MODULE III

EQUILIBRIUM OF FORCES AND FRICTION

1. Determine the reactions at contact points for spheres A, B and C as shown in Fig. it is given that $W_A = W_B = 4$ kN, $d_A = d_B = 500$mm, $d_C = 800$mm **(Dec2014 /Jan 2015)**

$$cos\theta = \frac{300}{650}$$

$$\theta = cos^{-1}\left(\frac{300}{650}\right)$$

$$\theta = 62.51$$

Using Lamis Theorem

$$\frac{6}{\sin 54.98} = \frac{R_{AC}}{\sin 152.51} = \frac{R_{BC}}{\sin 152.51}$$

$$R_{AC} = R_{BC} = 3.38\,KN$$

$$\sum F_x = 0$$

$$3.38\cos 62.51 - R_L cos\,25 = 0$$

$$R_L = 1.72\,KN$$

$$\sum F_y = 0$$

$$R_k - 4 + 1.72\sin 25 - 3.38\sin 62.51 = 0$$

$$R_k = 6.27\,KN$$

$$\sum F_x = 0$$

$$R_N \cos 15 - 3.38 \cos 62.51 = 0$$

$$R_N = 1.61\ KN$$

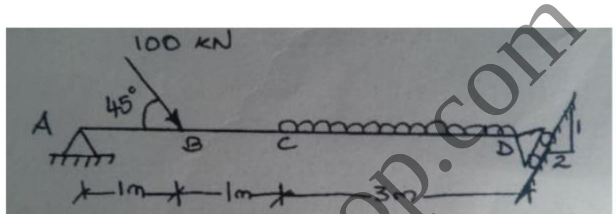
$$\sum F_y = 0$$

$$R_M - 4 + 1.61 \sin 15 - 3.38 \sin 62.51 = 0$$

$$R_M = 6.58\ KN$$

2. For the beam with loading shown in Fig. determine the reactions at the supports

(Dec2014 /Jan 2015)



$$\tan\theta = \frac{1}{2}$$

$$\theta = \tan^{-1}\frac{1}{2}$$

$$\Theta = 26.56$$

Take a moment about point A

$M_A = 100\sin45 + 150 \text{ X } 3.5 - Rd \sin63.43 \text{ X } 5 = 0$

Hence Rd = 133.26kN

$\sum Fx = 0$,

$H_A + 100\cos45 - 133.26 \text{ X } \cos63.43 = 0$

$H_A = -11.10$kN

$\sum Fy = 0$,

$V_A + 100\sin45 - 150 + 133.26 \text{ X } \sin63.43 = 0$

$V_A = 101.52$kN

$R_A = \sqrt{H_A^2 + V_A^2}$

= 102.12kN

$\theta = \tan^{-1}\frac{V_A}{H_A} = 83.76$

3. State and prove Lami‟s theorem

Statement: "If three coplanar forces acting simultaneously at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two forces".

Proof: Fig (a) shows three coplanar forces P, Q and R acting at point O in equilibrium. Let α, β and γ be the corresponding opposite angles then, $(P/\sin\alpha) = (Q/\sin\beta) = (R/\sin\gamma)$

From fig (b) in   OAC,

$AOC = 180 - \beta$

$ACO = BOC = 180 - \alpha$

$CAO = 180 - ACO - AOC$

$= 180 - (180-\alpha) - (180-\beta)$

$= \alpha+\beta-180$ ………………………(1)

We know that $\alpha+\beta+\gamma = 360^{o}$

Subtracting both sides by 180^{o}

$\alpha+\beta+\gamma-180 = 360-180$

$\alpha+\beta-180 = 180-\gamma$ ……………………….(2)

hence From (1) and (2)

$CAO = 180-\gamma$ ………………………..(3)

Apllying sine rule.

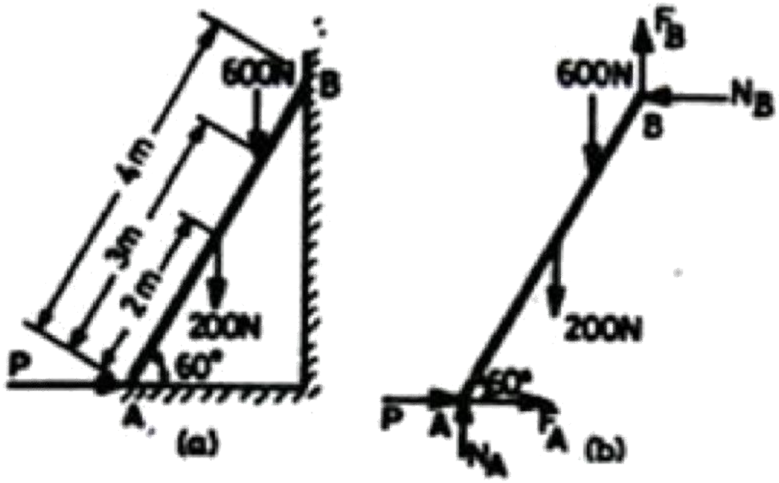
$$\frac{}{sinACO} = \frac{AC}{sinAOC} = \frac{OC}{sinCAO}$$

$$\frac{}{\sin(180-\alpha)} = \frac{Q}{\sin(180-\beta)} = \frac{R}{\sin(180-\gamma)}$$

Or since, $\sin(180-\alpha) = \sin\alpha$

$$\frac{}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

4. A ladder of length 4m weighing 200N is placed against a vertical wall as shown in fig. The coefficient of friction between wall and the ladder is 0.2 and that between the floor and the ladder is 0.3. the ladder in addition to its own weight has to support a man weighing 600N at a distance of 3m from A. Calculate the minimum horizontal force to be applied at A to prevent Slipping. **(Dec2014 /Jan 2015)**



$\varepsilon Fx = 0$

$0.5N_A - N_B = 0$

Hence $N_A = 2\ N_B$

$\varepsilon Fx = 0$

$N_A - 0.25N_B - 200 - 1000 = 0$

Hence $N_A = 1066.67N$

$N_B = 533.33N$

Take a moment about point A $200(2\cos\alpha)+1000(3\cos\alpha)-(N_B \times 4\sin\alpha)-(0.25N_B \times 4\cos\alpha) = 0$

$2866.67\cos\alpha = 2133.32\sin\alpha$

$\tan\alpha = 2866.67/2133.32$

Hence $\alpha = 53.34^o$

5. State laws of friction **(Dec2014 /Jan 2015)**

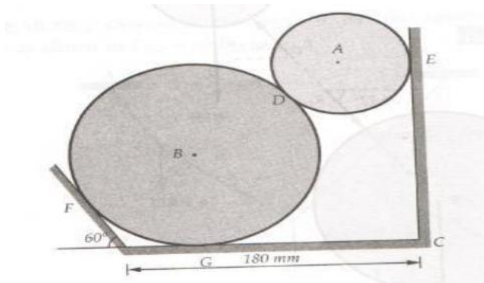
Solution

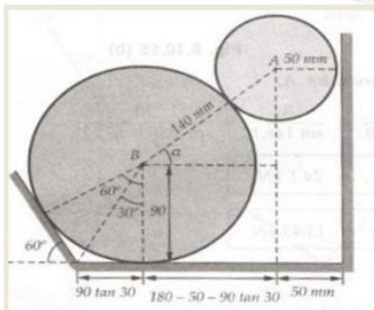
i) Force of friction acts opposite to direction of movement of body ii) Magnitude is equal to applied force that just moves the body iii) Force of friction depends upon roughness/smoothness of the surface

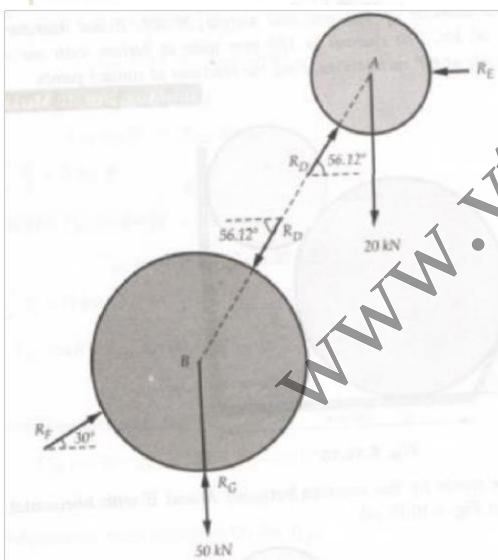
iv) Force of friction is independent of area of contact

v) When body moves dynamic friction comes into play

6. **Two cylinders A and B rest in a channel as shown in fig. A has a diameter of 100mm and weighs 20 kN, B has diameter of 180 mm and weighs 50kN. The channel is 180mm wide at bottom with one side vertical and the other side at 60^0 inclinations. Find the reactions at contact points. (Jul/Aug 2011)**







Using Lami‟s theorem

R_D/ Sin 90 = R_E/ sin 146.12 = 20/ sin(180- 56.12)

R_D=24.1kN
R_E=13.43kN

For B, ∑ Fx = 0 ;

$R_F \cos 30 - R_D \cos 56.12 = 0$

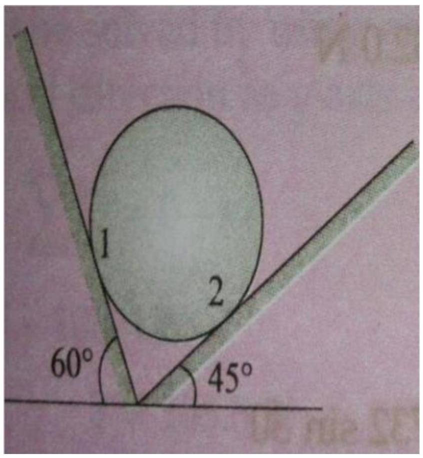
$R_F = 15.51kN$

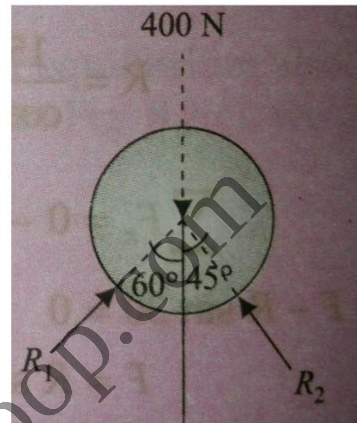
$\sum Fy = 0$

$R_F \sin 30 + R_G – 50 - R_D \sin 56.12 = 0$

$R_G = 62.25kN$

7. A 200 N sphere is resting in at rough as shown in fig. determine the reactions developed at contact surfaces. Assume all contact surfaces are smooth. (June 2012)





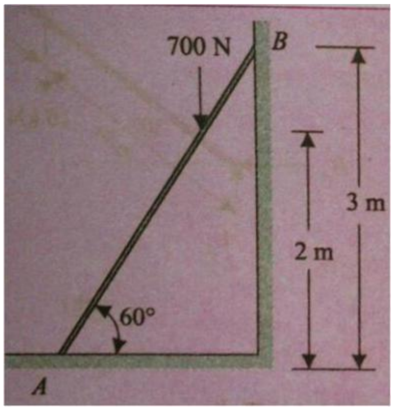
Soln. At contact point 1, the surface contact is making 60^0 to horizontal. Hence the reaction R_1 which is normal to it makes 60^0 with vertical. Similarly the reaction R_2 at contact point 2 makes 45^0 to the vertical. FBD as shown in figure.

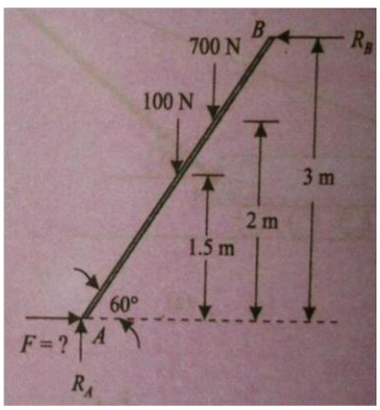
Applying lami"s theorem to the system of forces, we get

$R_1/ \sin (180 – 45) = R_2/ \sin (180 –60)= 400/ \sin (60+45)$

$R_1= 292.8N$ $R_2= 358.6N$

8. A ladder weighing 100N is to be kept in the position shown in figure. Resting on a smooth floor and leaning on a smooth wall. Determine the horizontal force required at floor level to prevent it from slipping when a man weighing 700 N is at 2 m above floor level. (Jan 2013)





Free body diagram of the ladder is as shown in figure. Ra is vertical and Rb is horizontal because the surface of contact is smooth. Self weight of 100N acts through centre point of ladder in vertical direction. Let F be the horizontal force required to be applied to prevent slipping. Then

$\sum M_A = 0$

$-R_B \times 3 + 700 \times 2 \cot 60 + 100 \times 1.5 \cot 60 = 0$

$R_B = 298.3N$

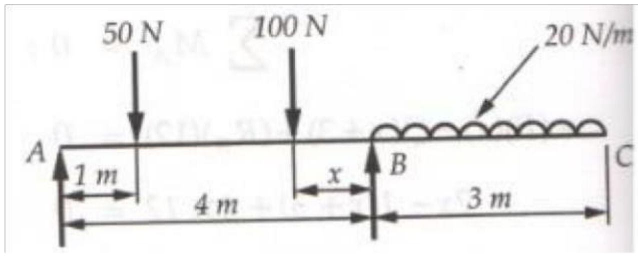
$\sum F_x = 0$

$F - R_B = 0$

$F - R_B = 298.3N$

9. **Determine the position of 10 N load on the beam such that reactions at the supports are equal for the beam loaded as shown in fig.**

(Jan/Feb 2012)



Using $\sum Fy = 0$;

$R_A + R_B – 50- 100- 20x\ 3= 0$

$R_A+ R_B= 210$

As $R_A=R_B$

$2R_A= 210$

$R_A= 105$ N

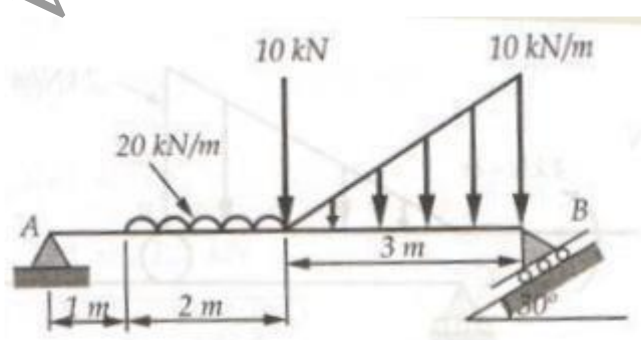
$\sum M_B=0$:

$100\ x + 50x3 - R_A\ x\ 4 – 20x3\ x\ 1.5= 0$ X

= 3.6m

10. **Determine the reactions at the supports for the beam loaded as shown in fig**.

(Jan/Feb 2012)



The Free body diagram of beam is shown in figure

$\sum M_A=0$:

$-40x2 - 10x3 - 15x5 + R_B \sin 60x6 = 0$

$R_B = 35.6kN, 60^0$

$\sum Fx = 0$

$A_x - R_B \cos 60 = 0$
$A_x = 17.8kN$

$\sum Fy = 0$

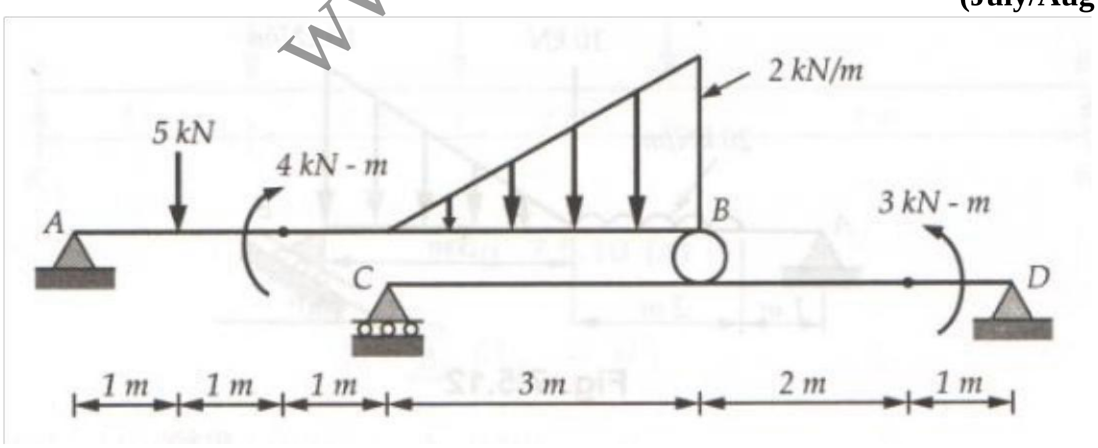
$A_y - 40 - 10 - 15 + R_B \sin 60 = 0$
$A_y = 34.17kN$

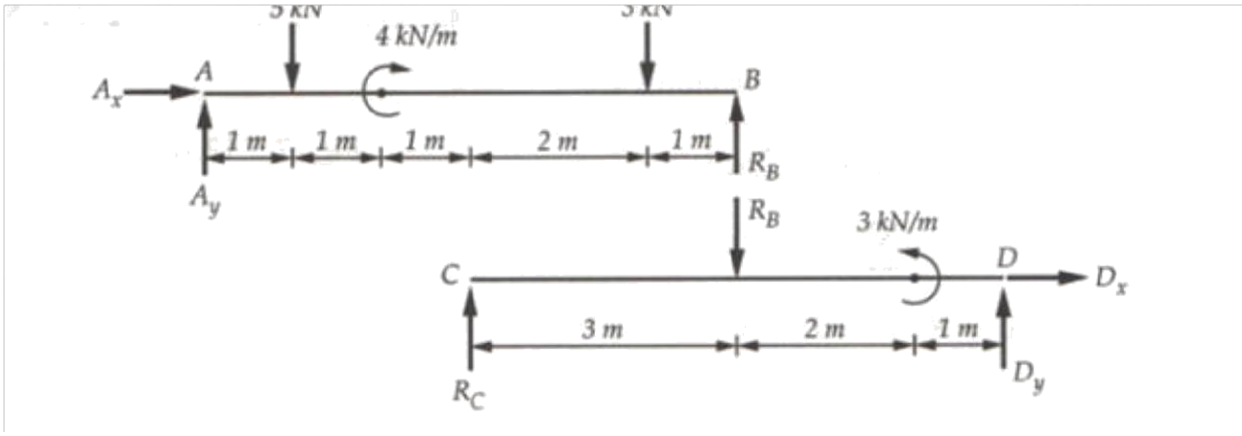
$R_A = \sqrt{A_{x2} + A_{y2}}$
$R_A = 38.53kN$

$\theta = \tan^{-1} (34.17/ 17.8)$
$\theta = 62.48^0$

11. Determine the reactions at the ends of the beam AB and CD as shown in fig. Neglect the self weight of the beams.

(July/Aug 2012)





For AB, $\sum Fx = 0$

$Ax = 0$

$\sum M_A = 0$

$-5x1 - 4 - 3x5 + R_B x6 = 0$

$R_B = 4kN$

$\sum F_y = 0$

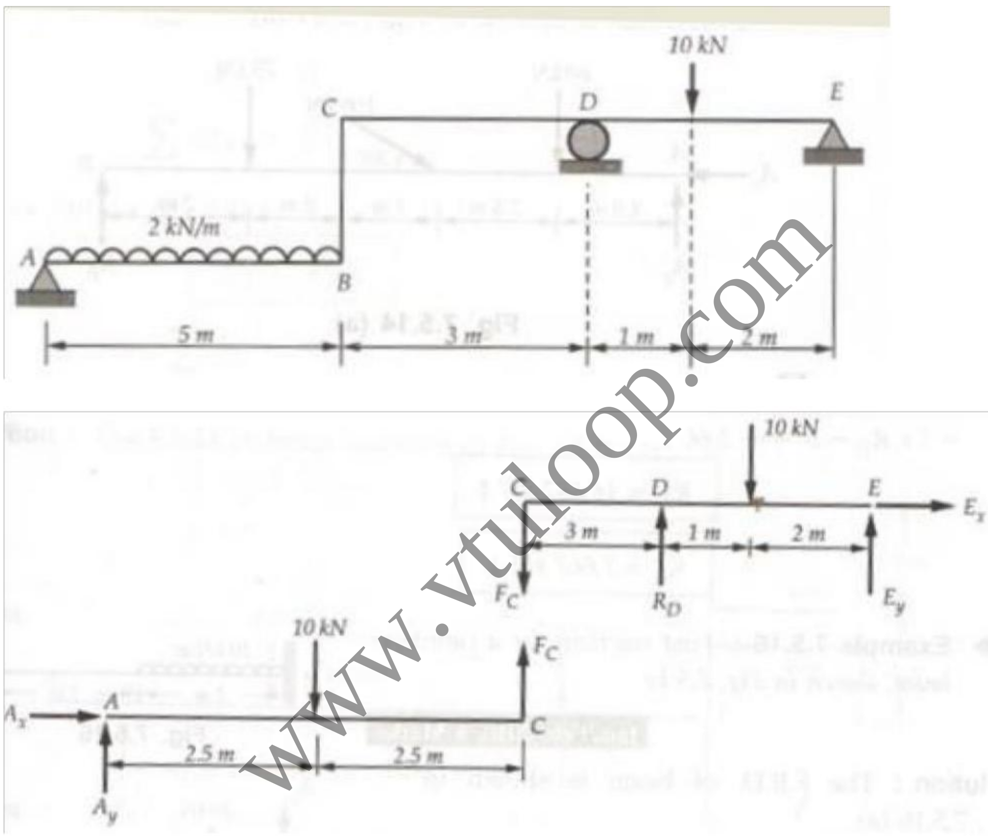
$\sum M_c = 0$

$\sum F_y = 0$

$R_C - R_B + D_y = 0$
$R_C = 2.5kN$

12. A beam ABCDE has a flexible link as shown in fig. determine the support reaction at A,D and E.

(July/Aug 2013)



For AC, $\sum M_A = 0$

$-\quad 10\text{x}2.5 + F_C\text{x } 5 = 0$

$F_C = 5kN$

$\sum F_x = 0$

$A_x = 0$

$\sum F_y = 0$

$A_y – 10 + F_c = 0$

$A_y = 5kN$

$R_A = 5kN$

For CDE,

$\sum M_D = 0$

$\sum F_x = 0$

$E_x = 0$

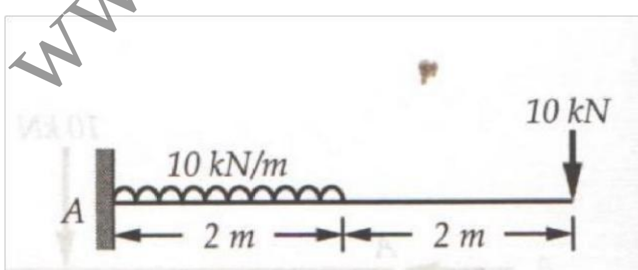
$\sum F_y = 0$

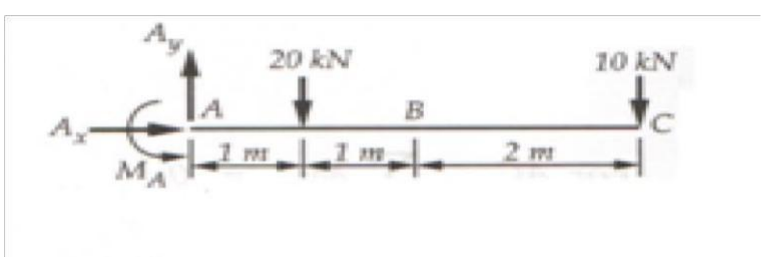
$- F_c + R_D – 10 + E_y = 0$

$-5 + R_D – 10 + (- 1.667) = 0$

$R_D = 16.667kN$, $R_e = 1.667kN$

13. Find reactions for a cantilever beam shown in the figure. (June 2014)





The free body diagram of beam is shown in

$\sum F_x = 0$

$A_x = 0$

$\sum F_y = 0$

$A_y - 20 - 10 = 0$

$A_y = 30kN$

$R_A = 30kN$

$\sum M_A = 0$

$M_A - 20x1 - 10x4 = 0$

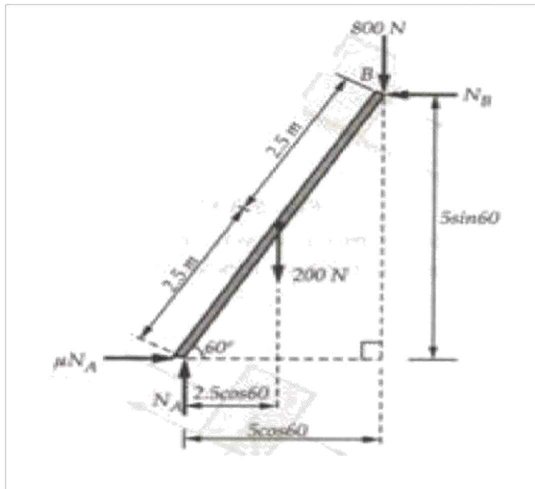
$M_A = 60kNm$

14. Explain Different types of supports? **(Jan, July2013)**

Supports are structures which prevent the beam or the body from moving and help to maintain equilibrium. A beam can have different types of supports as follows. The support reactions developed at each support are represented as follows.

1) **Simple support:** This is a support where a beam rests freely on a support. The beam is free to move only horizontally and also can rotate about the support. In such a support one reaction, which is perpendicular to the plane of support, is developed.
2) **Roller support:** This is a support in which a beam rests on rollers, which are frictionless. At such a support, the beam is free to move horizontally and as well rotate about the support. Here one reaction which is perpendicular to the plane of rollers is developed.
3) **Fixed support:** This is a support which prevents the beam from moving in any direction and also prevents rotation of the beam. In such a support a horizontal reaction, vertical reaction and a Fixed End Moment are developed to keep the beam in equilibrium.

15. A ladder 5m in length is resting against a smooth vertical wall and a rough horizontal floor. The ladder makes an angle of 60^0 with the horizontal. When a man of weight 800N is at the top of the rung, what is the coefficient of friction required at the bottom o the ladder and the floor such that the ladder does not slip? Take the weight of the ladder as 200N. (Jan/Feb 2012)



$\sum M_A = 0$

$-200 \times 2.5\cos 60 - 800 \times 5\cos 60 + N_B \times 5\sin 60 = 0$

$N_B = 519.615N$

$\sum F_y = 0$

$N_A - 200 - 800 = 0$

$N_A = 1000N$

$\sum F_x = 0$

$\mu N_A - N_B = 0$

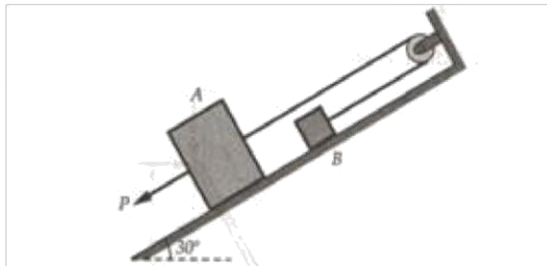
$\mu = N_B / N_A$

$= 519.615 / 1000$

$\mu = 0.52$

16. Determine the force P required to cause motion of blocks to impend. Take the weight of A as 90N and weight of as 45 N. Take the coefficient of friction for all contract surfaces as 0.25 as in figure. Consider the pulley being frictionless.

(Jan/Feb 2012)



As block A tends to move down the incline B tends to move up the incline . The free body diagrams of the two blocks are shown in figure. For FBD of B,

$\sum F_y = 0$

$N_B - 45\cos 30 = 0$

$N_B = 45\cos 30$

$\sum F_X = 0$

$T - 45 \sin 30 - 0.25\ N_B = 0$

$T = 32.243N$

For FBD of A,

$\sum F_y = 0$

$N_A - 90\cos 30 = 0$

$N_A = 90\cos 30$

$\sum F_X = 0$

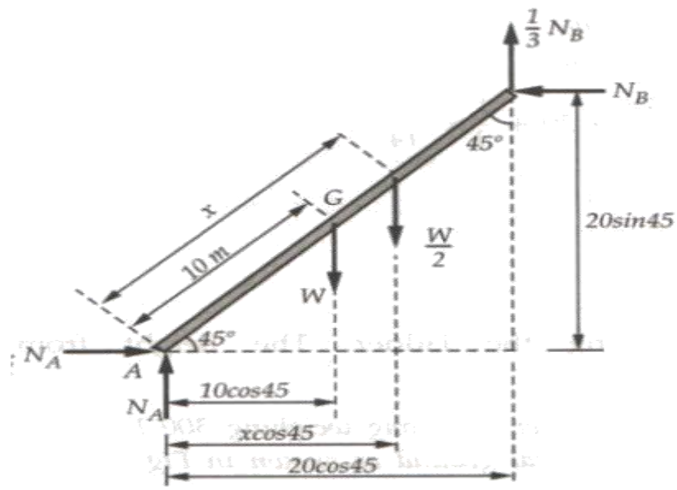
$T - P - 90\cos 30 + 0.25\ N_A = 0$

$P = T + 0.25\ N_A - 90\sin 30$

$P = 6.73N$

17. A uniform ladder of length 20m, rests against a vertical wall which it makes an angle of 45^0, the coefficient of friction between the ladder and the wall and ground respectively being 1/3 and ½. If a man, whose weight is one half of the ladder, ascends the ladder, how high will he be, when the ladder slips?

(July/Aug 2012)



$\sum F_X = 0$

½ $N_A - N_B = 0$

$\sum F_y = 0$

N_A – W- W/2 + 1/3 N_B = 0

7/3 N_B = 3/2 W

N_B = 9/14W

$\sum M_A = 0$

- W x 10cos45- (W/2)x cos 45+ N_B 20sin45+ 1/3 N_B 20cos45 = 0 As sin45 = cos45= $1/\sqrt{2}$

  Substituting N_B = 9/14W
  x= 14.29m
  This distance is along the ladder. The height from the floor will be 14.9sin45= 10.1m

18. State the laws of static friction. **(June/July 2013, 14)**

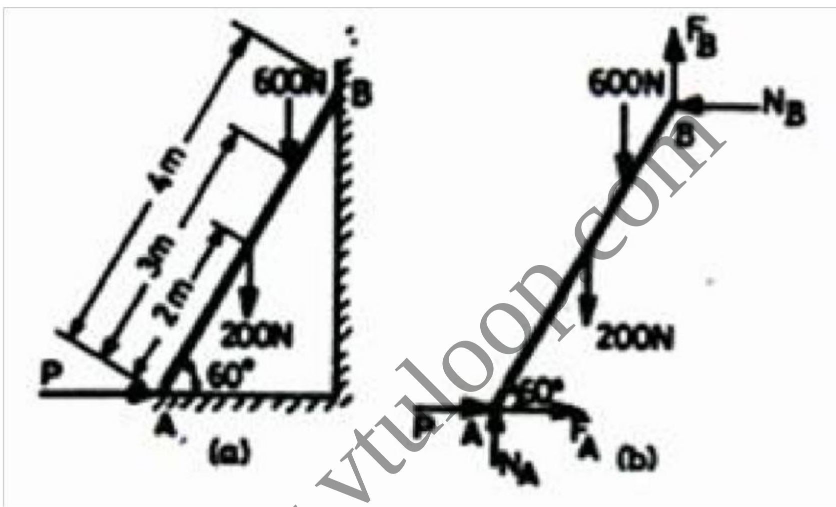
The frictional resistance developed between bodies having dry surfaces of contact obeys certain laws called laws of dry friction. They are as follows.

1) The frictional resistance depends upon the roughness or smoothness of the surface.
2) Frictional resistance acts in a direction opposite to the motion of the body.

3) The frictional resistance is independent of the area of contact between the two bodies.
4) The ratio of the limiting friction value (F) to the normal reaction (N) is a constant (coefficient of friction, μ)
5) The magnitude of the frictional resistance developed is exactly equal to the applied force till limiting friction value is reached or where the bodies is about to move.

19. A ladder of length 4m weighing 200N is placed against a vertical wall as shown in fig. The coefficient of friction between wall and the ladder is 0.2 and that between the floor and the ladder is 0.3. the ladder in addition to its own weight has to support a man weighing 600N at a distance of 3m from A. Calculate the minimum horizontal force to

be applied at A to prevent Slipping. **(June/July 2013, June 2012)**



The free body diagram of the ladder is as shown in Fig.

$$\Sigma M_A = 0$$

$$N_B\, 4 \sin 60° + F_B\, 4 \cos 60° - 600 \times 3 \cos 60° - 200 \times 2 \cos 60° = 0$$

Dividing throughout by 4 and rearranging,

$$N_B\, 0.866 + 0.5\, F_B = 275$$

From the law of friction, $F_B = 0.2\, N_B$

$$\therefore \quad N_B\,(0.866 + 0.5 \times 0.2) = 275$$

$$N_B = 284.68 \text{ N}$$

$$\therefore \quad F_B = 56.934 \text{ N}$$

$$\Sigma V = 0$$
$$N_A - 200 - 600 + 56.934 = 0$$
$$N_A = 743.066 \text{ N}$$
$$\therefore F_A = 0.3 N_A$$
$$\therefore F_A = 222.92 \text{ N}$$
$$\Sigma H = 0$$
$$P + F_A - N_B = 0$$
$$P = N_B - F_A = 284.68 - 222.92$$
$$\mathbf{P = 61.76\ N} \qquad \textit{Ans.}$$

20. A block weighing 800 N rests on an inclined plane at 12^o to the horizontal. If the coefficient of friction is 0.4, find the force required to pull the body up the plane, when the line of the force is (i) parallel to the plane (ii) horizontal (June 2014)

Sol: $N_1 = 800 \text{Sin } 78^o$
$= 782.52$ N

$P_1 - 0.4N_1 = 800 \text{ Cos } 78^o$
$P_1 = 479.34$ N

$P_2 \text{Cos } 12^o - 0.4N_2 = 800 \text{ Cos} 78^o$
$-P_2 \text{ Sin } 12^o + N_2 = 800 \text{ Sin} 78^o$
$P_2 = 535.69$

Stone (2)

$U = 25$ m/sec, $h=h^2$, $g= -9.81$ m/sec^2

$h_2 = 25t - 0.5 * 9.81\, t_2$ $h_2 = 25t -$

$4.905\, t_2$

Substituting $t=25$, $h_1 = 19.6$ m

4. What is projectile? Define the following terms briefly)Angle of projection ii) Horizontal range iii) Vertical height iv) Time of fight (Dec2014 /Jan 2015)

Solution.

Angle of projection (α): It is the angle with which the projectile is projected with respect to horizontal.

Time of flight (T): It is the total time required for the projectile to travel from the point of projection to the point of target.

Horizontal range (R): It is the horizontal distance between the point of projection and target point.

Vertical height (h): It is the vertical distance/height reached by the projectile from the point of projection.

5. A burglar's car starts at an acceleration of 2m/s2. A police vigilant party came after 5s and continued to chase the burgler's car with a uniform velocity of 20m/s. find the time taken in which the police van will overtake the car. (Dec2014 /Jan 2015)

Solution.

For burglar"s car $u=o$, $a=2$m/sec^2

Let t= time taken by police van and overtaking burglar"s car

As the police van came after 5sec , hence burglar"s car will be in motion for (t+5)sec uniform velocity of police van = 20 m/sec.

When police van will overtake burglar"s car , then distance travelled by both vans will be same.

Therefore distance travelled by police van = uniform velocity *t

$S=20t$

The distance travelled by burglar‟s car in (t+5)sec is

$$S= ut +0.5\ at^2$$
$$S= 0+(t+5)^2$$

Then equating both distance i.e

$$20t=(t+5)^2$$
$$20t= t2+25t +10t$$

$(t-5)^2=0$

t=5sec