CS 561 - Artificial Intelligence

Assignment 1

Abhrajyoti Kundu 234101003
Gorachand Mondal 234101015

Exercise 2: The Metropolis-Hastings Algorithm.

Introduction:

This exercise centres on the implementation and analysis of the Metropolis-Hastings algorithm. This algorithm is a fundamental tool within the Markov Chain Monte Carlo (MCMC) framework, employed for sampling from complex probability distributions. The primary focus will be on demonstrating the impact of varying proposal distribution variances (σ) on the efficiency and overall behaviour of the sampling process.

Target Distribution:

The target probability distribution is denoted by P(x). We aim to generate samples from this distribution.The mathematical expression for P(x) is proportional to $e^{-x^4} * (2 + sin(5x) + sin(-2x^2))$. It is noteworthy that P(x) exhibits a complex, multimodal structure. As a consequence, the Metropolis-Hastings algorithm emerges as a highly advantageous approach for this task. A function, target_distribution(x), has been implemented to efficiently evaluate the value of P(x) for any given input value x.

Proposal Distribution:

The algorithm utilizes a normal distribution, $Q(x'|x)$, centred at the current sample x with a variance of σ^2 for proposing new samples. This choice is favoured due to the simplicity and efficiency of sampling from the normal distribution. The function *proposal_distribution(x, σ)* is employed to generate a new proposed state x' from the current state x.

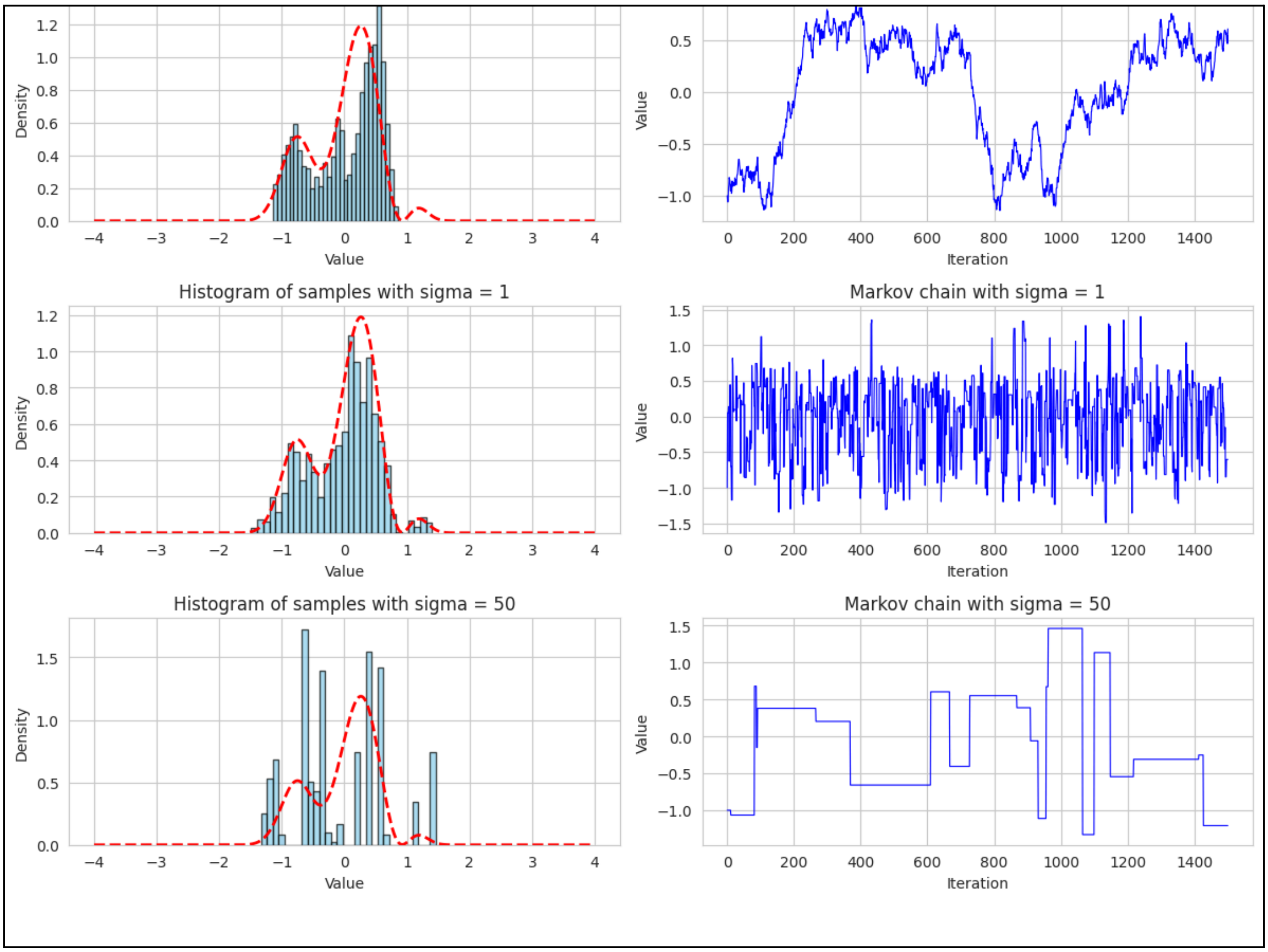
The Metropolis-Hastings Algorithm:

1. Start from an initial state: Begin with an initial state *initial_value*.
2. Propose a new state: Generate a candidate state $x*$ using the proposal distribution $Q(x'|x)$.
3. Calculate acceptance: Compute the acceptance ratio as *min(1,* $P(x*)/P(x)$*)*.
4. Move to the new state: Transition to $x*$ with probability equal to the acceptance ratio.
5. Repeat: Iterate for a specified number of steps to generate a Markov chain.

This generates a sequence of samples approximating the target distribution $P(x)$.

Observations:

- A low value of σ results in a slow exploration of the distribution, where consecutive samples are closely related.
- Conversely, a high value of σ encourages rapid jumps across the distribution, potentially leading to a higher rejection rate but aiding in overcoming local modes.
- An optimal σ strikes a balance, allowing for efficient exploration of the distribution while maintaining a reasonable acceptance rate.



Conclusion :

The Metropolis-Hastings algorithm's flexibility and efficiency are demonstrated through its ability to approximate complex distributions by adjusting the proposal distribution's variance. This analysis underscores the importance of tuning σ to optimize sampling behaviour, a critical consideration for practitioners using MCMC methods in diverse fields.