



DIGITAL IMAGE PROCESSING

IMAGE DIGITIZATION

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Section 1-Preface

I have chosen “Image Digitization” as my topics of dissertation. Simply because digitization is of crucial importance to data processing, storage and transmission. It allows information of all kinds in all formats to be carried with the same efficiency and also intermingled. In future I am interested to study in Ai. My topics would be how AI Image Processing Services combine advanced algorithmic technology with machine learning and computer vision to process large volumes of pictures easily and quickly. I will try to elaborate topics related with “Image Digitization”.

Section 2 -Abstract

Image is made by number of pixels and different major parameters like color and monochrome. Image is processed and executed by an image processing techniques. So image processing is the major part of signal processing.

Image is a visual representation of any object or we can say that the image or picture is created, copied and stored in a electronics form. Mathematically image is the form of two dimensional signal define by $f(x,y)$, where f is the intensity property like brightness and contrast.

Interest in digital image processing methods stems from two principal application areas: improvement of pictorial information for human interpretation; and processing of image data for storage, transmission, and representation for autonomous machine perception. It has several objectives: (1) to define the scope of the field that we call image processing; (2) to give a historical perspective of the origins of this field; (3) to give an idea of the state of the art in image processing by examining some of the principal areas in which it is applied; (4) to discuss briefly the principal approaches used in digital image processing; (5) to give an overview of the components contained in a typical, general-purpose image processing system.

Section 3 -Introduction

Image digitization represents an optical image as a set of numbers with no distortion, and is also convenient for computer analysis

An image captured by a sensor is expressed as a continuous function $f(x, y)$ of two co-ordinates in the plane. In Image digitization the function $f(x, y)$ is sampled into a matrix with n columns and m rows. An integer value is assigns to each continuous sample in the image quantization. The continuous range of the image $f(x, y)$ is split into k intervals. When finer the sampling (i.e. the larger m and n) and quantization (the larger k) the better the approximation of the continuous image function $f(x, y)$.

Space and **magnitude** discretization of the continuous image function $f(x, y)$

Image sampling: Discretization of spatial continuous coordinates (x, y) .

Grayscale level quantization: Discretization of magnitude $f(x, y)$ (Integer)

Section 4 -Digital Image Fundamentals :

The field of digital image processing refers to processing digital images by means of digital computer.

Digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are called picture elements, image elements, pels and pixels. Pixel is the term used most widely to denote the elements of digital image. An image is a two-dimensional function that represents a measure of some characteristic such as brightness or color of a viewed scene. An image is a projection of a 3- D scene into a 2D projection plane



Section 5 -Digitization

Digitization is the conversion of a continuous-tone and spatially continuous brightness distribution $f[x, y]$ to an discrete array of integers $f_q[n, m]$ by two operations which will be discussed in turn:

(A) SAMPLING — a function of continuous coordinates $f[x, y]$ is evaluated on a discrete matrix of samples indexed by $[n, m]$. You probably saw some discussion of sampling in the course on linear and Fourier mathematics.

(B) QUANTIZATION — the continuously varying brightness f at each sample is converted to a one of set of integers f_q by some nonlinear thresholding process. The digital image is a matrix of picture elements, or pixels if your ancestors are computers. Video descendants (and imaging science undergraduates) often speak of pels (often misspelled pels). Each matrix element is an integer which encodes the brightness at that pixel. The integer value is called the gray value or digital count of the pixel.

Computers store integers as Binary digits, or bits (0,1)

2 bits can represent: $00_{\Delta} = 0$, $01_{\Delta} = 1$, $10_{\Delta} = 2$, $11_{\Delta} = 3$; a total of $2^2 = 4$ numbers.

(where the symbol " $_{\Delta}$ " denotes the binary analogue to the decimal point "." and thus may be called the "binary point," which separates the ordered bits with positive and negative powers of 2).

m BITS can represent 2^m numbers

$\Rightarrow 8 \text{ BITS} = 1 \text{ BYTE} \Rightarrow 256 \text{ decimal numbers, } [0, 255]$

$\Rightarrow 12 \text{ BITS} = 4096 \text{ decimal numbers, } [0, 4095]$

$\Rightarrow 16 \text{ BITS} = 2^{16} = 65536 \text{ decimal numbers, } [0, 65535]$

Digitized images contain finite numbers of data "bits" and it probably is apparent that the process of quantization discards some of the content of the image, i.e., the quantized image differs from the unquantized image, so errors have been created. Beyond that, we can consider the "amount" of "information" in the quantized image, which is defined as the number of bits required to store the image. The number of bits of "information" usually is smaller than the the number of bits of "data" (which is merely the product of the number of image pixels and the number of bits per pixel). The subject of information content is very important in imaging and will be considered in the section on image compression. We will discuss digitizing and reconstruction errors after describing the image display process.

Image digitization :

(sampling + quantization)

Continuous image —————> Digital image

**Original
image**



(a)

**Blurred
image**



(b)

**Sharpened
image**



(c)

**Smoothed
image**



(d)

Section 6 -What Is Digital Image Processing

An image may be defined as a two-dimensional function, $f(x, y)$, where x and y are spatial (plane) coordinates, and the amplitude of f at any pair of coordinates (x, y) is called the intensity or gray level of the image at that point. When x , y , and the amplitude values of f are all finite, discrete quantities, we call the image a digital image. The field of digital image processing refers to processing digital images by means of a digital computer.

Digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are referred to as picture elements, image elements, pels, and pixels. Pixel is the term most widely used to denote the elements of a digital image.

Thus, digital image processing encompasses a wide and varied field of applications. There is no general agreement among authors regarding where image processing stops and other related areas, such as image analysis and computer vision, start.

A mid-level process is characterized by the fact that its inputs generally are images, but its outputs are attributes extracted from those images (e.g., edges, contours, and the identity of individual objects). Finally, higher-level processing involves “making sense” of an ensemble of recognized objects, as in image analysis, and, at the far end of the continuum, performing the cognitive functions normally associated with vision.

Thus, what we call digital image processing encompasses processes whose inputs and outputs are images and, in addition, encompasses processes that extract attributes from images, up to and including the recognition of individual objects.

The Origins of Digital Image Processing

One of the first applications of digital images was in the newspaper industry, when pictures were first sent by submarine cable between London and New York. Introduction of the Bartlane cable picture transmission system in the early 1920s reduced the time required to transport a picture across the Atlantic from more than a week to less than three hours. Specialized printing equipment coded pictures

for cable transmission and then reconstructed them at the receiving end. It was transmitted in this way and reproduced on a telegraph printer fitted with typefaces simulating a halftone pattern. Some of the initial problems in improving the visual quality of these early digital pictures were related to the selection of printing procedures and the distribution of intensity levels. The printing method used to obtain was abandoned toward the end of 1921 in favor of a technique based on photographic reproduction made from tapes perforated at the telegraph receiving terminal. Evident, both in tonal quality and in resolution.

The idea of a computer goes back to the invention of the abacus in Asia Minor, more than 5000 years ago. More recently, there were developments in the past two centuries that are the foundation of what we call a computer today. However, the basis for what we call a modern digital computer dates back to only the 1940s with the introduction by John von Neumann of two key concepts: (1) a memory to hold a stored program and data, and (2) conditional branching. These two ideas are the foundation of a central processing unit (CPU), which is at the heart of computers today.

Intel in the early 1970s; (6) introduction by IBM of the personal computer in 1981; and (7) progressive miniaturization of components, starting with large scale integration (LI) in the late 1970s, then very large scale integration (VLSI) in the 1980s, to the present use of ultra large scale integration (ULSI). Concurrent with these advances were developments in the areas of mass storage and display systems, both of which are fundamental requirements for digital image processing.

From the 1960s until the present, the field of image processing has grown vigorously. In addition to applications in medicine and the space program, digital image processing techniques now are used in a broad range of applications.

Examples of Fields that Use Digital Image Processing

Today, there is almost no area of technical endeavor that is not impacted in some way by digital image processing. Digital image processing has a broad spectrum of applications, such as 1. Remote sensing via satellites and others space crafts 2. Image transmission and storage for business applications 3. Medical processing 5. SONAR 7. Robotics and etc.

IMAGE PROCESSING TOOLBOX (IPT)

It is a collection of functions that extend the capability of the MATLAB numeric computing environment. These functions, and the expressiveness of the MATLAB language, make many image-processing operations easy to write in a compact, clear manner, thus providing an ideal software prototyping environment for the solution of image processing problem.

Components of an Image Processing System

As recently as the mid-1980s, numerous models of image processing systems being sold throughout the world were rather substantial peripheral devices that attached to equally substantial host computers. Late in the 1980s and early in the 1990s, the market shifted to image processing hardware in the form of single boards designed to be compatible with industry standard buses and to fit into engineering workstation cabinets and personal computers. In addition to lowering costs, this market shift also served as a catalyst for a significant number of new companies whose specialty is the development of software written specifically for image processing.

Although large-scale image processing systems still are being sold for massive imaging applications, such as processing of satellite images, the trend continues toward miniaturizing and blending of general-purpose small computers with specialized image processing hardware.

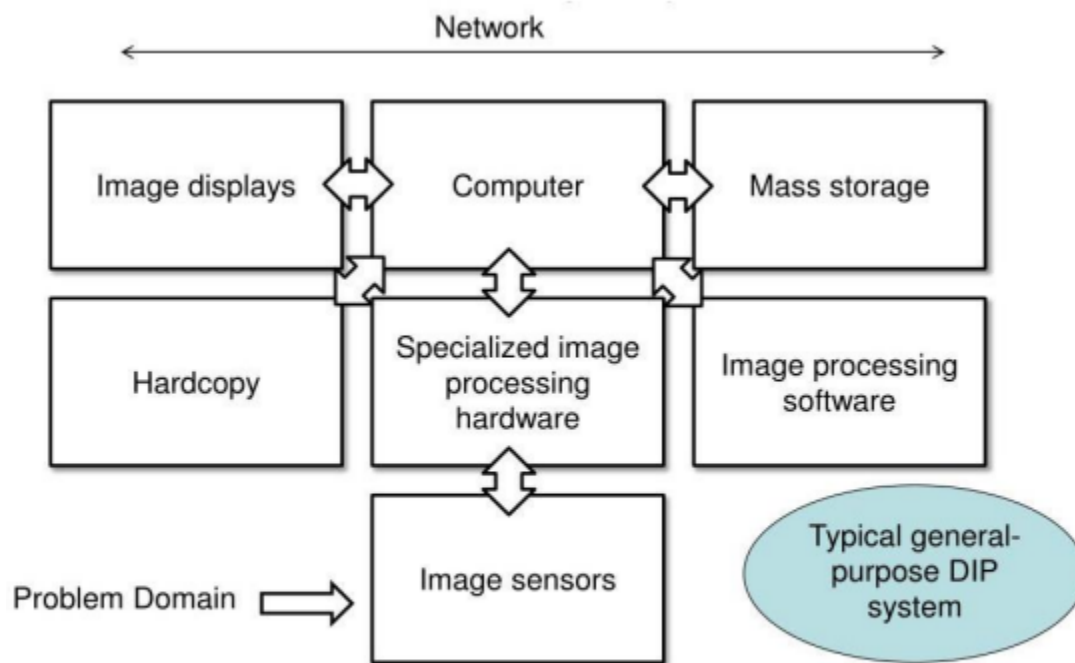


Fig: Components of Image processing System

Image sensors, Specialize Image Processing Hardware, computer, software, mass storage, image display, hardware devices, networking.

Section 7 -Why Image Digitization Is Necessary

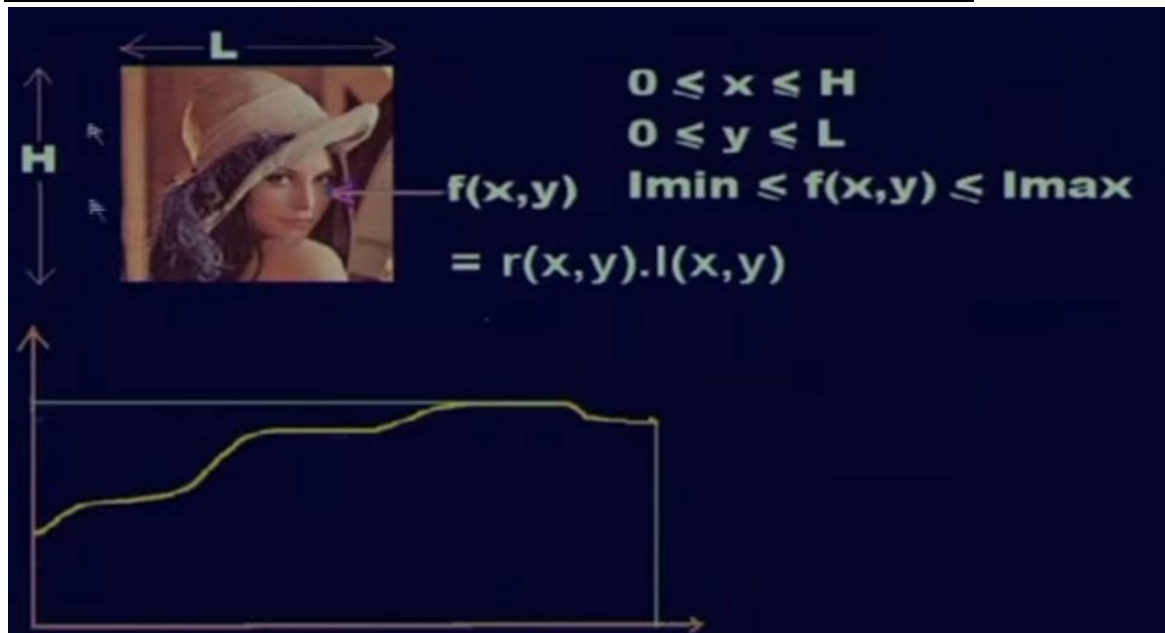


Image can be viewed as a 2 dimensional function given in the form of $f(x,y)$. In the picture above the girl has certain length L and height H . both has units of distance and length. Now, any point in this 2D image identified by image co-ordinates $f(x,y)$. Conveniently x axis is taken vertically downward and y axis taken horizontal.

So, every coordinates in this 2D space has limit like this. Here the value of x varies from 0 to H and the value of y varies from 0 to L . Any point x, y in this image the point x, y is the intensity or color values at point x, y which can be re, presented as a function of x and y , where x, y is at the point of image face. It is actually multiplication of two terms.

$r(x, y), I(x, y)$.

$r(x, y)$ is the reflectance of the surface point of which this particular image corresponds to. $I(x, y)$ is the intensity of light that is falling on the object surface.

Theoretically the $r(x, y)$ can vary from 0 to 1 and $i(x, y)$ can vary from 0 to infinity. So at point $f(x, y)$ in the image can have a value between 0 to infinity. But practically, the intensity or color at a particular point is given by (x, y) which varies from certain minimum represents by I_{\min} and certain maximum I_{\max} .

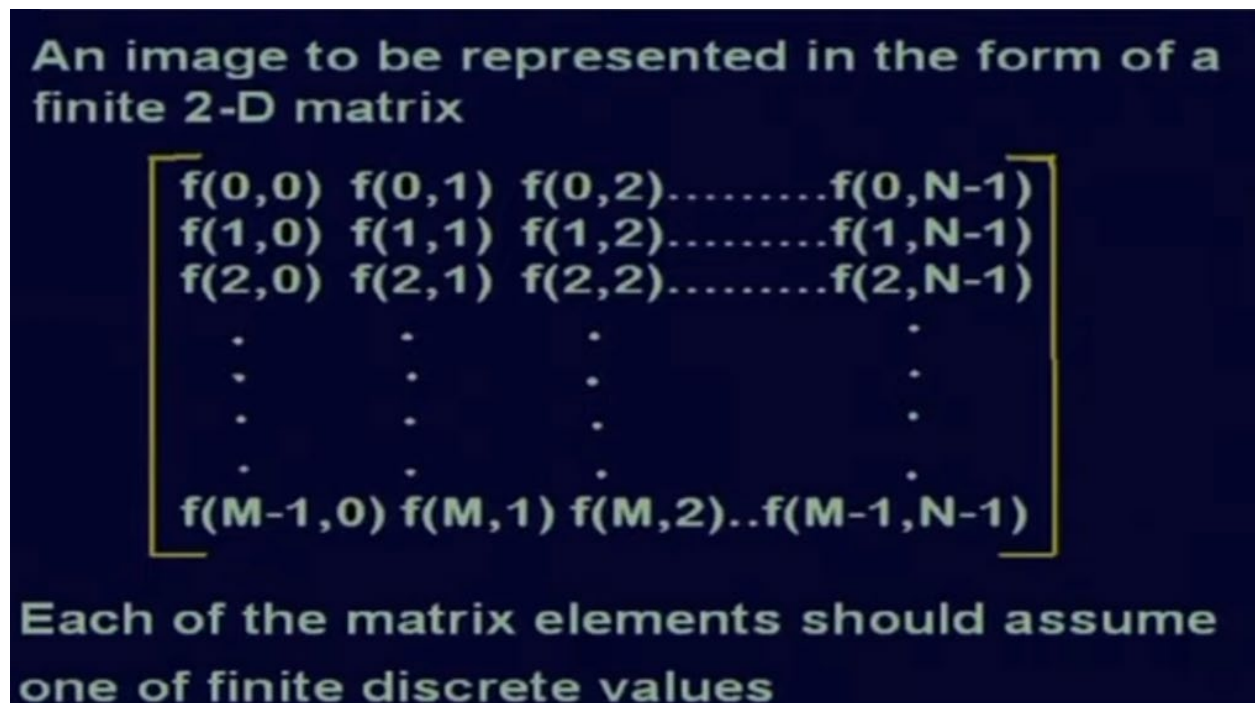
Intensity at the point x, y which is represented by (x, y) minimum intensity value to a certain maximum intensity value. Now in the graph figure it shows that if I take a horizontal line on the inner space and if I plot the intensity values along that line, the intensity profile will be like the graph. It again shows the minimum intensity value and maximum intensity value along the line. Or intensity along a line whether horizontal or vertical can assume any value between the maximum and minimum.

Here, lies the **problem**. When we consider a continuous image which can assume any value and intensity can be certain minimum or maximum. And the coordinates point x and y can also some value x can vary 0 to H and y can vary from 0 to L .

Now from the theory of real numbers, we know that given at any two points there are infinite number of points. So the image x varies from 0 to H , there can be infinite possible values of x between 0 and H . Similarly, there are infinite possible values of y between 0 to L . It means if I want to represent this image in a computer, then this image has to be represented by infinite number of points. Secondly, when I consider the intensity value at a particular point, we have seen that the intensity value $f(x, y)$ varies from I_{\min} to I_{\max} .

Again, if I take these two I_{\min} to I_{\max} to be maximum and minimum values possible. But the problem is the number of intensity values between minimum and maximum is again infinite number. It means if I represent an intensity value in a digital computer I have to have infinite number of bits to represent an intensity value. Obviously such a representation is not possible in any digital computer.

Therefore, we have to find out a way. Our requirement is we have to represent this image in a digital computer. So, instead of considering every possible point in an inner space, we will take some discrete set of points. And those discrete set points are decided by grid. If we have an uniform rectangular grid then at each of the grid location we can take a particular point. We will consider the intensity at that particular point. This is a process known as sampling.



So what desired in an image should be represented in the form of a finite 2D matrix like the above figure. This is a matrix representation of an image. The matrix has got finite number of elements.

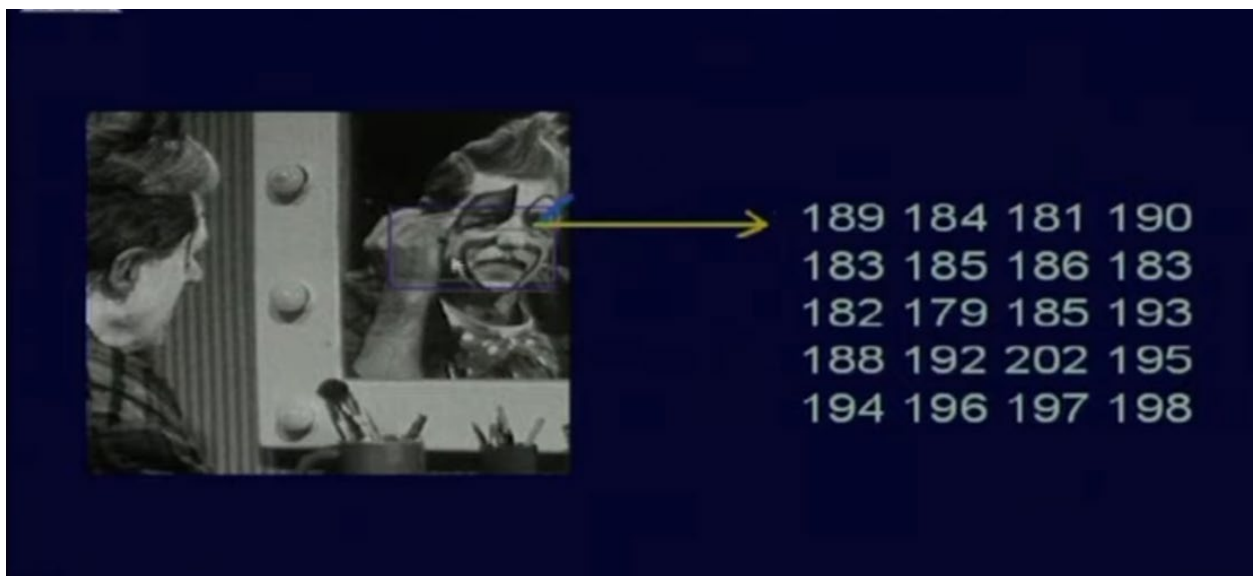
If we look at the matrix, we will find the matrix has M number rows varying from 0 to M – 1 and it has got N number of columns varying from 0 to N-1. Typically, for image processing application the dimension is usually taken as either 256/256 or 512/512 or 1k/1k and so on. But still whatever size of the matrix, the matrix is still be finite. We have finite number of rows and columns and we get image in the form of a matrix like the above picture.

Now, the second requirement is, if I don't do any other processing on this matrix elements. Every matrix elements represents an intensity value in the corresponding image location. The number of intensity values can again be infinite between certain minimum and maximum which is again not possible to be represent in a digital computer.

So, here we want is each of the matrix elements should also assume one of the finite discrete values.

At first we have to do sampling to represent the image in the finite 2D matrix. And each of the matrix elements again has to digitalized so that the intensity value at a particular element or a particular element in a matrix can assume only values from a finite set of discrete values. These two together complete the image digitization process.

Example:



We find the image in the above figure. And if I take a small rectangle in this image and try to find the values in the small rectangle. These values are finite in the form of a matrix. And every element in the

rectangular in the small rectangle or matrix assumes an integer. So an image when digitized will be represented in the form of a matrix like as the above picture.

Therefore, it indicates that by digitization, what means is an image representation by 2D finite matrix which is sampling. And secondly, each matrix element represented by one of the finite set of discrete values known as quantization.

Section 8 -Basic Concepts in Sampling and Quantization

The basic idea behind sampling and quantization is illustrated in Fig. 2.16. Figure 2.16(a) shows a continuous image, $f(x, y)$, that we want to convert to digital form. An image may be continuous with respect to the x - and y -coordinates, and also in amplitude. To convert it to digital form, we have to sample the function in both coordinates and in amplitude. Digitizing the coordinate values is called sampling. Digitizing the amplitude values is called quantization. The one-dimensional function shown in Fig. 2.16(b) is a plot of amplitude (gray level) values of the continuous image along the line segment AB in Fig. 2.16(a). The random variations are due to image noise. To sample this function, we take equally spaced samples along line AB, as shown in Fig. 2.16(c). The location of each sample is given by a vertical tick mark in the bottom part of the figure. The samples are shown as small white squares superimposed on the function. The set of these discrete locations gives the sampled function. However, the values of the samples still span

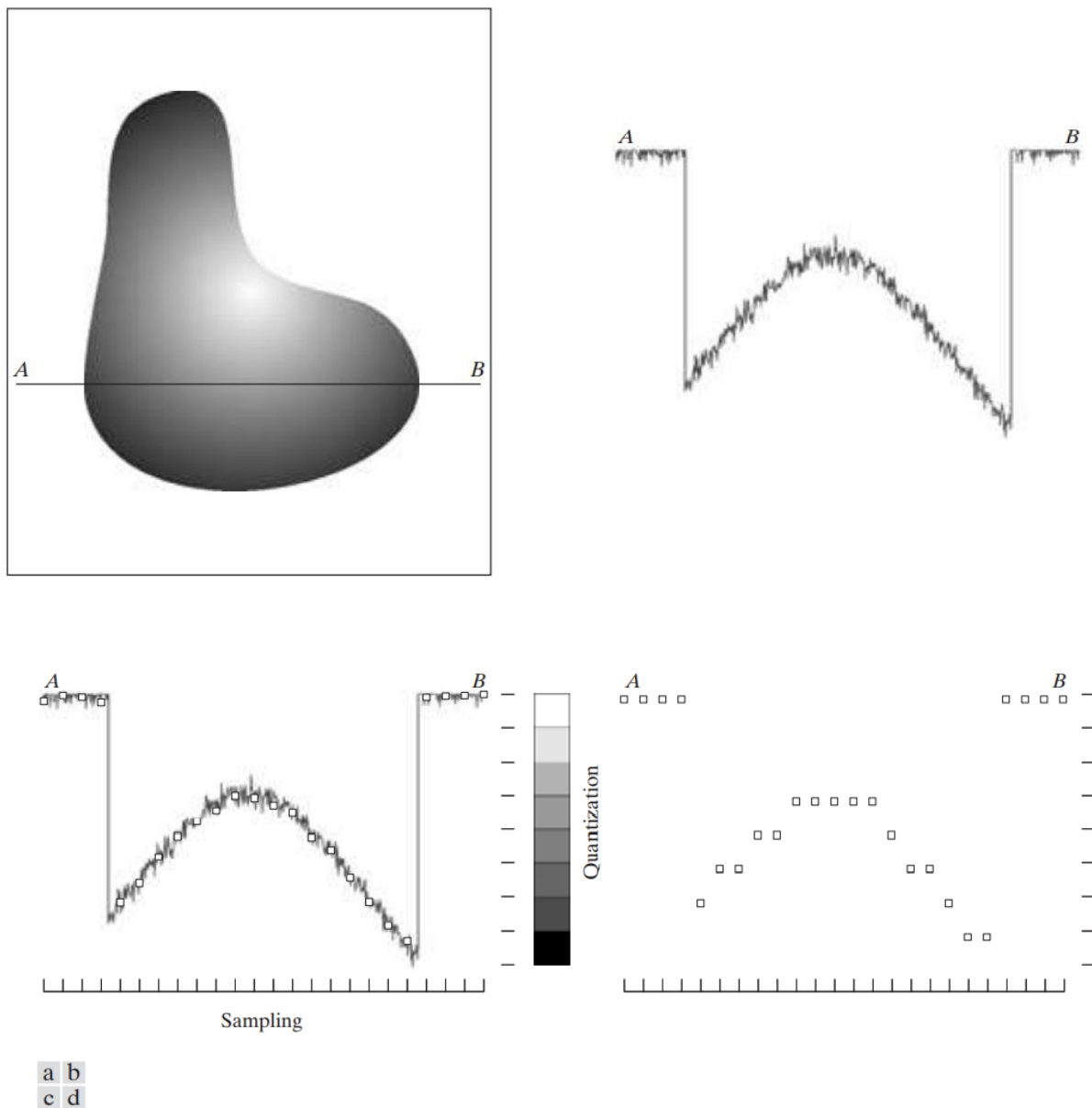


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

(vertically) a continuous range of gray-level values. In order to form a digital function, the gray-level values also must be converted (quantized) into discrete quantities. The right side of Fig. 2.16(c) shows the gray-level scale divided into eight discrete levels, ranging from black to white. The vertical tick marks indicate the specific value assigned to each of the eight gray levels. The continuous gray levels are

quantized simply by assigning one of the eight discrete gray levels to each sample. The assignment is made depending on the vertical proximity of a sample to a vertical tick mark. The digital samples resulting from both sampling and quantization are shown in Fig. 2.16(d). Starting at the top of the image and carrying out this procedure line by line produces a two-dimensional digital image.

Sampling in the manner just described assumes that we have a continuous image in both coordinate directions as well as in amplitude. In practice, the method of sampling is determined by the sensor arrangement used to generate the image. When an image is generated by a single sensing element combined with mechanical motion, as in Fig. 2.16(a), the output of the sensor is quantized in the manner described above. However, sampling is accomplished by selecting the number of individual mechanical increments at which we activate the sensor to collect data. Mechanical motion can be made very exact so, in principle, there is almost no limit as to how fine we can sample an image. However, practical limits are established by imperfections in the optics used to focus on the sensor an illumination spot that is inconsistent with the fine resolution achievable with mechanical displacements.

When a sensing strip is used for image acquisition, the number of sensors in the strip establishes the sampling limitations in one image direction. Mechanical motion in the other direction can be controlled more accurately, but it makes little sense to try to achieve sampling density in one direction that exceeds the sampling limits established by the number of sensors in the other. Quantization of the sensor outputs completes the process of generating a digital image.

When a sensing array is used for image acquisition, there is no motion and the number of sensors in the array establishes the limits of sampling in both directions. Quantization of the sensor outputs is as before.

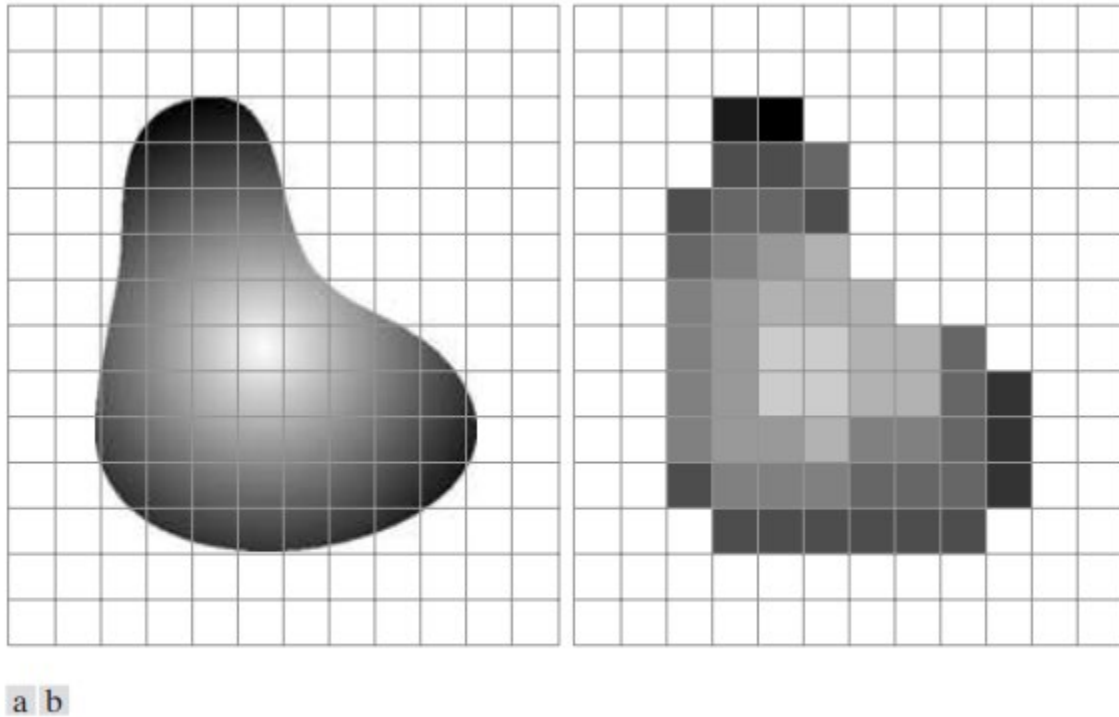


FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Figure 2.17 illustrates this concept. Figure 2.17(a) shows a continuous image projected onto the plane of an array sensor. Figure 2.17(b) shows the image after sampling and quantization. Clearly, the quality of a digital image is determined to a large degree by the number of samples and discrete gray levels used in sampling and quantization.

Section 9-IMAGE SAMPLING AND RECONSTRUCTION

CONCEPTS

In the design and analysis of image sampling and reconstruction systems, input images are usually regarded as deterministic fields (1–5). However, in some situations it is advantageous to consider the input to an image processing system, especially a noise input, as a sample of a two-dimensional random process (5–7). Both viewpoints are developed here for the analysis of image sampling and

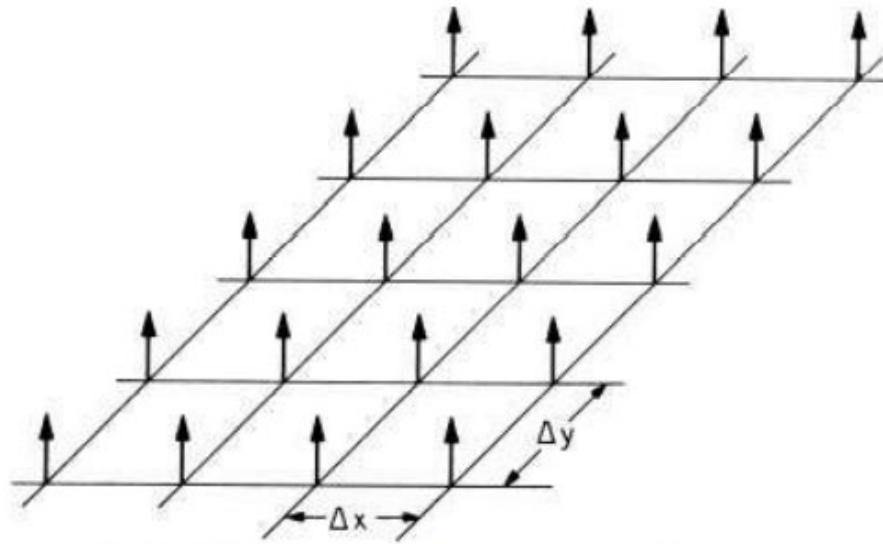


FIGURE 4.1-1. Dirac delta function sampling array.

4.1.1. Sampling Deterministic Fields

Sampling Deterministic Fields

Let denote a continuous, infinite-extent, ideal image field representing the luminance, photographic density, or some desired parameter of a physical image. In a perfect image sampling system, spatial samples of the ideal image would, in effect, be obtained by multiplying the ideal image by a spatial sampling function

$$S(x, y) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(x - j \Delta x, y - k \Delta y) \quad (4.1-1)$$

composed of an infinite array of Dirac delta functions arranged in a grid of spacing $(\Delta x, \Delta y)$ as shown in Figure 4.1-1. The sampled image is then represented as

$$F_P(x, y) = F_I(x, y)S(x, y) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} F_I(j \Delta x, k \Delta y) \delta(x - j \Delta x, y - k \Delta y) \quad (4.1-2)$$

where it is observed that may be brought inside the summation and evaluated only at the sample points $(j \Delta x, k \Delta y)$. It is convenient, for purposes of analysis, to consider the spatial frequency domain representation (ω_x, ω_y) of the sampled image obtained by taking the continuous two-dimensional Fourier transform of the sampled image. Thus

$$\mathcal{F}_P(\omega_x, \omega_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_P(x, y) \exp\{-i(\omega_x x + \omega_y y)\} dx dy \quad (4.1-3)$$

By the Fourier transform convolution theorem, the Fourier transform of the sampled image can be expressed as the convolution of the Fourier transforms of the ideal image and the $f(\omega_x, \omega_y)$ sampling function $S(\omega_x, \omega_y)$ as expressed by

$$\mathcal{F}_P(\omega_x, \omega_y) = \frac{1}{4\pi^2} \mathcal{F}_I(\omega_x, \omega_y) \circledast S(\omega_x, \omega_y) \quad (4.1-4)$$

The two-dimensional Fourier transform of the spatial sampling function is an infinite array of Dirac delta functions in the spatial frequency domain as given by

$$S(\omega_x, \omega_y) = \frac{4\pi^2}{\Delta x \Delta y} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(\omega_x - j \omega_{xs}, \omega_y - k \omega_{ys}) \quad (4.1-5)$$

Where ω_x and ω_y represent the Fourier domain sampling frequencies. It will be assumed that the spectrum of the ideal image is band limited to some bounds such that for ω_x and ω_y . Performing the convolution of Eq. 4.1-4 yields

$$\begin{aligned} \mathcal{F}_P(\omega_x, \omega_y) &= \frac{1}{\Delta x \Delta y} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{F}_I(\omega_x - \alpha, \omega_y - \beta) \\ &\times \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(\omega_x - j \omega_{xs}, \omega_y - k \omega_{ys}) d\alpha d\beta \end{aligned} \quad (4.1-6)$$

Upon changing the order of summation and integration and invoking the sifting property of the delta function, the sampled image spectrum becomes

$$\mathcal{F}_P(\omega_x, \omega_y) = \frac{1}{\Delta x \Delta y} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \mathcal{F}_I(\omega_x - j \omega_{xs}, \omega_y - k \omega_{ys}) \quad (4.1-7)$$

As can be seen from Figure 4.1-2, the spectrum of the sampled image consists of the spectrum of the ideal image infinitely repeated over the frequency plane in a grid of resolution. It should be noted that if Δx and Δy are chosen too large with respect to the spatial frequency limits of \mathcal{F}_I , the individual spectra will overlap.

A continuous image field may be obtained from the image samples of linear spatial interpolation or by linear spatial filtering of the sampled image. Let denote the continuous domain impulse response of an interpolation filter and) represent its transfer function. Then the reconstructed image is obtained

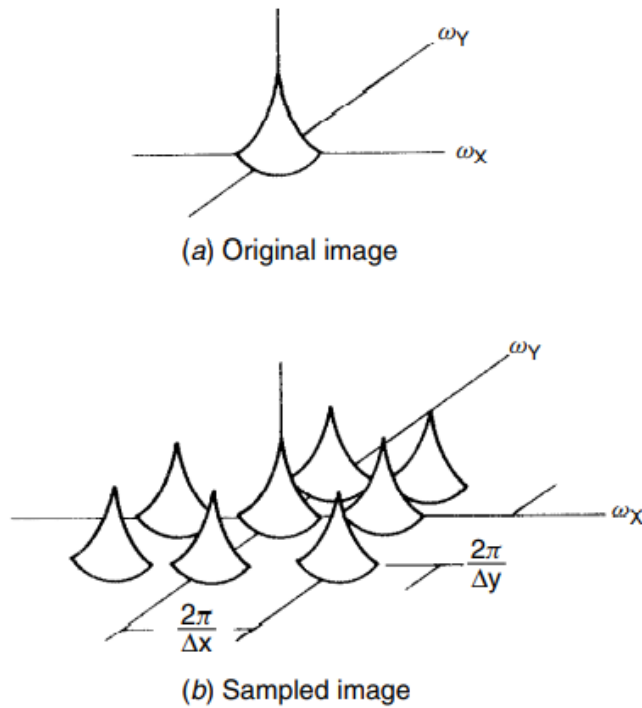


FIGURE 4.1-2. Typical sampled image spectra.

by a convolution of the samples with the reconstruction filter impulse response. The reconstructed image then becomes

$$F_R(x, y) = F_P(x, y) * R(x, y) \quad (4.1-8)$$

Upon substituting for) from Eq. 4.1-2 and performing the convolution, one obtains

$$F_R(x, y) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} F_I(j \Delta x, k \Delta y) R(x - j \Delta x, y - k \Delta y) \quad (4.1-9)$$

Thus it is seen that the impulse response function acts as a two-dimensional interpolation waveform for the image samples. The spatial frequency spectrum of the reconstructed image obtained from Eq. 4.1-8 is equal to the product of the reconstruction filter transform and the spectrum of the sampled image,

$$\mathcal{F}_R(\omega_x, \omega_y) = \mathcal{F}_P(\omega_x, \omega_y) \mathcal{R}(\omega_x, \omega_y) \quad (4.1-10)$$

or, from Eq. 4.1-7,

$$\mathcal{F}_R(\omega_x, \omega_y) = \frac{1}{\Delta x \Delta y} \mathcal{R}(\omega_x, \omega_y) \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \mathcal{F}_I(\omega_x - j \omega_{xs}, \omega_y - k \omega_{ys}) \quad (4.1-11)$$

It is clear from Eq. 4.1-11 that if there is no spectrum overlap and if filters out all spectra for the spectrum of the reconstructed image can be made equal to the spectrum of the ideal image, and therefore the images themselves can be made identical. The first condition is met for a band limited image if the sampling period is chosen such that the rectangular region bounded by the image cutoff frequencies lies within a rectangular region defined by one-half the sampling frequency. Hence

$$\omega_{xc} \leq \frac{\omega_{xs}}{2} \quad \omega_{yc} \leq \frac{\omega_{ys}}{2} \quad (4.1-12a)$$

or, equivalently,

$$\Delta x \leq \frac{\pi}{\omega_{xc}} \quad \Delta y \leq \frac{\pi}{\omega_{yc}} \quad (4.1-12b)$$

In physical terms, the sampling period must be equal to or smaller than one-half the period of the finest detail within the image. This sampling condition is equivalent to the one-dimensional sampling theorem constraint for time-varying signals that requires a time-varying signal to be sampled at a rate of at least twice its highest-frequency component. If equality holds in Eq. 4.1-12, the image is said to be sampled at its *Nyquist* rate; if Δx and Δy are smaller than required by the *Nyquist* criterion, the image is called oversampled; and if the opposite case holds, the image is under sampled.

If the original image is sampled at a spatial rate sufficient to prevent spectral overlap in the sampled image, exact reconstruction of the ideal image can be achieved by spatial filtering the samples with an appropriate filter. For example, as shown in Figure 4.1-3, a filter with a transfer function of the form

$$\mathcal{R}(\omega_x, \omega_y) = \begin{cases} K & \text{for } |\omega_x| \leq \omega_{xL} \text{ and } |\omega_y| \leq \omega_{yL} \\ 0 & \text{otherwise} \end{cases} \quad (4.1-13a)$$

$$(4.1-13b)$$

where K is a scaling constant, satisfies the condition of exact reconstruction. The point-spread function or impulse response of this reconstruction filter is

$$R(x, y) = \frac{K \omega_{xL} \omega_{yL}}{\pi^2} \frac{\sin \{ \omega_{xL} x \}}{\omega_{xL} x} \frac{\sin \{ \omega_{yL} y \}}{\omega_{yL} y} \quad (4.1-14)$$

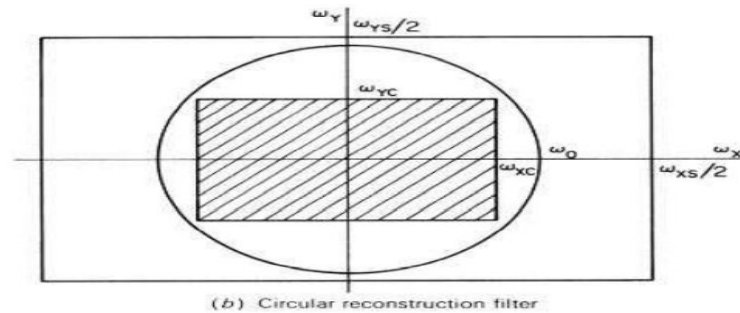
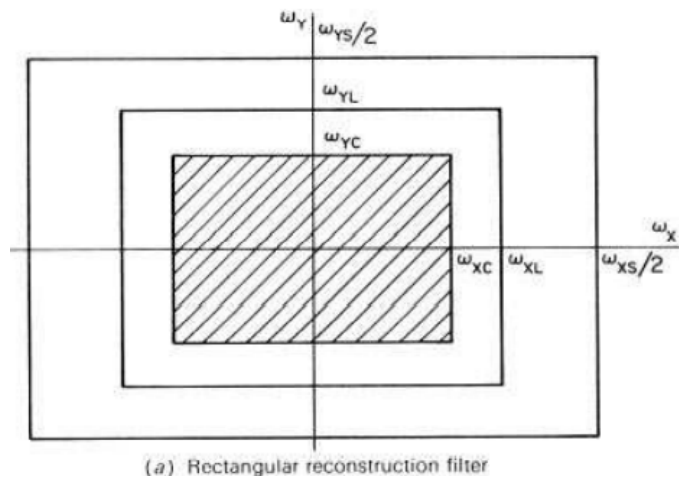


FIGURE 4.1-3. Sampled image reconstruction filters.

With this filter, an image is reconstructed with an infinite sum of $\sin \theta$ functions, called sinc functions. Another type of reconstruction filter that could be employed is the cylindrical filter with a transfer function

$$\mathcal{R}(\omega_x, \omega_y) = \begin{cases} K & \text{for } \sqrt{\omega_x^2 + \omega_y^2} \leq \omega_0 \\ 0 & \text{otherwise} \end{cases} \quad (4.1-15a)$$

$$(4.1-15b)$$

provided that $\omega_0^2 > \omega_{xc}^2 + \omega_{yc}^2$. The impulse response for this filter is

$$R(x, y) = 2\pi\omega_0 K \frac{J_1\left\{\omega_0\sqrt{x^2 + y^2}\right\}}{\sqrt{x^2 + y^2}} \quad (4.1-16)$$

Where J_j is a first-order Bessel function. There are a number of reconstruction filters, or equivalently, interpolation waveforms, that could be employed to provide perfect image reconstruction. In practice, however, it is often difficult to implement optimum reconstruction filters for imaging systems.

Section 10 -Image Quantization

Any analog quantity that is to be processed by a digital computer or digital system must be converted to an integer number proportional to its amplitude. The conversion process between analog samples and discrete-valued samples is called quantization. The following section includes an analytic treatment of the quantization process, which is applicable not only for images but for a wide class of signals encountered in image processing systems. The processing of quantized variables. The last section discusses the subjective effects of quantizing monochrome and color images

SCALAR QUANTIZATION

Figure 6.1-1 illustrates a typical example of the quantization of a scalar signal. In the quantization process, the amplitude of an analog signal sample is compared to a set of decision levels. If the sample amplitude falls between two decision levels, it is quantized to a fixed reconstruction level lying in the quantization band. In a digital system, each quantized sample is assigned a binary code. An equal-length binary code is indicated in the example.

For the development of quantitative scalar signal quantization techniques, let f and f' represent the amplitude of a real, scalar signal sample and its quantized value, respectively. It is assumed that f is a sample of a random process with known probability density $p(f)$. Furthermore, it is assumed that f is constrained to lie in the range

$$a_L \leq f \leq a_U \quad (6.1-1)$$

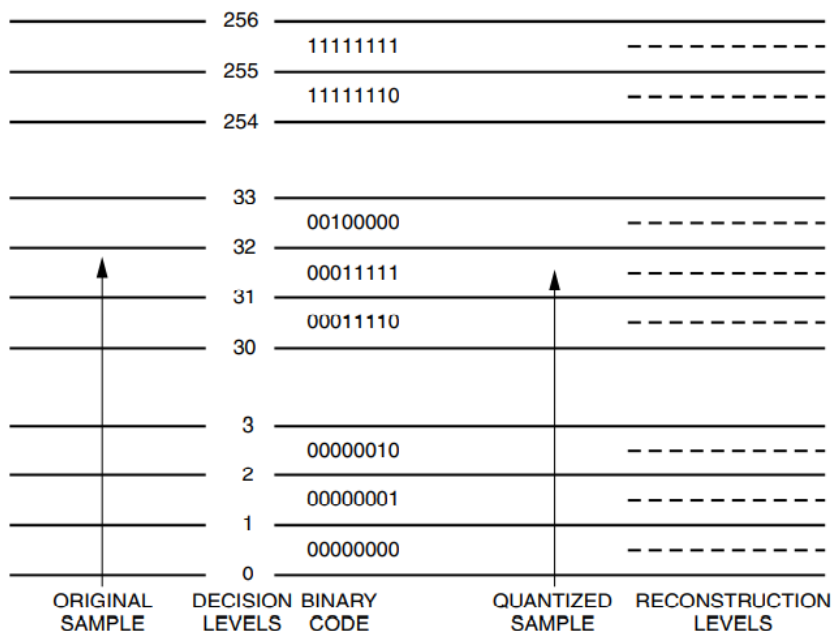


FIGURE 6.1-1. Sample quantization.

where a_U and a_L represent upper and lower limits.

Quantization entails specification of a set of decision levels d_j and a set of reconstruction levels r_j such that if

$$d_j \leq f < d_{j+1} \quad (6.1-2)$$

the sample is quantized to a reconstruction value r_j . Figure 6.1-2a illustrates the placement of decision and reconstruction levels along a line for J quantization levels. The staircase representation of Figure 6.1-2b is another common form of description.

Decision and reconstruction levels are chosen to minimize some desired quantization error measure between f and \hat{f} . The quantization error measure usually employed is the mean-square error because this measure is tractable, and it usually correlates reasonably well with subjective criteria. For J quantization levels, the mean-square quantization error is

$$\mathcal{E} = E\{(f - \hat{f})^2\} = \int_{a_L}^{a_U} (f - \hat{f})^2 p(f) df = \sum_{j=0}^{J-1} (f - r_j)^2 p(f) df \quad (6.1-3)$$

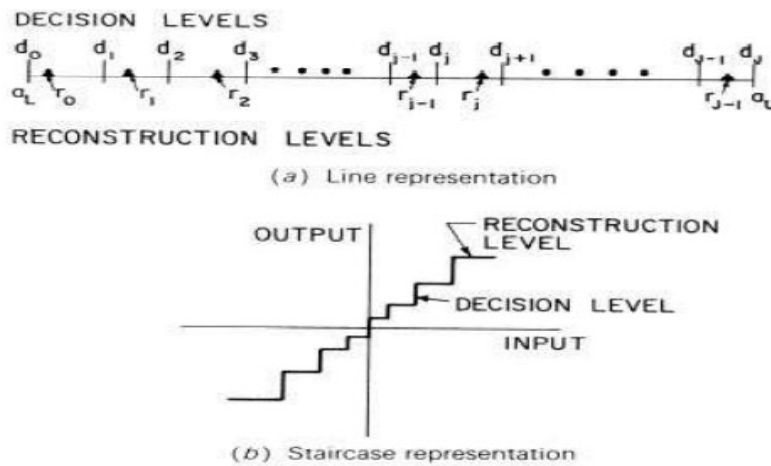


FIGURE 6.1-2. Quantization decision and reconstruction levels.

For a large number of quantization levels J , the probability density may be represented as a constant value $p(r_j)$ over each quantization band. Hence

$$\mathcal{E} = \sum_{j=0}^{J-1} p(r_j) \int_{d_j}^{d_{j+1}} (f - r_j)^2 df \quad (6.1-4)$$

which evaluates to

$$\mathcal{E} = \frac{1}{3} \sum_{j=0}^{J-1} p(r_j) [(d_{j+1} - r_j)^3 - (d_j - r_j)^3] \quad (6.1-5)$$

The optimum placing of the reconstruction level r_j within the range d_{j-1} to d_j can be determined by minimization of \mathcal{E} with respect to r_j . Setting

$$\frac{d\mathcal{E}}{dr_j} = 0 \quad (6.1-6)$$

yields

$$r_j = \frac{d_{j+1} + d_j}{2} \quad (6.1-7)$$

Therefore, the optimum placement of reconstruction levels is at the midpoint between each pair of decision levels. Substitution for this choice of reconstruction levels into the expression for the quantization error yields

$$\mathcal{E} = \frac{1}{12} \sum_{j=0}^{J-1} p(r_j)(d_{j+1} - d_j)^3 \quad (6.1-8)$$

The optimum choice for decision levels may be found by minimization of \mathcal{E} in Eq. 6.1-8 by the method of Lagrange multipliers. Following this procedure, Panter and Dite (1) found that the decision levels may be computed to a good approximation from the integral equation

$$d_j = \frac{(a_U - a_L) \int_{a_L}^{a_j} [p(f)]^{-1/3} df}{\int_{a_L}^{a_U} [p(f)]^{-1/3} df} \quad (6.1-9a)$$

where

$$a_j = \frac{j(a_U - a_L)}{J} + a_L \quad (6.1-9b)$$

for $j = 0, 1, \dots, J$. If the probability density of the sample is uniform, the

decision levels will be uniformly spaced. For non uniform probability densities, the spacing of decision levels is narrow in large-amplitude regions of the probability density function and widens in low-amplitude portions of the density. Equation 6.1-9 does not reduce to closed form for most probability density functions commonly encountered in image processing systems models, and hence the decision levels must be obtained by numerical integration.

If the number of quantization levels is not large, the approximation of Eq. 6.1-4 becomes inaccurate, and exact solutions must be explored. From Eq. 6.1-3, setting the partial derivatives of the error expression with respect to the decision and reconstruction levels equal to zero yields

$$\frac{\partial \mathcal{E}}{\partial d_j} = (d_j - r_j)^2 p(d_j) - (d_j - r_{j-1})^2 p(d_j) = 0 \quad (6.1-10a)$$

$$\frac{\partial \mathcal{E}}{\partial r_j} = 2 \int_{d_j}^{d_{j+1}} (f - r_j) p(f) df = 0 \quad (6.1-10b)$$

$$r_j = 2d_j - r_{j-1} \quad (6.1-11a)$$

$$r_j = \frac{\int_{d_j}^{d_{j+1}} f p(f) df}{\int_{d_j}^{d_{j+1}} p(f) df} \quad (6.1-11b)$$

Upon simplification, the set of equations as above. (6.1-11a) and (6.1-11b)

is obtained. Recursive solution of these equations for a given probability distribution $p(f)$ provides optimum values for the decision and reconstruction levels. Max (2) has developed a solution for optimum decision and reconstruction levels for a Gaussian density and has computed tables of optimum levels as a function of the number of quantization steps. Table 6.1-1 lists placements of decision and quantization levels for uniform, Gaussian, Laplacian, and Rayleigh densities for the *Max quantizer*.

If the decision and reconstruction levels are selected to satisfy Eq. 6.1-11, it can easily be shown that the mean-square quantization error becomes

$$\mathcal{E}_{\min} = \sum_{j=0}^{J-1} \left[\int_{d_j}^{d_{j+1}} f^2 p(f) df - r_j^2 \int_{d_j}^{d_{j+1}} p(f) df \right] \quad (6.1-12)$$

In the special case of a uniform probability density, the minimum mean-square quantization error becomes

$$\mathcal{E}_{\min} = \frac{1}{12J^2} \quad (6.1-13)$$

Quantization errors for most other densities must be determined by computation.

It is possible to perform nonlinear quantization by a companding operation, as shown in Figure 6.1-3, in which the sample is transformed nonlinearly, linear quantization is performed, and the inverse nonlinear transformation is taken (3). In the companding system of quantization, the probability density of the transformed samples is forced to be uniform. Thus, from Figure 6.1-3, the transformed sample value is

$$g = T\{f\} \quad (6.1-14)$$

where the nonlinear transformation is chosen such that the probability density of g is uniform. Thus,



FIGURE 6.1-3. Companding quantizer.

$$p(g) = 1 \quad (6.1-15)$$

For $-1/2 \leq g \leq 1/2$. If f is a zero mean random variable, the proper transformation function is (4)

$$T\{f\} = \int_{-\infty}^f p(z) dz - 1/2 \quad (6.1-16)$$

That is, the nonlinear transformation function is equivalent to the cumulative probability distribution of f . Table 6.1-2 contains the companding transformations and inverses for the Gaussian, Rayleigh, and Laplacian probability densities. It should be noted that nonlinear quantization by the companding

technique is an approximation to optimum quantization, as specified by the Max solution. The accuracy of the approximation improves as the number of quantization levels increases.

Quantizing for Minimum Distortion

Many data-transmission systems, analog input signals are first converted to digital form at the transmitter, transmitted in digital form, and finally reconstituted at the receiver as analog signals. The resulting output normally resembles the input signal but is not precisely the same since the quantizer at the transmitter produces the same digits for all input amplitudes which lie in each of a finite number of amplitude ranges. The receiver must assign to each combination of digits a single value which will be the amplitude of the reconstituted signal for an original input anywhere within the quantized range. The difference between input and output signals, assuming errorless transmission of the digits, is the quantization error. Since the digital transmission rate of any system is finite, one has to use a quantizer which sorts the input into a finite number of ranges, N .

Section 11 -Spatial and Gray-Level Resolution

Sampling is the principal factor determining the spatial resolution of an image. Basically, spatial resolution is the smallest discernible detail in an image. Suppose that we construct a chart with vertical lines of width W , with the space between the lines also having width W . A line pair consists of one such line and its adjacent space. Thus, the width of a line pair is $2W$, and there are $1/2W$ line pairs per unit distance. A widely used definition of resolution is simply the smallest number of discernible line pairs per unit distance; for example, 100 line pairs per millimeter. Gray-level resolution similarly refers to the smallest discernible change in gray level.

In gray level is a highly subjective process. We have considerable discretion regarding the number of samples used to generate a digital image, but this is not true for the number of gray levels. Due to hardware considerations, the number of gray levels is usually an integer power of 2, as mentioned in the previous section. The most common number is 8 bits, with 16 bits being used in some applications where enhancement of specific gray-level ranges is necessary.

When an actual measure of physical resolution relating pixels and the level of detail they resolve in the original scene are not necessary, it is not uncommon to refer to an L-level digital image of size M x N as having a spatial resolution of M*N pixels and a gray-level resolution of L levels. We will use this terminology from time to time in subsequent discussions, making a reference to actual resolvable detail only when necessary for clarity.

Figure 2.19 shows an image of size 1024*1024 pixels whose gray levels are represented by 8 bits. The other images shown in Fig. 2.19 are the results of sub sampling the 1024 x 1024 image. The sub sampling was accomplished by deleting the appropriate number of rows and columns



FIGURE 2.19 A 1024×1024 , 8-bit image subsampled down to size 32×32 pixels. The number of allowable gray levels was kept at 256.

8-bit image sub sampled down to size 32*32 pixels. The number of allowable gray levels was kept at 256. from the original image. For example, the 512 x 512 image was obtained by deleting every other row and column from the 1024 x 1024 image. The 256 x 256 image was generated by deleting every other row and column in the 512 x 512 image, and so on. The number of allowed gray levels was kept at 256.

Zooming and Shrinking Digital Images

The treatment of sampling and quantization with a brief discussion on how to zoom and shrink a digital image. This topic is related to image sampling and quantization because zooming may be viewed as oversampling, while shrinking may be viewed as under sampling. The key difference between these two operations and sampling and quantizing an original continuous image is that zooming and shrinking are applied to a digital image.

$$v(x', y') = ax' + by' + cx'y' + d \quad (2.4-6)$$

where the four coefficients are determined from the four equations in four unknowns that can be written using the four nearest neighbors of point (x', y') .

Image shrinking is done in a similar manner as just described for zooming. The equivalent process of pixel replication is row-column deletion. For example, to shrink an image by one-half, we delete every other row and column.

Section 12 -Some Basic Relationships Between Pixels

There are several important relationships between pixels in a digital image. As mentioned before, an image is denoted by $f(x, y)$. When referring in this section to a particular pixel, we use lowercase letters, such as p and q .

Neighbors of a Pixel

A pixel p at coordinates (x, y) has four horizontal and vertical neighbors whose coordinates are given by

$$(x+1, y), (x-1, y), (x, y+1), (x, y-1)$$

This set of pixels, called the 4-neighbors of p , is denoted by $N4(p)$. Each pixel is a unit distance from (x, y) , and some of the neighbors of p lie outside the digital image if (x, y) is on the border of the image. The four diagonal neighbors of p have coordinates

$$(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$$

and are denoted by $ND(p)$. These points, together with the 4-neighbors, are called the 8-neighbors of p , denoted by $N8(p)$. As before, some of the points in $ND(p)$ and $N8(p)$ fall outside the image if (x, y) is on the border of the image.

Adjacency, Connectivity, Regions, and Boundaries

Connectivity between pixels is a fundamental concept that simplifies the definition of numerous digital image concepts, such as regions and boundaries. To establish if two pixels are connected, it must be

determined if they are neighbors and if their gray levels satisfy a specified criterion of similarity (say, if their gray levels are equal). For instance, in a binary image with values 0 and 1, two pixels may be 4-neighbors, but they are said to be connected only if they have the same value.

Section 13 -Study of Grayscale Image In Image Processing

When a picture is to be processed by computer, it is often described as a matrix, or some other discrete data structure. But a picture is primarily a signal that conveys information to an observer, and there are many applications where this consideration is particularly important. We shall devote this and the next chapter to the discussion of such problems, especially for gray scale (class 1) images. The first problem is the conversion of a continuous picture into a discrete form, and this involves two processes: sampling, which is the selection of a discrete grid to represent an image, and quantization, which is the mapping of the brightness and color values into integers. In graphics one is concerned with similar problems: specifically the choice of the display resolution and number of gray levels or colors. These processes are also relevant in one-dimensional data and have been studied thoroughly in that case, but two-dimensional data present new problems. We shall devote the first part of this chapter to a review of transform techniques and then we shall discuss sampling for the one-dimensional case, followed by sampling for pictures. Quantization will be treated in the last section.

Color information is made by RGB color format. Human has an ability to describe any type of colors and also identify colors but machine has no capacity to do those things like humans. Same problem will also arise in the gray scale images So we need a that type of system who can identify the gray scale information.

The use of color in image processing is motivated by two principal factors; First color is a powerful descriptor that often simplifies object identification and extraction from a scene. Second, human can discern thousands of color shades and intensities, compared to about only two dozen shades of gray. In RGB model, each color appears in its primary spectral components of red, green and blue. This model is based on Cartesian coordinate system. Images represented in RGB color model consist of three component images. One for each primary, when fed into an RGB monitor, these three images combines on the phosphor screen to produce a composite color image. The number of bits used to represent each pixel in RGB space is called the pixel depth. Consider an RGB image in which each of the red, green and blue images is an 8-bit image. Under these conditions each RGB color pixel is said to have a depth of 24 bit. MATLAB 7.0 2007b was used for the implementation of all results.

Proposed Technique

In this technique we proposed a system, We know that the Image is made by number of pixels and different major parameters like color and monochrome (sometimes also known as black & white image or property). Image is processed and executed by an image processing techniques. So image processing is the major part of signal processing. Gray scale conversion is also a vital part of image processing. RGB or color information has a 3 dimensional property which makes signal processing so much bulky and heavy to remove this drawbacks gray scale conversion is necessary.

Grayscale images are those images where color information is missing and all color information is converted into gray scale format.

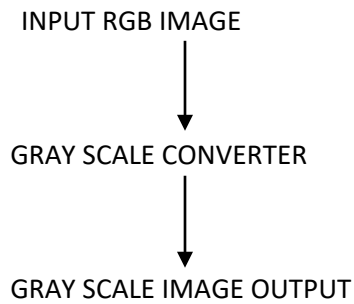


Fig : Gray scale Image converter

Grayscale images are distinct from one-bit bitonal black-and-white images, which in the context of computer imaging are images with only the two colors, black, and white .Grayscale images have many shades of gray in between. Matlab supports large amount of image formats i.e jpg, tif, bitmap, png, gif etc.

IV. MATLAB CODES FOR GRAY SCALE CONVERSION

```
I = rgb2gray(RGB);  
gray = rgb2gray(map);  
imshow(gray);  
//Another codes//  
I = rgb2gray(RGB);  
Gray =(I,0.2989 * R + 0.5870 * G + 0.1140 *  
B);  
imshow(Gray);
```




(a) original Image



(a) original Image



(b) Gray Scale Image



(b) Gray Scale Image

Fig.2 Converion of RGB to Gray scale

Often, the grayscale intensity is stored as an 8-bit integer giving 256 possible different shades of gray from black to white. If the levels are evenly spaced then the difference between successive gray levels is significantly better than the gray level resolving power of the human eye.

Grayscale images are very common, in part because much of today's display and image capture hardware can only support 8-bit images. In addition, grayscale images are entirely sufficient for many tasks and so there is no need to use more complicated and harder-to-process color images.

Section 14 -Conclusion

It was a huge experience for me to research about image digitization. I never thought that only three words DIP has so much to learn. When I was going through several books, my happiness leaves no bounds. So many things I have learnt which was beyond my imagination. I have learnt about digital image processing techniques including representation, sampling and quantization, image acquisition, image transforms, image enhancement, image smoothing and sharpening, image restoration. grey scale image in image processing, pixels, how to rearrange channels from BGR to RGB. It gave improved pictorial information for human clarification and processing of image data for storage, transmission, and representation for machine view. I came to know the pixels in the image can be manipulated to any desired density and contrast. Images can be stored and retrieved easily. It gave improved pictorial information for human clarification and processing of image data for storage, transmission, and representation for machine view



It was like an ocean of information which I have tried to accumulate in a short span. I can imagine what would be the future world for the coming generation. In near future this research will help me to move ahead in my future study. I am very grateful to my teacher for providing me and others such an opportunity.

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