The 8th Homework

Page 266, chapter 10, problem 9, 17, 25, 37, 45, 47, 49, 53, 65, 75

Page 292, chapter 11, problem 19, 21, 25, 35

10-9 (a) $\omega = \dfrac{2500 \times 2\pi}{60} = 262\text{rad/s}$

(b) $v = \omega R = 262 \times \dfrac{0.35}{2} = 45.85\text{m/s}$

(c) $a = \dfrac{v^2}{R} = \omega^2 R = 262^2 \times \dfrac{0.35}{2} = 1.2 \times 10^4 \text{m/s}^2$

10-17 (a) $\omega = \int (5t^2 - 3.5t)dt = \dfrac{5}{3}t^3 - \dfrac{7}{4}t^2$

(b) $\theta = \int (\dfrac{5}{3}t^3 - \dfrac{7}{4}t^2)dt = \dfrac{5}{12}t^4 - \dfrac{7}{12}t^3$

(c) at t=2s, $\omega = \dfrac{40}{3} - 7 = \dfrac{19}{3}\text{rad/s}$, $\theta = \dfrac{20}{3} - \dfrac{14}{3} = 2\text{rad}$

10-25 $\tau = rF\sin 90°$, $90 = 0.26F$, $F = 346\text{N}$

$$6 \times \frac{15}{2} \times 10^{-3} \times F_{\text{p}} = 90, \quad F_{\text{p}} = 2 \times 10^{-3}\text{N}$$

10-37 (a) As shown in the textbook on page981.

(b) According to Newton's II Law $\sum F = ma$

$$F_1 - m_1 g \sin 30° = m_1 a$$
$$F_{\text{T1}} = m_1(g\sin 30° + a) = 8 \times (9.8 \times 0.5 + 1) \approx 47\text{N}$$
$$m_2 g \sin 60° - F_{\text{T2}} = m_2 a$$
$$F_{\text{T2}} = m_2(g\sin 60° - a) = 10 \times (9.8 \times 0.866 - 1) \approx 75\text{N}$$

(c) $\tau = F_{\text{T2}}r - F_{\text{T1}}r = 75 \times 0.25 - 47 \times 0.25 = 7\text{m}\cdot\text{N}$

(d) $\tau = I\alpha = I\dfrac{a}{r}$, $I = \dfrac{\tau r}{a} = \dfrac{7 \times 0.25}{1} = 1.75\text{kg}\cdot\text{m}^2$

10-45 (a) $I = \left[\dfrac{2}{5}MR_0^2 + (\dfrac{3}{2}R_0)^2 M\right] = 5.3MR_0^2$

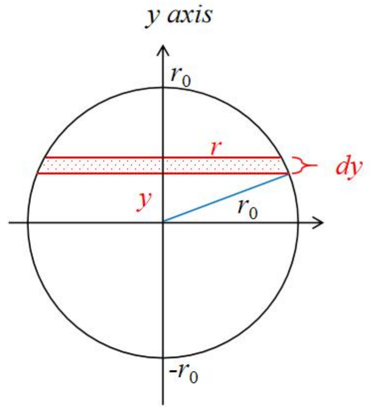
(b) $I' = 2M(\dfrac{3}{2}R_0)^2 = 4.5MR_0^2$ $\qquad (I - I')/I \approx 15\%$

10-47 (a) $I = I_{CM} + Md^2 = \dfrac{1}{2}MR_0^2 + M(0.25R_0)^2 = \dfrac{9}{16}MR_0^2$

(b) $I_z = I_x + I_y = 2I_x = 2I_y = \dfrac{1}{2}MR_0^2$, $I_x = I_y = \dfrac{1}{4}MR_0^2$

(c) $I = \dfrac{1}{4}MR_0^2 + MR_0^2 = \dfrac{5}{4}MR_0^2$

10-49 The mass density of the sphere is $\rho = \dfrac{M}{\frac{4}{3}\pi r_0^3}$



Divide the sphere into infinitesimally thin disks of thickness dy as shown in figure, the radius is r, so the inertia of the infinitesimally thin disk is

$$dI = \frac{1}{2}mr^2 = \frac{1}{2}[\rho(\pi\, r^2)dy]\,r^2$$

$$= \frac{1}{2}\left(\frac{M\pi}{\frac{4}{3}\pi r_0^3}\right) r^4 dy = \frac{3M}{8r_0^3} r^4 dy \qquad\qquad r^2 = r_0^2 - y^2$$

Then integrate over these disks:

$$I = 2\int_{y=0}^{y=r_0} dI = 2\int_{y=0}^{y=r_0} \frac{3M}{8r_0^3} r^4 dy = \frac{3M}{4r_0^3}\int_{y=0}^{y=r_0}(r_0^2 - y^2)^2 dy$$

$$= \frac{3M}{4r_0^3}\int_{y=0}^{y=r_0}(r_0^4 - 2r_0^2 y^2 + y^4)dy = \frac{3M}{4r_0^3}(r_0^4 y - \frac{2}{3}r_0^2 y^3 + \frac{1}{5}y^5)\Big|_{y=0}^{y=r_0}$$

$$= \frac{3M}{4r_0^3}(\frac{8}{15}r_0^5) = \frac{2}{5}Mr_0^2$$

10-53 The angular momentum is conserved.

$$I_1\omega_1 = I_2\omega_2$$

$$\omega_1 = \frac{I_2\omega_2}{I_1} = \frac{1}{3.5}\frac{2}{1.5} = 0.38\text{rev/s}$$

10-65 Mechanical energy is conserved. Choose the ground is zero potential point.

$$m_2gh = m_1gh + \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2 \qquad I = mR_0^2 \qquad \omega = v/R_0$$

$$m_2gh = m_1gh + \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}\left(mR_0^2\right)\left(\frac{v}{R_0}\right)^2$$

$$m_2gh = m_1gh + \frac{1}{2}(m_1 + m_2 + m)v^2$$

$$v = \sqrt{\frac{2(m_2 - m_1)gh}{m_1 + m_2 + m}} = \sqrt{\frac{2\times(38.0-35.0)\times 9.8\times 2.5}{35.0+38.0+4.8}} \approx 1.4\text{m/s}$$

10-75 Mechanical energy is conserved. Choose the ground is zero potential point.

$$mgR_0 = mgr_0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mg(R_0 - r_0) = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{5}mr_0^2)\left(\frac{v}{r_0}\right)^2$$

$$mg(R_0 - r_0) = \frac{7}{10}mv^2$$

$$v = \sqrt{\frac{10}{7}g(R_0 - r_0)}$$

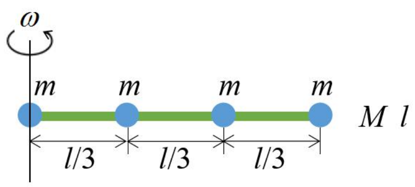
11-19 $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{P} = m\vec{v} = 7.6(-5\hat{i} - 4.5\hat{j} - 3.1\hat{k}) = -38.0\hat{i} - 34.2\hat{j} - 23.56\hat{k}$

$$\vec{L} = \vec{r} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -38.0 & -34.2 & -23.56 \end{vmatrix}$$

$$= [2\times(-23.6) - 3\times(-34.2)]\hat{i} + [3\times(-38.0) - 1\times(-23.56)]\hat{j} + [1\times(-34.2) - 3\times(-38.0)]\hat{k}$$

$$= 55.48\hat{i} - 90.44\hat{j} + 41.8\hat{k} \ \ (\mathrm{kg\cdot m^2/s})$$

11-21 (a) $K = K_1 + K_2 + K_3 + K_4 + K_5$



$$= \frac{1}{2}(I_1 + I_2 + I_3 + I_4 + I_5)\omega^2$$

$$= \frac{1}{2}(0 + m(\frac{l}{3})^2 + m(\frac{2l}{3})^2 + ml^2 + \frac{1}{3}Ml^2)\omega^2$$

$$= (\frac{7m}{9} + \frac{M}{6})l^2\omega^2$$

(b) $L = L_1 + L_2 + L_3 + L_4 + L_5$

$$= (I_1 + I_2 + I_3 + I_4 + I_5)\omega$$

$$= (0 + m(\frac{l}{3})^2 + m(\frac{2l}{3})^2 + ml^2 + \frac{1}{3}Ml^2)\omega$$

$$= (\frac{14m}{9} + \frac{M}{3})l^2\omega$$

11-25 (a) $L = I\omega + R_0M_2v + R_0M_1v = I\dfrac{v}{R_0} + R_0M_2v + R_0M_1v = (\dfrac{I}{R_0} + R_0M_2 + R_0M_1)v$

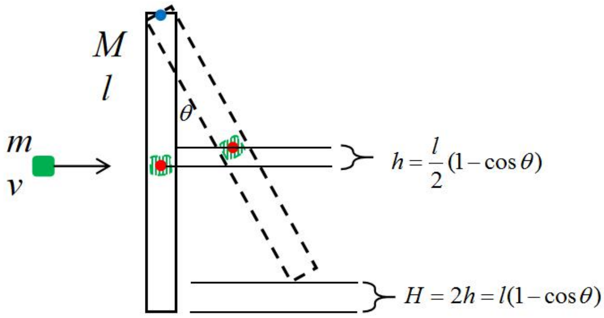
(b) $\tau = \dfrac{dL}{dt} = M_2gR_0$ $\quad (\dfrac{I}{R_0} + R_0M_2 + R_0M_1)a = M_2gR_0$; $a = \dfrac{M_2g}{\dfrac{I}{R_0^2} + M_2 + M_1}$

11-35 The angular momentum is conserved when the putty just sticks in the rod, so

$$mvr = I_1\omega + I_2\omega$$

$$mv\frac{l}{2} = m(\frac{l}{2})^2\omega + \frac{1}{3}Ml^2\omega$$

$$\omega = \frac{6mv}{l(4M+3m)}$$



The mechanical energy is conserved when the rod and putty swing together.

$$\frac{1}{2}(I_1 + I_2)\omega^2 = (M+m)gh$$

$$\frac{1}{2}[m(\frac{l}{2})^2 + \frac{1}{3}Ml^2][\frac{6mv}{l(4M+3m)}]^2 = (M+m)gh$$

$$H = 2h = \frac{3m^2v^2}{(M+m)(4M+3m)g}$$