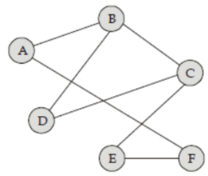
DISCRETE MATHEMATICS (GATE 2023) - REPORTS

Q. 21 Have any Doubt ?

Consider the simple graph G given in figure given below:



Then, which of the following statement(s) is/are correct?

A. Diameter of the graph is 3. — Your option is **Correct**

B. (A, F, E, C, D, B, A) is the only Hamiltonian cycle present in the graph.

C. Graph is 3-colorable. — Your option is **Correct**

D. Graph doesn't have any Eulerian cycle. — Your option is **Correct**

YOUR ANSWER - a,c,d | CORRECT ANSWER - a,c,d | STATUS - ✔

Solution :

(a, c, d)
1. Recall that the diameter is the maximum of all shortest path lengths between pairs of vertices. Note that the shortest path length between D and F is 3, and all other pairs of non-adjacent vertices share a neighbor.
2. One possible solution is (A, F, E, C, D, B, A). This cycle and its reverse should constitute all possible solutions.
3. One possible 3-coloring is: {A, D, E} red; {B, F} green; C blue. Because there exists an oddlength cycle (e.g. (B, D, C)), no 2-coloring exists. Therefore the given coloring uses the least possible number of colors.
4. True. This follows from the fact that there exist vertices with odd degree; e.g. B.

QUESTION ANALYTICS +

Q. 22 Have any Doubt ?

Alice at (0, 0) and walks to (3, 3) on the Cartesian plane without crossing the line $y = x$ (though she might just touch it). At each step, Alice either walks one step to the right or one step up wards. Then number of valid paths.

10 — Correct Option

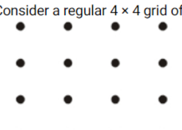
Solution :
10
We can manually find the number of paths. At any instant number of upward instance $\leq$ Number of rightward.
5 for above $y = x$, and 5 for below.

Your Answer is 9

QUESTION ANALYTICS +

Q. 23 Have any Doubt ?

Consider a regular 4 × 4 grid of sixteen points, as in this picture:



How many triangles can be formed whose corners lie on the grid? (A triangle has to have nonzero area)

516 — Your answer is **Correct**516

Solution :
516

The number of ways of picking three points from the grid is ${}^{16}C_3 = 560$. These three points will form a degenerate triangle (one with zero area) if and only if they all lie in a straight line. This straight line may be one of the rows, one of the columns, or one of the diagonals. The number of ways of picking three points, all from one row of the grid, is $4 \times {}^4C_3 = 16$ (pick the row, then pick 3 points within the row). Symmetrically, the number of ways of picking three points, all from the same column, is 16. Now let's try the diagonals. There are three "bottom-left" to "top-right" diagonals with at least three points the middle one has 4 points and the others have 3 each. The number of ways of picking three points, all from one of these diagonals, is ${}^4C_3 + 2 \times {}^3C_3 = 6$. The situation for "bottom-right" to "top-left" diagonals is identical and also gives 6 possibilities.
So the total number of ways of picking a degenerate triangle is $2 \times 16 + 2 \times 6 = 44$, and the number of ways of picking a non-degenerate triangle is $560 - 44 = 516$.

QUESTION ANALYTICS +

Q. 24

Have any Doubt ?

Consider a relation α on the set of function from N^+ to R, such that f α g iff $f = O(g)$. Then which of the following is correct?

A α is an equivalence relation

B α is an partial order relation

C α is an total order relation

D α is reflexive but not symmetric. Correct Option

Solution :
(d)

Recall that an equivalence relation is reflexive, symmetric and transitive. α is reflexive and transitive but not symmetric let $f(n) = n$, $g(n) = n^2$.

Here, $f = O(g)$ but $g \neq O(f)$

It is also clearly not antisymmetric, if $f(n) = n$, $g(n) = 2n$, $f = O(g)$ and $g = O(g)$ but $f \neq g$. This prevents α from being a partial order and thus it is not a total order also.

QUESTION ANALYTICS +

Q. 25

Have any Doubt ?

Which of the following statements is correct?

A If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$. Your option is Correct

B If $A \subseteq B$ and $B \subseteq C$, then $A \subset C$. Your answer is IN-CORRECT

C If $A \in B$ and $B \in C$, then $A \in C$. Your answer is IN-CORRECT

D If $A \in B$ and $B \in C$, then $A \notin C$. Correct Option

YOUR ANSWER - a,b,c

CORRECT ANSWER - a,d

STATUS -

Solution :
(a, d)

- Consider any element $a \in A$. Since $A \subseteq B$ every element of A is also an element of B, so $a \in B$. By same reasoning $a \in C$ since $B \subseteq C$. Thus, every element of A is an element of C, so $A \subseteq C$.
- Let $A = \{b\}$, $B = \{b, \{1\}\}$ and $C = \{\{b, \{1\}\}, \{4\}\}$ these satisfy $A \in B$ and $B \in C$ but $A \notin C$.

QUESTION ANALYTICS +

Q. 26

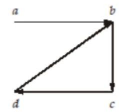
Have any Doubt ?

Let $A = \{a, b, c, d\}$ and let $R = \{(a, b), (b, c), (c, d), (d, b)\}$ be a relation on A. Then what will be the transitive closure R^* of R.

A $R^* = \{(a, b), (a, c), (a, d), (b, b), (b, c), (b, d), (c, b), (c, d), (d, b), (d, c), (d, d)\}$

B $R^* = \{(a, b), (a, c), (a, d), (b, b), (b, c), (b, d), (c, b), (c, c), (c, d), (d, b), (d, c), (d, d)\}$ Your answer is Correct

Solution :
(b)
Directed graph representing R:



If there is direct/indirect path between two elements then they will be in transitive closure.
Only option (b) satisfies.

C R^{*} = {(a, b), (b, c), (c, d), (d, b)}

D R^{*} = {(a, c), (a, d), (b, b), (b, d), (c, b), (c, c), (d, c), (d, d)}

QUESTION ANALYTICS +

Q. 27

Have any Doubt ?

A cop goes into a donut store and wishes to get a dozen. How many options does the officer have if she/he can choose from 5 different types of donuts and wishes to get at least one of each?

330 — Your answer is **Correct**330

Solution :
330
If the cop wants at least one of each type, he/she is really only choosing 7 donuts, not 12.
Thus this is equivalent to choosing 7 unordered items from 5 types with repetition; the answer is

$$^{7+5-1}C_7 = {}^{11}C_7 = 330$$

QUESTION ANALYTICS +

Q. 28

Have any Doubt ?

Finalphobia is a rare disease in which the victim has the delusion that he or she is being subjected to an intense mathematical examination:

I. A person selected uniformly at random has finalphobia with probability $\frac{1}{100}$.

II. A person with finalphobia has shaky hands with probability $\frac{9}{10}$.

III. A person without finalphobia has shaky hands with probability $\frac{1}{20}$.

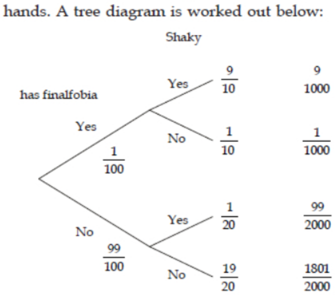
What is the probability that a person selected uniformly at random has finalphobia, given that he or she has shaky hands?

A $\frac{18}{1000}$

B $\frac{99}{1000}$

C $\frac{18}{117}$ — Your answer is **Correct**

Solution :
(c)
Let F be the event that the randomly-selected person has finalphobia, and let S be the event that he or she has shaky hands. A tree diagram is worked out below:



The probability that a person has finalphobia, given that he/she has shaky hands is

$$\Pr\left(\frac{F}{S}\right) = \frac{\Pr(F \cap S)}{\Pr(S)} = \frac{\frac{9}{1000}}{\frac{9}{1000} + \frac{99}{2000}} = \frac{18}{117}$$

D $\frac{99}{117}$

QUESTION ANALYTICS +

Q. 29

Have any Doubt ?

Let $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$ be a matrix representing a relation R on a set A. Then which of the following is correct?

A R is reflexive — Your answer is **Correct**

Solution :
(a)
Since if all $(a, a) \in R$ thus reflexive.

B R is symmetric

C R is irreflexive

D R is antisymmetric

QUESTION ANALYTICS +

Q. 30

Have any Doubt ?

Consider the sequence of the first $2n$ positive integers. In how many ways can you order it such that no two consecutive terms have a sum divisible by 2?

A $2(n!)^2$ — Your answer is **Correct**

Solution :
(a)
Two consecutive terms will sum to an even number if both are odd or both are even, so consecutive terms must always have opposite parities. Since there will be n numbers of each parity, the only way to arrange them is to have all odd-numbered terms be odd and all even-numbered terms even, or vice-versa, so 2 possibilities. After that decision, you can have any arrangement of the n odd and n even numbers, so the total number of ways is $2(n!)^2$.

B $(n!)^2$

C $2^n(n!)^2$

D $1 + 2(n!)^2$

QUESTION ANALYTICS +