21/08/2017

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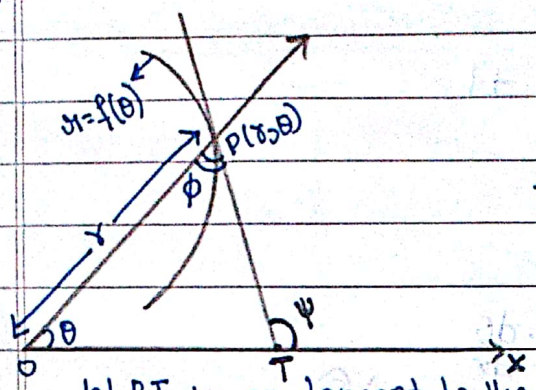
Polar Curves

*** Angle b/w radius vector and tangent to a curve.



let $r = f(\theta)$ be a polar curve and $P(r,\theta)$ be any point on it. Join OP, OP = r represents a radius vector which makes an angle θ with x axis.

let PT be an tangent to the curve which makes any angle ψ with x-axis as shown. let φ be the angle b/w the radius vector OP and tangent PT.

from fig.

$$\psi = \theta + \phi$$

$$\Rightarrow \tan\psi = \tan(\theta+\phi)$$

$$\tan\psi = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} \Rightarrow (1)$$

w.k.t

$$\frac{dy}{dx} = \tan\psi$$

$$\tan\psi = \frac{dy/d\theta}{dx/d\theta}$$

w.k.t

$x = r\cos\theta$

$$\frac{dx}{d\theta} = -r\sin\theta + (\cos\theta)\frac{dr}{d\theta}$$

$y = r\sin\theta$

$$\frac{dy}{d\theta} = r\cos\theta + (\sin\theta)\left(\frac{dr}{d\theta}\right)$$

$$\tan\psi = \frac{r\cos\theta + (\sin\theta)\left(\frac{dr}{d\theta}\right)}{-r\sin\theta + (\cos\theta)\left(\frac{dr}{d\theta}\right)}$$

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dividing both: Nr and Dr by $\left(\cos\theta\right)\cdot\left(\frac{dr}{d\theta}\right)$

$$\tan\psi = \frac{\frac{r}{(dr/d\theta)} + \tan\theta}{-(\tan\theta)(r)\left(\frac{dr}{d\theta}\right) + 1}$$

$$\tan\psi = \frac{\tan\theta + r\cdot\frac{d\theta}{dr}}{1-(\tan\theta)\left(r\cdot\frac{d\theta}{dr}\right)} \rightarrow (2)$$

on comparing eqn (1) and (2)

we get.

$$\boxed{\tan\phi = r\cdot\frac{d\theta}{dr}} \quad (\text{or}) \quad \boxed{\cot\phi = \frac{1}{r}\cdot\frac{dr}{d\theta}}$$

22/08/2014

* Angle of Intersection b/w polar curves.

$$\phi = |\phi_1 + \phi_2|$$

(or)

$$\tan\phi = \left|\frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\tan\phi_2}\right|$$

where ϕ_1 and ϕ_2 respectivly represents the angle of intersection b/w radius vector and tangent to the first curve and 2 curve.

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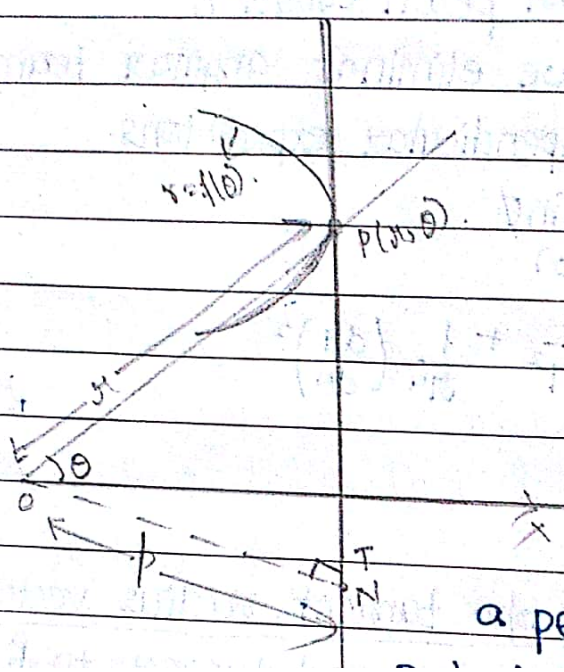
* condition of orthagonality.

$\tan\phi_1 \cdot \tan\phi_2 = -1$

* length of perpendicular form pole to tangent.



let $r = f(\theta)$ be a polar curve and $P(r,\theta)$ be any point on it. $OP = r$ repsents a radius vector which makes an angle θ with x-axis as shown. let pt be a tangent to the curve and ϕ is the angle b/w. radius vector and tangent. let $ON = p$ be a perpendicular drawn from the pole

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Mangal
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* pedal equation or p-r equation

- an equation containing p-r as variables where p is length of perpendicular from pole to tangent and r is radius vector. is called p-r equation (or) pedal equation.
- To find pedal equation we eliminat angular team θ or ϕ from the length of perpendicular expressions.

$$p = r\sin\phi$$

(or)

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$$

by using given curve.

* problems type one (angle b/w tangent, radius vector)

- find the angle b/w radius vector and tangent to the following polar curves.

01. $r^2\cos 2\theta = a^2$
02. $r = a(1-\cos\theta)$
03. $2a = 1+\cos\theta$
(04) $r^n = a^n \sin n\theta$
(05) $r^n = a^n \cos n\theta$
(06) $r^n = a^n \sec(n\theta+\alpha)$
** (07) $r = a(1+\sin\theta)$
(08) $r^2 \sin 2\theta = a^2$
(09) $r^m = a^m(\cos m\theta + \sin m\theta)$
(10) $r = ae^{\theta\cot\alpha}$, α is constant.

(01) Given $r^2\cos 2\theta = a^2$

$$\log(r^2\cos 2\theta) = \log a^2$$

$$\log r^2 + \log\cos 2\theta = \log a^2$$

$$2\log r = \log a^2 - \log\cos 2\theta$$

Diff^n w.r.t θ.

$$\cancel{2}\times\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 - \frac{1}{\cos 2\theta}(-\sin 2\theta)\times\cancel{2}$$

$$\cot\phi = \tan 2\theta$$

$$\cot\phi = \cot(\pi/2 - 2\theta)$$

$$\Rightarrow \phi = \frac{\pi}{2} - 2\theta \,//.$$

***

$$\tan\theta = \cot(\pi/2 - \theta$$

$$-\tan\theta = \cot(\pi/2 + \theta)$$

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$1-\cos\theta = 2\sin^2\theta/2$

$1+\cos\theta = 2\cos^2\theta/2$

$\sin\theta = 2\sin\theta/2 \cos\theta/2$

02. $r = a(1-\cos\theta)$.

$\log r = \log(a(1-\cos\theta))$.

$\log r = \log a + \log(1-\cos\theta)$

~~$\frac{1}{r}\frac{dr}{d}$~~ diff w.r.to θ

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{1-\cos\theta}(\sin\theta)$$

$$\cot\phi = \frac{\cancel{2}\sin\theta/2 \cos\theta/2}{\cancel{2}\sin^2\theta/2}$$

$$\cot\phi = \cot\theta/2$$

$$\boxed{\phi = \theta/2}$$

03. $\frac{2a}{r} = 1+\cos\theta$.

$$\log\left(\frac{2a}{r}\right) = \log(1+\cos\theta)$$

$$\log(2a) - \log(r) = \log(1+\cos\theta)$$

Diff w.r.to θ

$$0 - \frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+\cos\theta}\,-\sin\theta$$

$$\cancel{-}\cot\phi = \cancel{-}\frac{\cancel{2}\sin\theta/2 \cos\theta/2}{\cancel{2}\cos^2\theta/2}$$

$$\cot\phi = \frac{\sin\theta/2}{\cos\theta/2}$$

$$\cot\phi = \tan\theta/2$$

$$= \cot\left(\pi/2 - \theta/2\right)$$

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04. $r^n = a^n \sin n\theta$

$\log(r^n) = \log(a^n \sin n\theta)$

$n \log r = \log(a^n) + \log(\sin n\theta)$.

Diff^n w.r.t θ.

$\cancel{n} \times \frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\sin n\theta} \cdot \cos n\theta \cdot \cancel{n}$

$\cot \phi = \frac{\cos n\theta}{\sin n\theta}$

$\cancel{\cot} \phi = \cancel{\cot} n\theta$

$\boxed{\phi = n\theta}$

05. $r^n = a^n \cos n\theta$

$\log(r^n) = \log(a^n \cos n\theta)$

$n \log(r) = \log(a^n) + \log(\cos n\theta)$

Diff^n w.r.t θ.

$\cancel{n} \cdot \frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta} - \sin n\theta \cancel{n}$

$\cot \phi = -\tan n\theta$

$\cot \phi = \cot(\pi/2 + n\theta)$

$\boxed{\phi = \pi/2 + n\theta}$

06. $r^n = a^n \sec(n\theta + \alpha)$

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$\cot\phi = \cot\left(\pi/2 - n\theta - \alpha\right)$

$\phi = \left(\pi/2 - n\theta - \alpha\right)$

Q7) $r = a(1+\sin\theta)$.

$\log r = \log\left(a(1+\sin\theta)\right)$

$\log r = \log a + \log(1+\sin\theta)$

Diff^n w.r.to θ.

$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{1+\sin\theta}\cos\theta$

$\cot\phi = \frac{\cos\theta}{1+\sin\theta}$

$\boxed{\cos\theta = \cos^2\theta/2 - \sin^2\theta/2}$

$= \frac{\cos^2\theta/2 - \sin^2\theta/2}{\cos^2\theta/2 + \sin^2\theta/2 + 2\sin\theta/2\cos\theta/2}$

$= \frac{(\cos\theta/2 - \sin\theta/2)(\cos\theta/2 - \sin\theta/2)}{(\cos\theta/2 + \sin\theta/2)^2}$

$= \frac{(\cos\theta/2 - \sin\theta/2)}{(\cos\theta/2 + \sin\theta/2)}$

$\div$ by $\cos\theta/2$. we get

$\cot\phi = \frac{1 - \tan\theta/2}{1 + \tan\theta/2}$.

$= \frac{\tan\pi/4 - \tan\theta/2}{1 + \tan\frac{\pi}{4}\cdot\tan\theta/2}$.

$\cot\phi = \tan\left[\pi/4 - \theta/2\right]$.

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(08) $r^2 \sin 2\theta = a^2$.

$\log(r^2 \sin 2\theta) = \log a^2$

$\log(r^2) + \log(\sin 2\theta) = \log a^2$

$2\log r + \log(\sin 2\theta) = 2\log a^2$

$2\log r = \log a^2 - \log(\sin 2\theta)$.

$\cancel{2} \cdot \frac{1}{r}\frac{dr}{d\theta} = 0 - \frac{1}{\sin 2\theta} \cos 2\theta \cdot \cancel{2}$

$\cot\phi = -\frac{\cos 2\theta}{\sin 2\theta}$

$\cancel{\cot}\ \phi = -\cancel{\cot} 2\theta$

$\boxed{\phi = -2\theta}$

(09) $r^m = a^m(\cos m\theta + \sin m\theta)$.

$\log r^m = \log(a^m(\cos m\theta + \sin m\theta))$.

$\log r^m = \log(a^m) + \log(\cos m\theta + \sin m\theta)$.

$m\log r = 0 + \frac{1}{\cos m\theta + \sin m\theta}(-\sin m\theta \cdot m + \cos m\theta \cdot m)$

$\cancel{m} \cdot \frac{1}{r}\frac{dr}{d\theta} = \frac{\cancel{m}(-\sin m\theta + \cos m\theta)}{(\cos m\theta + \sin m\theta)}$.

$\cot\phi =$ divid by $\cos m\theta$.

$-\tan m\theta + 1$

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10) Given, $r = ae^{\theta \cot\alpha}$ α is constant.

$\log r = \log(ae^{\theta\cot\alpha})$

$= \log a + \log e^{\theta\cot\alpha}$

diff w.r.t θ

$\frac{1}{r}\frac{dr}{dt} = 0 + \theta\cot\alpha$

$\cot\phi = 1\times\cot\alpha$

$\phi = \alpha$.

II Find the angle of intersection b/w the following pair of curves:-

01. $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$

02. $r = a(1-\cos\theta)$ and $r = 2a\cos\theta$

03. $r = \frac{a}{1+\cos\theta}$ and $r = \frac{b}{1-\cos\theta}$

04. $r = a\theta$ and $r = a/\theta$

05. $a\log\theta = r$ and $r = a/\log\theta$

06. $r = \frac{a\theta}{1+\theta}$ and $r = \frac{a}{1+\theta^2}$

07. $r^n = a^n \sec(n\theta+\alpha)$ and $r^n = b^n\sec(n\theta+\beta)$

* trigonometric problems:- $\phi = |\phi_1 - \phi_2|$

* other problems:- $\tan\phi = \left|\frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\cdot\tan\phi_2}\right|$

01. $r = \sin\theta + \cos\theta$

$\log r = \log(\sin\theta + \cos\theta)$

$\frac{1}{r}\frac{dr}{dt} = \frac{1}{\sin\theta+\cos\theta}(\cos\theta - \sin\theta)$

$\cot\phi_1 = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$= \frac{1-\tan\theta}{} = \tan\pi/4 - \tan\theta$

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$\cot\phi_1 = \cot(\pi/2 - \pi/4 - \theta)$

$\boxed{\phi_1 = \frac{\pi}{4} + \theta} \rightarrow (1)$

$r = 2\sin\theta$

$\log r = \log(2\sin\theta)$

$= \log 2 + \log(\sin\theta)$

diff w.r.t θ

$\frac{1}{r}\frac{dr}{dt} = 0 + \frac{1}{\sin\theta}(\cos\theta)$

$\cot\phi_2 = \cot\theta$

$\boxed{\phi_2 = \theta} \rightarrow (2)$

$(1) - (2) = \phi = |\phi_1 - \phi_2|$

$= \frac{\pi}{4} + \theta - \theta = \pi/4$

2. Given $r = a(1-\cos\theta)$

$\log r = \log[a(1-\cos\theta)]$

$\log r = \log a + \log(1-\cos\theta)$

$\frac{1}{r}\cdot\frac{dr}{dt} = 0 + \frac{1}{1-\cos\theta}\sin\theta$

$\cot\phi_1 = \frac{\sin\theta}{1-\cos\theta}$

$= \frac{2\sin\theta/2\cos\theta/2}{2\sin^2\theta/2}$

$\cot\phi_1 = \cot\theta/2$

$\boxed{\phi_1 = \theta/2} \rightarrow (1)$

$r = 2a\cos\theta$

$\log r = \log(2a\cos\theta)$

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$\log r = \log 2a + \log \cos\theta$

$\frac{1}{r}\cdot\frac{dr}{dt} = 0 + \frac{1}{\cos\theta}(-\sin\theta)$

$\cot\phi_1 = -\tan\theta$

$\cot\phi_2 = \cot(\pi/2 + \theta) \Rightarrow ②$

$\phi = |\phi_1 - \phi_2|$

$= |\theta/2 - \pi/2 - \theta|$ $= \left|\frac{\theta}{2} - \frac{\pi}{2} - \frac{2\theta}{2}\right|$

$\cancel{= |\pi/2 - \theta/2|}$ $= \left|-\frac{\pi}{2} - \left[\frac{2\theta}{2} - \frac{\theta}{2}\right]\right|$

$= \left|-\frac{\pi}{2} - \frac{\theta}{2}\right|$

$\phi = \pi/2 + \theta/2$ $= \frac{\pi}{2} + \frac{\theta}{2}$

07. $r^n = a^n \sec(n\theta + \alpha)$

$\log(r^n) = \log(a^n) + \log(\sec(n\theta + \alpha))$

~~diff w.r.t.~~

$n\log r = \log a^n + \log\sec(n\theta + \alpha)$

$\cancel{n} \times \frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\sec(n\theta + \alpha)} \times \sec(n\theta + \alpha)\tan(n\theta + \alpha) \times \cancel{n}$

$\cot\phi_1 = \frac{\cancel{\sec(n\theta + \alpha)}\tan(n\theta + \alpha)}{\cancel{\sec(n\theta + \alpha)}}$

$\cot\phi_1 = \tan(n\theta + \alpha)$

$\cot\phi_1 = \cot(\pi/2 - n\theta - \alpha)$

$\phi_1 = \left(\frac{\pi}{2} - n\theta - \alpha\right)$

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3. $r = \frac{a}{1+\cos\theta}$ $\qquad$ $r = \frac{b}{1-\cos\theta}$

$\log r = \log\left(\frac{a}{1+\cos\theta}\right)$

$\log r = \log a - \log(1+\cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 - \frac{1}{1+\cos\theta}(-\sin\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{\sin\theta}{1+\cos\theta}$

$\cot\phi_1 = \frac{\cancel{2}\sin\theta/2\ \cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$

$\cot\phi_1 = \tan\theta/2.$

$\phi_1 = \frac{\pi}{2} - \frac{\theta}{2}$

$\log r = \log\left(\frac{b}{1-\cos\theta}\right)$

$\log r = \log b - \log(1-\cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 - \frac{1}{1-\cos\theta}(\sin\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = -\frac{\sin\theta}{1-\cos\theta}$

$\cot\phi_2 = -\frac{\cancel{2}\sin\theta/2\ \cos\theta/2}{\cancel{2}\sin^{\cancel{2}}\theta/2}$

$\cot\phi_2 = -\cot\theta/2$

$\phi_2 = -\theta/2$

$\phi = |\phi_1 - \phi_2|$

$= \frac{\pi}{2} - \theta/2 + \theta/2$

$\phi = \pi/2$ curve out orthogonally.

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4. Given $r = a\theta$ and $r = \frac{a}{\theta}$

$\log r = \log(a\theta)$

$\log r = \log a + \log\theta$

Diffn w.r.t θ

$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{\theta}$

$\cot\phi_1 = 1/\theta$

$\tan\phi_1 = \theta$

$\log r = \log\left(\frac{a}{\theta}\right)$

$\log r = \log a - \log\theta$

Diffn w.r.t θ

$\frac{1}{r}\frac{dr}{d\theta} = 0 - \frac{1}{\theta}$

$\cot\phi_2 = \frac{-1}{\theta}$

$\tan\phi_2 = -\theta$

w.k.t

$$\tan\phi = \left|\frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\cdot\tan\phi_2}\right|$$

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$\tan\phi = \left|\frac{2\theta}{1-\theta^2}\right| \Rightarrow (1)$

we have

$r = a\theta$ and $r = \frac{a}{\theta}$

$\cancel{\log r = \log a - \log\theta}$ $a\theta = \frac{a}{\theta} \Rightarrow \theta^2 = 1$

$\Rightarrow \theta = 1$.

sub in (1)

$\tan\phi = \left|\frac{2}{0}\right| = \phi = \tan^{-1}(\infty)$

$\phi = \tan^{-1}\left(\tan\frac{\pi}{2}\right)$

$\phi = \frac{\pi}{2}$ //.

5. $r = a\log\theta$ and $r = \frac{a}{\log\theta}$

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$$a \log\theta = \frac{a}{\log\theta} \Rightarrow (\log\theta)^2 = 1.$$

$$\log\theta = 1 \Rightarrow \theta = e$$

sub $\ln(1)$

$$\tan\phi = \left|\frac{2e}{1-e^2}\right|$$

$$\phi = \tan^{-1}\left(\frac{2e}{1-e^2}\right)$$

(6) $r = \frac{a\theta}{1+\theta}$ and $r = \frac{a}{1+\theta^2}$

$\log r = \log\frac{a\theta}{1+\theta}$ | $\log r = \log\frac{a}{1+\theta^2}$

$\log r = \log(a\theta) - \log(1+\theta)$.

diff w.r.t θ

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = \frac{1\cancel{a}}{\cancel{a}\theta} - \frac{1}{1+\theta}$$

$$\cot\phi_1 = \frac{1+\theta-\theta}{\theta(1+\theta)}$$

$$\frac{1+1}{\theta(1+\theta)}$$

$$\tan\phi_1 = \theta(1+\theta)$$

$\log r = \log a - \log(1+\theta^2)$

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = \frac{1\cdot a}{a} - \frac{1\,2\theta}{(1+\theta^2)}$$

$$\cot\phi_2 = \frac{-1\,2\theta}{(1+\theta^2)}$$

$$\tan\phi_2 = -\frac{(1+\theta^2)}{2\theta}$$

$$\tan\phi = \frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\tan\phi_2}$$

$$\tan\phi = \frac{\theta(1+\theta) + \frac{(1+\theta^2)}{2\theta}}{1 - \theta(1+\theta)\cdot\left(\frac{1+\theta^2}{2\theta}\right)}$$

$$\left|\frac{1(1+1) + \left(\frac{1+(1)^2}{2(1)}\right)}{1 - 1(1+1)\left(\frac{1+(1)^2}{\ldots}\right.}\right| = \tan\phi = \left|\frac{1(2)+(1)}{1-2}\right|$$

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$$\frac{a\theta}{1+\theta}, \quad \frac{a}{1+\theta^2}$$

$$\frac{a\theta}{1+\theta} = \frac{a}{1+\theta^2}$$

$$a\theta(1+\theta^2) = a(1+\theta)$$

$$a\theta + a\theta^3 = a + a\theta$$

$$\theta^3 = \frac{a}{a}$$

$$\boxed{\theta = 1}$$

$$\phi = \tan^{-1}(3)$$

(8) prove that the pair of curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ intersect orthogonality

$r = a(1+\cos\theta)$

$\log r = \log a + \log(1+\cos\theta)$

$\frac{1}{r}\cdot\frac{dr}{d\theta} = \frac{1}{a} + \frac{1}{1+\cos\theta}(-\sin\theta)$

$\cot\phi_1 = 0 + \frac{(-\sin\theta)}{1+\cos\theta}$

$\cot\phi_1 = \frac{-2\sin\theta/2 \cdot \cos\theta/2}{2\cos^2\theta/2}$

$= -\tan\theta/2$

$r = b(1-\cos\theta)$

$\log r = \log b + \log(1-\cos\theta)$

Diffn w.r.t θ

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{\sin\theta}{1-\cos\theta}$

$\cot\phi_2 = \frac{2\sin\theta/2 \cos\theta/2}{2\sin^2\theta/2}$

$= \cot\theta/2$

$\cot\phi_2 = \cot\theta/2$

$\tan\phi_2 = \frac{1}{\cot\theta/2} = \tan\theta/2$

$\cot\phi_1 = -\tan\theta/2$

$\tan\phi_1 = \frac{-1}{\tan\theta/2}$

$\tan\phi_1 \cdot \tan\phi_2 = -1$

The curves intersects orthogonally

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$\tan\phi_1 \cdot \tan\phi_2 = -1$

$1 \cdot -1 = -1$

$-1 = -1$

the curves intersect orthogonally.

Type-III

* find the pedal equation or P-R equation for the following polar curves

01. $r = a\theta$

02. $r^n = a^n \sin n\theta$

03. $r = a(1+\cos\theta)$

04. $2a/r = 1-\cos\theta$

05. $r^m \cos m\theta = a^m$

06. $r^n = a^n(\cos n\theta + \sin n\theta)$

07. $\frac{l}{r} = 1 + e\cos\theta$.

(2) Given $r^n = a^n \sin\theta \rightarrow (1)$

$\log(r^n) = \log a^n + \log \sin\theta$

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$$\frac{1}{p^2} = \frac{\operatorname{cosec}^2 n\theta}{r^2}$$

$$p^2 = \frac{r^2}{\operatorname{cosec}^2 n\theta}$$

$$p^2 = r^2 \sin^2 n\theta$$

$$p = r\sin n\theta$$

Since from (1)

$$\sin n\theta = \frac{r^n}{a^n}$$

$$p = r\left(\frac{r^n}{a^n}\right)$$

$$a^n p = r^{n+1}.$$

5. $r^m \cos m\theta = a^m$.

$$\log r^m + \log\cos m\theta = \log a^m$$

$$m\log r + \log\cos m\theta = \log a^m$$

diff w.r.t θ

$$\cancel{m}\cdot\frac{1}{r}\frac{dr}{d\theta} + \frac{1}{\cos m\theta}\cdot -\sin m\theta\cdot\cancel{m} = 0$$

$$\cancel{m}\cdot\frac{1}{r}\cdot\frac{dr}{d\theta} + -\tan m\theta = 0$$

$$\frac{1}{r}\frac{dr}{d\theta} = \tan m\theta\cdot r$$

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$$

$$= \frac{1}{r^2} + \frac{1}{r^4}\left(\tan m\theta\cdot r\right)^2$$

$$\frac{1}{r^2} + \frac{1}{r^{\cancel{4}}}\tan^2 m\theta\cdot r^{\cancel{2}}$$

$$\frac{1}{r^2} + \frac{1}{r^2}\tan^2 m\theta$$

$$= \frac{1}{r^2}\left(1 + \tan^2 m\theta\right)$$

$$\frac{1}{p^2} = \frac{1 + \tan^2 m\theta}{r^2} = \frac{\sec^2 m\theta}{r^2}$$

$$p^2 = r^2\cos^2 m\theta$$

$$p = r\cos m\theta$$

Since from (1)

$$\cos m\theta = \frac{a^m}{r^m}$$

$$p = r\left(\frac{a^m}{r^m}\right)$$

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03. $r = a(1+\cos\theta) \rightarrow (1)$

$\log r = \log\{a(1+\cos\theta)\}$.

$\log r = \log a + \log(1+\cos\theta)$

Diffn w.r.t θ.

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{(-\sin\theta)}{1+\cos\theta}$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{-\cancel{2}\sin\theta/2\ \cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$$

$$\frac{dr}{d\theta} = -r\tan\theta/2 \ //.$$

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$$

$$\frac{1}{r^2} + \frac{1}{r^4}\left(r^2\tan^2\theta/2\right)$$

$$= \frac{1}{r^2} + \frac{1}{r^2}\left(\tan^2\theta/2\right)$$

$$\frac{1}{r^2}\left\{1+\tan^2\theta/2\right\}.$$

$$\frac{1}{p^2} = \frac{1}{r^2}\sec^2\theta/2$$

$$p^2 = r^2\cos^2\theta/2 \rightarrow (1)$$

from (1) $1+\cos\theta = r/a$

$2\cos^2\theta/2 = r/a$

$\cos^2\theta/2 = r/2a$

Sub in eqn (1)

$p^2 = r^2(r/2a)$

$2ap^2 = r^3$.

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(4) $\frac{2a}{r} = (1-\cos\theta)$.

$$\frac{1}{r} = \frac{(1-\cos\theta)}{2a}.$$

$$r = \frac{2a}{(1-\cos\theta)}$$

taking log on both side we get-

$$\log r = \log\left(\frac{2a}{1-\cos\theta}\right)$$

$$\log r = \log 2a - \log(1-\cos\theta)$$

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 - \frac{1}{1-\cos\theta}(\sin\theta)$$

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = \frac{\cancel{2}\sin\theta/2\,\cos^2\theta/2}{\cancel{2}\sin^2\theta/2}$$

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = \cot\theta/2.$$

$$\frac{dr}{d\theta} = r\cot\theta/2.$$

w.k.t

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} = (r\cot\theta/2)^2$$

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(r^2\cot^2\theta/2\right)$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left(1+\cot^2\theta/2\right)$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left(\operatorname{cosec}^2\theta/2\right)$$

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$$p^2 = r^2\left(\sin^2 \theta/2\right)$$

from 1 → $1-\cos\theta = \frac{2a}{r}$.

$$\cancel{2}\sin^2\theta/2 = \frac{\cancel{2}a}{r}$$

$$\sin^2\theta/2 = \frac{a}{r}.$$

$$p^2 = r^{\cancel{2}}\left(\frac{a}{\cancel{r}}\right)$$

$$\boxed{p^2 = ar}$$

06. Given.

$$r^n = a^n(\cos n\theta + \sin n\theta) \rightarrow (1)$$

$$\log r^n = \log\{a^n(\cos n\theta + \sin n\theta)\}.$$

$$n\log r = \log a^n + \log(\cos n\theta + \sin n\theta).$$

$$n \times \frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{(-\sin n\theta \times n) + (\cos n\theta \times n)}{\cos n\theta + \sin n\theta}.$$

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = \cancel{n}\left\{\frac{\cos n\theta - \sin n\theta}{\cos n\theta + \sin n\theta}\right\}$$

$$\frac{dr}{d\theta} = r\left\{\frac{\cos n\theta - \sin n\theta}{\cos n\theta + \sin n\theta}\right\}.$$

we have.

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2.$$

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\cancel{r^2}\left\{\frac{(\cos n\theta - \sin n\theta)^2}{(\cos n\theta + \sin n\theta)^2}\right\}$$

$$\frac{1}{r^2}\left\{1 + \frac{(\cos n\theta - \sin n\theta)^2}{(\cos n\theta + \sin n\theta)^2}\right\}.$$

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$$\frac{1}{r^2}\left\{\frac{(\cos n\theta+\sin n\theta)^2+(\cos n\theta-\sin n\theta)^2}{(\cos n\theta+\sin n\theta)^2}\right\}$$

$$\frac{1}{r^2}\left\{\frac{\cos^2 n\theta+\sin^2 n\theta+\cancel{2\cos n\theta\sin\theta}+\cos^2 n\theta+\sin^2 n\theta-\cancel{2\cos n\theta\sin\theta}}{\left(\frac{r^n}{a^n}\right)^2}\right\}$$

$$=\frac{1}{r^2}=\frac{2\times a^{2n}}{r^{2n}}.$$

$$\frac{1}{p^2}=\frac{2a^{2n}}{r^{2n+2}}\Rightarrow r^{2n+2}=2a^{2n}p^2.$$

(7) $\frac{l}{r}=1+\cos\theta$.

$\frac{1}{r}=\frac{1+\cos\theta}{l}$ $\qquad$ $\frac{1}{r}=\frac{1+e\cos\theta}{l}$

$r=\frac{l}{1+\cos\theta}$ $\qquad$ $r=\frac{l}{1+e\cos\theta}$

$\log r=\log l-\log(1+\cos\theta)$

$\frac{1}{r}\cdot\frac{dr}{d\theta}=0-\frac{1}{1+\cos\theta}(-\sin\theta)$

$=\frac{\cancel{2}\sin\theta/2\cos\theta/2}{\cancel{2}\sin^2\theta/2}$

$=\tan\theta/2$

$\frac{dr}{d\theta}=r\tan\theta/2$.

$\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{r^2}(r^2\tan^2\theta/2)$

$\frac{1}{r^2}\left(1+\tan^2\theta/2\right)$

$\frac{1}{r^2}\left(\sec^2\theta/2\right)$.

$\log r=\log l-\log(1+e\cos\theta)$.

$\frac{1}{r}\cdot\frac{dr}{d\theta}=0-\frac{1}{1+e\cos\theta}(-e\sin\theta)$

$=\frac{e\sin\theta}{1+e\cos\theta}$.

$\frac{dr}{d\theta}=r\left(\frac{e\sin\theta}{1+e\cos\theta}\right)$.

$\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{r^2}\left(\frac{r^2e^2\sin^2\theta}{(1+e\cos\theta)^2}\right)$

$=\frac{1}{r^2}\left[\frac{1+e^2\sin^2\theta}{(1+e\cos\theta)^2}\right]$.

$=\frac{1}{r^2}\frac{(1+e\cos\theta)^2+e^2\sin^2\theta}{(1+e\cos\theta)^2}$

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$p^2 = r^2(\cos^2\theta/2)$

from (1) $1+\cos\theta = \frac{l}{r}$

$2\cos^2\theta/2 = \frac{l}{r}$

$\cos^2\theta/2 = \frac{l}{2r}$

Sub in eqn (I)

$p^2 = r^2\left(\frac{l}{2r}\right)$

$2rp^2 = r^2 l$

$\frac{1}{r^2}\left[\frac{1+e^2\cos^2\theta + 2e\cos\theta + e^2\sin^2\theta}{(1+e\cos\theta)^2}\right]$

$\frac{1}{r^2}\left(\frac{1+e^2+2e\cos\theta}{(1+e\cos\theta)^2}\right)$

$\frac{1}{r^2}\left(\frac{1+e^2+2\left(\frac{l}{r}-1\right)}{\left(\frac{l}{r}\right)^2}\right)$ $\cos\theta = \frac{l}{r}-1$

$= \frac{1}{r^2}\left(\frac{1+e^2+2\left(\frac{l}{r}-1\right)}{l^2}\cdot r^2\right)$

$= \frac{1}{p^2} = \left(\frac{re^2+2l-r}{rl^2}\right)$

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**(8) If $r^n = a^n \cos n\theta + b^n \sin n\theta \rightarrow (1)$ then prove that the pedal equation is $p^2(a^{2n}+b^{2n}) = r^{2n+2}$.

$$r^n = a^n \cos n\theta + b^n \sin\theta \rightarrow (1)$$

$$\log(r^n) = \log(a^n \cos n\theta + b^n \sin n\theta).$$

$$n \log r = \log(a^n \cos n\theta + b^n \sin n\theta)$$

Diff^n w.r.t θ.

$$n \times \frac{1}{r}\frac{dr}{d\theta} = \frac{-a^n \sin n\theta \times n + b^n \cos n\theta \times n}{a^n \cos n\theta + b^n \sin n\theta}.$$

$$\cancel{n} \times \frac{1}{r}\frac{dr}{d\theta} = \cancel{n}\left\{\frac{b^n \cos n\theta - a^n \sin n\theta}{a^n \cos n\theta + b^n \sin n\theta}\right\}.$$

$$\frac{dr}{d\theta} = r\left\{\frac{b^n \cos n\theta - a^n \sin n\theta}{a^n \cos n\theta + b^n \sin n\theta}\right\}.$$

W.K.T

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$$

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \times r^2\left\{\frac{(b^n \cos n\theta - a^n \sin n\theta)^2}{(a^n \cos n\theta + b^n \sin n\theta)^2}\right\}.$$

$$= \frac{1}{r^2}\left[1 + \frac{(b^n \cos n\theta - a^n \sin n\theta)^2}{(a^n \cos n\theta + b^n \sin n\theta)^2}\right\}.$$

$$= \frac{1}{r^2}\left[\frac{(a^n \cos n\theta + b^n \sin n\theta)^2 + (b^n \cos\theta - a^n \sin n\theta)^2}{(a^n \cos n\theta + b^n \sin n\theta)^2}\right\}$$

$$= \frac{1}{r^2}\left[\frac{a^{2n}\cos^2 n\theta + b^{2n}\sin^2 n\theta + \cancel{2a^n b^n \cos n\theta \sin n\theta} + a^{2n}\sin^2 n\theta + b^{2n}\cos^2 n\theta - \cancel{2a^n b^n \cos n\theta \sin n\theta}}{(a^n \cos n\theta + b^n \sin n\theta)^2}\right]$$

$$\frac{1}{r^2}\left[\frac{a^{2n}(\cos^2 n\theta + \sin^2 n\theta) + b^{2n}(\cos^2 n\theta + \sin^2 n\theta)}{(a^n \cos n\theta + b^n \sin n\theta)^2}\right].$$

$$\frac{1}{r^2}\left[\frac{a^{2n}(\cos^2 n\theta + \sin^2 n\theta) + b^{2n}(\cos^2 n\theta + \sin^2 n\theta)}{r^{2n}}\right]$$

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$$\frac{1}{p^2} = \frac{a^{2n}+b^{2n}}{r^{2n+2}}$$

$$p^2 = (a^{2n}+b^{2n}) = r^{2n+2}$$

~~(9) If $r\cos\left(\frac{1}{a}\sqrt{a^2-b^2}\right)\theta$ then prove that the peda~~

** (9) Show that for the curve $r\cos\left(\frac{1}{a}\sqrt{a^2-b^2}\right)\theta = \sqrt{a^2-b^2}$ the pedal equation is $p^2 = \frac{a^2r^2}{r^2+b^2}$

$$r\cos\left(\frac{1}{a}\sqrt{a^2-b^2}\right)\theta = \sqrt{a^2-b^2}$$

put $\frac{1}{a}\sqrt{a^2-b^2} = k \Rightarrow \sqrt{a^2-b^2} = ak$

$$r\cos k\theta = ak.$$

$$\log(r\cos k\theta) = \log(ak).$$

$$\log r + \log\cos k\theta = \log ak.$$

Diffn w.r.t θ.

$$\frac{1}{r}\cdot\frac{dr}{d\theta} + \frac{(-\sin k\theta)}{\cos k\theta}\times k = 0$$

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = k\tan k\theta$$

$$\frac{dr}{d\theta} = rk\tan k\theta$$

w.k.t $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$

$$\frac{1}{r^2} + \frac{1}{r^4}\left(r^2k^2\tan^2 k\theta\right).$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left\{1 + k^2\tan^2 k\theta\right\} \rightarrow (1)$$

we have

$$r\cos k\theta = ak \Rightarrow \cos k\theta = ak/r \Rightarrow \sec k\theta = \frac{r}{ak}.$$

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$$\sec^2 k\theta = \frac{r^2}{a^2k^2} = \sec^2 k\theta - 1 = \frac{r^2}{a^2k^2} - 1$$

$$\tan^2 k\theta = \frac{r^2}{a^2k^2} - 1$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left\{1 + k^2\left(\frac{r^2}{a^2k^2} - 1\right)\right\} = \frac{1}{r^2}\left\{1 + \frac{r^2}{a^2} - k^2\right\}$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left\{1 + \frac{r^2}{a^2} - \frac{1}{a^2}(a^2 - b^2)\right\}$$

$$= \frac{1}{r^2}\left\{1 + \frac{r^2}{a^2} - 1 + \frac{b^2}{a^2}\right\}$$

$$\frac{1}{p^2} = \frac{r^2 + b^2}{a^2r^2} \Rightarrow p^2 = \frac{a^2r^2}{r^2 + b^2} \quad //$$

06/09/2017

* Radius of curvature.

Curvature of a curve

- The curvature of a curve represents the amount of bending
- of curve it is denoted by (k)

Mathematically the curve is defined as.

$$k = \lim_{\delta s \to 0} \frac{\delta\psi}{\delta s} = \frac{d\psi}{ds}.$$

- If K is not equal to zero then the resciprocal of curvature is called the radius of curvature denoted by (ρ)
- That is $\rho = 1/k = \frac{ds}{d\psi}$

OT. Radius of curvature in cartesian form. (x and y).

$$\rho = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{d^2y/dx^2} \quad (\text{or}) \quad \rho = \frac{\left\{1 + \left(\frac{dx}{dy}\right)^2\right\}^{3/2}}{\frac{d^2x}{dy^2}}$$

$$\rho = \frac{\{1 + (y')^2\}^{3/2}}{y''}$$

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Q1 Find the radius of curvature for the curve $y = a\cosh(x/a)$

Given

$y = a\cosh(x/a) \rightarrow (1)$

$$\rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{d^2y/dx^2} \rightarrow (2)$$

Diff^n eq^n (1) w.r.t x.

$$\frac{dy}{dx} = \cancel{a} \times \sinh(x/a) \times \frac{1}{\cancel{a}}$$

$$\frac{dy}{dx} = \sinh(x/a)$$

Diff^n again w.r.t x.

$$\frac{d^2y}{dx^2} = \cosh(x/a) \times \frac{1}{a}.$$

Sub $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in eq^n (2)

$$\rho = \frac{\{1+\sinh^2(x/a)\}^{3/2}}{1/a\cosh(x/a)}$$

$$= \frac{\cosh^3(x/a)\}^{3/2}}{1/a\cosh(x/a)}.$$

$$= \frac{a\cosh^3(x/a)}{\cancel{\cosh(x/a)}}$$

$$\rho = a\cosh^2(x/a)$$

from (1), $\cosh(x/a) = y/a$

$$\rho = \cancel{a}\left(y^2/a^{\cancel{2}}\right)$$

$$\rho = \left(y^2/a\right).$$

Q2 Show that the radius of curvature for the curve $y = 4\sin x - \sin 2x$ at $x = \pi/2$ is $\frac{5\sqrt{5}}{4}$.

Given $y = 4\sin x - \sin 2x \rightarrow 1$

$$\rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{d^2y/dx^2} \rightarrow (2)$$

Diff^n eq^n (1) w.r.t x.

$$\frac{dy}{dx} = 4\cos x - 2\cos 2x \rightarrow (3)$$

at $x = \pi/2$, $dy/dx = 4\cos\pi/2 - 2\cos\pi$

$$\frac{dy}{dx} = 2.$$

Diff^n eq^n (3) w.r.t x.

$$\frac{d^2y}{dx^2} = -4\sin x + 4\sin 2x$$

at $x = \pi/2$ $\frac{d^2y}{dx^2} = -4\sin\pi/2 + 4\sin\pi$

$$\frac{d^2y}{dx^2} = -4.$$

Sub dy/dx and $\frac{d^2y}{dx^2}$ in eq^n (2)

$$\rho = \frac{\{1+4\}^{3/2}}{-4} = \frac{5^{3/2}}{-4} = \frac{\sqrt{5^3}}{-4}.$$

$$\rho = \frac{-5\sqrt{5}}{4} = |\rho| = \left|\frac{-5\sqrt{5}}{4}\right|$$

$$= \rho = 5\sqrt{5}.$$

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**03. Find the radius of curvature for the curve $x^3+y^3=3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

Given $x^3+y^3=3axy \rightarrow (1)$

$$\rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}} \rightarrow (2)$$

Diff^n eq^n (1) w.r.t x.

$$3x^2+3y^2\frac{dy}{dx}=3ax\frac{dy}{dx}+3ay.$$

$$3y^2\frac{dy}{dx}-3ax\frac{dy}{dx}=3ay-3x^2$$

$$\cancel{3}\frac{dy}{dx}(y^2-ax)=\cancel{3}(ay-y^2)$$

$$\frac{dy}{dx}=\frac{ay-x^2}{y^2-ax} \rightarrow (3)$$

at $(3a/2, 3a/2)$.

$$\frac{dy}{dx}=\frac{a\left(\frac{3a}{2}\right)-\left(\frac{3a}{2}\right)^2}{\left(\frac{3a}{2}\right)^2-a\left(\frac{3a}{2}\right)}=\frac{dy}{dx}=-1$$

Diff^n eq^n (3) w.r.t x.

$$\frac{d^2y}{dx^2}=\frac{(y^2-ax)\left(a\frac{dy}{dx}-2x\right)-(ay-x^2)\left(2y\frac{dy}{dx}-a\right)}{(y^2-ax)^2}$$

at $(3a/2, 3a/2)$

$$\frac{d^2y}{dx^2}=\frac{\left(\frac{9a^2}{4}-\frac{3a^2}{2}\right)(-a-3a)-\left(\frac{3a^2}{2}-\frac{9a^2}{4}\right)(3a-a)}{\left(\frac{9a^2}{4}-\frac{3a^2}{2}\right)^2}$$

$$\frac{d^2y}{dx^2}=\frac{\left(\frac{9a^2}{4}-\frac{3a^2}{2}\right)(-4a-4a)}{\left(\frac{9a^2}{4}-\frac{3a^2}{2}\right)^2}$$

$$=\frac{-8a}{\frac{9a^2-6a^2}{4}}$$

$$\frac{d^2y}{dx^2}=\frac{-8a\times 4}{3a^2}$$

$$\frac{d^2y}{dx^2}=\frac{-32}{3a}$$

sub in (2)

$$\rho=\frac{(1+1)^{3/2}}{\frac{-32}{3a}}=(2)^{3/2}\times\frac{-3a}{32}.$$

$$|\rho|=\left|\frac{-3a}{32}\times\sqrt{2^3}\right|$$

$$=\rho=\frac{3a}{\cancel{32}}\times\frac{\cancel{2}\sqrt{2}}{16}$$

$$\rho=\frac{3a\sqrt{2}}{16}$$

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**04) Find radius of curvature for the curve $x^4+y^4=2$.

Given $= x^4+y^4=2 \rightarrow (1)$

$$\rho=\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{d^2y/dx^2} \rightarrow (2)$$

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Diff^n eq ① w.r.t x

$$4x^3 + 4y^3 \frac{dy}{dx} = 0$$

$$4y^3 \frac{dy}{dx} = -4x^3$$

$$\frac{dy}{dx} = \frac{-x^3}{y^3} \rightarrow ③$$

at $(2,2)$ $\frac{dy}{dx} = -\frac{2^3}{2^3} = -1$

Diff^n eq ③ w.r.t x.

$$\frac{d^2y}{dx^2} = -\frac{\left\{y^3(3x^2) - x^3\left(3y^2\frac{dy}{dx}\right)\right\}}{(y^3)^2}$$

at $(2,2)$, $\frac{d^2y}{dx^2} = -\left\{\frac{96+96}{64}\right\}$

$$d^2y/dx^2 = -3.$$

Sub dy/dx and d^2y/dx^2 in ②

$$\rho = \frac{\{1+(-1)^2\}^{3/2}}{-3}$$

$$\frac{2^{3/2}}{-3}.$$

$$|\rho| = \left|\frac{-\sqrt{2^3}}{3}\right|$$

$$\rho = \frac{2\sqrt{2}}{3}$$

find the R.C for the curve $y^2 = \frac{a^2(a-x)}{x}$. at $(a,0)$

(or)

Find the RC for the curve

$$y^2 = \frac{a^2(a-x)}{x}$$

Diff^n eq ① w.r.t x.

$$2y\, dy/dx = a^2\left\{\frac{x(-1)-(a-x)}{x^2}\right\}$$

$$\frac{dy}{dx} = \frac{-a^3}{2x^2y}.$$

at $(a,0)$, $\frac{dy}{dx} = \infty$

hence, $\frac{dy}{dx} = \frac{-2x^2y}{a^3} \rightarrow ②$

at $(0,0)$ $\frac{dx}{dy} = 0$

Diff^n eq ② w.r.t y.

$$\frac{d^2x}{dy^2} = -\frac{2}{a^3}\left\{x^2 + 1 + y \times 2x \times \frac{dx}{dy}\right\}$$

at $(0,0)$, $\frac{d^2y}{dx^2} = \frac{-2}{a^3}(a^2) = -2/a.$

w.k.t $\rho = \frac{\{1+(dx/dy)^2\}^{3/2}}{d^2x/dy^2}$

$$= \frac{\{1+0\}^{3/2}}{-2/a}$$

$$\rho = -a/2 \times 1 \Rightarrow |\rho| = |-a/2|$$

$$\rho = a/2.$$

6. St for the parabola $y^2 = 4ax$, the square of the radius of curvature at any point varies

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a cube of the focal distance from that point.

Given $y^2 = 4ax \rightarrow (1)$

Diff^n eq (1) w.r.t x

$$\cancel{2}y\frac{dy}{dx} = \cancel{4}^{2}ax1$$

$$\frac{dy}{dx} = \frac{2ax1}{y} \rightarrow (2)$$

Diff^n eq (2) w.r.t x.

$$\frac{d^2y}{dx^2} = 2ax - \frac{1}{y^2} \times \frac{dy}{dx}$$

$$= \frac{-2a}{y^2} \times \frac{dy}{dx} = \frac{-2a}{y^2} \times \frac{2a}{y}$$

$$\frac{d^2y}{dx^2} = \frac{-4a^2}{y^3}.$$

w.k.t $\rho = \dfrac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{d^2y/dx^2}$

$$= \frac{\left\{1+\frac{4a^2}{y^2}\right\}^{3/2}}{\frac{-4a^2}{y^3}}$$

$$\rho = \left(\frac{y^2+4a^2}{y^2}\right)^{3/2} \times \frac{-y^3}{4a^2}$$

$$= \frac{-y^3}{4a^2} \times \frac{(y^2+4a^2)^{3/2}}{(y^2)^{3/2}}.$$

$$\rho = \frac{-(y^2+4a^2)^{3/2}}{4a^2}$$

$$\rho^2 = \frac{[(y^2+4a^2)^{3/2}]^2}{(4a^2)^2}$$

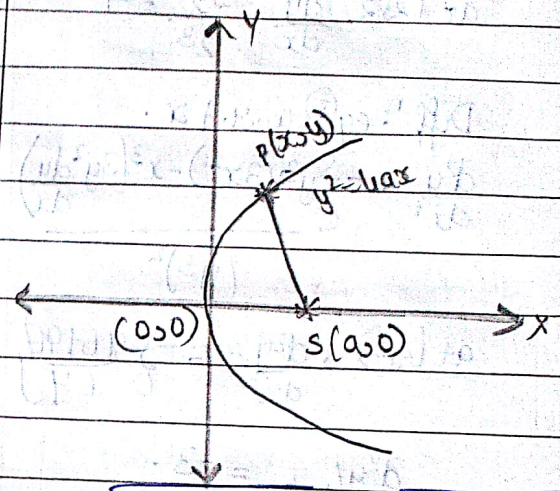
$$= \frac{(4ax+4a^2)^3}{16a^4}$$

$$\rho^2 = \frac{(4a)^3(x+a)^3}{16a^4}$$

$$= \frac{\cancel{16}\times 4\times \cancel{a^3}(x+a)^3}{\cancel{16}\times \cancel{a^4}}$$

$$\rho^2 = 4/a\,(x+a)^3 \rightarrow (3)$$

By distance formula.



$$SP = \sqrt{(x-a)^2+(y-0)^2} = \sqrt{x^2+a^2-2ax+y^2}$$

$$= \sqrt{x^2+a^2-2ax+4ax}$$

$$= \sqrt{x^2+a^2+2ax}$$

$$= \sqrt{(x+a)^2}$$

$$SP = (x+a)$$

Sub in (3)

$$\rho^2 = 4/a\,(SP)^3$$

$$\Rightarrow \rho^2 \propto (SP)^3 //.$$

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7. Find the radius of curvature for the curve $x = a(t + \sin t)$ and $y = a(1 - \cos t)$.

$\frac{dx}{dy} = a(1+\cos t)$ $\quad \frac{dy}{dt} = a\sin t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cancel{a}\sin t}{\cancel{a}(1+\cos t)}$

$= \frac{\cancel{2}\sin t/2 \cancel{\cos t/2}}{\cancel{2}\cos^{\cancel{2}} t/2}$

$\frac{dy}{dx} = \tan t/2 \Rightarrow (1)$

Diffn eqn (1) w.r.t x

$\frac{d^2y}{dx^2} = \sec^2 t/2 \times \frac{1}{2} \times \frac{dt}{dx}$

$= \frac{\sec^2 t/2}{2 * a(1+\cos t)}$

$= \frac{1}{2a} \frac{\sec^2 t/2}{2\cos^2 t/2}$

$\frac{d^2y}{dx^2} = \frac{1}{4a} \sec^4 t/2$

w.k.t $\rho = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{d^2y/dx^2}$

$= \frac{\left\{1 + \tan^2 t/2\right\}^{3/2}}{\frac{1}{4a}\sec^4 t/2}$

$= \frac{(\sec^2 t/2)^{3/2}}{\frac{1}{4a} \cdot \sec^4 t/2}$

$= \frac{1}{1/4a \sec t/2}$

$\rho = 4a \cos t/2$

8. Find the radius of curvature for the curve $x = a(\cos t + \log \tan t/2)$ and $y = a\sin t$

$\frac{dx}{dy} = a\left(-\sin t + \frac{\sec^2 t/2}{\tan t/2} \times \frac{1}{2}\right)$

$= a\left(-\sin t + \frac{\cancel{\cos t/2}}{\sin t/2} \times \frac{1}{\cos^{\cancel{2}} t/2} \times 1/2\right)$

$= a\left(-\sin t + \frac{1}{2\sin t/2 \cos t/2}\right)$

$= a\left(-\sin t + \frac{1}{\sin t}\right)$

$a\left(\frac{1-\sin^2 t}{\sin t}\right)$

$\frac{dx}{dt} = \frac{a\cos^2 t}{\sin t}$ //.

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cancel{a}\sin t}{\frac{\cancel{a}\cos^2 t}{\sin t}}$

$\frac{dy}{dx} = \tan t \Rightarrow (1)$

Diffn eqn (1) w.r.t x.

$\frac{d^2y}{dx^2} = \sec^2 t \times \frac{dt}{dx}$

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$$= \sec^2 t \times \frac{\sin t}{a\cos^2 t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{a}\sec^4 t \cdot \sin t.$$

w.k.t

$$\rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\frac{\{1+\tan^2 t)^{3/2}}{\frac{1}{a}\sec^4 t \cdot \sin t}$$

$$= \frac{\{\sec^2 t)^{3/2}}{\frac{1}{a}\sec^4 t \cdot \sin t}$$

$$\rho = \frac{1}{\frac{1}{a}\sec t \cdot \sin t}$$

$$= a\frac{\cos t}{\sin t}$$

$$\rho = a\cot t.$$

9. Find the radius of curvature for $x = a\cos^3 t$, $y = a\sin^3 t$ at $t = \pi/4$.

$$\frac{dx}{dt} = -3a\cos^2 t \sin t.$$

$$\frac{dy}{dt} = 3a\sin^2 t \cos t.$$

w.k.t $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t}$

$$\frac{dy}{dx} = -\tan t \rightarrow (1)$$

at $t = \pi/4 = \frac{dy}{dx} = -\tan\pi/4 = -1$

Diff^n eq^n (1) w.r.t x.

$$\frac{d^2y}{dx^2} = -\sec^2 t \times dt/dx.$$

$$= \frac{+\sec^2 t}{+3a\cos^2 t\sin t}$$

at $t = \pi/4$ $\frac{d^2y}{dx^2} = \frac{\sec^2\pi/4}{3a\cos^2\pi/4 \cdot \sin\pi}$

$$= \frac{2}{3a \times \frac{1}{2} \times \frac{1}{\sqrt{2}}}$$

$$\frac{d^2y}{dx^2} = \frac{4\sqrt{2}}{3a}$$

$$\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{d^2y/dx^2}$$

$$= \frac{\{1+1\}^{3/2}}{4\sqrt{2}/3a}.$$

$$= (2)^{3/2} \times \frac{3a}{4\sqrt{2}}$$

$$= \frac{\cancel{2}\sqrt{2} \times 3a}{\cancel{4}_2\sqrt{2}}$$

$$\rho = \frac{3a}{2}$$

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* Radius of curvature in pedal form.

$$\boxed{\rho = r\frac{dr}{dp}}$$

① Find the R.C. of $8ap^2 = r^3$

$$8ap^2 = r^3 \Rightarrow ①$$

Diff ① w.r.t. p

$$3r^2 \times dr/dp = 8a(2p)$$

$$r\frac{dr}{dp} = \frac{16ap}{3r^2}$$

$$\rho = \frac{16ap}{3r^2}$$

② Find the R.C. of $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2b^2}$

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2b^2} \Rightarrow ①$$

Diff ① w.r.t. p.

$$-\frac{2}{p^3} = 0 + 0 - \frac{1}{a^2b^2}\left(2r\frac{dr}{dp}\right)$$

$$= -\frac{2}{p^3} = -\frac{2}{a^2b^2}\left(r\frac{dr}{dp}\right)$$

$$\frac{a^2b^2}{p^3} = r\frac{dr}{dp}$$

$$\rho = \frac{a^2b^2}{p^3}$$

* Derivatives of arc length

① Cartesian form:-

$$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\frac{ds}{dy} = \sqrt{1+\left(\frac{dx}{dy}\right)^2}$$

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find ds/dx, ds/dy for $ay^2 = x^3$

Eg: $ay^2 = x^3$

Diff^n w.r.t x

$$a(2y)\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2ay}$$

$$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$= \sqrt{1+\left(\frac{3x^2}{2ay}\right)}$$

$$\sqrt{1+\frac{9x^4}{4a^2y^2}}$$

$$\frac{ds}{dy} = \sqrt{1+\left(\frac{dx}{dy}\right)^2}$$

$$= \sqrt{1+\frac{4a^2y^2}{9x^4}}.$$

② find ds/dt, ds/dy given $x = at^2$ and $y = 2at$.

$$\frac{dx}{dt} = 2at, \quad dy/dt = 2a.$$

$$dy/dx = \frac{dy/dt}{dx/dt} = \frac{2a}{2at}$$

$$dy/dt = \frac{1}{t}$$

$$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+1/t^2}$$

$$\frac{ds}{dy} = \sqrt{1+\left(\frac{dx}{dy}\right)^2}$$

$$= \sqrt{1+t^2}$$

formula.

$$\frac{ds}{dr} = \sqrt{1+\left(r\frac{d\theta}{dr}\right)^2} \quad (or)$$

$$\frac{ds}{dr} = \sec\phi$$

$$\frac{ds}{d\theta} = \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2} \quad (or)$$

$$\frac{ds}{d\theta} = r\operatorname{cosec}\phi$$

1. Find ds/dr, $ds/d\theta$ for $r = a(1-\cos\theta)$.

Given $r = a(1-\cos\theta)$

$$\frac{dr}{d\theta} = a(+\sin\theta)$$

$$\frac{ds}{dr} = \sqrt{1+\left(r\frac{ds}{dr}\right)^2}$$

$$= \sqrt{1+\left(\frac{r}{a\sin\theta}\right)^2}$$

$$= \sqrt{1+\left(\frac{a(1-\cos\theta)}{a\sin\theta}\right)^2}$$

$$\frac{ds}{dr} = \sqrt{1+\left(\frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2}\right)^2}$$

$$\phi = \sqrt{1+\tan^2\theta/2}$$

$$= \sqrt{\sec^2\theta/2}$$

$$\frac{ds}{dr} = \sec\theta/2$$

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$$\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$$

$$= \sqrt{a^2(1-\cos\theta)^2 + a^2\sin^2\theta}$$

$$= a\sqrt{1+\cos^2\theta - 2\cos\theta + \sin^2\theta}$$

$$\frac{ds}{d\theta} = a\sqrt{2 - 2\cos\theta}$$

$$= a\sqrt{2(1-\cos\theta)}$$

$$= a\sqrt{2 \times 2\sin^2\theta/2}$$

$$a = \sqrt{4\sin^2\theta/2}$$

$$\frac{ds}{d\theta} = 2a\sin\theta/2.$$

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Radius of curvature in polar form.

$$\rho = \frac{(r^2+r_1^2)^{3/2}}{r_1^2+2r_1^2 - rr_2} \quad \text{where } r_1 = dr/d\theta \text{ ; } r_2 = \frac{d^2r}{d\theta^2}$$

Q1. Find the radius of curvature from the curve $r = ae^{\theta\cot\alpha}$ where α = constant

Given

$$r = ae^{\theta\cot\alpha}$$

$$\log r = \log(ae^{\theta\cot\alpha})$$

$$= \log a + \log e^{\theta\cot\alpha}.$$

$$\log r = \log a + \theta\cot\alpha.$$

Diff w.r.t θ

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \cot\alpha(1)$$

$$r_1 = r\cot\alpha$$

again w.r.t θ.

$$r_2 = \cot\alpha \quad r_1 = (\cot\alpha)(r\cot\alpha)$$

$$r_2 = r\cot^2\alpha$$

$$\rho = \frac{(r^2+r_1^2)^{3/2}}{(r^2+2r_1^2 - rr_2}$$

Sub r_1 and r_2

$$\rho = \frac{(r^2+r^2\cot^2\alpha)^{3/2}}{r^2+2r^2\cot^2\alpha - r^2\cot^2\alpha.}$$

$$= \frac{(r^2)^{3/2}(1+\cot^2\alpha)^{3/2}}{(r^2+r^2\cot^2\alpha.}$$

$$= \frac{r^3(\operatorname{cosec}^2\alpha)^{3/2}}{r^2(1+\cot^2\alpha)}$$

$$= \frac{r\operatorname{cosec}^3\alpha}{\operatorname{cosec}^2\alpha.}$$

$$\rho = r\operatorname{cosec}\alpha.$$

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2. PT for a curve $r^n = a^n \cos n\theta$, radius of curvature at any points varies invesly as r^{n-1}

Given $r^n = a^n \cos n\theta \rightarrow (1)$

$\log r^n = \log(a^n \cos n\theta)$

$n \log r = \log a^n + \log \cos n\theta$

Diffn w.r.t θ.

$\cancel{n} \times \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{(-\sin\theta)}{\cos n\theta} \times \cancel{n}$

$\frac{1}{r}(r_1) = -\tan n\theta$

$r_1 = -r \tan n\theta$

Diffn again w.r.t θ

$r_2 = -r(\sec^2 n\theta \times n) - \tan n\theta (r_1)$

$= -nr \sec^2 n\theta + r \tan^2 n\theta$

$r_2 = r(\tan^2 n\theta - n\sec^2 n\theta)$

w.k.t

$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$

sub r_1 and r_2

$\rho = \frac{(r^2 + r^2 \tan^2 n\theta)^{3/2}}{r^2 + 2r^2 \tan^2 n\theta - r^2(\tan^2 n\theta - n\sec^2 n\theta)}$

$= \frac{(r^2)^{3/2}(1 + \tan^2 n\theta)^{3/2}}{r^2(1 + 2\tan^2 n\theta - \tan^2 n\theta + n\sec^2 n\theta)}$

$= \frac{r^3 (\sec^2 n\theta)^{3/2}}{r^2(1 + \tan^2 n\theta + n\sec^2 n\theta)}$

$\rho = \frac{r\sec^3 n\theta}{\sec^2 n\theta + n\sec^2 n\theta}$

$= \frac{r\sec^3 n\theta}{\sec^2 n\theta(1+n)}$

$\rho = \frac{r}{1+n} \times \sec n\theta \rightarrow (2)$

from (1) $\cos n\theta = \frac{r^n}{a^n} = \sec n\theta$

sub in (2)

$\rho = \frac{r}{1+n}\left(\frac{a^n}{r^n}\right) \Rightarrow \rho = \left(\frac{a^n}{1+n}\right)\left(\frac{1}{r^{n-1}}\right)$

$\rho \propto \frac{1}{r^{n-1}}$

03. Prove that the curve $r(1-\cos\theta) = 2a$ ρ^2 varies as r^3.

Given

$r(1-\cos\theta) = 2a$.

$\log[r(1-\cos\theta)] = \log 2a$

$\log r + \log(1-\cos\theta) = \log 2a$

$\log r = \log 2a - \log(1-\cos\theta)$.

Diffn w.r.t θ

$\frac{1}{r} \cdot \frac{dr}{d\theta} = 0 - \frac{\sin\theta}{1-\cos\theta}$.

$= -\frac{\cancel{2}\sin\theta/2 \cos\theta}{\cancel{2}\sin^2\theta/2}$

$= -\cot\theta/2$

or $r_1 = -r\cot\theta/2$.

Diffn again w.r.t θ.

$r_2 = -r(-\operatorname{cosec}^2\theta/2 \times 1/2) - \cot\theta/2 (r_1)$

$r_2 = r/2 \operatorname{cosec}^2\theta/2 + r\cot^2\theta/2$.

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$$r_2 = r\left\{\tfrac{1}{2}\operatorname{cosec}^2\theta/2 + \cot^2\theta/2\right\}.$$

$$\rho = \frac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}.$$

Sub r_1 and r_2

$$\rho = \frac{(r^2+r^2\cot^2\theta/2)^{3/2}}{r^2+2r^2\cot^2\theta/2 - r^2\left(\tfrac{1}{2}\operatorname{cosec}^2\theta/2 + \cot^2\theta/2\right)}$$

$$= \frac{(r^2)^{3/2}\left[1+\cot^2\theta/2\right)^{3/2}}{r^2\left\{1+2\cot^2\theta/2 - \tfrac{1}{2}\operatorname{cosec}^2\theta/2 - \cot^2\theta/2\right\}}.$$

$$= \frac{r^3\left(\operatorname{cosec}^2\theta/2\right)^{3/2}}{r^2\left\{1+\cot^2\theta/2 - \tfrac{1}{2}\operatorname{cosec}^2\theta/2\right\}}.$$

$$= \frac{r\operatorname{cosec}^3\theta/2}{\operatorname{cosec}^2\theta/2 - \tfrac{1}{2}\operatorname{cosec}^2\theta/2}$$

$$\rho = \frac{r\operatorname{cosec}^3\theta/2}{\tfrac{1}{2}\operatorname{cosec}^2\theta/2} = \rho = 2r\operatorname{cosec}\theta/2.$$

$$= \rho^2 = 4r^2\operatorname{cosec}^2\theta/2 \rightarrow (2)$$

from (1) $1-\cos\theta = \frac{2a}{r}$

$$2\sin^2\theta/2 = \frac{2a}{r}$$

$$\operatorname{cosec}^2\theta/2 = r/a$$

Sub in eqn (2)

$$\rho^2 = 4r^2(r/a) = (4/a)r^3$$

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Q.4 Show that for the curve $r = a(1+\cos\theta) \Rightarrow 1$

(i) $\frac{\rho^2}{r}$ is constant.

(ii) ρ_1 and ρ_2 are the radii of curvature at 2 excitiments of polar chord of a cardiode. Then $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$

$r = a(1+\cos\theta) \Rightarrow (1)$

$\log r = \log a + \log(1+\cos\theta)$

Diffn w.r.t θ

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{(-\sin\theta)}{(1+\cos\theta)}$$

$$\frac{1}{r}(r_1) = -\frac{\cancel{2}\sin\theta/2\ \cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$$

$$r_1 = -r\tan\theta/2$$

Diffn again w.r.t θ.

$$r_2 = -r\left(\sec^2\theta/2 \times \frac{1}{2}\right) - \tan\theta/2\,(r_1)$$

$$= -r/2\ \sec^3\theta/2 + r\tan^2\theta/2$$

$$r_2 = r\left\{\tan^2\theta/2 - 1/2\ \sec^2\theta/2\right\}$$

$$\rho = \frac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}$$

sub r_1 and r_2.

$$\rho = \frac{\{r^2 + r^2\tan^2\theta/2\}^{3/2}}{r^2 + 2r^2\tan^2\theta/2 - r^2\left[\tan^2\theta/2 - 1/2\ \sec^2\theta/2\right]}$$

$$= \frac{(r^2)^{3/2}\left(\sec^2\theta/2\right)^{3/2}}{r^2\left\{1+2\tan^2\theta/2 - \tan^2\theta/2 + 1/2\ \sec^2\theta/2\right\}}$$

$$= \frac{r^{\cancel{3}}\sec^3\theta/2}{\cancel{r^2}\left[1+\tan^2\theta/2 + 1/2\ \sec^2\theta/2\right]}$$

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$$\rho = \frac{r\sec^3\theta/2}{\sec^2\theta/2 + 1/2\sec^2\theta/2} = \frac{r\sec^3\theta/2}{3/2\sec^2\theta/2}$$

$$\rho = \frac{2r}{3}\sec\frac{\theta}{2} \Rightarrow \rho^2 = \frac{4r^2}{9}\sec^2\theta/2 \Rightarrow (2)$$

(i) from (1) $1+\cos\theta = r/a$.

$2\cos^2\theta/2 = r/a \Rightarrow \cos^2\theta/2 = r/2a$.

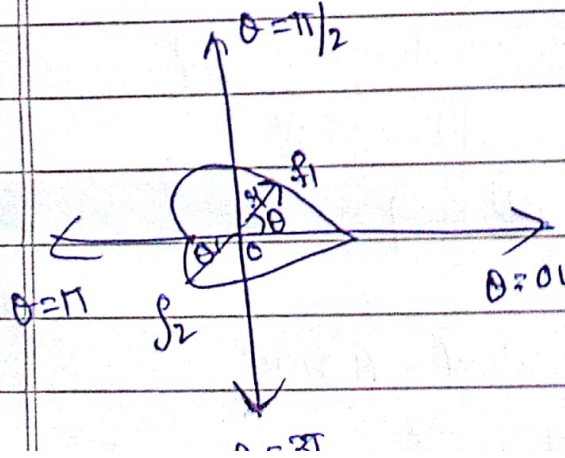
$\sec^2\theta/2 = 2a/r$.

Sub in (2)

$$\rho^2 = \frac{4r^2}{9} \times \frac{2a}{r} \Rightarrow \rho^2 = \frac{8a}{9} \times r.$$

$$\frac{\rho^2}{r} = \frac{8a}{9} = \text{constant}.$$

(ii) we have, $\rho^2 = \frac{8a}{9} \times r = \rho^2 = \frac{8a^2}{9}(1+\cos\theta) \Rightarrow (3)$



$$\rho_1^2 = \frac{8a^2}{9}(1+\cos\theta).$$

put $\theta = \pi + \theta$ in 3.

$$\rho_2^2 = \frac{8a^2}{9}(1+\cos(\pi+\theta)).$$

$$\rho_2^2 = 8a^2(1-\cos\theta)$$

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(1)

(2) Find the R.C of $r^n = a^n \sin n\theta$.

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Derivative formulas

01 $\frac{d}{dx}(x^n) = nx^{n-1}$

02 $\frac{d}{dx}(x) = 1$

03 $\frac{d}{dx}(ax) = a$

04 $\frac{d}{dx}(x^2) = 2x$

05 $\frac{d}{dx}(e^x) = e^x$

06. $\frac{d}{dx}(e^{ax}) = e^{ax} \cdot a$

07. $\frac{d}{dx}(a^x) = a^x \cdot \log a$

08. $\frac{d}{dx}(\log x) = \frac{1}{x}$

09. $\frac{d}{dx}(|x|) = \frac{|x|}{x}$ $+1$ if $x>0$, -1 if $x<0$

$\left\{ |x| \Rightarrow \begin{matrix} x \text{ if } x>0 \\ -x \text{ if } x<0 \end{matrix} \right\}$

10. $\frac{d}{dx}(x^x) = x^x(1+\log x)$

11. $\frac{d}{dx}(\sin x) = \cos x$

12. $\frac{d}{dx}(\cos x) = -\sin x$

13. $\frac{d}{dx}(\tan x) = \sec^2 x$

14. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

15. $\frac{d}{dx}(\sec x) = \sec x \tan x$.

16. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$.

17. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

18. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

19. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

20. $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

21. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

22. $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$

23. $\frac{d}{dx}(uv) = u'v + v'u$

24. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2}$

Trignometric formula

01. $\sin(A+B) = \sin A \cos B + \cos A \sin B$

02. $\sin(A-B) = \sin A \cos B - \cos A \sin B$

03. $\cos(A+B) = \cos A \cos B - \sin A \sin B$

04. $\cos(A-B) = \cos A \cos B + \sin A \sin B$

05. $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

06. $\cos(A+B) + \cos(A-B) = 2\cos A \cos B$

07. $\cos(A-B) - \cos(A+B) = 2\sin A \sin B$

08. $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$

09. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

10. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

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