Permutation and Combination

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Chapter 1

Permutation

1.1 Formulae for Permutation

ways of arranging n thing in r places is

$$^{n}P_{r} = \frac{n!}{(n-r)!} \tag{1.1}$$

Trick

$$^{10}P_{3} = 10 \times 9 \times 8$$

3 Factors from 10 in decending order.
Common Values

$$^{n}P_{0} = 1 \tag{1.2}$$

$$^{n}P_{n} = n \tag{1.3}$$

1.2 Repeated

If n things are arranged and p are of one kind, q are of one kind etc. then no of ways of arranging is

$$\frac{n!}{p!q!\dots} \tag{1.4}$$

1.3 Alternative arragement

4 Boys and 4 girls sit alternatively

$$4! \times 4! \times 2$$

3 Boys and 4 girls sit alternatively

$$3! \times 4!$$

This kind of arrangement is only possible when numbers are either equal or consequitive.

Chapter 2

Combination

2.1 Formulae for Combination

Ways of selecting r things out of n things is

$$^{n}C_{r} = \frac{n!}{(n-r)!r!}, \text{ where } n > r \tag{2.1}$$

Trick

$$^{10}C_{3} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} \tag{2.2}$$

3 terms in decending order starting from 10 divided by factorial 3.
Relation to Permutaiton

$$^{n}P_{r} = r!\,^{n}C_{r} \tag{2.3}$$

Relations in Combination Formulae

$$^{n}C_{r} = \,^{n}C_{n-r} \tag{2.4}$$

$$^{n}C_{x} = \,^{n}C_{y} \tag{2.5}$$

$$\implies n = x + y$$

Common Values

$$^{n}C_{0} = 1 = \,^{n}C_{n} \tag{2.6}$$

$$^{n}C_{1} = n \tag{2.7}$$

2.2 Polygon

2.2.1 Number of Triangles

Number of triangles in a polygon having n sides is

$$^nC_3$$

Number of equilateral triangles in hexagon = 2

2.2.2 Number of Diagonals

Number of diagonals in a polygon having n sides is

$$^nC_3 - n = \frac{n(n-3)}{2}$$

2.3 Number of Squares and Rectangles

For a figure having m horizontal and n vertical lines,

$$\text{Number of Rectangles} = {}^mC_2 \times {}^nC_2 \tag{2.8}$$

Number of Squares can be found using the number of boxes instead of lines
For a grid having m rows and n columns

$$\text{Number of squares} = mn + (m-1)(n-1) + (m-2)(n-2) + \dots \tag{2.9}$$

until one of the terms becomes 1

2.4 Dearrangement

If *all* n things in wrong place then number of ways is

$$n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots \text{upto n terms}\right] \tag{2.10}$$

2.5 Sum of digit on unit place

Sum of all the digits in unit place of all the numbers formed with the help of given digits is

$$(n-1)![\text{sum of digits}] \tag{2.11}$$

Chapter 3

Circlular Permutation

3.1 Formulae

Number of ways of arranging n thing in a circle is

$$(n-1)! \tag{3.1}$$

If the arrangement is the same both clockwise and anti clockwise then the number of ways of arranging n things in a circle is

$$\frac{1}{2}(n-1)! \tag{3.2}$$