Measuring moment inertia of rigid body via home-made trilinear torsion pendulum

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1. Experiment requirements

a. Learn the method to measure moment inertia (or inertia of moment) with trilinear torsion pendulum, understand the basic working conditions of trilinear torsion pendulum and the optimization principles of elements.

b. By employing current conditions at home, design and make a proper trilinear torsion pendulum.

c. Complete two measurement tasks: moment inertia of single rigid body via home-made trilinear torsion pendulum, moment inertia of off-center small cylinder around the center of trilinear torsion pendulum.

d. Complete data process: calculate moment inertia in the two measurements and compare with the theoretical values, achieve the percentage error.

e. Complete a PPT report for oral defense, containing the production progress of trilinear torsion pendulum, photos of entire self-made equipment, key elements and rigid body as well as a group photo of experimenter and equipment, writing answers of questions 1, 2, 3 and 4 in the section "error analysis and discussion".

f. Understand the selection principles of elements.

g. Understand and grasp the cumulative error amplification method.

2. Experiment principles

(1) Definition and characteristics of moment inertia
(refer to https://www.britannica.com/science/moment-of-inertia)

(2) Fundamental law of moment inertia J

a. Law of the fixed-axis rotation (see the attached PDF document)

If a torque M is applied to a body that is constrained to rotate around a fixed point and then the body will undergo an angular acceleration given by β, the moment inertia has a relation with torque

$$M = J\beta \tag{1}$$

b. Sum theorem

If the rigid body 1 and 2 have moment of inertia J_1 and J_2 around a given axis respectively. The moment inertia of whole system is the sum of J_1 and J_2

$$J = J_1 + J_2 \tag{2}$$

c. Parallel axis theorem

If a rigid body with a quality m has moment inertia J_c around the mass center axis, the moment inertia of this rigid body around the axis with a distance x away from the mass center axis (parallel to the mass center axis) can written as

$$J_x = J_c + mx^2 \tag{3}$$

(3) Structural properties of trilinear torsion pendulum (see figures below)

a. A small disc with uniform structure is horizontally fixed on a support, and can be rotated round their own vertical center axis. (horizontal and rotatable)

b. A large disc hangs under the small disc with three identical lines and make the entire system have the same vertical center axis. (three lines with same length, original height H >> changed height ΔH)

c. After slowly rotating the small disc N, the large disc M will periodically rotate (in the form of torsional pendulum) around vertical center axis of the two discs. (rotate the small disc to start trilinear torsion pendulum with a rotating angle of < 5°)

Figure 1. Schematic of trilinear torsion pendulum

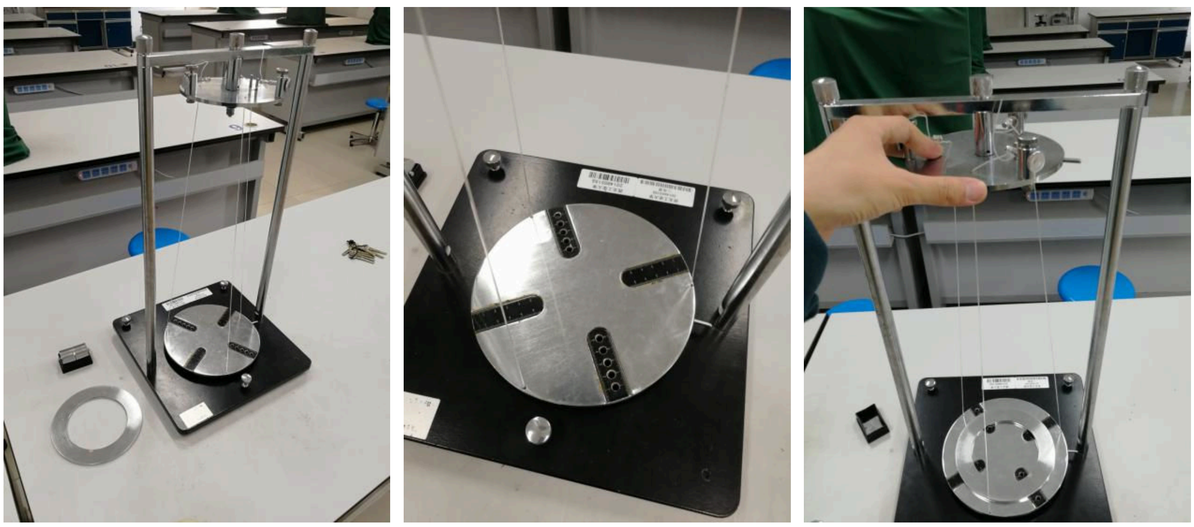
Figure 2. Trilinear torsion pendulum in our lab

(4) Theoretic equations of moment inertia of trilinear torsion pendulum

$$J_0 = \frac{m_0 g R r}{4\pi^2 H} T_0^2 \tag{4}$$

$$\text{or} \qquad J_0 = \frac{m_0 g D d}{12\pi^2 H} T_0^2 \tag{5}$$

Here, m_0 is the mass of large disc M, which can be measured by a balance. g is gravitational acceleration. T_0 is the cycle of large disc, which can be measured by stopwatch. d and D are the side lengths of small and large discs, respectively. D, d and H can be measured by a meter ruler. Thus, the moment inertia can be achieved using Eq. (5).

3. Experiment tasks

(1) Measuring moment inertia of a rigid body

The object should be a rigid body with unique shape (mass center locates at center axis, as shown in Fig. 3). For convenient theoretical calculations, the circular rings or discs are better. Based on sum theorem of moment inertia, the rigid body with mass m_1 can be obtained by subtraction of the moment inertia of the compound structure (the rigid body and large disc) and the moment inertia of large disc.

$$J = J_1 - J_0 = \frac{(m_0 + m_1) g D d}{12\pi^2 H} T_1^2 - J_0 \tag{6}$$

The theoretical value of moment inertia of an equal thickness ring rotating around center axis can be calculated by

$$J' = \frac{1}{8} m_1 (D_1^2 + D_2^2) \tag{7}$$

Here, D_1 and D_2 are the inner and external diameters of the ring, respectively. If the object is equal thickness disc with a diameter D, the theoretical moment inertia can be obtained by

$$J' = \frac{1}{8} m_1 D^2 \tag{8}$$

Through comparing the measured value in Eq. (6) with the theoretical value, the percentage error of moment inertia can be calculated.

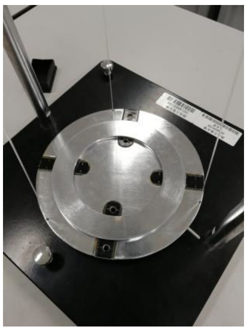

Figure 3. Sum theorem

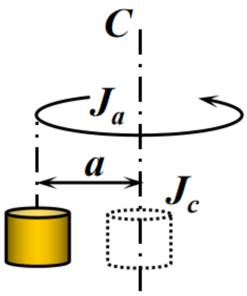

Figure 4. Parallel axis theorem

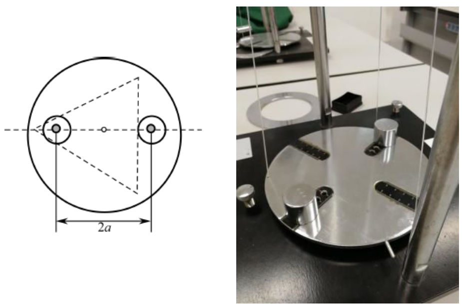

Figure 5. Symmetrical small cylinders

(2) Verifying parallel axis theorem

A small cylinder with the mass m and diameter d_x rotates around an axis with a distance a away from rotating center axis, as shown in Fig. 4. The moment inertia is expressed as

$$J_a^{'}=J_c+ma^2=\frac{1}{8}md_x^{\ 2}+ma^2 \tag{9}$$

When the moment inertia is measured, two identical small cylinders are symmetrically placed on the large disc, as shown in Fig. 5. (Why?) The distance between cylinder center to rotating center axis is a. Thus, the moment inertia of small cylinder around the rotating center axis can be written as

$$J_a=\frac{1}{2}\left[\frac{(m_0+2m)gDd}{12\pi^2 H}T_a^2-J_0\right] \tag{10}$$

Through comparing the measured value in Eq. (10) with the theoretical value in Eq. (9), the percentage error of moment inertia can be calculated.

4. Experiment procedures

(1) Search for proper elements and materials, design the frame structure.

(2) Prepare a large disc of trilinear torsion pendulum, get the mass m_0 of large disc (firstly get m_0).

(3) Produce the whole system of trilinear torsion pendulum, adjust the system to the best situation. The upper and down discs are parallel; The lines have the same length; H>>ΔH. The APP in phones can work as a **gradienter**.

(4) Measure D, d and H with meter rules many times. Measure cycle of torsion pendulum T_0.

(5) Task 1: weight the mass m_1 of an object, namely circular ring or disc, measure the inner and external diameters of ring (or diameter of disc), measure the cycle of torsion pendulum T_1.

(6) Task 2: weight the mass m_x of a small cylinder, measure diameter d_x of small cylinder and distance $2a$ between two centers of symmetrical cylinders.

(The measurement order is adjustable according to the practical conditions.)

✧ Selection criterion of instruments: Employ proper instruments according to the size of physical quantities and make the physical quantities have same significant digits. For example, for the length measurement, the meter ruler can be used to measure larger quantities (> 10 cm) and get four significant digits. For the quantities between 1 to 10 cm (e.g., d_x), the caliper can be used to get four significant digits. But, when the caliper is not available, only three significant digits can be got with a meter ruler. Therefore, d_x should not be too small to increase the relative error.

✧ An example of cumulative error amplification method: The human-controlled stopwatch error is about 0.2 s. However, the cycle of trilinear torsion pendulum is about 1~2 s. The measurement error of one cycle is very large. If 50 cycles are recorded at a time, the total time of 50 cycles is measured, thus the error 0.2 s is equally distributed into the 50 cycles. The error of one cycle is 0.004 s. Therefore, the third decimal place can be retained.

Measured values of trilinear torsion pendulum

	No. / quantity	1	2	3	Average values
	H(cm)				
	D(cm)				
	d(cm)				
disc M	m_0(g)				
	$50T_0$(s)				

Task 1：Measured values of circular ring

No.	1	2	3	Average values
disc M+ ring $50T_1$(s)				
D_1= (cm)		D_2= (cm)		m_1= (g)

Task 2：Measured values for verifying parallel axis theorem

quantity	$2a$ (cm)	d_x(cm)	m(g)	disc M+ small cylinder $50\ T_a$(s)
values				

5. Design key points of self-made instrument

(1) Instrument structure: As depicted in Fig. 6, the instrument consists of supporting structure, upper small disc, hanging lines and down large disc. On referring to the trilinear torsion pendulum in lab, analyze the difference and play particular attention to the linking method of upper disc and supporting structure. The upper disc can rotate around an axis above or below the supporting structure for starting the trilinear torsion pendulum.

(2) Simplified instrument structure: If the conditions are limited, the supporting structure and upper disc can be fixed together. It is better to use two hands to rotate and start the trilinear torsion pendulum.

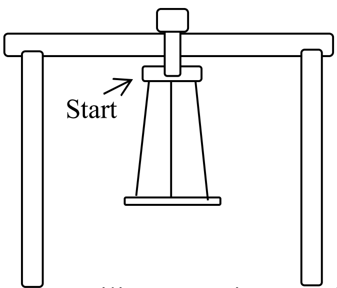

Figure 6 Standard trilinear torsion pendulum

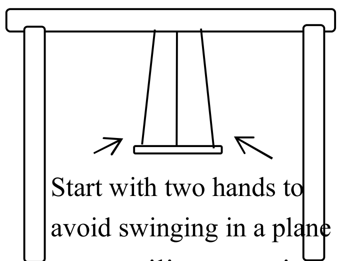

Figure 7 Simplified trilinear torsion pendulum

(3) Selection of elements: The down large disc connects the upper small disc with hanging lines. The lines should be made of low deformation material. The upper and down discs should be regular disc-shaped objects, such as grill, drawer and cardboard. The circular rings (or disc) and small cylinder could be objects with regular shaped objects. If there are not regular shaped objects, the pot cover, small tank of sand (or rice), water bottle with frozen ice, or others can be chosen. The object thickness should be uniform.

(4) Weighting of mass: Masses of discs and objects can be weighted using an electric balance. If there is no balance, a self-made balance can be used (The water can work as weight).

(5) The mass of selected object should not be too small enough.

(6) Refer to the self-made trilinear torsion pendulum in Fig. 8. Please note the function of every elements and rotating axis, wire-coiled pencils and holes in the box.

6. Questions

(1) The height H is measured in the experiment when there is no object on the disc. But, when the object is placed on the disc, the hanging lines will become longer. Please analyze the influence of this effect on the measured results.

(2) In experiments, the cycle is measured by recording 50 T time. Is it OK for 5 T，20 T，100 T，500 T? What’s its principle?

(3) In task 1, the object is chosen as a pot of water, is it OK? Why?

Figure 8. A self-made trilinear torsion pendulum