Guarantee of Integrity

I, m informed of all the regulations of exams and the corresponding punishments while violating them. I will abide by all the rules for exams and stay honest. Signature：_____

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Examination Paper of Northwestern Polytechnical University

Score:	

1st Semester of the Academic Year 2016-2017

Course School 2016 School of Computer Science

Course Name Discrete Mathematics

Date of Exam 2017.1.10 Duration and time 2 hours

NOTICE：Write all answers on answer sheet.

I. Choose the right answer（1 points for each, total 20 points）

1. Which one is proposition?:
   A. Do not pass go. B. The moon is made of green cheese.
   C. What time is it? D. $4 + x = 5$
2. Let p and q be the propositions
   p: Charry is good at Chinese: q: Charry is good at Mathematics
   Which one represent Charry is good at Chinese and Mathematics?
   A. $p \vee q$ B. $p \leftrightarrow q$ C. $p \rightarrow q$ D. $p \wedge q$
3. Which is not a tautology?
   A. $(p \wedge q) \rightarrow p$ B. $p \rightarrow (p \vee q)$ C. $(p \vee \neg p) \rightarrow F$ D. $F \rightarrow p$
4. Let P(x) denote the statement "$x \leq 4$." What is false?
   A. P(x) B. P(3) C. P(4) D. P(5)
5. Let P(x) denote the statement "x passed the exam", which represent someone in the class passed the exam?
   A. P(David) B. $\exists x P(x)$ C. $\forall x P(x)$ D. $\exists x \neg P(x)$
6. What is the cardinality of $\{\varnothing, \{\varnothing\}, \varnothing\}$?
   A. 0 B. 1 C. 2 D. 3
7. Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 3x + 2$ and $g(x) = 2x + 3$. What is the composition of f and g?
   A. 5x+5 B. 6x+5 C. 6x+7 D. 6x+11
8. Which function grows fastest?
   A. x B. logx C. x! D. 2^x
9. Decide which integer is congruent to 3 modulo 7?
   A. *37* B. *45* C. *-37* D. *-10*
10. Determine whether the integers in each of these sets are pairwise relatively prime.
    a) 21, 34, 55 b) 14, 17, 85 c) 28, 41, 49, 64 d) 17, 18, 19, 22
11. What are the quotient and remainder when −111 is divided by 11??
    A.10, 1 B. -11,10 C. -10,-1 D. 11,-10

Class:

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Name:

12. Which is a valid USPS? The United States Postal Service (USPS) sells money orders identified by an 11-digit number x1x2 . . . x11. The first ten digits identify the money order; x11 is a check digit that satisfies x11 = x1 + x2 + · · · + x10 mod 9.
A. 7555618873X B. 69661334215 C. 80189274359 D. 32897441440

13. Decide which integer is remainder of -101 mod 13?
A. 3 B. -3 C. 10 D. -10

14. Convert the binary expansion of $(1010001001)_2$ into hexadecimal expansion.
A. $(1211)_{16}$ B. $(289)_{16}$ C. $(149)_{16}$ D. $(649)_{16}$

15. How many bit strings are there of length five or less, not counting the empty string?
A. 2^5 B. 2^6 C. 2^6-1 D. 2^6-2

16. How many strings are there of three lowercase letters that have the letter y in them?
A. 26^3 B. 26^3-24^3 C. 26^3-25^3 D. 25^3

17. How many binary numbers are there of length six or less?
A. 2^6 B. 2^6-1 C. 2^7 D. 2^7-1

18. Let $A = \{0, 2, 4\}$, $B = \{0, 1, 2\}$, and $C = \{2, 4, 6\}$. Find $(A \cap B) \cup C$
A. {0,2} B. {0,2,4,6} C. {0,2,6} D. {2,4,6}

19. Which number is NOT a prime?
A. 21 B. 19 C. 5 D. 2

20. What is the coefficient of x^5 in the series $(1 + x)^{10}$?
A. 10! B. 10!/5! C. 10!/(5!5!) D. 5!*5!

II. Answer the question(5 points for each, total 40 points)

1. Let S = {−1, 0, 2, 4, 7}. Find f (S) if
a) $f(x) = 1$. b) $f(x) = x^2-2x$
2. Find the inverse of 7 modulo 33.
3. Solve the system of congruence $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 6$ using the method of back substitution.
4. Find the greatest common divisor of 234 and 455.
5. How many different strings can be made from the letters in HELLOWORLD, using all the letters?
6. How many different combinations of pennies, nickels, dimes and quarters can a piggy bank contain if it has 17 coins in it?
7. How many strings are there of three lowercase letters that have the letter x or y in them?（tip：count number of set without x and y）
8. How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a1, b1) and (a2, b2) such that a1 mod 5 = a2 mod 5 and b1 mod 7 = b2 mod 7?

III. Proof(8 points for each, SELECT 5 from all, total 40 points)

1. Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.
2. Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."
3. Show that if $a \equiv b \pmod m$ and $c \equiv d \pmod m$, where a, b, c, d are integers with $m \geq 2$, then $a-2*c \equiv b-2*d \pmod m$.
4. Prove when $n>4$, $n!$ grows faster than 2^n.
5. Prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n + 1)! - 1$ whenever n is a positive integer.
6. Solve these recurrence relations together with the initial conditions given. $a_n = a_{n-1} + 6a_{n-2}$ for $n \geq 2$, $a0 = 3$, $a1 = 6$
7. Find $f(n)$ when $n = 2^k$, where f satisfies the recurrence relation $f(n) = f(n/2) + 2$ with $f(1) = 1$.