

Questions & Answers 001 for compiler course

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Question & Answer

【Q1】 left-most derivation, syntax tree

【Q1】 Given $G[E]$:

$E \rightarrow TE'$ $E' \rightarrow ATE' \mid \varepsilon$ $T \rightarrow FT'$

$T' \rightarrow MFT' \mid \varepsilon$ $F \rightarrow (E) \mid i$

$A \rightarrow + \mid -$ $M \rightarrow * \mid /$

① Give the left-most derivation sequence for $i+i*i$

② Compute the First Set, Follow Set and LL(1) Table.

Then give the LL(1) table based left-most derivation for $(i+i)*i$

③ From the steps of above derivation from ②,

write the syntax tree that could represent $(i+i) MFT'E'$

Answer:

(1) left most derivation for " $i+i*i$ ".

Handwritten left-most derivation for $i+i*i$:

$$\begin{aligned} E &\Rightarrow TE' \Rightarrow FT'E' \Rightarrow iT'E' \Rightarrow iE' \quad [\because T' \Rightarrow \varepsilon] \\ &\Rightarrow iATE' \\ &\Rightarrow i+TE' \\ &\Rightarrow i+FT'E' \\ &\Rightarrow i+iT'E' \\ &\Rightarrow i+iMFT'E' \\ &\Rightarrow i+i*FT'E' \\ &\Rightarrow i+i*iT'E' \\ &\Rightarrow i+i*iE' \quad (T' \Rightarrow \varepsilon) \\ &\Rightarrow i+i*i \quad (E' \Rightarrow \varepsilon) \end{aligned}$$

(2) left most derivation for "(i+i)*i"

(2) Left-most derivation for "(i+i)*i"

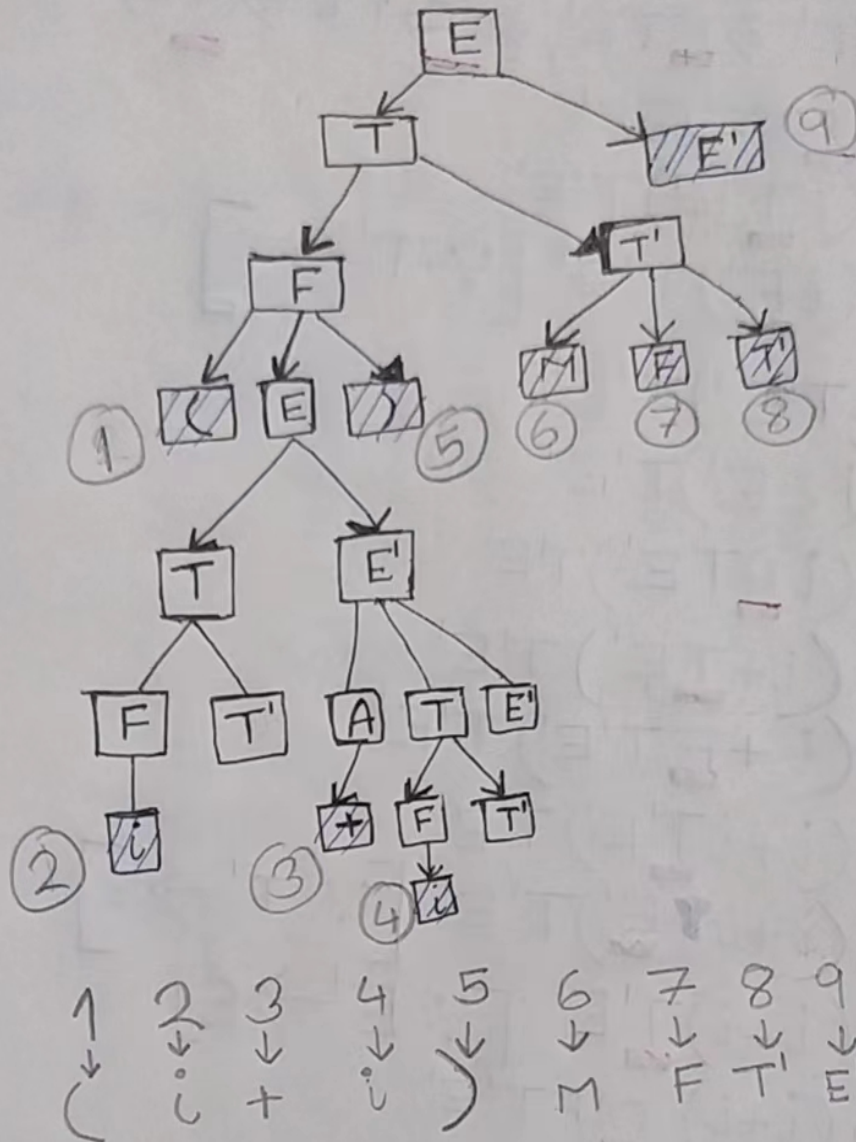
$$\begin{aligned} E &\Rightarrow \underline{T}E' \Rightarrow \underline{FT'E'} \Rightarrow \underline{(E)}T'E' \Rightarrow \underline{(TE')}T'E' \\ &\Rightarrow \underline{(FT'E')}T'E' \\ &\Rightarrow \underline{(iT'E')}T'E' \\ &\Rightarrow \underline{(iE')}T'E' \quad [\because T' \rightarrow \varepsilon] \\ &\Rightarrow \underline{(iT'E')}T'E' \\ &\Rightarrow \underline{(i\varepsilon E')}T'E' \\ &\Rightarrow \underline{(iATE')}T'E' \\ &\Rightarrow \underline{(i+TE')}T'E' \\ &\Rightarrow \underline{(i+FT'E')}T'E' \\ &\Rightarrow \underline{(i+iT'E')}T'E' \\ &\Rightarrow \underline{(i+iE')}T'E' \quad [\because T' \rightarrow \varepsilon] \\ &\Rightarrow \underline{(i+i)T'E'} \quad [\because E' \rightarrow \varepsilon] \\ &\Rightarrow \underline{(i+i)FT'E'} \\ &\Rightarrow \underline{(i+i)*FT'E'} \\ &\Rightarrow \underline{(i+i)*iT'E'} \\ &\Rightarrow \underline{(i+i)*iE'} \\ &\Rightarrow \underline{(i+i)*i} \end{aligned}$$

(3) Syntax tree for $(i+i)MFT'E'$

```
graph TD
    E[E] --> T1[T]
    E --> E1[E']
    T1 --> F1[F]
    T1 --> T2[T']
    F1 --> LP["("]
    F1 --> E2[E]
    F1 --> RP[")"]
    E2 --> T3[T]
    E2 --> E3[E']
    T3 --> F2[F]
    T3 --> T4[T']
    F2 --> i1["i"]
    T4 --> P["+"]
    E3 --> A["A"]
    E3 --> T5[T]
    E3 --> E4[E']
    T5 --> F3[F]
    T5 --> T6[T']
    F3 --> i2["i"]
    T6 --> M["M"]
    T6 --> F4[F]
    T6 --> T7[T']
    E4 --> T8[T']
    T8 --> E5[E']
```

1 $($ 2 i 3 $+$ 4 i 5 $)$ 6 M 7 F 8 T' 9 E'

(3) Syntax tree for $(i+i) \text{MFT}'E'$



【Q 2】 Ambiguous / Syntax Tree, etc.

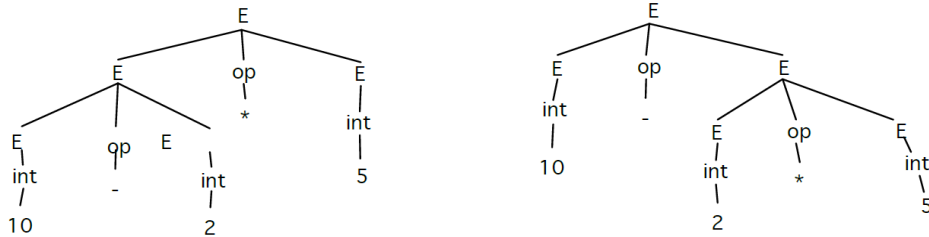
【Q2】 Given $G[E]$

$E \rightarrow E \text{ op } E \mid (E) \mid \text{int}$

$\text{op} \rightarrow + \mid - \mid * \mid /$

$\text{int} \rightarrow 0 \mid 1 \mid \dots \mid 9$

- ① Judge whether $G[E]$ is ambiguous. Give two different reasons.
- ② For string $10-2*5$, two different syntax tree could be left-most derived.



Give your computation sequence for the two trees.

Answer:

- (1) It is ambiguous.

Ans No: 2

(1) It's ambiguous

Reason 1: Both left recursive & right recursive are defined in the BNF rules.

Reason 2: For $2+3*4$, there will be at least two different syntax tree.

Reason 3: For $2+3*4$, there will be at least two different left most derivation.

Reason 4: For $2+3*4$, there will be at least two different right-most derivation.

(2)

left tree $\Rightarrow (10-2)*5$

right tree $\Rightarrow 10-(2*5)$

【Q3】 Write BNF syntax rules from given language

【Q3】 Given language as $\{a^n b^m c^m \mid n \geq 1, m \geq 0\}$, write BNF syntax rules.

Answer:

Ans No: 3

$G[S]$, S is the start symbol

$S \rightarrow AB$

$A \rightarrow aAb \mid ab$ (Since $n \geq 1$, so $A \rightarrow \epsilon$ is not included in the syntax)

$B \rightarrow cb \mid \epsilon$ (another BNF is equal: $B \rightarrow Bc \mid \epsilon$)

Checking

Minimum length string is $ab \mid abc$

Set of possible strings are $a^2b^2c, a^3b^3c^2$

Case 1

String ab

$S \rightarrow \underline{A}B$

$\rightarrow ab\underline{B}$

$\rightarrow ab\underline{\epsilon}$

$\rightarrow ab$

Case 2

String abc

$S \rightarrow \underline{A}B$

$\rightarrow ab\underline{B}$

$\rightarrow abc\underline{B}$

$\rightarrow abc\underline{\epsilon}$

$\rightarrow abc$

Case 3

String a^2b^2c

$S \rightarrow \underline{A}B$

$\rightarrow a\underline{A}bB$

$\rightarrow aa \underline{bb}B$

$\rightarrow aa \underline{bb}cB$

$\rightarrow a^2b^2c\underline{B}$

$\rightarrow a^2b^2c\underline{\epsilon}$

$\rightarrow a^2b^2c$

【Q4】 Write BNF from language

【Q4】 Suppose a language contains all positive even integers (0, 2, 4, 6, 8, ...)

① Write BNF for the above language supposing that the number could begin with number 0 ----

e.g: 03, 09 is allowed.

② If all number could not begin with 0. Write BNF.

Answer:

(1)

Ans No: 4

(1) Positive even number is 2, 4, 6, 8, 0

Define as symbol E

$$E \rightarrow 2 \mid 4 \mid 6 \mid 8 \mid 0$$

① $S \rightarrow E$

$$D \rightarrow 0 \mid 1 \mid 2 \mid \dots \mid 9$$

② $S \rightarrow D E$

③ $S \rightarrow D' E$

$$D' \rightarrow D D' \mid D \mid \epsilon$$

All of them can start with '0'

(2)

(2)

$$S \rightarrow D' D E$$

$\quad \quad \quad 1 \quad 0 \quad 2$

$$D' \rightarrow 1|2|3|\dots|9 \quad (\text{without } 0)$$
$$D \rightarrow 0|1|2|\dots|9|\epsilon$$
$$E \rightarrow 2|4|6|8|0$$

【Q5】 Eliminate left recursive and left factor

【Q5】 Given $G[A]$:

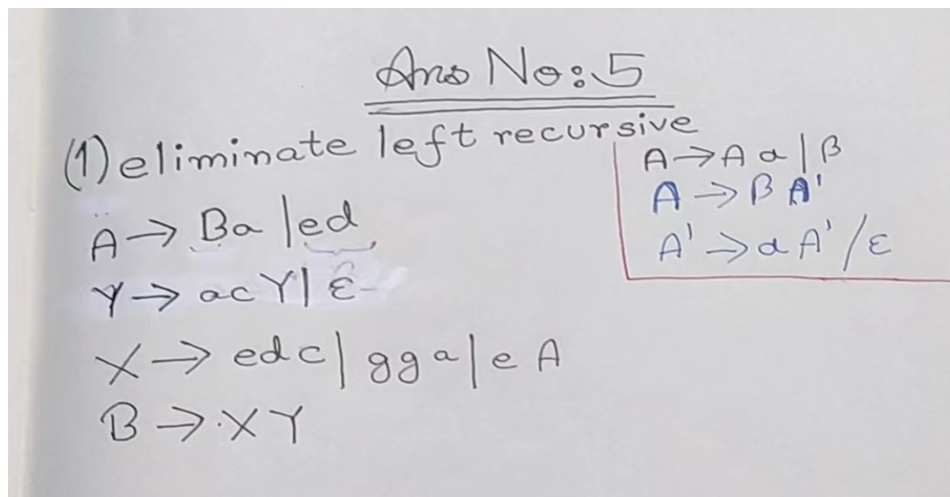
$A \rightarrow Ba \mid ed$

$B \rightarrow Bac \mid edc \mid ggA \mid eA$

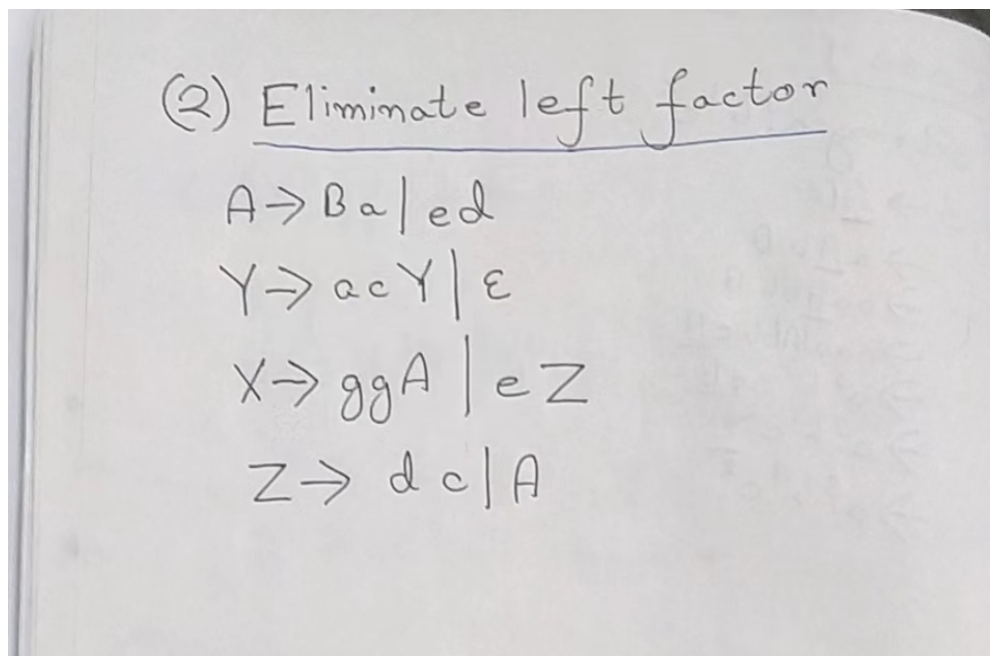
Please eliminate direct left recursive and left factor.

Answer:

(1) eliminate left recursive



(2)



【Q6】LL(1) parsing table, First/Follow set

【Q6】Given $G[S]$:

$S \rightarrow XY$

$X \rightarrow Ya | \epsilon$

$Y \rightarrow Zb | Z$

$Z \rightarrow d | \epsilon$

Calculate First set, Follow set and LL(1) parsing table. Then judge whether the syntax is or is not LL(1) grammar.

Answer:

(1) First and Follow

	First	Explain	Follow	explain
S→XY	ε,a,b,d,	First(S)=First(XY) (1) ε: XY=>εY=>εZ=>εε (2) a: XY=>YaY=>ZaY =>εaY=>aY (3) b: XY=>εY=>εZb=>εεb Or XY=>YaY=>ZbaY=>εbaY=>baY (4) d: XY=>YaY=>ZbaY=>dbaY Or XY=>YaY=>ZaY=>daY Or XY=>εY=>εZb=>εdb Or XY=>εY=>εZ=>εd	#	S#
X→Ya	a,b,d	First(Ya) (1) a: Ya=>εa (2) b: Ya=>Zba=>εba (3) d: Ya=>Zba=>dba Or Ya=>Za=>da	#,b,d	X only appear in S→XY 【Thinking path】 S#=>XY# (1) #: XY#=>XZ#=>Xε#=>X# (2) b: XY#=>XZb# =>Xεb#=>Xb# (3) d: XY#=>XZb#=>Xdb# Or XY#=>XZ#=>Xd#
X→ε	ε			
Y→Zb	b,d	First(Zb) (1) b: Zb=>εb (2) d: Zb=>db	#,a	Y only appear in S→XY and X→Ya (1) #: S#=>XY# (2) a: S#=>XY#=>YaY#
Y→Z	ε,d	First(Z)		
Z→d	d		#,a,b	Z only appear in: Y→Zb and Y→Z. Then consider scenario that Y appears. To cover all scenarios, each time Y appeared, the two appearances of Z must be considered. (1) #: S#=>XY#=>XZ# (2) a: X#=>Ya#=>Za (3) b: Y#=>Zb
Z→ε	ε			

(2) LL(1) parsing table----cells with color contain conflicts

	A	b	d	#
S	S→XY (according to First(XY))	S→XY (First(XY))	S→XY (according to First(XY))	S→XY (Follow(S). Reason: S=>XY=>ε)
X	X→Ya	X→Ya (First(Ya)) X→ε (Follow(X))	X→Ya (First(Ya)) X→ε (Follow(X))	X→ε (Follow(X))
Y	Y→Z (Follow)	Y→Zb (First(Zb))	Y→Zb (according to First(Zb)) Y→Z (according to Follow)	Y→Z (Follow)
Z	Z→ε (Follow)	Z→ε (Follow)	Z→d (according to First(d))	Z→ε (Follow)

(3) There are conflicts. It is not LL(1) grammar.

【Q7】LL(1) and ambiguous

【Q7】 Given $G[S]$:

$S \rightarrow \text{if } e \text{ then } S \mid \text{if } e \text{ then } S \text{ else } S$

Eliminate left recursive and get $G'[S]$:

$S \rightarrow AB$

$A \rightarrow \text{if } e \text{ then } S$

$B \rightarrow \text{else } S \mid \epsilon$

(1) Write the First set Follow Set and LL(1) table for $G'[S]$.

(2) Judge if it is LL(1).

(3) Is the grammar ambiguous?

Answer:

(1) $G'[S]$ is First、Follow set

	First	explain	Follow	explain
$S \rightarrow AB$	if	First(AB) , A begin with if	#,else	<p>(1) #: $S\# \text{ or } S\# \Rightarrow AB\# = A \text{ else } S\#$</p> <p>(2) else:</p> <p>$S\# \Rightarrow AB\# \Rightarrow \text{if } e \text{ then } S \text{ B}\# \Rightarrow \text{if } e \text{ then } S \text{ else } S\#$</p> <p>(3) if could not follow S directly</p> <p><u>Reason1:</u></p> <p>'If' is the first symbol of A ($A \rightarrow \text{if } e \text{ then } S$)</p> <p>But it is impossible to appear If e then S A.</p> <p>So since SA could not appear, "S if" is impossible to appear.</p> <p><u>Reason2:</u></p> <p>SS is impossible to appear too. That is "if e then S S" is impossible. Since generally there must be a ';' between S (S;S).</p>
$A \rightarrow \text{if } e \text{ then } S$	if		#,else	<p>(1) #: $S\# \Rightarrow AB\# \Rightarrow A\epsilon\# = A\#$</p> <p>(2) else: $S\# \Rightarrow AB\# \Rightarrow A \text{ else } S\#$</p> <p>(3) if is not in the follow set. Reason is as above rule.</p>
$B \rightarrow \text{else } S$	else		#,else	<p>(1) #: $S\# \Rightarrow AB\#$</p> <p>(2) else:</p> <p>$S\# \Rightarrow AB\# \Rightarrow \text{if } e \text{ then } S \text{ B}\# \Rightarrow \text{if } e \text{ then } AB \text{ B}\#$</p> <p>$\Rightarrow \text{if } e \text{ then } A \text{ B else } S\#$</p>
$B \rightarrow \epsilon$	ϵ		<p>#,else</p> <p>(B \rightarrow else S and B \rightarrow ϵ conflict)</p>	

LL(1) Parsing Table:

	If	then	else	#
S	$S \rightarrow AB$			
A	$A \rightarrow \text{if } e \text{ then } S$			
B			<p>$B \rightarrow \text{else } S$</p> <p>$B \rightarrow \epsilon$</p>	$B \rightarrow \epsilon$

(2) It is not LL(1). Reason: in the LL(1) table, there is conflict in cell of [B,else].

(3) Is the grammar ambiguous?

Yes.

Proof:

The conflict in [B,else] cell is caused by the famous ambiguous phenomena named if-else dangling.

Classic example : if e then if e then S else S

$S \Rightarrow AB\#$

\Rightarrow if e then S B#

\Rightarrow if e then A B B#

\Rightarrow if e then if e then S B B#

For the statement "if e then if e then S else S#", we could find two different left-most derivations:

First, if e then if e then S ϵ B#

Second, if e then if e then S else S ϵ #

【Q8】 Left recursive grammar must not be LL(1)

【Q8】 Proof the conclusion: if a grammar has left recursive, it must not be LL(1) grammar.

Answer:

Proof:

Suppose there is left recursive. Then there must be the two derivation segments as following:

$X \rightarrow \beta_1$

$X \rightarrow \beta_2$

In the two segments, we suppose that $X \rightarrow \beta_1$ is left recursive (then $X \rightarrow \beta_2$ must be able to derivate to a string only containing terminal symbols. Otherwise the grammar will contain useless symbol).

That is $X \Rightarrow \beta_1 \Rightarrow X \dots$

On the same time, in $X \Rightarrow \beta_2$, β_1 can begin with any terminal symbol (we represent as a).

Then $\text{First}(\beta_1) \cap \text{First}(\beta_2)$ will at least contain a.

Thus it is not a LL(1) grammar.

【Q9】 Left factor and LL(1)

【Q9】 Proof: grammar containing left factor must not be LL(1)

Answer:

Proof:

Suppose there is left factoring. Then there must be the two derivation segments as following:

$X \rightarrow a\beta_1$

$X \rightarrow a\beta_2$

In the two segments, we suppose that $X \rightarrow a\beta_1$ is left factoring (then $X \rightarrow a\beta_2$ must be able to derivate to a string only containing terminal symbols. Otherwise, the grammar will contain useless

symbol).

That is $X \Rightarrow \beta_1 \Rightarrow X \dots$

On the same time, in $X \Rightarrow \beta_2$, β_1 can begin with any initial terminal symbol (we represent as a).

Then $\text{First } X \cap \text{Second } X$ will at least contain initial terminal value a .

X won't know which production to consider because $X \rightarrow a\beta_1$ and $X \rightarrow a\beta_2$ must share some elements

Thus it is not a LL(1) grammar.

Note

We can see that initial symbol of each production is same. Therefore, it's having left factoring.

X won't know which production to consider, because of having same symbol at the right hand side.

The reason is that the first sets of $X \rightarrow a\beta_1$ and $X \rightarrow a\beta_2$ must share some elements among themselves.