ELASTICITY

Elasticity is the branch of science that deals with the elastic property of materials.

Deforming force is the force that produces deformation(change in dimensions- length, volume, shape) to the object.

Restoring force is the force equal and opposite to the deforming force that come in to play which tries to restore the body to its original state. On removal of the deforming force restoring force bring the body to its original state.

Elasticity/Elastic property is the property of the materials by virtue of which they regain their original shape and size after the removal of deforming force acting on them.

Plasticity/Plastic property is the property of the materials by virtue of which they retain their deformed shape and size after the removal of deforming force acting on them.

Classification of materials:

Materials are classified in to two groups based on their elastic property namely:

1. **Elastic materials** are the materials which regain their original state on removal of deforming force. Ex: Quartz , steel ,rubber
2. **Plastic materials** are the materials which are not able to regain their original state on removal of deforming force. Ex: Putty, clay, mud, wax, lead, chewing gum

Note: In general no material is perfectly elastic or perfectly plastic .

Load is the external force acting on the body that produces change in the dimensions of the body.

Stress is the force acting per unit area of cross section of the wire or rod.

OR

Stress is the restoring force per unit area of cross section of the wire or rod.

$$\text{Stress}(\sigma) = \frac{\text{Force}}{\text{Area of cross section}} = \frac{F}{A} \quad Nm^{-2}$$

Types of stress:

There are three types of stress namely:-

1. **Longitudinal/Tensile stress** is the stress in which the restoring force act perpendicular to the area of cross section and along the length of the wire.(Longitudinal stress produce changes in length only but not in shape and volume)
2. **Bulk/Volume stress** is the stress in which the body is subjected to equal forces normally on all the faces. (Bulk stress produce changes in Volume of the body only but not in shape)

3. **Shear stress/ Tangential stress** is the stress in which the restoring forces are parallel to the surface. (Shear stress produce changes in shape of the body only but not in Volume)

Strain is the ratio of the change in dimension of the material to the original dimension.

Strain$(\varepsilon) = \frac{\text{Change in dimension}}{\text{Original dimension}}$

Types of strain:

There are three types of strain namely:-

1. **Longitudinal strain** is the ratio of change in length (Δl) to the original length(L)
   Longitudinal strain $= \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{L}$

2. **Volume strain** is the ratio of change in volume (ΔV) to the original volume(V)
   Volume strain $= \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$,

3. **Shear strain** is the ratio of lateral displacement between two layers(Δx) to the perpendicular distance between the two layers (x)

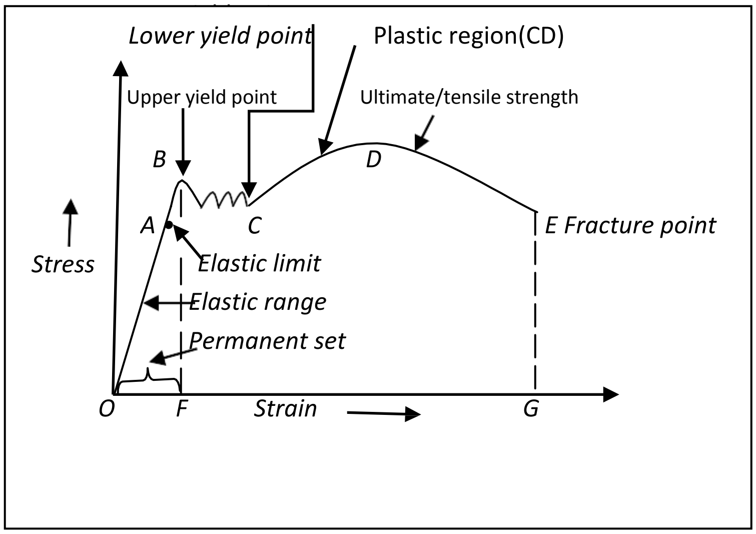
   Shear strain $= \frac{\text{lateral displacement between two layers}}{\text{perpendicular distance between the two layers}} = \frac{(\Delta x)}{x}$

HOOKE's LAW:

Hooke's law states that within elastic limit (of a body), stress varies directly as strain. ie: Stress α Strain

ie: $\frac{\text{stress}}{\text{strain}}$ = constant(E) ,where E is called modulus of elasticity or coefficient of elasticity.

Stress –Strain diagram :



Consider a material subjected to continuously varying load/stress .If we plot a graph of stress along Y-axis and strain along X-axis we get stress –strain graph OABCDE as shown in the diagram.

From O to A stress varies directly as strain .When load is removed the material regain its original dimension. The point A is called **elastic limit** up to which Hook's law is obeyed. If the stress is increased beyond A ,the strain increases and reaches B. Now the material is partly elastic and partly plastic. If stress is removed ,the material returns to the original state along BF. The region OF is the residual strain acquired by the material called **permanent set**.

The strain increases from B to C without further increase of stress. The region BC is in irregular shape. B is called **Upper yield point** and the stress is called **yield stress**.

The sudden increase in strain gets stopped at C called **Lower yield point**.

If stress is gradually increased beyond C, the strain increases and follow the path CD. Region CD is called **plastic region.** In this region the thickness of the material decreases without change in volume. The point D is the maximum stress the material can with stand and the corresponding deforming force is called **ultimate /tensile strength**.

In the region DE strain increases without increase of stress. In this region **neck** is formed in the material(wire),Due to the **neck** the material breaks even though the stress is decreased. The stress corresponding to the point E is called **breaking stress**.

The area under the curve OABCDEGO gives work done per unit volume.

Factors affecting elastic properties:

The elastic properties of the material are affected by the following factors, namely:

1. **Rolling and hammering** on a material generally breaks the bonding in the material which in turn increase the elastic property of the property.
2. **Temperature** increase of the material generally decrease the elasticity of the material.
   (Elastic property of invar steel is unaffected by temperature)
3. **Impurities** added to the material may increase or decrease the elastic property of the material.
   - The elasticity of the material increases if the elasticity of impurity is more than that of the material.
   - The elasticity of the material decreases if the elasticity of impurity is less than that of the material.
4. **Annealing**)increase the particle size and the material hardened, which in turn reduce the elasticity of the material.(**annealing** is the process of heating at a particular temperature and then cooling gradually)

What are the engineering importances of elastic materials?

The knowledge of elasticity is very very important in engineering applications as follows:

By knowing the elastic properties of different materials such as tensile strength, ductility, elastic limit, plasticity, stress –strain relationship, various modulii of elasticity and how the above properties can be altered etc Engineers can make better choice of materials for their use in different branches of engineering.

Ex: Pure metals are soft hence they are not preferred in engineering applications. Whereas Alloys are harder they find wide applications in engineering.

Elastic Modulus:

There are three types of elastic modulus, namely:-

Young's modulus(Y) is defined as the ratio of longitudinal(linear) stress to longitudinal(linear) strain with in elastic limit.

$$\text{Young's modulus} = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{\text{Force/Area}}{\text{Change in length/original length}} = \frac{F/A}{\Delta L/L}$$

$$Y = \frac{FL}{A\Delta L} \quad .. \quad N/m^2$$

Bulk modulus(K) is defined as the ratio of bulk stress to bulk strain .

Bulk modulus $=\frac{\text{Bulk stress}}{\text{Bulk strain}}.=\frac{\text{Force/Area}}{\text{Change in volume/original volume}}=\frac{\text{F/A}}{\Delta\text{V/V}}$

$\text{K}=\frac{\text{FV}}{\Delta\text{V.A}}=\frac{\text{PV}}{\Delta\text{V}}$ .. N/m^2 or Pa ,where P = F/A called bulk pressure.

Note: Compressibility is the reciprocal of bulk modulus, hence bulk modulus represents the incompressibility.

Rigidity(Shear) modulus ($\boldsymbol{n}$) is defined as the ratio of shear stress to shear strain

Rigidity modulus $=\frac{\text{Shear stress}}{\text{Shear strain}}$

$$\mathbf{n}=\frac{\text{Tangential force/Area}}{\text{lateral displacement between two layers/perpendicular distance between the two layers}}$$

$\mathbf{n}=\frac{\text{F/A}}{\Delta\text{x/x}}=\frac{\text{F/A}}{\theta}$ …… N/m^2 ,where Δx/x = tanθ ≈ θ for small θ

Poisson's Ratio($\boldsymbol{\sigma}$):

Longitudinal strain is the ratio of increase in length(ΔL) to the original length(L).

Longitudinal strain $=\frac{\text{Increase in length}}{\text{original length}}=\frac{\Delta\text{L}}{\text{L}}$

Lateral strain is the ratio of decrease in diameter(Δd) to the original diameter(d).

Lateral strain $=\frac{\text{decrease in diameter}}{\text{original diameter}}=\frac{\Delta\text{d}}{\text{d}}$

Poisson's Ratio($\boldsymbol{\sigma}$) is defined as the ration of lateral strain to the longitudinal strain.

Poisson's Ratio(σ) $=\frac{\text{Lateral strain}}{\text{Longitudinal strain}}=\frac{\Delta\text{d/d}}{\Delta\text{L/L}}=\frac{\Delta\text{d .L}}{\text{d .}\Delta\text{L}}$

Longitudinal strain coefficient($\boldsymbol{\alpha}$) is the longitudinal strain per unit stress.

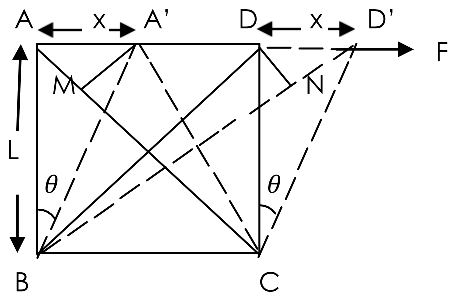
Lateral strain coefficient($\boldsymbol{\beta}$) is the ration of lateral strain per unit stress.

Poisson's Ratio($\boldsymbol{\sigma}$) is defined as the ration of lateral strain coefficient(β) $to\ the$ to the longitudinal strain coefficient(α).

Poisson's Ratio(σ) $=\frac{\text{Lateral strain coefficient}}{\text{Longitudinal strain coefficiernt}}=\frac{\beta}{\alpha}$

Relation between shear strain. longitudinal strain and compression(Bulk) strain/

ST (Show that) Longitudinal strain+$Compression\ strain = shear\ strain$



Consider a cube of each side length'L'.
Let ABCD be the front surface of the cube
When a force 'F' is applied along AD ,let ABCD is Displaced to position A'BCD' through a shearing angle 'θ'. Draw A'M and DN perpendicular to dioganals AC and BD' respective then A'C =MC and BD=BN

Now ND' is the extension in BD and AM is the compression in length AC

∴ Longitudinal strain $along\ BD = \frac{ND'}{BD}$..(1) and

Compression strain $along\ AC = \frac{AM}{AC}$..(2)

By Pythogara's theorem, AC $= BD = \sqrt{2}$ L ….(3)

From right angled triangle DND' ,D'N = DD' Cos $\angle DD'N$

= DD' Cos $\angle AD'B$ …(4)

But from right angled isosceles triangle ADB, $\angle ADB = 45°$

Since θ is very small , $\angle AD'B = \angle ADB = 45°$

∴ from eqn (4) ,we get ND' = DD' Cos °45 = $\frac{DD'}{\sqrt{2}}$ …….(5)

From eqns 1,3 and 5 ,we get Longitudinal strain = $\frac{DD'}{2L}$ but $\theta = \frac{DD'}{L}$ fromΔ^{le} CDD'

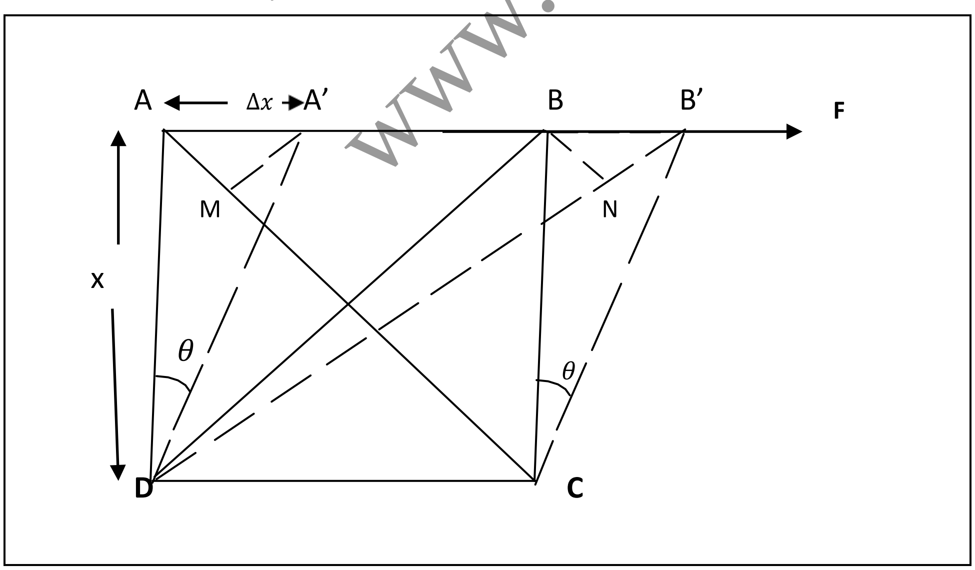
∴ Longitudinal strain = $\frac{\theta}{2}$ …….(6)

Similarly we can show that , Compression strain = $\frac{\theta}{2}$ ….(7)

From equns 6 & 7 , Longitudinal strain + Compression strain = θ ,the shearing strain

Thus , **Longitudinal strain**+$\boldsymbol{Compression\ strain = shear\ strain}$

Relation between Y, n $and\sigma$.



Let a force F acting tangentially on the upper surface ABQP of a cube of each side ' x ', displace it through a distance 'Δx'. A'B'Q'P' be the new position of the upper surface.

Then, shear stress applied $= \frac{Force}{Area} = \frac{F}{x^2}$ = T, tensile stress/Tension

Shear strain $= \frac{\text{Displacement of upper surface}}{\text{Side of cube}}$

$$\theta = \frac{\Delta x}{x} \quad \ldots(1)$$

Rigidity modulus $= \frac{\text{Shear stress}}{\text{shear strain}}$

$$n = \frac{T}{\theta} \quad \ldots.(2)$$

Due to the applied tangential force, tensile stress acts along the diagonal DB and compressive stress along the diagonal AC.

If **α** is the longitudinal strain coefficient and **β** is the lateral strain coefficient

Then elongation of the diagonal DB due to tensile stress = DB.T.α

Also, elongation of the diagonal DB due to compressive stress = DB.T.β

∴ Total elongation in length of DB = DB.T$(\alpha + \beta)$ ……(3)

Draw BN perpendicular to DB' , then NB' is the elongation in length of DB .

From the triangle BNB' , $NB' = BB' \cos(BB'N) = BB' \cos 45° = \Delta x . \frac{1}{\sqrt{2}}$ ….(4)

From eqn 3 & 4 ,we get DB.T$(\alpha + \beta) = \Delta x . \frac{1}{\sqrt{2}}$

But DB = $\sqrt{2}$ x ∴ $\sqrt{2}$ x T$(\alpha + \beta) = \Delta x . \frac{1}{\sqrt{2}}$

$$\sqrt{2}\, T(\alpha + \beta) = \frac{\Delta x}{x} . \frac{1}{\sqrt{2}}$$

From eqn(2) $\frac{\Delta x}{x} = \theta$ and from eqn(3) T= $n\,\theta$

$$\therefore \sqrt{2}\, n\, \theta\ (\alpha + \beta) = \theta . \frac{1}{\sqrt{2}}$$

$n\,(\alpha + \beta) = \frac{1}{2}$, on re arranging we get

$$n = \frac{1}{2\alpha(1+\beta/\alpha)}$$

$$\boldsymbol{n} = \frac{\boldsymbol{Y}}{\boldsymbol{2(1+\sigma)}} \quad \because \frac{1}{\alpha} = Y \text{ and } \frac{\beta}{\alpha} = \sigma$$

Relation between K, n and Y.

WKT Y= 3K(1−2σ) and on rearranging we get $\sigma = \frac{1}{2} - \frac{Y}{6K}$ ....(1)

Also , Y = $2n(1+\sigma)$ and on rearranging we get $\sigma = \frac{Y}{2n} - 1$ ....(2)

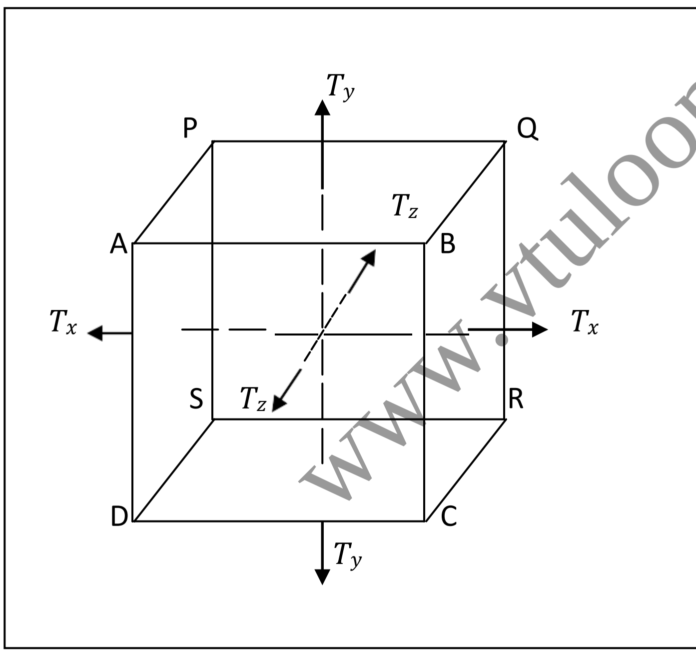
From eqns 1 & 2 we get , $\frac{1}{2} - \frac{Y}{6K} = \frac{Y}{2n} - 1$

$$\frac{1}{2} + 1 = \frac{Y}{2n} + \frac{Y}{6K}$$

$$\frac{3}{2} = Y\left(\frac{1}{2n} + \frac{1}{6K}\right)$$

$$\frac{3}{2} = Y\,\frac{(3K+n)}{6nK} \quad \text{or} \quad \mathbf{Y} = \frac{\mathbf{9nK}}{(\mathbf{3K+n})}$$

Relation between, K ,Y andσ.



Consider a unit cube ABCDPQRS of each side ‘1’ unit. Let T_x, T_y and T_z be the tensile stresses acting normal to parallel surfaces perpendicular to x,y and z axes respectively.

The side AB is elongated due to T_x and contracted due to T_y & T_z

The side AD is elongated due to T_y and contracted due to T_z & T_x

The side AP is elongated due to T_z and contracted due to T_x & T_y

If α is the elongation strain coefficient and β is the contraction strain coefficient, then the new lengths of the sides :

AB $= 1 + T_x\,\alpha - T_y\,\beta - T_z\,\beta$

AD $= 1 + T_y\,\alpha - T_z\,\beta - T_x\,\beta$

AP $= 1 + T_z\,\alpha - T_x\,\beta - T_y\,\beta$

$\therefore$ New cube volume= AB.AD.AP

$$= (1 + T_x\alpha - T_y\beta - T_z\beta).\,(1 + T_y\alpha - T_z\beta - T_x\beta).\,(1 + T_z\alpha - T_x\beta - T_y\beta)$$

$$= 1 + (\alpha - 2\beta)(T_x + T_y + T_z \quad \text{(neglecting higher order terms)}$$

If $T_x = T_y = T_z = T$

New volume $= 1 + 3T(\alpha - 2\beta)$

$\therefore$ Change in volume ,$\Delta V = 1 + 3T(\alpha - 2\beta) - 1$

$$= 3T(\alpha - 2\beta)$$

$$\text{Bulk strain} = \frac{\text{Change in volume}}{\text{Initial volume}} = \frac{\Delta V}{V} = \frac{3T(\alpha-2\beta)}{1} = 3T(\alpha - 2\beta)$$

$$\text{Bulk modulus} = \frac{\text{Bulk stress}}{\text{Bulk strain}} = \frac{T}{3T(\alpha-2\beta)}$$

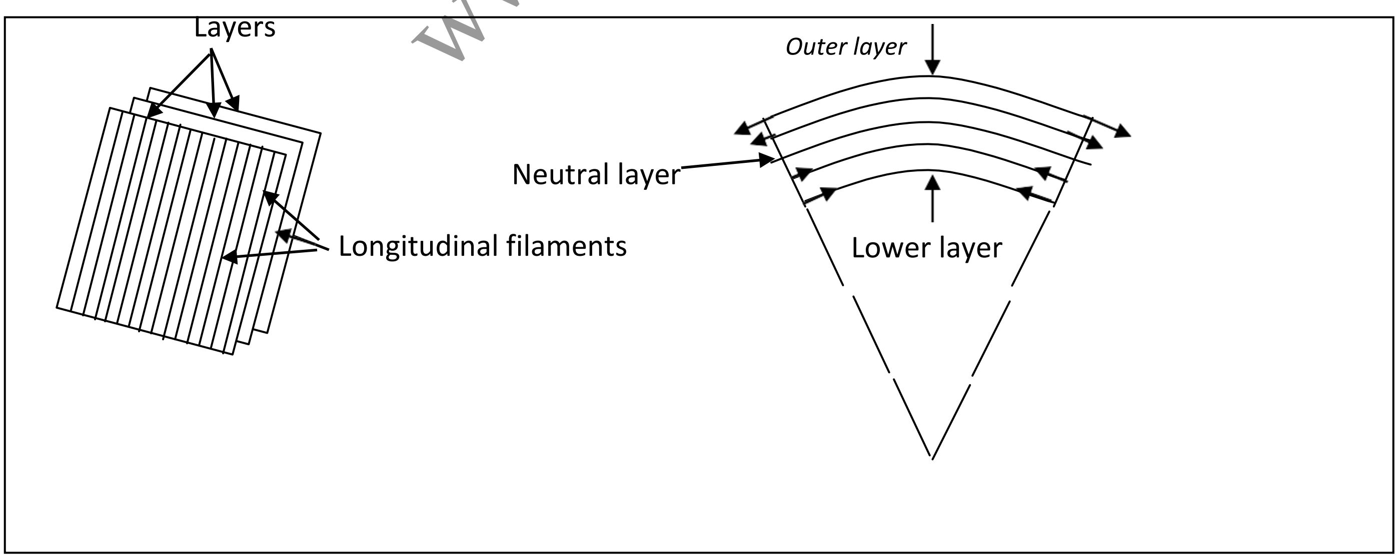
$$K = \frac{1}{3(\alpha-2\beta)} = \frac{1}{3\alpha(1-2\beta/\alpha)}$$

$$\boldsymbol{K} = \frac{\boldsymbol{Y}}{\boldsymbol{3(1-2\sigma)}} \quad \text{or} \because \frac{1}{\alpha} = Y \text{ and } \frac{\beta}{\alpha} = \sigma$$

Bending of beams:

A Beam is defined as a rod or (circular or rectangular) bar of uniform cross section whose length is very large than its thickness.

The beam may be considered to be made up of a large number of thin plane horizontal layers placed one over the other. Each layer in turn consist of a number of thin parallel longitudinal metallic **fibers/filaments** arranged side by side as shown in the diagram.



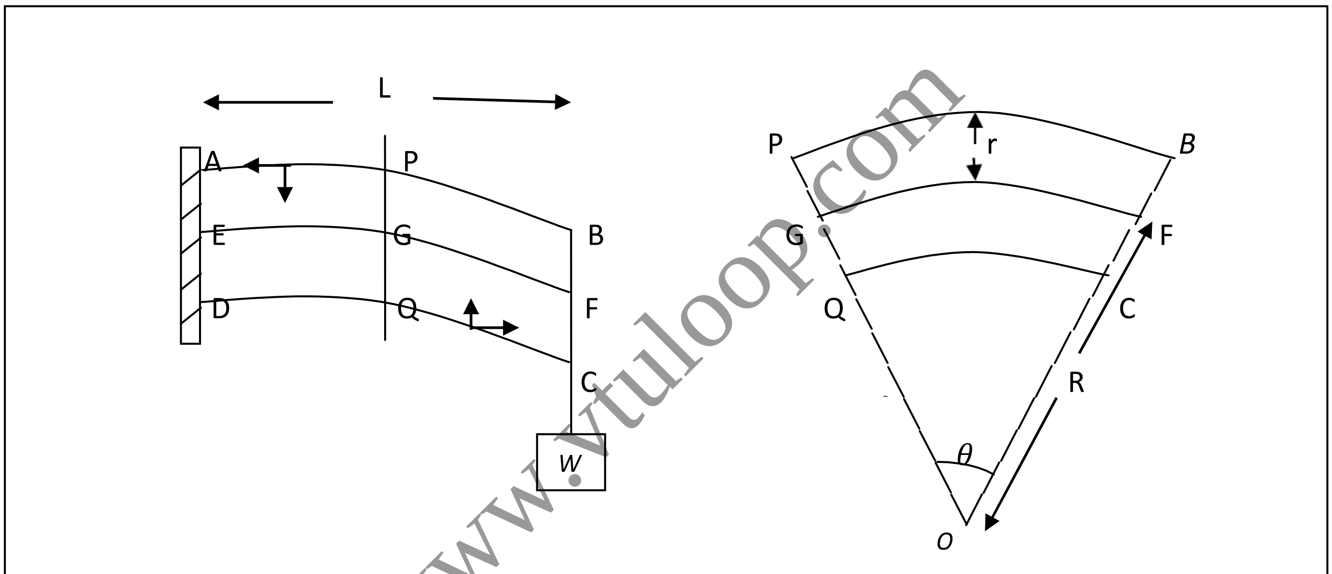
When a straight bar is bent ,the outer layers get elongated and Lower layers get contracted but one layer between them is neither elongated nor contracted, this layer is called **neutral layer/surface.**

Neutral axis is the axis in the cross section of the beam /Shaft along which there is no longitudinal stress or strain.

The elongation or contraction of any layer is proportional to its distance from neutral surface. Hence outer layers are maximum elongated or contracted.

Bending Moment of a Beam is defined as the total moment of the forces acting on the upper and lower layers/surfaces of the beam.

Expression for bending moment in terms of moment of inertia ie; BM $= \frac{Y}{R} I_g$



Consider a beam ABCD of length 'L', fixed at the end AD. When a load 'W' is attached at the end BC. The upper portion AB gets elongated & experience an inward force. The lower portion DC gets contracted & experience an outward force. Whereas the neutral filament EF is neither elongated nor contracted. The beam experiences two opposite couples, as a result the beam comes to rest.

At equilibrium,

Bending moment of the beam = Restoring couple acting on the beam.

Consider small portion of the beam PBCQ which gets curved due to load. Let PB,QC and GF be the outer, inner and neutral filaments respectively.

Let O be the centre and R the radius of curvature of the arc of neutral filament GF and 'r' be the distance of PB from GF.

Then, the length of filament, GF = $R\theta$, θ *is* the angle subtended by GF at O.

The length of the outer filament, PB = $(R + r)\theta$

$\therefore$ Linear strain = $\frac{\text{Increase in length}}{\text{Initial length}} = \frac{(R+r)\theta - R\theta}{R\theta} = \frac{r}{R}$

But , Young's modulus (Y)$=\frac{\text{Linear stress}}{\text{Linear strain}}$

Linear stress = Young's modulus x Linear strain= Y . $\frac{r}{R}$

Also Force = (Linear) Stress x area of cross section= Y . $\frac{r}{R} . a$

Moment of the force $= Force\ x\ distance =$ Y . $\frac{r}{R} a . r = \frac{Y}{R} ar^2$

∴ The total moment of the force acting on the upper and lower surfaces of the beam $= \frac{Y}{R}\sum ar^2$

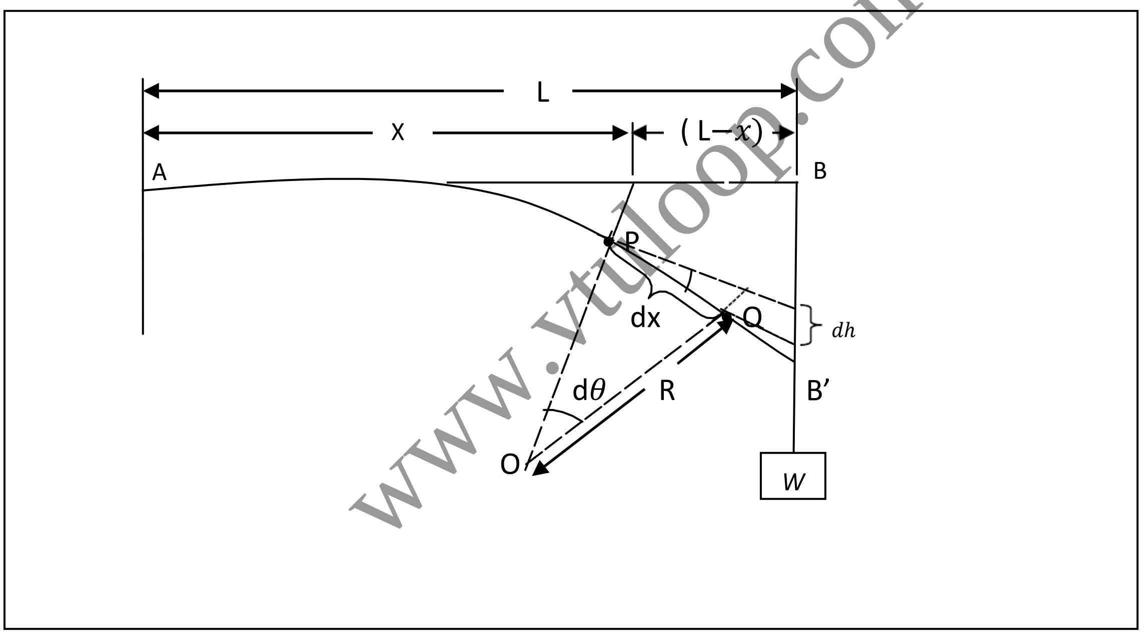
But Bending moment of the beam = {total moment of the force acting on the upper and lower surfaces of the beam.

$= \frac{Y}{R}\sum ar^2$ but $\sum ar^2 = I_g$,geometrical moment of inertia **,Bending moment of the beam** $= \frac{Y}{R} I_g$

Note: Bending moment is equal to restoring couple $= \frac{Y}{R} I_g$

Expression for Young's modulus in case of single cantilever(rectangular beam).

Y$= \frac{MgL^3}{3hI_g}$



Cantilever is a weight less beam whose length is very large compared to its thickness, which is fixed at end and the other end is free.

Consider a cantilever of length 'L' fixed at one end and a load 'W' is attached to the other end .

Let the neutral axis AB gets deflected to AB'. Consider a section P(close to the free end)at a distance 'x' from the fixed end so that PB≈ $PB' = (L - x)$

Then bending moment produced at P = W$(L - x)$ and

Restoring couple acting at P = $\frac{Y}{R} I_g$

At equilibrium, Restoring couple = Bending moment

$$\frac{Y}{R} I_g = W(L-x) \quad .....(1)$$

Consider another section Q close to P, so that the radius of curvatures of sections P&Q are same.

Let 'R' be the radius of curvature and 'dθ' be the angle between the radii of curvatures at P&Q. If 'dx' be the distance between P and Q, then dx =R dθ or

R = $\frac{dx}{d\theta}$ ......(2)

From eqns ,1&2, we get $Y I_g \frac{d\theta}{dx} = W(L-x)$

$$d\theta = \frac{W(L-x)}{Y I_g} dx \quad .....(3)$$

Also $d\theta$ be the angle between the tangents drawn at P and Q respectivelyand 'dh' be the depression between the points P and Q.

Then $d\theta = \frac{dh}{(L-x)}$ ....(4)

From eqns 3&4 ,we get

$$dh = \frac{W(L-x)^2}{Y I_g} dx \quad ....(5)$$

The depression 'h' of the cantilever is obtained by integrating eqn (5) w.r.t 'x'

$$\int dh = \int \frac{W(L-x)^2}{Y I_g} dx$$

$$h = \frac{W}{Y I_g} \int (L^2 + x^2 - 2Lx) dx$$

$$= \frac{W}{Y I_g} \left(L^2 x + \frac{x^3}{3} - 2L\frac{x^2}{2}\right) + C$$

where integration constant C=0 at x=0 & h=0

When L≈ x , $\left(L^2 x + \frac{x^3}{3} - 2L\frac{x^2}{2}\right) = \frac{L^3}{3}$

$\therefore$ h = $\frac{WL^3}{3Y I_g}$ and

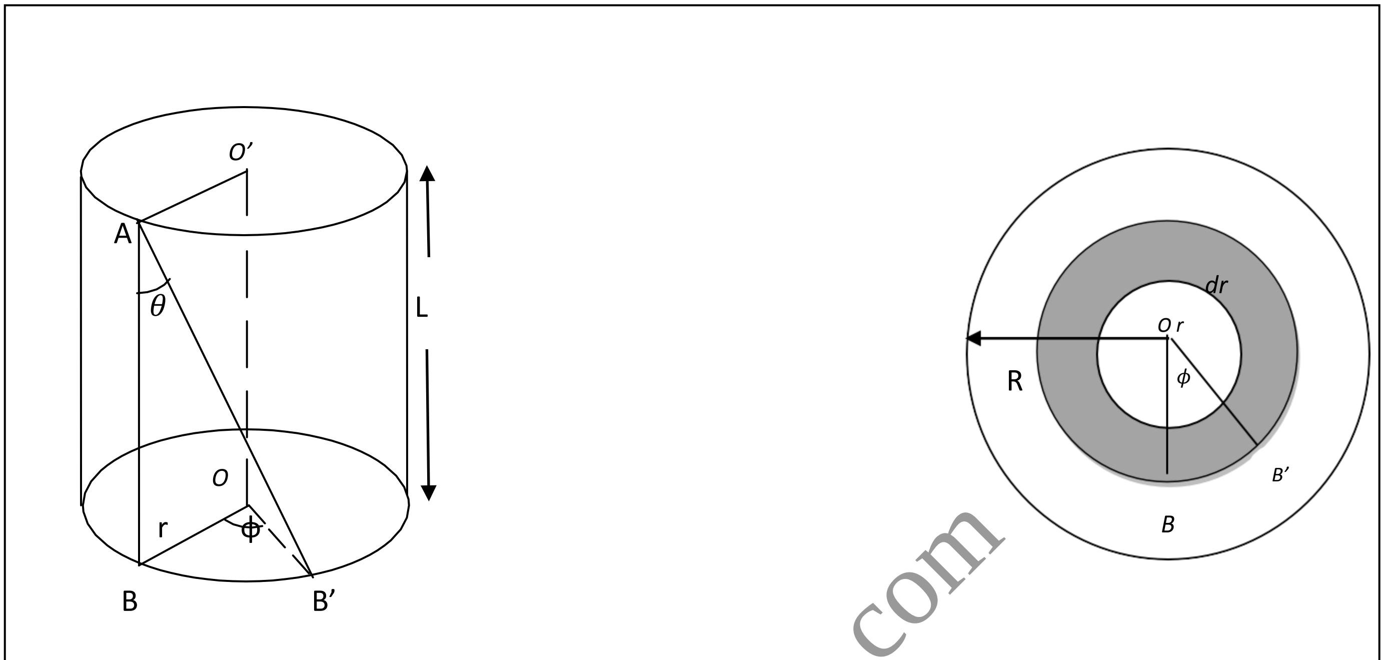
Young's modulus , **Y** = $\boldsymbol{\frac{WL^3}{3h\, I_g}}$

Note: For a rectangular single cantilever of width 'b' and thickness'd',

$$I_g = \frac{bd^3}{12} \text{ and } W = Mg, M = \text{mass and } g = \text{acceleration due to gravity}$$

$$\therefore \; \mathbf{Y} = \boldsymbol{\frac{4MgL^3}{h\, bd^3}}$$

Expression for twisting couple in a wire, C $= \dfrac{n\pi R^4}{2L}$



Consider a cylindrical wire of radius 'R ' and length' L' which is fixed at one end and the other end is twisted.
The wire is imagined to be made up of a large number of concentric hollow cylindrical wires/cylinders of each thickness 'dr'
Let the line AB of the hollow cylindrical wire of radius 'r' and thickness 'dr' gets twisted to AB'. If OO' be the axis of the wire and 'ϕ' be the angle of twisting, then from the sector OBB', arc BB' = $r\phi$ ....(1)
Consider the flat surface AB sheared to AB' due to twisting .

If 'θ' be the shearing angle,then arc BB' = $L\theta$ ...(2)

From eqns 1&2,we get $L\theta = r\phi$

$\therefore$ Shearing strain = $\frac{r\phi}{L}$

But, Shearing stress = Rigidity modulus x Shearing strain = n $\frac{r\phi}{L}$

If 'a' be the area of cross section of the (hollow) wire, then shearing force acting on it is given by , Shearing force = Shearing stress x area of cross section

$= n\frac{r\phi}{L}$a

Also area over which shearing force acts, a = Circumference x thickness = $2\pi r.\,dr$

shearing force acting $= n\frac{r\phi}{L}2\pi r.\,dr$

But, moment of the force $acting on the wire about axis OO' = Force\ x\ distance$

$$= n\frac{r\phi}{L}2\pi r.\,dr\,.\,r$$

$$= n\frac{\phi}{L}2\pi r^3 dr \ ....(4)$$

The total moment of the force acting on the cylindrical wire = $\int_0^R n\frac{\phi}{L}2\pi r^3 dr$

ie; Couple acting on the wire due to torque, C = $\frac{2n\pi\phi}{L}\left[\frac{r^4}{4}\right]_0^R$

$$= \frac{n\pi\varphi R^4}{2L}$$

$\therefore$ Couple per unit twist, c = $\frac{\text{Couple acting on the wire}}{\text{Angle of twisting}}$

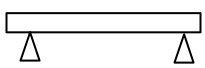
$$c = \frac{\frac{n\pi\phi R^4}{2L}}{\phi}$$

$$\mathbf{c = \frac{n\pi R^4}{2L}}$$

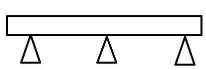
TYPES OF BEAMS:

There are four types of beams namely:-

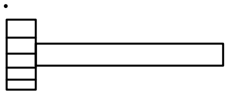
1. Simple beam is a bar resting upon supports at its ends.



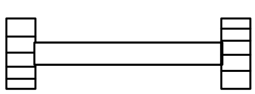
2. Continuous beam is a bar resting upon more than two supports.



3. Cantilever beam is a bar which is fixed at one end and free at the other end.



4. Fixed beam is a bar whose both ends are fixed.



Application of beams :-

Beams are used in:-

1. the fabrication of trolley ways
2. the chassis/frame as truck beds.
3. The elevators.
4. The construction of platform and bridges.
5. The buildings and bridges as GIRDERS.

Torsional Pendulum:

Torsional pendulum is a heavy body suspended from a rigid support with string and the body execute turning oscillations about the wire as axis.

Torsional oscillations are the turning oscillations executed by the torsinal pendulum about the wire as axis.

Time Period of torsional pendulum(T) is the time taken by the torsional pendulum to complete one to and fro turning oscillation and is given by the relation

T= $\boldsymbol{2\pi\sqrt{\frac{I}{c}}}$ where I= Moment of inertia of the body about the wire as axis and c = couple per unit twist for the wire.

Uses of torsional pendulum:

Torsional pendulum is used to find: –

1. the moment of inertia of irregular bodies.
2. The rigidity modulus of the material using the torsional pendulum of the material wire with regular body.

Limitations of Poisson's Ratio:

From the relations Y= $2n(1+\sigma)$ and Y= $3K(1-2\sigma)$

we get 3K(1−2σ) = $2n(1+\sigma)$ ….(1)

Both Positive and negative values are possible for σ.

For $+ve\ \sigma$, (1−2σ) > 0 ,this shows that $\sigma < 0.5$

For $-ve\ \sigma$, (1+σ) > 0 ,this shows that $\sigma > -1$

From the above cases it is clear that the value of σ lies between 0.5 and -1.

Case(1) Substituting $\sigma = 0.5$ in eqn(1), we get K $= \infty$, which is not practically possible.

Case(2) Substituting $\sigma = -1$ in eqn(1) , we get n = ∞, This value for σ is not practically possible.

Poisson derived a relation and showed that the value of $\sigma = 0.25$, but experimental value for σ lies between 0.2 and 0.4

@@end@@