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Module - 3

Vector Calculas.

* Definations and formula

01. $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \Rightarrow$ position vector of particle

02. $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow$ represents velocity of the particle. and $\vec{a} = \frac{d\vec{v}}{dt}$ (or) $\frac{d^2\vec{r}}{dt^2} \Rightarrow$ represents accelarations. of a particle

further $|\vec{v}| = \left|\frac{d\vec{r}}{dt}\right| = \frac{ds}{dt}$ represents speed of a particle.

03. derivative of scalar product and vector product of two vector

(i) $\frac{d}{dt}(\vec{F}\cdot\vec{G}) = \vec{F}\cdot\frac{d\vec{G}}{dt} + \vec{G}\cdot\frac{d\vec{F}}{dt}.$

(ii) $\frac{d}{dt}(\vec{F}\times\vec{G}) = \vec{F}\times\frac{d\vec{G}}{dt} + \frac{d\vec{F}}{dt}\times\vec{G}$

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*** 04. The component of velocity $\vec{v}$ along a given vector $\vec{B}$ (in the direction of $\vec{B}$) is $\vec{v}\cdot\hat{n}$ where $\hat{n} = \frac{\vec{B}}{|\vec{B}|}$ it is also called as the tangental component of velocity.

05. The component of accelorationalong with a give $\vec{B}$ is $\vec{a}\cdot\hat{n}$ (tangential component of accelenation)

06. The component velocity and acceleration perpendicular to the vector $\vec{B}$ that is normal component of velocity and acceleration are respectly given by $|\vec{v} - (\vec{v}\cdot\hat{n})\hat{n}|$

$|\vec{a} - (\vec{a}\cdot\hat{n})\hat{n}|$

07. Unit tangent vector $\hat{T} = \frac{\vec{T}}{|\vec{T}|} = \frac{d\vec{r}/dt}{|d\vec{r}/dt|}$

08. Unit normal vector. $\hat{N} = \frac{d\hat{T}/ds}{|d\hat{T}/ds|}$ where $d\hat{T}/ds = \frac{d\hat{T}}{dt} \times \frac{dt}{ds}$

$= \frac{\frac{d\hat{T}}{dt}}{\frac{ds}{dt}} = \frac{d\hat{T}}{ds}$

$$d\hat{T}/ds = \frac{d\hat{T}/dt}{\left|d\vec{r}/dt\right|}$$

09. Angle of intersection b/w two vectors $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

1. If $x = t^2+1, y = 4t-3, z = 2t^2-6t$ represents the parametric equation of a curve than find
(a) Unit tangent vector at any point ~~of~~ on the curve
(b) The angle b/w tangents at $t=1$ and $t=2$.

Given $x = t^2+1, y = 4t-3$ and $z = 2t^2-6t$
w.k.t

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = (t^2+1)\hat{i} + (4t-3)\hat{j} + (2t^2-6t)\hat{k} \rightarrow (1)$$

(a) w.k.t the tangent vector is given by.

$$\vec{T} = \frac{d\vec{r}}{dt} = \vec{T} = 2t\,\hat{i} + 4\hat{j} + (4t-6)\hat{k}$$

$$= |\vec{T}| = \sqrt{(2t)^2+(4)^2+(4t-6)^2}$$

$$= \sqrt{4t^2+16+16t^2+36-48t}$$

$$|\vec{T}| = \sqrt{20t^2-48t+52}$$

w.k.t the unit tangent vector is given by

$$\hat{T} = \frac{\vec{T}}{|\vec{T}|} = \frac{2t\hat{i} + 4\hat{j} + (4t-6)\hat{k}}{\sqrt{20t^2-48+52}}$$

(b) we have

$$\vec{T} = 2t\hat{i} + 4\hat{j} + (4t-6)\hat{k}$$

at $t=1$, $\vec{T} = 2\hat{i} + 4\hat{j} - 2\hat{k} = \vec{A}$ (say)

at $t=2$ $\vec{T} = 4\hat{i} + 4\hat{j} + 2k = \vec{B}$ (say)

w.k.t

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{8+16-4}{\sqrt{4+16+4}\cdot\sqrt{16+16+4}}$$

$$= \frac{20}{\sqrt{24}\cdot\sqrt{36}} = \frac{\cancel{20}^{10}}{\cancel{6}_3\sqrt{24}} = \theta = \cos^{-1}\left(\frac{10}{3\sqrt{24}}\right)$$

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Q2. Find the angle of intersection b/w tangents ~~and~~ to the curve
$\vec{r} = \{1 - t^2/3\}\hat{i} + t^2\hat{j} + \{t + \frac{t^2}{3}\}\hat{k}$ at $t = \pm 3$

$\Rightarrow$ we know that tangent vector ~~of~~ $\vec{T}$ is given by
$$\vec{T} = \frac{d\vec{r}}{dt} = (1 - 2t/3)\hat{i} + 2t\hat{j} + (1 + 2t/3)\hat{k}$$

at $t = 3$ $\vec{T} = -\hat{i} + 6\hat{j} + 3\hat{k} = \vec{A}$ (say)
at $t = -3$ $\vec{T} = 3\hat{i} - 6\hat{j} - \hat{k} = \vec{B}$ (say)
w.k.t
$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|} = \frac{-3-36-3}{\sqrt{1+36+9}\cdot\sqrt{9+36+1}}$$
$$= \frac{-42}{\sqrt{46}\cdot\sqrt{46}} = \frac{-\cancel{42}\,21}{\cancel{46}\,23}$$
$$\theta = \cos^{-1}\left(\frac{-21}{23}\right)$$

Q3. Find the angle of intersection b/w tangents to the curve
$\vec{r} = (t^2\hat{i} + 2t\hat{j} - t^3\hat{k})$. at ± 1

$\Rightarrow$ we know that tangent vector $\vec{T}$ is given by
$$\vec{T} = \frac{d\vec{r}}{dt} = (2t)\hat{i} + (2)j\,(-3t^2)k.$$
$$= 2t\hat{i} + 2\hat{j} - 3t^2\hat{k}$$
at $t = 1$ = $2\hat{i} + 2\hat{j} - 3\hat{k} = \vec{A}$ (say)
at $t = -1$ = $-2\hat{i} + 2\hat{j} - 3\hat{k} = \vec{B}$ (say
w.k.t
$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|} = \frac{-\cancel{4}+\cancel{4}+9}{\sqrt{4+4+9}\cdot\sqrt{4+4+9}}$$
$$= \frac{9}{\sqrt{17}\cdot\sqrt{17}} = \frac{9}{17}$$
$$\theta = \cos^{-1}(9/17).$$

Q4 Find the unit tangent and unit normal vector to the curve
$\vec{r} = 4\sin t\,\hat{i} + 4\cos\hat{j} + 3t\hat{k}$

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Given = $\vec{r} = 4\sin t\,\hat{i} + 4\cos t\,\hat{j} + 3t\hat{k} \rightarrow (1)$

(a) Unit tangent vector:-

$$\hat{T} = \frac{\vec{T}}{|\vec{T}|} = \frac{d\vec{r}/dt}{|d\vec{r}/dt|} \rightarrow (2)$$

Diffn eqn (1) w.r.t t.

$$\vec{T} \text{ or } \frac{d\vec{r}}{dt} = 4\cos t\,\hat{i} - 4\sin t\,\hat{j} + 3\hat{k}$$

$$|\vec{T}| \text{ or } \left|\frac{d\vec{r}}{dt}\right| = \sqrt{16\cos^2 t + 16\sin^2 t + 9}$$

$$= \sqrt{16(\cos^2 t + \sin^2 t) + 9}$$

$$= \sqrt{16+9} = \sqrt{25}.$$

$$|\vec{T}| \text{ (or) } \left|\frac{d\vec{r}}{dt}\right| = 5$$

sub $\vec{T}$ and $|\vec{T}|$ ~~in~~ eqn (2)

$$\hat{T} = \frac{4\cos t\,\hat{i} - 4\sin t\,\hat{j} + 3\hat{k}}{5} \rightarrow (3)$$

(b) Unit normal vector:-

$$\hat{N} = \frac{d\hat{T}/ds}{|d\hat{T}/ds|} \qquad \cancel{\hat{N} = \frac{d\hat{T}/ds}{|d\hat{T}/ds|}} \rightarrow (4) \qquad \frac{d\hat{T}}{ds} = \frac{d\hat{T}/dt}{|d\vec{r}/dt|}$$

Diffn eqn (3) w.r.t t.

$$\frac{d\hat{T}}{dt} = 1/5\left[-4\sin t\,\hat{i} - 4\cos t\,\hat{j}\right)$$

and

$$\left|\frac{d\vec{r}}{dt}\right| = 5$$

Sub in eqn (4)

$$\frac{d\hat{T}}{ds} = \frac{1/5(-4\sin t\,\hat{i} - 4\cos t\,\hat{j})}{5}$$

$$\frac{d\hat{T}}{ds} = 1/25\left(-4\sin t\,\hat{i} - 4\cos t\,\hat{j}\right)$$

sub in (4) we get

$$\hat{N} = \frac{\cancel{1/25}(-4\sin t\,\hat{i} - 4\cos t\,\hat{j})}{\cancel{1/25}\sqrt{16\sin^2 t + 16\cos^2 t}}$$

$$\hat{N} = \frac{-\cancel{4}(\sin t\,\hat{i} + \cos t\,\hat{j})}{\cancel{4}}$$

$$\hat{N} = -(\sin t\,\hat{i} + \cos t\,\hat{j})$$

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5. Find unit tangent and unit normal vector at $t=1$ for curve whose parametric equations are $x = t - \frac{t^3}{3}$, $y = t^2$ and $z = t + \frac{t^3}{3}$

Given

$$x = t - t^3/3, \; y = t^2 \text{ and } z = t + t^3/3$$

w.k.t

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = (t - t^3/3)\hat{i} + t^2\hat{j} + (t + t^3/3)\hat{k} \rightarrow (1)$$

* unit tangent vector

$$\hat{T} = \frac{\vec{T}}{|\vec{T}|} = \frac{d\vec{r}/dt}{|d\vec{r}/dt|} \rightarrow (2)$$

Diffⁿ eqⁿ (1) w.r.t t, we get

$$\frac{d\vec{r}}{dt} = (1-t^2)\hat{i} + 2t\hat{j} + (1+t^2)\hat{k}$$

at $t=1$ $\frac{d\vec{r}}{dt} = 2\hat{j} + 2\hat{k}$

$$\Rightarrow \left|\frac{d\vec{r}}{dt}\right| = \sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8}$$

$$= 2\sqrt{2}$$

sub in eq (2)

$$\hat{T} = \frac{2\hat{j} + 2\hat{k}}{2\sqrt{2}} = \frac{\cancel{2}(\hat{j}+\hat{k})}{\cancel{2}\sqrt{2}}$$

$$= \hat{T} = \frac{\hat{j}+\hat{k}}{\sqrt{2}} //$$

(b) Unit Normal vector

$$\hat{N} = \frac{d\hat{T}/ds}{|d\hat{T}/ds|} \rightarrow (3)$$

w.k.t $\frac{d\hat{T}}{ds} = \frac{d\hat{T}/dt}{|d\vec{r}/dt|}$

there $d\hat{T}/dt = d/dt \left\{ \frac{d\vec{r}/dt}{|d\vec{r}/dt|} \right\}$

$$\frac{d\hat{T}}{dt} = \frac{d}{dt}\left\{ \frac{(1-t^2)\hat{i} + 2t\hat{j} + (1+t^2)\hat{k}}{2\sqrt{2}} \right\}$$

$$= \frac{1}{2\sqrt{2}}\left\{ -2t\,\hat{i} + 2\hat{j} + 2t\hat{k} \right\}$$

$$\frac{d\hat{T}}{dt} = \frac{-t\hat{i} + \hat{j} + t\hat{k}}{\sqrt{2}} \;//.$$

$$\frac{d\hat{T}}{ds} = \frac{\frac{-t\hat{i} + \hat{j} + t\hat{k}}{\sqrt{2}}}{2\sqrt{2}} = \frac{-t\hat{i} + \hat{j} + t\hat{k}}{4}$$

at $t=1$, $d\hat{T}/ds = \frac{-\hat{i} + \hat{j} + \hat{k}}{4}$ //.

$$\left| d\hat{T}/ds \right| = 1/4\sqrt{1+1+1} = \sqrt{3}/4.$$

sub in (3)

$$\hat{N} = \frac{\frac{-\hat{i} + \hat{j} + \hat{k}}{\cancel{4}}}{\sqrt{3}/\cancel{4}} = \frac{-\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} \;//.$$

6. Find the unit ~~vec~~ tangent and unit Normal vector of the curve vector $\vec{r} = \cos 2t\,\hat{i} + \sin 2t\,\hat{j} + t\hat{k} \rightarrow (1)$ at $x = 1/\sqrt{2}$

Given

$$\vec{r} = \cos 2t\,\hat{i} + \sin 2t\,\hat{j} + \hat{k} \rightarrow (1)$$

$$x = 1/\sqrt{2}$$

unit tangent vector

$$\hat{T} = \frac{\vec{T}}{|\vec{T}|} = \frac{d\vec{r}/dt}{|d\vec{r}/dt|} \rightarrow (2)$$

diffⁿ eqⁿ ① w.r.t t, we get.

$$\frac{d\vec{r}}{dt} = -2\sin 2t\,\hat{i} + 2\cos 2t\,\hat{j} + \hat{k}$$

$$\left|\frac{d\vec{r}}{dt}\right| = \sqrt{4\sin^2 2t + 4\cos^2 2t + 1}$$
$$= \sqrt{4(\sin^2 2t + \cos^2 2t) + 1}$$
$$= \sqrt{5}$$

sub in eqⁿ ②

$$\hat{T} = \frac{-2\sin 2t\,\hat{i} + 2\cos 2t\,\hat{j} + \hat{k}}{\sqrt{5}} \rightarrow ③$$

(b) Unit Normal vector

$$\hat{N} = \frac{d\hat{T}/ds}{|d\hat{T}/ds|} \rightarrow ④$$

$$\frac{d\hat{T}}{ds} = \frac{d\hat{T}/dt}{|d\vec{r}/dt|} \rightarrow ⑤$$

there $d\hat{T}/dt = \frac{1}{\sqrt{5}}(-4\cos 2t\,\hat{i} - 4\sin 2t\,\hat{j})$

Sub $d\hat{T}/dt$ and $|d\vec{r}/dt|$ in eqⁿ (5)

$$d\hat{T}/ds = 1/\sqrt{5}\,\frac{(-4\cos 2t\,\hat{i} - 4\sin 2t\,\hat{j})}{\sqrt{5}}$$

$$d\hat{T}/ds = 1/5\,(-4\cos 2t\,\hat{i} - 4\sin 2t\,\hat{j})$$

$$= |d\hat{T}/ds| = 1/5\sqrt{16\cos^2 2t + 16\sin^2 2t} = 4/5$$

$d\hat{T}/ds$ and $|d\hat{T}/ds|$ in eqⁿ (4)

$$\hat{N} = \frac{\cancel{1/5}(-4\cos 2t\,\hat{i} - 4\sin 2t\,\hat{j})}{4/\cancel{5}}$$

$$\hat{N} = \frac{-\cancel{4}(\cos 2t\,\hat{i} + \sin 2t\,\hat{j})}{\cancel{4}}$$

$$\hat{n} = -(\cos 2t\,\hat{i} + \sin 2t\,\hat{j})$$

Given $x = 1/\sqrt{2}$

w.k.t $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = \cos 2t\,\hat{i} + \sin 2t\,\hat{j} + t\hat{k}$

$x = \cos 2t \Rightarrow \cos 2t = 1/\sqrt{2}$

$= 2t = \cos^{-1}(1/\sqrt{2}) = \pi/4$

$= t = \pi/8$

at $t = \pi/8$ $\hat{T} = \dfrac{-2\sin(2\pi/8)\,\hat{i} + 2\cos(2\pi/8)\hat{j} + \hat{k}}{\sqrt{5}}$

$= \dfrac{-2(1/\sqrt{2})\hat{i} + 2(1/\sqrt{2})\hat{j} + \hat{k}}{\sqrt{5}}$

$\hat{T} = \dfrac{-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \hat{k}}{\sqrt{5}}$

|||ly

$\hat{N} = -(\cos(2\pi/8)\,\hat{i} + \sin(2\pi/8)\hat{j})$

$\hat{N} = -\left[1/\sqrt{2}\,\hat{i} + 1/\sqrt{2}\,\hat{j}\right]$

***
*(4) A partical moves along the curve $C: x = 1-t^3,\ y = 1+t^2,\ z = 2t-5$
(a) determine its velocity and acceleration
(b) Find the components of velocity and acceleration in the direction $2\hat{i} + \hat{j} + 2\hat{k}$

Given C: $x = 1-t^3,\ y = 1+t^2$ and $z = 2t-5$.

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r} = (1-t^3)\,\hat{i} + (1+t^2)\hat{j} + (2t-5)\hat{k}$

(a) velocity and acceleration

velocity $(\vec{v}) = \dfrac{d\vec{r}}{dt} = -3t^2\hat{i} + 2t\hat{j} + 2\hat{k}$

accelaration $(\vec{a}) = \dfrac{d^2\vec{r}}{dt^2} = -6t\hat{i} + 2\hat{j}$

(b) at $t = 1$,

$\vec{V} = -3\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{a} = -6\hat{i} + 2\hat{j}$

let $\vec{B} = 2\hat{i} + \hat{j} + 2\hat{k}$

$|\vec{D}| = \sqrt{4+1+4} = \sqrt{9} = 3$

$\hat{n} = \frac{\vec{D}}{|\vec{D}|} = \frac{2\hat{i}+\hat{j}+2\hat{k}}{3}$ //.

∴ The components of velocity and accelaration is given by

$\vec{v}\cdot\hat{n} = (-3\hat{i}+2\hat{j}+2\hat{k})\cdot\frac{(2\hat{i}+\hat{j}+2\hat{k})}{3}$

$= 1/3(-6+2+4) = 0$ //

$\vec{a}\cdot\hat{n} = (-6\hat{i}+2\hat{j})\cdot\left(\frac{2\hat{i}+\hat{j}+2\hat{k}}{3}\right)$

$= 1/3(-12+2) = -10/3$.

(8) The partical moves along the curve $c: x=t^3-4t, y=t^2+4t$ and $z=8t^2-3t^3$. find the components of acceleration at $t=2$ ~~(i)~~ along the tangent and normal

Given $c: x=t^3-4t, y=t^2+4t$ and $z=8t^2-3t^3$.

$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$\vec{r} = (t^3-4t)\hat{i} + (t^2+4t)\hat{j} + (8t^2-3t^3)\hat{k}$

Diffⁿ w.r.t t.

$d\vec{r}/dt = (3t^2-4)\hat{i}+(2t+4)\hat{j}+(16t-9t^2)\hat{k}$

$d^2\vec{r}/dt^2 = (6t)\hat{i}+(2)\hat{j}+(16-18t)\hat{k}$

$= 6t\,\hat{i}+2\hat{j}+(16-18t)\hat{k}$

∴ The acceleration at $t=2$ is

$\vec{a} = (d^2\vec{r}/dt^2)_{t=2} = 12\hat{i}+2\hat{j}+16-20\hat{k}$ //

w.r.t the tangent vector is $t=2$

$d\vec{r}/dt = (3t^2-4)\hat{i}+(2t+4)\hat{j}+(16t-9t^2)\hat{k}$

$= (8\hat{i}+8\hat{j}-4\hat{k}) = \vec{D}$ (say)

$|\vec{D}| = \sqrt{64+64+16} = \sqrt{144} = 12$ //.

$\hat{n} = \frac{\vec{D}}{|\vec{D}|} = \frac{8\hat{i}+8\hat{j}-4\hat{k}}{12}$.

$= \frac{2\hat{i}+2\hat{j}-\hat{k}}{3}$

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The ~~nom~~ tangential component is

$\vec{a}\cdot\hat{n} = (12\hat{i}+2\hat{j}-20\hat{k})\cdot\left(\frac{2\hat{i}+2\hat{j}-\hat{k}}{3}\right)$

$= 1/3(24+4+20) = 16$.

The Normal component is.

$|\vec{a}-(\vec{a}\cdot\hat{n})\hat{n}| = |(12\hat{i}+2\hat{j}-20\hat{k}) - 16\left(\frac{2\hat{i}+2\hat{j}-\hat{k}}{3}\right)|$

$= |4/3\,\hat{i} - \frac{26}{3}\hat{j} - 44/3\,\hat{k}|$

$= \sqrt{\frac{16+676+1936}{9}}$

$= 17.08$

(9) A partical moves along the curve ~~x=~~ C: $x = 2t^3$, $y = t^2-4t$ $z = 3t-5$, Find the components of velocity and acceleration. at $t=1$ in the direction $\hat{i}-3\hat{j}+2\hat{k}$

$C: x = 2t^2, y = t^2-4t, z = 3t-5$.

$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$\vec{r} = 2t^2\hat{i}+(t^2-4t)\hat{j}+(3t-5)\hat{k}$

(a) Velocity and acceleration

Velocity $\vec{v} = \frac{d\vec{r}}{dt} = 4t\hat{i}+(2t-4)\hat{j}+(3)\hat{k}$

acceleration $(\vec{a})$ $\frac{d\vec{r}}{dt^2} = 4\hat{i}+2\hat{j}$

at $t=1$

$\vec{v} = 4\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{a} = 4\hat{i}+2\hat{j}$

let $\vec{B} = \hat{i}-3\hat{j}+2\hat{k}$

$|\vec{B}| = \sqrt{1+9+4} = \sqrt{14}$.

$\hat{n} = \frac{\vec{B}}{|\vec{B}|} = \frac{\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{14}}$.

The components of velocity and acceleration is givenby.

$\vec{v} \cdot \hat{n} = (4\hat{i} - 2\hat{j} + 3\hat{k}) \cdot \left(\frac{\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{14}}\right)$

$= \frac{1}{\sqrt{14}}(4+6+6) = \frac{16}{\sqrt{14}}$ //

$\vec{a} \cdot \hat{n} = (4\hat{i} + 2\hat{j}) \cdot \left(\frac{\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{14}}\right)$

$= \frac{1}{\sqrt{14}}(4-6) = -2/\sqrt{14}$ //.

(10) A partical moves along the curve $\vec{r} = 2\cos t\,\hat{i} - 2\sin t\,\hat{j} + t^2\hat{k}$ find the tangential and normal components of acceleration at $t=0$.

Given $\vec{r} = 2\cos t\,\hat{i} - 2\sin t\,\hat{j} + t^2\hat{k}$

Diffⁿ w.r.t t.

$\frac{d\vec{r}}{dt} = -2\sin t\,\hat{i} - 2\cos t\,\hat{j} + 2t\,\hat{k}$

$\frac{d^2\vec{r}}{dt^2} = -2\cos t\,\hat{i} + 2\sin t\,\hat{j} + 2\hat{k}$

∴ The acceleration at $t=0$ is.

$\vec{a} = \left(\frac{d^2\vec{r}}{dt^2}\right)_{t=0} = -2\hat{i} + 2\hat{k}$ //

w.k.t the tangent vector is.

$\frac{d\vec{r}}{dt} = -2\sin t\,\hat{i} - 2\cos t\,\hat{j} + 2t\,\hat{k}$

at $t=0$

$\frac{d\vec{r}}{dt} = -2\hat{j} = \vec{B}$

$|\vec{B}| = \sqrt{(-2)^2} = \sqrt{4} = 2$ //.

w.k.t

$\hat{n} = \frac{\vec{B}}{|\vec{B}|} = \frac{-2\hat{j}}{2} = -\hat{j}$ //.

The tangential component is

$\vec{a} \cdot \hat{n} = (-2\hat{i} + 2\hat{k}) \cdot (-\hat{j}) = 0$

the normal component is.

$$|\vec{a} - (\vec{a}\cdot\hat{n})\hat{n}| = |-2\hat{i}+2\hat{k}-0|$$
$$= |-2\hat{i}+2\hat{k}|$$
$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

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Group 2.

1. Gradient of scalar field.

(a) if $\phi(x,y,z)$ is a continuously differentiable function then

$$\text{grad}\,\phi \text{ (or) } \nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

2 (b) Divergence of vector

- If vector $\vec{F} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$ then $\text{div}\,\vec{F}$

$$\text{dive}\,\vec{F} \text{ or } \nabla\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

- If $\text{dive}\,\vec{F}$ or $\nabla\vec{F} = 0$ then $\vec{F}$ is called as solenoidal vector

3. ~~Curve~~ Curl of a vector

- If vector $\vec{F} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$

$$\nabla\times\vec{F} \text{ (or) } \text{curl}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{vmatrix}$$

- If $\text{curl}\,\vec{F}$ (or) $\nabla\times\vec{F} = 0$ then $\vec{F}$ is called irrotational vector.
- If $\vec{F}$ is irrotational then there exists ~~the~~ the scalar point function ϕ such that $\vec{F} = \nabla\phi$

  Here ϕ is called as scalar potential

- A irrotational vector is also called as conservative force field (or) potential field.

4. Unit Vector normal to the surface is $\hat{n} = \frac{\nabla\phi}{|\nabla\phi|}$

5. Angle b/w two surfaces $\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$

- If $\nabla\phi \cdot \nabla\phi_2 = 0$ then we can say that the surface intersectes orthogonally.

**6. Directional derivative

If $\phi(x,y,z)$ is a scalar function and $\vec{a}$ is a given directional vector. than

· Directional derivative $= \nabla\phi \cdot \hat{n}$ where $\hat{n} = \frac{\vec{a}}{|\vec{a}|}$

7. · tangent plane and normal line

tagent plane

$$A(x-x_1) + B(y-y_1) + c(z-z_1) = 0$$

Normal line

$$\frac{x-x_1}{A} = \frac{y-y_1}{B} = \frac{z-z_1}{c}$$

where $A = \left(\frac{\partial\phi}{\partial x}\right)_{(x,y,z)}$ $B = \left(\frac{\partial\phi}{\partial y}\right)_{(x,y,z)}$ $c = \left(\frac{\partial\phi}{\partial z}\right)_{(x,y,z)}$

Group 3 Vector Identities

1. Prove that curl (gard ϕ) $= \vec{0}$ (or) $\nabla \times (\nabla\phi) = \vec{0}$

Proof w.k.t

$$\text{grade (or) } \nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$\text{curl}(\text{grad}\,\phi) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial\phi}{\partial x} & \frac{\partial\phi}{\partial y} & \frac{\partial\phi}{\partial z} \end{vmatrix}$$

$$= \sum i\left(\frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial y}\right)\right)$$

$$= \sum i\left[\frac{\partial^2\phi}{\partial y\partial z} - \frac{\partial^2\phi}{\partial y\partial z}\right] = \vec{0}.$$

$$= \text{curl}(\text{grad}\,\phi) = \vec{0} \,//$$

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2. Prove that $div(curl \vec{A}) = 0$ or $\nabla.(\nabla \times \vec{A}) = 0$

proof If $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ then

$$curl = \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$curl \vec{A} = \hat{i}\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) - \hat{j}\left(\frac{\partial A_3}{\partial x} - \frac{\partial A_1}{\partial z}\right) + \hat{k}\left(\frac{\partial A_2}{\partial x} = \frac{\partial A_1}{\partial y}\right)$$

w.r.t $div \vec{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$

$$div(curl \vec{A}) = \frac{\partial}{\partial x}\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial A_3}{\partial x} + \frac{\partial A_1}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_2}{\partial x} - \right.$$

$$= \cancel{\frac{\partial^2 A_3}{\partial x \partial y}} - \cancel{\frac{\partial^2 A_2}{\partial x \partial z}} - \cancel{\frac{\partial^2 A_3}{\partial x \partial y}} + \cancel{\frac{\partial^2 A_1}{\partial y \partial z}} + \cancel{\frac{\partial^2 A_2}{\partial x \partial z}} - \cancel{\frac{\partial^2 A_1}{\partial y \partial z}}$$

$div(curl \vec{A}) = 0$

3. Prove that $div(\phi \vec{A}) = \phi(div \vec{A}) + grad\ \phi . \vec{A}$

$\nabla.(\phi\vec{A}) = \phi(\nabla.\vec{A}) + (\nabla\phi).\vec{A}$

proof :- let $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

$\phi\vec{A} = \phi A_1\hat{i} + \phi A_2\hat{j} + \phi A_3\hat{k}$

$$div(\phi\vec{A}) = \frac{\partial}{\partial x}(\phi A_1) + \frac{\partial}{\partial y}(\phi A_2) + \frac{\partial}{\partial z}(\phi A_3)$$

$$= \phi\frac{\partial A_1}{\partial x} + A_1\frac{\partial \phi}{\partial x} + \phi\frac{\partial A_2}{\partial y} + A_2\frac{\partial \phi}{\partial y} + \phi\frac{\partial A_3}{\partial z} + A_3\frac{\partial \phi}{\partial z}$$

$$= \phi\left[\frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}\right) + \left\{A_1\frac{\partial \phi}{\partial x} + A_2\frac{\partial \phi}{\partial y} + A_3\frac{\partial \phi}{\partial z}\right\}$$

$div(\phi\vec{A}) = \phi(div \vec{A}) + (grad\ \phi).\vec{A}$.

4. P.T $curl(\phi\vec{A}) = \phi(curl \vec{A}) + (grad\ \phi) \times \vec{A}$

$\nabla \times (\phi\vec{A}) = \phi(\nabla \times \vec{A}) + (\nabla\phi) \times \vec{A}$

let $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

$\phi\vec{A} = \phi A_1\hat{i} + \phi A_2\hat{j} + \phi A_3\hat{k}$

$$\text{curl}(\phi\vec{A}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \phi A_1 & \phi A_2 & \phi A_3 \end{vmatrix}$$

$$= \sum i \left[\frac{\partial}{\partial y}(\phi A_3) - \frac{\partial}{\partial z}(\phi A_2) \right]$$

$$= \sum i \left\{ \phi \frac{\partial A_3}{\partial y} + A_3 \frac{\partial \phi}{\partial y} - \phi \frac{\partial A_2}{\partial z} - A_2 \frac{\partial \phi}{\partial z} \right\}$$

$$= \sum i \left[\phi \left[\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z} \right] \right] + \sum i \left[A_3 \frac{\partial \phi}{\partial y} - A_2 \frac{\partial \phi}{\partial z} \right]$$

$$\text{curl}(\phi\vec{A}) = \phi(\text{curl}\,\vec{A}) + (\text{grad}\,\phi) \times \vec{A}.$$

* **Problem (Group ②):**

Q. **Problems on angle b/w surfaces**

***1. Find the angle b/w the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at $(2,-1,2)$.

⇒ Given $x^2+y^2+z^2-9=0$ and $x^2+y^2-z-3=0$

let $\phi_1 = x^2+y^2+z^2-9$ and $\phi_2 = x^2+y^2-z-3$

w.k.t

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$\nabla\phi_1 = 2x\hat{i} + 2y\hat{j} + 4\hat{k}$ | $\nabla\phi_2 = 2x\hat{i} + 2y\hat{j} - \hat{k}$

at $(2,-1,2)$

$\nabla\phi_1 = 4\hat{i} - 2\hat{j} + 4\hat{k}$ and $\nabla\phi_2 = 4\hat{i} - 2\hat{j} - \hat{k}$

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|} = \frac{16+4-4}{\sqrt{16+4+16}\cdot\sqrt{16+4+1}} = \frac{\cancel{16}^{\,8}}{\cancel{6}_{\,3}\sqrt{21}}$$

$$\cos\theta = \left(\frac{8}{3\sqrt{21}}\right) \Rightarrow \theta = \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$$

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2. Find the angle b/w normals to the surface $xy = z^2$ at the points $(4,1,2)$ and $(3,3,-3)$

sol Given $xy - z^2 = 0$

let $\phi_1 = xy - z^2$

w.k.t

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$\nabla\phi = y\hat{i} + x\hat{j} - 2z\hat{k}$$

at $(4,1,2)$

$$\nabla\phi_1 = \hat{i} + y\hat{j} - 4\hat{k}$$

at $(3,3,-3)$

$$\nabla\phi_2 = 3\hat{i} + 3\hat{j} + 6\hat{k}$$

w.k.t

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|} = \frac{3+12-24}{\sqrt{1+16+16}\cdot\sqrt{9+9+36}} = \frac{-9}{\sqrt{33}\sqrt{54}}.$$

$$\cos\theta = \frac{-9}{\sqrt{33}\times 3\sqrt{6}} = \theta = \cos^{-1}\left(\frac{-3}{\sqrt{198}}\right)$$

*3. verify that the surfaces $4x^2 + z^3 = 4$ and $5x^2 - 2yz - 9x = 0$ intersect orthogonally at $(1,-1,2)$.

let $\phi_1 = 4x^2 + z^3 - 4$ $\quad \phi_2 = 5x^2 - 2yz - 9x$.

$\nabla\phi_1 = 8x\hat{i} + 3z^2\hat{j}$ and $\nabla\phi_2 = (10x-9)\hat{i} - 2z\hat{j} - 2y$

at $(1,-1,2)$

$\nabla\phi_1 = (8\hat{i} + 12\hat{k})$ and $\nabla\phi_2 = (\hat{i} - 4\hat{j} + 2\hat{k})$ //

consider

$$\nabla\phi_1 \cdot \nabla\phi_2 = 8 + 24 = 32 \neq 0$$

∴ Thus surfaces will not intersect orthogonally problem on grad, div, curl, solenoidal and irrotational vector and problems on directional derivative.

(1) If $\vec{F} = grad(x^3+y^3+z^3-3xyz)$ then find div $\vec{F}$ and curl $\vec{F}$.

$\Rightarrow$ w.k.t grad $\phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$

$$\vec{F} = (3x^2-3yz)\hat{i} + (3y^2-3xz)\hat{j} + (3z^2-3xy)\hat{k}$$

w.k.t

i) $$\text{div}\vec{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial y}.$$

$$= \frac{\partial(3x^2-3yz)}{\partial x} + \frac{\partial(3y^2-3xz)}{\partial y} + \frac{\partial(3z^2-3xy)}{\partial z}$$

$$= 6x+6y+6z$$

$$= 6(x+y+z)$$

(ii) curl F =
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2-3yz & 3y^2-3xz & 3z^2-3xy \end{vmatrix}$$

$$i[-\cancel{3x}+\cancel{3x}] - j[-\cancel{3y}+\cancel{3y}] + \hat{k}[-\cancel{3z}+\cancel{3z}]$$

$$=0$$

(2) S.T $\vec{F} = \frac{x\hat{i}+y\hat{j}}{x^2+y^2}$ is both solenoidal and irrotational.

Given $\vec{F} = \frac{x}{x^2+y^2}\hat{i} + \frac{y}{x^2+y^2}\hat{j} = f_1\hat{i}+f_2\hat{j}$

$$\text{div}\vec{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$\text{div}\vec{F} = \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) + \partial\left(\frac{y}{x^2+y^2}\right) + 0$$

$$= \frac{(x^2+y^2)(1)-(x(2x))}{(x^2+y^2)^2} + \frac{(x^2+y^2)(1)-y(2y)}{(x^2+y^2)^2}$$

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$$= \frac{x^2+y^2-2x^2+x^2+y^2-2y^2}{(x^2+y^2)^2} = 0$$

$\operatorname{div} \vec{F} = 0 = \vec{F}$ is solenoidal.

(ii) $\text{curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{x^2+y^2} & \frac{y}{x^2+y^2} & 0 \end{vmatrix}$

$$= i[0-0] - \hat{j}[0-0] + \hat{k}\left[\frac{-y(2x)}{(x^2+y^2)^2} - \frac{-x(2y)}{(x^2+y^2)^2}\right]$$

$$= \text{curl } \vec{F} = \left[\frac{-2xy}{(x^2+y^2)^2} + \frac{2xy}{(x^2+y^2)^2}\right]\hat{k} = \vec{0}$$

$\text{curl } \vec{F} = \vec{0} \Rightarrow \vec{F}$ is irrotational.

3. Find a, b and c such that $\vec{F} = (x+y+az)\hat{i} + (bx+2y-z)\hat{j} + (x+cy+2z)\hat{k}$

given $\vec{F}$ is irrotational $\Rightarrow \text{curl } \vec{F} = \vec{0}$

$$\text{curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x+y+az) & (bx+2y-z) & (x+cy+2z) \end{vmatrix} = \vec{0}$$

$$\hat{i}(c+1) - \hat{j}(1-a) + \hat{k}(b-1) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$c+1=0$,	$-(1-a)=0$	$b-1=0$
$c=-1$	$-1+a=0$	$b=1$
	$a=1$	$b=1$ //

4. Find the direction derivative of $\phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the direction $2\hat{i} - \hat{j} - 2\hat{k}$

$\phi = x^2yz + 4xz^2 \quad (1,-2,-1)$

$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$\nabla\phi = (2xyz + 4z^2)\hat{i} + (x^2z)\hat{j} + (x^2y + 8xz)\hat{k}$

at $(1,-2,-1)$

$= (4+4)\hat{i} + (-1)\hat{j} + (-2-8)\hat{k}$

$\nabla\phi = 8\hat{i} - \hat{j} - 10\hat{k}$

$\vec{d} = 2\hat{i} - \hat{j} - 2\hat{k}$

$|\vec{d}| = \sqrt{4+1+4} = \sqrt{9} = 3$

$\hat{n} = \frac{\vec{d}}{|\vec{d}|} = \frac{2\hat{i} - \hat{j} - 2\hat{k}}{3}$

w.k.t, the directional derivative is

$\nabla\phi \cdot \hat{n} = (8\hat{i} - \hat{j} - 10\hat{k}) \cdot \frac{(2\hat{i} - \hat{j} - 2\hat{k})}{3}$

$= \frac{16+1+20}{3} = \frac{37}{3}$ //.

5. Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2,-1,2)$ along the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$

$\phi = 4xz^3 - 3x^2y^2z \quad (2,-1,2)$.

$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$\nabla\phi = (4z^3 - 6xy^2z)\hat{i} + (-6yx^2z)\hat{j} + (12z^2x - 3x^2y^2)\hat{k}$

$+6(-1)(4)$

at $(2,-1,2)$

$12(4)(2)$

$= (32-24)\hat{i} + (48)\hat{j} + (96-12)\hat{k}$

$= 8\hat{i} + 48\hat{j} + 84\hat{k}$

$\vec{d} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$|\vec{d}| = \sqrt{4+9+36} = \sqrt{49} = 7$ //.

$\hat{n} = \frac{\vec{d}}{|\vec{d}|} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$

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w.k.t the directional derivative is

$$\nabla\phi \cdot \hat{n} = 8\hat{i}+48\hat{j}+84\hat{k} \cdot \frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}$$

$$\frac{16-144+504}{7} = \frac{376}{7}$$

H.W Find the D.D of $\phi = \frac{xz}{x^2+y^2}$ at $(1,-1,1)$ ~~along the~~ in the ~~dem~~ direction of vector $\vec{R} = \hat{i}-2\hat{j}+\hat{k}$.

Given $\phi = \frac{xz}{x^2+y^2}$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$= \left(\frac{xz \cdot 2x + y^2 - x^2+y^2 \cdot z}{(x^2+y^2)^2}\right)\hat{i} + \left(-\frac{xz}{2y}\right)\hat{j} + x\hat{k}$$

$$= \left[\frac{xz(2x+0) - (x^2+y^2)z}{(x^2+y^2)^2}\right]\hat{i} + \left[\frac{xz(2y) - (x^2+y^2)(0)}{(x^2+y^2)^2}\right]\hat{j}$$

$$+ \left[\frac{xz(0) - (x^2+y^2)z}{(x^2+y^2)^2}\right]\hat{k}$$

$$= \frac{2x^2z - z(x^2+y^2)}{(x^2+y^2)^2}\hat{i} + \frac{2xyz}{(x^2+y^2)^2}\hat{j} + \frac{-z(x^2+y^2)}{(x^2+y^2)^2}\hat{k}$$

(7) Find the directional derivative of $\phi = x \cdot y^2 + y \cdot z^3$ at $(2,-1,1)$ in the direction ~~towards~~ towards the point $(1,2,2)$

Given $\phi = x \cdot y^2 + y \cdot z^3$ $(2,-1,1)$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$= y^2\hat{i} + (2xy + z^3)\hat{j} + 3yz^2\hat{k}$$

at point $(2,-1,1)$.

$$\hat{i} + (-4+1)\hat{j} + (-3\hat{k})$$

$$\nabla\phi = \hat{i} - 3\hat{j} - 3\hat{k}.$$

let $\vec{a} = (1,2,2)$

$$= \hat{i} + 2\hat{j} + 2\hat{k}.$$

$$|\vec{a}| = \sqrt{4+4+1} = \sqrt{9} = 3.$$

$$\hat{n} = \frac{\hat{i} + 2\hat{j} + 2\hat{k}}{3}.$$

The directional derivative is:

$$\nabla\phi \cdot \hat{n} = \hat{i} - 3\hat{j} - 3\hat{k} \cdot \frac{\hat{i} + 2\hat{j} + 2\hat{k}}{3}$$

$$\frac{1 - 6 - 6}{3} = \frac{-11}{3}$$

(8) find the directional derivative of $\phi f(x,y,z) = x^2y^2z^2$ at $(1,1,-1)$ in the direction of the tangent to the curve.

$x = e^t, y = 1 + 2\sin t, z = t - \cos t, -1 \le t \le 1$

$$F = x^2y^2z^2$$

$$\nabla F = \frac{\partial F}{\partial x}\hat{i} + \frac{\partial F}{\partial y}\hat{j} + \frac{\partial F}{\partial z}\hat{k}$$

$$\nabla F = (2xy^2z^2)\hat{i} + (2yx^2z^2)\hat{j} + (2x^2y^2z)\hat{k}.$$

at $(1,1,-1)$

$$= 2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\nabla F = 2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{r} = e^t\hat{i} + (1 + 2\sin t)\hat{j} + (t - \cos t)\hat{k}$$

w.k.t = the tangent vector is

$$\frac{d\vec{r}}{dt} = e^t\hat{i} + (2\cos t)\hat{j} + (1 + \sin t)\hat{k}$$

$e^t = 1$

$t = \log(1)$

$t = 0.$

at $t=0$

$$\frac{d\vec{r}}{dt} = \hat{i} + 2\hat{j} + \hat{k} = \vec{a}\ (\text{say}).$$

$$|\vec{a}| = \sqrt{1+4+1} = \sqrt{6}.$$

$$\nabla f \cdot \hat{n} = 2\hat{i} + 2\hat{j} - 2\hat{k} \cdot \frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{6}}$$

$$\frac{2+4-2}{\sqrt{6}} = \frac{4}{\sqrt{6}} \ //$$

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9. Find the directional derivative of xyz along the direction ~~a. at~~ of normal to the surface. $xy^2 + yz^2 + zx^2 = 3$ at the point $(1,1,1)$ also find the equation of the tangent plain and normal line.

let $\phi = xyz$

$$\nabla\phi = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial y}\hat{k}$$

$$\nabla\phi = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$$

at given point $(1,1,1)$

$$= \hat{i} + \hat{j} + \hat{k}$$

let $\psi = xy^2 + yz^2 + zx^2 - 3.$

$$\nabla\psi = (y^2 + 2xz)\hat{i} + (2yx + z^2)\hat{j} + (2zy + x^2)\hat{k}$$

at point $(1,1,1)$

$$3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\vec{a} = 3(\hat{i} + \hat{j} + \hat{k})$$

$$\vec{a} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

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$|\vec{d}|$ $\sqrt{9+9+9} = \sqrt{27}$

$= 3\sqrt{3}$

$\hat{n} = \frac{3\hat{i}+3\hat{j}+3\hat{k}}{3\sqrt{3}}$ $\frac{\vec{d}}{|\vec{d}|}$

$= \frac{\cancel{3}(\hat{i}+\hat{j}+\hat{k})}{\cancel{3}\sqrt{3}}$

D.D =

$\nabla\phi\cdot\hat{n} = \hat{i}+\hat{j}+\hat{k} \cdot \frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$

$\frac{1}{\sqrt{3}}(1+1+1) = \frac{3}{\sqrt{3}} = \sqrt{3}$

(i) tangent plane $= A(x-1) + B(y-y_1) + C((z-z_1)$:

$(x_1, y_1, z_1)(1,1,1)$

$A = \left(\frac{\partial\psi}{\partial x}\right) = 3$, $B = \left(\frac{\partial\psi}{\partial y}\right) = 3$, $B = \left(\frac{\partial\psi}{\partial y}\right) = 3.$

10. If the directional derivative at $axy^2+byz+cz^2x^3$ at $(-1,1,2)$ has a max magnitude at 32 units in the direction parallel to y-axis, the find a, b, and c.

$\phi = axy^2 + byz + cz^2x^3$

$\nabla\phi = (y^2+3x^2z^2)\hat{i}\ (2yxa+ bz)\hat{j} + (by + c2zx^3)\hat{k}$

at $(-1,1,2)$.

$\therefore \nabla\phi = (a-12c)\hat{i} + (-2a+b)\hat{j} + (b-4c)\hat{k} \rightarrow (1)$

Given that $\nabla\phi\cdot\hat{j} = 32 \rightarrow (2)$

sub eq. 1 into 2

$[(a+12c)\hat{i} + (-2a+b)\hat{j} + (b-4c)\hat{k}]\cdot\hat{j} = 32$

$a+12c=0 \quad -2a+b=32. \quad b-4c=0$

solving we get $a = -12$

$b = 4$

$c = 1$.

11. show that vector $\vec{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ is irrotational also find scalar potential ϕ such that vector $\vec{F} = \nabla\phi$

$\vec{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$

$$\text{curl}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y+z & z+x & x+y \end{vmatrix}$$

$$\hat{i}[1-1] - \hat{j}[1-1] + \hat{k}[1-1]$$

$\text{curl}\,\vec{F} = 0$

$\vec{F}$ = irrotation.

$\vec{F} = \nabla\phi$

$$(y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k} = \nabla \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

equating the components

$\frac{\partial\phi}{\partial x} = y+z \qquad \frac{\partial\phi}{\partial y} = z+x \qquad \frac{\partial\phi}{\partial z} = (x+y)$

$\int\frac{\partial\phi}{\partial x} = \int y+z\,\partial x\; c_1 \qquad \int\frac{\partial\phi}{\partial y} = \int z+x\;\partial y\; c_2 \qquad \int\frac{\partial\phi}{\partial y} = \int x+y\,\partial z$

$\phi = (y+z)x + c_1 \qquad \phi = (z+x)y\; c_2 \qquad \phi = (x+y)z\; c_3$

$\phi = xy + xz + c_1 \qquad \phi = yz + yx + c_2 \qquad \phi = xz + yz + c_3$

choosing

$c_1 = yz,\; c_2 = xz,\; c_3 = xy$

$\phi = xy + xz + yz$

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12 Show that vector $\vec{F} = (2xy^2+yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is irrotational also find scalar potential ϕ such that vector $\vec{F} = \nabla\phi$.

$$\vec{F} = (2xy^2+yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$$

$$\text{curl}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy^2+yz & 2x^2y+xz+2yz^2 & 2y^2z+xy \end{vmatrix}$$

$$i\left[[4yz + x] - [x + 4yz]\right] - \hat{j}\left[[y] - [y]\right] + \hat{k}\left[[4xy + z] - [4xy+z]\right]$$

$$i[0] - j[0] + \hat{k}[0]$$

$$\text{curl}\,\vec{F} = 0$$

$\vec{F}$ = irrotation.

$$\vec{F} = \nabla\phi$$

$$(2xy^2+yz)\hat{i} + (2x^2y+xz+2yz^2)\hat{j} + (2y^2z+xy)\hat{k} = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

equating the component

$$\frac{\partial\phi}{\partial x} = 2xy^2 + yz \qquad \frac{\partial\phi}{\partial y} = 2x^2y + xz + 2yz^2 \qquad \frac{\partial\phi}{\partial z} = 2y^2z + xy.$$

$$\int\frac{\partial\phi}{\partial x} = \int 2xy^2 + yz\,\partial x\, C_1$$

$$\phi = (2xy^2+yz)x + C_1$$

$$\phi = x^2y^2 + xyz + C_1$$

$$\int\frac{\partial\phi}{\partial y} = \int 2x^2y + xz + 2yz^2\,\partial y\, C_2$$

$$\phi = (2x^2y + xz + 2yz^2)y + C_2$$

$$\phi = x^2y^2 + xyz + y^2z^2 + C_2$$

$$\int\frac{\partial\phi}{\partial z} = \int 2y^2z + xy\,\partial z$$

$$\phi = (2y^2z + xy)z + C_3$$

$$\phi = y^2z^2 + xyz$$

Chossing.

$C_1 = y^2z^2 \quad C_2 = 0 \quad C_3 = x^2y^2.$

$$\phi = x^2y^2 + xyz + y^2x^2$$

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(13) If $\vec{F} = \nabla(xy^3z^2)$ find $div\,\vec{F}$ and $curl\,\vec{F}$ at $(1,-1,1)$.

$grad\,\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$\vec{F} = (y^3z^2)\hat{i} + (3y^2xz^2)\hat{j} + (2zxy^3)\hat{k}$

$div\,\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

$= 0 + 6yxz^2 + 2xy^3$

at $(1,-1,1)$

$=$

$6(-1)(1)(1)^2 + 2(1)(-1)^3$

$-6-2$

$div\,\vec{F} = -8$

$$curl\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^3z^2 & 3y^2xz^2 & 2zxy^3 \end{vmatrix}$$

$\hat{i}\,(6xy^2z - 6xy^2z) - \hat{j}\,(2zy^3 - 2y^3z)\ \hat{k}\,(3y^2z^2 - 3y^2z^2)$

$= \hat{i}(0) - \hat{j}(0) + \hat{k}(0)$

$= 0$

14) If $\vec{F} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$ then S.T $\vec{F} \cdot curl\,\vec{F} = 0$.

$\vec{F} = (x+y+1)\,\hat{i} + \hat{j} - (x+y)\hat{k}$

$$curl\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (x+y+1) & 1 & -x-y \end{vmatrix}$$

$\hat{i}\left[\cancel{-1}-0\right]-\hat{j}\left[-1\,\cancel{0}\right]+\hat{k}\left[0-1\right]$

$\text{curl}\vec{F} \quad -1\hat{i}+2\hat{j}-\hat{k}.$

$$\left((x+y+1)\hat{i}+\hat{j}-(x+y)\hat{k}\right)\cdot\left[-\hat{i}+\hat{j}-\hat{k}\right].$$

$$\vec{F}\cdot\text{curl}\vec{F} = (x+y+1)-1+(1)(1)+(x+y)$$

$$\cancel{-x}\,\cancel{-y}\,\cancel{-1}+\cancel{1}+\cancel{x}+\cancel{y} = 0$$

15. Find $\nabla\times(\nabla\times\vec{A})$, where

$$\vec{A} = xy\hat{i}+y^2z\hat{j}+z^2y\hat{k}.$$

$$\text{curl}\vec{A} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & y^2z & z^2y \end{vmatrix}$$

$\hat{i}(z^2-y^2)-\cancel{0}+0\,\cancel{0}+\hat{k}\,[0-x)$

$(z^2-y^2)\hat{i}+(0-x)\hat{k}$

$\nabla\times\vec{A} = (z^2-y^2)\hat{i}-x\hat{k}$

$$\nabla(\nabla\times\vec{A}) \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z^2-y^2 & 1 & -x \end{vmatrix}$$

$\hat{i}(0-0)\,\hat{j}(-1-2z)+\hat{k}(\cancel{2y}-2y)$

$(-1-2z)\hat{j}+2y\hat{k}$

$\nabla(\nabla\times\vec{A}) = (1+2z)\hat{j}+2y\hat{k}$

16. If $\vec{F} = (3x^2y - z)\hat{i} + (xz^3 + y^4)\hat{j} - 2(x^3z^2)\hat{k}$, find grad(div $\vec{F}$) at $(2,-1,0)$

$$\vec{F} = (3x^2y - z)\hat{i} + (xz^2 + y^4)\hat{j} - 2(x^3z^2)\hat{k}.$$

$$\text{div}\,\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$= 6xy + 4y^3 - 4x^3z$$

$$\nabla \text{div}\,\vec{F} = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

$$= (6y - 12x^2z)\hat{i} + (6x + 12y^2)\hat{j} + (-4x^3)\hat{k}$$

at $(2,-1,0)$

$$(-6)\hat{i} + (12+12)\hat{j} + (-32)\hat{k}$$

$$-6\hat{i} + 24\hat{j} - 32\hat{k}$$

17. P.T $\nabla^2 \cdot f(r) = f''(r) + 2/r\, f'(r)$

proof :- $\nabla^2 f(r) = \sum \frac{\partial^2}{\partial x^2}(f(r)) = \sum \frac{\partial}{\partial x}\left(f'(r)\frac{\partial r}{\partial x}\right) \rightarrow (1)$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = \vec{r}\cdot\vec{r} = x^2 + y^2 + z^2$$

$$r^2 = x^2 + y^2 + z^2$$

$$= 2r \cdot \frac{\partial r}{\partial x} = 2x$$

$$r \cdot \frac{\partial r}{\partial x} = x.$$

$$= \frac{\partial r}{\partial x} = x/r$$

Sub in (1)

$$\nabla^2 f(r) = \sum \frac{\partial}{\partial x}\left(f'(r) \cdot x/r\right)$$

$$= \sum \left\{ x/r \times f''(r) \times \partial r/\partial x - f'(r) \left[\frac{r \times 1 - x \times \frac{\partial r}{\partial x}}{r^2}\right]\right.$$

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$$= \Sigma \left\{ \left[(x/r) f''(r) (x/r) - f'(r) \left(\frac{r - x(x/r)}{r^2} \right) \right\} \right.$$

$$= \nabla^2 f(r) = \Sigma \frac{x^2}{r^2} f''(r) - f'(r) \left(\frac{r^2 - x^2}{r^3} \right) \Big\}.$$

$$= \frac{1}{r^2} f''(r) \, \Sigma x^2 - \frac{f'(r)}{r^3} \, \Sigma (r^2 - x^2)$$

$$= \frac{1}{r^2} f''(r) (x^2 + y^2 + z^2) - \frac{f'(r)}{r^3} \left\{ r^2 - x^2 + r^2 - y^2 + r^2 - z^2 \right\}$$

$$= \frac{1}{\cancel{r^2}} f''(r) (\cancel{r^2}) - \frac{f'(r)}{(r^3)} \left\{ 3r^2 - (x^2 + y^2 + z^2) \right\}$$

$$= f''(r) - \frac{f'(r)}{(r^3)} \times 2r^2$$

$$\nabla^2 f(r) = f''(r) - 2/r \, f'(r).$$