Probability

Abhyas Kumar Kanaujia

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Chapter 1

Probability

1.1 Types

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1.1.1 Mutually Exclusive Event

No common points between E_1 and E_2

$$E_1 \cap E_2 = \phi \tag{1.1}$$

Formula

$$P(A \cup B) = P(A) + P(B) \tag{1.2}$$

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\ P(A \cup B) &= P(A \text{ or } B) = P(A + B) \end{aligned}$$

1.1.2 Non-Mutually Exclusive Event

At least one common point between E_1 and E_2

$$E_1 \cap E_2 \neq \phi \tag{1.3}$$

Fromula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{1.4}$$

$$\begin{aligned} P(A \cup B \cup C) =& P(A) + P(B) + P(C) - P(A \cap B) \\ & - P(B \cap C) - P(C \cap A) + P(A \cup B \cap C) \\ P(A \cup B) =& P(A) + P(B) - P(A \cup B) \end{aligned}$$

Happening ofexactly one out of two event

$$P(\text{One out of two}) = P(A) + P(B) - 2P(A \cap B) \tag{1.5}$$

1.1.3 Independent Event

$$P(A \cap B) = P(A) \cdot P(B) \tag{1.6}$$

1.1.4 Dependent Event (Conditional Probability)

Probability of happening of A based on previous event B.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \tag{1.7}$$

Similarly,

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

1.2 Formulae for Probability

$$\text{Probability of Happening} = \frac{\text{No. of Fav. Cases}}{\text{No. of Total Cases}} \tag{1.8}$$

$$P(H) = \frac{\text{Fav. Cases}}{\text{Fav. Cases + Unfav. Cases}} \tag{1.9}$$

$$\text{Probability of not Happening} = \frac{\text{No. of Unfav. Cases}}{\text{No. of Total Cases}} \tag{1.10}$$

$$P(\overline{H}) = \frac{\text{UnFav. Cases}}{\text{Fav. Cases + UnFav. Cases}} \tag{1.11}$$

$$P(H) + P(\overline{H}) = 1 \tag{1.12}$$

$$0 \leq P(H) \leq 1$$

where 0 is impossible event and 1 is sure event.

$$P(\text{At least once Happening}) = 1 - P(\overline{H}) \tag{1.13}$$

1.3 Concept of coin

1.3.1 Coins and $p = q = \frac{1}{2}$

n coins thrown at random, getting exactly r successes

$$P(x = r) = \frac{{}^nC_r}{2^r} \tag{1.14}$$

At least one

$$1 - q^n \tag{1.15}$$

1.3.2 Denominator

Total number of outcomes for tossing n coins is

$$2^n \tag{1.16}$$

Either head or tail in each coin.

1.4 Concept of Dice

1.4.1 Sum Of Dice Problem

Two Dice

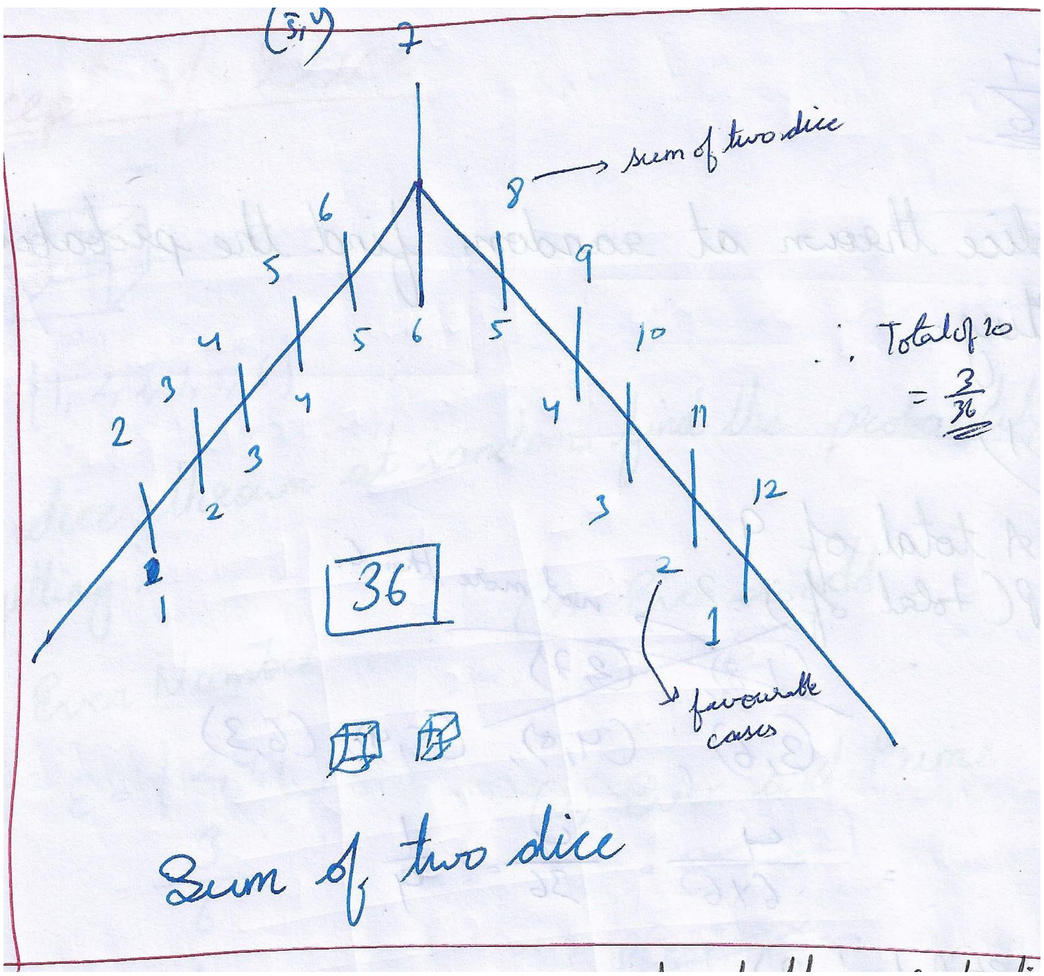

Figure 1.1: Pyramid of Sum of Two Dice

Three Dice

Sum	**Favourable Cases**	**Sum**
3	1	18
4	3	17
5	6	16
6	10	15
7	15	14
8	21	13
9	25	12
10	27	11

Sum= $6 \times 6 \times 6 = 216$

1.4.2 Denominator

Total number of outcomes for rolling n dice is

$$6^n \tag{1.17}$$

Either one of six values in each die.

Common denominators

- 1 die = 6 outcomes
- 2 dice = 36 outcomes
- 3 dice = 216 outcomes

1.5 Concept of Leap Year

Non Leap Year $\rightarrow$ 365 Days $\rightarrow$ 1 odd Days
Leap Year $\rightarrow$ 366 Days $\rightarrow$ 2 odd Days

$$P(52 \text{ Sundays}) = 1 \rightarrow \text{Sure event}$$
$$P(54 \text{ Sundays}) = 0 \rightarrow \text{Impossible event}$$

Each week day at least 52 times
Non Leap Year $\rightarrow$ One week day 53 times, rest 52 times
Leap Year $\rightarrow$ 2 week days 53 times, rest 52 times

Non Leap Year

Sun/Mon/Tue/Wed/Thur/Fri/Sat = 1

$$\therefore\ P(53 \text{ Sun}) = \frac{1}{7} \text{ etc.}$$

Leap Year

Sun-Mon/Mon-Tue/Tue-Wed/Wed-Thur/Thur-Fri/Fri-Sat/Sat-Sun = 1

$$\therefore\ P(53 \text{ Sun}) = \frac{1}{7}$$
$$P(53 \text{ Sun or Mon}) = \frac{3}{7}$$

1.6 Concept of Cards

Total 52 Cards:

1. by Color

   26 Red (Heart and Diamond)

   26 Black (Spade and Club)

2. By Suite

   13 Hearts (Red)

   13 Diamond (Red)

   13 Club (Black)

   13 Spade (Black)

3. By Type in each suite

   10 Number Cards (A, 2, ...,10)

   3 Face Cards (Jack(J), Queen(Q), King(K))

   4 Winning/Power/Respective Cards (A, J, Q, K)

Value of Power Cards

- A $\rightarrow$ 1

- J → 11
- Q → 12
- K → 13

1.6.1 Denominator

Total number of ways of drawing 1 card at random = ${}^{52}C_1 = 52$ Total number of ways of drawing 2 cards at random = ${}^{52}C_2 = 1326$

1.7 Concept of Truth and Lie

At least one of A and B speaks the truth = $P(A)P(\overline{B}) + P(\overline{A})P(B)$

Same formula for when they contradict eachother.

1.8 Concpet of Turn

If A and B take turn doing a task then Probability that A will win first is

$$\begin{aligned} P(\text{A wins first}) =& P(A_1) + P(\overline{A_1})P(\overline{B_1})P(A_2) + \\ & P(\overline{A_1})P(\overline{B_1})P(\overline{A_2})P(\overline{B_2})P(A_3) + \dots \end{aligned}$$

Use infintie GP.

General Formula for n people

$$P(A) = \frac{2^{n-1}}{2^n - 1} \tag{1.18}$$

$$P(B) = \frac{P(A)}{2} \tag{1.19}$$

$$P(C) = \frac{P(B)}{2} \tag{1.20}$$

until n-th Person.

See notes for Map making.

1.9 Concept of Lottery

See notes

1.10 Concept of Chess Board

See notes

Memorize Answers

Two squares are choosen at random on a chess-board, the Probability that they have a side in common is $\frac{1}{18}$

1.11 AP Formula

See notes.

Out of $(2n+1)$

$$P(AP) = \frac{3n}{4n^2-1} \tag{1.21}$$

Out of $(2n+2)$

$$P(AP) = \frac{3}{4n+2} \tag{1.22}$$

1.12 Odds

These formulae are derived from

$$P(H) = \frac{FC}{FC+UFC}$$
$$P(\overline{H}) = \frac{UFC}{FC+UFC}$$

1.12.1 Odds in Favour

$$\text{Odds in Favour of an event} = \frac{\text{Num. of Fav. Cases}}{\text{Num. of Unfav Cases}} \tag{1.23}$$

1.12.2 Odds Against

$$\text{Odds in Favour of an event} = \frac{\text{Num. of Unfav. Cases}}{\text{Num. of Fav Cases}} \tag{1.24}$$

1.13 Formula for Binomial Distribution

Only applicable for Independent event.

From n trials, r success, p Probability of happening and q Probability of not happening then

$$P(X = r) = {}^nC_r q^{n-r} p^r \tag{1.25}$$

This is obtianed from $(q + p)^n$

1.14 Bayes Theorem

Probability of occurrence of an event related to any condition.

$$P(A/B) = \frac{P(A)P(B/A)}{P(B)} \tag{1.26}$$

1.15 Binomial Distribution

$$(q + p)^n \tag{1.27}$$

$$P(X = r) = {}^nC_r q^{n-r} p^r \tag{1.28}$$

$$\text{Mean} = np \tag{1.29}$$

$$S.D = \sigma = npq \tag{1.30}$$

$$Var = \sigma^2 = \sqrt{npq} \tag{1.31}$$