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6.5 → 13, 14, 16, 32, 38

13) A book publisher has 3000 copies of a discrete mat books. How many ways are there to store these books in there three ware house. If the copies of the book are indistinguishable?

$$\binom{3000+3-1}{3000} = \frac{3002!}{2! \times 3000!} = \frac{\overset{1501}{\cancel{3002}} \times 3001 \times \cancel{3000!}}{\cancel{2} \times 1 \times \cancel{3000!}} = 4,504,501$$

14) How many solutions are there to the equation $n_1 + n_2 + n_3 + n_4 = 17$, where n_1, n_2, n_3 & n_4 are nonnegative integers?

$$\binom{17+4-1}{17} = \binom{20}{17} = \frac{20!}{17! \times 3!} = \frac{20 \times 19 \times \cancel{18} \times \cancel{17!}}{\cancel{3 \times 2} \times \cancel{17!}} = 1140$$

16) How many solutions are there to the equation

$n_1 + n_2 + n_3 + n_4 + n_5 + n_6 = 29$,

where, n_i, $i = 1, 2, 3, 4, 5, 6$ is a nonnegative integers

Such that

(a) $n_i > 1$ for $i = 1, 2, 3, 4, 5, 6$? $n_i \geq 2 \Rightarrow \underbrace{n_i - 2}_{n_i'} \geq 0$

$\boxed{n_i = n_i' + 2}$

$n_1' + 2 + n_2' + 2 + n_3' + 2 + n_4' + 2 + n_5' + 2 + n_6' + 2 = 29$

$n_1' + n_2' + n_3' + n_4' + n_5' + n_6' = 17$

$$\boxed{\binom{22}{17} = 26334}$$

(b) $n_1 \geq 1$, $n_2 \geq 2$, $n_3 \geq 3$, $n_4 \geq 4$, $n_5 \geq 5$ and $n_6 \geq 6$?

$(n_1' + 1) + (n_2' + 2) + (n_3' + 3) + (n_4' + 4) + (n_5' + 5) + (n_6' + 6) = 29$

$x_1' + x_2' + x_3' + x_4' + x_5' + x_6' = 8$

$\Rightarrow \binom{13}{8} = 1287$

(c) $x_1 \leq 5$? Number of solutions with $x_1 \geq 6 / x'$

$x_1' + x_2 + x_3 + x_4 + x_5 + x_6 = 29 - 6 = 23 \rightarrow \binom{23+6-1}{23}$

Number of solutions for all scenarios $\binom{29+6-1}{29}$

$$\binom{34}{29} - \binom{28}{23} = \boxed{179976}$$

(d) $x_1 < 8$ and $x_2 > 8$?

$x_2 - 9 = x_2'$

$x_2 \geq 9 \Rightarrow x_2 = x_2' + 9$

$x_1 + x_2' + x_3 + x_4 + x_5 + x_6 = 29 - 9 \Rightarrow \binom{20+6-1}{20} = C(29, 20)$

We count the complimentary set for $x_1 < 8$ which is $x_1 \geq 8$

$\boxed{x_1 = x_1' + 8}$

$\Rightarrow \binom{12+6-1}{12} = C(17, 12)$

then $\Rightarrow C(25, 5) - C(17, 5) = \boxed{46962}$

32) How many different strings can be made from the letters in MISSISSIPPI, using all the letters?

$$\frac{11!}{4! \times 4! \times 2!} = \frac{11 \times 10 \times 9 \times \cancel{8} \times 7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{4!} \times \cancel{2} \times \cancel{1}} = \boxed{34650}$$

38) How many different bit strings can be formed using six 1s and eight 0s?

$$C(14,6) \times C(8,8) = \frac{14!}{6! \times 8!} \times 1 = \frac{\cancel{14}^{7} \times 13 \times \cancel{12} \times 11 \times \cancel{10} \times \cancel{9}^{3} \times \cancel{8!}}{\cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{8!}}$$

$$= \boxed{3003}$$

8.1 2, 12

2) (a) Find a recurrence relation for the number of permutaion of a set with n elements.

Permutation of $nPR = \frac{n!}{(n-R)!}$

$P(n) = nP(n-1)$

$P(1) = 1$

(b) use the recurence relations la Line/ the number of permutations of a set with n elements using iterately

$\boxed{P(n) = nP(n-1)}$ $P(1) = 1$ for $n \geq 1$

$P(2) = 2(P(1)) = 2 \times 1$

$P(3) = 3(P(2)) = 3 \times (2 \times 1)$

$P(4) = 4(P(3)) = 4 \times (3 \times (2 \times 1))$

$P(5) = 5 \times (P(4)) = 5 \times 4 \times 3 \times 2 \times 1$

$P(n) = n(P(n-1)) = \boxed{n!}$

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12) (a) Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one, two or three stairs at a time.

(b) what are the initial conditions?

(c) in how many ways can this person climb a flight of eight stairs?

a $S(n) = S(n-1) + S(n-2) + S(n-3)$

b $S(-2) = S(-1) = 0$, $S(0) = 1$

n	0	1	2	3	4	5	6	7	8
$s(n)$	1	1	2	4	7	13	24	44	81

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8.2 — 4(c,d,e), 14, 18, 21, 22, 25

4 Solve these recurrence relations together with the given initial conditions.

(c) $a_n = 6a_{n-1} - 8a_{n-2}$ for $n \geq 2$, $a_0 = 4$, $a_1 = 10$

$a_n - 6a_{n-1} + 8a_{n-2} = 0$

$r^2 - 6r + 8 = 0$

$(r-2)(r-4) = 0$

$r = 2,\ r = 4$

$$\boxed{a_n = \alpha(2)^n + \beta(4)^n}$$

☆ $a_0 = 4 \Rightarrow \cancel{\alpha} + \beta = 4$ $\quad (-2)$

$a_1 = 10 \Rightarrow 2\alpha + 4\beta = 10$

$2\beta = 2$

$\boxed{\beta = 1}$

$\alpha + 1 = 4$

$\boxed{\alpha = 3}$

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$\Rightarrow a_n = 3 \times 2^4 + 4^n$

(d) $a_n = 2a_{n-1} - \alpha_{n-2}$ for $n \geq 2$, $a_0 = 4$, $a_1 = 1$

$a_n - 2a_{n-1} - a_{n-2} = 0$

$r^2 - 2r - r = 0$

$r = 1 \pm \sqrt{2}$

$a_n = \alpha(1+\sqrt{2})^n + \beta(1-\sqrt{2})^n$

$a_0 = 4 \Rightarrow \alpha + \beta = 4$

$a_1 = 1 \Rightarrow \alpha(1+\sqrt{2}) + \beta(1-\sqrt{2}) = 1$

$\beta = \frac{3\sqrt{2}+8}{2}$ $\alpha = \frac{24-3\sqrt{2}}{4}$

$\Rightarrow a_n = \left(\frac{24-3\sqrt{2}}{4}\right)(1+\sqrt{2})^n + \left(\frac{3\sqrt{2}+8}{2}\right)(1-\sqrt{2})^n$

(e) $a_n = \alpha_{n-2}$ for $n \geq 2$, $a_0 \geq 5$, $a_1 \geq 1$

$a_n - a_{n-2} = 0$

$r^2 - 1 = 0$

$r = \pm 1$

$a_0 = 5 \Rightarrow \alpha + \cancel{\beta} = 5$

$a_1 = -1 \Rightarrow +\alpha - \cancel{\beta} = -1$

$2\alpha = 4$

$\boxed{\alpha = 2}$

$a_n = \alpha(1)^n + \beta(-1)^n \Rightarrow a_n = 2(1)^n + 3(-1)^n$ $\boxed{\beta = 3}$

14) Find the solution to $a_n = 5a_{n-2} - 4a_{n-4}$ with $a_0 = 3$, $a_1 = 2$, $a_2 = 6$ and $a_3 = 8$.

$$a_n - 5a_{n-2} + 4a_{n-4} = 0$$

$$r^4 - 5r^2 + 4 = 0$$

$$r^2 = t \Rightarrow (t^2 - 5t + 4) = 0$$

$$t = 4 \rightarrow r^2 = 4 \Rightarrow \boxed{r = \pm 2}$$

$$t = 1 \rightarrow r^2 = 1 \Rightarrow \boxed{r = \pm 1}$$

$$\boxed{\begin{aligned} \alpha_0 &= 3 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \\ \alpha_1 &= 2 = \alpha_1 - \alpha_2 + 2\alpha_3 - 2\alpha_4 \\ \alpha_2 &= 6 = \alpha_1 + \alpha_2 + 4\alpha_3 - 4\alpha_4 \\ \alpha_3 &= 8 = \alpha_1 - \alpha_2 + 8\alpha_3 - 8\alpha_4 \end{aligned}}$$

$$a_n = \alpha_1(1)^n + \alpha_2(-1)^n + \alpha_3(2)^4 + \alpha_4(-2)^n$$

$$\boxed{\alpha_1 = 3 - \alpha_2 - \alpha_3 - \alpha_4}$$

$$2 = (3 - \alpha_2 - \alpha_3 - \alpha_4) - \alpha_2 + 2\alpha_3 - 2\alpha_4 \Rightarrow \alpha_2 = \boxed{\frac{1 + \alpha_2 - 3\alpha_4}{2}}$$

$$6 = 3 + 3\alpha_3 + 3\alpha_4 \Rightarrow \boxed{\alpha_3 = 1 - \alpha_4}$$

$$\Rightarrow 8 = (3 - \alpha_2 - \alpha_3 - \alpha_4) - \left(\frac{1 + \alpha_3 - 3\alpha_4}{2}\right) + 8\alpha_3 - 8\alpha_4$$

$$= 8 - 12\alpha_4 \Rightarrow \boxed{\alpha_4 = 0}, \ \alpha_1 = 1, \ \alpha_2 = 1, \ \alpha_3 = 1$$

$$\Rightarrow a_n = \alpha_1(1)^n + \alpha_2(-1)^n + \alpha_3(2^n) + \alpha_4(-2)^n$$

$$\boxed{= 1 + (-1)^n + 2^n}$$

18 Solve the recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$ with $a_0 = -5$, $a_1 = 4$ and $a_2 = 88$

$$a_n - 6a_{n-1} + 12a_{n-2} - 8a_{n-3} = 0$$

$$r^3 - 6r^2 + 12r - 8 = 0$$

$$(r-2)^3 \Rightarrow r_2 = r = 2 \quad r = 2$$

$$a_n = \alpha_1 2^n + \alpha_2 n_2^n + \alpha_3 n^2 2^n$$

$$a_0 = -5 = \alpha_1$$

$$a_1 = 4 = 2\alpha_1 + 2\alpha_2 + 2\alpha_3$$

$$a_2 = 88 = 4\alpha_1 + 8\alpha_2 + 16\alpha_3$$

$$\Rightarrow \alpha_1 = -5, \quad \alpha_2 = 1/2, \quad a_3 = \frac{13}{2}$$

$$a_n = -5 \times 2^n + \left(\frac{n}{2}\right) 2^n + \left(\frac{13n^2}{2}\right) \times 2^4 = -5 \times 2^n + n \times 2^{n-1} + 13n^2 \times 2^{n-1}$$

21 What is the general form of the solutions of a linear homogeneous recurrence solution if it's characteristic equation has roots 1, 1, 1, 1, −2, −2, −2, 3, 3, −4?

$r_1 = 1$ with multiplicity 4, $r_2 = -2$ with multiplicity 3, $r_3 = 3$ with multiplicity 2 and $r_4 = -4$ with muliplicity 1.

$$a_n = (a_{1.0} + a_{1.1} n + a_{1.2} n^2 + a_{1.3} n^3)(1)^n + (a_{2.0} + a_{2.1} n + a_{2.2} n^2)(-2)^n + (a_{3.0} + a_{3.1} n)(3)^n + (a_{4.0})(-4)^n$$

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25)

(a) Determine values of the constants A and B such that $a_n = An + B$ is a solution of recurrence relation $a_n = 2a_{n-1} + n + 5$

(b) use Th. 5 to find all solutions of this recurrence relation.

(c) Find the solution of this recurrence relation with $a_0 = 4$

$a_n - 2a_{n-1} = n - 5$ $\qquad$ $a_n = a_n^h + a_n^p$

$r - 2 = n + 5$

$(a_n^h) \Rightarrow a_n - 2a_{n-1} = 0$ $\qquad$ $a_n^p \Rightarrow a_n - 2a_{n-1} = n + 5$

$An + B$

$r - 2 = 0$

$\boxed{r = 2}$ $\qquad$ $a_n = 2a_{n-1} + n + 5$

$\boxed{a_n = \alpha 2^n}$ $\qquad$ $\Rightarrow An + B = 2(A(n-1) + B) + n + 5$

$\Rightarrow An + B = n(2A + 1) - 2A + 2B + 5$

$\boxed{A = 2A + 1 \Rightarrow A = -1}$

$\boxed{B = 2 + 2B + 5 \Rightarrow B = -7}$

$\Rightarrow a_n = a_n^h + a_n^p$ $\qquad$ $S\ a_n^p = An + B = -n - 7$

$a_n = \alpha 2^n - n - 7$

$a_0 = 4 \Rightarrow \alpha - 7 = 4$

$\boxed{\alpha = 11}$

$\Rightarrow \boxed{a_n = 11 \cdot 2^n - n - 7}$

8.4 8(a, c, e), 12(a, c, e), 17, 23, 36

For each of these generating functions provide a closed formula for the sequence determines

(a) $(x^2+1)^3$

$$(x^2+1)^3 = \binom{3}{0}(x^2)^{3-0}(1)^0$$
$$= \binom{3}{1}(x^2)^{3-1}(1)^1$$
$$= \binom{3}{2}(x^2)^{3-2}(1)^2$$
$$= \binom{3}{3}(x^2)^{3-3}(1)^3$$
$$= 1+3x^2+3x^4+x^6$$

Therefore,

$a_0 = 1$ $a_4 = 3$

$a_1 = 0$ $a_5 = 0$

$a_2 = 3$ $a_6 = 6$

$a_3 = 0$

(c) $\frac{1}{(1-2x^2)}$

$$\frac{1}{(1-2x^2)} = \sum_{W=0}^{+\infty}(2x^2)^k$$
$$= \sum_{W=0}^{+\infty} 2^k x^{2k}$$

Therefore,

$a_n = 2^{n}$ $(n = \text{even}, n \geq 0)$

$a_n = 0$ $(n = \text{odd}, n \geq 0)$

(e) $x - 1 + \left(\frac{1}{(1-3x)}\right)$

$x - 1 + \left(\frac{1}{(1-3x)}\right)$

$= x - 1 + \sum_{K=0}^{\infty} (3x)^K$

$= x - 1 + 1 + 3x + \sum_{K=2}^{\infty} 3^K x^K$

$= 4x + \sum_{K=2}^{+\infty} 3^K x^K$

Therefore,

$a_0 = 0$ $\quad n = 2, 3, \dots$

$a_1 = 4$ $\quad a_n = 3^n$

$\boxed{12}$ Find the coefficient of x^{12} in the power series of each of these functions

(a) $\frac{1}{(1+3x)}$

$\frac{1}{(1+3x)} = \sum_{K=0}^{\infty} (-3x)^4$

$\frac{1}{1+3x} = \sum_{K=0}^{\infty} (-3)^K x^4$

general coefficient

$a_K = (-3)^K$

x^{12} when $K = 12$

$a_{12} = (-3)^{12} = 531,441$

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(c) $\dfrac{1}{(1+x)^8}$

$$\frac{1}{(1+x)^8} = \frac{1}{(1-(-x))^8}$$

$$\frac{1}{(1+x)^8} = \sum_{K=0}^{\infty} \binom{8+K-1}{K} (-x)^4$$

$$\frac{1}{(1+x)^8} = \sum_{K=0}^{+\infty} \binom{7+k}{k} (-1)^4 x^k$$

general coefficient

$$a_K = \binom{7+k}{k} (-1)^4$$

x^{12} when $K=12$

$$a_{12} = \binom{7+12}{12} (-1)^{12}$$

$$a_{12} = 50{,}389$$

(e) $\dfrac{x^3}{(1+4x)^2}$

$$\frac{x^3}{(1+4x)^2} = x^3 \frac{1}{(1-(-4x))^2}$$

$$\frac{x^3}{(1+4x)^2} = x^3 \cdot \sum_{K=0}^{+\infty} \binom{2+K-1}{K} (-4x)^2$$

$$\frac{x^3}{(1+4x)^2} = \sum_{k=0}^{+\infty} \binom{1+k}{k} (-4)^k x^{k+3}$$

general coefficient

$$a_{k+3} = \binom{1+k}{k} (-4)^k$$

x^4 when $k = 9$

$$a_{9+3} = \binom{1+9}{9} (-4)^9$$

$$a_n = \binom{10}{9} (-4)^9$$

$$a_{12} = -2,621,440$$

$\boxed{17}$ In how many ways can 25 identical donuts be distributed to four police officers so that each officer gets atleast three but no more than seven donuts?

atleast three, no more than seven

$$3 \leq a \leq 7$$

$$x^3 + x^4 + x^5 + x^6 + x^7$$

$$(x^3)(1 + x + x^2 + x^3 + x^4)$$

$$(x^3 + x^4 + x^5 + x^6 + x^7)^4 = x^{12} \left(\sum_{k=0}^{4} x^k \right)^4$$

$$(x^3 + x^4 + x^5 + x^6 + x^7)^4 = x^{12} \left(\frac{1 - x^5}{1 - x} \right)^4$$

$$(x^3+x^4+x^5+x^6+x^7) = x^{12}(1+(-x^5))^4$$
$$(1+(-x))^{-4}$$

extended bionomical theorem:

$$(x^3+x^4+x^5+x^6+x^7)^4 = x^{12}\sum_{k=0}^{\infty}\binom{4}{m}(-1)^n x^{5m} \cdot \sum_{K=0}^{\infty}\binom{-4}{K}(-1)^K \cdot x^K$$

$$b_m = \binom{4}{m}(-1)^m \qquad 6_K = \binom{-6}{K}(-1)^K$$

$$(x^3+x^4+x^5+x^6+x^7)^4 = x^{12}\sum_{m=0}^{\infty} b_m x^{5m} \sum_{K=0}^{\infty} C_K x^K$$

Finding coefficient of x^{25}

$$12+5m+k = 25$$
$$5m+k = 13$$

- $m=0$ $m=2$
  $K=13$ $K=3$

- $m=1$
  $K=8$

$$a_{25} = b_0 C_{13} + b_1 C_8 + b_2 C_3$$

$$a_{25} = \binom{4}{0}(-1)^a\binom{-4}{13}(-1)^8 + \binom{-4}{1}(-1)^1\binom{-4}{8}(-1)^8 + \binom{4}{2}(-1)^2\binom{-4}{3}(-1)^3$$

$$a_{25} = 560 - 660 + 120$$
$$= 20 \text{ ways}$$

23

(a) What is the generating function for $\{a_K\}$, where a_K is the number of solutions of $x_1+x_2+x_3=k$, when x_1, x_2, and x_3 are integers with $x_1 \geq 2$, $0 \leq x_2 \leq 3$ and $2 \leq x_3 \leq 5$?

(b) (answer to part (a) to find a_6

<u>Answer:</u>

(a) $x_1 \geq 2 \longrightarrow x^2 + x_3 + \dots\dots = a$

$0 \leq x_2 \leq 3 \longrightarrow 1+x+x^2+x^3 = b$

$2 \leq x_3 \leq 5 \longrightarrow x^2+x^3+x^4+x^5 = c$

$abc = x^2(1+x+x^2+x^3)^2(x^2)(1+x+\cdots)$

$abc = x^4(1+x+x^2+x^3)^2(1+x+x^2\cdots)$

$abc = x^4(1+x+x^2+x^3)^2\left(\sum_{K=0}^{\infty} x^K\right)$

$abc = x^4(1+x+x^2+x^3)^2 \cdot \frac{1}{1-x}$

$$abc = \frac{x^4(1+x+x^2+x^3)^2}{1-x} \quad \dots\dots (i)$$

↳ generating function for $\{a_K\}$

(b) From (i)

$abc = x^4\left(\sum_{K=0} x^K\right)^2 \frac{1}{1-x}$

$$abc = x^4\left(\frac{1-x^4}{1-x}\right)^2 \cdot \frac{1}{1-x}$$

$$abc = x^4(1+(-x^4))^2\,(1+(-x))^{-3}$$

$$abc = x^4 \sum_{m=0}^{\infty} \binom{2}{m}(-1)^m x^{4m} \sum_{k=0}^{\infty} \binom{-3}{k}(-1)^m x^k$$

$$b_m = \binom{2}{m}(-1)^m$$

$$C_k = \binom{-3}{k}(-1)^k$$

$$abc = x^4 \sum_{n=0}^{\infty} b_m x^{4m} \sum_{k=0}^{\infty} 4x^4$$

$m = 0$, $k = 2$

$$a_6 = b_0 C_2$$

$$a_6 = \binom{2}{0}(-1)^0 \binom{-3}{2}(-1)^2$$

$$a_6 = 6$$

$\boxed{36}$ Useing generating function to solve the recurrence relation $a_k = 3a_{k-1} + 4^{k-1}$ with the initial condition $a_0 = 1$

$$F(x) = \sum_{k=0}^{\infty} a_k x^4$$

$$F(x) - a_0 = \sum_{k\geq 1}^{\infty} a_k x^k$$

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$$F(x) - a_0 = \sum_{k=1}^{\infty} (3a_{k-1} + 4^{k-1}) x^k$$

$$n = k-1$$

$$F(x) - a_0 = 3x \sum_{n=0}^{+\infty} a_n x^n + x \sum_{n=0}^{+\infty} (4x)^n$$

$$\sum_{k=1}^{+\infty} x^4 = \frac{1}{1-x}$$

$$F(x) - a_0 = 3x\, F(x) + \frac{x}{1-4x}$$

$$F(x) - 1 = 3x F(x) = \frac{x}{1-4x}$$

$$(1-3x) F(x) = \frac{-3x+1}{1-4x}$$

$$F(x) = \frac{1}{1-4x}$$

$$\sum_{k=1}^{+\infty} x^k = \frac{1}{1-x}$$

$$F(x) = \sum_{k=1}^{+\infty} 4^k x^k \qquad a_K = 4^4$$

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8.5 9, 10

9) How many students are enrolled in a course where in calculas, discrete mathematics, data structures, programming languages at a school. If there are 507, 292, 312 and 344 students in those courses respectively; 14 in both calculas and data structure, 213 in both calculus and programming languages; 211 in both discrete mathematics and data structures, 43 in both discrete mathematics and programming languages; no student may take calculus and discrete mathematics or data structures and programming languages concurrently

Students enrolled in

A = Calculus

B = Discrete mathematics

C = Data structures

D = programming languages

$|A| = 507$ $|C| = 312$

$|B| = 292$ $|D| = 344$

(1) $\leftarrow |A \cap B| = 0 \quad |B \cap C| = 211$

$|A \cap C| = 14 \quad |B \cap D| = 43$

$|A \cap D| = 213 \quad |C \cap D| = 0 \rightarrow$ (2)

(1) and (2) $= 0$

$|A \cap B \cap C| = 0$

$|A \cap B \cap D| = 0$

$|A \cap C \cap D| = 0$

$|B \cap C \cap D| = 0$

$|A \cap B \cap C \cap D| = 0$

$|A \cup B \cup C \cup D|$

$= |A| + |B| + |C| + |D| -$

$(|A \cap B| + |A \cap C| + |A \cap D| + |B \cap C| + |B \cap D| + |C \cap D| +$

$+ (|A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| +$

$+ |A \cap B \cap C \cap D|$

$= (507 + 292 + 312 + 344) - (0 + 14 + 213 + 211 + 43 + 0) +$

$+ 0 - 0$

$= 974$ students

10 Find the number of positive integers not exceeding 100 that are not divisible by 5 or by 7.

$|A| = \mathbb{Z}$ not exceeding 100

$|B| =$ numbers divisible by 5 or 7

$5 \downarrow 1$, $7 \downarrow 2$

$|A| = 100$

$d_1 = 5$

$|B_1| = \frac{|A|}{d_1} = \frac{100}{5} = 20$

$d_2 = 7$

$|B_2| = \frac{|A|}{d_2} = \frac{100}{7} = 14, \ldots \approx 14$

$d = 7 \times 5 = 35$

$|B_1 \cap B_2| = \frac{|A|}{d} = \frac{100}{35} = 2 \ldots\ldots \approx 2$

$|B_1 \cup B_2| = |B_1| + |B_2| - |B_1 \cap B_2|$

$= 20 + 14 - 2$

$|B_1 \cup B_2| = 34 - 2 = 32$

↳ divisible by 5 or 7

not divisible by 5 or 7

$|B_1 \cup B_2|^c = |A| - |B_1 \cup B_2|$

$|B_1 \cup B_2|^c = 100 - 32 = 68$ integers