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Computer Science & IT (CS)

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DISCRETE MATHEMATICS-2 (GATE 2023) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) | CORRECT(12) | INCORRECT(5) | SKIPPED(0)

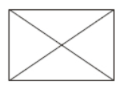
Q. 1

Have any Doubt ?

Consider the following statements:
S_1 : There is always Hamilton path in a directed complete graph.
S_2 : There is always Hamilton path in a undirected complete graph.
S_3 : There is always Euler path in a undirected complete graph.
Which of the above statements are correct?

A. S_1 and S_2 only — Your answer is **Correct**

Solution :
(a)
Complete graph says, for a node always an edge to reach another node.
Let a simple path created with nodes in sequence V_0, V_1, V_2 ....., V_i, now some nodes are left. Graph is complete, so V_i can reach any node which is not in V_0 to V_i and let select V_k. now edge (V_i, V_k) is not in created path. This way we can get Hamilton path. Above procedure valid for both directed and undirected complete graph.
So, S_1 and S_2 are correct.
If number of nodes in complete graph is odd then there is always a Euler path, but for n = even and $n > 2$ then complete graph has no euler path.
Let $n = 4$, then no euler path.



B. S_1 and S_3 only

C. S_2 only

D. S_1, S_2, S_3 all

QUESTION ANALYTICS

Level : Difficult

Correct Marks : 1

Negative Marks : 0

Accuracy: 14.29%

Topper's Time: hrs

Average Time: 02:15 min

Your Time: 00:00:15 hrs

Q. 2

Have any Doubt ?

Let a_n denote the number of edges in a complete simple undirected graph with n vertices, where $n > 1$ and $a_1 = 0$. Which of the following is correct recurrence relation for a_n?

A. $a_n = a_{n-1} + n$, for $n > 1$ and $a_1 = 0$

B. $a_n = (a_{n-1} - a_{n-2}) \times 3$, for $n > 2$, $a_1 = 0$ and $a_2 = 1$

C. $a_n = (n-1)a_{n-1} + (n-2)$, for $n > 1$, $a_1 = 0$

D. $a_n = a_{n-1} + n - 1$, for $n > 1$ and $a_1 = 0$ — Your answer is **Correct**

Solution :
(d)
Number of edges in complete graph with n vertices $(a_n) = \dfrac{n(n-1)}{2}$

So,
$$a_n = \frac{n(n-1)}{2} = \frac{n^2-n}{2}$$
$$= \frac{n^2-3n+2+2n-2}{2}$$
$$= a_{n-1} + n - 1, \text{ for } n > 1$$

QUESTION ANALYTICS

Level : Easy

Correct Marks : 1

Negative Marks : 0

Accuracy

66.67%

Topper's Time

00:00:00 hrs

Average Time

02:57 min

Your Time

00:01:17 hrs

Q. 3

Have any Doubt ?

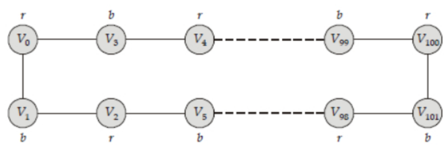
Consider a simple undirected graph $G(V, E)$ with 102 nodes labelled $V_0, V_1, V_2, \ldots V_{101}$. For node V_i, $0 \le i \le 99$, if i is even then $(V_i, V_{i+3}) \in E$, else if i is odd then $(V_i, V_{i+1}) \in E$. Other than this, (V_0, V_1) and (V_{101}, V_{100}) are in E. What is the chromatic number of graph G?

A 1

B 2 — Your answer is Correct

Solution :
(b)

G is look like



where r is red and b is black.
So, chromatic number is 2.
Basically, G is a cycle graph with even number of nodes.

C 3

D 4

QUESTION ANALYTICS

Level : Medium

Correct Marks : 1

Negative Marks : 0

Accuracy

47.62%

Topper's Time

00:00:00 hrs

Average Time

03:26 min

Your Time

00:07:53 hrs

Q. 4

Have any Doubt ?

Consider a complete undirected simple graph G with 4 nodes. Each vertex and each edge in G are labelled (different). How many different maximal matchings possible for above graph G?

A 4

B 6

C 2

D 3 — Your answer is Correct

Solution :
(d)

Size of maximal matching (MM) in G with n nodes $\le \left\lfloor \frac{4}{2} \right\rfloor$

Here, $n = 4$

$$\text{Maximum size of MM} = \left\lfloor \frac{4}{2} \right\rfloor = 2$$

$$n = 4$$

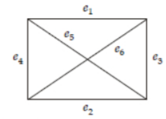
$$\text{Number of edges } (e) = \frac{n(n-1)}{2} = 6$$

Steps:
1. Selecting one edge $(e_1) = {}^6C_1 = 6$
2. On selecting one edge, two vertices has selected, now two vertices remains.
3. Only one edge (e_2) possible from two vertices. So, no need to consider it.

So, matching is (e_1, e_2).
But (e_2, e_1) also possible from above process.

So, Total number of MM = $\frac{C_1}{2} = \frac{6}{2} = 3$

Let,



MMs are $\{e_1, e_2\}$, $\{e_3, e_4\}$, $\{e_5, e_6\}$.

QUESTION ANALYTICS +

Q. 5

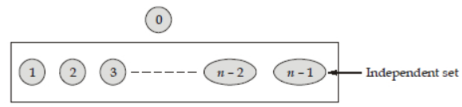
Have any Doubt ?

Consider a simple undirected graph $G(V, E)$, where V is set of vertices and E is set of edges. Size (cardinality) of a maximal independent set of G is $|V| - 1$. The minimum and maximum number of edges possible in G respectively are

A $1, |V| - 1$ Your answer is **Correct**

Solution :
(a)

Let $|V| = n$, nodes named as 0, 1, 2, ... $n - 1$, let independent set consists nodes named 1, 2, ... $n - 1$, and node 0 is not in this set. Consider following representation



Let given independent set is S.

For any V_i and V_j node from S, there is no edge. And S is maximal independent set, so you cannot add node 0 in S.

So, there is atleast one edge from node 0 to any node of S.

So minimum number of edge is 1. And for maximum, every node of S has edge with node 0.

So, $|V| - 1$ maximum edges possible.

B $1, |V|$

C $0, |V| - 1$

D 0, 1

QUESTION ANALYTICS +

Q. 6

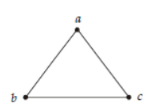
Have any Doubt ?

Number of subgraphs of k_3 with atleast one vertex are _______. (Assume all vertices are different and labelled)

17 Correct Option

Solution :
17

Let k_3 is



Cases:

(i) One vertex in subgraph, 3 vertices so 3 possibilities, and there is no edge possible here.

(ii) Two vertices in subgraph,

$\Rightarrow$ $^3C_2 = 3$ ways to select the vertices

$\Rightarrow$ $^2C_2 = 1$ edge possible in a subgraph

$\Rightarrow$ 2 possibilities for an edge (select or not)

So, Subgraph possible $= {}^3C_2 \times 2^1 = 6$

(iii) Three vertices in subgraph,

$\Rightarrow$ $^3C_3 = 1$ way to select vertices

$\Rightarrow$ $^3C_2 = 3$ edges possible in a subgraph

$\Rightarrow$ 2 possibilities for an edge.

So, Possible subgraphs $= 1 \times 2^3 = 8$

From (i), (ii), (iii)

Total number of subgraphs $= 3 + 6 + 8 = 17$

Your Answer is 10

QUESTION ANALYTICS +

QUESTION ANALYTICS

Q. 7

Have any Doubt ?

Let $(G, \square)$ be a group, where G is a set, $\square$ is a binary operation on G, and $|G| = 24$. Let x is the minimum possible order of an element in G and $x > 0$. Then the number of elements in G whose order is x are _______.

1 Correct Option

Solution :
1
Minimum possible order of an element is 1 and only identity element has order 1.

Your Answer is 12

QUESTION ANALYTICS +

Q. 8

Have any Doubt ?

Consider a sequence $\{a_n\}$, which satisfy following recurrence relation:

$$a_n = 4a_{n-2}$$

where $a_0 = 0$, $a_1 = 1$.
Which of the following is/are correct for given sequence? {Assume $\{a_n\} = a_0, a_1, a_2$ .....}

A $a_n = \frac{1}{4}(2)^n - \frac{1}{4}(-2)^n$ Your option is Correct

B $a_{2n} = 4^n + 1, n \geq 1$

C $a_{2n+1} = 4^n, n \geq 0$ Your option is Correct

D $a_{128} = 4^{64}$

YOUR ANSWER - a,c	CORRECT ANSWER - a,c	STATUS - ✓

Solution :
(a, c)
Apply homogeneous equation solution.

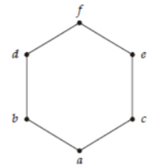
$$a_n = 4a_{n-2}$$
$$\Rightarrow \quad x^2 - 4 = 0$$
$$x = \pm 2$$
So,
$$a_n = C_1(2)^n + C_2(-2)^n$$
$$a_0 = 0 = C_1(2)^0 + C_2(-2)^0$$
$$\Rightarrow \quad 0 = C_1 + C_2 \quad ...(i)$$
$$a_1 = 1 = C_1(2)^1 + C_2(-2)^1$$
$$\Rightarrow \quad 1 = 2C_1 - 2C_2 \quad ...(ii)$$
From (i) and (ii)
$$C_1 = \frac{1}{4}, \; C_2 = -\frac{1}{4}$$
$$a_n = \frac{1}{4}(2)^n - \frac{1}{4}(-2)^n$$
$$= \frac{1}{4}(2^n - (-2)^n)$$
If, n = Even
$$a_n = 0, \text{ and so, } a_{128} = 0$$
If, n = Odd
$$a_n = \frac{1}{4}(2^n + 2^n)$$
$$= \frac{2.2^n}{4} = 2^{n-1} = (4)^{\frac{n-1}{2}}$$
Let, $n = 2k + 1$
$$a_{2k+1} = (4)^{\frac{2k+1-1}{2}} = 4^k, \text{ for } k \geq 0$$

QUESTION ANALYTICS +

Q. 9

Have any Doubt ?

Consider the following Hasse diagram of a poset (S, R), where $S = \{a, b, c, d, e, f\}$ and R is a relation:



Which of the following is/are correct?

A 4 pairs of elements are non-comparable in poset (S, R) — Correct Option

B $(d, b) \in R$ — Your answer is IN-CORRECT

C $(a, e) \in R$ — Your option is Correct

D (S, R) is a lattice — Your option is Correct

YOUR ANSWER - b,c,d

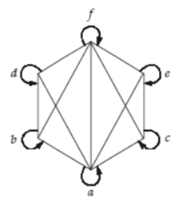
CORRECT ANSWER - a,c,d

STATUS -

Solution :

(a, c, d)

(a) Full graph for given Hasse diagram



Total number of undirected edges (without loops) = (Comparable edge) + (Non-comparable edge)

$$\frac{6(6-1)}{2} = 11 + x$$

$$x = 15 - 11 = 4$$

Here (b, e), (b, c), (d, e), (d, c) are incomparable pairs.

(b) $(d, b) \in R$, so $(d, b) \notin R$

(c) (a, c), $(c, e) \in R$, so $(a, e) \in R$

QUESTION ANALYTICS +

Q. 10

Have any Doubt ?

Let $(S, *)$ is a semi-group. For every a and b in S, if $a * b = b * a$, then $a = b$. Consider the following statements:

$S_1 : \forall a \in S\ (a * a = a)$

S_2 : For every a, b in S,

$$a * b * a = a$$

S_3 : For every a, b, c in S,

$$a * b * c = b * a$$

Which of the above statements is/are correct?

A Only S_1 and S_2 are correct — Your answer is Correct

Solution :

(a)

$S_1 \Rightarrow \quad (a * a) * a = a * (a * a)$ {Associativity}

So, $\quad a * a = a$ {from given condition consider $a * a = b$} ...(i)

$S_2 \Rightarrow \quad (a * b * a) * a = a * b * (a * a)$ {Associativity}

$= a * b * a$ {from (i)} ...(ii)

$a * (a * b * a) = (a * a) * b * a$ {Associativity}

$= a * b * a$ {from (i)} ...(iii)

(ii) and (iii) are equal.

$$(a * b * a) * a = a * (a * b * a)$$

Using given condition

$a * b * a = a$ ...(iv)

$S_3 \Rightarrow \quad (a * b * c) * (b * a) = a * (b * c * b) * a$ {Associativity}

$= a * b * a$ {from (iv)}

$= a$ {from (iv)} ...(v)

$\Rightarrow \quad (b * a) * (a * b * c)$

$= b * (a * a) * b * c$ {Associativity}

$= (b * a * b) * c$ {from (i)}

$= b * c$ {from (iv)} ...(vi)

(v) $\neq$ (vi)

So S_3 is incorrect.

B Only S_2 and S_3 are correct

C Only S_1 is correct

D All S_1, S_2, S_3 are correct

QUESTION ANALYTICS +