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Computer Science & IT (CS)
HOME
MY TEST
BOOKMARKS
MY PROFILE
REPORTS
BUY PACKAGE
NEWS
TEST SCHEDULE

ENGINEERING MATHEMATICS-2: (GATE 2022) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT
ALL(17) CORRECT(9) INCORRECT(3) SKIPPED(5)

Q. 11 Solution Video | Have any Doubt ?

For real number, x and y with $y = 3x^2 + 3x + 1$ the maximum and minimum value of y for $x \in [-2, 0]$ are respectively _______.

A. 7 and $\frac{1}{4}$ — Your answer is **Correct**

Solution :
(a)

$$y = 3x^2 + 3x + 1$$
$$\frac{dy}{dx} = 6x + 3, \quad \frac{dy}{dx} = 0$$
$$\Rightarrow \quad 6x + 3 = 0$$
$$x = -\frac{1}{2}$$
$$\frac{d^2y}{dx^2} = 6$$
Also, $\frac{d^2y}{dx^2} = 6 > 0$ so minimum

So maximum value of y in $[-2, 0]$ is maximum $\{f(-2), f(0)\}$ i.e. max $\{7, 1\} = 7$.
Minimum value of y in $[-2, 0]$

$$\min\left\{\begin{matrix} f(-2), & f(0), & f\left(-\frac{1}{2}\right) \\ \downarrow & \downarrow & \downarrow \\ 7 & 1 & \frac{1}{4}\end{matrix}\right\} = \frac{1}{4}$$

So, maximum value 7 and minimum value $\frac{1}{4}$.

B. 7 and 1

C. -2 and $-\frac{1}{2}$

D. 1 and $\frac{1}{4}$

QUESTION ANALYTICS +

Q. 12 Solution Video | Have any Doubt ?

A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale?

A. $\frac{25}{46}$

B. $\frac{4}{25}$

C. $\frac{44}{91}$ — Your answer is **Correct**

Solution :
(c)

Let, A = First drawn orange is good
B = Second drawn orange is good
C = Third drawn orange is good

The oranges are not replaced.

Thus, $P(A) = \frac{12}{15}, P(B) = \frac{11}{14}, P(C) = \frac{10}{13}$

The box is approved for sale, if all three oranges are good.
Thus, the probability of getting all the oranges good

$$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}$$

D $\frac{4}{81}$

QUESTION ANALYTICS +

Q. 13

Solution Video | Have any Doubt ?

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \, dx$ is

A $\frac{\pi}{2}$

B $\frac{3\pi}{4}$

C $\frac{3\pi}{8}$

Your answer is **Correct**

Solution :
(c)

$$f(x) = \sin^4 x$$

Also,

$$f(-x) = \sin^4(-x) = \sin^4 x = f(x)$$

So,

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \, dx = 2\int_0^{\frac{\pi}{2}} \sin^4 x \, dx = 2\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (1+\cos^2 2x - 2\cos 2x)dx$$

$$\Rightarrow \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(1-\cos 2x + \frac{1+\cos 4x}{2}\right)dx$$

$$\Rightarrow \frac{1}{4}\int_0^{\frac{\pi}{2}} (3-4\cos 2x + \cos 4x)dx$$

$$\Rightarrow \frac{1}{4}\left[3x - \frac{4\sin 2x}{2} + \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$$

On solving we get $\frac{3\pi}{8}$.

D $\frac{\pi}{8}$

QUESTION ANALYTICS +

Q. 14

Solution Video | Have any Doubt ?

A random variable X has probability density function

$$f(x) = \begin{cases} a+bx & \text{for } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

Given the expected value $E(X) = \frac{2}{3}$ the $\Pr[X < 0.5]$ is ______. (Upto 2 decimal places)

0.25

Correct Option

Solution :
0.25

$$f(x) = \begin{cases} a+bx, & \text{for } 0<x<1 \\ 0, & \text{otherwise} \end{cases}$$

$$E(X) = \frac{2}{3}$$

$$\Rightarrow \int_0^1 x\,f(x)dx = \frac{2}{3}$$

$$\Rightarrow \int_0^1 x(a+bx)dx = \frac{2}{3}$$

$$\Rightarrow \quad a\left(\frac{x^2}{2}\right)_0^1 + b\left(\frac{x^3}{3}\right)_0^1 = \frac{2}{3}$$

$$a\left(\frac{1}{2}\right) + b\left(\frac{1}{3}\right) = \frac{2}{3}$$

$$\Rightarrow \quad 3a + 2b = 4 \quad \ldots(i)$$

Now, $\int_0^1 f(x)dx = 1$

Total probability is always equal to 1.

$$\int_0^1 (a+bx)dx$$

$$\left(ax + \frac{bx^2}{2}\right)_0^1 = 1$$

$$a + \frac{b}{2} = 1$$

$$2a + b = 2 \quad \ldots(ii)$$

On solving equation (i) and (ii)

$$a = 0, \quad b = 2$$

So, $f(x) = \begin{cases} 2x, & \text{for } 0 < x < 1 \\ 0, & \end{cases}$

Now, we need $\int_0^{\frac{1}{2}} 2x dx = \frac{1}{4} = 0.25$

QUESTION ANALYTICS +

Q. 15

Solution Video | Have any Doubt ?

Out of all possible four digit numbers, a number is picked at random, the probability that the number does not contain the digit 6 is ______. (Upto 2 decimal places)

0.64 [0.64 - 0.65] — Correct Option

Solution :
0.64 [0.64 - 0.65]

Total number of 4 digit numbers = 9000 (1000 to 9999)

- At first place 0 and 6 are not allowed, so 8 choices for filling first place.
- For second, third and fourth place all digits are allowed except 6. So 9 choices.

So, n(not containing dight 6) = $8 \times 9 \times 9 \times 9$

$$P(\text{not containing the digit } 6) = \frac{8 \times 9 \times 9 \times 9}{9000} = 0.648$$

Your Answer is 0.13

QUESTION ANALYTICS +

Q. 16

? FAQ | Solution Video | Have any Doubt ?

Which of the following is/are true?

A. The function x^3 has neither global minima nor global maxima. — Your option is Correct

B. The function $|x|$ has the global minima at $x = 0$. — Your option is Correct

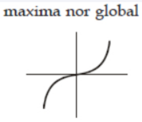
C. The function $\sqrt[x]{x}, (x < 0)$, has the global minima at $x = e$. — Your answer is IN-CORRECT

D. The function $\sqrt[x]{x}, (x > 0)$, has the global maxima at $x = e$. — Correct Option

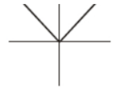
YOUR ANSWER - a,b,c	CORRECT ANSWER - a,b,d	STATUS - ✗

Solution :
(a, b, d)

- For graph of x^3, neither global maxima nor global minima is defined.



- For graph of $|x|$ we can see that global minima is 0.



- Option (c)

Let, $$y = x^{1/x}$$

$$\log y = \frac{\log x}{x}$$

$$y = e^{\frac{\log x}{x}}$$

y is maximum or minimum when

$$f(x) = \frac{\log x}{x} \text{ is maximum or minimum}$$

Therefore, $$f'(x) = \frac{x\left(\frac{1}{x}\right) - \log x}{x^2} = \frac{1-\log x}{x^2}$$

$$f'(x) = 0$$

$$1 - \log x = 1$$

$$\log x = 1 \Rightarrow x = e$$

Now, $$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - 2x(1-\log x)}{x^4} = \frac{-3x + 2x\log x}{x^4}$$

$$\Rightarrow \quad f''(e) = -\frac{e}{e^4} < 0 \text{ (max)}$$

Therefore, $y = \sqrt[x]{x}$ is maximum at $x = e$

Thus, (a), (b) and (d) are correct.

QUESTION ANALYTICS +

Q. 17

FAQ | Solution Video | See your Answers

Consider a function f which is continuous and differentiable in the interval [−4, 4] such that for every x in the interval $f'(x) \le 5$. If $f(-4)$ is 9, then f(4) can be ______.

A 49 — Correct Option

B 50

C 40 — Correct Option

D 60

YOUR ANSWER - NA | CORRECT ANSWER - a,c | STATUS - SKIPPED

Solution :

(a, c)

Using Lagrange,

$$f'(x) = \frac{f(b) - f(a)}{b - a}$$

Here, $b = 4$, $a = -4$ and $f(-4) = 9$

$$f'(x) = \frac{f(4) - 9}{4 - (-4)}$$

$$f'(x) = \frac{f(4) - 9}{8}$$

Also, $$f'(x) \le 5$$

So, $$\frac{f(4) - 9}{8} \le 5$$

$$f(4) \le 40 + 9$$

$$f(4) \le 49$$

So, $f(4)$ can be atmost 49.

QUESTION ANALYTICS +