31/10/2017

Module-5

Linear algebra.

* linear transformations

- A linear transformations in two dimension is defined as

$$y_1 = a_1x_1 + a_2x_2$$
$$y_2 = b_1x_1 + b_2x_2$$

The matrix form of the above transmission is given by

$$Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- Similarly a linear transformation in three dimension is defined as

$$y_1 = a_1x_1 + a_2x_2 + a_3x_3$$
$$y_2 = b_1x_1 + b_2x_2 + b_3x_3 \;; y_3 = c_1x_1 + c_2x_2 + c_3x_3.$$

The matrix form is.

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

- If $Y = AX$ representes a linear transformation than $X = A^{-1}Y$ representes inverse linear transformation.
- A linear transformation $Y = AX$ is said to Regular or Non-Singular if $|A| \neq 0$

1. Show that the linear transformation

$$y_1 = 2x_1 + x_2 + x_3$$
$$y_2 = x_1 + x_2 + x_3$$
$$y_3 = x_1 - 2x_3$$

is regular and hence find its inverse.

Given $y_1 = 2x_1 + x_2 + x_3$

$y_2 = x_1 + x_2 + 2x_3$

$y_3 = x_1 - 2x_3$

the matrix form of a LT is

$Y = AX$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \quad A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

(1) <u>To p.T L.T is regular.</u>

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{vmatrix} = -1 \neq 0$$

The L.T is regular (or) Non-Singular.

(2) To find Inverse L.T:-

$$A^{-1} = \frac{adj(A)}{|A|} = \begin{vmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{vmatrix}$$

hence $X = A^{-1}Y$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$x_1 = 2y_1 - 2y_2 - y_3$

$x_2 = -4y_1 + 5y + 3y_3$

$x_3 = y_1 - y_2 - y_3$.

Q. Show that the following linear transformation is non similar and find its inverse.

$$y_1 = 2x_1 - 2x_2 - x_3$$
$$y_2 = -4x_1 + 5x_2 + 3x_3$$
$$y_3 = x_1 - x_2 - x_3.$$

The matrix form of a LT is.

$Y = AX$ where

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \quad A = \begin{bmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

① To P.T L.T is regular

$$|A| = \begin{vmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{vmatrix} = -1 \neq 0.$$

the L.T is regular (or) Non-Singular.

② To find inverse L.T

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix}$$

hence $X = A^{-1}Y$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$x_1 = 2y_1 + y_2 + y_3$$
$$x_2 = y_1 + y_2 + 2y_3$$
$$x_3 = y_1 - 2y_3.$$

3. Show that the following linear transformation is non similar and find its inverse

$y_1 = 8x_1 - 6x_2 + 2x_3$

$y_2 = -6x_1 + 7x_2 - 4x_3$

$y_3 = 2x_1 - 4x_2 + 3x_3$

The matrix form of a LT is

$Y = AX$ where

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

① To P.T L.T is regular

$$|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix} = -40 \neq 0$$

The L.T is regular (or) Non-Singular

② To find Inverse L.T

$$\begin{bmatrix} 0.025 & -0.25 & -0.35 \\ -0.25 & -0.5 & -0.5 \\ -0.35 & -0.5 & -0.1 \end{bmatrix}$$

hence $X = A^{-1}Y$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.025 & -0.25 & -0.35 \\ -0.25 & -0.5 & -0.5 \\ -0.35 & -0.5 & -0.1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$x_1 = 0.025y_1 - 0.25y_2 - 0.35y_3$

$x_2 = -0.25y_1 - 0.5y_2 - 0.5y_3$

$x_3 = -0.35y_1 - 0.5y_2 - 0.1y_3.$

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* Method of finding largest / Dominent Eigen value and Eigen vector

*** Rayleigh - Power

Working rule

(01) Step 1 :- Given a square matrix A, we choose the initial the eigen vector X_0 in the forms like $[1,0,0]^T$ (or) $[0,1,0]^T$ (or) $[0,0,0]^T$ (or) $[1,1,1]^T$

02. Step 2 we compute AX_0, which is being a column matrix. In is matrix we take the largest element as a common factor and we write $AX_0 = \lambda_1 X_1$. This process is called as normallisation.

example
$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \underset{\lambda_1}{2} \underset{X_1}{\begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix}} = \lambda_1 X_1$$
highest

03. Step 3 we ~~conclude~~ AX_1 compute AX_1. By the process normallisation we write $AX_1 = \lambda_2 X_2$

04. Step 4 we repeat the above procedure till we get the root of desired accuracy.

***01. Find the largest (or) Dominent eigen value and eigen vector of the following matries by Raleigh - Power method.

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$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

let $X_0 = (1,0,0)^T$ be the initial eigen vector.

$$\Rightarrow A \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \lambda_1 X_1$$

$$\lambda_1 = 2, X_1 = \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix}$$

$$AX_1 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 2.0 \end{bmatrix} = 2.5 \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \lambda_2 X_2.$$

$$AX_2 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 0 \\ 2.6 \end{bmatrix} = 2.8 \begin{bmatrix} 1 \\ 0 \\ 0.9285 \end{bmatrix} = \lambda_3 X_3.$$

0.9285.

$$AX_3 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.9285 \end{bmatrix} = \begin{bmatrix} 2.9285 \\ 0 \\ 2.857 \end{bmatrix} = 2.9285 \begin{bmatrix} 1 \\ 0 \\ 0.9755 \end{bmatrix} = \lambda_4 X_4.$$

$$AX_4 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.9755 \end{bmatrix} = \begin{bmatrix} 2.9755 \\ 0 \\ 2.955 \end{bmatrix} = 2.9755 \begin{bmatrix} 1 \\ 0 \\ 0.9931 \end{bmatrix} = \lambda_5 X_5.$$

$$AX_5 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.9931 \end{bmatrix} = \begin{bmatrix} 2.9931 \\ 0 \\ 2.9862 \end{bmatrix} = 2.9931 \begin{bmatrix} 1 \\ 0 \\ 0.9976 \end{bmatrix} = \lambda_6 X_6$$

$$\lambda = 2.9931 \approx 3$$
$$X = (1, 0, 0.9976)^T \approx [1, 0, 1]^T$$ //.

Q2. $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ by taking $(1, 0.5, 0.5)^T$ as inital eigen vector.

$x_0 = (1, 0.5, 0.5)^T$ be the initial eigen vector.

$$AX_0 = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ -4.5 \end{bmatrix} = -4.5 \begin{bmatrix} -0.5 \\ 0.6 \\ -1 \end{bmatrix} = \lambda_1 X_1$$

$$AX_1 = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix} \begin{bmatrix} -0.5 \\ 0.6 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.1 \\ 0.4 \\ -0.6 \end{bmatrix} = 0.6 \begin{bmatrix} -0.67 \\ 0.67 \\ -1 \end{bmatrix} = \lambda_2 X_2$$

$$AX_2 = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix} \begin{bmatrix} -0.67 \\ 0.67 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.696 \\ 0.68 \\ -1.02 \end{bmatrix} = 1.02 \begin{bmatrix} -0.676 \\ 0.67 \\ -1 \end{bmatrix} = \lambda_3 X_3$$

$$AX_3 = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix} \begin{bmatrix} -0.676 \\ 0.67 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.696 \\ 0.68 \\ -1.02 \end{bmatrix} = 1.02 \begin{bmatrix} -0.688 \\ 0.68 \\ -1 \end{bmatrix} \lambda_4 X_4.$$

$$\lambda = 1.02 \simeq 1$$
$$X = [0.688, 0.68, -1]$$

3. $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ by taking $X_0 = (1,1,1)^T$

X_0 is $(1,1,1)^T$.

$$AX_0 \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix} \begin{bmatrix} 1 \\ 0.8 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2.2 \\ 1.4 \\ 2 \end{bmatrix} = 2.2 \begin{bmatrix} 1 \\ 0.6363 \\ 0.9090 \end{bmatrix}$$

$$AX_1 = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix} \begin{bmatrix} 1 \\ 0.6363 \\ 0.9090 \end{bmatrix} = \begin{bmatrix} 2.0918 \\ 1.1824 \\ 2.0008 \end{bmatrix} = 2.0918 \begin{bmatrix} 1 \\ 0.565 \\ 0.956 \end{bmatrix} = \lambda_2 X_2.$$

$$AX_2 = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix} \begin{bmatrix} 1 \\ 0.565 \\ 0.956 \end{bmatrix} = \begin{bmatrix} 2.048 \\ 1.09 \\ 2.004 \end{bmatrix} = 2.048 \begin{bmatrix} 1 \\ 0.532 \\ 0.978 \end{bmatrix} = \lambda_3 X_3$$

$$AX_3 \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix} \begin{bmatrix} 1 \\ 0.532 \\ 0.978 \end{bmatrix} = \begin{bmatrix} 2.026 \\ 1.046 \\ 2.004 \end{bmatrix} = 2.026 \begin{bmatrix} 1 \\ 0.516 \\ 0.989 \end{bmatrix} = \lambda_4 X_4$$

$\lambda = 2.026 \simeq 2$

$x = (1, 0.516, 0.989)^T \simeq (1, 0.5, 1)^T$

4. $A \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

x^0 is $[1, 0, 0]^T$.

$$AX_0 = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ -6 \\ 2 \end{bmatrix} = 8 \begin{bmatrix} 1 \\ -0.75 \\ 0.25 \end{bmatrix} = \lambda_1 X_1$$

$$AX_1 = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.75 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 13 \\ -12.25 \\ 5.75 \end{bmatrix} = 13 \begin{bmatrix} 1 \\ -0.942 \\ 0.442 \end{bmatrix} = \lambda_2 X_2.$$

$$AX_2 = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.942 \\ 0.442 \end{bmatrix} = \begin{bmatrix} 14.536 \\ -14.362 \\ 7.094 \end{bmatrix} = 14.536 \begin{bmatrix} 1 \\ -0.988 \\ 0.488 \end{bmatrix} = \lambda_3 X_3$$

$$AX_3 = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.988 \\ 0.488 \end{bmatrix} = \begin{bmatrix} 14.904 \\ -14.868 \\ 7.416 \end{bmatrix} = 14.904 \begin{bmatrix} 1 \\ -0.997 \\ 0.497 \end{bmatrix} = \lambda_4 X_4.$$

$$AX_4 = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.997 \\ 0.497 \end{bmatrix} = \begin{bmatrix} 14.976 \\ -14.967 \\ 7.479 \end{bmatrix} = 14.976 \begin{bmatrix} 1 \\ -0.999 \\ 0.499 \end{bmatrix}$$

$\lambda = 14.976 \simeq 15$ $\quad x = (1, -0.999, 0.499)^T \simeq (1, -1, 0.5)^T$

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* Rank of martix

* Row reduce Echelon foom

A matrix 'A' is said to in row reduce Echelon form if it satifies the following conditions.

01. The leading entre of each row must be non-zero element.
02. The element below the leading entre are zeroes.
03. The number of zeroes in each row must be greater than its previous row
04. If there is a zero row it should be write below the non-zero rows.

- In other word Echelon form martix represents an upper triangular martix. (element below the principle diagonal are zero).

* Rank

The Rank of matrix 'A' denoted by $\rho(A)$ representes the no. of non-zero rows in Echelon form matrix.

Q. problems

1. Find the rank of following matries appling elementary row transmition or by reducing them to row reduced enchlen form.

(i) $A \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$

(iii) $A \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$ (iv) $A \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & -4 & 3 & 10 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 0 \end{bmatrix}$

(i) $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

In = $R_2 \to R_2 - 2R_1$ $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$

$R_3 \to R_3 - R_1$

$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 + R_2$.

$\rho(A) = 2$ //.

2(A) = $\begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$ $R_1 \leftrightarrow R_3$.

$\begin{bmatrix} 4 & 8 & 13 & 12 \\ 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 8 & 13 & 12 \\ 0 & -2 & -3 & -4 \\ 0 & 2 & 3 & 4. \end{bmatrix}$ $R_2 \to 2R_2 - R_1$

$\begin{bmatrix} 4 & 8 & 13 & 12 \\ 0 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 + R_2$

$\rho(A) = 2$ //.

3 $A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$

$n = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 0 & 2 & 4 & 7 \\ 0 & 6 & 6 & 13 \\ 0 & 6 & 0 & 7 \end{bmatrix}$ $\begin{matrix} R_2 \to 2R_2 - R_1 \\ R_3 \to 2R_3 - R_1 \\ R_4 \to 2R_4 - R_1 \end{matrix}$

$n = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 0 & 2 & 4 & 7 \\ 0 & 0 & -6 & -8 \\ 0 & 0 & -12 & -14 \end{bmatrix}$ $\begin{matrix} R_3 \to R_3 - 3R_2 \\ R_4 \to R_4 - 3R_2 \end{matrix}$

$n = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 0 & 2 & 4 & 7 \\ 0 & 0 & -6 & -8 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ $R_4 \to R_4 - 2R_3$

$\rho(A) = 4$ //.

4 = $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$n = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 0 & 5 & 9 & -1 \\ 0 & 1 & 5 & 3 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ $\begin{matrix} R_2 \to 2R_2 - R_1 \\ R_3 \to 2R_3 - R_1 \end{matrix}$

$$n = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 0 & 5 & 9 & -1 \\ 0 & 5 & 16 & 16 \\ 0 & 0 & -4 & -4 \end{bmatrix} \quad \begin{matrix} R_3 \to 5R_3 - R_2 \\ R_4 \to 5R_4 - R_2 \end{matrix}$$

$$n = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 0 & 5 & 9 & -1 \\ 0 & 0 & 16 & 16 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_4 \to 4R_4 + R_3.$$

$$f(A) = 3.$$

(5) $$\begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 0 \end{bmatrix}$$

$-4+8$

$$n = \begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 0 & 0 & -1 & -9 & 4 \\ 0 & 1 & -1 & 3 & 1 \\ 0 & 1 & -2 & +4 & -4 \end{bmatrix} \quad \begin{matrix} R_2 \to 2R_2 - R_1 \\ R_4 \to R_4 - 2R_1 \end{matrix}$$

$$n = \begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 0 & 1 & -2 & +4 & -4 \\ 0 & 0 & -1 & 3 & 1 \\ 0 & 0 & -1 & -9 & 4 \end{bmatrix} \quad R_2 \leftrightarrow R_4.$$

$-1+2$

$$n = \begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 0 & 1 & -2 & +4 & -4 \\ 0 & 0 & +1 & -1 & 5 \\ 0 & 0 & -1 & -9 & 4 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$n=\begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 0 & 1 & 2 & -4 & -4 \\ 0 & 0 & -3 & 5 & 5 \\ 0 & 0 & 0 & -32 & 7 \end{bmatrix} \quad R_4 \to 3R_4 + R_3$$

$\rho(A) = 4$

$$n=\begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 0 & 1 & -2 & +4 & 4 \\ 0 & 0 & +1 & -1 & 5 \\ 0 & 0 & -1 & -9 & 4 \end{bmatrix}$$

$$n=\begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 0 & 1 & -2 & 4 & 4 \\ 0 & 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & -10 & 9 \end{bmatrix} \quad R_4 \to R_4 + R_3$$

$\rho(A) = 4.$

* Solution of linear. simultaneous equation

(i) Gauss- Elimination method

- Step-1 Given a system of equation, we write the Augmented matrix [A:B]
- Step-2 We apply eliminator row ~~elemen~~ operation we reduce augmented matrix into row reduce echelon form.
- Step3 We write the equation for the echelon form matrices. and solving. we get the required solution.

Q1 Solve the following system of equation using Gauss-elimination method.

(i) $x+y+z=9$

$x-2y+3z=8$

$2x+y-z=3.$

The augmented matrix is.

$$[A:B] \begin{bmatrix} 1 & 1 & 1 & : 9 \\ 1 & -2 & 3 & : 8 \\ 2 & 1 & -1 & : 3 \end{bmatrix}$$

$$ Tn = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{array} $$

$$ Tn = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & -11 & -44 \end{array}\right] \begin{array}{l} \\ R_3 \to 3R_3 - R_2 \\ \\ \end{array} $$

$x + y + z = 9$

$-3y + 2z = -1$

$-11z = -44.$

$x = 2, \; y = 3, \; z = 4.$

(2) Given $2x_1 + x_2 + 4x_3 = 12$

$4x_1 + 11x_2 - x_3 = 33$

$8x_1 - 3x_2 + 2x_3 = 20.$

The augmented matrix is.

$$ (A:B) \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 4 & 11 & -1 & 33 \\ 8 & -3 & 2 & 20 \end{array}\right] $$

$$ Tn = \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 0 & 9 & -9 & 9 \\ 0 & -7 & 14 & -28 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{array} $$

$$ Tn = \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 0 & 9 & -9 & 9 \\ 0 & 0 & -189 & -189 \end{array}\right] \begin{array}{l} \\ R_3 \to 9R_3 + 7R_2 \\ \\ \end{array} $$

$2x + y + 4z = 12.$

$9y - 9z = 9,$

$-189z = -189.$

Solving we get.

$x = 3$

$y = 2$

$z = 1$

(iii) Given

$10x+y+z=12$

$2x+10y+z=13$

$x+y+5z=7$

The augmented matrix is.

$$(A:B)=\begin{bmatrix} 10 & 1 & 1 & : & 12 \\ 2 & 10 & 1 & : & 13 \\ 1 & 1 & 5 & : & 7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 10 & 1 & 1 & : & 12 \\ 0 & 49 & 4 & : & 53 \\ 0 & 9 & 49 & : & 58 \end{bmatrix} \begin{array}{l} \\ R_2 \to 5R_2 - R_1 \\ R_3 \to 10R_3 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 10 & 1 & 1 & : & 12 \\ 0 & 49 & 4 & : & 53 \\ 0 & 0 & 2365 & : & 2365 \end{bmatrix} R_3 \to 49R_3 - 9R_2.$$

$10x+y+z=12$

$49y+4z=53$

$2365z=2365$

solving we get

$x=1, y=1, z=1.$

(iv) $5x_1+x_2+x_3+x_4=4.$

$x_1+7x_2+x_3+x_4=12$

$x_1+x_2+6x_3+x_4=-5$

$x_1+x_2+x_3+4x_4=-6$

$-2, -1, 1, 2$

The augmented matrix is

$$(A:B)=\begin{bmatrix} 5 & 1 & 1 & 1 & : & 4 \\ 1 & 7 & 1 & 1 & : & 12 \\ 1 & 1 & 6 & 1 & : & -5 \\ 1 & 1 & 1 & 4 & : & -6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 5 & 1 & 1 & 1 & : & 4 \\ 0 & 34 & 4 & 4 & : & 54 \\ 0 & 4 & 29 & 4 & : & -29 \\ 0 & 4 & 4 & 19 & : & -34 \end{bmatrix} \begin{array}{l} \\ R_2 \to 5R_2 - R_1 \\ R_3 \to 5R_3 - R_1 \\ R_4 \to 5R_4 - R_1 \end{array}$$

$$\text{Sn}= \left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 0 & 34 & 4 & 4 & 54 \\ 0 & 0 & & & \\ 0 & 0 & & & \end{array}\right] \quad \begin{array}{l} R_3 \to 34R_3 - 4R_2 \\ R_4 \to 34R_4 - 4R_2 \end{array}$$

II Gauss-Jordan Method

working rule.

Step 1. Give a system of equations we write the ~~a~~ augmented matrix

Step 2. we apply the elementary row operation and reduce into Echelon form

Step 3. again by applying row operation we the matrix into diagonal form.

Step 4. we write the equations for diagonal form matrix and solving we get the required solution.

Q.1 Solve the following system by Gauss-Jordan Method.

(i) $x+y+z=9$

$x-2y+3z=8$

$2x+y-z=3$.

The augmented matrix is.

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$$\text{Sn}= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

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$$\text{II}n = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & -11 & -44 \end{array}\right] \quad R_3 \to 3R_3 - R_2.$$

$$n \left[\begin{array}{ccc:c} 11 & 11 & 0 & 55 \\ 0 & -33 & 0 & -99 \\ 0 & 0 & -11 & -44 \end{array}\right] \quad \begin{array}{l} R_1 \to 11R_1 + 11_3. \\ R_2 \to 11R_2 + 2R_3. \end{array}$$

$$n \left[\begin{array}{ccc:c} 33 & 0 & 0 & 66 \\ 0 & -33 & 0 & -99 \\ 0 & 0 & -11 & -44 \end{array}\right] \quad R_1 \to 3R_1 + R_2.$$

$33x = 66 \quad x = 2$

$-33y = -99 \quad y = 3.$

$-11z = -44 \quad z = 4.$

(ii) $2x_1 + x_2 + 4x_3 = 12$

$4x_1 + 11x_2 - x_3 = 33$

$8x_1 - 3x_2 + 2x_3 = 20$

The augmented matrix is

$$[A:B] = \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 4 & 11 & -1 & 33 \\ 8 & -3 & 2 & 20 \end{array}\right]$$

$$\text{I}n = \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 0 & 9 & -9 & 9 \\ 0 & -7 & -14 & -28 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{array}$$

$$\text{II}n = \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 0 & 9 & -9 & 9 \\ 0 & 0 & -189 & -189 \end{array}\right] \quad R_3 \to 9R_3 + 7R_2$$

$R_2/9 - R_3/-189$

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$$\sim \left[\begin{array}{ccc:c} 2 & 1 & 4 & 12 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 2 & 1 & 0 & 8 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - 4R_3 \\ R_2 \to R_2 + R_3 . \end{array}$$

$$\sim \left[\begin{array}{ccc:c} 2 & 0 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad R_1 \to R_1 - R_2$$

$2x = 6 \Rightarrow x = 3$

$y = 2$

$z = 1$

(iii) $10x + y + z = 12$

$2x + 10y + z = 13$

$x + y + 5z = 7$

The augmented matrix is

$$[A:B] = \left[\begin{array}{ccc:c} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{array}\right]$$

$$\text{I}\sim \left[\begin{array}{ccc:c} 10 & 1 & 1 & 12 \\ 0 & 49 & 4 & 53 \\ 0 & 9 & 49 & 58 \end{array}\right] \quad \begin{array}{l} R_2 \to \cancel{5}R_2 - R_1 \\ R_3 \to 10R_3 - R_1 \end{array}$$

$$\text{II}\sim \left[\begin{array}{ccc:c} 10 & 1 & 1 & 12 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & 2365 & 2365 \end{array}\right] \quad R_3 \to 49R_3 - 9R_2$$

$R_3 / 2365$

$$\approx \left[\begin{array}{ccc:c} 10 & 1 & 1 & 12 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\approx \left[\begin{array}{ccc:c} 10 & 1 & 0 & 11 \\ 0 & 49 & 0 & 49 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - 4R_3. \end{array}$$

$$\approx \left[\begin{array}{ccc:c} 490 & 0 & 0 & 490 \\ 0 & 49 & 0 & 49 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad R_1 \to 49R_1 - R_2.$$

$$490x = 490 \Rightarrow x = 1$$
$$49y = 49 \Rightarrow y = 1$$
$$z = 1 //.$$

7/11/2017

Gauss-Siedel Iterative method

Suppose we have a system of linear ~~eq~~ simealtaneous equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3.$$ such that
$$|a_{11}| > |a_{12}| + |a_{13}|$$
$$|a_{22}| > |a_{21}| + |a_{23}|$$
$$|a_{33}| > |a_{31}| + |a_{32}|$$ then the system is said to be in diagonal dominat form and hence we can apply Gauss-Siedel Method as follows

- If the system of eqaation are not in diagonal dominat form then we rearrange the equations, we express them as dominate system and hence we apply gass-seidel method.

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Q.1 Solve the following system of equation using Gauss-siedel iterative method.

(i) $10x + y + z = 12$ $\quad x = 1/10[12 - y - z)$

$x + 10y + z = 12$ $\quad y = 1/10[12 - x - z)$

$x + y + 10z = 12.$ $\quad z = 1/10[12 - x - y)$

let $(x, y, z) = (0, 0, 0)$ be the trail solution or initial approximation.

I^{st} app :- $(x)' = \frac{1}{10}(12 - 0 - 0] = 1.2$

$(y)' = 1/10(12 - 1.2 - 0] = 1.08$

$(z)' = 1/10(12 - 1.2 - 1.08) = 0.972.$

II^{nd} app = $(x)'' = 1/10(12 - 1.08 - 0.972) = 0.9948$

$(y)'' = 1/10(12 - 0.9948 - 0.972) = 1.0033$

$(z)^2 = 1/10(12 - 0.9948 - 1.0033) = 1.0002.$

III^{rd} app = $(x)''' = 1/10(12 - 1.0033 - 1.0002) = 0.9997$

$(y)''' = 1/10(12 - 0.9997 - 1.0002) = 1.0000$

$(z)''' = 1/10(12 - 0.9997 - 1.0000) = 1.0000$

$(x, y, z) = (1, 1, 1).$

(ii) $5x + 2y + z = 12$

$x + 4y + 2z = 15$

$x + 2y + 5z = 20$

by taking (1, 0, 3) as initial approximation.

$x = 1/5[12 - 2y - z)$

$y = 1/4[15 - x - 2z)$

$z = 1/5(20 - x - 2y)$

let $(x, y, z) = (1, 0, 3)$ be the initial approximation.

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1^{st} app. $(x)' = 1/5(12 - 0 - 3) = 1.8$

$(y)' = 1/4(15 - 1.8 - (2\times3)) = 1.8$

$(z)' = 1/5(20 - 1.8 - (2\times1.8)) = 2.95.$

2^{nd} app $(x)'' = 1/5(12 - 1.8^{\times 2} - 2.95) = 1.096.$

$(y)'' = 1/4(12 - 1.096 - 2.95^{\times 2}) = 2.016$

$(z)'' = 1/5(12 - 1.096 - 2.016^{\times 2}) = 2.9744.$

III app $(x)''' = 1/5(12 - (2\times2.016) - 2.9744) = 0.9987$

$(y)''' = 1/4(15 - 0.9987 - (2\times2.9744)) = 2.0131$

$(z)''' = (1/5)(20 - 0.9987 - (2\times2.0131)) = 2.9950.$

$(x, y, z) = (1, 2, 3).$

3. $x + y + 54z = 110$

$27x + 6y - z = 85.$

$6x + 15y + 2z = 72.$

Here, the given eqn are not in diagonally dominant form
Hence, we rearrange the eqn as follows.

$27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110.$

$x = 1/27(85 - 6y + z)$

$y = 1/15(72 - 6x - 2z)$

$z = 1/54(110 - x - y)$

let $(x, y, z) = (0, 0, 0)$ be the trail solution.

1^{st} app $(x)' = 1/27(85 - 0 + 0) = 3.1481$

$(y)' = 1/15(72 - (6\times3.1481) - 0) = 3.5408$

$(z)' = 1/54(110 - 3.1481 - 3.5408) = 1.9132.$

2nd app $(x)^2 = 1/27(85-(6\times3.5408)+1.9132) = 2.4322$

$(y)^2 = 1/15[72-(6\times2.4322)-(2\times1.9132)] = 3.5720.$

$(z)^2 = 1/54[110-2.4322-3.5720) = 1.9258$

III rd app $(x)^3 = 1/27[85-(6\times3.5720)+1.9258] = 2.4256$

$(y)^3 = 1/15(72-(6\times2.4856)-2(1.9258)) = 3.5729.$

$(z)^3 = 1/54(110-2.4256-3.5729) = 1.9259$

$(x,y,z) = (2.42, 3.57, 1.92)$.

9/11/2021

* Eigen values and Eigen vectors.

working rule

Step 1 Give a square matrix A, we write the characterist equation. $|A-\lambda I|=0$

Step 2 To expand the determinate we take a polynomial in λ, solving the polynomial we get. the roots which are called as Eigen values.

Step 3 For each Eigen value we solve the system of equations obtained from the characterist martix we get the Eigen vectors.

Q1. Find the Eigen values and corresponding Eigen vectors of the following marties.

(i) $A = \begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix}$ (ii) $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ (iii) $A \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

(iv) $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (v) $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(i) $A = \begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix}$

The ch. eq^n is $|A - \lambda I| = 0$

$$\begin{vmatrix} -3-\lambda & 8 \\ -2 & 7-\lambda \end{vmatrix} = 0$$

$(-3-\lambda)(7-\lambda) + 16 = 0$

$-21 + 3\lambda - 7\lambda + \lambda^2 + 16 = 0$

$\lambda^2 - 4\lambda - 5 = 0$

$\lambda = 5$ & -1 are the eigen value.

consider $\begin{bmatrix} -3-\lambda & 8 \\ -2 & 7-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\left.\begin{aligned} (-3-\lambda)x_1 + 8x_2 &= 0 \\ -2x_1 + (7-\lambda)x_2 &= 0 \end{aligned}\right\} \Rightarrow (1)$$

case (i) for $\lambda = 5$, eq$^n \Rightarrow (1)$

$-8x_1 + 8x_2 = 0 \Rightarrow 8x_1 = 8x_2 = x_1 = x_2$

$-2x + 2x_2 = 0 \Rightarrow \cancel{2}x_1 = \cancel{2}x_2 \Rightarrow x_1 = x_2$

let $x_2 = 1$ $\quad x_1 = 1$

$(x_1, x_2)^T = (1, 1)^T$

case (ii) for $\lambda = 1$ $(1) \Rightarrow$

$-2x_1 + 8x_2 = 0 \Rightarrow \cancel{2}x_1 = \cancel{8}^{4}x_2 = x_1 = 4x_2$

$-2x_1 + 8x_2 = 0 \Rightarrow \cancel{2}x_1 = \cancel{8}^{4}x_2 = x_1 = 4x_2$

let $x_2 = 1$ $\therefore x_1 = 4$.

$(x_1, x_2)^T = (4, 1)^T$

(ii) $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$

The ch. eq^n is $|A - \lambda I| = 0$

$$\begin{vmatrix} -1-\lambda & 3 \\ -2 & 4-\lambda \end{vmatrix} = 0.$$

$(-1-\lambda)(4-\lambda) + 6 = 0$

$-4 + \lambda - 4\lambda + \lambda^2 + 6 = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda = 2$ & 1. are the eigen value.

consider.

$$\begin{bmatrix} -1-\lambda & 3 \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(-1-\lambda)x_1 + 3(x_2) = 0$

$-2x_1 + (4-\lambda)x_2 = 0$

case 1 for $\lambda = 2$.

$-3x_1 + 3x_2 = 0 \Rightarrow \cancel{3}x_1 = \cancel{3}x_2 = x_1 = x_2$

$-2x_1 + 2x_2 = 0 \Rightarrow \cancel{2}x_1 = \cancel{2}x_2 = x_1 = x_2.$

let $x_2 = 1 \therefore x_1 = 1$

$(x_1, x_2)^T = (1, 1)^T$

case 2 for $\lambda = 1$

$-2x_1 + 3x_2 = 0 \quad 2x_1 = 3x_2 = x_1 = 3/2 x_2$

$-2x + 3x_2 = 0 \quad 2x_1 = 3x_2 = x_1 = 3/2 x_2$

let $x_2 = 1 \quad x_1 = 3/2.$

$(x_1, x_2)^T = (3/2, 1)^T$

(iii) $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

$(2-\lambda)\left[4-2\lambda-2\lambda+\lambda^2\right]$

$8-4\lambda-4\lambda+2\lambda^2-4\lambda+2\lambda^2+2\lambda^2-\lambda^3$

$-2+\lambda$

$\cancel{8}-\cancel{8\lambda}+2\lambda^2-4\lambda+2\lambda^2+2\lambda^2-\lambda^3-\cancel{2}+\cancel{\lambda}$

$6-7\lambda+6\lambda^2-\lambda^3$

$6-11\lambda+6\lambda^2-\lambda^3$.

$-\lambda^3+6\lambda^2-11\lambda+6$.

The ch. eqⁿ is $|A-\lambda I|=0$

$$\begin{vmatrix} 2-\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & 0 & 2-\lambda \end{vmatrix} = 0.$$

$(2-\lambda)\left[(2-\lambda)(2-\lambda)\right]+1\left[-(2-\lambda)(1)\right]$.

$(2-\lambda)\left[4-2\lambda-2\lambda+\lambda^2\right]+1\left[-2+\lambda\right]$

$8-4\lambda-4\lambda-2\lambda^2-4\lambda+2\lambda^2+2\lambda^2+\lambda^3 \quad -2+\lambda$

$\cancel{8-8\lambda}-\ 8-12\lambda+2\lambda^2+\lambda^3-2+\lambda$

$6-11\lambda+2\lambda^2+\lambda^3$

$\lambda^3+2\lambda^2-11\lambda+6=0$

$(2-\lambda)^3-(2-\lambda)=0$

$(2-\lambda)\left[(2-\lambda)^2-1\right]=0$

$2-\lambda=0$

$\lambda=2$

$4+\lambda^2-4\lambda-1=0$

$\lambda^2-4\lambda+3=0$

$\lambda=3,1$.

$\lambda = 1, 2, 3.$ are the egigen values.

consider.

$$\begin{bmatrix} 2-\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & 0 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{matrix} 0 \\ 0 \\ 0. \end{matrix}$$

$$\left.\begin{aligned} (2-\lambda)x_1 + x_3 &= 0 \\ (2-\lambda)x_2 &= 0 \\ x_1 + (2-\lambda)x_3 &= 0 \end{aligned}\right\} (1)$$

for (i) $\lambda = 1$.

$x_1 + x_3 = 0 \Rightarrow x_1 = -x_3$

$x_2 = 0$

$x_1 + x_3 = 0 \Rightarrow x_1 = -x_3$

let $x_3 = 1$ $\therefore x_1 = -1$

$(x_1, x_2, x_3)^T = (-1, 0, 1)^T$

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for $\lambda = 2$.

$x_3 = 0$

$x_1 = 0$ let $x_2 = 1$ because If it becomes $(0,0,0)$ it is non zero

$(x_1, x_2, x_3)^T = (0, 1, 0)^T$

for $\lambda = 3$.

$-1x_1 + x_3 = 0 \Rightarrow -x_1 + x_3 = 0 \quad x_1 = x_3$

$-1x_2 = 0 \Rightarrow -x_2 = 0 \quad +x_2 = 0$

$x_1 - 1x_3 = 0. \Rightarrow x_1 - x_3 = 0 \quad x_1 = x_3$

let $x_3 = 1 \therefore x_1 = 1$

$(x_1, x_2, x_3)^T = (1, 0, 1)^T$ //.

$$6(A) = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

The ch eq^n is $(A - \lambda I) = 0$.

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0.$$

$(8-\lambda)[(7-\lambda)(3-\lambda) - (-4)(-4)] + 6[-6(3-\lambda) + 8] + 2[24 - 2(7-\lambda)] = 0$

$(8-\lambda)[21 - 3\lambda - 7\lambda + \lambda^2 - 16] + 6[-18 + 6\lambda + 8] + 2(24 - 12 + 2\lambda)$

$8-\lambda[\lambda^2 - 10\lambda + 5] + 36\lambda - 60 + 4\lambda + 20 = 0$

$8\lambda^2 - 80\lambda + \cancel{40} - \lambda^3 + 10\lambda^2 - 5\lambda + 40\lambda - \cancel{40} = 0$

$-\lambda^3 + 18\lambda^2 - 45\lambda = 0$

$\lambda = 15, 3, 0$ are the eigen values.

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{aligned} (8-\lambda)x_1 - 6x_2 + 2x_3 &= 0 \\ -6x_1 + (7-\lambda)x_2 - 4x_3 &= 0 \\ 2x_1 - 4x_2 + (3-\lambda)x_3 &= 0 \end{aligned}$$

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case 1.

For $\lambda = 0 \Rightarrow$.

$8x_1 - 6x_2 + 2x_3 = 0$

$-6x_1 + 7x_2 - 4x_3 = 0$

$2x_1 - 4x_2 + 3x_3 = 0$

case 2.

for $\lambda = 3$

$5x_1 - 6x_2 + 2x_3 = 0$

$-6x_1 + 4x_2 - 4x_3 = 0$

$2x_1 - 4x_2 = 0$

case 3

for $\lambda = 15$.

$-7x_1 - 6x_2 + 2x_3 = 0$

$-6x_1 - 8x_2 - 4x_3 = 0$

$2x_1 - 4x_2 - 12x_3 = 0$.

using RCM,

case 1 $\dfrac{x_1}{\begin{vmatrix} -6 & 2 \\ 7 & 4 \end{vmatrix}} = \dfrac{-x_2}{\begin{vmatrix} 8 & 2 \\ -6 & -4 \end{vmatrix}} = \dfrac{x_3}{\begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}} = 1$

$\dfrac{x_1}{10} = \dfrac{+x_2}{+20} = \dfrac{x_3}{20} = 1 \Rightarrow \dfrac{x_1}{1} = \dfrac{x_2}{2} = \dfrac{x_3}{2} = 1$

$(x_1, x_2, x_3)^T = (1, 2, 2)^T$

case 2

$\dfrac{x_1}{\begin{vmatrix} -6 & 2 \\ 4 & -4 \end{vmatrix}} = \dfrac{-x_2}{\begin{vmatrix} 5 & 2 \\ -6 & -4 \end{vmatrix}} = \dfrac{x_3}{\begin{vmatrix} 5 & -6 \\ -6 & 4 \end{vmatrix}}$

$\dfrac{x_1}{16} = \dfrac{+x_2}{+8} = \dfrac{x_3}{-16} = 1$

$\dfrac{x_1}{2} = \dfrac{x_2}{1} = \dfrac{x_3}{-2} = 1$

$(x_1, x_2, x_3) = (2, 1, -2)^T$.

case 3 $\dfrac{x_1}{\begin{vmatrix} -6 & 2 \\ -8 & -4 \end{vmatrix}} = \dfrac{-x_2}{\begin{vmatrix} -7 & 2 \\ -6 & -4 \end{vmatrix}} = \dfrac{x_3}{\begin{vmatrix} -7 & -6 \\ -6 & -8 \end{vmatrix}}$

$\dfrac{x_1}{40} = \dfrac{-x_2}{40} = \dfrac{x_3}{20} = 1$

$\dfrac{x_1}{2} = \dfrac{x_2}{-2} = \dfrac{x_3}{1} = 1$

$(x_1, x_2, x_3) = (2, -2, 1)^T$.

V $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ The ch eqn is $|A - \lambda I| = 0$ $= \begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{bmatrix} = 0$.

$(1-\lambda)[(5-\lambda)(1-\lambda) - 1] - 1[3 - (1-\lambda)] + 3[1 - (5-\lambda)(3)]. = 0$

$(1-\lambda)[5 - 5\lambda - \lambda + \lambda^2 - 1] - 1[3 - 1 + \lambda] + 3[1 - 15 + 3\lambda]. = 0$

$(1-\lambda)[4 - 6\lambda + \lambda^2] - 1[2+\lambda] + 3[-14 + 3\lambda] = 0$

$4 - 6\lambda + \lambda^2 - 4\lambda + 6\lambda^2 - \lambda^3 + 2 - \lambda - 42 + 9\lambda. = 0$

$-\lambda^3 - 10\lambda + 7\lambda^2 + 2 - \lambda + 40 + 9\lambda.$

$-\lambda^3 + 7\lambda^2 - 2\lambda + 44 = 0.$

$= -\lambda^3 + 7\lambda^2 - 36 = 0$

$\lambda = -2, 3$ and 6 are the eigenvalues.

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$$\begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{matrix} (1-\lambda)x_1 + x_2 + 3x_3 = 0 \\ x_1 + (5-\lambda)x_2 + x_3 = 0 \\ 3x_1 + x_2 + (1-\lambda)x_3 = 0. \end{matrix}$$

case.1 for = -2.

$3x_1 + x_2 + 3x_3 = 0$

$x_1 + 7x_2 + x_3 = 0$

$3x_1 + x_2 + 3x_3 = 0$

case.2 for = 3.

$-2x_1 + x_2 + 3x_3 = 0$

$x_1 + 2x_2 + x_3 = 0$

$3x_1 + x_2 - 2x_3 = 0.$

case.3 for 6.

$-5x_1 + x_2 + 3x_3 = 0$

$x_1 - x_2 + x_3 = 0$

$3x_1 + x_2 - 5x_1 = 0.$

case1.
$$\frac{x_1}{\begin{vmatrix} 1 & 3 \\ 7 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 3 & 1 \\ 1 & 7 \end{vmatrix}}$$

$$\frac{x_1}{-20} = \frac{-x_2}{0} = \frac{x_3}{20}.$$

$$(x_1, x_2, x_3)^T = (-1, 0, 1)^T$$

case2.
$$\frac{x_1}{\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix}}$$

$$\frac{x_1}{-5} = \frac{\cancel{-}x_2}{\cancel{-}5} = \frac{x_3}{|5|}$$

$$(x_1, x_2, x_3)^T = (-1, 1, 1)^T$$

case 3
$$\frac{x_1}{\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -5 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -5 & 1 \\ 1 & -1 \end{vmatrix}}$$

$$\frac{x_1}{4} = \frac{\cancel{-}x_2}{\cancel{-}8} = \frac{x_3}{4}$$

$$(x_1, x_2, x_3)^T = (1, 2, 1)^T$$

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* Diagonialisation of a matrix

Work rule

Step1 :- Given a square matrix A, we find the Eigen value and eigen vector.

Step2 :- we find a modal matrix P which is a matrix of eigen vector.

Step3. We find $P^{-1} = \frac{adj(P)}{|P|} =$

Step4 :- We compute $D = P^{-1}AP =$ we get the required diagonalisable matrix

Note The diagonal matrix D is also called as spectral matrix.

1\. Diagonalise $A = \begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix}$

repeat the process from the problem no.1 of eigen values of eigen vectors.

Eigen values:

$\lambda = 5$ and -1

Eigen vectors:

$X_1 = (1, 1)^T$

$X_2 = (4, 1)^T$

Model matrix is $P = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$

$P^{-1} = \frac{adj(P)}{|P|} = \begin{bmatrix} -1/3 & 4/3 \\ 1/3 & -1/3 \end{bmatrix}$

$D \; P^{-1}AP = \begin{bmatrix} -1/3 & 4/3 \\ 1/3 & -1/3 \end{bmatrix} \begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$

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2. Reduce the matrix into diagonal form.

$$A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$$

Eigen values :- $\lambda = 2, 1$

Eigen vectors

$$X_1 = (1, 1)^T$$
$$X_2 = (3/2, 1)^T$$

Modal matrix is $P = \begin{bmatrix} 1 & 3/2 \\ 1 & 1 \end{bmatrix}$

$$P^{-1} = \frac{adj(P)}{|P|} = \begin{bmatrix} -2 & 3 \\ 2 & -2 \end{bmatrix}$$

$$D = P^{-1}AP = \begin{bmatrix} -2 & 3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3/2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

3. Find the modal and spectral matix of

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Eigen values

$$\lambda = 1, 2, 3$$

Eigen vectrons :-

$$x_1 = (-1, 0, 1)^T$$
$$x_2 = (0, 1, 0)^T$$
$$x_3 = (1, 0, 1)^T$$

Model matrix is $P = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

The spectral matrix is $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

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* Quadratic forms:-

A homogenous equation in any number of variables of 2° degree is called as a quadratic forms
In general, $a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2$ represents a quadratic form in two variables similarly a quadratic form in three variables can be return as.

$$a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{23}x_2x_3 + 2a_{31}x_3x_1$$

* Martix of Quadratic form

(a) for two variables.

$$A = \begin{bmatrix} \text{coeffic of } x_1^2 & \tfrac{1}{2}\text{ coff } x_1x_2 \\ \tfrac{1}{2}\text{ coeff of } x_1x_2 & \text{coffic of } x_2^2 \end{bmatrix}$$

(b) for three variables

$$A = \begin{bmatrix} \text{coeff of } x_1^2 & \tfrac{1}{2}\text{coff } x_1x_2 & \tfrac{1}{2}\text{ coff } x_1x_3 \\ \tfrac{1}{2}\text{coeff of } x_1x_2 & \text{coff of } x_2^2 & \tfrac{1}{2}\text{ coff } x_2x_3 \\ \tfrac{1}{2}\text{coff } x_1x_3 & \tfrac{1}{2}\text{ coff } x_2x_3 & \text{coff } x_3^2 \end{bmatrix}$$

* Reducing of Quadractic form x cannonical form. or Sum of squares

Step1 Given QF, we write its matrices
Step2. we find Eigen values of the matrix
Step3. If $\lambda_1, \lambda_2, \lambda_3$ are eigen the values of matrix then the required cannonical form or sum of square is $\lambda_1 y^2 + \lambda_2 y_2^2 + \lambda_3 y_3^3$

* Rank, Index, signature and nature of QF

1. Rank = No. of non zero eigen values.
2. Index = No. of positive Eigen values.
3. Signature = No. of positive Eigen values − no. of negative Eigen value
4. nature of QF = (i) if all eigen values are positive ⇒ "+ve definite"
   (ii) if all eigen values are negative ⇒ "−ve definite"
   (iii) if any one eigen value is zero and all other eigen value are positive ⇒ "positive semi definite"
   (iv) if any one eigen value is zero and all other eigen value are negative ⇒ "negative semi definite"
   (v) If few eigen value are positive and eigen value negative ⇒ "in definite"

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* Reduce the Quadratic form to.

$2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$ into cannonical form. Hence find its rank, index, signature and nature.

1. $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$

The matrix of QF is

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

From problem no ③ of Eigen values and Eigen vectors, we have.

$(\lambda_1, \lambda_2, \lambda_3) = (1, 2, 3)$

The cannonical form (or) $= y_1^2 + 2y_2^2 + 3y_3^2$.
sum of square.

(i) Rank = 3.
(ii) Index = 3
(iii) Signature = 3 − 0 = 3.
(iv) Nature of QF = Positive definite.

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2. Reduce the Quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4xz = 0$. sum of squares using orthogonal tranfermassion method. find its rank, index, signature, nature.

The matrix of QF is.

$$A = \begin{bmatrix} 8 & -6 & +2 \\ -6 & 7 & -4 \\ +2 & -4 & 3 \end{bmatrix}$$

from problem on ④ of Eigen values an Eigen vectors, we have

$$(\lambda_1, \lambda_2, \lambda_3) = (0, 3, 15)$$

$$3y_2^2 + 15y_3^3 = 0$$

Rank = 2.

Index = 2

signature = 2-1 = 1

nature = positive semi definite.

3. Reduce the Quadratic form. $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 6x_1x_3 + 2x_2x_3$. into cannonical form and find rank, index signature nature.

$$A \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

from problem on ③ of Eigen values on Eigen vectors we have

$$(\lambda_1, \lambda_2, \lambda_3) = (-2, 3, 6)$$

$$-2y_1^2 + 3y_2^2 + 6y_3^3 = 0$$

Rank = 3

Index = 2

Signature = 2-1 = 1

nature = ~~po~~ indefinite.

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