Technical Appendix I: Energy Solution for Embedded Walls

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This document has been superseded by the documentation and python code available at: https://github.com/AbiBateman/MSD_Walls.

This technical appendix should be used in conjunction with the calculation detailed in Technical Appendix II: MSD Solution for Rigid Wall Rotation.

Section 1 of this appendix details the calculation of (1) the elastoplastic solution in inhomogeneous soil, (2) the potential energy loss increments at each excavation stage and (3) the internal plastic work increment at each excavation stage. Section 2 details the general solution and closed form solution for the MSD solution for rigid wall rotation.

Energy Solution for Embedded Walls

1. Elastoplastic solution in Inhomogeneous Soil (linear variation with depth)

1a. Elastic Strain Energy Increment at excavation stage m, ΔU_m

The elastic strain energy stored in the length of the wall below the lowest prop at excavation stage *m* is given by the following equation

$$U_m = \int_0^{\lambda_m} dU = \frac{1}{2} EI_w \int_0^{\lambda_m} w''(y)^2 \, dy \tag{A1}$$

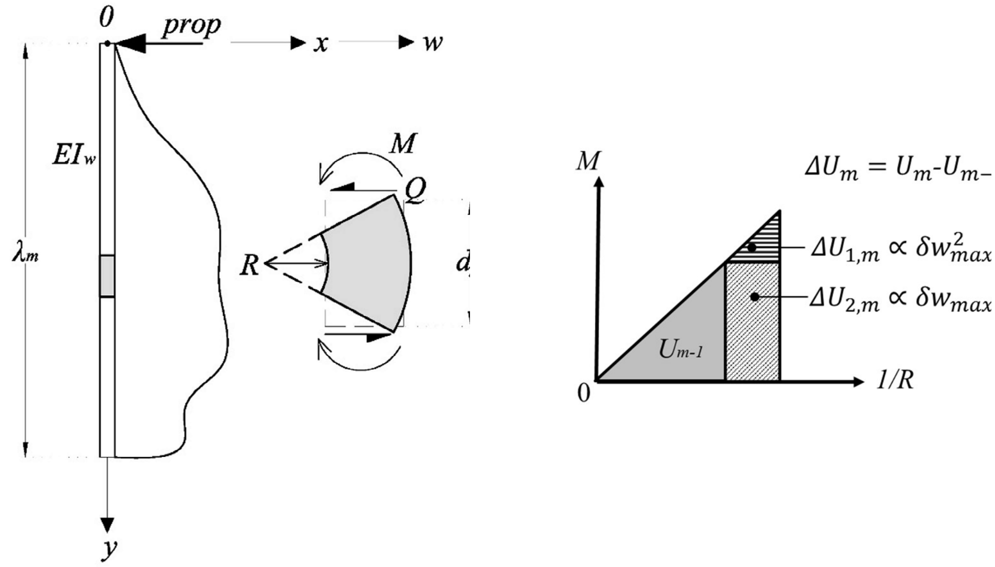

Figure A1. Calculation approach for incremental elastic strain energy ΔU_m stored in the wall during excavation stage *m*.

Figure A1 illustrates the concept of incremental strain energy ΔU_m stored in the wall during excavation stage *m*. To this end, wall deflection *w(y)* at stage *m* as the sum of all the previous increments of δw the elastic strain energy increment at a given stage *m.* This is then calculated as

$$\Delta U_m = U_m - U_{m-1} = \frac{1}{2} EI_w \left[\int_0^{\lambda_m} [\left(\sum_{i=1}^{m} \delta w_i'' \right)^2 - \left(\sum_{i=1}^{m-1} \delta w_i'' \right)^2] dy \right] \tag{A2}$$

which may be written as

$$\Delta U_m = \frac{1}{2} EI_w \left[\int_0^{\lambda_m} [\delta w_m''(y)^2 + 2\delta w_m''(y) \sum_{i=1}^{m-1} \delta w_i''(y)] dy \right] \tag{A3}$$

where

$$\delta w_m(y) = \frac{1}{2}\left[1-\cos\left(\frac{2\pi y}{\lambda_m}\right)\right]\delta w_{\max,m} = \frac{1}{2}\left[1-\cos\left(\frac{2\pi y}{\lambda_m}\right)\right](\delta w_{\max,m}=1)\cdot\delta \overline{w}_{\max,m} \tag{A4}$$

Thus the incremental elastic strain energy may be written as the following sum of two terms relative to the first and second power of $\delta w_{\max,m}$

$$\Delta U_m = C_1 \delta \overline{w}_{max,m}^2 + C_2 \delta \overline{w}_{max,m} \tag{A5}$$

where

$$C_1 = \pi^4 \frac{EI_w}{\lambda_m^3} \left(\delta w_{max,m} = 1\right) \tag{A6}$$

and

$$C_2 = 2\pi^3 \frac{EI_w}{\lambda_m^3} \left(\delta w_{max,m} = 1\right) \sum_{i=1}^{m-1} \delta w_{max,i} \frac{\sin(2\pi\lambda_m/\lambda_i)}{\lambda_i/\lambda_m - (\lambda_i/\lambda_m)^3} \tag{A7}$$

1b. Potential Energy Loss Increment at excavation stage m, ΔP_m

The potential energy loss increment at stage *m* can be written, in two dimensions, as

$$\Delta P_m = \int_{area} \vec{\gamma}_{sat}(x,y) \cdot \overrightarrow{\delta u}(x,y) dA \tag{A8}$$

Calculation of the above integral for each of the four zones of the considered mechanism is given below:

i. Rectangular Zone ABCD

The potential energy loss increment at stage *m* in the rectangular zone *ABCD* is calculated following the approach shown in Figure A2 as

$$\Delta P_m = \int_{area} d\Delta P_m \tag{A9}$$

where

$$d\Delta P_m = \gamma_{sat} \delta w_m(x) dx dy \tag{A10}$$

and

$$\delta w_m(x) = \frac{1}{2}\left[1-\cos\left(\frac{2\pi x}{\lambda_m}\right)\right]\delta w_{\max,m} \tag{A11}$$

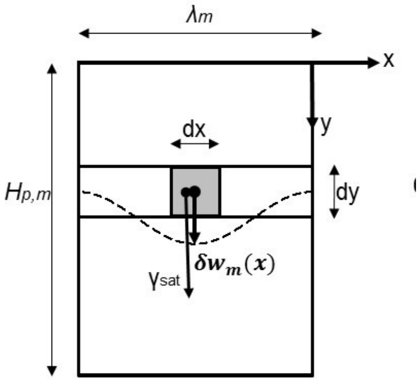

Figure A2. Calculation approach of Potential Energy drop *ΔP* during excavation stage *m* in the rectangular *ABCD* zone of the deformation mechanism

Integration yields the following result for area *ABCD*

$$\Delta P_{m,ABCD} = \frac{1}{2}\gamma_{sat} H_{p,m} \lambda_m \delta w_{\max,m} \tag{A12a}$$

or written in terms of normalised deflection

$$\Delta P_{m,ABCD} = \frac{1}{2}\gamma_{sat} H_{p,m} \lambda_m (\delta w_{\max,m} = 1) \cdot \delta \overline{w}_{\max,m} \tag{A12b}$$

ii. Circular Zone CDE

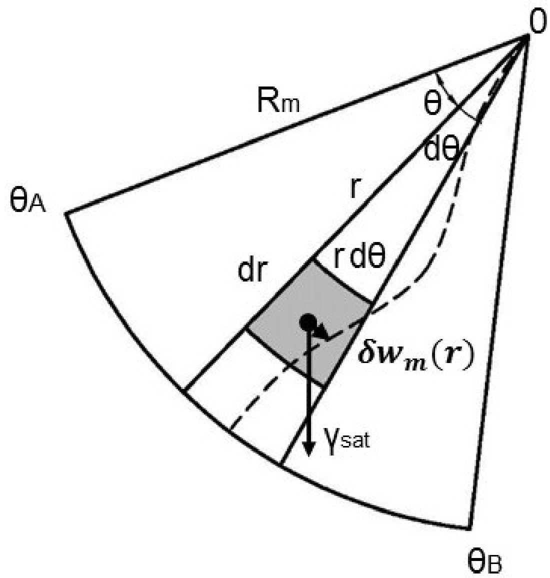

Figure A3. Calculation approach of Potential Energy drop *ΔP* during excavation stage *m* in the circular zones *CDE* and *EFH* of the deformation mechanism.

The circular area is treated using polar coordinates as shown in Figure 3A and the potential energy loss is now described as

$$d\Delta P_m = \gamma_{sat} \cos\theta\, \delta w_m(r)\, r\, d\theta\, dr \tag{A13}$$

and

$$\delta w_m(r) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi r}{\lambda_m}\right)\right]\delta w_{\max,m} \tag{A14}$$

integrating over the area *CDE*

$$\Delta P_m = \gamma_{sat} \int_0^{\lambda_m} \delta w_m(r) r dr \int_0^{\pi/2} \cos\theta d\theta \tag{A15}$$

which yields

$$\Delta P_{m,CDE} = \gamma_{sat} \frac{{\lambda_m}^2}{4} \delta w_{\max,m} \tag{A16a}$$

or

$$\Delta P_{m,CDE} = \gamma_{sat} \frac{{\lambda_m}^2}{4} (\delta w_{\max,m} = 1) \cdot \delta \overline{w}_{\max,m} \tag{A16b}$$

iii. Circular Zone FHE

In a similar manner as for zone *CDE* the integral now is written for *FHE* as

$$\Delta P_m = \gamma_{sat} \int_0^{\lambda_m - h_p} \delta w_m(r) r dr \int_{\pi/2}^{3\pi/4} \cos\theta d\theta \tag{A17}$$

which yields

$$\Delta P_{m,FHE} = -\gamma_{sat} \frac{\sqrt{2}-1}{\sqrt{2}} \frac{\lambda_m\left(\lambda_m - h_p\right)}{4} \delta w_{\max,m} \tag{A18a}$$

or

$$\Delta P_{m,FHE} = -\gamma_{sat} \frac{\sqrt{2}-1}{\sqrt{2}} \frac{\lambda_m\left(\lambda_m - h_p\right)}{4} (\delta w_{\max,m} = 1) \cdot \delta \overline{w}_{\max,m} \tag{A18b}$$

iv. *Triangular Zone FHI*

The circular area is treated using coordinate system (*q,s*) as shown in Figure 4A and the potential energy loss is now described by Eq. (A19)

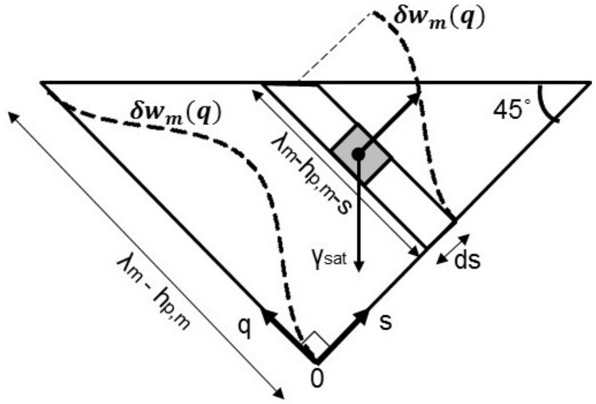

Figure A4. Calculation approach of Potential Energy drop *ΔP* during excavation stage *m* in triangular zone *FHI* of the deformation mechanism.

$$\delta\Delta P_m = -\gamma_{sat}\delta w_m(q)\cos\frac{\pi}{4}dqds \tag{A19}$$

and

$$\delta w_m(q) = \frac{1}{2}\left[1-\cos\left(\frac{2\pi q}{\lambda_m - h_p}\right)\right]\delta w^*_{\max,m} \tag{A20}$$

with

$$\delta w^*_{\max,m} = \frac{\lambda_m}{\lambda_m - h_p}\delta w_{\max,m} \tag{A21}$$

integrating over the area *FHI*

$$\Delta P_m = -\gamma_{sat}\int_0^{\lambda_m - h_p}\int_0^{\lambda_m - h_p - s}\delta w_m(q)\cos\frac{\pi}{4}dqds \tag{A22}$$

which yields

$$\Delta P_{m,FHI} = -\gamma_{sat}\frac{\sqrt{2}}{2}\frac{\lambda_m\left(\lambda_m - h_p\right)}{4}\delta w_{\max,m} \tag{A23a}$$

or

$$\Delta P_{m,FHI} = -\gamma_{sat}\frac{\sqrt{2}}{2}\frac{\lambda_m\left(\lambda_m - h_p\right)}{4}(\delta w_{\max,m}=1)\cdot\delta\overline{w}_{\max,m} \tag{A23b}$$

The sum of the Potential Energy Loss increments in all areas of the mechanism yields the Potential Energy Loss Increment at the given excavation stage *m*

$$\Delta P_m = \Delta P_{m,ABCD} + \Delta P_{m,CDE} + \Delta P_{m,EFH} + \Delta P_{m,FHI} \tag{A24}$$

which can be written as

$$\Delta P_m = \left[a\gamma_{sat}\lambda_m^2 (\delta w_{\max,m} = 1) \right] \cdot \delta \overline{w}_{\max,m} = A\delta \overline{w}_{\max,m} \tag{A25}$$

where

$$a = \frac{1}{2}\frac{H_{p,m}}{\lambda_m} + \frac{1}{4} - \frac{\sqrt{2}-1}{4\sqrt{2}}\frac{\lambda_m - h_p}{\lambda_m} - \frac{1}{4\sqrt{2}}\frac{\lambda_m - h_p}{\lambda_m} = \frac{1}{2}\frac{H_{p,m}}{\lambda_m} + \frac{1}{4}\frac{h_p}{\lambda_m} \tag{A26}$$

1c. Internal Plastic Work Increment at excavation stage m, ΔW_m

The internal plastic work increment at a given excavation stage *m*, is calculated as

$$\delta \Delta W = (\sigma_{xx}\delta\varepsilon_{xx} + 2\sigma_{xy}\delta\varepsilon_{xy} + \sigma_{yy}\delta\varepsilon_{yy})dxdy \geq 0 \tag{A27}$$

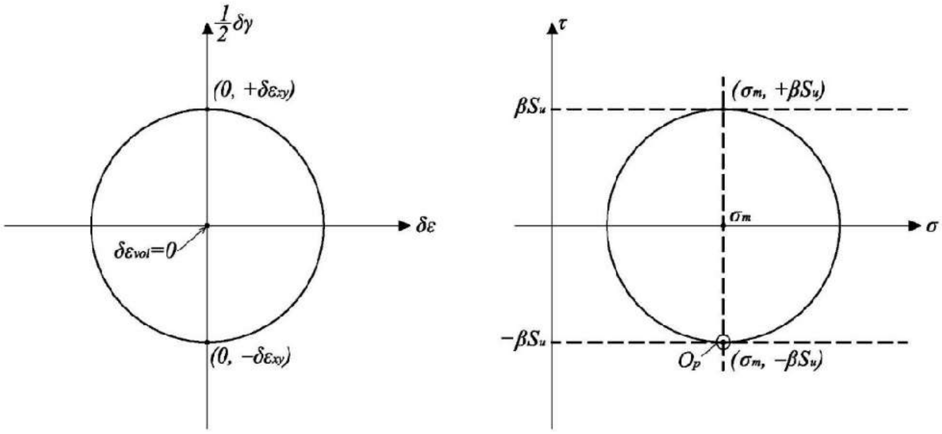

Figure A5. Mohr circle representation of plastic strains and stresses in the proposed mechanism.

under undrained conditions and for the considered mechanism as shown in Figure A5

$$\sigma_{xx} = \sigma_{yy} = \sigma_m \tag{A28a}$$

$$\delta\varepsilon_{xx} = \delta\varepsilon_{yy} = 0 \tag{A28b}$$

$$\sigma_{xy} = \beta_m S_u \tag{A28c}$$

$$\delta\varepsilon_{xy} = \frac{1}{2}\delta\gamma \tag{A28d}$$

therefore

$$\delta\Delta W = \left|\beta_m S_u \delta\gamma\right| dxdy \tag{A29}$$

and overall in a mechanism section

$$\Delta W = \int_{area} \left|\beta_m S_u \delta\gamma\right| dxdy \geq 0 \tag{A30}$$

which should be positive according to Drucker's stability postulate.

For the case of inhomogeneous soil, and specifically undrained shear strength linearly increasing with depth as $S_u(y)=S_u+cy$, the calculation of the above integral for each of the four zones of the considered mechanism is given below:

i. Rectangular Zone ABCD

In the realm of the rectangular zone *ABCD*, described in Figure A6, the shear strain increment $\delta\gamma$ is calculated as

$$\delta\gamma_m = \frac{d\delta w_{y,m}}{dx} + \frac{d\delta w_{x,m}}{dy} = \frac{d\delta w_{y,m}}{dx} \tag{A31}$$

and

$$\delta w_m(x) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi x}{\lambda_m}\right)\right]\delta w_{\max,m} \tag{A32}$$

yielding

$$\delta\gamma_m = \frac{\pi}{\lambda_m}\sin\left(\frac{2\pi x}{\lambda_m}\right)\delta w_{\max,m} \tag{A33}$$

which is positive for $-\lambda_m \leq x \leq -\lambda_m/2$.

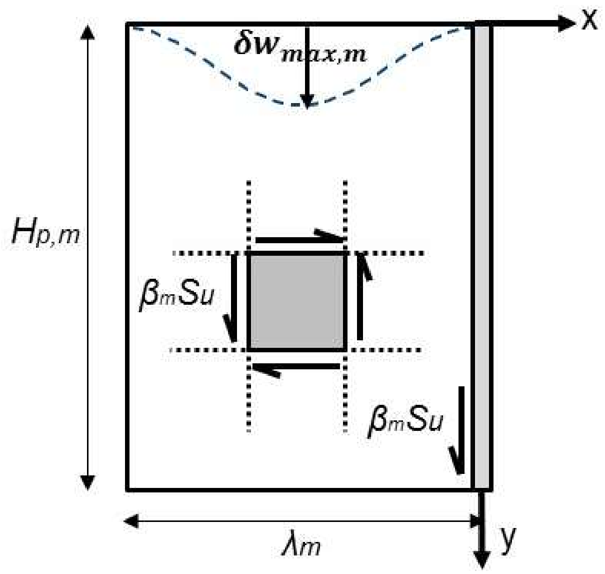

Figure A6. Calculation approach of Internal Elasto-Plastic Work *ΔW* during excavation stage *m* in the rectangular zone *ABCD* of the mechanism.

The undrained shear strength variation in this section is

$S_u(y)=S_{uo}+cy$ (A34)

where S_{uo} the value at the surface and c a dimensional constant.

Then the internal plastic work increment in the zone can be evaluated from the following integral

$$\Delta W_{m,ABCD} = 2\int_{-\lambda_m}^{-\lambda_m/2}\int_{0}^{H_p}\beta_m(S_{uo}+cy)\frac{\pi}{\lambda_m}\sin\left(\frac{2\pi x}{\lambda_m}\right)\delta w_{\max,m}dydx \tag{A35}$$

which yields

$$\Delta W_{m,ABCD} = 2\beta_m(S_{uo}+\frac{c}{2}H_p)H_p\delta w_{\max,m} \tag{A36a}$$

or

$$\Delta W_{m,ABCD} = 2\beta_m(S_{uo}+\frac{c}{2}H_p)H_p(\delta w_{\max,m}=1)\cdot\delta\overline{w}_{\max,m} \tag{A36b}$$

The first part of the above equation corresponds to the solution for homogeneous soil profiles

$$\Delta W_{m,ABCD,\hom} = 2\beta_m S_{uo}H_p\delta w_{\max,m} \tag{A36c}$$

ii. Circular Zone CDE

In the realm of the circular zone *CDE*, described in Figure A7, the shear strain increment $\delta\gamma$ is calculated as

$$\delta\gamma_m = \frac{d\delta w_{\theta,m}}{dr} + \frac{d\delta w_{r,m}}{d\theta} = \frac{d\delta w_{\theta,m}}{dr} \tag{A37}$$

and

$$\delta w_m(r) = \frac{1}{2}\left[1-\cos\left(\frac{2\pi r}{\lambda_m}\right)\right]\delta w_{\max,m} \tag{A38}$$

yielding

$$\delta\gamma_m = \frac{\pi}{\lambda_m}\sin\left(\frac{2\pi r}{\lambda_m}\right)\delta w_{\max,m} \tag{A39}$$

which is positive for $0 \leq r \leq \lambda_m/2$.

The undrained shear strength variation in this section is

$$S_u(r,\theta) = S_u(H_p) + c\, r \sin\theta \tag{A40}$$

where $S_u(H_p)$ the value at depth H_p (top of zone *CDE*) and c as before.

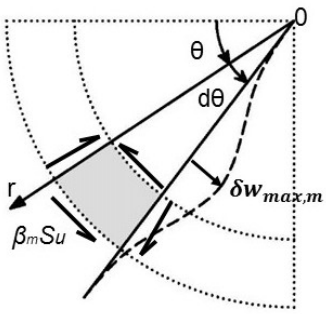

Figure A7. Calculation approach of Internal Elasto-Plastic Work ΔW during excavation stage m in the circular zones *CDE* and *EFH* of the mechanism.

Then the internal plastic work increment in the zone can be evaluated from the following integral

$$\Delta W_{m,CDE} = 2\int_0^{\lambda_m/2}\int_0^{\pi/2} \beta_m[S_u(H_p) + cr\sin\theta]\frac{\pi}{\lambda_m}\sin\left(\frac{2\pi r}{\lambda_m}\right)\delta w_{\max,m} r\, d\theta\, dr \tag{A41}$$

which yields

$$\Delta W_{m,CDE} = \beta_m[S_u(H_p) + c\lambda_m\frac{\pi^2-4}{\pi^3}]\frac{\pi}{4}\lambda_m\delta w_{\max,m} \tag{A42a}$$

or

$$\Delta W_{m,CDE} = \beta_m [S_u(H_p) + c\lambda_m \frac{\pi^2 - 4}{\pi^3}] \frac{\pi}{4} \lambda_m (\delta w_{\max,m} = 1) \cdot \delta \overline{w}_{\max,m} \tag{A42b}$$

The first part of the above equation corresponds to the solution for homogeneous soil profiles

$$\Delta W_{m,CDE,\hom} = \beta_m S_u(H_p) \frac{\pi}{4} \lambda_m \delta w_{\max,m} \tag{A42c}$$

iii. Circular Zone EFH

In a similar manner as for zone *CDE* the integral now is written for *FHE* as

$$\Delta W_{m,EFH} = 2 \int_0^{(\lambda_m - h_p)/2} \int_0^{\pi/2} \beta_m [S_u(H_m) + cr\sin\theta] \frac{\pi\lambda_m}{(\lambda_m - h_p)^2} \sin\left(\frac{2\pi r}{\lambda_m - h_p}\right) \delta w_{\max,m} r d\theta dr \tag{A43}$$

which yields

$$\Delta W_{m,EFH} = \beta_m [S_u(H_m) + c(\lambda_m - h_p)\sqrt{2} \frac{\pi^2 - 4}{\pi^3}] \frac{\pi}{8} \lambda_m \delta w_{\max,m} \tag{A44a}$$

or

$$\Delta W_{m,EFH} = \beta_m [S_u(H_m) + c(\lambda_m - h_p)\sqrt{2} \frac{\pi^2 - 4}{\pi^3}] \frac{\pi}{8} \lambda_m (\delta w_{\max,m} = 1) \cdot \delta \overline{w}_{\max,m} \tag{A44b}$$

The first part of the above equation corresponds to the solution for homogeneous soil profiles

$$\Delta W_{m,EFH,\hom} = \beta_m S_u(H_m) \frac{\pi}{8} \lambda_m \delta w_{\max,m} \tag{A44c}$$

iv. Triangular Zone FHI

In the realm of the rectangular zone *FHI,* shown in Figure A8, the shear strain increment $\delta\gamma$ is calculated as

$$\delta\gamma_m = \frac{d\delta w_{s,m}}{dq} + \frac{d\delta w_{q,m}}{ds} = \frac{d\delta w_{s,m}}{dq} \tag{A45}$$

and

$$\delta w_m(q) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi q}{\lambda_m - h_p}\right)\right] \delta w_{\max,m} \tag{A46}$$

yielding

$$\delta\gamma_m = \frac{\pi}{\lambda_m - h_p}\sin\left(\frac{2\pi q}{\lambda_m - h_p}\right)\delta w_{\max,m} \tag{A47}$$

The undrained shear strength variation in this section is

$$S_u(q,s) = S_u(H) + c\frac{\sqrt{2}}{2}\,[(\lambda_m - h_p) - (q+s)] \tag{A48}$$

where $S_u(H)$ the value at depth H (top of zone FHI) and c as before.

The product $S_u \cdot \delta\gamma_m$ is positive for $0 \le q \le (\lambda_m - h_p)/2$ when $0 \le s \le (\lambda_m - h_p)/2$, and for $0 \le q \le \lambda_m - h_p - s$ when $(\lambda_m - h_p)/2 \le s \le \lambda_m - h_p$

Then the internal plastic work increment in the zone can be evaluated from the following integral

$$\Delta W_{m,FHI} = \int_0^{(\lambda_m-h_p)/2}\left[\int_0^{(\lambda_m-h_p)/2}\beta_m S_u(q,s)\delta\gamma_m(q)dq - \int_{(\lambda_m-h_p)/2}^{\lambda_m-h_p-s}\beta_m S_u(q,s)\delta\gamma_m(q)dq\right]ds + \int_{(\lambda_m-h_p)/2}^{\lambda_m-h_p}\int_0^{\lambda_m-h_p-s}\beta_m S_u(q,s)\delta\gamma_m(q)dqds \tag{A49}$$

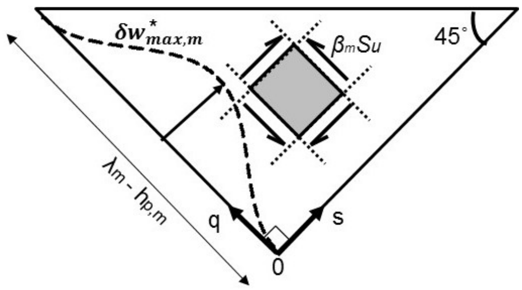

Figure A8. Calculation approach of Internal Elasto-Plastic Work ΔW during excavation stage m in the triangular zone FHI of the mechanism.

which yields

$$\Delta W_{m,FHI} = \beta_m[S_u(H) + c(\lambda_m - h_p)\sqrt{2}\frac{3\pi^2-4}{16\pi^2}]\lambda_m\delta w_{\max,m} \tag{A50a}$$

or

$$\Delta W_{m,FHI} = \beta_m[S_u(H) + c(\lambda_m - h_p)\sqrt{2}\frac{3\pi^2-4}{16\pi^2}]\lambda_m(\delta w_{\max,m} = 1)\cdot\delta\overline{w}_{\max,m} \tag{A50b}$$

The first part of the above equation corresponds to the solution for homogeneous soil profiles

$$\Delta W_{m,FHI,\hom} = \beta_m S_u(H)\lambda_m\delta w_{\max,m} \tag{A50c}$$

The sum of the Internal Plastic Work increments in all areas of the mechanism yields the Internal Plastic Work Increment at the given excavation stage m

$$\Delta W_m = \Delta W_{m,ABCD} + \Delta W_{m,CDE} + \Delta W_{m,EFH} + \Delta W_{m,FHI} \tag{A51}$$

which can be written as

$$\Delta W_m = \left[\beta_m \lambda_m \left(b_{\text{hom}} S_u + b_{inh} c \right) (\delta w_{\max,m} = 1) \right] \cdot \delta \overline{w}_{\max,m} = B \delta \overline{w}_{\max,m} \tag{A52}$$

where

$$b_{\text{hom}} = 2 \frac{H_{p,m}}{\lambda_m} + \frac{\pi}{4} + \frac{\pi}{8} + 1 \tag{A53a}$$

$$b_{inh} = \frac{H_{p,m}^2}{\lambda_m} + \frac{\pi^2 - 4}{4\pi^2} \lambda_m + \frac{\pi}{4} H_{p,m} + \frac{\pi^2 - 4}{8\pi^2} \sqrt{2}(\lambda_m - h_p) + \frac{\pi}{8} H_m + \frac{3\pi^2 - 4}{16\pi^2} \sqrt{2}(\lambda_m - h_p) + H_m \tag{A53b}$$

General Solution

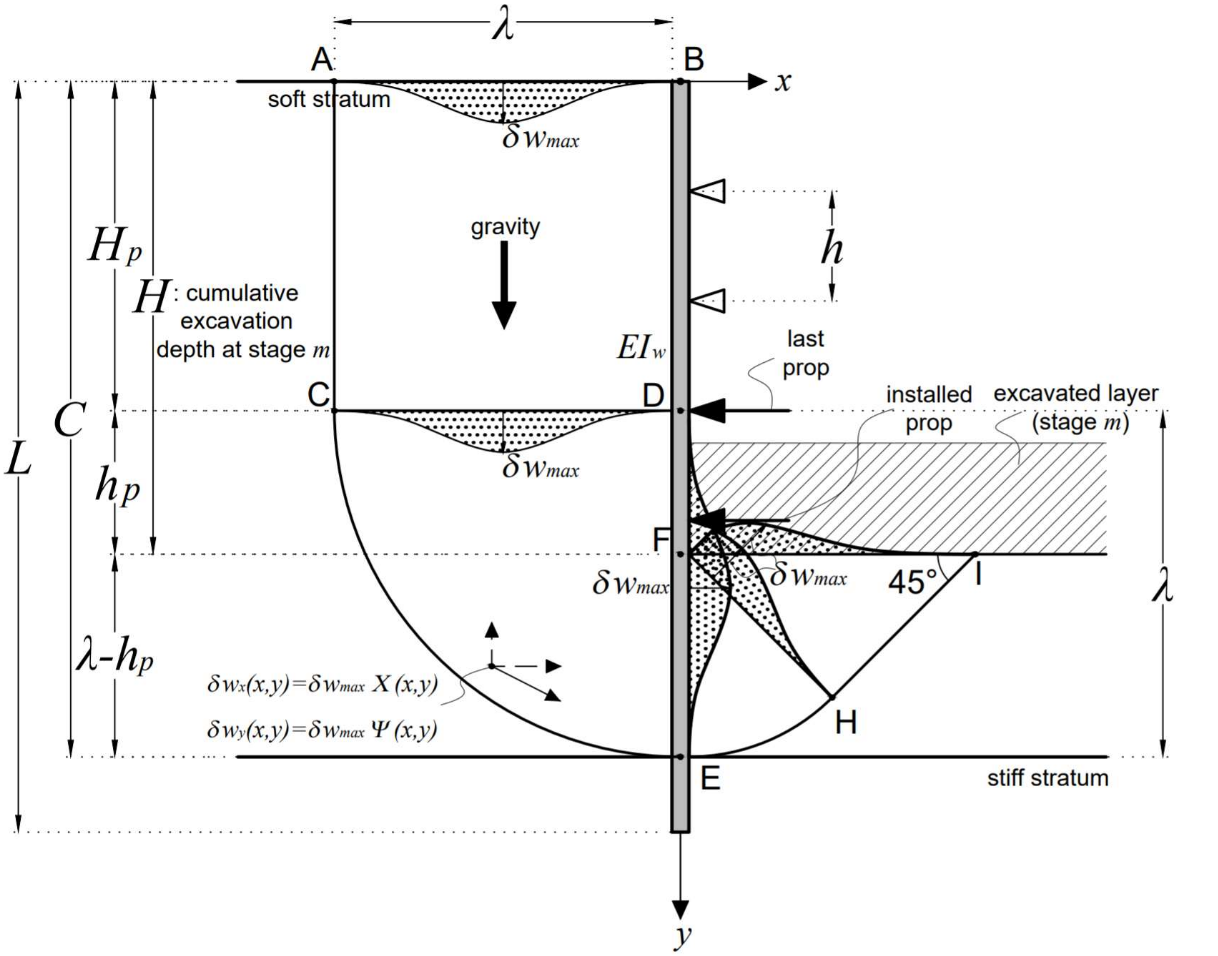

Figure A9. Incremental wall bulging at excavation stage m, plus plastic deformation mechanism considered for propped wall.

Considering the elastoplastic mechanism depicted in Fig A9, the potential energy loss equals the sum of the internal elastoplastic work plus the elastic strain energy stored in the wall in a given excavation stage *m*

$$\Delta P_m = \Delta W_m + \Delta U_m \tag{A54}$$

Substituting Eqs. (A5), (A25) and (A52) into Eq. (A54) yield the dimensionless incremental peak deformation of the wall at stage *m*

$$\delta\overline{w}_{max,m} = (A - B - C_2)/C_1 \tag{A55}$$

The above solution requires assuming a value for the mobilisation factor β_m contributing in coefficient *B.* The mobilisation factor β_m can be described as in Vardanega & Bolton (2011) using the following simple formula

$$\beta_m = \chi \left(\frac{\gamma_{ave,m}}{\gamma_{M=2}} \right)^b \tag{A56}$$

where χ, b are dimensionless parameters, $\gamma_{M=2}$ is the strain at half the peak undrained strength and $\gamma_{ave,m}$ the average shear strain in the mechanism at excavation stage *m* which can be evaluated as (*modified after* Lam & Bolton 2011)

$$\gamma_{ave,m} = \sum_{i=1}^{m} 2\frac{\delta w_{max,i}}{\lambda_i} \tag{A57}$$

which requires assuming a value for $\delta w_{max,m}$.

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Closed – Form solution (for b=0.5)

For the special case of exponent b=0.5 a closed-form solution is derived that allows solving directly for $\delta w_{max,m}$ without iterations and assumptions for β_m which in this case is written as

$$\beta_m = \chi \sqrt{\frac{2}{\gamma_{M=2}} \left(\sum_{i=1}^{m-1} \frac{\delta w_{max,i}}{\lambda_i} + \frac{\delta w_{max,m}}{\lambda_m} \right)} \tag{A58}$$

or as

$$\beta_m = \sqrt{B_1 + B_2 \delta\overline{w}_{\max,m}} \tag{A59a}$$

where

$$B_1 = \frac{2\chi^2}{\gamma_{M=2}} \sum_{i=1}^{m-1} \frac{\delta w_{max,i}}{\lambda_i} \tag{A59b}$$

$$B_2 = \frac{2\chi^2}{\lambda_m \gamma_{M=2}} (\delta w_{max,m} = 1) \tag{A59c}$$

The Eq. (45) can become by substituting Eqs. (A5), (A25), (A52) and (A59)

$$A\delta\bar{w}_{max,m} = B * \sqrt{B_1 + B_2\delta\bar{w}_{max,m}} \cdot \delta\bar{w}_{max,m} + C_1\delta\overline{w}^2{}_{max,m} + C_2\delta\overline{w}_{max,m} \tag{A60}$$

where $B^*=B/\beta_m$. Eq. (A60) can be reduced to a second order polynomial equation

$$\left(A - C_2 - C_1 \delta\overline{w}_{max,m}\right)^2 = B*^2 \left(B_1 + B_2 \delta\overline{w}_{max,m}\right) \tag{A61}$$

which yields the following closed form solution for $\delta\overline{w}_{\max,m}$

$$\delta\overline{w}_{max,m} = \frac{1}{2C_1^2}\left[B*^2 B_2 + 2(A - C_2)C_1 - \sqrt{B*^4 {B_2}^2 + 4B*^2 B_2 C_1 A + 4B*^2 B_1 {C_1}^2 - 4B*^2 B_2 C_1 C_2}\right] \tag{A62}$$