Fundamentals of Electric Circuit 2020.5

Chapter 11
AC Power Analysis

Chapter 11 AC Power Analysis

11.1 Introduction

Our effort in ac circuit analysis so far has been focused mainly on calculating voltage and current. Our major concern in this chapter is power analysis.

Power analysis is very important. Power is the most important quantity in electric utilities, electronic, and communication systems, because such systems involve transmission of power from one point to another.

11.1 Introduction

Also, every industrial and household electrical device—every fan, motor, lamp, pressing iron, TV, personal computer—has a power rating that indicates how much power the equipment requires; exceeding the power rating can do permanent damage to an appliance.

The most common form of electric power is 50- or 60-Hz ac power. The choice of ac over dc allowed high-voltage power transmission from the power generating plant to the consumer.

11.2 INSTANTANEOUS AND AVERAGE POWER

The instantaneous power $p(t)$ absorbed by an element is the product of the instantaneous voltage $v(t)$ across the element and the instantaneous current $i(t)$ through it.

$$\boxed{p(t) = v(t)i(t)}$$

The instantaneous power is the power at any instant of time.

$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

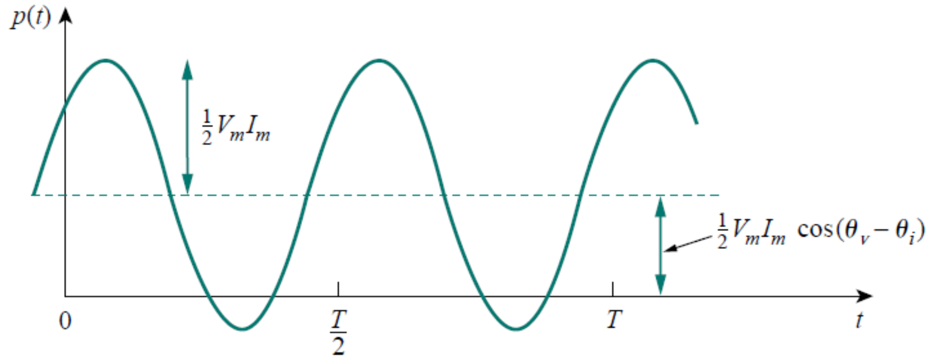
11.2 INSTANTANEOUS AND AVERAGE POWER

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

The instantaneous power has two parts.

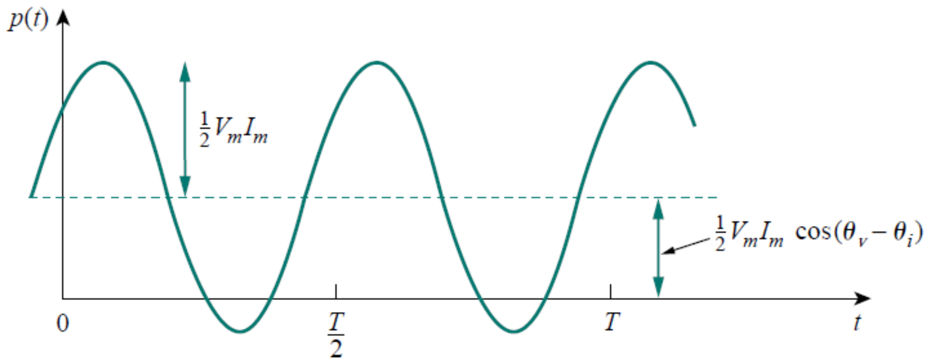
The first part is constant or time independent. Its value depends on the phase difference between the voltage and the current.

The second part is a sinusoidal function whose frequency is 2ω, which is twice the angular frequency of the voltage or current.



11.2 INSTANTANEOUS AND AVERAGE POWER

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$



We observe that $p(t)$ is periodic, $p(t) = p(t + T_0)$, and has a period of $T_0 = T/2$, since its frequency is twice that of voltage or current.

When p(t) is positive, power is absorbed by the circuit. When p(t) is negative, power is absorbed by the source; that is, power is transferred from the circuit to the source.

This is possible because of the storage elements (capacitors and inductors) in the circuit.

11.2 INSTANTANEOUS AND AVERAGE POWER

The instantaneous power changes with time and is therefore difficult to measure. The average power is more convenient to measure.

The average power is the average of the instantaneous power over one period.

$$P = \frac{1}{T}\int_0^T p(t)\,dt \qquad p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$P = \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\,dt + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt + \frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \qquad \mathbf{V} = V_m \underline{/\theta_v} \qquad \mathbf{I} = I_m \underline{/\theta_i}$$

$$\frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2} V_m I_m \underline{/\theta_v - \theta_i}$$

$$= \frac{1}{2} V_m I_m [\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i)]$$

$$\boxed{P = \frac{1}{2} \mathrm{Re}\left[\mathbf{V}\mathbf{I}^*\right] = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}$$

When $\theta_v = \theta_i$,

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} |\mathbf{I}|^2 R$$

a purely resistive circuit absorbs power at all times

When $\theta_v - \theta_i = \pm 90^\circ$,

$$P = \frac{1}{2} V_m I_m \cos 90^\circ = 0$$

a purely reactive circuit absorbs no average power

A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power.

EXAMPLE 11.1

Given that

$v(t) = 120\cos(377t + 45^\circ)$ V and $i(t) = 10\cos(377t - 10^\circ)$ A

find the instantaneous power and the average power absorbed by the passive linear network of Fig. 11.1.

Solution:

The instantaneous power is given by

$$p = vi = 1200\cos(377t + 45^\circ)\cos(377t - 10^\circ)$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

$$p = 600[\cos(754t + 35^\circ) + \cos 55^\circ]$$

$$p(t) = 344.2 + 600\cos(754t + 35^\circ) \text{ W}$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}120(10)\cos[45^\circ - (-10^\circ)]$$

$$= 600\cos 55^\circ = 344.2 \text{ W}$$

EXAMPLE 11.2

Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70\ \Omega$ when a voltage $\mathbf{V} = 120\underline{/0^\circ}$ is applied across it.

Solution:

The current through the impedance is

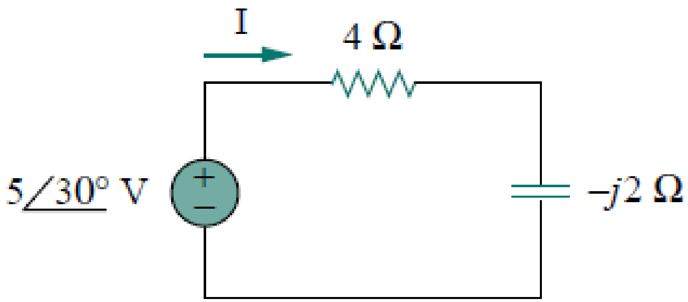
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{120\underline{/0^\circ}}{30 - j70} = \frac{120\underline{/0^\circ}}{76.16\underline{/-66.8^\circ}} = 1.576\underline{/66.8^\circ}\ \text{A}$$

The average power is

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(1.576)\cos(0 - 66.8^\circ) = 37.24\ \text{W}$$

EXAMPLE 11.3

For the circuit shown in Fig. 11.3, find the average power supplied by the source and the average power absorbed by the resistor.



Solution:

The current **I** is given by

$$\mathbf{I} = \frac{5\angle 30^\circ}{4 - j2} = \frac{5\angle 30^\circ}{4.472\angle -26.57^\circ} = 1.118\angle 56.57^\circ \text{ A}$$

The average power supplied by the voltage source is

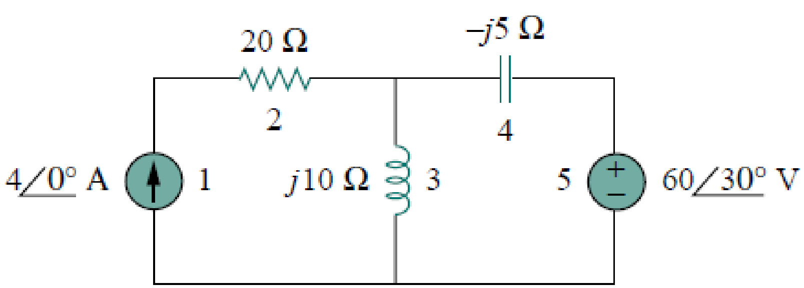
$$P = \frac{1}{2}(5)(1.118)\cos(30^\circ - 56.57^\circ) = 2.5 \text{ W}$$

$$\mathbf{I} = \mathbf{I}_R = 1.118\angle 56.57^\circ \text{ A} \qquad \mathbf{V}_R = 4\mathbf{I}_R = 4.472\angle 56.57^\circ \text{ V}$$

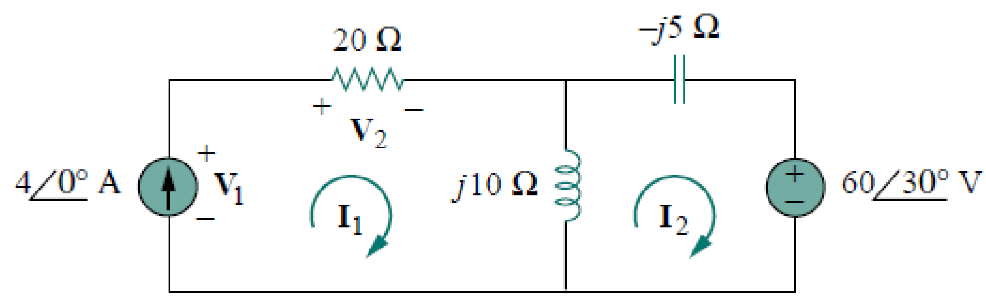
$$P = \frac{1}{2}(4.472)(1.118) = 2.5 \text{ W}$$

EXAMPLE 11.4

Determine the power generated by each source and the average power absorbed by each passive element in the circuit of Fig. 11.5(a).



(a)



(b)

Solution:

We apply mesh analysis as shown in Fig. 11.5(b). For mesh 1,

$$\mathbf{I}_1 = 4 \text{ A}$$

For mesh 2,

$$(j10 - j5)\mathbf{I}_2 - j10\mathbf{I}_1 + 60\angle 30^\circ = 0, \qquad \mathbf{I}_1 = 4 \text{ A}$$

$$\mathbf{I}_2 = -12\angle -60^\circ + 8$$
$$= 10.58\angle 79.1^\circ \text{ A}$$

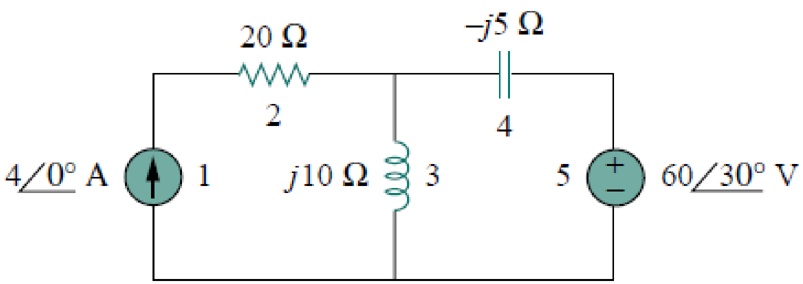
For the voltage source,

$$P_5 = \frac{1}{2}(60)(10.58)\cos(30^\circ - 79.1^\circ) = 207.8 \text{ W}$$

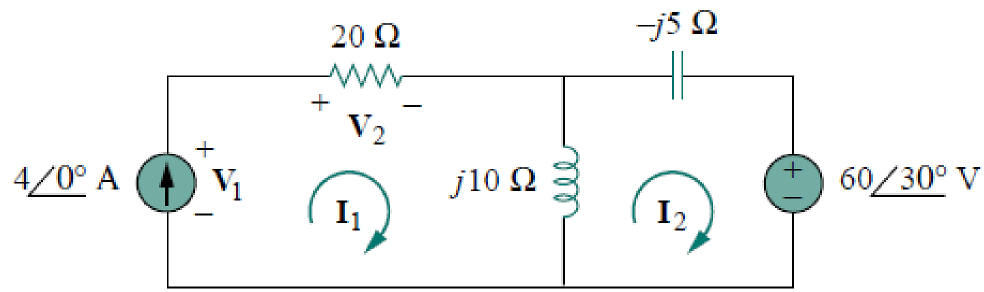
$$P_1 + P_2 + P_3 + P_4 + P_5 = -367.8 + 160 + 0 + 0 + 207.8 = 0$$

EXAMPLE 11.4

Determine the power generated by each source and the average power absorbed by each passive element in the circuit of Fig. 11.5(a).



(a)



(b)

For the current source, the current through it is $\mathbf{I}_1 = 4\angle 0^\circ$ and the voltage across it is

$$\begin{aligned}\mathbf{V}_1 = 20\mathbf{I}_1 + j10(\mathbf{I}_1 - \mathbf{I}_2) &= 80 + j10(4 - 2 - j10.39)\\ &= 183.9 + j20 = 184.984\angle 6.21^\circ \text{ V}\end{aligned}$$

The average power supplied by the current source is

$$P_1 = -\frac{1}{2}(184.984)(4)\cos(6.21^\circ - 0) = -367.8 \text{ W}$$

For the resistor,

$$P_2 = \frac{1}{2}(80)(4) = 160 \text{ W}$$

For the capacitor,

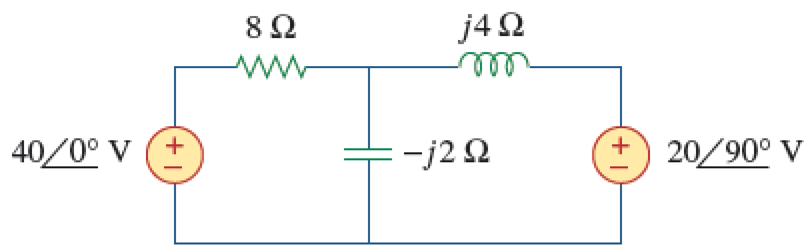
$$P_4 = \frac{1}{2}(52.9)(10.58)\cos(-90^\circ) = 0$$

For the inductor,

$$P_3 = \frac{1}{2}(105.8)(10.58)\cos 90^\circ = 0$$

Practice Problem 11.4

Calculate the average power absorbed by each of the five elements in the circuit of Fig. 11.6.

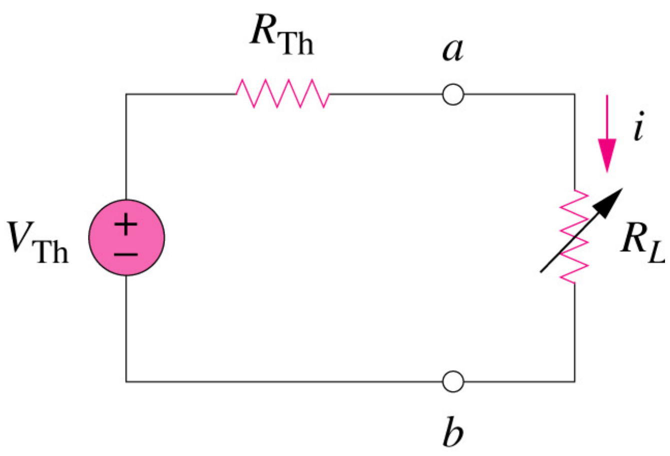


Answer: 40-V Voltage source: -60 W; $j20$-V Voltage source: -40 W; resistor: 100 W; others: 0 W.

11.3 MAXIMUM AVERAGE POWER TRANSFER

DC circuits

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).



$$\boxed{R_L = R_{Th}}$$

$$\boxed{p_{max} = \frac{V_{Th}^2}{4R_{Th}}}$$

EXAMPLE 4.13

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

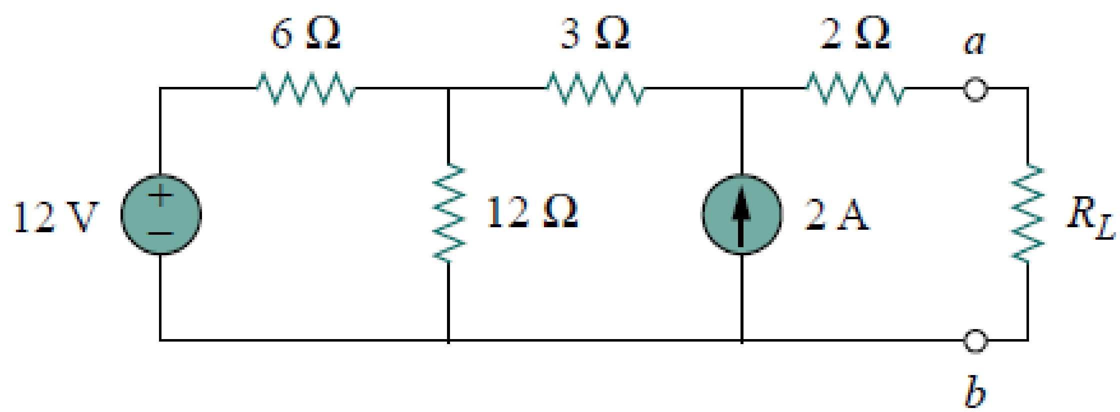
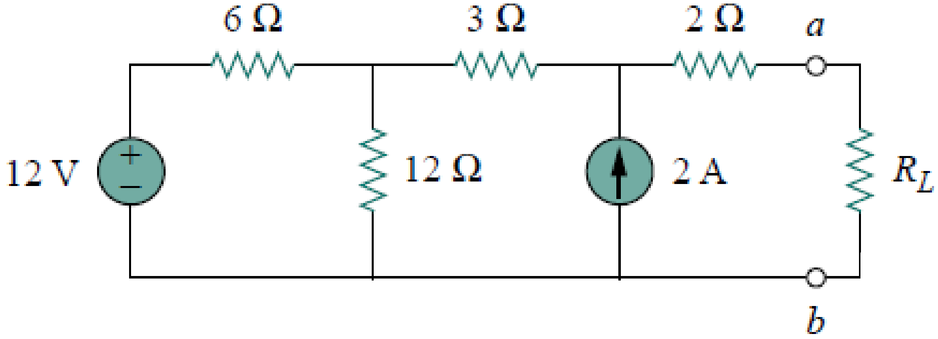
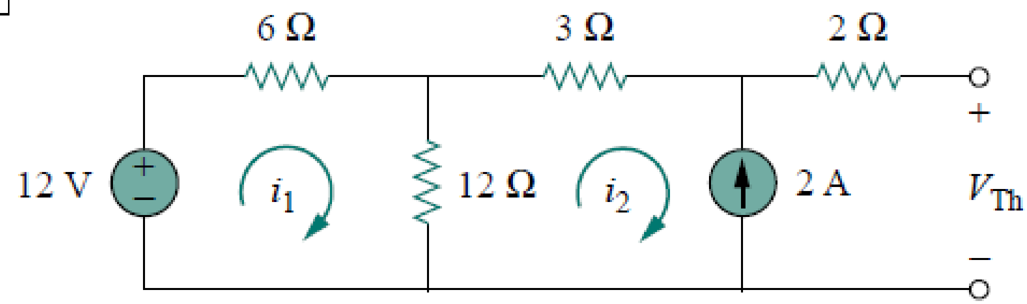


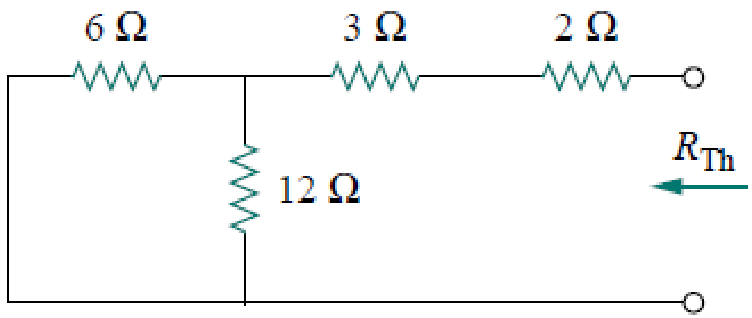
Figure 4.50 For Example 4.13.

Solution:

We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals a-b. To get R_{Th}, we use the circuit in Fig. 4.51(a) and obtain







(a)

$$R_{Th} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

$$-12 + 18i_1 - 12i_2 = 0,$$

$$i_2 = -2\ \text{A}$$

$$V_{Th} = 22\ \text{V}$$

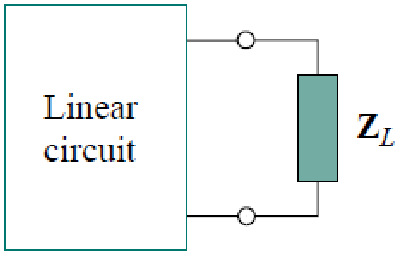
For maximum power transfer,

$$R_L = R_{Th} = 9\ \Omega$$

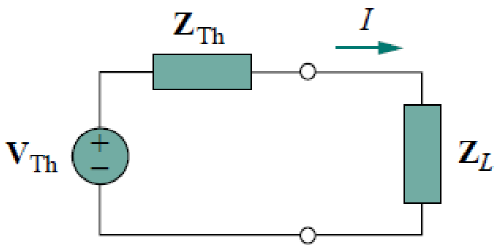
$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\ \text{W}$$

11.3 MAXIMUM AVERAGE POWER TRANSFER

AC circuits



(a)



(b)

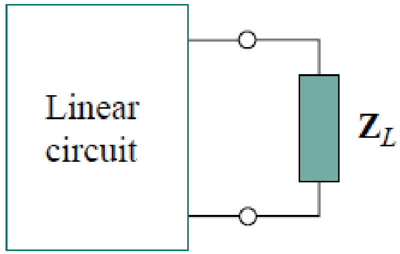
$$\mathbf{Z}_L = R_L + jX_L \qquad \mathbf{Z}_{Th} = R_{Th} + jX_{Th}$$

$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} = \frac{\mathbf{V}_{Th}}{(R_{Th} + jX_{Th}) + (R_L + jX_L)}$$

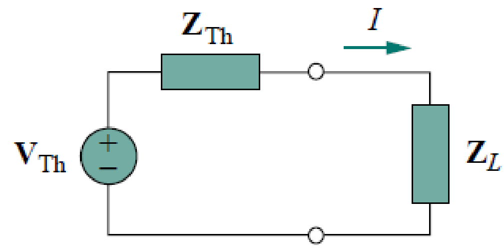
$$P = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{|\mathbf{V}_{Th}|^2 R_L/2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

Our objective is to adjust the load parameters R_L and X_L so that P is maximum. To do this we set $\partial P/\partial R_L$ and $\partial P/\partial X_L$ equal to zero. From

11.3 MAXIMUM AVERAGE POWER TRANSFER



(a)



(b)

$$\frac{\partial P}{\partial X_L} = -\frac{|\mathbf{V}_{\mathrm{Th}}|^2 R_L (X_{\mathrm{Th}} + X_L)}{[(R_{\mathrm{Th}} + R_L)^2 + (X_{\mathrm{Th}} + X_L)^2]^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{|\mathbf{V}_{\mathrm{Th}}|^2[(R_{\mathrm{Th}} + R_L)^2 + (X_{\mathrm{Th}} + X_L)^2 - 2R_L(R_{\mathrm{Th}} + R_L)]}{2[(R_{\mathrm{Th}} + R_L)^2 + (X_{\mathrm{Th}} + X_L)^2]^2}$$

Setting $\partial P/\partial X_L$ to zero gives

$$X_L = -X_{\mathrm{Th}}$$

and setting $\partial P/\partial R_L$ to zero results in

$$R_L = \sqrt{R_{\mathrm{Th}}^2 + (X_{\mathrm{Th}} + X_L)^2}$$

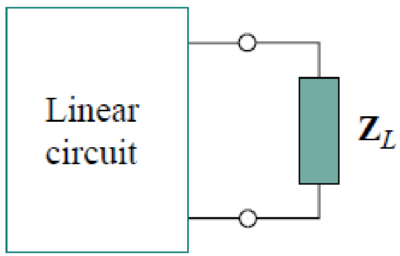
$$R_L = R_{\mathrm{Th}}$$

$$\boxed{\mathbf{Z}_L = R_L + jX_L = R_{\mathrm{Th}} - jX_{\mathrm{Th}} = \mathbf{Z}_{\mathrm{Th}}^*}$$

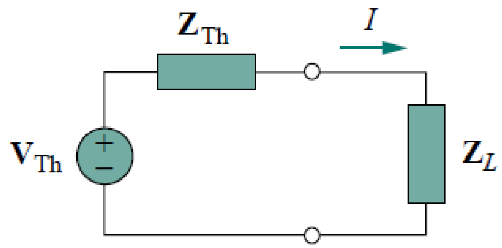
$$\boxed{P_{\max} = \frac{|\mathbf{V}_{\mathrm{Th}}|^2}{8R_{\mathrm{Th}}}}$$

For maximum average power transfer, the load impedance $\mathbf{Z}_L$ must be equal to the complex conjugate of the Thevenin impedance $\mathbf{Z}_{\mathrm{Th}}$.

11.3 MAXIMUM AVERAGE POWER TRANSFER



(a)



(b)

$$\boxed{\mathbf{Z}_L = R_L + jX_L = R_{\text{Th}} - jX_{\text{Th}} = \mathbf{Z}_{\text{Th}}^*}$$

$$\boxed{P_{\max} = \frac{|\mathbf{V}_{\text{Th}}|^2}{8R_{\text{Th}}}}$$

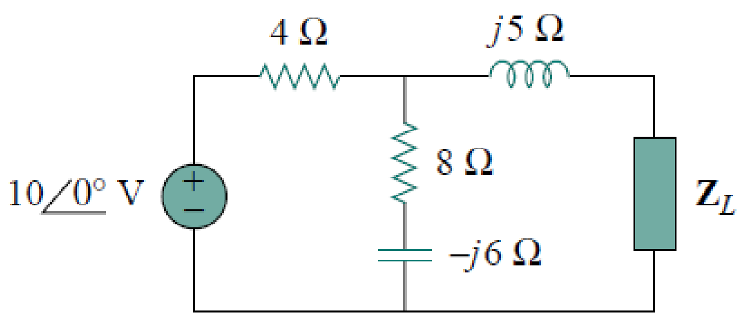
$X_L = 0$ the load is purely real（a purely resistive load）

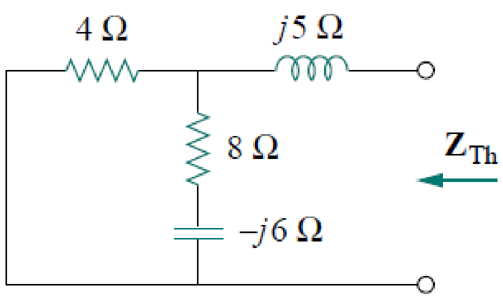
and setting $\partial P/\partial R_L$ to zero results in

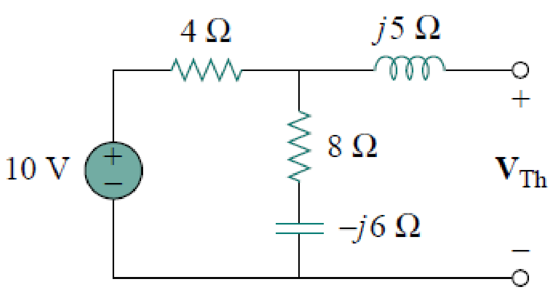
$$R_L = \sqrt{R_{\text{Th}}^2 + (X_{\text{Th}} + X_L)^2} \qquad R_L = \sqrt{R_{\text{Th}}^2 + X_{\text{Th}}^2} = |\mathbf{Z}_{\text{Th}}|$$

This means that for maximum average power transfer to a purely resistive load, the load impedance (or resistance) is equal to the magnitude of the Thevenin impedance.

Example 11.5 Determine the load impedance $\mathbf{Z}_L$ that maximizes the average power drawn from the circuit of Fig. 11.8. What is the maximum average power?







Solution:

First we obtain the Thevenin equivalent at the load terminals. To get $\mathbf{Z}_{Th}$, consider the circuit shown in Fig. 11.9(a). We find

$$\mathbf{Z}_{Th} = j5 + 4 \parallel (8 - j6) = j5 + \frac{4(8 - j6)}{4 + 8 - j6} = 2.933 + j4.467\ \Omega$$

To find $\mathbf{V}_{Th}$, consider the circuit in Fig. 11.8(b). By voltage division,

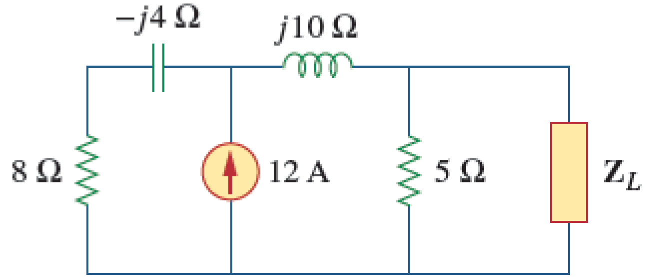
$$\mathbf{V}_{Th} = \frac{8 - j6}{4 + 8 - j6}(10) = 7.454\underline{/-10.3^\circ}\ \text{V}$$

The load impedance draws the maximum power from the circuit when

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = 2.933 - j4.467\ \Omega \qquad P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}} = \frac{(7.454)^2}{8(2.933)} = 2.368\ \text{W}$$

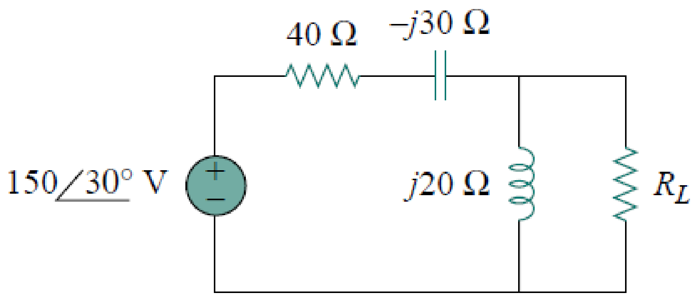
Practice Problem 11.5

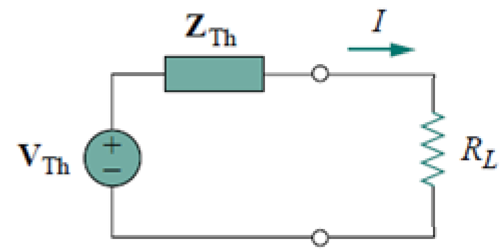
For the circuit shown in Fig. 11.10, find the load impedance $\mathbf{Z}_L$ that absorbs the maximum average power. Calculate that maximum average power.



Answer: $3.415 - j0.7317\ \Omega$, 51.47 W.

Example 11.6 In the circuit in Fig. 11.11, find the value of R_L that will absorb the maximum average power. Calculate that power.





Solution:

We first find the Thevenin equivalent at the terminals of R_L.

$$\mathbf{Z}_{Th} = (40 - j30) \parallel j20 = \frac{j20(40 - j30)}{j20 + 40 - j30} = 9.412 + j22.35\ \Omega$$

By voltage division,

$$\mathbf{V}_{Th} = \frac{j20}{j20 + 40 - j30}(150\angle 30^\circ) = 72.76\angle 134^\circ\ \text{V}$$

The value of R_L that will absorb the maximum average power is

$$R_L = |\mathbf{Z}_{Th}| = \sqrt{9.412^2 + 22.35^2} = 24.25\ \Omega$$

The current through the load is $$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + R_L} = \frac{72.76\angle 134^\circ}{33.39 + j22.35} = 1.8\angle 100.2^\circ\ \text{A}$$

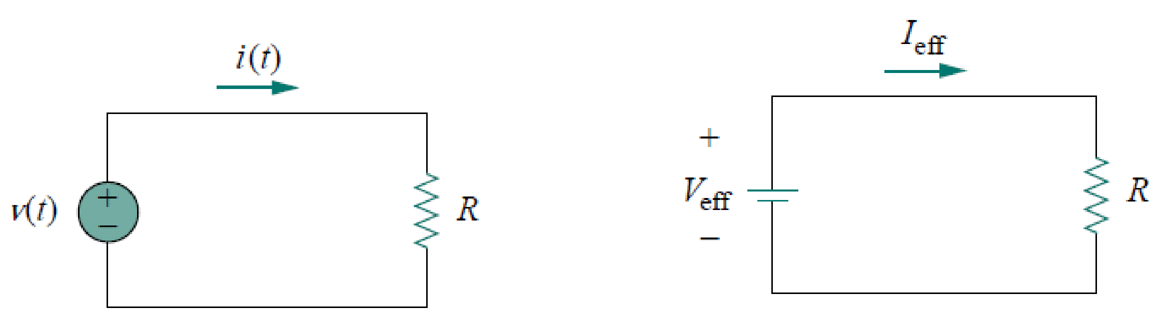
The maximum average power absorbed by R_L is $$P_{max} = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{1}{2}(1.8)^2(24.25) = 39.29\ \text{W}$$

11.4 EFFECTIVE OR RMS VALUE

- The idea of effective value arises from the need to measure the effectiveness of a voltage or current source in delivering power to a resistive load.

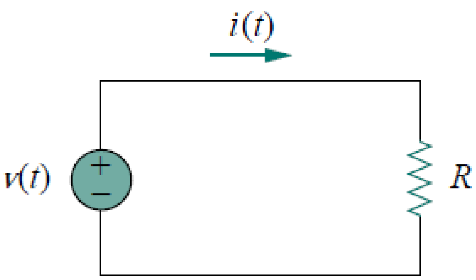
> The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

In Fig. 11.13, the circuit in (a) is ac while that of (b) is dc. Our objective is to find I_{eff} that will transfer the same power to resistor R as the sinusoid i. The average power absorbed by the resistor in the ac circuit is

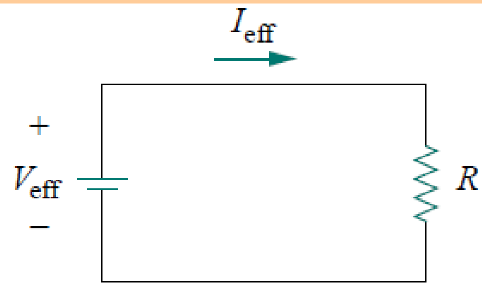


(a) ac circuit, (b) dc circuit.

11.4 EFFECTIVE OR RMS VALUE



(a) ac circuit,



(b) dc circuit.

$$P = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{R}{T}\int_0^T i^2\,dt \qquad P = I_{\text{eff}}^2 R$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} \qquad V_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

the effective value is the (square) *root* of the *mean* (or average) of the *square* of the periodic signal.

the *root-mean-square* value, or *rms* value $I_{\text{eff}} = I_{\text{rms}}, \qquad V_{\text{eff}} = V_{\text{rms}}$

> The effective value of a periodic signal is its root mean square (rms) value.

For any periodic function *x(t)* in general, the rms value is given by

$$\boxed{X_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2\,dt}}$$

For the sinusoid $i(t) = I_m \cos \omega t$, the effective or rms value is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t \, dt} \quad = \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t)\, dt} = \frac{I_m}{\sqrt{2}}$$

Similarly, for $v(t) = V_m \cos \omega t$, $\quad V_{\text{rms}} = \dfrac{V_m}{\sqrt{2}}$

The average power can be written in terms of the rms values.

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i) = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

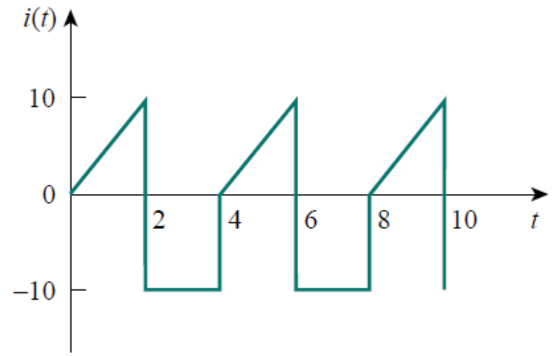
Similarly, the average power absorbed by a resistor R can be written as

$$P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

- When a sinusoidal voltage or current is specified, it is often in terms of its maximum (or peak) value or its rms value, since its average value is zero.
- The power industries specify phasor magnitudes in terms of their rms values rather than peak values.
- For instance, the 220 V available at every household is the rms value of the voltage from the power company.

EXAMPLE 11.7

Determine the rms value of the current waveform in Fig. 11.14. If the current is passed through a 2-Ω resistor, find the average power absorbed by the resistor.



Solution:

The period of the waveform is $T = 4$. Over a period, we can write the current waveform as

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

The rms value is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (-10)^2\,dt\right]}$$

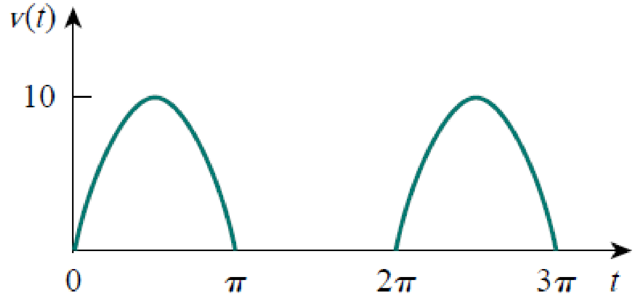
$$= \sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\bigg|_0^2 + 100t\bigg|_2^4\right]} = \sqrt{\frac{1}{4}\left(\frac{200}{3} + 200\right)} = 8.165 \text{ A}$$

The power absorbed by a 2-Ω resistor is

$$P = I_{\text{rms}}^2 R = (8.165)^2(2) = 133.3 \text{ W}$$

EXAMPLE 11.8

The waveform shown in Fig. 11.16 is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a 10-Ω resistor.



Solution:

The period of the voltage waveform is $T = 2\pi$, and

$$v(t) = \begin{cases} 10 \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

The rms value is obtained as

$$V_{\text{rms}}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{2\pi}\left[\int_0^\pi (10\sin t)^2\,dt + \int_\pi^{2\pi} 0^2\,dt\right] = \frac{50}{2\pi}\left(\pi - \frac{1}{2}\sin 2\pi - 0\right) = 25,$$

The average power absorbed is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{5^2}{10} = 2.5\text{ W}$$

11.5 APPARENT POWER AND POWER FACTOR

$$v(t) = V_m \cos(\omega t + \theta_v) \qquad \text{and} \qquad i(t) = I_m \cos(\omega t + \theta_i)$$

or, in phasor form, $\mathbf{V} = V_m \underline{/\theta_v}$ and $\mathbf{I} = I_m \underline{/\theta_i}$, the average power is $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

$$\boxed{S = V_{\text{rms}} I_{\text{rms}}}$$

- The average power is a product of two terms. The product $V_{\text{rms}} \times I_{\text{rms}}$ is known as the *apparent power S*. The factor $\cos(\theta_v - \theta_i)$ is called the *power factor* (pf).

The apparent power (in VA) is the product of the rms values of voltage and current.

$$\boxed{\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)}$$

The power factor is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedance.

The angle $\theta_v - \theta_i$ is called the *power factor angle*,

11.5 APPARENT POWER AND POWER FACTOR

$$\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

- For a purely resistive load, the voltage and current are in phase, so that $\theta_v - \theta_i = 0$ and pf = 1. This implies that the apparent power is equal to the average power.
- For a purely reactive load, $\theta_v - \theta_i = \pm 90^\circ$ and pf = 0. In this case the average power is zero.
- In between these two extreme cases, pf is said to be leading or lagging.
- Leading power factor means that current leads voltage, which implies a capacitive load.
- Lagging power factor means that current lags voltage, implying an inductive load.

EXAMPLE 11.9

A series-connected load draws a current $i(t) = 4\cos(100\pi t + 10°)$ A when the applied voltage is $v(t) = 120\cos(100\pi t - 20°)$ V. Find the apparent power and the power factor of the load. Determine the element values that form the series-connected load.

Solution:

The apparent power is

$$S = V_{\text{rms}} I_{\text{rms}} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240 \text{ VA}$$

The power factor is

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos(-20° - 10°) = 0.866 \qquad \text{(leading)}$$

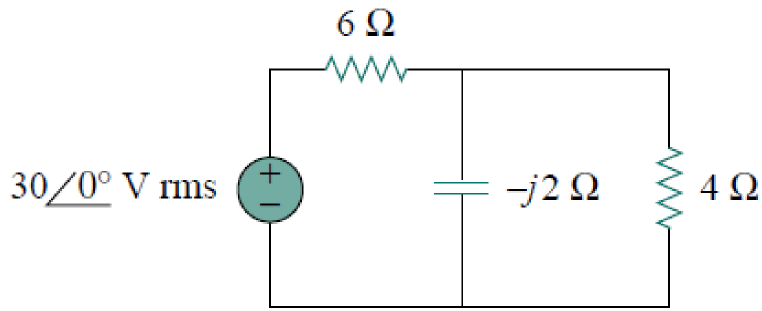
The pf is leading because the current leads the voltage.

The load impedance **Z** can be modeled by a 25.98-Ω resistor in series with a capacitor with

$$X_C = -15 = -\frac{1}{\omega C} \qquad C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2\ \mu\text{F}$$

EXAMPLE 11.10

Determine the power factor of the entire circuit of Fig. 11.18 as seen by the source. Calculate the average power delivered by the source.



Solution:

The total impedance is

$$\mathbf{Z} = 6 + 4 \parallel (-j2) = 6 + \frac{-j2 \times 4}{4 - j2} = 6.8 - j1.6 = 7\angle{-13.24}\ \Omega$$

The power factor is

$$\text{pf} = \cos(-13.24) = 0.9734 \qquad \text{(leading)} \qquad \text{since the impedance is capacitive.}$$

$$\mathbf{I}_{\text{rms}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{Z}} = \frac{30\angle{0^\circ}}{7\angle{-13.24^\circ}} = 4.286\angle{13.24^\circ}\ \text{A}$$

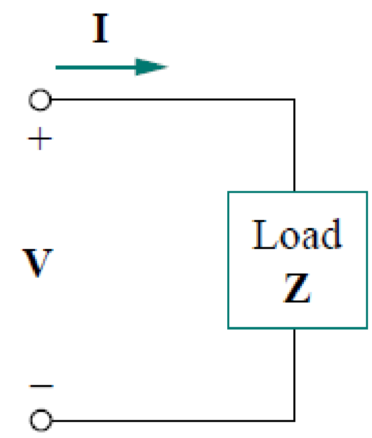
The average power supplied by the source is $P = V_{\text{rms}} I_{\text{rms}}\, \text{pf} = (30)(4.286)0.9734 = 125\ \text{W}$

$$P = I_{\text{rms}}^2 R = (4.286)^2(6.8) = 125\ \text{W}$$

11.6 COMPLEX POWER

- Considerable effort has been expended over the years to express power relations as simply as possible. Power engineers have coined the term *complex power*, which they use to find the total effect of parallel loads.
- Complex power is important in power analysis because it contains *all* the information pertaining to the power absorbed by a given load.

$$\mathbf{V} = V_m \underline{/\theta_v} \text{ and } \mathbf{I} = I_m \underline{/\theta_i},$$



the *complex power* S absorbed by the ac load is the product of the voltage and the complex conjugate of the current, $\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^*$

In terms of the rms values, $\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}}$

$$\mathbf{V}_{\text{rms}} = \frac{\mathbf{V}}{\sqrt{2}} = V_{\text{rms}} \underline{/\theta_v}$$

$$\mathbf{I}_{\text{rms}} = \frac{\mathbf{I}}{\sqrt{2}} = I_{\text{rms}} \underline{/\theta_i}$$

$$\mathbf{S} = V_{\text{rms}} I_{\text{rms}} \underline{/\theta_v - \theta_i}$$

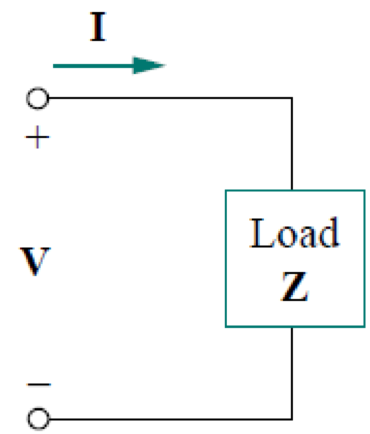
$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) + j V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

11.6 COMPLEX POWER

$$\mathbf{V} = V_m \angle \theta_v \text{ and } \mathbf{I} = I_m \angle \theta_i,$$

$$\mathbf{V}_{\text{rms}} = \frac{\mathbf{V}}{\sqrt{2}} = V_{\text{rms}} \angle \theta_v$$

$$\mathbf{I}_{\text{rms}} = \frac{\mathbf{I}}{\sqrt{2}} = I_{\text{rms}} \angle \theta_i$$



$$\mathbf{S} = V_{\text{rms}} I_{\text{rms}} \angle \theta_v - \theta_i$$
$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) + j V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

$$\mathbf{S} = I_{\text{rms}}^2 (R + jX) = P + jQ$$

$$P = \text{Re}(\mathbf{S}) = I_{\text{rms}}^2 R$$
$$Q = \text{Im}(\mathbf{S}) = I_{\text{rms}}^2 X$$

P is the average or real power and it depends on the load's resistance R. Q depends on the load's reactance X and is called the *reactive* (or quadrature) power.

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i), \qquad Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

The real power P is the average power in watts delivered to a load; it is the only useful power. It is the actual power dissipated by the load. The reactive power Q is a measure of the energy exchange between the source and the reactive part of the load. The unit of Q is the *volt-ampere reactive* (VAR) to distinguish it from the real power, whose unit is the watt.

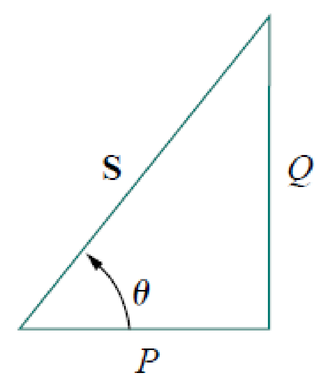
$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i), \qquad Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).

> Complex power (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity, its real part is real power P and its imaginary part is reactive power Q.

Introducing the complex power enables us to obtain the real and reactive powers directly from voltage and current phasors.

$$\begin{aligned}
\text{Complex Power} &= \mathbf{S} = P + jQ = \frac{1}{2}\mathbf{V}\mathbf{I}^* \\
&= V_{\text{rms}} I_{\text{rms}} \underline{/\theta_v - \theta_i} \\
\text{Apparent Power} &= S = |\mathbf{S}| = V_{\text{rms}} I_{\text{rms}} = \sqrt{P^2 + Q^2} \\
\text{Real Power} &= P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i) \\
\text{Reactive Power} &= Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i) \\
\text{Power Factor} &= \frac{P}{S} = \cos(\theta_v - \theta_i)
\end{aligned}$$



power triangle

EXAMPLE 11.11

The voltage across a load is $v(t) = 60\cos(\omega t - 10^\circ)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5\cos(\omega t + 50^\circ)$ A. Find: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

Solution:

(a) For the rms values of the voltage and current, we write

$$\mathbf{V}_{\text{rms}} = \frac{60}{\sqrt{2}}\angle -10^\circ, \qquad \mathbf{I}_{\text{rms}} = \frac{1.5}{\sqrt{2}}\angle +50^\circ$$

The complex power is

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = \left(\frac{60}{\sqrt{2}}\angle -10^\circ\right)\left(\frac{1.5}{\sqrt{2}}\angle -50^\circ\right) = 45\angle -60^\circ \text{ VA}$$

The apparent power is $S = |\mathbf{S}| = 45$ VA

(b) We can express the complex power in rectangular form as

$$\mathbf{S} = 45\angle -60^\circ = 45[\cos(-60^\circ) + j\sin(-60^\circ)] = 22.5 - j38.97$$

Since $\mathbf{S} = P + jQ$, the real power is $P = 22.5$ W the reactive power is $Q = -38.97$ VAR

(c) The power factor is pf $= \cos(-60^\circ) = 0.5$ (leading) $\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{60\angle -10^\circ}{1.5\angle +50^\circ} = 40\angle -60^\circ\ \Omega$

EXAMPLE 11.12

A load **Z** draws 12 kVA at a power factor of 0.856 lagging from a 120-V rms sinusoidal source. Calculate: (a) the average and reactive powers delivered to the load, (b) the peak current, and (c) the load impedance.

Solution:

(a) Given that pf $= \cos\theta = 0.856$, we obtain the power angle as $\theta = \cos^{-1} 0.856 = 31.13^\circ$. If the apparent power is $S = 12{,}000$ VA, then the average or real power is

$$P = S\cos\theta = 12{,}000 \times 0.856 = 10.272 \text{ kW}$$

while the reactive power is

$$Q = S\sin\theta = 12{,}000 \times 0.517 = 6.204 \text{ kVA}$$

(b) Since the pf is lagging, the complex power is

$$\mathbf{S} = P + jQ = 10.272 + j6.204 \text{ kVA}$$

From $\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}}$, we obtain

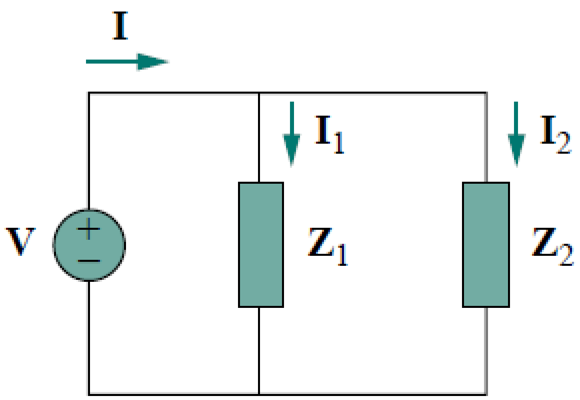
$$\mathbf{I}^*_{\text{rms}} = \frac{\mathbf{S}}{\mathbf{V}_{\text{rms}}} = \frac{10{,}272 + j6204}{120\underline{/0^\circ}} = 85.6 + j51.7 \text{ A} = 100\underline{/31.13^\circ} \text{ A}$$

Thus $\mathbf{I}_{\text{rms}} = 100\underline{/-31.13^\circ}$ and the peak current is

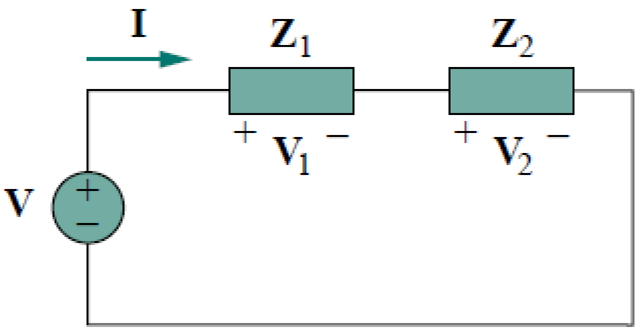
$$I_m = \sqrt{2}I_{\text{rms}} = \sqrt{2}(100) = 141.4 \text{ A}$$

(c) The load impedance $\mathbf{Z} = \dfrac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \dfrac{120\underline{/0^\circ}}{100\underline{/-31.13^\circ}} = 1.2\underline{/31.13^\circ}\ \Omega$

11.7 CONSERVATION OF AC POWER



(a)



(b)

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\begin{aligned} \mathbf{S} &= \frac{1}{2}\mathbf{V}\mathbf{I}^* \\ &= \frac{1}{2}\mathbf{V}(\mathbf{I}_1^* + \mathbf{I}_2^*) \\ &= \frac{1}{2}\mathbf{V}\mathbf{I}_1^* + \frac{1}{2}\mathbf{V}\mathbf{I}_2^* \\ &= \mathbf{S}_1 + \mathbf{S}_2 \end{aligned}$$

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$$

$$\begin{aligned} \mathbf{S} &= \frac{1}{2}\mathbf{V}\mathbf{I}^* \\ &= \frac{1}{2}(\mathbf{V}_1 + \mathbf{V}_2)\mathbf{I}^* \\ &= \frac{1}{2}\mathbf{V}_1\mathbf{I}^* + \frac{1}{2}\mathbf{V}_2\mathbf{I}^* \\ &= \mathbf{S}_1 + \mathbf{S}_2 \end{aligned}$$

We conclude from Eqs. (11.53) and (11.55) that whether the loads are connected in series or in parallel (or in general), the total power *supplied* by the source equals the total power *delivered* to the load. Thus, in general, for a source connected to N loads,

$$\boxed{\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_1 + \cdots + \mathbf{S}_N}$$

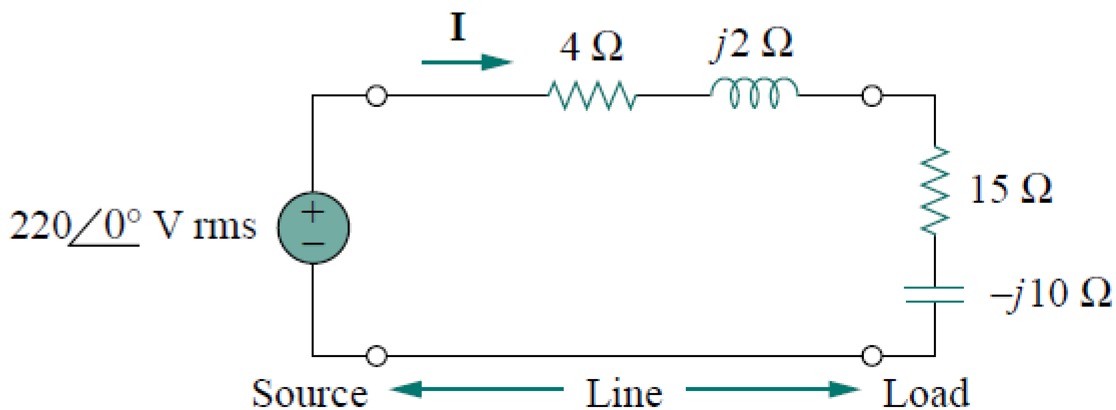
This means that the total complex power in a network is the sum of the complex powers of the individual components. (This is also true of real power and reactive power, but not true of apparent power.) This expresses the principle of conservation of ac power:

> The complex, real, and reactive powers of the sources equal the respective sums of the complex, real, and reactive powers of the individual loads.

From this we imply that the real (or reactive) power flow from sources in a network equals the real (or reactive) power flow into the other elements in the network.

EXAMPLE 11.13

Figure 11.24 shows a load being fed by a voltage source through a transmission line. The impedance of the line is represented by the $(4 + j2)\ \Omega$ impedance and a return path. Find the real power and reactive power absorbed by: (a) the source, (b) the line, and (c) the load.

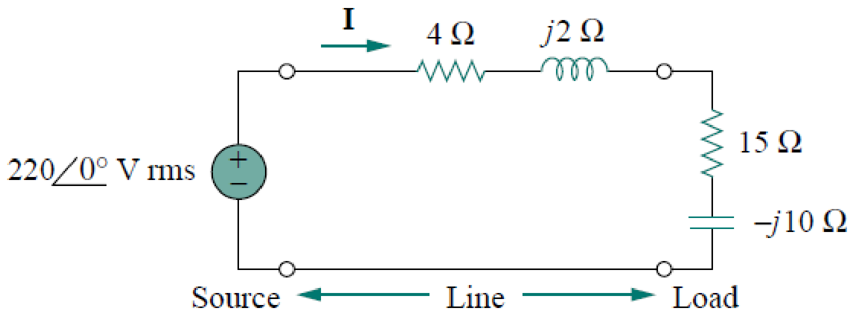


Solution:

The total impedance is

$$\mathbf{Z} = (4 + j2) + (15 - j10) = 19 - j8 = 20.62\underline{/-22.83^\circ}\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{220\underline{/0^\circ}}{20.62\underline{/-22.83^\circ}} = 10.67\underline{/22.83^\circ}\ \text{A rms}$$



(a) For the source, the complex power is

$$\mathbf{S}_s = \mathbf{V}_s\mathbf{I}^* = (220\angle 0^\circ)(10.67\angle -22.83^\circ)$$

$$= 2347.4\angle -22.83^\circ = (2163.5 - j910.8)\text{ VA}$$

From this, we obtain the real power as 2163.5 W and the reactive power as 910.8 VAR (leading).

(b) For the line, the voltage is

$$\mathbf{V}_{\text{line}} = (4 + j2)\mathbf{I} = (4.472\angle 26.57^\circ)(10.67\angle 22.83^\circ)$$

$$= 47.72\angle 49.4^\circ \text{ V rms}$$

The complex power absorbed by the line is

$$\mathbf{S}_{\text{line}} = \mathbf{V}_{\text{line}}\mathbf{I}^* = (47.72\angle 49.4^\circ)(10.67\angle -22.83^\circ)$$

$$= 509.2\angle 26.57^\circ = 455.4 + j227.7 \text{ VA}$$

(c) For the load, the voltage is

$$\mathbf{V}_L = (15 - j10)\mathbf{I} = (18.03\angle -33.7^\circ)(10.67\angle 22.83^\circ)$$
$$= 192.38\angle -10.87^\circ \text{ V rms}$$

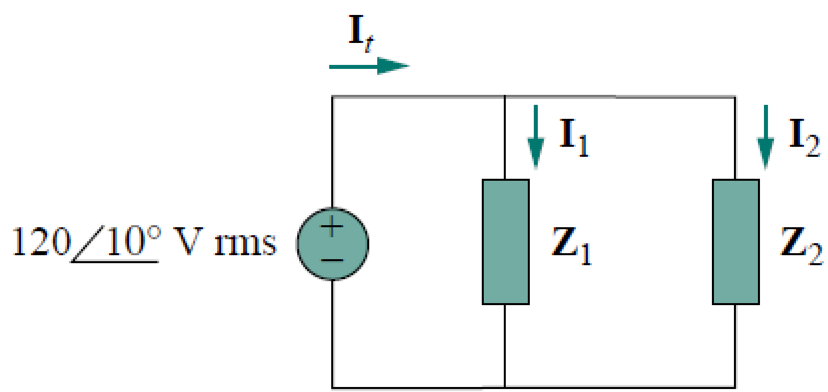
The complex power absorbed by the load is

$$\mathbf{S}_L = \mathbf{V}_L\mathbf{I}^* = (192.38\angle -10.87^\circ)(10.67\angle -22.83^\circ)$$
$$= 2053\angle -33.7^\circ = (1708 - j1139) \text{ VA}$$

The real power is 1708 W and the reactive power is 1139 VAR (leading). Note that $\mathbf{S}_s = \mathbf{S}_{\text{line}} + \mathbf{S}_L$, as expected. We have used the rms values of voltages and currents.

EXAMPLE 11.14

In the circuit of Fig. 11.26, $\mathbf{Z}_1 = 60\underline{/-30^\circ}\ \Omega$ and $\mathbf{Z}_2 = 40\underline{/45^\circ}\ \Omega$. Calculate the total: (a) apparent power, (b) real power, (c) reactive power, and (d) pf.



Solution:

The current through $\mathbf{Z}_1$ is
$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_1} = \frac{120\underline{/10^\circ}}{60\underline{/-30^\circ}} = 2\underline{/40^\circ}\ \text{A rms}$$

while the current through $\mathbf{Z}_2$ is
$$\mathbf{I}_2 = \frac{\mathbf{V}}{\mathbf{Z}_2} = \frac{120\underline{/10^\circ}}{40\underline{/45^\circ}} = 3\underline{/-35^\circ}\ \text{A rms}$$

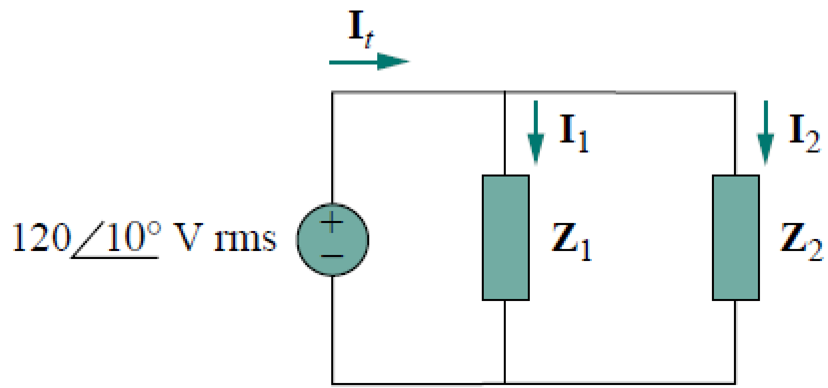
The complex powers absorbed by the impedances are

$$\mathbf{S}_1 = \frac{V_{\text{rms}}^2}{\mathbf{Z}_1^*} = \frac{(120)^2}{60\underline{/30^\circ}} = 240\underline{/-30^\circ} = 207.85 - j120\ \text{VA}$$

$$\mathbf{S}_2 = \frac{V_{\text{rms}}^2}{\mathbf{Z}_2^*} = \frac{(120)^2}{40\underline{/-45^\circ}} = 360\underline{/45^\circ} = 254.6 + j254.6\ \text{VA}$$

EXAMPLE 11.14

In the circuit of Fig. 11.26, $\mathbf{Z}_1 = 60\angle -30^\circ\ \Omega$ and $\mathbf{Z}_2 = 40\angle 45^\circ\ \Omega$. Calculate the total: (a) apparent power, (b) real power, (c) reactive power, and (d) pf.



The total complex power is $\quad \mathbf{S}_t = \mathbf{S}_1 + \mathbf{S}_2 = 462.4 + j134.6$ VA

(a) The total apparent power is $\quad |\mathbf{S}_t| = \sqrt{462.4^2 + 134.6^2} = 481.6$ VA.

(b) The total real power is $\quad P_t = \mathrm{Re}(\mathbf{S}_t) = 462.4$ W or $P_t = P_1 + P_2$.

(c) The total reactive power is $\quad Q_t = \mathrm{Im}(\mathbf{S}_t) = 134.6$ VAR or $Q_t = Q_1 + Q_2$.

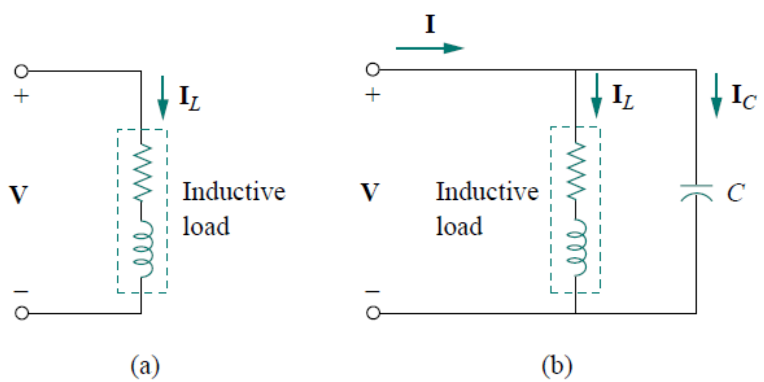
(d) The pf $= P_t/|\mathbf{S}_t| = 462.4/481.6 = 0.96$ (lagging).

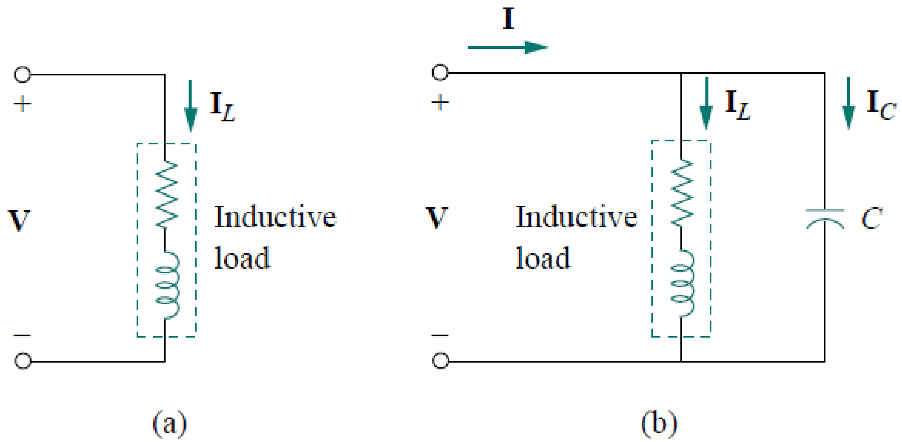
11.8 POWER FACTOR CORRECTION

1. Most domestic loads (such as washing machines, air conditioners, and refrigerators) and industrial loads (such as induction motors) are inductive and operate at a low lagging power factor. Although the inductive nature of the load cannot be changed, we can increase its power factor.

> The process of increasing the power factor without altering the voltage or current to the original load is known as power factor correction.

2. Since most loads are inductive, as shown in Fig. 11.27(a), a load's power factor is improved or corrected by deliberately installing a capacitor in parallel with the load, as shown in Fig. 11.27(b).





it is assumed that the circuit in Fig. 11.27(a) has a power factor of $\cos\theta_1$, while the one in Fig. 11.27(b) has a power factor of $\cos\theta_2$.

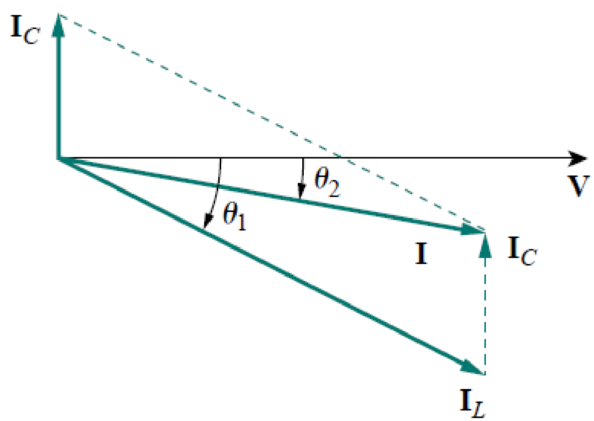
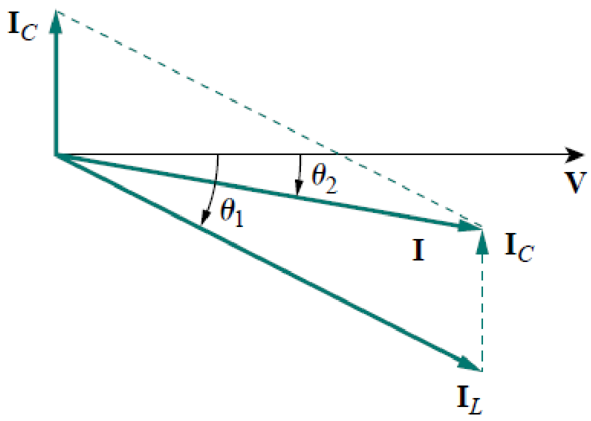
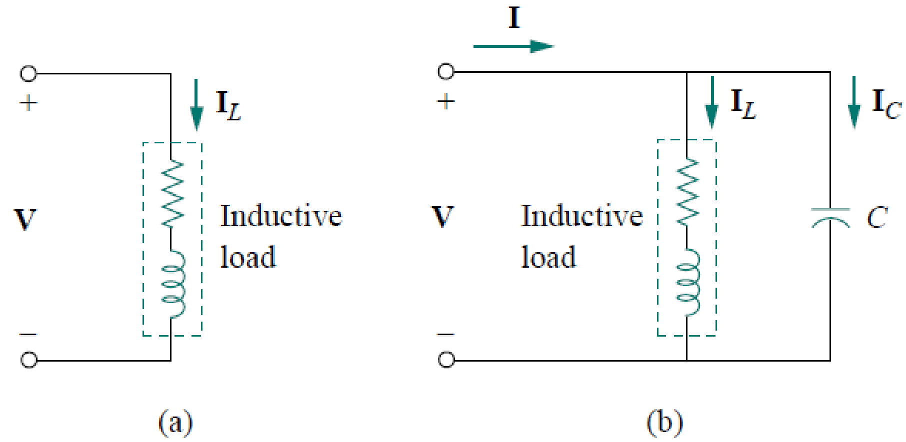


Fig. 11.28 shows that adding the capacitor has caused the phase angle between the supplied voltage and current to reduce from θ_1 to θ_2, thereby increasing the power factor.

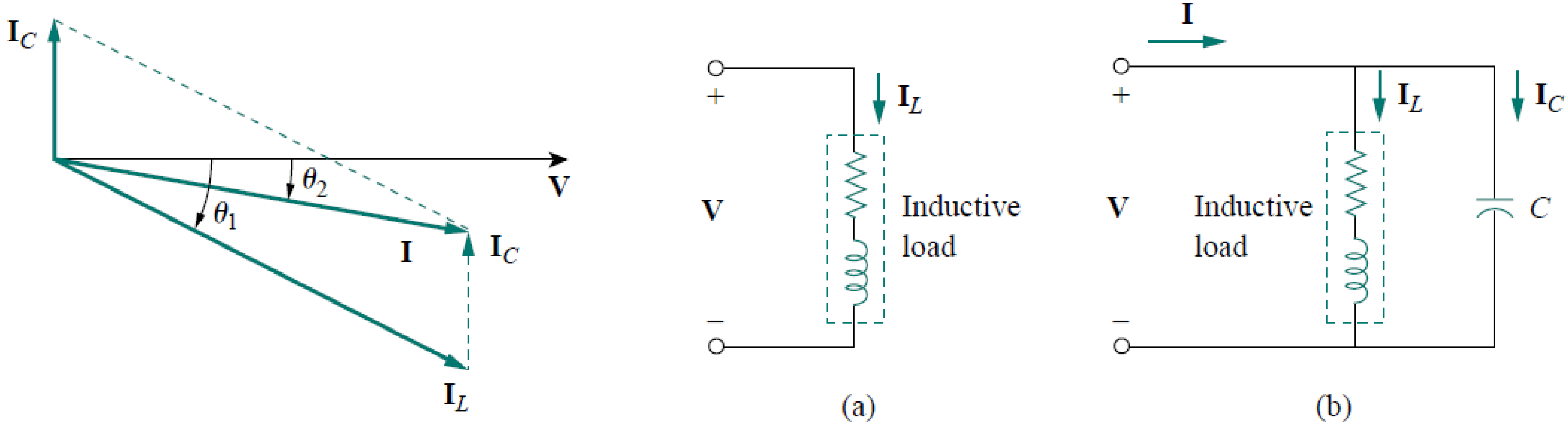




We also notice from the magnitudes of the vectors in Fig. 11.28 that with the same supplied voltage, the circuit in Fig. 11.27(a) draws larger current I_L than the current I drawn by the circuit in Fig. 11.27(b). Power companies charge more for larger currents, because they result in increased power losses (by a squared factor, since $P = I^2{}_L R$).

Therefore, it is beneficial to both the power company and the consumer that every effort is made to minimize current level or keep the power factor as close to unity as possible.

By choosing a suitable size for the capacitor, the current can be made to be completely in phase with the voltage, implying unity power factor.



If we desire to increase the power factor from $\cos\theta_1$ to $\cos\theta_2$ without altering the real power (i.e., $P = S_2 \cos\theta_2$),

$$\boxed{C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}}$$

Note that the real power P dissipated by the load is not affected by the power factor correction because the average power due to the capacitance is zero.

EXAMPLE 11.15

When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}$$

Solution:

If the pf $= 0.8$, then $\quad \cos\theta_1 = 0.8 \quad \Longrightarrow \quad \theta_1 = 36.87^\circ$

$$S_1 = \frac{P}{\cos\theta_1} = \frac{4000}{0.8} = 5000 \text{ VA} \qquad Q_1 = S_1 \sin\theta = 5000 \sin 36.87 = 3000 \text{ VAR}$$

When the pf is raised to 0.95, $\quad \cos\theta_2 = 0.95 \quad \Longrightarrow \quad \theta_2 = 18.19^\circ$

The real power P has not changed. But the apparent power has changed; its new value is

$$S_2 = \frac{P}{\cos\theta_2} = \frac{4000}{0.95} = 4210.5 \text{ VA} \qquad Q_2 = S_2 \sin\theta_2 = 1314.4 \text{ VAR}$$

EXAMPLE 11.15

When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

$$\boxed{C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}}$$

The difference between the new and old reactive powers is due to the parallel addition of the capacitor to the load. The reactive power due to the capacitor is

$$Q_C = Q_1 - Q_2 = 3000 - 1314.4 = 1685.6 \text{ VAR}$$

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{1685.6}{2\pi \times 60 \times 120^2} = 310.5\ \mu\text{F}$$

Summary and Review

1. The instantaneous power absorbed by an element is the product of the element's terminal voltage and the current through the element: $p = vi$.

2. Average or real power P (in watts) is the average of instantaneous power p:

$$P = \frac{1}{T}\int_0^T p\, dt$$

If $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$, then $V_{\text{rms}} = V_m/\sqrt{2}$, $I_{\text{rms}} = I_m/\sqrt{2}$, and

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

Inductors and capacitors absorb no average power, while the average power absorbed by a resistor is $1/2\ I_m^2 R = I_{\text{rms}}^2 R$.

3. Maximum average power is transferred to a load when the load impedance is the complex conjugate of the Thevenin impedance as seen from the load terminals, $\mathbf{Z}_L = \mathbf{Z}^*_{\text{Th}}$.

4. The effective value of a periodic signal $x(t)$ is its root-mean-square (rms) value.

$$X_{\text{eff}} = X_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2\,dt}$$

For a sinusoid, the effective or rms value is its amplitude divided by $\sqrt{2}$.

5. The power factor is the cosine of the phase difference between voltage and current:

$$\text{pf} = \cos(\theta_v - \theta_i)$$

It is also the cosine of the angle of the load impedance or the ratio of real power to apparent power. The pf is lagging if the current lags voltage (inductive load) and is leading when the current leads voltage (capacitive load).

6. Apparent power S (in VA) is the product of the rms values of voltage and current:

$$S = V_{\text{rms}} I_{\text{rms}}$$

It is also given by $S = |\mathbf{S}| = \sqrt{P^2 + Q^2}$, where Q is reactive power.

7. Reactive power (in VAR) is:

$$Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i) = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

8. Complex power $\mathbf{S}$ (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. It is also the complex sum of real power P and reactive power Q.

$$\mathbf{S} = \mathbf{V}_{\text{rms}} \mathbf{I}_{\text{rms}}^{*} = V_{\text{rms}} I_{\text{rms}} \underline{/\theta_v - \theta_i} = P + jQ$$

Also,

$$\mathbf{S} = I_{\text{rms}}^2 \mathbf{Z} = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*}$$

9. The total complex power in a network is the sum of the complex powers of the individual components. Total real power and reactive power are also, respectively, the sums of the individual real powers and the reactive powers, but the total apparent power is not calculated by the process.

10. Power factor correction is necessary for economic reasons; it is the process of improving the power factor of a load by reducing the overall reactive power.

Assignment (page 492)

Problems 11.15, 11.21, 11.51, 11.52

1、In the circuit of Fig. 1, load *A receives 4 kVA* at 0.8 pf leading. Load *B receives 2.4 kVA at 0.6 pf* lagging. Box *C is an inductive load that consumes* 1 kW and receives 500 VAR.

(a) Determine **I.**

(b) Calculate the power factor of the combination.

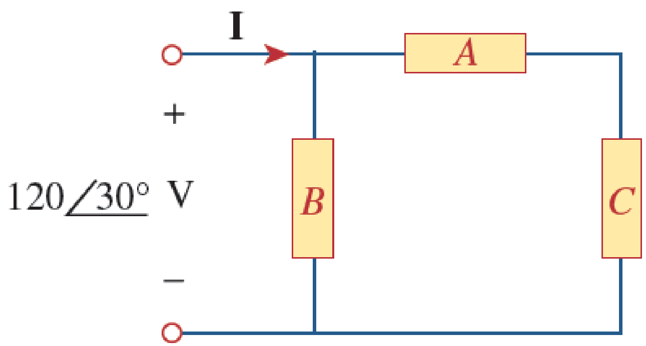


Fig. 1

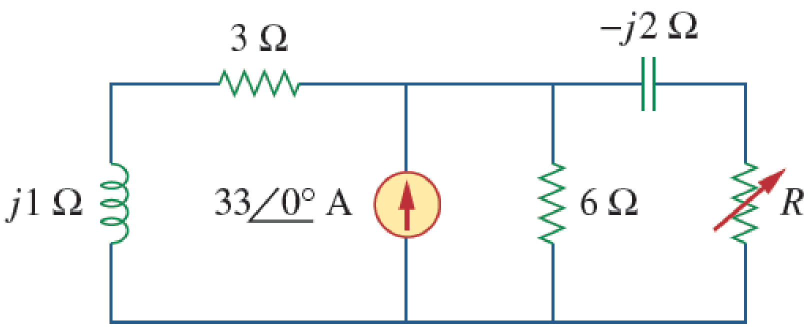


Fig. 2

2、The variable resistor *R* in the circuit of Fig. 2 is adjusted until it absorbs the maximum average power. Find *R* and the maximum average power absorbed.

11.4 EFFECTIVE OR RMS VALUE

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

the effective value is the (square) *root* of the *mean* (or average) of the *square* of the periodic signal.

the *root-mean-square* value, or *rms* value $I_{\text{eff}} = I_{\text{rms}},$ $V_{\text{eff}} = V_{\text{rms}}$

The effective value of a periodic signal is its root mean square (rms) value.

For any periodic function *x(t)* in general, the rms value is given by

$$\boxed{X_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2\,dt}}$$

For the sinusoid $i(t) = I_m \cos \omega t$, the effective or rms value is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t \, dt} \quad = \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t)\, dt} = \frac{I_m}{\sqrt{2}}$$

Similarly, for $v(t) = V_m \cos \omega t$, $\quad V_{\text{rms}} = \dfrac{V_m}{\sqrt{2}}$

The average power can be written in terms of the rms values.

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i) = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

Similarly, the average power absorbed by a resistor R can be written as

$$P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

11.5 APPARENT POWER AND POWER FACTOR

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

$$\boxed{S = V_{\text{rms}} I_{\text{rms}}}$$

$$\boxed{\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)}$$

- The average power is a product of two terms. The product $V_{rms} * I_{rms}$ is known as the *apparent power* S. The factor $\cos(\theta_v - \theta_i)$ is called the *power factor* (pf).

The apparent power (in VA) is the product of the rms values of voltage and current.

The power factor is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedance.

The angle $\theta_v - \theta_i$ is called the *power factor angle*,

11.5 APPARENT POWER AND POWER FACTOR

$$\boxed{\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)}$$

- For a purely resistive load, the voltage and current are in phase, so that $\theta_v - \theta_i = 0$ and pf = 1. This implies that the apparent power is equal to the average power.
- For a purely reactive load, $\theta_v - \theta_i = \pm 90^\circ$ and pf = 0. In this case the average power is zero.
- In between these two extreme cases, pf is said to be leading or lagging.
- Leading power factor means that current leads voltage, which implies a capacitive load.
- Lagging power factor means that current lags voltage, implying an inductive load.

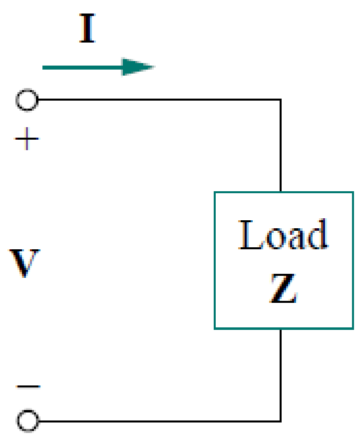
11.6 COMPLEX POWER

$$\mathbf{V} = V_m \angle \theta_v \text{ and } \mathbf{I} = I_m \angle \theta_i,$$

the *complex power* S absorbed by the ac load is the product of the voltage and the complex conjugate of the current,

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^*$$

$$\mathbf{V}_{\text{rms}} = \frac{\mathbf{V}}{\sqrt{2}} = V_{\text{rms}} \angle \theta_v \qquad \mathbf{I}_{\text{rms}} = \frac{\mathbf{I}}{\sqrt{2}} = I_{\text{rms}} \angle \theta_i$$



In terms of the rms values, $\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}}$

$$\begin{aligned}\mathbf{S} &= V_{\text{rms}} I_{\text{rms}} \angle \theta_v - \theta_i \\ &= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) + j V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)\end{aligned}$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i), \qquad Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

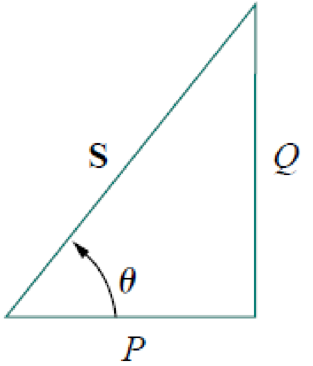
$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i), \qquad Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).

Complex power (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity, its real part is real power P and its imaginary part is reactive power Q.

Introducing the complex power enables us to obtain the real and reactive powers directly from voltage and current phasors.

$$\text{Complex Power} = \mathbf{S} = P + jQ = \frac{1}{2}\mathbf{V}\mathbf{I}^*$$
$$= V_{\text{rms}} I_{\text{rms}} \underline{/\theta_v - \theta_i}$$
$$\text{Apparent Power} = S = |\mathbf{S}| = V_{\text{rms}} I_{\text{rms}} = \sqrt{P^2 + Q^2}$$
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$
$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$
$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$



power triangle