Homework -2

1.2 Applications of Propositional Logic

Question No.2

You can see the movie only if you are over 18 years old or you have the permission of a parent. Express your answer in terms of *m*: “You can see the movie,” *e*: “You are over 18 years old,” and *p*: “You have the permission of a parent.”

Answer No.2

Given:

m: "You can see the movie"
e: "You are over 18 years old"
p: "You have the permission of a parent"

INTERPRETATION SYMBOLS

Negation —p: not p
Disjunction p V q: p or q
Conjunction p A q: p and q
Conditional statement p —5 q: if p, then q
Biconditional statement p ++ q: p if and only if q

SOLUTION

"p only if q" can be rewritten as "if p, then q".
"You can see the movie only if you are over 18 years old or you have the permission of a parent" can then be
rewritten as "if m, then (e or p)", or rewritten using the above symbols:

Ans: (eV p)⟶m

Reference: https://youtu.be/M_WV3_0OUyc

Question No.4

To use the wireless network in the airport you must pay the daily fee unless you are a subscriber to the service.

Express your answer in terms of

w: “You can use the wireless network in the airport,”

d: “You pay the daily fee,” and

s: “You are a subscriber to the service.”

Answer No.4

A necessary condition for one to use the wireless network in the airport (w) is to either pay the daily fee (d) or be a subscriber to the service (s)

Ans: $w \rightarrow (d \lor s)$

Question No.36

The police have three suspects for the murder of Mr.Cooper: Mr. Smith, Mr. Jones, and Mr. Williams. Smith,Jones, and Williams each declare that they did not kill Cooper. Smith also states that Cooper was a friend of Jones and that Williams disliked him. Jones also states that he did not know Cooper and that he was out of town the day Cooper was killed. Williams also states that he saw both Smith and Jones with Cooper the day of the killing and that either Smith or Jones must have killed him. Can you determine who the murderer was if

a) one of the three men is guilty, the two innocent menare telling the truth, but the statements of the guilty
man may or may not be true?
b) innocent men do not lie?

Answer No.36

Case-1:-

There can not be 3 truth tellers, since that would mean that nobody is the killer. But someone has to be the killer so there has to be a liar .

Smith is the killer?

Smith can not be the killer because that would mean that Smith is lying. smith can only be lying, if he's "and" statement is false. (cooper was not a friend of jones or williams was a friend of jones or both) if smith is lying (cooper was not a friend of jones) then that would mean that Jones is telling the truth(he did not know cooper), and williams is also lying (he did not see jones with cooper)

or
if smith is lying (cooper was a friend of williams, and jones was a friend of cooper) then jones is also lying(he did know cooper) and williams is telling the truth(he saw jones with cooper).
There can not be 2 liars so smith can not be the killer.

Case-2:-

Jones is the killer?

Jones is lying, (he knew cooper)
smith, and williams can both be telling the truth.

Case-3:-

Williams is the killer?

Williams is lying.(he did not see jones and cooper together) Then Jones is telling the truth, he does not know cooper, but smith is lying he (cooper is not a friend of jones.) There can not be 2 liars so Williams can not be the killer.

Ans: Jones is the killer

Question No.40

Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authorities. Alice said,“ Carlos did it.” John said, “I did not do it.” Carlos said, “Diana did it.” Diana said, “Carlos lied when he said that I did it.”

a) If the authorities also know that exactly one of the four suspects is telling the truth, who did it? Explain your reasoning.
b) If the authorities also know that exactly one is lying, who did it? Explain your reasoning.

Answer No.40

Part A of the Question

Carlos and Diana cannot be both lying. If Carlos is lying, then Diana is telling the truth and if Carlos is telling the truth, then Diana is lying. It is either Carlos or Diana who is telling the truth. John is lying, and John said "I did not do it".Therefore, John had the unauthorized access.

Part B of the Question

Carlos and Diana can not be both be telling the truth, so the liar is either Carlos or Diana. This means that Alice is telling the truth. Alice said: "Carlos did it" Therefore it was Carlos who had the unauthorized access.

Ans: A) John had the unauthorized access
B) Carlos had the unauthorized access

1.3 Propositional Equivalences

Question No.12

Show that each of these conditional statements is a tautology by using truth tables.
a) $[\neg p \wedge (p \vee q)] \rightarrow q$
b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
c) $[p \wedge (p \rightarrow q)] \rightarrow q$
d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Answer No.12

DEFINITIONS

A proposition is a tautology, if the proposition is always true (thus true for any combination of truth values for the variables p, q, r, ...).

TRUTH TABLE

A conjunction p A q is true, if both (sub)propositions (p and q) are true.
A disjunction p V q is true, if either of the (sub)propositions (p or q) are true.
A negation —p is true, if the (sub)proposition p is false.
A conditional statement p —5 q is true, if p is false, or if both (sub)propositions are true.
A biconditional statement p ++ q is true, if both (sub)propositions are true or if both (sub)propositions are false.

(a) The conditional statement is a tautology, because the last column of the following truth table contains only true T (and thus does not contain false F).

p	q	p v q	¬ p	¬ p ^ (p v q)	$[\neg p \land (p \lor q)] \to q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

(b) The conditional statement is a tautology, because the last column of the following truth table contains only true T (and thus does not contain false F).

p	q	r	p->q	q->r	(p->q)^(q->r)	p->r	$[(p \to q) \land (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

(c) The conditional statement is a tautology, because the last column of the following truth table contains only true T (and thus does not contain false F).

p	q	p->q	p^(p->q)	$[p \wedge (p \rightarrow q)] \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(d) The conditional statement is a tautology, because the last column of the following truth table contains only true T (and thus does not contain false F).

p	q	r	$p \vee q$	p->q	q->r	$(p \vee q)$^ (p->r)	$(p \vee q)$^ (p->r)^ (q->r)	$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	T
F	F	T	F	T	T	F	F	T
F	F	F	F	T	T	F	F	T

Ans All conditional statements are tautologies

Question No.20

Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent

Answer No.20

p	q	¬p	¬q	$(p \wedge q)$	$(\neg p \wedge \neg q)$	$(p \wedge q) \vee (\neg p \wedge \neg q)$	$(p \leftrightarrow q)$
T	T	F	F	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	F	T	T	T

Therefore, $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent

Question No.21

Show that , $\neg (p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.

Answer No.21

p	q	¬q	(p ↔ q)	$\neg (p \leftrightarrow q)$	p ↔ ¬q
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	F

Therefore, $\neg (p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent

Question No.23

Show that $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent.

Answer No.23

p	q	¬p	¬q	¬p ↔ q	p ↔ ¬q
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

Therefore, $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent.

Question No.24

Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

Answer No.24

p	q	p ↔ q	$(p \oplus q) = \neg(p \leftrightarrow q)$	$\neg(p \oplus q)$
T	T	T	F	T
T	F	F	T	F
F	T	F	T	F
F	F	T	F	T

Therefore, $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

Question No.27

Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

Answer No.27

p	q	r	(p → q)	(q→ r)	$(p \rightarrow r) \wedge (q \rightarrow r)$	$(p \vee q)$	(p ∨ q) → r
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	F
T	F	T	F	T	F	T	F
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	Y

Therefore, $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

Question No.30

Show that, $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

Answer No.30

p	q	r	$(q \rightarrow r)$	¬p	¬p → (q → r)	(p ∨ r)	q → (p ∨ r)
T	T	T	T	F	T	T	T
T	T	F	F	F	T	T	T
T	F	T	T	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

Therefore, $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent

Question No.31

Show that $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent.

Answer No.31

p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Therefore, $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent.