Name : ABID ALI
Student_No : 2019380141

Assignment 8

7.1 Suppose the agent has progressed to the point shown in Figure 7.4(a), page 239, having perceived nothing in [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned with the contents of [1,3], [2,2], and [3,1]. Each of these can contain a pit, and at most one can contain a wumpus. Following the example of Figure 7.5, construct the set of possible worlds. (You should find 32 of them.) Mark the worlds in which the KB is true and those in which each of the following sentences is true: $\alpha 2$ = "There is no pit in [2,2]." $\alpha 3$ = "There is a wumpus in [1,3]."
Hence show that KB |= $\alpha 2$ and KB |= $\alpha 3$.
Answer: To save space, we'll show the list of models as a table (Figure S7.1) rather than a collection of diagrams. There are eight possible combinations of pits in the three squares, and four possibilities for the wumpus location (including nowhere).

We can see that KB |= $\alpha 2$ because every line where KB is true also has $\alpha 2$ true. Similarly, for$\alpha 3$.

7.4 Which of the following are correct?
a. False |= True.
Answer:
False |= True is true because False has no models and hence entails every sentence AND because True is true in all models and hence is entailed by every sentence.

b. True |= False.
Answer:
True |= False is false.

c. (A ∧ B) |=(A ⇔ B).
Answer:
(A ∧ B) |= (A ⇔ B) is true because the left-hand side has exactly one model that is one of the two models of the right-hand side.

d. A ⇔ B |= A ∨ B.
Answer:
A ⇔ B |= A ∨ B is false because one of the models of A ⇔ B has both A and B false, which does not satisfy A ∨ B.

e. A ⇔ B |= ¬A ∨ B.
Answer:
A ⇔ B |= ¬A ∨ B is true because the RHS is A ⇒ B, one of the conjuncts in the definition of A ⇔ B.

f. (A ∧ B) ⇒ C |=(A ⇒ C) ∨ (B ⇒ C).
Answer:

$(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$ is true because the RHS is false only when both disjuncts are false, i.e., when A and B are true and C is false, in which case the LHS is also false. This may seem counterintuitive, and would not hold if $\Rightarrow$ is interpreted as "causes."

g. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$.

Answer:

$(C \vee(\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$ is true; proof by truth table enumeration, or by application of distributivity (Fig 7.11).

h. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$.

Answer:

$(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$ is true; removing a conjunct only allows more models.

i. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.

Answer:

$(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$ is false; removing a disjunct allows fewer models.

j. $(A \vee B) \wedge \neg (A \Rightarrow B)$ is satisfiable.

Answer:

$(A \vee B) \wedge \neg(A \Rightarrow B)$ *is* satisfiable; model has A and $\neg B$.

k. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable.

Answer:

$(A \Leftrightarrow B) \wedge (\neg A \vee B)$ *is* satisfiable; RHS is entailed by LHS so models are those of $A \Leftrightarrow B$.

l. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

Answer:

$(A \Leftrightarrow B) \Leftrightarrow C$ does have the same number of models as $(A \Leftrightarrow B)$; half the models of $(A \Leftrightarrow B)$ satisfy $(A \Leftrightarrow B) \Leftrightarrow C$, as do half the non-models, and there are the same numbers of models and non-models.

7.13 This exercise looks into the relationship between clauses and implication sentences.

a. Show that the clause $(\neg P1 \vee \cdots \vee \neg Pm \vee Q)$ is logically equivalent to the implication sentence $(P1 \wedge \cdots \wedge Pm) \Rightarrow Q$.

Answer:

$P \Rightarrow Q$ is equivalent to $\neg P \vee Q$ by implication elimination (Figure 7.11), and $\neg(P_1 \wedge \cdots \wedge P_m)$ is equivalent to $(\neg P_1 \vee \cdots \vee \neg P_m)$ by de Morgan's rule, so $(\neg P_1 \vee \cdots \vee \neg P_m \vee Q)$ is equivalent to $(P_1 \wedge \cdots \wedge P_m) \Rightarrow Q$.

b. Show that every clause (regardless of the number of positive literals) can be written in the form $(P1 \wedge \cdots \wedge Pm) \Rightarrow (Q1 \vee \cdots \vee Qn)$, where thePs andQs are proposition symbols. A knowledge base consisting of such sentences is in implicative normal form or Kowalski form (Kowalski, 1979).

Answer:

A clause can have positive and negative literals; let the negative literals have the form $\neg P_1, \ldots, \neg P_m$ and let the positive literals have the form $Q_1, \ldots, Q_n$, where the P_is and Q_js are symbols. Then the clause can be written as $(\neg P_1 \vee \cdots \vee \neg P_m \vee Q_1 \vee \cdots \vee Q_n)$.

By the previous argument, with $Q = Q_1 \vee \cdots \vee Q_n$, it is immediate that the clause is equivalent to

$(P_1 \wedge \cdots \wedge P_m) \Rightarrow Q_1 \vee \cdots \vee Q_n$.

c. Write down the full resolution rule for sentences in implicative normal form.

Answer:

For atoms p_i, q_i, r_i, s_i where $p_j = q_k$: