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(1)

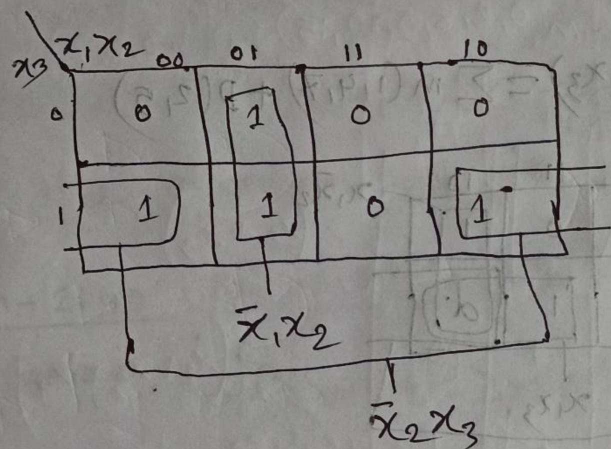
Assignment - 5

K-map part

Problem - 37

Soln: Given

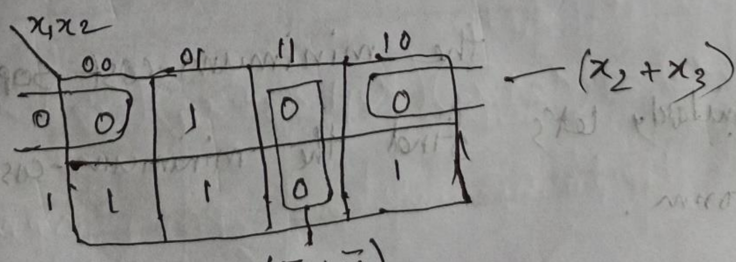
$$f(x_1, x_2, x_3) = \sum m(1,2,3,5)$$



Mapping the 1's from the k-map as follows.

$f(x_1, x_2, x_3) = \bar{x}_1 x_2 + \bar{x}_2 x_3$ Which is the minimum cost SOP form.

Again,



$\therefore f(x_1, x_2, x_3) = (\bar{x}_1 + \bar{x}_2)(x_2 + x_3)$

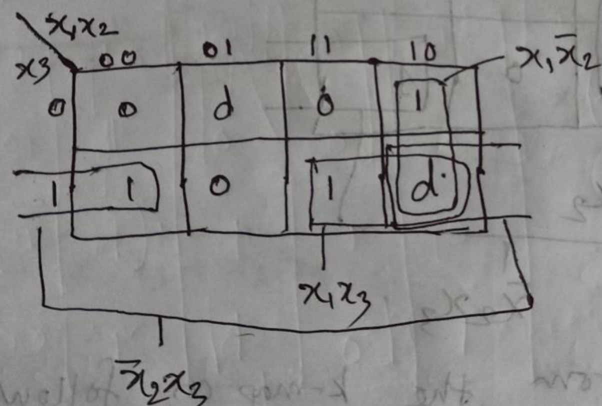
Mapping the 0's from the K-map as follows.

$f(x_1, x_2, x_3) = (\bar{x}_1 + \bar{x}_2)(x_2 + x_3)$ which is the minimum-cos POS form.

Problem-2.38

Soln:

$$f(x_1, x_2, x_3) = \sum m(1, 4, 7) + D(2, 5)$$

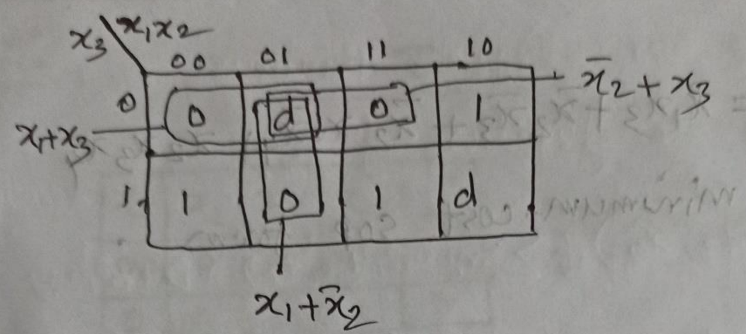


Mapping the 1's from the K-map as follows.

$f(x_1, x_2, x_3) = \bar{x}_2 x_3 + x_1 x_3 + x_1 \bar{x}_2$ which is the minimum-cost SOP form

Again, similarly let's find the minimum-cost POS form.

$f(x_1, x_2, x_3) = \sum m(1,4,7) + D(2,5)$

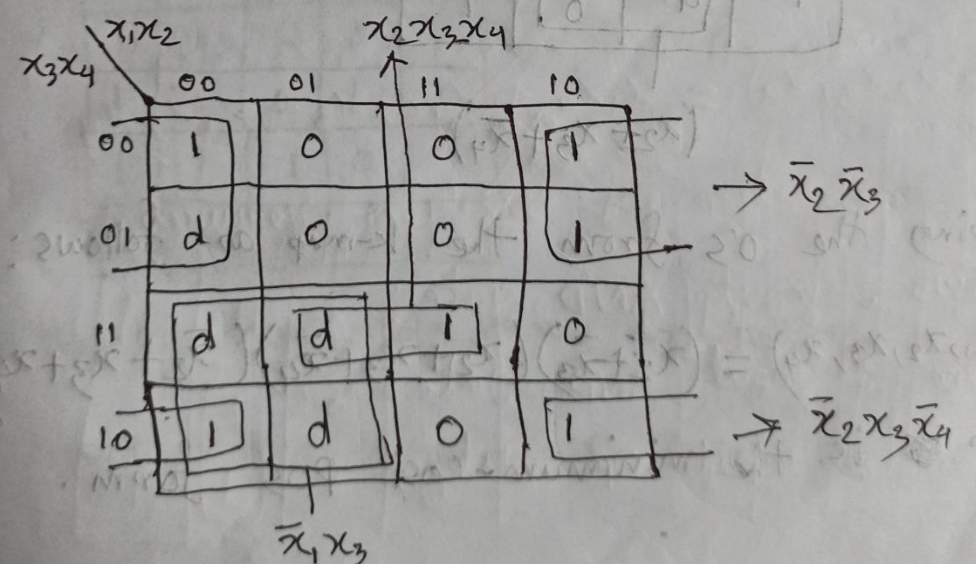


Mapping the 0's from the K-map as follows:

$f(x_1, x_2, x_3) = (\bar{x}_2 + x_3)(x_1 + x_3)(x_1 + \bar{x}_2)$ which is the minimum-cost POS form.

Problem-2.40

Soln: Given, $f(x_1, \dots, x_4) = \sum m(0,2,8,9,10,15) + D(1,3,6,7)$



Mapping the ones from the k-map as follows.

$$f(x_1, x_2, x_3, x_4) = \bar{x}_1 x_3 + \bar{x}_2 \bar{x}_3 + \bar{x}_2 x_3 \bar{x}_4 + x_2 x_3 x_4$$

This is the minimum-cost SOP form.

Again,

$$f(x_1, x_2, x_3, x_4) = \sum m(0, 2, 8, 9, 10, 15) + D(1, 3, 6, 7)$$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1	0	0	1
01	d	0	0	1
11	d	d	1	0
10	1	d	0	1

$(x_2 + \bar{x}_3)$

$(\bar{x}_2 + x_3 + x_4)$

$(x_2 + x_3 + \bar{x}_4)$

Mapping the 0's from the k-map as follows:

$$f(x_1, x_2, x_3, x_4) = (x_2 + \bar{x}_3)(x_2 + x_3 + \bar{x}_4)(\bar{x}_2 + x_3 + x_4)$$

This is the minimum-cost POS form.

Problem-2.45

Solⁿ:

Truth Table

x_1	x_2	x_3	x_4	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$$f(x_1, x_2, x_3, x_4) = \sum m(7, 11, 13, 14, 15)$$

$x_1x_2x_4$

x_3x_4 \ x_1x_2	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	0	1	1	1
10	0	0	1	0

$x_1x_3x_4$

$x_2x_3x_4$ $\quad$ $x_1x_2x_3$

Mapping the 1's from the k-map as follows.

$$f = x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4$$

Problem-2.48

Solⁿ: The K-map for getting the minimum cost SOP form for the function f is:

$$f(x_1, x_2, x_3, x_4) = \sum m(0,2,4,6,7,9) + D(10,11)$$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1	1	0	0
01	0	0	0	1
11	0	1	0	d
10	1	1	0	d

$x_1\bar{x}_2x_4$

$\bar{x}_1x_2x_3$

$\bar{x}_1\bar{x}_4$

∴ minimum cost SOP expression is

$$f(x_1, x_2, x_3, x_4) = \bar{x}_1\bar{x}_4 + \bar{x}_1x_2x_3 + x_1\bar{x}_2x_4$$

The realization of function requires,

Two 3-input AND gate;

One 2-input AND gate,

One 3-input OR gate.

The number of gates needed to implementation the function is :

The cost of f = total no. of gates + total no. of inputs

$$= (2+1+1) + \{(2)(3) + (2)(1) + (3)(1)\}$$

$$= 15$$

$$g(x_1, x_2, x_3, x_4) = \sum m(2, 4, 9, 10, 15) + D(0, 13, 14)$$

x_3x_4 \ x_1x_2	00	01	11	10
00	d	1	0	0
01	0	0	d	1
11	0	0	1	0
10	1	0	d	1

$\bar{x}_1\bar{x}_3\bar{x}_4$

$x_1\bar{x}_3x_4$

$x_1x_2x_3$

$\bar{x}_2x_3\bar{x}_4$

$\therefore$ The minimum cost SOP expression is,

$$g(x_1, x_2, x_3, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2x_3\bar{x}_4 + x_1\bar{x}_3x_4 + x_1x_2x_3$$

The realization of this function requires,

Four 3-input AND gate ;

One 4- input OR gate ;

$\therefore$ The cost of g = total no. of gates + total no. of inputs

$$= (4+1) + \{(3)(4) + (4)(1)\} = 21$$

The total cost of realization of function f and g is $= 15 + 21 = 36$

$\therefore$ The resultant modified k-map is,

$f(x_1, x_2, x_3, x_4)$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1	1	0	0
01	0	0	0	1
11	0	1	0	d
10	1	1	0	d

$g(x_1, x_2, x_3, x_4)$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	d	1	0	0
01	0	0	d	1
11	0	0	1	0
10	1	0	d	1

Modified minimum cost SOP forms are

$$f(x_1, x_2, x_3, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2 x_3 \bar{x}_4 + x_1\bar{x}_2\bar{x}_3 x_4 + \bar{x}_1 x_2 x_3$$

$$g(x_1, x_2, x_3, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2 x_3 \bar{x}_4 + x_1\bar{x}_2\bar{x}_3 x_4 + x_1 x_2 x_3$$

Hence the first three terms are shared between the two functions.

The implementation of both the functions require:

Four 3-input AND gate;

One 4-input AND gate;

Two 4-input OR gate;

$\therefore$ The cost of system = total no. of gates + total no. of inputs

$$= (4+1+2) + \{(3)(4) + (4)(1) + (4)(2)\}$$

$$= 31$$

Problem - 2.69

Soln:

SOP, $f = x_1\bar{x}_2\bar{x}_3 + \bar{x}_2\bar{x}_4 + \bar{x}_2\bar{x}_3x_4 + x_1x_2x_3x_4$

K-map

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	0	0	1	0
10	1	1	0	1

$x_1x_2x_3x_4$

$\bar{x}_2x_4$

$x_1\bar{x}_2\bar{x}_3$ $\bar{x}_2\bar{x}_3x_4$

Fig: 1

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	0	0	1	0
10	1	1	0	1

$x_1x_2x_3x_4$

$\bar{x}_2\bar{x}_3x_4$ $\bar{x}_2x_3\bar{x}_4$

Fig: 2

$f = \bar{x}_2x_3\bar{x}_4 + x_1x_2x_3x_4 + \bar{x}_2\bar{x}_3$

SOP, $g = x_1\bar{x}_3x_4 + x_1x_2x_4 + \bar{x}_1x_3x_4 + \bar{x}_2x_3\bar{x}_4$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	0	0	1	1
01	0	0	1	0
11	0	1	1	0
10	0	1	0	1

$\bar{x}_1x_3x_4$ $x_1\bar{x}_3x_4$

$x_1x_2x_4$ $\bar{x}_2x_3\bar{x}_4$

Fig: 3

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	0	0	1	1
01	0	0	1	0
11	0	1	1	0
10	0	1	0	1

$\bar{x}_1x_3x_4$ $\bar{x}_2x_3\bar{x}_4$

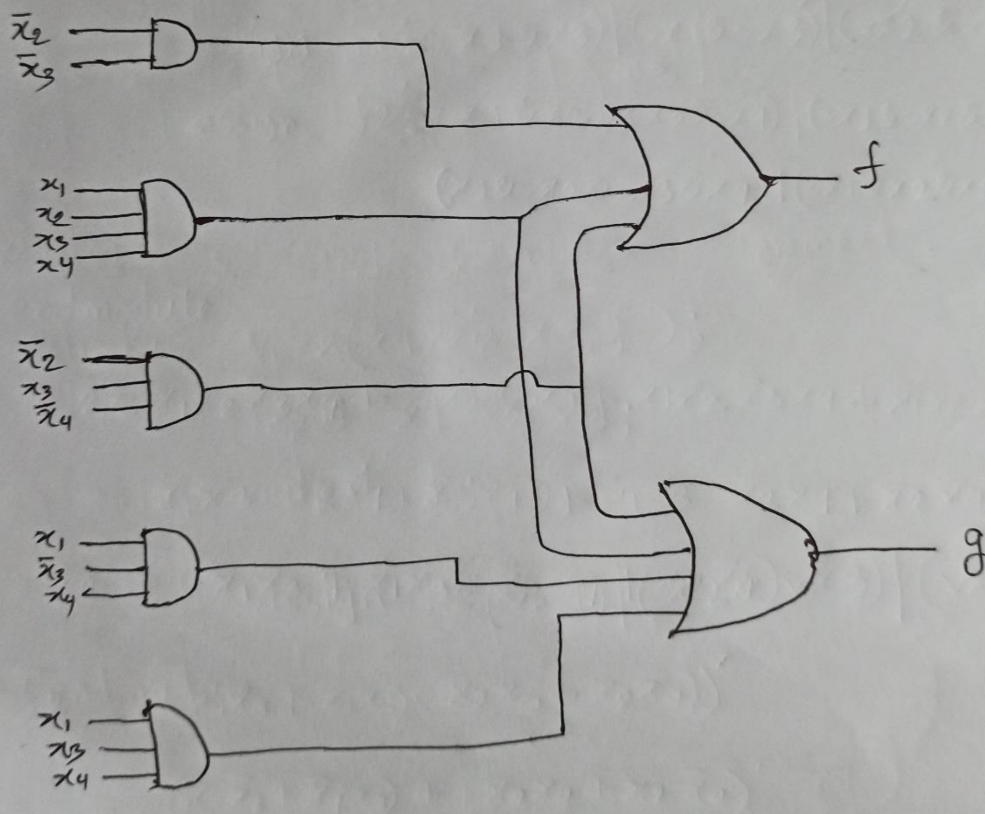
$x_1x_2x_3x_4$ $x_1\bar{x}_3x_4$

Fig: 4.

$$g = \bar{x}_2 x_3 \bar{x}_4 + x_1 x_2 x_3 x_4 + x_1 \bar{x}_3 x_4 + \bar{x}_1 x_3 x_4$$

$\therefore$ f has 4 gates and 12 inputs.

g has 5 gates and 17 inputs.

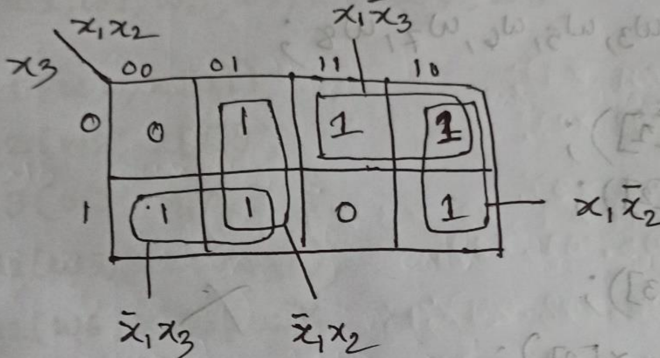


minimum cost - 7 gates and 22 inputs.

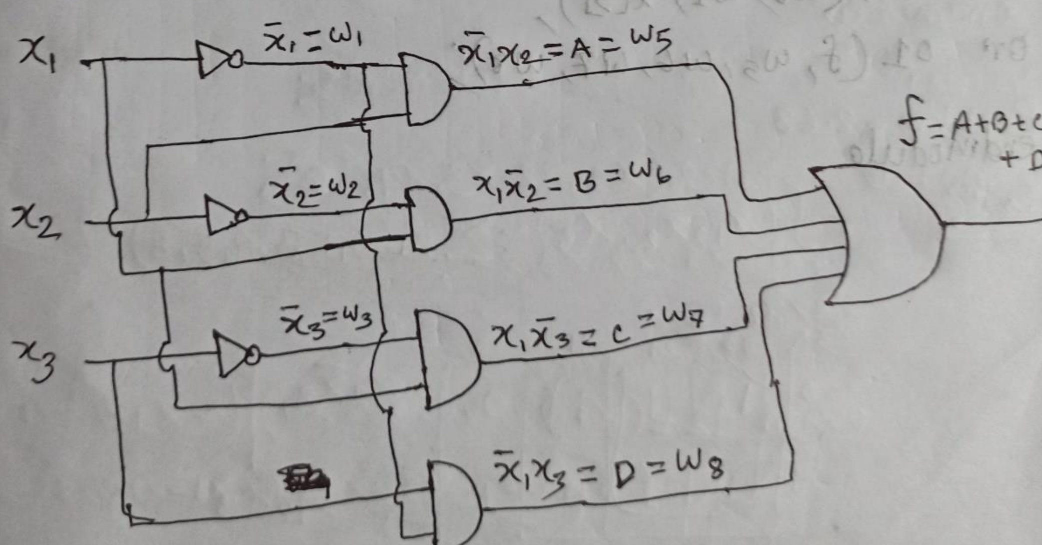
Verilog Part

Problem- 2.62

Soln: $f(x_1, x_2, x_3) = \sum m(1,2,3,4,5,6)$



$\therefore f = \bar{x}_1x_3 + \bar{x}_1x_2 + x_1\bar{x}_2 + x_1\bar{x}_3$



Verilog code using gate-level primitives

```
module  mux4to1 (x1, x2, x3, [redacted], f);
        input x1, x2, x3, [redacted];
        output f;

        not(w1, x1);
        not (w2, x2);
        not (w3, x3);
        and (w5, w1, x2);
        and (w6, w2, x1);
        and (w7, w3, x1);
        and (w8, w1, x3);
        or (f, w5, w6, w7, w8);
endmodule
```

Problem - 2.63

Solⁿ:

$$f(x_1, x_2, x_3) = \sum m(0,1,3,4,5,6)$$

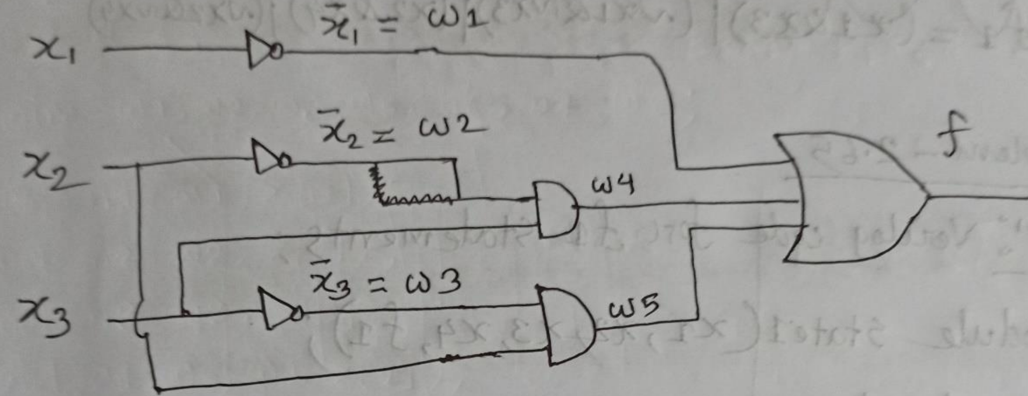
$x_3 \backslash x_1x_2$	00	01	11	10
0	1	1	1	0
1	1	1	0	1

$x_2\bar{x}_3$

$\bar{x}_2 x_3$

$\bar{x}_1$

$\therefore f = \bar{x}_1 + x_2\bar{x}_3 + \bar{x}_2 x_3$



Verilog code using continuous assignment

```
module ver2(x1,x2,x3,f);
        input x1,x2,x3;
        output f;

        assign f=(~x1)|(x2&~x3)|(~x2&x3);

endmodule
```

Problem - 2.65

Soln: (a) verilog code is given below:

```
module State(x1,x2,x3,x4,f1,f2);
        input x1,x2,x3,x4;
        output f1, f2;

        assign f1 = (x1&x3)|(~x1&~x3)|(~x2&~x4);
        assign f2 = (x1&x2&~x3&~x4)|(~x1&~x2&x3&x4)|
                    (x1&~x2&~x3&x4)|(~x1&x2&x3&~x4);

endmodule
```

(b) $f2 = x_1x_2\bar{x}_3\bar{x}_4 + \bar{x}_1\bar{x}_2x_3x_4 + x_1\bar{x}_2\bar{x}_3x_4 + \bar{x}_1x_2x_3\bar{x}_4$

$= x_1\bar{x}_3(x_2\bar{x}_4 + \bar{x}_2x_4) + \bar{x}_1x_3(\bar{x}_2x_4 + x_2\bar{x}_4)$

$\Rightarrow f2 = (x_1\bar{x}_3 + \bar{x}_1x_3)(\bar{x}_2x_4 + x_2\bar{x}_4)$

$\therefore \bar{f}2 = ((x_1\bar{x}_3 + \bar{x}_1x_3)(\bar{x}_2x_4 + x_2\bar{x}_4))'$

$= (x_1\bar{x}_3 + \bar{x}_1x_3)' + (\bar{x}_2x_4 + x_2\bar{x}_4)'$

$= (x_1\bar{x}_3)'(\bar{x}_1x_3)' + (\bar{x}_2x_4)'(x_2\bar{x}_4)'$

$= (\bar{x}_1 + x_3)(x_1 + \bar{x}_3) + (x_2 + \bar{x}_4)(\bar{x}_2 + x_4)$

$$= (\cancel{\bar{x}_1 x_1} + x_3 x_1 + \bar{x}_1 \bar{x}_3 + \cancel{x_3 \bar{x}_3}) + (\cancel{x_2 \bar{x}_2} + \bar{x}_4 \bar{x}_2 + x_2 x_4 + \cancel{\bar{x}_4 x_4})$$

$$= x_3 x_1 + \bar{x}_1 \bar{x}_3 + \bar{x}_2 \bar{x}_4 + x_2 x_4$$

$$= f_1$$

$\therefore \overline{f_2} = f_1$ (proved)