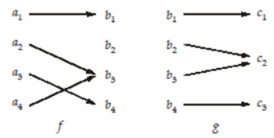
DISCRETE MATHEMATICS-1 (GATE 2023) - REPORTS

Q. 1 Have any Doubt ?

Consider three sets $A = \{a_1, a_2, a_3, a_4\}$, $B = \{b_1, b_2, b_3, b_4\}$ and $C = \{c_1, c_2, c_3\}$. Consider function $f: A \to B$ and $g: B \to C$, following diagram shows the function f and g.

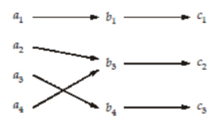


Where if arrow from a_i to b_j then $f(a_i) = b_j$ and if arrow from b_i to c_j then $g(b_i) = c_j$.
Which of the following is correct for left composition of g with f (g o f)?

A g o f is injective

B g o f is surjective — Your answer is **Correct**

Solution :
(b)
On combining diagrams:



$$g \circ f\,(a_1) = g(f(a_1)) = g(b_1) = c_1$$
$$g \circ f\,(a_2) = g(f(a_2)) = g(b_3) = c_2$$
$$g \circ f\,(a_3) = g(f(a_3)) = g(b_4) = c_3$$
$$g \circ f\,(a_4) = g(f(a_4)) = g(b_3) = c_2$$

$\forall x \in C$ (x has preimage for $g \circ f$)
So, $g \circ f$ is surjective.
$$g \circ f\,(a_2) = g \circ f\,(a_4) = c_2$$
So, $g \circ f$ is not injective, and not bijective also.

C g o f is bijective

D g o f is neither injective nor surjective

QUESTION ANALYTICS +

Q. 2 Have any Doubt ?

Consider a box contains 7 red balls and 7 blue balls. A man selects some balls from this bag randomly. How many balls must be select to be sure of having atleast 2 red balls?

A 3

B 4

C 7

D 9 — Your answer is **Correct**

Solution :
(d)
If he selects 9 balls, then maximum 7 balls can be blue and rest 2 must be red.
If he selects 8 balls or less, then 1 or 0 balls can be red and rest are blue.

QUESTION ANALYTICS +

Q. 3 Have any Doubt ?

Let A and B are two non-empty sets. A relation R from A to B satisfy following statement:
$\forall x \in A(\exists y \in B ((x, y) \in R \text{ and } ((x, z) \in R \to (y = z))) \leftrightarrow x \neq a)$
where a is an element of A.
Consider the following statements:
S_1 : R is a function.
S_2 : For any element e in B, (a, e) cannot be in R.
Which of the above statements is/are correct?

A Only S_1 is correct

B Only S_2 is correct — Your answer is **IN-CORRECT**

C Both S_1 and S_2 are correct

D Neither S_1 nor S_2 are correct — Correct Option

Solution :
(d)

Let, $p \equiv \exists y \in B ((x, y) \in R \text{ and } ((x, z) \in R \to (y = z)))$
$q \equiv x \neq a$

Let above given statement of propositions is s, then
$s \equiv \forall x \in A\ (p \leftrightarrow q)$

There are two cases for s

1. p and q both are correct, then $x \neq a$ and for every x there is unique y in B such that $(x, y) \in R$.
   If $(x, z) \in R$ then $z = y$. So, x maps to unique y in B for $x \neq a$.
2. Both p and q are incorrect.
   So, $x = a$ and p' is true
   $p' = \neg(\exists y \in B ((x, y) \in R \text{ and } ((x, z) \in R \to (z = y))))$
   $= \forall y \in B ((x, y) \notin R \text{ or } ((x, z) \in R \to (z = y))')$
   $= \forall y \in B ((x, y) \notin R \text{ or } ((x, z) \in R \wedge (z \neq y)))$
   So, $x = a$ and p' says

(i) For any y in $B (a, y) \notin B$

or

(ii) 'a' relates to more than one element of B on relation R

According to (2) case and its subcases (i) and (ii), R is not function. So, S_1 is incorrect.
And from sub case (ii) of case (2), $(a, e) \in R$ possible for $e \in B$. So, S_2 is incorrect.

QUESTION ANALYTICS +

Q. 4 Have any Doubt ?

Consider a plane with 10 different points, out of which 4 points are collinear (points can placed on a line). Number of lines obtained after joining all pair of points using a line are

A 40 — Your answer is **Correct**

Solution :
(a)

All points are distinct, so number of pair of points = ${}^{10}C_2$.
So, there can be ${}^{10}C_2$ lines.
But 4 points are collinear, some lines are overlapped.
Number of lines created using 4 points = ${}^{4}C_2$.
So, number of lines created using no pair in which both points from collinear points = ${}^{10}C_2 - {}^{4}C_2$
But we have to consider the longest line created by these collinear points on which some other lines overlapped.

So, Total number of lines created $= {}^{10}C_2 - {}^{4}C_2 + 1$

$$= \left(\frac{10\times 9}{2}\right) - \left(\frac{4\times 3}{2}\right) + 1$$
$$= 45 - 6 + 1 = 40$$

B 45

C 41

D 21

QUESTION ANALYTICS +

Q. 5 Have any Doubt ?

Let the following statements S_1 and S_2 are true:
S_1 : "If Neha qualified GATE 2023 then she will get admission in IISc Bangalore".
S_2 : "Neha qualified GATE 2023".
Consider the following conclusions:
C_1 : "Neha will get admission in IISc Bangalore".
C_2 : "Neha will get admission in IISc Bangalore and she qualified GATE 2023".

Which of the above conclusions are correct?

(A) Only C_1 is correct

(B) Only C_2 is correct

(C) Both C_1 and C_2 are correct — Your answer is **Correct**

Solution :
(c)

Consider following propositions:

p : "Neha qualified GATE 2023".

q : "Neha will get admission in IISc Bangalore".

S_1 and S_2 are correct/true

$$S_1 \equiv p \to q$$
$$S_2 \equiv p$$

According to Modus Ponens inference rule,

q is true. And $C_1 \equiv q$, so C_1 is true.

$\Rightarrow$ p is true (given) and q is true (shown above), using conjunction inference rule, $p \wedge q$ is true.

And $C_2 = p \wedge q$, so C_2 is correct.

(D) Neither C_1 nor C_2 is correct

QUESTION ANALYTICS +

Q. 6

Have any Doubt ?

Let A and B are two sets where $|B| = 6$ and $A \subset B$. The number of possible values of $|A|$ are ______.

6 — Correct Option

Solution :
6

$A \subset B$, it says atleast one element of B is not an element of A.

Cases:

(i) Only 1 element of B is not element of A then $|A| = 5$.

(ii) Only 2 element of B are not in A, then $|A| = 4$.

$\vdots$

(iii) 6 element of B are not in A, then $A = \phi$ and $|A| = 0$.

So, possible value of $|A|$ are 0, 1, 2, 3, 4, 5 and they are 6.

Your Answer is 63

QUESTION ANALYTICS +

Q. 7

Have any Doubt ?

p, q and r be propositions as given below:
p : Program requested a page
q : Page fault occurred
r : Error occurred
Consider a system with following specifications:
S_1 : "If program requested a page, then page fault occurred or error occurred".
S_2 : "Error occurred if and only if program requested a page and page fault occurred".
S_3 : "Error occurred".
For how many possible truth assignments to p, q and r such that the above system specifications are consistent?

1 — Correct Option

Solution :
1

$$S_1 \equiv p \to (q + r)$$
$$S_2 \equiv r \leftrightarrow (p \cdot q)$$
$$S_3 \equiv r$$

System specifications S_1, S_2 and S_3 are consistent if and only if $S_1 \wedge S_2 \wedge S_3$ is not contradiction.

So, $[p \to (q + r)] \wedge [r \leftrightarrow (q \cdot q)] \wedge [r]$ must be true for atleast one true assignment.

Let, $z = [p \to (q + r)] \wedge [r \leftrightarrow (p \cdot q)] \wedge r$ and z is true iff $r = 1$

So, $r = 1$ ...(i)

If $r = 1$, then $p \cdot q$ must be true from S_2 and then

$p = 1$ and $q = 1$ ...(ii)

And if $p = 1$, $q = 1$ and $r = 1$

Then S_1 is true.

So, for only one truth assignment, given specifications are consistent.

You can solve this using truth table also. You will get $S_1 \wedge S_2 \wedge S_3$ as true only for one truth assignment.

Your Answer is 5

QUESTION ANALYTICS +

Q. 8

Have any Doubt ?

Let A and B are sets, and P is power set of a set. Which of the following is/are correct?

A $P(A \cap B) = P(A) \cap P(B)$ — Your option is Correct

B $P(A \cup B) = P(A) \cup P(B)$

C $P(A \times B) = P(A) \times P(B)$

D If $A = \phi$ then $|A| = |P(A)|$, where ϕ is null set

YOUR ANSWER - a

CORRECT ANSWER - a

STATUS -

Solution :

(a)

Option (d):

$$A = \phi \text{ then } |A| = 0$$
$$|P(A)| = 2^{|A|} = 2^0 = 1 \neq 0$$
$$P(A) = \{\phi\}$$

Option (b):

If $A = \{1\}$ and $B = \{2\}$, then

$$P(A \cup B) = \{\{\ \}, \{1\}, \{2\}, \{1, 2\}\}$$
$$P(A) = \{\{\ \}, \{1\}\}$$
$$P(B) = \{\{\ \}, \{2\}\}$$
$$P(A) \cup P(B) = \{\{\ \}, \{1\}, \{2\}\}$$

And $P(A \cup B) \neq P(A) \cup P(B)$

Option (c):

Here domain of LHS set is not equal to domain RHS set.

Let, $A = \{1\}$ and $B = \{2\}$

$$A \times B = \{(1, 2)\}$$
$$P(A \times B) = \{\{\ \}, \{1, 2\}\}$$
$$P(A) \times P(B) = \{(\{\ \}, \{\ \}), (\{\ \}, \{2\}), (\{1\}, \{\ \}), (\{1\}, \{2\})\}$$

So, $P(A) \times P(B) \neq P(A \times B)$

Option (a) is a property. You can prove it also.

Refer a standard method/book for proof.

Or try like this

1. Let x is an arbitrary element of $P(A \cap B)$ then x is also an element of $P(A) \cap P(B)$.
2. Let x is an arbitrary element of $P(A) \cap P(B)$ then x is also an element of $P(A \cap B)$.

QUESTION ANALYTICS +

Q. 9

Have any Doubt ?

Let p, q and r be propositions and the expression $(p \to q) \to (q \to r)$ is false for given conditions.
Then which of the following is/are correct for given conditions.

A $q \to r$ is false — Your option is Correct

B $r \to p$ is false

C $p \to q$ is true — Correct Option

D $p \to r$ is true

YOUR ANSWER - a

CORRECT ANSWER - a,c

STATUS -

Solution :

(a, c)

$$\begin{aligned} s &= (p \to q) \to (q \to r) \\ &= (p' \vee q)' \vee (q' \vee r) \\ &= (p \wedge q') \vee (q' \vee r) \\ &= ((p \wedge q') \vee q') \vee r && \{\text{Associativity}\} \\ &= (q') \vee r && \{\text{Absorption law}\} \\ &= q \to r \end{aligned}$$

$(p \to q) \to (q \to r) \equiv (q \to r)$

So, $(q \to r)$ is also false.
s is false for

p	q	r	s	$r \to p$	$p \to q$	$p \to r$
0	1	0	0	1	1	1
1	1	0	0	1	1	0

From above table, (b), (c) are incorrect and (c) is correct.

QUESTION ANALYTICS +

Q. 10

Have any Doubt ?

Consider the following statement S:

"Someone has visited every city in India except Delhi"

Let visited (x, y) = "x has visited city y", where domain of x is all people and domain of y is all cities in India. Which of the following statement is equivalent to s?

A $\exists x \forall y$ (visited (x, y)) $\vee$ ($y \neq$ Delhi))

B $\exists x \forall y$ (visited (x, y)) $\wedge$ ($y \neq$ Delhi)) — Your answer is **IN-CORRECT**

C $\exists x \forall y$ (visited (x, y)) $\rightarrow$ ($y \neq$ Delhi))

D $\exists x \forall y$ (visited (x, y)) $\leftrightarrow$ ($y \neq$ Delhi)) — Correct Option

Solution :
(d)

Let visited $(x, y) \equiv A$ and $(y \neq \text{Delhi}) \equiv B$.

$\Rightarrow$ If S is true then that person is x and
(i) If we select $y \neq$ Delhi then A and B both are true.
(ii) If we select y = Delhi then A and B both are incorrect.
In both above situation, statement in (d) is correct.

$\Rightarrow$ Option (a): If a person visited all cities of India, then it will also come in statement of (a).

$\Rightarrow$ Option (b): Let (visited $(x, y) \wedge (y \neq \text{Delhi})) \equiv P(x, y)$
If S is true, then for a x if we select y = Delhi then $p(x, y)$ is false, so, statement in (b) is false, but it should true.

$\Rightarrow$ Option (c): Let (visited $(x, y) \to y \neq \text{Delhi}) \equiv P(x, y)$
If S is true and if there is a person which visited all cities in India except Delhi and Mumbai will also come in statement in (c).
For example, for this person.
(i) If y = Delhi, then $P(x, y)$ is true.
(ii) If $y \neq$ Delhi and $y \neq$ Mumbai, then $P(x, y)$ is true.
(iii) If y = Mumbai, then visited (x, y) is false and $y \neq$ Delhi is true, so $P(x, y)$ is true.

From above cases, we can say above given person will also come in solution.
So statement in (c) is not equivalent to S.

QUESTION ANALYTICS +