Bootcamp 6: Reinforcement Learning

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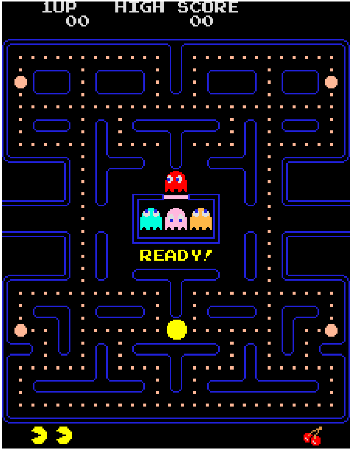
Overview

Problem: ML for Pacman.

How would you solve pacman with machine learning?

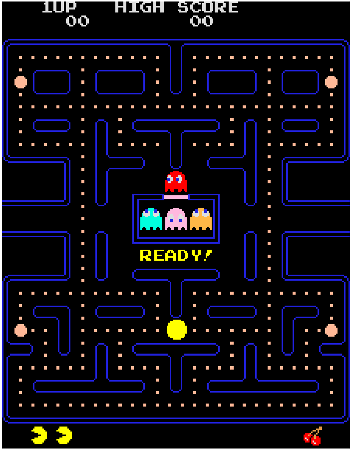


Problem: ML for Pacman.

How would you solve pacman with machine learning?

Find a model which takes screen pixels to actions:

$$\pi_\theta : s_t \mapsto a_t.$$



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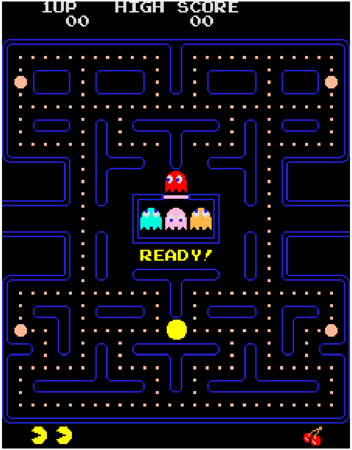
What is your loss function? Data?



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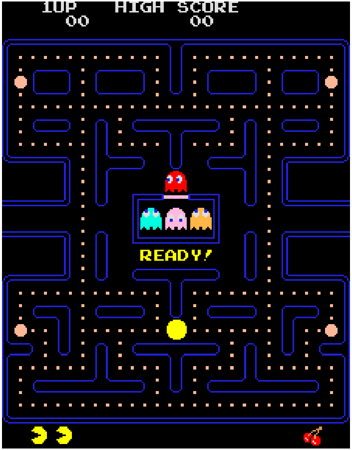
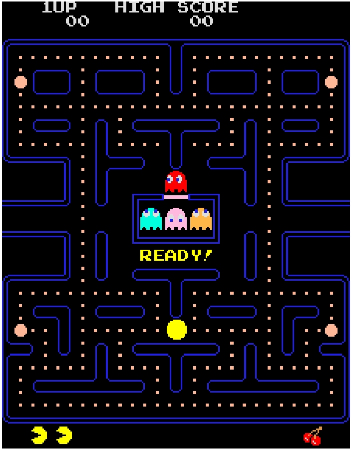
$\Longrightarrow$



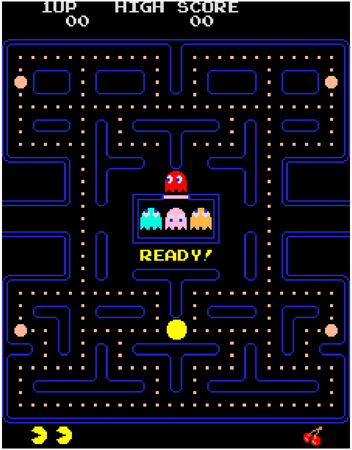
Solution: Reinforcement Learning

- Supervised learning is *not* the most general formulation of learning.



Solution: Reinforcement Learning

- Supervised learning is *not* the most general formulation of learning.
- Humans learn through reward and penalty



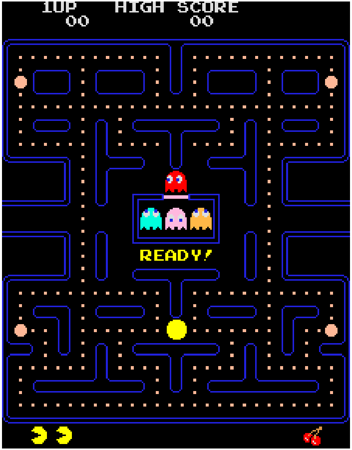
Solution: Reinforcement Learning

- Can we make algorithms which improve with crude reward signals?

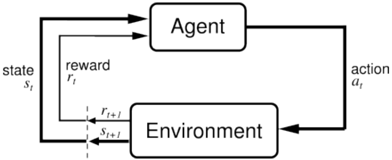
Machine learning without explicit objective functions

$\Downarrow$

Reinforcement Learning (RL)



The Core Idea



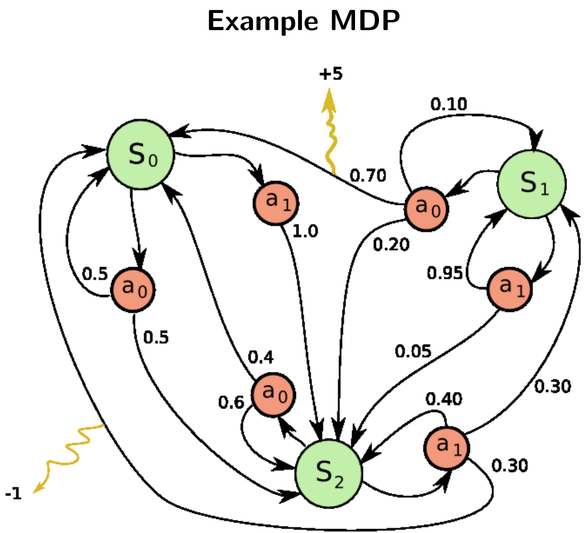
- Models (agents) take action a_t in some environment.
- Environment provides state s_t, reward r_t.
- Models learn to maximize reward r_t, $\forall t$.

Markov Decision Process (MDP)

Environment, $E = (\mathcal{S}, \mathcal{A}, \mathcal{R}, \rho, r)$.

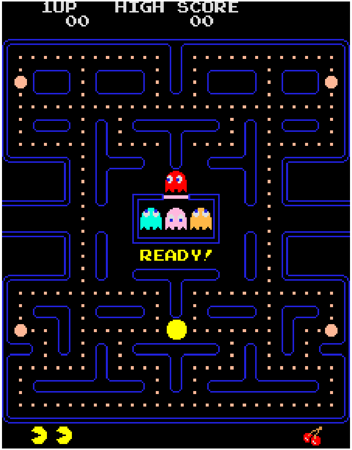
1. State space, $\mathcal{S}$
2. Action space, $\mathcal{A}$
3. Reward space, $\mathcal{R}$
4. Transition distribution, $\rho(s' \mid s, a)$. Given a previous state s and action a, environment gives s'.
5. Reward function $r(s, a) \in \mathcal{R}$.

Markov Property: $\rho(s' \mid s, a)$ depends only on s, a not previous states!


Example MDP
+5
0.10
0.70
S0
a1
1.0
a0
S1
0.20
0.5
a0
0.95
a1
0.5
0.05
0.4
a0
0.30
0.6
0.40
a1
S2
0.30
-1

Pacman as an MDP

- $\mathcal{S} = \mathbb{R}^{256\times256}$, images as state space.
- $\mathcal{A} = \{\uparrow, \downarrow, \rightarrow, \leftarrow\}$, joystick as action space.
- $r(s_t, a_t)$ = change in score.
- $\rho(s_{t+1} \mid s_t, a_t)$ = next frame of game after joystick action a_t.



Policies/Agents

Two different types of agents

- Deterministic policy $a = \pi(s)$ acts in E.
- Stochastic policy $a \sim \pi(a|s)$ gives a probability distibution over actions.

Policy Trajectories

$$s_1 \xrightarrow{\pi} a_1 \xrightarrow{\rho, r} s_2, r_2 \xrightarrow{\pi} a_2 \xrightarrow{\rho, r} \cdots$$

Value under a policy

The **state value** is a function of a given state for an agent π defined as

$$V^{\pi}(s_t) = \mathbb{E}\left[\sum_{n=t+1}^{\infty} \gamma^n r(s_n, \pi(s_n))\right]$$

1. γ is the discount factor
2. $\pi(s_n)$ is the action the agent π makes after seeing state s_n.
3. $r(s_n, \pi(s_n))$ is the reward the agent gets from taking that action.

Value under a policy

The **state-action value** for an agent π is defined such that

$$Q^\pi(s_t, a_t) = \mathbb{E}\left[\underbrace{r(s_t, a_t)}_{\text{reward for } a_t} + V^\pi(s_t)\right]$$

- Given some state s_t, the *best* agent, π^* is one that take action

$$a_t = \operatorname*{argmax}_a Q(s_t, a).$$

Problems in Reinforcement Learning

- **Policy Optimization:** maximize the expected reward with respect to a policy π;

$$\pi^* = \underset{\pi}{\operatorname{argmax}}\, \mathbb{E}\left[\sum_{t=0}^{\infty} r_t\right]$$

- **Policy Evaluation:** Given some fixed policy π compute expected return.
  - Computing Q^π, V^π, and other expectations on policy rollout.
  - Lets us perform policy optimization!

Algorithms

Behavioral Cloning

Behavioral Cloning: Supervised learning in MDPs using and expert agent expert π^*!

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Given expert examples $\mathcal{D} = (s_t, a_t = \pi^*(s_t))$ and a model π_θ find θ^* st

$$\theta^* = \operatorname*{argmin}_\theta \mathcal{L}(a_t, \pi_\theta(s_t)).$$

where $\mathcal{L}$ is some loss function.

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- Show, don't tell!

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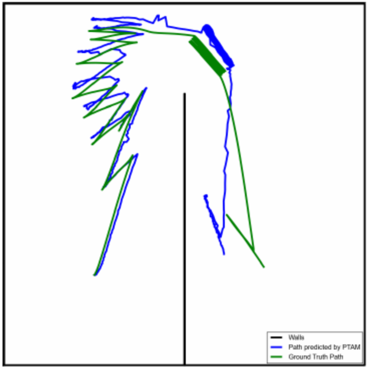
where $\mathcal{L}$ is some loss function.

- Show, don't tell!
- No complicated machinery, just standard ML.

Issue: Compounding Error

Given some irreducible error $\epsilon = 0.001$

- $\mathcal{L}(a_0, \pi_\theta(s_0)) = \epsilon$
- $\mathcal{L}(a_1, \pi_\theta(s_1)) = 2\epsilon$
- $\mathcal{L}(a_2, \pi_\theta(s_2)) = 3\epsilon$
- $\mathcal{L}(a_3, \pi_\theta(s_3)) = 4\epsilon$
- $\mathcal{L}(a_4, \pi_\theta(s_4)) = 5\epsilon$



Issue: Distribution Mismatch

- States expert dataset $\mathcal{D}$ generated by π^* have different distribution than those generated by π_θ.

  $\implies$ No self correction.

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Solution: DAgger.

- Do BC on $\mathcal{D}$ and generate E_0 states generated by π_θ.
- Label E_0 with expert level actions and add to $\mathcal{D}$.

Q-Learning (State-action Value Iteration)

Goals of Q-learning

1. Approximate Q^{π^*}, the Q function of the optimal agent, as $Q(s_t, a_t)$.

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1. Approximate Q^{π^*}, the Q function of the optimal agent, as $Q(s_t, a_t)$.
2. Using Q, find the agent, π, that best approximates the optimal agent, π^*.

Q-Learning (State-action Value Iteration)

How do we define best?

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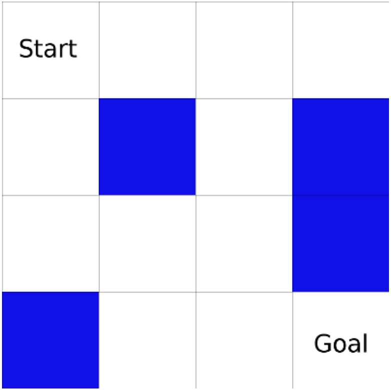
Given some state s_t, the **best** agent, π^* is one that takes action

$$a_t = \arg\max_a Q(s_t, a).$$

Q-Learning (State-action Value Iteration)

An example: Frozen Lake Problem



Q-Learning (State-action Value Iteration)

- 100 reward for reaching the goal
- 0 otherwise

How do we keep track of this long term reward?

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Q **function**

Q-Learning (State-action Value Iteration)

How do we actually calculate the Q function?

Q-Learning (State-action Value Iteration)

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The Bellman Equation.

Q-Learning (State-action Value Iteration)

How do we actually calculate the Q function?

The Bellman Equation.

$$Q^{\pi}(s_t, a_t) = r_t + \gamma Q^{\pi}(s_{t+1}, \pi(s_{t+1}))$$

Q-Learning (State-action Value Iteration)

One Q-Learning Algorithm: Tabular Q-Learning

- Explore the environment
- On the way, use the Bellman equation to store a table of expected future reward (Q) for each state-action pair.
- Use this table to pick the best possible action for any given state.

Q-Learning (State-action Value Iteration)

An example update for Frozen Lake.

Suppose our stored Q table looks like so:

Up	Down	Left	Right
0	65	0	40
0	0	0	0
0	0	0	0
0	0	0	0
50	75	30	20
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

Q-Learning (State-action Value Iteration)

An example update for Frozen Lake.

Then suppose our agent moves **down** from the starting square

Q-Learning (State-action Value Iteration)

An example update for Frozen Lake.

Then we update using the Bellman equation.

$$Q(s_{t+1}, a_{t+1}) = Q(s_t, a_t) + \alpha(r_t + \gamma(\max_a Q(s_t, a) - Q(s_t, a_t))$$

Up	Down	Left	Right
0	65	0	40
0	0	0	0
0	0	0	0
0	0	0	0
50	75	30	20
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

Q-Learning (State-action Value Iteration)

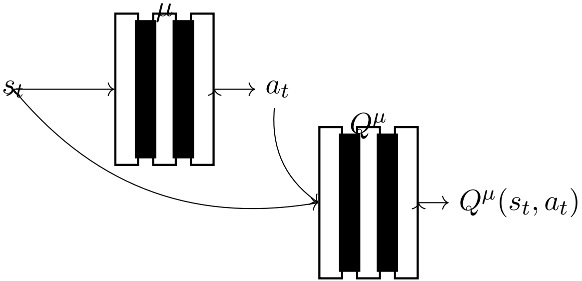
An example update for Frozen Lake.
The table now looks like so:

Up	Down	Left	Right
0	70	0	40
0	0	0	0
0	0	0	0
0	0	0	0
50	75	30	20
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

Policy Iteration

Deep Determisitic Policy Gradient

1. Actor neural network $\mu : \mathcal{S} \to \mathcal{A}$
2. Critic network $Q^\mu : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$
3. Performance of μ is $Q^\mu(s_t, \mu(s_t))$. **Maximize performance!** $\nabla_W Q^\mu(s_t, a_t) = \nabla_a Q^\mu(s_t, a) \cdot \nabla_W \mu(s_t)$



Questions?