Homework-5

1.6 Rules of Inference(Pg-82)

Question No.18

What is wrong with this argument? Let $S(x, y)$ be "x is shorter than y." Given the premise $\exists s S(s, \text{Max})$, it follows that $S(\text{Max}, \text{Max})$. Then by existential generalization it follows that $\exists x S(x, x)$, so that someone is shorter than himself.

Answer No.18

Let us assume:
S(T, y) = "x is shorter than y"
The premise ∃sS(s, Max) is given and thus we assume that BsS(s, Max) is true.
We then know that there exists a person y such that S(y, Max) is true. However, we do not know if the person y is Max (it is possible for the person y to be somebody who is not Max), thus we cannot conclude S(Max, Max). Moreover, S(Max, Max) cannot be true, because Max cannot be shorter than himself. This also applies to everybody and thus the statement ∃sS(x, x) is always false.

We do not know if Max is the person for which S(s,Max) is true.

Question No.24

Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then
$\forall x P(x) \vee \forall x Q(x)$ is true.

1. $\forall x(P(x) \vee Q(x))$	Premise
2. $P(c) \vee Q(c)$	Universal instantiation from (1)
3. $P(c)$	Simplification from (2)
4. $\forall x P(x)$	Universal generalization from (3)
5. $Q(c)$	Simplification from (2)

6. $\forall x Q(x)$ Universal generalization from (5)
7. $\forall x(P(x) \vee \forall x Q(x))$ Conjunction from (4) and (6)

Answer No.24

Rules of Inference

**

p ^ q

q Simplification

**

p
q

∴ p∧q Conjunction

**

$\forall x P(x)$

∴ P(c) Universal instantiation

**

P(c) for an arbitrary c

∴ $\forall x P(x)$

Solution:

Text in red has been edited from the given solution.

	Step	Reason
1.	$\forall x(P(x) \vee Q(x))$	Premise
2.	$P(c) \vee Q(x)$	Universal instantiation from (1)
3.		Simplification cannot be used,because it requires a conjunction ^ instead of a disjunction v

The same error occurs in step (3) and step (5) of the solution in the exercise prompt,namely that we cannot use simplification.

Question No.27

Use rules of inference to show that if $\forall x(P(x) \to (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

Answer No.27

We assume that $\forall x\,(P(x) \to (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, thus that these two statements are the premises.
Let c be arbitrary.

	Step	Reason
1.	$\forall x\,(P(x) \to (Q(x) \wedge S(x)))$	Premise
2.	$\forall x(P(x) \wedge R(x))$	Premise
3.	$P(c) \to (Q(c) \wedge S(c)$	Universal instantitation from (1)
4.	$P(c) \wedge R(c)$	Universal instantitation from (2)
5.	$P(c)$	Simplification from (4)
6. .	$Q(c) \wedge S(c)$	Modus ponens from (3) and (5)
7.	$S(c)$	Simplification from (6)

8.	R(c)	Simplification from (4)
9. .	$R(c) \wedge S(c)$	Conjunction from (7) and (8)
10.	$\forall x(R(x) \wedge S(x))$	Universal generalization from (9)

Thus have shown that if premises $\forall x\,(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then the conclusion $\forall x(R(x) \wedge S(x))$ is also true.

Question No.29

Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

Answer No.29

We can set this up in two-column format.The proof is rather long but straightforward if we go one step at a time.

	Step	Reason
1.	$\exists x \neg P(x)$	Premise
2.	$\neg P(c)$	Existential instantiation using (1)
3.	$\forall x(P(x) \vee Q(x))$	Premise
4.	$P(c) \vee Q(c)$	Universal instantiation using (3)
5.	$Q(c)$	Disjunctive syllogism using (4) and (2)
6.	$\forall x(\neg Q(x) \vee S(x))$	Premise
7.	$\neg Q(c) \vee S(c)$	Universal instantiation using (6)
8.	$S(c)$	**Disjunctive syllogism using (5) and (7), since $\neg\neg Q(c) \equiv Q(c)$**
9.	$\forall x(R(x) \rightarrow \neg S(x))$	Premise
10.	$R(c) \rightarrow \neg S(c)$	Universal instantiation using (9)
11.	$\neg R(c)$	**Modus tollens using (8) and (10), since $\neg\neg S(c) \equiv S(c)$**

12. $\exists x \neg R(x)$ Existential generalization using (11)

Question No.31

Use resolution to show that the hypotheses "It is not raining or Yvette has her umbrella," "Yvette does not have her umbrella or she does not get wet," and "It is raining or Yvette does not get wet" imply that "Yvette does not get wet."

Answer No.31

Idempotent laws:

$$p \vee p \equiv p$$

$$p \wedge p \equiv p$$

Rules of Inference:

Resolution:

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

Solution:

Let us assume:

p = It is raining (Note: the negation $\neg p$ then means that "It is not raining").

q = Yvette has her umbrella (Note: the negation -nq then means that "Yvette does not have her umbrella").

r = Yvette gets wet (Note: the negation —x then means that "Yvette does not get wet").

We can translate the given sentences to mathematical propositions using the above interpretations.

	Step	**Reason**
1.	$\neg p \vee q$	Premise
2.	$\neg q \vee \neg r$	Premise
3.	$p \vee \neg r$	Premise
4.	$q \vee \neg r$	Resolution from (1) and (3)
5.	$\neg r \vee \neg r$	Resolution from (2) and (4)
6.	$\neg r$	Idempotent law from (5)

Statement (6) means “Yvette does not get wet”, thus we have derived the conclusion.

Final,Result “Yvette does not get wet”