Today we will study integrals

$$\int \arctan x dx = \frac{1}{1+x^2} \arctan x + C$$

$$\frac{d}{dx}(4x^2 + 3x) = 8x + 3$$

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Problem 7.1.6

$$\int x^2 e^{3x} dx = ?$$

We can solve this integral by using integration by parts. Let $u = x^2$ and $dv = e^{3x}dx$. Then $du = 2xdx$ and $v = \frac{1}{3}e^{3x}$. Substituting into the formula for integration by parts, we get:

$$\int x^2 e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \int \frac{2}{3} x e^{3x} dx$$

Now, we can use integration by parts again to evaluate the remaining integral. Let $u = \frac{2}{3}x$ and $dv = e^{3x}dx$. Then $du = \frac{2}{3}dx$ and $v = \frac{1}{3}e^{3x}$. Substituting into the formula for integration by parts, we get:

$$\int \frac{2}{3} x e^{3x} dx = \frac{1}{3} x e^{3x} - \int \frac{2}{9} e^{3x} dx$$

Combining this result with the previous one, we get:

$$\int x^2 e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \frac{1}{3} x e^{3x} + \int \frac{2}{9} e^{3x} dx$$

Now, we can evaluate the remaining integral by using the power rule of integration. We get:

$$\int x^2 e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \frac{1}{3} x e^{3x} + \frac{2}{9} \cdot \frac{1}{3} e^{3x} + C$$

Simplifying, we get:

$$\int x^2 e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \frac{1}{3} x e^{3x} + \frac{2}{9} e^{3x} + C$$

where C is the constant of integration.