DISCRETE MATHEMATICS (GATE 2023) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(33) | CORRECT(26) | INCORRECT(4) | SKIPPED(3)

Q. 1 Have any Doubt ?

The number of generators of cyclic group of order 219 is ______.

144 — Your answer is Correct144

Solution :
144
The number of generators of a cyclic group of order n is equal to the number of integers between 1 and n that are relatively prime to n. Namely, the number of generators is equal to φ(n), where is the Euler totient function. We know that G is a cyclic group of order 219. Hence, the number of generators of G is Φ(219) = Φ(3) Φ(73) = 2 × 72 = 144.

QUESTION ANALYTICS +

Q. 2 Have any Doubt ?

If a is an integer lying in [−5, 30], find the probability that the graph $y = x^2 + 2(a+4)x - 5a + 64$ is strictly above x-axis is P. Then the value of $\frac{2}{P}$ is ______.

9 — Correct Option

Solution :
9

$$x^2 + 2(a+4)x - 5a + 64 \geq 0$$

If $D \leq 0$

Then, $(a+4)^2 = [-5a + 64] < 0$

or $a^2 + 13a - 48 < 0$

or $(a+16)(a-3) < 0$

or $-5 \leq a < 2$

Then favourable cases : 8 and total case : 36.

$$P = \frac{2}{9}$$

Hence, $\frac{2}{P} = \frac{2}{2} \times 9 = 9$

QUESTION ANALYTICS +

Q. 3 Have any Doubt ?

How many nonnegative integers less than or equal to 300 are coprime with 144?

A. 300

B. 150

C. 100 — Your answer is Correct

Solution :
(c)
144 has a prime factorization of all 2's and 3's.
So, by inclusion-exclusion, the answer is 300 – (Number divisible by 2) – (Number divisible by 3) + (Number divisible by 2 and 3). Of course, the last is the same as the number divisible by 6.

Since 300 is divisible by 2, 3 and 6, the formula is $300 - \frac{300}{2} - \frac{300}{3} + \frac{300}{6} = 300 - 150 - 100 + 50 = 100$

D. 50

QUESTION ANALYTICS +

Q. 4

Have any Doubt ?

Which of the following statement(s) is/are correct for a graph G on a finite set of n vertices?

A. If every vertex of G has degree 2, then G contains a cycle. — Your option is Correct

B. If G is disconnected, then its complement is connected — Your option is Correct

C. If T is a non-cyclic tour in G, and no strictly longer tour in G contains T, then both endpoints of T have odd degree. — Your option is Correct

D. If every vertex of G has degree 2, then G may not contains a cycle.

YOUR ANSWER - a,b,c

CORRECT ANSWER - a,b,c

STATUS -

Solution :

(a, b, c)

1. **True.** Assume, for contradiction, that G has no cycle, and consider the longest path P in G (one must exist, since the graph is finite). Let v be the final vertex in P — since v has degree 2, it must have two edges e_1 and e_2 incident on it, of which one, say e_1, is the last edge of the path P. Then e_2 cannot be incident on any other vertex of P since that would create a cycle (v, e_2, [section of P ending in e_1], v). So e_2 and its other endpoint are not part of P, and can be appended to P to give a strictly longer path, which contradicts our choice of P. Hence G must contain a cycle.
2. **True.** Let G' denote the complement of G. Consider any two vertices u, v in G. If u and v are in different connected components in G, then no edge of G connects them, so there will be an edge $\{u, v\}$ in G'. If u and v are in the same connected component in G, then consider any vertex w in a different connected component (since G is disconnected, there must be at least one other connected component). By our first argument, the edges $\{u, w\}$ and $\{v, w\}$ exist in G', so u and v are connected by the path (u, w, v). Hence any two vertices are connected in G', so the whole graph is connected.
3. **True.** At each vertex of the tour T, label the incident edges as "entering" or "leaving" as in the lecture notes. At each endpoint of the tour, there is one leaving edge for every entering edge, except for exactly one edge (since the tour is non-cyclic) which is either leaving (if the vertex is the first one of the tour) or entering (if the vertex is the last one). This means an odd number of edges of the tour are incident at each endpoint. Assume, for contradiction, that an endpoint has even degree — since the tour contributes an odd number of edges to this count, there must be at least one non-tour edge incident at this vertex. This edge can be added to T to give a strictly longer tour that contains T, which is a contradiction. Hence both endpoints of T have odd degree.

QUESTION ANALYTICS +

Q. 5

Have any Doubt ?

A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is

A. 6

B. 4

C. $\frac{6}{25}$

D. $\frac{12}{5}$ — Your answer is Correct

Solution :

(d)

Probability of getting a green ball, $p = \frac{15}{25}$.

Probability of getting a yellow ball, $q = \frac{10}{25}$.

$$
\begin{aligned}
n &= 10 \\
\text{var}(X) &= n \times p \times q \\
&= 10 \times \left(\frac{3}{5}\right) \times \left(\frac{2}{5}\right) \\
&= 10 \times \left(\frac{6}{25}\right) = \frac{12}{5}
\end{aligned}
$$

Hence option (d) is the answer.

QUESTION ANALYTICS +

Q. 6

Have any Doubt ?

Let A and B be two sets containing 4 and 2 elements respectively. Then number of subsets of the set $A \times B$, each having atleast 3 element is ______.

219 — Your answer is **Correct** 219

Solution :
219

$$n(A) = 4 \text{ and } n(B) = 2$$

$$\Rightarrow \quad n(A \times B) = 8$$

Thus, $2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2 = 219$

QUESTION ANALYTICS

+

Q. 7

Have any Doubt ?

A phone number is a 7-digit sequence that does not start with 0. A phone number is very lucky if its digits are strictly increasing, such as with 1235689. How many very lucky phone numbers are there?

A. 45

B. 36 — Your answer is **Correct**

Solution :
(b)

A very lucky phone number is uniquely defined by the choice of seven distinct digits. We can ignore the digit 0. The number of ways to pick seven distinct non-zero digits is ${}^9C_7 = 36$, which is the number of very lucky phone numbers.

C. 49

D. 62

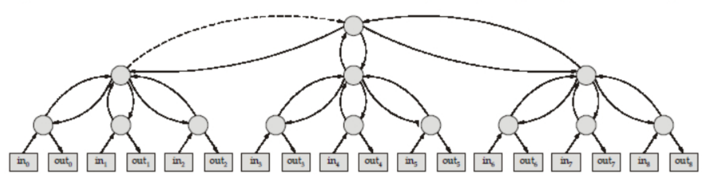
QUESTION ANALYTICS

+

Q. 8

Have any Doubt ?

Consider the complete ternary-tree network with 9 inputs and 9 outputs shown below where packets are routed randomly. The route each packet takes is the shortest path between input and output. Let I_0, I_1 and I_2 be indicator random variables for the events that a packet originating at in0, in_1 and in_2, respectively, crosses the dashed edge in the figure. Let $T = I_0 + I_1 + I_2$ be a random variable for the number of packets passing through the dashed edge.



Now, suppose that each input sends a single packet to an output selected uniformly at random; the packet destinations are mutually independent. (Note that outputs may receive packets from multiple inputs including their corresponding input.) Then, the value of $\dfrac{\text{Expectation }[T]}{\text{Variance }[T]}$ is ______.

[2 Marks, NAT]

3 — Correct Option

Solution :
3

A packet will pass through the dashed edge if it originates in inputs 0 – 2 and is destined for outputs 3 – 8. For $j \in \{0, 1, 2\}$. Let I_j be an indicator random variable for the event that a packet leaving input j passes through the dashed edge. The probability of this event is $\frac{2}{3}$. It follows that:

$$\begin{aligned} T &= I_1 + I_2 + I_3 \\ \text{Ex }[T] &= \text{Ex }[I_1 + I_2 + I_3] \\ &= \text{Ex }[I_1] + \text{Ex }[I_2] + \text{Ex }[I_3] \\ &= 3 \cdot \frac{2}{3} = 2 \end{aligned}$$

Similarly, but the linearity of variance for independent variables.

$$\begin{aligned} T &= I_1 + I_2 + I_3 \\ \text{Var }[T] &= \text{Var }[I_1] + \text{Var }[I_2] + \text{Var }[I_3] \end{aligned}$$

$$= 3 \cdot \frac{2}{3}\left(2-\frac{2}{3}\right)=\frac{2}{3}$$

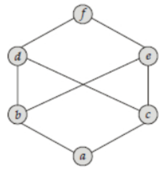
Hence, $$\frac{\text{Ex}[T]}{\text{Var}[T]} = \frac{2}{\frac{2}{3}}=3$$

QUESTION ANALYTICS +

Q. 9

Have any Doubt ?

The graph given below is an example of



A Non lattice poset — Your answer is Correct

Solution :
(a)
b and c have common upper bounds but none of these are LUB.

B Semi lattice

C Partial lattice

D Bounded lattice

QUESTION ANALYTICS +

Q. 10

Have any Doubt ?

The function $f : R \to R$ defined by $f(x) = e^x$. Then f is

A One-one and onto

B One-one but not onto — Your answer is Correct

Solution :
(b)

Let $x_1, x_2 \in R$

and $f(x_1) = f(x_2)$

or $e^{x_1} = e^{x_2}$

or $x_1 = x_2$

Therefore f is one-one.

Let, $f(x) = e^x = y$

$x = \log y$

We know that negative real numbers have no pre-image or the function is not onto and zero is not the image of any real number.

Therefore, f is not onto.

C Onto but not one-one

D Neither one-one nor onto

QUESTION ANALYTICS +