ABID ALI
ABIDALI — 2019380141

6.1, 16, 51, 55
6.2, 8, 16, 21
6.3, 11, 24, 28, 30
6.4, 7, 8

11.2, 20, 21, 22, 23, 24
11.3, 8, 14, 18,
23(b), 24(b)

6.1

16. $26^n - 25^n$: 66351

51. 64 + 256 + 352 : 352

55. 32 positive integers

6.2

8. Pigeon hole Principle:
Objects : number of integers : $d+1$
holes : number of reminders : d

$$\left\lceil \frac{d+1}{2} \right\rceil : 2$$

16. (a) Divide the first ten positive integers into the following five groups : {1, 10}, {2, 9}, {3, 8}, {4, 7}, {5, 6}.
the sum of the two numbers in any group is 11.
Proof: Once we choose 7 numbers out of 10, only 3 numbers remain unchoosen. They can be in a maximum of 3 groups. Therefore, at least 2 groups are choosen completely, So, we have at least 4 parts sums equal to 11.

(b) no.

21. (a) no,
if there are less than or equal 8 freshman, less than or equal 8 juniors in the class, then altogether there are no more than 24 students or does, which is not the case.

(b) no,
there are either at least 3 freshman, at least 19 sophomores, or at least 5 juniors in the class

6.3

11. a) $C(10,4) : 10!/(4! \times 6!)$
$: (10 \times 9 \times 8 \times 7)/4! : 210$

b) $C(10,0) + C(10,1) + C(10,2) + C(10,3) + C(10,4)$
$: 10!/(0! \times 10!) + 10!/(1! \times 9!) + 10!/(2! \times 8!) + 10!/(3! \times 7!) + 10!(4! \times 5!)$
$: 1 + 10 + 45 + 120 + 210 : 386$

c) $2^{10} - [C(10,0) + C(10,1) + C(10,2) + C(10,3)]$
$: 1024 - 1 - 10 - 45 - 120 : 848$

d) $C(10,5) : 10!/(5! \times 5!) : 252$

24. the possible way to arrange ten women in a row is
$10P10 = 10! : 3628800$
We need to find how many ways we can arrange 66

max in the possible 1111 possible places

11P6 : 11 x 10 x 9 x 8 x 7 x 6 : 332640

11P6 : 11 x 10 x 9 x 8 x 7 x 6 : 332640

the answer:

3628800 x 332640 : 1, 209,084,032,000

28.

(a)

$C(13,10) : \frac{13!}{10!\,3!} : \frac{13.12.11}{1.2.3} : 13.2.11 : 286$

(b)

$P(13,10) : \frac{13!}{(13-10)!} : \frac{13!}{(13-10)!} : \frac{13!}{3!}$

$: 13.12.11.10.9.8.7.6.5.4$

$: 1.037.836.800$

(c)

One women choosen

$C(10,9)\,C(3,1) : \frac{10!}{8!\,1!} \cdot \frac{3!}{1!\,2!} = 10.3 : 30$ ways

2 women choosen

$C(10,8)\,C(3,2) : \frac{10!}{8!\,2!} \cdot \frac{3!}{2!\,7!} : 45.3 : 135$

3 women choosen

$C(10,7)\,C(3,3) : \frac{10!}{7!\,3!} \cdot \frac{3!}{3!\,0!} = \frac{10.9.8}{1.2.3} \cdot 1$

$: 120$

altogether : 30 + 135 + 120 : 285

30.

each key is determined by the set of 19 true questions.

Therefore $\binom{40}{19}$ subsets of size 19 tower from a set of size 40. the number of ways are

$\frac{40!}{(7!(40-(7)!} : 88732378800$

6.4

(7) Binomial theorem

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

We are interested in the term,

$$x^9 \text{ in } (2-x)^{19} = (2+(-x))^{19}$$

$$n = 19$$

$$j = 9$$

The corresponding term is then:

$$\binom{n}{j} 2^{n-j} (-x)^j = \binom{19}{9} 2^{19-9} (-x)^9$$

$$= \frac{19!}{9!(19-9)!} 2^{10} (-x)^9$$

$$= \frac{19!}{9!\,10!} 2^{10} x^9$$

$$= -92,378 \cdot 1024 x^9$$

$$= -94,595,072 x^9$$

Thus the coefficient of x^9 is then, $-94,595,072$

$\frac{6.4}{(8)}$

Binomial theorem,

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

We are interested in the term, x^8y^9 in $(3x+2y)^{17}$

$$n = 17$$
$$j = 9$$

The corresponding term is then:

$$\binom{n}{j}(3x)^{n-j}(2y)^j = \binom{17}{9}(3x)^{17-9}(2y)^9$$

$$= \frac{17!}{9!(17-9)!}(3x)^8(2y)^9$$

$$= \frac{17!}{9!\,8!} 3^8 x^8 2^9 y^9$$

$$= 24310 \cdot 3^8 2^9 x^8 y^9$$

$$= 81,662,929,920\, x^8 y^9$$

Thus the coefficient of x^8y^9 is then, 81,662,929,920