DISCRETE MATHEMATICS-1 (GATE 2023) - REPORTS

Q. 11 Have any Doubt ?

Consider a binary relation R on a set A and A is a set of Laptops in a shop. (x, y) is in R if cost of x is more than cost of y or RAM size in y is more than RAM size of x. Which of the following is correct?

(A) R is always reflexive and symmetric.

(B) R is always transitive and antisymmetric.

(C) R is always transitive. — Your answer is **IN-CORRECT**

(D) None of the above — Correct Option

Solution :
(d)

Let $c(\ )$ and $s(\ )$ are cost and RAM size functions respectively.
Counter examples for each case

(i) Reflexive:

$$c(x) < c(x), \text{ always false}$$
$$s(x) < s(x), \text{ always false}$$

$\forall a \in A\ ((a, a) \notin R)$
So, R is no reflexive

(ii) Symmetric:
Let $x \neq y$, $c(x) > c(y)$ and $s(x) < s(y)$ then $(x, y) \in R$, but $(y, x) \notin R$.
So, there may be the case when R is not symmetric.

(iii) Transitive:
Let $x \neq y \neq z$ and $(x, y), (y, z) \in R$. And

$$c(x) > c(y) \qquad c(y) < c(z)$$
$$s(x) > s(y) \qquad s(y) < s(z)$$

Also, $c(x) < c(z)$ and $s(x) > s(z)$.
From above conditions, $(x, y), (y, z) \in R$ but $(x, z) \notin R$. So, there can be case where R is not transitive.

(iv) Antisymmetric:
Let $x \neq y$, $c(x) > c(y)$ and $s(x) > s(y)$.
Then $(x, y), (y, x) \in R$. So, R may not be antisymmetric.

QUESTION ANALYTICS +

Q. 12 Have any Doubt ?

Consider the following statements:
S_1 : If a binary relation R on set A is symmetric, transitive, and for every a in A there exists b in A such that (a, b) is in R, then R is an reflexive relation.
S_2 : If a binary relation R is antisymmetric, then transitive closure of R is also antisymmetric.
Which of the above statements is/are correct?

(A) Only S_1 is correct — Correct Option

Solution :
(a)

S_1 : Consider any a in A, so $(a, b) \in R$ (given), now two cases:

(i) $b = a$
So, $(a, a) \in R$

(ii) $b \neq a$
R is symmetric and $(a, b) \in R$, so $(b, a) \in R$.
R is transitive and $(a, b), (b, a) \in R$.
So, $(a, a) \in R$
From (i) and (ii) cases, for every a in A, $(a, a) \in R$, So, R is reflexive.
S_1 is correct.

S_2 : Let $R = \{(a, b), (b, c), (c, a)\}$ is a relation on set $A = \{a, b, c\}$. R is antisymmetric.
Let R' is transitive closure of R, so

$$R' = \{(a, b), (b, a), (b, c), (c, b), (a, c), (c, a)\}$$

And R' is not antisymmetric.
So, S_2 is incorrect.

(B) Only S_2 is correct — Your answer is **IN-CORRECT**

(C) Both S_1 and S_2 are correct

C Both S_1 and S_2 are correct

D Neither S_1 nor S_2 is correct

QUESTION ANALYTICS +

Q. 13 Have any Doubt ?

Let $Q(x, y)$ is a statement on variable x and y, where $x, y \in \{0, 1\}$. Consider a statement $s = \forall x \exists y\ Q(x, y)$. Which of the following proposition is equivalent to statement s?

A $(Q(0, 0) \wedge Q(0, 1)) \vee (Q(1, 0) \wedge Q(1, 1))$

B $(Q(0, 0) \vee Q(1, 0)) \vee (Q(0, 1) \vee Q(1, 1))$

C $(Q(0, 0) \wedge Q(1, 0)) \vee (Q(0, 1) \wedge Q(1, 1))$ Correct Option

Solution :
(c)
Try to figure out the answer using given s in form of English statement.
Let say, for all value of x, which are 0 and 1, there is a y which can be 0 or 1, or both.
Now compare this statement with option (c). Others options are

$$\text{Option (a)} = \exists x \forall y\ Q(x, y)$$
$$\text{Option (b)} = \exists x \exists y\ Q(x, y)$$
$$\text{Option (d)} = \forall x \forall y\ Q(x, y)$$

D $(Q(0, 0) \wedge Q(1, 0)) \wedge (Q(0, 1) \wedge Q(1, 1))$

QUESTION ANALYTICS +

Q. 14 Have any Doubt ?

Consider a three groups A, B and C, where groups A, B and C consist 3, 4 and 2 different teachers respectively. Now, principal of school wants to make a new group G which consists atmost one teacher from group A and atleast one teacher from group B and C each. Number of ways to create group G are ______.

180 Your answer is Correct180

Solution :
180
Number of ways to select atleast one teacher from B = (Total ways to select from B) – (no-one selected from B)

$$= 2^4 - 1 \qquad \text{...(i)}$$

Number of ways to select at least one teacher from C = (Total ways to select from C) – (no-one selected from C)

$$= 2^2 - 1 \qquad \text{...(ii)}$$

Two cases for selection from A with other
(i) No-one selected from A and atleast one from B and C

$$= 1 \cdot (2^4 - 1)(2^2 - 1)$$
$$= 15 \times 3 = 45 \qquad \text{...(iii)}$$

(ii) One selected from A with others

$$= {}^3C_1 \cdot (2^4 - 1)(2^2 - 1)$$
$$= 3 \times 15 \times 3 = 135 \qquad \text{...(iv)}$$

From (iii) and (iv)
Total number of ways to create G = 45 + 135 = 180

QUESTION ANALYTICS +

Q. 15 Have any Doubt ?

Let U is a universal set and A, B are subsets of U. And $|A \cap B| = 5$, $|(A \cup B)^C| = 18$, $|A^C| = 20$ and $|B^C| = 22$. Let $|A \cup B| = x$ and $|A - B| = y$, then $x + y$ is ______. (A^C is complement of A with respect to U and same for others)

15 Your answer is Correct15

Solution :
15

$$\text{Let,} \quad |U| = t$$
$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$t - |(A \cup B)^C| = (t - |A^C|) + (t - |B^C|) - |A \cap B|$$

$$
\begin{aligned}
t - 18 &= (t - 20) + (t - 22) - 5 \\
t &= 29 \\
|A \cup B| &= t - 18 = 29 - 18 = 11
\end{aligned}
$$

So, $x = 11$

$$
\begin{aligned}
|A - B| &= |A| - |A \cap B| \\
&= t - |A^C| - |A \cap B| \\
&= 29 - 20 - 5 = 4
\end{aligned}
$$

So, $y = 4$

$$x + y = 11 + 4 = 15$$

QUESTION ANALYTICS +

Q. 16

Have any Doubt ?

Let p, q and r are propositions. Which of the following is/are correct for logical equivalence?

A $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$ Your option is Correct

B $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$ Your option is Correct

C $(p \to r) \wedge (q \to r) \equiv (p \wedge q) \to r$

D $(p \to r) \vee (q \to r) \equiv (p \vee q) \to r$

YOUR ANSWER - a,b

CORRECT ANSWER - a,b

STATUS -

Solution :

(a, b)

(a) $(p \to q) \wedge (p \to r) = (p' \vee q) \wedge (p' \vee r)$

$= p' \vee (q \wedge r)$ {Using distributive property}

$= p \to (q \wedge r)$

$=$ RHS

(b) $(p \to q) \vee (p \to r) = (p' \vee q) \vee (p' \vee r)$

$= p' \vee q \vee r$

$= p' \vee (q \vee r)$ {Using associative property}

$= p \to (q \vee r)$

$=$ RHS

(c) $(p \to r) \wedge (q \to r) = (p' \vee r) \wedge (q' \vee r)$

$= (p' \wedge q') \vee r$ {Using distributive property} ...(i)

$(p \wedge q) \to r = (p \wedge q)' \vee r$

$= (p' \vee q') \vee r$ {De Morgan's law} ...(ii)

Equation (i) $\neq$ (ii)

(d) $(p \to r) \vee (q \to r) = (p' \vee r) \vee (q' \vee r)$

$= (p' \vee q') \vee r$ {Associative property} ...(iii)

$(p \wedge q) \to r = (p \vee q)' \vee r$

$= (p' \wedge q') \vee r$ {De Morgan's law} ...(iv)

Equation (iii) $\neq$ (iv)

QUESTION ANALYTICS +

Q. 17

Have any Doubt ?

Let $P(x, y, z)$ be the statement "$x + y = z$", where x, y and z are variables and domain of all variables consists of all real numbers. Which of the following statements is/are true?

A $\forall x\, \forall y\, \exists z\, P(x, y, z)$ Your option is Correct

B $\exists z\, \forall x\, \forall y\, P(x, y, z)$

C $\exists z\, \forall x\, \forall y\, P(x, y, z)$ Correct Option

D $\forall x\, \exists y\, \exists z\, P(x, y, z)$ Your option is Correct

YOUR ANSWER - a,d

CORRECT ANSWER - a,c,d

STATUS -

Solution :

(a, c, d)

R = Real number set

(a) Statement says "for all x and for all y there is a z for which $x + y = z$", is true.

Because, if we select any x and a and y from R then $(x + y) \in R$ and it will be only one number, so there is a z.

(b) It says "there is a z for which all x and all y, then $x + y = z$", is false. Because, for a fixed one $z \in R$, $x + y = z$ will not satisfy for all x and all y.

(c) We know, $\forall t\ Q(t)$ is true, then $\exists t\ Q(t)$ also true.

So, let $\forall y\ \exists z\ P(x, y, z) = Q(x, y, z)$ then from (a), $\forall x\ \forall y\ \exists z\ P(x, y, z)$ is true, so $\forall x\ Q(x, y, z)$ is also true, then $\exists x\ Q(x, y, z)$ is also true, then $\exists x\ \forall y\ \exists z\ P(x, y, z)$ is also true.

(d)
$$S = \forall x\ \exists y\ \exists z\ P(x, y, z)$$

Let
$$\exists y\ \exists z = Q(x, y, z)$$
$$S = \forall x\ Q(x, y, z)$$

From (a), $\forall x\ \forall y\ \exists z\ P(x, y, z)$ is true,

Let
$$\forall y\ \exists z\ P(x, y, z) = T(x, y, z)$$

Let $\forall y\ \exists z\ P(x, y, z)$ is true, then $\exists y\ \exists z\ P(x, y, z)$ is also true.

From (a),

$\Rightarrow$
$$\begin{aligned} S &= \forall x\ \forall y\ \exists z\ P(x, y, z) \\ &= \forall x[\forall y\ (\exists z\ P(x, y, z))] \\ &= \text{True} \end{aligned}$$

So, $S_2 = \forall x[\exists y\ (\exists z\ P(x, y, z))]$ is also true using above statement

QUESTION ANALYTICS +