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Class Assignment - Maths

Find Z-transform:-

① 2^K

$$Z[\{f(K)\}] = F(z) = \sum_{-\infty}^{\infty} f(K) z^{-K}$$

$$= \sum_{0}^{\infty} 2^K \cdot z^{-K}$$

$$= \sum_{0}^{\infty} \frac{2^K}{z^K}$$

$$= \sum_{0}^{\infty} \left(\frac{2}{z}\right)^K$$

Series:- $1 + \left(\frac{2}{z}\right) + \left(\frac{2}{z}\right)^2 + \left(\frac{2}{z}\right)^3 + \cdots$

$a = 1, \quad r = \frac{2}{z}$

By geometric series sum $= S = \frac{a}{1-r}$

$$= \frac{1}{1-\frac{2}{z}} = \boxed{\frac{z}{z-2}}$$

② $\sin 2K$

$$Z[\{\sin 2K\}] = \sum_{0}^{\infty} \sin 2K \cdot z^{-K}$$

$$\sin(2K) = \frac{e^{j2K} - e^{-j2K}}{2j}$$

$$= \frac{1}{2j}\left(e^{j2K} - e^{-j2K}\right)$$

$$= \frac{1}{2j}\left[\sum_{0}^{\infty} e^{j2K} z^{-K} - \sum_{0}^{\infty} e^{-j2K} z^{-K}\right]$$

$\because e^{j2K} = (e^{j2})^K, \quad e^{-j2K} = (e^{-j2})^K$

$$= \frac{1}{2j}\left[\sum_{0}^{\infty}\left(\frac{e^{j2}}{z}\right)^{K} - \sum_{0}^{\infty}\left(\frac{e^{-j2}}{z}\right)^{K}\right]$$

① $\sum_{0}^{\infty}\left(\frac{e^{j2}}{z}\right)^{K} = \left[1 + \frac{e^{j2}}{z} + \left(\frac{e^{j2}}{z}\right)^{2} + \left(\frac{e^{j2}}{z}\right)^{3} \cdots\right]$

② $\sum_{0}^{\infty}\left(\frac{e^{-j2}}{z}\right)^{K} = \left[1 + \left(\frac{e^{-j2}}{z}\right) + \left(\frac{e^{-j2}}{z}\right)^{2} + \cdots\right]$

For ① $a_1 = 1$, $r_1 = \frac{e^{j2}}{z}$

For ② $a_1 = 1$, $r_2 = \frac{e^{-j2}}{z}$

So by geometric sum, $S = \frac{a}{1-r}$

$$= \left[\frac{a_1}{1-r_1}\right] - \left[\frac{a_2}{1-r_2}\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1-\frac{e^{j2}}{z}} - \frac{1}{1-\frac{e^{-j2}}{z}}\right]$$

$$= \frac{1}{2j}\left[\frac{z}{z-e^{j2}} - \frac{z}{z-e^{-j2}}\right]$$

$$= \frac{z}{2j}\left[\frac{1}{z-e^{j2}} - \frac{1}{z-e^{-j2}}\right]$$

$\because \frac{1}{z-e^{j\theta}} - \frac{1}{z-e^{-j\theta}} = \frac{(e^{-j\theta} - e^{j\theta})}{(z-e^{j\theta})(z-e^{-j\theta})} = \frac{-2j\sin\theta}{z^2 - 2z\cos\theta + 1}$

$$= \frac{z}{2\cancel{j}}\left(\frac{-2\cancel{j}\sin 2}{z^2 - 2z\cos 2 + 1}\right)$$

$$\boxed{= \frac{z\sin 2}{z^2 - 2z\cos 2 + 1}}$$

③ $c^k \sin\alpha k$

$$Z[c^k \sin\alpha k] = \sum_0^\infty c^k \sin(\alpha k)\cdot z^{-k}$$

$$= \sum_0^\infty (cz^{-1})^k \sin(\alpha k)$$

$$\because \sin\alpha k = \frac{e^{j\alpha k} - e^{-j\alpha k}}{2j}$$

$$= \frac{c^k(e^{j\alpha k} - e^{-j\alpha k})}{2j} = \frac{1}{2j}(c^k e^{j\alpha k} - c^k e^{-j\alpha k})$$

$$= \frac{1}{2j}((ce^{j\alpha})^k - (ce^{-j\alpha})^k)$$

$$= \frac{1}{2j}\left[\sum_0^\infty (ce^{j\alpha})^k z^{-k} - \sum_0^\infty (ce^{-j\alpha})^k z^{-k}\right]$$

$$= \frac{1}{2j}\left[\sum_0^\infty \left(\frac{ce^{j\alpha}}{z}\right)^k - \sum_0^\infty \left(\frac{ce^{-j\alpha}}{z}\right)^k\right]$$

① $\sum_0^\infty \left(\frac{ce^{j\alpha}}{z}\right)^k = \left[1 + \frac{ce^{j\alpha}}{z} + \left(\frac{ce^{j\alpha}}{z}\right)^2 + \cdots\right]$

② $\sum_0^\infty \left(\frac{ce^{-j\alpha}}{z}\right)^k = \left[1 + \frac{ce^{-j\alpha}}{z} + \left(\frac{ce^{-j\alpha}}{z}\right)^2 + \cdots\right]$

$a_1 = 1$, $r_1 = \frac{ce^{j\alpha}}{k}$

$a_2 = 1$, $r_2 = \frac{ce^{-j\alpha}}{k}$

By Geometric Sum,

$$\frac{1}{z - ce^{j\alpha}} - \frac{1}{z - ce^{-j\alpha}} = \frac{ce^{-j\alpha} - ce^{j\alpha}}{(z - ce^{j\alpha})(z - ce^{-j\alpha})}$$

$$= \frac{-2jc\sin\alpha}{z^2 - 2cz\cos\alpha + c^2}$$

$$X(z) = \frac{z}{2j}\left(\frac{-2jc\sin\alpha}{z^2 - 2cz\cos\alpha + c^2}\right)$$

$$\boxed{= \frac{cz\sin\alpha}{z^2 - 2cz\cos\alpha + c^2}}$$

(4) $\cos\alpha K$

$$Z[\cos\alpha k] = \sum_{0}^{\infty} \cos(\alpha k) \cdot z^{-k}$$

$$\because \cos\alpha K = \frac{e^{j\alpha K} + e^{-j\alpha K}}{2}$$

$$= \sum_{0}^{\infty} \left(\frac{e^{j\alpha K} + e^{-j\alpha K}}{2}\right) z^{-K}$$

$$= \frac{1}{2} \sum_{0}^{\infty} \left(e^{j\alpha K} + e^{-j\alpha k}\right) z^{-K}$$

$$= \frac{1}{2}\left[\sum_{0}^{\infty} \left(\frac{e^{j\alpha}}{z}\right)^{K} + \sum_{0}^{\infty} \left(\frac{e^{-j\alpha}}{z}\right)^{K}\right]$$

(1) $\sum_{0}^{\infty} \left(\frac{e^{j\alpha}}{z}\right)^{K} = 1 + \frac{e^{i\alpha}}{z} + \left(\frac{e^{i\alpha}}{z}\right)^{2} + \cdots$

(2) $\sum_{0}^{\infty} \left(\frac{e^{-j\alpha}}{z}\right)^{K} = 1 + \frac{e^{-j\alpha}}{z} + \left(\frac{e^{-j\alpha}}{z}\right)^{2} + \cdots$

$a_1 = 1$, $r_1 = \frac{e^{j\alpha}}{z}$

$a_2 = 1$, $r_2 = \frac{e^{-j\alpha}}{z}$

By geometric sum, $S = \frac{a}{1-r}$

$$= \frac{1}{2}\left[\frac{1}{1 - \frac{e^{j\alpha}}{z}} + \frac{1}{1 - \frac{e^{-j\alpha}}{z}}\right]$$

$$= \frac{1}{2}\left[\frac{z}{z - e^{j\alpha}} + \frac{z}{1 - e^{-j\alpha}}\right]$$

$$= \frac{z}{2}\left[\frac{1}{z - e^{j\alpha}} + \frac{1}{1 - e^{-j\alpha}}\right]$$

$$= \frac{z}{2}\left(\frac{2(z - \cos\alpha)}{z^2 - 2z\cos\alpha + 1}\right)$$

$$\boxed{= \frac{z(z - \cos\alpha)}{z^2 - 2z\cos\alpha + 1}}$$