

No: _____

Makeup Examination Paper of

Northwestern Polytechnical University

1st Semester of the Academic Year 2020-2021

Course School _____ 2020 School of Computer Science

Course Name _____ Discrete Mathematics

Date of Exam _____ 2020.12.9 Duration and time _____ 2 hours

NOTICE: Write all answers on answer sheet.

I. Choose the right answer (2 points for each, total 20 points)

- How many propositions in the following statements?
1. Do not pass go. 2. The moon is made of green cheese.
3. What time is it? 5. $4 + x = 5$
6. Today is rain. 7. You are later for the exam.
A. 1 B. 2 C. 3 D. 4
- Let p and q be the propositions
 p : Charry is good at Chinese; q : Charry is good at Mathematics
Which one represent Charry is good at Chinese or she is good at Mathematics?
A. $p \wedge q$ B. $p \vee q$ C. $p \rightarrow q$ D. $p \leftrightarrow q$
- Let $P(x)$ denote the statement "x passed the exam", which represent at least one student didn't in the class pass the exam?
A. $P(\text{David})$ B. $\exists x P(x)$ C. $\forall x \neg P(x)$ D. $\exists x \neg P(x)$
- What is the cardinality of $\{\emptyset, \{\emptyset\}, \{\emptyset, \emptyset\}\}$?
A. 0 B. 1 C. 2 D. 3
- Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 7$. What is the inverse function of f ?
A. $2x+7$ B. $7-2x$ C. $1/2*(7-x)$ D. does not exist
- How many rows appear in a truth table for the compound propositions?
 $(p \vee \neg q) \wedge (r \vee s)$
A. 8 B. 16 C. 32 D. 24
- What is the negation of $2+2=5$?
A. $2+2=5$ B. $2+3=5$ C. $2+2=4$ D. $2+2 < 5$
- Decide which integer is remainder of $-101 \bmod 11$?
A. -2 B. 2 C. -9 D. 9
- Convert the binary expansion of $(10000000001)_2$ into hexadecimal expansion.
A. $(101)_{16}$ B. $(201)_{16}$ C. $(301)_{16}$ D. $(401)_{16}$
- How many functions are $O(x)$
a) $f(x) = 10$ b) $f(x) = 3x + 7$ c) $f(x) = x^2 + x + 1$ d) $f(x) = 5 \log x$ e) $f(x) = |x|$
A. 2 B. 3 C. 4 D. 5

II. Answer the question (6 points for each, total 36 points)

- Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Express each of these sets with bit strings where the i th bit in the string is 1 if i is in the set

and 0 otherwise.

- a) {4, 5} b) { 1, 3, 10} c) {2, 4, 7, 8, 9}
2. Let $S = \{-1, 0, 1, 2, 4, 7\}$. Find $f(S)$ if
a) $f(x) = 1$. b) $f(x) = 3x + 1$. C) $f(x) = x^2 + 2x + 1$
3. Find the inverse of 7 modulo 26.
4. Periodicals are identified using an International Standard Serial Number (ISSN). An ISSN consists of two blocks of four digits. The last digit in the second block is a check digit. This check digit is determined by the congruence $d_8 \equiv 3d_1 + 4d_2 + 5d_3 + 6d_4 + 7d_5 + 8d_6 + 9d_7 \pmod{11}$. When $d_8 \equiv 10 \pmod{11}$, we use the letter X to represent d_8 in the code.
- 1) Determine the last number of 1570-868.
- 2) Check if 1059-1027 is an valid ISSN.
5. Use the Euclidean algorithm to find $\gcd(1529, 14039)$.
6. How many solutions does the equation $x_1 + x_2 + x_3 = 15$ have, where x_1, x_2 and x_3 are natural numbers?

III. Proof(4 points for No.1, 8 points for each of others, total 44 points)

1. Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.
2. Devise an algorithm that finds the min number of all the integers in a list.
3. Prove that 5 divides $n^5 - n$ whenever n is a nonnegative integer.
4. Use mathematical induction to prove following statement.

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n},$$

5. Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 16$.
6. Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even.