Homework-4

Chapter 8 Relational Database Design (Pg-369)

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8.4

Use Armstrong's axioms to prove the soundness of the union rule. (Hint: Use the augmentation rule to show that, if $\alpha \rightarrow \beta$, then $\alpha \rightarrow \alpha \beta$. Apply the augmentation rule again, using $\alpha \rightarrow \gamma$
, and then apply the transitivity rule.)

Solution:

We are to prove that: if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$
then $\alpha \rightarrow \beta \gamma$

Using the hint, we get:

$\alpha \rightarrow \beta$ (given)

$\alpha \alpha \rightarrow \alpha \beta$ (augmentation rule)

$\alpha \rightarrow \alpha \beta$ (union of identical sets)

$\alpha \rightarrow \gamma$ (given)

$\alpha \beta \rightarrow \gamma \beta$ (augmentation rule)

$\alpha \rightarrow \beta \gamma$ (transitivity rule and set union commutativity)

8.5

Use Armstrong's axioms to prove the soundness of the pseudotransitivity rule.

Solution:

We are to proof using Armstrong's axioms the soundness of the Pseudotransitivity Rule:
if $\alpha \rightarrow \beta$ and $\gamma \beta \rightarrow \delta$, then $\alpha \gamma \rightarrow \delta$.

$\alpha \rightarrow ß$ (given)

$\alpha \gamma \rightarrow \gamma ß$ (augmentation rule and set union commutativity)

$\gamma ß \rightarrow \delta$ (given)

$\alpha \gamma \rightarrow \delta$ (transitivity rule)

8.6

Compute the closure of the following set F of functional dependencies for relation schema R = (A, B, C, D, E).

A → BC

CD → E

B → D

E → A

List the candidate keys for R

Solution:

Some shorthand representation of the result are shown with the primary target of getting the nontrivial members of F^+

Starting with A → BC, we can conclude: A → B and A → C.

Since A → B and B → D, A → D (decomposition, transitive)

Since A → C D and C D → E, A → E (union, decomposition, transitive)

Since A → A ,(reflexive) we have,

A → ABCDE from the above steps (union)

Since E → A, E → ABCDE (transitive)

Since C D → E, C D → ABCDE (transitive)

Since $B \rightarrow D$ and $BC \rightarrow C\,D$, $BC \rightarrow ABC\,DE$ (augmentative, transitive)

Also, $C \rightarrow C$, $D \rightarrow D$, $B\,D \rightarrow D$, etc.

Therefore, any functional dependency with A, E, BC, or C D on the left hand side of the arrow is in F^+, no matter which other attributes appear in the FD.

Allow * to represent any set of attributes in R, then F^+ is $B\,D \rightarrow B$, $B\,D \rightarrow D$, $C \rightarrow C$, $D \rightarrow D$, $B\,D \rightarrow B\,D$, $B \rightarrow D$, $B \rightarrow B$, $B \rightarrow B\,D$, and all FDs of the form $A * \rightarrow \alpha$, $BC * \rightarrow \alpha$, $C\,D * \rightarrow \alpha$, $E * \rightarrow \alpha$ where α is any subset of {A, B, C, D, E}.

The candidate keys for R are A, BC, C D, and E.

8.7

Using the functional dependencies of Practice Exercise 8.6,

compute the canonical cover F_c .

Solution:

We have the given set of FDs F :-

$A \rightarrow BC$

$CD \rightarrow E$

$B \rightarrow D$

$E \rightarrow A$

We see that the left side of each FD in F is unique.

Also, none of the attributes in the left side or right side of any of the FDs is extraneous.

Therefore, the canonical cover F_c is equal to F.

8.17

A functional dependency $\alpha \rightarrow ß$ is called a partial dependency if there is a proper subset ɣ of a such that $ɣ \rightarrow ß$. We say that ß is partially dependent on α. A relation schema R is in second normal form (2NF) if each attribute A in R meets one of the following criteria:

- It appears in a candidate key.

• It is not partially dependent on a candidate key.

Show that every 3NF schema is in 2NF. (Hint: Show that every partial dependency is a transitive dependency)

Solution:

A relation schema R is said to be in 3NF if there is no non-prime attribute A in R for which A is transitively dependent on a key for R. We can also rewrite the definition of 2NF given here as :

"A relation schema R is in 2NF if no non-prime attribute A is partially dependent on any candidate key for R."

To prove that every 3NF schema is in 2NF, it suffices to show that if a non-prime attribute A is partially dependent on a candidate key α, then A is also transitively dependent on the key α. Let A be a non-prime attribute in R. Let a be a candidate key for R. Suppose A is partially dependent on α

• From the definition of a partial dependency, we know that for some proper subset ß of α, ß → A.

• Since ß ⊂ α, α → ß. Also, ß → α does not hold, since α is a candidate key.

• Finally, since A is non-prime, it cannot be in either ß or α.

Thus we conclude that α → A is a transitive dependency. Hence we have proved that every 3NF schema is also in 2NF.

8.29

Consider the following set F of functional dependencies on the relation schema r(A, B, C, D, E, F):

A → BCD

BC → DE

B → D

D → A

a. Compute B^+.

b. Prove (using Armstrong's axioms) that AF is a superkey.

c. Compute a canonical cover for the above set of functional dependencies F; give each step of your derivation with an explanation.

d. Give a 3NF decomposition of r based on the canonical cover.

e. Give a BCNF decomposition of r using the original set of functional dependencies.

f. Can you get the same BCNF decomposition of r as above, using the canonical cover?

Solution:

a.

B → BD (third dependency)

BD → ABD (fourth dependency)

ABD → ABCD (first dependency)

ABCD → ABCDE (second dependency)

Thus, B^+ = ABCDE

b. Prove (using Armstrong's axioms) that AF is a superkey.

A → BCD (Given)

A → ABCD (Augmentation with A)

BC → DE (Given)

ABCD → ABCDE (Augmentation with ABCD)

A → ABCDE (Transitivity)

AF → ABCDEF (Augmentation with F)

c.

We see that D is extraneous in dep. 1 and 2, because of dep. 3.

Removing these two, we get the new set of rules

A → BC

BC → E

B → D

D → A

Now notice that B^+ is ABCDE, and in particular, the FD $B \rightarrow E$ can be determined from this set. Thus, the attribute C is extraneous in the third dependency. Removing this attribute, and combining with the third FD, we get the final canonical cover as :

$A \rightarrow BC$

$B \rightarrow DE$

$D \rightarrow A$

Here, no attribute is extraneous in any FD.

d.

We see that there is no FD in the canonical cover such that the set of attributes is a subset of any other FD in the canonical cover. Thus, each FD gives rise to its own relation, giving

$r_1(A, B, C)$

$r_2(B, D, E)$

$r_3(D, A)$

Now the attribute F is not dependent on any attribute. Thus, it must be a part of every superkey. Also, none of the relations in the above schema have F, and hence, none of them have a superkey. Thus, we need to add a new relation with a superkey.

$r_4(A, F)$

e.

We start with $r(A, B, C, D, E, F)$

We see that the relation is not in BCNF because of the first FD. Hence, we decompose it accordingly to get

$r_1(A, B, C, D)$ $r_2(A, E, F)$

Now we notice that $A \rightarrow E$ is an FD in F^+, and causes r_2 to violate BCNF. Once again, decomposing r_2 gives $r_1(A, B, C, D)$ $r_2(A, F)$ $r_3(A, E)$

This schema is now in BCNF.

f.

If we use the functional dependencies in the preceding canonical cover directly, we cannot get the above decomposition. However, we can infer the original dependencies from the canonical cover, and if we use those for BCNF decomposition, we would be able to get the same decomposition.

8.33

Given a relational schema r(A, B,C, D), does $A \rightarrow\rightarrow BC$ logically imply $A \rightarrow\rightarrow B$ and $A \rightarrow\rightarrow C$? If yes prove it, else give a counter example.

Solution:

$A \rightarrow\rightarrow BC$ holds on the following table:

r :

A	B	C	D
a_1	b_1	c_1	d_1
a_1	b_2	c_2	d_2
a_1	b_1	c_1	d_2
a_1	b_2	c_2	d_1

If $A \rightarrow\rightarrow B$, then we know that there exists t_1 and t_3 such that $t_1[B] = t_3[B]$. Thus, we must choose one of the following for t_1 and t_3:

- $t_1 = r_1$ and $t_3 = r_3$, or $t_1 = r_3$ and $t_3 = r_1$:

Choosing either $t_2 = r_2$ or $t_2 = r_4$, $t_3 [C] \neq t_2[C]$.

- $t_1 = r_2$ and $t_3 = r_4$, or $t_1 = r_4$ and $t_3 = r_2$:

Choosing either $t_2 = r_1$ or $t_2 = r_3$, $t3[C] \neq t_2[C]$.

Therefore, the condition $t_3 [C] = t2[C]$ cannot be satisfied, so the conjecture is false.