I. **Choose the right answer（2 points for each, total 30 points）**
BACDB CCBDD CBDBC

II. **Answer the question(6 points for each, total 30 points)**

1. Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.
   Q: $\because 2x^2 <= 14$ $\therefore$ $x \subset \{-2，-1,0,1,2\}$ （2 points）
   $\because 5y^2 <= 14$ $\therefore$ $y \subset \{-1,0,1\}$ （2 points）
   when y=0, no x can meet $2x^2 + 5y^2 = 14$ （1 point）
   y=-1 or 1, no x can meet $2x^2 + 5y^2 = 14$ （1 point）
   so there are no solutions in integers x,y to the equation $2x^2 + 5y^2 = 14$

2. Let $A = \{0, 2,4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find.
   a) $A \cap B \cap C$. b) $A \cup B \cup C$. c) $(A \cup B) \cap C$.
   (2 points per question)
   Q: a) $A \cap B \cap C = \{4,6\}$
   b) $A \cup B \cup C=\{0,1,2,3,4,5,6,7,8,9,10\}$
   c) $(A \cup B) \cap C=\{4,5,6,8,10\}$
   d) $(A \cap B) \cup C = \{0,2,4,5,6,7,8,9,10\}$

3. Let S = {−1, 0, 2, 4, 7}. Find f (S) if
   a) $f(x) = 1$. b) $f(x) = 2x + 1$. C) $f(x) = x^2+2x$
   (2 points per question)

   Q: a) {1} b) {-1,1,5,9,15} c) {-1,0,8,24,63}

4. Find the inverse of 7 modulo 26.
   Q:
   26=3*7+5
   7=5+2
   5=2*2+1
   1=5-2*2=5-2*(7-5)=5-2*7+2*5=3*5-7*2=3*(26-3*7)-2*7=3*26-11*7
   So -11 is inverse of 7 modulo 26. So do 15,41…
   You should write how to get the answer, if you only write the answer, you will lose half score. And inverse is a set of integers.

5. A((BC)D)
   10*40*50+10*50*30+30*10*30=44000

III. Proof(8 points for each, total 40 points)

1. Show that ¬p → (q → r) and q → (p ∨ r) are logically equivalent.
   - Use truth table total 8 rows, 1 point one row
   - Use inference rules to show it.

- Use Conjunctive Normal Form to show it.

2. Devise an algorithm that finds the sum of all the integers in a list.

```
procedure sumup(x: integer, a1, a2, . . . , an: distinct integers)
sum := 0
while (i ⩽ n)
    i := i + 1
    sum=sum+x[i]
return sum
```

3. Use rules of inference to show that the hypotheses

Step Reason

1. ¬t Hypothesis
2. s → t Hypothesis
3. ¬s Modus tollens using (1) and (2)
4. (¬r ∨ ¬f) → (s ∧ l) Hypothesis
5. (¬(s ∧ l)) → ¬(¬r ∨ ¬f) Contrapositive of (4)
6. (¬s ∨ ¬l) → (r ∧ f) De Morgan's law and double negative
7. ¬s ∨ ¬l Addition, using (3)
8. r ∧ f Modus ponens using (6) and (7)
9. r Simplification using (8)

4. Prove when n>4, n! grows faster than 2^n.

Q: Let *P (n)* be the statement when n>4, n! grows faster than 2^n.

basis step:

when n=5, n!=120 2^5=32<n!

inductive step: if P(k) is true, it will implies P(k+1) is true.

P(k) is true, then $k!>2^k$

$(k+1)!=k!*(k+1)>k!*2>2^k*2=2^{(k+1)}$

So P(k)->P(k+1)

For any n>4, n! grows faster than 2^n.

5. If f and f ◦ g are one-to-one, does it follow that g is one-to-one?

Yes

To clarify the setting, suppose that g : A → B and f : B → C , so that f ◦ g: A → C . We will prove that if f ◦ g is one-to-one, then g is also one-to-one, so not only is the answer to the question "yes," but part of the hypothesis is not even needed. Suppose that g were not one-to-one. By definition this means that there are distinct elements a1 and a2 in A such that g(a1) = g(a2). Then certainly f(g(a1)) = f(g(a2)), which is the same statement as (f ◦ g)(a1) = (f ◦ g)(a2). By definition this means that f ◦ g is not one-to-one, and our proof is complete.