Bootcamp 6: Reinforcement Learning

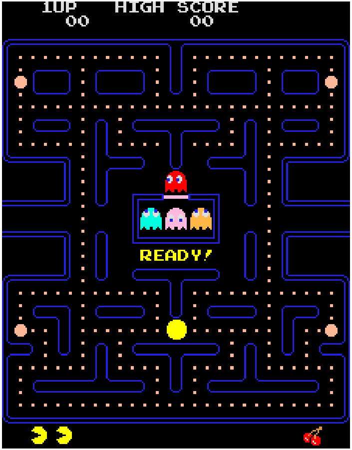
William H. Guss, James Bartlett
{wguss, james}@ml.berkeley.edu
Machine Learning at Berkeley

April 22, 2016

Overview

Problem: ML for Pacman.

How would you solve pacman with machine learning?

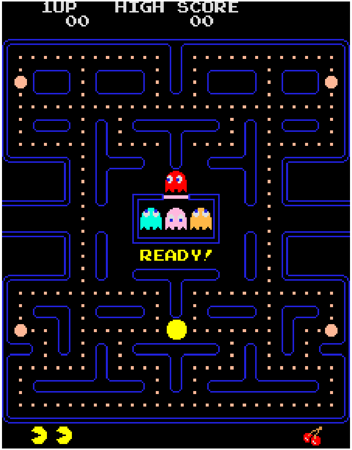


Problem: ML for Pacman.

How would you solve pacman with machine learning?

Find a model which takes screen pixels to actions:

$$\pi_\theta : s_t \mapsto a_t.$$



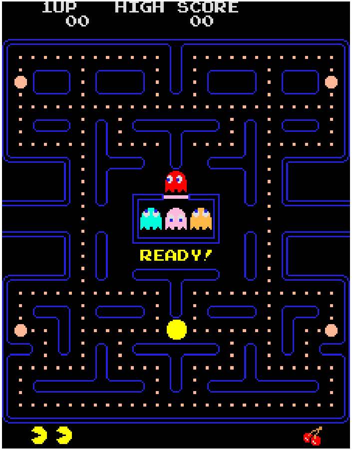
Problem: ML for Pacman.

How would you solve pacman with machine learning?

Find a model which takes screen pixels to actions:

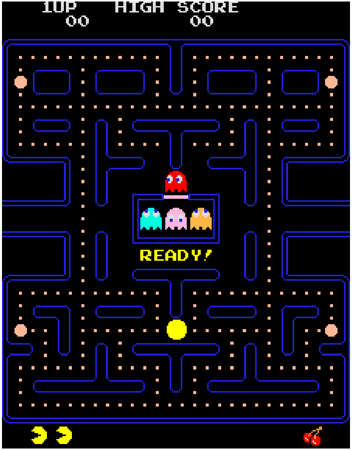
$$\pi_\theta : s_t \mapsto a_t.$$

What is your loss function? Data?



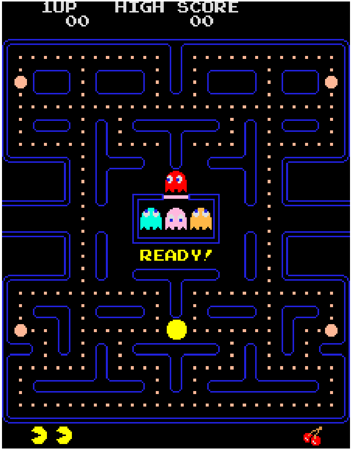
Problem: ML for Pacman.

$\Longrightarrow$



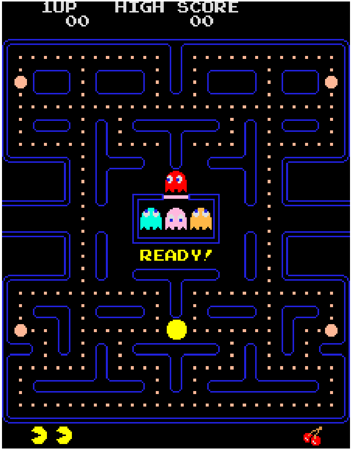
Solution: Reinforcement Learning

- Supervised learning is *not* the most general formulation of learning.



Solution: Reinforcement Learning

- Supervised learning is *not* the most general formulation of learning.
- Humans learn through reward and penalty



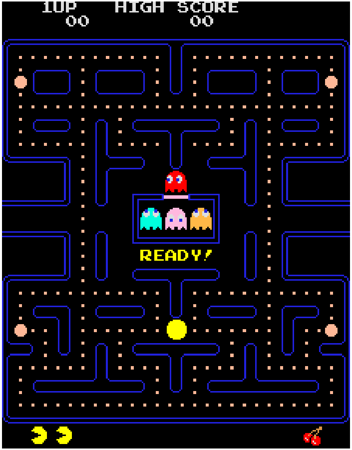
Solution: Reinforcement Learning

- Can we make algorithms which improve with crude reward signals?

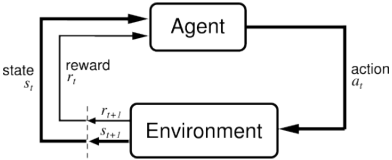
Machine learning without explicit objective functions

$\Downarrow$

Reinforcement Learning (RL)



The Core Idea



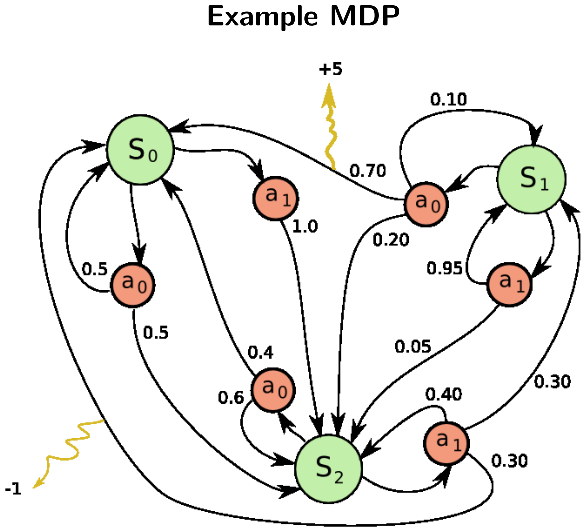
- Models (agents) take action a_t in some environment.
- Environment provides state s_t, reward r_t.
- Models learn to maximize reward r_t, $\forall t$.

Markov Decision Process (MDP)

Environment, $E = (\mathcal{S}, \mathcal{A}, \mathcal{R}, \rho, r)$.

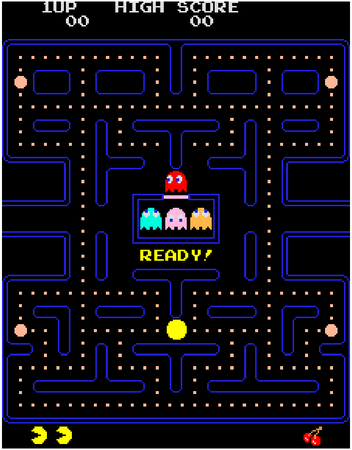
1. State space, $\mathcal{S}$
2. Action space, $\mathcal{A}$
3. Reward space, $\mathcal{R}$
4. Transition distribution, $\rho(s' \mid s, a)$. Given a previous state s and action a, environment gives s'.
5. Reward function $r(s, a) \in \mathcal{R}$.

Markov Property: $\rho(s' \mid s, a)$ depends only on s, a not previous states!


Example MDP
+5
0.10
0.70
1.0
0.20
0.95
0.5
0.5
0.4
0.6
0.05
0.40
0.30
0.30
-1
S_0
S_1
S_2
a_0
a_1

Pacman as an MDP

- $\mathcal{S} = \mathbb{R}^{256\times 256}$, images as state space.
- $\mathcal{A} = \{\uparrow, \downarrow, \rightarrow, \leftarrow\}$, joystick as action space.
- $r(s_t, a_t) =$ change in score.
- $\rho(s_{t+1} \mid s_t, a_t) =$ next frame of game after joystick action a_t.



Policies/Agents

Two different types of agents

- Deterministic policy $a = \pi(s)$ acts in E.
- Stochastic policy $a \sim \pi(a|s)$ gives a probability distibution over actions.

Policy Trajectories

$$s_1 \xrightarrow{\pi} a_1 \xrightarrow{\rho, r} s_2, r_2 \xrightarrow{\pi} a_2 \xrightarrow{\rho, r} \cdots$$

Value under a policy

The **state value** is a function of a given state for an agent π defined as

$$V^\pi(s_t) = \mathbb{E}\left[\sum_{n=t+1}^{\infty} \gamma^n r(s_n, \pi(s_n))\right]$$

1. γ is the discount factor
2. $\pi(s_n)$ is the action the agent π makes after seeing state s_n.
3. $r(s_n, \pi(s_n))$ is the reward the agent gets from taking that action.

Value under a policy

The **state-action value** for an agent π is defined such that

$$Q^\pi(s_t, a_t) = \mathbb{E}\left[\underbrace{r(s_t, a_t)}_{\text{reward for } a_t} + V^\pi(s_t)\right]$$

- Given some state s_t, the *best* agent, π^* is one that take action

$$a_t = \underset{a}{\operatorname{argmax}}\, Q(s_t, a).$$

Problems in Reinforcement Learning

- **Policy Optimization:** maximize the expected reward with respect to a policy π;

$$\pi^* = \underset{\pi}{\operatorname{argmax}} \, \mathbb{E}\left[\sum_{t=0}^{\infty} r_t\right]$$

- **Policy Evaluation:** Given some fixed policy π compute expected return.
  - Computing Q^π, V^π, and other expectations on policy rollout.
  - Lets us perform policy optimization!

Q-Learning (State-action Value Iteration)

The action-value function (simplified).

1. The future expected reward of an agent π is

$$Q^\pi(s_t, a_t) = \underbrace{r(s_t, a_t)}_{\text{reward for } a_t} + \sum_{n=t+1}^{\infty} \gamma^n r(s_n, \pi(s_n))$$

2. The Bellman equation gives us

$$Q^\pi(s_t, a_t) = r_t + \gamma Q^\pi(s_{t+1}, \pi(s_{t+1}))$$

''

3. Given some state s_t, the **best** agent, π^* is one that take action

$$a_t = \arg\max_a Q(s_t, a).$$

Q-Learning (State-action Value Iteration)

TODO: DO THESE SLIDES The action-value function (simplified).

1. The Q function for π^* is

$$Q^*(s_t, a_t) = r_t + \gamma \arg\max_a Q^\pi(s_{t+1}, a).$$

2. We can *approximate* this with deep learning!
   1. Make a neural network $\mathcal{N} : \mathcal{S} \to \mathbb{R}^n$ which predicts the future reward of taking each possible action

$$\mathcal{N}(s_t) = \begin{pmatrix} Q^*(s_t, a_1) \\ Q^*(s_t, a_2) \\ \vdots \\ Q^*(s_t, a_n) \end{pmatrix}$$

Reinforcement Learning

Guss & Bartlett

Introduction

Theory

Algorithms

Questions

Deep Q-Learning

$$s_t \rightarrow \mathcal{N} \rightarrow$$

$$Q^*(s_t, a_1) = -12$$

$$Q^*(s_t, a_2) = 0.17$$

$$Q^*(s_t, a_3) = 0.22$$

$$Q^*(s_t, a_4) = 0.03$$

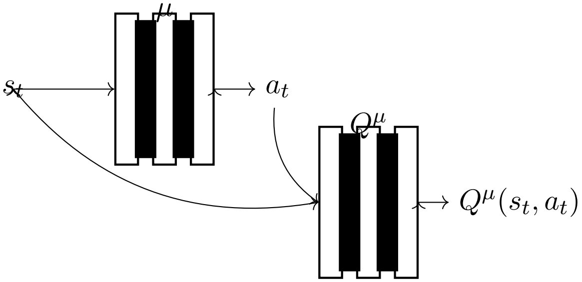
$$Q^*(s_t, a_5) = -3.2$$

$$\pi(s_t) = \arg\max_a \mathcal{N}(s_t)$$

Policy Iteration

Deep Determisitic Policy Gradient

1. Actor neural network $\mu : \mathcal{S} \to \mathcal{A}$
2. Critic network $Q^\mu : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$
3. Performance of μ is $Q^\mu(s_t, \mu(s_t))$. **Maximize performance!** $\nabla_W Q^\mu(s_t, a_t) = \nabla_a Q^\mu(s_t, a) \cdot \nabla_W \mu(s_t)$



Questions?