DISCRETE MATHEMATICS-2 (GATE 2023) - REPORTS

Q. 11

Have any Doubt ?

Consider the following recurrence relation

$$na_n + na_{n-1} - a_{n-1} = 2^n$$

where $a_0 = 2$

Which of the following is correct for a_n?

A. $a_n = -\frac{3}{2}(-1)^n + \frac{3}{2}\cdot 2^n$ for $n > 0$

B. $a_n = -\frac{2}{3}(-1)^n + \frac{2}{3}\cdot 2^n$ for $n > 0$

C. $a_n = \frac{1}{n}\left(\frac{3}{2}(-1)^n - \frac{2}{3}2^n\right)$ for $n > 0$

D. $a_n = -\frac{1}{n}\left(\frac{2}{3}(-1)^n - \frac{2}{3}2^n\right)$ for $n > 0$

Your answer is **Correct**

Solution :

(d)

$$\Rightarrow \quad na_n + na_{n-1} - a_{n-1} = 2^n$$

$$\Rightarrow \quad na_n + (n-1)\,a_{n-1} = 2^n$$

Let, $$b_n = n \cdot a_n$$

$$\Rightarrow \quad na_n + (n-1)\,a_{n-1} = 2^n$$

$$b_n + b_{n-1} = 2^n \qquad \ldots(i)$$

Homogeneous solution is equation (i)

$$b_n + b_{n-1} = 0$$

$$x + 1 = 0$$

$$x = -1$$

Homogeneous solution

$$b_n(H) = C(-1)^n$$

Particular solution of equation (i)

Let $d\ 2^n$ is solution then

$$b_n + b_{n-1} = 2^n$$

$$d2^n + d2^{n-1} = 2^n$$

$$2^n \cdot d\left(1 + \frac{1}{2}\right) = 2^n$$

$$d = \frac{2}{3}$$

$$b_n\,(P) = \frac{2}{3}\cdot 2^n$$

So, combined solution is

$$b_n = C(-1)^n + \frac{2}{3}2^n$$

$$b_0 = (0)\cdot a_0 = 0$$

$$b_0 = C(-1)^0 + \frac{2}{3}2^0$$

$$0 = C + \frac{2}{3}$$

$$C = -\frac{2}{3}$$

So, $$b_n = \left(-\frac{2}{3}\right)(-1)^n + \frac{2}{3}2^n$$

$$n\cdot a_n = \left(-\frac{2}{3}\right)(-1)^n + \frac{2}{3}2^n$$

$$a_n = \frac{1}{n}\left(-\frac{2}{3}(-1)^n + \frac{2}{3}2^n\right)$$

$$= -\frac{1}{n}\left(\frac{2}{3}(-1)^n - \frac{2}{3}2^n\right)$$

QUESTION ANALYTICS +

Q. 12

Have any Doubt ?

Consider the following statements:

S_1 : In a distributive lattice, if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$, then $x = y$, where a, x, y are elements of set of this distributive lattice.

S_2 : In a distributive lattice L, a given element a can have at most one complement.
Which of the above statements are correct?

A Only S_1

B Only S_2

C Both S_1 and S_2 Your answer is **Correct**

Solution :
(c)

S_1 :
$$\begin{aligned} x &= x \vee (x \wedge a) && \{\text{Absorption law of lattice}\} \\ &= x \vee (a \wedge y) \\ &= (x \vee a) \wedge (x \vee y) && \{\text{Distribution law}\} \\ &= (y \vee a) \wedge (y \vee x) && \{\text{Commutative law}\} \\ &= y \vee (a \wedge x) && \{\text{Distributive law}\} \\ &= y \vee (a \wedge y) \\ &= y && \{\text{Absorption law}\} \end{aligned}$$

So, S_1 is correct.

S_2 :

Let x and y are complement of a, then
$$a \wedge x = 0, \quad a \vee x = 1$$
$$a \wedge y = 0, \quad a \vee y = 1$$
Where 0 and 1 are least and greatest element respectively of given lattice
$$a \wedge x = a \wedge y = 0$$
$$a \vee x = a \vee y = 1$$
From statement S_1, conclude $x = y$.

So, if complement exist in distributive lattice then it will be unique.

S_2 is correct.

D Neither S_1 nor S_2

QUESTION ANALYTICS +

Q. 13 Have any Doubt ?

Let S be a set and * is a binary operation on S. 'e' is an element of S and it is an identity element for operation * on set S. Consider, for every element a of S, $a^2 = e$. Consider following statements.

S_1 : If (S, *) is a group then (S, *) is commutative.

S_2 : If (S, *) is a monoid then (S, *) is commutative.

Which of the above statements is/are correct?

A Only S_1 is correct

B Only S_2 is correct

C Both S_1 and S_2 are correct Your answer is **Correct**

Solution :
(c)

Assume $(S, *)$ is a monoid.

Let $a, b, c \in S$ and $a * b = c$
$$\begin{aligned} (a * a) * (b * b) &= e * e = e && \{\because a^2 = e = b^2\} && \ldots(i) \\ \Rightarrow \quad c * c &= e \\ \Rightarrow \quad (a * b) * (a * b) &= e \\ \Rightarrow \quad a * (b * a) * b &= e && \{\text{Associativity in monoid}\} \\ \Rightarrow \quad a * (b * a) * b &= (a * a) * (b * b) && && \{\text{from (i)}\} \\ \Rightarrow \quad a * (b * a) * b &= a * (a * b) * b && \{\text{Associativity}\} \end{aligned}$$
Left * of a on both sides.
$$\begin{aligned} \Rightarrow \quad a * a * (b * a) * b &= a * a * (a * b) * b \\ \Rightarrow \quad e * (b * a) * b &= e * (a * b) * b \end{aligned}$$
Right * of b on both sides
$$\begin{aligned} \Rightarrow \quad (b * a) * b * b &= (a * b) * b * b \\ \Rightarrow \quad (b * a) * e &= (a * b) * e \\ \Rightarrow \quad (b * a) &= (a * b) \end{aligned}$$
So, if $(S, *)$ is monoid, then it is commutative.

All groups are monoid. So, same property hold if $(S, *)$ is a group.

So, both S_1 and S_2 are correct.

D Neither S_1 nor S_2 is correct

QUESTION ANALYTICS +

Q. 14 Have any Doubt ?

Consider a sequence $\{a_k\}$ whose generating function is $G(x) = x^2 + 3x + 7 + \left(\frac{1}{1-x^2}\right)$. a_0 and a_7 are 1st and 8th term of given sequence respectively. The value of $a_0 + a_7 =$ ______.

8

Correct Option

Solution :
8

Let,

$$G(x) = x^2 + 3x + 7 + f(x)$$

$$f(x) = \frac{1}{1-x^2} = \frac{1}{2}\left[\frac{1}{1-x} + \frac{1}{1+x}\right]$$

$$= \frac{1}{2}\left[\underbrace{1+x+x^2+x^3+\ldots\ldots}_{\frac{1}{1-x}} + \underbrace{1-x+x^2-x^3+\ldots\ldots}_{\frac{1}{1+x}}\right]$$

$$= \frac{1}{2}\left[2+2x^2+2x^4+2x^6\ldots\ldots\right]$$

$$= 1 + x^2 + x^4 + x^6 + \ldots\ldots$$

$$G(x) = 7 + 3x + x^2 + f(x)$$

$$= 7 + 3x + x^2 + 1 + x^2 + x^4 + \ldots\ldots$$

$$= 8x^0 + 3x + 2x^2 + x^4 + x^6 + \ldots\ldots \quad \ldots(i)$$

From generating function is $G(x)$,

$$a_0 = 8,\ a_1 = 3,\ a_2 = 2,\ a_3 = 0,\ a_4 = 1,\ a_5 = 0,\ a_6 = 1$$

$$a_n = \begin{cases} 0, \text{ if } n > 2 \text{ and } n \text{ is odd} \\ 1, \text{ if } n > 2 \text{ and } n \text{ is even} \end{cases}$$

So,

$$a_7 = 0$$

$$a_0 + a_7 = 8 + 0 = 8$$

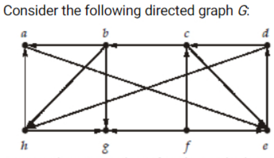
Your Answer is 11

QUESTION ANALYTICS

Level : Difficult	Accuracy	Topper's Time	
	0%	00:00:00 hrs	
Correct Marks : 0	Average Time	Your Time	
Negative Marks : 0	01:36 min	00:02:22 hrs	

Q. 15 Have any Doubt ?

Consider the following directed graph *G*:



Let *n* and *e* are number of nodes and edges respectively in maximal strongly-connected component of *G*. The value of $n + e$ is ______.

15

Correct Option

Solution :
15

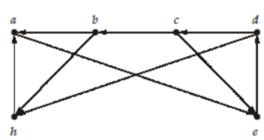
There is an algorithm for finding strongly-connected components from directed graph using DFS.

But, to save time you can use some properties of strongly connected component.

In strongly-connected component, a node do not have 0 indegree or outdegree.

Here g has outdegree = 0, and f has indegree = 0. So remove g, f and edges associated with them.

Now graph is



And this graph is strongly connected.

So,

$$n = 6 \text{ and } e = 9$$

$$n + e = 9 + 6 = 15$$

Your Answer is 13

QUESTION ANALYTICS +

Q. 16 Have any Doubt ?

Q. 16

Let $s = \{a, b, c\}$ and $P(s)$ is power set of s. $X = (P(s), \subseteq)$ is partially ordered set (poset) with partial order as $\subseteq$. Which of the following is/are correct for poset X.

A. Poset X is a lattice. — Your option is Correct

B. For every element z of $P(s)$, s − z is complement of z in poset X. — Your option is Correct

C. Poset X is a distributive lattice. — Your option is Correct

D. Hasse diagram of X has 9 edges.

YOUR ANSWER - a,b,c | CORRECT ANSWER - a,b,c | STATUS - ✓

Solution :

(a, b, c)

(a) Let $x, y \in P(s)$, then using subset properties

$$x \vee y = x \cup y$$

and $$x \wedge y = x \cap y$$

So, X is lattice.

(c) For any $x, y, z \in P(s)$,

$$\begin{aligned} x \wedge (y \vee z) &= x \wedge (y \cup z) \\ &= x \cap (y \cup z) \\ &= (x \cap y) \cup (x \cap z) \quad \text{\{Distribution in set operations\}} \\ &= (x \wedge y) \vee (x \wedge z) \end{aligned}$$

$$\begin{aligned} x \vee (y \wedge z) &= x \cup (y \cap z) \\ &= (x \cup y) \cap (x \cup z) \quad \text{\{Distribution in set\}} \\ &= (x \vee y) \wedge (x \vee z) \end{aligned}$$

So, X is distributive lattice.

$O = \{\ \}$ is least element and $I = (a, b, c)$ is greatest element of X.

(b) Let $z \in P(s)$, then $s - z$ is complement of z in sets. And $s - z \in P(s)$.

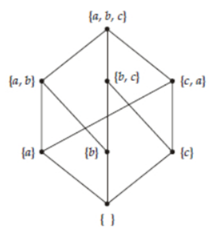
$$z \cup (s - z) = s = (a, b, c)$$

So, $$z \vee (s - z) = z \cup (s - z) = s = (a, b, c) = I$$

and $$z \wedge (s - z) = z \cap (s - z) = \{\ \} = 0$$

So, $s - z$ is complement of z in X.

(d) Hasse diagram of X.



It has 12 edges.

QUESTION ANALYTICS +

Q. 17 — Have any Doubt ?

Let $(G, \Box)$ be a group where G is a set and $\Box$ is a binary operation on G. Consider G' is a nonempty finite subset of G. And G' is closed under $\Box$. Then which of the following is/are correct?

A. $(G', \Box)$ is subgroup of $(G, \Box)$. — Your option is Correct

B. Identity element of group $(G, \Box)$ is also an element of G'. — Your option is Correct

C. There may be an element in G' whose inverse is not in G'.

D. Associative property hold for algebraic structure $(G', \Box)$. — Your option is Correct

YOUR ANSWER - a,b,d | CORRECT ANSWER - a,b,d | STATUS - ✓

Solution :

(a, b, d)

Theorem: $(G, \Box)$ is a group, $G' \subseteq G$ and G' is finite. If $(G', \Box)$ closed under $\Box$, then $(G', \Box)$ is subgroup of $(G, \Box)$. So, (a) is correct.

If $(G', \Box)$ is subgroup then it has an identity element of $(G, \Box)$. And every element in G' has its inverse in G'. So, (b) is correct and (c) is incorrect.

If $(G', \Box)$ is subgroup, then it definitely follow associative property.

Note: For proof of theorem, refer a standard book or video solution will be provided.

QUESTION ANALYTICS

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