5.2 Young's Modulus of Steel Wire

Objectives

1. Understand the meaning of Young's Modulus
2. Learn to use the optical level for the measurement of micro variation.
3. Learn the method of successive subtracting method for data process.

Theory

Any object shape will change under an external force. We usually call such shape change as deformation. Given the external force is limited within a proper threshold, the deformation will disappear and the shape will return to the original case for the removal of exerted force, hence, this type of deformation is referred as plastic deformation. Plastic deformation can be observed in most materials including metals, soils, rocks, concrete, foams, bone and skin, even though it is commonly linked with spring in most case. In the experiment, the elastic deformation for the thin metal wire, which is also the object under investigation, will lead to the creation of an internal stress for the shape restoration.

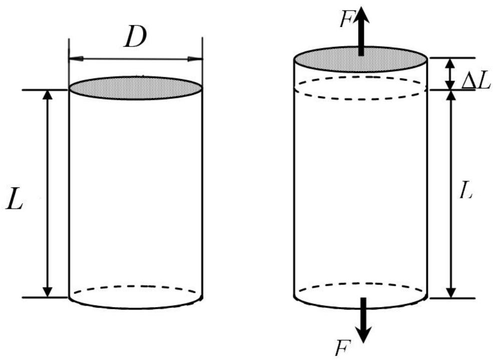

Figure 5-2-1 Schematic view of length extension of steel wire.

The elastic deformation can be characterized in terms of stress and strain, which refers to the force and shape change, respectively. When a force applied to an object, internal force, i.e., stress, is induced to resist the distortion. The stress causing the deformation is defined by the ration of internal force to the cross section area over which the force is applied.

Figure 5-2-1 depicts the schematic view for elastic deformation of steel wire. The stress can be expressed by

$$\text{stress} = \frac{F}{A} \tag{5-2-1}$$

where F is the force applied and A is the cross-sectional area of the material (as shown in Fig. 5-2-1) Notice that the standard units of stress are [N/m^2, or Pa].

When the stress is applied to an object, the deformation will definitely occurs no matter how the quantity is. The quantity of deformation is measured by using strain, which is uniquely defined for various type of deformations. In general, it is some deformation per unit dimension. For the steel wire stretched by external force, the quantity strain (for tensile and compressive types of stress) can be defined as follows:

$$\text{strain} = \frac{\Delta L}{L} \tag{5-2-2}$$

where L is the original length of the steel, and ΔL is the length variation due to the external stress imposed. Notice that strain is a dimensionless quantity. (In the cases of shear and hydraulic stress, strain is defined slightly differently).

Hooke's Law is usually written for linear systems and states that the applied force is directly proportional to the resulting displacement. The more general form of Hooke's Law states that the applied stress is directly proportional to the related strain. In the case of steel wire as discussed above, the Hooke's Law can be written as

$$\frac{F}{A} = E\frac{\Delta L}{L} \tag{5-2-3}$$

where E, named as Young's Modulus, is a constant that describes the ratio of stress to strain. One thing to be noted that the Hooke's law is only valid within the elastic limit, which is the threshold for elastic deformation.

As shown in Figure 2-1, for the case of a steel wire with diameter, D of cross section. The force is applied from the a weight with mass, m, the young's modulus from Eq. 2-3 can vbe rewritten by

$$E = \frac{FL}{S\Delta L} = \frac{4mgL}{\pi D^2 \Delta L} \tag{5-2-4}$$

In this experiment, the weight used possesses only 1kg, resulting in extremely tiny elongation.

Obviously, this variation can not be measured by using common meter ruler. Here, an optical level system is employed to obtain the elongation.

Figures 5-2-2 and 5-2-3 shows the Young's modulus measurement and the optical level systems, respectively. Optical level is made up of telescope and one small circular plane mirror supported by three metal toes which form isosceles triangle The length of the back toe can be adjusted if necessary. The distance from the back toe to the line connecting the two front toes is called the length of optical level.

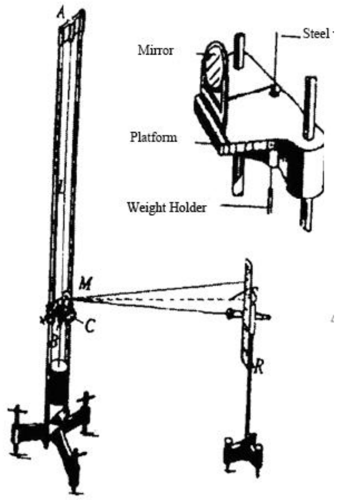

Figure 5-2-2 Young's modulus measuring apparatus

As shown in Fig. 5-2-3, the plane mirror surface normal is in accordance with the axes of the telescope when there is no force on the wire. Suppose the plane mirror is perfectly perpendicular with respect to the horizontal axis, thus the reading at the telescope is zero. When adding the weight, the wire is stretched by ΔL. The back toe of the optical level is dropped accordingly so that the flat form is inclined by α. Now the reading of measuring scale from the telescope is l. From the geometrical optics analysis, we can have the following expression for inclination angle

$$\tan\theta = \frac{\Delta L}{b} \qquad \tan 2\theta = \frac{l}{R} \tag{5-2-5}$$

where R is the distance between the plane mirror and the telescope. The elongation ΔL is rather small as stated above, so is the inclination angle, then an approximation can be done.

$$\theta = \frac{\Delta L}{b} \qquad 2\theta = \frac{l}{R} \tag{5-2-6}$$

Thus, we can get the expression to measure ΔL

$$\Delta L = \frac{bl}{2R} \tag{5-2-7}$$

From such process, the elongation ΔL is obtained by the reading on the telescope, l. The magnification of optical level, M, is the ratio between l and ΔL

$$M = \frac{l}{\Delta L} = \frac{2R}{b} \tag{5-2-8}$$

Clearly, the magnification can be increased by either enhance R or reduce b. As reducing b will destroy the approximation condition used in the calculation above, it is feasible to increase R to improve the M.

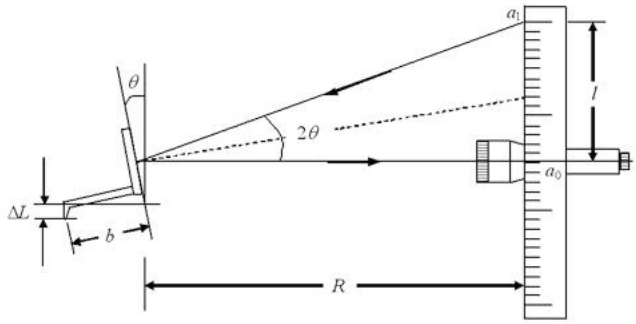

Figure 5-2-3 Schematic view of optical level.

For this experimental, the telescope possesses magnification by 30, stadia constant of 100. The minimum stadia is 2 mm. The distance between the upper and lower reticles, H, mutiplid by the stadia constant of 100 is the distance between the ruler and its image, which is doubled times of R.

$$R = \frac{100H}{2} = 50H \tag{5-2-9}$$

Substituting Eq. 2-9 into Eqs. 2-7 and 2-4, we can get the expression for Young's modulus

$$E = \frac{400mgLH}{\pi D^2 bl} \tag{5-2-10}$$

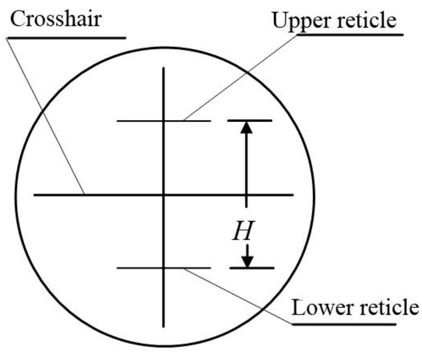

Figure 5-2-4 Reticles inside the telescope sight

Apparatus

Young's modulus device, Optical level and telescope, micrometer caliper, meter, a group of weights

Procedures

At the beginning of experiment, study the apparatus carefully and work with it until you understand ho to measure elongation. Adjust the height of the telescope to the same height of the optical lever. The two front metal toes of the optical lever are put in the transverse groove of the flat while the back metal toe of the optical lever is put on the clamp.

NOTES: Do not touch the wire and do not put the back toe into the crack between the clamp and the flat

1. Put three weights with each mass of 1 Kg firstly to pull the steel wire.
2. Move the telescope to the point to the plane mirror and post on the telescope and the plane mirror in one line;
3. Put the eye outside the telescope in the direction of gap-post; looking for the image of the measuring scale from the outside of telescope. If you can not find the object, adjust the telescope position and the angle of the plane mirror.
4. Put your eyes on the telescope, adjust the ocular to make the cross clear; adjust the focus length of the telescope to make the image of the measuring scale.
5. Add weights to the weight holder; the telescope will give reading of the measuring scale. Take the telescope reading and find the original length of the wire from its support to the point of attachment to the clamp. Loa the measured wire with successive weights, taking the telescope

reading for each addition. Unload one weight per time, and record the data for each step.

6. Measure the diameter of the wire, D, at various points along its length for ten times via micrometer. Measure the original length of steel wire, L, via a meter. Record b, H during the measurement of l.

Data Record

Table 5-2-1. The diameter of steel wire

<table>
<tr><td>No.</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>D（mm）</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>$\overline{D}$ (mm)</td><td colspan="10"></td></tr>
<tr><td>ΔD(mm)</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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Table 5-2-2 The length elongation of steel wire

<table>
<tr><th>No of weights</th><th>M(kg)</th><th>a_1(cm) (Loading)</th><th>a_2(cm) (Unloading)</th><th>$\overline{a}$ （cm） (Mean)</th><th>$x=a_{i+4}-a_i$</th><th>$\overline{x}$ (cm)</th><th>Δx_i（cm）</th></tr>
<tr><td>1</td><td>3</td><td></td><td></td><td></td><td rowspan="2"></td><td rowspan="8"></td><td></td></tr>
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Table 5-2-3 The parameters (L, b, H) of optical level

Number	1	2	3	4	5
x_1 (cm) (Position of the upper reticle)					
x_2 (cm) (Position of the lower reticle)					
$H = \lvert x_1 - x_2 \rvert$					
$\overline{H}$					
L(cm)					
b (cm)					

Questions

1. How to determine whether the wire is elastic deformation or not in the whole process of adding or reducing weight?

2. To reduce the measurement error of **E**, which quantities are the key factors according to error analysis? What measures are adopted in this experiment?

4. Whether any even-number set of data could be processed by successive minus method? What prerequisite should be met?

5. Could the data be processed by graphic method instead of successive minus? If yes, how?

6. Try to design some principles and methods of measuring the thickness of solid films (such as paper) with the optical lever and telescope scale system which are used in this experiment.