Honesty Guaranty

I know the examination rules, promise to be honest and abide by the rules. Signature:

Examination of Northwestern Polytechnical University

2021— 2022 School Year 1 Semester

School of Computer Science, Course: Discrete Mathematics, Class Hours:56
Exam. Date: 2021.12.28, Exam. Duration:2Hours,Written Exam.(closed-book)

Item	1	2	3	4	Total Score
Score					

Class		Student ID.		Name	

1. **Choose the right answer among A, B, C, and D and fill the blanks(2 points each，20 points in all).**

(1) Which sentence is proposition? (   )

A. How are you? B. x+5<y. C. Write this carefully. D. 2+2=3.

(2) Express the statement“ All computer science students study discrete mathematics.” using predicates and quantifiers where P(x) represents the statement that x is computer science student, and Q(x) represents the statement that x studies discrete mathematics. (   )

A. $\forall x\,(P(x)\wedge Q(x))$ B. $\forall x\,(P(x)\rightarrow Q(x))$

C. $\exists x\,(P(x)\wedge Q(x))$ D. $\exists x\,(P(x)\rightarrow Q(x))$

(3) Let A ={a,{a}}, its power set is (   ).

A. {{a}, {{a}},{a,{a}}} B. {a,{a}, {a,{a}}}

C. {Ø, a, {a},{a,{a}}} D. {Ø, {a}, {{a}},{a,{a}}}

(4) The relation R={(3,4)} on {1,2,3,4} is (   ).

A. reflexive and antisymmetric B. symmetric and transitive

C. antisymmetric and transitive D. reflexive and symmetric

(5) Let A={1,2,3,4}. R_1={(1,1),(2,2),(2,3),(4,4)} and R_2 = {(1,1),(2,2),(2,3),(3,2),(4,4)} are the relations on the set A, then R_2 is (　　) of R_1.

A. reflexive closure　　　B. symmetric closure

C. transitive closure　　　D. symmetric and transitive closure

(5) LetA={1,2,3,4,5}. R={(1,3),(1,1),(3,1),(3,3),(2,2),(5,2),(5,5),(2,5),(4,2),(4,4),(4,5),(5,4),(2,4)} is equivalence relation, partition produced using R is (　　).

A. {{1,3},{2},{4,5}}　　　B. {{1,3},{2,4,5}}

C. {{1},{3},{2,4,5}}　　　D. {{1},{3},{4},{2,5}}

(6) R is a relation from A={a,b,c } to B={d,e,f,g}, which one is a function (　　).

A. R={(a,e),(b,e),(c,e),(b,f)}

B. R={(a,e),(c,f)}

C. R={(a,e),(b,e),(c,e)}

D. R={(a,e),(c,e),(c,d)}

(7) Let f be the function from N to N with f(x)=x+1, f is (　　).

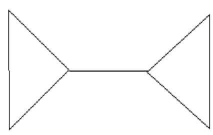
A. one-to-one　　　B. one-to-one and onto

C. onto　　　D. not one-to-one and not onto

(8) A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them. How many balls must she select to be sure of having at least three balls of the same color?(　　)

A. 4　　B. 3　　C. 5　　D. 13

(9) Consider the graph below. The following statement (　　) is false.

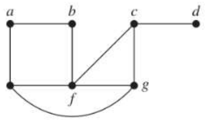


A. It is undirected graph.　　　B. It is connected graph.

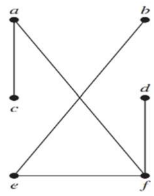
C. It is an Euler graph.　　　D. It is not Hamilton graph.

(10) which graph is a tree? (　　)

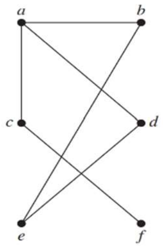
A.



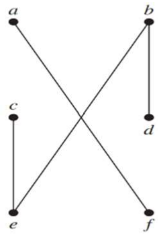
B.



C.



D.



2. Fill the answer in the blanks (20 points in all).

(1) Logical expressions $p \to (q \to r)$ and $(p \to q) \to r$ are ___________ (equivalent or not equivalent).

(2) Assume that P(x, y) means "x + y = 0", where x and y are integers. Then the truth value of the statement $\exists y \forall x\, P(x, y)$ is ____________.

(3) Let A and B be sets. If $|A|=m$, $|B|=n$, then the number of $A \times B$ is _______________.

(4) A relation on a set is an equivalence relation if it has three properties: __________

____________________.

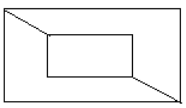
(5) Suppose that A={1,2,3}. Then the matrix of R={(1,1),(1,2),(2,2), (2,3),(3,3)} on the set A is _________________________.

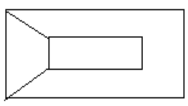
(6) Let A={1,2,3,4}, and R={(1,1), (2,1),(3,2),(4,3)} is a relation on set A, then R^2 =__________________________________.

(7) There are____________ways to select three students from a 10-member team to participate in International Collegiate Programming Contest.

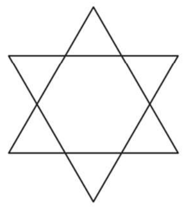
(8) There are____________solutions to the equation $x_1 + x_2 + x_3 + x_4 = 17$, where x_1, x_2, x_3, and x_4 are nonnegative integers.

(9) The following two graphs are _____________ (isomorphic/not isomorphic).





(10) The following graph________________(has or hasn't) an Euler circuit.



4. Answer the questions (60 points in all).

(1) Construct an argument to show that the premises $p \vee q$, $p \rightarrow \neg r$, $s \rightarrow \neg t$, $\neg s \rightarrow r$ and t lead to the conclusion q. (10 points)

(2) Suppose that A={1,2,3,4,6,12}, and R={(a,b) | a divides b}is a partial ordering on set A. (10 points)

a) Draw the Hasse diagram of R.

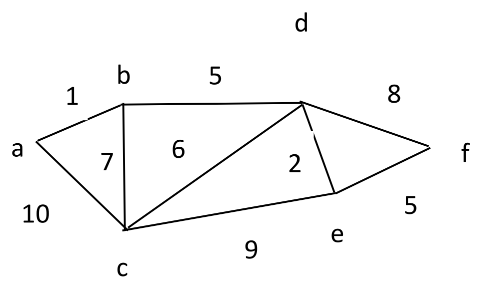
b) Find the minimal element, the maximal element, the greatest element, the least element, the least upper bound and the greatest lower element of the subset {2,3,4,6}.

(3) Find the solutions of the recurrence relation $a_n = -5a_{n-1} - 6a_{n-2} + 42 \cdot 4^n$ with $a_1=56$ and $a_2=278$. (10 points)

Hint: The recurrence relation has a solution of the form $a_n = C \cdot 4^n$, where C is constant.

(4) How many positive integers not exceeding 1000 are not divisible by not only 5 and 6 but also 8? (10 points)

(5) Use Dijkstra's algorithm to find the shortest path from a to any other vertices. (10 points)



(6) Use Huffman coding to encode these symbols with given frequencies: A: 0.10, B: 0.25, C: 0.05, D: 0.15, E: 0.30, F: 0.07, G: 0.08.

a) draw its binary tree and produce every letter's prefix code.

b) What is the average number of bits required to encode a symbol? (10 points)