

Number Systems :-

→ A set of value used to represent quantity.

Name	Base (r)	Radix
Binary	2	(0, 1)
Octal	8	(0, 1, 2, 3, 4, 5, 6, 7).
Decimal	10	(0, 1, 2, ..., 9)
Dodecimal	12	(0, 1, 2, 3, ..., 9, A, B)
Hexadecimal	16	(0, 1, ..., 9, A, B, C, D, E, F)
	4	(0 to 3) for (0, 1, 2, 3)

Weighted and Unweighted case :-

$$7392 = 7 \times 1000 + 3 \times 100 + 9 \times 10 + 2$$

$$= 7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

Coefficients,

weight

Number Systems

weighted
 decimal
 binary
 octal
 BCD etc.

unweighted
 ex. gray code
 excess-3 etc.

Binary Number System

→ 0 to (r-1).

r = base

0 to (2-1) → 0 to 1 = 0, 1. (Binary digits)

→ Binary digit is also called 'bit'.

Ex: $\begin{matrix} & 1 & 0 & 1 & 0 & 1 \\ & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{matrix}$

$$= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 16 + 4 + 1 = 21$$

weights

Ex: 10101.11

$$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$

$$= 16 + 4 + 1 + 0.5 + 0.25 = 21.75$$

MSB and LSB

MSB - Most significant bit

LSB - Least significant bit

MSB: $\begin{matrix} 1 & 0 & 1 & 0 & 1 \\ \uparrow & & & & \uparrow \\ \text{MSB} & & & & \text{LSB} \end{matrix}$

$$= 21$$

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ & & & & \uparrow \\ & & & & \text{LSB} \end{matrix}$

$$= 20$$

difference 1

$\begin{matrix} 0 & 0 & 1 & 0 & 0 \\ & & & & \uparrow \\ & & & & \text{LSB} \end{matrix}$

$$= 5$$

difference 16

So left side is MSB, because by changing left side diff. is more.

→ Bit is smallest unit of data.

→ 1 Nibble = 4 bits

used in BCD and Hexadecimal.

1 byte = 8 bits.

1 word = 16 bits = 2 bytes

1 double word = 32 bits = 4 bytes.

Decimal to binary Conversion:-

Q: $(13)_{10} \rightarrow ()_2$

~~201032~~

method - 1:-

$$13 = 8 + 4 + 1$$

$$= 2^3 + 2^2 + 2^0$$

$$2^0 = 1$$

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

1	1	0	1
$\frac{1}{2^3}$	$\frac{1}{2^2}$	$\frac{0}{2^1}$	$\frac{1}{2^0}$
↑	↑	↑	↑
2^3	2^2	2^1	2^0

method - 2:-

	dividend.	reminders
2 13	1	$(1101)_2$
2 6	0	
2 3	1	
2 1	1	
divisor		

Q: $(25.625)_{10} \rightarrow ()_2$

$$\begin{array}{r} 2 \overline{) 25} \\ 2 \overline{) 12} - 1 \\ 2 \overline{) 6} - 0 \\ 2 \overline{) 3} - 0 \\ 1 - 1 \end{array}$$

$$(1001)_2$$

$$0.625 \times 2 = 1.25$$

$$0.25 \times 2 = 0.5$$

$$0.5 \times 2 = 1.0$$

$$0.0 \times 2 = 0.0$$

$$(.1010)_2$$

$$\underline{\text{Ans}} \text{ is } (1001.101)_2$$

Decimal to Octal Conversion:-

Q) $(112)_{10} \rightarrow (?)_8$

$$\begin{array}{r} 8 \overline{) 112} \\ 8 \overline{) 14} \rightarrow 0 \\ 1 \rightarrow 6 \end{array} \uparrow (160)_2$$

Q) $(25.625)_{10} \rightarrow (?)_8$

$$\begin{array}{r} 8 \overline{) 25} \\ 8 \overline{) 3} \rightarrow 1 \\ 0 \rightarrow 3 \end{array} \uparrow (31)_8$$

$$\begin{aligned} 0.625 \times 8 &= 5.000 \\ 0.000 \times 8 &= 0.000 \end{aligned} \downarrow (05)_2$$

$(31.5)_2$ is Ans.

Decimal to Hexadecimal Conversion:-

Q) $(254)_{10} \rightarrow (?)_{16}$

$$\begin{array}{r} 16 \overline{) 254} \\ 16 \overline{) 14} \rightarrow 14 \\ 0 \rightarrow 15 \end{array} \uparrow$$

$$\begin{aligned} (15 \ 14)_{16} \\ = (F \ E)_{16} \end{aligned}$$

Q) $(25.625)_{10} \rightarrow (?)_{16}$

$$\begin{array}{r} 16 \overline{) 25} \\ 16 \overline{) 1} \rightarrow 1 \\ 0 \rightarrow 1 \end{array} \uparrow (19)_{16}$$

$$\begin{aligned} 0.625 \times 16 &= 10.000 = A \\ 0.000 \times 16 &= 0.000 \end{aligned}$$

$(19.A)_{16}$

Q) $(274)_{10} \rightarrow (?)_4$

$$\begin{array}{r} 4 \overline{) 27} \\ 4 \overline{) 3} \rightarrow 3 \\ 1 \rightarrow 2 \end{array} \uparrow (123)_4$$

$$\begin{aligned} 0.4 \times 4 &= 1.6 \\ 0.6 \times 4 &= 2.4 \\ 0.4 \times 4 &= 1.6 \\ 0.6 \times 4 &= 2.4 \end{aligned} \downarrow (0.1212)$$

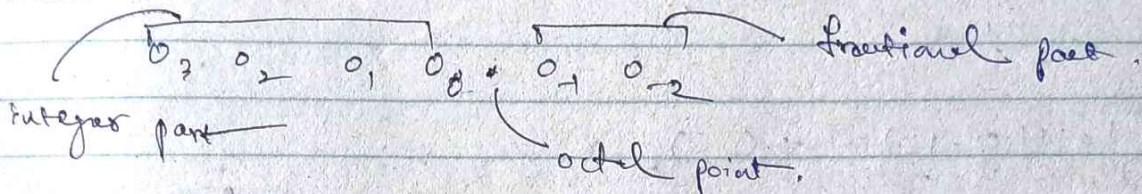
$(123.1212)_4$

Binary to decimal Conversion:-

Ex: $(10101.11)_2 \rightarrow (?)_{10}$

$$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$
$$= 21 + 0.75 = (21.75)_{10}$$

Octal to decimal Conversion:-



Ans: $(57.4)_8 \rightarrow (?)_{10}$

$$5 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = 40 + 7 + 0.5 = 47.5$$

Ex: $(4507.44)_8 \rightarrow (?)_{10}$

$$4 \times 8^3 + 5 \times 8^2 + 0 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} + 4 \times 8^{-2}$$

$$2048 + 320 + 0 + 7 + 0.5 + 0.0625 = (2375.5625)_{10}$$

Hexadecimal to Decimal Conversion:-

Ex: $(BAD)_{16} \rightarrow (?)_{10}$

B - 11

A - 10

D - 13

$$11 \times 16^2 + 10 \times 16^1 + 13 \times 16^0$$

$$= 2816 + 160 + 13 = (2989)_{10}$$

Ex: $(57.4)_{16} \rightarrow (?)_{10}$

$$5 \times 16^1 + 7 \times 16^0 + 4 \times 16^{-1}$$

$$= 80 + 7 + 0.25 = (87.25)_{10}$$

Octal to binary & binary to Octal Conversion:-

→

0	-	000
1	-	001
2	-	010
3	-	011
4	-	100
5	-	101
6	-	110
7	-	111

Ex (37.45)₈ → ()₂

011 111 100 101

(011111100101)₂ ✓

Ex (10110.11)₂ → ()₈

010 110

2 6

110

6

grouping

→ (26.6)₈ ✓

Ex (1001.1)₂

001 001 100

1 1 4

(11.4)₈

Hexadecimal to binary & binary to Hexadecimal:-

Ex (2594)₁₆ → ()₂

0010 0101 1001 1010

Ex (ADEF.30)₁₆ → ()₂

1101 1010 1111 1110 . 0011 0000

Ex (10001001.11)₂ → ()₁₆

(8.9.e)₁₆ ✓

grouping

Hexadecimal to Octal and Octal to Hexadecimal Conversion:-

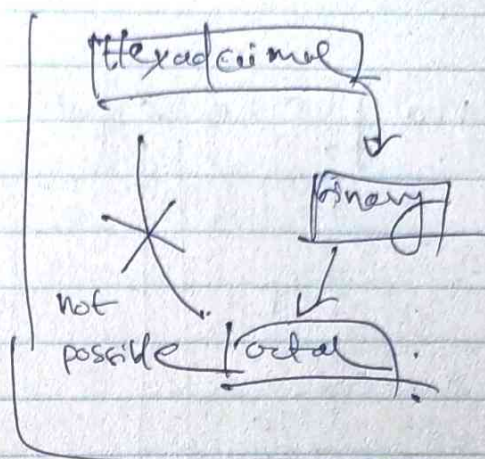
Ex $(CAD)_{16} \rightarrow (?)_8$

A $CAD \rightarrow$ binary

$$\begin{array}{ccc} 1100 & 1010 & 1101 \\ \hline \end{array}_2$$

↓ octal

$(625)_8$



Ex $(685)_8 \rightarrow (?)_{16}$

A octal

↓
binary

$$\begin{array}{ccc} 6 & 8 & 5 \\ \downarrow & \downarrow & \downarrow \\ 110 & 1000 & 101 \\ \hline \end{array}_2$$

$$\begin{array}{ccc} 110 & 1010 & 101 \\ \hline \end{array}_2$$

$(195)_{16}$

Binary addition:-

	Sum	Carry
0 + 0	0	0
0 + 1	1	0
1 + 0	1	0
1 + 1	0	1

Ex

$$\begin{array}{r} (1)(0)(0) \\ 110 \\ + 101 \\ \hline 1011 \end{array}$$

Binary multiplication:-

$$\begin{array}{r} 1010 \\ \times 101 \\ \hline 1010 \\ 0000 \\ 1010 \\ \hline 110010 \end{array}$$

Binary Subtraction:-

	sub	borrow
0 - 0	0	0
0 - 1	1	1
1 - 0	1	0
1 - 1	0	0

Ex

$$\begin{array}{r} (-1) \\ 11011 \\ - 10110 \\ \hline 00101 \end{array}$$

Ex

$$\begin{array}{r} (-)(-)(-) \\ 1110 \\ \times 111 \\ \hline 0111 \end{array}$$

Binary division :-

$$\begin{array}{r}
 110 \overline{) 101010.000111} \\
 \underline{0} \\
 10 \\
 \underline{0} \\
 101 \\
 \underline{000} \\
 1010 \\
 \underline{110} \\
 01001 \\
 \underline{110} \\
 00110 \\
 \underline{110} \\
 0
 \end{array}$$

Octal Addition :-

$$\begin{array}{r}
 \text{Ex) } \begin{array}{r} 00 \\ 243 \\ 212 \\ \hline 455 \end{array}
 \end{array}$$

if the sum within 7 then
no carry.

$$\begin{array}{r}
 \text{Ex) } \begin{array}{r} 111 \\ 567 \\ 943 \\ \hline 1032 \end{array}
 \end{array}$$

$$11 > 7$$

$$\begin{array}{r}
 11 = 1 \times 8 + 3 \\
 \text{Carry} \quad \text{sum}
 \end{array}$$

$$8 > 7$$

$$\begin{array}{r}
 8 = 1 \times 8 + 0 \\
 \text{Carry} \quad \text{sum}
 \end{array}$$

$$7 + 3 = 10$$

$$\text{and } 10 > 7$$

$$\begin{array}{r}
 10 = 1 \times 8 + 2 \\
 \downarrow \quad \downarrow \\
 \text{Carry} \quad \text{sum}
 \end{array}$$

Octal Subtraction:-

Ques 1

$$\begin{array}{r} 743 \\ - 564 \\ \hline 157 \end{array}$$

borrow = 8

$$8 + 3 = 11$$

$$11 - 4 = 7$$

in place of 4 there is 3 now.

$$3 < 6$$

borrow from 8

$$8 + 3 = 11$$

$$11 - 6 = 5$$

in place of 7 there is 6

$$6 - 5 = 1$$

Ques 2

$$\begin{array}{r} 512 \\ - 624 \\ \hline 265 \\ 337 \end{array}$$

Octal Multiplication:-

Ques 1

$$\begin{array}{r} 22 \\ \times 12 \\ \hline 44 \\ + 22 \\ \hline 1104 \end{array}$$

$$2 \times 2 = 4$$

$$2 \times 2 = 4 > 8$$

$$14 = 1 \times 8 + 6$$

Carry sum

$$6 + 2 = 8$$

$$38 = 1 \times 8 + 0$$

Ques 2

$$2 + 2 = 4$$

$$9 = 1 \times 8 + 1$$

$$1 \times 3 = 3 \quad 9 \times 1 = 2 \times 8 + 5$$

$$15 + 2 = 17 \rightarrow 2 \times 8 + 1$$

$$11 = 1 \times 8 + 3$$

Ques 3

$$\begin{array}{r} 11 \\ \times 122 \\ \hline 22 \\ 222 \\ + 1111 \\ \hline 1315 \\ 736 \\ \hline 10675 \end{array}$$

1's and 2's Complement :-

$$(r-1)' \text{ comp} = r^4 - N - 1 \\ = (r^4 - 1) - N.$$

if $r=10$, then $r^4 - 1 = 9999$

if $r=8$, then $r^4 - 1 = 7777$

if $r=16$, then $r^4 - 1 = \text{ffff}$

if $r=2$, then $r^4 - 1 = 1111$

Q1. Obtain 1's of $(1010)_2$.

Subtract from 1111

$$\begin{array}{r} 1111 \\ 1010 \\ \hline 0101 \end{array} \Delta$$

we see 1's complement is just inverse.

Q2. Obtain 2's comp of $(1011010)_2$.

find 1's complement and add 1.

$$\begin{array}{r} 1's \text{ comp} = 01000101 \\ \hline 01000110 \end{array} \Delta$$

Shortcut for 2's Complement :-

Step-1: write given number.

Step-2: Starting from LSP, copy all the 0's till the first 1.

Step-3: Copy the first 1.

Step-4: Complement all the remaining bits.

Q3. 10111000

$$\begin{array}{r} 01001000 \\ \hline \end{array} \Delta$$

Q4. $(101100)_2$

$$\begin{array}{r} 101100 \\ \hline 010100 \\ \hline \end{array} \Delta$$