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### 1. Introduction

## 1.1 Introduction for physics Experiment

The laboratory experiment for university physics is the first experimental course during undergraduate study. As pedagogical tool the physics laboratory experiment is very important. It provides a tangible manifestation of the abstract physic concepts that are discussed in other lectures; an opportunity to connect the theoretical analysis with the real. It also provides students chances to learn to communicate effectively about the difficult quantitative concepts. Being able to write a sentence that sufficiently describes a phenomenon is a skill that enhances anyone's resume. Besides, this course will cultivate students to extract meaningful information from numbers.

This edition of *physics experiments* includes 10 preliminary experiments, 5 multidisciplinary and modern physics experiments and one self-design experiment. Besides to preliminary physics and computer-aided experiments, this course will introduce some more interesting experiments, most of which are integrated with more concept beyond one field, such as various types of sensors. Also, high level requirements for the course preparation, ability for practical handling, the essential part of basic scientific quality will be emphasized. Finally, we hope that this course achieves these objectives and does so in a way that is fun.

The main purpose of an introductory physics laboratory is to provide "hand-on" experiences of various physical principles. In so doing, one becomes familiar with laboratory equipment, procedures, and the scientific method. In general, the theory of a physical principle will be presented in an experiment, and the predicted results will be tested by experimental measurements. Of course, these well-known principle have been tested many times before, and there are accepted values for certain physical quantities. Basically you will be comparing you experimentally measured values to accepted theoretical or measured values. Even so, you will experience the excitement of the scientific method. Imagine that you are first person to perform an experiment to test a scientific theory.

## 1.2 General Laboratory Procedures

#### **Advance Study**

Students often come to the laboratory unprepared, even though they should have read the experiment before the lab period to familiarize themselves with it. To address this problem, an advance study assignment precedes each experiment. The assignment consists of a set of questions drawn from the theory and experimental Procedures sections of the Experiment. To answer the

questions, students must read the experiment before the lab period; consequently, they will be better prepared.

#### **Experiment Operation**

First of all, carefully check the status of instruments and tools. Then, you should always arrange, assembly or adjust the instruments (including proper circuit connection, optical beam adjustment) to give the best performance possible following the advisement of instructor/textbook. Observe experimental phenomena to obtain a better understating of underlying physics mechanism. At last, make and record readings as precisely as the apparatus will permit. Always estimate one significant figure beyond the smallest graduation on the instrument being read.

Data should not be recorded on scrap paper and then transferred to your recorded form. If, after you have recorded a reading, you decide that it is in error and should be discarded, mark through it and recorded the corrected reading below it. Always record the proper unit beside the number or at the heading of a column when a whole column of readings use the same unit.

#### **Laboratory Reports**

A laboratory report form is provided for each experiment in which experimental data are accorded. This should be done neatly. Calculations of experimental results should be included. Remember, the neatness, organization, and explanations of your measurements and calculations in the laboratory report represent the quality of your work. A complete report should includes as follows, (1)Experiment objective,

- (2)Experiment instruments: recording each device's name, technical parameters.
- (3) Key points for operation process.
- (4) Experiment data
- (5) Data process: including data table, uncertainty and results calculation.
- (6) Complete expression for experiment results
- (7) Discussion: including analysis or summary for experiment, some discussions on some phenomena observed, answers to questions drawn from the theory and experiment procedures.

## 1.3 Fundamental Requirements

#### **Advance Preparation**

Every student is about to perform physics experiment should complete advance study by reading the textbook. Without being advance prepared, it is not permitted to beginning the operation procedure.

#### **Safety**

The most important thing in the laboratory is your safety and that of others. Experiments are designed to be done safely, but proper caution should always be exercised.

A potential danger comes from a lack of knowledge of the equipment and procedures. Upon entering the physics lab at the beginning of the lab period, you will probably find the equipment for an experiment on the laboratory table. Restrain your curiosity and do not play with the equipment. You may hurt yourself and/or the equipment. A good general rule is:

Do not touch or turn on laboratory equipment until it has been explained and permission has been given by the instructor.

Also, certain items in various experiments can be particularly dangerous, for example, hot objects, electricity, mercury lamps, and radioactive sources. In some instances, such as with hot objects and electricity, basic common sense and knowledge are required.

However, in other instances, such as with mercury lamps and radioactive sources, you may not be aware of the possible dangers. Mercury lamps may emit ultraviolet radiation that can be harmful to your eyes. Consequently, some sources need to be properly shielded. Some radioactive sources are solids and are encapsulated to prevent contact. Others are in liquid form and are transferred during an experiment, so there is a danger of spillage. Proper handing is therefore important.

In general, necessary precautions will be given in the experiment descriptions. *Note them well*. When you see the arrow symbol in the margin as illustrated here, you should take extra care to follow the procedure carefully and adhere to the precautions described in the text. As pointed out earlier, experiments are designed to be done safely. Yet a common kitchen match can be dangerous if used improperly. Another good rule for the laboratory is:

If you have any questions about the safety of a procedure, ask your instructor before doing it. The physics lab is a place to learn and practice safety.

#### **Apparatus Care**

The apparatus provided for the laboratory experiments is often expensive and in some instances quite delicate, If used improperly, certain pieces—of apparatus can be damaged. The general rules given above concerning personal safety also apply to equipment care. Even after familiarizing oneself with the equipment, it is often advisable or required to have an experimental setup checked and approved by—the instructor before putting it into operation. This is particularly true for electrical experiments. Applying power to improperly wired circuits can cause serious damage to meters and other pieces of apparatus.

If a piece of equipment is broken or does not function properly, it should be reported to the laboratory instructor. Also, after you complete an experiment, the experimental setup should be

disassembled and left neatly as found, unless you are otherwise instructed.

If you accidentally break some equipment or the equipment stops working properly during an experiment, report it to your instructors. Otherwise, the next time the equipment is used, a great deal of time may be wasted trying to get good results.

## 2. Error Theory and Data Process

## 2.1 Basic Concept of Measurement

Measurement plays a fundamental role in our modern world. In commerce, goods are priced by volume, mass, or something length or area; services such as the transportation are billed according to quantity of materials as well as according to the distance it is delivered. In commercial transactions, errors in measurement have a direct bearing on profits and costs.

In the engineering technologies, every project begins and ends with measurements. The design of a highway or skyscraper starts with a survey; the design of a power transformer starts with measurement of the electrical and magnetic properties of the wire, the insulation, and the magnetic core. The final product must then be tested to see if it actually measures up to its theoretical performance.

Mathematics is a scheme for dealing with numbers and with functions, or sets of numbers. This scheme tells us how to operate on numbers to get new numbers, and how to operate on functions to get new functions. But mathematics itself does not tell us what these numbers or functions mean, as far as anything physical is concerned.

Numbers and mathematical functions acquire physical meaning only when engineers and scientists become involved. Their job is to find ways of expressing properties of the real world in numerical form. Only then does mathematics become a practical tool.

Measurement is process of comparison. Measurement is determination of the magnitude of a quantity by comparison with a standard for that quantity. Quantities frequently measured include time, length, area, volume, press, mass, force, and energy. To express a measurement, there must be a basic unit of the quantity involved, e.g., the inch (1 in=2.54 cm) or second, and a standard of measurement calibrated in such units, e.g., a ruler or clock. Had we compared the radio tower's height to something we call a "yard", the result of the measurement would have been 99 instead of 297. Had we compared it to something we call a "meter", the result would have been 90.5. Have we use inch, the measurement's numerical result would have been 3 560.

#### 2.1.1 Direct/indirect measurement

According to the differences of the measurement methods, measurements can be divided into **direct** and **indirect measurements**. Generally, **direct measurement** refers to measuring exactly the thing that you're looking to measure. For example, measure length with ruler, measure time by a stopwatch, current by an ammeter.

**Indirect measurement** means that one wants to measure something by measuring something else. Actually, is a process of determination of the magnitude of a quantity through calculating with formula, for example, if one intends to measure gravitational acceleration via simple pendulum, stop watch and meter ruler can be used to measure directly period, T, and length of pendulum, T, Then gravitational acceleration, T, and length of pendulum, T are direct measurement while gravitational acceleration is indirect measurement.

### 2.1.2 Equally/unequally accurate measurement

**Equally accurate measurement** means repeated measurements under the same experimental conditions (same devices, same operator, same temperature....).

**Unequally accurate measurement** refers to pursuit higher measurement accuracy via comparison of different measurement approaches (devices, methods, or operators). The final results is usually calculated with weighted approaches.

#### 2.1.3 Unit

Obviously, it is just as important to specify the thing we are comparing with as it is to quote the number itself. This is why we say that the numbers by themselves are meaningless. In the technologies, it is absolute essential that we have a unit associated with each number we use. You will notice that units are specified for all numbers through the book.

We will use many different kinds of units: for example, units of distance (inches, centimeters, feet, yards, meters, kilometers, miles, and so on) and units of time (milliseconds, seconds, minutes, hours, days, years, etc.). In addition, we use combinations of units, or compound units for some quantities: speed, for instance, may be expressed in feet per second, kilometer per hour, miles per hour, or any of a number f other combination. And everytime we make a measurement, we are comparing the known size of one of these units with the size of the quantity we are measuring. Even if a comparison must be made indirectly, it is still a comparison. It is important to keep this point in mind whenever undertaking measurements.

In 1960, 36 nations signed a treaty to create an international system of units. The entire system is officially called "Le Système International d' Unites" (SI), and is called the International System in the English- speaking countries. The SI is essentially the same as what e have come to know as the metric system. Table I. lists the several SI base units. The SI base units are the official international units for the seven different kinds of physical quantities that can be measured. We can measure length, time, mass, electric current, temperature, luminous intensity, and amount of

substance.

Table I The SI Base Units

Quantity	Unit
Length	Meter (m)
Time	Second (s)
Mass	Kilogram (kg)
Electric current	Amphere (A)
Temperature	Kelvin (K)
Luminous intensity	Candela (cd)
Amount of substance	Mole (mol)

Of course, we use many units other than the base units listed in the table I. There are many other such defined units. Defined units are not part of International System. Rather they have been related to the corresponding SI units through numerical definitions.

Units that can be expressed as combinations of the SI base units are called compound units or derived units. Compound, or derived units are units for all quantities other than length, time, mass, electric current, temperature, luminous intensity, and amount of substance. In the international System, all such units can be expressed as a combination of some of the base units.

## 2.2 Basic Concept of Measurement Error

### 2.2.1 Definition of Error

What we have to do is to examine how wrong we can allow them to be. And instead of calling them wrong, we will talk about the measurement error, and how big this error is likely to be.

**Error** is the difference between the measured value and true value, as expressed in Eq. 2-1.

$$\varepsilon_{x} = x - \mu \tag{2-1}$$

Where x,  $\mu$ ,  $\varepsilon_x$  refer to measured value, true value and error, respectively.

Error can be negative  $(x < \mu)$  or positive  $(x > \mu)$ , presenting the deviation range of measured value with respect to the true value. Smaller error indicates more close between true value and measured value. It is therefore that one can see the reliability of measured data from the value of measured error.

Notice that we are now using the word "error" in a very special, technical sense. In the sciences of

technologies, an error is not the same as a mistake. Mistakes can avoided, or at least corrected, whereas errors in measurement can never be eliminated completely. The best we can do is to do try our best to keep the errors small enough that the results can still be used for its intended purposes. First, we have to understand "true value" of an object in nature.

True value is consistent with the definition of a given particular quantity. The value would be obtained by a perfect measurement or by nature indeterminate. Since we can never know the value of any physical quantity unless we measure it., and since no measurement is absolutely accurate, if follows that we can never know that the "true value of any physical quantity. This is why "true value" is enclosed in quotation marks. When we make a measurement, we usually assume that there is such a thing as a true value, yet at the same time we recognize that we will never know exactly what this "true value" is. This severely limits the usefulness of our definition of measurement error, for it means we can never calculate exactly what or measurement error is. Therefore, we can substitute "true value" with "conventional true value". A conventional trure value is the value attributed to a particular quantity and accepted, sometimes by convention, as having an uncertainty appropriated for a given purpose. Conventional true value is sometimes called "assigned value" or "target value". In measurement, theorectically value, empirical value and average measurement value would be taken as conventional true value. Accordingly, the difference between a computed or measured value and a conventional true value is a called bias.

In practice, we can take enough repeated measurements to obtain enough information to make good estimates of the average size of these errors in the data. The mean value of measured data is often used to present true value. Thus, error definition can be expressed as follows,

$$\varepsilon_{x} = x - \overline{x} \tag{2-2}$$

where  $\bar{x}$  is the average value of measured data.

#### **Absolute Error**

When the size of the estimated error in a quantity is expressed in the same units as that quantity it is called an absolute *error*. The quantity and its absolute error are conventionally expressed in the standard form of this example

Length = 
$$2.25\pm0.03$$
 cm

The value 0.03 after the  $\pm$  sign in this expression is the estimated absolute error in the value of 2.25.

#### 2.2.2 Percent Error

The error in a measurement can also be expressed as a percent.

$$E_x = \frac{\left|\mathcal{E}_x\right|}{\mu} \times 100\% \tag{2-3}$$

Similar to the expression of absolute error, percent error in the length would be

$$E_{Length} = \frac{0.03}{2.25} \times 100\% = 1.33\%$$

#### 2.2.3 Deviation

There are cases when we want to compare the results of two measurements assumed equally reliable, that is, to find the absolute or percent difference between the two. For example, you might want to compare two independent determinations you made of a quantity, or to compare your experimental results with that obtained independently by someone else in the class. Suppose two measurements of a length give 3.6 and 3.8 mm, respectively, the exact value not being known. The percent difference is found by comparing the deviation (or difference) with the average of the two. Hence, we have,

Percent difference=
$$\frac{3.8-3.6}{(3.8+3.6)/2} \times 100\% = 5.4\%$$

Similarly, deviation can be written as absolute amounts or as percents.

### 2.2.4 Experimental Discrepancy

When an experimental result is compared with another which is assumed more reliable, it is customary to call the difference between the two the experimental discrepancy. These may be expressed as absolutes or as percents.

An obvious questions is when should errors be expressed as absolute errors and when should they be expressed as percent errors? Common sense and good judgment should be used in choosing which form to use to represent an error. Consider a temperature measurement with a thermometer known to be reliable to +0.5 °C. This causes a 0.5 % error in the measurement of the boiling point of water (100 °C) but a 10% error in the measurement of cold water at a temperature of 5 °C. Clearly in this case the use of absolute errors is more meaningful.

There are situations in which errors expressed as percents are more meaningful. For instances, suppose one measures the distance between two streets to be 380 m, while a professional surveyor's record shows the distance as 400 m. In another case, a person estimates the width of a table as 1.8 m when it should be 2.0 m. The absolute error in the first case is 10m, and in the latter case, 0.2 m. The one who measured the table made an error of 0.2 m in each meter measured. On

this basis they would have made an error of 40 m in the street measurement. The percent error in the first case is 2.5 % whereas the percent error in the second case is 10 %.

#### 2.2.5 Significant Figures

The digits required to express a number to the same accuracy as the measurement it represents are known as *significant figures*. The number of significant figures in a value can be defined as all the digits between and including the first non-zero digit from the left, through the last digit.

For instance, 0.44 has two significant figures, and the number 66.770 has 5 significant figures. Zeroes are significant except when used to locate the decimal point, as in the number 0.00030, which has 2 significant figures. Zeroes may or may not be significant for numbers like 1200, where it is not clear whether two, three, or four significant figures are indicated. To avoid this ambiguity, such numbers should be expressed in scientific notation to (e.g. 1.2 × 10<sup>3</sup> clearly indicates two significant figures). If the above measurement is made with a meter stick, the last digit recorded is an estimated figure representing a fractional part of a millimeter division. *All recorded data should include the last estimated figure in the result, even though it may be zero*. If this measurement had appeared to be exactly 20 cm, it should have been recorded as 20.00 cm, since lengths can be estimated by means of this apparatus to about 0.01 cm. When the measurement is written as 20 cm it indicates that the value is known to be somewhere between 19 and 21 cm, whereas the value is actually known to be between 19.99 and 20.01 cm.

Referring again to the 20.64 cm measurement, the possible error in this measurement is  $\pm$  0.01 cm and was recorded as being nearer to 20.64 than to 20.63 or 20.65. Hence, the uncertainty is less than one part in two thousand. Besides, it should be understood that the measurement error is dependent on the measuring apparatus and the technique of the experimenter. For instance, it is possible that 20.64 cm measurement is known to lie between 20.635 and 20.645cm so that the error is  $\pm$  0.005 cm. As a result, it is important to specify the measurement error.

### 2.2.6 Systematic Error

We can now make a difference between two basic type of measurement errors—systematic error and random error. We need to make this distinction because these errors are handled in difference ways. A systematic error, which is also called determinate error, remains the same change throughout a set of measurement trials. A random error varies from trial to trial and is equally likely to be positive or negative.

Of the two types, systematic errors are usually the more difficult to detect and account for.

Systematic errors generally originate in one of the two ways:

- (1) **Errors of calibration**. If the measuring instrument is not brought into precise agreement with a standard, or if a standard itself is not a faithful reproduction of a primary standard, then all readings from the instrument will be affected in the same way, giving rise to a systematic error. For instance, any measurement of a time interval on a clock that gains time will be too large.
- (2) **Errors of use**. If the instrument is not used under condition identical to those prevailing when it was calibrated. The change of conditions may affect the way the instrument responds to the quantity being measured. Again, all the measurement in a set of trials will be affected in the same way, and the error is systematic. For instance, if a steel tape measure was calibrated at temperature of 20 °C but is being used at a temperature of -10 °C, thermal contraction causes all the measurements to come out slightly too high.

Once we know that a systematic error exists in a measurement, we can often figure how to eliminate it, or at least make it small enough to neglect. Of course, discovering a systematic error is not always easy, so it is wise to be on guard against them constantly.

What can be done to minimize systematic errors? First, it is important to fully understand the instrument and the physics of its operation. We should know how the instrument's accuracy is likely to be affected by temperature, humidity, and barometric pressure. We should know exactly how to calibrate the instrument, and how often the instrument usually needs to be recalibrated. If the instrument was last calibrated under conditions different from those that currently prevail, we may have to perform a recalibration on the spot. If a recalibration is not practical, we may have to correct our readings mathematically.

Since the range of instruments is so enormous, it is often necessary to evaluate systematic errors on many instruments we have never encountered in formal training. To do so, we have to rely on the manufacturer's operating manuals. We also need to have a reasonable understanding of basic physics, or else we can never judge which factors are likely to affect the equipment's operation.

With systematic errors, we can sometimes (but not always) make a correction to the measurement based on this estimate. The following example shows one way to do so.

Example: Thermal contraction of a tape measure A 30-meter steel tape measure is designed for use at a temperature of 20.0 °C. Suppose that we need to use the tape outdoors when the temperature is only -9.0 °C. Since the steel tape will contract in the cold, we expect a systematic error to result. And because the scale divisions are getting closer together, we expect the measured results to be too high.

How much contraction is there? The handbooks tell us that steel contracts by 11 millionths of its

length for each 1 °C drop in temperature. We have a total temperature drop of 29 °C, so the total fractional contraction is

 $(29)\times(11\times10^{-6})=319\times10^{-6}$  where the notation  $10^{-6}$  represents one millionth.

We can how calculate the total contraction in centimeters:

$$(319\times10^{-6})\times(30m)\times(100cm/1m)=0.96cm$$

This would be the systematic error for each 30 metre measurement, if we neglected the contraction due to temperature.

If we have reason to believe that this is the only systematic error, we can easily correct our measurements by subtracting this 0.96 centimeters for each measured 30 meters. In other words, once we have taken the time to figure out how large our systematic error is, we no longer need to have the error.

Correction for systematic error is an essential part of many measurement procedures.

**Parallax** (systematic or random) - This error can occur whenever there is some distance between the measuring scale and the indicator used to obtain a measurement. If the observer's eye is not squarely aligned with the pointer and scale, the reading may be too high or low (some analog meters have mirrors to help with this alignment).

**Instrument drift** (systematic) - Most electronic instruments have readings that drift over time. The amount of drift is generally not a concern, but occasionally this source of error can be significant and should be considered.

Lag time and hysteresis (systematic) - Some measuring devices require time to reach equilibrium, and taking a measurement before the instrument is stable will result in a measurement that is generally too low. The most common example is taking temperature readings with a thermometer that has not reached thermal equilibrium with its environment. A similar effect is *hysteresis* where the instrument readings lag behind and appear to have a "memory" effect as data are taken sequentially moving up or down through a range of values. Hysteresis is most commonly associated with materials that become magnetized when a changing magnetic field is applied.

#### 2.2.7 Random Error

Random errors, is also called indeterminate error. We classify errors that are not deterministic and that may affect each data point in the experiment in a different way as random errors. These may result from finite instrument precision and from intrinsic or external "noise". Errors of this type may be reduced by optimizing the experimental setup or averaging large number of repeated measurements, but can never be completely eliminated. Although their origin may also be very

subtle, we at least have ways of dealing with them mathematically. In most measurement, only random errors will contribute to estimates of probable error.

Random errors arise because of either uncontrolled variables or specimen variations.

- (1) Uncontrolled variables. These variables are minor fluctuation in environmental or operating conditions that cause the instrument to respond differently from one measurement trial to the next.
- (2) Specimen variations. If the measurement trial are being made on a number of presumably "Identical" samples, minor differences in chemistry, physical structure, optical properties, etc., between one measurement specimen and another, will give rise to random errors.

#### 2.2.8 Gross Personal Error

Gross personal errors, sometimes called **mistakes** or **blunders**, should be avoided and corrected if discovered. As a rule, gross personal errors are excluded from the error analysis discussion because it is generally assumed that the experimental result was obtained by following correct procedures. The term **human error** should also be avoided in error analysis discussions because it is too general to be useful.

### 2.2.9 Precision and Accuracy

The words precision and accuracy are frequently used when discussing measurement errors and it is important that the student understand what they mean. A measurement with relatively small random error is said to have high precision. A measurement which has small systematic error is said to have high accuracy. A measurement which has both high precision and high accuracy is sometimes called highly *reliable*. These words can be tricky. A precise measurement may be inaccurate if it has a systematic error. An accurate measurement may be imprecise if its random error is large.

It is most important for students to learn how to estimate the experimental errors and to see how these errors affect the reliability of the final result. Entire books have been written on this subject.

### 2.3 Statistic of Random Error

#### 2.3.1 Gaussian Distribution

#### 2.3.1.1 Gaussian Distribution Probability

Earlier it was suggested that random errors lead to a limit in the precision of a measurement. It was also mentioned that the effects of random errors may be reduced by repeating an experiment many tomes and averaging the results. Understanding how this happens how this happens requires some discussion of "Gaussian distribution of error".

The galton board, also known as a quincunx or beam machine, is a device for statistical experimental named after English scientist Francis Galton. It consists of an upright board with evenly spaced nails (or pegs) driven into its upper half, while the nails are arranged win staggered order, and a lower half divided into a number of evenly-spaced rectangular slots. The front of the device is covered with a glass cover to allow viewing of both nails and slots. In the middle of the upper edge, there is a funnel into which balls can be poured, where the diameter of the balls must be much smaller than the distance between the nails. The funnel is located precisely above the central nail of the second row so that each ball, if perfectly centered, would fall vertically and directly onto the uppermost point of this nail's surface. The figure above shows a variant of the board in which only the nails that can potentially be hit by a ball dropped from the funnel are included, leading to a triangular array instead of a rectangular one.

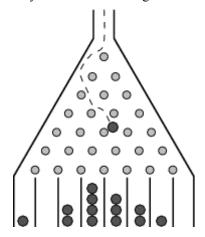


Figure 2-1 Galton Board Experiment

If the number of balls is sufficiently large, then the distribution of the heights of the balls heaps will approximate a normal distribution. A normal distribution in a variation x, the standard deviation  $\sigma$  is a statistic distribution with probability density function.

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
(2-4)

Here, standard deviation  $\sigma$  shows how much variation or dispersion from the average exits. A low standard deviation indicates that the data points tend to be very close to the mean; a high standard deviation indicates that the data points are spread out over a large range of values.

As shown in Figure 2-2, measured data distribution obeys normal distribution with respect to the true value.

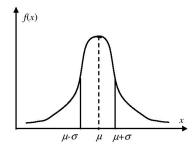


Figure 2-2 Normal distribution of measured data, x

As the error  $\varepsilon_x = x - \mu$  , we can obtain the distribution formula of measured error.

$$\psi(\varepsilon) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{\varepsilon^2}{2\sigma^2}} \tag{2-5}$$

where  $\psi(\varepsilon)$  is the probably density function of error. According to the normalization condition, the area under  $\psi(\varepsilon)$  curve is unity, which means the probability of error is 100% during confidence interval  $(-\infty, +\infty)$ . Obviously, smaller value of  $\sigma$ , more steep of  $\psi(\varepsilon)$  peak, indicating high concentration of random error (Fig. 2-3). It refers to small value of absolute error dominates random distribution, i.e., deviation of measured data is quite small and repetition is well.

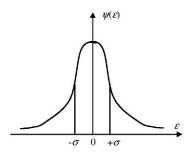


Figure 2-3 Normal distribution of random error

The probability of random error in the interval  $(-\sigma, +\sigma)$  can be calculated

$$p = \int_{-\sigma}^{+\sigma} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{\varepsilon^2}{2\sigma^2}} d\varepsilon = 0.683$$
 (2-6)

If a data distribution is approximately normal then about 68.3 percent of the data values are within one standard deviation of the mean (mathematically,  $\mu \pm \sigma$ , where  $\mu$  is the arithmetic mean), about 95 percent are within two standard deviations ( $\mu \pm 2\sigma$ ), and about 99.7 percent lie within three standard deviations ( $\mu \pm 3\sigma$ ). This is known as the 68-95-99.7 rule, or the empirical rule. For normal distribution,  $\sigma$ , corresponding to probability of 68.3 percent, can be calculated by dividing error limit  $3\sigma$  (probability is close to unity) by constant C=3. In this case, C is named as confidence factor for normal distribution.

#### 2.3.1.2 Property of Normal Distribution

From the random error distribution curve, one can see three properties of normal distribution

#### (1) Single peak

Probability density increase as approach to zero error, which indicates the probability of small error is higher than that of low error.

#### (2) Symmetry

The probabilities of positive and negative value of error are the same

#### (3) Bound limitation

Under proper experimental condition, absolute error will not break a specific region.

#### 2.3.1.3 Best Estimated True Value

As the true value can not be measured accurately, the mean value of repeated measurement is generally used as best estimated value.

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{2-7}$$

#### 2.3.1.4 Standard Deviation

From theoretical point of view, standard deviation for infinite repeated measurement can be expressed,

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2} \quad (n \to \infty)$$
 (2-8)

However, due to limited repetition during actual experiment, standard error can be estimated as

follow,

$$s_x = t_p \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$
 (2-9)

where Sx is called standard deviation.  $t_p$  is modified parameters varying with repetition times.

Table 2-2  $t_p$  as a function of repetition time

n-1	1	2	3	4	5	6	7	8	9	10	15	20	30	40	$\infty$
$t_P$	1.84	1.32	1.20	1.14	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.01	1

In fact, mean value,  $\bar{x}$ , random variable, is much closer to true value than any other measured dat. Its standard deviation is

$$S_{\overline{x}} = \frac{S_x}{\sqrt{n}} = t_p \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (x_i - \overline{x})^2}$$
 (2-10)

#### 2.3.2 Uniform Distribution

In probability theory and statistics, the continuous uniform distribution or rectangular distribution is a family of symmetric probability distributions such that for each member of the family, all intervals of the same length on the distribution's support are equally probable. The support is defined by the two parameters, -  $\Delta$  and +  $\Delta$ , which are its minimum and maximum values. The distribution is often abbreviated U(-  $\Delta$ , +  $\Delta$ ).

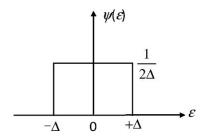


Figure 2-4 Uniform distribution of random error

The probability density function of the continuous uniform distribution is:

$$\psi(\varepsilon) = \begin{cases}
\frac{1}{2\Delta} & -\Delta \leqslant \varepsilon \leqslant +\Delta \\
0 & \varepsilon < -\Delta, \varepsilon > +\Delta
\end{cases}$$
(2-11)

where  $\Delta$  is error bound for normal distribution.

### 2.4 Uncertainty

In metrology, measurement uncertainty is a non-negative parameter characterizing the dispersion of the values attributed to a measured quantity. The uncertainty has a probabilistic basis and reflects incomplete knowledge of the quantity. All measurements are subject to uncertainty and a measured value is only complete if it is accompanied by a statement of the associated uncertainty. All measurements have some degree of uncertainty that may come from a variety of sources. The process of evaluating this uncertainty associated with a measurement result is often called uncertainty analysis or error analysis.

When reporting a measurement, the measured value should be reported along with an estimate of the total **combined standard uncertainty** of the value. The total uncertainty is found by combining the uncertainty components based on the two types of uncertainty analysis:

#### Type A

It refers to the method of evaluation of uncertainty by the statistical analysis of a series of observations. Type A components, labeled as  $u_A(x)$ , is the statistic distribution of error in multiple repeated measurement on the same experimental conditions. This method primarily includes random errors.

Type B refers to errors components arising from apparatus, environments and related reasons.

### 2.4.1 Uncertainty of Direct Measurement

#### 2.4.1.1 Type A Component

For a specific quantity, x, type A component of uncertainty for multiple measurement by n times equals its standard deviation

$$u_{A}(x) = s_{\bar{x}} = \frac{s_{x}}{\sqrt{n}} = t_{p} \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (x_{i} - \bar{x})^{2}}$$
(2-12)

#### 2.4.1.2 Type B Component

This method includes *systematic* errors and any other uncertainty factors that the experimenter believes are important. It is noted that in our physics experiment, type B component only refers to apparatus error and can be expressed as follows,

$$u_{\rm B}(x) = \frac{\Delta}{C} \tag{2-13}$$

where  $\Delta$  is the instrumental error, C is 3 when the apparatus error has a normal distribution and  $\sqrt{3}$  for uniform distribution.

Instrumental error of several common apparatus are given as follows,

- (1) Micrometer caliper.  $\Delta$ =0.004 mm for the scale range of 0~100 mm.
- (2) Vernier caliper. Error limitation equals to the division. For instance, for vernier caliper of 50 divisions,  $\Delta$ =0.02mm.
- (3) Metre ruler,  $\Delta$ =0.5mm.
- (4) Stopwatch. Accuracy of common stopwatch can be 0.01 s. However, considering human's response during handling stopwatch, error limitation is  $\Delta = 0.2$ s
- (5) Ammeter/Voltmeter.  $\Delta$ =Full Scale× a %, where a is the meter's accuracy class and often has seven different values.
- (6) Resistance box.  $\Delta$ =Reading value  $\times a$  %, where a is resistance box class. For the common resistance box, it has five classes. For instance, ZX21 has a class of 0.1 and ZX-35 has a class of 0.2.

Table 2-3 Histrumental error distribution								
Instruments	Micrometer	Vernier Caliper	Meter	Stopwatch	Ammeter			
Error	Normal, 0.004mm	Rectangular 0.02/0.05mm	Normal, 0.5mm	Normal, 0.2s	Rectangular Range ×a %			

Table 2-3 Instrumental error distribution

#### 2.4.1.2 Absolute Uncertainty

The total uncertainty for measured quantity x is the combination for A and B components of uncertainty

$$u_{\rm c}(x) = \sqrt{u_{\rm A}(x)^2 + u_{\rm B}(x)^2}$$
 (2-14)

### 2.4.1.2 Complete Express of Direct Measurement

$$\begin{cases} x = \overline{x} \pm u_{c}(x) \text{ (Unit)} \\ u_{r}(x) = \frac{u_{c}(x)}{\overline{x}} \times 100\% \end{cases}$$
 (2-15)

where  $u_c(x)$  is the absolute uncertainty,  $u_r(x)$  is relative uncertainty (percent uncertainty). The underlying physics of Eq. 2-15 is the probability of true value of x located in confidence interval  $\bar{x} \pm u_c(x)$  is 0.683.

### 2.4.2 Uncertainty of Indirect Measurement

We are not directly interested in a measured value, but we want to use it in a formula to calculate another quantity. In many cases, we measure many of the quantities in the formula and each has an associated uncertainty. We deal here with how to propagate uncertainties to obtain a well-defined uncertainty on a computed quantity. Take N = f(x, y, z) for example, N is an indirect measured quantity. x, y, z are the direct quantities, whose have respective absolute uncertainty  $x = \overline{x} \pm u_c(x)$ ,  $y = \overline{y} \pm u_c(y)$ ,  $z = \overline{z} \pm u_c(z)$ , where  $u_c(x)$ ,  $u_c(y)$   $u_c(z)$  are the related absolute uncertainty.

#### 2.4.2.1 Best Value of Indirect Measurement

The best estimated value for indirect measured quantity N is calculated by the mean value for direct quantities.

$$\overline{N} = f(\overline{x}, \overline{y}, \overline{z}) \tag{2-16}$$

#### 2.4.2.2 Absolute uncertainty of Indirect Measurement

The absolute uncertainty  $u_N$  can be expressed as follows,

$$u_{c}(N) = \sqrt{\left(\frac{\partial f}{\partial x}\right)^{2} u_{c}(x)^{2} + \left(\frac{\partial f}{\partial y}\right)^{2} u_{c}(y)^{2} + \left(\frac{\partial f}{\partial z}\right)^{2} u_{c}(z)^{2}}$$
(2-17)

#### 2.4.2.3 Percent Uncertainty of Indirect Measurement

The percent uncertainty is given by

$$u_{\rm r}(N) = \sqrt{\left(\frac{\partial \ln f}{\partial x}\right)^2 u_{\rm c}(x)^2 + \left(\frac{\partial \ln f}{\partial y}\right)^2 u_{\rm c}(y)^2 + \left(\frac{\partial \ln f}{\partial z}\right)^2 u_{\rm c}(z)^2}$$
(2-18)

#### 2.4.2.4 Complete Expression of Indirect Measurement

$$\begin{cases}
N = \overline{N} \pm u_{c}(N) \text{ (Unit)} \\
u_{r}(N) = \frac{u_{c}(N)}{\overline{N}} \times 100\%
\end{cases}$$
P=0.683 (2-19)

#### **NOTES**

(1) Generally,  $u_c(N)$  keeps one or two significant figures (when the first significant figure is 1 or 2 keep two, otherwise keep one).  $\overline{N}$  should keep the same accuracy with  $u_c(N)$ .  $u_r(N)$  remains one or two significant figures.

(2) When quantity is too large or too small, but with few significant figure, express form can be changed by multiply by  $10^k$ , where k is integer. For instance,  $0.00253\pm0.00002$  can be written as  $(2.53\pm0.02)\times10^{-3}$ .

### **Uncertainty and Significant Figures**

For the same reason that it is dishonest to report a result with more significant figures than are reliably known, the uncertainty value should also not be reported with excessive precision. For example, if we measure the density of copper, it would be unreasonable to report a result like: measured density =  $8.93 \pm 0.4753$  g/cm<sup>3</sup>.

The uncertainty in the measurement cannot be known to that precision. In most experimental work, the confidence in the uncertainty estimate is not much better than about 50 % because of all the various sources of error, none of which can be known exactly. Therefore, to be consistent with this large uncertainty in the uncertainty, the uncertainty value should be stated to only one significant figure (or perhaps 2 sig. figs. if the first digit is a 1). Experimental uncertainties should be rounded to one (or at most two) significant figures.

To help give a sense of the amount of confidence that can be placed in the standard deviation, the following table indicates the relative uncertainty associated with the standard deviation for various sample sizes. Note that in order for an uncertainty value to be reported to 3 significant figures, more than 10,000 readings would be required to justify this degree of precision.

## 3. Data Process

Data Processing refers to the procedure from raw experimental data to final result. To obtain a reliable and accurate result, it is essential to scientifically process and analyze original data. That is, data processing is integral part of physics experiment.

### 3.1 Table

Table is often used to arrange original data, which are recorded from equipment directly and without any mathematical process. With ordered arrangement of rows and columns, data table can facilitate data category and perform preliminary data process.

#### **Advantage of Table Method**

Table method has following advantages

- 1. Enhance the proficiency of data process, reduce or even avoid mistakes
- 2. Facilitate the check whether the measured data is reasonable
- 3. Exhibit directly and specifically the relationship between various quantities
- 4. Assist to find the various trends between related quantities.

#### **Requirements of Table Method**

- 1. Data table should have a specific name and number.
- 2. Each row and each column have intersected one cell.
- Row and column represents quantities or signal. Each signal should be labeled with unit, order.
- 4. Data recorded in the table should have correct significant figures.

## 3.2 Graphical Method

From an examination of the tabulated values of a number of measurements of related quantities, it is often difficult to grasp the real relationship existing between the numbers. A method widely used to discover such relationships is the graphical method, which gives a pictorial view of the results and makes it possible to interpret the data by a quick balance.

#### **Advantage of Graphical Method**

- (1) Directly display the relationship and varying trend between quantities. This property becomes more visible for the case whose function is unknown.
- (2) It can be straight to calculate the slop, intercept, differential, and integration (area), peak value or the value of some quantity via interpolation, extrapolation and asymptote, etc.

- (3) Curve plotted to present the trend of discrete points has the advantage to reduce random error and help to find and analyze systematic error.
- (4) It is very useful to build up empirical functions via trend between quantities displayed by graph.

#### **Graphical Method and rules**

#### (1) Selection of graph paper

It is essential to plot graph on the coordinate paper. Based on the requirement, various graph papers including rectangular Cartesian axes paper, logarithmic paper can be used.

#### (2) Choice of Scale

To complete a graph, the independent variable, is usually plotted on the abscissa scale (X axis) and the dependent variable is plotted on the ordinate scale (Y axis). Each axis has arrow at the end point.

Choose a size of graph that bears some relation to the accuracy with which the plotted data are known. In general, the curve should fill most of the sheet. It is important to check the significant figures of measured data when choosing proper scale and division of each axis. In principle, estimated digit, at the end of significant figure, should also be estimated on the graph sheet. This requires the smallest square represent the last reliable digit of significant figure. As a consequence, the accuracy does not change during graphing process.

Note the range of values of independent variable (X quantity), and the number of spaces along the X axis. Choose a scale for main divisions on the graph paper that are easily subdivided and such that the entire range of values may be included. Subdivisions such as 1, 2, 5, and 10 are best, but 4 is sometimes used; never use 3, 7, 9, because these make it very difficult to read values from the graph. The same procedures should be sued for the ordinate scale, but the divisions on the ordinate and abscissa scales need not be alike. In many cases, it is not necessary that the intersection of the two axes represent the zero value of both variables. If the values to be plotted are exceptionally large or small, use some multiplying factor that permits using a maximum of two or three digits to indicate the value of the main division. A multiplying factor such as  $\times 10^{-2}$  or  $\times 10^{-6}$  placed at the right of the largest value on the scale may be used.

#### (3) Labeling

After you have decided which variable is to be plotted on which axis, neatly letter the name of quantity being plotted together with proper unit. Abbreviate all units in standard form. Then write the numbers along main divisions on the graph paper., using an appropriate scale as explained in the preceding paragraph. The title should be neatly labeled on the body of the graph paper, but it is

usually best to do this after the points have been plotted so the title will not interfere with the curve. Explanatory legends should also be shown.

#### (4) Plotting Curve

Use a sharp pencil and make small dots to locate the points. DO NOT write the coordinates of the points on the graph paper. Carefully encircle each point with a circle about 2 or 3 mm in diameter. In drawing the graph it is not always possible to make all of the points lie on a smooth curve. In such cases, a smooth curve should be drawn through the series of points to follow the general trend and thus represent an average. Suppose the plotted points show a straight line trend. To draw the straight line which best represents the relationship which produced the series of points, proceed as follows. First, cover the lower half of the points and draw a faint sharp cross in the centroid of the points in the top half of series. Next cover the top half and mark a cross at the centroid of the lower group. Then draw a line straight through the two crosses and it will represent a true average. If the series of points appears to represent a function which is not a straight line, the points should lies on both sides of the curve along all parts of the curve.

When more than one curve is drawn on a graph, and it is desirable to distinguish the points associated with one curve from those associated with another, crosses ( $\times$ ), triangles ( $\Delta$ ), squares ( $\square$ ), and circles ( $\bigcirc$ ) may be used.

#### (5) Analysis and Interpretation of Graphs

One of the principal advantages afforded by graphical representation is the simplicity with which new information can be obtained directly from the graph by observing its shape and intercepts.

The sharp of a graph immediately tells one whether the dependent variable increases or decreases with an increase of the independent variable. It also shows something of the rate of change. If the points lie along a straight line, there is a linear relationship between the variables. If the variables are directly proportional to each other, they approach zero simultaneously, and the line passes through the origin. Curves which are straight line and do not pass through the origin do not indicate direct proportion.

In discussing the slope of a graph we must distinguish between physical slop and geometric slope. The geometric slope is usually the angular inclination of the line with respect to the X axis. In plotting physical data, there may be an enormous difference in the size of the units on the two axes. The physical slop is found by dividing  $\Delta y$  by  $\Delta x$ , using for each the scales and units that have been chosen for those axes. The unit of the slope will be the ratio of the units on the respective axes. With the physical slope, one is not usually concerned at all with the angle the line makes with the

X axis, the tangent of the angle has no meaning.

Significant information is often revealed y the intersections of the graph with the coordinate axes. This is true for other types of curves as well as for straight lines. The true interpretation of the intercept can be obtained only if the scales used begin at zero. In many cases, there are no data available for drawing the curve to the axes. If the plotted points indicate the trend of the curve, one may be justified in extrapolating the curve to the intercept desired.

*Extrapolation* is accompanied by extending the curve in the desired direction by a dotted line rather than a solid line, thus indicating that data are not available for this portion of the curve. Intercepts obtained by extrapolation my serve as clues to aid in the theoretical interpretation of the phenomena being observed.

## 3.3 Successive Subtracting Method

Successive subtracting method is used for the linear relationship between dependent variable, y, and independent variable, x, which varies with the same interval. The independent variable has even number data.

In physics experiment, it often uses one time successive subtracting method, that is, dependent variable, y, has a linear relationship with x.

$$y = a_0 + a_1 x$$

Suppose there are n group of data satisfy above condition (n is even number, set n = 2m, m is an integer), measured data are given as follows,

$$y_1 = a_0 + a_1 x, y_2 = a_0 + a_1 (2x), y_3 = a_0 + a_1 (3x), \dots, y_m = a_0 + a_1 (mx)$$
  
 $y_{m+1} = a_0 + a_1 (m+1)x, y_{m+2} = a_0 + a_1 (m+2)x, \dots, y_n = a_0 + a_1 (nx)$ 

Divide all the measured data by two groups following the consequence of independent variable, the number of data in each group is m. Subtract independent variable to its counterpart in the same order of another group, we have,

$$\delta_1 = y_{m+1} - y_1$$

$$\delta_2 = y_{m+2} - y_2$$

$$\vdots$$

$$\delta_m = y_n - y_m$$

 $\delta_i$  is variation of y when x has changed by m. Due to linear relationship,  $\delta_1 \sim \delta_m$  should be equal to each other from theoretical point of view.

$$\overline{\delta} = \frac{1}{m} \sum_{i=1}^{m} \delta_i$$

The mean value of variation of y corresponding to one x change on the independent variable is

$$\overline{\delta}_{y} = \frac{\overline{\delta}}{m} = \frac{1}{m^{2}} \sum_{i=1}^{m} \delta_{i}$$

With respect to estimation of uncertainty, type A of  $\delta$  is

$$u_{\rm A}(\delta) = s_{\bar{\delta}} = t_P \sqrt{\frac{1}{m(m-1)} \sum_{i=1}^{m} (\delta_i - \bar{\delta})^2}$$

Type B uncertainty of  $\delta$  is determined by instrumental error, Suppose the limitation of instrumental error is  $\Delta$ , followed by uniform distribution, B components of uncertainty is

$$u_{\rm B}(\delta) = \frac{\Delta}{\sqrt{3}}$$

Thus, the absolute uncertainty is written as

$$u_{\rm C}(\delta) = \sqrt{u_{\rm A}^2(\delta) + u_{\rm B}^2(\delta)}$$

From the uncertainty propagation formula, uncertainty of  $\delta_y$  can be expressed

$$u_{\rm C}(\delta_{\rm y}) = \frac{u_{\rm C}(\delta)}{m}$$

Take the Young's modulus's measurement of steel wire experiment for instance. The length variation of steel wire under external force is given in following table.

Table 3-1 length variation of steel wire as a function of loading

No	1	2	3	4	5	6	7	8
Loading (×9.8N)	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00
Length of wire (cm)	0.00	1.34	2.72	4.06	5.43	6.80	8.16	9.51

It is well known that the elongation of steel wire has linear relationship with external force. In actual experiment, one 1kg weight is used to stretch the wire for each time measurement, which makes sure length variation is the same for consecutive interval. In this case, n=8, m=4. We can divide eight data of wire length into two groups, i.e., the first four data forms one group and remaining four data for second group. After subtracting length data of second group by the first one in the same sequence, we can have,

$L_1$	$L_2$	$L_3$	$L_4$	$\overline{L}$ /cm
5.43	5.46	5.44	5.45	5.44

Type A uncertainty is  $(t_P=1.20)$ 

$$u_{\rm A}(L) = s_{\overline{L}} = t_P \sqrt{\frac{1}{4(4-1)} \sum_{i=1}^{4} (L_i - \overline{L})^2} = 0.009$$
 cm

As the meter ruler is used to measure the length, type B uncertainty for  $\overline{L}$ 

$$u_{\rm B}(L) = \frac{\Delta}{3} \approx 0.02$$
 cm

Thus, the absolute uncertainty is

$$u_{\rm C}(L) = \sqrt{u_{\rm A}^2(L) + u_{\rm B}^2(L)} = \sqrt{0.009^2 + 0.02^2} = 0.03 \text{ cm}$$

The mean value of elongation of steel wire under 1 N force is

$$\bar{l} = \frac{\overline{L}}{4 \times 9.8} = 0.139$$
 cm

The absolute uncertainty of  $\bar{l}$  is

$$u_{\rm C}(l) = \frac{u_{\rm C}(L)}{4 \times 9.8} = 0.001$$
 cm

Finally, complete expression of length variation of steel wire under 1 N force is as follows,

$$\begin{cases} l = 0.139 \pm 0.001 \text{ cm/N} \\ u_r(l) = \frac{0.001}{0.139} \times 100 \times \% = 0.72\% \end{cases}$$
 (p=0.683)

## 3.4 Linear Regression

If the relationship between two sets of data (x and y) is linear, when the data is plotted (y versus x) the result is a straight line. This relationship is known as having a **linear correlation** and follows the equation of a straight line y = mx + b. Below is an example of a sample data set and the plot of a "best-fit" straight line through the data.

If we expect a set of data to have a linear correlation, it is not necessary for us to plot the data in order to determine the constants m (slope) and b (y-intercept) of the equation y = mx + b. Instead, we can apply a statistical treatment known as **linear regression** to the data and determine these constants.

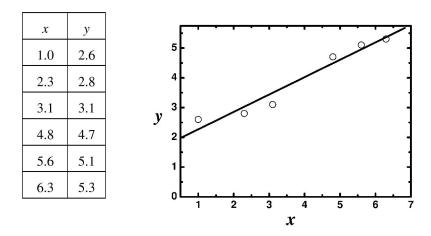


Figure 3-1 schematic graph of linear regression approach.

Given a set of data  $(x_i, y_i)$  with n data points, the slope and y-intercept can be determined using the following:

$$m = \frac{n\sum(xy) - \sum x\sum y}{n\sum(x^2) - (\sum x)^2} \equiv \frac{\overline{xy} - \overline{xy}}{\overline{x}^2 - \overline{x}^2}$$
(3-1)

$$b = \frac{\sum y - m \sum x}{n} \tag{3-2}$$

Note that the limits of the summation, which are i to n, and the summation indices on x and y have been omitted.

It is also possible to determine the correlation coefficient, r, which gives us a measure of the reliability of the linear relationship between the x and y values. A value of r = 1 indicates an exact linear relationship between x and y. Values of r close to 1 indicate excellent linear reliability. If the correlation coefficient is relatively far away from 1, the predictions based on the linear relationship, y = mx + b, will be less reliable.

Given a set of data  $(x_i, y_i)$  with n data points, the correlation coefficient, r, can be determined by

$$r = \frac{n\sum(xy) - \sum x\sum y}{\sqrt{[n\sum(x^2) - (\sum x)^2][n\sum(y^2) - (\sum y)^2]}}$$
(3-3)

## 4. General Apparatus

Before beginning the experiment, we would like to advise students to understand the operation rule of some general apparatus, such as vernier caliper, micrometer caliper, balance, stopwatch, etc.

## 4.1 Micrometer Caliper

A micrometer caliper sometimes known as a micrometer screw gauge, is a device incorporating a calibrated screw used widely for precise measurement of small distances in mechanical engineering and machining as well as most mechanical trades, along with other metrological instruments such as dial, vernier, and digital calipers. Micrometers are often, but not always, in the form of calipers.

Micrometer screw-gauge is one of most widely used micrometers used for measuring accurately the diameter of a thin wire or the thickness of a sheet of metal. It consists of a U-shaped frame fitted with a screwed spindle which is attached to a thimble.

A micrometer caliper is composed of following parts.

#### Frame

The C-shaped body holds the anvil and barrel in constant relation to each other. It is thick because it needs to minimize flexion, expansion, and contraction, which would distort the measurement. The frame is heavy and consequently has a high thermal mass, to prevent substantial heating up by the holding hand/fingers. It is often covered by insulating plastic plates which further reduce heat transference. Explanation: if you hold the frame long enough so that it heats up by 10°C, then the increase in length of any 10 cm linear piece of steel is of magnitude 1/100 mm. For micrometers this is their typical accuracy range. Micrometers typically have a temperature specified, at which the measurement is correct.

#### Anvil

The shiny part is face to spindle, and that the sample rests against.

#### Sleeve / barrel / stock

The stationary round part has the linear scale on it. Sometimes vernier markings.

#### Lock nut / lock-ring / thimble lock

The knurled part (or lever), one can tighten to hold the spindle stationary, such as when momentarily holding a measurement.

#### Screw

The heart of the micrometer. It is inside the barrel.

#### **Spindle**

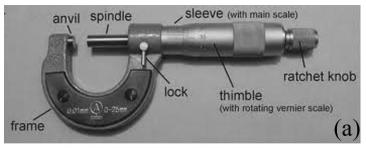
The shiny cylindrical part can be moved by the thimble toward the anvil.

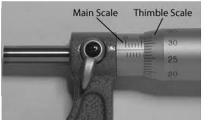
#### **Thimble**

The part that one's thumb turns. Graduated markings.

#### Ratchet stop

(not shown in illustration) Device on end of handle limits applied pressure by slipping at a calibrated torque.





(b)

Figure 4-1 (a) The schematic view of micrometer screw gauge and (b) the magnification view of reading part.

The screw has a known pitch such as 0.5 mm. Pitch of the screw is the distance moved by the spindle per revolution. Hence in this case, for one revolution of the screw the spindle moves forward or backward 0.5 mm. This movement of the spindle is shown on an engraved linear millimeter scale on the sleeve. On the thimble there is a circular scale which is divided into 50 or 100 equal parts.

When the anvil and spindle end are brought in contact, the edge of the circular scale should be at the zero of the sleeve (linear scale) and the zero of the circular scale should be opposite to the datum line of the sleeve. If the zero is not coinciding with the datum line, there will be a positive or negative zero error as shown in figure below.

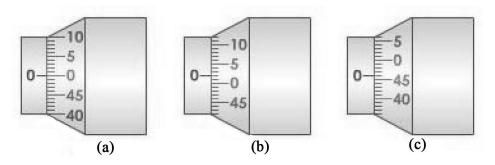


Figure 4-2 Reading error (a) non zero (b) negative zero error (c) positive error **How to read the micrometer screw gauge** 

As shown in Fig. 4-3, the spindle of an ordinary metric micrometer has 2 threads per millimeter, and thus one complete revolution moves the spindle through a distance of 0.5 millimeter. The longitudinal line on the frame is graduated with 1 millimeter divisions and 0.5 millimeter subdivisions. The thimble has 50 graduations, each being 0.01 millimeter (one-hundredth of a millimeter). Thus, the reading is given by the number of millimeter divisions visible on the scale of the sleeve plus the particular division on the thimble which coincides with the axial line on the sleeve.



Figure 4-3 The reading for micrometer screw gauge

- Choose the correct micrometer for your application, and set the screw on the object you are measuring. Make sure it is well touched but not over tightened. To make sure this, rotate knob instead of the thimble directly.
- 2. Use the micrometer's locking device to hold the spindle in place.
- 3. Remove the micrometer and read the linear gauge. This gauge will most likely be abbreviated so that "24" equals "0.24 inches" or something similar, depending on the micrometer you are using.

4. Using the horizontal line on the linear scale as a pointer, read the scale on the barrel. The number of increments on the barrel will depend on the size of the step between increments on the linear scale. For example, if the linear gauge is divided into quarter steps, the barrel will have 25 increments.

#### Combine the readings.

From Figure 4-3, the thimble was screwed out so that graduation 5, and one additional 0.5 subdivision were visible (as shown in the image), and that graduation 27 on the thimble coincided with the axial line on the sleeve. Please DO NOT forget the estimated digit, which is around 0.9 for this case. This gauge reads "27.9," which is an abbreviation for "0.279 mm". The combined reading, then, is "5.000+0.500+0.279=5.729 mm".

## 4.2 Vernier Caliper

The vernier caliper is a precision instrument that can be used to measure internal and external distances extremely accurately. The example shown below is a manual caliper. Measurements are interpreted from the scale by the user. This is more difficult than using a digital vernier caliper which has an LCD digital display on which the reading appears. The manual version has both an imperial and metric scale. Manually operated vernier calipers can still be bought and remain popular because they are much cheaper than the digital version. Also, the digital version requires a small battery whereas the manual version does not need any power source.

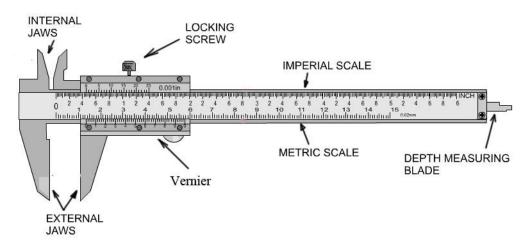


Figure 4-4 The configuration of the vernier caplier.

As shown in Fig. 4-4, a vernier caliper consists of the following eight parts

- 1. External jaws: used to measure external diameter or width of an object
- 2. Internal jaws: used to measure internal diameter of an object
- 3. Depth probe: used to measure depths of an object or a hole
- 4. Main scale: gives measurements of up to one decimal place(in cm).
- 5. Main scale: gives measurements in fraction(in inch)
- 6. Vernier gives measurements up to two decimal places(in cm)
- 7. Vernier gives measurements in fraction(in inch)
- 8. Locking screw: used to block movable part to allow the easy transferring a measurement

A caliper must be properly applied against the part in order to take the desired measurement. For example, when measuring the thickness of a plate a vernier caliper must be held at right angles to the piece. Some practice may be needed to measure round or irregular objects correctly. Accuracy of measurement when using a caliper is highly dependent on the skill of the operator. Regardless of type, a caliper's jaws must be forced into contact with the part being measured. As both part and caliper are always to some extent elastic, the amount of force used affects the indication. A consistent, firm touch is correct. Too much force results in an under indication as part and tool distort; too little force gives insufficient contact and an over indication. This is a greater problem with a caliper incorporating a wheel, which lends mechanical advantage. This is especially the case with digital calipers, calipers out of adjustment, or calipers with a poor quality beam.

Simple calipers are uncalibrated. The measurement taken must be compared against a scale. Whether the scale is part of the caliper or not, all analog calipers—verniers and dials—require good eyesight in order to achieve the highest precision. Digital calipers have the advantage in this area. Calibrated calipers may be mishandled, leading to loss of zero. When a calipers' jaws are fully closed, it should of course indicate zero. If it does not, it must be recalibrated or repaired. It might seem that a vernier caliper cannot get out of calibration but a drop or knock can be enough. Digital calipers have zero set buttons.

Vernier, dial and digital calipers can be used with accessories that extend their usefulness. Examples are a base that extends their usefulness as a depth gauge and a jaw attachment that allows measuring the center distance between holes. Since the 1970s a clever modification of the moveable jaw on the back side of any caliper allows for step or depth measurements in addition to external caliper measurements, in similar fashion to a universal micrometer.

For The external measurement (diameter) of a round section piece of steel is measured using a vernier caliper with metric scale.

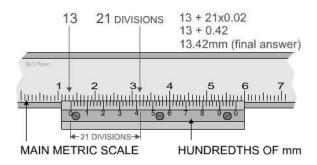


Figure 4-5 The reading diagram for the vernier caplier.

Figure 4-5 shows the configuration for the vernier caliper reading. The correct reading is composed of the following steps.

- 1. The main metric scale is read first and this shows that there are 13 whole divisions before the 0 on the vernier. Therefore, the first number is 13.
- 2. The vernier scale is then read. The best way to do this is to count the number of divisions until you get to the division that lines up with the main metric scale. This is 21 divisions.
- 3. This 21 is multiplied by 0.02 giving 0.42 as the answer (each division on the hundredths scale is equivalent to 0.02mm).
- 4. The 13 and the 0.42 are added together to give the final measurement of 13.42mm (the diameter of the piece of round section steel).

## 4.3 Electronic stopwatch

A **stopwatch** is a handheld timepiece designed to measure the amount of time elapsed from a particular time when activated to when the piece is deactivated. A large digital version of a stopwatch designed for viewing at a distance, as in a sports stadium, is called a **stopclock**. The timing functions are traditionally controlled by two buttons on the case. Pressing the top button starts the timer running, and pressing the button a second time stops it, leaving the elapsed time displayed. A press of the second button then resets the stopwatch to zero. The second button is also used to record *split times* or *lap times*. When the split time button is pressed while the watch is running, the display freezes, allowing the elapsed time to that point to be read, but the watch mechanism continues running to record total elapsed time. Pressing the split button a second time allows the watch to resume display of total time.

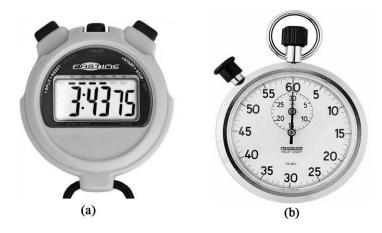


Figure 4-6 The photograph of (a) electronic stopwatch (b) Mechanical stopwatch

**Mechanical stopwatches** are powered by a mainspring, which must be periodically wound up by turning the knurled knob at the top of the watch.

**Digital electronic stopwatches** are available which, due to their crystal oscillator timing element, are much more accurate than mechanical timepieces. Because they contain a microchip, they often include date and time-of-day functions as well. Some may have a connector for external sensors, allowing the stopwatch to be triggered by external events, thus measuring elapsed time far more accurately than is possible by pressing the buttons with one's finger. The first digital timer used in organized sports was the digitimer, developed by Cox Electronic Systems, Inc. of Salt Lake City Utah (1971). It utilized a Nixie-tube readout and provided a resolution of 1/1000 second. Its first use was in ski racing, but was later used by the World University Games in Moscow, Russia, the U.S. NCAA, and in the Olympic trials.

The device is used when time periods must be measured precisely and with a minimum of complications. Laboratory experiments and sporting events like sprints are good examples.

The stopwatch function is also present as an additional function of many digital wristwatches, cell phones, and portable music players.

It is noted that the accuracy for the electronic stopwatch offered by the lab has an accuracy of 0.01s. Considering the time delay of human being when pushing the button of stopwatch, the device error is limited by 0.2s.

## 4.4 Digital Balance

In physics experiment, digital balance based on sensor and single chip technology, is widely used for the weight measurement. Various types of sensors such as strain, capacitance and electromagnetic balanced were used. The scale and increment are the two key parameters of

balance. In this experiment, JY strain senor electronic balance employing. The maximum capacity is 1000g and the smallest increment is 0.1g.

#### **Operation instructions**

Tune the bolt underneath the bottom of balance to make sure the air bubble located at the center.

Turn on the power, then balance goes to zero condition after few seconds self diagnose process then it is ready for measurement.

During the operation, for the non-zero initial case, push the button "reset/zeroing to let the balance go back to zero condition.



Figure 4-7 Photograph of electronic balance

#### **NOTES:**

- 1. The object to be measured should not be in excessive of the maximum capacity of the balance.
- 2. Place the object in the center of balance.
- 3. Keep dry of the surface of balance.

### 4.5 Resistance Box

A resistance box is a typically compact piece of equipment that contains multiple resistors hooked up to one or multiple switches and is designed to provide multiple electrical resistances. The primary benefit of having a compact way to alter electrical resistance is that it moves the need to actually change resistors or unit design just to change resistance. Many resistors can be set at many levels, providing immediate access to a combination of resistances, a possibility useful for many applications requiring electrical manipulation. The complexity of the resistance box will probably change based on the application, though many of these units are set to standard values.

One important aspect of resistance is the variety of available designs and functions. High-end resistance boxes may have tighter electrical ranges and a multitude of available resistances. More advance units may contain many swithes meant to be used separately or in combination to manage a current and provide that resistance to an outside source. A simple resistance box, on the other

hand, may only have a single switch or two, perhaps three, and may only be capable of providing standard resistance levels within less strict limitations. The overall design and functionality of a resistance box will, of course, depend on its functions.

Resistance boxes can be used in experimental, developmental and laboratory work. Generally, a resistance box is used to assist in the design and manufacturing of a circuit. During the configuration of the circuit, it might be necessary to test which resistance will have the best effect given certain circuit characteristics.



Figure 4-8 photograph of decade resistance box

A decade resistance box (As shown in Fig. 4-8), is widely used to alter the characteristics of laboratory circuits. A decade resistance box is piece of laboratory equipment that allows the user to dial in a precise amount of electrical resistance to be inserted into a circuit. For example, it might have a dial for 100s of Ohms, 10s of Ohms, and 1s of Ohms, and allow the user to set any value from 999 Ohms to 0 Ohms in 1 ohm steps. It often has several switch dials, each of which can switch from zero to nine. Scale of switch panels change by one order. Take a ZX-35 resistance box for instance, it is a six-dial decade resistance box. There are totally four contact terminals, one zero reading is used to circuit connection while the remaining three terminals represent different resistance output range  $(0.9, 9.9, \text{ and } 99999.9 \, \Omega)$ . It totally has six switch dials with different scale. When a decade resistance box is used, general instructions are given as follows

- 1. Select proper resistance range as needed, then, you can decide which terminal of scale range is required;
- 2. Before connecting to external circuit, make sure each switch is turn to "zero" reading, especially the switch dial with large scale, i.e, X 10000.
- 3. Connect zero-reading terminal and anther terminal with proper range to external circuit. The whole output resistance value can be determined by the sum of the reading of each dial multiplied by related scale.