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HOME
MY TEST
BOOKMARKS
MY PROFILE
REPORTS
BUY PACKAGE
NEWS
TEST SCHEDULE

DISCRETE MATHEMATICS (GATE 2023) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(33) | CORRECT(26) | INCORRECT(4) | SKIPPED(3)

Q. 11 Have any Doubt ?

We have n married couples who are to sit at a round table of 2n spots. How many arrangements are possible if all $2n$ rotations of a given arrangement are considered equivalent and each person sits next to his/her spouse?

A. $2^n(n-1)!$ — Your answer is Correct

Solution :
(a)
Consider this as a problem of n things at a table instead of $2n$, treating each couple as a single unit. From the lecture notes, there are $(n-1)!$ arrangements of couples. Then each couple can be arranged in one of two ways, so multiplying n times by 2 gives $2^n(n-1)!$.

B. $3^n(n-2)!$

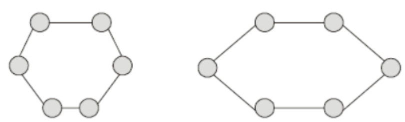
C. $3^n(n-1)!$

D. $2^n(n-2)!$

QUESTION ANALYTICS +

Q. 12 Have any Doubt ?

Suppose we have a simple undirected graph G with 2n vertices and $2n$ edges, where $n \geq 3$. The graph consists of two disjoint cycles with n edges.
For example if $n = 6$, the graph would look like this:



Now, A pair of vertices u and v from G is selected uniformly at random from the pairs of distinct vertices with no edge between them. A new graph G' is constructed to be the same as G, except that there is an edge between u and v. What is the probability that G' is connected?

A. $\frac{n}{2}$

B. $\frac{2n}{n-3}$

C. $\frac{n}{2n-3}$ — Your answer is Correct

Solution :
(c)
G' is connected iff u and v wins from different cycles. There are n^2 pairs of vertices consisting of vertices in different cycles in all there are $\left({}^{2n}C_2 - 2n\right)$ pair of vertices with no edge between them.

$$\Rightarrow \quad P = \frac{n^2}{{}^{2n}C_2 - 2n}$$

$$\Rightarrow \quad P = \frac{n}{2n-3}$$

D. $\frac{1}{n-3}$

QUESTION ANALYTICS +

Q. 13 Have any Doubt ?

How many possible six figure salaries (in whole dollar amounts) are there that contain some digit at least twice?

763920 — Your answer is Correct763920

Solution :
763920

The total number of possible six figure salaries, without any restrictions, is 9×10^5 (digits 1 ... 9 in the first place and digits 0 ... 9 in the rest). Now let's find the number of salaries that contain no digit more than once. This is $9 \times {}^9C_5$, since there are 9 possibilities for the first digit (0 is not allowed) and for the remaining 5 we need to choose 5 digits out of of 9 in order without repetition. Subtract the number of distinct-digit salaries from the overall number of possible salaries gives us the desired number of salaries that contain some digit at least twice.

Specifically, $9 \times 10^5 - 9 \times {}^9C_5 = 763920$

QUESTION ANALYTICS +

Q. 14

Have any Doubt ?

Consider these two propositions:

$P : (A \vee B) \Rightarrow C$

$Q : (\neg C \Rightarrow \neg A) \vee (\neg C \Rightarrow \neg B)$

Which of the following best describes the relationship between P and Q?

A $P \rightarrow Q$ — Your answer is Correct

Solution :
(a)

A	B	C	P	Q
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	1
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

Observe from the last two columns of the table that $P \rightarrow Q$ is true only.

B $Q \rightarrow P$

C P and Q are equivalent

D $(P \wedge Q) \vee Q$ is tautology

QUESTION ANALYTICS +

Q. 15

Have any Doubt ?

Let T be a tree having n vertices. Let $V_1, V_2 \in V(T)$. How many distinct paths are there from V_1 and V_2 is T______.

1 — Your answer is Correct1

Solution :
1

Tree always has only 1 pair between any two vertices.

QUESTION ANALYTICS +

Q. 16

Have any Doubt ?

You are given the following predicate on the set P of all people who ever lived:

Parent(x, y), true iff x is the parent of y

Then which of the following represents the mathematical logic corresponding to: "All people have two parents".

A $\forall x \in P\ \exists y, z \in P : (\text{Parent}(y, x) \wedge \text{Parent}(z, x))$

B $\exists x \in P\ \forall y, z \in P : (\text{Parent}(y, x) \to \text{Parent}(z, x) \wedge y \neq z))$

C $\forall x \in P\ \exists y, z \in P : (\text{Parent}(y, x) \wedge \text{Parent}(z, x) \wedge y \neq z)$ Your answer is Correct

Solution :
(c)

D $\exists x \in P\ \forall y, z \in P : (\text{Parent}(y, x) \wedge \text{Parent}(z, x) \wedge y \neq z))$

QUESTION ANALYTICS +

Q. 17

Have any Doubt ?

A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is

A $\frac{1}{5}$

B $\frac{3}{4}$

C $\frac{3}{10}$

D $\frac{2}{5}$ Your answer is Correct

Solution :
(d)
There are 2 possible cases: First ball drawn is red or is black.

$$P(\text{first ball black}) = \frac{6}{10}$$

$$P(\text{first ball red}) = \frac{4}{10}$$

$$P(\text{second ball red} - \text{first ball drawn is black}) = \frac{4}{12}$$

$$P(\text{second ball red} - \text{first ball drawn is red}) = \frac{6}{12} = \frac{1}{2}$$

From total Probability Theorem,

The probability of the second ball being red, $P(R) = \left(\frac{4}{10}\right)\times\left(\frac{1}{2}\right)+\left(\frac{6}{10}\right)\times\left(\frac{4}{12}\right)$

$$= \left(\frac{1}{5}\right)+\left(\frac{1}{5}\right)=\frac{2}{5}$$

Hence option (d) is the answer.

QUESTION ANALYTICS +

Q. 18

Have any Doubt ?

A phone number is a 7-digit sequence that does not start with 0. Call a phone number lucky if its digits are in non-decreasing order. For example, 1112234 is lucky, but 1112232 is not. How many lucky phone numbers are there?

6435 Your answer is Correct6435

Solution :
6435
A lucky phone number is uniquely identified by the choice of seven digits, with possible repetitions, since these digits must then be placed in sorted order. We can ignore the digit 0 altogether, since in a non-decreasing sequence 0 would have to occur in the first place and we explicitly rule out this possibility. The number of ways to pick an ordered sequence of seven non-zero digits with possible repetitions is thus ${}^{7+9-1}C_7 = 6435$.

QUESTION ANALYTICS +

Q. 19

Have any Doubt ?

What is the number of integer solutions of the equation $x_1 + x_2 + x_3 = 50$, such that $x_i \geq 0$ for each $1 \leq i \leq 3$?

1326 Your answer is Correct1326

Solution :
1326

We need to distribute 50 units between 3 variables. The situation is equivalent to distributing 50 balls into 3 bias the number of options for this is

$$^{3+50-1}C_{50} = {}^{52}C_{50} = 1326$$

QUESTION ANALYTICS +

Q. 20

Have any Doubt ?

Which of the following is/are represents number of simple directed (unweighted) graphs on the set of vertices $\{V_1, V_2, \ldots, V_n\}$ are there that have atmost one edge between any pair of vertices?

A $3^{{}^nC_2}$ Correct Option

B $3^{\frac{n(n-1)}{2}}$ Correct Option

C $\sum_{k=0}^{{}^nC_2} \left({}^{{}^nC_2}C_k\right)2^k$ Your option is Correct

D $\sum_{k=0}^{{}^nC_2} \left({}^{{}^nC_2}C_k\right)3^k$

YOUR ANSWER - c

CORRECT ANSWER - a,b,c

STATUS -

Solution :
(a, b, c)

There are ${}^nC_2$ ordered pairs of vertices. For each pair $\{a, b\}$ we may have only the edge (a, b), or only the edge (b, a) or no edge at all, giving a total of 3 multiply exclusive possibilities. So, the required number of graph $3^{{}^nC_2} = 3^{\frac{n(n-1)}{2}}$.

Note: Another idea can be if we remove the directionality of the edges, the resulting graph is a simple undirected graph by the given conditions. Lets its have k edges, out of a possible maximum of ${}^nC_2$. There are $\left({}^{{}^nC_2}C_k\right)$ ways to pick these edges and 2^k ways to assign directions so,

$$\sum_{k=0}^{{}^nC_2} \left({}^{{}^nC_2}C_k\right)2^k.$$

QUESTION ANALYTICS +