Review

Lecture 9: Logical Agents

- Knowledge-based agents
- Wumpus world
- Logic in general - models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - Forward chaining/backward chaining/resolution
- Summary

Review

Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions
- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences wrt models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Resolution is complete for propositional logic
  Forward, backward chaining are linear-time, complete for Horn clauses
  Propositional logic lacks expressive power, so it does not scale to environment of unbounded size

Lecture 12: First-Order Logic

Chapter 8

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Outline

- Why FOL?
- Syntax and semantics of FOL
- Using FOL
- Wumpus world in FOL
- Knowledge engineering in FOL
- Summary

Pros and cons of propositional logic

- ☺ Propositional logic is declarative

- ☺ Propositional logic allows partial/disjunctive/negated information
  - (unlike most data structures and databases)
- ☺ Propositional logic is compositional:
  - meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- ☺ Meaning in propositional logic is context-independent
  - (unlike natural language, where meaning depends on context)
- ☹ Propositional logic has very limited expressive power
  - (unlike natural language)

First-order logic

- propositional logic assumes the world contains facts
- first-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Syntax of FOL: Basic elements

- Constants KingJohn, 2, NUS,...
- Predicates Brother, Spouse， >,...
- Functions Sqrt, LeftLegOf,...
- Variables x, y, a, b,...
- Connectives $\neg, \Rightarrow, \wedge, \vee, \Leftrightarrow$
- Equality $=$
- Quantifiers* $\forall, \exists$

Atomic sentences

Atomic sentence = *predicate* ($term_1$,...,$term_n$)
or $term_1$ = $term_2$

Term = *function* ($term_1$,...,$term_n$)
or *constant* or *variable*

E.g., *Brother(KingJohn,RichardTheLionheart)* → *(Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))*

Complex sentences

- Complex sentences are made from atomic sentences using connectives

  $\neg S$, $S_1 \wedge S_2$, $S_1 \vee S_2$, $S_1 \Rightarrow S_2$, $S_1 \Leftrightarrow S_2$,

  E.g. *Sibling(KingJohn,Richard)* $\Rightarrow$ *Sibling(Richard,KingJohn)*

  $>(1,2) \vee \leq (1,2)$

  $>(1,2) \wedge \neg >(1,2)$

Truth in first-order logic

- Sentences are true with respect to a model and an interpretation
- **Model** contains objects (domain elements) and relations among them
- Interpretation specifies referents for

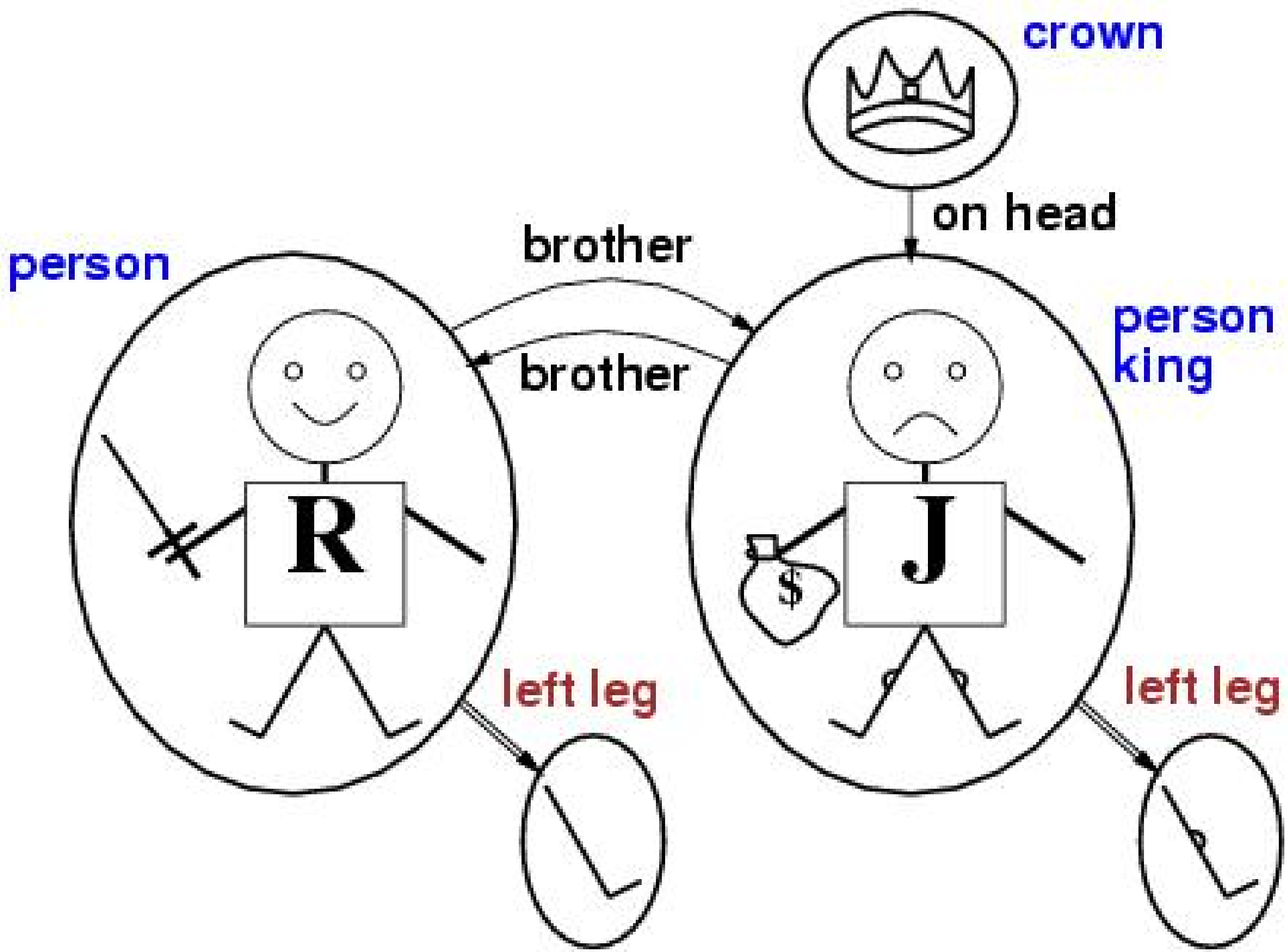
  constant symbols → objects

  predicate symbols → relations

  function symbols → functional relations

- An atomic sentence *predicate*($term_1,\ldots,term_n$) is true iff the objects referred to by $term_1,\ldots,term_n$ are in the relation referred to by *predicate*

Models for FOL: Example



Universal quantification*

- $\forall$<*variables*> <*sentence*>

  Everyone at NUS is smart:

  $\forall x\ At(x,NUS) \Rightarrow Smart(x)$

- $\forall x$ *P* is true in a model *m* iff *P* is true with *x* being each possible object in the model

- Roughly speaking, equivalent to the conjunction of instantiations of *P*

e.g

$$
\begin{aligned}
& At(KingJohn,NUS) \Rightarrow Smart(KingJohn) \\
\wedge\ & At(Richard,NUS) \Rightarrow Smart(Richard) \\
\wedge\ & At(NUS,NUS) \Rightarrow Smart(NUS)
\end{aligned}
$$

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$

- Common mistake:

  using $\wedge$ as the main connective with $\forall$:

  $\forall x\ At(x,NUS) \wedge Smart(x)$

  means "Everyone is at NUS and everyone is smart"

Existential quantification*

- $\exists$<*variables*> <*sentence*>

- Someone at NUS is smart:
  $\exists x$ At(x,NUS) $\wedge$ Smart(x)

- $\exists x\ P$ is true in a model *m* iff *P* is true with *x* being some possible object in the model

- Roughly speaking, equivalent to the disjunction of instantiations of *P*

  At(KingJohn,NUS) $\wedge$ Smart(KingJohn)
  $\vee$ At(Richard,NUS) $\wedge$ Smart(Richard)
  $\vee$ At(NUS,NUS) $\wedge$ Smart(NUS)
  $\vee$ ...

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake:

  using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x\ At(x,NUS) \Rightarrow Smart(x)$$

is true if there is anyone who is not at NUS!

Properties of quantifiers

- $\forall x\ \forall y$ is the same as $\forall y\ \forall x$
- $\exists x\ \exists y$ is the same as $\exists y\ \exists x$
- $\exists x\ \forall y$ is not the same as $\forall y\ \exists x$

  e.g. $\exists x\ \forall y$ Loves(x,y)
  - "There is a person who loves everyone in the world"

  e.g. $\forall y\ \exists x$ Loves(x,y)
  - "Everyone in the world is loved by at least one person"

- **Quantifier duality**: each can be expressed using the other

- $\forall x$ Likes(x,IceCream) $\neg\exists x\ \neg$Likes(x,IceCream)

- $\exists x$ Likes(x,Broccoli) $\neg\forall x\ \neg$Likes(x,Broccoli)

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object

E.g., definition of *Sibling* in terms of *Parent*:

$$\forall x,y\ Sibling(x,y) \Leftrightarrow [\neg(x = y) \land \exists m,f\ \neg(m = f) \land Parent(m,x) \land Parent(f,x) \land Parent(m,y) \land Parent(f,y)]$$

Using FOL

The kinship domain:

- Brothers are siblings

  Given: x,y, *Brother(x,y), Sibling(x,y)*

  $\forall$x,y *Brother(x,y)* $\Leftrightarrow$ *Sibling(x,y)*

- One's mother is one's female parent

  $\forall$m,c *Mother(c)* = m $\Leftrightarrow$ *(Female(m)* $\wedge$ *Parent(m,c))*

- "Sibling" is *symmetric*

  $\forall$x,y *Sibling(x,y)* $\Leftrightarrow$ *Sibling(y,x)*

Using FOL

The set domain: ***Axioms***

- Set: $\forall s\ Set(s) \Leftrightarrow (s = \{\}\ ) \vee (\exists x,s_2\ Set(s_2) \wedge s = \{x|s_2\})$
- Empty set: $\neg\exists x,s\ \{x|s\} = \{\}$
- Adjoin: $\forall x,s\ x \in s \Leftrightarrow s = \{x|s\}$
- Members: $\forall x,s\ x \in s \Leftrightarrow \exists y,s_2\ (s = \{y|s_2\} \wedge (x = y \vee x \in s_2))$
- Subset: $\forall s_1,s_2\ s_1 \subseteq s_2 \Leftrightarrow (\forall x\ x \in s_1 \Rightarrow x \in s_2)$
- Equality: $\forall s_1,s_2\ (s_1 = s_2) \Leftrightarrow (s_1 \subseteq s_2 \wedge s_2 \subseteq s_1)$
- Intersection: $\forall x,s_1,s_2\ x \in (s_1 \cap s_2) \Leftrightarrow (x \in s_1 \wedge x \in s_2)$
- Union: $\forall x,s_1,s_2\ x \in (s_1 \cup s_2) \Leftrightarrow (x \in s_1 \vee x \in s_2)$

Interacting with FOL KBs

- Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at *t=5*:

  `Tell`(KB,Percept([Smell,Breeze,None],5))

  `Ask`(KB,∃a BestAction(a,5))

I.e., **Does the KB entail some best action at *t=5*?**

Answer: *Yes*, {*a/Shoot*} ← substitution (binding list)

- Given a sentence *S* and a substitution σ,

  *S*σ denotes the result of plugging σ into *S*;

e.g.,

S = Smarter(x,y)

σ = {x/Hillary,y/Bill}

*S*σ = Smarter(Hillary,Bill)

- `Ask`(KB,S) returns some/all σ such that KB ⊨ σ

Knowledge base for the wumpus world

- Perceptions

  e.g. $\forall t,s,b\ Percept([s,b,Glitter],t) \Rightarrow Glitter(t)$

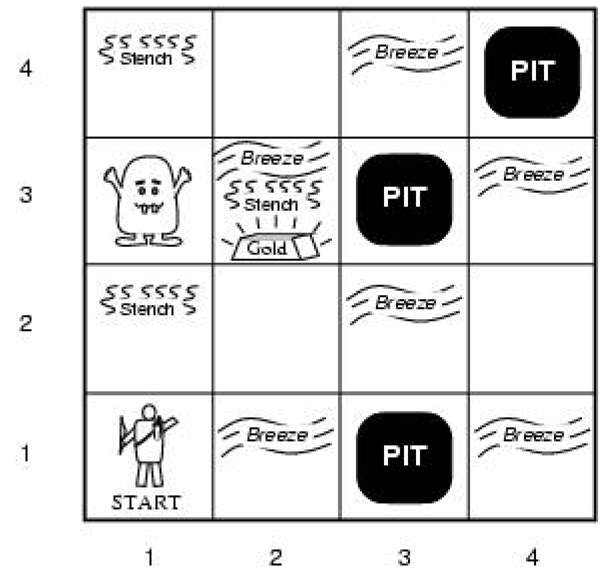
- Reflex

  e.g. $\forall t\ Glitter(t) \Rightarrow BestAction(Grab,t)$

- Representation of environment?

Wumpus World PEAS description

- **Wumpus World** story
- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- Environment
  - Squares **adjacent** to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square



- Sensors: Stench, Breeze, Glitter, Bump, Scream
- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot

Deducing hidden properties

- Adjacency of any two squares

  $\forall x,y,a,b\ \mathit{Adjacent}([x,y],[a,b])$

  $\Leftrightarrow [a,b] \in \{[x+1,y], [x-1,y],[x,y+1],[x,y-1]\}$

  $\Leftrightarrow ((a=x+1 \vee a=x-1) \wedge b=y) \vee (a=x \wedge (b=y+1 \vee b=y-1))$

- Properties of squares:

  $\forall s,t\ \mathit{At}(\text{Agent},s,t) \wedge \text{Breeze}(t) \Rightarrow \text{Breezy}(s)$

- Squares are breezy near a pit:
  - Diagnostic rule---infer cause from effect

    $\forall s\ \text{Breezy}(s) \Rightarrow \exists\ r\ \text{Adjacent}(r,s) \wedge \text{Pit}(r)$
  - Causal rule---infer effect from cause

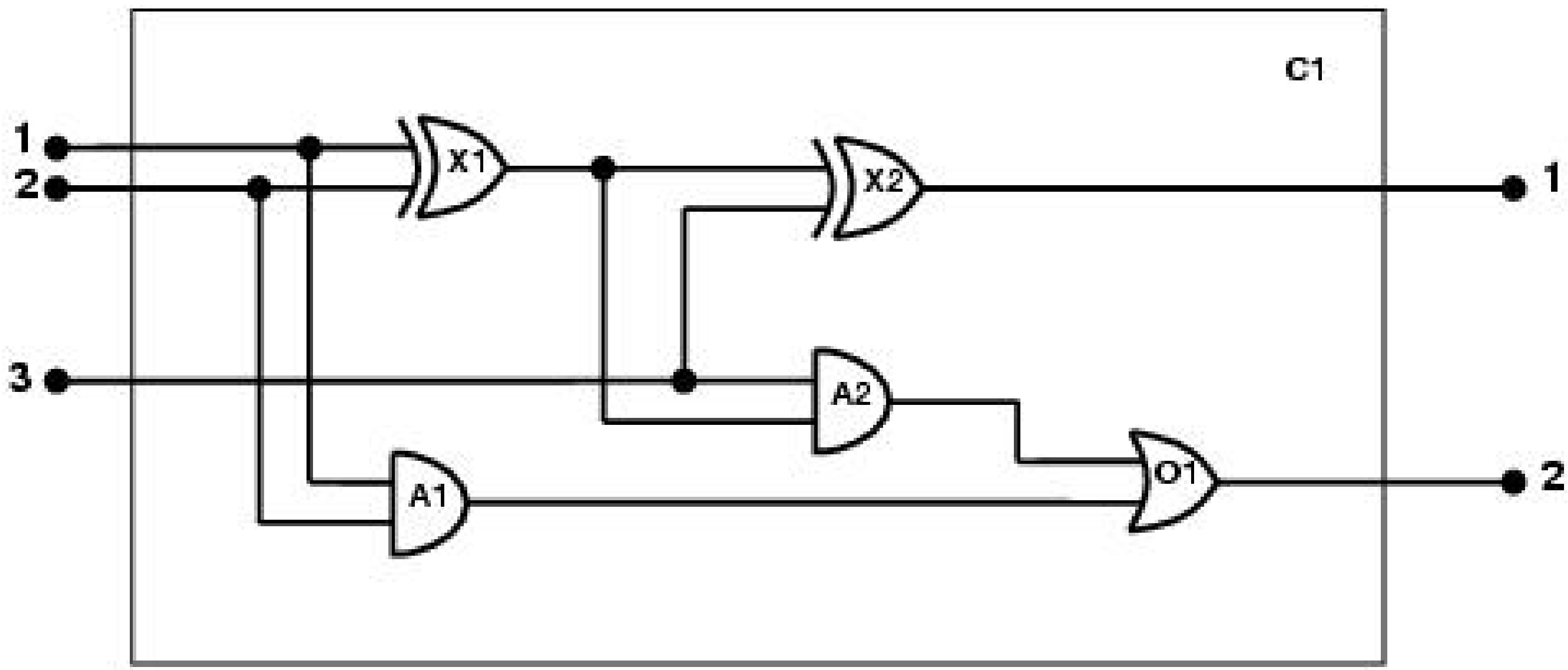
    $\forall r\ \text{Pit}(r) \Rightarrow \forall s\ \text{Adjacent}(r,s) \Rightarrow \text{Breezy}(s)$

Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

The electronic circuits domain

One-bit full adder



The electronic circuits domain

1. Identify the task
   – Does the circuit actually add properly? (circuit verification)
2. Assemble the relevant knowledge
   – Composed of wires and gates; Types of gates (AND, OR, XOR, NOT)
   – Irrelevant: size, shape, color, cost of gates
3. Decide on a vocabulary
   – Alternatives:

     $Type(X_1) = XOR$

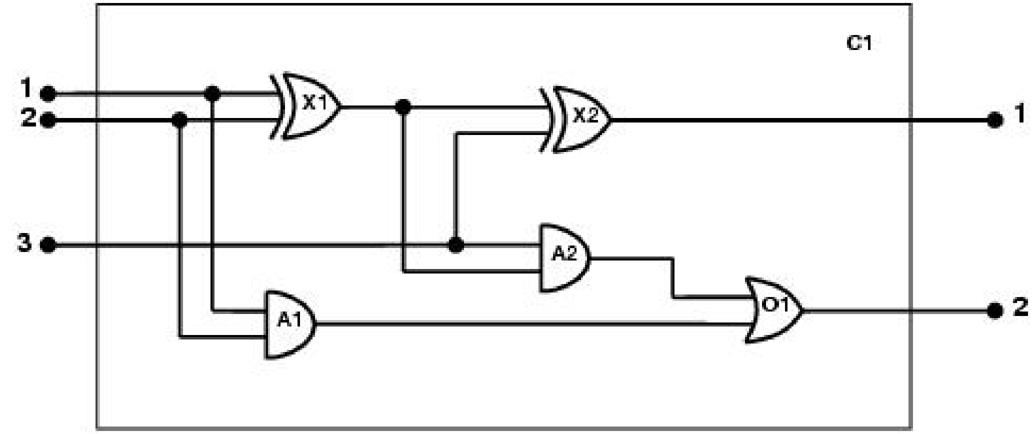
     $Type(X_1, XOR)$

     $XOR(X_1)$

The electronic circuits domain

4. Encode general knowledge of the domain: *axioms*
   - $\forall t_1,t_2$ Connected$(t_1, t_2) \Rightarrow$ Signal(t_1) = Signal(t_2)
   - $\forall t$ Signal$(t) = 1 \vee$ Signal$(t) = 0$
   - $1 \neq 0$
   - $\forall t_1,t_2$ Connected$(t_1, t_2) \Rightarrow$ Connected(t_2, t_1)
   - $\forall g$ Type(g) = OR $\Rightarrow$ Signal(Out(1,g)) = 1 $\Leftrightarrow \exists n$ Signal(In(n,g)) = 1
   - $\forall g$ Type(g) = AND $\Rightarrow$ Signal(Out(1,g)) = 0 $\Leftrightarrow \exists n$ Signal(In(n,g)) = 0
   - $\forall g$ Type(g) = XOR $\Rightarrow$ Signal(Out(1,g)) = 1 $\Leftrightarrow$ Signal(In(1,g)) $\neq$ Signal(In(2,g))
   - $\forall g$ Type(g) = NOT $\Rightarrow$ Signal(Out(1,g)) $\neq$ Signal(In(1,g))

The electronic circuits domain



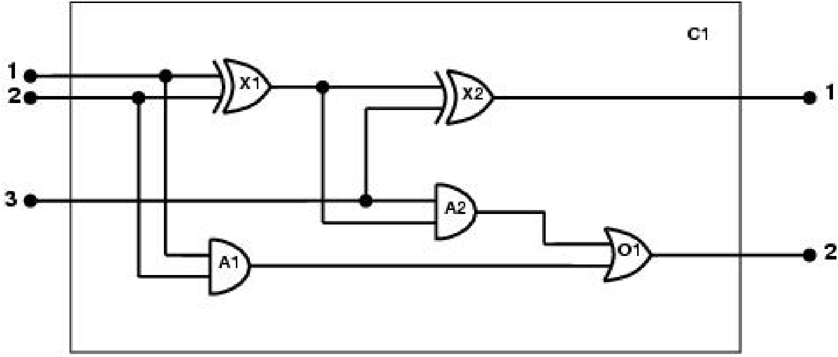
5. Encode the specific problem instance

Type(X_1) = XOR
Type(X_2) = XOR
Type(A_1) = AND
Type(A_2) = AND
Type(O_1) = OR

Connected(Out(1,X_1),In(1,X_2))
Connected(Out(1,X_1),In(2,A_2))
Connected(Out(1,A_2),In(1,O_1))
Connected(Out(1,A_1),In(2,O_1))
Connected(Out(1,X_2),Out(1,C_1))
Connected(Out(1,O_1),Out(2,C_1))

Connected(In(1,C_1),In(1,X_1))
Connected(In(1,C_1),In(1,A_1))
Connected(In(2,C_1),In(2,X_1))
Connected(In(2,C_1),In(2,A_1))
Connected(In(3,C_1),In(2,X_2))
Connected(In(3,C_1),In(1,A_2))

The electronic circuits domain



6. Pose queries to the inference procedure

What are the possible sets of values of all the terminals for the adder circuit?

$\exists i_1,i_2,i_3,o_1,o_2$ Signal(In(1, C_1)) = i_1 $\wedge$ Signal(In(2,C_1)) = i_2 $\wedge$ Signal(In(3,C_1)) = i_3 $\wedge$ Signal(Out(1,C_1)) = o_1 $\wedge$ Signal(Out(2,C_1)) = o_2

7. Debug the knowledge base

May have omitted assertions like $1 \neq 0$

Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power: sufficient to define wumpus world

Assignment

- Exercise 8.10, 8.1