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CHAPTER 2.2: EXERCISES

Exe 4:

a) $A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$

b) $A \cap B = \{3\}$

c) $A - B = \{1, 2, 4, 5\}$

d) $B - A = \{0, 6\}$

Exe 19:

Let's prove: 1- $\overline{A \cap B \cap C} \subseteq \bar{A} \cup \bar{B} \cup \bar{C}$

$x \in \overline{A \cap B \cap C}$ by assumption

$\neg(x \notin A \cap B \cap C)$ definetion of complement

$\neg\{(x \in A) \wedge (x \in B) \wedge (x \in C)\}$ definetion of intersection

$(x \notin A) \vee (x \notin B) \vee (x \notin C)$ De Morgan Law

$(x \in \bar{A}) \vee (x \in \bar{B}) \vee (x \in \bar{C})$ definextion of complement

$x \in \bar{A} \cup \bar{B} \cup \bar{C}$ definetion of union

2_ $\bar{A} \cup \bar{B} \cup \bar{C} \subseteq \overline{A \cap B \cap C}$

$x \in \bar{A} \cup \bar{B} \cup \bar{C}$ by assumption

$(x \in \bar{A}) \cup (x \in \bar{B}) \cup (x \in \bar{C})$ definetion of union

$\neg(x \notin A) \cap (x \notin B) \cap (x \notin C)$ De Morgan Law

$\neg(x \notin A \cap B \cap C)$ definetion of intersection

$x \in \overline{A \cap B \cap C}$ definetion of complement

So, we proved that: $\overline{A \cap B \cap C}, \bar{A} \cup \bar{B} \cup \bar{C}$

Exe 26:

$$(A-B)-C = (A-B)\cap\bar{C} = (A\cap\bar{B})\cap\bar{C} = A\cap\bar{B}\cap\bar{C}$$

And,

$$(A-C)-(B-C) = (A\cap\bar{C})-(B\cap\bar{C}) = (A\cap\bar{C})\cap\overline{B\cap C}$$

$$= (A\cap\bar{C})\cap(\bar{B}\cup C) = (A\cap\bar{C}\cap\bar{B})\cup(A\cap C\cap\bar{C})$$

$$= (A\cap\bar{B}\cap\bar{C})\cup\varnothing = A\cap\bar{B}\cap\bar{C}$$

So, we proved that, $(A-B)-C = (A-C)-(B-C)$

Exe 27:

a) $A\cap B\cap C = \{4, 6\}$

b) $A\cup B\cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

c) $(A\cup B)\cap C = \{5, 4, 6, 8, 10\}$

d) $(A\cap B)\cup C = \{0, 2, 4, 5, 6, 7, 8, 9, 10\}$

Exe 32:

a) No, Because A, B can be subsets of C, but different, but $A\cup C = B\cup C = C$

b) No. For example C is $\varnothing$

c) Proof $A\subseteq B$:

Let, $x\in A$, Here is 2 cases =

$x\in C$, so, $x\in A\cap C = B\cap C$	$x\notin C$, so, $x\in A\cup C$
$\therefore\ x\in B\cap C$	so $x\in B\cup C$ and $x\notin C$
$\therefore\ x\in B$	so, $x\in B$

So, that conclude that $A = B$

Exe 35:

$$\overline{(\bar{A} \cup B)} \cap \overline{(B \cup C)} \cap \overline{(\bar{A} \cup C)} = (\bar{A} \cap \bar{B}) \cap (\bar{B} \cap \bar{C}) \cap (\bar{A} \cap \bar{C})$$

$$= \bar{A} \cap \bar{A} \cap \bar{B} \cap \bar{B} \cap \bar{C} \cap \bar{C} = \bar{A} \cap \bar{B} \cap \bar{C}$$

Exe 56:

a) $\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}^+$, set of positive integers

$\bigcap_{i=1}^{\infty} A_i = \phi$

b) $\bigcup_{i=1}^{\infty} A_i = \mathbb{N}$, natural numbers

$\bigcap_{i=1}^{\infty} A_i = \{0, \}$

c) $\bigcup_{i=1}^{\infty} A_i = \mathbb{R}^+$

$\bigcap_{i=1}^{\infty} A_i = (0, 1)$

d) $\bigcup_{i=1}^{\infty} A_i = (1, \infty)$

$\bigcap_{i=1}^{\infty} A_i = \phi$

CH 2.3 HOME WORK

Exe 8 :

a) 1
b) 2
c) -1
d) 0
e) 3
f) -2
g) 1
h) 2

Exe 10:

a) YES, its one-to-one
b) No, b is image to both a & b.
c) No, d is image to both.

Exe 11 :

a) YES
b) No, a don't have preimage
c) No, a don't have preimage

Exe 22:

a) It's a bijection, because the inverse function is, $f^{-1}(x)$, $\frac{4-x}{3}$
b) $f(4) = f(-4)$, so it's not one-to-one, so not bijective
c) f is bijective only from $\mathbb{R}-\{-2\}$ to $\mathbb{R}-\{1\}$, so it's not bijective from $\mathbb{R}$ to $\mathbb{R}$
d) It is a bijection and $f^{-1}(x) = \sqrt[3]{x-1}$ but not from $\mathbb{R}$ to $\mathbb{R}$ it's from $\mathbb{R}^{+}$ to $(1, \infty)$

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Exe 38:

$f\circ g(x) = f(g(x)) = (g(x))^2 + 1 = (x+2)^2+1 = x^2+4x+5$

$g\circ f(x) = g(f(x)) = f(x)+2 = x^2+1+2 = x^2+3$

Ex 41:

* Let $x, y \in \mathbb{R}$ and $x \Rightarrow y$

  Suppose $f(x) = f(y) \Rightarrow ax+b = ay+b$

  $\Rightarrow ax = ay \Rightarrow \boxed{x=y}$

  So, f is injective

* Let $y \in \mathbb{R}$ such that $f(x)=y$, for some x

  So, $ax+b=y \Rightarrow ax = y-b \Rightarrow \boxed{x = \frac{y-b}{a}}$

  So, there exist x such that $f(x)=y$

  So, f is subjective

  $\Rightarrow$ We, conclude that f is invertible

  and $f^{-1}(x) = \frac{x-b}{a}$