Solution 1.1

(a) $q = 6.482\text{x}10^{17}$ x $[-1.602\text{x}10^{-19}$ C$]$ = **–103.84 mC**

(b) $q = 1.\ 24\text{x}10^{18}$ x $[-1.602\text{x}10^{-19}$ C$]$ = **–198.65 mC**

(c) $q = 2.46\text{x}10^{19}$ x $[-1.602\text{x}10^{-19}$ C$]$ = **–3.941 C**

(d) $q = 1.628\text{x}10^{20}$ x $[-1.602\text{x}10^{-19}$ C$]$ = **–26.08 C**

Solution 1.2

Determine the current flowing through an element if the charge flow is given by

(a) $q(t) = (3)$ mC

(b) $q(t) = (4t^2 + 20t - 4)$ C

(c) $q(t) = (15e^{-3t} - 2e^{-18t})$ nC

(d) $q(t) = 5t^2(3t^3 + 4)$ pC

(e) $q(t) = 2e^{-3t}\sin(20\pi t)$ μC

(a) $i = dq/dt = 0$ mA

(b) $i = dq/dt = (8t + 20)$ A

(c) $i = dq/dt = (-45e^{-3t} + 36e^{-18t})$ nA

(d) $i = dq/dt = (75t^4 + 40t)$ pA

(e) $i = dq/dt = \{-6e^{-3t}\sin(20\pi t) + 40\pi e^{-3t}\cos(20\pi t)\}$ μA

Solution 1.3

(a) $q(t) = \int i(t)dt + q(0) = \underline{(3t + 1)\ \ C}$

(b) $q(t) = \int (2t + s)\, dt + q(v) = \underline{(t^2 + 5t)\ \ mC}$

(c) $q(t) = \int 20 \cos\ \left(10t + \pi / 6\right) + q(0) = \underline{(2\sin(10t + \pi / 6) + 1)\mu C}$

(d) $$q(t) = \int 10e^{-30t} \sin 40t + q(0) = \frac{10e^{-30t}}{900 + 1600}(-30 \sin 40t - 40 \cos t)$$

$$= \underline{- e^{-30t} (0.16 \cos 40\, t + 0.12\, \sin 40t)\ C}$$

Solution 1.4

Since i is equal to $\Delta q/\Delta t$ then $i = 300/30 =$ **10 amps**.

Solution 1.5

$$q = \int i dt = \int_0^{10} \frac{1}{2} t dt = \frac{t^2}{4}\Bigg|_0^{10} = \underline{25 \text{ C}}$$

Solution 1.6

(a) At t = 1ms, $i = \frac{dq}{dt} = \frac{30}{2} =$ **<u>15 A</u>**

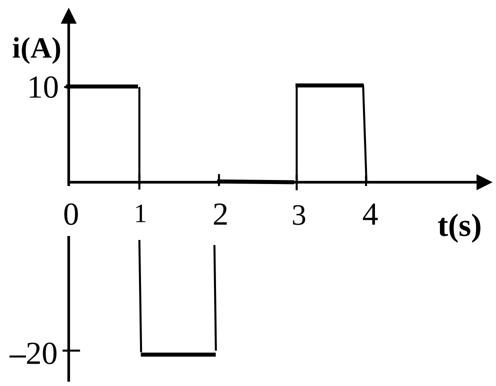
(b) At t = 6ms, $i = \frac{dq}{dt} =$ **<u>0 A</u>**

(c) At t = 10ms, $i = \frac{dq}{dt} = \frac{-30}{4} =$ **<u>–7.5 A</u>**

Solution 1.7

$$i = \frac{dq}{dt} = \begin{cases} 10A, & 0 < t < 1 \\ -20A, & 1 < t < 2 \\ 0A, & 2 < t < 3 \\ 10A, & 3 < t < 4 \end{cases}$$

which is sketched below:



Solution 1.8

$$q = \int i\,dt = \frac{10 \times 1}{2} + 10 \times 1 = \underline{15\ \mu C}$$

Solution 1.9

(a) $q = \int i dt = \int_0^1 10\, dt = \underline{10\ C}$

(b) $q = \int_0^3 i dt = 10 \times 1 + \left(10 - \frac{5 \times 1}{2}\right) + 5 \times 1$

$= 15 + 7.5 + 5 = \underline{22.5C}$

(c) $q = \int_0^5 i dt = 10 + 10 + 10 = \underline{30\ C}$

Solution 1.10

$q = it = 10 x 10^3 x 15 x 10^{-6} =$ **150 mC**

Solution 1.11

$q = it = 90 \times 10^{-3} \times 12 \times 60 \times 60 =$ **3.888 kC**

$E = pt = ivt = qv = 3888 \times 1.5 =$ **5.832 kJ**

Solution 1.12

For 0 < t < 6s, assuming q(0) = 0,

$$q(t) = \int_0^t idt + q(0) = \int_0^t 3tdt + 0 = 1.5t^2$$

At t=6, q(6) = $1.5(6)^2$ = 54
For 6 < t < 10s,

$$q(t) = \int_6^t idt + q(6) = \int_6^t 18dt + 54 = 18t - 54$$

At t=10, q(10) = 180 – 54 = 126
For 10<t<15s,

$$q(t) = \int_{10}^t idt + q(10) = \int_{10}^t (-12)dt + 126 = -12t + 246$$

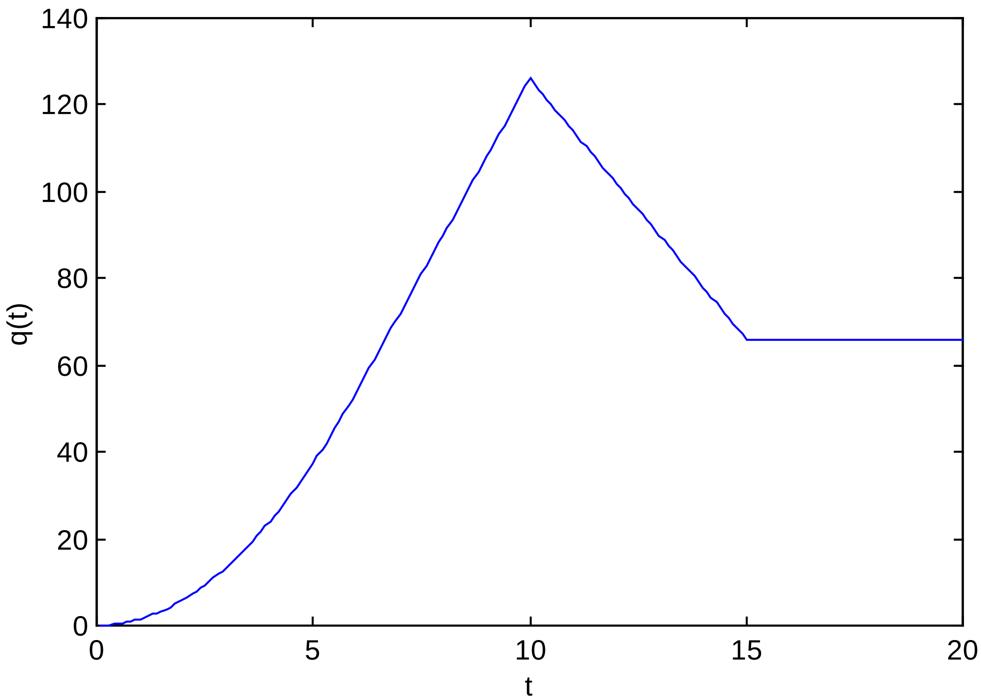
At t=15, q(15) = -12x15 + 246 = 66
For 15<t<20s,

$$q(t) = \int_{15}^t 0dt + q(15) = 66$$

Thus,

$$q(t) = \begin{cases} 1.5t^2 \ \mathbf{C,} \ \mathbf{0 < t < 6s} \\ 18t - 54 \ \mathbf{C,} \ \mathbf{6 < t < 10s} \\ -12t + 246 \ \mathbf{C,} \ \mathbf{10 < t < 15s} \\ 66 \ \mathbf{C,} \ \mathbf{15 < t < 20s} \end{cases}$$

The plot of the charge is shown below.


140
120
100
80
60
40
20
0
q(t)
0
5
10
15
20
t

Solution 1.13

(a) $i = [dq/dt] = 20\pi\cos(4\pi t)$ mA

$p = vi = 60\pi\cos^2(4\pi t)$ mW

At t=0.3s,

$$p = vi = 60\pi\cos^2(4\pi 0.3) \text{ mW} = \mathbf{123.37\ mW}$$

(b) $W =$ $\int p dt = 60\pi \int_0^{0.6} cos^2(4\pi t)dt = 30\pi \int_0^{0.6}$

$$W = 30\pi[0.6+(1/(8\pi))[\sin(8\pi 0.6)-\sin(0)]] = \mathbf{58.76\ mJ}$$

Solution 1.14

The voltage *v(t)* across a device and the current *i(t)*
through it are

$v(t) = 20\sin(4t)$ volts and $i(t) = 10(1 + e^{-2t})$ m-amps.

Calculate:

(a) the total charge in the device at $t = 1$ s, assume $q(0) = 0$.

(b) the power consumed by the device at $t = 1$ s.

(a) $q = \int i dt = \int_0^1 0.01\left(1 + e^{-2t}\right) dt = 0.01\left(t - 0.5e^{-2t}\right)\Big|_0^1 = 0.01\left(1 - 0.5e^{-2} + 0.5\right)$

$= 0.01(1 - 0.135335 + 0.5) =$ **13.647 mC**.

(b) $p(t) = v(t)i(t)$; $v(1) = 20\sin(4) = 20\sin(229.18°) = -15.135$ volts;
and $i(1) = 10(1+e^{-2})(10^{-3}) = 10(1.1353)(10^{-3}) = 11.353$ m-amps
$p(1) = (-15.125)(11.353)(10^{-3}) =$ **–171.71 mW**

Solution 1.15

(a) $q = \int idt = \int_0^2 0.006e^{-2t}dt = \frac{-0.006}{2}e^{2t}\Big|_0^2$

$= -0.003\left(e^{-4} - 1\right) =$

2.945 mC

(b) $v = \frac{10di}{dt} = -0.012e^{-2t}(10) = -0.12e^{-2t}$ V this leads to p(t) = v(t)i(t) =

$(-0.12e^{-2t})(0.006e^{-2t})$ = $\mathbf{-720e^{-4t}}$ **μW**

(c) $w = \int pdt = -0.72\int_0^3 e^{-4t}\ dt = \frac{-720}{-4}e^{-4t}10^{-6}\Big|_0^3$ = **–180 μJ**

Solution 1.16

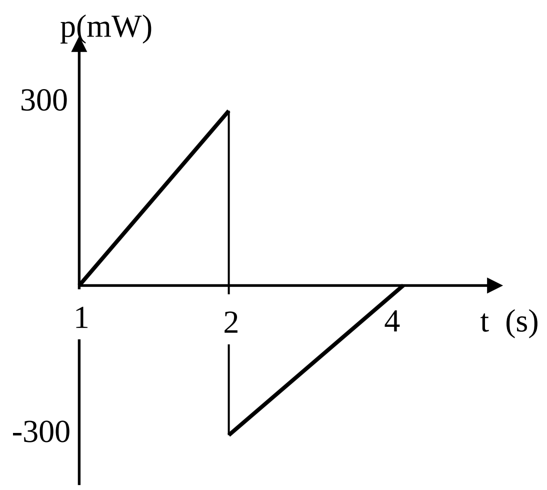
(a)

$$i(t) = \begin{cases} 30t \text{ mA}, & 0 < t < 2 \\ 120\text{-}30t \text{ mA}, & 2 < t < 4 \end{cases}$$

$$v(t) = \begin{cases} 5 \text{ V}, & 0 < t < 2 \\ -5 \text{ V}, & 2 < t < 4 \end{cases}$$

$$p(t) = \begin{cases} 150t \text{ mW}, & 0 < t < 2 \\ -600+150t \text{ mW}, & 2 < t < 4 \end{cases}$$

which is sketched below.



(b) From the graph of p,

$$W = \int_0^4 pdt = \underline{0 \text{ J}}$$

Solution 1.17

Figure 1.28 shows a circuit with four elements, p_1 = 60 watts absorbed, p_3 = –145 watts absorbed, and p_4 = 75 watts absorbed. How many watts does element 2 absorb?

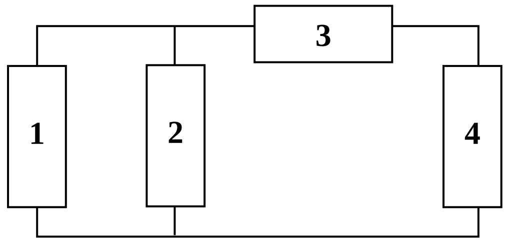


Figure 1.28
For Prob. 1.17.

Σ p = 0 = 60 + p_2 – 145 + 75 = 0 or p_2 = –60 + 145 – 75 = **10 watts absorbed**.

Solution 1.18

p_1 = 30(-10) = **-300 W**
p_2 = 10(10) = **100 W**
p_3 = 20(14) = **280 W**
p_4 = 8(-4) = **-32 W**
p_5 = 12(-4) = **-48 W**

Solution 1.19

Find I and the power absorbed by each element in the network of Fig. 1.30.

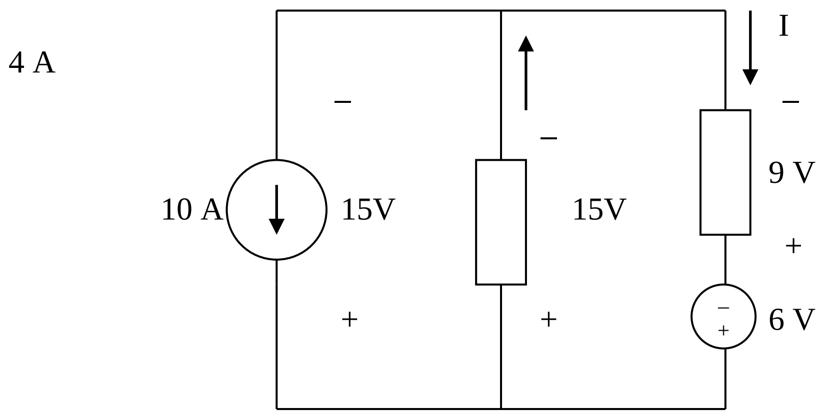


Figure 1.30
For Prob. 1.19.

I = –10 + 4 = **–6 amps**

Calculating the power absorbed by each element means we need to find vi (being careful to use the passive sign convention) for each element.

$P_{\text{10 amp source}}$ = –10x15 = **–150 W**
$p_{\text{element with 15 volts across it}}$ = 4x15 = **60 W**
$p_{\text{element with 9 volts across it}}$ = –(–6x9) = **54 W**
$p_{\text{6 volt source}}$ = –(–6x6) = **36 W**

One check we can use is that the sum of the power absorbed must equal zero which is what it does.

Solution 1.20

$p_{\text{30 volt source}}$ = 30x(–6) = **–180 W**
$p_{\text{12 volt element}}$ = 12x6 = **72 W**
$p_{\text{28 volt e.ement with 2 amps flowing through it}}$ = 28x2 = **56 W**
$p_{\text{28 volt element with 1 amp flowing through it}}$ = 28x1 = **28 W**
$p_{\text{the 5Io dependent source}}$ = 5x2x(–3) = **–30 W**

Since the total power absorbed by all the elements in the circuit must equal zero,
or 0 = –180+72+56+28–30+$p_{\text{into the element with Vo}}$ or

$p_{\text{into the element with Vo}}$ = 180–72–56–28+30 = **54 W**

Since $p_{\text{into the element with Vo}}$ = V_ox3 = 54 W or V_o = **18 V**.

Solution 1.21

$p = vi \quad \longrightarrow \quad i = \frac{p}{v} = \frac{60}{120} = 0.5 \text{ A}$

q = it = 0.5x24x60x60 = **43.2 kC**

$N_e = qx6.24x10^{18} = \underline{2.696x10^{23} \text{ electrons}}$

Solution 1.22

$q = it = 40x10^{3}x1.7x10^{-3}$ = **68 C**

Solution 1.23

$W = pt = 1.8x(15/60)\ x30\ kWh = 13.5kWh$
$C = 10cents\ x13.5 =$ **$1.35**

Solution 1.24

W = pt = 60 x24 Wh = 0.96 kWh = 1.44 kWh

C = 8.2 centsx0.96 = **11.808 cents**

Solution 1.25

A 1.2–kW toaster takes roughly 4 minutes to heat four slices of bread. Find the cost of operating the toaster twice per day for 2 weeks (14 days). Assume energy costs 9 cents/kWh.

$$\text{Cost} = 1.2\ \text{kW} \times \frac{4}{60}\,\text{hr} \times 14 \times 9\ \text{cents/kWh}\ = \mathbf{10.08\ cents}$$

Solution 1.26

(a) Clearly 10.78 watt-hours = (voltage)(current)(time) = 3.85I(3) or
I = 10.78/[(3.85)(3)] = **933.3 mA**

(b) p = energy/time = 10.78/3 = **3.593 W**

(c) amp-hours = energy/voltage = 10.78/3.85 = **2.8 amp-hours**

Solution 1.27

(a) Let $T = 4h = 4 \times 3600$

$$q = \int idt = \int_0^T 3dt = 3T = 3 \times 4 \times 3600 = \underline{43.2\ \text{kC}}$$

(b) $W = \int pdt = \int_0^T vidt = \int_0^T (3)\left(10 + \frac{0.5t}{3600}\right)dt$

$$= 3\left(10t + \frac{0.25t^2}{3600}\right)\Bigg|_0^{4\times 3600} = 3\left[40 \times 3600 + 0.25 \times 16 \times 3600\right]$$

$$= \underline{475.2\ \text{kJ}}$$

(c) $W = 475.2\ \text{kWs}, \quad (\text{J} = \text{Ws})$

$$\text{Cost} = \frac{475.2}{3600}\ \text{kWh} \times 9\ \text{cent} = \underline{1.188\ \text{cents}}$$

Solution 1.28

A 150-W incandescent outdoor lamp is connected to a 120-V source and is left burning continuously for an average of 12 hours per day. Determine:

(a) the current through the lamp when it is lit,
(b) the cost of operating the light for one non-leap year if electricity costs 9.5 cents per kWh.

(a) $i = \dfrac{P}{V} = \dfrac{150}{120}$
= **1.25 A**

(b) $\text{w} = \text{pt} = 150 \times 365 \times 12 \text{ Wh} = 657 \text{ kWh}$

$\text{Cost} = \$0.095 \times 657$
= **$62.42**

Solution 1.29

$$w = pt = 1.2\text{kW}\,\frac{(20+40+15+45)}{60}\,\text{hr} + 1.8\,\text{kW}\left(\frac{30}{60}\right)\text{hr}$$

$$= 2.4 + 0.9 = 3.3\,\text{kWh}$$

$$\text{Cost} = 12\text{ cents} \times 3.3 = \underline{39.6\text{ cents}}$$

Solution 1.30

Monthly charge = \$6

First 250 kWh @ \$0.02/kWh = \$5

Remaining 2,436–250 kWh = 2,186 kWh @ \$0.07/kWh= \$153.02

Total = **\$164.02**

Solution 1.31

In a household, a business is run for an average of 6 hours per day. The total power consumed by the computer and its printer is 230 watts. In addition, a 75-W light runs during the same 6 hours. If their utility charges 11.75 cents per kWhr, how much do the owners pay every 30 days?

$$\text{Total energy consumed over every 30 day period} = 30[(230+75)6] = 54.9 \text{ kWhr}$$

$$\text{Cost per 30 day period} = \$0.1175\text{x}54.9 = \mathbf{\$6.451}$$

Solution 1.32

$i = 20\ \mu A$

$q = 15\ C$

$t = q/i = 15/(20x10^{-6}) =$ **$750x10^3$ hrs**

Solution 1.33

$$i = \frac{dq}{dt} \rightarrow q = \int i dt = 2000 \times 3 \times 10^{-3} = \underline{6\ C}$$

Solution 1.34

(a) Energy = $\sum pt$ = 200 x 6 + 800 x 2 + 200 x 10 + 1200 x 4 + 200 x 2
= **10 kWh**

(b) Average power = 10,000/24 = **416.7 W**

Solution 1.35

energy = (5x5 + 4x5 + 3x5 + 8x5 + 4x10)/60 = **2.333 MWhr**

Solution 1.36

A battery can be rated in ampere-hours or watt hours. The ampere hours can be obtained from the watt hours by dividing watt hours be a nominal voltage of 12 volts. If an automobile battery is rated at 20 ampere-hours,

(a) what is the maximum current that can be supplied for 15 minutes?
(b) how many days will it last if it is discharged at a rate of 2 mA?

(a) I = 20/0.25 = **80 amps**.
(b) days = (20/0.002)/24 = **416.7 days**.

Solution 1.37

A total of 2 MJ are delivered to an automobile battery (assume 12 volts) giving it an additional charge. How much is that additional charge? Express your answer in ampere-hours.

Solution

2,000,000 = w = pt = vit = 12it = 12(charge) or

charge = $2x10^6/12 = 1.666710^5$ coulomb = 1.666710^5 Coulomb x 1 hour/3,600 seconds = 46.3 ampere-hour.

charge = **46.3 ampere-hours**.

Solution 1.38

P = 10 hp = 7460 W

W = pt = 7460 × 30 × 60 J = $\mathbf{13.43 \times 10^6}$ **J**

Solution 1.39

W = pt = 600x4 = 2.4 kWh
C = 10cents x2.4 = **24 cents**

Solution 2.1

Design a problem, complete with a solution, to help students to better understand Ohm's Law. Use at least two resistors and one voltage source. Hint, you could use both resistors at once or one at a time, it is up to you. Be creative.

Although there is no correct way to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The voltage across a 5-kΩ resistor is 16 V. Find the current through the resistor.

Solution

$$v = iR \qquad i = v/R = (16/5)\text{ mA} = \mathbf{3.2\ mA}$$

Solution 2.2

$p = v^2/R \rightarrow \mathbf{R} = v^2/p = 14400/60 =$ **240 ohms**

Solution 2.3

For silicon, $\rho = 6.4x10^2$ Ω-m. $A = \pi r^2$. Hence,

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} \longrightarrow r^2 = \frac{\rho L}{\pi R} = \frac{6.4x10^2 x4x10^{-2}}{\pi x240} = 0.033953$$

r = **184.3 mm**

Solution 2.4

(a) $\mathbf{i} = 40/100 = \mathbf{400\ mA}$
(b) $\mathbf{i} = 40/250 = \mathbf{160\ mA}$

Solution 2.5

n = **9**; l = **7**; **b** = n + l – 1 = **15**

Solution 2.6

n = 8; l = 8; **b** = n + l –1 = **<u>15</u>**

Solution 2.7

6 branches and 4 nodes

Solution 2.8

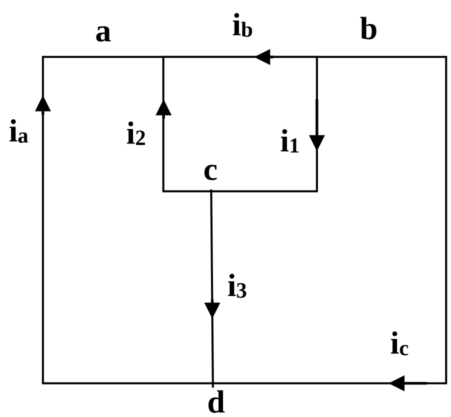
Design a problem, complete with a solution, to help other students to better understand Kirchhoff's Current Law. Design the problem by specifying values of i_a, i_b, and i_c, shown in Fig. 2.72, and asking them to solve for values of i_1, i_2, and i_3. Be careful specify realistic currents.

Although there is no correct way to work this problem, this is one of the many possible solutions. Note that the solution process must follow the same basic steps.

Problem

Use KCL to obtain currents i_1, i_2, and i_3 in the circuit shown in Fig. 2.72 given that $i_a = 2$ amps, $i_b = 3$ amps, and $i_c = 4$ amps.

Solution



At node a, $-i_a - i_2 - i_b = 0$ or $i_2 = -2 - 3 =$ **-5 amps**
At node b, $i_b + i_1 + i_c = 0$ or $i_1 = -3 - 4 =$ **-7 amps**
At node c, $i_2 + i_3 - i_1 = 0$ or $i_3 = -7 + 5 =$ **-2 amps**

We can use node d as a check, $i_a - i_3 - i_c = 2 + 2 - 4 = 0$ which is as expected.

Solution 2.9

Find i_1, i_2, and i_3 in Fig. 2.73.

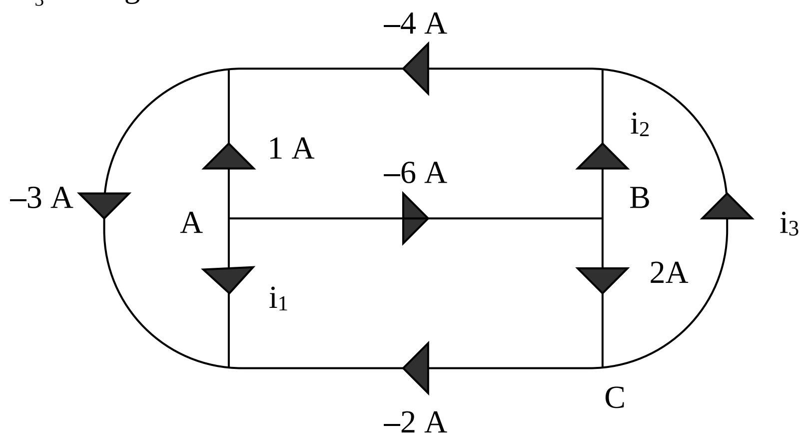


Figure 2.73
For Prob. 2.9.

Solution

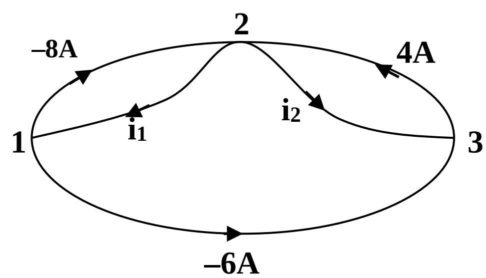
Step 1. We can apply Kirchhoff's current law to solve for the unknown currents.

Summing all of the currents flowing out of nodes A, B, and C we get,

at A, $1 + (–6) + i_1 = 0$;
at B, $–(–6) + i_2 + 2 = 0$; and
at C, $(–2) + i_3 – 2 = 0$.

Step 2. We now can solve for the unknown currents, $i_1 = –1 + 6 =$ **5 amps**; $i_2 = –6 – 2 =$ **–8 amps**; and $i_3 = 2 + 2 =$ **4 amps**.

Solution 2.10



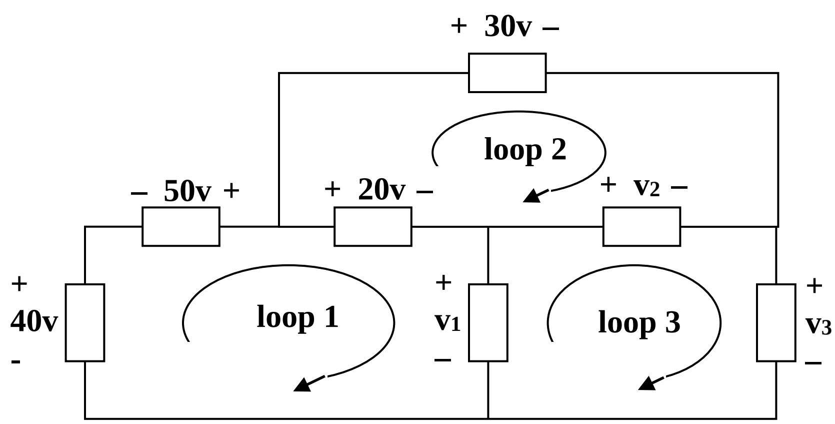
At node 1, $-8-i_1-6 = 0$ or $i_1 = -8-6 = \mathbf{-14\ A}$

At node 2, $-(-8)+i_1+i_2-4 = 0$ or $i_2 = -8-i_1+4 = -8+14+4 = \mathbf{10\ A}$

Solution 2.11

$$-V_1 + 1 + 5 = 0 \quad \longrightarrow \quad V_1 = \underline{6 \text{ V}}$$
$$-5 + 2 + V_2 = 0 \quad \longrightarrow \quad V_2 = \underline{3 \text{ V}}$$

Solution 2.12

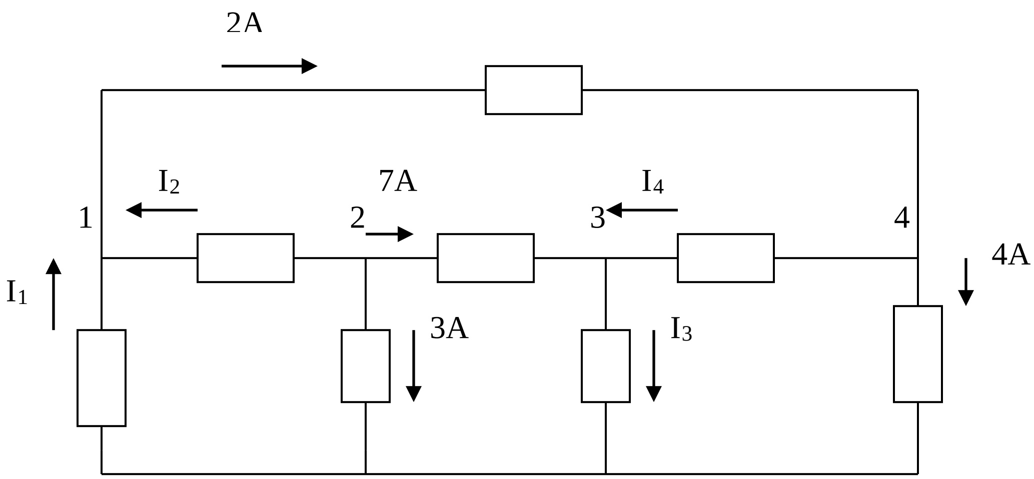


For loop 1, $-40 - 50 + 20 + v_1 = 0$ or $v_1 = 40 + 50 - 20 =$ **70 V**

For loop 2, $-20 + 30 - v_2 = 0$ or $v_2 = 30 - 20 =$ **10 V**

For loop 3, $-v_1 + v_2 + v_3 = 0$ or $v_3 = 70 - 10 =$ **60 V**

Solution 2.13



At node 2,

$$3+7+I_2=0 \quad \longrightarrow \quad I_2=-10A$$

At node 1,

$$I_1+I_2=2 \quad \longrightarrow \quad I_1=2-I_2=12A$$

At node 4,

$$2=I_4+4 \quad \longrightarrow \quad I_4=2-4=-2A$$

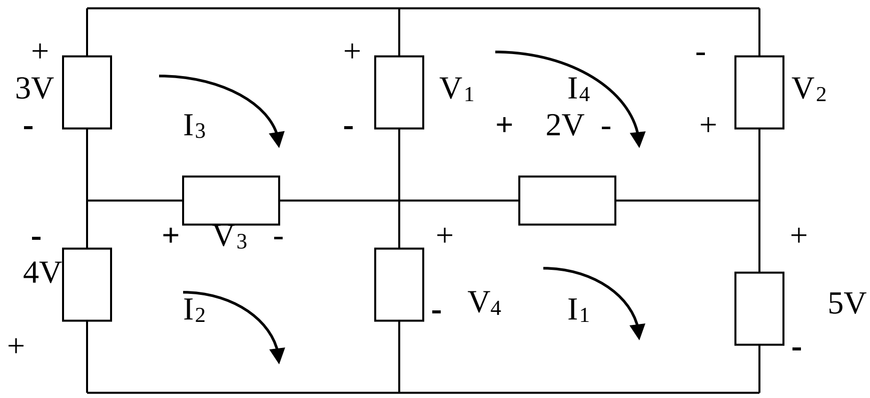
At node 3,

$$7+I_4=I_3 \quad \longrightarrow \quad I_3=7-2=5A$$

Hence,

$$\underline{I_1=12A, \quad I_2=-10A, \quad I_3=5A, \quad I_4=-2A}$$

Solution 2.14



For mesh 1,

$-V_4 + 2 + 5 = 0 \quad \longrightarrow \quad V_4 = 7V$

For mesh 2,

$+4 + V_3 + V_4 = 0 \quad \longrightarrow \quad V_3 = -4 - 7 = -11V$

For mesh 3,

$-3 + V_1 - V_3 = 0 \quad \longrightarrow \quad V_1 = V_3 + 3 = -8V$

For mesh 4,

$-V_1 - V_2 - 2 = 0 \quad \longrightarrow \quad V_2 = -V_1 - 2 = 6V$

Thus,

$\underline{V_1 = -8V, \quad V_2 = 6V, \quad V_3 = -11V, \quad V_4 = 7V}$

Solution 2.15

Calculate v and i_x in the circuit of Fig. 2.79.

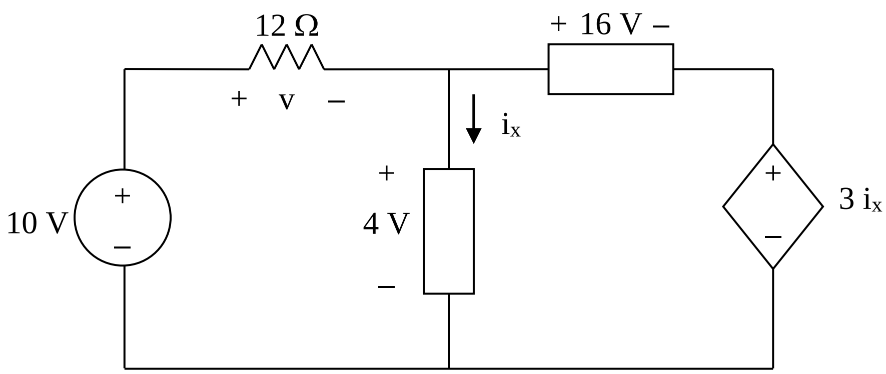


Figure 2.79
For Prob. 2.15.

Solution

For loop 1, $-10 + v + 4 = 0$, $v =$ **6 V**

For loop 2, $-4 + 16 + 3i_x = 0$, $i_x =$ **–4 A**

Solution 2.16

Determine $\boldsymbol{V_o}$ in the circuit in Fig. 2.80.

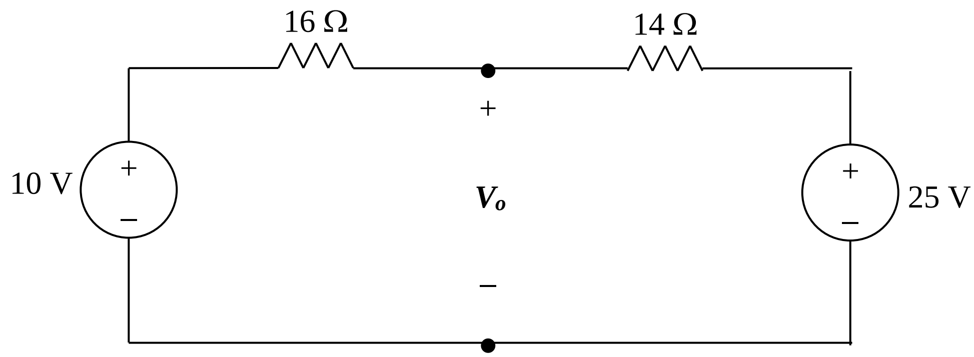


Figure 2.80
For Prob. 2.16.

Solution

Apply KVL,

$$-10 + (16+14)I + 25 = 0 \text{ or } 30I = 10-25 = - \text{ or } I = -15/30 = -500 \text{ mA}$$

Also,

$$-10 + 16I + \boldsymbol{V_o} = 0 \text{ or } \boldsymbol{V_o} = 10 - 16(-0.5) = 10+8 = \mathbf{18\ V}$$

Problem 2.17

Obtain v_1 through v_3 in the circuit in Fig. 2.81.

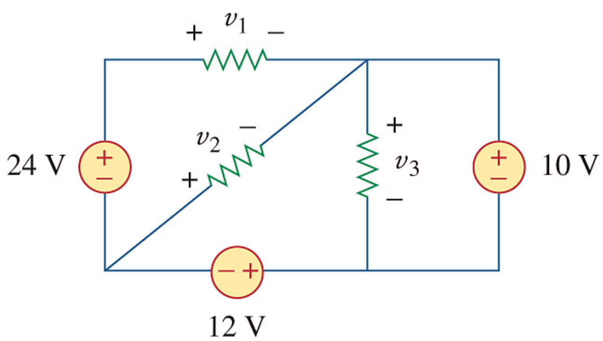


Figure 2.81
For Prob. 2.17.

Solution 2.18

Find *I* and *V* in the circuit of Fig. 2.82.

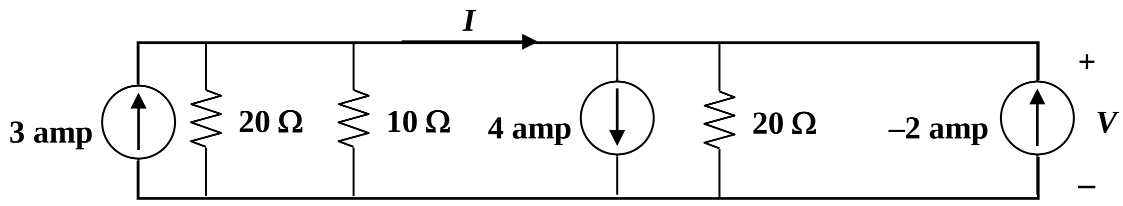


Figure 2.82
For Prob. 2.18.

Solution.

Step 1. We can make use of both Kirchhoff's KVL and KCL. KVL tells us that the voltage across all the elements of this circuit is the same in every case. Ohm's Law tells us that the current in each resistor is equal to V/R. Finally we can use KCL to find I.

Applying KCL and summing all the current flowing out of the top node and setting it to zero we get, $-3 + [V/20] + [V/10] + 4 + [V/20] - [-2] = 0$.

Finally at the node to the left of I we can write the following node equation which will give us I, $-3 + [V/20] + [V/10] + I = 0$.

Step 2. $[0.05+0.1+0.05]V = 0.2V = 3-4-2 = -3$ or $V =$ **–15 volts**.

$I = 3-V[0.05+0.1] = 3-[-15]0.15 =$ **5.25 amps**.

Solution 2.19

Applying KVL around the loop, we obtain

$$-(-8) - 12 + 10 + 3i = 0 \longrightarrow \mathbf{i = -2A}$$

Power dissipated by the resistor:

$$p_{3\Omega} = i^2R = 4(3) = \mathbf{12W}$$

Power supplied by the sources:

$$p_{12V} = 12\,((-2)) = \mathbf{-24W}$$

$$p_{10V} = 10\,(-(-2)) = \mathbf{20W}$$

$$p_{8V} = (-8)(-2) = \mathbf{16W}$$

Solution 2.20

Determine i_o in the circuit of Fig. 2.84.

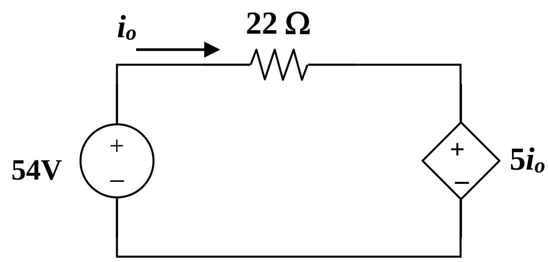


Figure 2.84
For Prob. 2.20

Solution

Applying KVL around the loop,

$-54 + 22\boldsymbol{i_o} + 5\boldsymbol{i_o} = 0 \longrightarrow \boldsymbol{i_o} = \mathbf{4A}$

Solution 2.21

Applying KVL,

$$-15 + (1+5+2)I + 2 V_x = 0$$

But $V_x = 5I$,

$$-15 + 8I + 10I = 0, \qquad I = 5/6$$

$$V_x = 5I = 25/6 = \mathbf{4.167\ V}$$

Solution 2.22

Find $\boldsymbol{V_o}$ in the circuit in Fig. 2.86 and the power absorbed by the dependent source.

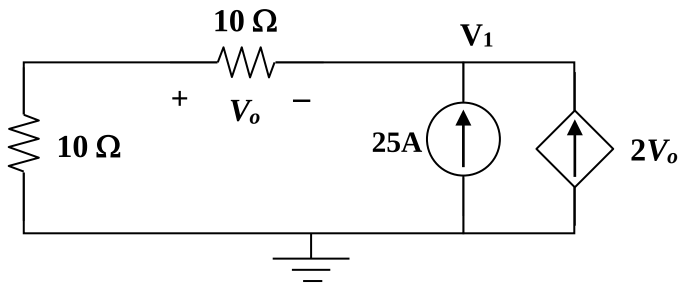


Figure 2.86
For Prob. 2.22

Solution

At the node, KCL requires that $[-V_o/10]+[-25]+[-2V_o] = 0$ or $2.1V_o = -25$

or V_o = **–11.905 V**

The current through the controlled source is $i = 2V_0 = -23.81$ A
and the voltage across it is $V_1 = (10+10)\ i_0$ (where $i_0 = -V_0/10$) = 20(11.905/10) = 23.81 V.

Hence,

$p_{\text{dependent source}} = V_1(-i) = 23.81\text{x}(-(-23.81))$ = **566.9 W**

Checking, $(25-23.81)^2(10+10) + (23.81)(-25) + 566.9 = 28.322 - 595.2 + 566.9 = 0.022$ which is equal zero since we are using four places of accuracy!

Solution 2.23

In the circuit shown in Fig. 2.87, determine v_x and the power absorbed by the 60-Ω resistor.

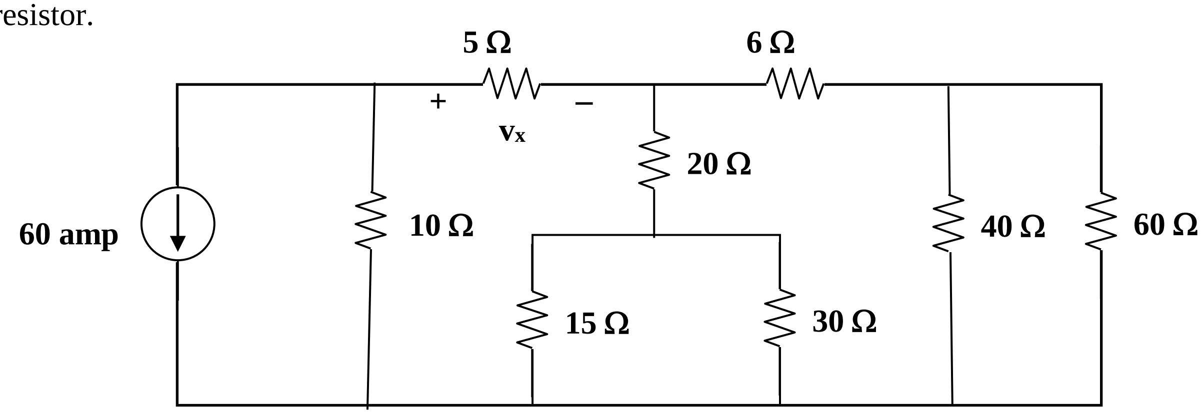
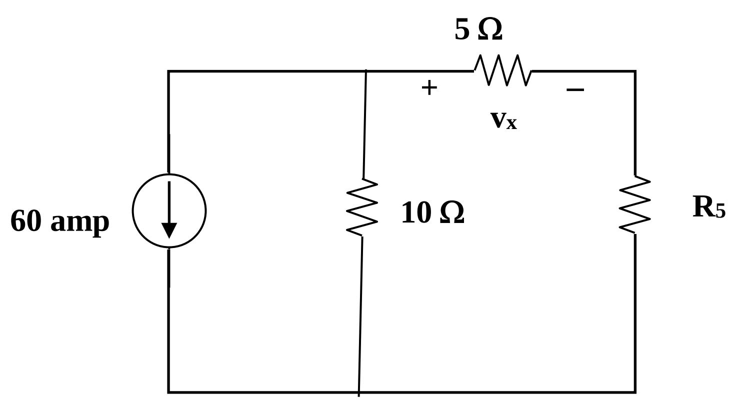


Figure 2.87
For Prob. 2.23.

Step 1. Although we could directly use Kirchhoff's current law to solve this, it will be easier if we reduce the circuit first.

The reduced circuit looks like this,



$R_1 = 40x60/(40+60)$
$R_2 = 6 + R_1$
$R_3 = 15x30/(15+30)$
$R_4 = 20+R_3$
$R_5 = R_2R_4/(R_2+R_4)$

Letting $V_{10} = v_x + V_{R5}$ and using Kirchhoff's current law, we get

$60 + V_{10}/10 + V_{10}/(5+R_5) = 0$
$60 + V_{10}/10 + V_{10}/20 = 0$
$V_{10} = –60x20/3 = –400$ volts

We could have also used current division to find the current through the 5 Ω resistor, however, $i_5 = V_{10} /(5+R_5)$ and $v_x = 5i_5$

Calculating the power delivered to the 60-ohm resistor requires that we find the voltage across the resistor. $V_{R5} = V_{10} – v_x$; using voltage division we get $V_{60} = [V_{R5} /(6+R_1)]R_1$. Finally $P_{60} = (V_{60})^2/60$.

Step 2.

$R_1 = 40x60/(40+60) = 2400/100 = 24$;
$R_2 = 6 + R_1 = 6+24 = 30$;
$R_3 = 15x30/(15+30)\ 450/45 = 10$;
$R_4 = 20+R_3 = 20+10 = 30$;
$R_5 = R_2R_4/(R_2+R_4) = 30x30/(30+30) = 15$.

Now, we have $60 + (V_{10}/10) + (V_{10}/(20)) = 0$ or $V_{10} = –60x20/3 = –400$ and $i_{10} = –400/20 = –20$ and

$$v_x = 5i_5 = 5(–20) = \mathbf{–100\ volts}.$$

$V_{R5} = V_{10} – v_x = –400 – (–100) = –300$; using voltage division we get $V_{60} = [V_{R5} /(6+R_1)]R_1. = [–300/30]24 = –240$. Finally,

$$P_{60} = (V_{60})^2/60 = (–240)^2/60 = \mathbf{960\ watts}.$$

Solution 2.24

(a) $$I_0 = \frac{V_s}{R_1 + R_2}$$

$$V_0 = -\alpha I_0 \left(R_3 \| R_4\right) = -\frac{\alpha V_s}{R_1 + R_2} \cdot \frac{R_3 R_4}{R_3 + R_4}$$

$$\boldsymbol{\frac{V_0}{Vs} = \frac{-\alpha R_3 R_4}{(R_1 + R_2)(R_3 + R_4)}}$$

(b) If $R_1 = R_2 = R_3 = R_4 = R$,

$$\left|\frac{V_0}{V_S}\right| = \frac{\alpha}{2R} \cdot \frac{R}{2} = \frac{\alpha}{4} = 10 \longrightarrow \alpha = \mathbf{40}$$

Solution 2.25

$V_0 = 5 \text{ x } 10^{-3} \text{ x } 10 \text{ x } 10^3 = 50V$

Using current division,

$$I_{20} = \frac{5}{5+20}(0.01x50) = \mathbf{0.1\ A}$$

$$V_{20} = 20 \text{ x } 0.1 \text{ kV} = \mathbf{2\ kV}$$

$$p_{20} = I_{20}\, V_{20} = \mathbf{0.2\ kW}$$

Solution 2.26

For the circuit in Fig. 2.90, $\boldsymbol{i_o}$ = 5 A. Calculate $\boldsymbol{i_x}$ and the total power absorbed by the entire circuit.

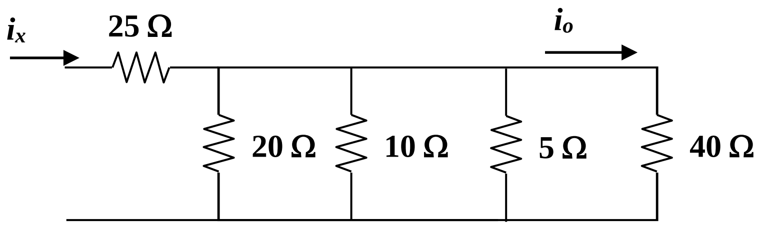


Figure 2.90
For Prob. 2.26.

Solution

Step 1. $V_{40} = 40i_o$ and we can combine the four resistors in parallel to find the equivalent resistance and we get $(1/R_{eq}) = (1/20) + (1/10) + (1/5) + (1/40)$.

This leads to $i_x = V_{40}/R_{eq}$ and $P = (i_x)^2(25+R_{eq})$.

Step 2. V_{40} = 40x5 = 200 volts and $(1/R_{eq}) = (1/20) + (1/10) + (1/5) + (1/40) = 0.05 + 0.1 + 0.2 + 0.025 = 0.375$ or $R_{eq} = 2.667\ \Omega$ and $i_x = 200/R_{eq} = 75$ amps.

$$P = (75)^2(25+2.667) = \mathbf{155.62\ kW}.$$

Solution 2.27

Calculate $\boldsymbol{I_o}$ in the circuit of Fig. 2.91.

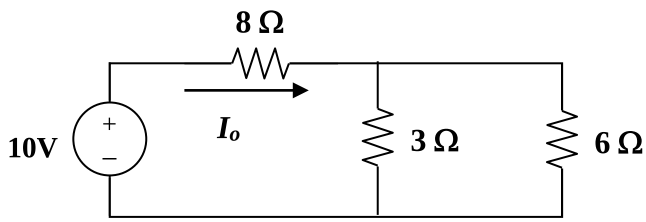


Figure 2.91
For Prob. 2.27.

Solution

The 3-ohm resistor is in parallel with the c-ohm resistor and can be replaced by a [(3x6)/(3+6)] = 2-ohm resistor. Therefore,

$\boldsymbol{I_o}$ = 10/(8+2) = **1 A**.

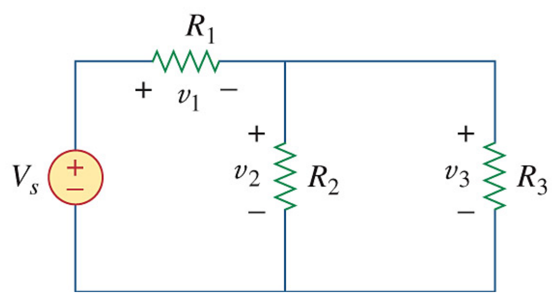
Solution 2.28

Design a problem, using Fig. 2.92, to help other students better understand series and parallel circuits.

Although there is no correct way to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find v_1, v_2, and v_3 in the circuit in Fig. 2.92.



Solution

We first combine the two resistors in parallel

$$15 \| 10 = 6\ \Omega$$

We now apply voltage division,

$$v_1 = \frac{14}{14+6}(40) = \underline{\mathbf{28\ V}}$$

$$v_2 = v_3 = \frac{6}{14+6}(40) = 12\ \text{V}$$

Hence, $v_1 = \mathbf{28\ V}$, $v_2 = \mathbf{12\ V}$, $v_s = \mathbf{12\ V}$

Solution 2.29

All resistors (R) in Fig. 2.93 are 10 Ω each. Find R_{eq}.

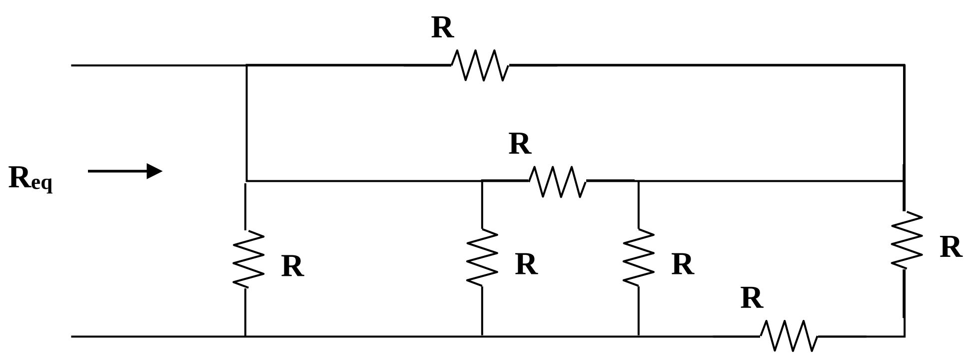


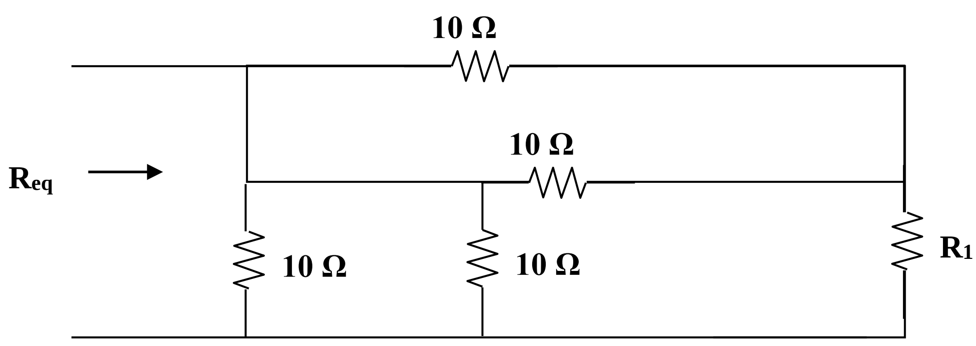
Figure 2.93
For Prob. 2.29.

Solution

Step 1. All we need to do is to combine all the resistors in series and in parallel.

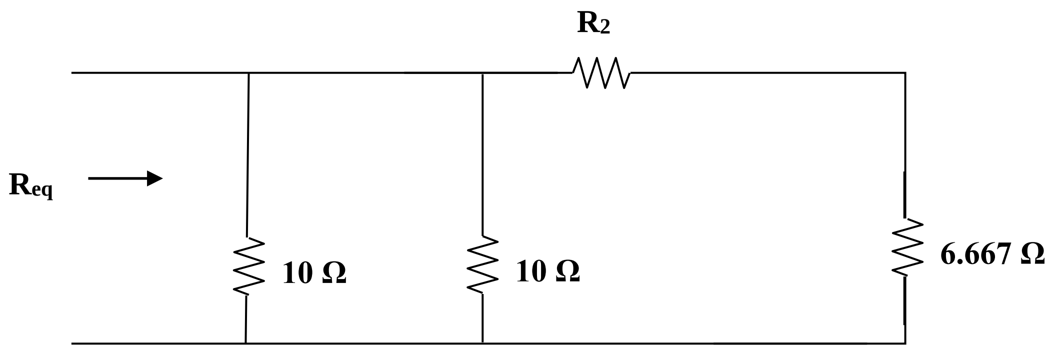
$$R_{eq} = \frac{\left(\frac{R(R)}{R+R}\right)\left(\frac{R(R)}{R+R}+\frac{R(R+R)}{R+R+R}\right)}{\left(\frac{R(R)}{R+R}\right)+\left(\frac{R(R)}{R+R}+\frac{R(R+R)}{R+R+R}\right)}$$ which can be derived by inspection. We will look at a simpler approach after we get the answer.

Step 2. $R_{eq} = \frac{5[(5+6.667)]}{5+5+6.667} = \frac{58.335}{16.667}$ = **3.5 Ω**.

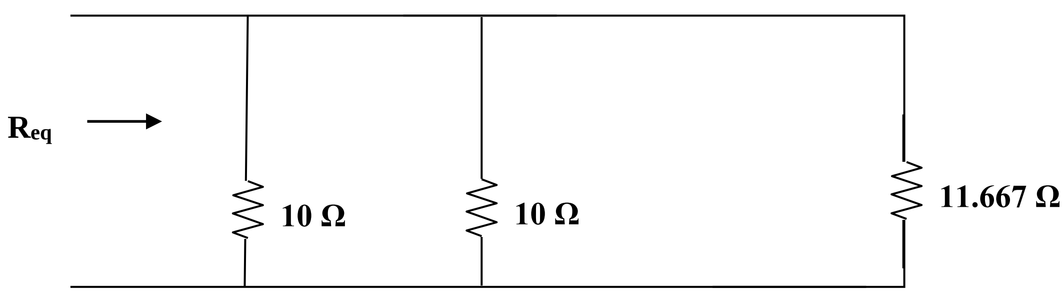


Checking we get,

$R_1 = 10(20)/(10+20) = 6.667\ \Omega$.



We get $R_2 = 10(10)/(10+10) = 5\ \Omega$.



Finally we get (noting that 10 in parallel with 10 gives us 5 Ω,

$R_{eq} = 5(11.667)/(5+11.667) = \mathbf{3.5\ \Omega}$.

Solution 2.30

Find $\boldsymbol{R_{eq}}$ for the circuit in Fig. 2.94.

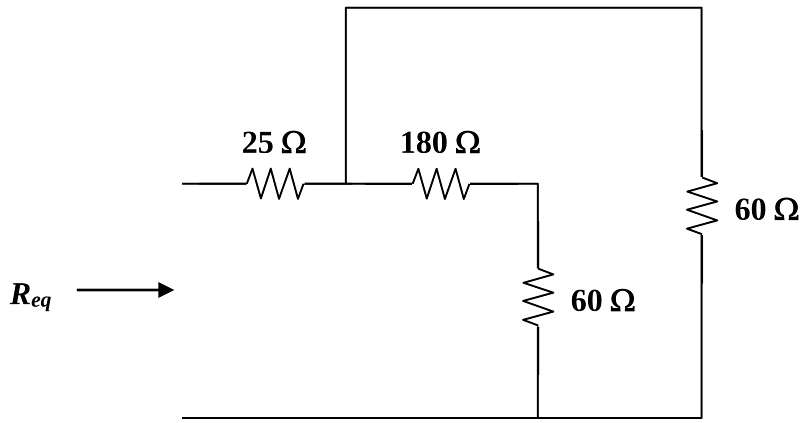


Figure 2.94
For Prob. 2.30.

Solution

We start by combining the 180-ohm resistor with the 60-ohm resistor which in turn is in parallel with the 60-ohm resistor or = [60(180+60)/(60+180+60)] = 48.

Thus,
R_{eq} = 25+48 = **73 Ω**.

Solution 2.31

$$R_{eq} = 3 + 2 // 4 // 1 = 3 + \frac{1}{1/2 + 1/4 + 1} = 3.5714$$

i_1 = 200/3.5714 = **56 A**

v_1 = 0.5714xi_1 = 32 V and i_2 = 32/4 = **8 A**

i_4 = 32/1 = **32 A**; i_5 = 32/2 = **16 A**; and i_3 = 32+16 = **48 A**

Solution 2.32

Find $\boldsymbol{i_1}$ through $\boldsymbol{i_4}$ in the circuit in Fig. 2.96.

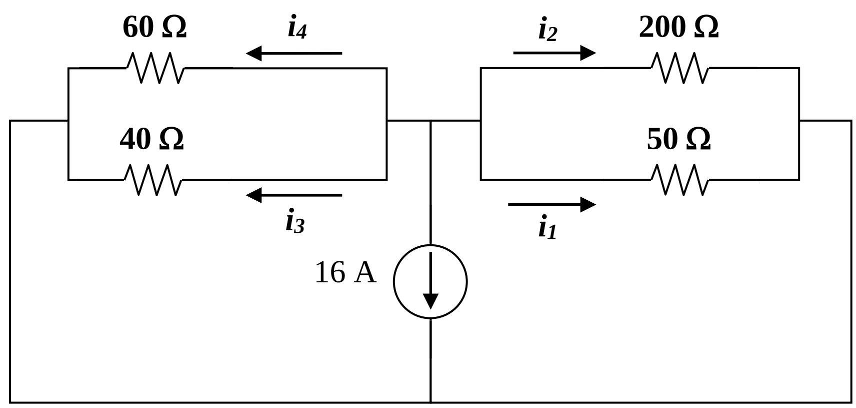


Figure 2.96
For Prob. 2.32.

Solution

We first combine resistors in parallel.

$$40\|60 = \frac{40x60}{100} = 24\ \Omega \text{ and } \ 50\|200 = \frac{50x200}{250} = 40\ \Omega$$

Using current division principle,

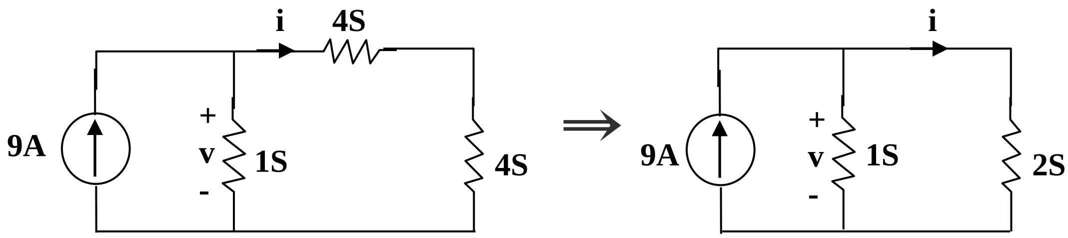
$$i_1 + i_2 = \frac{24}{24+40}(-16) = -6A, i_3 + i_4 = \frac{40}{64}(-16) = -10A$$

$$i_1 = \frac{200}{250}(6) = \mathbf{-4.8\ A} \text{ and } i_2 = \frac{50}{250}(-6) = \mathbf{-1.2\ A}$$

$$i_3 = \frac{60}{100}(-10) = \mathbf{-6\ A} \text{ and } i_4 = \frac{40}{100}(-10) = \mathbf{-4\ A}$$

Solution 2.33

Combining the conductance leads to the equivalent circuit below



$$6S \| 3S = \frac{6x3}{9} = 2\text{S} \text{ and } 2\text{S} + 2\text{S} = 4\text{S}$$

Using current division,

$$\text{i} = \frac{1}{1+\frac{1}{2}}(9) = \textbf{6 A}, \text{ v} = 3(1) = \textbf{3 V}$$

Solution 2.34

Using series/parallel resistance combination, find the equivalent resistance seen by the source in the circuit of Fig. 2.98. Find the overall absorbed power by the resistor network.

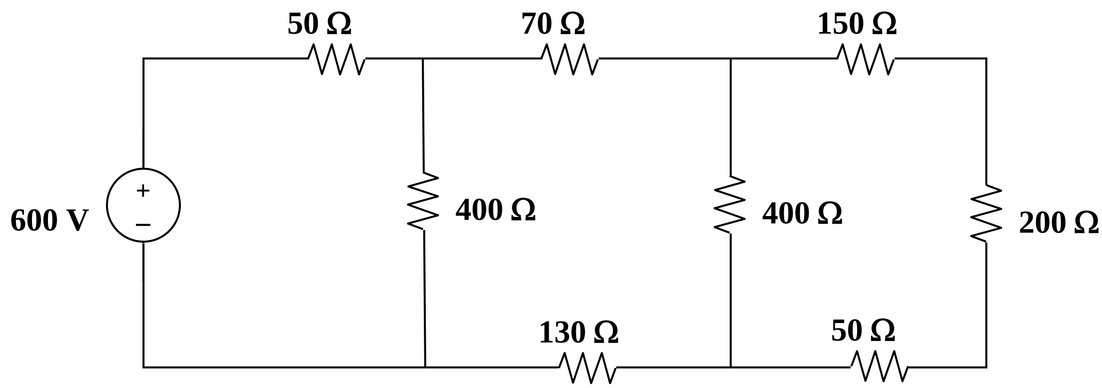


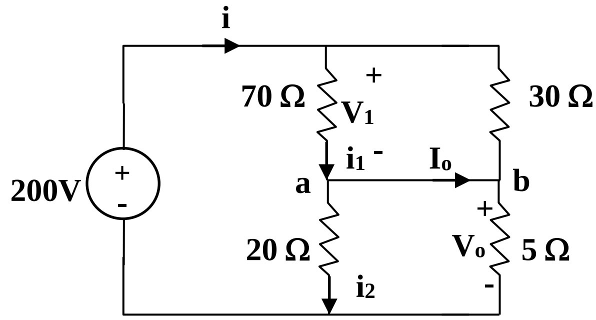
Figure 2.98
For Prob. 2.34.

Step 1. Let $R_1 = 400(150+200+50)/(400+150+200+50)$ and $R_2 = 400(70+R_1+130)/(400+70+R_1+130)$. Thus, the resistance seen by the source is equal to $R_{eq} = 50+R_2$ and total power delivered to the circuit = $(600)^2/R_{eq}$.

Step 2. $R_1 = 400\text{x}400/800 = 200$ and $R_2 = 400\text{x}400/800 = 200$ and $R_{eq} = 50+200 = \mathbf{250\ \Omega}$.

$$P = 360{,}000/250 = \mathbf{1.44\ kW}.$$

Solution 2.35



Combining the resistors that are in parallel,

$$70 \| 30 = \frac{70x30}{100} = 21\Omega \ , \qquad 20 \| 5 = \frac{20x5}{25} = 4\ \Omega$$

$$i = \frac{200}{21+4} = 8\ A$$

$v_1 = 21i = 168$ V, $v_o = 4i = 32$ V

$$i_1 = \frac{v_1}{70} = 2.4\ A,\ i_2 = \frac{v_o}{20} = 1.6\ A$$

At node a, KCL must be satisfied

$$i_1 = i_2 + I_o \longrightarrow 2.4 = 1.6 + I_o \longrightarrow I_o = 0.8\ A$$

Hence,

v_o = **32 V** and I_o = **800 mA**

Solution 2.36

$$20//(30+50) = 16, \quad 24 + 16 = 40, \quad 60//20 = 15$$
$$R_{eq} = 80+(15+25)40 = 80+20 = 100\ \Omega$$

$$i = 20/100 = 0.2\ A$$

If i_1 is the current through the 24-Ω resistor and i_o is the current through the 50-Ω resistor, using current division gives

$$i_1 = [40/(40+40)]0.2 = 0.1 \text{ and } i_o = [20/(20+80)]0.1 = 0.02\ A \text{ or}$$

$v_o = 30i_o = 30x0.02 =$ **600 mV**.

Solution 2.37

Given the circuit in Fig. 2.101 and that the resistance, R_{eq}, looking into the circuit from the left is equal to 100 Ω, determine the value of R_1.

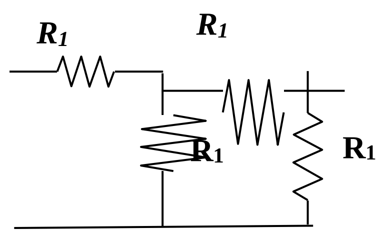


Figure 2.101
For Prob. 2.37.

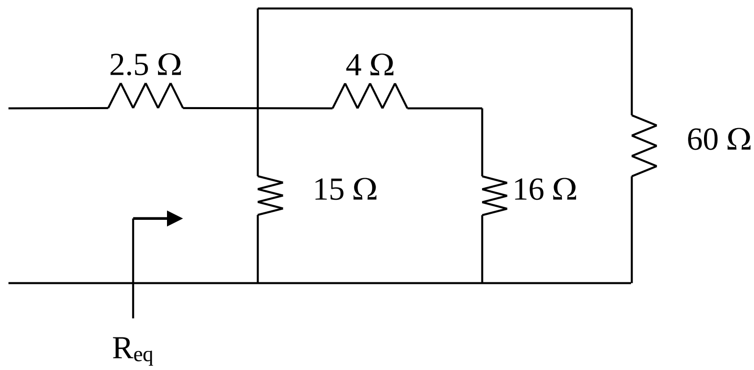
Step 1. First we calculate R_{eq} in terms of R_1. Then we set R_{eq} to 100 ohms and solve for R_1.

$R_{eq} = R_1 + R_1(R_1+R_1)/(R_1+R_1+R_1) = R_1[1+1(2)/3]$

Step 2. $100 = R_1(3+2)/3$ or $R_1 =$ **60 Ω**.

Solution 2.38

$20//80 = 80x20/100 = 16$, $6//12 = 6x12/18 = 4$
The circuit is reduced to that shown below.



$(4 + 16)//60 = 20x60/80 = 15$

$R_{eq} = 2.5+15||15 = 2.5+7.5 =$ **$10\ \Omega$** and

$i_o = 35/10 =$ **3.5 A**.

Solution 2.39

Evaluate R_{eq} looking into each set of terminals for each of the circuits shown in Fig. 2.103.

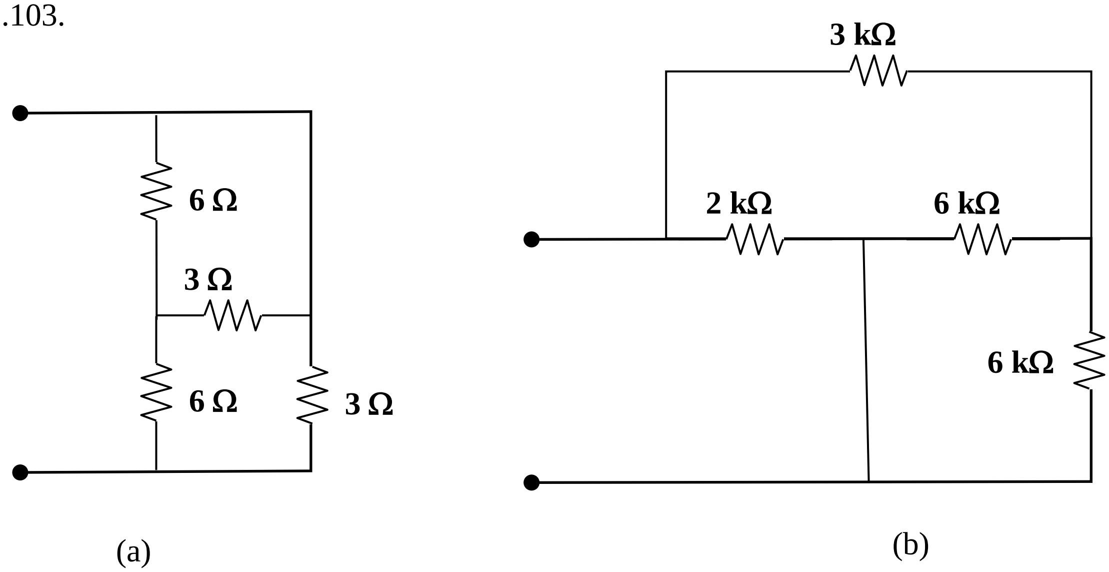


Figure 2.103
For Prob. 2.39.

Step 1. We need to remember that two resistors are in parallel if they are connected together at both the top and bottom and two resistors are connected in series if they are connected only at one end with nothing else connected at that point. With that in mind we can calculate each of the equivalent resistances.

(a) $R_{eqa} = \dfrac{3\left(6+\left(\dfrac{3x6}{(3+6)}\right)\right)}{3+6+\left(\dfrac{3x6}{(3+6)}\right)}$ and (b) $R_{eqb} = \dfrac{2k\left(3k+\left(\dfrac{6kx6k}{(6k+6k)}\right)\right)}{2k+3k+\left(\dfrac{6kx6k}{(6k+6k)}\right)}$.

Step 2. (a) R_{eqa} = 3x8/11 = **2.182 Ω** and (b) R_{eqb} = **1.5 kΩ**.

Solution 2.40

Req = $8 + 4 \| (2 + 6 \| 3) = 8 + 2 =$ **10 Ω**

$$I = \frac{15}{R_{eq}} = \frac{15}{10} = \mathbf{1.5\ A}$$

Solution 2.41

Let R_0 = combination of three 12Ω resistors in parallel

$$\frac{1}{R_o} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} \longrightarrow R_o = 4$$

$$R_{eq} = 30 + 60 \| (10 + R_0 + R) = 30 + 60 \| (14 + R)$$

$$50 = 30 + \frac{60(14 + R)}{74 + R} \longrightarrow 74 + R = 42 + 3R$$

or R = **16 Ω**

Solution 2.42

(a) $R_{ab} = 5 \| (8 + 20 \| 30) = 5 \| (8 + 12) = \dfrac{5x20}{25} = \mathbf{4\ \Omega}$

(b) $R_{ab} = 2 + 4 \| (5 + 3) \| 8 + 5 \| 10 \| 4 = 2 + 4 \| 4 + 5 \| 2.857 = 2 + 2 + 1.8181 = \mathbf{5.818\ \Omega}$

Solution 2.43

(a) $R_{ab} = 5\|20 + 10\|40 = \dfrac{5x20}{25} + \dfrac{400}{50} = 4 + 8 = \mathbf{12\ \Omega}$

(b) $60\|20\|30 = \left(\dfrac{1}{60} + \dfrac{1}{20} + \dfrac{1}{30}\right)^{-1} = \dfrac{60}{6} = 10\Omega$

$R_{ab} = 80\|(10 + 10) = \dfrac{80 + 20}{100} = \mathbf{16\ \Omega}$

Solution 2.44

For the circuits in Fig. 2.108, obtain the equivalent resistance at terminals *a-b*.

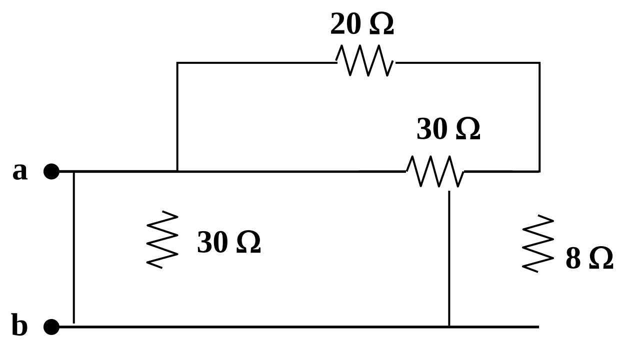


Figure 2.108
For Prob. 2.44

Solution

Step 1. First we note that the 20 Ω and 30 Ω resistors are in parallel and can be replaced by a [(20x30)/(20+30)] resistor which in now in series with the 8 Ω resistor which gives R_1. Now we R_1 in parallel with the 30 Ω which gives us R_{ab} = [(R_1x30)/(R_1+30)].

Step 2. $R_1 = (600/50) + 8 = 12 + 8 = 20\ \Omega$ and

$R_{ab} = 20\text{x}30/(20+30) = \mathbf{12\ \Omega}$.

Solution 2.45

(a) 10//40 = 8, 20//30 = 12, 8//12 = 4.8

$R_{ab} = 5 + 50 + 4.8 = \underline{59.8\Omega}$

(b) 12 and 60 ohm resistors are in parallel. Hence, 12//60 = 10 ohm. This 10 ohm and 20 ohm are in series to give 30 ohm. This is in parallel with 30 ohm to give 30//30 = 15 ohm. And 25//(15+10) = 12.5. Thus,

$R_{ab} = 5 + 12.8 + 15 = \underline{32.5\Omega}$

Solution 2.46

Find I in the circuit of Fig. 2.110.

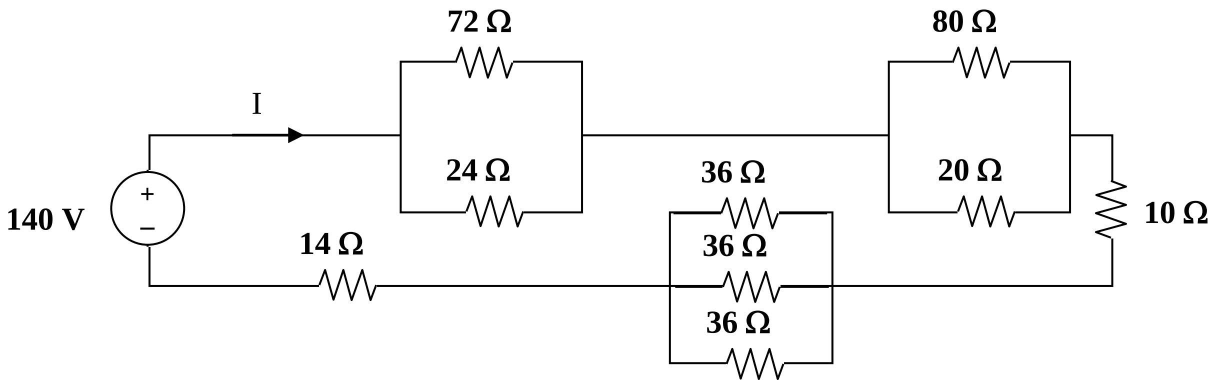


Figure 2.110
For Prob. 2.46.

Solution

Step 1. First we need to determine the total resistance that the source sees.

$R_{eq} = \frac{24x72}{24+72} + \frac{20x80}{20+80} + 10 + \frac{1}{\frac{1}{36}+\frac{1}{36}+\frac{1}{36}} + 14$ and $I = 140/R_{eq}$.

Step 2. R_{eq} = 18+16+10+12+14 = 70 Ω and I = 140/70 = **2 amps**.

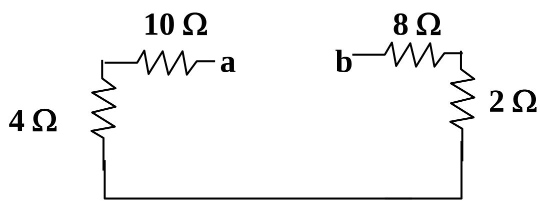
R_{eq} = 12 + 5||20 + [1/((1/15)+(1/15)+(1/15))] + 5 + 24||8
= 12 + 4 + 5 + 5 + 6 = 32 Ω

I = 80/32 = **2.5 A**

Solution 2.47

$$5 \| 20 = \frac{5x20}{25} = 4\Omega$$

$$6 \| 3 = \frac{6x3}{9} = 2\Omega$$



$R_{ab} = 10 + 4 + 2 + 8 = \mathbf{24\ \Omega}$

Solution 2.48

(a) $$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{100+100+100}{10} = 30$$

$R_a = R_b = R_c =$ **30 Ω**

(b) $$R_a = \frac{30x20 + 30x50 + 20x50}{30} = \frac{3100}{30} = 103.3\Omega$$

$$R_b = \frac{3100}{20} = 155\Omega, \quad R_c = \frac{3100}{50} = 62\Omega$$

$R_a =$ **103.3 Ω**, R_b = **155 Ω**, R_c = **62 Ω**

Solution 2.49

Transform the circuits in Fig. 2.113 from Δ to Y.

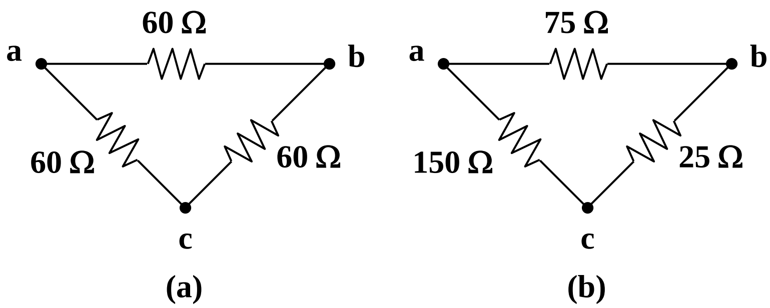


Figure 2.113
For Prob. 2.49.

Step 1. (a) $R_{an} = \frac{60x60}{60+60+60} = R_{bn} = R_{cn}$ and

(b) $R_{an} = \frac{150x75}{150+75+25}$; $R_{bn} = \frac{25x75}{150+75+25}$; $R_{cn} = \frac{150x25}{150+75+25}$.

Step 2. (a) R_{an} = **20 Ω**= $R_{bn} = R_{cn}$ and

(b) R_{an} = 11250/250 = **45 Ω**; R_{bn} = 1875/250 = **7.5 Ω**; and
R_{cn} = 3750/250 = **15 Ω**.

R_{an} R_{bn} R_{cn}

Solution 2.50

Design a problem to help other students better understand wye-delta transformations using Fig. 2.114.

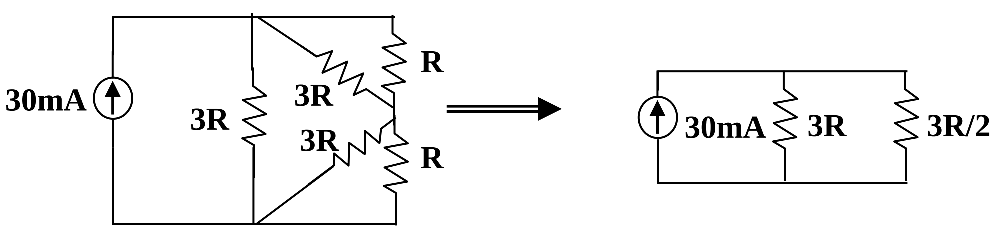
Although there is no correct way to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

What value of *R* in the circuit of Fig. 2.114 would cause the current source to deliver 800 mW to the resistors.

Solution

Using $R_\Delta = 3R_Y = 3R$, we obtain the equivalent circuit shown below:



$$3R \| R = \frac{3R\text{x}R}{4R} = \frac{3}{4}R$$

$$3R \| \left(\frac{3}{4}R + \frac{3}{4}R\right) = 3R \| \frac{3}{2}R = \frac{3R\text{x}\frac{3}{2}R}{3R + \frac{3}{2}R} = R$$

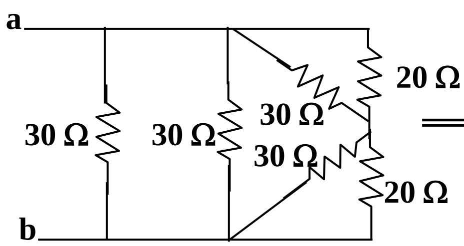
$P = I^2R \longrightarrow 800 \text{ x } 10^{-3} = (30 \text{ x } 10^{-3})^2 R$

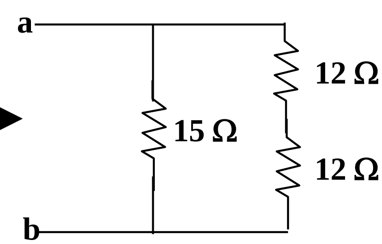
R = <u>**889 Ω**</u>

Solution 2.51

(a) $30\|30 = 15\Omega$ and $30\|20 = 30x20/(50) = 12\Omega$

$R_{ab} = 15\|(12+12) = 15x24/(39) = \mathbf{9.231\ \Omega}$





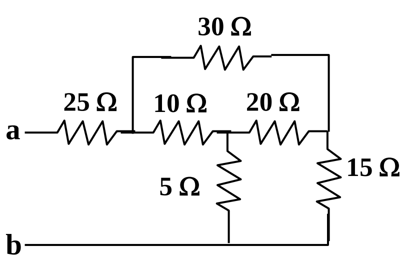
(b) Converting the T-sub network into its equivalent Δ network gives

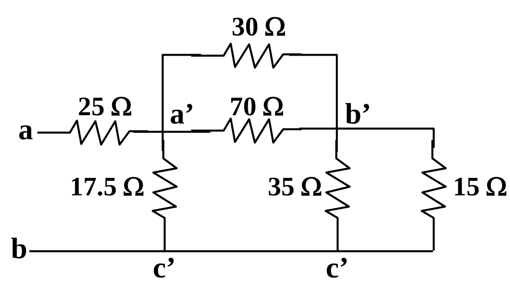
$R_{a'b'}$ = 10x20 + 20x5 + 5x10/(5) = 350/(5) = 70 Ω
$R_{b'c'}$ = 350/(10) = 35Ω, Ra'c' = 350/(20) = 17.5 Ω

Also $30\|70 = 30x70/(100) = 21\Omega$ and 35/(15) = 35x15/(50) = 10.5

$R_{ab} = 25 + 17.5\|(21+10.5) = 25 + 17.5\|31.5$

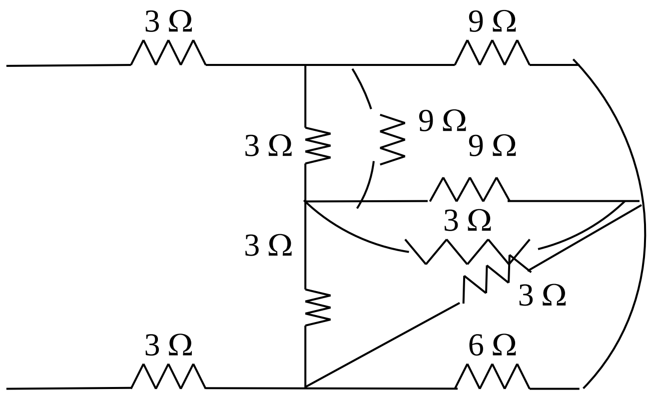
$R_{ab} = \mathbf{36.25\ \Omega}$



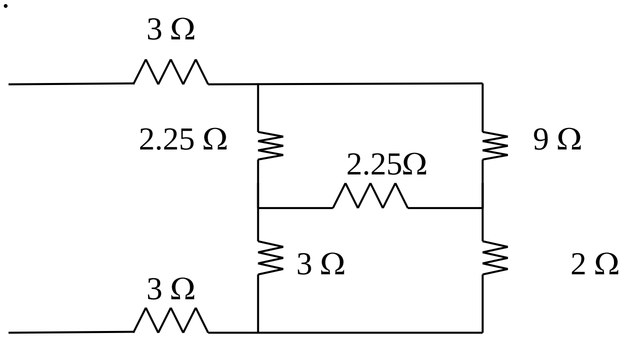


Solution 2.52

Converting the wye-subnetwork to delta-subnetwork, we obtain the circuit below.



3//1 = 3x1/4 = 0.75, 2//1 =2x1/3 = 0.6667. Combining these resistances leads to the circuit below.

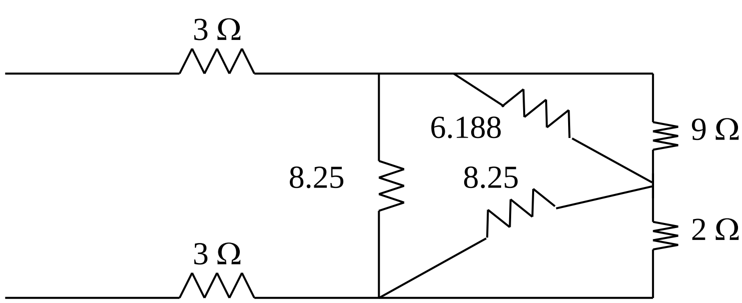


We now convert the wye-subnetwork to the delta-subnetwork.

R_a = [(2.25x3+2.25x3+2.25x2.25)/3] = 6.188 Ω

R_b = R_c = 18.562/2.25 = 8.25 Ω

This leads to the circuit below.

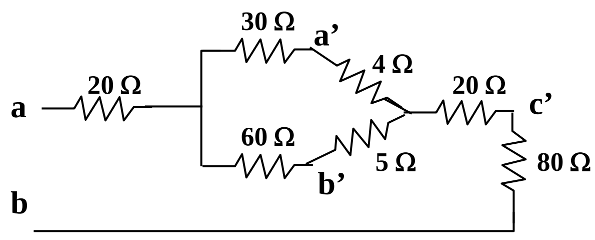


$R = 9||6.188+8.25||2 = 3.667+1.6098 = 5.277$

$R_{eq} = 3+3+8.25||5.277 = \mathbf{9.218\ \Omega}$.

Solution 2.53

(a) Converting one Δ to T yields the equivalent circuit below:



$$R_{a'n} = \frac{40x10}{40+10+50} = 4\Omega,\ \ R_{b'n} = \frac{10x50}{100} = 5\Omega,\ \ R_{c'n} = \frac{40x50}{100} = 20\Omega$$

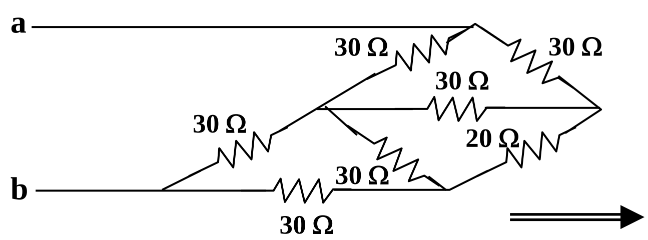
R_{ab} = 20 + 80 + 20 + $(30+4)\|(60+5) = 120 + 34\|65$

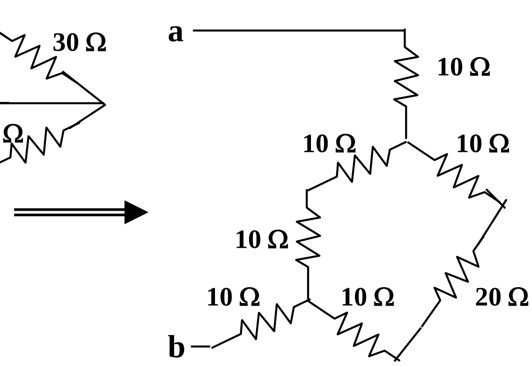
R_{ab} = **142.32 Ω**

(b) We combine the resistor in series and in parallel.

$$30\|(30+30) = \frac{30x60}{90} = 20\Omega$$

We convert the balanced Δs to Ts as shown below:





R_{ab} = 10 + $(10+10)\|(10+20+10) + 10 = 20 + 20\|40$

R_{ab} = 33.33 Ω

Solution 2.54

Consider the circuit in Fig. 2.118. Find the equivalent resistance at terminals:
(a) a-b, (b) c-d.

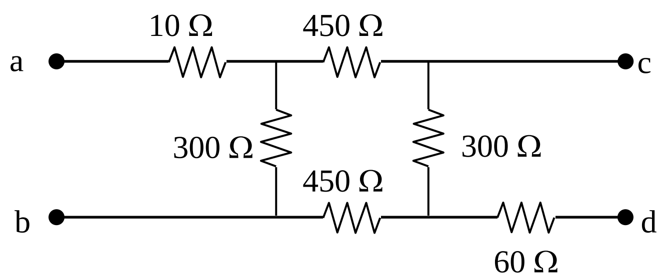


Figure 2.118
For Prob. 2.54.

Step 1. $R_{ab} = 10 + \dfrac{300x(450+300+450)}{300+450+300+450}$ nd $R_{cd} = \dfrac{300(450+300+450)}{300+450+300+450}$ 60.

Step 2. R_{ab} = 10+240 = **250 Ω** and R_{cd} = 240+60 = **300 Ω**.

Solution 2.55

Calculate I_o in the circuit of Fig. 2.119.

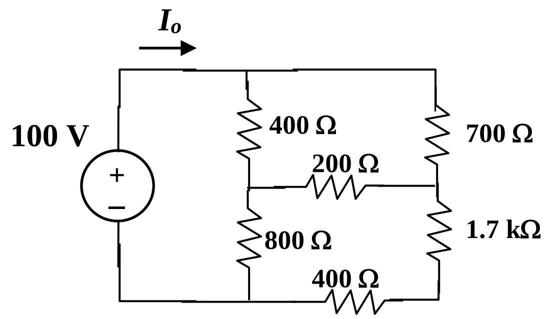
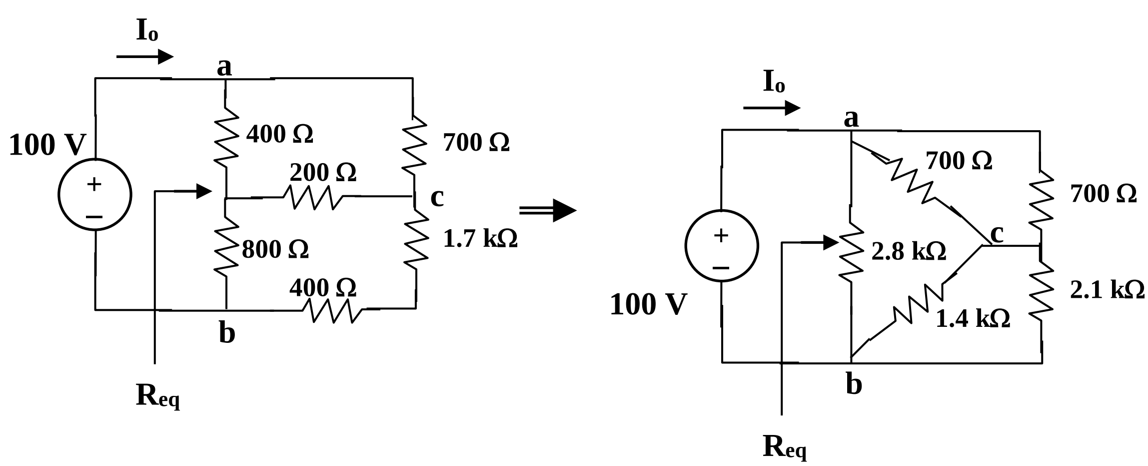


Figure 2.119
For Prob. 2.55.

Solution

Step 1. First we convert the T to Δ.



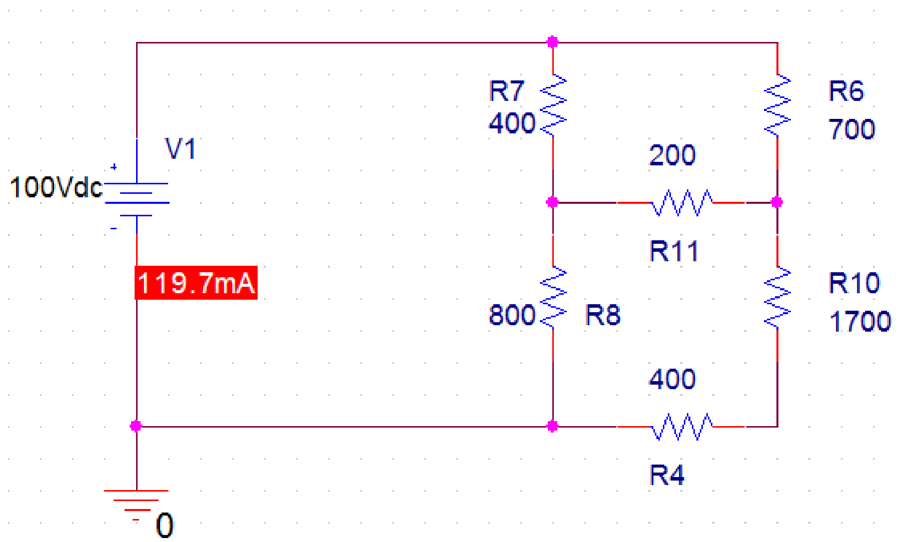
Next we let R_1 = 400; R_2 = 800; and R_3 = 200. Now we can calculate the values of the delta circuit. Let R_{num} = 400x800+800x200+200x400 and then we get R_{ab} = R_{num}/R_3; R_{bc} = R_{num}/R_1; R_{ac} = R_{num}/R_2. Finally $R_{eq} = \frac{2.8k\left[\frac{R_{ac}700}{R_{ac}+700}+\frac{R_{bc}1.7k}{R_{bc}+1.7k}\right]}{2.8k+\frac{R_{ac}700}{R_{ac}+700}+\frac{R_{bc}1.7k}{R_{bc}+1.7k}}$ and $I_o = 100/R_{eq}$.

Step 2. R_{num} = 400x800+800x200+200x400 = 560,000 and then we get R_{ab} = 560,000/200 = 2,800; R_{bc} = 560,000/400 = 1,400; R_{ac} = 560,000/800 = 700.

Let $R_{acb} = \left[\frac{R_{ac}700}{R_{ac}+700}+\frac{R_{bc}2.1k}{R_{bc}+2.1k}\right] = \left[\frac{700x700}{700+700}+\frac{1.4k2.1k}{1.4k+2.1k}\right] = 350+840 =$1190.

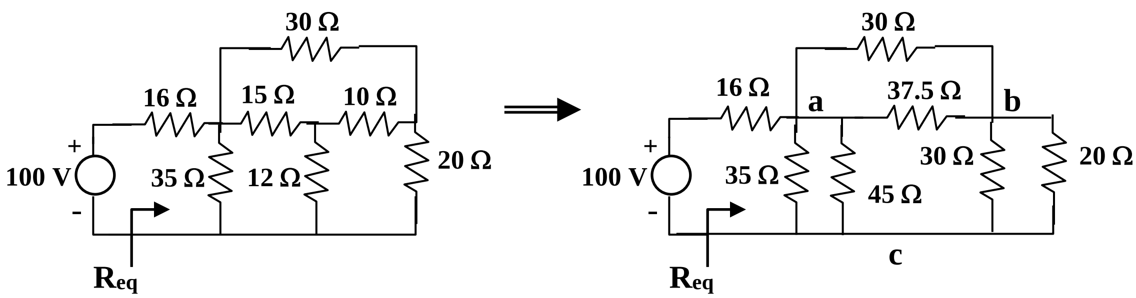
$R_{eq} = \frac{2.8k[1190]}{2.8k+1190} = \frac{3.332k}{3.99} = 835.1\ \Omega$ and $I_o = 100/835.1 =$ **119.75 mA**.

Checking with PSpice we get,



Solution 2.56

We need to find R_{eq} and apply voltage division. We first transform the Y network to Δ.



$$R_{ab} = \frac{15x10 + 10x12 + 12x15}{12} = \frac{450}{12} = 37.5\Omega$$

R_{ac} = 450/(10) = 45Ω, R_{bc} = 450/(15) = 30Ω

Combining the resistors in parallel,

$$30||20 = (600/50) = 12\ \Omega,$$

$$37.5||30 = (37.5x30/67.5) = 16.667\ \Omega$$

$$35||45 = (35x45/80) = 19.688\ \Omega$$

$$R_{eq} = 19.688||(12 + 16.667) = 11.672\Omega$$

By voltage division,

$$v = \frac{11.672}{11.672 + 16}100 = \underline{\mathbf{42.18\ V}}$$

Solution 2.57

Find R_{eq} and I in the circuit of Fig. 2.121.

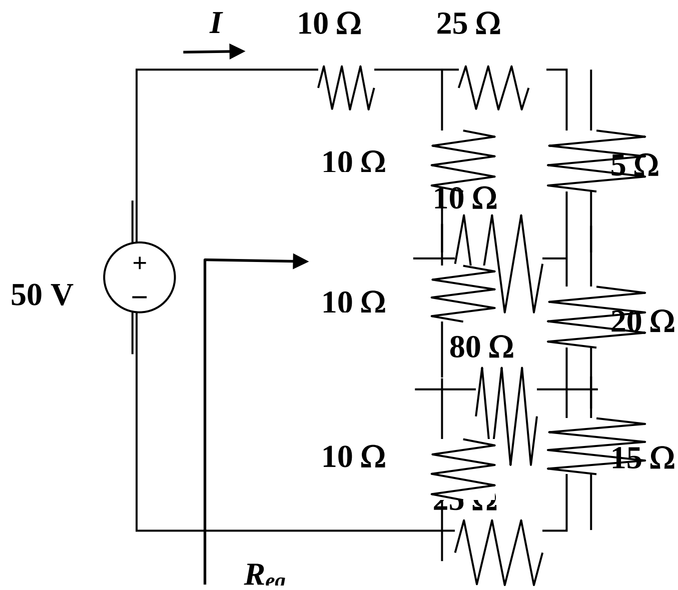
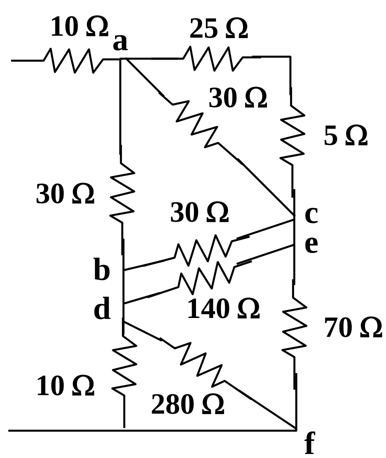


Figure 2.121
For Prob. 2.57.

Solution



$$R_{ab} = \frac{10x10+10x10+10x10}{10} = \frac{300}{10} = 30\ \Omega$$

R_{ac} = 216/(8) = 27Ω, R_{bc} = 36 Ω

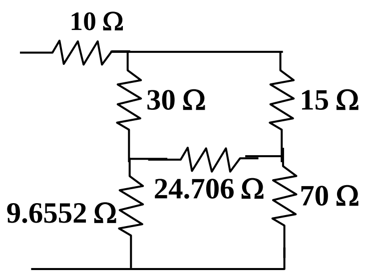
$$R_{de} = \frac{40x20+20x80+80x40}{40} = \frac{5600}{40} = 140\ \Omega$$

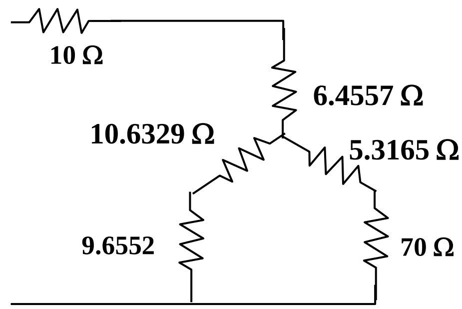
R_{ef} = 5600/(80) = 70 Ω, R_{df} = 5600/(20) = 280 Ω

Combining resistors in parallel,

$$30\|30 = \frac{900}{60} = 15\,\Omega,\quad 30\|140 = \frac{4200}{170} = 24.706\,\Omega$$

$$10\|280 = \frac{2800}{290} = 9.6552\,\Omega$$





$$R_{an} = \frac{30x15}{30+15+24.706} = \frac{450}{69.706} = 6.4557\ \Omega$$

$$R_{bn} = \frac{30x24.706}{69.706} = 10.6329\ \Omega$$

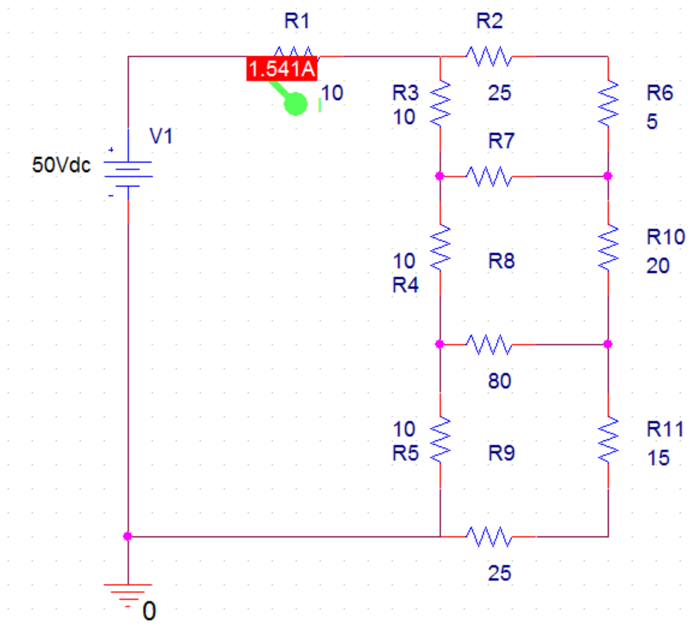
$$R_{cn} = \frac{24.706x15}{69.706} = 5.3165\ \Omega$$

$$R_{eq} = 10 + 6.4557 + (10.6329 + 9.6552)\|(5.3165 + 70)$$

$$= 16.4557 + 20.2881 \,||\, 75.3165 = 16.4557 + 1528.03/95.605$$

R_{eq} = **32.44 Ω** and I = 50/(R_{eq}) = **1.5413 A**

Checking with PSpice we get,



Solution 2.58

The 150 W light bulb in Fig. 2.122 is rated at 110 volts. Calculate the value of V_s to make the light bulb operate at its rated conditions.

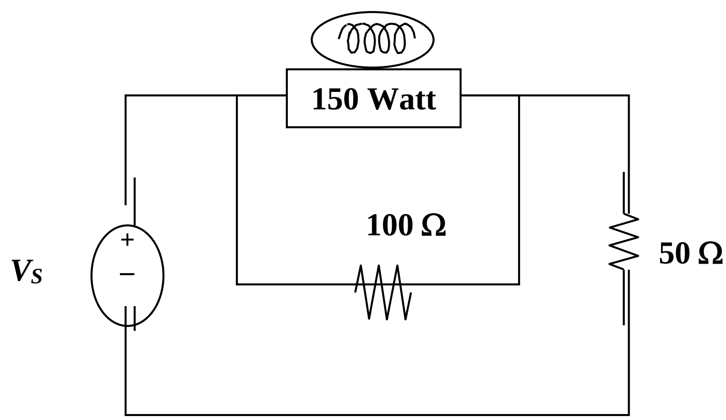
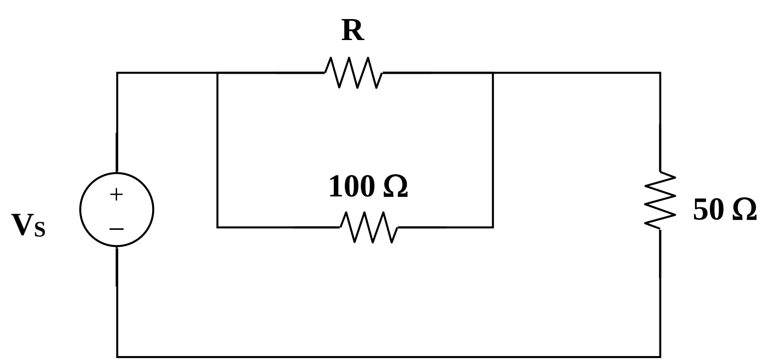


Figure 2.112
For Prob. 2.58.

Solution

Step 1. First we need to calculate the value of the resistance of the lightbulb. $150 = (110)^2/R$ or $R = (110)^2/150$. Now we have an equivalent circuit as shown below.



Next we note that $V_R = 110$ volts. The equivalent parallel resistance is equal to $100R/(100+R) = R_{eq}$. Now we have a simple voltage divider or $110 = V_s[R_{eq}/(R_{eq}+50)]$ and $V_s = 110(R_{eq}+50)/R_{eq}$.

Step 2. $R = 80.667$ and $R_{eq} = 8066.7/180.667 = 44.65$. This leads to,

$V_s = 110(94.65)/44.65 =$ **233.2 volts**.

Solution 2.59

An enterprising young man travels to Europe carrying three lightbulbs he had purchased in North America. The lightbulbs he has are a 100 watt lightbulb, a 60 watt lightbulb, and a 40 watt lightbulb. Each lightbulb is rated at 110 volts. He wishes to connect these to a 220 volt system that is found in Europe. For reasons we are not sure of, he connects the 40 watt lightbulb in series with a parallel combination of the 60 watt lightbulb and the 100 watt lightbulb as shown Fig. 2.123. How much power is actually being delivered to each lightbulb? What does he see when he first turns on the lightbulbs?

Is there a better way to connect these lightbulbs in order to have them work more effectively?

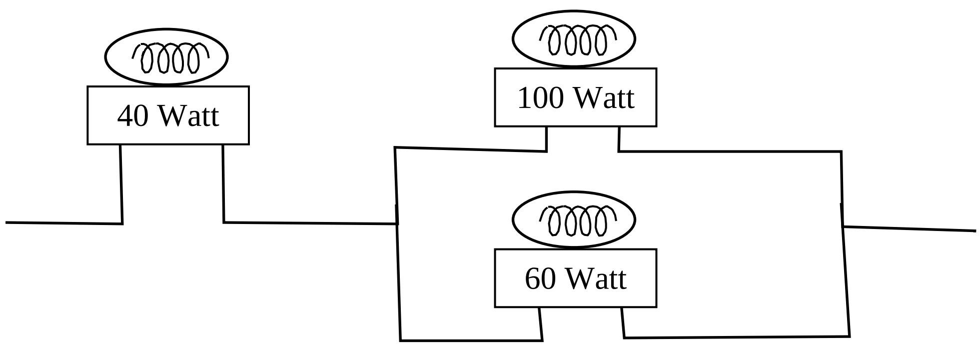


Figure 2.123
For Prob. 2.59.

Solution

Step 1. Using $p = v^2/R$, we can calculate the resistance of each bulb.

$R_{40W} = (110)^2/40$
$R_{60W} = (110)^2/60$
$R_{100W} = (110)^2/100$

The total resistance of the series parallel combination of the bulbs is
$R_{Tot} = R_{40W} + R_{100W}R_{60W}/(R_{100W} + R_{60W})$.

We can now calculate the voltage across each bulb and then calculate the power delivered to each. $V_{40W} = (220/R_{Tot})R_{40W}$ and the voltage across the other two, $V_{60||100}$, will equal $220 – V_{40W}$. $P_{40W} = (V_{40W})^2/R_{40W}$, $P_{60W} = (V_{60||100})^2/R_{60W}$, and $P_{100W} = (V_{60||100})^2/R_{100W}$.

Step 2.
$R_{40W} = (110)^2/40 = 12{,}100/40 = 302.5\ \Omega$
$R_{60W} = (110)^2/60 = 12{,}100/60 = 201.7\ \Omega$

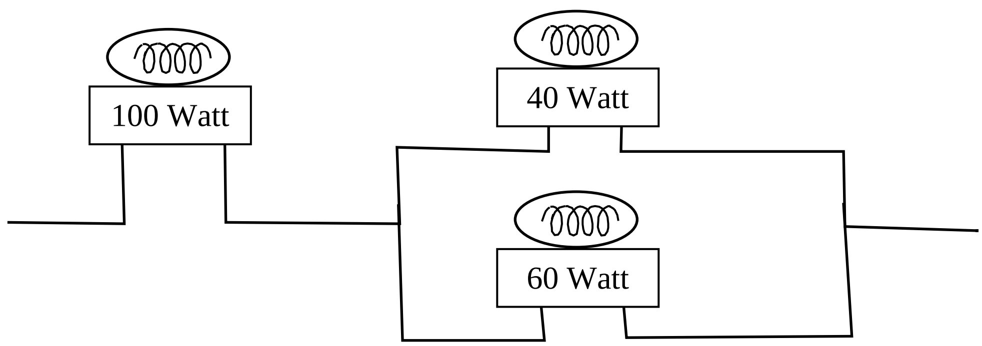
$R_{100W} = (110)^2/100 = 12,100/100 = 121\ \Omega$

$R_{Tot} = 302.5 + 121x201.7/(121+201.7) = 302.5 + 24,406/322.7 = 302.5+75.63 = 378.1\ \Omega$.

$V_{40W} = (220/R_{Tot})R_{40W} = 0.5819x302.5 = 176$ volts and the voltage across the other two, $V_{100||40}$, will equal 220 – V_{40W} = 44 volts. $P_{40W} = (V_{40W})^2/R_{40W} = 30976/302.5 =$ **102.4 watts**, $P_{60W} = (V_{60||100})^2/R_{60W} = 1936/201.7 =$ **9.6 watts**, and $P_{100W} = (V_{60||100})^2/R_{100W} = 1936/121 =$ **16 watts**.

Clearly when he flips the switch to light the bulbs the 40 watt bulb will flash bright as it burns out! Not a good thing to do!

Is there a better way to connect them? There are two other possibilities. However what if we place the bulb with the lowest resistance in series with a parallel combination of the other two what happens? Logic would dictate that this might give the best result. So, let us try the 100 watt bulb in series with the parallel combination of the other two as shown below.



Now we get, $R_{Tot} = 121 + 302.5x201.7/(302.5+201.7) = 121 + 61,014/504.2 = 121+121.01 = 242\ \Omega$. Without going further we can see that this will work since the resistances are essentially equal which means that each bulb will work as if they were individually connected to a 110 volt system.

$V_{100W} = (220/R_{Tot})121 = 110$ and the voltage across the other two, $V_{60||40}$, will equal 220 – V_{100W} = 110. $P_{100W} = (V_{100W})^2/121 =$ **100 watts**, $P_{60W} = (V_{60||40})^2/201.7 =$ **60 watts**, and $P_{40W} = (V_{60||40})^2/302.5 =$ **40 watts**. This will work!

Answer: P_{40W} = 102.4 W (means that this immediately burns out), P_{60W} = 9.6 W, P_{100W} = 16 W. The best way to wire the bulbs is to connect the 100 W bulb in series with a parallel combination of the 60 W bulb and the 40 W bulb.

Solution 2.60

If the three bulbs of Prob. 2.59 are connected in parallel to the 120-V source, calculate the current through each bulb.

Solution

Using $p = v^2/R$, we can calculate the resistance of each bulb.

$R_{30W} = (120)^2/30 = 14{,}400/30 = 480\ \Omega$

$R_{40W} = (120)^2/40 = 14{,}400/40 = 360\ \Omega$

$R_{50W} = (120)^2/50 = 14{,}400/50 = 288\ \Omega$

The current flowing through each bulb is 120/R.

$i_{30} = 120/480 =$ **250 mA**.

$i_{40} = 120/360 =$ **333.3 mA**.

$i_{30} = 120/288 =$ **416.7 mA**.

Unlike the light bulbs in 2.59, the lights will glow brightly!

Solution 2.61

There are three possibilities, but they must also satisfy the current range of 1.2 + 0.06 = 1.26 and 1.2 – 0.06 = 1.14.

(a) Use R_1 and R_2:

$R = R_1 \| R_2 = 80 \| 90 = 42.35\Omega$

$p = i^2R = 70W$

$i^2 = 70/42.35 = 1.6529$ or $i = 1.2857$ (which is outside our range)

cost = \$0.60 + \$0.90 = \$1.50

(b) Use R_1 and R_3:

$R = R_1 \| R_3 = 80 \| 100 = 44.44\ \Omega$

$i^2 = 70/44.44 = 1.5752$ or $i = 1.2551$ (which is within our range),

cost = \$1.35

(c) Use R_2 and R_3:

$R = R_2 \| R_3 = 90 \| 100 = 47.37\Omega$

$i^2 = 70/47.37 = 1.4777$ or $i = 1.2156$ (which is within our range),

cost = \$1.65

Note that cases (b) and (c) satisfy the current range criteria and (b) is the cheaper of the two, hence the correct choice is:

R_1 and R_3

Solution 2.62

$p_A = 110x8 = 880$ W, $p_B = 110x2 = 220$ W

Energy cost = \$0.06 x 365 x10 x (880 + 220)/1000 = **\$240.90**

Solution 2.63

Use eq. (2.61),

$$R_n = \frac{I_m}{I - I_m} R_m = \frac{2x10^{-3} x100}{5 - 2x10^{-3}} = 0.04\Omega$$

$$I_n = I - I_m = 4.998 \text{ A}$$

$$p = I_n^2 R = (4.998)^2 (0.04) = 0.9992 \ \cong \mathbf{1\ W}$$

Solution 2.64

The potentiometer (adjustable resistor) R_x in Fig. 2.126 is to be designed to adjust current I_x from 10 mA to 1 A. Calculate the values of R and R_x to achieve this.

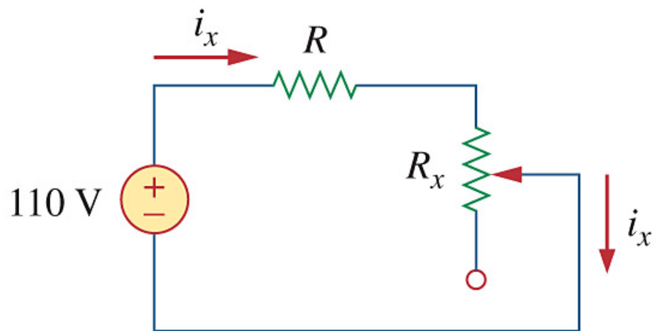


Figure 2.126
For Prob. 2.64.

Solution

Step 1. Even though there are an infinite number of combinations that can meet these requirements, we will focus on making the potentiometer the most sensitive.

First we will determine the value of R by setting the potentiometer equal to zero.
$i_x = 110/R = 1$ A. Next we set the potentiometer to its maximum value or
$0.01 = 110/(R+R_x)$ or $R_x = (110/0.01) – R$. We now have enough equations to solve for R and R_x.

Step 2.

$$R = \mathbf{110\ \Omega} \text{ and } R_x = 11{,}000 – 110 = \mathbf{10.89\ k\Omega}.$$

Solution 2.65

Design a circuit that uses a d'Arsonval meter (with an internal resistance of 2 kΩ that requires a current of 5 mA to cause the meter to deflect full scale) to build a voltmeter to read values of voltages up to 100 volts.

Solution.

Step 1. Since 100 volts across the meter will cause the current through the meter to be 100/2,000 = 0.05 amps, a way must be found to limit the current to 0.005 amps. Clearly adding a resistance in series with the meter will accomplish that. The value of the resistance can be found by solving for $100/R_{Tot} = 0.005$ amps where $R_{Tot} = 2{,}000 + R_s$.

Step 2. $R_{Tot} = 100/0.005 = 20$ kΩ. $R_s = 20{,}000 - 2{,}000 = 18$ kΩ. So, our **circuit consists of the meter in series with an 18 kΩ resistor.**

Solution 2.66

$$20 \text{ k}\Omega/\text{V} = \text{sensitivity} = \frac{1}{I_{fs}}$$

$$\text{i.e., } I_{fs} = \frac{1}{20}\text{k}\Omega/\text{V} = 50\ \mu\text{A}$$

The intended resistance $R_m = \frac{V_{fs}}{I_{fs}} = 10(20\text{k}\Omega/\text{V}) = 200\text{k}\Omega$

(a) $$R_n = \frac{V_{fs}}{i_{fs}} - R_m = \frac{50\ \text{V}}{50\,\mu\text{A}} - 200\ \text{k}\Omega = \mathbf{800\ k\Omega}$$

(b) $$p = I_{fs}^2 R_n = (50\ \mu\text{A})^2(800\ \text{k}\Omega) = \mathbf{2\ mW}$$

Solution 2.67

(a) By current division,

$$i_0 = 5/(5 + 5)\ (2\text{ mA}) = 1\text{ mA}$$
$$V_0 = (4\text{ k}\Omega)\ i_0 = 4 \text{ x } 10^3 \text{ x } 10^{-3} = \mathbf{4\ V}$$

(b) $4\text{k}\| 6\text{k} = 2.4\text{k}\Omega.$ By current division,

$$i_0' = \frac{5}{1+2.4+5}(2\text{mA}) = 1.19\text{ mA}$$
$$v_0' = (2.4\text{ k}\Omega)(1.19\text{ mA}) = \mathbf{2.857\ V}$$

(c) % error = $\left|\frac{v_0 - v_0'}{v_0}\right|$ x 100% = $\frac{1.143}{4}\text{x}100 = \mathbf{28.57\%}$

(d) $4\text{k}\| 36\text{ k}\Omega = 3.6\text{ k}\Omega.$ By current division,

$$i_0' = \frac{5}{1+3.6+5}(2\text{mA}) = 1.042\text{mA}$$
$$v_0'(3.6\,\text{k}\Omega)(1.042\,\text{mA}) = 3.75\text{V}$$
$$\%\text{ error} = \left|\frac{v - v_0'}{v_0}\right|\text{x}100\% = \frac{0.25\text{x}100}{4} = \mathbf{6.25\%}$$

Solution 2.68

(a) $40 = 24 \| 60\Omega$

$$i = \frac{4}{16+24} = \mathbf{100\ mA}$$

(b) $$i' = \frac{4}{16+1+24} = \mathbf{97.56\ mA}$$

(c) $$\%\ \text{error} = \frac{0.1 - 0.09756}{0.1} \text{x}100\% = \mathbf{2.44\%}$$

Solution 2.69

A voltmeter is used to measure V_o in the circuit in Fig. 2.129. The voltmeter model consists of an ideal voltmeter in parallel with a 250-kΩ resistor. Let V_s = 95 V, R_s = 25 kΩ, and R_1 = 40 kΩ. Calculate V_o with and without the voltmeter when

(a) R_2 = 5 kΩ (b) R_2 = 25 kΩ

(c) R_2 = 250 kΩ

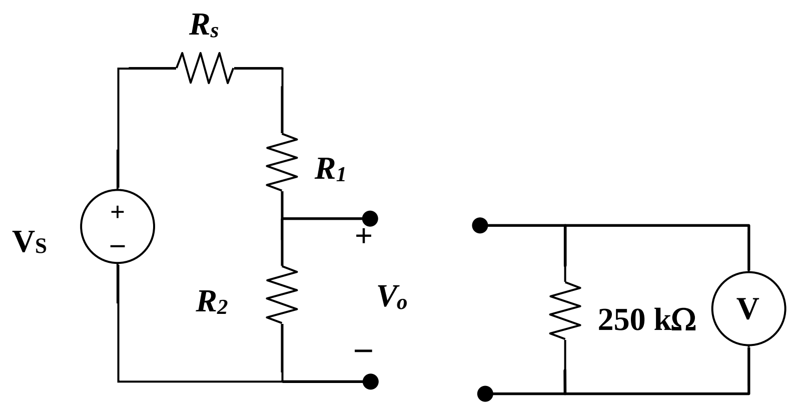


Figure 2.129
For Prob. 2.69

Solution

Step 1. $V_o = V_s \dfrac{\left(\frac{250kR_2}{250k+R_2}\right)}{R_s+R_1+\frac{250kR_2}{250k+R_2}} = 95\dfrac{\left(\frac{250kR_2}{250k+R_2}\right)}{65k+\frac{250kR_2}{250k+R_2}}$ and

$V_o = V_s\dfrac{R_2}{R_s+R_1+R_2} = 95\dfrac{R_2}{65k+R_2}$.

Step 2. (a) $V_o = 95\dfrac{\left(\frac{250kR_2}{250k+R_2}\right)}{65k+\frac{250kR_2}{250k+R_2}}$ = 95(4.902/69.902) = **6.662 volts** and

$V_o = 95\dfrac{R_2}{65k+R_2}$ = 95(5k/70k) = **6.786 volts**

(b) $V_o = 95\dfrac{\left(\frac{250kR_2}{250k+R_2}\right)}{65k+\frac{250kR_2}{250k+R_2}}$ = 95(22.727/87.727) = **24.61 volts** and

$V_o = 95\dfrac{R_2}{65k+R_2}$ = 95(25/90) = **26.39 volts**

(c) $V_o = 95\dfrac{\left(\frac{250kR_2}{250k+R_2}\right)}{65k+\frac{250kR_2}{250k+R_2}}$ = 95(125/190) = **62.5 volts** and

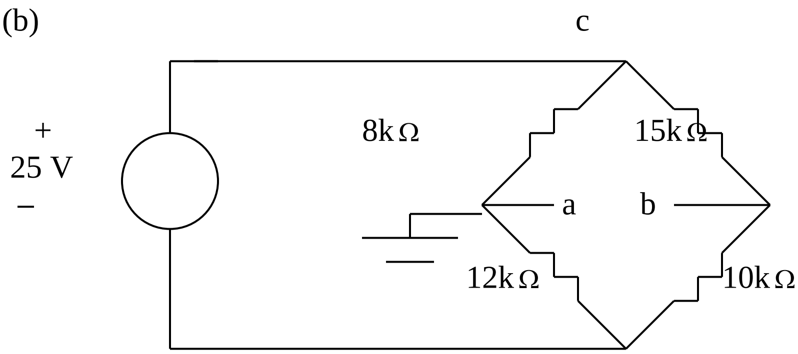
$V_o = 95\dfrac{R_2}{65k+R_2}$ = 95(250/315) = **75.4 volts**

Solution 2.70

(a) Using voltage division,

$$v_a = \frac{12}{12+8}(25) = \underline{15V}$$
$$v_b = \frac{10}{10+15}(25) = \underline{10V}$$
$$v_{ab} = v_a - v_b = 15 - 10 = \underline{5V}$$

(b)



v_a = $\underline{\mathbf{0}}$; v_{ac} = –(8/(8+12))25 = –10V; v_{cb} = (15/(15+10))25 = 15V.

v_{ab} = v_{ac} + v_{cb} = –10 + 15 = $\underline{\mathbf{5V}}$.

v_b = –v_{ab} = $\underline{\mathbf{–5V}}$.

Solution 2.71

Figure 2.131 represents a model of a solar photovoltaic panel. Given that V_s = 95 **V**, R_1 = 25 Ω, i_L = 2 **A**, find R_L.

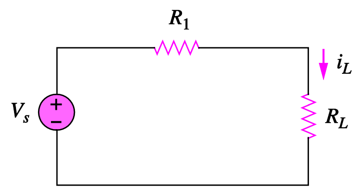


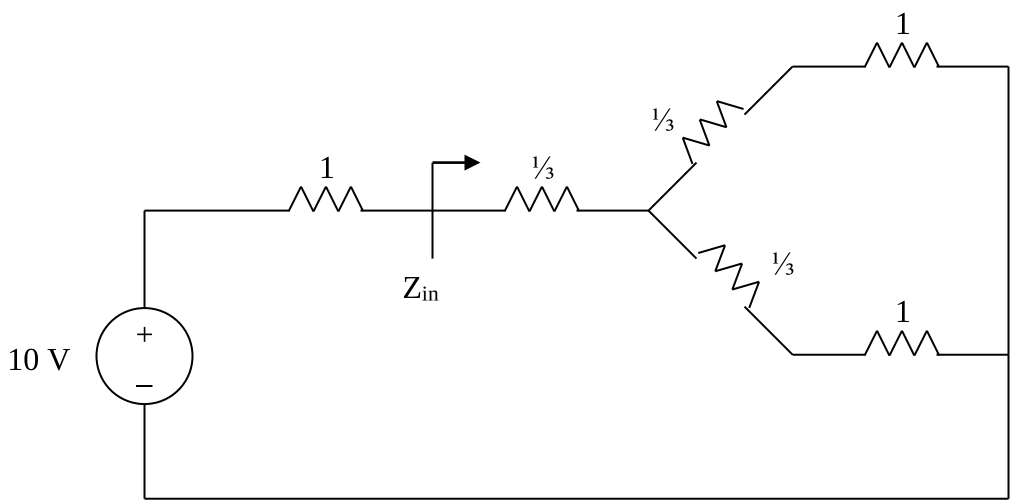
Figure 2.131
For Prob. 2.71.

Step 1. $V_s = i_L(R_1+R_L)$ or $R_L = (95/2) – 25$

Step 2. $R_L = 47.5 – 25 =$ **22.5 Ω**

Solution 2.72

Converting the delta subnetwork into wye gives the circuit below.



$$Z_{in} = \frac{1}{3} + (1 + \frac{1}{3}) // (1 + \frac{1}{3}) = \frac{1}{3} + \frac{1}{2}(\frac{4}{3}) = 1\ \Omega$$

$$V_o = \frac{Z_{in}}{1 + Z_{in}}(10) = \frac{1}{1+1}(10) = \underline{5\ \text{V}}$$

Solution 2.73

By the current division principle, the current through the ammeter will be one-half its previous value when

$$R = 20 + R_x$$
$$65 = 20 + R_x \longrightarrow R_x = \mathbf{45\ \Omega}$$

Solution 2.74

With the switch in high position,

$$6 = (0.01 + R_3 + 0.02) \times 5 \longrightarrow R_3 = \mathbf{1.17\ \Omega}$$

At the medium position,

$$6 = (0.01 + R_2 + R_3 + 0.02) \times 3 \longrightarrow R_2 + R_3 = 1.97$$

or $R_2 = 1.97 - 1.17 = \mathbf{0.8\ \Omega}$

At the low position,

$$6 = (0.01 + R_1 + R_2 + R_3 + 0.02) \times 1 \longrightarrow R_1 + R_2 + R_3 = 5.97$$

$R_1 = 5.97 - 1.97 = \mathbf{4\ \Omega}$

Solution 2.75

Find R_{ab} in the four-way power divider circuit in Fig. 2.135. Assume each $R = 4\ \Omega$.

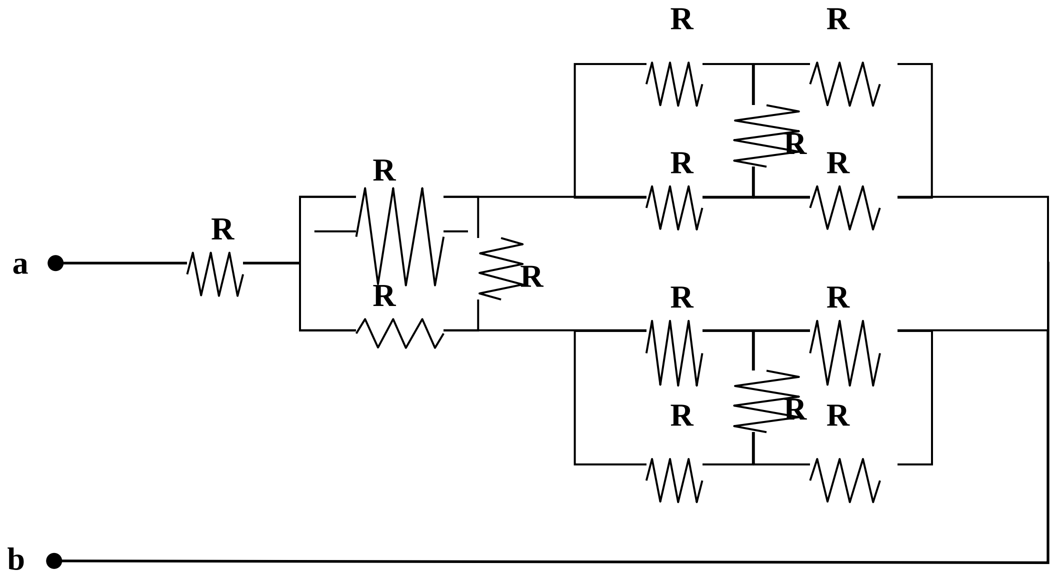
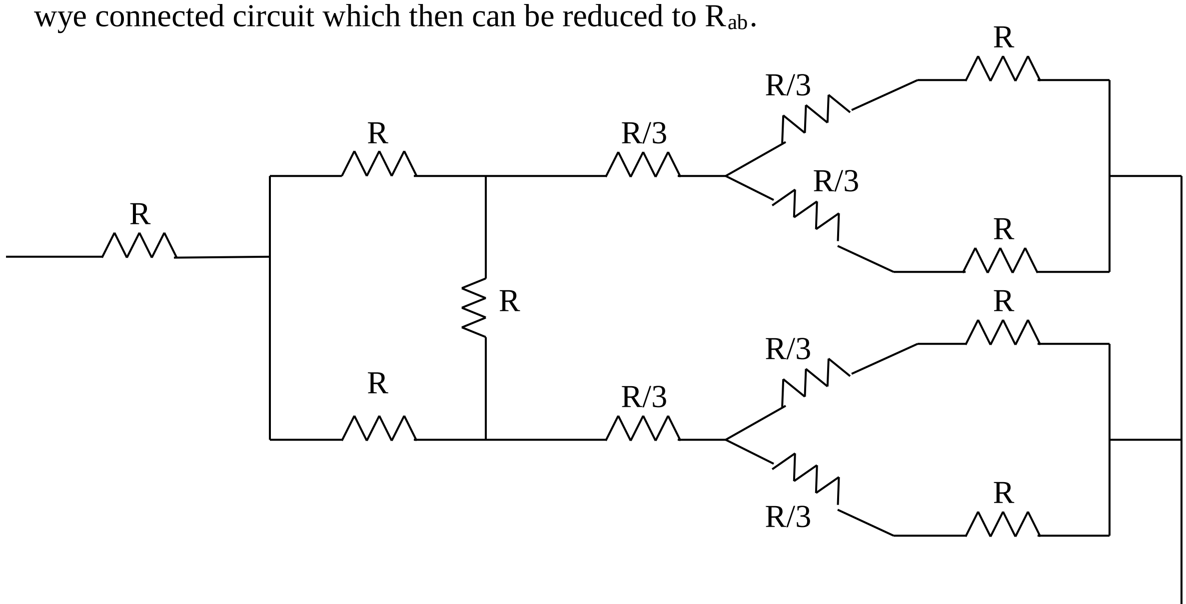


Figure 2.135
For Prob. 2.75.

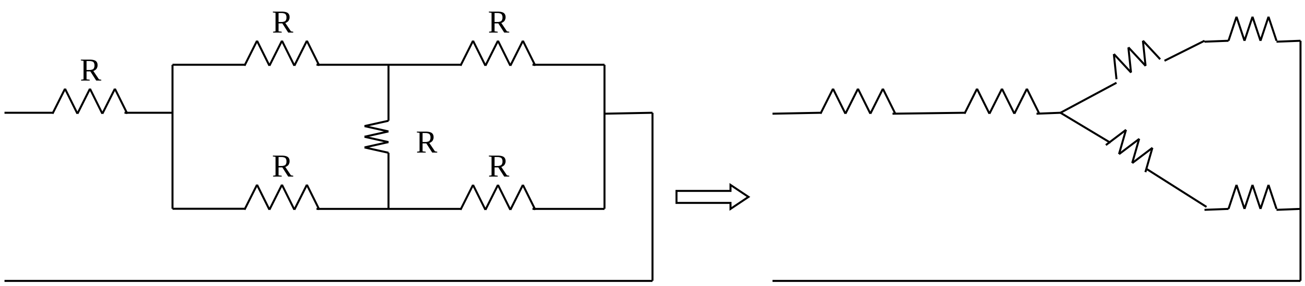
Step 1. There are two delta circuits that can be converted to a wye connected circuit. This then allows us to combine resistances together in series and in parallel. This will yield a new circuit with one remaining delta connected circuit that can be converted to a wye connected circuit which then can be reduced to R_{ab}.



Step 2. Converting delta-subnetworks to wye-subnetworks and combining resistances leads to the circuit below.

$$\frac{R}{3} + \frac{(4R/3)(4R/3)}{(4R/3)+(4R/3)} = R\left(\frac{1}{3} + \frac{\frac{16}{9}}{\frac{8}{3}}\right) = R$$

With this combination, the circuit is further reduced to that shown below.



Again we convert the delta to a wye connected circuit and the values of the wye resistances are all equal to R/3 and combining all the series and parallel resistors gives us R in series with R. Thus,

$$R_{ab} = R+R = 4+4 = \mathbf{8\ \Omega}$$

Solution 2.76

$Z_{ab} = 1 + 1 = \mathbf{2\ \Omega}$

Solution 2.77

(a) $5\ \Omega = 10 \| 10 = 20 \| 20 \| 20 \| 20$

i.e., **four 20 Ω resistors in parallel.**

(b) $311.8 = 300 + 10 + 1.8 = 300 + 20 \| 20 + 1.8$

i.e., **one 300Ω resistor in series with 1.8Ω resistor and a parallel combination of two 20Ω resistors.**

(c) $40\text{k}\Omega = 12\text{k}\Omega + 28\text{k}\Omega = (24 \| 24k) + (56k \| 56k)$

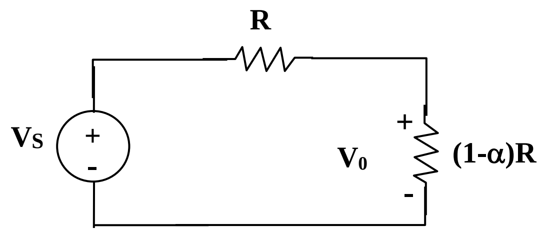
i.e., **Two 24kΩ resistors in parallel connected in series with two 56kΩ resistors in parallel.**

(d) $52.32\text{k}\Omega = 28\text{k}+24\text{k}+300+20 = 56\text{k}||56\text{k}+24\text{k}+300+20$

i.e., **A series combination of a 20Ω resistor, 300Ω resistor, 24kΩ resistor, and a parallel combination of two 56kΩ resistors.**

Solution 2.78

The equivalent circuit is shown below:



$$V_0 = \frac{(1-\alpha)R}{R+(1-\alpha)R}V_S = \frac{1-\alpha}{2-\alpha}V_S$$

$$\frac{V_0}{V_S} = \frac{1-\alpha}{2-\alpha}$$

Solution 2.79

Since $p = v^2/R$, the resistance of the sharpener is

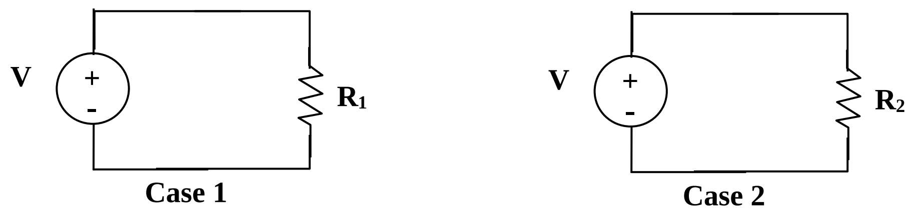
$$R = v^2/(p) = 6^2/(240 \text{ x } 10^{-3}) = 150\Omega$$
$$I = p/(v) = 240 \text{ mW}/(6\text{V}) = 40 \text{ mA}$$

Since R and R_x are in series, I flows through both.

$$IR_x = V_x = 9 - 6 = 3 \text{ V}$$
$$R_x = 3/(I) = 3/(40 \text{ mA}) = 3000/(40) = \mathbf{75\ \Omega}$$

Solution 2.80

The amplifier can be modeled as a voltage source and the loudspeaker as a resistor:



Hence $p = \frac{V^2}{R}, \quad \frac{p_2}{p_1} = \frac{R_1}{R_2} \longrightarrow p_2 = \frac{R_1}{R_2} p_1 = \frac{10}{4}(12) = \mathbf{30\ W}$

Solution 2.81

For a specific application, the circuit shown in Fig. 2.140 was designed so that I_L = 83.33 mA and that R_{in} = 5 kΩ. What are the values of R_1 and R_2?

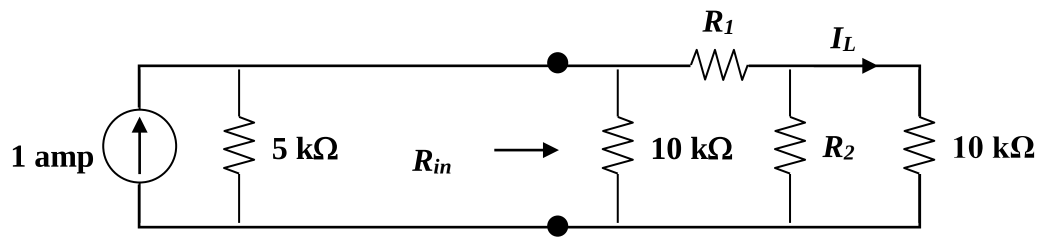


Figure 2.140
For Prob. 2.81.

Solution

Step 1. Calculate R_{in} in terms of R_1 and R_2. Next calculate the value of I_L in terms of R_1 and R_2.

$R_{in} = \dfrac{10k\left(R_1 + \dfrac{R_2 10k}{R_2 + 10k}\right)}{10k + R_1 + \dfrac{R_2 10k}{R_2 + 10k}}$ = 5k and since R_{in} = 5k, the current by current division entering R_{in} has to equal 500 mA. Again using current division, the current through R_1 = 250 mA. Finally we can use current division to obtain I_L.

$I_L = 0.25xR_2/(R_2+10k) = 0.08333$ A.

Step 2. First we can calculate R_2. $0.25R_2 = 0.08333(R_2 + 10{,}000)$ or

$(0.25–0.08333)R_2 = 833.3$ or $R_2 = 833.3/0.16667 = 5{,}000 =$ **5 kΩ**.

Next $5k = \dfrac{10k\left(R_1 + \dfrac{5kx10k}{5k + 10k}\right)}{10k + R_1 + \dfrac{5k10k}{5k + 10k}} = \dfrac{10k(R_1 + 3.3333k)}{10k + R_1 + 3.3333k}$ or

$5k(R_1+13.3333k) = 10k(R_1+3.3333)$ or $R_1+13.3333 = 2R_1 + 6.6666$ or

$R_1 = 13.3333k–6.6666k = 6.6667k =$ **6.667 kΩ**.

Solution 2.82

The pin diagram of a resistance array is shown in Fig. 2.141. Find the equivalent resistance between the following:

(a) 1 and 2 (b) 1 and 3 (c) 1 and 4

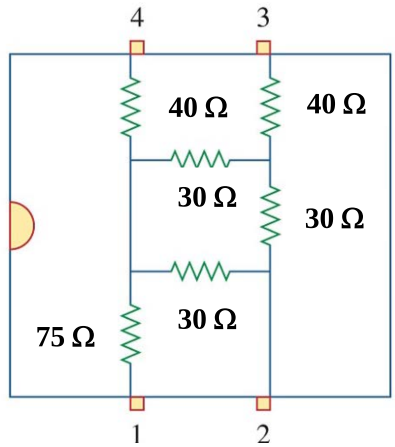
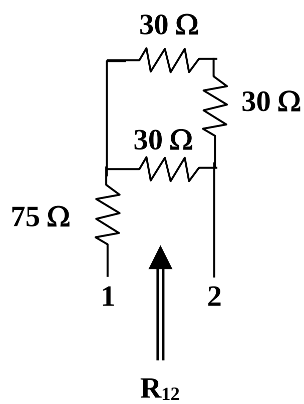


Figure 2.141
For Prob. 2.82.

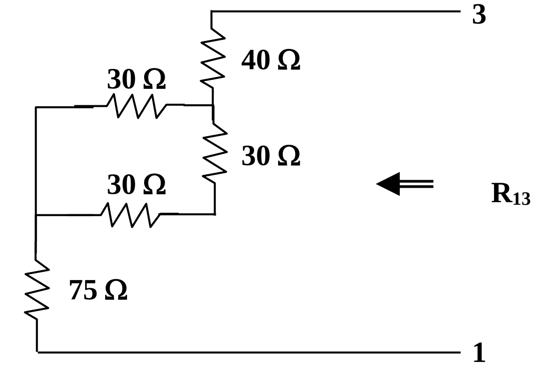
Solution

Step 1. Each pair of contacts will connect a specific circuit where we can use the variety of wye-delta, series, and paralleling of resistances to obtain the desired results.

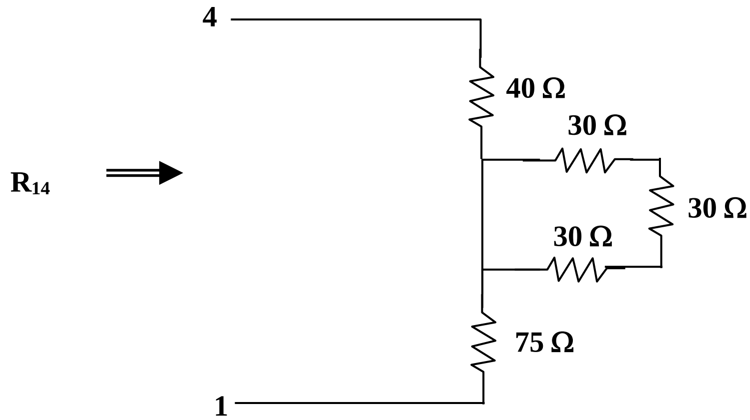
(a)



(b)



(c)



Step 2. (a) R_{12} = 75 + 30x60/(30+60) = 75+20 = **95 Ω**

(b) R_{13} = 75 + [30x60/(30+60)] + 40 = 135 Ω

(c) R_{14} = 40 + 75 = **105 Ω**

Solution 2.83

Two delicate devices are rated as shown in Fig. 2.142. Find the values of the resistors R_1 and R_2 needed to power the devices using a 36-V battery.

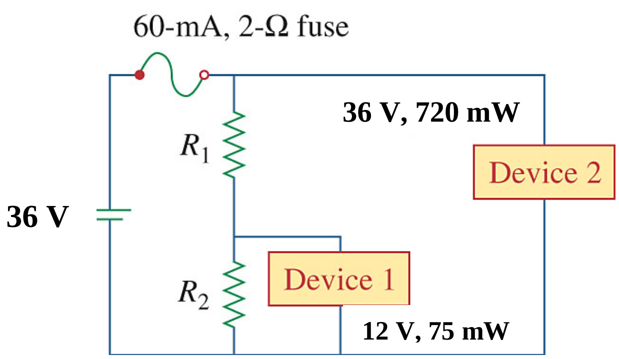


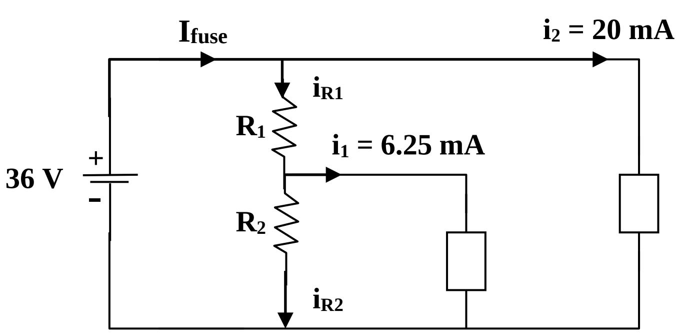
Figure 2.142
For Prob. 2.83.

Solution

The voltage across the fuse should be negligible when compared with 24 V (this can be checked later when we check to see if the fuse rating is exceeded in the final circuit). We can calculate the current through the devices.

$$I_1 = \frac{p_1}{V_1} = \frac{75mW}{12V} = 6.25mA$$

$$I_2 = \frac{p_2}{V_2} = \frac{720mW}{36} = 20mA$$



Let R_3 represent the resistance of the first device, we can solve for its value from knowing the voltage across it and the current through it.

$$R_3 = 12/0.00625 = 1{,}920\ \Omega$$

This is an interesting problem in that it essentially has two unknowns, R_1 and R_2 but only one condition that need to be met and that is that the voltage across R_3 must equal 12 volts. Since the circuit is powered by a battery we could choose the value of R_2 which draws the least current, $R_2 = \infty$. Thus we can calculate the value of R_1 that gives 12 volts across R_3.

$$12 = (36/(R_1 + 1920))1920 \text{ or } R_1 = (36/12)1920 - 1920 = \mathbf{3.84\ k\Omega}$$

This value of R_1 means that we only have a total of 26.25 mA flowing out of the battery through the fuse which means it will not open and produces a voltage drop across it of 0.0525 mV. This is indeed negligible when compared with the 36-volt source.

Solution 3.1

Using Fig. 3.50, design a problem to help other students to better understand nodal analysis.

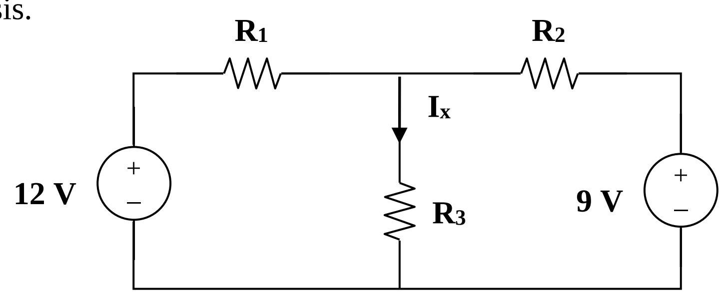


Figure 3.50
For Prob. 3.1 and Prob. 3.39.

Solution

Given $R_1 = 4$ kΩ, $R_2 = 2$ kΩ, and $R_3 = 2$ kΩ, determine the value of I_x using nodal analysis.

Let the node voltage in the top middle of the circuit be designated as V_x.

$[(V_x–12)/4k] + [(V_x–0)/2k] + [(V_x–9)/2k] = 0$ or (multiply this by 4 k)

$(1+2+2)V_x = 12+18 = 30$ or $V_x = 30/5 = 6$ volts and

$I_x = 6/(2k) =$ **3 mA**.

Solution 3.2

At node 1,

$$\frac{-v_1}{10} - \frac{v_1}{5} = 6 + \frac{v_1 - v_2}{2} \longrightarrow 60 = -8v_1 + 5v_2 \qquad (1)$$

At node 2,

$$\frac{v_2}{4} = 3 + 6 + \frac{v_1 - v_2}{2} \longrightarrow 36 = -2v_1 + 3v_2 \qquad (2)$$

Solving (1) and (2),

v_1 = **0 V**, v_2 = **12 V**

Solution 3.3

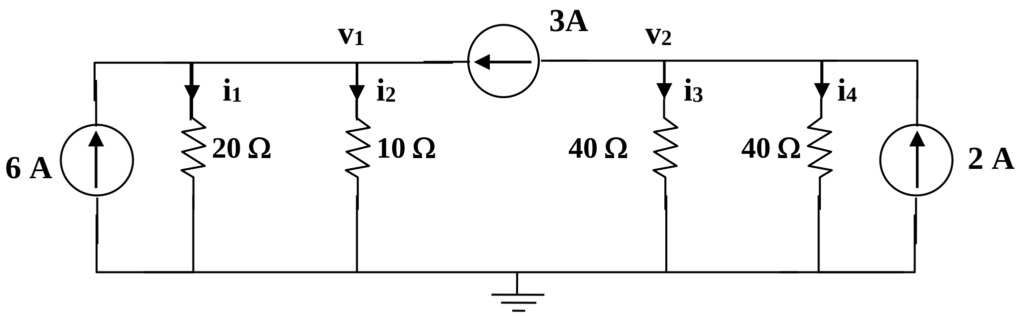
Applying KCL to the upper node,

$$-8+\frac{v_0}{10}+\frac{v_o}{20}+\frac{v_o}{30}+20+\frac{v_0}{60}=0 \text{ or } v_0 = \mathbf{-60\ V}$$

$i_1 = \dfrac{v_0}{10} = \mathbf{-6\ A}$, $i_2 = \dfrac{v_0}{20} = \mathbf{-3\ A}$,

$i_3 = \dfrac{v_0}{30} = \mathbf{-2\ A}$, $i_4 = \dfrac{v_0}{60} = \mathbf{1\ A}$.

Solution 3.4



At node 1,

$$-6 - 3 + v_1/(20) + v_1/(10) = 0 \text{ or } v_1 = 9(200/30) = 60 \text{ V}$$

At node 2,

$$3 - 2 + v_2/(10) + v_2/(5) = 0 \text{ or } v_2 = -1(1600/80) = -20 \text{ V}$$

$i_1 = v_1/(20) =$ **3 A**, $i_2 = v_1/(10) =$ **6 A**,
$i_3 = v_2/(40) =$ **–500 mA**, $i_4 = v_2/(40) =$ **–500 mA**.

Solution 3.5

Obtain v_o in the circuit of Fig. 3.54.

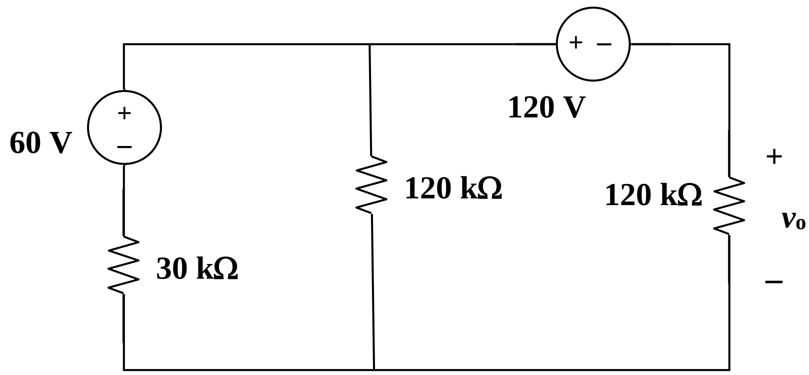
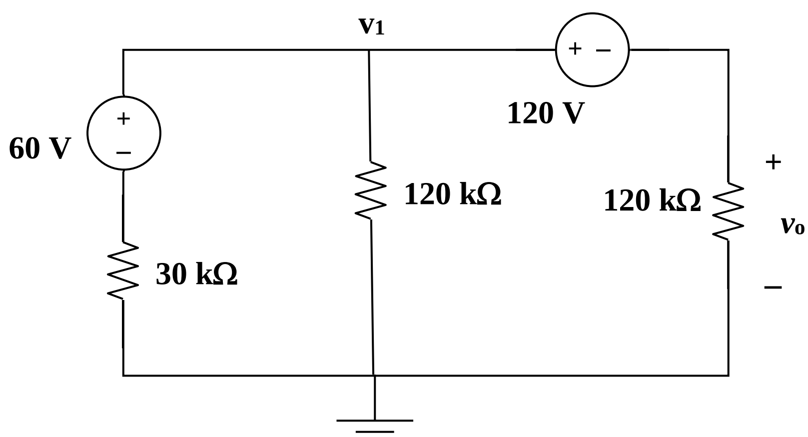


Figure 3.54
For Prob. 3.5.

Step 1. First you need to pick a reference, so we place a ground at the bottom of the circuit. Then we identify the unknown node and then write our nodal equations. Next we apply a constraint equation to solve for v_o.



At node 1, $[((v_1–60)–0)/30k] + [(v_1–0)/120k] + [((v_1–120)/120k] = 0$ and $v_o = (v_1–120)–0$.

Step 2. $[(1/30k)+(1/120k)+(1/120k)]v_1 = (60/30k)+(120/120k) = 0.002+0.001 = 0.003 = (6/120k)v_1$ or $v_1 = 0.003x20k = 60$ volts.

Therefore,

$$v_o = 60–120 = \mathbf{–60\ V}.$$

Solution 3.6

Solve for V_1 using nodal analysis.

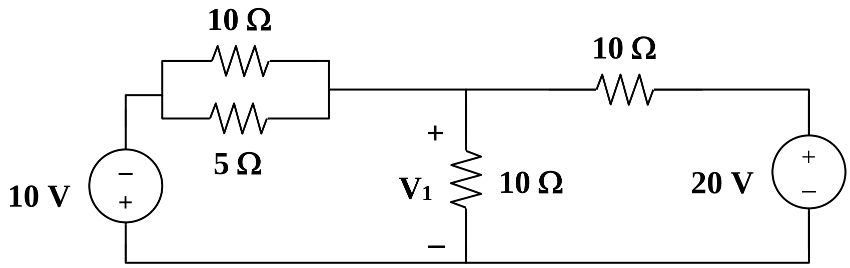


Figure 3.55
For Prob. 3.6.

Step 1. The first thing to do is to select a reference node and to identify all the unknown nodes. We select the bottom of the circuit as the reference node. The only unknown node is the one connecting all the resistors together and we will call that node V_1. The other two nodes are at the top of each source. Relative to the reference, the one at the top of the 10-volt source is –10 V. At the top of the 20-volt source is +20 V.

Step 2. Setup the nodal equation (there is only one since there is only one unknown).

$$\frac{(V_1-(-10))}{5}+\frac{(V_1-(-10))}{10}+\frac{(V_1-0)}{10}+\frac{(V_1-20)}{10}=0$$

Step 3. Simplify and solve.

$$\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}\right)V_1=-\frac{10}{5}-\frac{10}{10}+\frac{20}{10}$$

$$(0.2+0.1+0.1+0.1)V_1=0.5V_1=-2-1+2=-1$$

or

$$V_1 = \mathbf{-2\ V}.$$

The answer can be checked by calculating all the currents and see if they add up to zero. The top two currents on the left flow right to left and are 0.8 A and 1.6 A respectively. The current flowing up through the 10-ohm resistor is 0.2 A. The current flowing right to left through the 10-ohm resistor is 2.2 A. Summing all the currents flowing out of the node, V_1, we get, +0.8+1.6 –0.2–2.2 = 0. The answer checks.

Solution 3.7

Apply nodal analysis to solve for V_x in the circuit in Fig. 3.56.

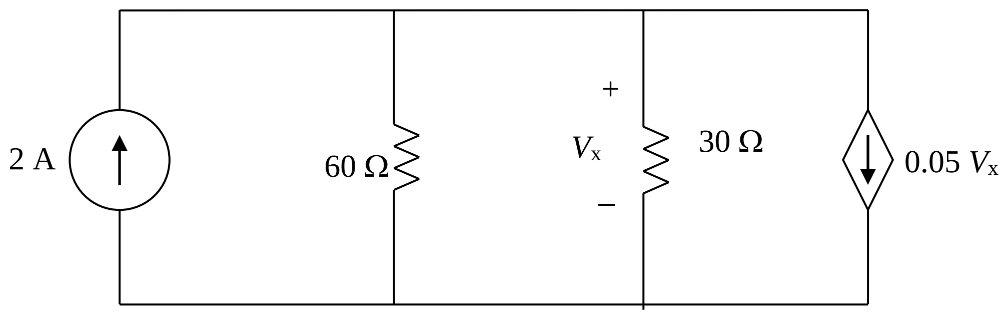


Figure 3.56
For Prob. 3.7.

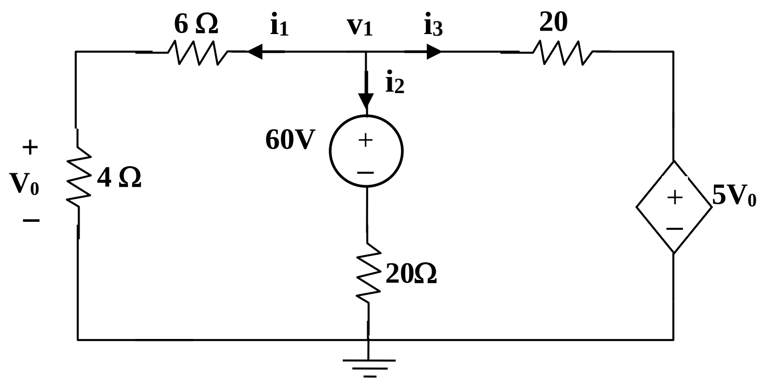
Step 1. First we identify all of the unknown nodes and in this case, we only have one and that is V_x. Next we write one nodal equation.

$$-2+\frac{V_x-0}{60}+\frac{V_x-0}{30}+0.05V_x=0\,.$$

Step 2. $[0.05+(1/60)+(1/30)]V_x = [0.05+0.05]V_x = 0.1V_x = 2$ or $V_x =$ **20 V**.

Substituting into the original equation for a check we get, –2 + (20/60) + (20/30) + (0.05)(20) = 0 = –2 + 0.33333 + 0.66667 + 1 = 0. The answer checks!

Solution 3.8



$i_1 + i_2 + i_3 = 0 \longrightarrow \dfrac{v_1}{10} + \dfrac{(v_1 - 60) - 0}{20} + \dfrac{v_1 - 5v_0}{20} = 0$

But $\quad v_0 = \dfrac{2}{5}v_1$ so that $2v_1 + v_1 – 60 + v_1 – 2v_1 = 0$

or $v_1 = 60/2 = 30$ V, therefore $v_o = 2v_1/5 =$ **12 V**.

Solution 3.9

Let V_1 be the unknown node voltage to the right of the 250-Ω resistor. Let the ground reference be placed at the bottom of the 50-Ω resistor. This leads to the following nodal equation:

$$\frac{V_1 - 24}{250} + \frac{V_1 - 0}{50} + \frac{V_1 - 60I_b - 0}{150} = 0$$

simplifying we get

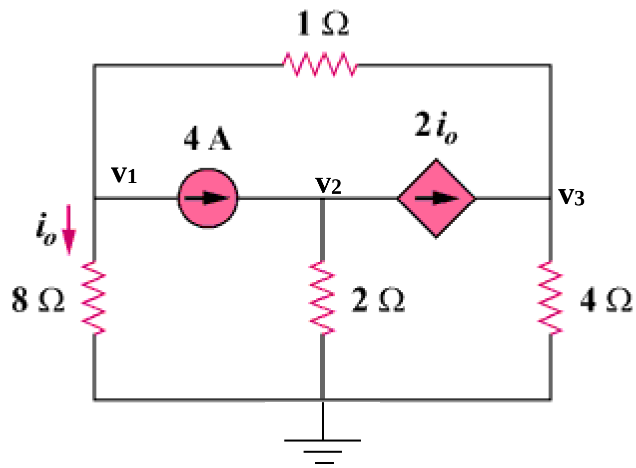
$$3V_1 - 72 + 15V_1 + 5V_1 - 300I_b = 0$$

But $I_b = \dfrac{24 - V_1}{250}$. Substituting this into the nodal equation leads to

$$24.2V_1 - 100.8 = 0 \quad \text{or } V_1 = 4.165 \text{ V.}$$

Thus, $I_b = (24 - 4.165)/250 =$ **79.34 mA**.

Solution 3.10



At node 1. $[(v_1–0)/8] + [(v_1–v_3)/1] + 4 = 0$

At node 2. $–4 + [(v_2–0)/2] + 2i_o = 0$

At node 3. $–2i_o + [(v_3–0)/4] + [(v_3–v_1)/1] = 0$

Finally, we need a constraint equation, $i_o = v_1/8$

This produces,

$$1.125v_1 – v_3 = –4 \quad (1)$$

$$0.25v_1 + 0.5v_2 = 4 \quad (2)$$

$$–1.25v_1 + 1.25v_3 = 0 \text{ or } v_1 = v_3 \quad (3)$$

Substituting (3) into (1) we get $(1.125–1)v_1 = –4$ or $v_1 = –4/0.125 = –32$ volts. This leads to,

$i_o = 32/8 =$ **–4 amps**.

Solution 3.11

Find V_o and the power absorbed by all the resistors in the circuit of Fig. 3.60.

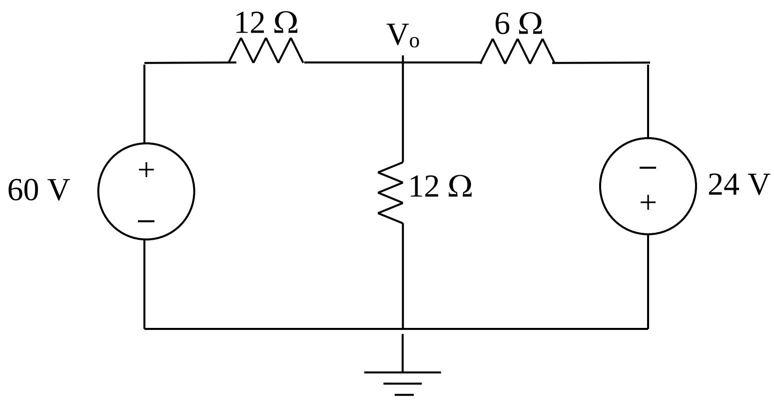


Figure 3.60
For Prob. 3.11.

Solution

At the top node, KCL produces $\frac{V_o - 60}{12} + \frac{V_o - 0}{12} + \frac{V_o - (-24)}{6} = 0$

$(1/3)V_o = 1$ or $V_o =$ **3 V**.

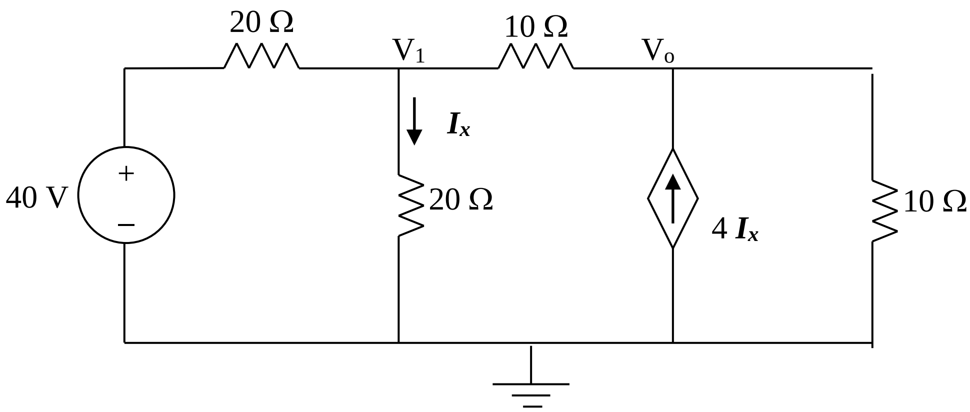
$P_{12\Omega} = (3–60)^2/1 =$ **293.9 W** (this is for the 12 Ω resistor in series with the 60 V source)

$P_{12\Omega} = (V_o)^2/12 = 9/12 =$ **750 mW** (this is for the 12 Ω resistor connecting V_o to ground)

$P_{4\Omega} = (3–(–24))^2/6 =$ **121.5 W**.

Solution 3.12

There are two unknown nodes, as shown in the circuit below.



At node 1,

$$\frac{V_1 - 40}{20} + \frac{V_1 - 0}{20} + \frac{V_1 - V_o}{10} = 0 \text{ or}$$

$$(0.05+0.05+.1)V_1 - 0.1V_o = 0.2V_1 - 0.1V_o = 2 \qquad (1)$$

At node o,

$$\frac{V_o - V_1}{10} - 4I_x + \frac{V_o - 0}{10} = 0 \text{ and } I_x = V_1/20$$

$$-0.1V_1 - 0.2V_1 + 0.2V_o = -0.3V_1 + 0.2V_o = 0 \text{ or} \qquad (2)$$

$$V_1 = (2/3)V_o \qquad (3)$$

Substituting (3) into (1),

$$0.2(2/3)V_o - 0.1V_o = 0.03333V_o = 2 \text{ or}$$

V_o = **60 V**.

Solution 3.13

Calculate v_1 and v_2 in the circuit of Fig. 3.62 using nodal analysis.

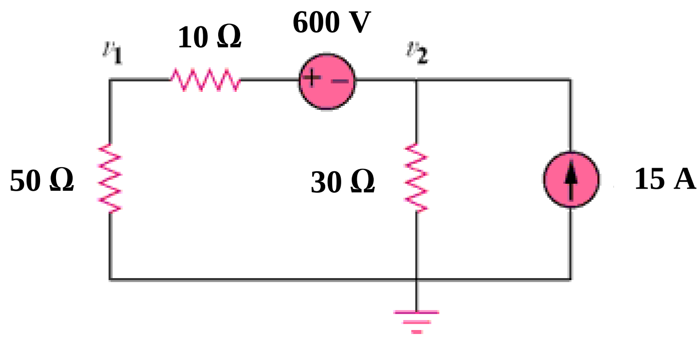


Figure 3.62
For Prob. 3.13.

Solution

Step 1. We note that the 10 ohm resistor is in series with the 50 ohm resistor which can be replaced by a 60 ohm resistor. The then gives us a circuit with one unknown node and we can write one nodal equation to let us solve for v_2.

Once we have v_2 we can use voltage division to solve for v_1.

$$\frac{(v_2+600)-0}{60}+\frac{v_2-0}{30}-15=0$$ and $v_2 = [(v_1+600)–0](50/60)$.

Step 2. $[(1/60)+(1/30)]v_2 = –(600/60)+15 = 5 = 0.05v_2$ or $v_2 =$ **100 V**.

Now $v_1 = 700(50/60) =$ **583.3 V**.

Solution 3.14

Using nodal analysis, find v_o in the circuit of Fig. 3.63.

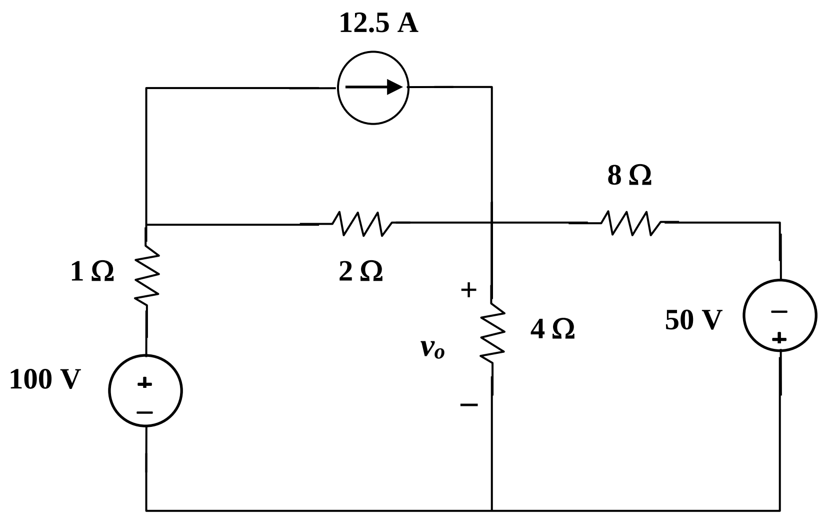
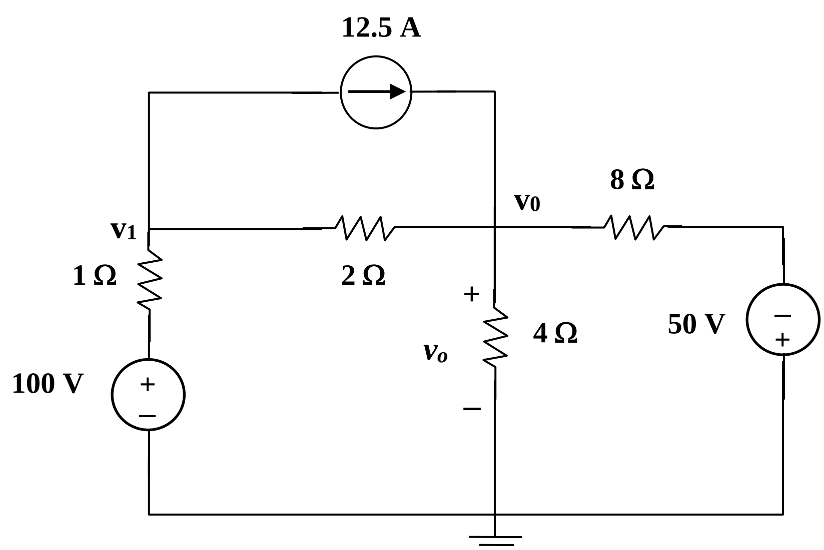


Figure 3.63
For Prob. 3.14.

Solution



At node 1,

$$[(v_1-100)/1] + [(v_1-v_o)/2] + 12.5 = 0 \text{ or } 3v_1 - v_o = 200-25 = 175 \qquad (1)$$

At node o,

$$[(v_o-v_1)/2] - 12.5 + [(v_o-0)/4] + [(v_o+50)/8] = 0 \text{ or } -4v_1 + 7v_o = 50 \qquad (2)$$

Adding 4x(1) to 3x(2) yields,

$$4(1) + 3(2) = -4v_o + 21v_o = 700 + 150 \text{ or } 17v_o = 850 \text{ or}$$

$$v_o = \mathbf{50\ V}.$$

Checking, we get $v_1 = (175+v_o)/3 = 75$ V.

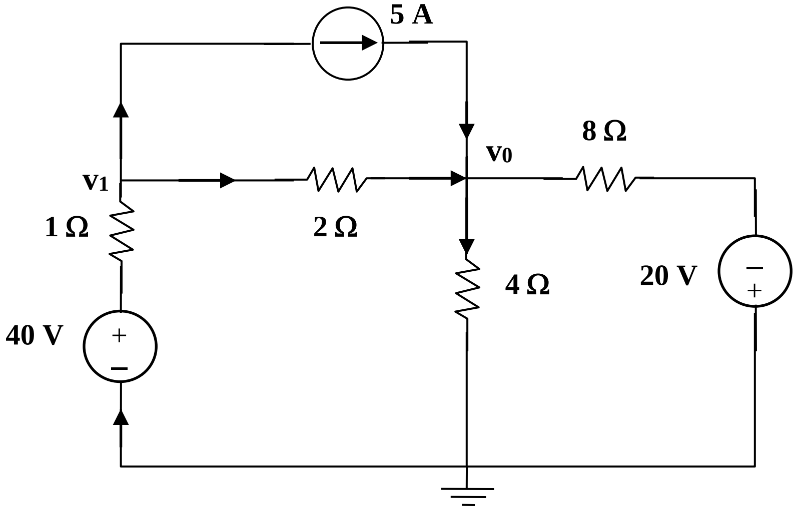
At node 1,

$$[(75-100)/1] + [(75-50)/2] + 12.5 = -25 + 12.5 + 12.5 = 0!$$

At node o,

$$[(50-75)/2] + [(50-0)/4] + [(50+50)/8] - 12.5 = -12.5 + 12.5 + 12.5 - 12.5 = 0!$$

Solution 3.15



Nodes 1 and 2 form a supernode so that $v_1 = v_2 + 10$ (1)

At the supernode, $2 + 6v_1 + 5v_2 = 3(v_3 - v_2) \longrightarrow \quad 2 + 6v_1 + 8v_2 = 3v_3$ (2)

At node 3, $2 + 4 = 3(v_3 - v_2) \longrightarrow \quad v_3 = v_2 + 2$ (3)

Substituting (1) and (3) into (2),

$$2 + 6v_2 + 60 + 8v_2 = 3v_2 + 6 \longrightarrow v_2 = \frac{-56}{11}$$

$$v_1 = v_2 + 10 = \frac{54}{11}$$

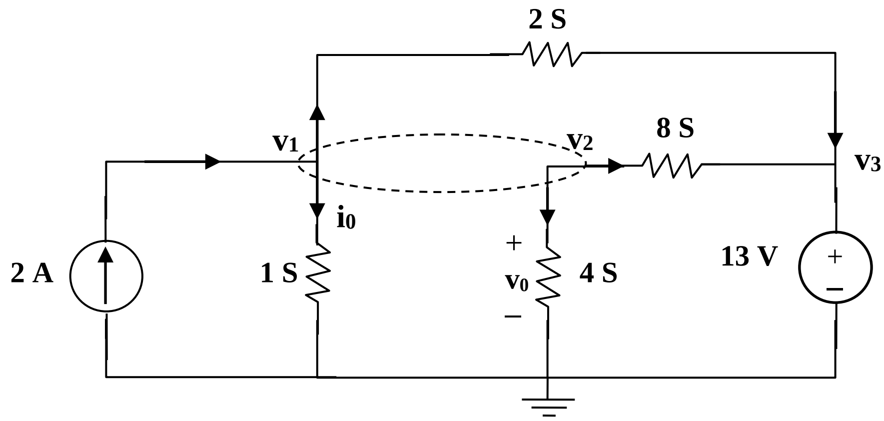
$i_0 = 6v_i =$ **29.45 A**

$$P_{65} = \frac{v_1^2}{R} = v_1^2 G = \left(\frac{54}{11}\right)^2 6 = \mathbf{144.6\ W}$$

$$P_{55} = v_2^2 G = \left(\frac{-56}{11}\right)^2 5 = \mathbf{129.6\ W}$$

$$P_{35} = (v_L - v_3)^2 G = (2)^2 3 = \mathbf{12\ W}$$

Solution 3.16



At the supernode,

$$2 = v_1 + 2\,(v_1 - v_3) + 8(v_2 - v_3) + 4v_2, \text{ which leads to } 2 = 3v_1 + 12v_2 - 10v_3 \quad (1)$$

But

$$v_1 = v_2 + 2v_0 \text{ and } v_0 = v_2.$$

Hence

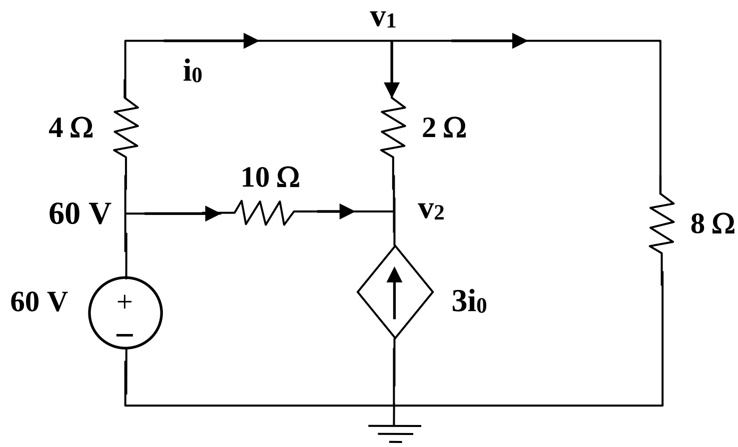
$$v_1 = 3v_2 \qquad (2)$$

$$v_3 = 13V \qquad (3)$$

Substituting (2) and (3) with (1) gives,

$$v_1 = \mathbf{18.858\ V},\ v_2 = \mathbf{6.286\ V},\ v_3 = \mathbf{13\ V}$$

Solution 3.17



At node 1, $\dfrac{60 - v_1}{4} = \dfrac{v_1}{8} + \dfrac{v_1 - v_2}{2}$ $\qquad 120 = 7v_1 - 4v_2 \qquad (1)$

At node 2, $3i_0 + \dfrac{60 - v_2}{10} + \dfrac{v_1 - v_2}{2} = 0$

But $i_0 = \dfrac{60 - v_1}{4}$.

Hence

$$\frac{3(60 - v_1)}{4} + \frac{60 - v_2}{10} + \frac{v_1 - v_2}{2} = 0 \longrightarrow 1020 = 5v_1 + 12v_2 \qquad (2)$$

Solving (1) and (2) gives $v_1 = 53.08$ V. Hence $i_0 = \dfrac{60 - v_1}{4} =$ **1.73 A**

Solution 3.18

Determine the node voltages in the circuit in Fig. 3.67 using nodal analysis.

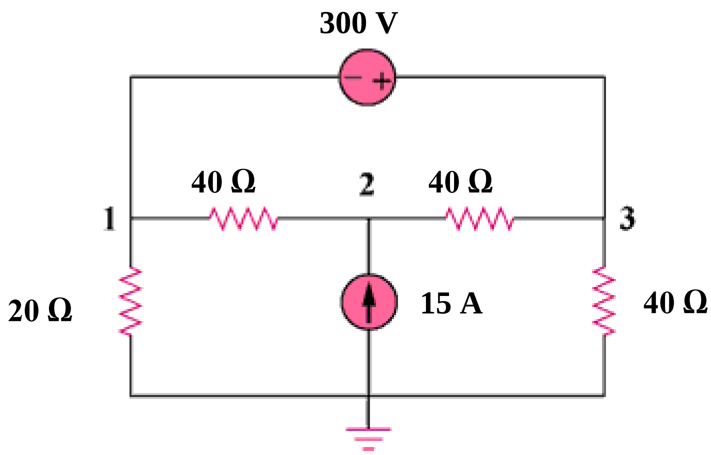


Figure 3.67
For Prob. 3.18.

Step 1. First we identify the unknown nodes and find that there really are only two unknown nodes, v_1 and v_2 since $v_3 = v_1 + 300$ V (essentially a supernode).

$$\frac{v_1 - 0}{20} + \frac{v_1 - v_2}{40} + \frac{(v_1 + 300) - v_2}{40} + \frac{(v_1 + 300) - 0}{40} = 0 \text{ and}$$

$$\frac{v_2 - v_1}{40} - 15 + \frac{v_2 - (v_1 + 300)}{40} = 0.$$ Finally we need, $v_3 = v_1 + 300$.

Step 2. $(0.05+0.025+0.025+0.025)v_1 - (0.025+0.025)v_2 = -15$ or $0.125v_1 - 0.05v_2 = -15$ and $-(0.025+0.025)v_1 + (0.025+0.025)v_2 = 22.5$ or $-0.05v_1 + 0.05v_2 = 22.5$. Adding the two equations together we get, $0.075v_1 = 7.5$ or $v_1 =$ **100 V**. Since $0.05v_2 = 0.05v_1 + 22.5 = 27.5$ or $v_2 =$ **550 V**.

Finally $v_3 = v_1 + 300 =$ **400 V**.

Solution 3.19

At node 1,

$$5 = 3 + \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{8} + \frac{V_1}{4} \quad \longrightarrow \quad 16 = 7V_1 - V_2 - 4V_3 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{8} = \frac{V_2}{2} + \frac{V_2 - V_3}{4} \quad \longrightarrow \quad 0 = -V_1 + 7V_2 - 2V_3 \qquad (2)$$

At node 3,

$$3 + \frac{12 - V_3}{8} + \frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{4} = 0 \quad \longrightarrow \quad -36 = 4V_1 + 2V_2 - 7V_3 \quad (3)$$

From (1) to (3),

$$\begin{pmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ 4 & 2 & -7 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 0 \\ -36 \end{pmatrix} \quad \longrightarrow \quad AV = B$$

Using MATLAB,

$$\boldsymbol{V = A^{-1}B} = \begin{bmatrix} \mathbf{10} \\ \mathbf{4.933} \\ \mathbf{12.267} \end{bmatrix} \quad \longrightarrow \quad \underline{\boldsymbol{V_1 = 10\ \mathrm{V},\ V_2 = 4.933\ \mathrm{V},\ V_3 = 12.267\ \mathrm{V}}}$$

Solution 3.20

For the circuit in Fig. 3.69, find v_1, v_2, and v_3 using nodal analysis.

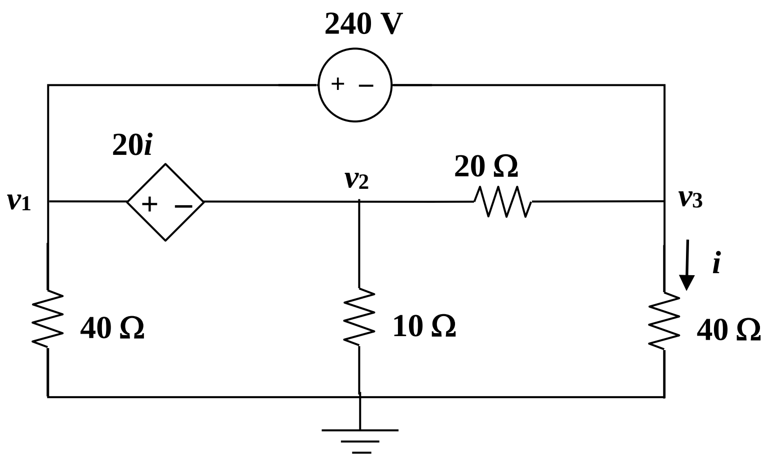


Figure 3.69
For Prob. 3.20.

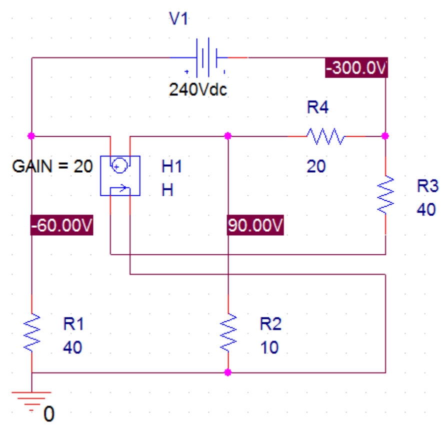
Step 1. This is an interesting problem, once we choose, say v_1, the other two nodes are really known i.e. $v_2 = v_1 - 20i$ and $v_3 = v_1 - 240$ volts.

Obviously we have one supernodes. Additionally we need the constraint equation, $i = (v_3–0)/40 = (v_1–240)/40$.

$[(v_1–0)/40] + [((v_1–20i)–0)/10] + [((v_1–240)–0)/40] = 0$

Step 2. $(0.025+0.1+0.025)v_1 - 0.05v_1 + 12 - 6 = 0$ or $0.1v_1 = -6$ or $v_1 =$ **–60 V**. Now $v_3 = -60-240 =$ **–300 V**. This leads to $i = -300/40 = -7.5$ and $v_2 = -60+150 =$ **90 V**.

Checking with PSpice we get,



Solution 3.21

For the circuit in Fig. 3.70, find v_1 and v_2 using nodal analysis.

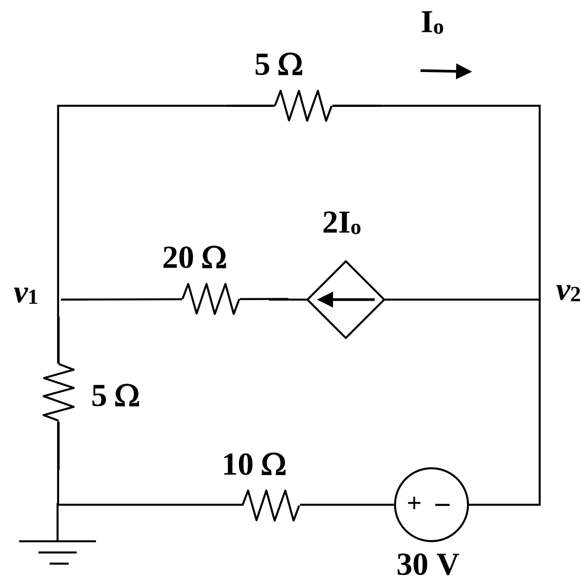
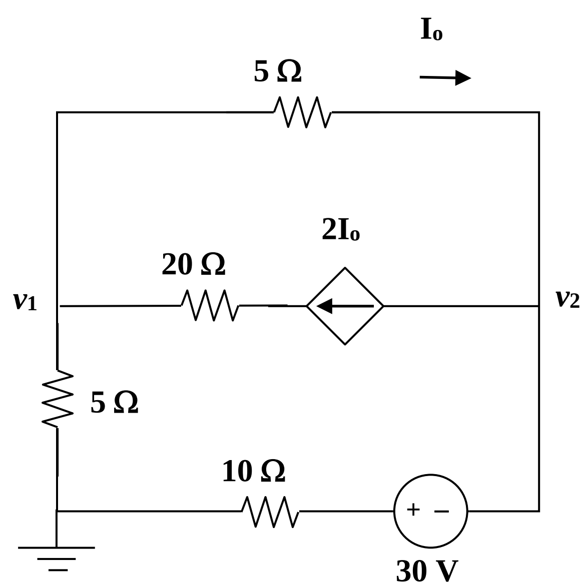


Figure 3.70
For Prob. 3.21.

Step 1. We start by writing the nodal equations. Then we need a constraint equation. This then will allow us to solve for v_1 and v_2.



Node 1. $[(v_1–0)/5] – 2I_o + [(v_1–v_2)/5] = 0$ or $(0.2+0.2)v_1 – 0.2v_2 – 2I_o = 0$

Node 2. $[(v_2–v_1)/5] + 2I_o + [(v_2+30–0)/10] = 0$ or $–0.2v_1 + 0.3v_2 + 2I_o = –3$

Constraint equation, $I_o = [(v_1–v_2)/5]$.

Step 2. $2I_o = 0.4(v_1–v_2)$ or $(0.2+0.2)v_1 – 0.2v_2 – 0.4(v_1–v_2) = 0$ or

$0v_1 + 0.2v_2 = 0$ or $v_2 = 0$. Now, $–0.2v_1 + 0.3v_2 + 0.4v_1 – 0.4v_2 = 0.2v_1 – 0.1v_2 = 0.2v_1 = –3$ or $v_1 = –15$ volts. Therefore,

$v_1 =$ **–15 V** and $v_2 =$ **0 V**.

Solution 3.22

Determine v_1 and v_2 in the circuit in Fig. 3.71.

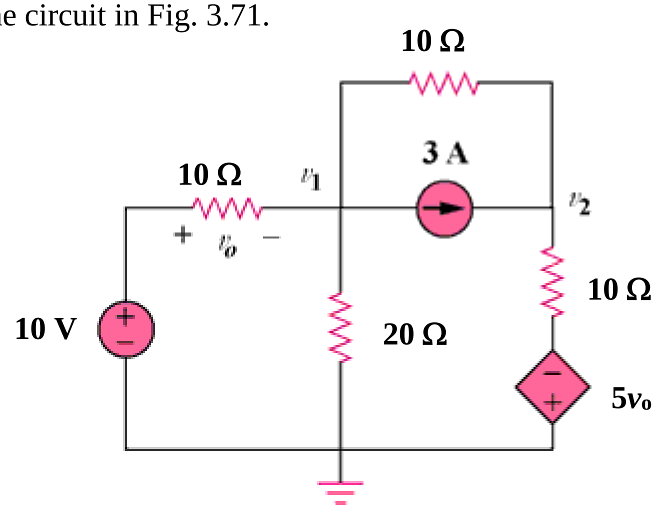


Figure 3.71
For Prob. 3.22.

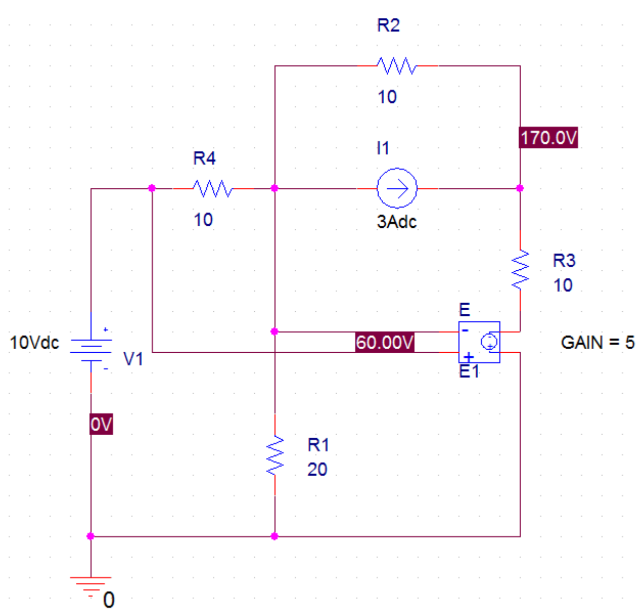
Solution

Step 1. We have two unknown so we end up with two nodal equations,
$[(v_1–10)/10] + [(v_1–0)/20] + 3 + [(v_1–v_2)/10] = 0$ and
$[(v_2–v_1)/10] – 3 + [(v_2–(–5v_o))/10] = 0$. We now have two equations with three unknowns so we need a constraint equation, $v_o = 10–v_1$ or $5v_o = 50–5v_1$.

Step 2. $(0.1+0.05+0.1)v_1 – 0.1v_2 = –2 = 0.25v_1 – 0.1v_2$ and
$–0.1v_1 + (0.1+0.1)v_2 + 5 – 0.5v_1 = 3$ or $–0.6v_1 + 0.2v_2 = –2$. Now we can combine the two equations after having multiplied the first one by 2.

$0.5v_1 –0.2v_2 = –4$ and
$–0.6v_1 + 0.2v_2 = –2$ or $–0.1v_1 = –6$ or $v_1 =$ **60 V** and $0.2v_2 = 0.6v_1 – 2 = 34$ or $v_2 =$ **170 V**.

Checking with PSpice we get,



Solution 3.23

Use nodal analysis to find V_o in the circuit of Fig. 3.72.

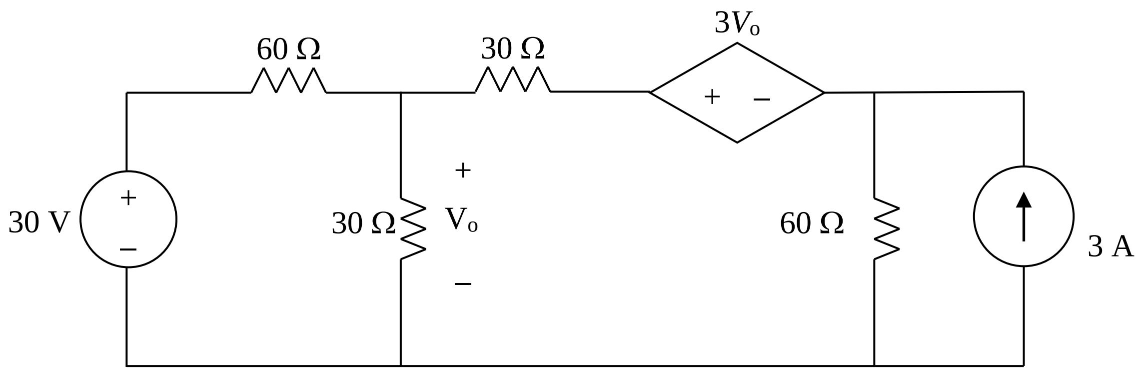
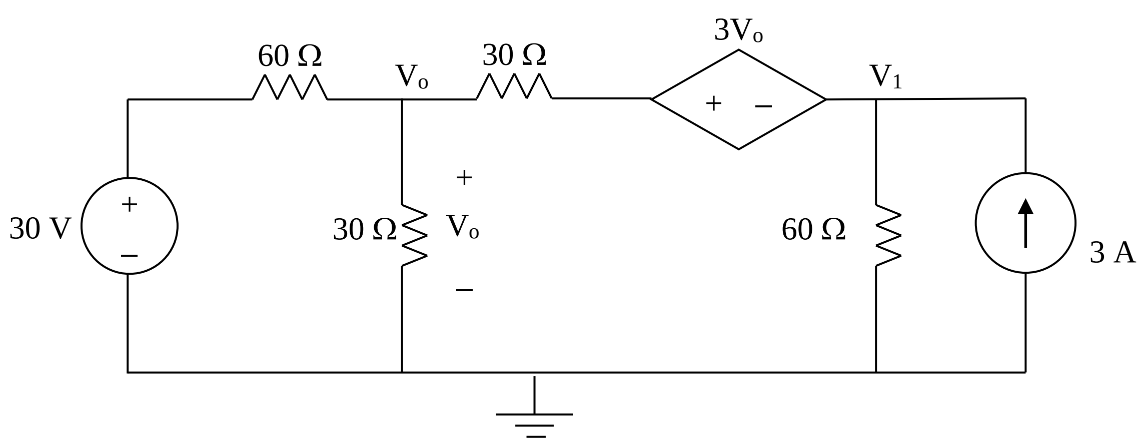


Figure 3.72
For Prob. 3.23.

Solution

Step 1. We apply nodal analysis to the circuit shown below.



At node o, $\frac{V_o - 30}{60} + \frac{V_o - 0}{30} + \frac{V_o - (3V_o + V_1)}{30} = 0$ and at node 1, we get,

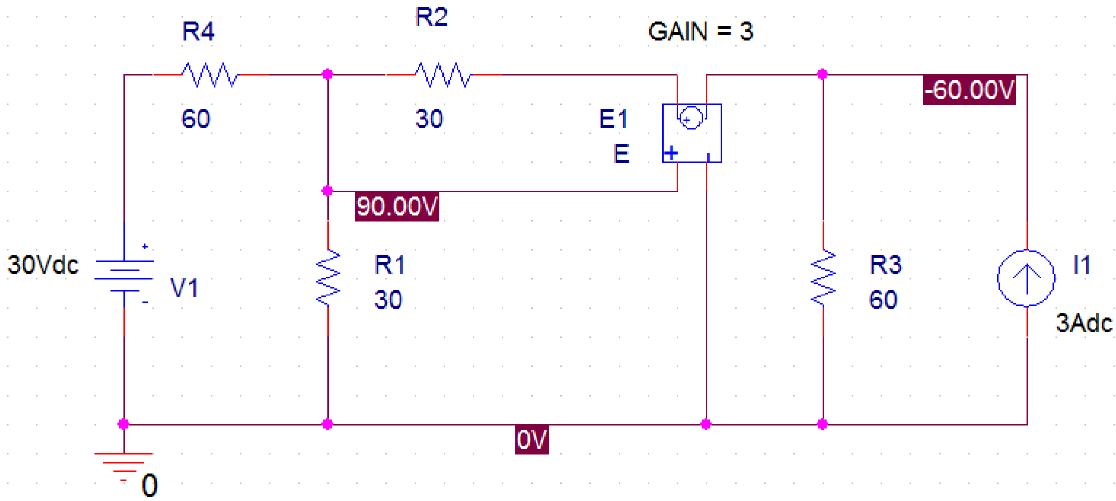
$$\frac{(3V_o + V_1) - V_o}{30} + \frac{V_1 - 0}{60} - 3 = 0$$

Step 2. $[(1/60)+(1/30)+(1/30)–(3/30)]V_o – (1/30)v_1 = 0.5$ or
$(0.08333–0.1)V_o – 0.033333V_1 = –0.0166667V_o – 0.033333V_1 = 0.5$ and

$(0.1–0.033333)V_o + 0.05V_1 = 0.06667V_o + 0.05V_1 = 3$. To find V_o all we need to do is to multiply the first equation by 3 and multiply the second equation by 2 and then combine them.

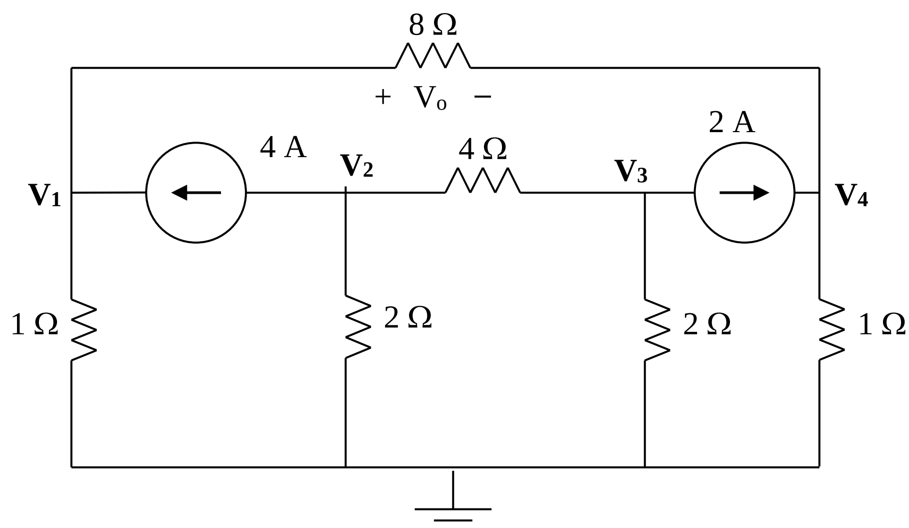
$–0.05V_o – 0.1V_1 = 1.5$ and $0.13333V_o + 0.1V_1 = 6$ which leads to $0.083333V_o = 7.5$ or $V_o = \mathbf{90\ V}$.

Checking with PSpice we get,



Solution 3.24

Consider the circuit below.



$$\frac{V_1 - 0}{1} - 4 + \frac{V_1 - V_4}{8} = 0 \rightarrow 1.125V_1 - 0.125V_4 = 4 \qquad (1)$$

$$+4 + \frac{V_2 - 0}{2} + \frac{V_2 - V_3}{4} = 0 \rightarrow 0.75V_2 - 0.25V_3 = -4 \qquad (2)$$

$$\frac{V_3 - V_2}{4} + \frac{V_3 - 0}{2} + 2 = 0 \rightarrow -0.25V_2 + 0.75V_3 = -2 \qquad (3)$$

$$-2 + \frac{V_4 - V_1}{8} + \frac{V_4 - 0}{1} = 0 \rightarrow -0.125V_1 + 1.125V_4 = 2 \qquad (4)$$

$$\begin{bmatrix} 1.125 & 0 & 0 & -0.125 \\ 0 & 0.75 & -0.25 & 0 \\ 0 & -0.25 & 0.75 & 0 \\ -0.125 & 0 & 0 & 1.125 \end{bmatrix} V = \begin{bmatrix} 4 \\ -4 \\ -2 \\ 2 \end{bmatrix}$$

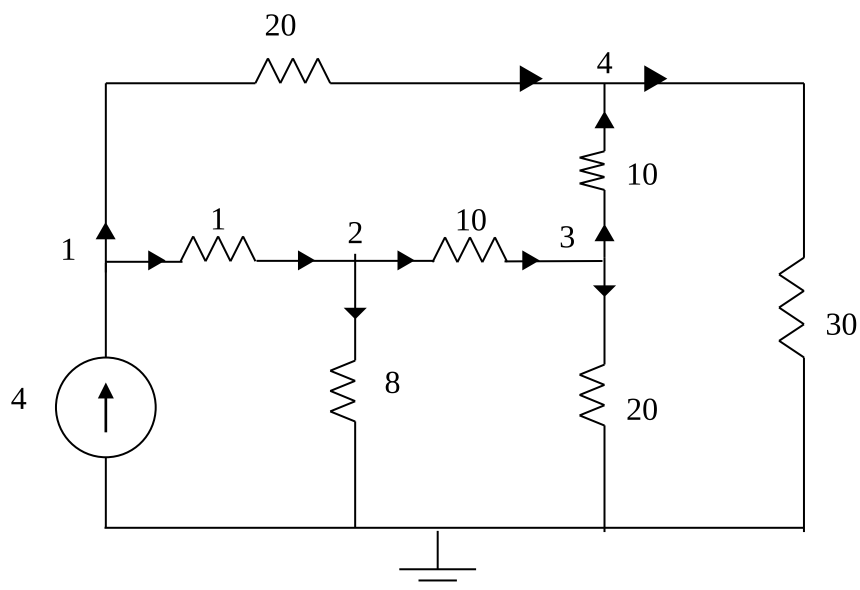
Now we can use MATLAB to solve for the unknown node voltages.

```
>> Y=[1.125,0,0,-0.125;0,0.75,-0.25,0;0,-0.25,0.75,0;-0.125,0,0,1.125]

Y =

    1.1250         0         0   -0.1250
         0    0.7500   -0.2500         0
         0   -0.2500    0.7500         0
   -0.1250         0         0    1.1250

>> I=[4,-4,-2,2]'

I =

     4
    -4
    -2
     2

>> V=inv(Y)*I

V =

    3.8000
   -7.0000
   -5.0000
    2.2000
```

$V_o = V_1 – V_4 = 3.8 – 2.2 =$ **1.6 V**.

Solution 3.25

Consider the circuit shown below.



At node 1.

$$4 = \frac{V_1 - V_2}{1} + \frac{V_1 - V_4}{20} \longrightarrow 80 = 21V_1 - 20V_2 - V_4 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{1} = \frac{V_2}{8} + \frac{V_2 - V_3}{10} \longrightarrow 0 = -80V_1 + 98V_2 - 8V_3 \qquad (2)$$

At node 3,

$$\frac{V_2 - V_3}{10} = \frac{V_3}{20} + \frac{V_3 - V_4}{10} \longrightarrow 0 = -2V_2 + 5V_3 - 2V_4 \qquad (3)$$

At node 4,

$$\frac{V_1 - V_4}{20} + \frac{V_3 - V_4}{10} = \frac{V_4}{30} \longrightarrow 0 = 3V_1 + 6V_3 - 11V_4 \qquad (4)$$

Putting (1) to (4) in matrix form gives:

$$\begin{bmatrix} 80 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 21 & -20 & 0 & -1 \\ -80 & 98 & -8 & 0 \\ 0 & -2 & 5 & -2 \\ 3 & 0 & 6 & -11 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$B = A\ V \longrightarrow V = A^{-1} B$$

Using MATLAB leads to

$$V_1 = \mathbf{25.52\ V},\quad V_2 = \mathbf{22.05\ V},\quad V_3 = \mathbf{14.842\ V},\quad V_4 = \mathbf{15.055\ V}$$

Solution 3.26

At node 1,

$$\frac{15-V_1}{20}=3+\frac{V_1-V_3}{10}+\frac{V_1-V_2}{5} \longrightarrow -45=7V_1-4V_2-2V_3 \qquad (1)$$

At node 2,

$$\frac{V_1-V_2}{5}+\frac{4I_o-V_2}{5}=\frac{V_2-V_3}{5} \qquad (2)$$

But $I_o=\dfrac{V_1-V_3}{10}$. Hence, (2) becomes

$$0=7V_1-15V_2+3V_3 \qquad (3)$$

At node 3,

$$3+\frac{V_1-V_3}{10}+\frac{-10-V_3}{15}+\frac{V_2-V_3}{5}=0 \longrightarrow 70=-3V_1-6V_2+11V_3 \qquad (4)$$

Putting (1), (3), and (4) in matrix form produces

$$\begin{pmatrix} 7 & -4 & -2 \\ 7 & -15 & 3 \\ -3 & -6 & 11 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}=\begin{pmatrix} -45 \\ 0 \\ 70 \end{pmatrix} \longrightarrow AV=B$$

Using MATLAB leads to

$$V=A^{-1}B=\begin{pmatrix} -7.19 \\ -2.78 \\ 2.89 \end{pmatrix}$$

Thus,

V_1 = **–7.19V**; V_2 = **–2.78V**; V_3 = **2.89V**.

Solution 3.27

At node 1,

$$2 = 2v_1 + v_1 - v_2 + (v_1 - v_3)4 + 3i_0, \quad i_0 = \quad 4v_2. \text{ Hence,}$$

$$2 = 7v_1 + 11v_2 - 4v_3 \qquad (1)$$

At node 2,

$$v_1 - v_2 = 4v_2 + v_2 - v_3 \longrightarrow \qquad 0 = - v_1 + 6v_2 - v_3 \qquad (2)$$

At node 3,

$$2v_3 = 4 + v_2 - v_3 + 12v_2 + 4(v_1 - v_3)$$

or

$$-4 = 4v_1 + 13v_2 - 7v_3 \qquad (3)$$

In matrix form,

$$\begin{bmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{vmatrix} = 176, \quad \Delta_1 = \begin{vmatrix} 2 & 11 & -4 \\ 0 & -6 & 1 \\ -4 & 13 & -7 \end{vmatrix} = 110$$

$$\Delta_2 = \begin{vmatrix} 7 & 2 & -4 \\ 1 & 0 & 1 \\ 4 & -4 & -7 \end{vmatrix} = 66, \quad \Delta_3 = \begin{vmatrix} 7 & 11 & 2 \\ 1 & -6 & 0 \\ 4 & 13 & -4 \end{vmatrix} = 286$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{110}{176} = 0.625\text{V}, \quad v_2 = \frac{\Delta_2}{\Delta} = \frac{66}{176} = 0.375\text{V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{286}{176} = 1.625\text{V}.$$

v_1 = **625 mV**, v_2 = **375 mV**, v_3 = **1.625 V**.

Solution 3.28

At node c,

$$\frac{V_d - V_c}{10} = \frac{V_c - V_b}{4} + \frac{V_c}{5} \longrightarrow 0 = -5V_b + 11V_c - 2V_d \qquad (1)$$

At node b,

$$\frac{V_a + 90 - V_b}{8} + \frac{V_c - V_b}{4} = \frac{V_b}{8} \longrightarrow -90 = V_a - 4V_b + 2V_c \quad (2)$$

At node a,

$$\frac{V_a - 60 - V_d}{4} + \frac{V_a}{16} + \frac{V_a + 90 - V_b}{8} = 0 \longrightarrow 60 = 7V_a - 2V_b - 4V_d \quad (3)$$

At node d,

$$\frac{V_a - 60 - V_d}{4} = \frac{V_d}{20} + \frac{V_d - V_c}{10} \longrightarrow 300 = 5V_a + 2V_c - 8V_d \quad (4)$$

In matrix form, (1) to (4) become

$$\begin{pmatrix} 0 & -5 & 11 & -2 \\ 1 & -4 & 2 & 0 \\ 7 & -2 & 0 & -4 \\ 5 & 0 & 2 & -8 \end{pmatrix}\begin{pmatrix} V_a \\ V_b \\ V_c \\ V_d \end{pmatrix} = \begin{pmatrix} 0 \\ -90 \\ 60 \\ 300 \end{pmatrix} \longrightarrow AV = B$$

We use MATLAB to invert A and obtain

$$V = A^{-1}B = \begin{pmatrix} -10.56 \\ 20.56 \\ 1.389 \\ -43.75 \end{pmatrix}$$

Thus,

V_a = **–10.56 V**; V_b = **20.56 V**; V_c = **1.389 V**; VC_d = **–43.75 V**.

Solution 3.29

At node 1,

$$5+V_1-V_4+2V_1+V_1-V_2=0 \quad \longrightarrow \quad -5=4V_1-V_2-V_4 \qquad (1)$$

At node 2,

$$V_1-V_2=2V_2+4(V_2-V_3)=0 \quad \longrightarrow \quad 0=-V_1+7V_2-4V_3 \qquad (2)$$

At node 3,

$$6+4(V_2-V_3)=V_3-V_4 \quad \longrightarrow \quad 6=-4V_2+5V_3-V_4 \qquad (3)$$

At node 4,

$$2+V_3-V_4+V_1-V_4=3V_4 \quad \longrightarrow \quad 2=-V_1-V_3+5V_4 \qquad (4)$$

In matrix form, (1) to (4) become

$$\begin{pmatrix} 4 & -1 & 0 & -1 \\ -1 & 7 & -4 & 0 \\ 0 & -4 & 5 & -1 \\ -1 & 0 & -1 & 5 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix}=\begin{pmatrix} -5 \\ 0 \\ 6 \\ 2 \end{pmatrix} \quad \longrightarrow \quad AV=B$$

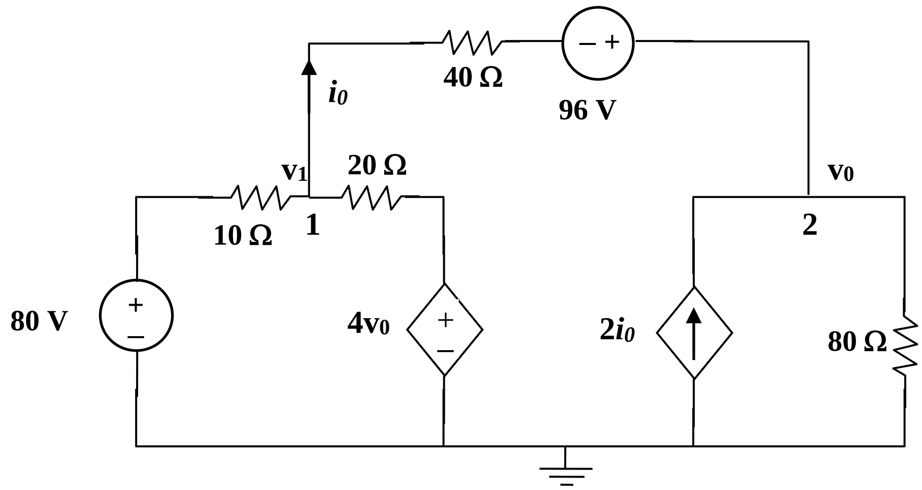
Using MATLAB,

$$V=A^{-1}B=\begin{pmatrix} -0.7708 \\ 1.209 \\ 2.309 \\ 0.7076 \end{pmatrix}$$

i.e.

$$\underline{\boldsymbol{V_1=-0.7708\text{ V},\ V_2=1.209\text{ V},\ V_3=2.309\text{ V},\ V_4=0.7076\text{ V}}}$$

Solution 3.30



At node 1,

$$[(v_1–80)/10]+[(v_1–4v_o)/20]+[(v_1–(v_o–96))/40] = 0 \text{ or}$$
$$(0.1+0.05+0.025)v_1 – (0.2+0.025)v_o =$$
$$0.175v_1 – 0.225v_o = 8–2.4 = 5.6 \qquad (1)$$

At node 2,

$$–2\boldsymbol{i_o} + [((v_o–96)–v_1)/40] + [(v_o–0)/80] = 0 \text{ and } \boldsymbol{i_o} = [(v_1–(v_o–96))/40]$$
$$–2[(v_1–(v_o–96))/40] + [((v_o–96)–v_1)/40] + [(v_o–0)/80] = 0$$
$$–3[(v_1–(v_o–96))/40] + [(v_o–0)/80] = 0 \text{ or}$$
$$–0.0.075v_1 + (0.075+0.0125)v_o = 7.2 =$$
$$–0.075v_1 + 0.0875v_o = 7.2 \qquad (2)$$

Using (1) and (2) we get,

$$\begin{bmatrix} 0.175 & -0.225 \\ -0.075 & 0.0875 \end{bmatrix}\begin{bmatrix} v_1 \\ v_o \end{bmatrix} = \begin{bmatrix} 5.6 \\ 7.2 \end{bmatrix} \text{ or}$$

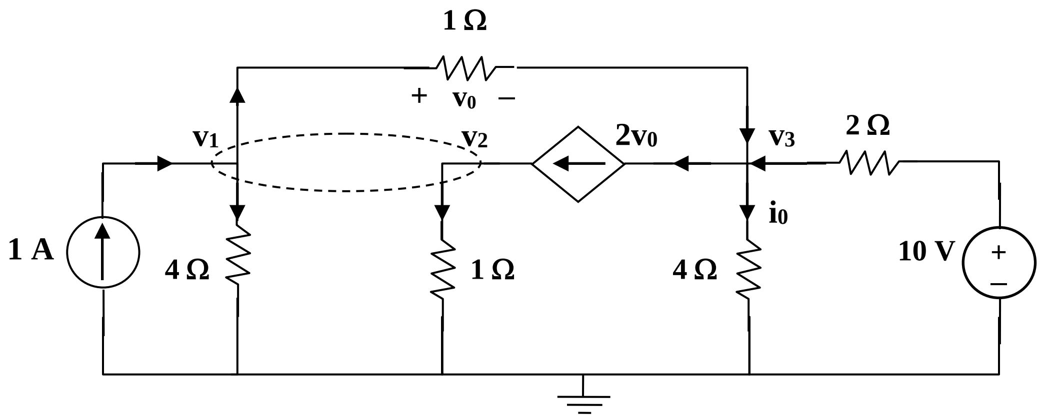
$$\begin{bmatrix} v_1 \\ v_o \end{bmatrix} = \frac{\begin{bmatrix} 0.0875 & 0.225 \\ 0.075 & 0.175 \end{bmatrix}}{0.0153125 - 0.016875}\begin{bmatrix} 5.6 \\ 7.2 \end{bmatrix} = \frac{\begin{bmatrix} 0.0875 & 0.225 \\ 0.075 & 0.175 \end{bmatrix}}{-0.0015625}\begin{bmatrix} 5.6 \\ 7.2 \end{bmatrix}$$

v_1 = –313.6–1036.8 = –1350.4

v_o = –268.8–806.4 = **–1.0752 kV**

and $\boldsymbol{i_o}$ = $[(v_1–(v_o–96))/40]$ = [(–1350.4–(–1075.2–96))/40] = **–4.48 amps**.

Solution 3.31



At the supernode,

$$1 + 2v_0 = \frac{v_1}{4} + \frac{v_2}{1} + \frac{v_1 - v_3}{1} \qquad (1)$$

But $v_o = v_1 - v_3$. Hence (1) becomes,

$$4 = -3v_1 + 4v_2 + 4v_3 \qquad (2)$$

At node 3,

$$2v_o + \frac{v_3}{4} = v_1 - v_3 + \frac{10 - v_3}{2}$$

or

$$20 = 4v_1 + 0v_2 - v_3 \qquad (3)$$

At the supernode, $v_2 = v_1 + 4i_o$. But $i_o = \frac{v_3}{4}$. Hence,

$$v_2 = v_1 + v_3 \qquad (4)$$

Solving (2) to (4) leads to,

$v_1 =$ **4.97V**, $v_2 =$ **4.85V**, $v_3 =$ **–0.12V**.

Solution 3.32

Obtain the node voltages $\boldsymbol{v_1}$, $\boldsymbol{v_2}$, and $\boldsymbol{v_3}$ in the circuit of Fig. 3.81.

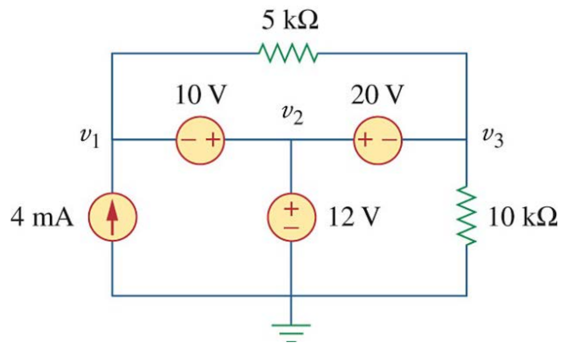
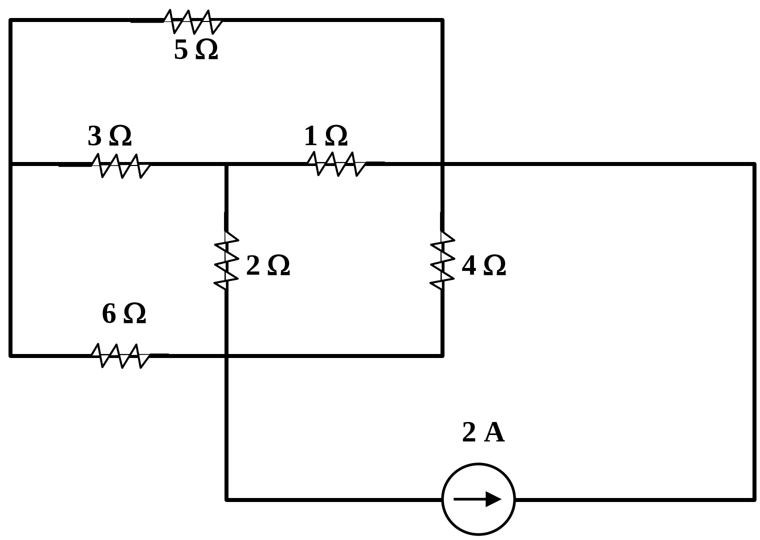


Figure 3.81
For Prob. 3.32.

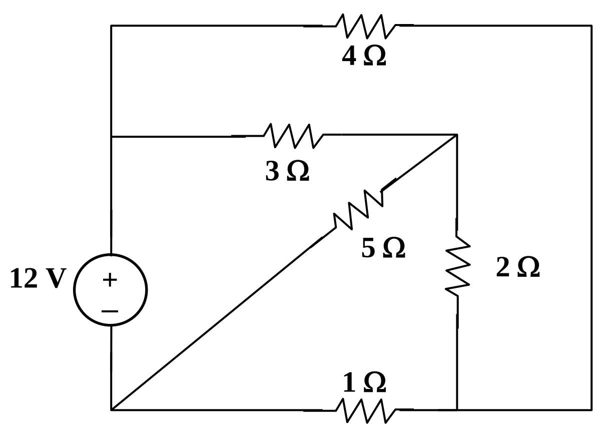
Step 1. and 2. This is an interesting problem. Clearly we have one supernode and that all the node voltages are known! From the circuit, v_2 = **120 V**; $v_1 = v_2 - 50$ = **70 V**; and $v_3 = v_2 - 75$ = **45 V**.

Solution 3.33

(a) This is a **planar** circuit. It can be redrawn as shown below.

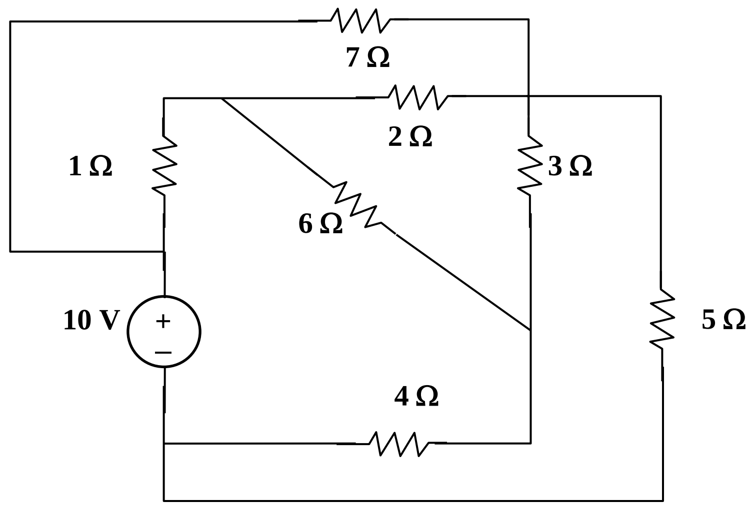


(b) This is a **planar** circuit. It can be redrawn as shown below.



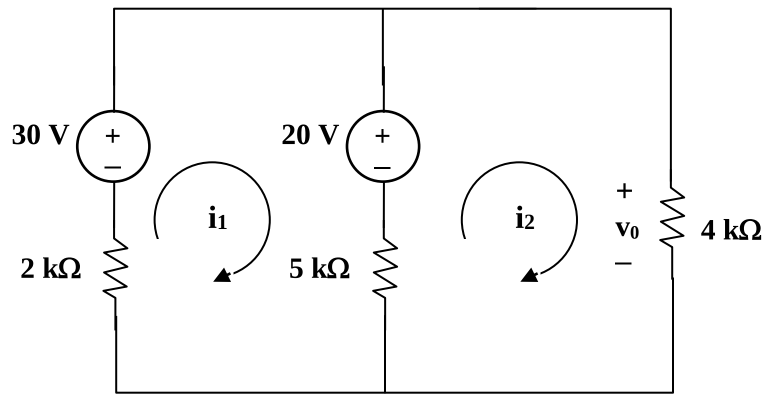
Solution 3.34

(a) This is a **planar** circuit because it can be redrawn as shown below,



(b) This is a **non-planar** circuit.

Solution 3.35



Assume that i_1 and i_2 are in mA. We apply mesh analysis. For mesh 1,

$$-30 + 20 + 7i_1 - 5i_2 = 0 \quad \text{or} \quad 7i_1 - 5i_2 = 10 \qquad (1)$$

For mesh 2,

$$-20 + 9i_2 - 5i_1 = 0 \quad \text{or} \quad -5i_1 + 9i_2 = 20 \qquad (2)$$

Solving (1) and (2), we obtain, $i_2 = 5$.

$$v_0 = 4i_2 = \mathbf{20\ volts}.$$

Solution 3.36

Use mesh analysis to obtain i_a, i_b, and i_c in the circuit shown in Fig. 3.84.

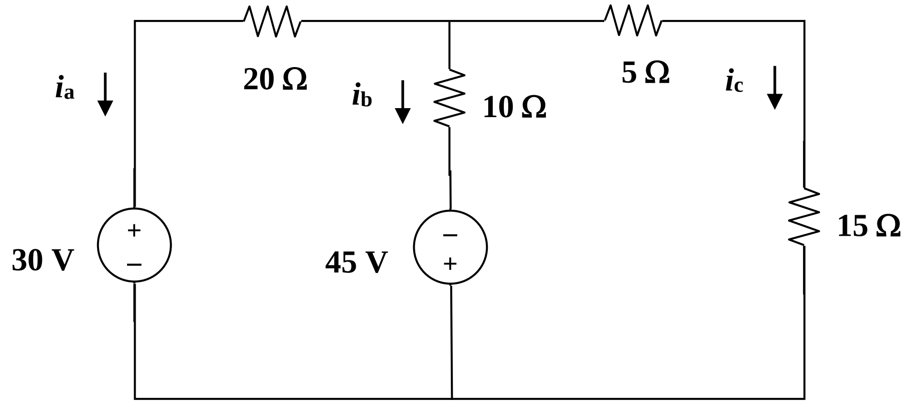
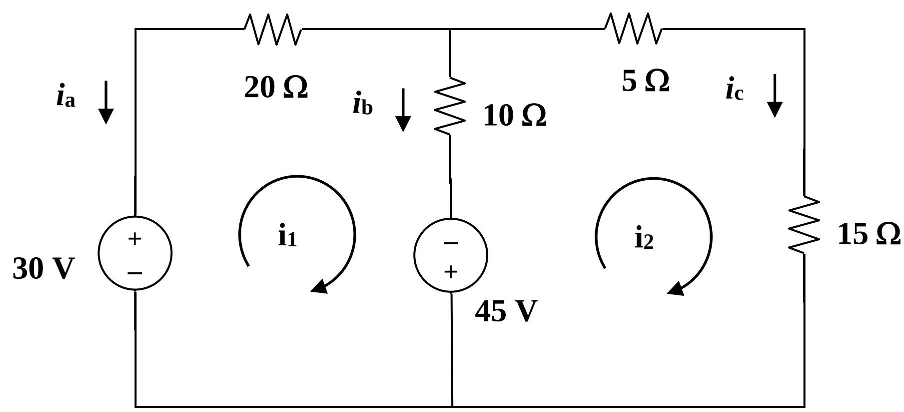


Figure 3.84
For Prob. 3.36.

Step 1. Establish two unknown loop currents and write the mesh equations. Then solve the mesh equations for the two unknown loop currents which will allow us to solve for the unknown branch currents.



Loop 1. $-30 + 20i_1 + 10(i_1-i_2) - 45 = 0$ or $30i_1 - 10i_2 = 75$

Loop 2. $45 + 10(i_2-i_1) + 5i_2 + 15i_2 = 0$ or $-10i_1 + 30i_2 = -45$

Finally $i_a = -i_1$; $i_b = i_1 - i_2$; and $i_c = i_2$.

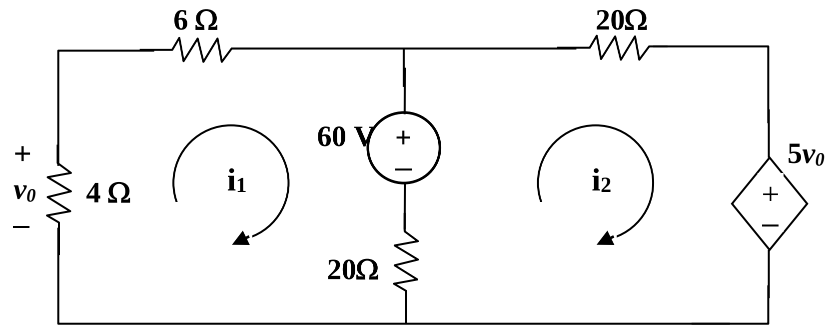
Step 2. The matrix equation is,

$$\begin{bmatrix} 30 & -10 \\ -10 & 30 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 75 \\ -45 \end{bmatrix} \text{or} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{\begin{bmatrix} 30 & 10 \\ 10 & 30 \end{bmatrix}}{900-100}\begin{bmatrix} 75 \\ -45 \end{bmatrix}$$

i_1 = (2250–450)/800 = 2.25 A and i_2 = (750–1350)/800 = –750 mA.

Finally, i_a = **–2.25 A**; i_b = **3 A**; and i_c = **–750 mA**.

Solution 3.37



Applying mesh analysis to loops 1 and 2, we get,

$$30i_1 - 20i_2 + 60 = 0 \text{ which leads to } i_2 = 1.5i_1 + 3 \quad (1)$$

$$-20i_1 + 40i_2 - 60 + 5v_0 = 0 \quad (2)$$

$$\text{But, } v_0 = -4i_1 \quad (3)$$

Using (1), (2), and (3) we get $-20i_1 + 60i_1 + 120 - 60 - 20i_1 = 0$ or

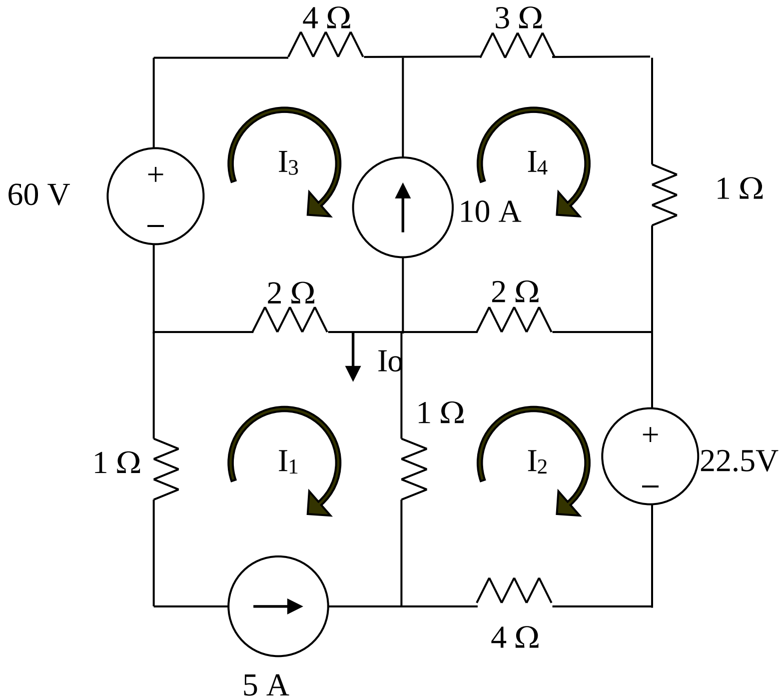
$$20i_1 = -60 \text{ or } i_1 = -3 \text{ amps and } i_2 = 7.5 \text{ amps.}$$

Therefore, we get,

$$v_0 = -4i_1 = \mathbf{12\ volts}.$$

Solution 3.38

Consider the circuit below with the mesh currents.



$I_1 = -5$ A (1)

$1(I_2 - I_1) + 2(I_2 - I_4) + 22.5 + 4I_2 = 0$

$7I_2 - I_4 = -27.5$ (2)

$-60 + 4I_3 + 3I_4 + 1I_4 + 2(I_4 - I_2) + 2(I_3 - I_1) = 0$ (super mesh)

$-2I_2 + 6\,I_3 + 6I_4 = +60 - 10 = 50$ (3)

But, we need one more equation, so we use the constraint equation $-I_3 + I_4 = 10$. This now gives us three equations with three unknowns.

$$\begin{bmatrix} 7 & 0 & -1 \\ -2 & 6 & 6 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -27.5 \\ 50 \\ 10 \end{bmatrix}$$

We can now use MATLAB to solve the problem.

```
>> Z=[7,0,-1;-2,6,6;0,-1,0]
```

```
Z =

     7     0    -1
    -2     6     6
     0    -1     0
>> V=[–27.5,50,10]'

V =
  –27.5
   50
   10
>> I=inv(Z)*V

I =
  –1.3750
  –10.0000
   17.8750
```

$I_o = I_1 – I_2 = –5 – 1.375 =$ **$–6.375$ A**.

Check using the super mesh (equation (3)):

$–2I_2 + 6\ I_3 + 6I_4 = 2.75 – 60 + 107.25 = 50!$

Solution 3.39

Using Fig. 3.50 from Prob. 3.1, design a problem to help other students to better understand mesh analysis.

Solution

Given $R_1 = 4$ kΩ, $R_2 = 2$ kΩ, and $R_3 = 2$ kΩ, determine the value of I_x using mesh analysis.

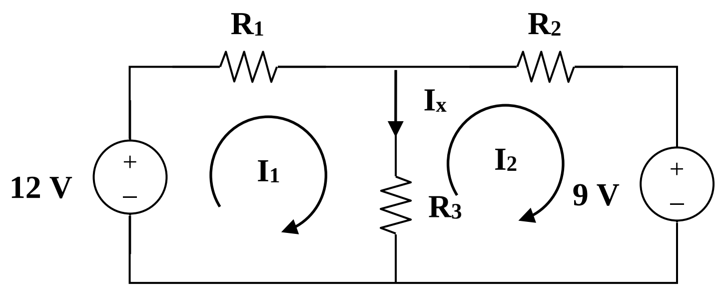


Figure 3.50
For Prob. 3.1 and 3.39.

For loop 1 we get $-12 + 4kI_1 + 2k(I_1-I_2) = 0$ or $6I_1 - 2I_2 = 0.012$ and at

loop 2 we get $2k(I_2-I_1) + 2kI_2 + 9 = 0$ or $-2I_1 + 4I_2 = -0.009$.

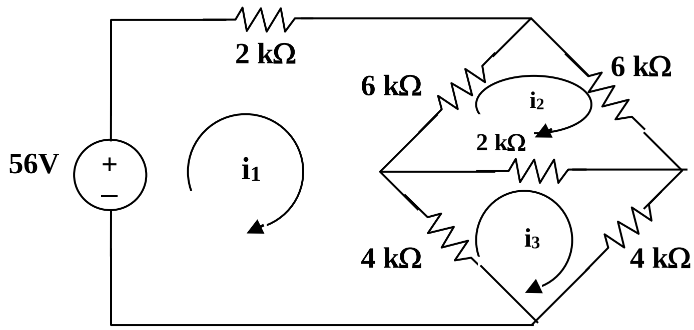
Now $6I_1 - 2I_2 = 0.012 + 3[-2I_1 + 4I_2 = -0.009]$ leads to,

$10I_2 = 0.012 - 0.027 = -0.015$ or $I_2 = -1.5$ mA and $I_1 = (-0.003+0.012)/6 = 1.5$ mA.

Thus,

$I_x = I_1-I_2 = (1.5+1.5)$ mA = **3 mA**.

Solution 3.40



Assume all currents are in mA and apply mesh analysis for mesh 1.

$$-56 + 12i_1 - 6i_2 - 4i_3 = 0 \text{ or } 6i_1 - 3i_2 - 2i_3 = 28 \qquad (1)$$

for mesh 2,

$$-6i_1 + 14i_2 - 2i_3 = 0 \text{ or } -3i_1 + 7i_2 - i_3 = 0 \qquad (2)$$

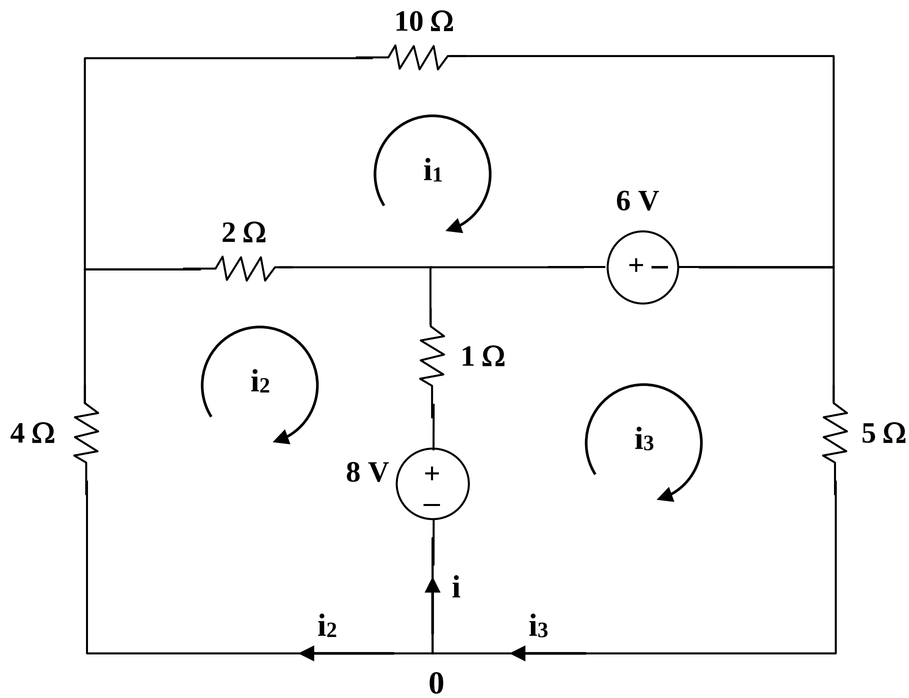
for mesh 3,

$$-4i_1 - 2i_2 + 10i_3 = 0 \text{ or } -2i_1 - i_2 + 5i_3 = 0 \qquad (3)$$

Solving (1), (2), and (3) using MATLAB, we obtain,

$$\boldsymbol{i_o} = i_1 = \mathbf{8\ mA}.$$

Solution 3.41



For loop 1,

$$6 = 12i_1 - 2i_2 \longrightarrow 3 = 6i_1 - i_2 \qquad (1)$$

For loop 2,

$$-8 = -2i_1 + 7i_2 - i_3 \qquad (2)$$

For loop 3,

$$-8 + 6 + 6i_3 - i_2 = 0 \longrightarrow 2 = -i_2 + 6i_3 \qquad (3)$$

We put (1), (2), and (3) in matrix form,

$$\begin{bmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{vmatrix} = -234, \quad \Delta_2 = \begin{vmatrix} 6 & 3 & 0 \\ 2 & 8 & 1 \\ 0 & 2 & 6 \end{vmatrix} = 240$$

$$\Delta_3 = \begin{vmatrix} 6 & -1 & 3 \\ 2 & -7 & 8 \\ 0 & -1 & 2 \end{vmatrix} = -38$$

At node 0, $i + i_2 = i_3$ or $i = i_3 - i_2 = \dfrac{\Delta_3 - \Delta_2}{\Delta} = \dfrac{-38 - 240}{-234} =$ **1.188 A**

Solution 3.42

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the mesh currents in the circuit of Fig. 3.88.

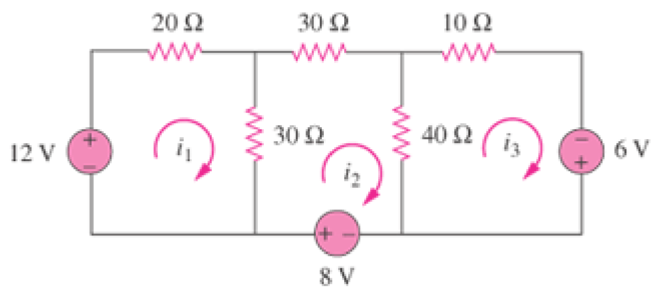


Figure 3.88

Solution

For mesh 1,

$$-12+50I_1-30I_2=0 \longrightarrow 12=50I_1-30I_2 \quad (1)$$

For mesh 2,

$$-8+100I_2-30I_1-40I_3=0 \longrightarrow 8=-30I_1+100I_2-40I_3 \quad (2)$$

For mesh 3,

$$-6+50I_3-40I_2=0 \longrightarrow 6=-40I_2+50I_3 \quad (3)$$

Putting eqs. (1) to (3) in matrix form, we get

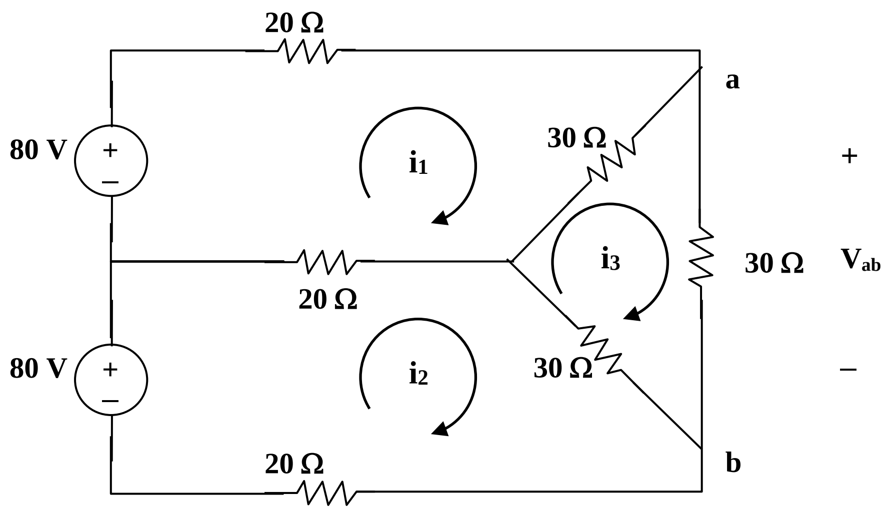
$$\begin{pmatrix} 50 & -30 & 0 \\ -30 & 100 & -40 \\ 0 & -40 & 50 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}=\begin{pmatrix} 12 \\ 8 \\ 6 \end{pmatrix} \longrightarrow AI=B$$

Using Matlab,

$$I=A^{-1}B=\begin{pmatrix} 0.48 \\ 0.40 \\ 0.44 \end{pmatrix}$$

i.e. I_1 = **480 mA**, I_2 = **400 mA**, I_3 = **440 mA**

Solution 3.43



For loop 1,

$$80 = 70i_1 - 20i_2 - 30i_3 \longrightarrow 8 = 7i_1 - 2i_2 - 3i_3 \qquad (1)$$

For loop 2,

$$80 = 70i_2 - 20i_1 - 30i_3 \longrightarrow 8 = -2i_1 + 7i_2 - 3i_3 \qquad (2)$$

For loop 3,

$$0 = -30i_1 - 30i_2 + 90i_3 \longrightarrow 0 = i_1 + i_2 - 3i_3$$

(3)

Solving (1) to (3), we obtain $i_3 = 16/9$

$$I_o = i_3 = 16/9 = \mathbf{1.7778\ A}$$

$$V_{ab} = 30i_3 = \mathbf{53.33\ V}.$$

Solution 3.44

Use mesh analysis to obtain $\boldsymbol{i_o}$ in the circuit of Fig. 3.90.

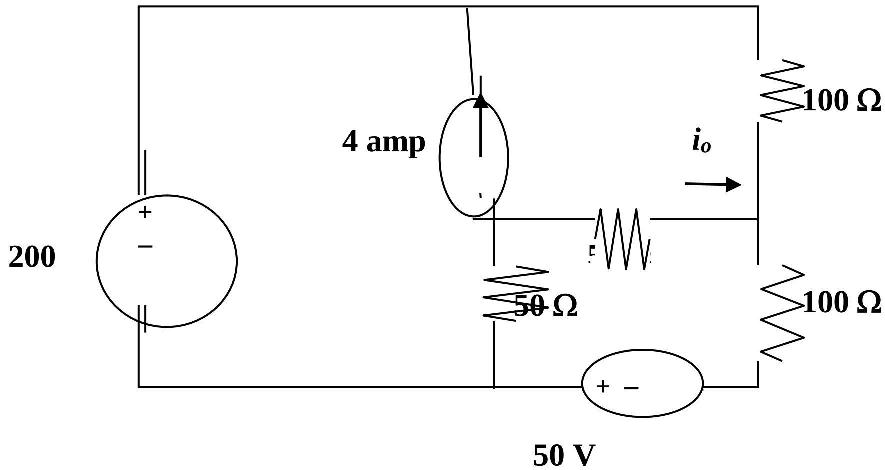
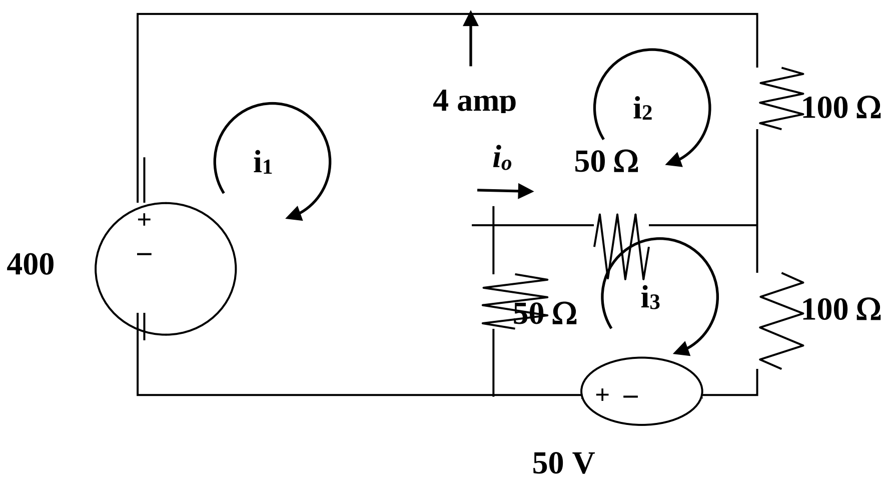


Figure 3.90
For Prob. 3.44.

Step 1. We need to redraw the circuit using a supermesh. Next we identify our unknown loop currents. Then we write our mesh equations and write the equation incorporating the current from the current source.



Supermesh (loop 1 and 2), $-400 + 100i_2 + 50(i_2 - i_3) + 50(i_1 - i_3) = 0$; loop 3 produces $50(i_3 - i_1) + 50(i_3 - i_2) + 100i_3 - 50 = 0$; and $i_2 - i_1 = 4$. We have three equations and three unknowns. Finally we note that $i_o = i_3 - i_2$.

Step 2. Since we need i_2 and i_3 let us use the constraint equation, $i_1 = i_2 – 4$, to allow us to solve for i_2 and i_3.

Using the first two equations we get,

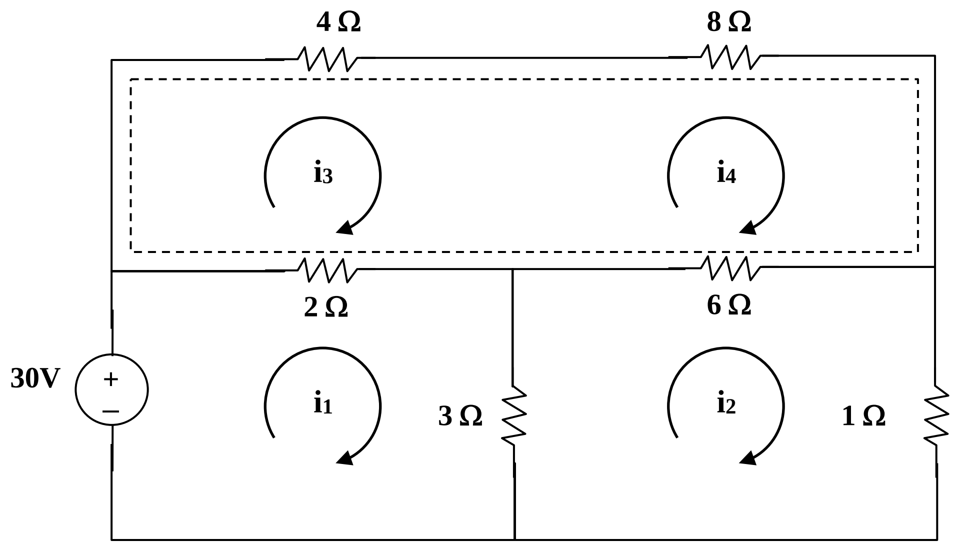
$100i_2 + 50(i_2–i_3) + 50((i_2–4)–i_3) = 400$ or $200i_2 – 100i_3 = 600$ and
$50(i_3–(i_2–4)) + 50(i_3–i_2) + 100i_3 = 50$ or $–100i_2 + 200i_3 = –150$. Thus,

$$\begin{bmatrix} 200 & -100 \\ -100 & 200 \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 600 \\ -150 \end{bmatrix} or \begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \frac{\begin{bmatrix} 200 & 100 \\ 100 & 200 \end{bmatrix}}{40,000-10,000}\begin{bmatrix} 600 \\ -150 \end{bmatrix}.$$

Finally, i_2 = [(120,000–15,000)/30,000] = 3.5 amps and i_3 = [(60,000–30,000)/30,000] = 1 amp. Now we get,

$i_o = i_3 – i_2 = 1–3.5 =$ **–2.5 amps**.

Solution 3.45



For loop 1, $30 = 5i_1 - 3i_2 - 2i_3$ (1)

For loop 2, $10i_2 - 3i_1 - 6i_4 = 0$ (2)

For the supermesh, $6i_3 + 14i_4 - 2i_1 - 6i_2 = 0$ (3)

But $i_4 - i_3 = 4$ which leads to $i_4 = i_3 + 4$ (4)

Solving (1) to (4) by elimination gives $i = i_1 =$ **8.561 A**.

Solution 3.46

Calculate the mesh currents i_1 and i_2 in Fig. 3.92.

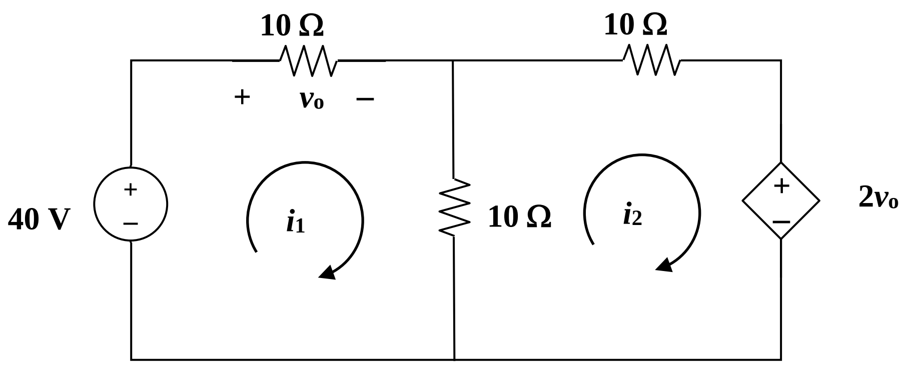


Figure 3.92
For Prob. 3.46.

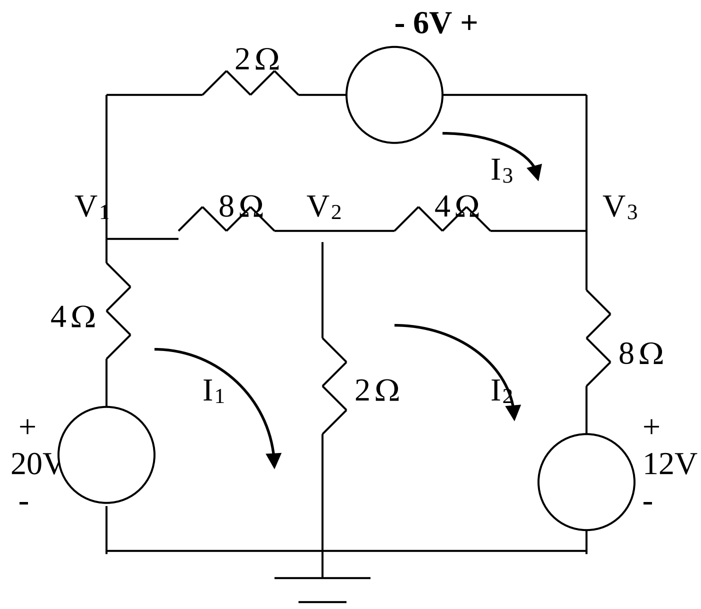
Step 1. Loop 1 $-40 + 10i_1 + 10(i_1 - i_2) = 0$ and for loop 2 $10(i_2 - i_1) + 10\, i_2 + 2v_o = 0$.

We now have two equations but three unknowns so we need a constraint equation or $v_o = 10i_1$.

Step 2. We now have $20i_1 - 10i_2 = 40$ and $-10i_1 + 20i_2 + 2(10i_1) = 0 = 10i_1 + 20i_2$ or $i_1 = -2i_2$ which leads to $20(-2i_2) - 10i_2 = -50i_2 = 40$ or $i_2 =$ **–800 mA**. Now we get $i_1 = -2(-0.8) =$ **1.6 A**.

Solution 3.47

First, transform the current sources as shown below.



For mesh 1,

$$-20 + 14I_1 - 2I_2 - 8I_3 = 0 \quad \longrightarrow \quad 10 = 7I_1 - I_2 - 4I_3 \qquad (1)$$

For mesh 2,

$$12 + 14I_2 - 2I_1 - 4I_3 = 0 \quad \longrightarrow \quad -6 = -I_1 + 7I_2 - 2I_3 \qquad (2)$$

For mesh 3,

$$-6 + 14I_3 - 4I_2 - 8I_1 = 0 \quad \longrightarrow \quad 3 = -4I_1 - 2I_2 + 7I_3 \qquad (3)$$

Putting (1) to (3) in matrix form, we obtain

$$\begin{pmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 10 \\ -6 \\ 3 \end{pmatrix} \quad \longrightarrow \quad AI = B$$

Using MATLAB,

$$I = A^{-1}B = \begin{bmatrix} 2 \\ 0.0333 \\ 1.8667 \end{bmatrix} \quad \longrightarrow \underline{\ } I_1 = 2.5,\ I_2 = 0.0333, I_3 = 1.8667$$

But

$$I_1 = \frac{20 - V}{4} \quad \longrightarrow \quad V_1 = 20 - 4I_1 = \mathbf{10\ V}$$

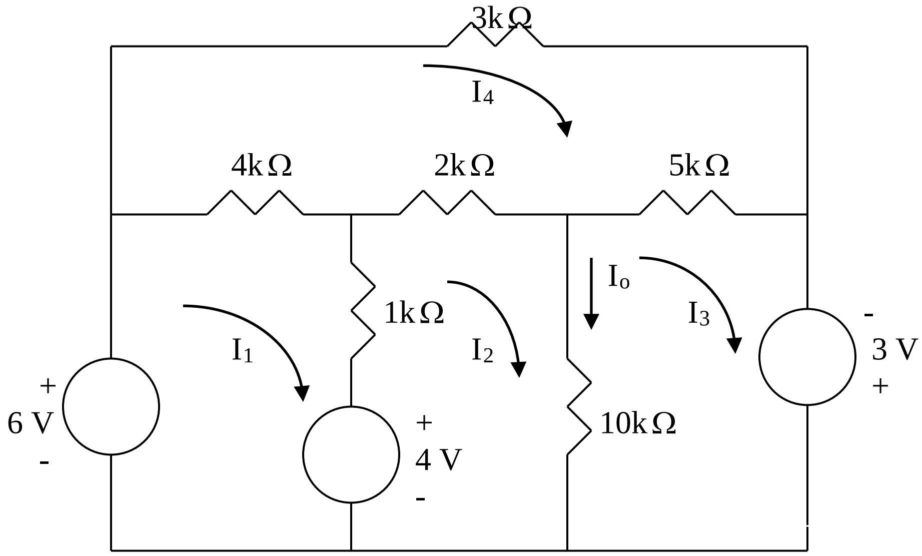
$$V_2 = 2(I_1 - I_2) = \mathbf{4.933\ V}$$

Also,

$$I_2 = \frac{V_3 - 12}{8} \quad \longrightarrow \quad V_3 = 12 + 8I_2 = \mathbf{12.267\ V}.$$

Solution 3.48

We apply mesh analysis and let the mesh currents be in mA.



For mesh 1,

$$-6+8+5I_1-I_2-4I_4=0 \longrightarrow 2=5I_1-I_2-4I_4 \qquad (1)$$

For mesh 2,

$$-4+13I_2-I_1-10I_3-2I_4=0 \longrightarrow 4=-I_1+13I_2-10I_3-2I_4 \qquad (2)$$

For mesh 3,

$$-3+15I_3-10I_2-5I_4=0 \longrightarrow 3=-10I_2+15I_3-5I_4 \qquad (3)$$

For mesh 4,

$$-4I_1-2I_2-5I_3+14I_4=0 \qquad (4)$$

Putting (1) to (4) in matrix form gives

$$\begin{pmatrix} 5 & -1 & 0 & -4 \\ -1 & 13 & -10 & -2 \\ 0 & -10 & 15 & -5 \\ -4 & -2 & -5 & 14 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix}=\begin{pmatrix} 2 \\ 4 \\ 3 \\ 0 \end{pmatrix} \longrightarrow AI=B$$

Using MATLAB,

$$I=A^{-1}B=\begin{pmatrix} 3.608 \\ 4.044 \\ 3.896 \\ 3 \end{pmatrix} 0.148$$

The current through the $10k\Omega$ resistor is $I_o = I_2 - I_3$ = **148 mA**.

Solution 3.49

Find v_o and i_o in the circuit of Fig. 3.94.

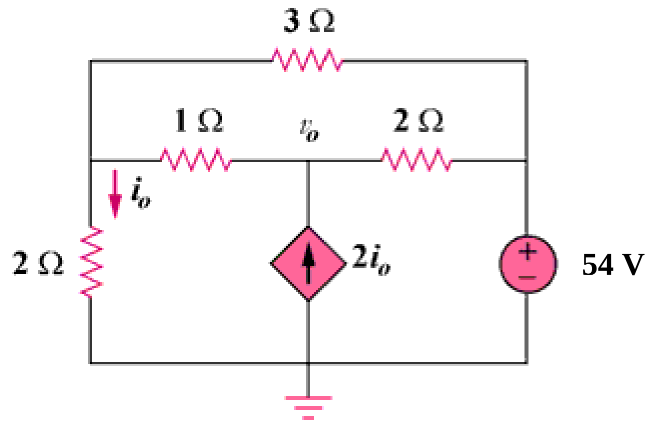
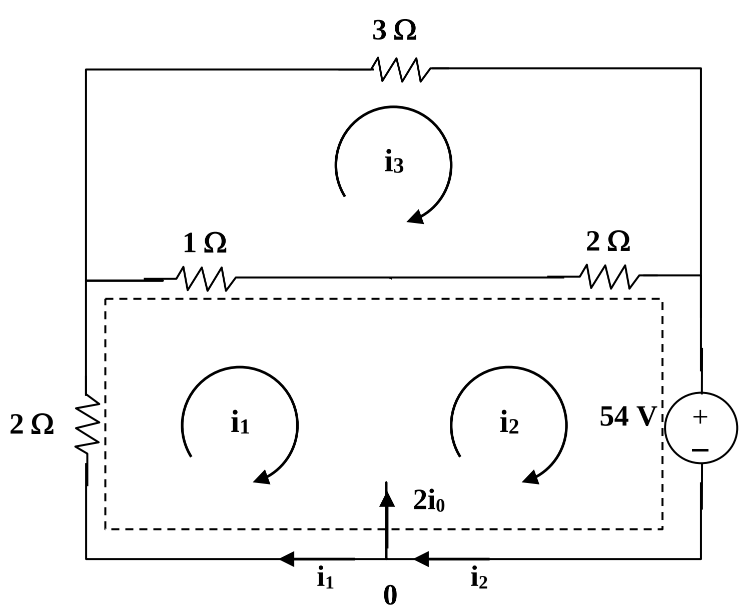


Figure 3.94
For Prob. 3.49.

Step 1. First we note that we have three unknown loop currents but we can only write two mesh equations (one is a supermesh). So we will need a constraint equation or $i_o = -i_1$ and $i_2 - i_1 = 2i_o = -2i_1 = i_2 - i_1$ or $i_2 = -i_1$. Now we have three equations and three unknowns.



(a)

The supermesh gives us $2i_1 + 1(i_1 - i_3) + 2(i_2 - i_3) + 54 = 0$ and loop 3 produces $1(i_3 - i_1) + 3i_3 + 2(i_3 - i_2) = 0$. Finally $v_o = 2(i_2 - i_3) + 54 = -2(i_1 + i_3) + 54$.

Step 2. $3i_1 + 2i_2 – 3i_3 = –54 = (3–2)i_1 – 3i_3$ or $i_1 – 3i_3 = –54$.

Next $–i_1 – 2i_2 + 6i_3 = 0 = (–1+2)i_1 + 6i_3 = i_1 + 6i_3 = 0$. This leads to $i_1 = –6i_3$ and $–6i_3 – 3i_3 = –54$ or $i_3 = 6$ and $i_1 = –36$ or i_o = **36 A**. Finally, $v_o = –2(–36+6) + 54$ $= 60 + 54 =$ **114 V**.

Solution 3.50

Use mesh analysis to find the current i_o in the circuit in Fig. 3.95.

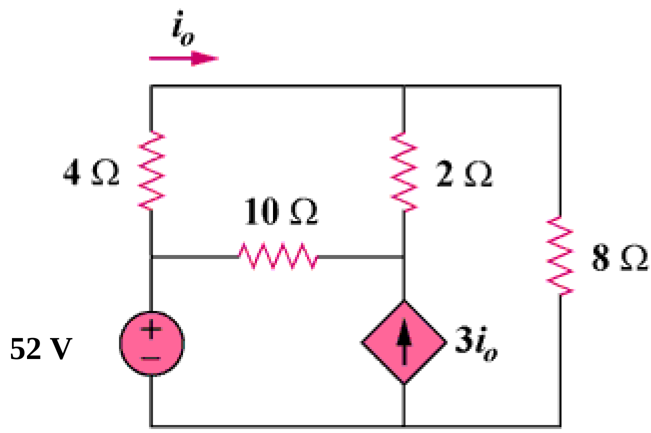
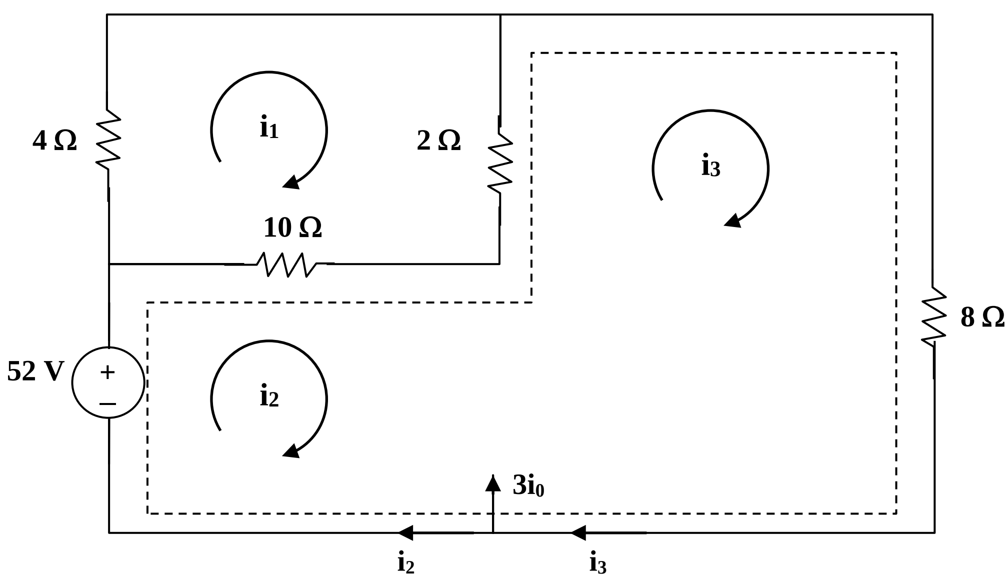


Figure 3.95
For Prob. 3.50.

Step 1. We note that we have three unknown loop currents but only two mesh equations (one is a supermesh). So we need a two constraint equations, one for i_2 and i_3 and i_o i_1.



For loop 1, $16i_1 - 10i_2 - 2i_3 = 0$ which leads to $8i_1 - 5i_2 - i_3 = 0$ (1)

For the supermesh, $-52 + 10i_2 - 10i_1 + 10i_3 - 2i_1 = 0$

or $$-6i_1 + 5i_2 + 5i_3 = 26 \quad (2)$$

Also, $3i_0 = i_3 - i_2$ and $i_0 = i_1$ which leads to $3i_1 = i_3 - i_2$ (3)

Using $i_3 = 3i_1 + i_2$ we get $8i_1 - 5i_2 - 3i_1 - i_2 = 0$ or $5i_1 = 6i_2$ or $i_1 = 1.2i_2$ and $-6i_1 + 5i_2 + 5i_3 = 26$ becomes $-6(1.2i_2) + 5i_2 + 5(3.6i_2+i_2) = 26 = (-7.2+5+23)i_2$ or $i_2 = 1.25$; $i_1 = 1.2x1.25 = 1.5$; and $i_3 = 3x1.5+1.25 = 5.75$. Finally,

$$i_o = \mathbf{1.5\ A}.$$

Solution 3.51

Apply mesh analysis to find v_o in the circuit in Fig. 3.96.

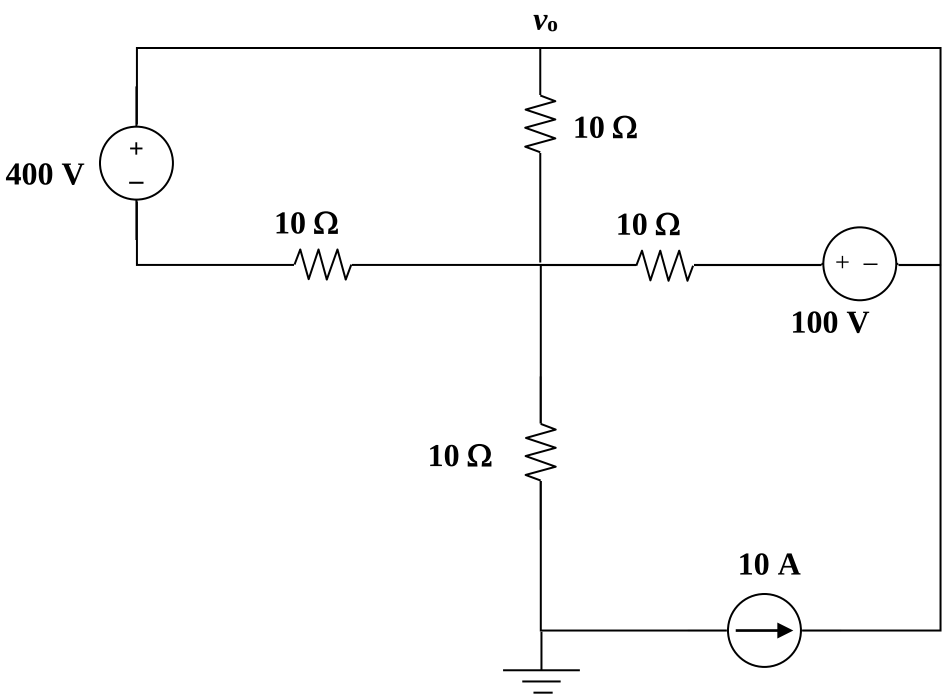
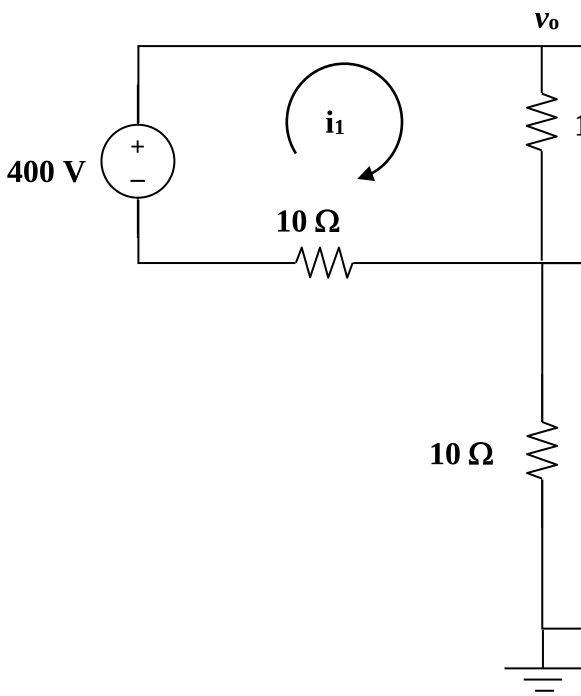


Figure 3.96
For Prob. 3.51.

Solution continued on the next page...

Step 1. First we identify the unknown loop currents and write the mesh equations.



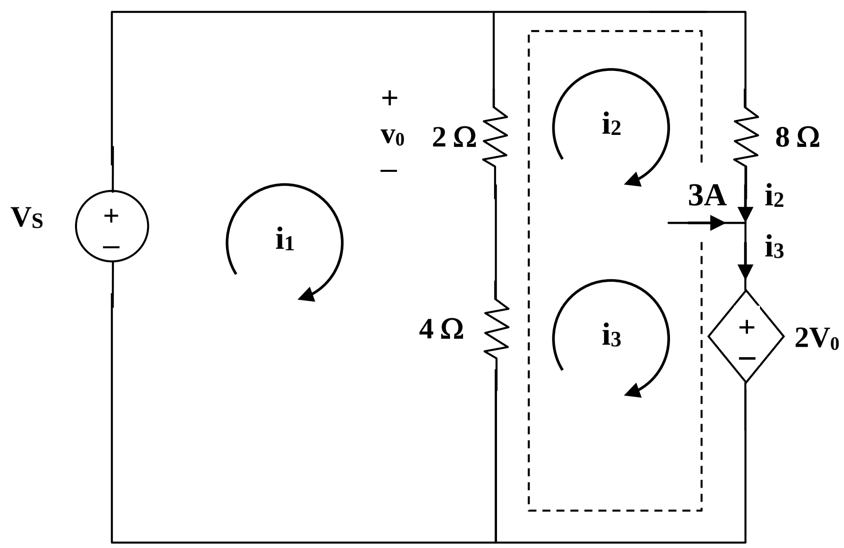
For loop 1 we get $-400 + 10(i_1-i_2) + 10i_1 = 0$ or $20i_1 - 10i_2 = 400$.
For loop 2 we get $10(i_2-i_1) - 100 + 10(i_2-(-10)) = 0$ or $-10i_1 + 20i_2 = 0$.

Finally we need v_o. Clearly $v_o = 10(i_1-i_2) + 10(10)$.

Step 2. From $-10i_1 + 20i_2 = 0$ we obtain $i_1 = 2i_2$ and from $20i_1 - 10i_2 = 400$ we obtain $40i_2 - 10i_2 = 30i_2 = 400$ or $i_2 = 13.333$ amps and $i_1 = 26.67$ amps. Now,

$v_o = 133.33 + 100 =$ **233.3 volts**.

Solution 3.52



For mesh 1,

$$2(i_1 - i_2) + 4(i_1 - i_3) - 12 = 0 \text{ which leads to } 3i_1 - i_2 - 2i_3 = 6 \qquad (1)$$

For the supermesh, $2(i_2 - i_1) + 8i_2 + 2v_0 + 4(i_3 - i_1) = 0$

But $v_0 = 2(i_1 - i_2)$ which leads to $-i_1 + 3i_2 + 2i_3 = 0$
(2)

For the independent current source, $i_3 = 3 + i_2$ (3)

Solving (1), (2), and (3), we obtain,

i_1 = **3.5 A**, i_2 = **-0.5 A**, i_3 = **2.5 A**.

Solution 3.53

Applying mesh analysis leads to;

$-12 + 4kI_1 - 3kI_2 - 1kI_3 = 0$ (1)

$-3kI_1 + 7kI_2 - 4kI_4 = 0$

$-3kI_1 + 7kI_2 = -12$ (2)

$-1kI_1 + 15kI_3 - 8kI_4 - 6kI_5 = 0$

$-1kI_1 + 15kI_3 - 6k = -24$ (3)

$I_4 = -3mA$ (4)

$-6kI_3 - 8kI_4 + 16kI_5 = 0$

$-6kI_3 + 16kI_5 = -24$ (5)

Putting these in matrix form (having substituted $I_4 = 3mA$ in the above),

$$\begin{bmatrix} 4 & -3 & -1 & 0 \\ -3 & 7 & 0 & 0 \\ -1 & 0 & 15 & -6 \\ 0 & 0 & -6 & 16 \end{bmatrix} k \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_5 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ -24 \\ -24 \end{bmatrix}$$

ZI = V

Using MATLAB,

```
>> Z = [4,-3,-1,0;-3,7,0,0;-1,0,15,-6;0,0,-6,16]

Z =

     4    -3    -1     0
    -3     7     0     0
    -1     0    15    -6
     0     0    -6    16

>> V = [12,-12,-24,-24]'

V =

    12
   -12
   -24
   -24
```

We obtain,

```
>> I = inv(Z)*V

I =
```

1.6196 mA
–1.0202 mA
–2.461 mA
3 mA
–2.423 mA

Solution 3.54

Let the mesh currents be in mA. For mesh 1,

$-12+10+2I_1-I_2=0 \quad \longrightarrow \quad 2=2I_1-I_2$ (1)

For mesh 2,

$-10+3I_2-I_1-I_3=0 \quad \longrightarrow \quad 10=-I_1+3I_2-I_3$ (2)

For mesh 3,

$-12+2I_3-I_2=0 \quad \longrightarrow \quad 12=-I_2+2I_3$ (3)

Putting (1) to (3) in matrix form leads to

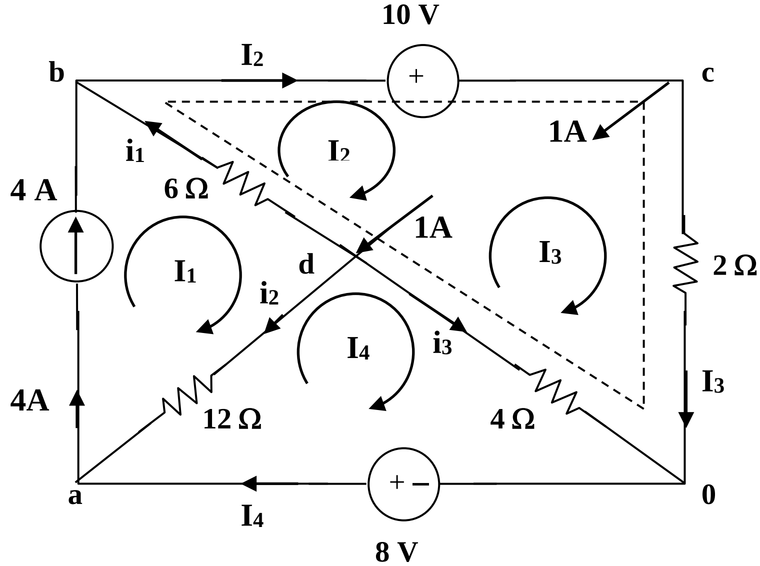
$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}=\begin{pmatrix} 2 \\ 10 \\ 12 \end{pmatrix} \quad \longrightarrow \quad AI=B$$

Using MATLAB,

$$I=A^{-1}B=\begin{bmatrix} 5.25 \\ 8.5 \\ 10.25 \end{bmatrix} \quad \longrightarrow \quad \underline{I_1=5.25\,\text{mA},\ I_2=8.5\,\text{mA}, I_3=10.25\,\text{mA}}$$

I_1 = **5.25 mA**, I_2 = **8.5 mA**, and I_3 = **10.25 mA**.

Solution 3.55



It is evident that $I_1 = 4$ (1)

For mesh 4, $12(I_4 – I_1) + 4(I_4 – I_3) – 8 = 0$ (2)

For the supermesh $6(I_2 – I_1) + 10 + 2I_3 + 4(I_3 – I_4) = 0$
or $-3I_1 + 3I_2 + 3I_3 – 2I_4 = -5$ (3)

At node c, $I_2 = I_3 + 1$ (4)

Solving (1), (2), (3), and (4) yields, $I_1 = 4A$, $I_2 = 3A$, $I_3 = 2A$, and $I_4 = 4A$

At node b, $i_1 = I_2 – I_1 =$ **-1A**

At node a, $i_2 = 4 – I_4 =$ **0A**

At node 0, $i_3 = I_4 – I_3 =$ **2A**

Solution 3.56

Determine $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ **i**n the circuit of Fig. 3.101.

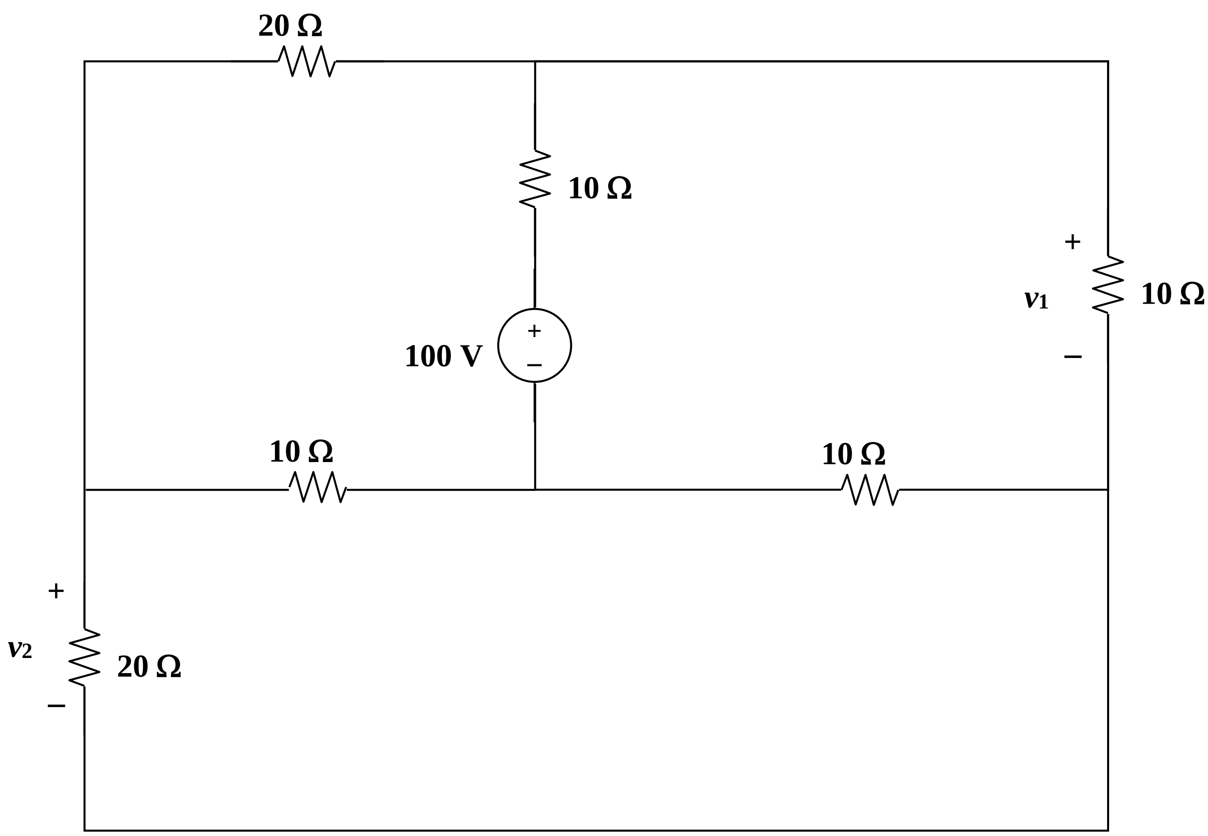
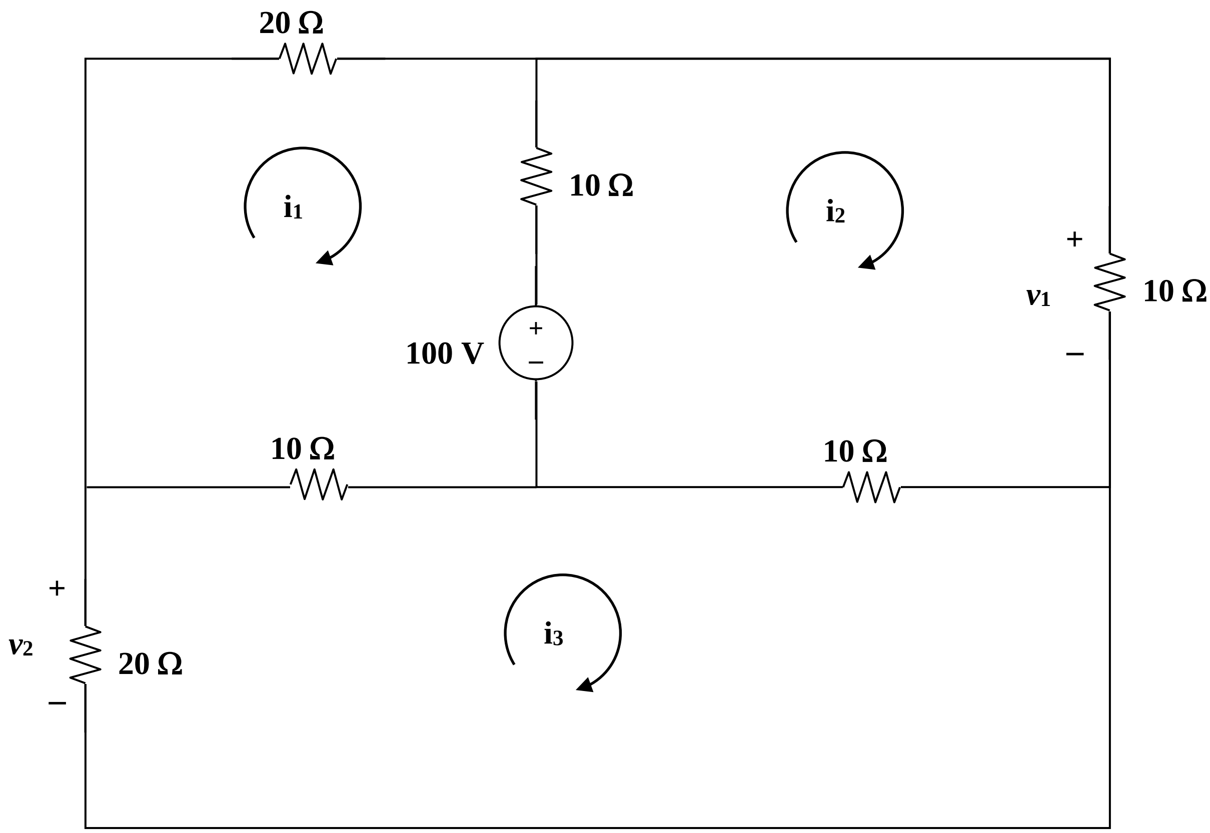


Figure 3.101
For Prob. 3.56.

Step 1. First we redraw the circuit and establish the unknown loop currents. Next we write the three mesh equations and put them into matrix form.

We will have a three by three matrix which we can invert and solve for the unknown loop currents. Finally we can solve for v_1 (= $10i_2$) and v_2 (= $20i_3$).

$20i_1 + 10(i_1–i_2) +100 + 10(i_1–i_3) = 0$ or $40i_1 – 10i_2 – 10i_3 = –100$
$–100 + 10(i_2–i_1) + 10i_2 + 10(i_2–i_3) = 0$ or $–10i_1 + 30i_2 – 10i_3 = 100$
$20i_3 + 10(i_3–i_1) + 10(i_3–i_2) = 0$ or $–10i_1 – 10i_2 + 40i_3 = 0$ which leads to,



Step 2.

$$\begin{bmatrix} 40 & -10 & -10 \\ -10 & 30 & -10 \\ -10 & -10 & 40 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -100 \\ 100 \\ 0 \end{bmatrix}$$ using MATLAB we get,

```
>> R=[40,-10,-10;-10,30,-10;-10,-10,40]

R =

   40  -10  -10
  -10   30  -10
  -10  -10   40

>> V=[-100;100;0]
```

V =

-100
100
0

>> I=inv(R)*V

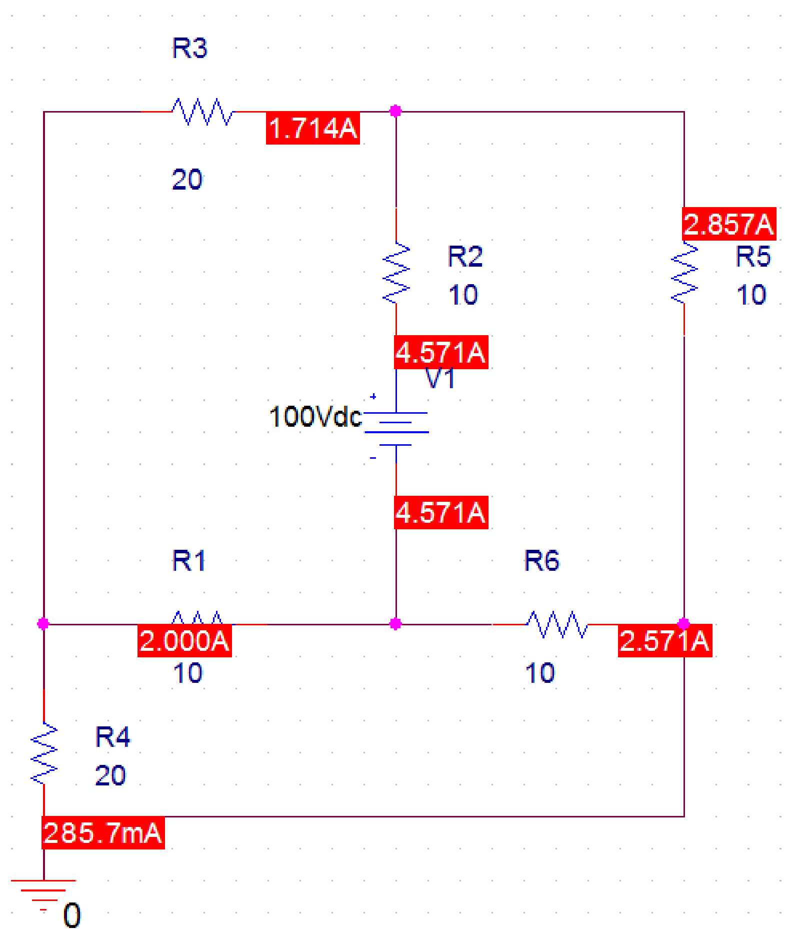
I =

-1.7143
2.8571
0.2857

Thus, $i_1 = –1.7143$ A, $i_2 = 2.8571$ A and $i_3 = 0.2857$ A which leads to,

$$v_1 = 10i_2 = 28.57 \text{ V and } v_2 = –20i_3 = –5.714 \text{ V}.$$

Checking with PSpice we get,



Solution 3.57

In the circuit in Fig. 3.102, find the values of R, V_1, and V_2 given that $i_o = 20$ mA.

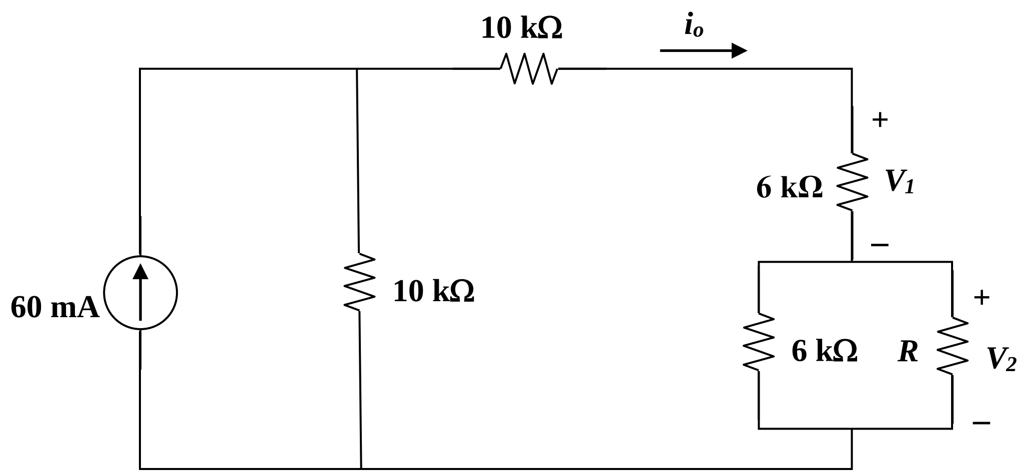


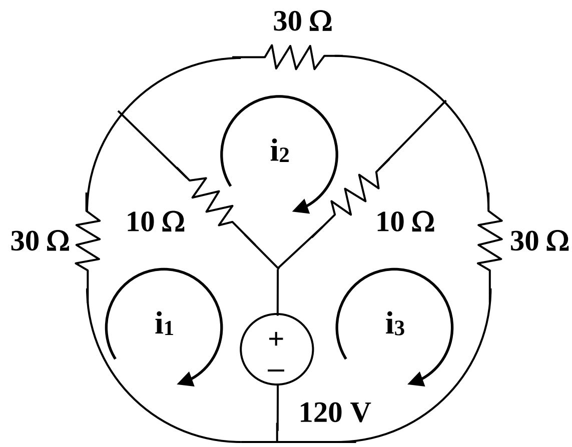
Figure 3.102
For Prob. 3.57.

Step 1. Since $i_o = 0.02$ A, $V_1 = 6{,}000 \text{x} 0.02$. By current division we get $V_2/R = 0.02[6k/(6k+R)]$ and $0.04 \text{x} 10{,}000 = 0.02[10k + 6k + 6k\text{x}R/(6k+R)]$. We can now solve for R, V_1, and V_2.

Step 2. $400 = 200 + 120 + 120R/(6k+R)$ or $R/(6k+R) = 80/120 = (2/3)$ or $1.5R = 6k + R$ or R = **12 kΩ**. V_1 = **120 V**. Now we can find V_2.

$V_2 = R\{0.02[6k/(6k+12k)]\} = 12k\{120/18k\}$ = **80 V**.

Solution 3.58



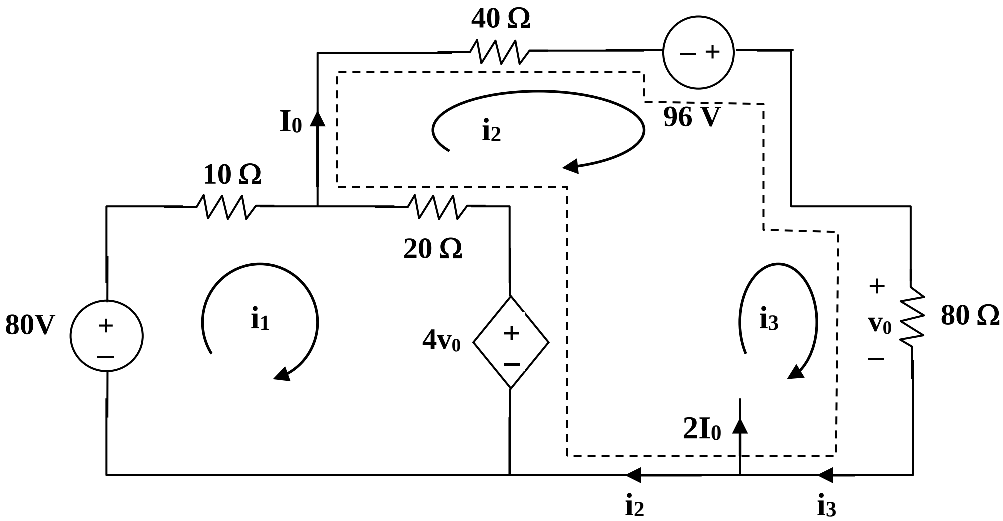
For loop 1, $120 + 40i_1 - 10i_2 = 0$, which leads to $-12 = 4i_1 - i_2$ (1)

For loop 2, $50i_2 - 10i_1 - 10i_3 = 0$, which leads to $-i_1 + 5i_2 - i_3 = 0$ (2)

For loop 3, $-120 - 10i_2 + 40i_3 = 0$, which leads to $12 = -i_2 + 4i_3$ (3)

Solving (1), (2), and (3), we get, i_1 = **-3A**, i_2 = **0**, and i_3 = **3A**

Solution 3.59



For loop 1, $-80 + 30i_1 - 20i_2 + 4v_0 = 0$, where $v_0 = 80i_3$

or $4 = 1.5i_1 - i_2 + 16i_3$ (1)

For the supermesh, $60i_2 - 20i_1 - 96 + 80i_3 - 4\,v_0 = 0$, where $v_0 = 80i_3$

or $4.8 = -i_1 + 3i_2 - 12i_3$ (2)

Also, $2I_0 = i_3 - i_2$ and $I_0 = i_2$, hence, $3i_2 = i_3$

(3)

From (1), (2), and (3),
$$\begin{bmatrix} 3 & -2 & 32 \\ -1 & 3 & -12 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 4.8 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -2 & 32 \\ -1 & 3 & -12 \\ 0 & 3 & -1 \end{vmatrix} = 5, \ \Delta_2 = \begin{vmatrix} 3 & 8 & 32 \\ -1 & 4.8 & -12 \\ 0 & 0 & -1 \end{vmatrix} = -22.4, \ \Delta_3 = \begin{vmatrix} 3 & -2 & 8 \\ -1 & 3 & 4.8 \\ 0 & 3 & 0 \end{vmatrix} = -67.2$$

$I_0 = i_2 = \Delta_2/\Delta = -28/5 =$ **-4.48 A**

$v_0 = 8i_3 = (-84/5)80 =$ **-1.0752 kvolts**

Solution 3.60

Calculate the power dissipated in each resistor in the circuit in Fig. 3.104.

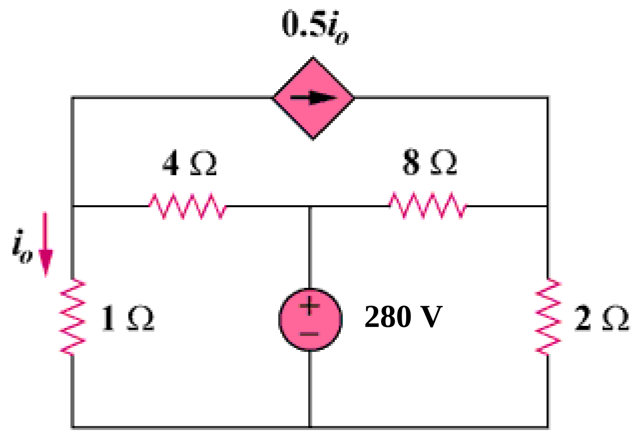
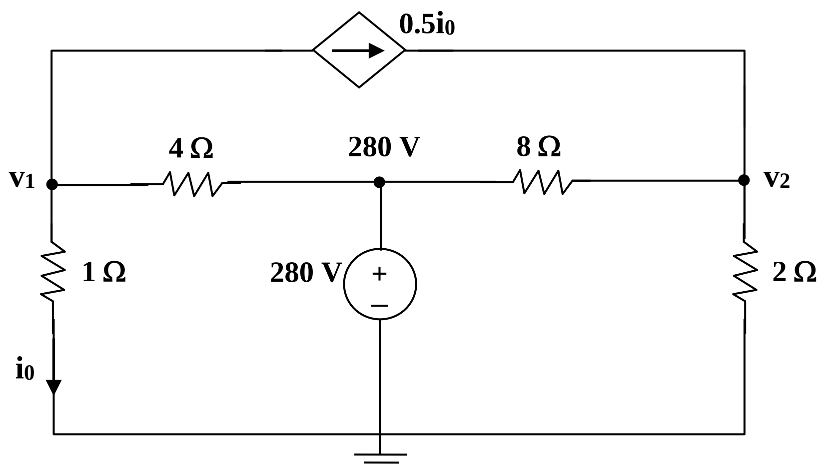


Figure 3.104
For Prob. 3.60.

Step 1. First we identify all of the unknown nodes of which we find two. Next we write two nodal equations. Since we have three unknowns but only two equations we need a constraint equation, $i_o = (v_1–0)/1 = v_1$.



At node 1, $[(v_1–0)/1] + [(v_1–280)/4] + 0.5i_o = 0$ and at node 2, $[(v_2–280)/8] – 0.5i_o + [(v_2–0)/2] = 0$. Finally $P_1 = (v_1)^2/1$; $P_4 = (v_1–280)^2/4$; $P_8 = (v_2–280)^2/8$; and $P_2 = (v_2)^2/2$.

Step 2. $(1+0.25)v_1 + 0.5v_1 = 1.75v_1 = 70$ or $v_1 = 40$ V. $(0.125+0.5)v_2 = 35+20 = 55$ or $v_2 = 55/0.625 = 88$ V. Finally,

P_1 = **1.6 kW**; P_4 = **14.4 kW**; P_8 = **4.608 kW**; and P_2 = **3.872 kW**.

Solution 3.61

Calculate the current gain i_o/i_s in the circuit of Fig. 3.105.

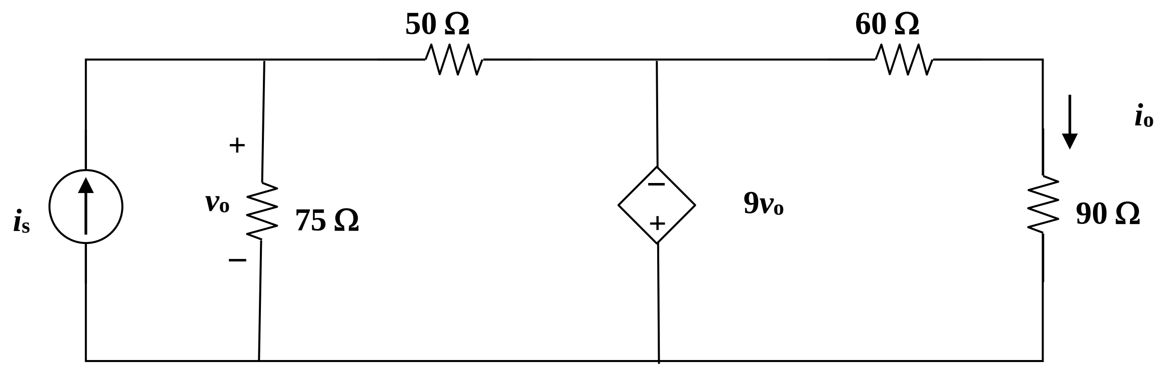
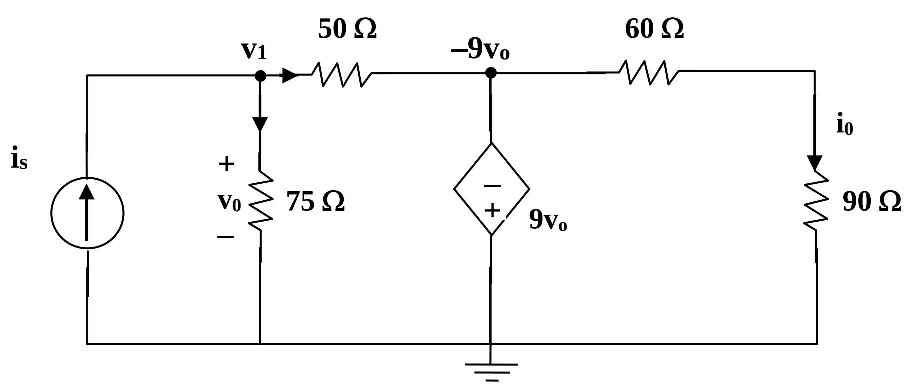


Figure 3.105
For Prob. 3.61.

Step 1. Since we wish to calculate the gain of this circuit we need to find i_o in terms of i_s. We can do this by using nodal analysis. First we identify the unknown nodes of which there is really only one. We can then write one nodal equation but we end up with two unknowns so we need a constraint equation, $v_o = v_1$.

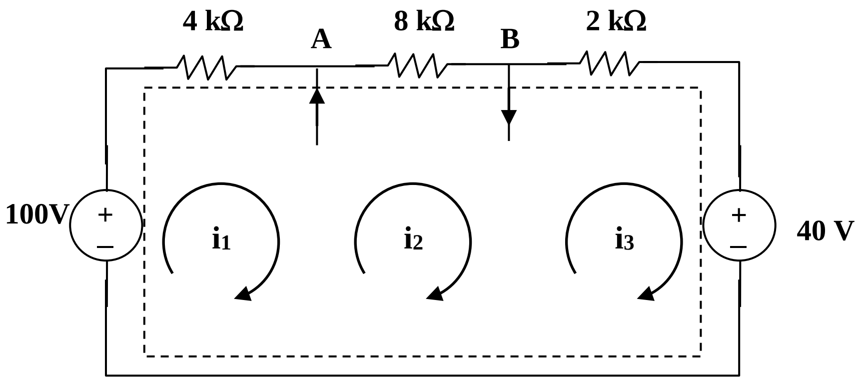


At node 1 we get, $-i_s + [(v_1-0)/75] + [(v_1-(-9v_o))/50] = 0$. Finally we can find $i_o = (-9v_o-0)/150 = -3v_o/50$.

Step 2. $(0.013333+0.2)v_1 = i_s$ or $v_1 = 4.688i_s$ and $i_o = -3(4.688i_s)/50 = -0.2813i_s$ which leads to,

$$i_o/i_s = \mathbf{-0.2813}.$$

Solution 3.62



We have a supermesh. Let all R be in kΩ, i in mA, and v in volts.

For the supermesh, $-100 + 4i_1 + 8i_2 + 2i_3 + 40 = 0$ or $30 = 2i_1 + 4i_2 + i_3$ (1)

At node A, $i_1 + 4 = i_2$ (2)

At node B, $i_2 = 2i_1 + i_3$ (3)

Solving (1), (2), and (3), we get i_1 = **2 mA**, i_2 = **6 mA**, and i_3 = **2 mA**.

Solution 3.63

Find v_x, and i_o in the circuit shown in Fig. 3.107.

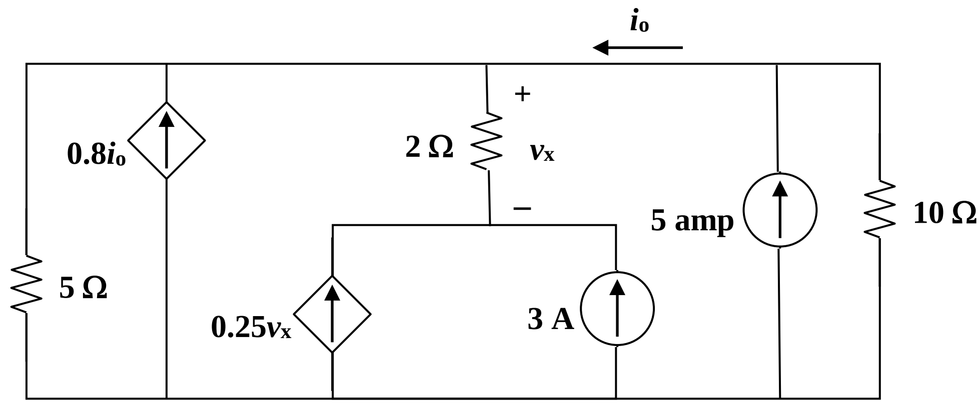
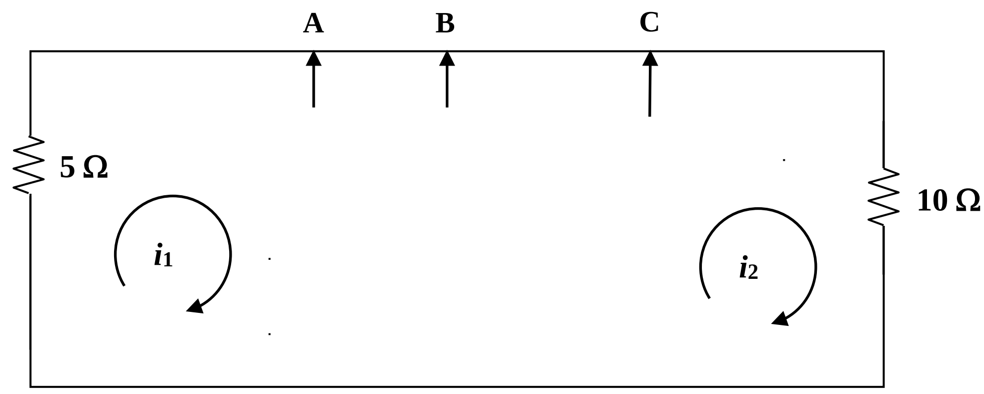


Figure 3.107
For Prob. 3.63.

Solution

Step 1. First we need to redraw the circuit to reflect the unknown currents.



For the supermesh, $5i_1 + 10i_2 = 0$.

At A, $-i_1 - 0.8i_o + I_{AB} = 0$. At B, $-I_{AB} - 0.25v_x - 3 + I_{BC} = 0$. Finally, at C, $-I_{BC} - 5 + i_2 = 0$.

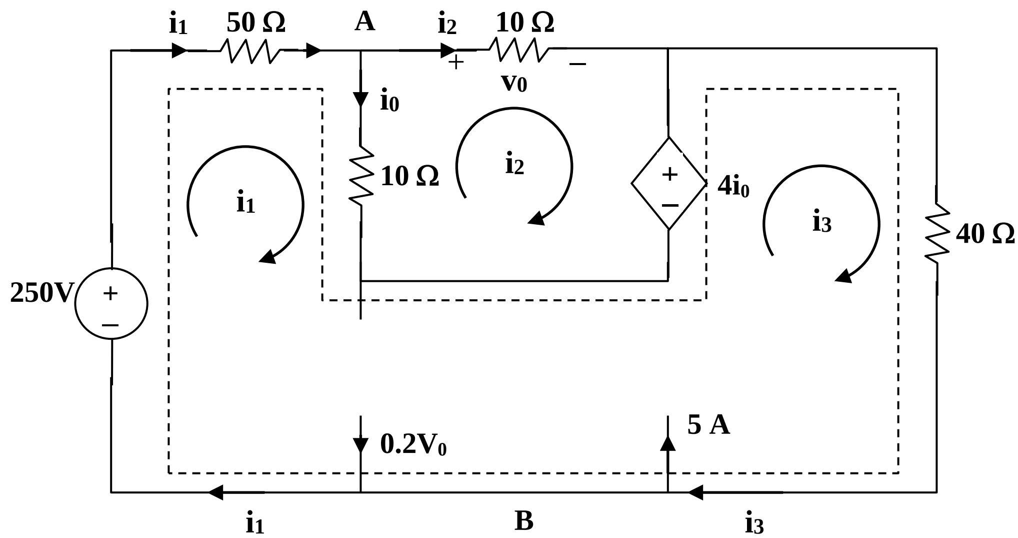
Our constraint equations are $v_x = 2(-0.25v_x-3)$ and $i_o = -I_{BC}$. We now have 5 unknowns and 5 equations.

Step 2. From $v_x = 2(–0.25v_x–3)$ we get $v_x = –6/1,5 =$ **–4 volts**.

From $5i_1 + 10i_2 = 0$, $i_o = –I_{BC}$, and $–I_{BC} – 5 + i_2 = 0$ we get $i_2 = 5 – i_o$ and $i_1 = –2i_2 = 2i_o – 10$.

From $–i_1 – 0.8i_o + I_{AB} = 0$, $–I_{AB} – 0.25v_x – 3 +I_{BC} = 0$ or $I_{AB} = –2 – i_o$, and $i_o = –I_{BC}$ or $I_{BC} = –i_o$, $i_1 = 2i_o – 10$, and $v_x = –4$ we get
$–(2i_o – 10) – 0.8i_o + (–2–i_o) = 0$ or
$–(2i_o – 10) – 0.8i_o + (–2 – i_o) = 0 = –3.8i_o +10 – 2$ or $i_o =$ **2.105 A**.

Solution 3.64



For mesh 2, $20i_2 - 10i_1 + 4i_0 = 0$ (1)

But at node A, $i_o = i_1 - i_2$ so that (1) becomes $i_1 = (16/6)i_2$ (2)

For the supermesh, $-250 + 50i_1 + 10(i_1 - i_2) - 4i_0 + 40i_3 = 0$

or $28i_1 - 3i_2 + 20i_3 = 125$ (3)

At node B, $i_3 + 0.2v_0 = 2 + i_1$ (4)

But, $v_0 = 10i_2$ so that (4) becomes $i_3 = 5 + (2/3)i_2$ (5)

Solving (1) to (5), $i_2 = 0.2941$ A,

$v_0 = 10i_2 =$ **2.941 volts**, $i_0 = i_1 - i_2 = (5/3)i_2 =$ **490.2mA**.

Solution 3.65

For mesh 1,

$-12 + 12I_1 - 6I_2 - I_4 = 0$ or

$12 = 12I_1 - 6I_2 - I_4 \qquad (1)$

For mesh 2,

$-6I_1 + 16I_2 - 8I_3 - I_4 - I_5 = 0 \qquad (2)$

For mesh 3,

$-8I_2 + 15I_3 - I_5 - 9 = 0$ or

$9 = -8I_2 + 15I_3 - I_5 \qquad (3)$

For mesh 4,

$-I_1 - I_2 + 7I_4 - 2I_5 - 6 = 0$ or

$6 = -I_1 - I_2 + 7I_4 - 2I_5 \qquad (4)$

For mesh 5,

$-I_2 - I_3 - 2I_4 + 8I_5 - 10 = 0$ or

$10 = -I_2 - I_3 - 2I_4 + 8I_5 \qquad (5)$

Casting (1) to (5) in matrix form gives

$$\begin{pmatrix} 12 & -6 & 0 & 1 & 0 \\ -6 & 16 & -8 & -1 & -1 \\ 0 & -8 & 15 & 0 & -1 \\ -1 & -1 & 0 & 7 & -2 \\ 0 & -1 & -1 & -2 & 8 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 9 \\ 6 \\ 10 \end{pmatrix} \longrightarrow \quad AI = B$$

Using MATLAB we input:
Z=[12,-6,0,-1,0;-6,16,-8,-1,-1;0,-8,15,0,-1;-1,-1,0,7,-2;0,-1,-1,-2,8]
and V=[12;0;9;6;10]

This leads to

```
>> Z=[12,-6,0,-1,0;-6,16,-8,-1,-1;0,-8,15,0,-1;-1,-1,0,7,-2;0,-1,-1,-2,8]

Z =

   12   -6    0   -1    0
   -6   16   -8   -1   -1
    0   -8   15    0   -1
   -1   -1    0    7   -2
    0   -1   -1   -2    8

>> V=[12;0;9;6;10]
```

```
V =

    12
     0
     9
     6
    10

>> I=inv(Z)*V

I =

    2.1701
    1.9912
    1.8119
    2.0942
    2.2489
```

Thus,

I = **[2.17, 1.9912, 1.8119, 2.094, 2.249] A**.

Solution 3.66

The mesh equations are obtained as follows.

$$-12+24+30I_1-4I_2-6I_3-2I_4=0$$

or

$30I_1 - 4I_2 - 6I_3 - 2I_4 = -12$ (1)

$$-24+40-4I_1+30I_2-2I_4-6I_5=0$$

or

$-4I_1 + 30I_2 - 2I_4 - 6I_5 = -16$ (2)

$-6I_1 + 18I_3 - 4I_4 = 30$ (3)

$-2I_1 - 2I_2 - 4I_3 + 12I_4 - 4I_5 = 0$ (4)

$-6I_2 - 4I_4 + 18I_5 = -32$ (5)

Putting (1) to (5) in matrix form

$$\begin{bmatrix} 30 & -4 & -6 & -2 & 0 \\ -4 & 30 & 0 & -2 & -6 \\ -6 & 0 & 18 & -4 & 0 \\ -2 & -2 & -4 & 12 & -4 \\ 0 & -6 & 0 & -4 & 18 \end{bmatrix} I = \begin{bmatrix} -12 \\ -16 \\ 30 \\ 0 \\ -32 \end{bmatrix}$$

ZI = V

Using MATLAB,

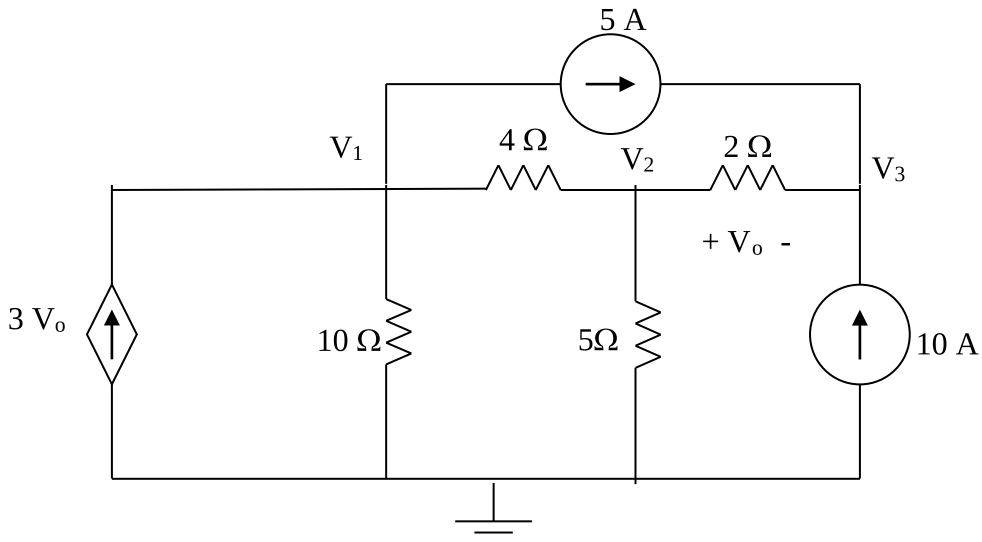
```
>> Z = [30,-4,-6,-2,0;
-4,30,0,-2,-6;
-6,0,18,-4,0;
-2,-2,-4,12,-4;
0,-6,0,-4,18]
```

```
Z =

    30    -4    -6    -2     0
    -4    30     0    -2    -6
    -6     0    18    -4     0
    -2    -2    -4    12    -4
     0    -6     0    -4    18

>> V = [-12,-16,30,0,-32]'

V =

   -12
   -16
    30
     0
   -32

>> I = inv(Z)*V

I =
```

-0.2779 A
-1.0488 A
1.4682 A
-0.4761 A
-2.2332 A

Solution 3.67

Consider the circuit below.



$$\begin{bmatrix} 0.35 & -0.25 & 0 \\ -0.25 & 0.95 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix} V = \begin{bmatrix} -5+3V_o \\ 0 \\ 15 \end{bmatrix}$$

Since we actually have four unknowns and only three equations, we need a constraint equation.

$$V_o = V_2 – V_3$$

Substituting this back into the matrix equation, the first equation becomes,

$$0.35V_1 – 3.25V_2 + 3V_3 = –5$$

This now results in the following matrix equation,

$$\begin{bmatrix} 0.35 & -3.25 & 3 \\ -0.25 & 0.95 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix} \mathrm{V} = \begin{bmatrix} -5 \\ 0 \\ 15 \end{bmatrix}$$

Now we can use MATLAB to solve for V.

```
>> Y=[0.35,-3.25,3;-0.25,0.95,-0.5;0,-0.5,0.5]

Y =

    0.3500   -3.2500    3.0000
   -0.2500    0.9500   -0.5000
         0   -0.5000    0.5000

>> I=[–5,0,15]'

I =

    –5
     0
     15

>> V=inv(Y)*I

V =

–410.5262
–194.7368
–164.7368
```

$V_o = V_2 - V_3 = -77.89 + 65.89 =$ **–30 V**.

Let us now do a quick check at node 1.

–3(–30) + 0.1(–410.5) + 0.25(–410.5+194.74) + 5 = 90–41.05–102.62+48.68+5 = 0.01; essentially zero considering the accuracy we are using. The answer checks.

Solution 3.68

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the voltage V_o in the circuit of Fig. 3.112.

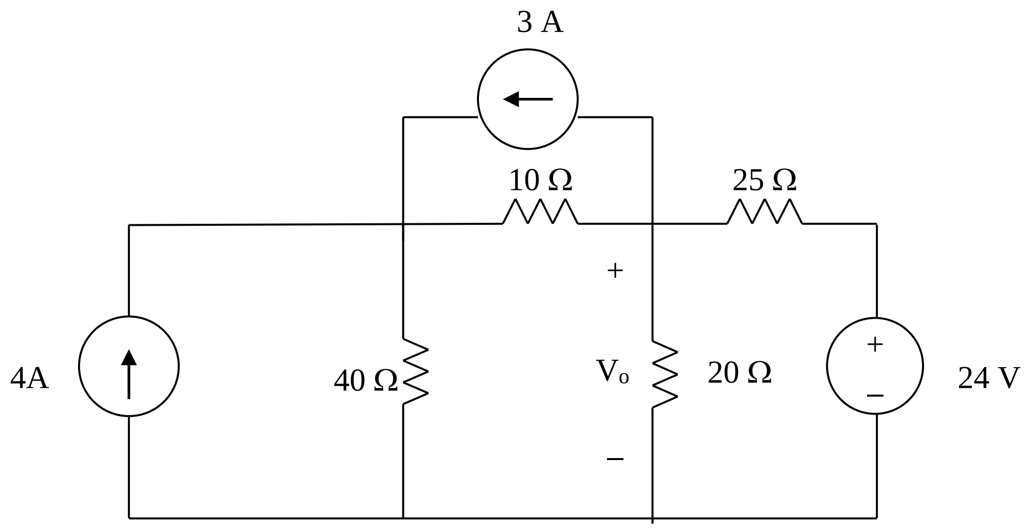
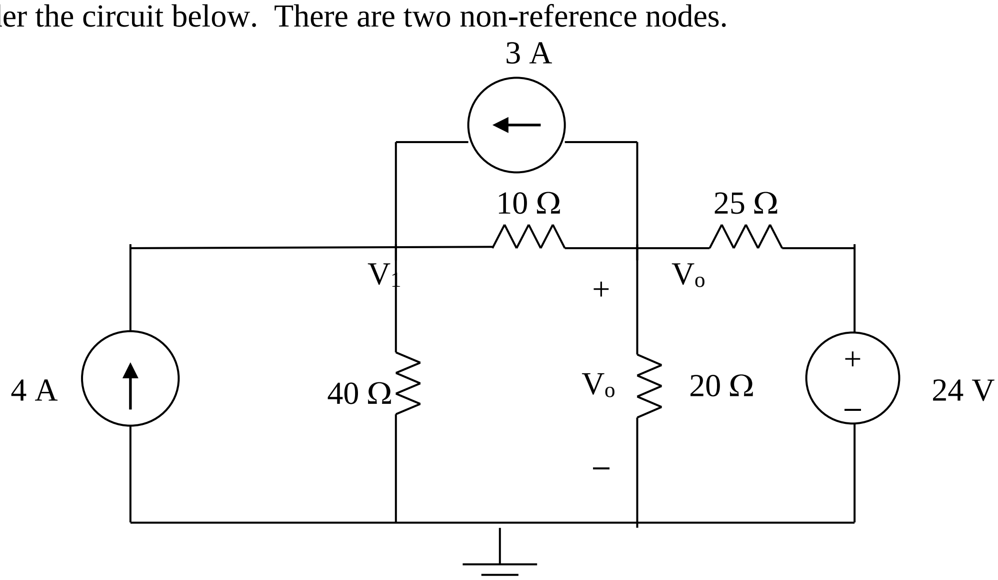


Figure 3.112
For Prob. 3.68.

Solution

Consider the circuit below. There are two non-reference nodes.



$$\begin{bmatrix} 0.125 & -0.1 \\ -0.1 & 0.19 \end{bmatrix} V = \begin{bmatrix} +4+3 \\ -3+24/25 \end{bmatrix} = \begin{bmatrix} 7 \\ -2.04 \end{bmatrix}$$

Using MATLAB, we get,

```
>> Y=[0.125,-0.1;-0.1,0.19]

Y =

   0.1250   -0.1000
  -0.1000    0.1900

>> I=[7,-2.04]'

I =

   7.0000
  -2.0400

>> V=inv(Y)*I

V =

  81.8909
  32.3636
```

Thus, V_o = **32.36 V**.

We can perform a simple check at node V_o,

3 + 0.1(32.36–81.89) + 0.05(32.36) + 0.04(32.36–24) =
3 – 4.953 + 1.618 + 0.3344 = – 0.0004; answer checks!

Solution 3.69

For the circuit in Fig. 3.113, write the node voltage equations by inspection.

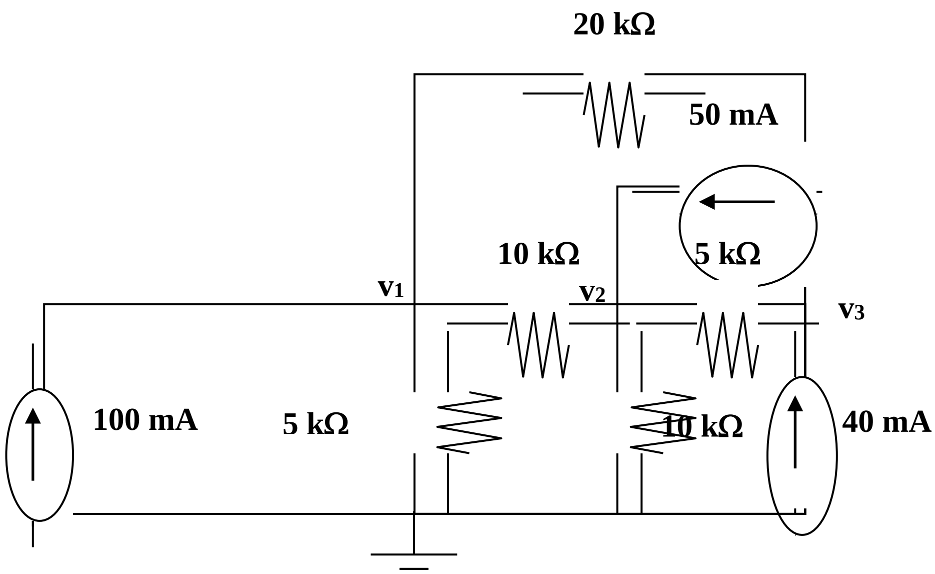


Figure 3.113
For Prob. 3.69.

Step 1. Assume that all conductance's are in mS, all currents are in mA, and all voltages are in volts.

G_{11} = (1/5) + (1/10) + (1/20) = 0.35, G_{22} = (1/10) + (1/10) + (1/5) = 0.4,
G_{33} = (1/5) + (1/20) = 0.25, G_{12} = –1/10 = –0.1, G_{13} = –0.05,
G_{21} = –0.1, G_{23} = –0.2, G_{31} = –0.05, G_{32} = –0.2

i_1 = 100, i_2 = 50, and i_3 = 30–50 = –10.

Step 2. The node-voltage equations are:

$$\begin{bmatrix} 0.35 & -0.1 & -0.05 \\ -0.1 & 0.4 & -0.2 \\ -0.05 & -0.2 & 0.25 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \\ -10 \end{bmatrix}$$

Solution 3.70

$$\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} V = \begin{bmatrix} 4I_x + 20 \\ -4I_x - 7 \end{bmatrix}$$

With two equations and three unknowns, we need a constraint equation,

$I_x = 2V_1$, thus the matrix equation becomes,

$$\begin{bmatrix} -5 & 0 \\ 8 & 5 \end{bmatrix} V = \begin{bmatrix} 20 \\ -7 \end{bmatrix}$$

This results in $V_1 = 20/(-5) =$ **–4 V** and
$V_2 = [-8(-4) - 7]/5 = [32 - 7]/5 =$ **5 V**.

Solution 3.71

$$\begin{bmatrix} 9 & -4 & -5 \\ -4 & 7 & -1 \\ -5 & -1 & 9 \end{bmatrix} I = \begin{bmatrix} 30 \\ -15 \\ 0 \end{bmatrix}$$

We can now use MATLAB solve for our currents.

```
>> R=[9,–4,–5;–4,7,–1;–5,–1,9]

R =

   9   –4   –5
  –4    7   –1
  –5   –1    9

>> V=[30,–15,0]'

V =

  30
  –15
   0

>> I=inv(R)*V

I =
```

6.255 A
1.9599 A
3.694 A

Solution 3.72

By inspection, write the mesh-current equations for the circuit in Fig. 3.116.

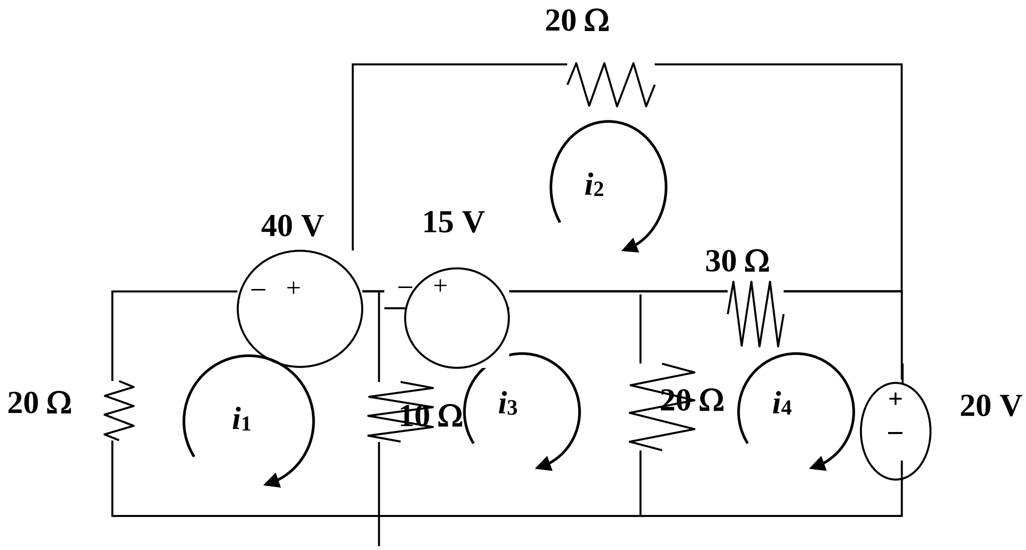


Figure 3.116
For Prob. 3.72.

Step 1. First we write the resistance equations by inspection.

R_{11} = 20 + 10 = 30, R_{22} = 20 + 30 = 50, R_{33} = 10 + 20 = 30, R_{44} = 20 + 30 = 50, R_{12} = 0, R_{13} = –10, R_{14} = 0, R_{21} = 0, R_{23} = 0, R_{24} = –30, R_{31} = –10, R_{32} = 0, R_{34} = –20, R_{41} = 0, R_{42} = –30, R_{43} = –20, we note that $R_{ij} = R_{ji}$ for all i not equal to j. Finally v_1 = 40; v_2 = –15; v_3 = 15; and v_4 = –20.

Step 2. Hence the mesh-current equations are:

$$\begin{bmatrix} 30 & 0 & -10 & 0 \\ 0 & 20 & 0 & -30 \\ -10 & 0 & 30 & -20 \\ 0 & -30 & -20 & 50 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 40 \\ -15 \\ 15 \\ -20 \end{bmatrix}$$

Solution 3.73

Write the mesh-current equations for the circuit in Fig. 3.117.

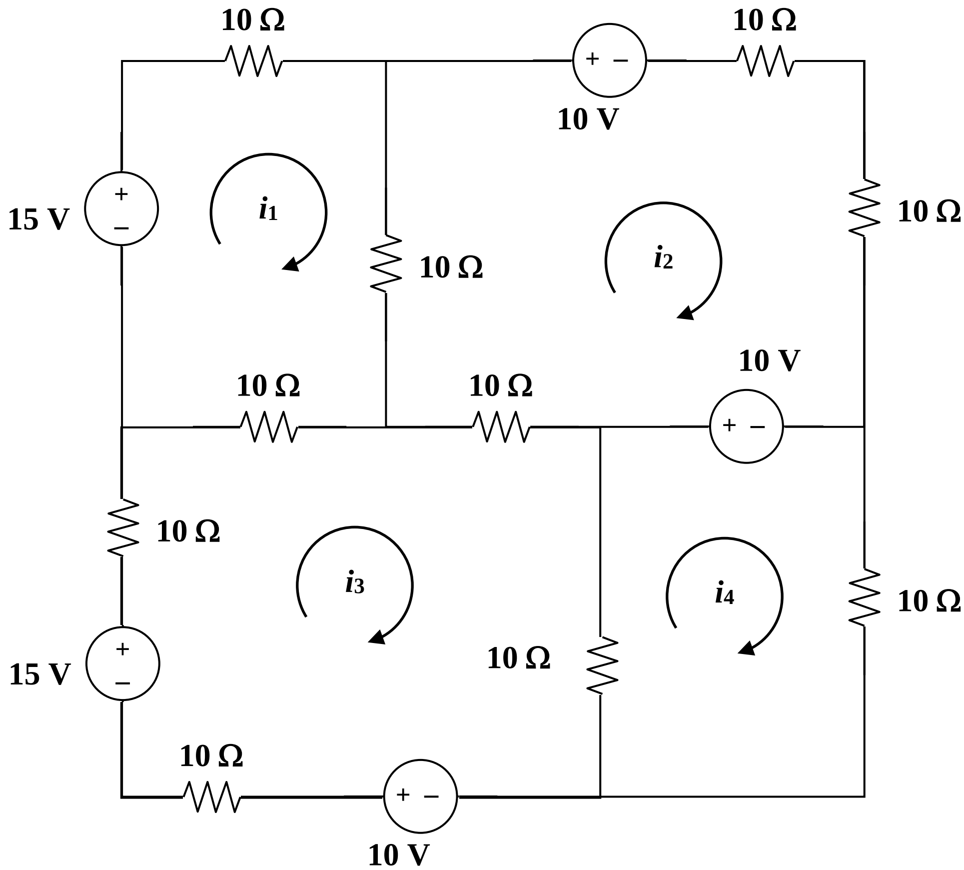


Figure 3.117
For Prob. 3.73.

Solution

Loop 1. $-15 + 10i_1 + 10(i_1 - i_2) + 10(i_1 - i_3) = 0$ or $30i_1 - 10i_2 - 10i_3 = 15$

Loop 2. $10(i_2 - i_1) + 10 + 20i_2 - 10 + 10(i_2 - i_3) = 0$ or $-10i_1 + 40i_2 - 10i_3 = 0$

Loop 3. $-10 + 20i_3 - 15 + 10(i_3 - i_1) + 10(i_3 - i_2) + 10(i_3 - i_4) = 0$ or
$-10i_1 - 10i_2 + 50i_3 - 10i_4 = 25$

Loop 4. $10(i_4 - i_3) + 10 + 10i_4 = 0$ or $-10i_3 + 20i_4 = -10$

Thus, $\begin{bmatrix} 30 & -10 & -10 & 0 \\ -10 & 40 & -10 & 0 \\ -10 & -10 & 50 & -10 \\ 0 & 0 & -10 & 20 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ 25 \\ -10 \end{bmatrix}$.

Solution 3.74

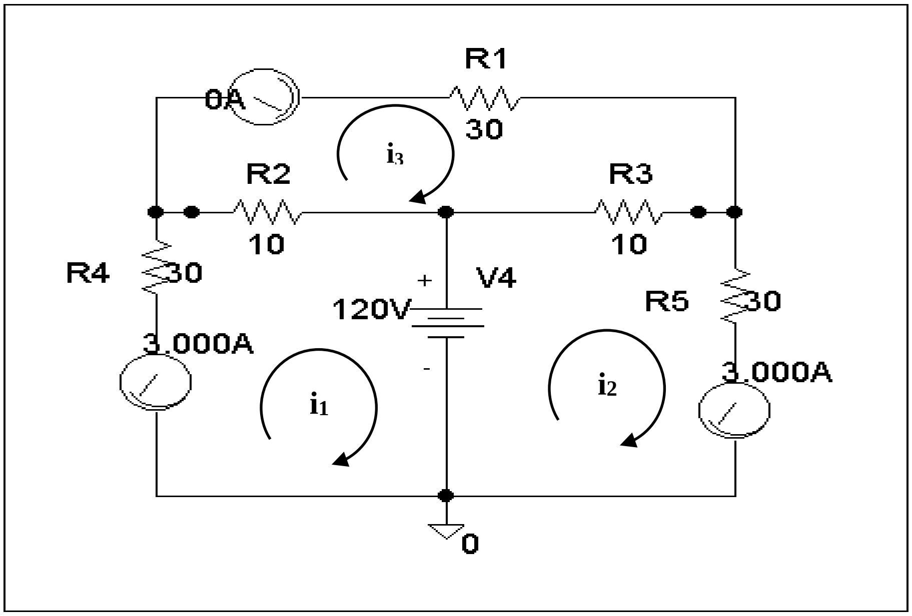
$R_{11} = R_1 + R_4 + R_6$, $R_{22} = R_2 + R_4 + R_5$, $R_{33} = R_6 + R_7 + R_8$,
$R_{44} = R_3 + R_5 + R_8$, $R_{12} = -R_4$, $R_{13} = -R_6$, $R_{14} = 0$, $R_{23} = 0$,
$R_{24} = -R_5$, $R_{34} = -R_8$, again, we note that $R_{ij} = R_{ji}$ for all i not equal to j.

The input voltage vector is = $\begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ -V_4 \end{bmatrix}$

$$\begin{bmatrix} R_1 + R_4 + R_6 & -R_4 & -R_6 & 0 \\ -R_4 & R_2 + R_4 + R_5 & 0 & -R_5 \\ -R_6 & 0 & R_6 + R_7 + R_8 & -R_8 \\ 0 & -R_5 & -R_8 & R_3 + R_5 + R_8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ -V_4 \end{bmatrix}$$

Solution 3.75

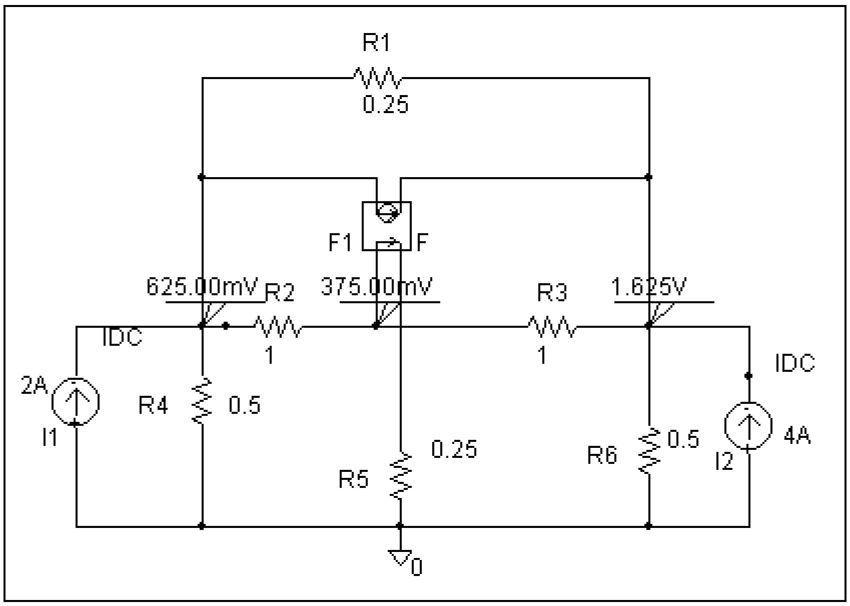
```
* Schematics Netlist *

R_R4          $N_0002 $N_0001  30
R_R2          $N_0001 $N_0003  10
R_R1          $N_0005 $N_0004  30
R_R3          $N_0003 $N_0004  10
R_R5          $N_0006 $N_0004  30
V_V4          $N_0003 0 120V
v_V3          $N_0005 $N_0001 0
v_V2          0 $N_0006 0
v_V1          0 $N_0002 0
```



Clearly, i_1 = **–3 amps**, i_2 = **0 amps**, and i_3 = **3 amps**, which agrees with the answers in Problem 3.44.

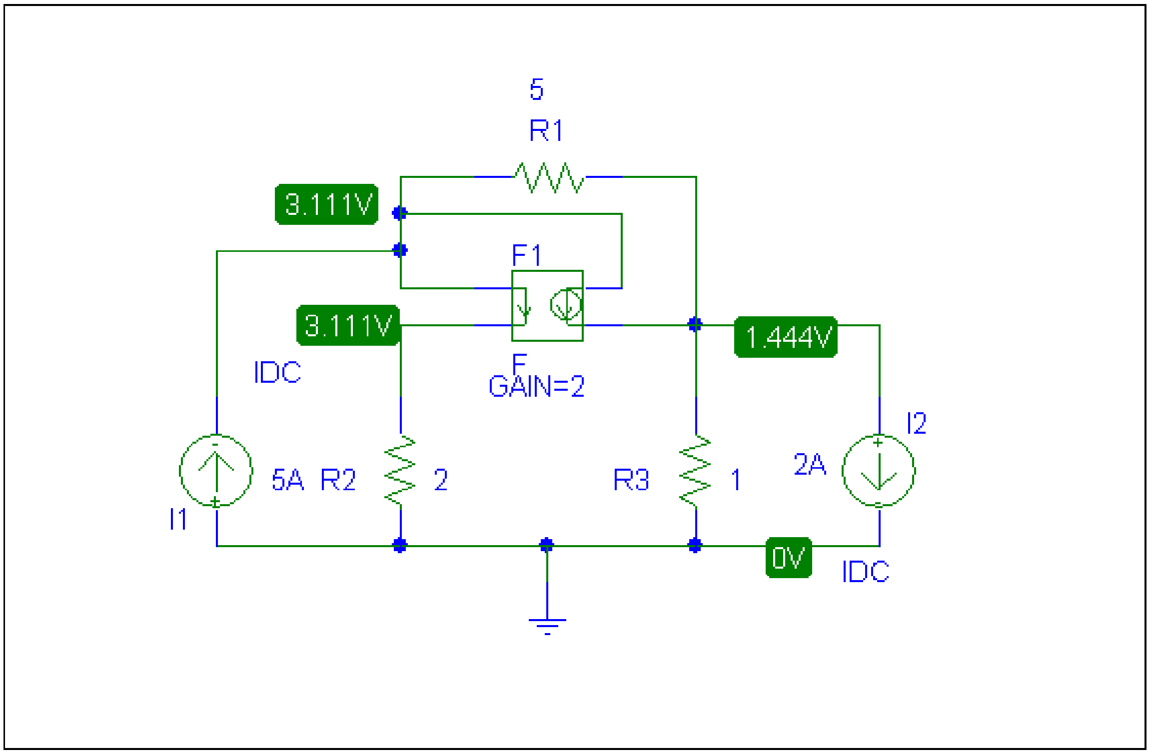
Solution 3.76

```
* Schematics Netlist *

I_I2         0 $N_0001 DC 4A
R_R1         $N_0002 $N_0001  0.25
R_R3         $N_0003 $N_0001  1
R_R2         $N_0002 $N_0003  1
F_F1         $N_0002 $N_0001 VF_F1 3
VF_F1        $N_0003 $N_0004 0V
R_R4         0 $N_0002  0.5
R_R6         0 $N_0001  0.5
I_I1         0 $N_0002 DC 2A
R_R5         0 $N_0004  0.25
```



Clearly, v_1 = **625 mVolts**, v_2 = **375 mVolts**, and v_3 = **1.625 volts**, which agrees with the solution obtained in Problem 3.27.

Solution 3.77



As a check we can write the nodal equations,

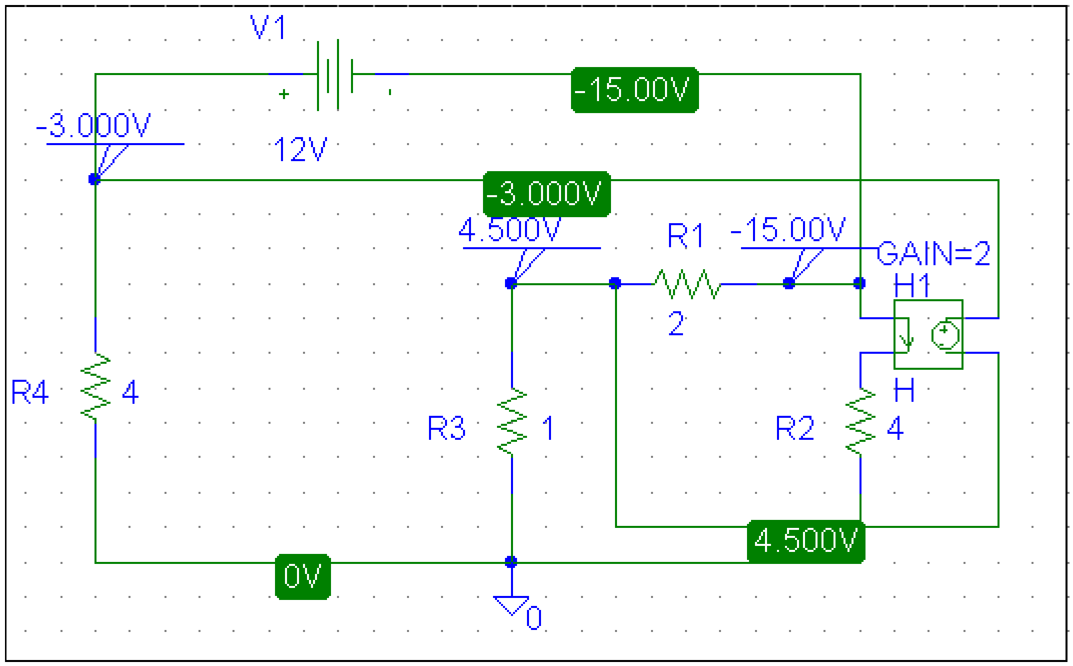
$$\begin{bmatrix} 1.7 & -0.2 \\ -1.2 & 1.2 \end{bmatrix} V = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

Solving this leads to V_1 = **3.111 V** and V_2 = **1.4444 V**. The answer checks!

Solution 3.78

The schematic is shown below. When the circuit is saved and simulated the node voltages are displayed on the pseudo components as shown. Thus,

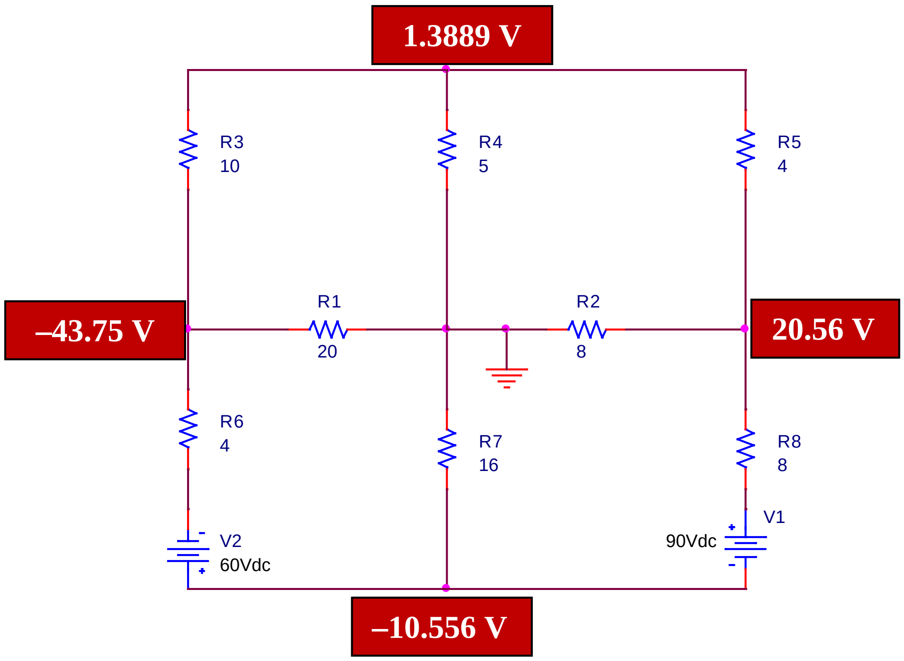
$\underline{V_1 = -3\text{V}, \quad V_2 = 4.5\text{V}, V_3 = -15\text{V},}$



Solution 3.79

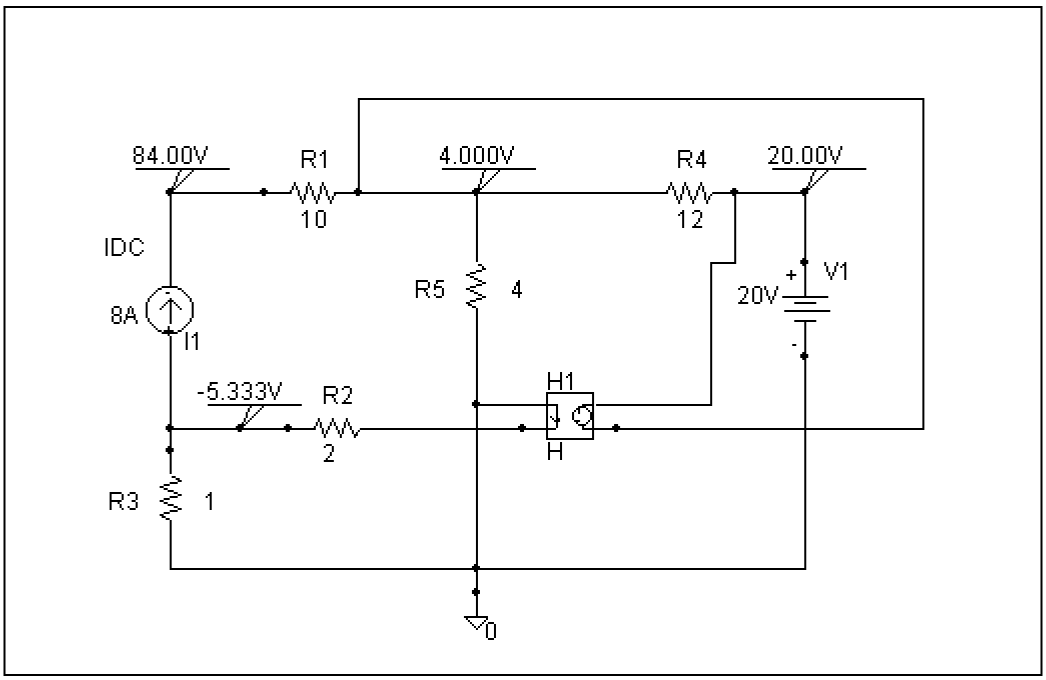
The schematic is shown below. When the circuit is saved and simulated, we obtain the node voltages as displayed. Thus,

V_a = **–10.556 volts**; V_b = **20.56 volts**; V_c = **1.3889 volts**; and V_d = **–43.75 volts**.



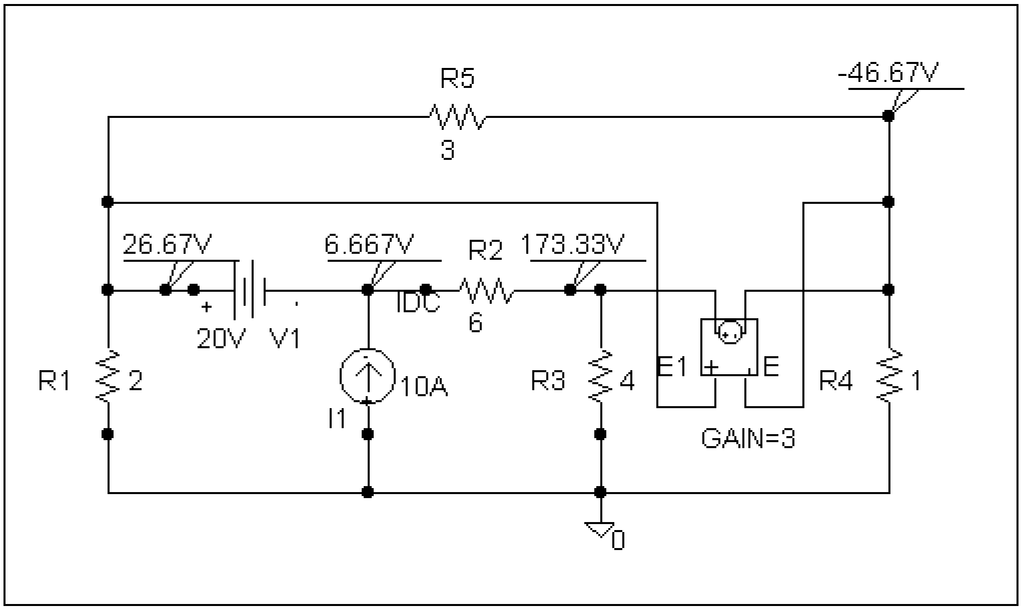
Solution 3.80

```
* Schematics Netlist *

H_H1          $N_0002 $N_0003 VH_H1 6
VH_H1         0 $N_0001 0V
I_I1          $N_0004 $N_0005 DC 8A
V_V1          $N_0002 0 20V
R_R4          0 $N_0003  4
R_R1          $N_0005 $N_0003  10
R_R2          $N_0003 $N_0002  12
R_R5          0 $N_0004  1
R_R3          $N_0004 $N_0001  2
```



Clearly, v_1 = **84 volts**, v_2 = **4 volts**, v_3 = **20 volts**, and v_4 = **-5.333 volts**

Solution 3.81

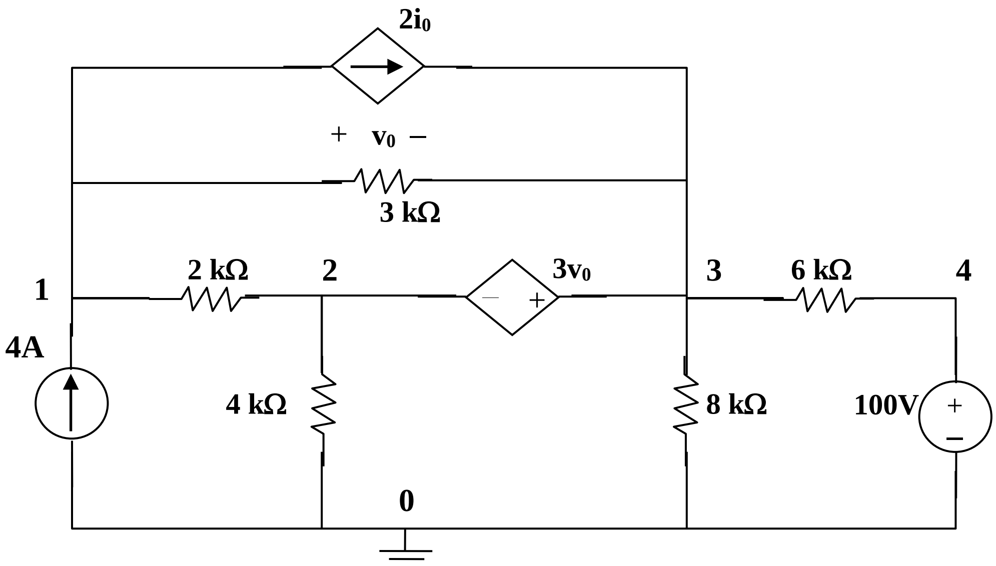


Clearly, v_1 = **26.67 volts**, v_2 = **6.667 volts**, v_3 = **173.33 volts**, and v_4 = **–46.67 volts** which agrees with the results of Example 3.4.

This is the netlist for this circuit.

```
* Schematics Netlist *

R_R1         0 $N_0001  2
R_R2         $N_0003 $N_0002  6
R_R3         0 $N_0002  4
R_R4         0 $N_0004  1
R_R5         $N_0001 $N_0004  3
I_I1         0 $N_0003 DC 10A
V_V1         $N_0001 $N_0003 20V
E_E1         $N_0002 $N_0004 $N_0001 $N_0004 3
```

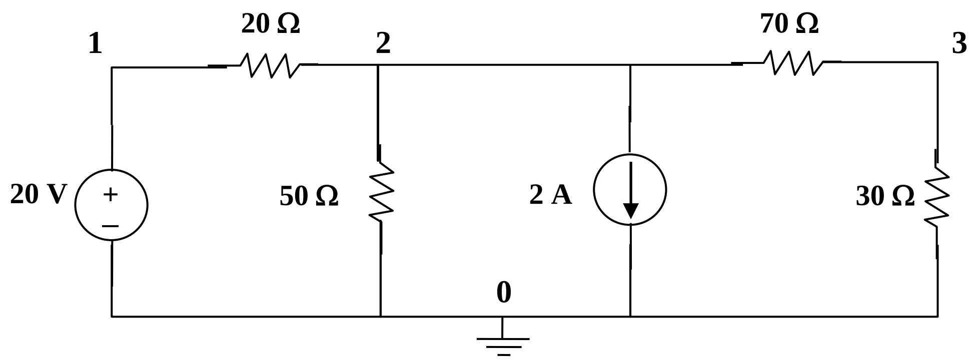
Solution 3.82



This network corresponds to the Netlist.

Solution 3.83

The circuit is shown below.



When the circuit is saved and simulated, we obtain v_2 = **–12.5 volts**

Solution 3.84

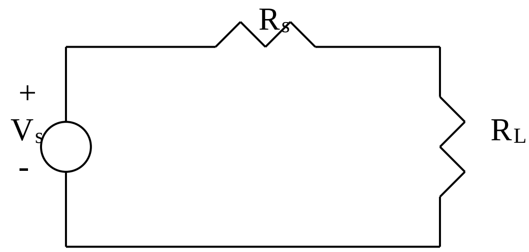
From the output loop, $v_0 = 50i_0 x 20x10^3 = 10^6 i_0$ (1)

From the input loop, $15x10^{-3} + 4000i_0 - v_0/100 = 0$ (2)

From (1) and (2) we get, i_0 = **2.5 μA** and v_0 = **2.5 volt**.

Solution 3.85

The amplifier acts as a source.



For maximum power transfer,

$$R_L = R_s = \underline{9\Omega}$$

Solution 3.86

Let v_1 be the potential across the 2 k-ohm resistor with plus being on top. Then,

Since $i = [(0.047–v_1)/1k]$
$[(v_1–0.047)/1k] – 400[(0.047–v_1)/1k] + [(v_1–0)/2k] = 0$ or

$401[(v_1–0.047)] + 0.5v_1 = 0$ or $401.5v_1 = 401x0.047$ or
$v_1 = 0.04694$ volts and $i = (0.047–0.04694)/1k = 60$ nA

Thus,

$v_0 = –5000x400x60x10^{-9}$ = **–120 mV**.

Solution 3.87

For the circuit in Fig. 3.123, find the gain $\boldsymbol{v_o/v_s}$.

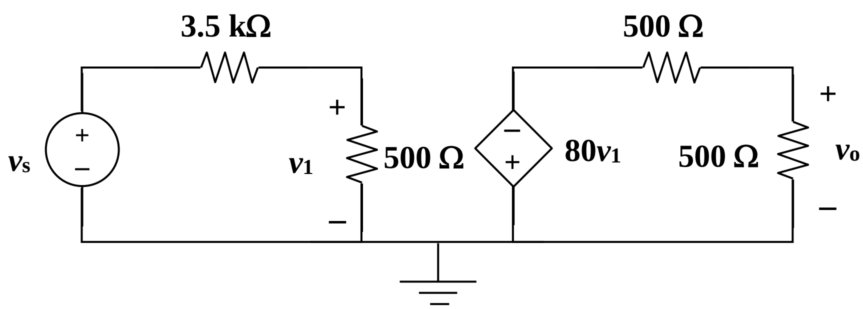


Figure 3.123
For Prob. 3.87.

Step 1. We can solve this using mesh analysis with two unknown mesh currents.

For the loop on the left we get, $-v_s + 3{,}500i_1 + 500i_1 = 0$ and $v_1 = 500i_1$.
For the loop on the right we get, $80v_1 + 500i_2 + 500i_2$ and $v_o = 500i_2$.

Step 2. $i_1 = v_s/4{,}000$ and $v_1 = 500v_s/4{,}000 = v_s/8$. $i_2 = -80(v_s/8)/1{,}000$ and $v_o = 500(-10v_s)/1{,}000 = -5v_s$. Therefore,

$$v_o/v_s = \mathbf{-5}.$$

Solution 3.88

Determine the gain $\boldsymbol{v_o/v_s}$ of the transistor amplifier circuit in Fig. 3.124.

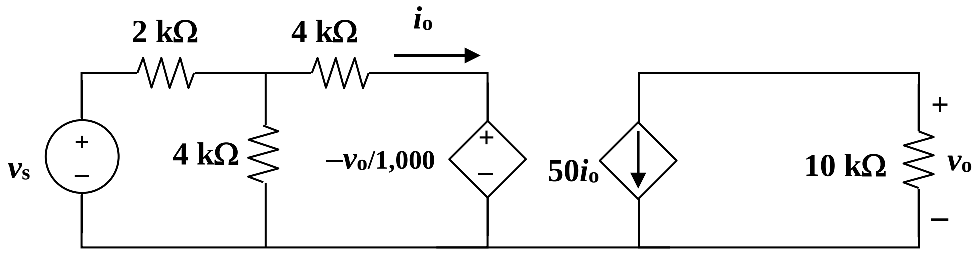


Figure 3.124
For Prob. 3.88.

Solution

Step 1. The loop on the right gives us $v_o = -10k(50i_o)$. We have two loops in the left hand circuit which produces $-v_s + 2ki_1 + 4k(i_1 - i_o) = 0$ and $4k(i_o - i_1) + 4ki_o - (v_o/1000) = 0$.

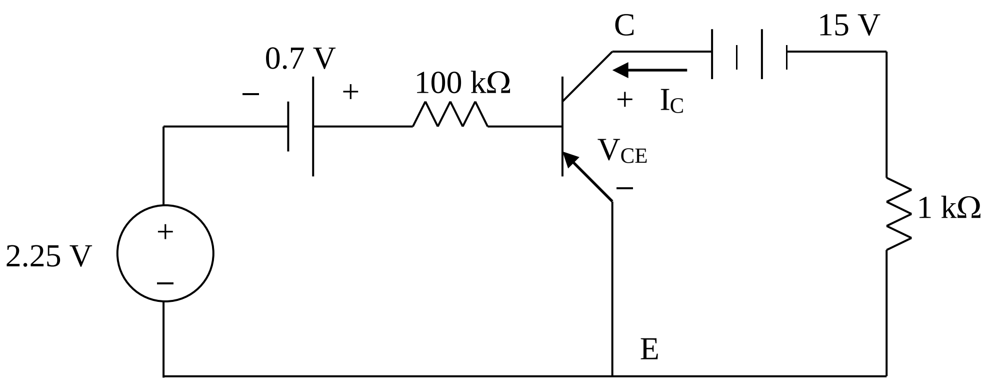
Step 2. $8ki_o - 4ki_1 - (-500ki_o/1000) = 0$ or $8.5ki_o = 4ki_1$ or $i_1 = 2.125i_o$ and $(2k+4k)i_1 - 4ki_o = v_s = 2.125(6k)i_o - 4ki_o = 8.75ki_o$ or $i_o = v_s/8.75k$.

Now we have $v_o = -500kv_s/8.75k = -57.14v_s$ or

$$v_o/v_s = \mathbf{-57.14}.$$

Solution 3.89

Consider the circuit below.



For the left loop, applying KVL gives

$$-2.25 - 0.7 + 10^5 I_B + V_{BE} = 0 \text{ but } V_{BE} = 0.7 \text{ V means } 10^5 I_B = 2.25 \text{ or}$$

$$I_B = \mathbf{22.5\ \mu A}.$$

For the right loop, $-V_{CE} + 15 - I_C x10^3 = 0$. Addition ally, $I_C = \beta I_B = 100x22.5x10^{-6} =$ 2.25 mA. Therefore,

$$V_{CE} = 15{-}2.25x10^{-3}x10^3 = \mathbf{12.75\ V}.$$

Solution 3.90

Calculate $\boldsymbol{v_s}$ for the transistor in Fig. 3.126, given that $\boldsymbol{v_o}$ = 6 V, β = 90, V_{BE} = 0.7V.

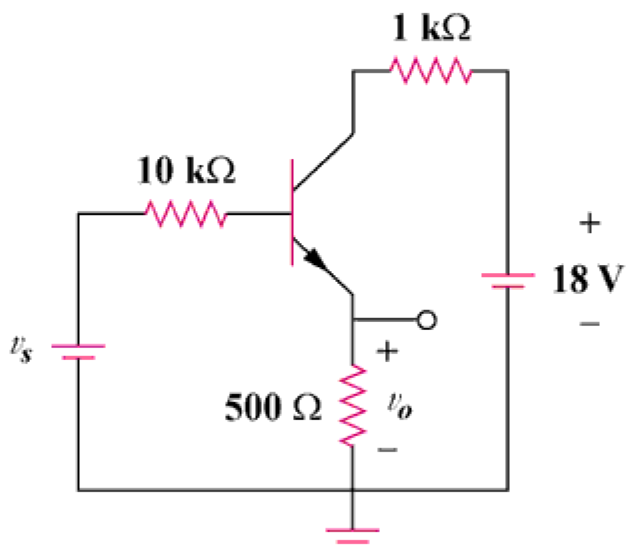
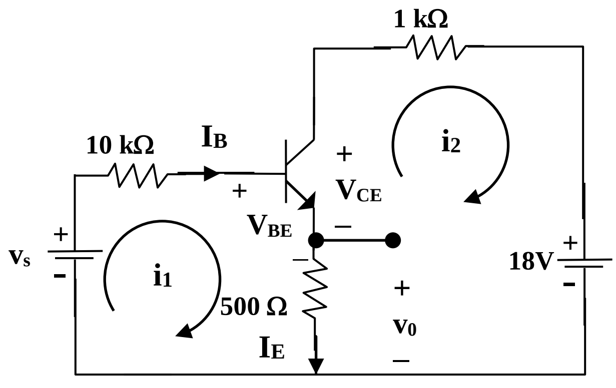


Figure 3.126
For Prob. 3.90.



For loop 1, $-v_s + 10k(I_B) + V_{BE} + I_E\,(500) = 0 = -v_s + 0.7 + 10{,}000I_B + 500(1 + \beta)I_B$

which leads to $v_s - 0.7 = 10{,}000I_B + 500(91)I_B = 55{,}500I_B$

But, $v_0 = 500I_E = 500\text{x}91I_B = 6$ which leads to $I_B = 1.318680\text{x}10^{-4}$

Therefore, $v_s = 0.7 + 55{,}500I_B$ = **8.019 volts**.

Solution 3.91

For the transistor circuit of Fig. 3.127, find $\boldsymbol{I_B}$, $\mathbf{V_{CE}}$, and $\boldsymbol{v_o}$. Take $\beta = 150$, $V_{BE} = 0.7$V.

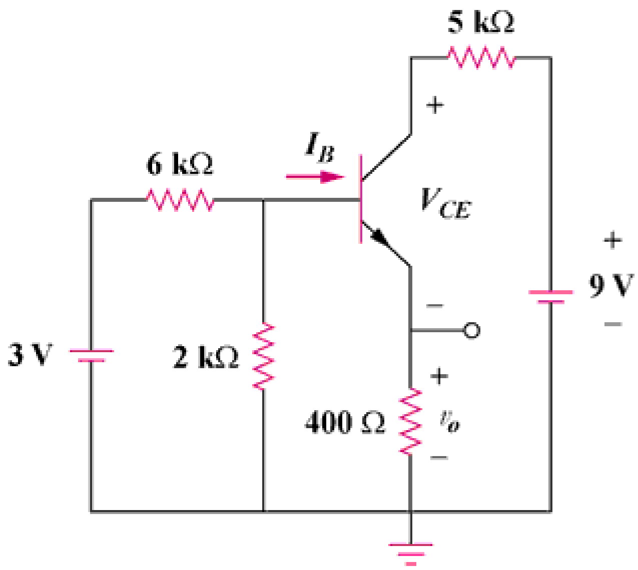
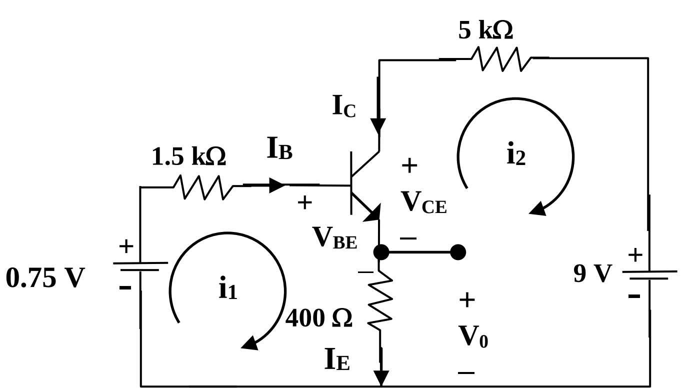


Figure 3.127
For Prob. 3.91.

Solution

We first determine the Thevenin equivalent for the input circuit.

$R_{Th} = 6||2 = 6x2/8 = 1.5$ kΩ and $V_{Th} = 2(3)/(2+6) = 0.75$ volts



For loop 1, $-0.75 + 1.5kI_B + V_{BE} + 400I_E = 0 = -0.75 + 0.7 + 1{,}500I_B + 400(1 + \beta)I_B$ or $(1{,}500 + 400x151)I_B = 61{,}900I_B = 0.05$ or

$$I_B = 0.05/61{,}900 = \mathbf{0.8078\ \mu A}.$$

$$v_0 = 400I_E = 400(1 + \beta)I_B = 400(151)0.8078 = \mathbf{48.49\ mV}$$

For loop 2, $-400I_E - V_{CE} - 5kI_C + 9 = 0$, but, $I_C = \beta I_B$ and $I_E = (1 + \beta)I_B$

$$V_{CE} = 9 - 5k\beta I_B - 400(1 + \beta)I_B = 9 - 0.60585 - 0.04879 = 9 - 0.6546 =$$

$$V_{CE} = \mathbf{8.345\ volts}.$$

Solution 3.92

Using Fig. 3.28, design a problem to help other students better understand transistors. Make sure you use reasonable numbers!

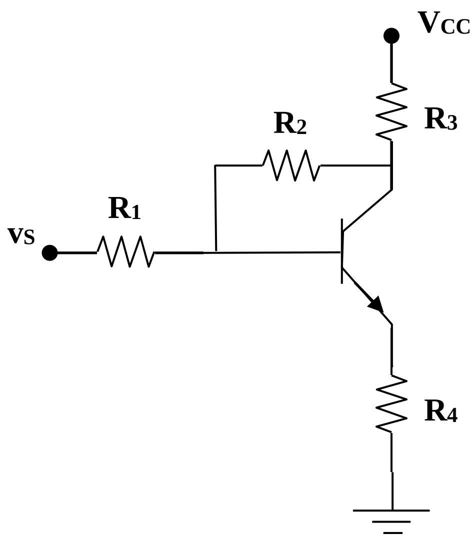


Figure 3.28
For Prob. 3.92.

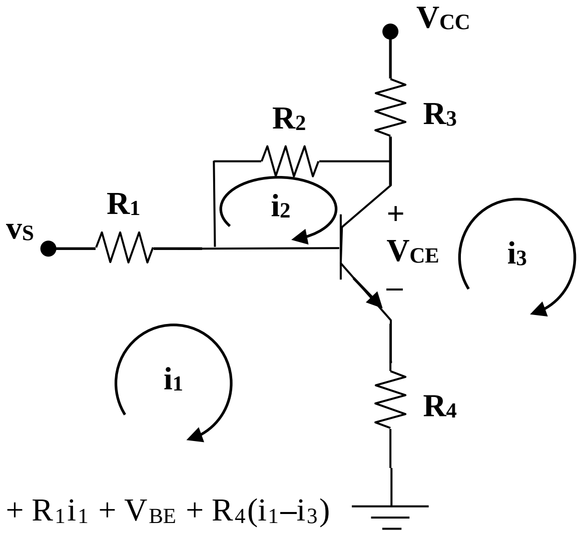
Although there are many ways to work this problem, this is just one example that could qualify as a solution.

Problem

Given the circuit shown in Fig. 3.28 and R_1 = 100 kΩ, R_2 = 1 kΩ, R_3 = 1 kΩ, R_4 = 100 Ω, β= 100, V_{CC} = 30 V, v_s = 20 V, and V_{BE} = 0.7. Determine V_{CE}.

Solution continued on the next page...

Solution



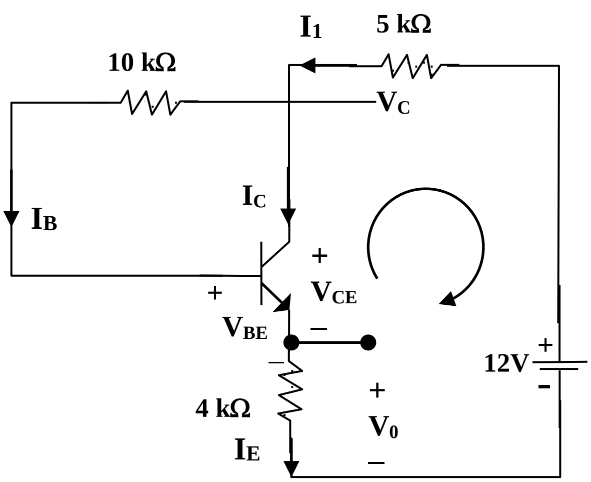
Loop 1, $-v_s + R_1 i_1 + V_{BE} + R_4(i_1 - i_3)$
Loop 2, $R_2 i_2 + V_{CB} = 0$
Loop 3, $R_4(i_3 - i_1) - V_{CE} + R_3 i_3 + V_{CC} = 0$.

We also have some constraint equations, $I_B = i_1 - i_2$, $I_C = i_2 - i_3 = \beta I_B$, and $V_{CE} = V_{BE} + V_{CB}$.

$100.1ki_1 - 0.1ki_3 = 20 - 0.7 = 19.3$
$1ki_2 + V_{CB} = 0$
$-0.1ki_1 + 1.1ki_3 - V_{CE} = -30$
$i_2 - i_3 = 100(i_1 - i_2)$ or $100i_1 - 101i_2 + i_3 = 0$ or $i_3 = -100i_1 + 101i_2$
$V_{CB} = V_{CE} - 0.7$

Substituting for values of i_3 and V_{CB} we get
$100.1ki_1 + 10ki_1 - 10.1ki_2 = 19.3$ or $110.1ki_1 - 10.1ki_2 = 19.2$ or $i_1 = 0.091735i_2 + 0.17439/k$
$1ki_2 + V_{CE} = 0.7$ or $i_2 = -[(V_{CE})/k] + 0.7/k$
$-0.1ki_1 - 110ki_1 + 111.1ki_2 - V_{CE} = -30$
$= -110.1ki_1 + 111.1ki_2 - V_{CE} = -10.1ki_2 - 19.2 + 111.1ki_2 - V_{CE} = 101ki_2 - 19.2 - V_{CE}$
$= -101V_{CE} + 70.7 - 19.2 - V_{CE} = -30$ or $70.7 + 30 - 19.2 = 102V_{CE}$ or
$V_{CE} = 81.5/102 =$ **799 mV**.

Solution continued on the next page...



$I_1 = I_B + I_C = (1 + \beta)I_B$ and $I_E = I_B + I_C = I_1$

Applying KVL around the outer loop,

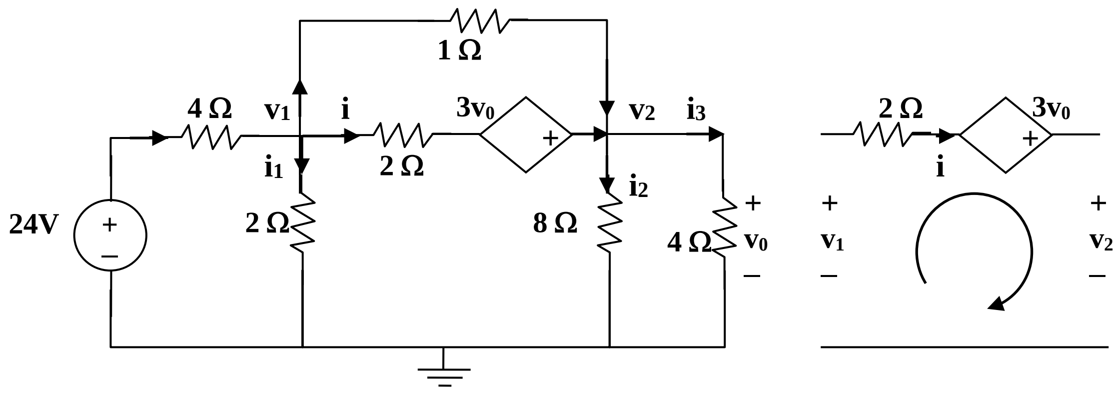
$4kI_E + V_{BE} + 10kI_B + 5kI_1 = 12$

$12 – 0.7 = 5k(1 + \beta)I_B + 10kI_B + 4k(1 + \beta)I_B = 919kI_B$

$I_B = 11.3/919k = 12.296\ \mu A$

Also, $12 = 5kI_1 + V_C$ which leads to $V_C = 12 – 5k(101)I_B =$ **5.791 volts**

Solution 3.93



From (b), $-v_1 + 2i - 3v_0 + v_2 = 0$ which leads to $i = (v_1 + 3v_0 - v_2)/2$

At node 1 in (a), $((24 - v_1)/4) = (v_1/2) + ((v_1 + 3v_0 - v_2)/2) + ((v_1 - v_2)/1)$, where $v_0 = v_2$

or $24 = 9v_1$ which leads to $v_1 = \mathbf{2.667\ volts}$

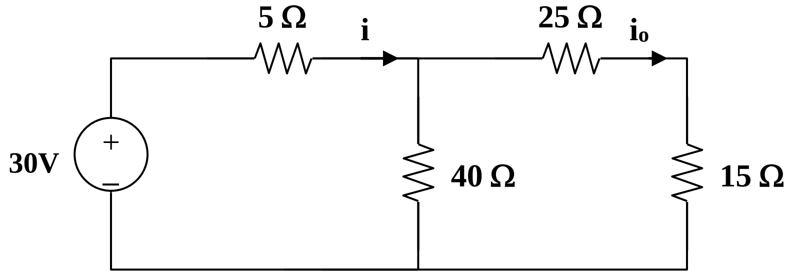
At node 2, $((v_1 - v_2)/1) + ((v_1 + 3v_0 - v_2)/2) = (v_2/8) + v_2/4$, $v_0 = v_2$

$v_2 = 4v_1 = \mathbf{10.66\ volts}$

Now we can solve for the currents, $i_1 = v_1/2 = \mathbf{1.333\ A}$, $i_2 = \mathbf{1.333\ A}$, and

$i_3 = \mathbf{2.6667}$ A.

Solution 4.1



$40 \| (25+15) = 20\Omega$, i = [30/(5+20)] = 1.2 and i_o = i20/40 = **600 mA**.

Since the resistance remains the same we get can use linearity to find the new value of the voltage source = (30/0.6)5 = **250 V**.

Solution 4.2

Using Fig. 4.70, design a problem to help other students better understand linearity.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find v_o in the circuit of Fig. 4.70. If the source current is reduced to 1 μA, what is v_o?

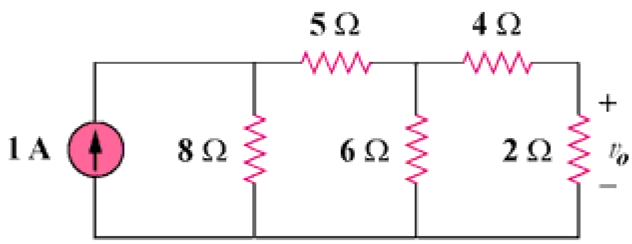
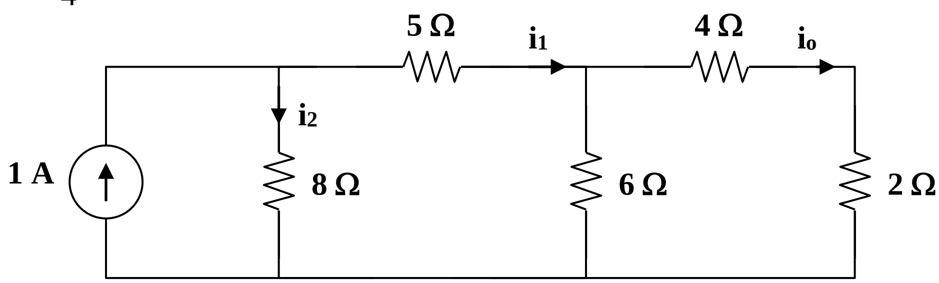


Figure 4.70

Solution

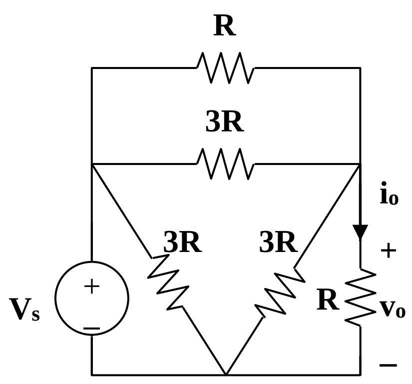
$$6\|(4+2) = 3\Omega, \quad i_1 = i_2 = \frac{1}{2}\text{A}$$

$$i_o = \frac{1}{2}i_1 = \frac{1}{4}, \quad v_o = 2i_o = \underline{\mathbf{0.5V}}$$

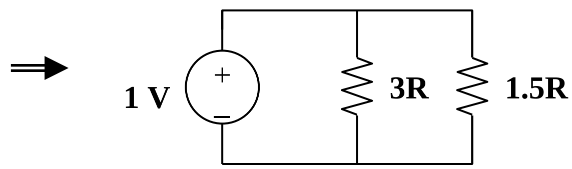


If $i_s = 1\mu A$, then v_o = **0.5μV**

Solution 4.3



(a)



(b)

(a) We transform the Y sub-circuit to the equivalent Δ.

$$R \| 3R = \frac{3R^2}{4R} = \frac{3}{4}R, \quad \frac{3}{4}R + \frac{3}{4}R = \frac{3}{2}R$$

$v_o = \dfrac{v_s}{2}$ independent of R

$i_o = v_o/(R)$

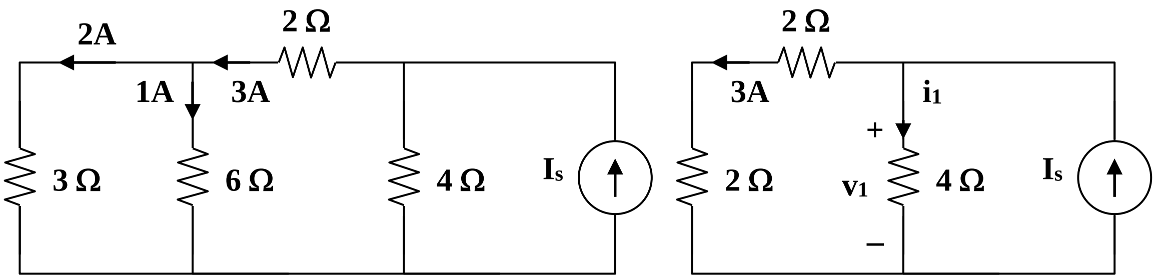
When $v_s = 1V$, $v_o =$ **0.5V**, $i_o =$ **0.5A**

(b) When $v_s = 10V$, $v_o =$ **5V**, $i_o =$ **5A**

(c) When $v_s = 10V$ and $R = 10\Omega$,
$v_o =$ **5V**, $i_o = 10/(10) =$ **500mA**

Solution 4.4

If $I_o = 1$, the voltage across the 6Ω resistor is 6V so that the current through the 3Ω resistor is 2A.



(a) (b)

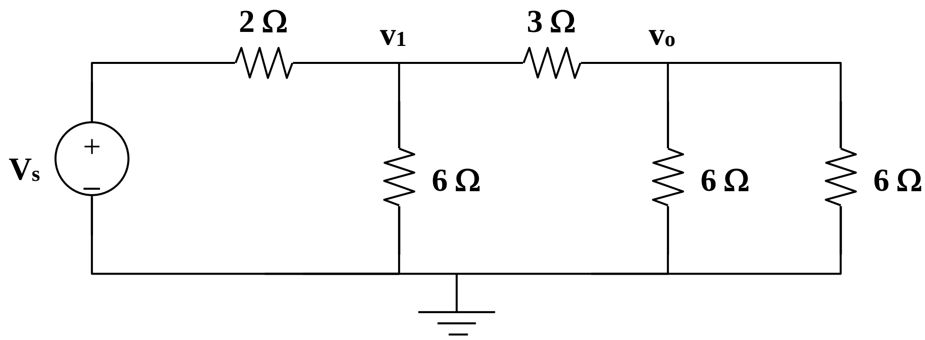
$3\|6 = 2\Omega$, $v_o = 3(4) = 12V$, $i_1 = \frac{v_o}{4} = 3A$.

Hence $I_s = 3 + 3 = 6A$

If $I_s = 6A \longrightarrow I_o = 1$

$I_s = 9A \longrightarrow I_o = 9/6 =$ **1.5A**

Solution 4.5



If v_o = 1V, $\quad V_1 = \left(\frac{1}{3}\right) + 1 = 2V$

$$V_s = 2\left(\frac{2}{3}\right) + v_1 = \frac{10}{3}$$

If $v_s = \frac{10}{3}$ $\longrightarrow$ $v_o = 1$

Then $v_s = 15$ $\longrightarrow$ $v_o = \frac{3}{10} x15 =$ **4.5V**

Solution 4.6

Due to linearity, from the first experiment,

$$V_o = \frac{1}{3}V_s$$

Applying this to other experiments, we obtain:

Experiment	V_s	V_o
2	48	16 V
3	1 V	0.333 V
4	-6 V	-2V

Solution 4.7

Use linearity and the assumption that $V_x = 1V$ to find the actual value of V_o in Fig. 4.75.

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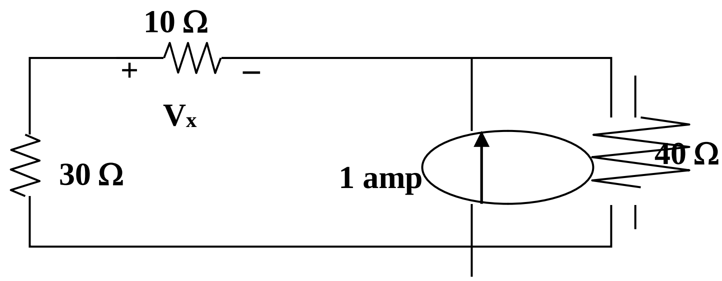


Figure 4.75
For Prob. 4.7.

Solution

Step 1. If we let V_x = 1 volt then I_{10} = 0.1 amp which leads to $V_{30\text{-}10}$ = 0.1x40 = 4 volts. Then I_{40} = 4/40 = 0.1 amp which would have required a current source equal to –0.1 – 0.1 = –0.2 amps.

Step 2. Since the current source is 1 amp which is –5(–0.2) then the voltage V_x = –5x1 or,

V_x = **–5volts**.

If V_o = 1V, then the current through the 2-Ω and 4-Ω resistors is ½ = 0.5. The voltage across the 3-Ω resistor is ½ (4 + 2) = 3 V. The total current through the 1-Ω resistor is 0.5 +3/3 = 1.5 A. Hence the source voltage

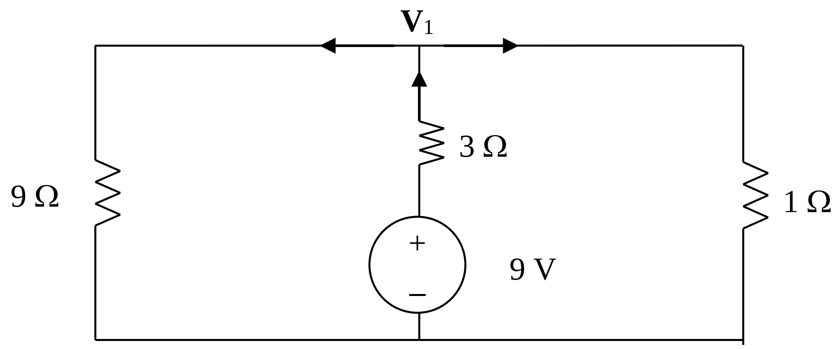
$$v_s = 1x1.5 + 3 = 4.5 \text{ V}$$

If $v_s = 4.5 \longrightarrow 1V$

Then $v_s = 4 \longrightarrow \frac{1}{4.5}x4 = \underline{0.8889\ \text{V}}$ = **888.9 mV**.

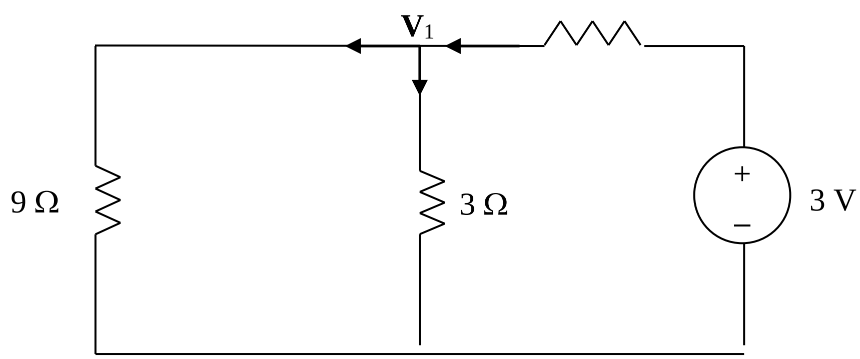
Solution 4.8

Let $V_o = V_1 + V_2$, where V_1 and V_2 are due to 9-V and 3-V sources respectively. To find V_1, consider the circuit below.



$$\frac{9-V_1}{3} = \frac{V_1}{9} + \frac{V_1}{1} \longrightarrow V_1 = 27/13 = 2.0769$$

To find V_2, consider the circuit below.



$$\frac{V_2}{9} + \frac{V_2}{3} = \frac{3-V_2}{1} \longrightarrow V_2 = 27/13 = 2.0769$$

$$V_o = V_1 + V_2 = \mathbf{4.1538\ \ V}$$

Solution 4.9

Given that I = 6 amps when V_s = 160 volts and I_s = –10 amps and I = 5 amp when V_s = 200 volts and I_s = 0, use superposition and linearity to determine the value of I when V_s = 120 volts and I_s = 5 amps.

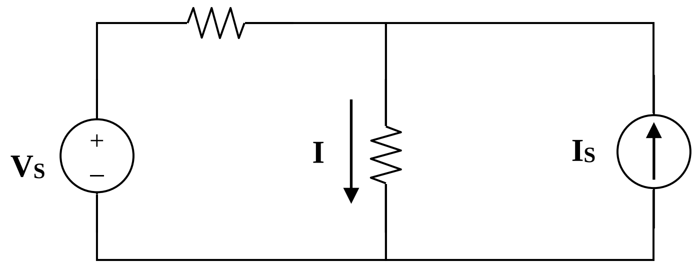


Figure 4.77
For Prob. 4.9.

Solution

At first this appears to be a difficult problem. However, if you take it one step at a time then it is not as hard as it seems. The important thing to keep in mind is that it is linear!

First superposition tells us that I = I' + I" where I' is the current contributed by the voltage source and I" is current contributed by the current source. Linearity tells us that I' = (V_s)5/200 = V_s/40. To find the relationship for I" we use superposition and linearity to find the value for I" = I_s(K) where I = 6 = (160/40) + (–10)(K) or –10K = 6 – 4 = 2 or K = –0.2.

This then leads to I = (120/40) – 5(0.2) = 3 – 1 = **2 A**.

Solution 4.10

Using Fig. 4.78, design a problem to help other students better understand superposition. Note, the letter *k* is a gain you can specify to make the problem easier to solve but must not be zero.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 4.78, find the terminal voltage V_{ab} using superposition.

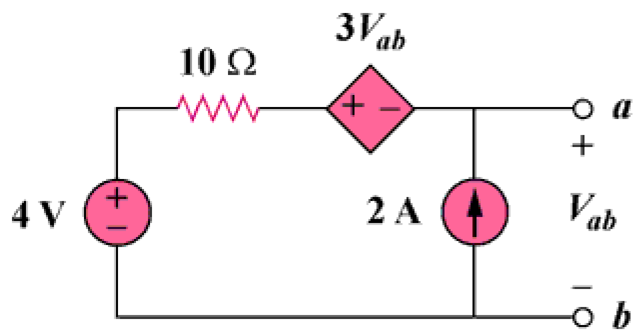
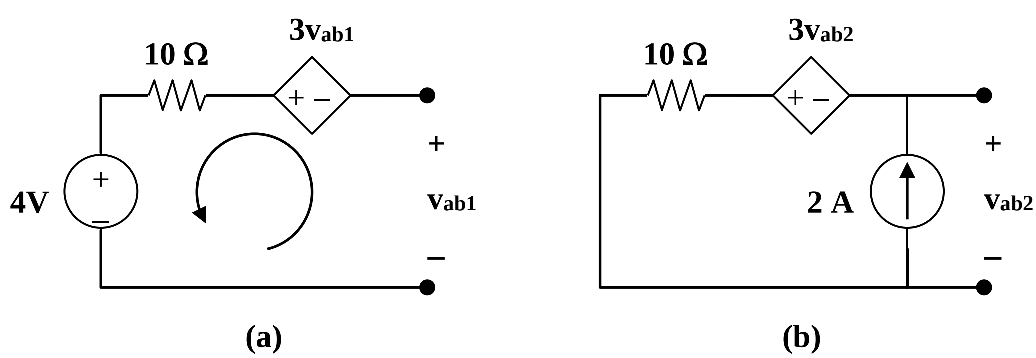


Figure 4.78
For Prob. 4.10.

Solution

Let $v_{ab} = v_{ab1} + v_{ab2}$ where v_{ab1} and v_{ab2} are due to the 4-V and the 2-A sources respectively.



For v_{ab1}, consider Fig. (a). Applying KVL gives,

$$- v_{ab1} - 3\, v_{ab1} + 10x0 + 4 = 0, \text{ which leads to } v_{ab1} = 1 \text{ V}$$

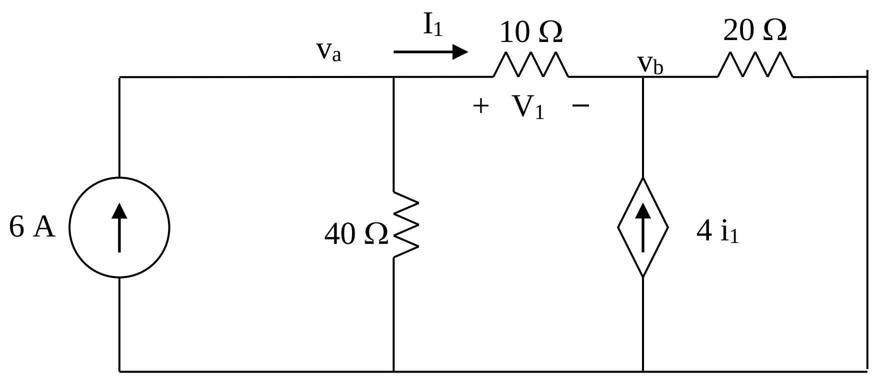
For v_{ab2}, consider Fig. (b). Applying KVL gives,

$$-v_{ab2} - 3v_{ab2} + 10x2 = 0, \text{ which leads to } v_{ab2} = 5$$

$$v_{ab} = 1 + 5 = \mathbf{6\ V}$$

Solution 4.11

Let $v_o = v_1 + v_2$, where v_1 and v_2 are due to the 6-A and 80-V sources respectively. To find v_1, consider the circuit below.



At node a,

$$6 = \frac{v_a}{40} + \frac{v_a - v_b}{10} \longrightarrow 240 = 5v_a - 4v_b \qquad (1)$$

At node b,

$$-I_1 - 4I_1 + (v_b - 0)/20 = 0 \text{ or } v_b = 100I_1$$

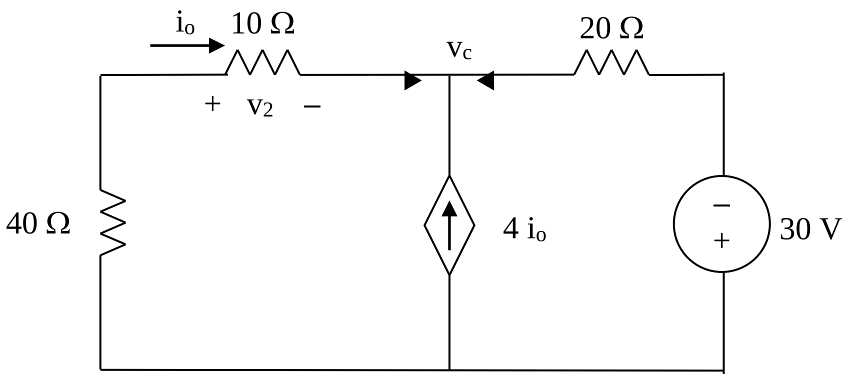
But $i_1 = \dfrac{v_a - v_b}{10}$ which leads to $100(v_a - v_b)10 = v_b$ or $v_b = 0.9091v_a$ (2)

Substituting (2) into (1),

$$5v_a - 3.636v_a = 240 \text{ or } v_a = 175.95 \text{ and } v_b = 159.96$$

However, $v_1 = v_a - v_b = 15.99$ V.

To find v_2, consider the circuit below.



$$\frac{0-v_c}{50}+4i_o+\frac{(-30-v_c)}{20}=0$$

But $\quad i_o=\dfrac{(0-v_c)}{50}$

$$-\frac{5v_c}{50}-\frac{(30+v_c)}{20}=0 \quad \longrightarrow \quad v_c=-10\ \text{V}$$

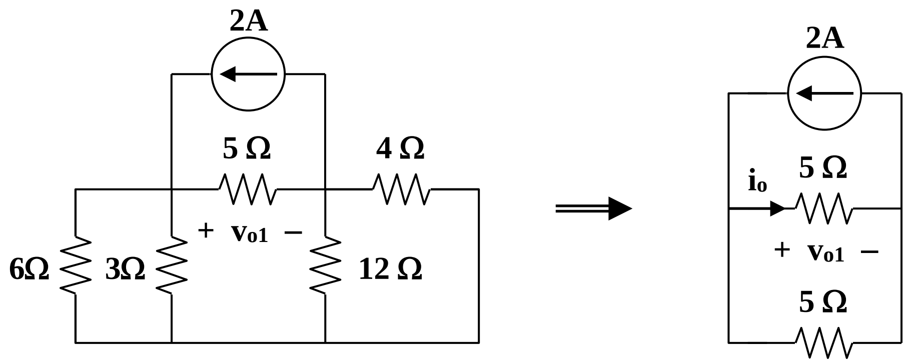
$$i_2=\frac{0-v_c}{50}=\frac{0+10}{50}=\frac{1}{5}$$

$$v_2=10i_2=2\ \text{V}$$

$v_o = v_1 + v_2$ =15.99 + 2 = **17.99 V** and $i_o = v_o/10$= **1.799 A**.

Solution 4.12

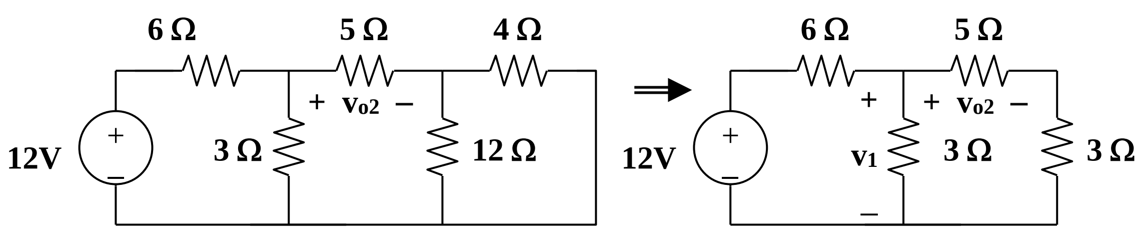
Let $v_o = v_{o1} + v_{o2} + v_{o3}$, where v_{o1}, v_{o2}, and v_{o3} are due to the 2-A, 12-V, and 19-V sources respectively. For v_{o1}, consider the circuit below.



$$6||3 = 2 \text{ ohms}, \ 4||12 = 3 \text{ ohms. Hence,}$$

$$i_o = 2/2 = 1, v_{o1} = 5io = 5 \text{ V}$$

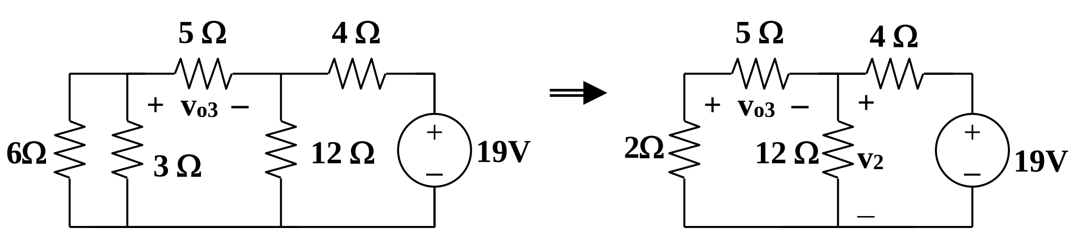
For v_{o2}, consider the circuit below.



$$3||8 = 24/11, \ v_1 = [(24/11)/(6 + 24/11)]12 = 16/5$$

$$v_{o2} = (5/8)v_1 = (5/8)(16/5) = 2 \text{ V}$$

For v_{o3}, consider the circuit shown below.

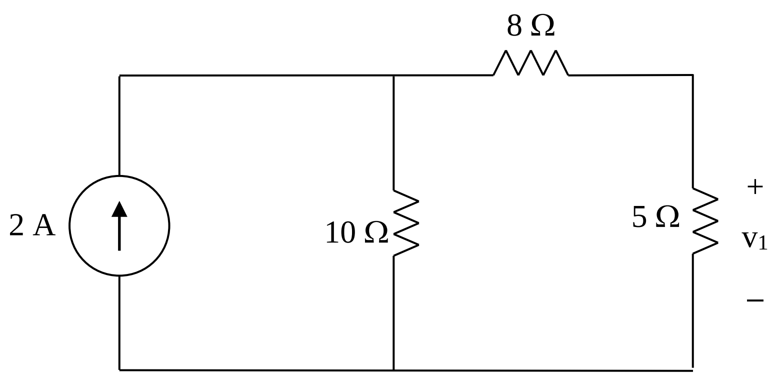


$$7||12 = (84/19) \text{ ohms}, \ v_2 = [(84/19)/(4 + 84/19)]19 = 9.975$$

$$v = (-5/7)v2 = -7.125$$
$$v_o = 5 + 2 - 7.125 = \mathbf{-125 \text{ mV}}$$

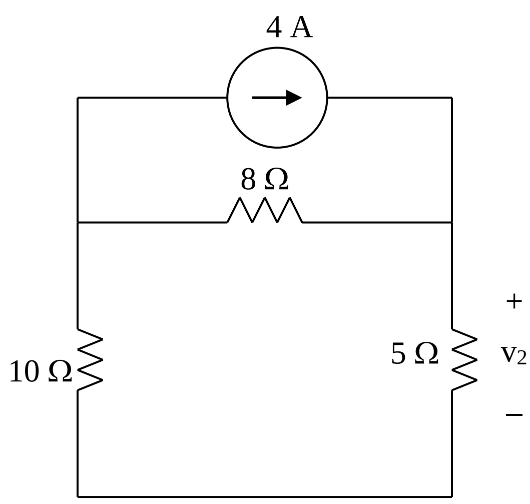
Solution 4.13

Let $v_o = v_1 + v_2 + v_3$, where v_1, v_2, and v_3 are due to the independent sources. To find v_1, consider the circuit below.



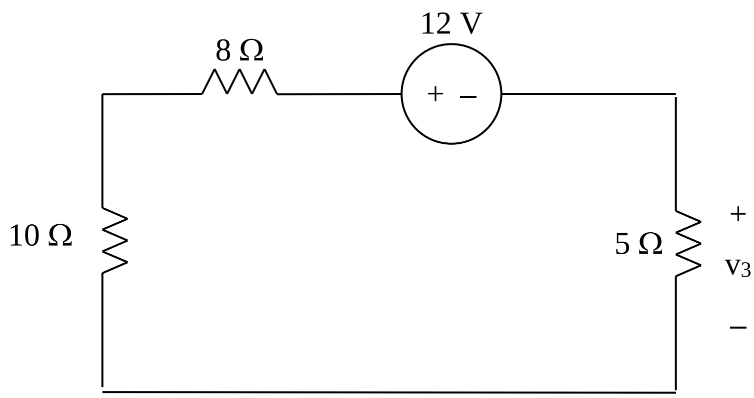
$$v_1 = 5x\frac{10}{10+8+5}x2 = 4.3478$$

To find v_2, consider the circuit below.



$$v_2 = 5x\frac{8}{8+10+5}x4 = 6.9565$$

To find v_3, consider the circuit below.

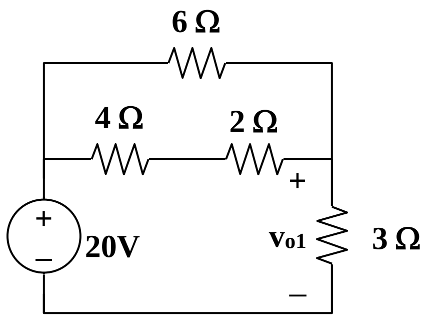


$$v_3 = -12\left(\frac{5}{5+10+8}\right) = -2.6087$$

$$v_o = v_1 + v_2 + v_3 = \underline{8.6956\ \text{V}} = \mathbf{8.696V}.$$

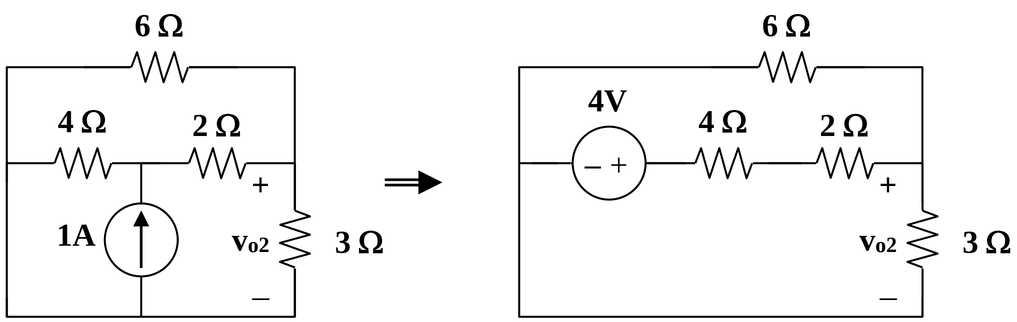
Solution 4.14

Let $v_o = v_{o1} + v_{o2} + v_{o3}$, where v_{o1}, v_{o2} , and v_{o3}, are due to the 20-V, 1-A, and 2-A sources respectively. For v_{o1}, consider the circuit below.



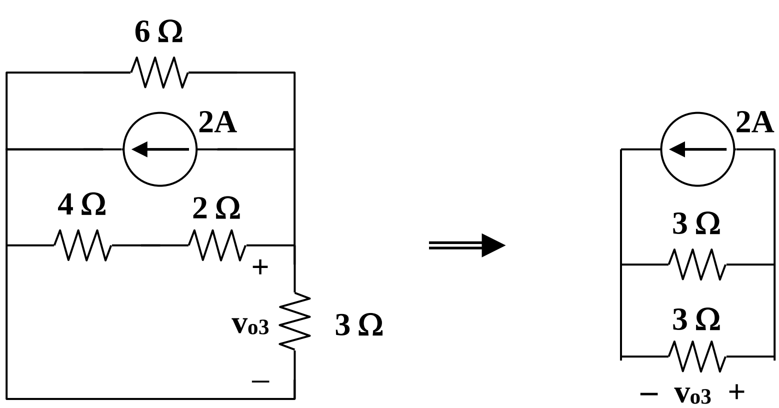
$$6||(4 + 2) = 3 \text{ ohms}, v_{o1} = (½)20 = 10 \text{ V}$$

For v_{o2}, consider the circuit below.



$$3||6 = 2 \text{ ohms}, v_{o2} = [2/(4 + 2 + 2)]4 = 1 \text{ V}$$

For v_{o3}, consider the circuit below.

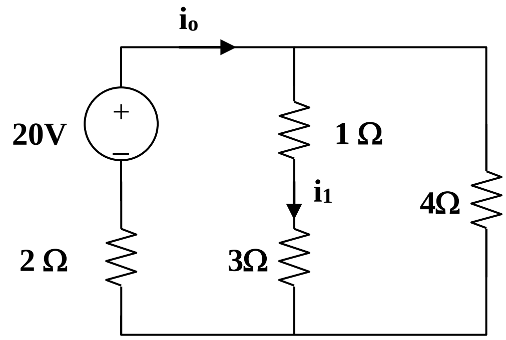


$$6||(4 + 2) = 3, v_{o3} = (-1)3 = –3$$

$$v_o = 10 + 1 – 3 = \mathbf{8\ V}$$

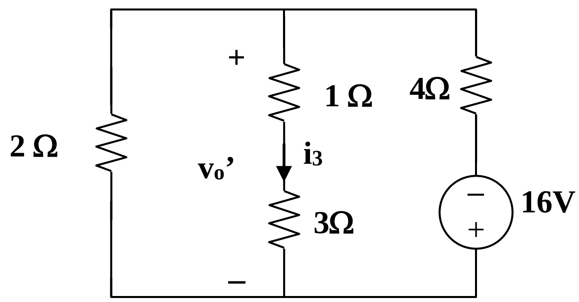
Solution 4.15

Let $i = i_1 + i_2 + i_3$, where i_1 , i_2 , and i_3 are due to the 20-V, 2-A, and 16-V sources. For i_1, consider the circuit below.



$$4\|(3 + 1) = 2 \text{ ohms, Then } i_o = [20/(2 + 2)] = 5 \text{ A, } i_1 = i_o/2 = 2.5 \text{ A}$$

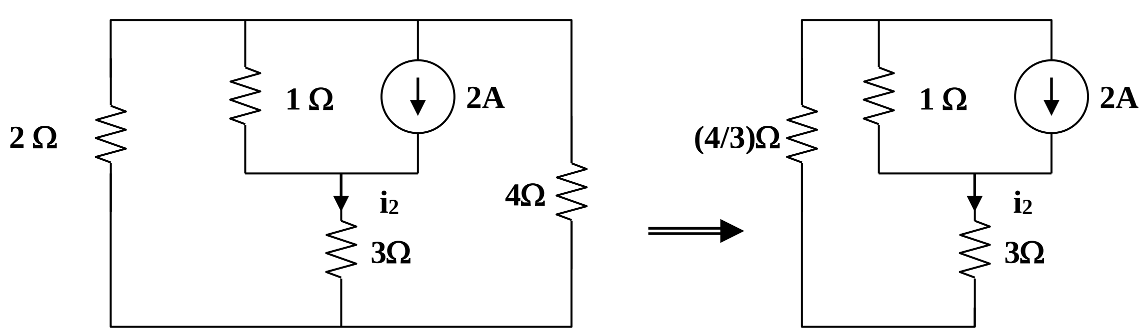
For i_3, consider the circuit below.



$$2\|(1 + 3) = 4/3, \; v_o' = [(4/3)/((4/3) + 4)](-16) = -4$$

$$i_3 = v_o'/4 = -1$$

For i_2, consider the circuit below.



$$2\|4 = 4/3, \ 3 + 4/3 = 13/3$$

Using the current division principle.

$$i_2 = [1/(1 + 13/2)]2 = 3/8 = 0.375$$

$$i = 2.5 + 0.375 - 1 = \mathbf{1.875\ A}$$

$$p = i^2R = (1.875)^2 3 = \mathbf{10.55\ watts}$$

Solution 4.16

Given the circuit in Fig. 4.84, use superposition to obtain $\boldsymbol{i_o}$.

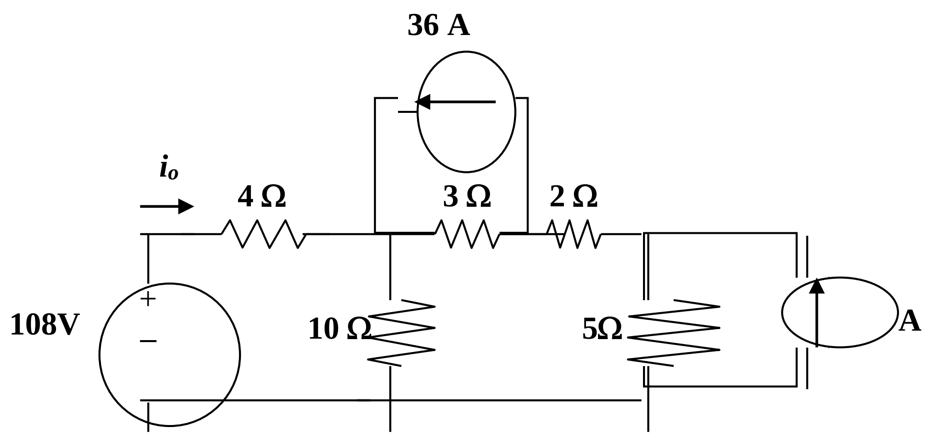
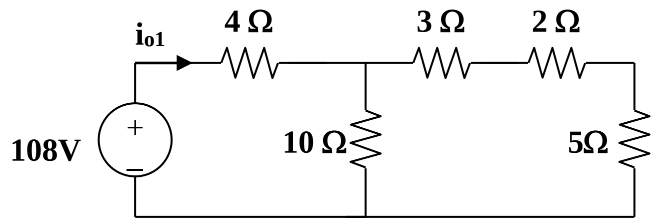


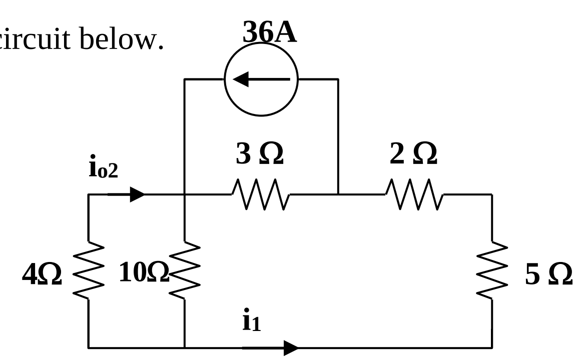
Figure 4.84
For Prob. 4.16.

Let $i_o = i_{o1} + i_{o2} + i_{o3}$, where i_{o1}, i_{o2}, and i_{o3} are due to the 108 V, 36 A, and 18 A sources. For i_{o1}, consider the circuit below.



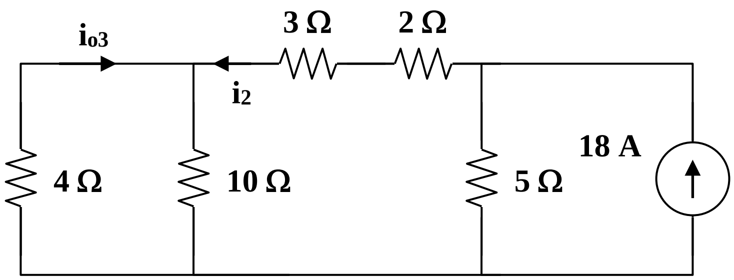
$$10||(3 + 2 + 5) = 5 \text{ ohms}, \quad i_{o1} = 108/(5 + 4) = 12 \text{ A}$$

For i_{o2}, consider the circuit below.



$$2 + 5 + 4||10 = 7 + 40/14 = 69/7$$
$$i_1 = [3/(3 + 69/7)]36 = 756/90 = 8.4, \quad i_{o2} = [-10/(4 + 10)]i_1 = -6 \text{ A}$$

For i_{o3}, consider the circuit below.



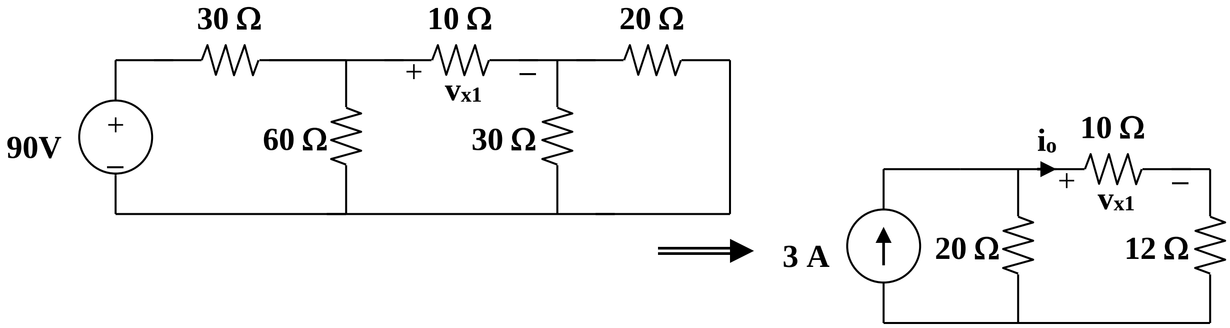
$$3 + 2 + 4\|10 = 5 + 20/7 = 55/7$$

$$i_2 = [5/(5 + 55/7)]18 = 7, \quad i_{o3} = [-10/(10 + 4)]i_2 = -5$$

$$i_o = 12 - 6 - 5 = 1 = \mathbf{1\ A}.$$

Solution 4.17

Let $v_x = v_{x1} + v_{x2} + v_{x3}$, where v_{x1}, v_{x2}, and v_{x3} are due to the 90-V, 6-A, and 40-V sources. For v_{x1}, consider the circuit below.

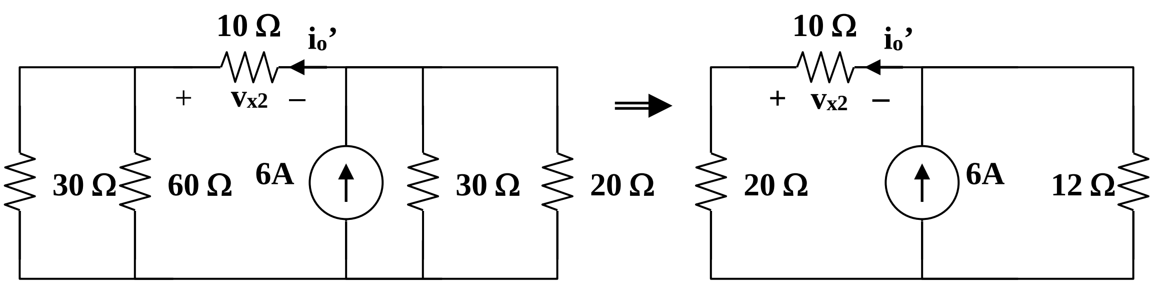


$$20||30 = 12 \text{ ohms}, \ 60||30 = 20 \text{ ohms}$$

By using current division,

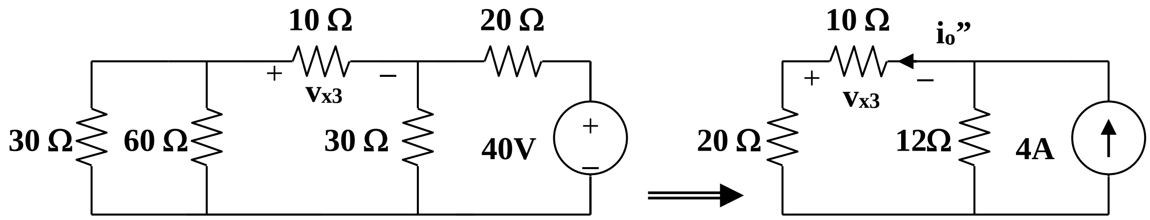
$$i_o = [20/(22 + 20)]3 = 60/42, \ v_{x1} = 10i_o = 600/42 = 14.286 \text{ V}$$

For v_{x2}, consider the circuit below.



$$i_o' = [12/(12 + 30)]6 = 72/42, \ v_{x2} = -10i_o' = -17.143 \text{ V}$$

For v_{x3}, consider the circuit below.



$$i_o" = [12/(12 + 30)]2 = 24/42, \ v_{x3} = -10i_o" = -5.714 = [12/(12 + 30)]2 = 24/42,$$
$$v_{x3} = -10i_o" = -5.714$$
$$= [12/(12 + 30)]2 = 24/42, \ v_{x3} = -10i_o" = -5.714$$
$$v_x = 14.286 - 17.143 - 5.714 = \mathbf{-8.571 \text{ V}}$$

Solution 4.18

Use superposition to find V_o in the circuit of Fig. 4.86.

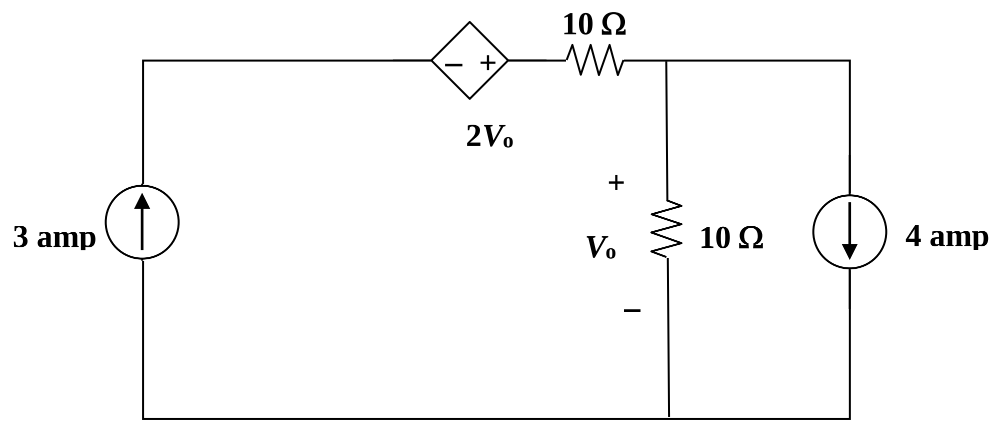
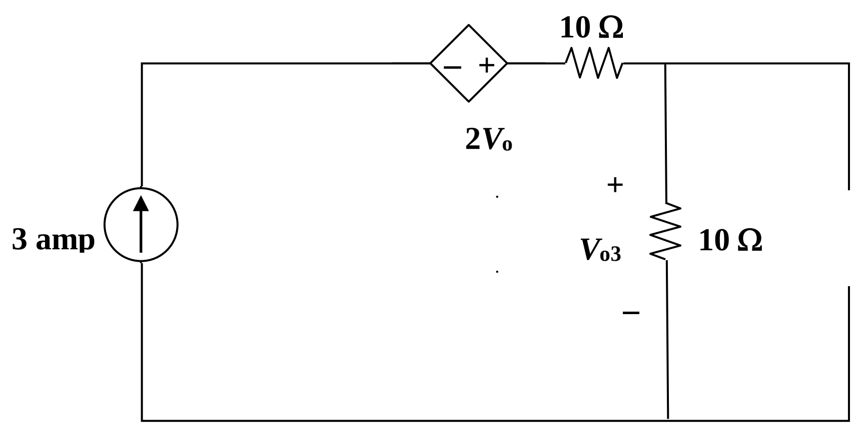
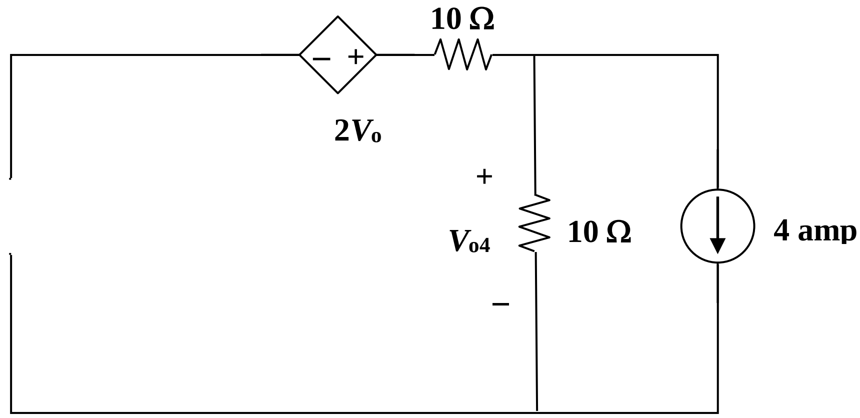


Figure 4.86
For Prob. 4.18.

Solution

Step 1. Since we only have two independent sources, we need to look at the contributions from each of these sources. Next we create two circuits.





Step 2. V_{o3} = 10x3 = 30 volts and V_{o4} = 10(–4) = –40 volts which leads to,

$V_o = V_{o3} + V_{o4}$ = 30 – 40 = **–10 volts**.

Solution 4.19

Use superposition to solve for $\boldsymbol{v_x}$ in the circuit of Fig. 4.87.

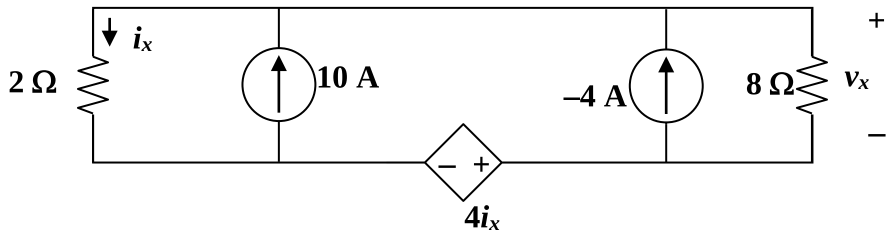
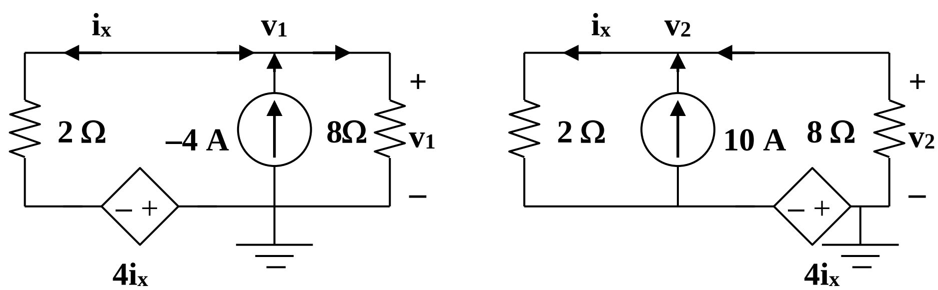


Figure 4.87
For Prob. 4.19.

Solution

Let $v_x = v_1 + v_2$, where v_1 and v_2 are due to the 4-A and 6-A sources respectively.



(a) **(b)**

To find v_1, consider the circuit in Fig. (a).

$$[(v_1–0)/8] – (–4) + [(v_1 – (–4i_x))/2] = 0 \text{ or } (0.125+0.5)v_1 = –4 – 2i_x \text{ or } v_1 = –6.4 – 3.2i_x$$

But, $i_x = (v_1 – (–4i_x))/2$ or $i_x = –0.5v_1$. Thus,

$$v_1 = –6.4 + 3.2(0.5v_1), \text{ which leads to } v_1 = 6.4/0.6 = 10.667$$

To find v_2, consider the circuit shown in Fig. (b).

$$v_2/8 – 10 + (v_2 – (–4i_x))/2 = 0 \text{ or } v_2 + 3.2i_x = 16$$

But $i_x = –0.5v_2$. Therefore,

$$v_2 + 3.2(–0.5v_2) = 16 \text{ which leads to } v_2 = –26.67$$

Hence, $v_x = 10.667 - 26.667 = \mathbf{-16\ V}$.

Checking,

$$i_x = -0.5v_x = 8\text{ A}$$

Now all we need to do now is sum the currents flowing out of the top node.

$$8 - 10 + 4 + (-16/8) = 0.$$

Solution 4.20

Use source transformations to reduce the circuit between terminals a and b shown in Fig. 4.88 to a single voltage source in series with a single resistor.

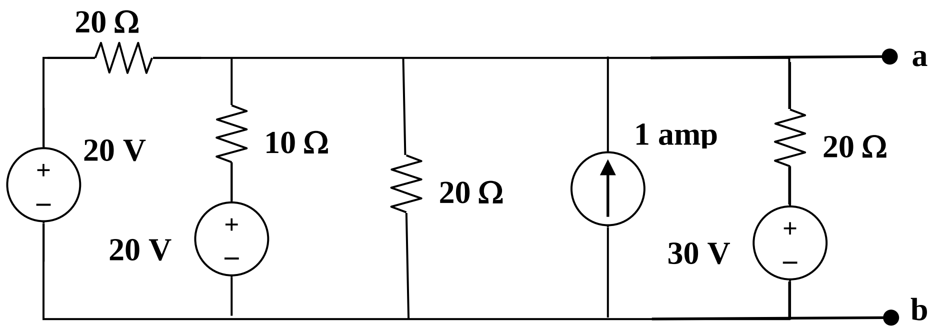
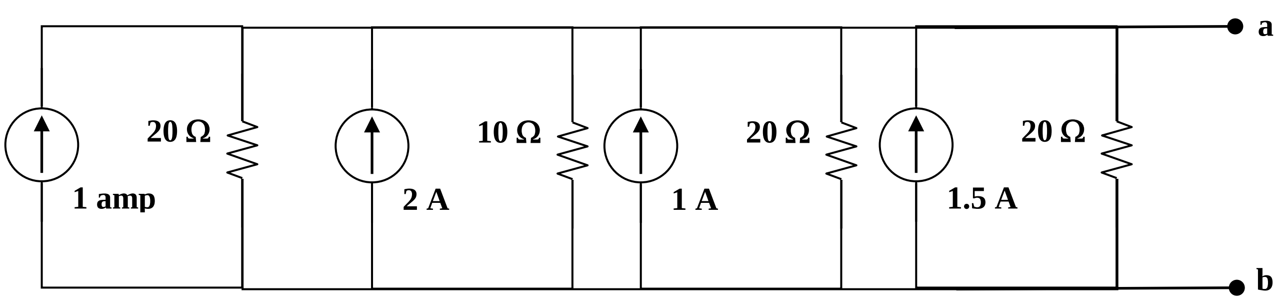


Figure 4.88
For Prob. 4.20.

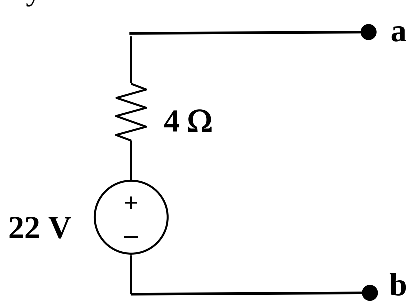
Solution

Step 1. This problem is most easily solved by converting all the voltage sources in series with resistors to current sources in parallel with resistors.



Now all we need is to add the current sources together algebraically and place all the resistors in parallel and combine them. Finally all we need to do is to change the current source and resistance back into a single voltage source in series with a resistor.

Step 2. I = 1+2+1+1.5 = 5.5 A and ($1/R_{eq}$) = 0.05+0.1+0.05+0.05 = 0.25 or R_{eq} = **4 Ω**. Finally V = 5.5x4 = **22 V**.



Solution 4.21

Using Fig. 4.89, design a problem to help other students to better understand source transformation.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Apply source transformation to determine v_o and i_o in the circuit in Fig. 4.89.

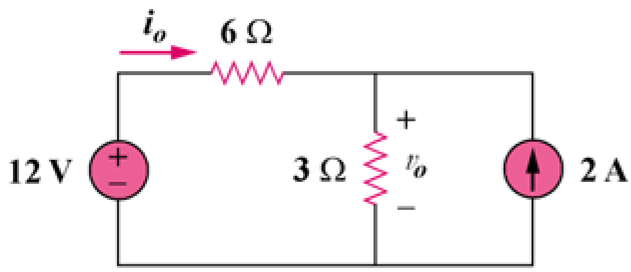
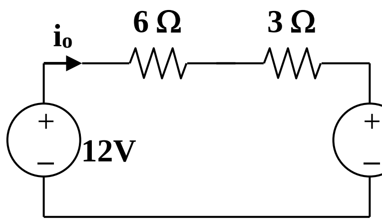
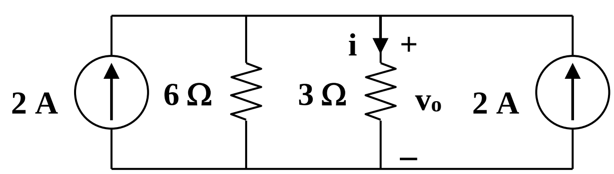


Figure 4.89

Solution

To get i_o, transform the current sources as shown in Fig. (a).





(a) **(b)**

From Fig. (a), $-12 + 9i_o + 6 = 0$, therefore i_o = **666.7 mA**

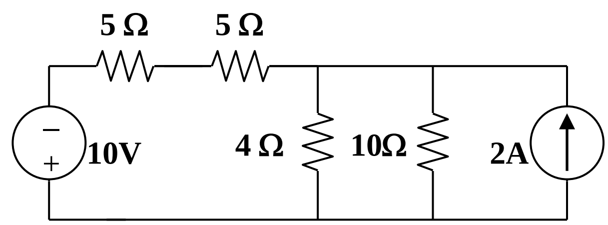
To get v_o, transform the voltage sources as shown in Fig. (b).

$$i = [6/(3 + 6)](2 + 2) = 8/3$$

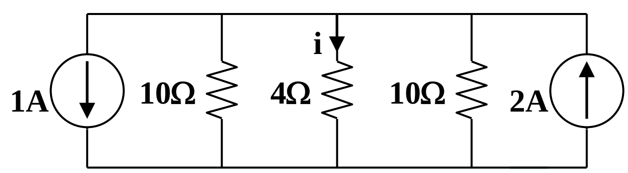
$$v_o = 3i = \mathbf{8\ V}$$

Solution 4.22

We transform the two sources to get the circuit shown in Fig. (a).



(a)



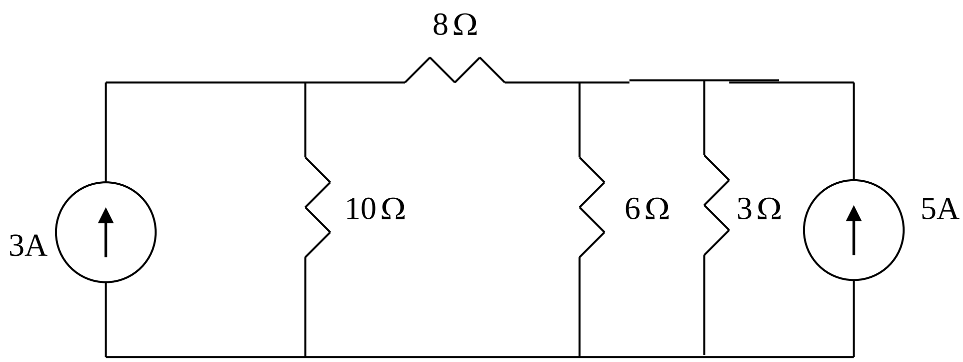
(b)

We now transform only the voltage source to obtain the circuit in Fig. (b).

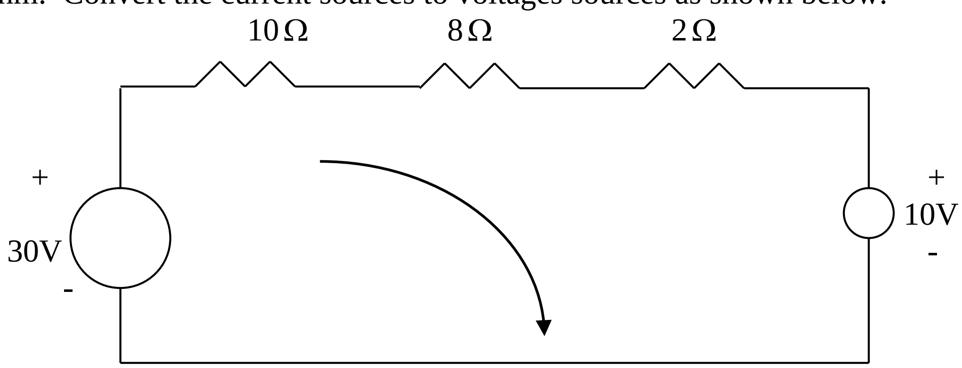
$$10||10 = 5 \text{ ohms}, \ i = [5/(5 + 4)](2 - 1) = 5/9 = \mathbf{555.5\ mA}$$

Solution 4.23

If we transform the voltage source, we obtain the circuit below.



3//6 = 2-ohm. Convert the current sources to voltages sources as shown below.



Applying KVL to the loop gives

$-30+10+I(10+8+2)=0 \quad \longrightarrow \quad$ I = **1 A**

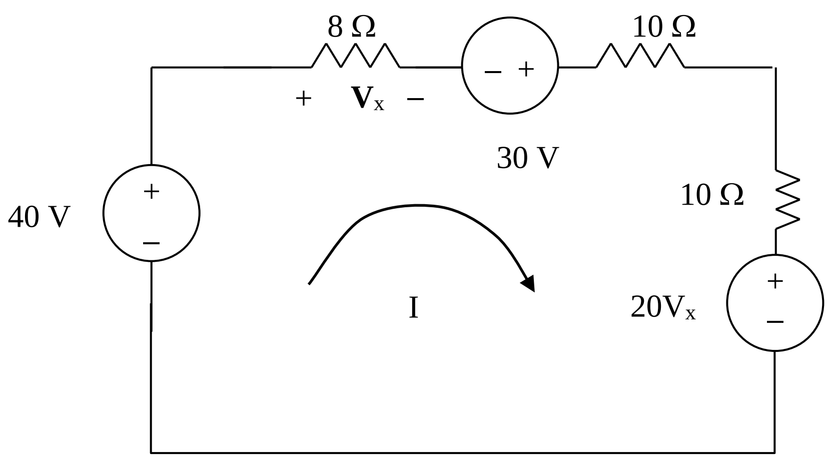
$$p = VI = I^2R = \mathbf{8\ W}$$

Solution 4.24

Transform the two current sources in parallel with the resistors into their voltage source equivalents yield,

> a 30-V source in series with a 10-Ω resistor and a $20V_x$-V sources in series with a 10-Ω resistor.

We now have the following circuit,

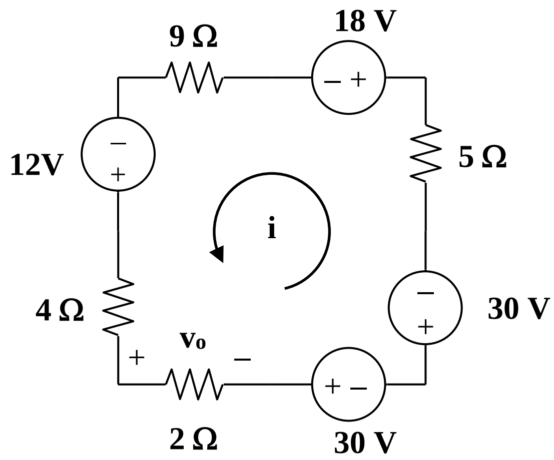


We now write the following mesh equation and constraint equation which will lead to a solution for V_x,

> $28I - 70 + 20V_x = 0$ or $28I + 20V_x = 70$, but $V_x = 8I$ which leads to
>
> $28I + 160I = 70$ or $I = 0.3723$ A or $V_x =$ **2.978 V**.

Solution 4.25

Transforming only the current source gives the circuit below.



Applying KVL to the loop gives,

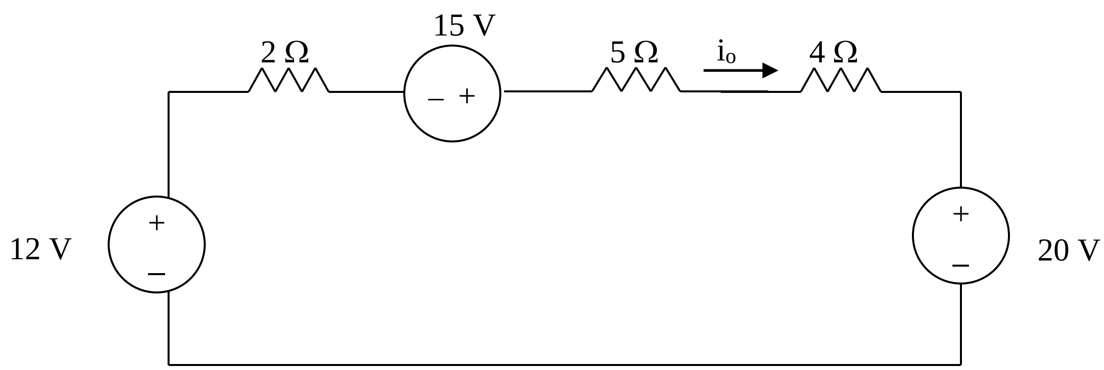
$$-(4 + 9 + 5 + 2)i + 12 - 18 - 30 - 30 = 0$$

$$20i = -66 \text{ which leads to } i = -3.3$$

$v_o = 2i =$ **-6.6 V**

Solution 4.26

Transforming the current sources gives the circuit below.



$-12 + 11i_o - 15 + 20 = 0$ or $11i_o = 7$ or $i_o =$ **636.4 mA**.

Solution 4.27

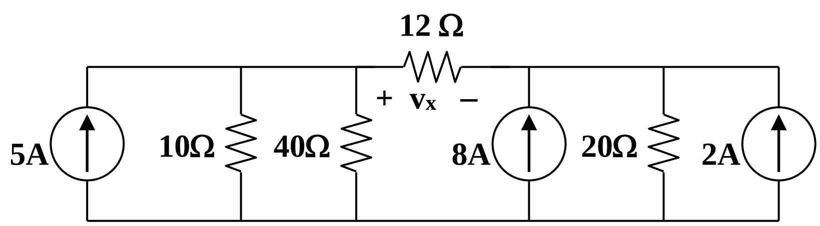
Transforming the voltage sources to current sources gives the circuit in Fig. (a).

$$10||40 \;=\; 8 \text{ ohms}$$

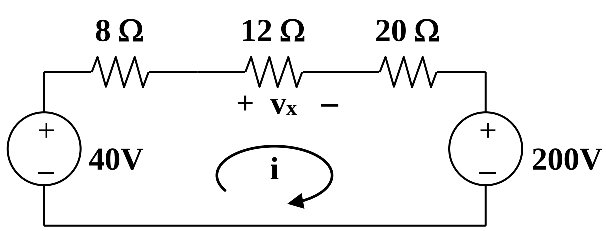
Transforming the current sources to voltage sources yields the circuit in Fig. (b). Applying KVL to the loop,

$$-40 \;+\; (8 + 12 + 20)i + 200 \;=\; 0 \;\;\text{leads to}\;\; i \;=\; -4$$

$$v_x \;\; 12i \;=\; \mathbf{-48\ V}$$



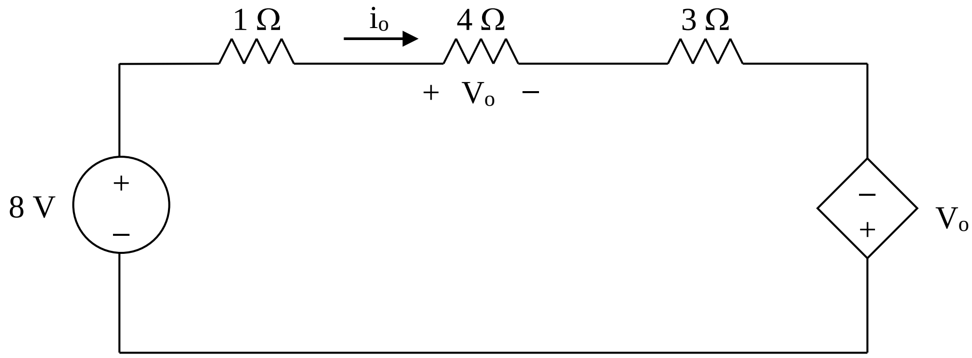
(a)



(b)

Solution 4.28

Convert the dependent current source to a dependent voltage source as shown below.



Applying KVL,

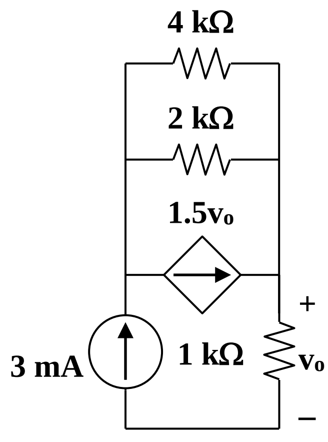
$$-8 + i_o(1 + 4 + 3) - V_o = 0$$

But $V_o = 4i_o$

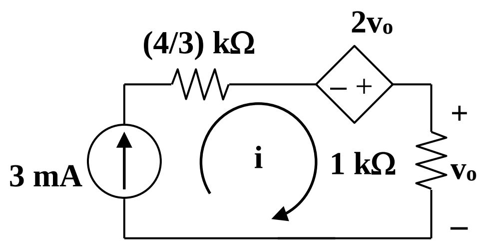
$$-8 + 8i_o - 4i_o = 0 \quad \longrightarrow \quad i_o = \underline{2\ \text{A}}$$

Solution 4.29

Transform the dependent voltage source to a current source as shown in Fig. (a). 2||4 = (4/3) k ohms



(a)



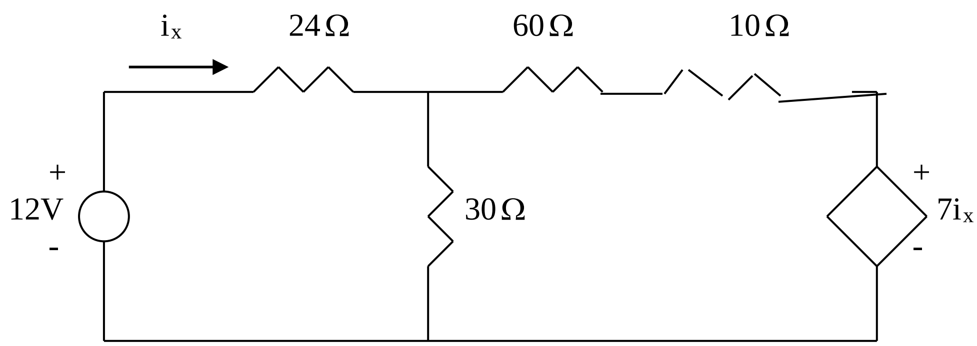
(b)

It is clear that i = 3 mA which leads to v_o = 1000i = **3 V**

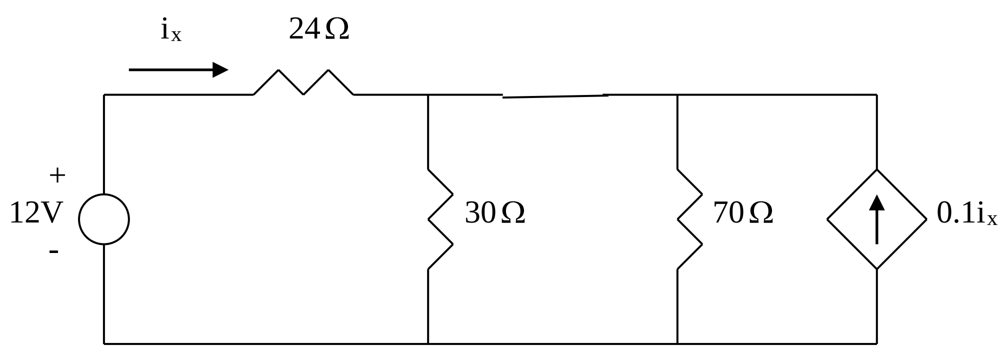
If the use of source transformations was not required for this problem, the actual answer could have been determined by inspection right away since the only current that could have flowed through the 1 k ohm resistor is 3 mA.

Solution 4.30

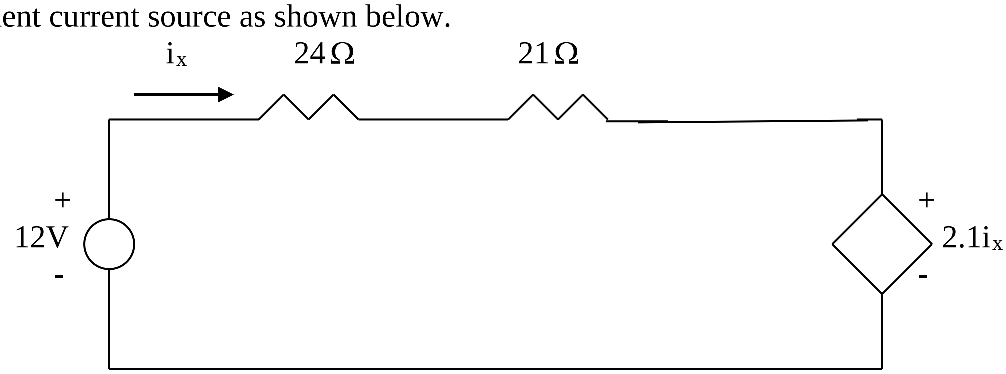
Transform the dependent current source as shown below.



Combine the 60-ohm with the 10-ohm and transform the dependent source as shown below.



Combining 30-ohm and 70-ohm gives 30//70 = 70x30/100 = 21-ohm. Transform the dependent current source as shown below.



Applying KVL to the loop gives

$$\mathbf{45}\boldsymbol{i}_x - \mathbf{12} + \mathbf{2.1}\boldsymbol{i}_x = \mathbf{0} \quad \longrightarrow \quad \boldsymbol{i}_x = \frac{\mathbf{12}}{\mathbf{47.1}} = \mathbf{254.8\ mA}.$$

Solution 4.31

Determine v_x in the circuit of Fig. 4.99 using source transformation.

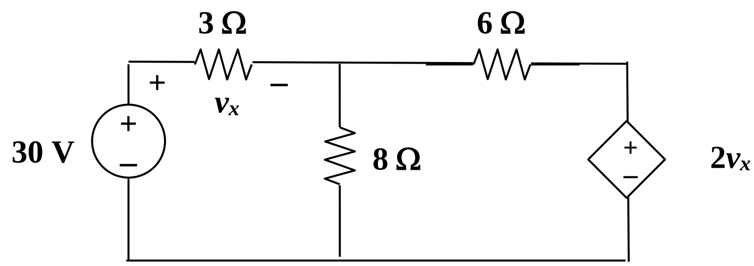
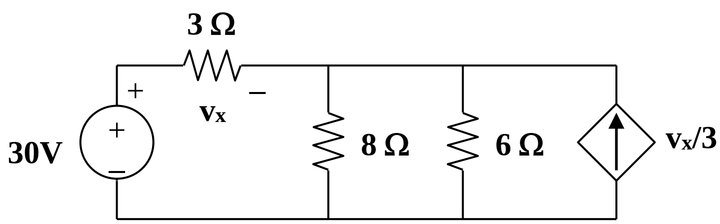
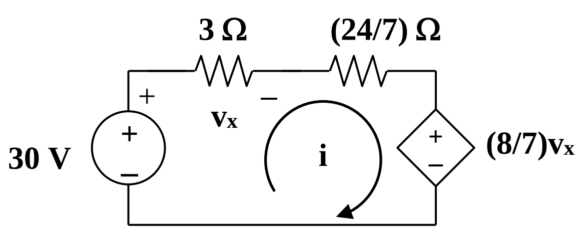


Figure 4.99
For Prob. 4.31.

Solution Transform the dependent source so that we have the circuit in Fig. (a). 6||8 = (24/7) ohms. Transform the dependent source again to get the circuit in Fig. (b).



(a)



(b)

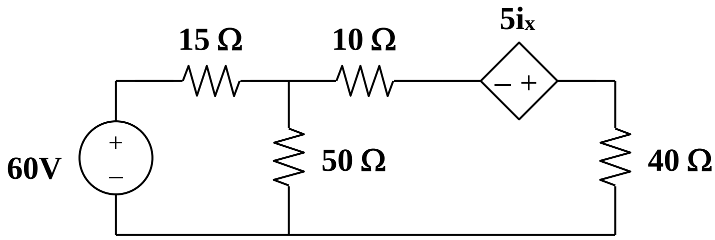
From Fig. (b), $v_x = 3i$, or $i = v_x/3$.

Applying KVL, $-30 + (3 + 24/7)i + (8/7)v_x = 0$

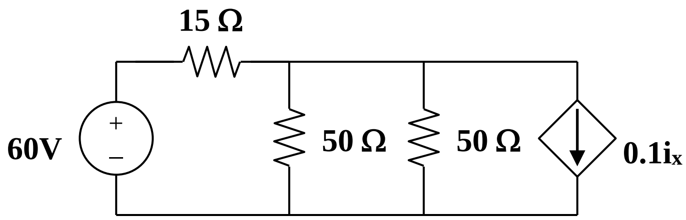
$$[(21 + 24)/7]v_x/3 + (8/7)v_x = 30 \text{ leads to } v_x = 30/3.2857 = \mathbf{9.13\ V}.$$

Solution 4.32

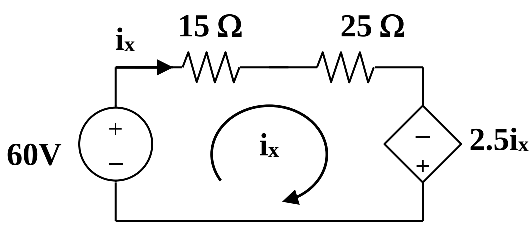
As shown in Fig. (a), we transform the dependent current source to a voltage source,



(a)



(b)



(c)

In Fig. (b), $50||50 = 25$ ohms. Applying KVL in Fig. (c),

$-60 + 40i_x - 2.5i_x = 0$, or $i_x =$ **1.6 A**

Solution 4.33

Determine the Thevenin equivalent circuit, shown in Fig. 4.101, as seen by the 7-ohm resistor. Then calculate the current flowing through the 7-ohm resistor.

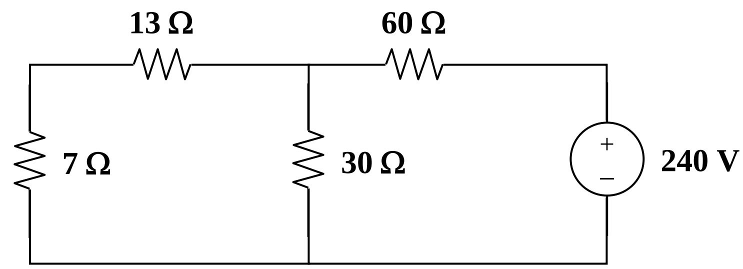
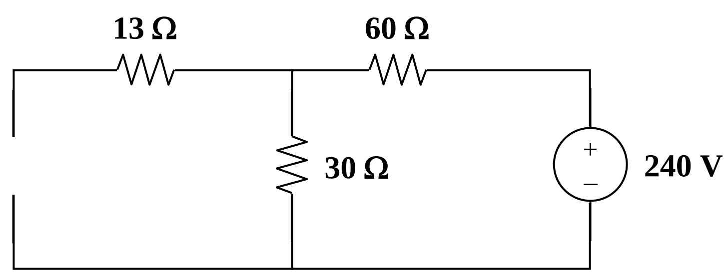
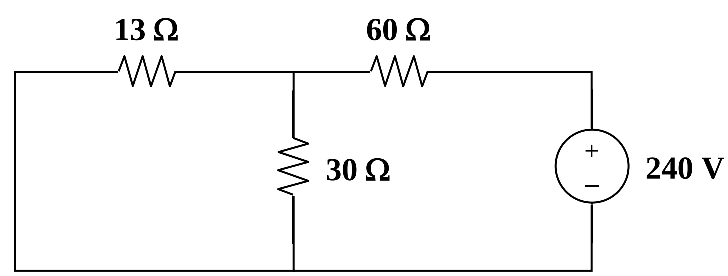


Figure 4.101
For Prob. 4.33.

Solution

Step 1. We need to find V_{oc} and I_{sc}. To do this, we will need two circuits, label the appropriate unknowns and solve for V_{oc}, I_{sc}, and then R_{eq} which is equal to V_{oc}/I_{sc}.





For the open circuit voltage all we need to do is to recognize that there is no voltage drop across the 13 Ω resistor so that V_{oc} = 240x30/(30+60).
Clearly I_{sc} is equal to the current through the 13 Ω resistor or
I_{sc} = [240/(60+(13x30/(13+30))][13x30/(13+30)]/13.

Step 2. $V_{oc} = V_{Thev}$ = **80 V**. I_{sc} = [240/69.07]9.07/13 = 2.4242 which leads to R_{eq} = **33 Ω**. Clearly,

$$I_7 = 80/(7+33) = \mathbf{2\ A}.$$

Solution 4.34

Using Fig. 4.102, design a problem that will help other students better understand Thevenin equivalent circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Thevenin equivalent at terminals ***a-b*** of the circuit in Fig. 4.102.

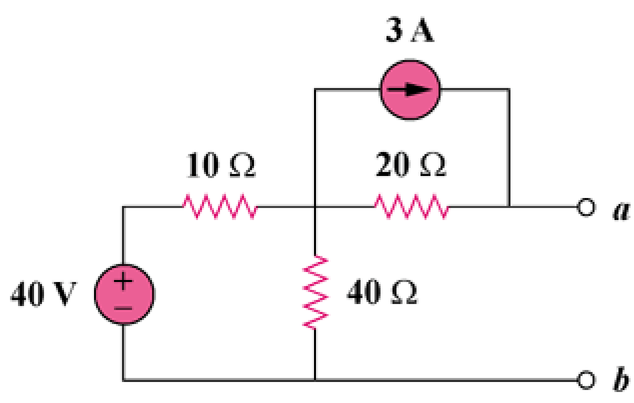
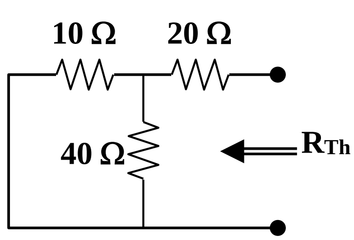


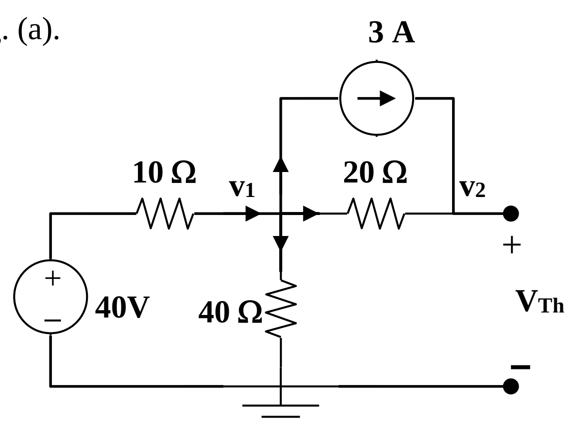
Figure 4.102

Solution

To find R_{Th}, consider the circuit in Fig. (a).



(a)



(b)

$$R_{Th} = 20 + 10||40 = 20 + 400/50 = \mathbf{28\ ohms}$$

To find V_{Th}, consider the circuit in Fig. (b).

At node 1, $(40 - v_1)/10 = 3 + [(v_1 - v_2)/20] + v_1/40, \; 40 = 7v_1 - 2v_2$ (1)

At node 2, $3 + (v1 - v2)/20 = 0$, or $v1 = v2 - 60$ (2)

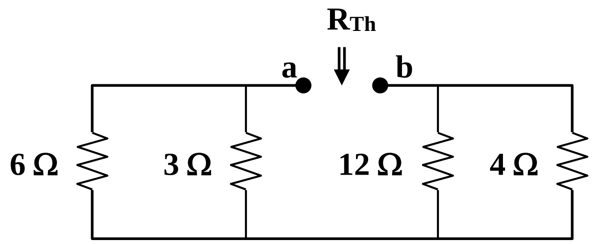
Solving (1) and (2), $v_1 = 32$ V, $v_2 = 92$ V, and $V_{Th} = v_2 =$ **92 V**

Solution 4.35

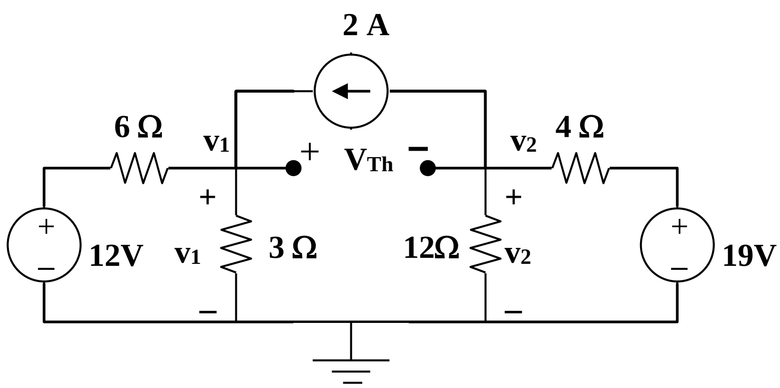
To find R_{Th}, consider the circuit in Fig. (a).

$$R_{Th} = R_{ab} = 6\|3 + 12\|4 = 2 + 3 = 5 \text{ ohms}$$

To find V_{Th}, consider the circuit shown in Fig. (b).



(a)

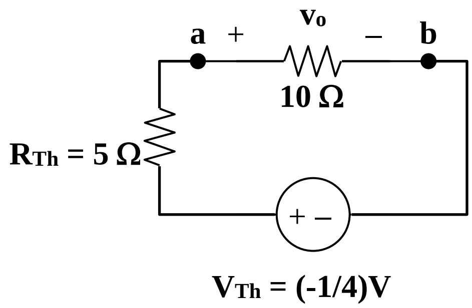


(b)

At node 1, $\quad 2 + (12 - v_1)/6 = v_1/3$, or $v_1 = 8$

At node 2, $\quad (19 - v_2)/4 = 2 + v_2/12$, or $v_2 = 33/4$

But, $\quad -v_1 + V_{Th} + v_2 = 0$, or $V_{Th} = v_1 - v_2 = 8 - 33/4 = -0.25$



$v_o = V_{Th}/2 = -0.25/2 = \mathbf{-125\ mV}$

Solution 4.36

Solve for the current ***i*** in the circuit of Fig. 4.103 using Thevenin's theorem. (*Hint:* Find the Thevenin equivalent as seen by the 12-Ω resistor.)

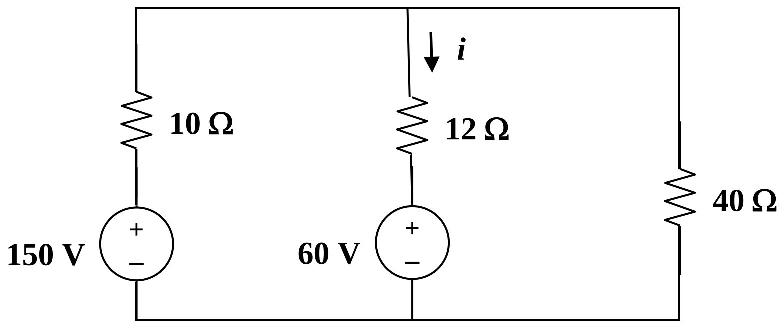
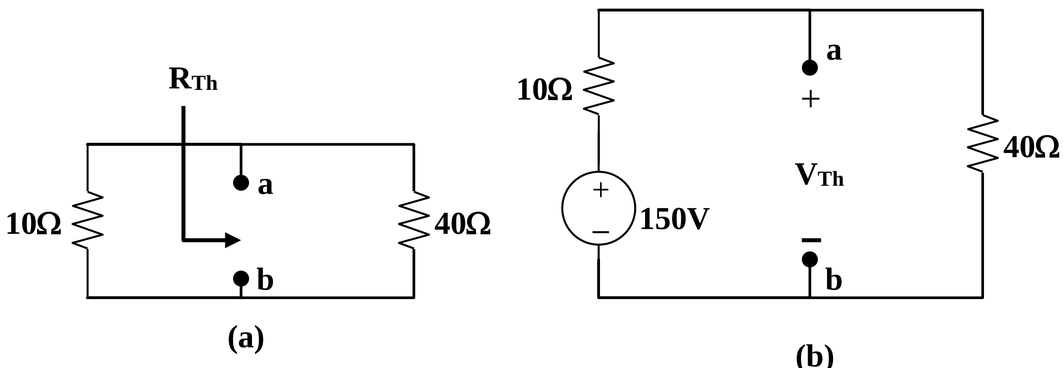


Figure 4.103
For Prob. 4.36.

Solution

Although we could just remove the 12 Ω resistor and find the Thevenin equivalent, let us remove the 60 V voltage source and the 12 Ω resistor (b) and then know that the Thevenin voltage is equal to the Thevenin voltage we find below –60 volts. To find the Thevenin resistance set the 150 V source to 0.

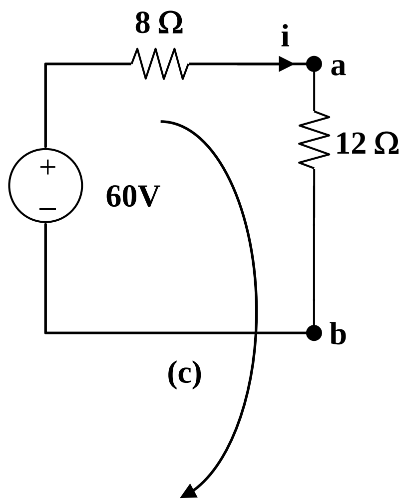


(a)

(b)

From Fig. (a), $R_{Th} = 10||40 = 8$ ohms

From Fig. (b), $V_{Th} = (40/(10 + 40))150 = 120$V or the actual Thevenin voltage is equal to 120–60 = 60 V.



(c)

The equivalent circuit of the original circuit is shown in Fig. (c). Applying KVL,

$$-60 + (8 + 12)i = 0, \text{ which leads to } i = \mathbf{3\ A}.$$

Solution 4.37

Find the Norton equivalent with respect to terminals *a*-*b* in the circuit shown in Fig. 4.104.

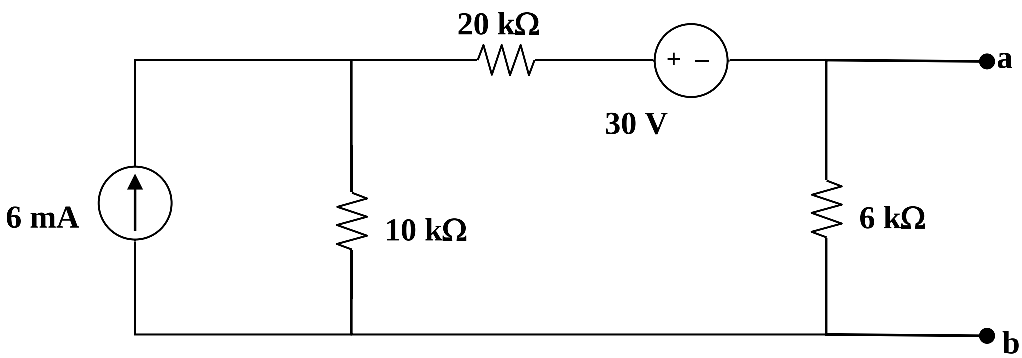


Figure 4.104
For Prob. 4.37.

Solution

Step 1. Since we do not have a dependent source we can find the equivalent resistance by setting the independent sources equal to zero. Therefore, $R_{eq} = 6k(30k)/(6k+30k)$.

Now all we need to do is to find the value of $I_{sc} = I_N$. $10k(I_{sc}–0.006) + 20kI_{sc} + 30 = 0$.

Step 2. $R_{eq} = 180k/36 =$ **5 kΩ**. $30kI_{sc} = 30$ or $I_{sc} = I_N =$ **1 mA**.

Solution 4.38

Apply Thevenin's theorem to find V_o in the circuit of Fig. 4.105.

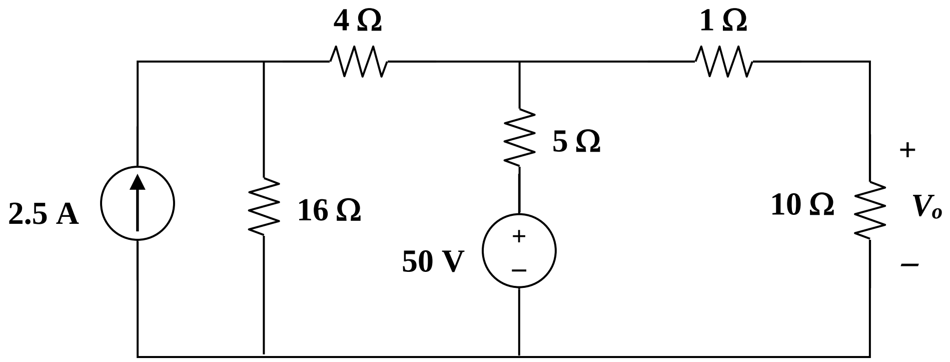
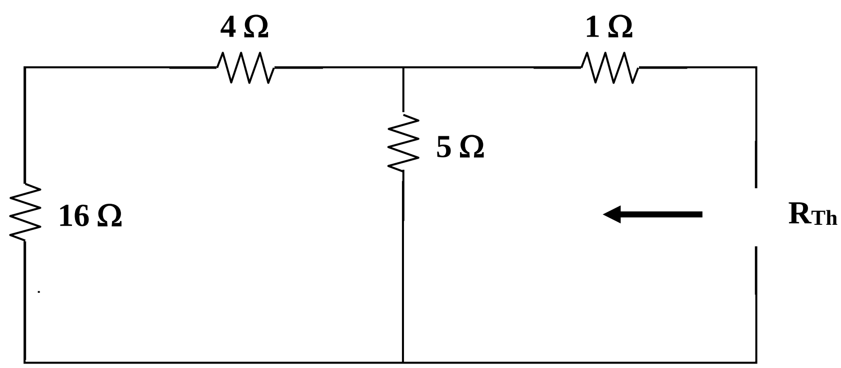


Figure 4.105
For Prob. 4.38.

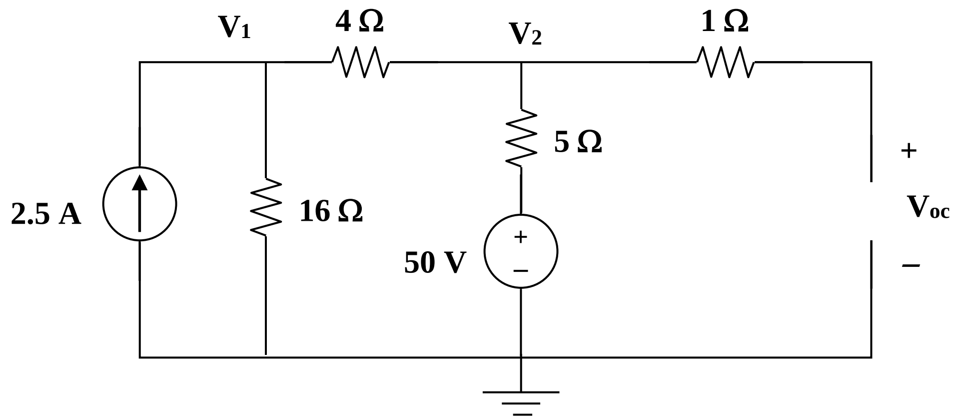
Solution

We find Thevenin equivalent at the terminals of the 10-ohm resistor. For R_{Th}, consider the circuit below.



$R_{Th} = 1 + 5//(4+16) = 1 + 4 =$ **5 Ω**.

For V_{Th}, consider the circuit below.



At node 1,

$$-2.5+\frac{V_1-0}{16}+\frac{V_1-V_2}{4}=0 \quad\longrightarrow\quad 0.3125V_1-0.25V_2=2.5 \qquad (1)$$

At node 2,

$$\frac{V_2-V_1}{4}+\frac{V_2-50}{5}=0 \quad\longrightarrow\quad -0.25V_1+0.45V_2=10 \qquad (2)$$

Solving (1) and (2) leads to $V_{Th}=V_2=48$ V. Thus we get,

V_o = 48[10/(5+10)] = **32 V**.

Solution 4.39

Obtain the Thevenin equivalent at terminals a-b of the circuit shown in Fig. 4.106.

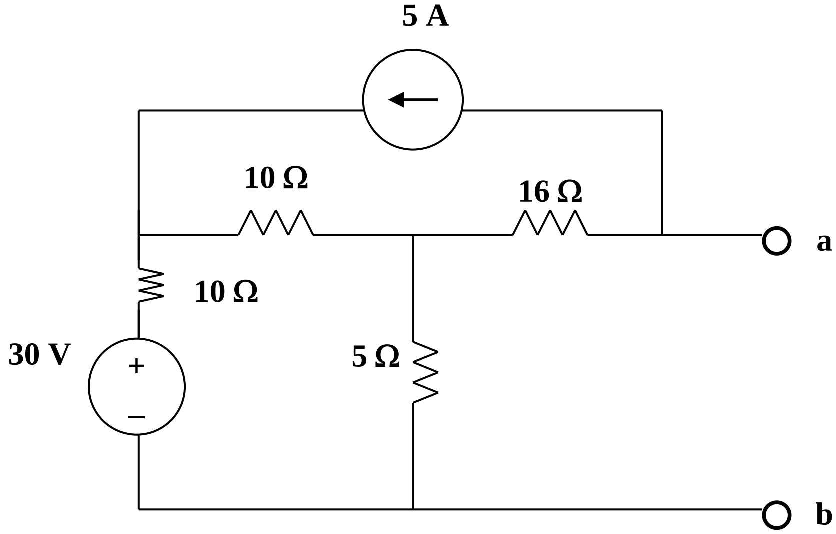
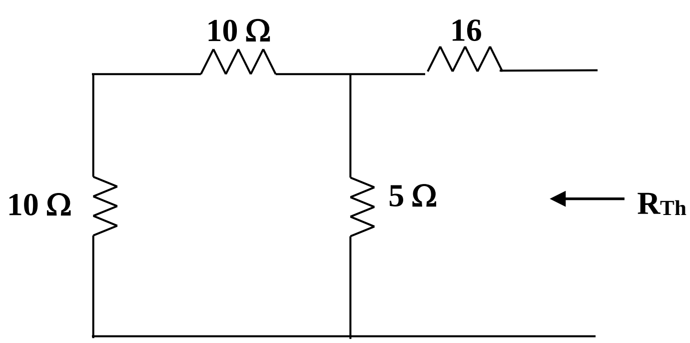


Figure 4.106
For Prob. 4.39.

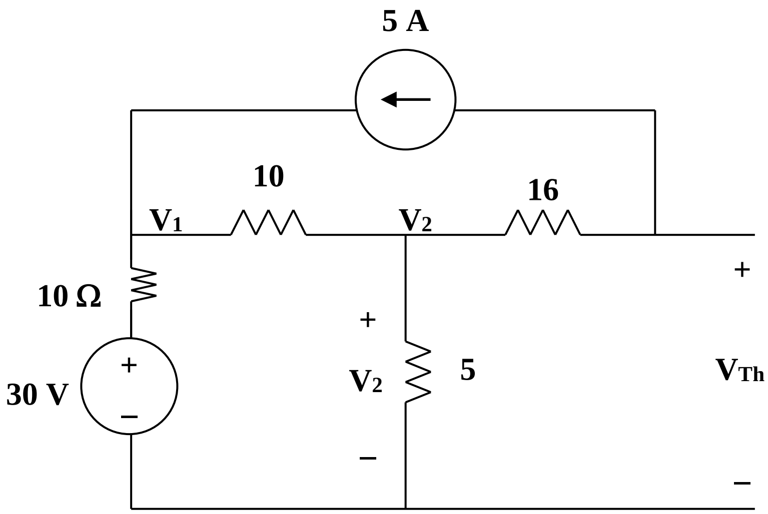
Solution

We obtain R_{Th} using the circuit below.



$$R_{Thev} = 16 + (20||5) = 16 + (20x5)/(20+5) = \mathbf{20\ \Omega}$$

To find V_{Th}, we use the circuit below.

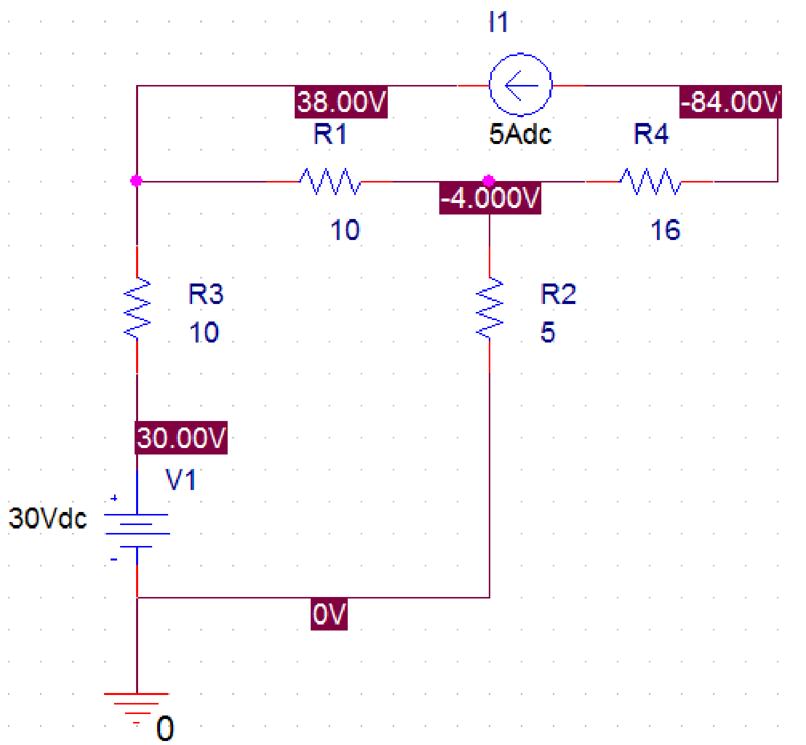


At node 1, $[(V_1–30)/10] + [(V_1–V_2)/10] – 5 = 0$ or
$[0.1+0.1]v_1 – 0.1V_2 = 8$ (1)

At node 2, $[(V_2–V_1)/10] + [(V_2–0)/5] + 5 = 0$ or
$–0.1V_1 + 0.3V_2 = –5$ (2)

Adding 3x(1) to (2) gives $(0.6–0.1)V_1 = 19$ or $V_1 = 19/0.5 = 38$ and $V_2 = (–5+0.1\text{x}38)/0.3 = –4$ V.

Finally, $V_{Th} = V_2 + (–5)16 –4–80 =$ **–84 V**. Checking with PSpice we get,



Solution 4.40

To obtain V_{Th}, we apply KVL to the loop.

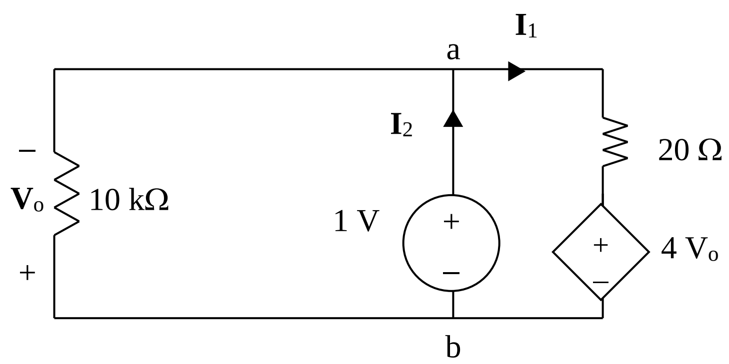
$$-70+(10+20)kI+4V_o=0$$

But $V_o=10kI$

$$70=70kI \longrightarrow I=1mA$$

$$-70+10kI+V_{Th}=0 \longrightarrow V_{Th}=\underline{60\text{ V}}$$

To find R_{Th}, we remove the 70-V source and apply a 1-V source at terminals a-b, as shown in the circuit below.



We notice that V_o = -1 V.

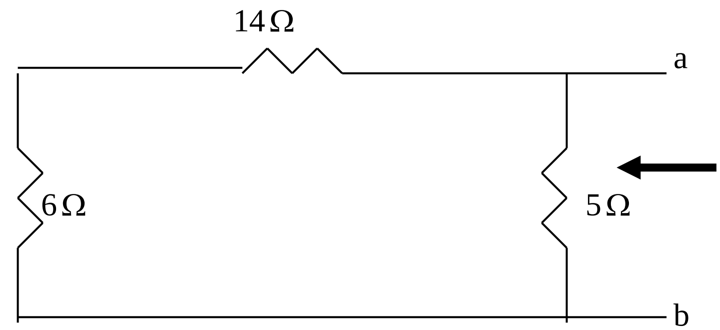
$$-1+20kI_1+4V_o=0 \longrightarrow I_1=0.25\text{ mA}$$

$$I_2=I_1+\frac{1V}{10k}=0.35\text{ mA}$$

$$R_{Th}=\frac{1V}{I_2}=\frac{1}{0.35}k\Omega=\underline{2.857\text{ k}\Omega}$$

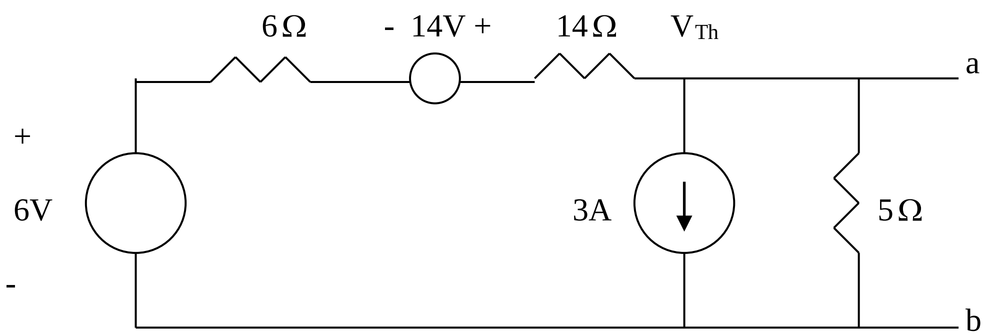
Solution 4.41

To find R_{Th}, consider the circuit below



$$R_{Th} = 5//(14+6) = 4\Omega = R_N$$

Applying source transformation to the 1-A current source, we obtain the circuit below.



At node a,

$$\frac{14+6-V_{Th}}{6+14} = 3 + \frac{V_{Th}}{5} \longrightarrow V_{Th} = -8\text{ V}$$

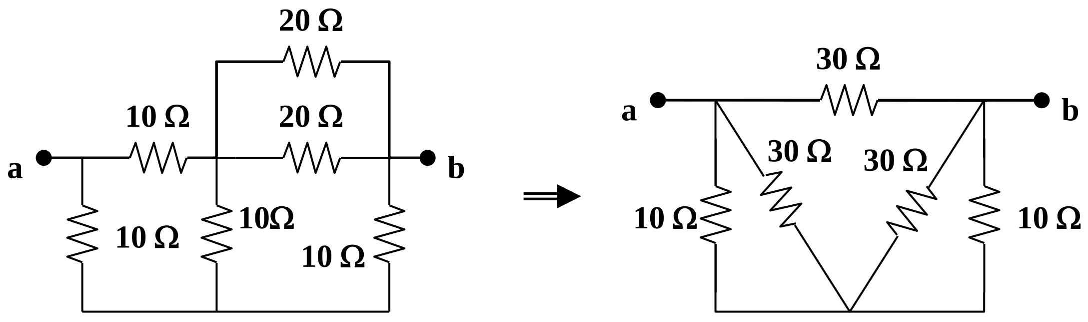
$$I_N = \frac{V_{Th}}{R_{Th}} = (-8)/4 = -2\text{ A}$$

Thus,

$$\underline{\boldsymbol{R_{Th} = R_N = 4\Omega, \quad V_{Th} = -8\text{V}, \quad I_N = -2\text{ A}}}$$

Solution 4.42

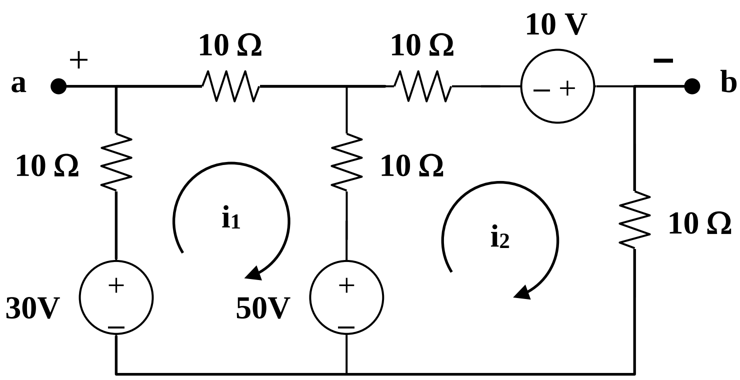
To find R_{Th}, consider the circuit in Fig. (a).



(a) **(b)**

$20||20 = 10$ ohms. Transform the wye sub-network to a delta as shown in Fig. (b).
$10||30 = 7.5$ ohms. $R_{Th} = R_{ab} = 30||(7.5 + 7.5) =$ **10 ohms**.

To find V_{Th}, we transform the 20-V (to a current source in parallel with the 20 Ω resistor and then back into a voltage source in series with the parallel combination of the two 20 Ω resistors) and the 5-A sources. We obtain the circuit shown in Fig. (c).



(c)

For loop 1, $-30 + 50 + 30i_1 - 10i_2 = 0$, or $-2 = 3i_1 - i_2$ (1)

For loop 2, $-50 - 10 + 30i_2 - 10i_1 = 0$, or $6 = -i_1 + 3i_2$ (2)

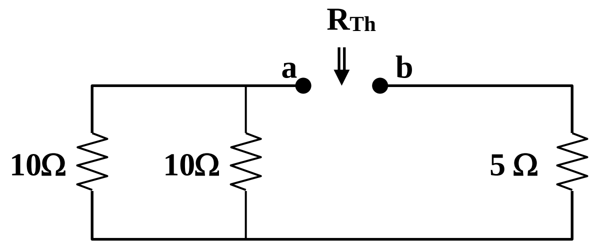
Solving (1) and (2), $i_1 = 0$, $i_2 = 2$ A

Applying KVL to the output loop, $-v_{ab} - 10i_1 + 30 - 10i_2 = 0$, $v_{ab} = 10$ V

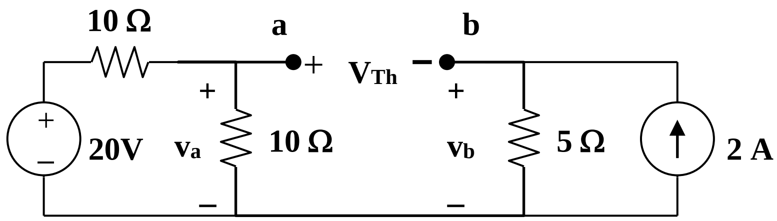
$$V_{Th} = v_{ab} = \textbf{10 volts}$$

Solution 4.43

To find R_{Th}, consider the circuit in Fig. (a).



(a)



(b)

$$R_{Th} = 10||10 + 5 = \mathbf{10\ ohms}$$

To find V_{Th}, consider the circuit in Fig. (b).

$$v_b = 2x5 = 10\text{ V},\ v_a = 20/2 = 10\text{ V}$$

But, $-v_a + V_{Th} + v_b = 0$, or $V_{Th} = v_a - v_b =$ **0 volts**

Solution 4.44

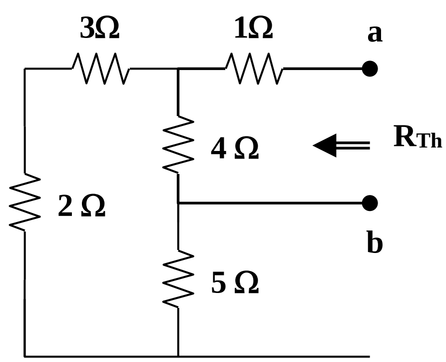
(a) For R_{Th}, consider the circuit in Fig. (a).

$$R_{Th} = 1 + 4||(3 + 2 + 5) = \mathbf{3.857\ ohms}$$

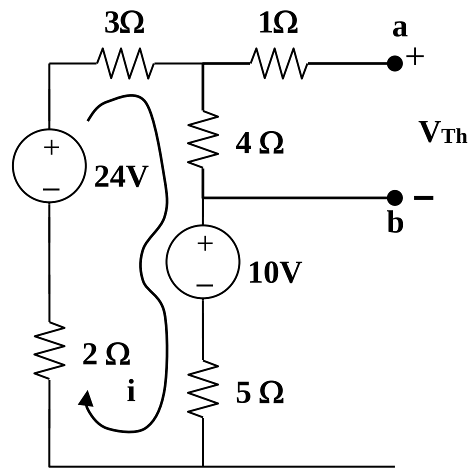
For V_{Th}, consider the circuit in Fig. (b). Applying KVL gives,

$$10 – 24 + i(3 + 4 + 5 + 2), \text{ or } i = 1$$

$$V_{Th} = 4i = \mathbf{4\ V}$$

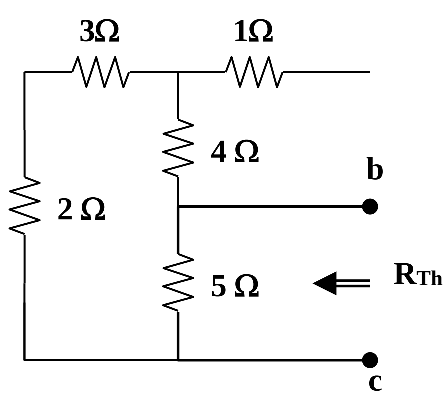


(a)

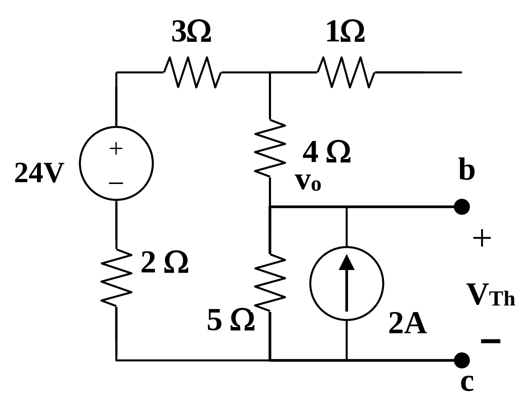


(b)

(b) For R_{Th}, consider the circuit in Fig. (c).



(c)



(d)

$$R_{Th} = 5||(2 + 3 + 4) = \mathbf{3.214\ ohms}$$

To get V_{Th}, consider the circuit in Fig. (d). At the node, KCL gives,

$$[(24 - vo)/9] + 2 = vo/5, \text{ or } vo = 15$$

$$V_{Th} = vo = \mathbf{15\ V}$$

Solution 4.45

Find the Thevenin equivalent of the circuit in Fig. 4.112 as seen by looking into terminals *a* and *b*.

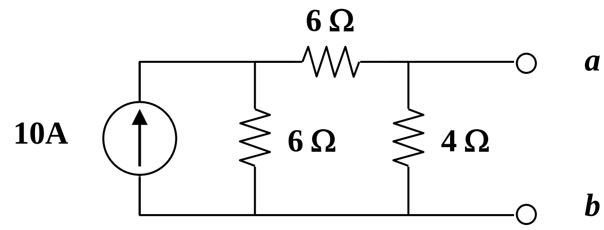
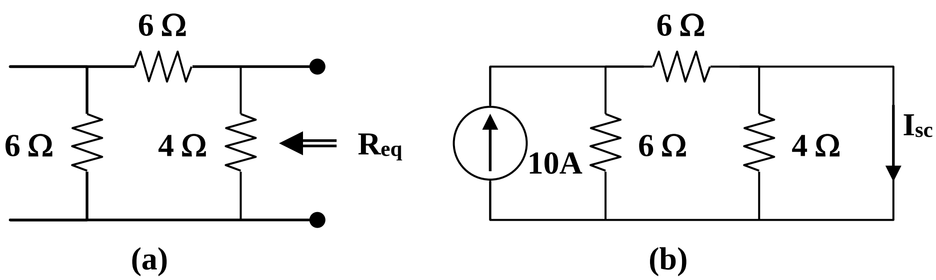


Figure 4.112
For Prob. 4.45.

Solution

For R_{eq}, consider the circuit in Fig. (a).



$$R_{eq} = (6 + 6)||4 = \mathbf{3\ \Omega}$$

For V_{Thev}, we first find I_{sc} and then $V_{Thev} = I_{sc}R_{eq}$. For I_{sc}, consider the circuit in Fig. (b). The 4-ohm resistor is shorted so that 10-A current is equally divided between the two 6-ohm resistors. Hence, $I_{sc} = 10/2 = 5$ A. Thus,

$$V_{Thev} = 5x3 = \mathbf{15\ V}.$$

Solution 4.46

Using Fig. 4.113, design a problem to help other students better understand Norton equivalent circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Norton equivalent at terminals a-b of the circuit in Fig. 4.113.

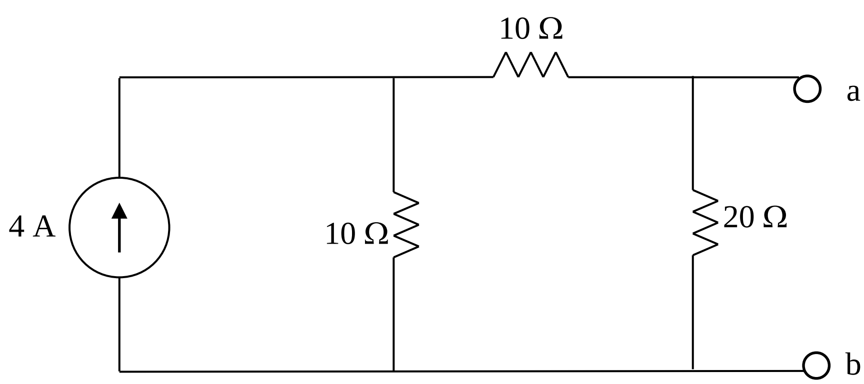
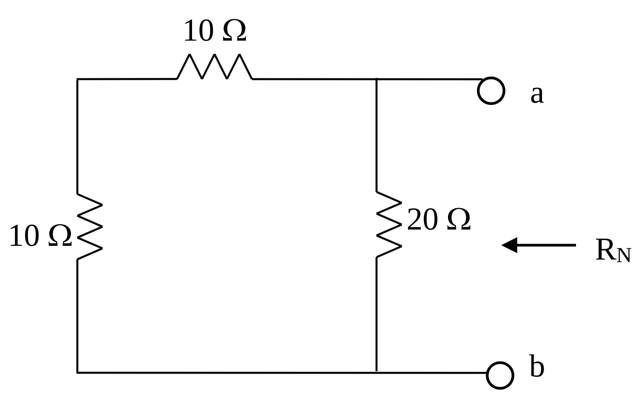


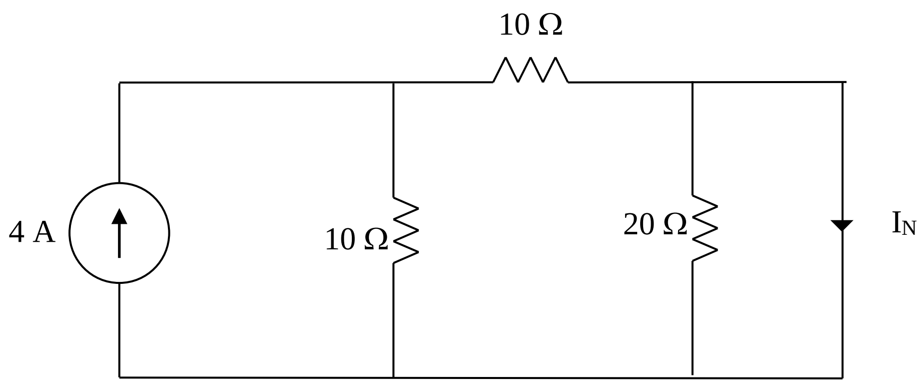
Figure 4.113 For Prob. 4.46.

Solution

R_N is found using the circuit below.



$R_N = 20//(10+10) = 10\ \Omega$
To find I_N, consider the circuit below.



The 20-Ω resistor is short-circuited and can be ignored.

$$I_N = \tfrac{1}{2} \times 4 = \mathbf{2\ A}$$

Solution 4.47

Obtain the Thevenin and Norton equivalent circuits of the circuit in Fig. 4.114 with respect to terminals *a* and *b*.

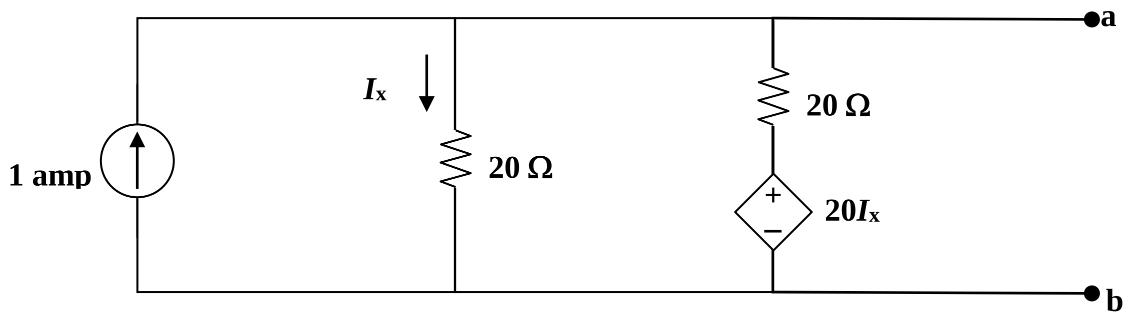


Figure 4.114
For Prob. 4.47.

Solution

Step 1. We note that there is a dependent source which means to best way to identify the equivalent circuits is to find $V_{oc} = V_{Thev}$ and $I_{sc} = I_N$ and $R_{eq} = V_{oc}/I_{sc}$.

$-1 + [(V_{oc}-0)/20] + [(V_{oc}-20I_x)/20] = 0$ and $I_x = V_{oc}/20$.

$I_{sc} = 1$ amp ($I_x = 0$ because of shorting a to b and the dependent voltage source is equal to zero because $I_x = 0$).

Step 2. $-1 + [V_{oc}/20] + [V_{oc}/20] - [V_{oc}/20] = 0$ or $V_{oc} = 20$ volts. Therefore,

$V_{Thev} =$ **20 V**, $I_N =$ **1 A**, and $R_{eq} =$ **20Ω**.

Solution 4.48

Determine the Norton equivalent at terminals ***a-b*** for the circuit in Fig. 4.115.

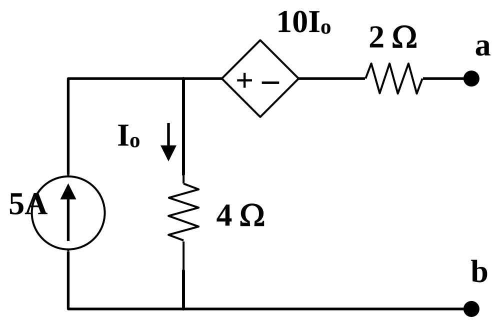
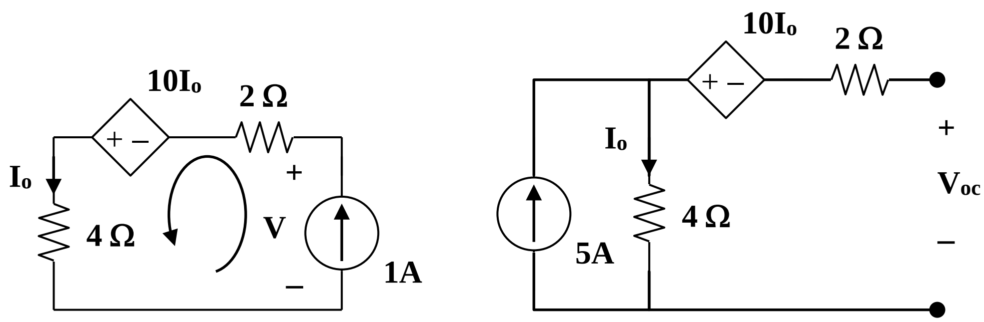


Figure 4.115
For Prob. 4.48.

Solution

To get R_{Th}, consider the circuit in Fig. (a).



(a) **(b)**

From Fig. (a), $I_o = 1,$ $6 - 10 - V = 0,$ or $V = -4$

$$R_{eq} = V/1 = \mathbf{-4\ ohms}$$

Note that the negative value of R_{eq} indicates that we have an active device in the circuit since we cannot have a negative resistance in a purely passive circuit.

To solve for I_N we first solve for V_{oc}, consider the circuit in Fig. (b),

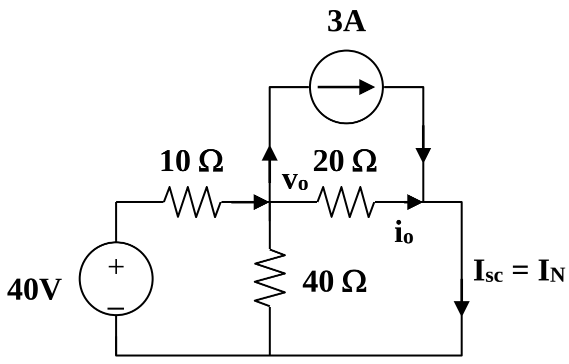
$$I_o = 5,\ V_{oc} = -10I_o + 4I_o = -30\text{ V}$$

$$I_N = V_{oc}/R_{eq} = \mathbf{7.5\ A}.$$

Solution 4.49

$R_N = R_{Th} =$ **28 ohms**

To find I_N, consider the circuit below,

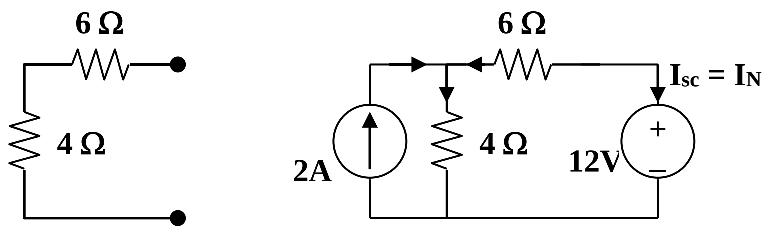


At the node, $(40 - v_o)/10 = 3 + (v_o/40) + (v_o/20)$, or $v_o = 40/7$

$i_o = v_o/20 = 2/7$, but $I_N = I_{sc} = i_o + 3 =$ **3.286 A**

Solution 4.50

From Fig. (a), $R_N = 6 + 4 =$ **10 ohms**

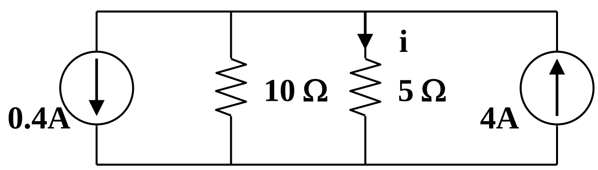


(a) (b)

From Fig. (b), $2 + (12 - v)/6 = v/4$, or $v = 9.6$ V

$-I_N = (12 - v)/6 = 0.4$, which leads to $I_N =$ **-0.4 A**

Combining the Norton equivalent with the right-hand side of the original circuit produces the circuit in Fig. (c).



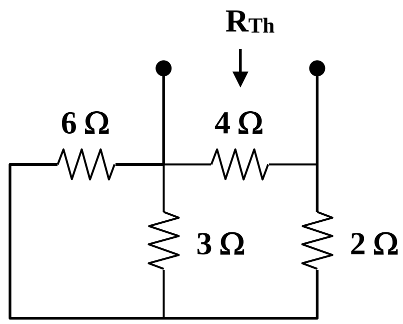
(c)

$i = [10/(10 + 5)]\,(4 - 0.4) =$ **2.4 A**

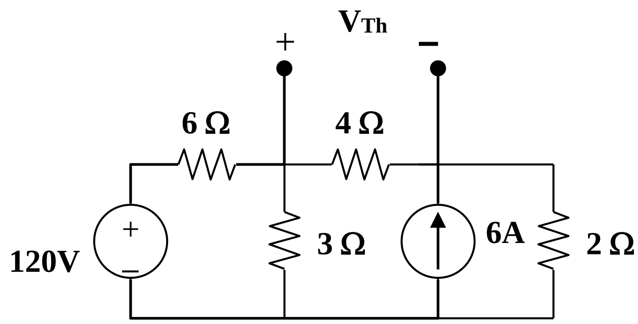
Solution 4.51

(a) From the circuit in Fig. (a),

$$R_N = 4||(2 + 6||3) = 4||4 = \mathbf{2\ ohms}$$

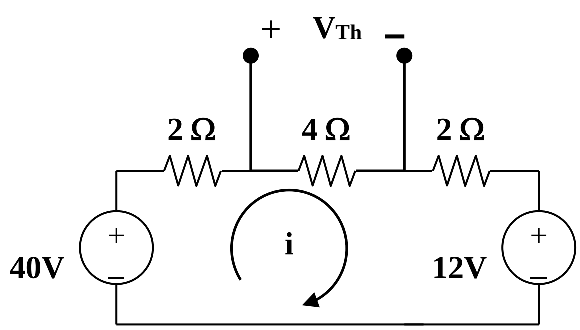


(a)



(b)

For I_N or V_{Th}, consider the circuit in Fig. (b). After some source transformations, the circuit becomes that shown in Fig. (c).



(c)

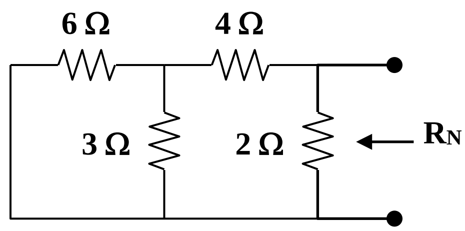
Applying KVL to the circuit in Fig. (c),

$$-40 + 8i + 12 = 0 \text{ which gives } i = 7/2$$

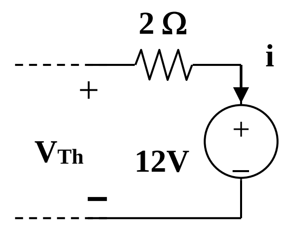
$$V_{Th} = 4i = 14 \text{ therefore } I_N = V_{Th}/R_N = 14/2 = \mathbf{7\ A}$$

(b) To get R_N, consider the circuit in Fig. (d).

$$R_N = 2||(4 + 6||3) = 2||6 = \mathbf{1.5\ ohms}$$



(d)



(e)

To get I_N, the circuit in Fig. (c) applies except that it needs slight modification as in Fig. (e).

$$i = 7/2,\ V_{Th} = 12 + 2i = 19,\ I_N = V_{Th}/R_N = 19/1.5 = \mathbf{12.667\ A}$$

Solution 4.52

For the transistor model in Fig. 4.118, obtain the Thevenin equivalent at terminals ***a-b***.

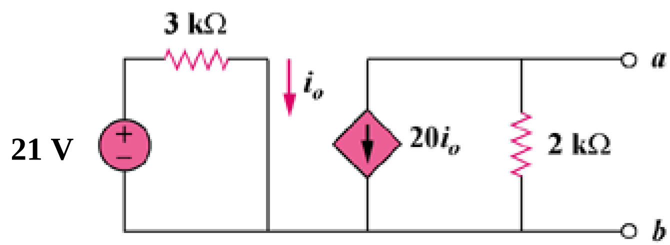
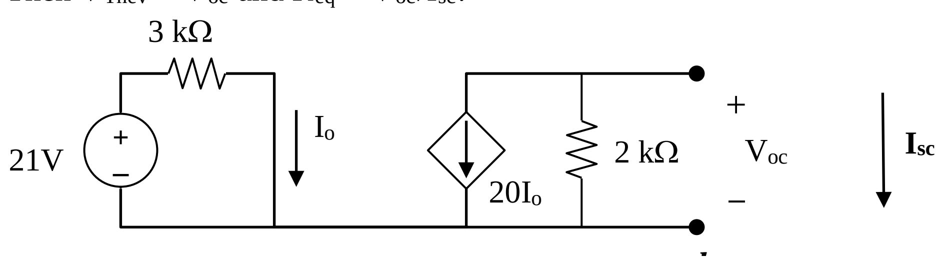


Figure 4.118
For Prob. 4.52.

Solution

Step 1. To find the Thevenin equivalent for this circuit we need to find V_{oc} and I_{sc}.

Then $V_{Thev} = V_{oc}$ and $R_{eq} = V_{oc}/I_{sc}$.



For V_{oc}, $I_o = (21–0)/3k = 7$ mA and $20I_o + (V_{oc}–0)/2k = 0$.

For I_{sc}, $I_{sc} = –20I_o$.

Step 2. $V_{oc} = –2k(20I_o) = –40x7 =$ **–280 volts** $= V_{Thev}$

$i_{sc} = –20x7x10^{–3} = –140$ mA or

$R_{eq} = –280/(–140x10^{–3}) =$ **2 kΩ**.

Solution 4.53

Find the Norton equivalent at terminals ***a-b*** of the circuit in Fig. 4.119.

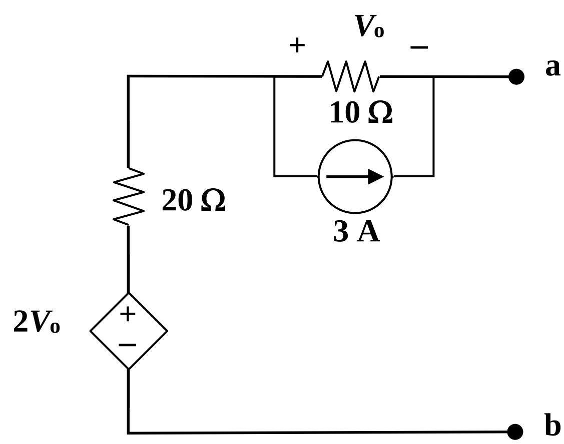


Figure 4.119
For Prob. 4.53.

Solution

Step 1. Since we have a dependent source, we need to determine $I_{sc} = I_N$ and $V_{oc} = V_{Thev}$ and $R_{eq} = V_{oc}/I_{sc}$. $V_{oc} = V_{ab} = 3(10) + 2V_o$ where $V_o = –3(10) = –30$ volts.

For I_{sc} we need to solve this mesh equation, $–2V_o + 20I_{sc} + 10(I_{sc}–3) = 0$ and $V_o = 10(I_{sc}–3)$.

Step 2. $V_{oc} = 30 – 60 = –30$ volts. $–20(I_{sc}–3) +20I_{sc} + 10I_{sc} – 30 = 0$ or

I_{sc} = **$–3$ amps**. Therefore $R_{eq} = –30/–3 =$ **10 Ω**.

Solution 4.54

To find $V_{Th} = V_x$, consider the left loop.

$$-3 + 1000i_o + 2V_x = 0 \longrightarrow 3 = 1000i_o + 2V_x \qquad (1)$$

For the right loop,

$$V_x = -50x40i_o = -2000i_o \qquad (2)$$

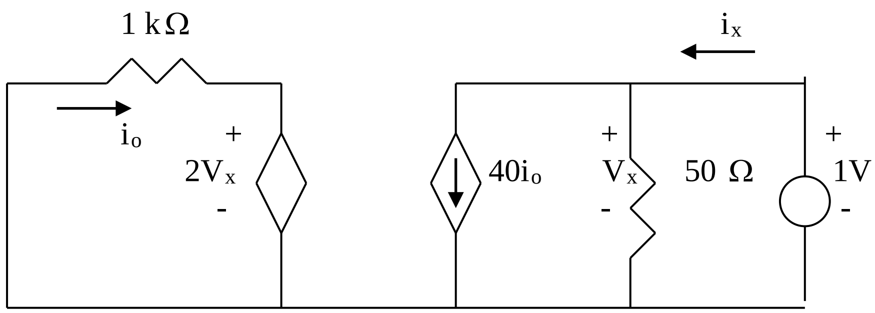
Combining (1) and (2),

$$3 = 1000i_o - 4000i_o = -3000i_o \longrightarrow i_o = -1\text{mA}$$

$$V_x = -2000i_o = 2 \longrightarrow \underline{V_{Th} = 2}$$

To find R_{Th}, insert a 1-V source at terminals a-b and remove the 3-V independent source, as shown below.

.



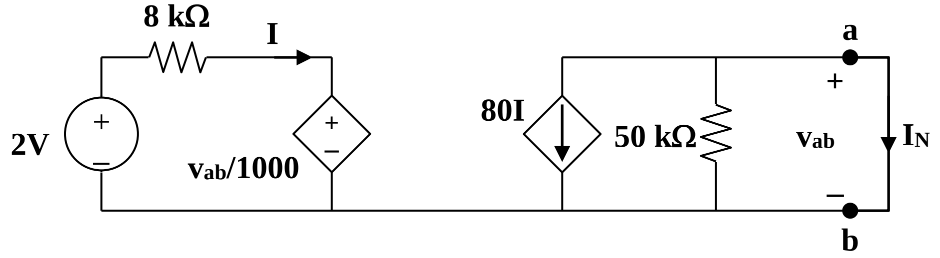
$$V_x = 1, \quad i_o = -\frac{2V_x}{1000} = -2\text{mA}$$

$$i_x = 40i_o + \frac{V_x}{50} = -80\text{mA} + \frac{1}{50}\text{A} = \text{-60mA}$$

$$R_{Th} = \frac{1}{i_x} = -1/0.060 = \underline{-16.67\Omega}$$

Solution 4.55

To get R_N, apply a 1 mA source at the terminals a and b as shown in Fig. (a).



(b)

We assume all resistances are in k ohms, all currents in mA, and all voltages in volts. At node a,

$$(v_{ab}/50) + 80I = 1 \qquad (1)$$

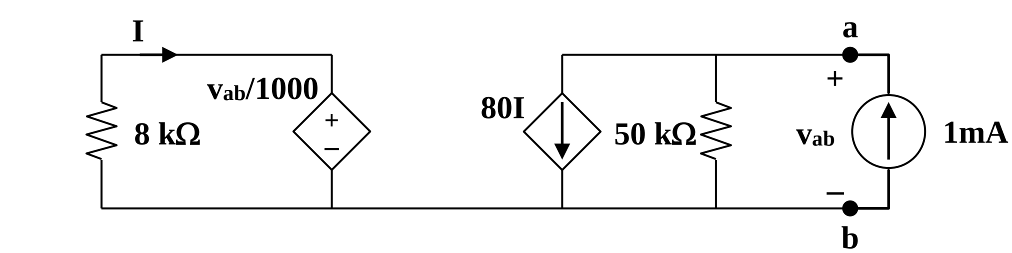
Also,

$$-8I = (v_{ab}/1000), \text{ or } I = -v_{ab}/8000 \qquad (2)$$

From (1) and (2), $(v_{ab}/50) – (80v_{ab}/8000) = 1$, or $v_{ab} = 100$

$$R_N = v_{ab}/1 = \mathbf{100\ k\ ohms}$$

To get I_N, consider the circuit in Fig. (b).



(a)

Since the 50-k ohm resistor is shorted,

$$I_N = -80I,\ v_{ab} = 0$$

Hence, $8i = 2$ which leads to $I = (1/4)$ mA

$$I_N = \mathbf{-20\ mA}$$

Solution 4.56

Use Norton's theorem to find V_o in the circuit of Fig. 4.122.

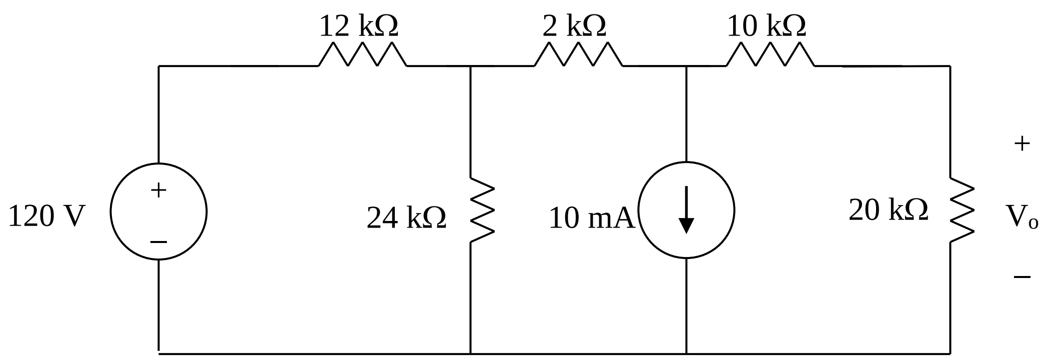
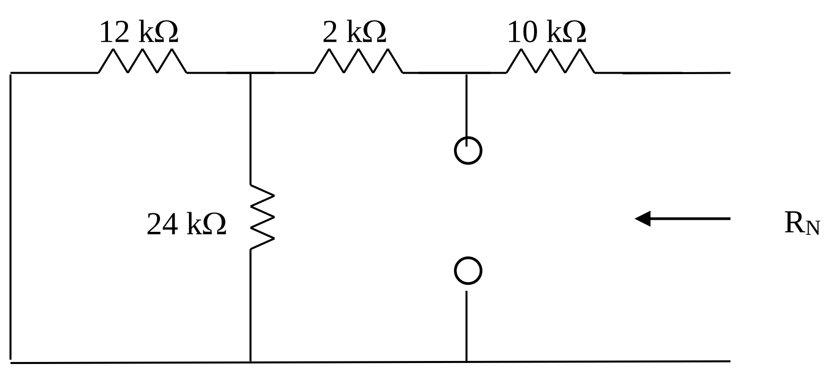


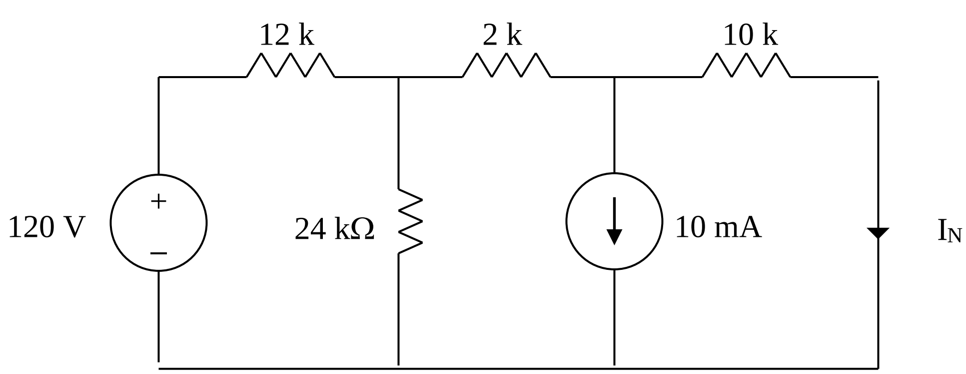
Figure 4.122
For Prob. 4.56.

Solution

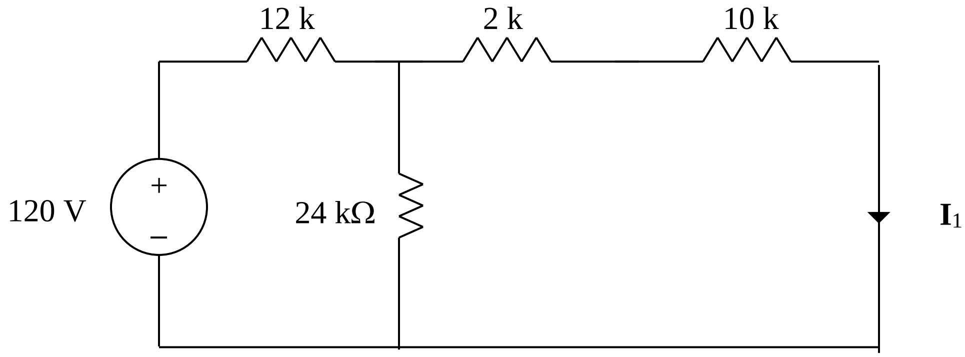
We remove the 20 kΩ resistor temporarily and find the Norton equivalent across its terminals. R_{eq} is obtained from the circuit below.



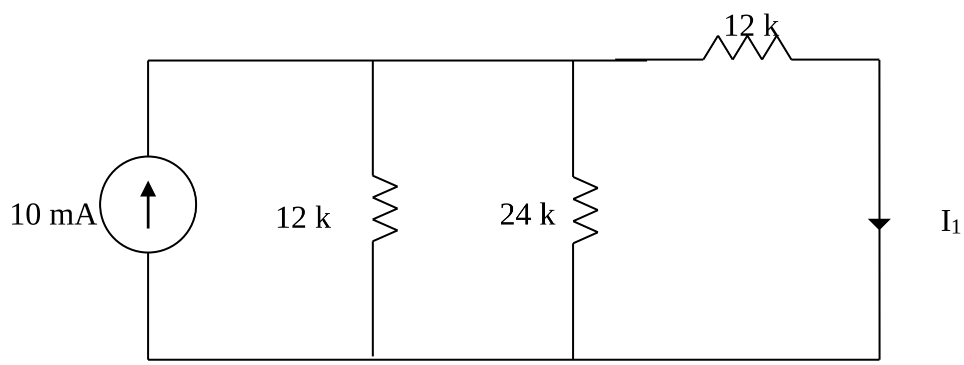
R_{eq} = 10 + 2 + (12//24) = 12+8 = 20 kΩ
I_N is obtained from the circuit below.



We can use superposition theorem to find I_N. Let $I_N = I_1 + I_2$, where I_1 and I_2 are due to 16-V and 3-mA sources respectively. We find I_1 using the circuit below.

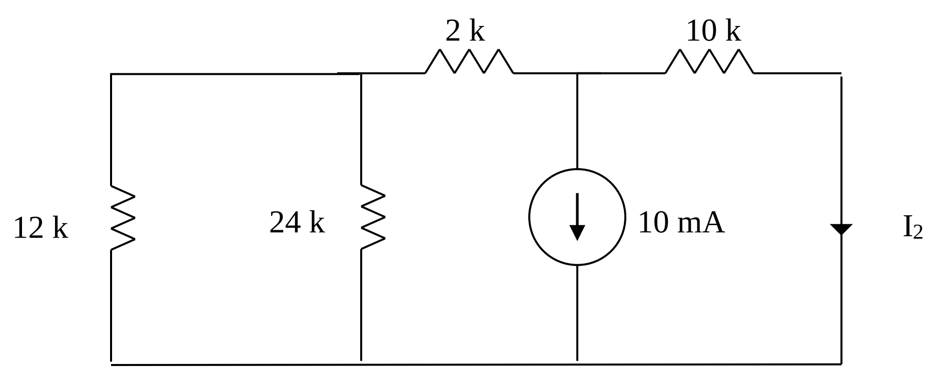


Using source transformation, we obtain the circuit below.



$12//24 = 8$ kΩ and $I_1 = [8k/(8k+12k)]0.01 = 4$ mA.

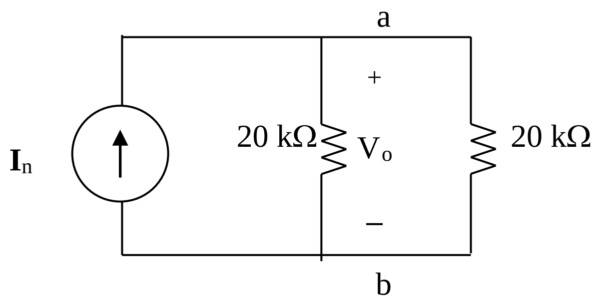
To find I_2, consider the circuit below.



$2k + 12k//24\text{ k} = 10$ kΩ
$I_2=0.5(-10\text{mA}) = -5$ mA

$I_N = 4–5 = –1$ mA

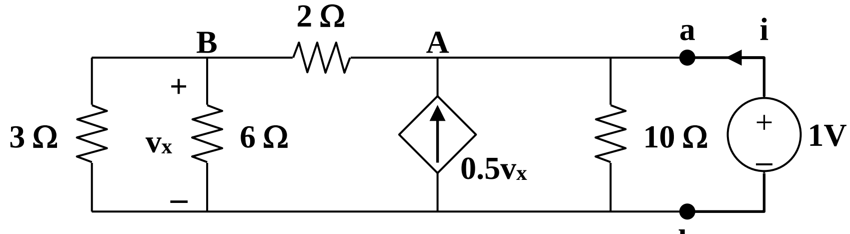
The Norton equivalent with the 20 kΩ resistor is shown below



$V_o = 20k(20k/(20k+20k))(–1\text{ mA}) = \mathbf{–10\ V}$.

Solution 4.57

To find R_{Th}, remove the 50V source and insert a 1-V source at a – b, as shown in Fig. (a).



(a)

We apply nodal analysis. At node A,

$$i + 0.5v_x = (1/10) + (1 - v_x)/2, \text{ or } i + v_x = 0.6 \qquad (1)$$

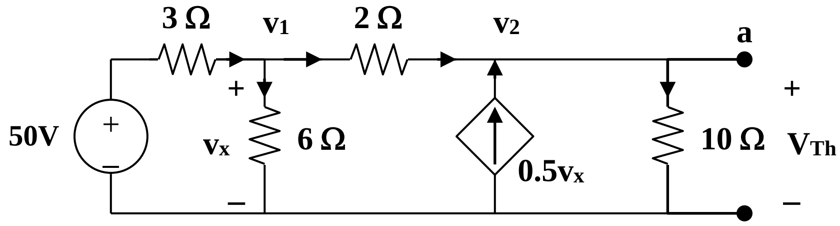
At node B,

$$(1 - v_o)/2 = (v_x/3) + (v_x/6), \text{ and } v_x = 0.5 \qquad (2)$$

From (1) and (2), $i = 0.1$ and

$$R_{Th} = 1/i = \mathbf{10\ ohms}$$

To get V_{Th}, consider the circuit in Fig. (b).



(b)

At node 1, $(50 - v_1)/3 = (v_1/6) + (v_1 - v_2)/2$, or $100 = 6v_1 - 3v_2$ (3)

At node 2, $0.5v_x + (v_1 - v_2)/2 = v_2/10$, $v_x = v_1$, and $v_1 = 0.6v_2$ (4)

From (3) and (4),

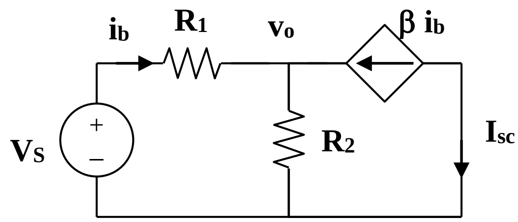
$$v_2 = V_{Th} = \mathbf{166.67\ V}$$

$$I_N = V_{Th}/R_{Th} = \mathbf{16.667\ A}$$

$$R_N = R_{Th} = \mathbf{10\ ohms}$$

Solution 4.58

This problem does not have a solution as it was originally stated. The reason for this is that the load resistor is in series with a current source which means that the only equivalent circuit that will work will be a Norton circuit where the value of R_N = **infinity**. I_N can be found by solving for I_{sc}.



Writing the node equation at node vo,

$$i_b + \beta i_b = v_o/R_2 = (1 + \beta)i_b$$

But $$i_b = (V_s - v_o)/R_1$$

$$v_o = V_s - i_b R_1$$

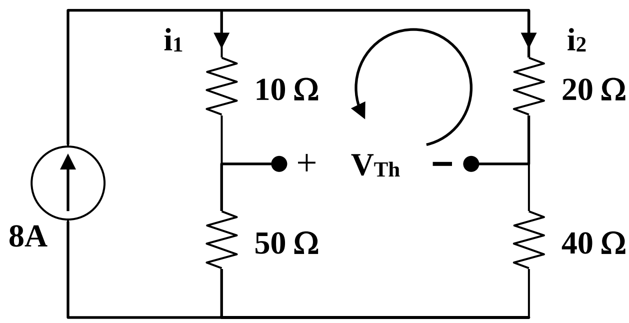
$$V_s - i_b R_1 = (1 + \beta)R_2 i_b, \text{ or } i_b = V_s/(R_1 + (1 + \beta)R_2)$$

$$I_{sc} = I_N = -\beta i_b = \mathbf{-\beta V_s/(R_1 + (1 + \beta)R_2)}$$

Solution 4.59

R_{Th} = (10 + 20)||(50 + 40) 30||90 = **22.5 ohms**

To find V_{Th}, consider the circuit below.

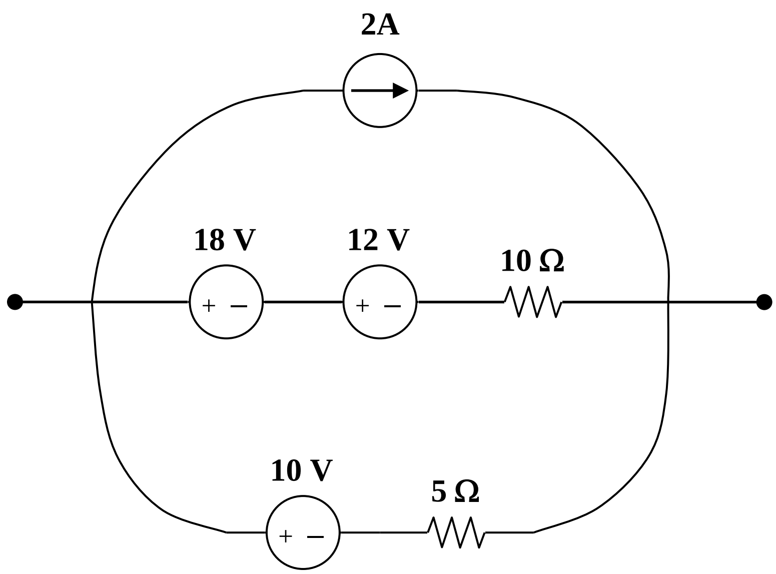


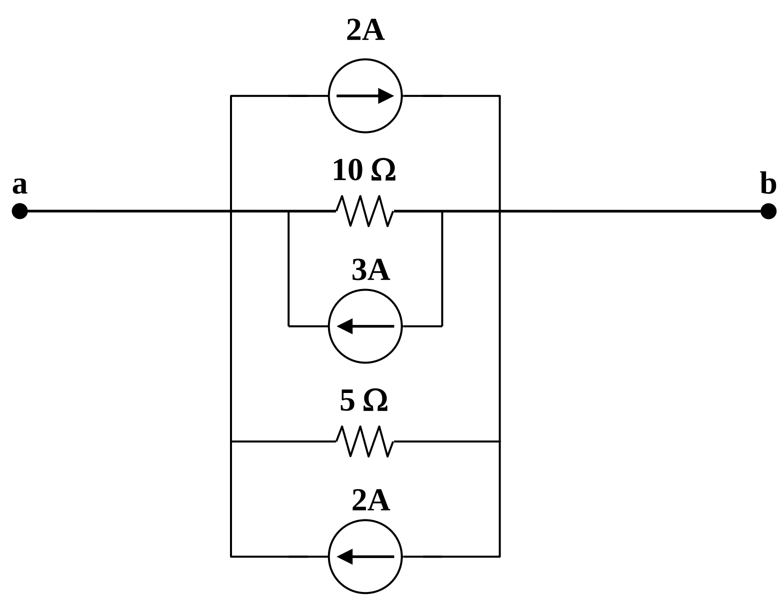
$i_1 = i_2 = 8/2 = 4$, $10i_1 + V_{Th} - 20i_2 = 0$, or $V_{Th} = 20i_2 - 10i_1 = 10i_1 = 10x4$

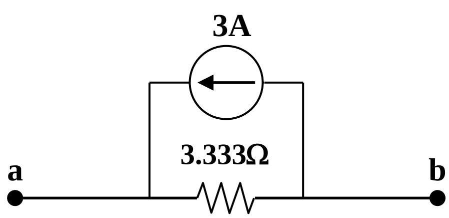
V_{Th} = **40V**, and $I_N = V_{Th}/R_{Th} = 40/22.5 =$ **1.7778 A**

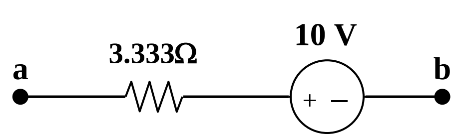
Solution 4.60

The circuit can be reduced by source transformations.









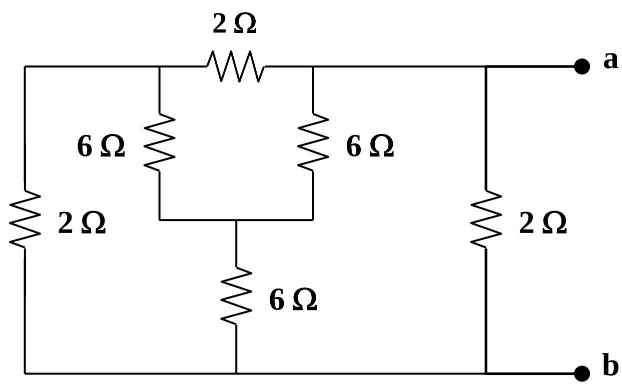
Norton Equivalent Circuit **Thevenin Equivalent Circuit**

Solution 4.61

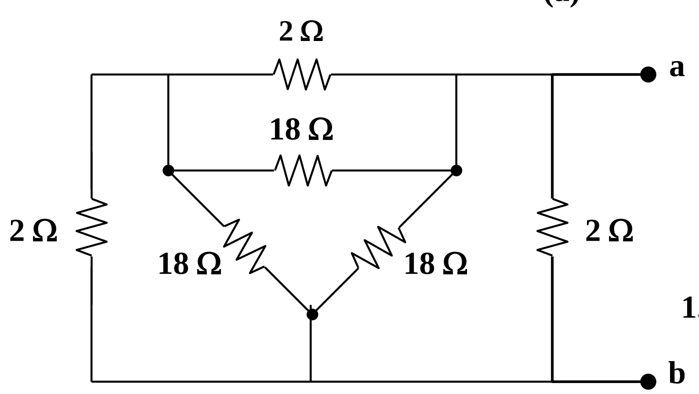
To find R_{Th}, consider the circuit in Fig. (a).

Let $R = 2||18 = 1.8$ ohms, $R_{Th} = 2R||R = (2/3)R =$ **1.2 ohms**.

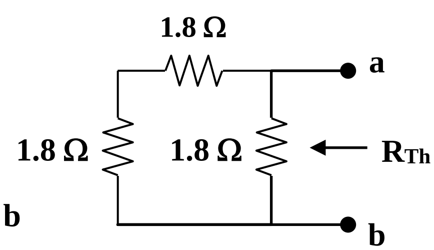
To get V_{Th}, we apply mesh analysis to the circuit in Fig. (d).



(a)

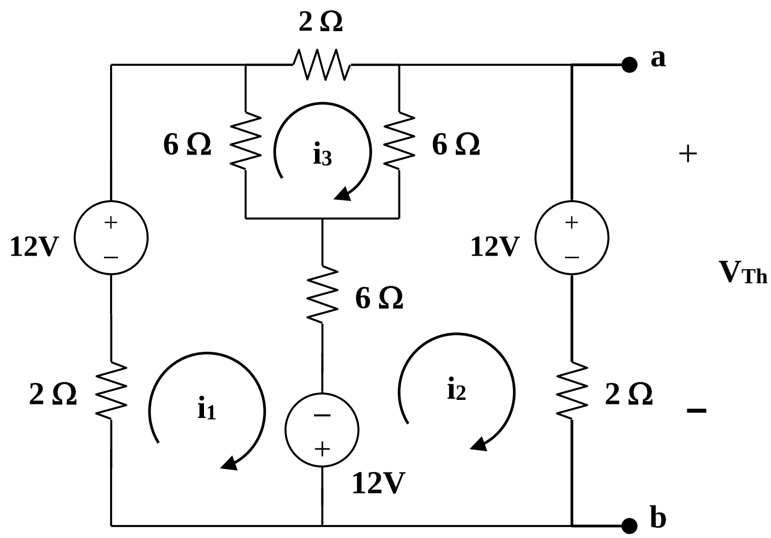


(b)



(c)

Solution continued on the next page...



(d)

$-12 - 12 + 14i_1 - 6i_2 - 6i_3 = 0$, and $7i_1 - 3i_2 - 3i_3 = 12$ (1)

$12 + 12 + 14i_2 - 6i_1 - 6i_3 = 0$, and $-3i_1 + 7i_2 - 3i_3 = -12$ (2)

$14i_3 - 6i_1 - 6i_2 = 0$, and $-3i_1 - 3i_2 + 7i_3 = 0$ (3)

This leads to the following matrix form for (1), (2) and (3),

$$\begin{bmatrix} 7 & -3 & -3 \\ -3 & 7 & -3 \\ -3 & -3 & 7 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -3 & -3 \\ -3 & 7 & -3 \\ -3 & -3 & 7 \end{vmatrix} = 100, \qquad \Delta_2 = \begin{vmatrix} 7 & 12 & -3 \\ -3 & -12 & -3 \\ -3 & 0 & 7 \end{vmatrix} = -120$$

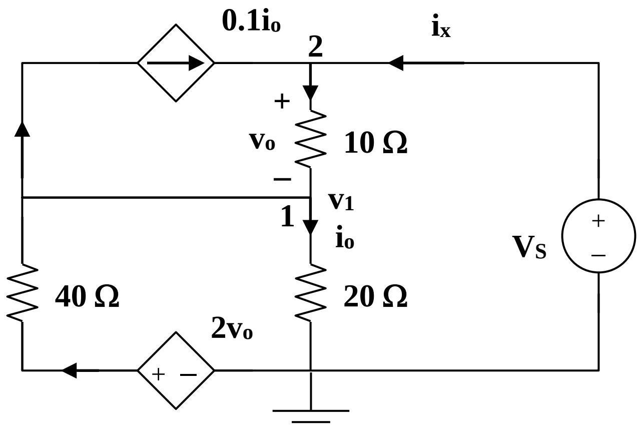
$$i_2 = \Delta/\Delta_2 = -120/100 = -1.2 \text{ A}$$

$$V_{Th} = 12 + 2i_2 = \mathbf{9.6\ V}, \text{ and } I_N = V_{Th}/R_{Th} = \mathbf{8\ A}$$

Solution 4.62

Since there are no independent sources, V_{Th} = 0 V

To obtain R_{Th}, consider the circuit below.



At node 2,

$$i_x + 0.1i_o = (1 - v_1)/10, \text{ or } 10i_x + i_o = 1 - v_1 \qquad (1)$$

At node 1,

$$(v_1/20) + 0.1i_o = [(2v_o - v_1)/40] + [(1 - v_1)/10] \qquad (2)$$

But $i_o = (v_1/20)$ and $v_o = 1 - v_1$, then (2) becomes,

$$1.1v_1/20 = [(2 - 3v_1)/40] + [(1 - v_1)/10]$$

$$2.2v_1 = 2 - 3v_1 + 4 - 4v_1 = 6 - 7v_1$$

or

$$v_1 = 6/9.2 \qquad (3)$$

From (1) and (3),

$$10i_x + v_1/20 = 1 - v_1$$

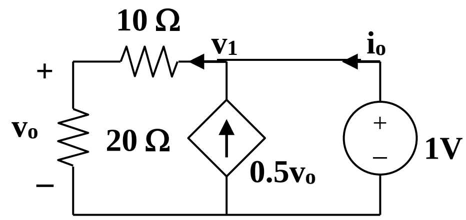
$$10i_x = 1 - v_1 - v_1/20 = 1 - (21/20)v_1 = 1 - (21/20)(6/9.2)$$

$$i_x = 31.52 \text{ mA}, \ R_{Th} = 1/i_x = \mathbf{31.73\ ohms.}$$

Solution 4.63

Because there are no independent sources, $I_N = I_{sc} =$ **0 A**

R_N can be found using the circuit below.



Applying KCL at node 1, $v_1 = 1$, and $v_o = (20/30)v_1 = 2/3$

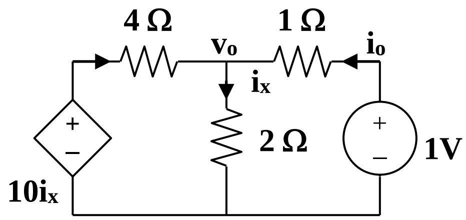
$i_o = (v_1/30) - 0.5v_o = (1/30) - 0.5x2/3 = 0.03333 - 0.33333 = -0.3$ A.

Hence,

$R_N = 1/(-0.3) =$ **–3.333 ohms**

Solution 4.64

With no independent sources, $V_{Th} = \mathbf{0\ V}$. To obtain R_{Th}, consider the circuit shown below.



$$i_x = [(1 - v_o)/1] + [(10i_x - v_o)/4], \text{ or } 5v_o = 4 + 6i_x \qquad (1)$$

But $i_x = v_o/2$. Hence,

$$5v_o = 4 + 3v_o, \text{ or } v_o = 2,\ i_o = (1 - v_o)/1 = -1$$

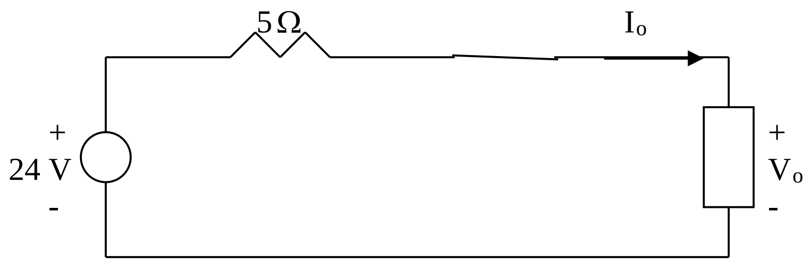
$$\text{Thus, } R_{Th} = 1/i_o = \mathbf{-1\ ohm}$$

Solution 4.65

At the terminals of the unknown resistance, we replace the circuit by its Thevenin equivalent.

$$R_{eq} = 2 + (4\,||\,12) = 2 + 3 = 5\Omega, \qquad V_{Th} = \frac{12}{12+4}(32) = 24\text{ V}$$

Thus, the circuit can be replaced by that shown below.



Applying KVL to the loop,

$$-24 + 5I_o + V_o = 0 \qquad \longrightarrow \qquad \mathbf{V_o = 24 - 5I_o.}$$

Solution 4.66

Find the maximum power that can be delivered to the resistor ***R*** in the circuit in Fig. 4.132.

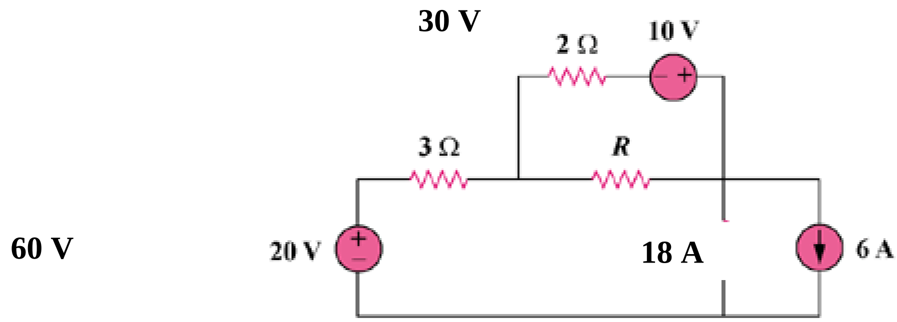
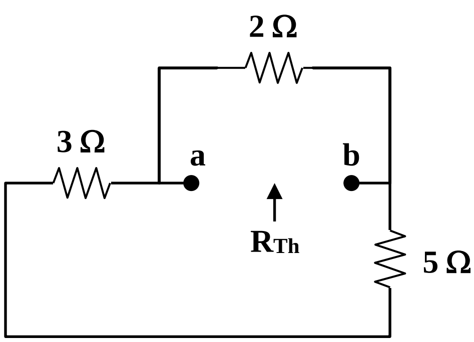


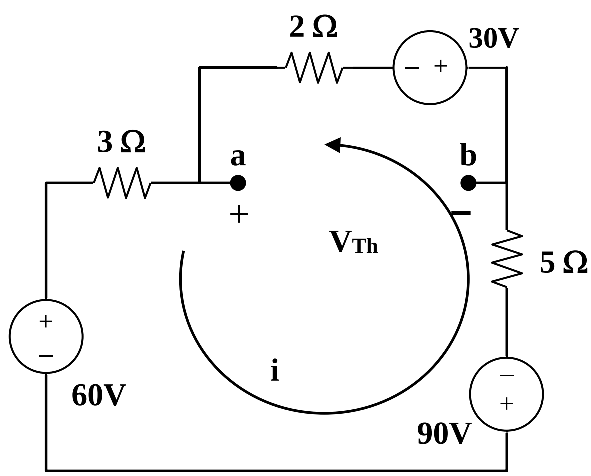
Figure 4.132
For Prob. 4.66.

Solution

We first find the Thevenin equivalent at terminals a and b. We find R_{Th} using the circuit in Fig. (a).



(a)



(b)

$$R_{Th} = 2||(3 + 5) = 2||8 = \mathbf{1.6\ \Omega}$$

By performing a source transformation on the given circuit, we obtain the circuit in (b). We now use this to find V_{Th}.

$$10i + 90 + 60 + 30 = 0, \text{ or } i = -18$$

$$V_{Th} + 30 + 2i = 0, \text{ or } V_{Th} = 6\text{ V}$$

$$p = V_{Th}^2/(4R_{Th}) = (6)^2/[4(1.6)] = \mathbf{5.625\ watts}.$$

Solution 4.67

The variable resistor R in Fig. 4.133 is adjusted until it absorbs the maximum power from the circuit. (a) Calculate the value of R for maximum power. (b) Determine the maximum power absorbed by R.

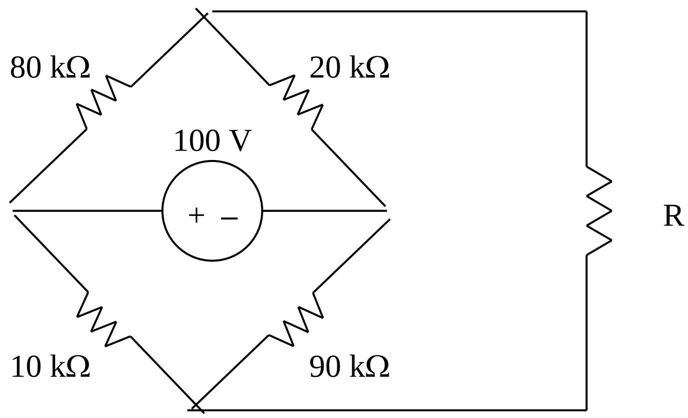
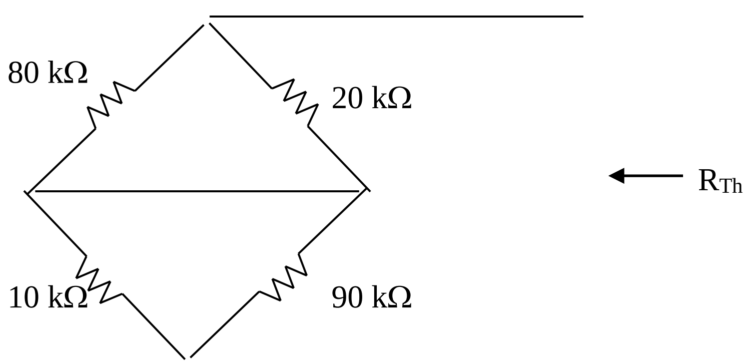


Figure 4.133
For Prob. 4.67.

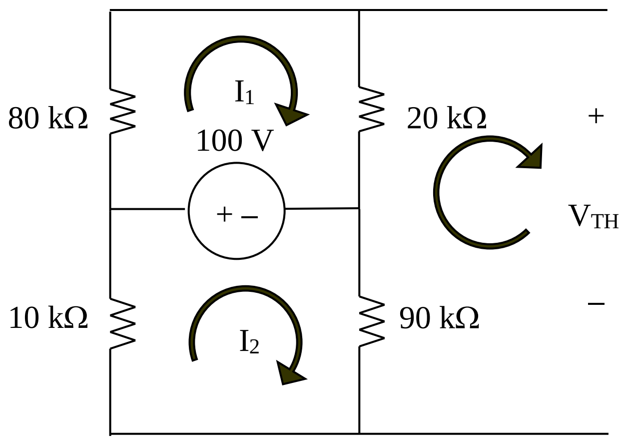
Solution

We first find the Thevenin equivalent. We find R_{Th} using the circuit below.



R_{Thev} = [80k20k/(80k+20k)] + [10k90k/(10k+90k)] = [(1600k/100)+(900k/100)]
= 16k+9k = 25 kΩ.

We find V_{Th} using the circuit below. We apply mesh analysis.



Loop 1, $-100 + (80k+20k)I_1 = 0$ or $I_1 = 100/100k = 1$ mA.

Loop 2, $(10k+90k)I_2 + 100 = 0$ or $I_2 = -100/100k = -1$ mA.

Finally, $V_{Thev} = 20k(0.001) + 90k(-0.001) = 20-90 = -70$ V.

(a) $R = R_{Th} =$ **25 kΩ**

(b) $P_{max} = (V_{Thev})^2/(4R_{Thev}) = (-70)^2/(4x25k) =$ **49 mW**.

Solution 4.68

Consider the 30 Ω resistor. First compute the Thevenin equivalent circuit as seen by the 30 Ω resistor. Compute the value of ***R*** that results in Thevenin equivalent resistance equal to the 30 Ω resistance and then calculate power delivered to the 30 Ω resistor. Now let R = 0 Ω, 110 Ω, and ∞, calculate the power delivered to the 30 Ω resistor in each case. What can you say about the value of R that will result in the maximum power that can be delivered to the 30 Ω resistor?

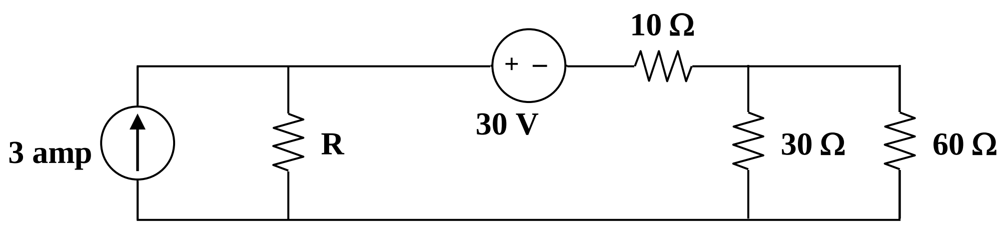
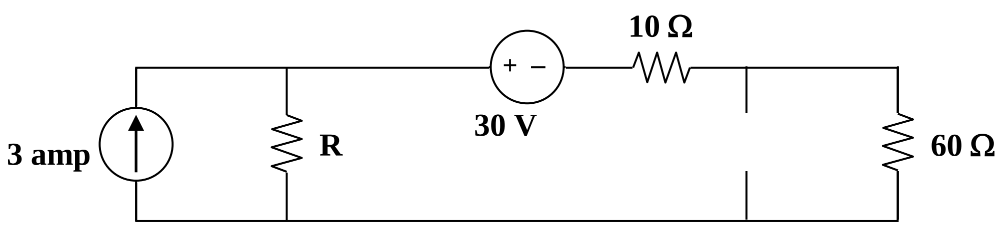


Figure 4.134
For Prob. 4.68.

Step 1. The first thing we need to do is to solve for V_{Thev} and R_{eq} as seen by the 30 ohm resistor.



We can replace the current source and parallel R into a voltage source of 3R in series with R. Now we can find the open circuit voltage equal to 60[(3R–30)/(R+10+60)]. Since we do not have any dependent sources we can calculate R_{eq} = 60(R+10)/(R+10+60). Finally the power delivered to the 30 ohm resistor is equal to $[V_{oc}/(R_{eq}+30)]^2(30)$. We also need to know the value of R that makes R_{eq} = 30 or 30 = [(60R+600)/(R+70)] or (R+70) = 2R+20 or R = **50 Ω**.

Step 2. Let us look at the Thevenin equivalent resistance first. When R = 0, R_{eq} = 600/70 = 8.571 ohms, when R = 110, R_{eq} = 40 Ω, and when R = ∞, R_{eq} = 60 Ω. Now look at the Thevenin voltage. When R = 0 we get V_{Thev} = –60x30/70 = –25.71 volts, when R = 50 Ω we get V_{Thev} = 60 volts, when R = 110 Ω, V_{Thev} = 100 volts, and when R = ∞ V_{Thev} = 180 volts.

Now we can calculate the values of power, R = 0 we get $P_{30} = [–25.71/38.571]^2 30$ = **13.329 watts**, when R = 50 we get $P_{30} = [60/60]^2 30$ = **30 watts**, $P_{30} = [100/70]^2 30$ = **61.22 watts**, and when R = ∞ $P_{30} = [180/90]^2 30$ = **120 watts**. It would appear that the value of R that causes the maximum power to be delivered to the 30 Ω resistor in R = ∞.

Solution 4.69

Find the maximum power transferred to resistor ***R*** in the circuit of Fig. 4.135.

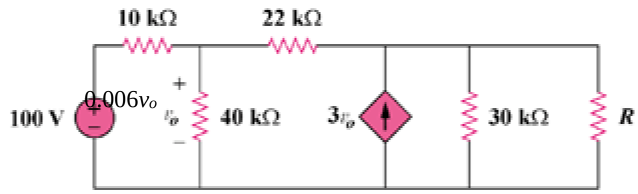
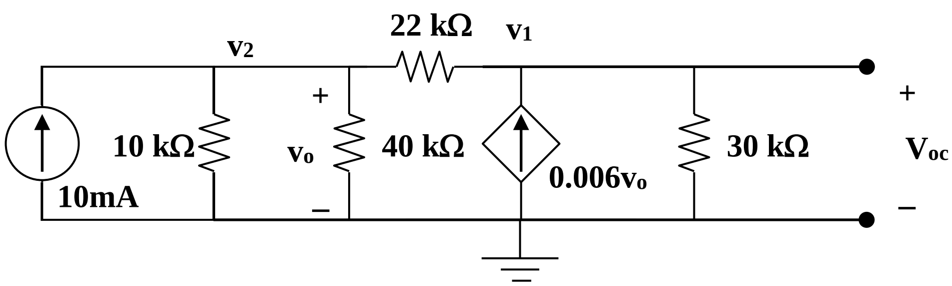


Figure 4.135
For Prob. 4.69.

Solution

First we need the Thevenin equivalent seen by the resistor R. To find the Thevenin equivalent circuit we only need to find V_{oc} and I_{sc}.



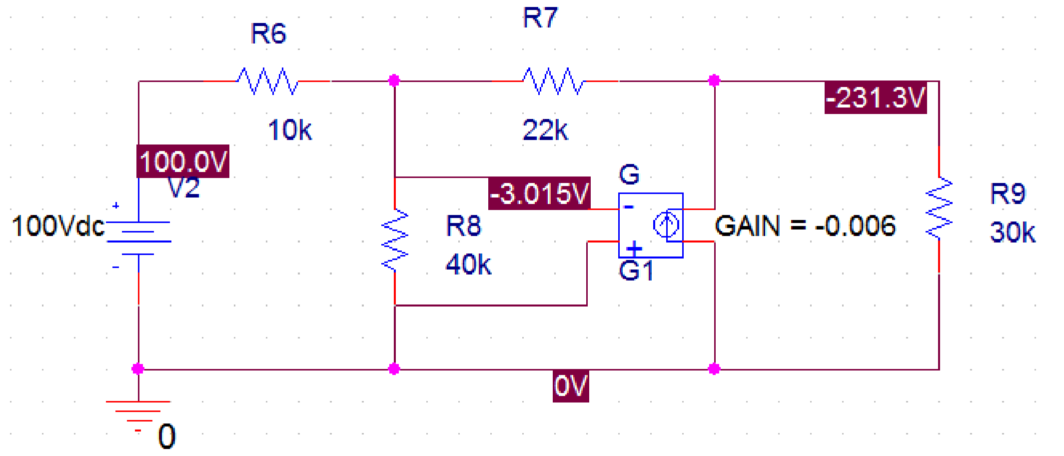
Now we have $[(v_1–v_2)/22k] – 0.006v_2 + [(v_1–0)/30k] = 0$ and
$–0.01 + [(v_2–0)/10k] + [(v_2–0)/40k] + [(v_2–v_1)/22k] = 0$ which leads to
$[0.0000454545+0.006]v_2 = 0.00604545v_2 = (0.0454545+0.03333)v_1/1000$ or
$v_2 = [0.0787878/0.00604545]v_1/1000 = 0.0130326v_1$. Finally,
$\{[(0.1+0.025+0.0454545)(0.0130326)/1000]–[0.0454545/1000]\}v_1 =$
$\{[(0.1704545)(0.0130326)–0.0454545]/1000\}v_1 = 0.01$ or v_1
$= 10/(–0.043233) = –231.3$ V.

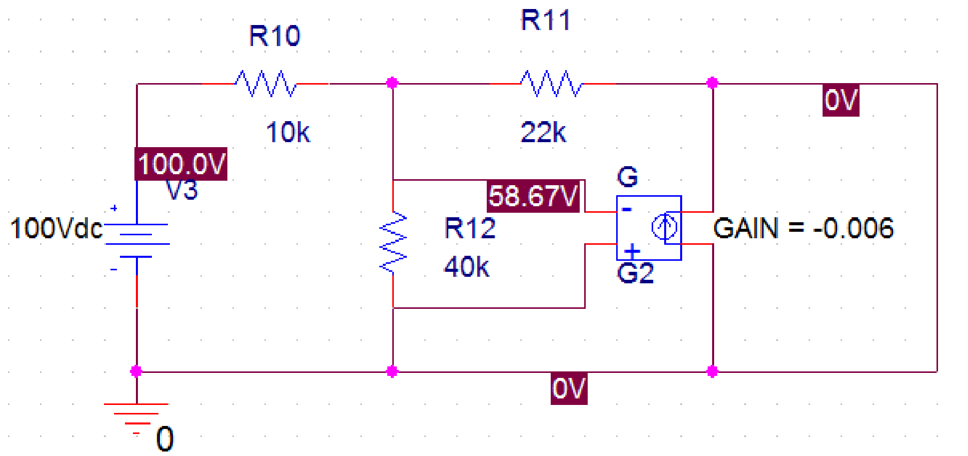
To determine I_{sc} we set $v_1 = 0$ and find the current through the short. We have
$–0.01 + [(v_2–0)/10k] + [(v_2–0/40k] + [(v_2–0)/22k] = 0$ or
$[(0.1+0.025+0.0454545)/1000]v_2 = 0.01$ or $v_2 = 10/0.1704545 = 58.667$ V.

Now we can find $I_{sc} = [(v_2–0)/22k] + 0.006(58.667) = [(58.667)/22k] + 0.006(58.667) =$
$0.0026667+0.352 = 0.35467$ A or
$R_{Thev} = –231.3/0.35467 = 16.9443/0.751414 =$ **–652.2 Ω**.

This is now an interesting problem since the equivalent resistance is negative. Obviously the correct answer is let R = 652.2 Ω which then means the current through R is infinite and the power delivered to R is infinity. The negative resistance for the equivalent circuit means that both the source and resistance effectively deliver power to the load. Please note that a negative equivalent resistance indicates that we have a dependent source.

We can check the voltages by using PSpice and we get,





Solution 4.70

Determine the maximum power delivered to the variable resistor *R* shown in the circuit of Fig. 4.136.

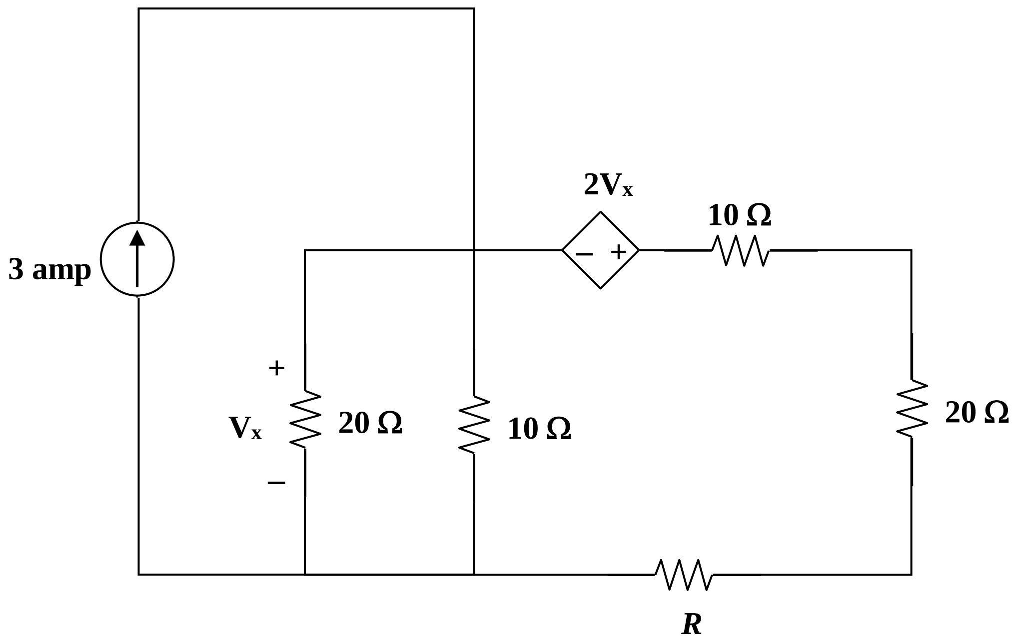
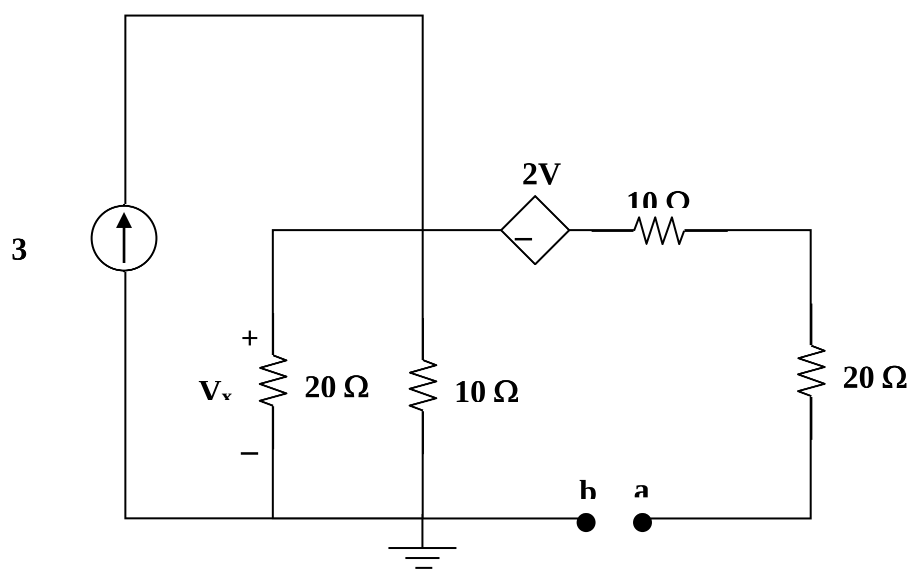


Figure 4.136
For Prob. 4.70.

Step 1. We need to start with finding the Thevenin equivalent circuit looking into the terminals connected to the resistor R. Once we find the equivalent circuit we know that the maximum power will be delivered to R when R = R_{eq}. To find the Thevenin equivalent we need to find V_{oc} and I_{sc} (see the circuit below).



Let $V_{oc} = V_{ab}$ and let $I_{sc} = I_{ab}$. We then need to analyze the separate circuits.

To solve for V_{oc}, $V_{ab} = 2V_x + V_x = 3V_x$. To solve for V_x we solve this nodal equation $-3 + [(V_x-0)/20] + [(V_x-0)/10] = 0$.

To solve for I_{sc} we know that $I_{ab} = [(V_x+2V_x-0)/(10+20)] =[(3V_x)/30] = V_x/10$ again we solve this nodal equation to find V_x, $-3 + [(V_x-0)/20] + [(V_x-0)/10] + [(3V_x-0)/30] = 0$.

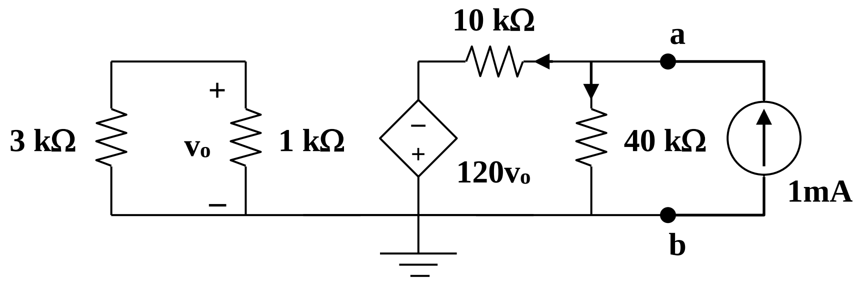
Step 2. For V_{oc}, $[(1/20)+(1/10)]V_x = 3$ or $V_x = 3(20/3) = 20$ volts which leads to $V_{oc} = 3V_x = 60$ volts $= V_{Thev}$.

For I_{sc} $[(1/20)+(1/10)+(1/10)]V_x = (5/20)V_x = 3$ or $V_x = 12$ volts. Therefore, $I_{sc} = V_x/10 = 1.2$ amps. Finally, $R_{eq} = R = 60/1.2 = 50\ \Omega$.

Now, $P_R = (60/100)^2 50 =$ **18 watts**.

Solution 4.71

We need R_{Th} and V_{Th} at terminals a and b. To find R_{Th}, we insert a 1-mA source at the terminals a and b as shown below.



Assume that all resistances are in k ohms, all currents are in mA, and all voltages are in volts. At node a,

$$1 = (v_a/40) + [(v_a + 120v_o)/10], \text{ or } 40 = 5v_a + 480v_o \qquad (1)$$

The loop on the left side has no voltage source. Hence, $v_o = 0$. From (1), $v_a = 8$ V.

$$R_{Th} = v_a/1 \text{ mA} = 8 \text{ kohms}$$

To get V_{Th}, consider the original circuit. For the left loop,

$$v_o = (1/4)8 = 2 \text{ V}$$

For the right loop, $v_R = V_{Th} = (40/50)(-120v_o) = -192$

The resistance at the required resistor is

$$R = R_{Th} = \mathbf{8\ k\Omega}$$

$$p = V_{Th}^2/(4R_{Th}) = (-192)^2/(4x8x10^3) = \mathbf{1.152\ watts}$$

Solution 4.72

(a) For the circuit in Fig. 4.138, obtain the Thevenin equivalent at terminals ***a-b***.
(b) Calculate the current in $R_L = 13\ \Omega$.
(c) Find R_L for maximum power deliverable to R_L.
(d) Determine that maximum power.

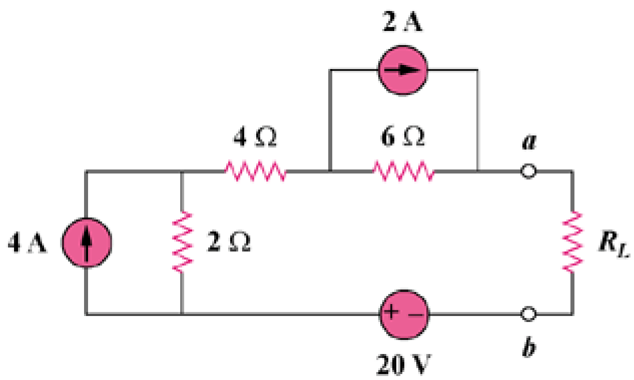


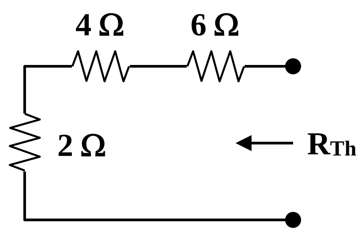
Figure 4.138
For Prob. 4.72.

Solution

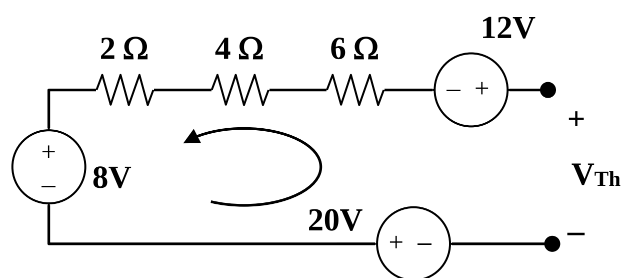
(a) R_{Th} and V_{Th} are calculated using the circuits shown in Fig. (a) and (b) respectively.

From Fig. (a), $R_{Th} = 2 + 4 + 6 =$ **12 ohms**

From Fig. (b), $-V_{Th} + 12 + 8 + 20 = 0$, or $V_{Th} =$ **40 V**



(a)



(b)

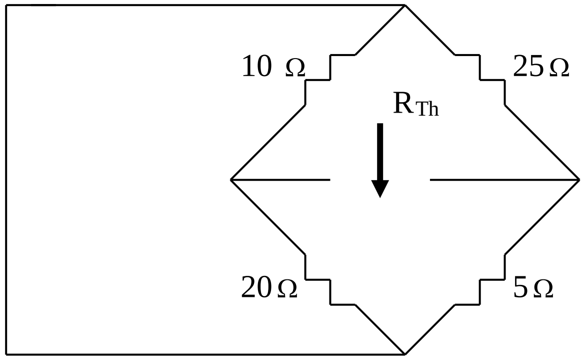
(b) $i = V_{Th}/(R_{Th} + R) = 40/(12 + 13) =$ **1.6 A**

(c) For maximum power transfer, $R_L = R_{Th} =$ **12 ohms**

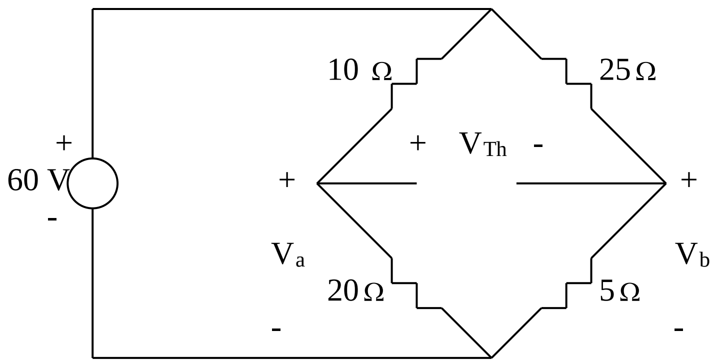
(d) $p = V_{Th}^2/(4R_{Th}) = (40)^2/(4\text{x}12) =$ **33.33 watts**.

Solution 4.73

Find the Thevenin's equivalent circuit across the terminals of R.



$$R_{Th} = 10//20 + 25//5 = 325/30 = 10.833\Omega$$



$$V_a = \frac{20}{30}(60) = 40, \qquad V_b = \frac{5}{30}(60) = 10$$

$$-V_a + V_{Th} + V_b = 0 \quad \longrightarrow \quad V_{Th} = V_a - V_b = 40 - 10 = 30 \text{ V}$$

$$\boldsymbol{p_{\max} = \frac{V_{Th}^{\ 2}}{4R_{Th}} = \frac{30^2}{4x10.833} = 20.77 \text{ W}}.$$

Solution 4.74

When R_L is removed and V_s is short-circuited,

$$R_{Th} = R_1 \| R_2 + R_3 \| R_4 = [R_1 R_2/(R_1 + R_2)] + [R_3 R_4/(R_3 + R_4)]$$

$$R_L = R_{Th} = \mathbf{(R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4)/[(R_1 + R_2)(R_3 + R_4)]}$$

When R_L is removed and we apply the voltage division principle,

$$V_{oc} = V_{Th} = v_{R2} - v_{R4}$$

$$= ([R_2/(R_1 + R_2)] - [R_4/(R_3 + R_4)])V_s = \{[(R_2R_3) - (R_1R_4)]/[(R_1 + R_2)(R_3 + R_4)]\}V_s$$

$$p_{max} = V_{Th}^2/(4R_{Th})$$

$$= \{[(R_2R_3) - (R_1R_4)]^2/[(R_1 + R_2)(R_3 + R_4)]^2\}V_s^2[(R_1 + R_2)(R_3 + R_4)]/[4(a)]$$

$$\text{where } a = (R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4)$$

$$p_{max} =$$

$$\mathbf{[(R_2R_3) - (R_1R_4)]^2V_s^2/[4(R_1 + R_2)(R_3 + R_4)(R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4)]}$$

Solution 4.75

For the circuit in Fig. 4.141, determine the value of ***R*** such that the maximum power delivered to the load is 12 mW.

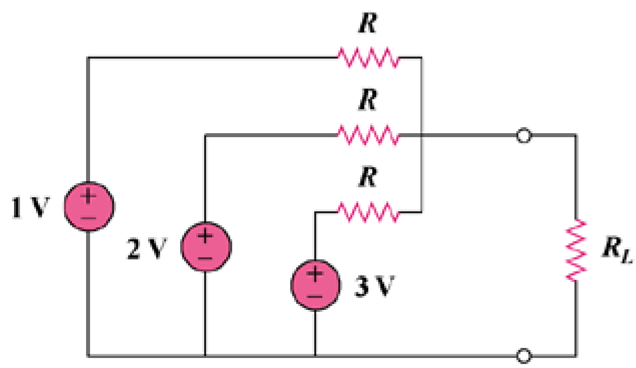
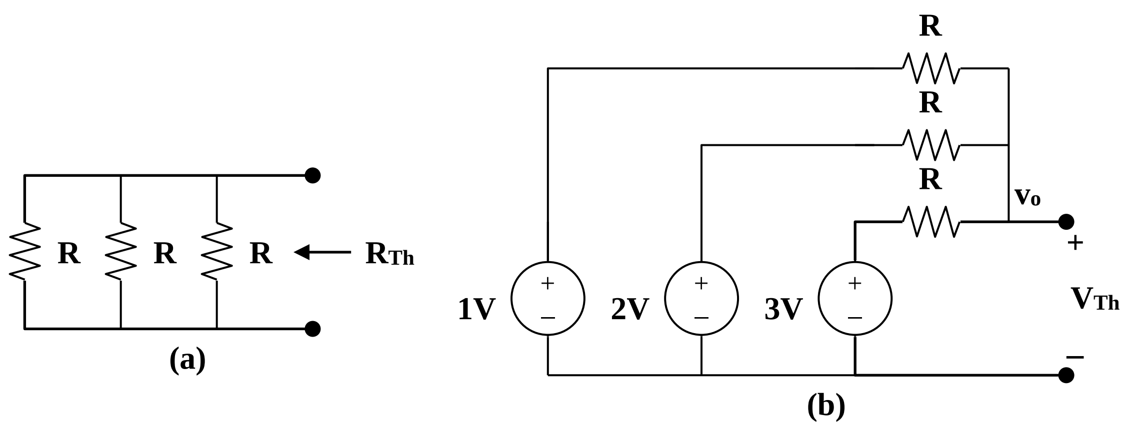


Figure 4.141
For Prob. 4.75.

Solution

We need to first find R_{Th} and V_{Th}.



Consider the circuit in Fig. (a).

$$(1/R_{eq}) = (1/R) + (1/R) + (1/R) = 3/R$$

$$R_{eq} = R/3$$

From the circuit in Fig. (b),

$$((1 - v_o)/R) + ((2 - v_o)/R) + ((3 - v_o)/R) = 0$$

$$v_o = 2 = V_{Th}$$

For maximum power transfer,

$$R_L = R_{Th} = R/3$$

$$P_{max} = [(V_{Th})^2/(4R_{Th})] = \mathbf{12\ mW}$$

$$R_{Th} = [(V_{Th})^2/(4P_{max})] = 4/(4xP_{max}) = 1/P_{max} = R/3$$

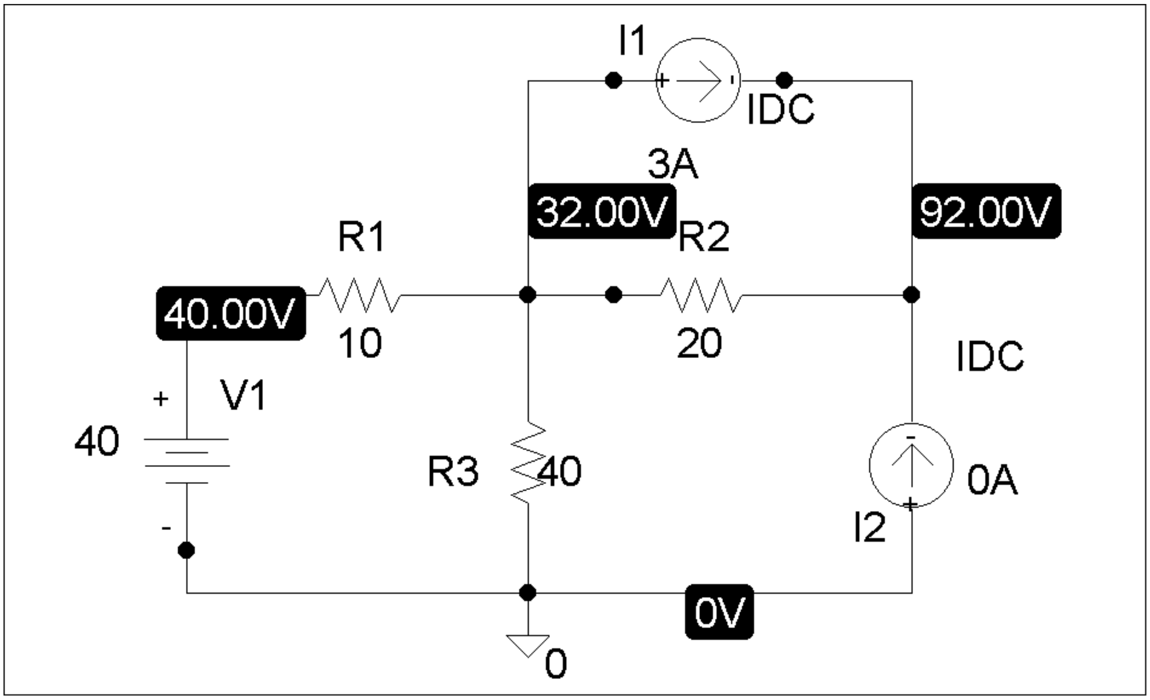
$$R = 3/(0.012) = \mathbf{250\ \Omega}.$$

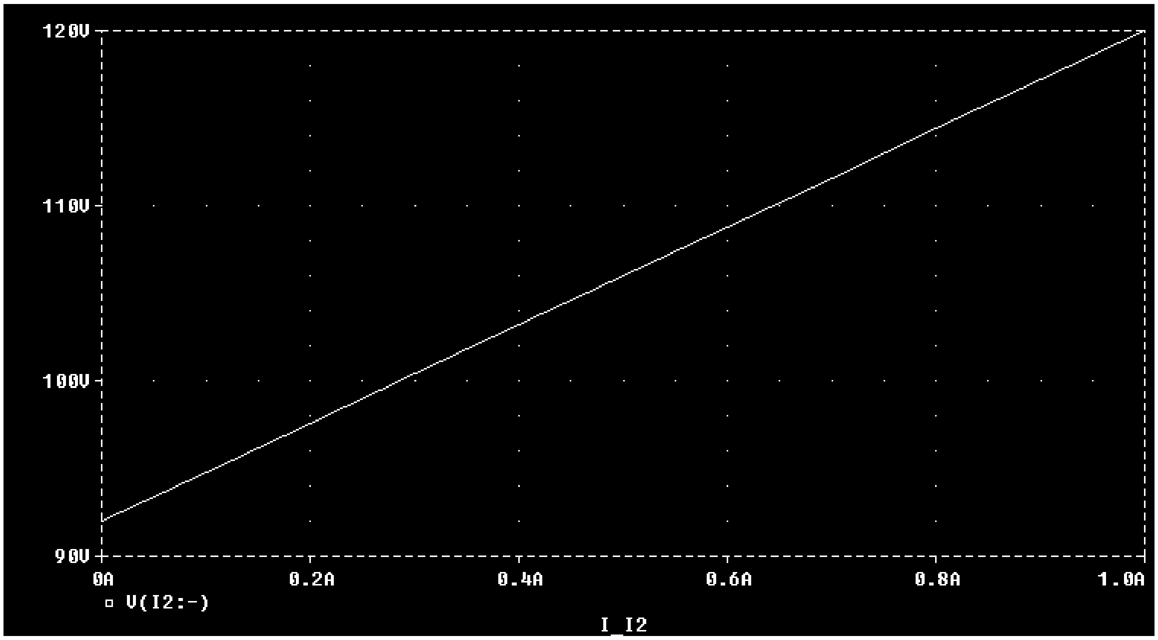
Solution 4.76

Follow the steps in Example 4.14. The schematic and the output plots are shown below. From the plot, we obtain,

V = **92 V** [i = 0, voltage axis intercept]

R = Slope = (120 – 92)/1 = **28 ohms**



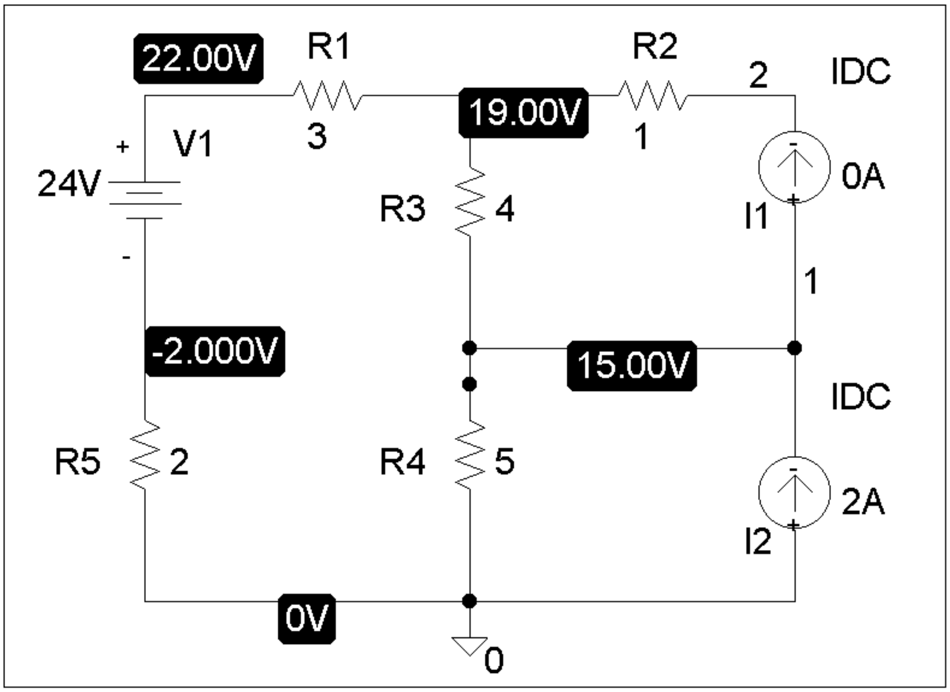

120V
110V
100V
90V
0A
0.2A
0.4A
0.6A
0.8A
1.0A
V(I2:-)
I_I2

Solution 4.77

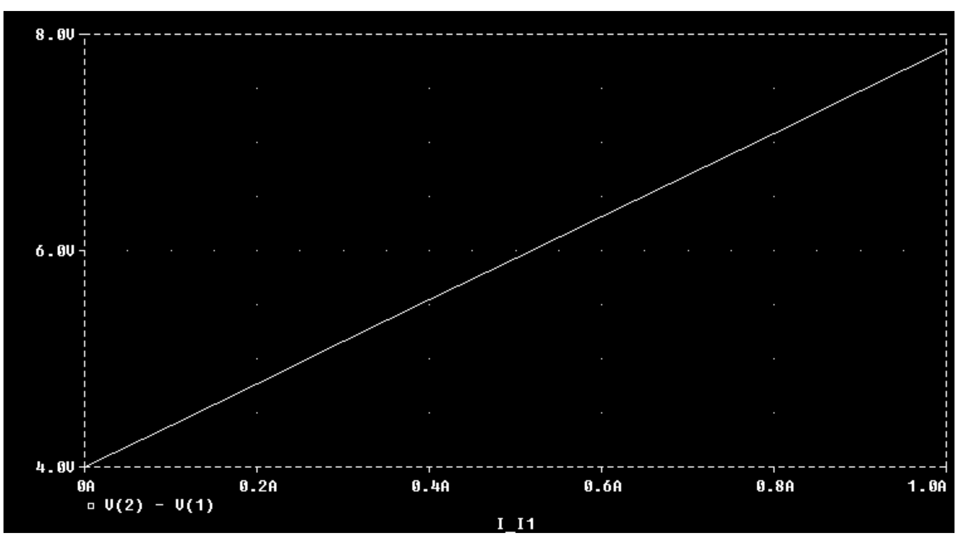
(a) The schematic is shown below. We perform a dc sweep on a current source, I1, connected between terminals a and b. We label the top and bottom of source I1 as 2 and 1 respectively. We plot V(2) – V(1) as shown.

$$V_{Th} = \mathbf{4\ V} \text{ [zero intercept]}$$

$$R_{Th} = (7.8 - 4)/1 = \mathbf{3.8\ ohms}$$

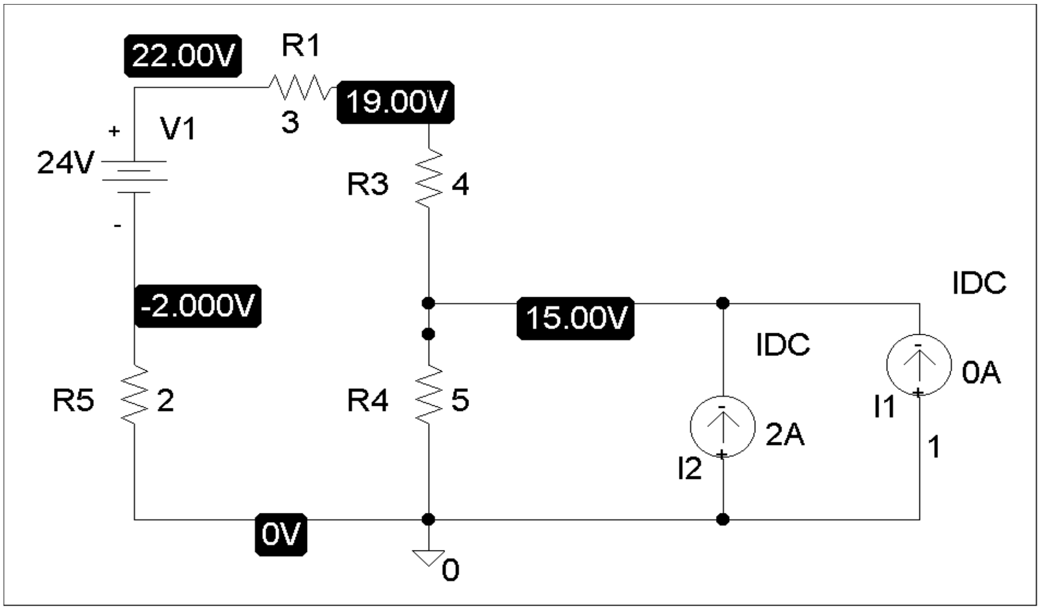


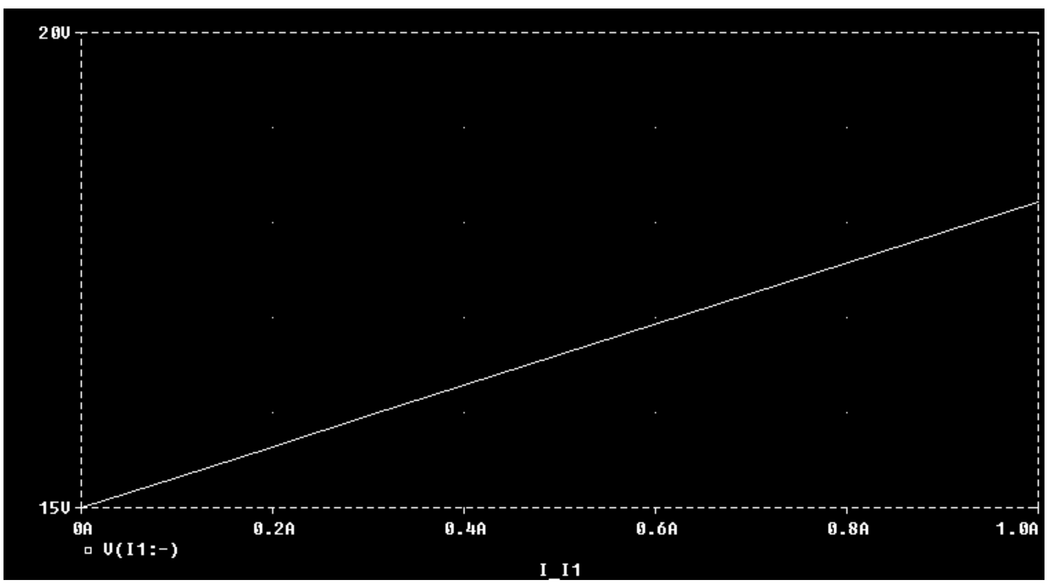
(b) Everything remains the same as in part (a) except that the current source, I1, is connected between terminals b and c as shown below. We perform a dc sweep on I1 and obtain the plot shown below. From the plot, we obtain,



V = **15 V** [zero intercept]

R = (18.2 – 15)/1 = **3.2 ohms**



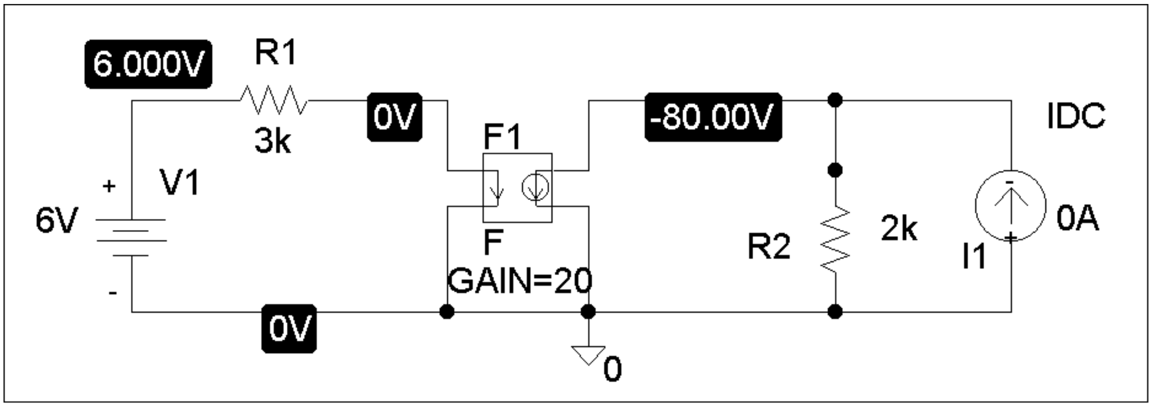


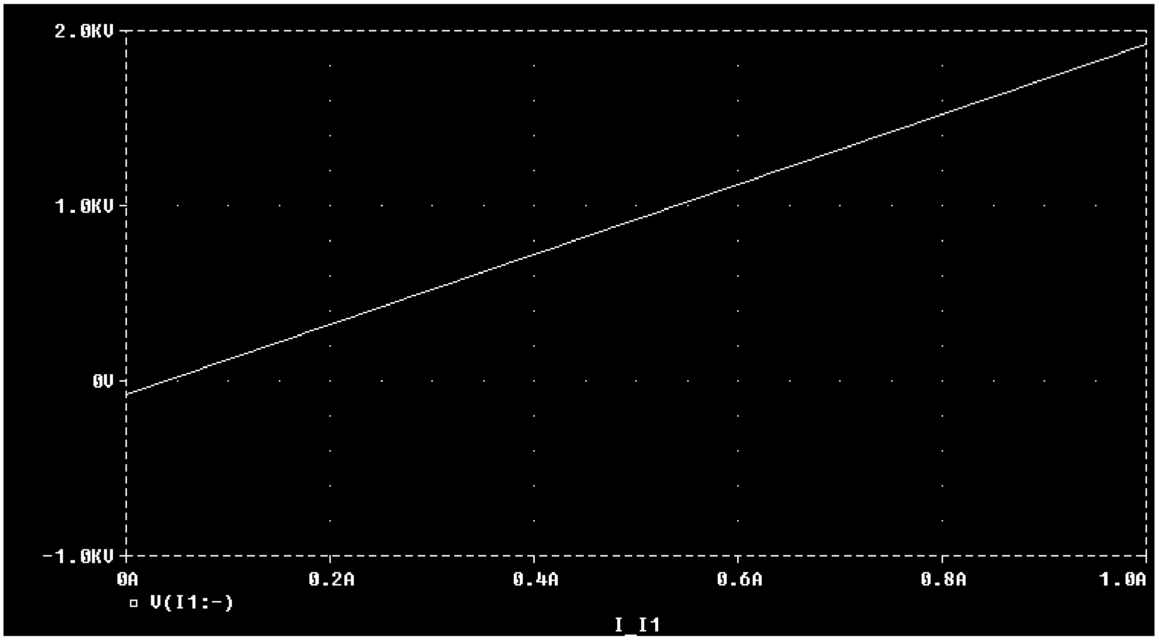
Solution 4.78

The schematic is shown below. We perform a dc sweep on the current source, I1, connected between terminals a and b. The plot is shown. From the plot we obtain,

$$V_{Th} \;=\; \mathbf{-80\ V} \;\; \text{[zero intercept]}$$

$$R_{Th} \;=\; (1920 - (-80))/1 \;=\; \mathbf{2\ k\ ohms}$$



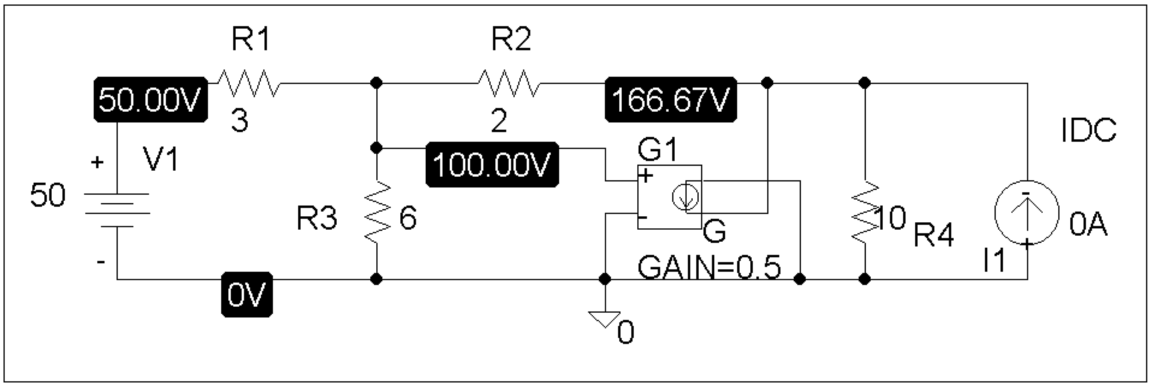


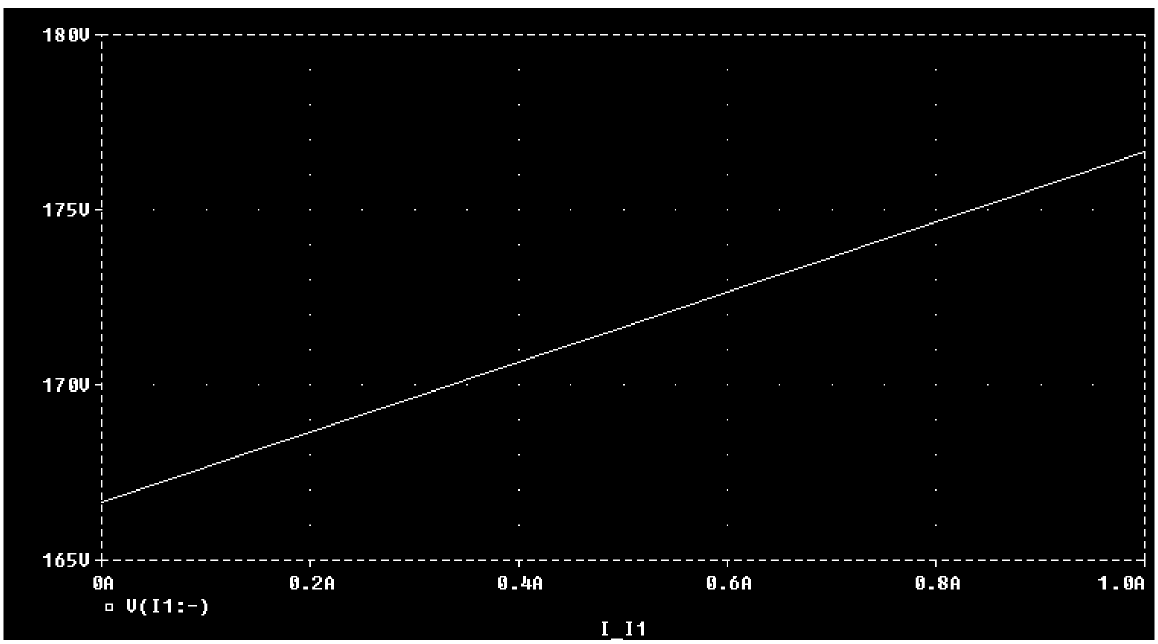
Solution 4.79

After drawing and saving the schematic as shown below, we perform a dc sweep on I1 connected across a and b. The plot is shown. From the plot, we get,

$$V = \mathbf{167\ V} \quad \text{[zero intercept]}$$

$$R = (177 - 167)/1 = \mathbf{10\ ohms}$$



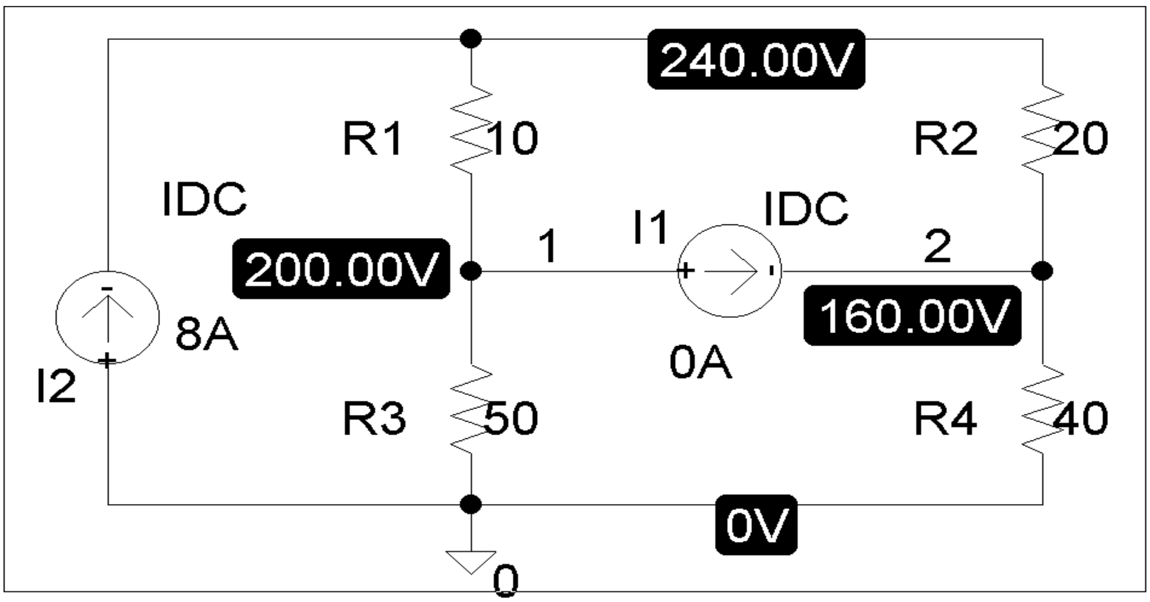


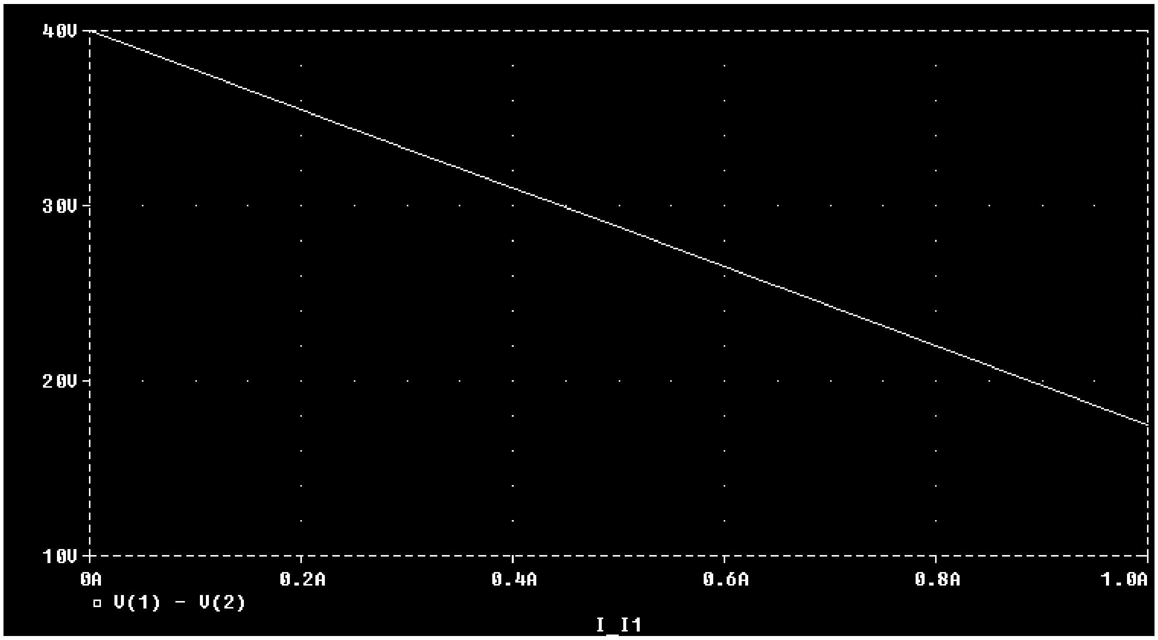
Solution 4.80

The schematic in shown below. We label nodes a and b as 1 and 2 respectively. We perform dc sweep on I1. In the Trace/Add menu, type v(1) – v(2) which will result in the plot below. From the plot,

$$V_{Th} = \mathbf{40\ V} \quad \text{[zero intercept]}$$

$$R_{Th} = (40 - 17.5)/1 = \mathbf{22.5\ ohms} \quad \text{[slope]}$$



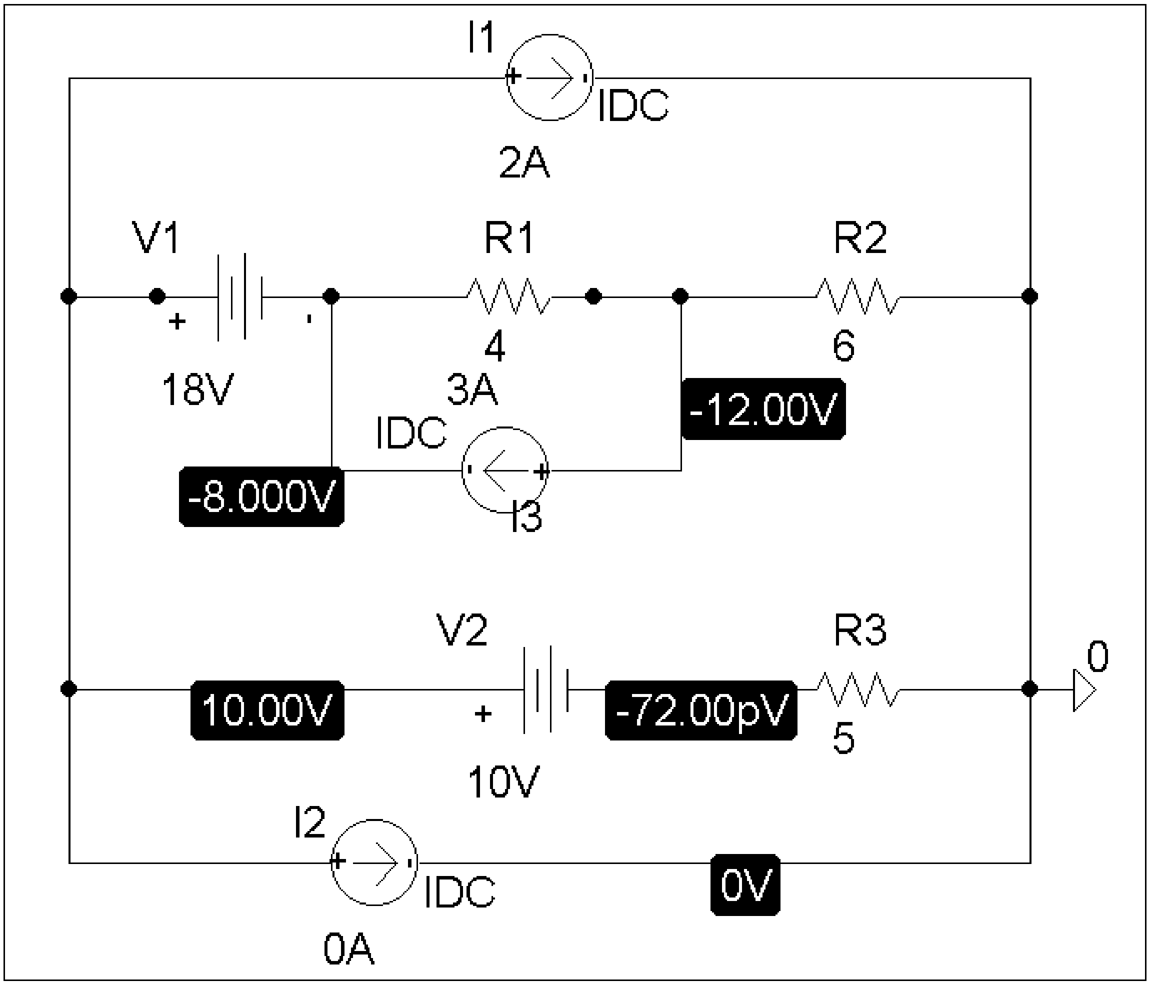

40V
30V
20V
10V
0A
0.2A
0.4A
0.6A
0.8A
1.0A
V(1) - V(2)
I_I1

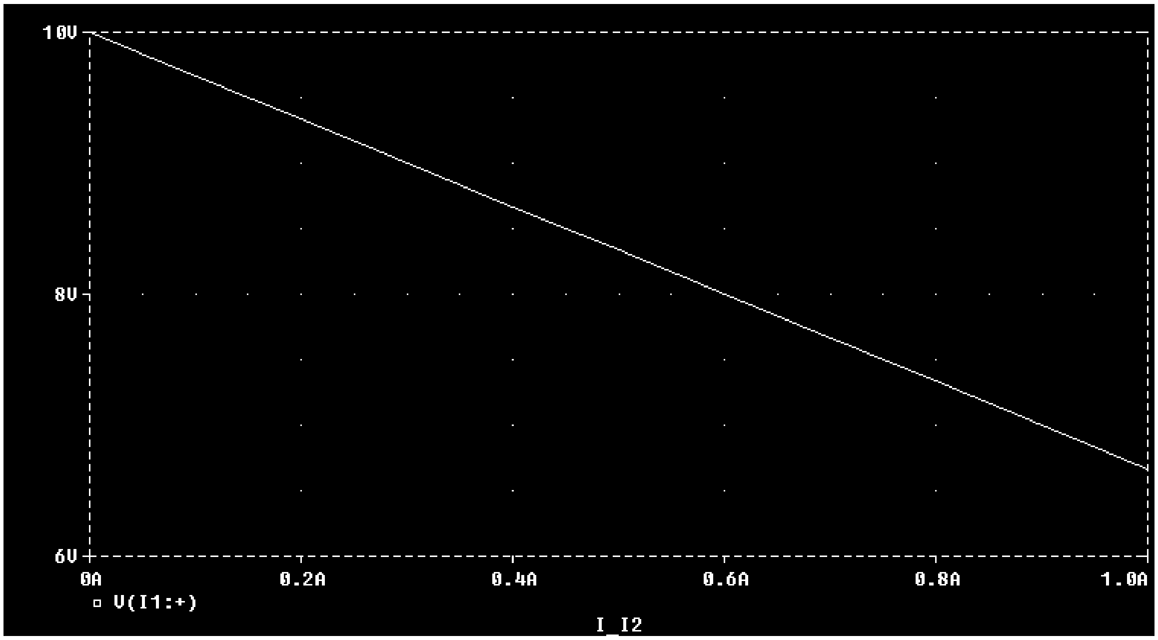
Solution 4.81

The schematic is shown below. We perform a dc sweep on the current source, I2, connected between terminals a and b. The plot of the voltage across I2 is shown below. From the plot,

$$V_{Th} = \mathbf{10\ V} \text{ [zero intercept]}$$

$R_{Th} = (10 – 6.7)/1 = \mathbf{3.3\ ohms}$. Note that this is in good agreement with the exact value of 3.333 ohms.




10V
8V
6V
0A
0.2A
0.4A
0.6A
0.8A
1.0A
V(I1:+)
I_I2

Solution 4.82

An automobile battery has an open circuit voltage of 14.7 volts which drops to 12 volts when connected to two 65 watt headlights. What is the resistance of each headlight and the value of the internal resistance of the battery?

Step 1. Basically we can treat this like a Thevenin equivalent circuit problem.

Clearly $V_{oc} = V_{Thev} = 14.7$ volts and R_{HP} is equal to the resistance of each bulb in parallel or $R_{HP} = R_B R_B/(R_B+R_B)$. In addition, 2x65 = 130 = 12i and
$R_{HB} = 12/i$ and $i = 14.7/(R_s + R_{HB})$.

Step 2. $i = 130/12 = 10.8333$ amps and $R_{HB} = 12/10.8333 = 1.107696$ ohms = $R_B/2$ or R_B = **2.215 Ω** per bulb. Finally $(R_s+1.107696) = 14.7/10.8333$ or

R_s = **249.2 mΩ**.

Solution 4.83

The following results were obtained from measurements taken between the two terminals of a resistive network.

Terminal Voltage	72 V	0 V
Terminal Current	0 A	9A

Find the Thevenin equivalent of the network.

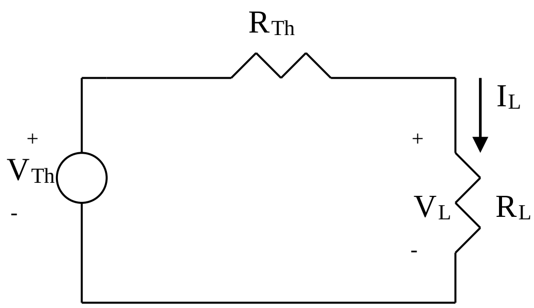
Solution

Step 1. We note that $V_{Thev} = V_{oc}$ and $I_N = I_{sc}$ and $R_{eq} = V_{oc}/I_{sc} = R_{Thev} = R_N$.

Step 2. V_{Thev} = **72 V** and $R_{Thev} = 72/9 =$ **8 Ω**.

Solution 4.84

Let the equivalent circuit of the battery terminated by a load be as shown below.



For open circuit,

$$R_L = \infty, \quad \longrightarrow \quad V_{Th} = V_{oc} = V_L = \underline{10.8\,\text{V}}$$

When R_L = 4 ohm, V_L=10.5,

$$I_L = \frac{V_L}{R_L} = 10.8/4 = 2.7$$

But

$$V_{Th} = V_L + I_L R_{Th} \quad \longrightarrow \quad R_{Th} = \frac{V_{Th} - V_L}{I_L} = \frac{12 - 10.8}{2.7} = \underline{0.4444\Omega}$$

= **444.4 mΩ**.

Solution 4.85

The Thevenin equivalent at terminals *a-b* of the linear network shown in Fig. 4.142 is to be determined by measurement. When a 10-kΩ resistor is connected to terminals *a-b*, the voltage V_{ab} is measured as 20 V. When a 30-kΩ resistor is connected to the terminals, V_{ab} is measured as 40 V. Determine: (a) the Thevenin equivalent at terminals *a-b*, (b) V_{ab} when a 20-kΩ resistor is connected to terminals *a-b*.

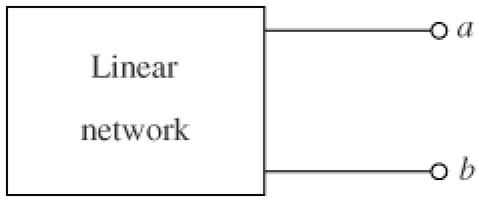
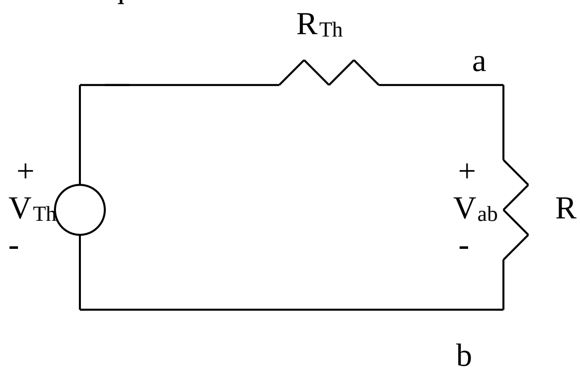


Figure 4.142
For Prob. 4.85.

Solution

(a) Consider the equivalent circuit terminated with R as shown below.



$$V_{ab} = \frac{R}{R+R_{Th}}V_{Th} \quad \longrightarrow \quad 20 = \frac{10}{10+R_{Th}}V_{Th}$$

or $200 + 20R_{Thev} = 10V_{Thev}$ where R_{Th} is in k-ohm.

Similarly, $40 = \frac{30}{30+R_{Th}}V_{Th}$ or $1200 + 40R_{Thev} = 30V_{Thev}$. We now have two equations with two unknowns or $V_{Thev} = 20+2R_{Thev}$ and $1200 + 40R_{Thev} = 30(20+2R_{Thev})$ or $20R_{Thev} = 1200–600$ or $R_{Thev} =$ **30 kΩ**. Next we get $V_{Thev} = 20 + 2(30) =$ **80 V**.

(b) Clearly $V_{ab} = 80[20/(20+30)] =$ **32 V**.

Solution 4.86

A black box with a circuit in it is connected to a variable resistor. An ideal ammeter (with zero resistance) and an ideal voltmeter (with infinite resistance) are used to measure current and voltage as shown in Fig. 4.143. The results are shown in the table below.

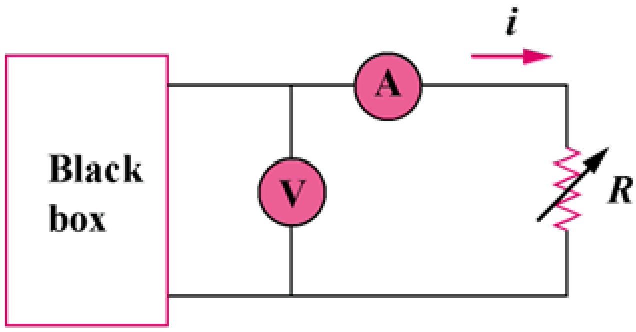


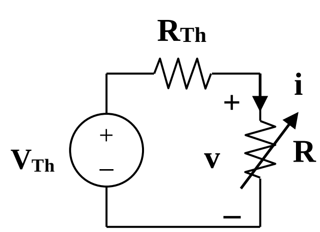
Figure 4.143
For Prob. 4.86.

(a) Find i when $R = 12\ \Omega$.
(b) Determine the maximum power from the box.

$R(\Omega)$	V(V)	i(A)
2	6	3
8	16	2
14	21	1.5

Solution

Step 1. We replace the box with the Thevenin equivalent.



$V_{Th} = v + iR_{Th}$

When i = 3, v = 6, which implies that $V_{Th} = 6 + 3R_{Th}$ (1)

When i = 2, v = 16, which implies that $V_{Th} = 16 + 2R_{Th}$ (2)

Step 2. From (1) – (2), we get $0 = -10 + R_{Thev}$ or $R_{Thev} = 10\ \Omega$ and $V_{Thev} = 6+30$ or 36 V. It is interesting to note that we really only need the two data points, the third is redundant.

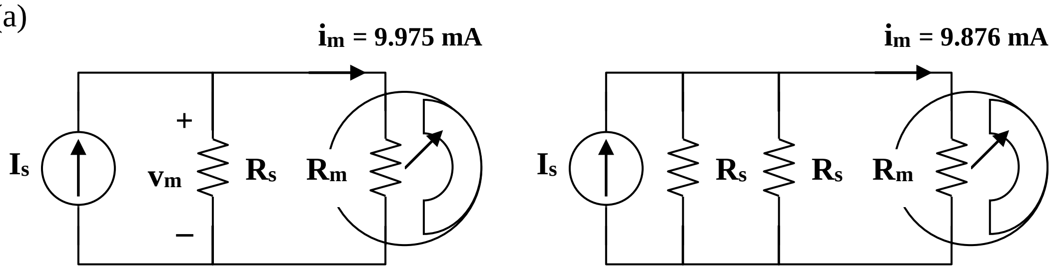
(a) When $R = 12\ \Omega$, $i = V_{Th}/(R + R_{Th}) = 36/(12 + 10) =$ **1.6364 A**

(b) For maximum power, $R = R_{TH}$

$$P_{max} = (V_{Th})^2/4R_{Th} = 36^2/(4x10) = \mathbf{32.4\ watts}.$$

Solution 4.87

(a)



From Fig. (a),

$$v_m = R_m i_m = 9.975 \text{ mA x } 20 = 0.1995 \text{ V}$$

$$I_s = 9.975 \text{ mA} + (0.1995/R_s) \qquad (1)$$

From Fig. (b),

$$v_m = R_m i_m = 20\text{x}9.876 = 0.19752 \text{ V}$$

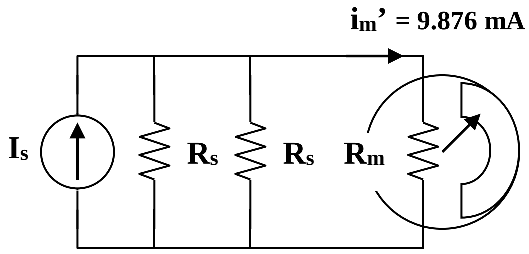
$$I_s = 9.876 \text{ mA} + (0.19752/2\text{k}) + (0.19752/R_s)$$

$$= 9.975 \text{ mA} + (0.19752/R_s) \qquad (2)$$

Solving (1) and (2) gives,

R_s = **8 k ohms**, I_s = **10 mA**

(b)

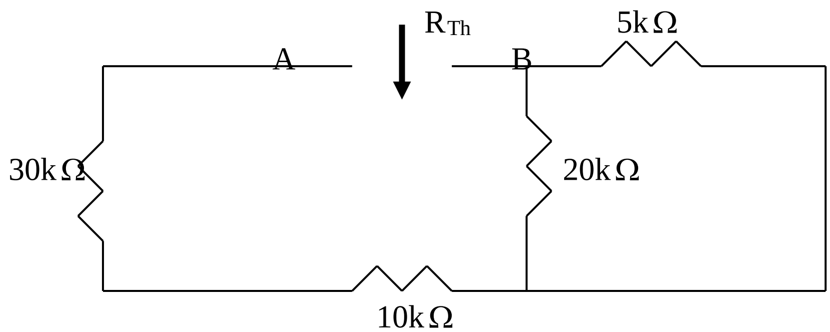


$$8\text{k}||4\text{k} = 2.667 \text{ k ohms}$$

i_m' = [2667/(2667 + 20)](10 mA) = **9.926 mA**

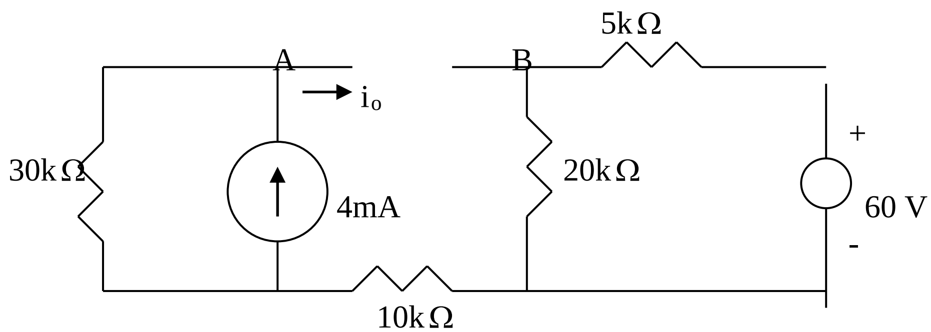
Solution 4.88

To find $R_{Th,}$ consider the circuit below.



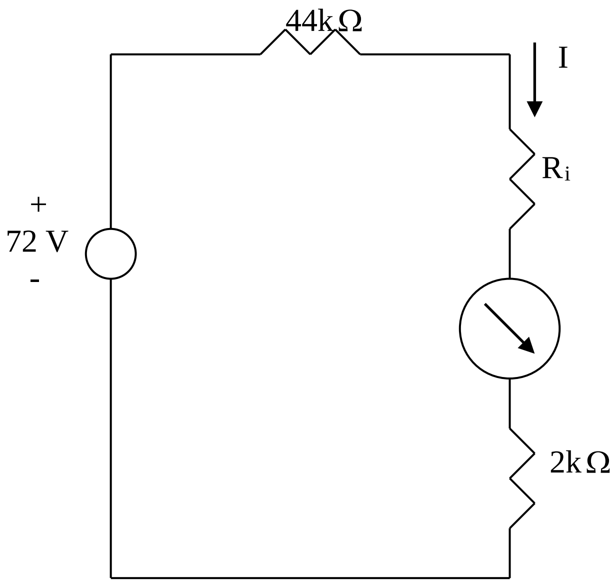
$R_{Th} = 30 + 10 + 20 // 5 = 44k\Omega$

To find V_{Th} , consider the circuit below.



$$V_A = 30x4 = 120, \quad V_B = \frac{20}{25}(60) = 48, \quad V_{Th} = V_A - V_B = 72 \text{ V}$$

The Thevenin equivalent circuit is shown below.



$$I = \frac{72}{44 + 2 + R_i} \text{ mA}$$

assuming R_i is in k-ohm.

(a) When $R_i = 500\,\Omega$,

$$\boldsymbol{I = \frac{72}{44 + 2 + 0.5} = \underline{1.548\,\text{mA}}}$$

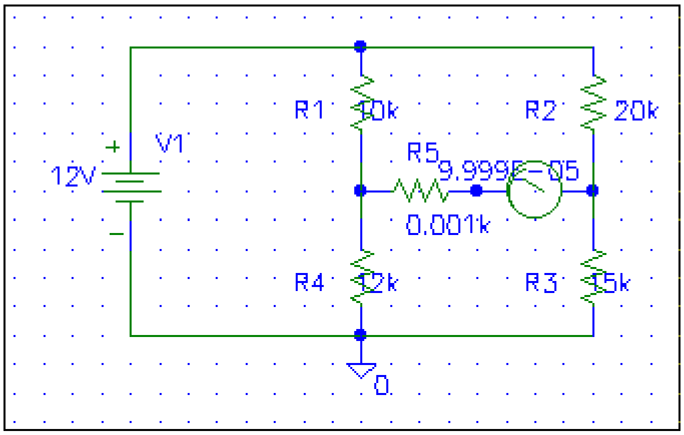
(b) When $R_i = 0\,\Omega$,

$$\boldsymbol{I = \frac{72}{44 + 2 + 0} = \underline{1.565\,\text{mA}}}$$

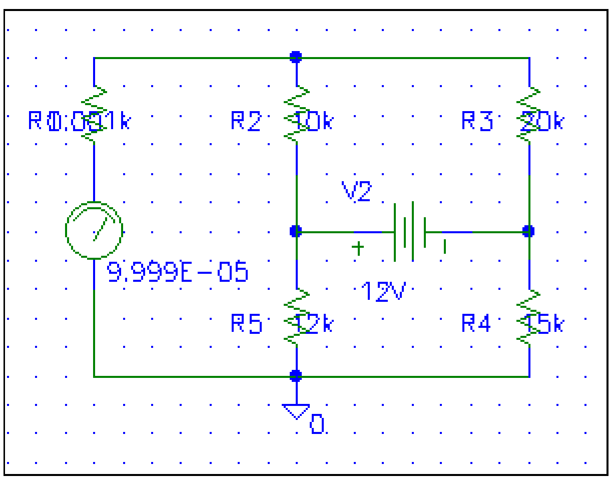
Solution 4.89

It is easy to solve this problem using Pspice.

(a) The schematic is shown below. We insert IPROBE to measure the desired ammeter reading. We insert a very small resistance in series IPROBE to avoid problem. After the circuit is saved and simulated, the current is displaced on IPROBE as **99.99μA** .



(b) By interchanging the ammeter and the 12-V voltage source, the schematic is shown below. We obtain exactly the same result as in part (a).



Solution 4.90

$R_x = (R_3/R_1)R_2 = (4/2)R_2 = 42.6, R_2 = 21.3$

which is (21.3ohms/100ohms)% = **21.3%**

Solution 4.91

(a) In the Wheatstone bridge circuit of Fig. 4.147 select the values of $\boldsymbol{R_a}$ and $\boldsymbol{R_b}$ such that the bridge can measure $\boldsymbol{R_x}$ in the range of 0-25 Ω.
(b) Repeat for the range of 0-250 Ω.

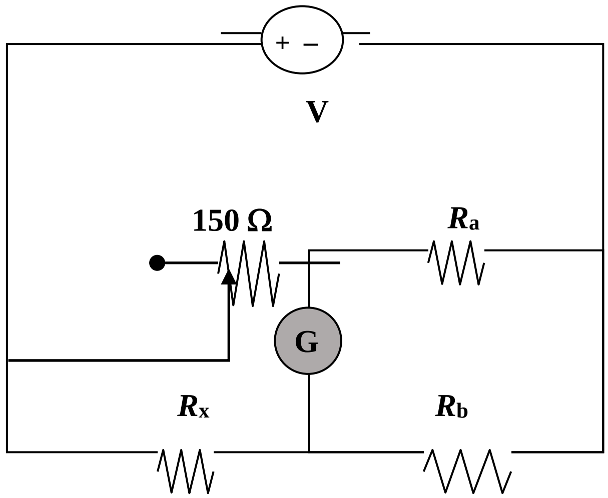


Figure 4.147
For Prob. 4.91.

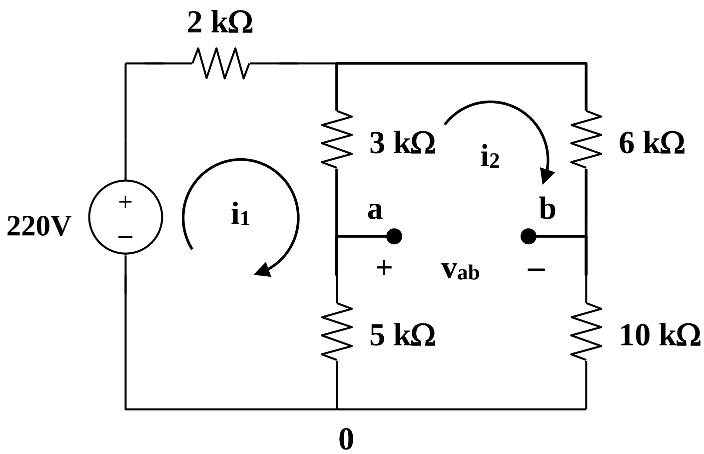
Step 1. We start from $[R_a/(k(150))] = [R_b/R_x]$ when the current through G is equal to zero where $0<k<1$. Since R_x can range from 0 to 25 Ω (a) and from 0 to 250 Ω (b), we can then calculate R_a and R_b. We will use the limits to do this, in other words when $R_x = 25\ \Omega$ then $k = 1$ and when $R_x = 250\ \Omega$ $k = 1$.

Step 2. (a) $R_a/150 = R_b/25$ gives one equation and two unknowns so we need to select one of them to solve for the other one. A good value to choose is $R_b = \mathbf{25\ \Omega}$ which leads to $R_a = \mathbf{150\ \Omega}$.

(b) Like (a), we can choose any value we wish but a good value might be $R_b = \mathbf{250\ \Omega}$ which leads to $R_a = \mathbf{150\ \Omega}$.

Solution 4.92

For a balanced bridge, $v_{ab} = 0$. We can use mesh analysis to find v_{ab}. Consider the circuit in Fig. (a), where i_1 and i_2 are assumed to be in mA.



(a)

$$220 = 2i_1 + 8(i_1 - i_2) \text{ or } 220 = 10i_1 - 8i_2 \quad (1)$$

$$0 = 24i_2 - 8i_1 \text{ or } i_2 = (1/3)i_1 \quad (2)$$

From (1) and (2),

$$i_1 = 30 \text{ mA and } i_2 = 10 \text{ mA}$$

Applying KVL to loop 0ab0 gives

$$5(i_2 - i_1) + v_{ab} + 10i_2 = 0 \text{ V}$$

Since $v_{ab} = 0$, the bridge is balanced.

When the 10 k ohm resistor is replaced by the 18 k ohm resistor, the bridge becomes unbalanced. (1) remains the same but (2) becomes

$$0 = 32i_2 - 8i_1, \text{ or } i_2 = (1/4)i_1 \quad (3)$$

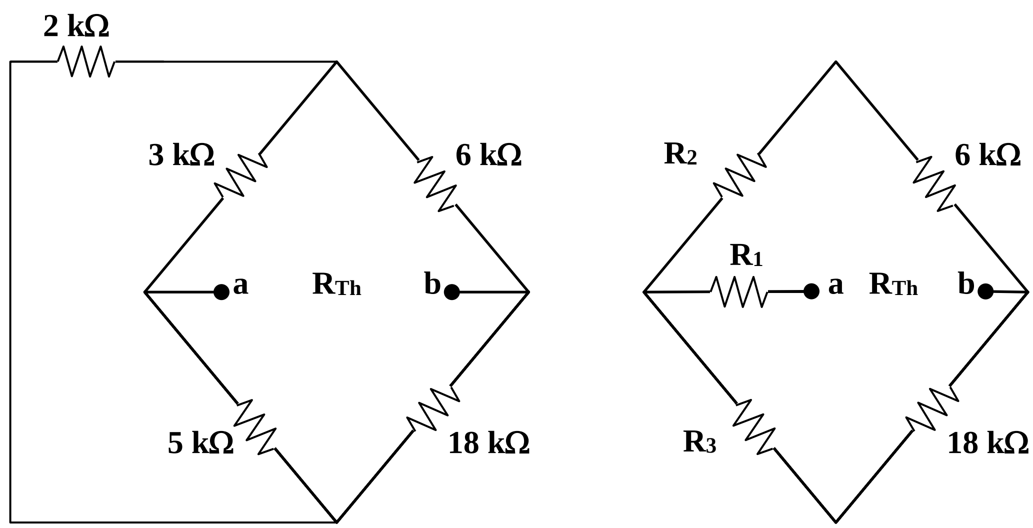
Solving (1) and (3),

$$i_1 = 27.5 \text{ mA}, \ i_2 = 6.875 \text{ mA}$$

$$v_{ab} = 5(i_1 - i_2) - 18i_2 = -20.625 \text{ V}$$

$$V_{Th} = v_{ab} = -20.625 \text{ V}$$

To obtain R_{Th}, we convert the delta connection in Fig. (b) to a wye connection shown in Fig. (c).



(b) **(c)**

$$R_1 = 3x5/(2 + 3 + 5) = 1.5 \text{ k ohms}, \quad R_2 = 2x3/10 = 600 \text{ ohms},$$

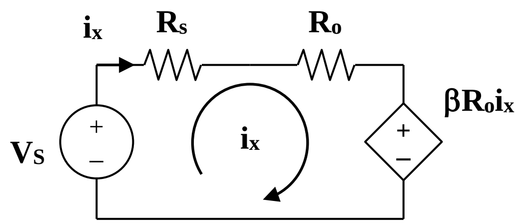
$$R_3 = 2x5/10 = 1 \text{ k ohm.}$$

$$R_{Th} = R_1 + (R_2 + 6)||(R_3 + 18) = 1.5 + 6.6||9 = 6.398 \text{ k ohms}$$

$$R_L = R_{Th} = \mathbf{6.398\ k\ ohms}$$

$$P_{max} = (V_{Th})^2/(4R_{Th}) = (20.625)^2/(4x6.398) = \mathbf{16.622\ mWatts}$$

Solution 4.93



$$-V_s + (R_s + R_o)i_x + \beta R_o i_x = 0$$

$$i_x = \mathbf{V_s/(R_s + (1 + \beta)R_o)}$$

Solution 4.94

(a) $$V_o/V_g = R_p/(R_g + R_s + R_p) \qquad (1)$$

$$R_{eq} = R_p \| (R_g + R_s) = R_g$$

$$R_g = R_p(R_g + R_s)/(R_p + R_g + R_s)$$

$$R_gR_p + R_g^2 + R_gR_s = R_pR_g + R_pR_s$$

$$R_pR_s = R_g(R_g + R_s) \qquad (2)$$

From (1), $$R_p/\alpha = R_g + R_s + R_p$$

$$R_g + R_s = R_p((1/\alpha) - 1) = R_p(1 - \alpha)/\alpha \qquad (1a)$$

Combining (2) and (1a) gives,

$$R_s = [(1 - \alpha)/\alpha]R_{eq} \qquad (3)$$

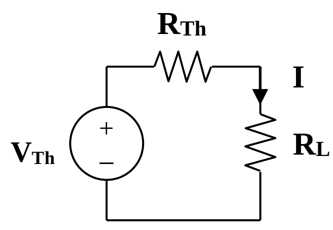
$$= (1 - 0.125)(100)/0.125 = \mathbf{700\ ohms}$$

From (3) and (1a),

$$R_p(1 - \alpha)/\alpha = R_g + [(1 - \alpha)/\alpha]R_g = R_g/\alpha$$

$$R_p = R_g/(1 - \alpha) = 100/(1 - 0.125) = \mathbf{114.29\ ohms}$$

(b)



$$V_{Th} = V_s = 0.125V_g = 1.5\ \text{V}$$

$$R_{Th} = R_g = 100\ \text{ohms}$$

$$I = V_{Th}/(R_{Th} + R_L) = 1.5/150 = \mathbf{10\ mA}$$

Solution 4.95

A dc voltmeter with a sensitivity of 10 kΩ/V is used to find the Thevenin equivalent of a linear network. Readings on two scales are as follows:

(a) 0-10 V scale: 8 V

(b) 0-50 V scale: 10 V

Obtain the Thevenin voltage and the Thevenin resistance of the network.

Solution

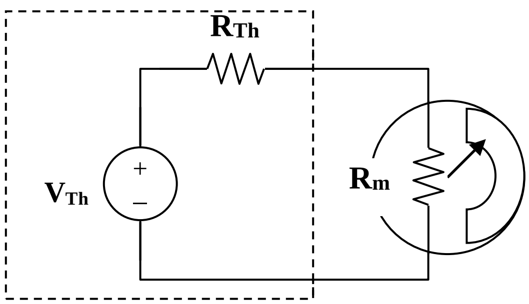
Let 1/sensitivity = 1/(10 k ohms/volt) = 100 μA

For the 0 – 10 V scale,

$$R_m = V_{fs}/I_{fs} = 10 \text{ V}/100 \text{ μA} = 100 \text{ kΩ}$$

For the 0 – 50 V scale,

$$R_m = 50 \text{ V}/100 \text{ μA} = 500 \text{ kΩ}$$



$$V_{Th} = I(R_{Th} + R_m)$$

(a) A 8 V reading corresponds to

$$I = (8/10)I_{fs} = 0.8\text{x}100 \text{ μA} = 80 \text{ μA}$$

$$V_{Th} = (80 \text{ μA}) R_{Th} + (80 \text{ μA}) 100 \text{ kΩ}$$

$$= 8 + (80 \text{ μA}) R_{Th} \qquad (1)$$

(b) A 10 V reading corresponds to

$$I = (10/50)I_{fs} = 0.2 \text{ x } 100 \text{ μA} = 20 \text{ μA}$$

$$V_{Th} = (20 \text{ μA}) R_{Th} + (20 \text{ μA}) 500 \text{ kΩ}$$

$$V_{Th} = 10 + (20 \text{ μA}) R_{Th} \qquad (2)$$

From (1) – (2)

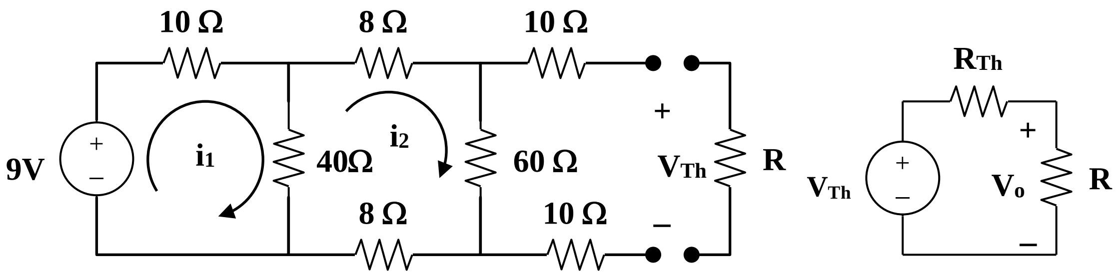
$$0 = -2 + (60\ \mu A)\, R_{Th} \quad \text{which leads to} \quad R_{Th} = \mathbf{33.33\ k\Omega}$$

From (1),

$$V_{Th} = 8 + (80\text{x}10^{-6}\text{x}33.333\text{x}10^{3}) = \mathbf{10.667\ V}$$

Solution 4.96

(a) The resistance network can be redrawn as shown in Fig. (a),



(a) (b)

$$R_{Th} = 10 + 10 + [60\|(8 + 8 + [10\|40])] = 20 + (60\|24) = 37.14 \text{ ohms}$$

Using mesh analysis,

$$-9 + 50i_1 - 40i_2 = 0 \qquad (1)$$
$$116i_2 - 40i_1 = 0 \text{ or } i_1 = 2.9i_2 \qquad (2)$$

From (1) and (2), $i_2 = 9/105 = 0.08571$

$$V_{Th} = 60i_2 = 5.143 \text{ V}$$

From Fig. (b),

$$V_o = [R/(R + R_{Th})]V_{Th} = 1.8 \text{ V}$$

$$R/(R + 37.14) = 1.8/5.143 = 0.35 \text{ or } R = 0.35R + 13 \text{ or } R = (13)/(1–0.35)$$

which leads to $R = \mathbf{20\ \Omega}$ (note, this is just for the $V_o = 1.8$ V)

(b) Asking for the value of R for maximum power would lead to $R = R_{Th} = \mathbf{37.14\ \Omega}$.

However, the problem asks for the value of R for maximum current. This happens when the value of resistance seen by the source is a minimum thus R = 0 is the correct value.

$$I_{max} = V_{Th}/(R_{Th}) = 5.143/(37.14) = \mathbf{138.48\ mA}.$$

Solution 4.97

A common-emitter amplifier circuit is shown in Fig. 4.152. Obtain the Thevenin equivalent to the left of points ***B*** and ***E***.

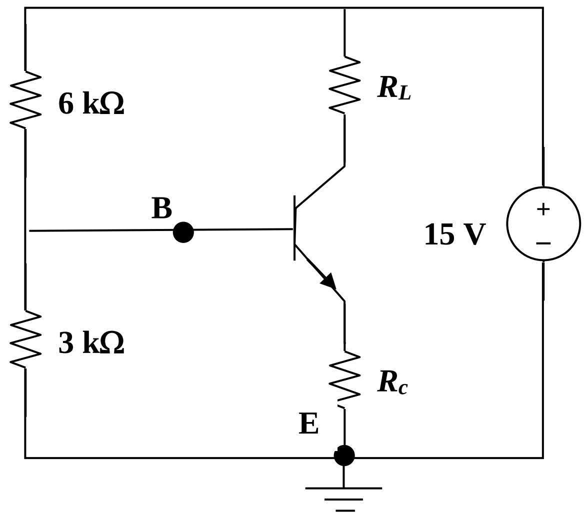
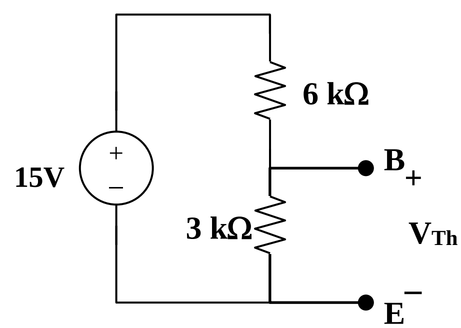


Figure 4.152
For Prob. 4.97.

Solution

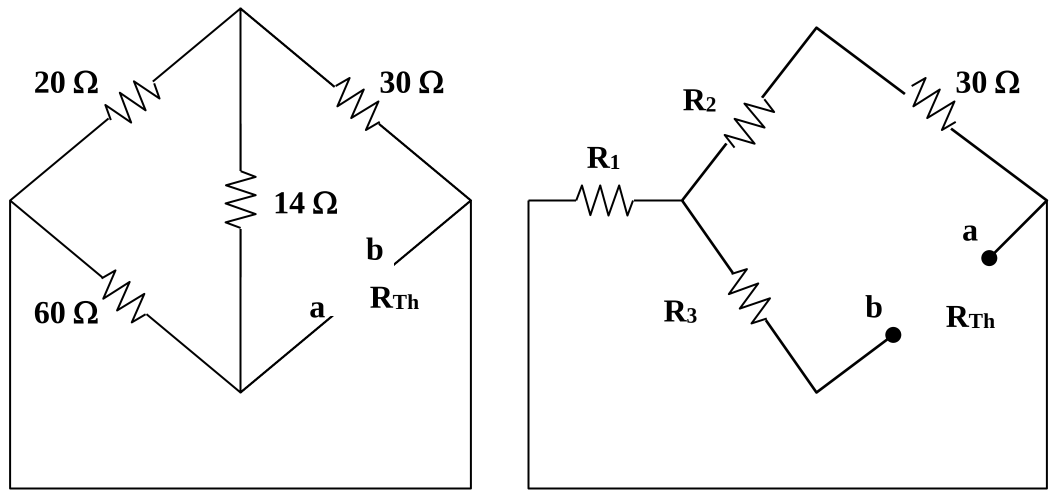


$$R_{Th} = R_1||R_2 = 6||3 = \mathbf{2\ k\ ohms}$$

$$V_{Th} = [R_2/(R_1 + R_2)]v_s = [3/(6 + 3)](15) = \mathbf{5\ V}$$

Solution 4.98

The 20-ohm, 60-ohm, and 14-ohm resistors form a delta connection which needs to be connected to the wye connection as shown in Fig. (b),



(a) (b)

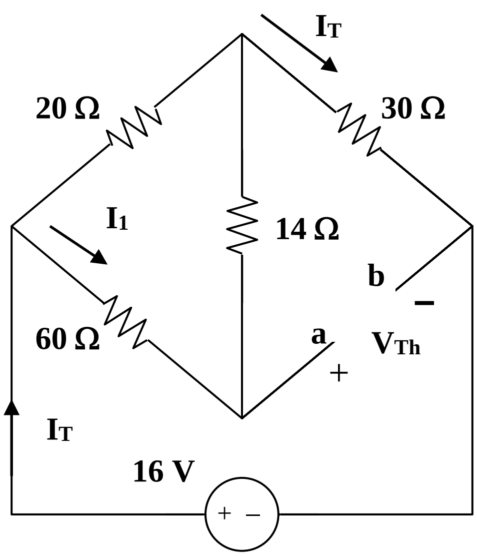
$$R_1 = 20x60/(20 + 60 + 14) = 1200/94 = 12.766 \text{ ohms}$$

$$R_2 = 20x14/94 = 2.979 \text{ ohms}$$

$$R_3 = 60x14/94 = 8.936 \text{ ohms}$$

$$R_{Th} = R_3 + R_1||(R_2 + 30) = 8.936 + 12.766||32.98 = 18.139 \text{ ohms}$$

To find V_{Th}, consider the circuit in Fig. (c).



(c)

$$I_T = 16/(30 + 15.745) = 349.8 \text{ mA}$$

$$I_1 = [20/(20 + 60 + 14)]I_T = 74.43 \text{ mA}$$

$$V_{Th} = 14I_1 + 30I_T = 11.536 \text{ V}$$

$$I_{40} = V_{Th}/(R_{Th} + 40) = 11.536/(18.139 + 40) = 198.42 \text{ mA}$$

$$P_{40} = I_{40}^2R = \mathbf{1.5748\ watts}$$

Solution 5.1

(a) R_{in} = **1.5 MΩ**
(b) R_{out} = **60 Ω**
(c) $A = 8x10^4$
Therefore $A_{dB} = 20 \log 8x10^4$ = **98.06 dB**

Solution 5.2

The open-loop gain of an op amp is 50,000. Calculate the output voltage when there are inputs of +10 μV on the inverting terminal and + 20 μV on the noninverting terminal.

Solution

$$v_0 = Av_d = A(v_2 - v_1)$$
$$= 2x10^4\ (20–10)\ x\ 10^{-6} = \mathbf{100\ mV}$$

Solution 5.3

Determine the voltage input to the inverting terminal of an op amp when –40 μvolts is applied to the noninverting terminal and the output through an open-loop gain of 150,000 is 15 volts.

Solution

$v_0 = Av_d = A(v_2 - v_1)$
$= 1.5 \text{ x } 10^5 (v_2 + 40\text{x } 10^{-6}) = 15 \text{ V}$ or

$v_2 = (15 - 6)/(0.15\text{x}10^6) = 9/(0.15\text{x}10^6) =$ **60 μV**.

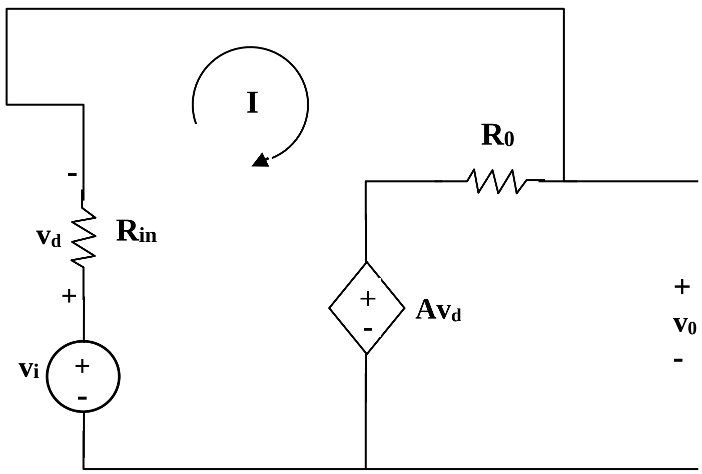
Solution 5.4

$v_0 = Av_d = A(v_2 - v_1)$

$$v_2 - v_1 = \frac{v_0}{A} = \frac{-4}{2x10^6} = -2\mu V$$

$v_2 - v_1 = -2\ \mu V = -0.002$ mV
$1\ mV - v_1 = -0.002$ mV
$v_1 =$ **1.002 mV**

Solution 5.5



$-v_i + Av_d + (R_i + R_0)\, I = 0 \qquad (1)$

But $\quad v_d = R_iI,$

$-v_i + (R_i + R_0 + R_iA)\, I = 0$

$$I = \frac{v_i}{R_0 + (1+A)R_i} \qquad (2)$$

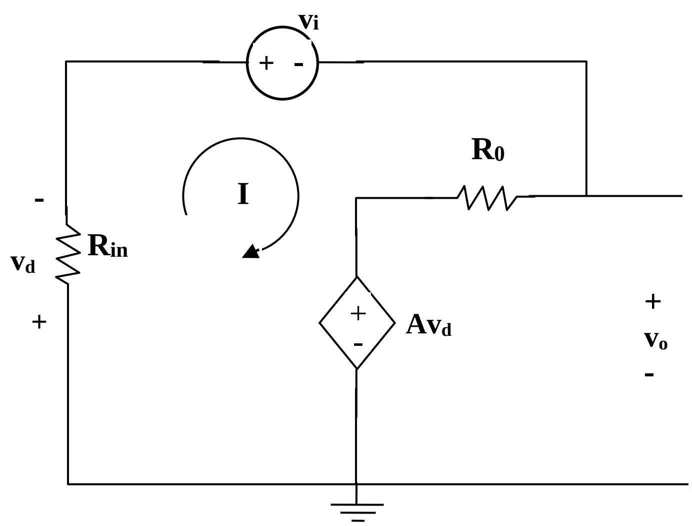
$-Av_d - R_0I + v_0 = 0$

$$v_0 = Av_d + R_0I = (R_0 + R_iA)I = \frac{(R_0 + R_iA)v_i}{R_0 + (1+A)R_i}$$

$$\frac{v_0}{v_i} = \frac{R_0 + R_iA}{R_0 + (1+A)R_i} = \frac{100 + 10^4\, x10^5}{100 + (1+10^5)} \cdot 10^4$$

$$\cong \frac{10^9}{(1+10^5)} \cdot 10^4 = \frac{100{,}000}{100{,}001} = \mathbf{0.9999990}$$

Solution 5.6



$(R_0 + R_i)R + v_i + Av_d = 0$

But $\quad v_d = R_i I,$

$v_i + (R_0 + R_i + R_i A)I = 0$

$$I = \frac{-v_i}{R_0 + (1 + A)R_i} \qquad (1)$$

$-Av_d - R_0 I + v_o = 0$

$v_o = Av_d + R_0 I = (R_0 + R_i A)I$

Substituting for I in (1),

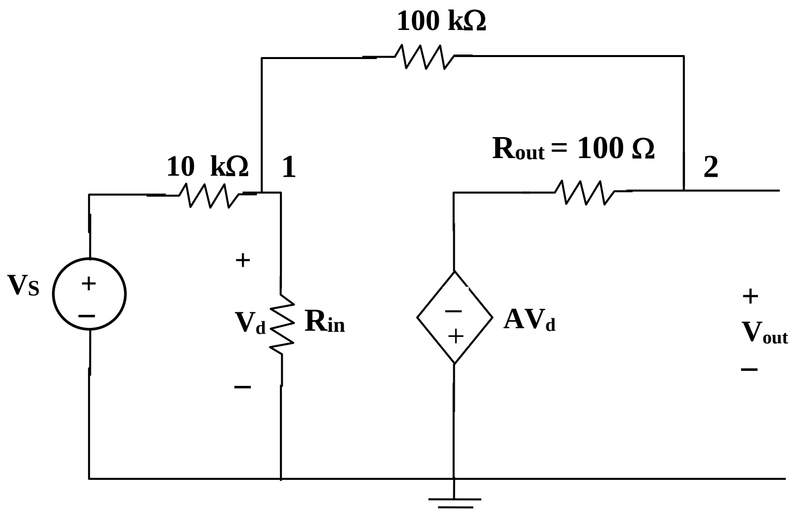
$$v_0 = -\left(\frac{R_0 + R_i A}{R_0 + (1 + A)R_i}\right)v_i$$

$$= -\frac{\left(50 + 2x10^6 x2x10^5\right)\cdot 10^{-3}}{50 + \left(1 + 2x10^5\right)x2x10^6}$$

$$\cong \frac{-200{,}000x2x10^6}{200{,}001x2x10^6} mV$$

$v_0 =$ **-0.999995 mV**

Solution 5.7



At node 1, $(V_S - V_1)/10\text{ k} = [V_1/100\text{ k}] + [(V_1 - V_0)/100\text{ k}]$

$10\,V_S - 10\,V_1 = V_1 + V_1 - V_0$

which leads to $V_1 = (10V_S + V_0)/12$

At node 2, $(V_1 - V_0)/100\text{ k} = (V_0 - (-AV_d))/100$

But $V_d = V_1$ and $A = 100{,}000$,

$V_1 - V_0 = 1000\,(V_0 + 100{,}000V_1)$

$0 = 1001V_0 + 99{,}999{,}999[(10V_S + V_0)/12]$

$0 = 83{,}333{,}332.5\,V_S + 8{,}334{,}334.25\,V_0$

which gives us $(V_0/V_S) = -10$ (for all practical purposes)

If $V_S = 1$ mV, then $V_0 =$ **–10 mV**

Since $V_0 = A\,V_d = 100{,}000\,V_d$, then $V_d = (V_0/10^5)$ V $=$ **–100 nV**

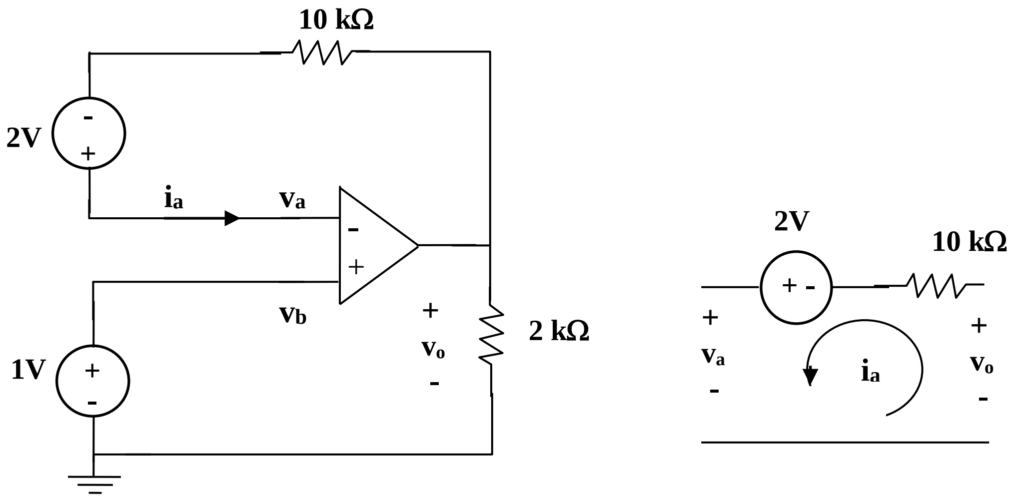
Solution 5.8

(a) If v_a and v_b are the voltages at the inverting and noninverting terminals of the op amp.

$v_a = v_b = 0$

$$1\text{mA} = \frac{0 - v_0}{2k} \longrightarrow v_0 = \mathbf{-2\ V}$$

(b)



(a) (b)

Since $v_a = v_b = 1\text{V}$ and $i_a = 0$, no current flows through the 10 kΩ resistor. From Fig. (b),

$$-v_a + 2 + v_0 = 0 \longrightarrow v_0 = v_a - 2 = 1 - 2 = \mathbf{-1V}$$

Solution 5.9

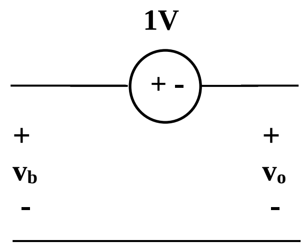
(a) Let v_a and v_b be respectively the voltages at the inverting and noninverting terminals of the op amp

$$v_a = v_b = 4\text{V}$$

At the inverting terminal,

$$1\text{mA} = \frac{4 - v_0}{2\text{k}} \longrightarrow v_0 = \mathbf{2V}$$

(b)



Since $v_a = v_b = 3\text{V}$,

$$-v_b + 1 + v_o = 0 \longrightarrow v_o = v_b - 1 = \mathbf{2V}$$

Solution 5.10

Since no current enters the op amp, the voltage at the input of the op amp is v_s. Hence

$$v_s = v_o\left(\frac{10}{10+10}\right) = \frac{v_o}{2} \longrightarrow \frac{v_o}{v_s} = \mathbf{2}$$

Solution 5.11

Using Fig. 5.50, design a problem to help other students to better understand how ideal op amps work.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find v_o and i_o in the circuit in Fig. 5.50.

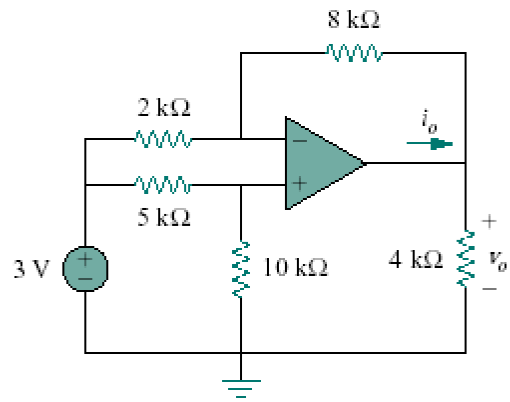
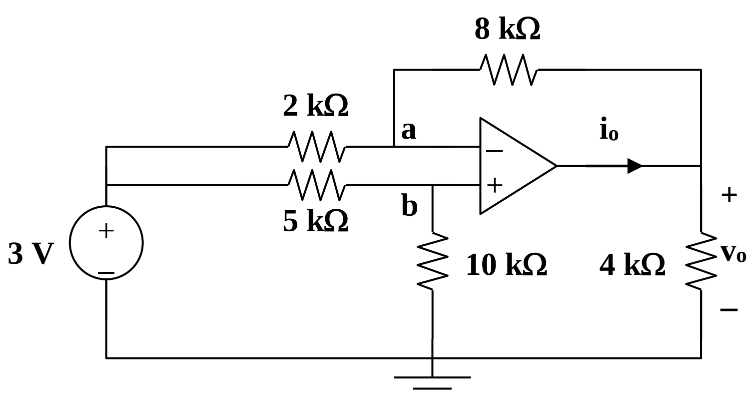


Figure 5.50 for Prob. 5.11

Solution



$$v_b = \frac{10}{10+5}(3) = 2V$$

At node a,

$$\frac{3 - v_a}{2} = \frac{v_a - v_o}{8} \longrightarrow 12 = 5v_a - v_o$$

But $v_a = v_b = 2V$,

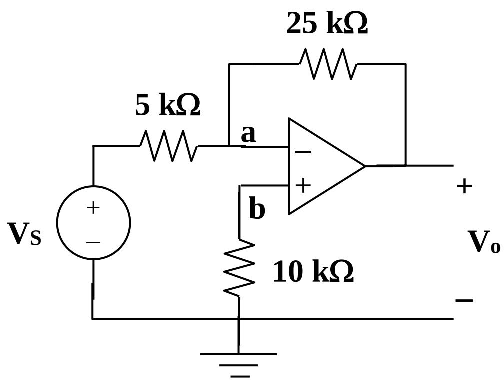
$$12 = 10 - v_o \longrightarrow v_o = \mathbf{-2V}$$

$$-i_o = \frac{v_a - v_o}{8} + \frac{0 - v_o}{4} = \frac{2+2}{8} + \frac{2}{4} = 1\text{mA}$$

$$i_o = \mathbf{-1mA}$$

Solution 5.12

Step 1. Label the unknown nodes in the op amp circuit. Next we write the node equations and then apply the constraint, $V_a = V_b$. Finally, solve for V_o in terms of V_s.



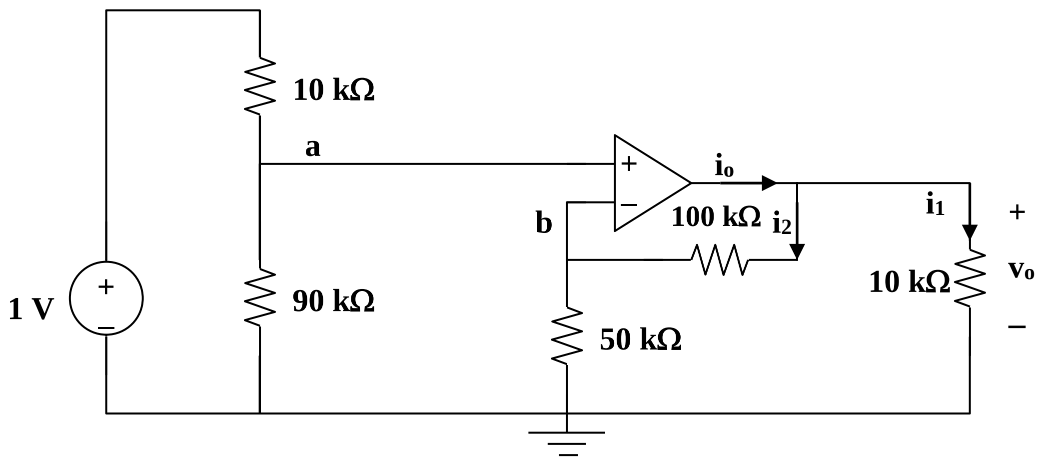
Step 2. $[(V_a-V_s)/5k] + [(V_a-V_o)/25k] + 0 = 0$ and

$[(V_b-0)/10k] + 0 = 0$ or $V_b = 0 = V_a$! Thus,

$[(-V_s)/5k] + [(-V_o)/25k] = 0$ or,

$$V_o = (-25/5)V_s \text{ or } V_o/V_s = \mathbf{-5}.$$

Solution 5.13



By voltage division,

$$v_a = \frac{90}{100}(1) = 0.9\text{V}$$

$$v_b = \frac{50}{150}v_o = \frac{v_o}{3}$$

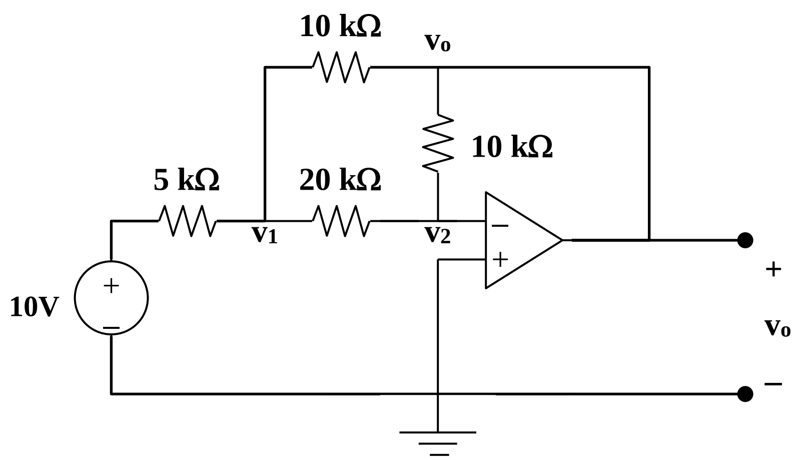
But $v_a = v_b \longrightarrow \frac{v_0}{3} = 0.9 \longrightarrow v_o = \mathbf{2.7V}$

$$i_o = i_1 + i_2 = \frac{v_o}{10k} + \frac{v_o}{150k} = 0.27\text{mA} + 0.018\text{mA} = \mathbf{288\ \mu A}$$

Solution 5.14

Transform the current source as shown below. At node 1,

$$\frac{10 - v_1}{5} = \frac{v_1 - v_2}{20} + \frac{v_1 - v_o}{10}$$



But $v_2 = 0$. Hence $40 - 4v_1 = v_1 + 2v_1 - 2v_o \longrightarrow$ $40 = 7v_1 - 2v_o$ (1)

At node 2, $\frac{v_1 - v_2}{20} = \frac{v_2 - v_o}{10}$, $v_2 = 0$ or $v_1 = -2v_o$ (2)

From (1) and (2), $40 = -14v_o - 2v_o \longrightarrow$ $v_o =$ $\mathbf{-2.5V}$

Solution 5.15

(a) Let v_1 be the voltage at the node where the three resistors meet. Applying KCL at this node gives

$$i_s = \frac{v_1}{R_2} + \frac{v_1 - v_o}{R_3} = v_1\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{v_o}{R_3} \qquad (1)$$

At the inverting terminal,

$$i_s = \frac{0 - v_1}{R_1} \longrightarrow v_1 = -i_s R_1 \qquad (2)$$

Combining (1) and (2) leads to

$$\boldsymbol{i_s\left(1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}\right) = -\frac{v_o}{R_3} \longrightarrow \underline{\frac{v_o}{i_s} = -\left(R_1 + R_3 + \frac{R_1 R_3}{R_2}\right)}}$$

(b) For this case,

$$\frac{v_o}{i_s} = -\left(20 + 40 + \frac{20x40}{25}\right)\text{k}\Omega = \underline{\text{-}92\ \text{k}\Omega}$$

= **–92 kΩ**

Solution 5.16

Using Fig. 5.55, design a problem to help students better understand inverting op amps.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Obtain i_x and i_y in the op amp circuit in Fig. 5.55.

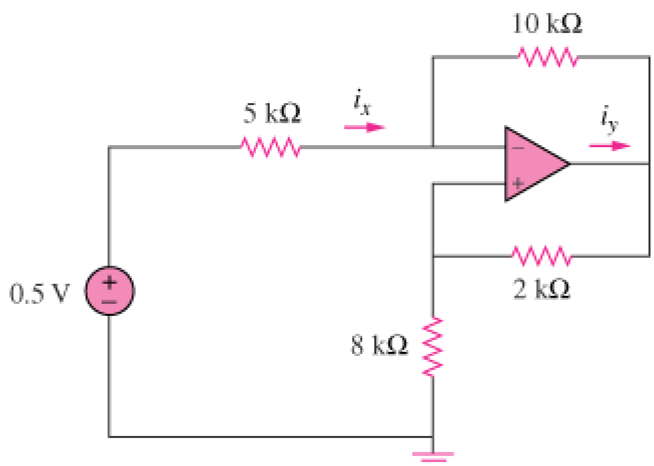
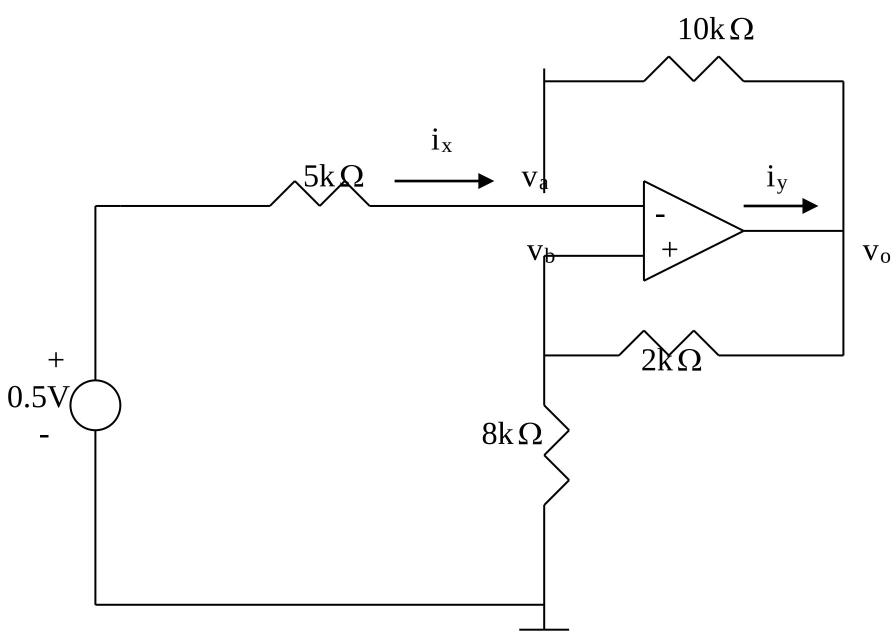


Figure 5.55

Solution



Let currents be in mA and resistances be in kΩ. At node a,

$$\frac{0.5 - v_a}{5} = \frac{v_a - v_o}{10} \longrightarrow 1 = 3v_a - v_o \qquad (1)$$

But

$$v_a = v_b = \frac{8}{8+2} v_o \longrightarrow v_o = \frac{10}{8} v_a \qquad (2)$$

Substituting (2) into (1) gives

$$1 = 3v_a - \frac{10}{8} v_a \longrightarrow v_a = \frac{8}{14}$$

Thus,

$$i_x = \frac{0.5 - v_a}{5} = -1/70 \text{ mA} = \underline{-14.28\ \mu\text{A}}$$

$$\boldsymbol{i_y = \frac{v_o - v_b}{2} + \frac{v_o - v_a}{10} = 0.6(v_o - v_a) = 0.6(\frac{10}{8} v_a - v_a) = \frac{0.6}{4}\ x\ \frac{8}{14}\ \textbf{mA}}$$

= **85.71 μA**

Solution 5.17

(a) $\quad G = \dfrac{v_o}{v_i} = -\dfrac{R_f}{R_i} = -\dfrac{12}{5} = \mathbf{-2.4}$

(b) $\quad \dfrac{\mathrm{v_o}}{\mathrm{v_i}} = -\dfrac{80}{5} = \mathbf{-16}$

(c) $\quad \dfrac{\mathrm{v_o}}{\mathrm{v_i}} = -\dfrac{2000}{5} = \mathbf{-400}$

(a) **–2.4**, (b) **–16**, (c) **–400**

Solution 5.18

For the circuit, shown in Fig. 5.57, solve for the Thevenin equivalent circuit looking into terminals A and B.

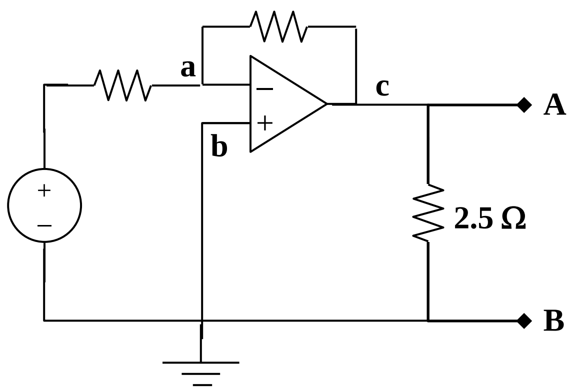


Figure 5.57
For Prob. 5.18.

Solution

Write a node equation at a. Since node b is tied to ground, $v_b = 0$. Since writing a node equation at c adds an additional unknown, the current from the op amp, we need to use the constraint equation, $v_a = v_b$. Once, we know v_c, we then proceed to solve for V_{oc} and I_{sc}. This will lead to $V_{Thev} = V_{oc}$ and $R_{eq} = V_{oc}/I_{sc}$.

$[(v_a – 9)/10k] + [(v_a – v_c)/10k] + 0 = 0$

Our constraint equation leads to,

$v_a = v_b = 0$ or $v_c = –9$ volts

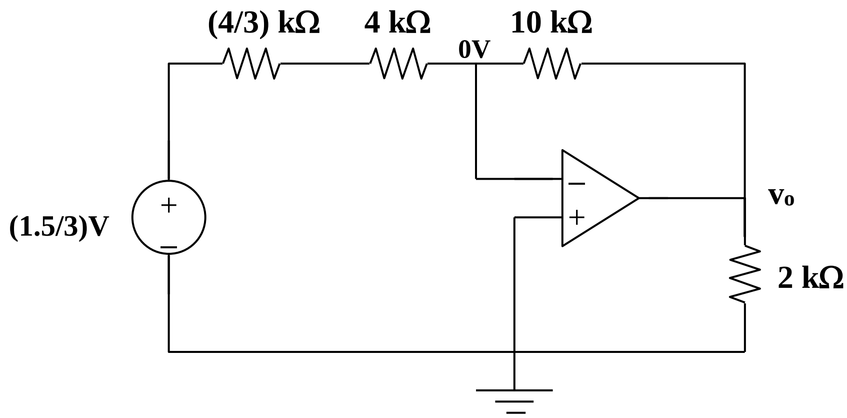
This is also the open circuit voltage (note, the op-amp keeps the output voltage at –9 volts in spite of any connection between A and B. Since this means that even a short from A to B would theoretically then produce an infinite current, $R_e = 0\ \Omega$. In real life, the short circuit current will be limited to whatever the op-amp can put out into a short circuited output.

V_{Thev} = **–9 volts**; R_{eq} = **0 Ω**.

Solution 5.19

We convert the current source and back to a voltage source.

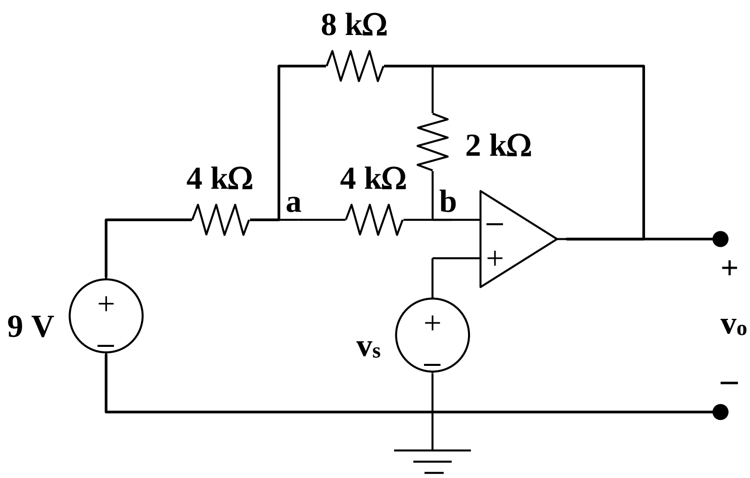
$$2\|4 = \frac{4}{3}$$



$$v_o = -\frac{10k}{\left(4+\frac{4}{3}\right)k}\left(\frac{1.5}{3}\right) = -937.5 \text{ mV}.$$

$$i_o = \frac{v_o}{2k} + \frac{v_o - 0}{10k} = \mathbf{-562.5\ \mu A}.$$

Solution 5.20



At node a,

$$\frac{9-v_a}{4} = \frac{v_a - v_o}{8} + \frac{v_a - v_b}{4} \longrightarrow 18 = 5v_a - v_o - 2v_b \qquad (1)$$

At node b,

$$\frac{v_a - v_b}{4} = \frac{v_b - v_o}{2} \longrightarrow v_a = 3v_b - 2v_o \qquad (2)$$

But $v_b = v_s = 2$ V; (2) becomes $v_a = 6 - 2v_o$ and (1) becomes

$$-18 = 30 - 10v_o - v_o - 4 \qquad v_o = -44/(-11) = \mathbf{4\ V}.$$

Solution 5.21

Calculate v_o in the op amp circuit of Fig. 5.60.

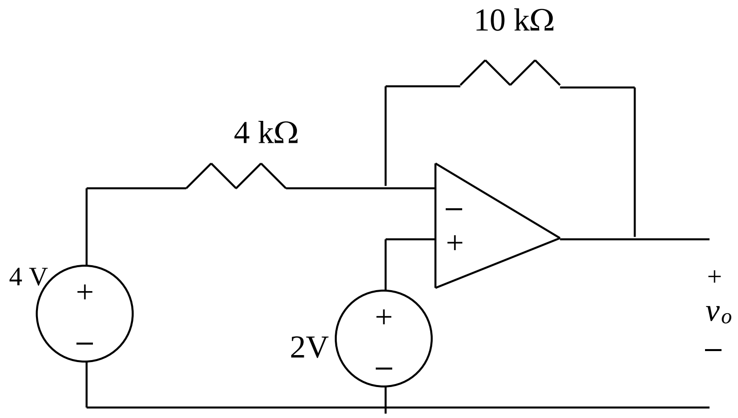


Figure 5.60
For Prob. 5.21.

Solution

Let the voltage at the inverting input of the op amp be v_a and at the noninverting input v_b. This leads to,

$[(v_a–4)/4k] + [(v_a–v_o)/10k] + 0 = 0$ with the constraint equation, $v_b = 2 = v_a$.

$v_o/10k = (–2/4k) + (2/10k) = (–0.5+0.2)/1k$ or

$$v_o = \mathbf{–3\ V}.$$

Solution 5.22

$A_v = -R_f/R_i = -15.$

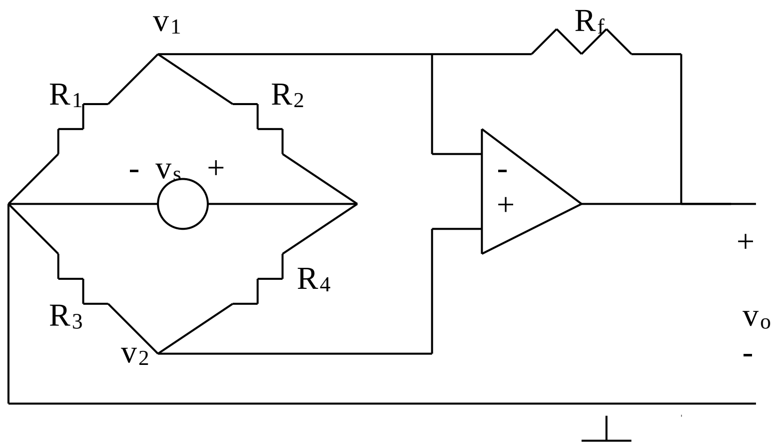
If $R_i = 10k\Omega$, then R_f = **150 kΩ.**

Solution 5.23

At the inverting terminal, v=0 so that KCL gives

$$\frac{v_s - 0}{R_1} = \frac{0}{R_2} + \frac{0 - v_o}{R_f} \longrightarrow \underline{\frac{v_o}{v_s} = -\frac{R_f}{R_1}}$$

Solution 5.24



We notice that $v_1 = v_2$. Applying KCL at node 1 gives

$$\frac{v_1}{R_1}+\frac{(v_1-v_s)}{R_2}+\frac{v_1-v_o}{R_f}=0 \quad \longrightarrow \quad \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_f}\right)v_1-\frac{v_s}{R_2}=\frac{v_o}{R_f} \qquad (1)$$

Applying KCL at node 2 gives

$$\frac{v_1}{R_3}+\frac{v_1-v_s}{R_4}=0 \quad \longrightarrow \quad v_1=\frac{R_3}{R_3+R_4}v_s \qquad (2)$$

Substituting (2) into (1) yields

$$v_o=R_f\left[\left(\frac{R_3}{R_1}+\frac{R_3}{R_f}-\frac{R_4}{R_2}\right)\left(\frac{R_3}{R_3+R_4}\right)-\frac{1}{R_2}\right]v_s$$

i.e.

$$\underline{\boldsymbol{k=R_f\left[\left(\frac{R_3}{R_1}+\frac{R_3}{R_f}-\frac{R_4}{R_2}\right)\left(\frac{R_3}{R_3+R_4}\right)-\frac{1}{R_2}\right]}}$$

Solution 5.25

Calculate v_o in the op amp circuit of Fig. 5.63.

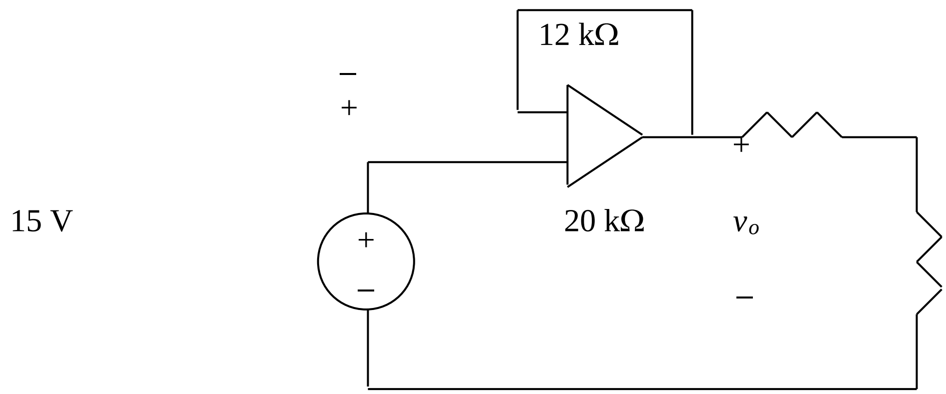


Figure 5.63
For Prob. 5.25.

Solution

This is a voltage follower. If v_c is the output of the op amp,

$v_c = 15$ V

$v_o = [20k/(20k+12k)]v_c = [20/32]15 =$ **9.375 V**.

Solution 5.26

Using Fig. 5.64, design a problem to help other students better understand noninverting op amps.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine $\boldsymbol{i_o}$ in the circuit of Fig. 5.64.

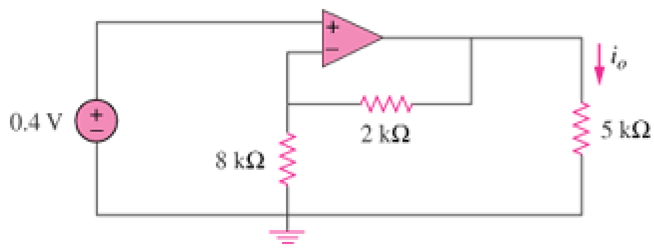
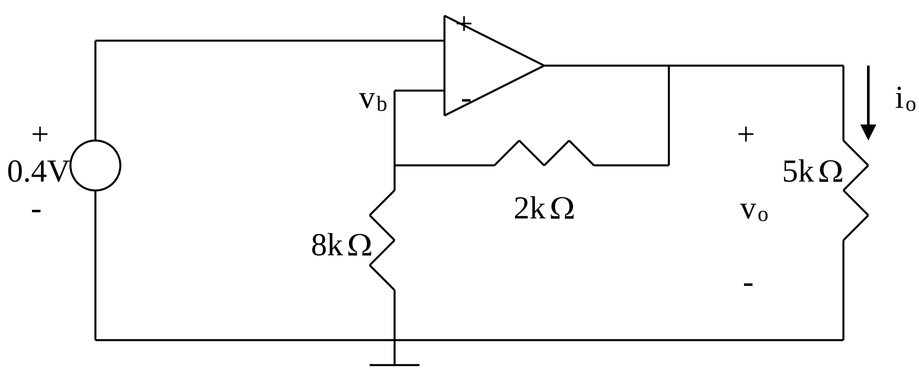


Figure 5.64

Solution



$$v_b = 0.4 = \frac{8}{8+2}v_o = 0.8v_o \longrightarrow v_o = 0.4/0.8 = 0.5\text{ V}$$

Hence,

$$\boldsymbol{i_o = \frac{v_o}{5k} = \frac{0.5}{5k} = \underline{0.1\,\text{mA}}}$$

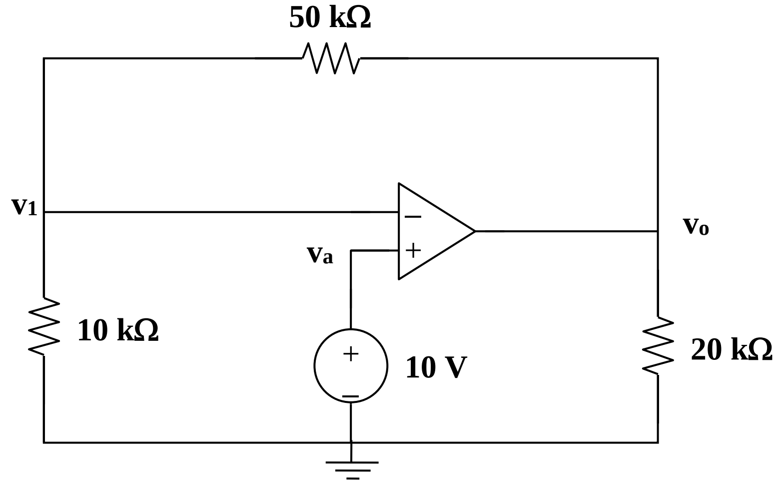
Solution 5.27

This is a voltage follower.

v_1 = [24/(24+16)]7.5 = 4.5 V; v_2 = v_1 = 4.5 V; and

v_o = [12/(12+8)]4.5 = **2.7 V**.

Solution 5.28



At node 1, $\dfrac{0 - v_1}{10k} = \dfrac{v_1 - v_o}{50k}$

But $v_1 = 10V$,

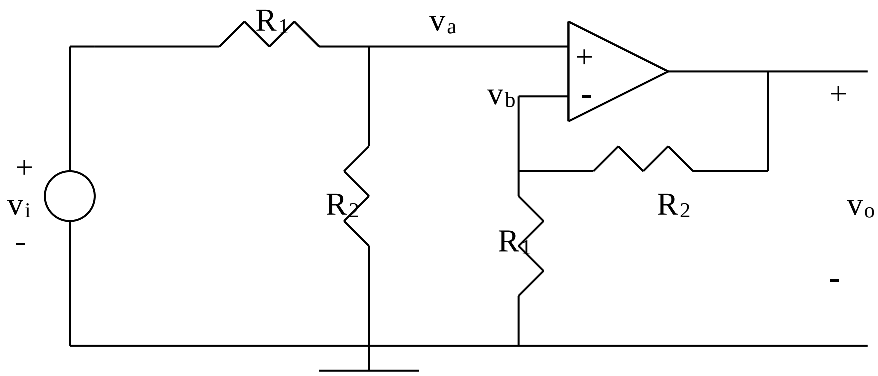
$-5v_1 = v_1 - v_o$, leads to $v_o = 6v_1 = \mathbf{60V}$

Alternatively, viewed as a noninverting amplifier,

$v_o = (1 + (50/10))\ (10V) = \mathbf{60V}$

$i_o = v_o/(20k) = 60/(20k) = \mathbf{3\ mA}$.

Solution 5.29



$$v_a = \frac{R_2}{R_1 + R_2} v_i, \qquad v_b = \frac{R_1}{R_1 + R_2} v_o$$

But $v_a = v_b$ $\longrightarrow$ $\frac{R_2}{R_1 + R_2} v_i = \frac{R_1}{R_1 + R_2} v_o$

Or

$$\underline{\boldsymbol{\frac{v_o}{v_i} = \frac{R_2}{R_1}}}$$

Solution 5.30

In the circuit shown in Fig. 5.68, find $\boldsymbol{i_x}$ and the power absorbed by the 20-kΩ resistor.

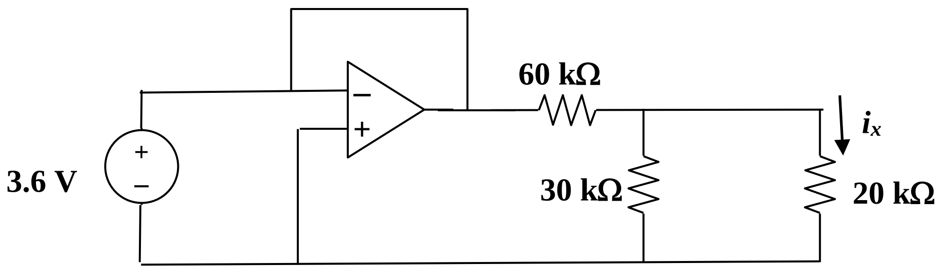


Figure 5.68
For Prob. 5.30.

Solution

The op amp is clearly a voltage follower with an output voltage equal to 3.6 V.

If we combine the parallel resistors together we get [20kx30k/(20k+30k)] = 12k.

The current through the 60 kΩ resistor is equal to 3.6/(60k+12k) = 50 μA. We now have a current divider and $i_x = 50\text{x}10^{-6}\text{x}30\text{k}/(30\text{k}+20\text{k})$ = **30 μA**.

Now we can calculate the power absorbed by the 20 kΩ resistor,

$p_{20} = (i_x)^2\text{x}20\text{k}$ = **18 μW**.

Solution 5.31

For the circuit in Fig. 5.69, find i_x.

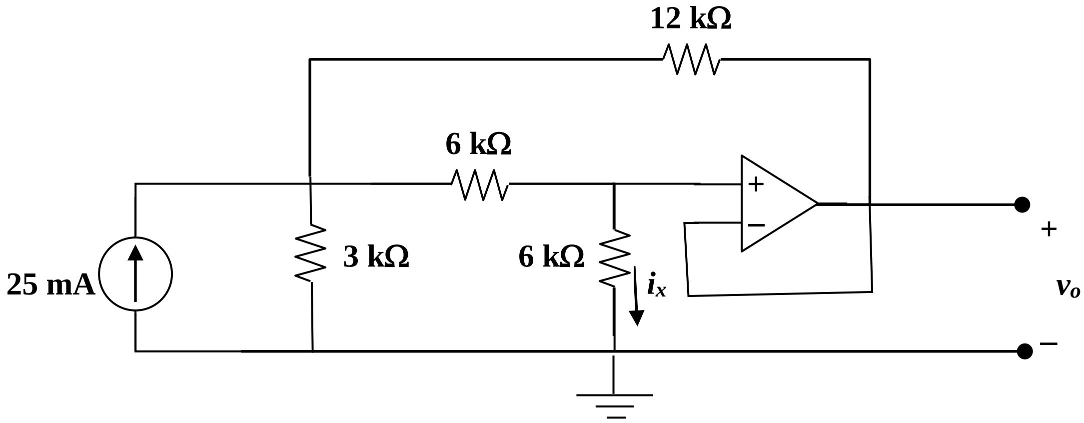
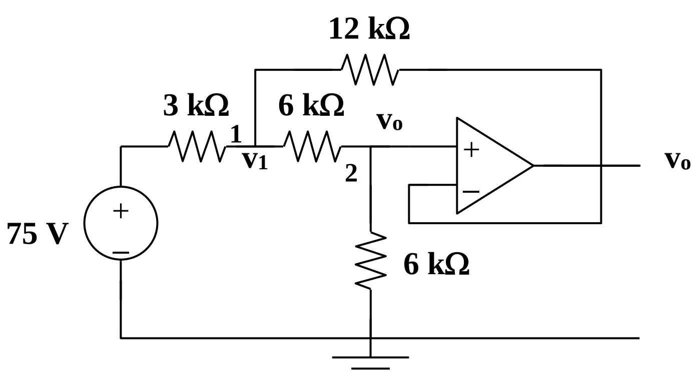


Figure 5.69
For Prob. 5.31.

Solution

After converting the current source to a voltage source, the circuit is as shown below:



At node 1, $[(v_1–75)/3k] + [(v_1–v_o)/6k] + [(v_1–v_o)/12k] = 0$ or

$7v_1 – 3v_o = 300$ (1)

At node 2, $[(v_o–v_1)/6k] + [(v_o–0)/6k] + 0 = 0$ or $v_1 = 2v_o$ (2)

Finally, $i_x = [(v_o–0)/6k]$ (3)

From (1), (2), and (3) $14v_o – 3v_o = 300$ or $v_o = 27.27$ V and

$i_x =$ **4.545 mA**.

Solution 5.32

Calculate i_x and v_o in the circuit of Fig. 5.70. Find the power dissipated by the 60-kΩ resistor.

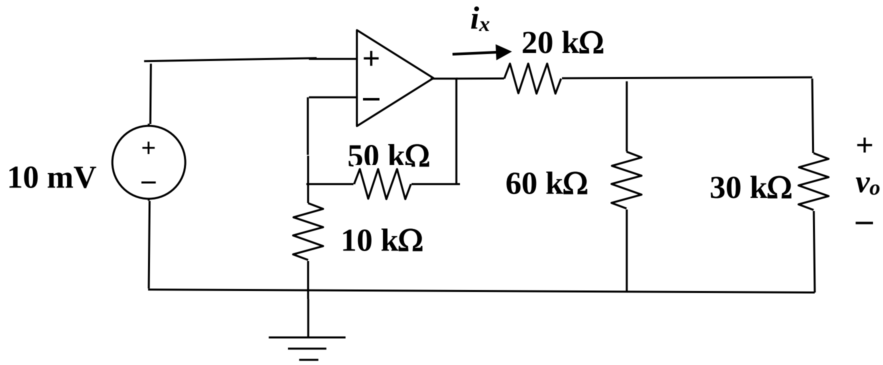


Figure 5.70
For Prob. 5.32.

Solution

Let
v_x = the voltage at the output of the op amp. The given circuit is a non-inverting amplifier.

$$v_x = \left(1 + \frac{50}{10}\right)(10 \text{ mV}) = 60 \text{ mV}$$

$$60 \| 30 = 20\text{k}\Omega$$

By voltage division,

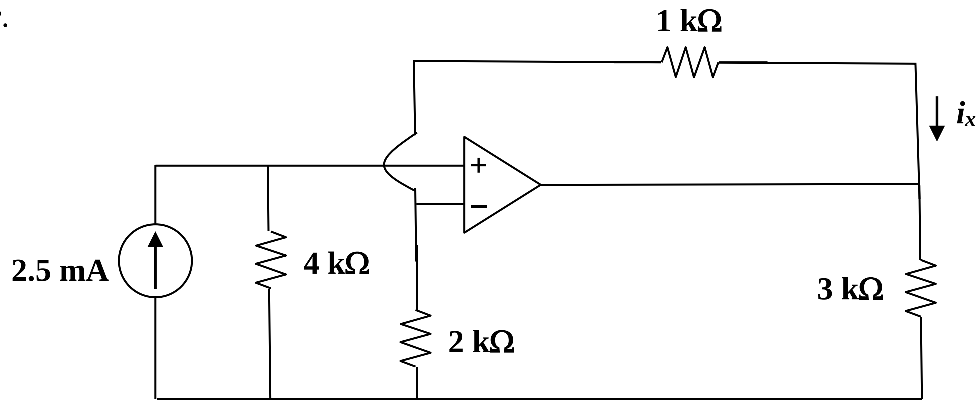
$$v_o = \frac{20}{20+20} v_x = \frac{v_x}{2} = 30 \text{ mV}$$

$$i_x = \frac{v_x}{(20+20)k} = \frac{60mV}{40k} = \mathbf{1.5\ \mu A}$$

$$p = \frac{v_o^2}{R} = \frac{900x10^{-6}}{60x10^3} = \mathbf{15\ nW}.$$

Solution 5.33

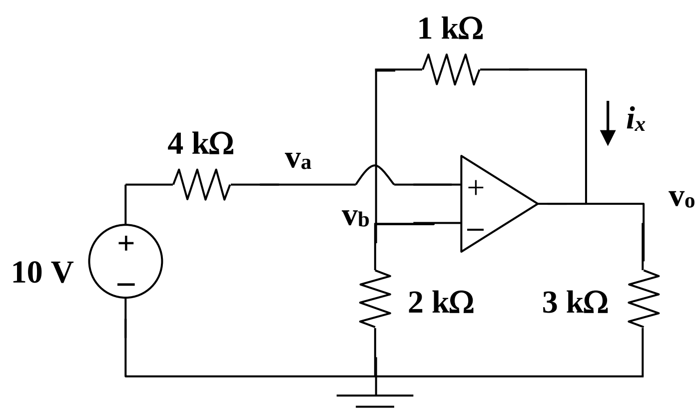
Refer to the op amp circuit in Fig. 5.71. Calculate $\boldsymbol{i_x}$ and the power absorbed by the 3-kΩ resistor.



'igure 5.71
For Prob. 5.33.

Solution

After transforming the current source, the current is as shown below:



This is a noninverting amplifier which means that $[(v_b–0)/2k] + [(v_b–v_o)/1k] + 0 = 0$ and $v_a = 10 = v_b$. Thus, $2v_o = 3v_b = 30$ or $v_o = 15$ V.

Now $i_x = [(v_b–v_o)/1k] = –5/1k =$ **–1 mA** and $p_{3k} = (v_o)^2/3k =$ **75 mW**.

Solution 5.34

$$\frac{v_1 - v_{in}}{R_1} + \frac{v_1 - v_{in}}{R_2} = 0 \qquad (1)$$

but

$$v_a = \frac{R_3}{R_3 + R_4} v_o \qquad (2)$$

Combining (1) and (2),

$$v_1 - v_a + \frac{R_1}{R_2} v_2 - \frac{R_1}{R_2} v_a = 0$$

$$v_a\left(1 + \frac{R_1}{R_2}\right) = v_1 + \frac{R_1}{R_2} v_2$$

$$\frac{R_3 v_o}{R_3 + R_4}\left(1 + \frac{R_1}{R_2}\right) = v_1 + \frac{R_1}{R_2} v_2$$

$$v_o = \frac{R_3 + R_4}{R_3\left(1 + \frac{R_1}{R_2}\right)}\left(v_1 + \frac{R_1}{R_2} v_2\right)$$

$$v_O = \underline{\boldsymbol{\frac{R_3 + R_4}{R_3(R_1 + R_2)}(v_1 R_2 + v_2)}}$$

Solution 5.35

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_f}{R_i} = 7.5 \longrightarrow R_f = 6.5R_i$$

If R_i = **60 kΩ**, R_f = **390 kΩ**.

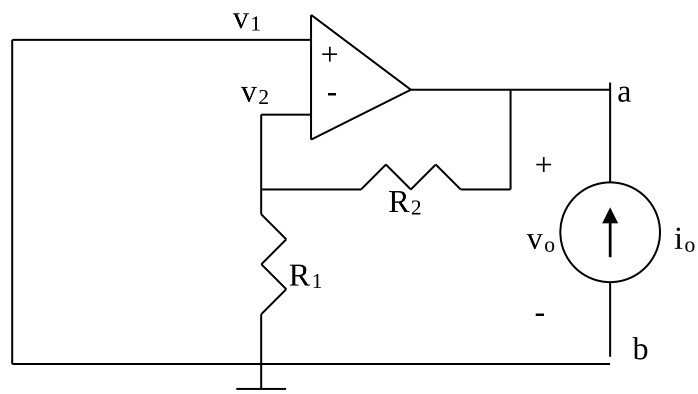
Solution 5.36

$V_{Th} = V_{ab}$

But $\quad v_s = \dfrac{R_1}{R_1 + R_2} V_{ab}$. Thus,

$$V_{Th} = V_{ab} = \frac{R_1 + R_2}{R_1} v_s = \underline{(1 + \frac{R_2}{R_1}) v_s}$$

To get R_{Th}, apply a current source I_o at terminals a-b as shown below.



Since the noninverting terminal is connected to ground, $v_1 = v_2 = 0$, i.e. no current passes through R_1 and consequently R_2 . Thus, $v_o = 0$ and

$$\underline{\boldsymbol{R_{Th} = \frac{v_o}{i_o} = 0}}$$

Solution 5.37

Determine the output of the summing amplifier in Fig. 5.74.

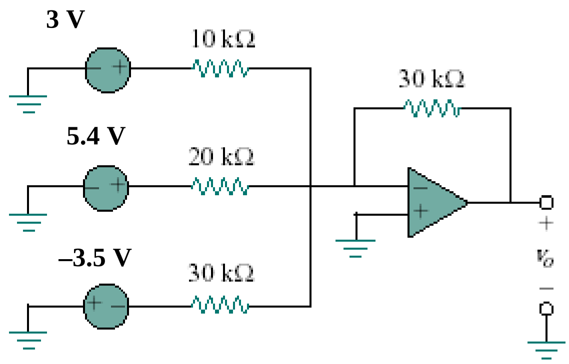


Figure 5.74
For Prob. 5.37.

Solution

$$v_o = -\left[\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right]$$

$$= -\left[\frac{30}{10}(3) + \frac{30}{20}(5.4) + \frac{30}{30}(-3.5)\right]$$

v_o = **–13.6 V**.

Solution 5.38

Using Fig. 5.75, design a problem to help other students better understand summing amplifiers.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Calculate the output voltage due to the summing amplifier shown in Fig. 5.75.

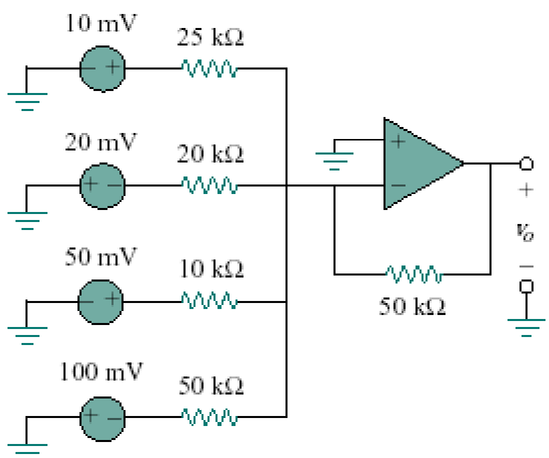


Figure 5.75

Solution

$$v_o = -\left[\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3 + \frac{R_f}{R_4}v_4\right]$$

$$= -\left[\frac{50}{25}(10) + \frac{50}{20}(-20) + \frac{50}{10}(50) + \frac{50}{50}(-100)\right]$$

$$= \mathbf{-120mV}$$

Solution 5.39

For the op amp circuit in Fig. 5.76, determine the value of v_2 in order to make v_o = –7.5 V.

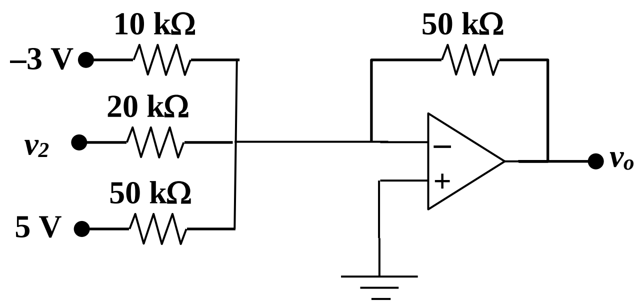


Figure 5.76
For Prob. 5.39.

Solution

This is a summing amplifier.

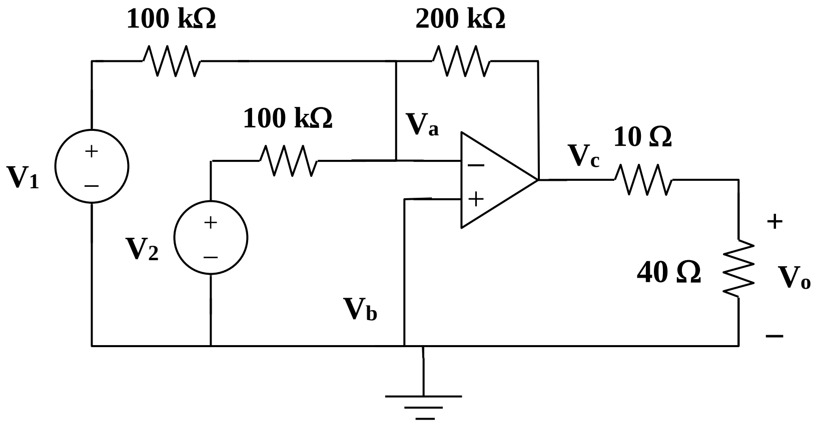
$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right) = -\left(\frac{50}{10}(-3) + \frac{50}{20}v_2 + \frac{50}{50}(5)\right) = 10 - 2.5v_2 =$$

15 – 2.5v_2 – 5 = 10 – 2.5v_2 = –7.5 or v_2 = 17.5/2.5 or

v_2 = **7 V**.

Solution 5.40

Determine V_o in terms of V_1 and V_2.



Step 1. Label the reference and node voltages in the circuit, see above. Note we now can consider nodes a and b, we cannot write a node equation at c without introducing another unknown. The node equation at a is $[(V_a–V_1)/10^5] + [(V_a–V_2)/10^5] + 0 + [(V_a–V_c)/2x10^5] = 0$. At b it is clear that $V_b = 0$. Since we have two equations and three unknowns, we need another equation. We do get that from the constraint equation, $V_a = V_b$. After we find V_c in terms of V_1 and V_2, we then can determine V_o which is equal to $[(V_c–0)/50]$ times 40.

Step 2. Letting $V_a = V_b = 0$, the first equation can be simplified to,

$$[–V_1/10^5] + [–V_2/10^5] + [–V_c/2x10^5] = 0$$

Taking V_c to the other side of the equation and multiplying everything by $2x10^5$, we get,

$$V_c = –2V_1 – 2V_2$$

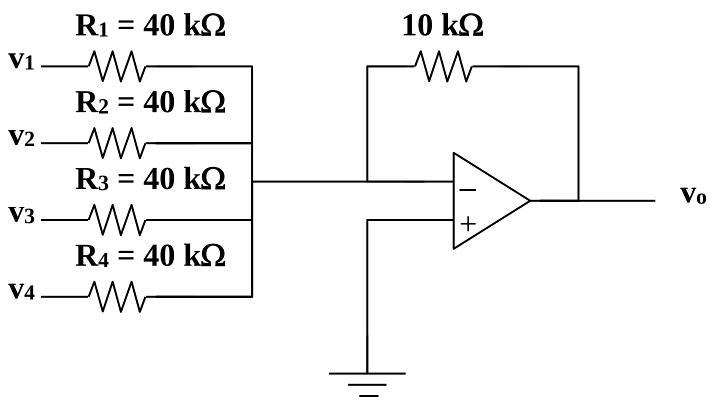
Now we can find V_o which is equal to $(40/50)V_c = 0.8[–2V_1–2V_2]$

V_o = $\mathbf{–1.6V_1–1.6V_2}$.

Solution 5.41

$R_f/R_i = 1/(4) \longrightarrow R_i = 4R_f = 40\text{k}\Omega$

The averaging amplifier is as shown below:



Solution 5.42

The feedback resistor of a three-input averaging summing amplifier is 50 kΩ. What are the values of R_1, R_2, and R_3?

Solution

Since the average of three numbers is the sum of those numbers divided by three, the value of the feedback resistor needs to be equal to one-third of the input resistors or, $R_i = 3R_f$ where i = 1, 2, and 3. Therefore,

$R_1 = R_2 = R_3 = 3 \times 50{,}000 =$ **150 kΩ**.

Solution 5.43

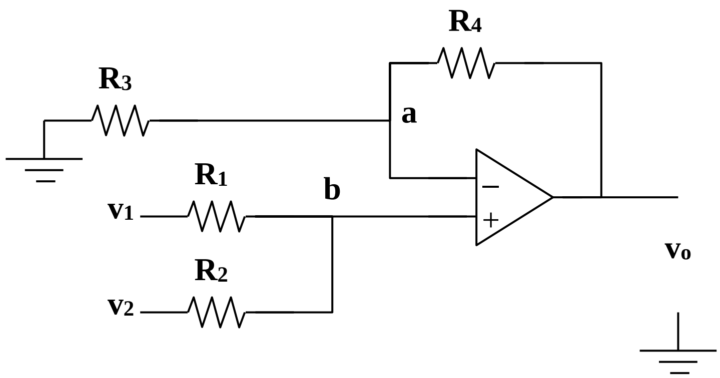
The feedback resistor of a five-input averaging summing amplifier is 40 kΩ. What are the values of R_1, R_2, R_3, R_4, and R_5?

Solution

In order to find the average of five inputs each input resistor needs to be five times the feedback resistor or,

$R_1 = R_2 = R_3 = R_4 = R_5 = 5x40{,}000 =$ **200 kΩ**.

Solution 5.44



At node b, $\dfrac{v_b - v_1}{R_1} + \dfrac{v_b - v_2}{R_2} = 0 \longrightarrow v_b = \dfrac{\dfrac{v_1}{R_1} + \dfrac{v_2}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$ (1)

At node a, $\dfrac{0 - v_a}{R_3} = \dfrac{v_a - v_o}{R_4} \longrightarrow v_a = \dfrac{v_o}{1 + R_4 / R_3}$ (2)

But $v_a = v_b$. We set (1) and (2) equal.

$$\frac{v_o}{1 + R_4 / R_3} = \frac{R_2 v_1 + R_1 v_2}{R_1 + R_2}$$

or

$$v_o = \frac{\left(\boldsymbol{R_3 + R_4}\right)}{\boldsymbol{R_3\left(R_1 + R_2\right)}}\left(\boldsymbol{R_2 v_1 + R_1 v_2}\right)$$

Solution 5.45

Design an op amp circuit to perform the following operation:

$v_o = 3.5v_1 - 2.5v_2$

All resistances must be ≤ 100 kΩ.

Solution

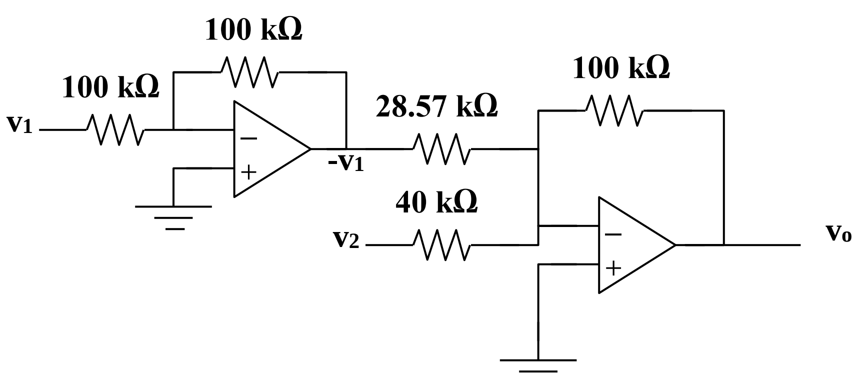
This can be achieved as follows:

$$v_o = -\left[\frac{R}{R/3}(-v_1) + \frac{R}{R/2}v_2\right]$$

$$= -\left[\frac{R_f}{R_1}(-v_1) + \frac{R_f}{R_2}v_2\right]$$

i.e. $R_f = R$, $R_1 = R/3.5$, and $R_2 = R/2.5$

Thus we need an inverter to invert v_1, and a summer, as shown below (R≤100kΩ). Let us pick R = 100 kΩ. Note, we can have an infinite number of values that satisfy the conditions. We will use the value of R = 100 kΩ for our design.



Solution 5.46

Using only two op amps, design a circuit to solve,

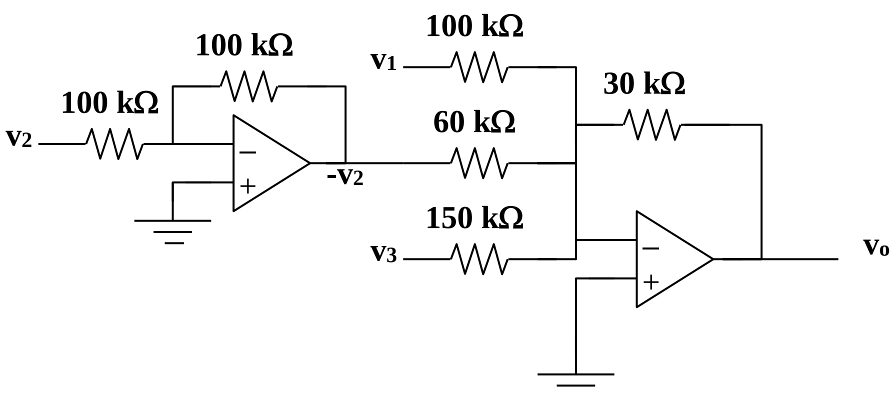
$$-v_{out} = \frac{v_3 - v_1}{5} + \frac{v_1 - v_2}{2}.$$

Solution

Although there are several ways to accomplish this, the easiest way is to simplify the above equation and then design the circuit. Thus,

$-v_{out} = (-0.2+0.5)v_1 - 0.5v_2 + 0.2v_3$ or $-v_{out} = 0.3v_1 - 0.5v_2 + 0.2v_3$.

Let us pick R_f = 30 kΩ. To obtain the proper gains, R_1 =100 kΩ, R_2 = 60 kΩ, and R_3 = 150 kΩ. (30,000/10,000) = 0.3, (30,000/60,000) = 0.5, and (30,000/150,000) = 0.2.



Solution 5.47

Using eq. (5.18), $R_1 = 2k\Omega,\ \mathrm{R}_2 = 30k\Omega,\ \ \mathrm{R}_3 = 2k\Omega,\ \ \mathrm{R}_4 = 20k\Omega$

$$v_o = \frac{30(1+2/30)}{2(1+2/20)}v_2 - \frac{30}{2}V_1 = \frac{32}{2.2}(2) - 15(1) = \underline{14.09\ \mathrm{V}}$$

= **14.09 V**.

Solution 5.48

The circuit in Fig. 5.80 is a differential amplifier driven by a bridge. Find v_o.

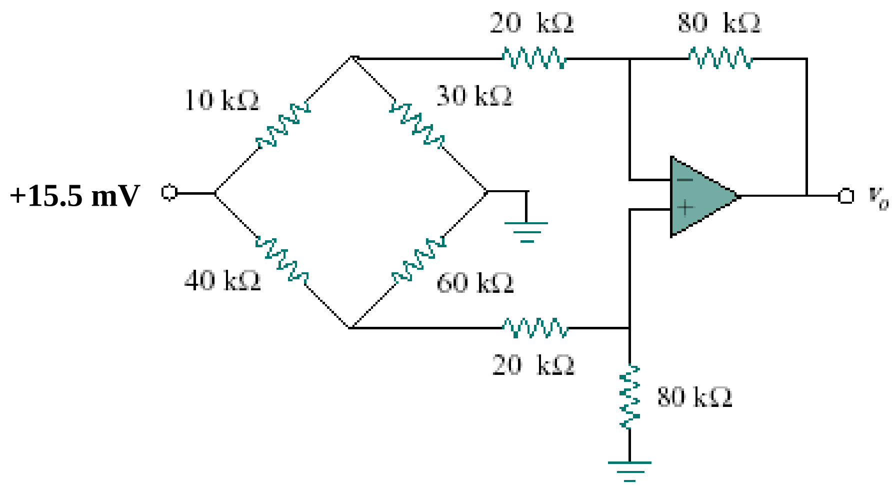


Figure 5.80
For Prob. 5.48.

Solution

We can break this problem up into parts. The 15.5 mV source separates the lower circuit from the upper. In addition, there is no current flowing into the input of the op amp which means we now have the 40-kohm resistor in series with a parallel combination of the 60-kohm resistor and the equivalent 100-kohm resistor.

Thus, $\quad 40\text{k} + (60\text{x}100\text{k})/(160) = 77.5\text{k}$

which leads to the current flowing through this part of the circuit,

$$i = 15.5\ \text{m}/77.5\text{k} = 200\text{x}10^{-9}\ \text{A}$$

The voltage across the 60k and equivalent 100k is equal to,

$$v = i\text{x}37.5\text{k} = 7.5\ \text{mV}$$

We can now calculate the voltage across the 80-kohm resistor.

$$v_{80} = 0.8\text{x}7.5\ \text{m} = 6\ \text{mV}$$

which is also the voltage at both inputs of the op amp and the voltage between the 20-kohm and 80-kohm resistors in the upper circuit. Let v_1 be the voltage to the left of the 20-kohm resistor of the upper circuit and we can write a node equation at that node.

$$(v_1 – 15.5m)/(10k) + v_1/30k + (v_1 – 6m)/20k = 0$$

or $6v_1 – 93 + 2v_1 + 3v_1 – 18 = 0$ or $v_1 = 10.091$ mV.

The current through the 20k-ohm resistor, left to right, is,

$$i_{20} = (10.091m – 6m)/20k = 204.55x10^{–9} \text{ A}$$

thus, $v_o = 6m – 204.55x10^{–9}x80k =$ **–10.364 mV**.

Solution 5.49

$R_1 = R_3 = 20k\Omega$, $R_2/(R_1) = 4$

i.e. $R_2 = 4R_1 = 80k\Omega = R_4$

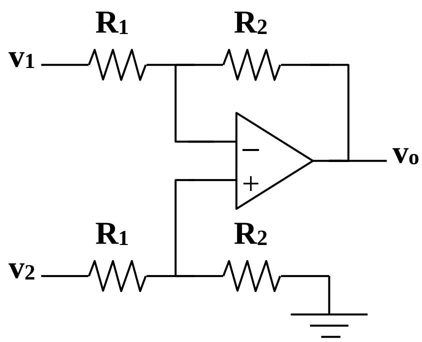
Verify: $$v_o = \frac{R_2}{R_1}\frac{1+R_1/R_2}{1+R_3/R_4}v_2 - \frac{R_2}{R_1}v_1$$

$$= 4\frac{(1+0.25)}{1+0.25}v_2 - 4v_1 = 4(v_2 - v_1)$$

Thus, $R_1 = R_3 =$ **20 kΩ**, $R_2 = R_4 =$ **80 kΩ**.

Solution 5.50

(a) We use a difference amplifier, as shown below:



$$v_o = \frac{R_2}{R_1}(v_2 - v_1) = 2.5(v_2 - v_1),$$ i.e. $R_2/R_1 = 2.5$

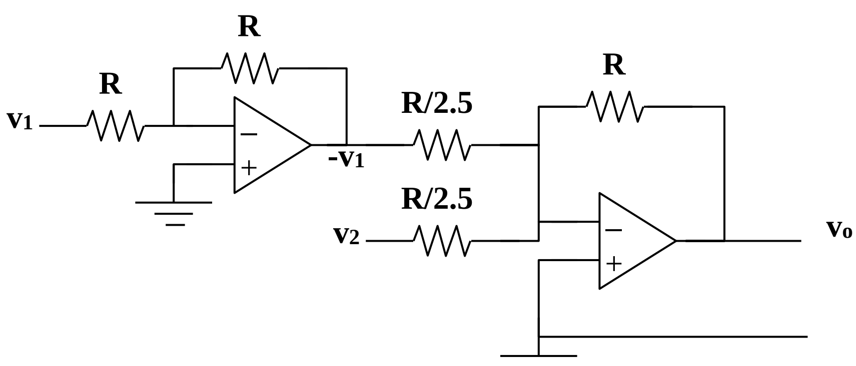
If R_1 = **100 kΩ** then R_2 = **250kΩ**

(b) We may apply the idea in Prob. 5.35.

$$\begin{aligned} v_0 &= 2.5v_1 - 2.5v_2 \\ &= -\left[\frac{R}{R/2}(-v_1) + \frac{R}{R/2}v_2\right] \\ &= -\left[\frac{R_f}{R_1}(-v_1) + \frac{R_f}{R_2}v_2\right] \end{aligned}$$

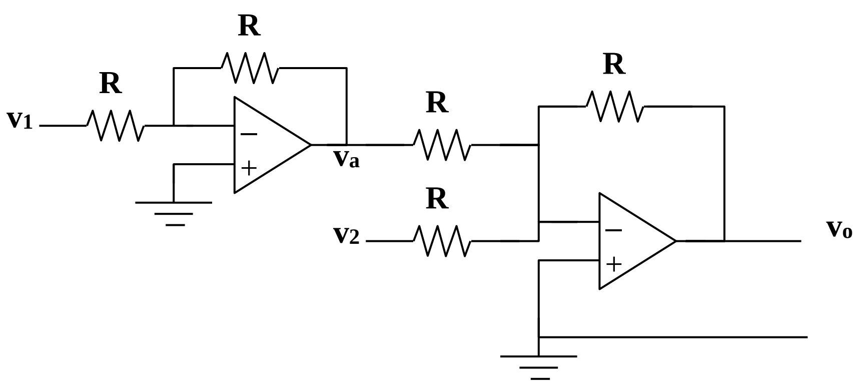
i.e. $R_f = R$, $R_1 = R/2.5 = R_2$

We need an inverter to invert v_1 and a summer, as shown below. We may let $R = 100$ kΩ.



Solution 5.51

We achieve this by cascading an inverting amplifier and two-input inverting summer as shown below:



Verify:

$$v_o = -v_a - v_2$$

But $\quad v_a = -v_1$. Hence

$$v_o = v_1 - v_2.$$

Solution 5.52

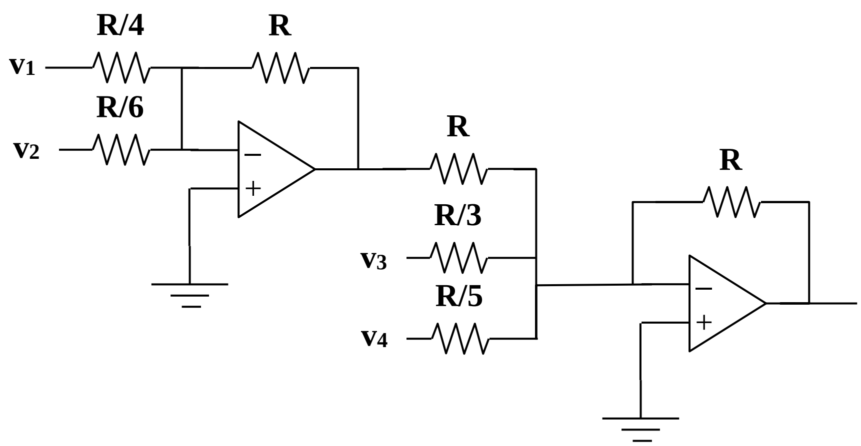
Design an op amp circuit such that

$v_o = 4v_1 + 6v_2 - 3v_3 - 5v_4$

Let all the resistors be in the range of 20 to 200 kΩ.

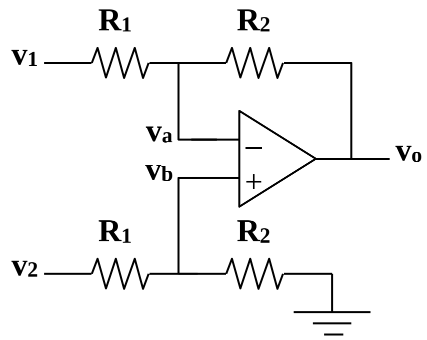
Solution

A summing amplifier shown below will achieve the objective. An inverter is inserted to invert v_2. Since the smallest resistance must be at least 20 kΩ, then let R/6 = 20kΩ therefore let R = **120 kΩ**.



Solution 5.53

(a)



At node a,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2} \longrightarrow v_a = \frac{R_2 v_1 + R_1 v_o}{R_1 + R_2} \qquad (1)$$

At node b, $$v_b = \frac{R_2}{R_1 + R_2} v_2 \qquad (2)$$

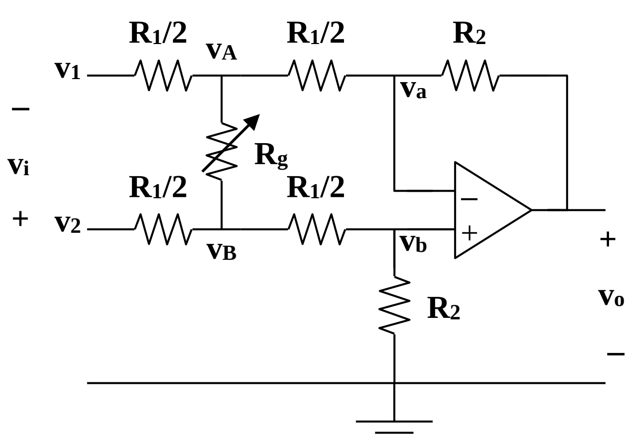
But $v_a = v_b$. Setting (1) and (2) equal gives

$$\frac{R_2}{R_1 + R_2} v_2 = \frac{R_2 v_1 + R_1 v_o}{R_1 + R_2}$$

$$v_2 - v_1 = \frac{R_1}{R_2} v_o = v_i$$

$$\frac{v_o}{v_i} = \underline{\frac{R_2}{R_1}}$$

(b)



At node A, $$\frac{v_1 - v_A}{R_1/2} + \frac{v_B - v_A}{R_g} = \frac{v_A - v_a}{R_1/2}$$

or $$v_1 - v_A + \frac{R_1}{2R_g}\left(v_B - v_A\right) = v_A - v_a \qquad (1)$$

At node B, $$\frac{v_2 - v_B}{R_1/2} = \frac{v_B - v_A}{R_1/2} + \frac{v_B - v_b}{R_g}$$

or $$v_2 - v_B - \frac{R_1}{2R_g}(v_B - v_A) = v_B - v_b \qquad (2)$$

Subtracting (1) from (2),

$$v_2 - v_1 - v_B + v_A - \frac{2R_1}{2R_g}\left(v_B - v_A\right) = v_B - v_A - v_b + v_a$$

Since, $v_a = v_b$,

$$\frac{v_2 - v_1}{2} = \left(1 + \frac{R_1}{2R_g}\right)\left(v_B - v_A\right) = \frac{v_i}{2}$$

or $$v_B - v_A = \frac{v_i}{2} \cdot \frac{1}{1 + \frac{R_1}{2R_g}} \qquad (3)$$

But for the difference amplifier,

$$v_o = \frac{R_2}{R_1/2}\left(v_B - v_A\right)$$

or $$v_B - v_A = \frac{R_1}{2R_2} v_o \qquad (4)$$

Equating (3) and (4), $$\frac{R_1}{2R_2} v_o = \frac{v_i}{2} \cdot \frac{1}{1 + \frac{R_1}{2R_g}}$$

$$\underline{\frac{v_o}{v_i} = \frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{R_1}{2R_g}}}$$

(c) At node a, $\dfrac{v_1 - v_a}{R_1} = \dfrac{v_a - v_A}{R_2/2}$

$$v_1 - v_a = \frac{2R_1}{R_2}v_a - \frac{2R_1}{R_2}v_A \qquad (1)$$

At node b, $v_2 - v_b = \dfrac{2R_1}{R_2}v_b - \dfrac{2R_1}{R_2}v_B \qquad (2)$

Since $v_a = v_b$, we subtract (1) from (2),

$$v_2 - v_1 = \frac{-2R_1}{R_2}(v_B - v_A) = \frac{v_i}{2}$$

or $\quad v_B - v_A = \dfrac{-R_2}{2R_1}v_i \qquad (3)$

At node A,

$$\frac{v_a - v_A}{R_2/2} + \frac{v_B - v_A}{R_g} = \frac{v_A - v_o}{R/2}$$

$$v_a - v_A + \frac{R_2}{2R_g}\left(v_B - v_A\right) = v_A - v_o \qquad (4)$$

At node B, $\dfrac{v_b - v_B}{R/2} - \dfrac{v_B - v_A}{R_g} = \dfrac{v_B - 0}{R/2}$

$$v_b - v_B - \frac{R_2}{2R_g}\left(v_B - v_A\right) = v_B \qquad (5)$$

Subtracting (5) from (4),

$$v_B - v_A + \frac{R_2}{R_g}\left(v_B - v_A\right) = v_A - v_B - v_o$$

$$2\left(v_B - v_A\right)\left(1 + \frac{R_2}{2R_g}\right) = -v_o \qquad (6)$$

Combining (3) and (6),

$$\frac{-R_2}{R_1}v_i\left(1 + \frac{R_2}{2R_g}\right) = -v_o$$

$$\boldsymbol{\frac{v_o}{v_i} = \frac{R_2}{R_1}\left(1 + \frac{R_2}{2R_g}\right)}$$

Solution 5.54

The first stage is a summer (please note that we let the output of the first stage be v_1).

$$v_1 = -\left(\frac{R}{R}v_s + \frac{R}{R}v_o\right) = -v_s - v_o$$

The second stage is a noninverting amplifier

$$v_o = (1 + R/R)v_1 = 2v_1 = 2(-v_s - v_o) \quad \text{or } 3v_o = -2v_s$$

$$v_o/v_s = \mathbf{-0.6667}.$$

Solution 5.55

Let $A_1 = k$, $A_2 = k$, and $A_3 = k/(4)$

$$A = A_1A_2A_3 = k^3/(4)$$

$$20\text{Log}_{10}A = 42$$

$$\text{Log}_{10}A = 2.1 \longrightarrow \quad A = 10^{2.1} = 125.89$$

$k^3 = 4A = 503.57$

$k = \sqrt[3]{503.57} = 7.956$

Thus

$A_1 = A_2 =$ **7.956**, $A_3 =$ **1.989**

Solution 5.56

Using Fig. 5.83, design a problem to help other students better understand cascaded op amps.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Calculate the gain of the op amp circuit shown in Fig. 5.83.

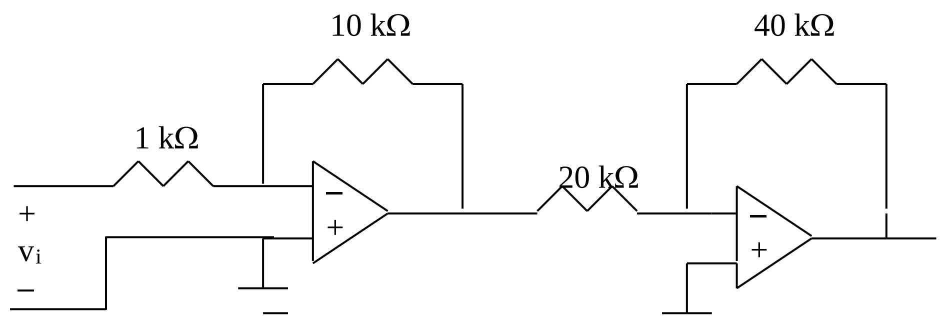


Figure 5.83 For Prob. 5.56.

Solution

Each stage is an inverting amplifier. Hence,

$$\frac{v_o}{v_s} = \left(-\frac{10}{1}\right)\left(-\frac{40}{20}\right) = \underline{20}$$

Solution 5.57

Let v_1 be the output of the first op amp and v_2 be the output of the second op amp.

The first stage is an inverting amplifier.

$$v_1 = -\frac{50}{25}v_{s1} = -2v_{s1}$$

The second state is a summer.

$$v_2 = -(100/50)v_{s2} - (100/100)v_1 = -2v_{s2} + 2v_{s1}$$

The third state is a noninverting amplifier

$$v_o = (1+\frac{100}{50})v_2 = 3v_2 = \underline{6v_{s1} - 6v_{s2}}$$

Solution 5.58

Calculate i_o in the op amp circuit of Fig. 5.85.

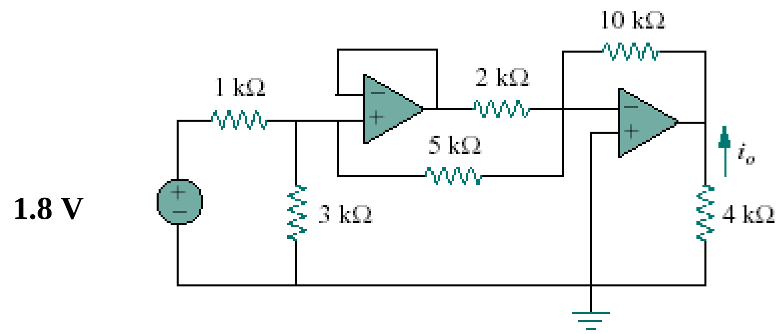


Figure 5.85
For Prob. 5.58.

Solution

Looking at the circuit, the voltage at the right side of the 5-kΩ resistor must be at 0V if the op amps are working correctly. Thus the 1-kΩ is in series with the parallel combination of the 3-kΩ and the 5-kΩ. By voltage division, the input to the voltage follower is:

$$v_1 = \frac{3\|5}{1+3\|5}(1.8) = 1.1739V = \text{to the output of the first op amp.}$$

Thus,

$$v_o = -10((1.1739/5)+(1.1739/2)) = -8.217 \text{ V}.$$

$$i_o = \frac{0 - v_o}{4k} = \mathbf{2.054\ mA}.$$

Solution 5.59

The first stage is a noninverting amplifier. If v_1 is the output of the first op amp,

$$v_1 = (1 + 2R/R)v_s = 3v_s$$

The second stage is an inverting amplifier

$$v_o = -(4R/R)v_1 = -4v_1 = -4(3v_s) = -12v_s$$

$v_o/v_s =$ **-12**.

Solution 5.60

Calculate v_o/v_i in the op amp circuit in Fig. 5.87.

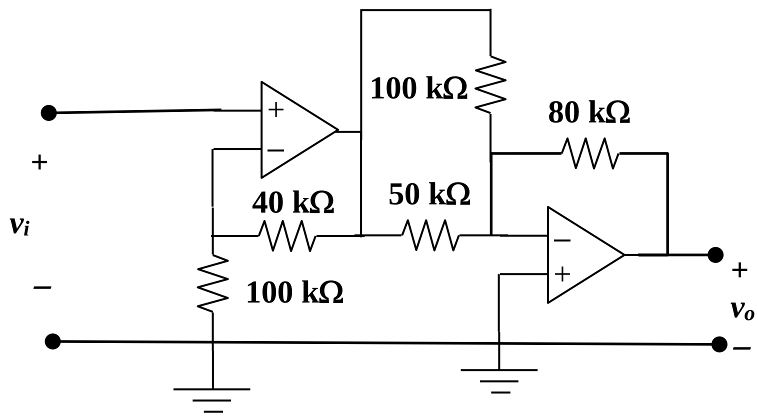


Figure 5.87
For Prob. 5.60.

Solution

The first stage is a noninverting amp with an output voltage equal to $[140k/100k]v_i = 1.4v_i$.

At the negative input of the second op amp we get $[(0–1.4v_i)/100k] + [(0–1.4v_i)/50k] + [(0–v_o)/80k] + 0 = 0$. Thus, $v_o = –[80k(3/100k)]1.4v_i = –3.36v_i$. This leads to,

$v_o/v_i =$ **–3.36**.

Solution 5.61

Determine v_o in the circuit of Fig. 5.88.

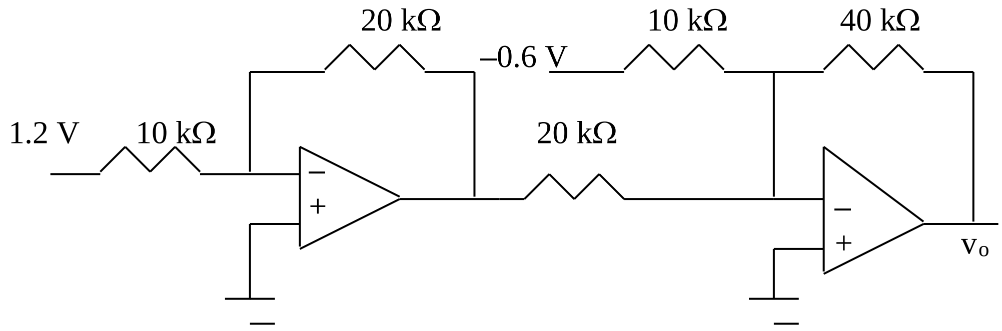


Figure 5.88
For Prob. 5.61.

Solution

The first op amp is an inverter. If v_1 is the output of the first op amp,

$$v_1 = -(20/10)(1.2) = -2.4 \text{ V}$$

The second op amp is a summer

$$V_o = -(40/10)(-0.6) - (40/20)(-2.4) = 2.4 + 4.8$$

$$= \mathbf{7.2\ V}.$$

Solution 5.62

Let v_1 = output of the first op amp
v_2 = output of the second op amp

The first stage is a summer

$$v_1 = -\frac{R_2}{R_1} v_i - \frac{R_2}{R_f} v_o \qquad (1)$$

The second stage is a follower. By voltage division

$$v_o = v_2 = \frac{R_4}{R_3 + R_4} v_1 \longrightarrow v_1 = \frac{R_3 + R_4}{R_4} v_o \qquad (2)$$

From (1) and (2),

$$\left(1 + \frac{R_3}{R_4}\right) v_o = -\frac{R_2}{R_1} v_i - \frac{R_2}{R_f} v_o$$

$$\left(1 + \frac{R_3}{R_4} + \frac{R_2}{R_f}\right) v_o = -\frac{R_2}{R_1} v_i$$

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \cdot \frac{1}{1 + \dfrac{R_3}{R_4} + \dfrac{R_2}{R_f}} = \boldsymbol{\frac{-R_2 R_4 R_f}{R_1\left(R_2 R_4 + R_3 R_f + R_4 R_f\right)}}$$

Solution 5.63

The two op amps are summers. Let v_1 be the output of the first op amp. For the first stage,

$$v_1 = -\frac{R_2}{R_1}v_i - \frac{R_2}{R_3}v_o \qquad (1)$$

For the second stage,

$$v_o = -\frac{R_4}{R_5}v_1 - \frac{R_4}{R_6}v_i \qquad (2)$$

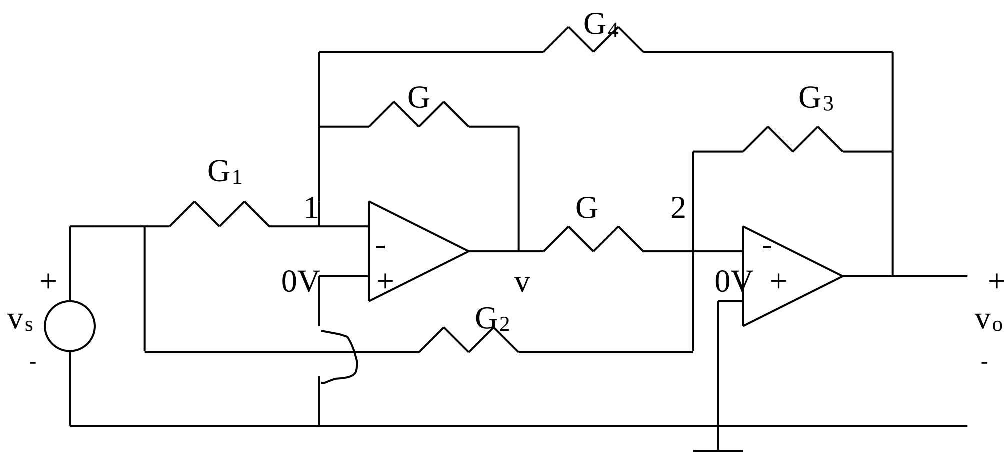
Combining (1) and (2),

$$v_o = \frac{R_4}{R_5}\left(\frac{R_2}{R_1}\right)v_i + \frac{R_4}{R_5}\left(\frac{R_2}{R_3}\right)v_o - \frac{R_4}{R_6}v_i$$

$$v_o\left(1 - \frac{R_2R_4}{R_3R_5}\right) = \left(\frac{R_2R_4}{R_1R_5} - \frac{R_4}{R_6}\right)v_i$$

$$\boldsymbol{\frac{v_o}{v_i} = \frac{\frac{R_2R_4}{R_1R_5} - \frac{R_4}{R_6}}{1 - \frac{R_2R_4}{R_3R_5}}}$$

Solution 5.64



At node 1, $v_1=0$ so that KCL gives

$$G_1 v_s + G_4 v_o = -Gv \qquad (1)$$

At node 2,

$$G_2 v_s + G_3 v_o = -Gv \qquad (2)$$

From (1) and (2),

$$G_1 v_s + G_4 v_o = G_2 v_s + G_3 v_o \longrightarrow (G_1 - G_2)v_s = (G_3 - G_4)v_o$$

or

$$\underline{\boldsymbol{\frac{v_o}{v_s} = \frac{G_1 - G_2}{G_3 - G_4}}}$$

Solution 5.65

Find v_o in the op amp circuit of Fig. 5.92.

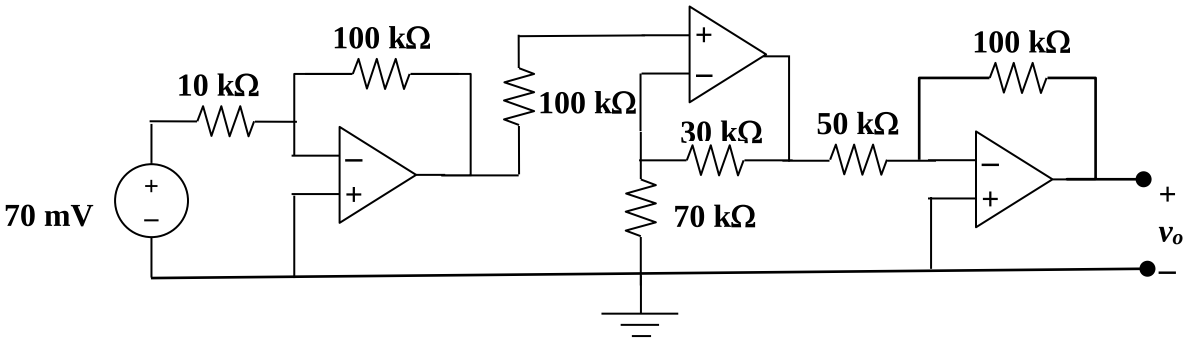


Figure 5.92
For Prob. 5.65.

Solution

The output of the first op amp (to the left) is –(100k/10k)0.07 = –700 mV. The output of the second op amp has to be equal to –0.7[100k/70k] = –1 V. Finally the output of the third op amp is –[100k/50k](–1) = **2 V**.

Solution 5.66

We can start by looking at the contributions to v_o from each of the sources and the fact that each of them go through inverting amplifiers.

The 6 V source contributes –[100k/25k]6; the 4 V source contributes –[40k/20k][–(100k/20k)]4; and the 2 V source contributes –[100k/10k]2 or

$$v_o = \frac{-100}{25}(6) - \frac{40}{20}\left(-\frac{100}{20}\right)(4) - \frac{100}{10}(2)$$

$$= -24 + 40 - 20 = \mathbf{-4V}$$

Solution 5.67

Obtain the output v_o in the circuit of Fig. 5.94.

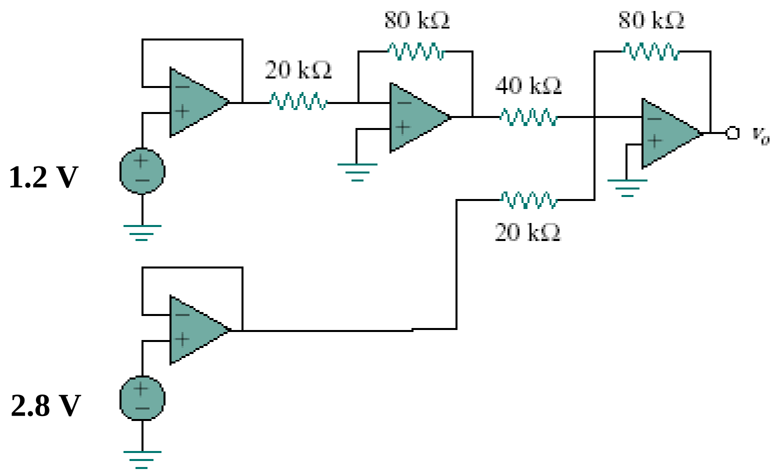


Figure 5.94
For Prob. 5.67.

Solution

$$v_o = -\frac{80}{40}\left(-\frac{80}{20}\right)(1.2) - \frac{80}{20}(2.8) = 9.6 - 11.2 = \mathbf{-1.6\ V}.$$

Solution 5.68

If $R_q = \infty$, the first stage is an inverter.

$$V_a = -\frac{15}{5}(15) = -45\,\text{mV}$$

when V_a is the output of the first op amp.

The second stage is a noninverting amplifier.

$$v_o = \left(1 + \frac{6}{2}\right)v_a = (1+3)(-45) = \mathbf{-180mV}.$$

Solution 5.69

In this case, the first stage is a summer

$$v_a = -\frac{15}{5}(15) - \frac{15}{10}v_o = -45 - 1.5v_o$$

For the second stage,

$$v_o = \left(1 + \frac{6}{2}\right)v_a = 4v_a = 4(-45 - 1.5v_o)$$

$$7v_o = -180 \qquad v_o = -\frac{180}{7} = \mathbf{-25.71\ mV}.$$

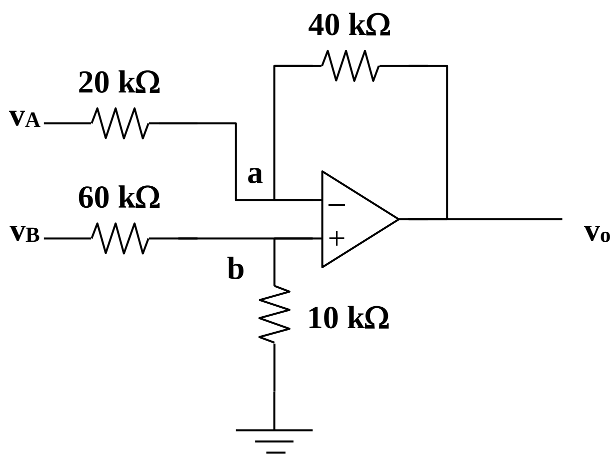
Solution 5.70

The output of amplifier A is

$$v_A = -\frac{30}{10}(1) - \frac{30}{10}(2) = -9$$

The output of amplifier B is

$$v_B = -\frac{20}{10}(3) - \frac{20}{10}(4) = -14$$



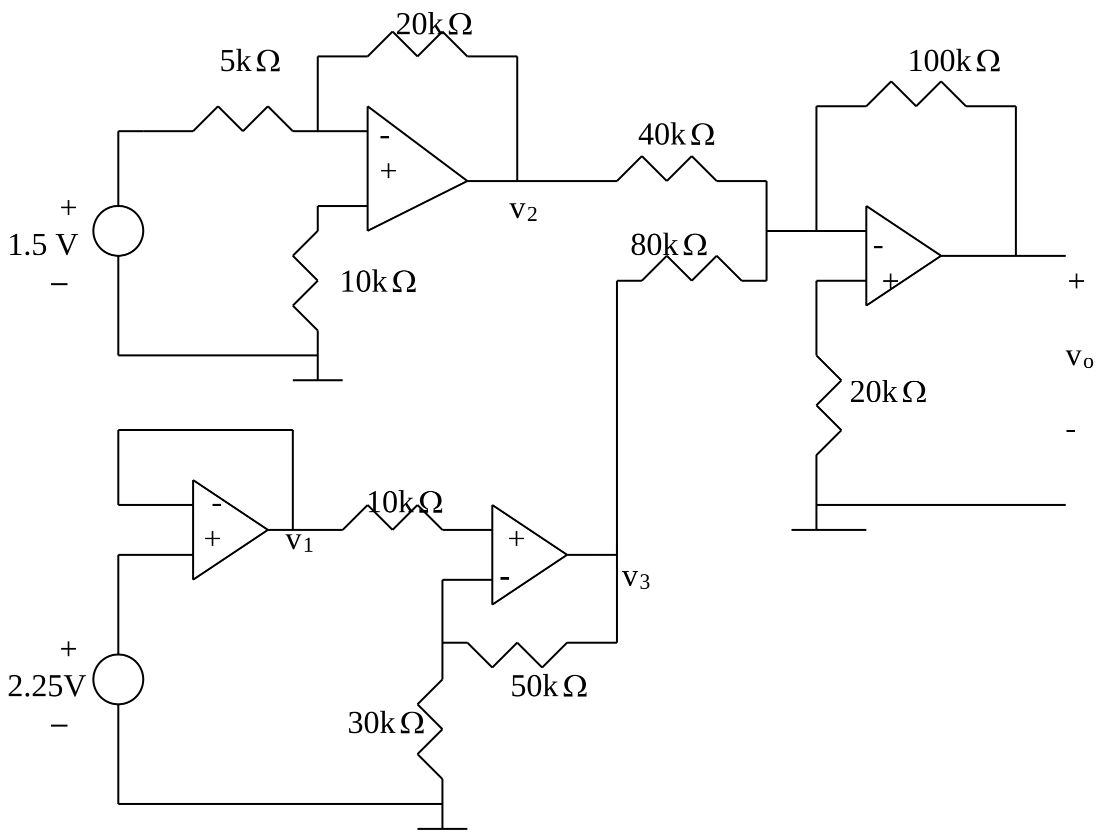
$$v_b = \frac{10}{60+10}(-14) = -2V$$

At node a, $\frac{v_A - v_a}{20} = \frac{v_a - v_o}{40}$

But $v_a = v_b = -2V$, $2(-9+2) = -2-v_o$

Therefore, $v_o =$ **12V**

Solution 5.71



$$v_1 = 2.25, \qquad v_2 = -\frac{20}{5}(1.5) = -6, \qquad v_3 = (1+\frac{50}{30})v_1 = 6$$

$$v_o = -\left(\frac{100}{40}v_2 + \frac{100}{80}v_3\right) = -(-15+7.5) = \mathbf{7.5\ V}.$$

Solution 5.72

Find the load voltage v_L in the circuit of Fig. 5.98.

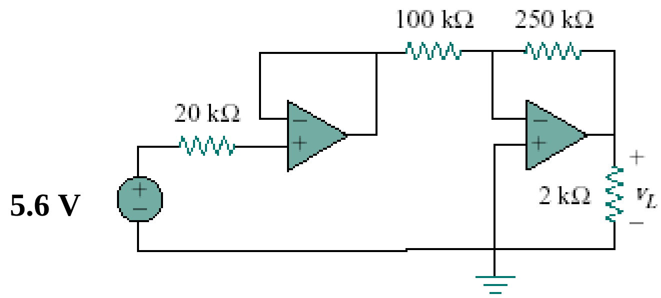


Figure 5.98
For Prob. 5.72.

Solution

Since no current flows into the input terminals of ideal op amp, there is no voltage drop across the 20 kΩ resistor. As a voltage follower, the output of the first op amp is

$$v_{01} = 5.6 \text{ V}$$

The second stage is an inverter

$$v_2 = -\frac{250}{100} v_{01}$$

$$= -2.5(5.6) = \mathbf{-14\ V}.$$

Solution 5.73

Determine the load voltage v_L in the circuit of Fig. 5.99.

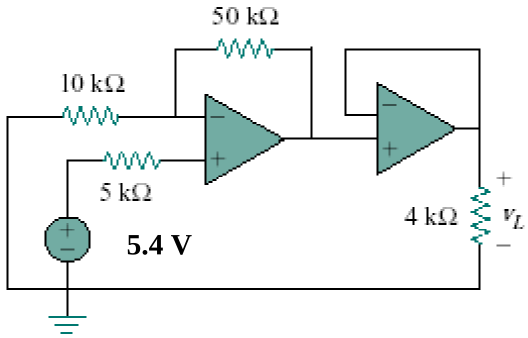


Figure 5.99
For Prob. 5.73.

Solution

The first stage is a noninverting amplifier. The output is

$$v_{o1} = \frac{50}{10}(5.4) + 5.4 = 32.4V$$

The second stage is a voltage follower whose output is

$$v_L = v_{01} = \mathbf{32.4\ V}.$$

Solution 5.74

Let v_1 = output of the first op amp
v_2 = input of the second op amp.

The two sub-circuits are inverting amplifiers

$$v_1 = -\frac{100}{10}(0.9) = -9V$$

$$v_2 = -\frac{32}{1.6}(0.6) = -12V$$

$$i_o = \frac{v_1 - v_2}{20k} = -\frac{-9+12}{20k} = \mathbf{150\ \mu A}.$$

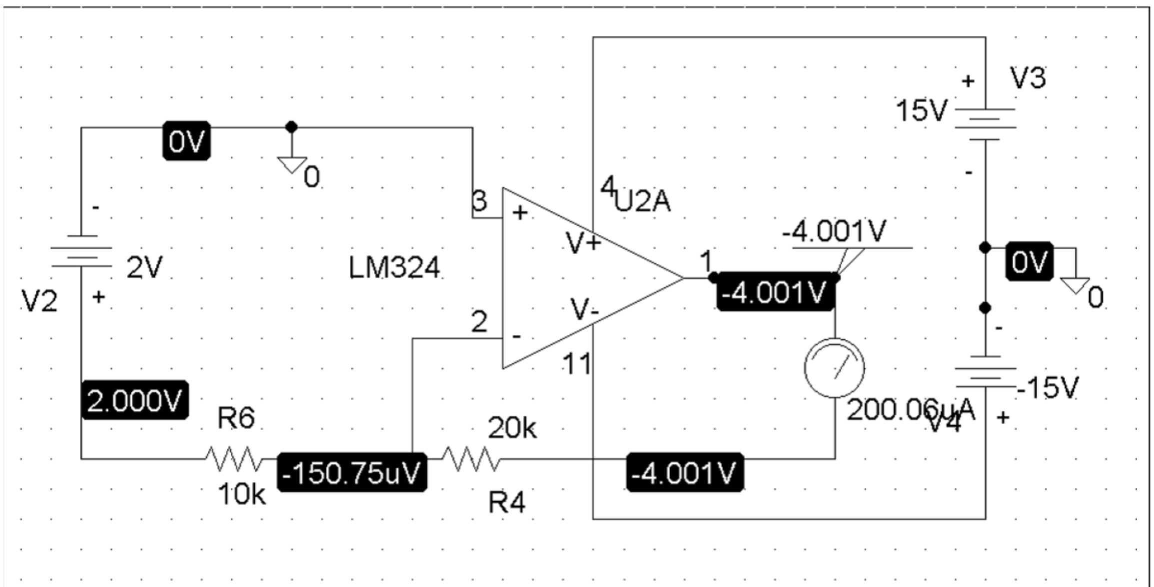
Solution 5.75

The schematic is shown below. Pseudo-components VIEWPOINT and IPROBE are involved as shown to measure v_o and i respectively. Once the circuit is saved, we click Analysis | Simulate. The values of v and i are displayed on the pseudo-components as:

$$i = \mathbf{200\ \mu A}$$

$$(v_o/v_s) = -4/2 = \mathbf{-2}$$

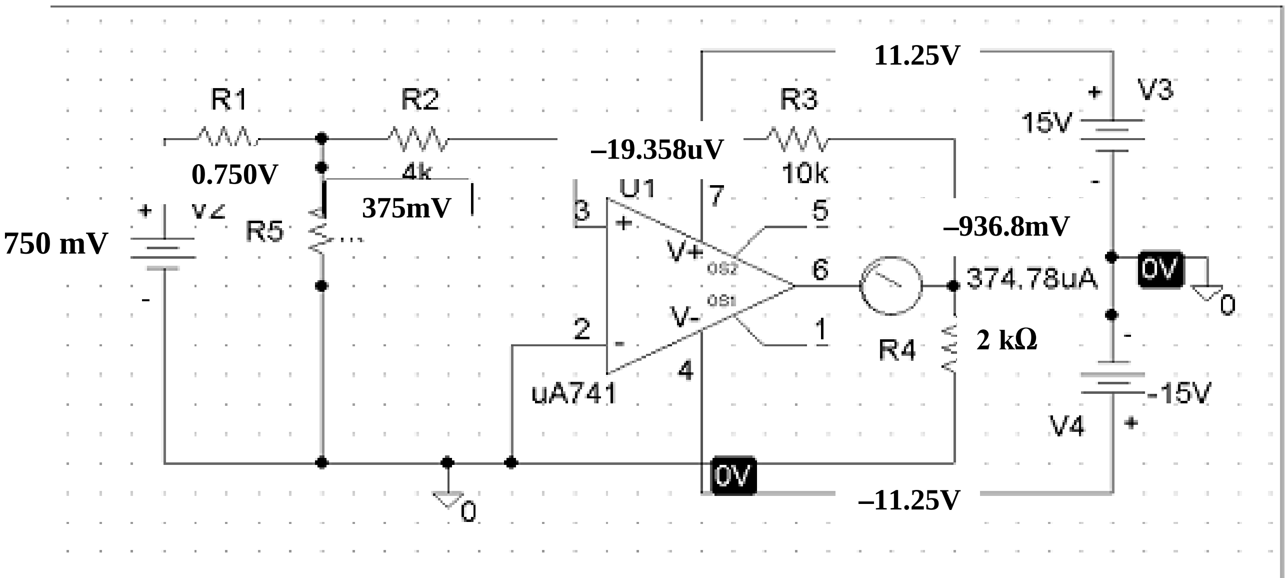
The results are slightly different than those obtained in Example 5.11.



Solution 5.76

The schematic is shown below. IPROBE is inserted to measure i_o. Upon simulation, the value of i_o is displayed on IPROBE as

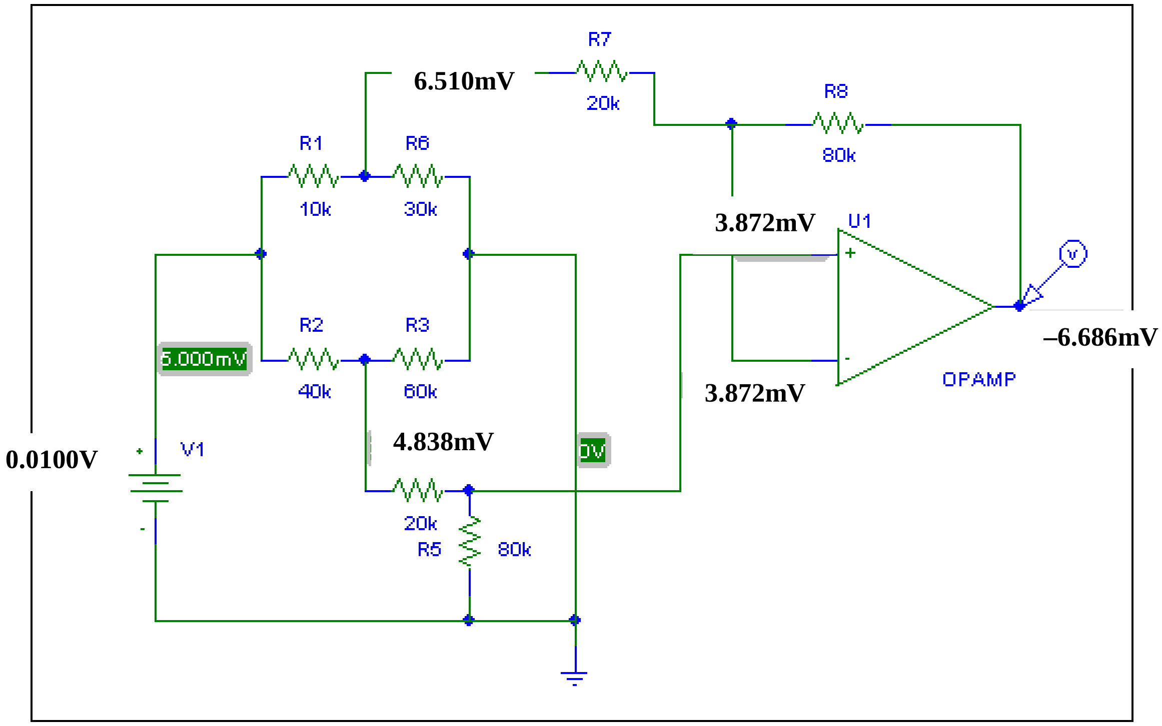
$$i_o = -562.5\ \mu A$$



Solution 5.77

The schematic for the PSpice solution is shown below.

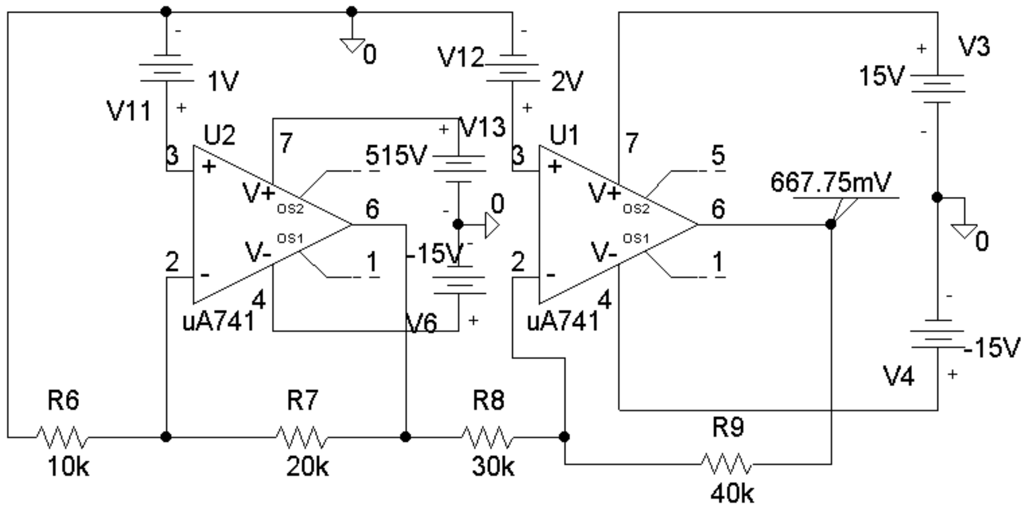
Note that the output voltage, **–6.686 mV**, agrees with the answer to problem, 5.48.



Solution 5.78

The circuit is constructed as shown below. We insert a VIEWPOINT to display v_o. Upon simulating the circuit, we obtain,

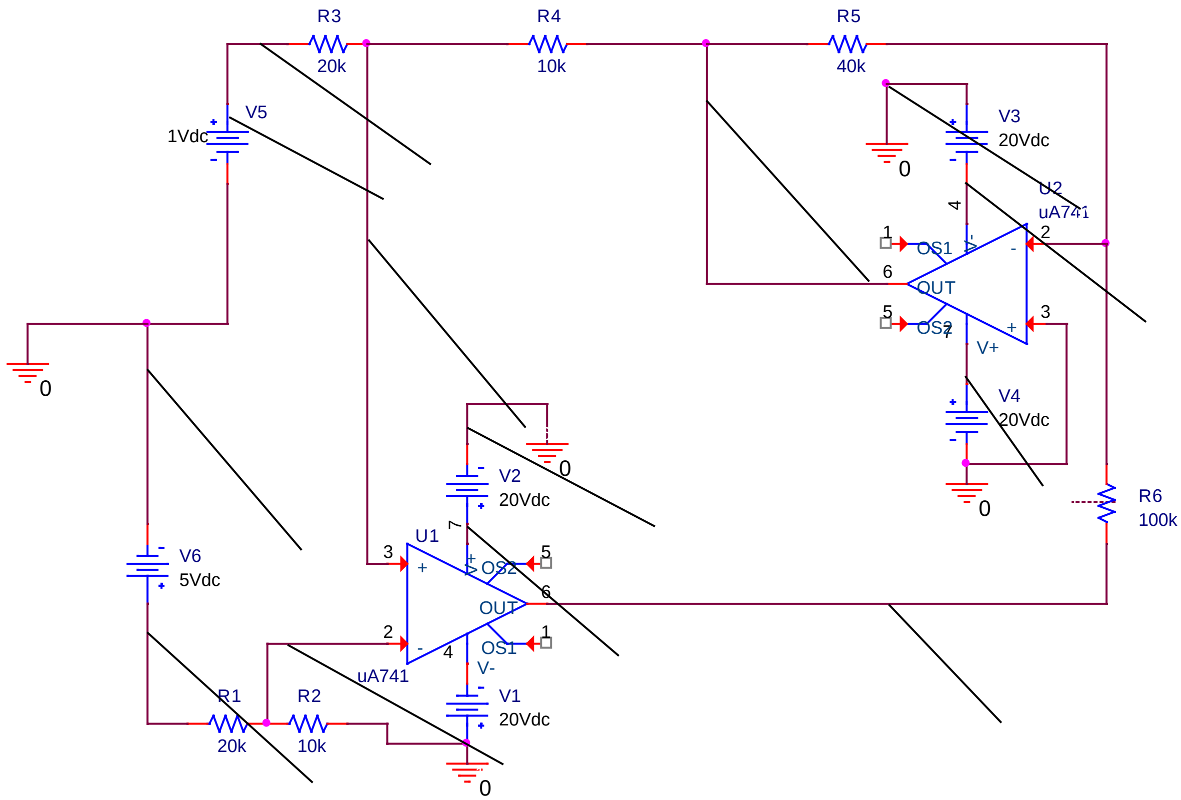
$$v_o = \mathbf{667.75\ mV}$$



Solution 5.79

The schematic is shown below.

$$v_o \;=\; \mathbf{-4.992\ V}$$



Checking using nodal analysis we get,

For the first op-amp we get $v_{a1} = [5/(20+10)]10 = 1.6667 \text{ V} = v_{b1}$.

For the second op-amp, $[(v_{b1} – 1)/20] + [(v_{b1} – v_{c2})/10] = 0$ or $v_{c2} = 10[1.6667–1)/20] + 1.6667 = 2 \text{ V}$;

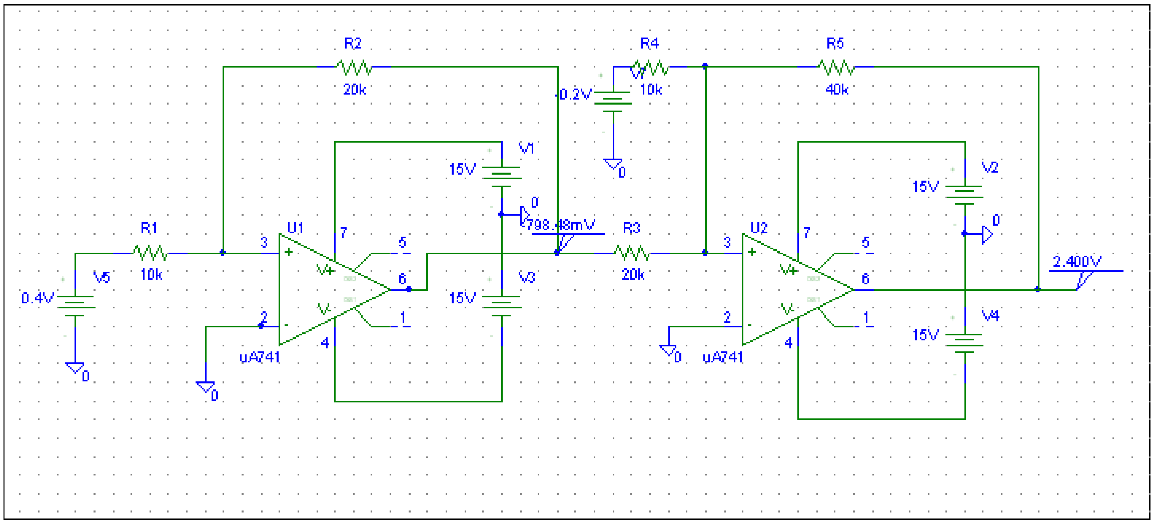
$[(v_{a2} – v_{c2})/40] + [(v_{a2} – v_{c1})/100] = 0$; and $v_{b2} = 0 = v_{a2}$. This leads to $v_{c1} = –2.5v_{c2}$. Thus,

$$= \mathbf{-5\ V}.$$

Solution 5.80

The schematic is as shown below. After it is saved and simulated, we obtain

$$v_o = \mathbf{2.4\ V}.$$

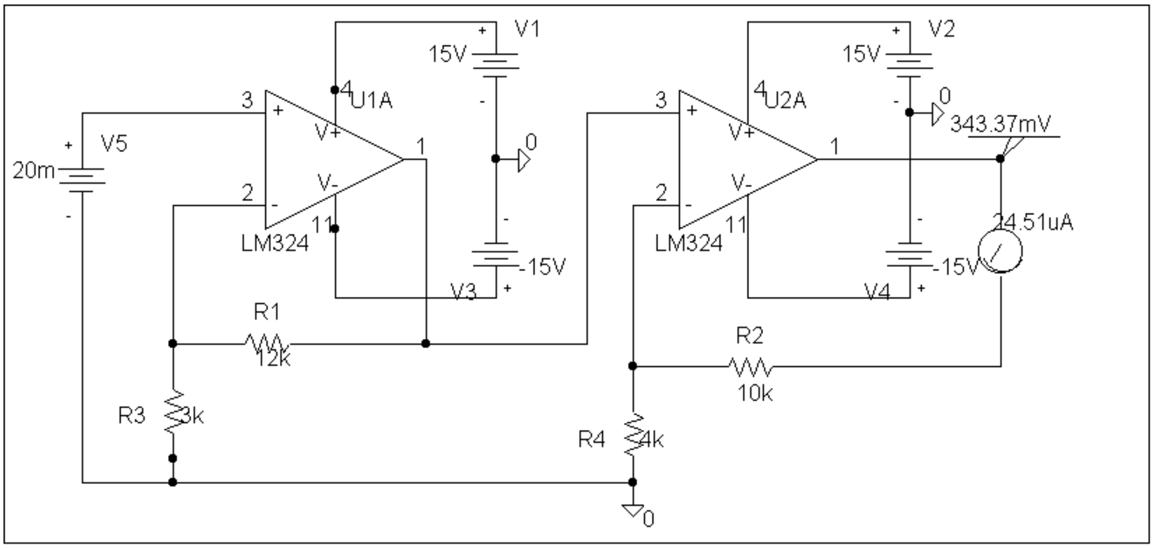


Solution 5.81

The schematic is shown below. We insert one VIEWPOINT and one IPROBE to measure v_o and i_o respectively. Upon saving and simulating the circuit, we obtain,

$$v_o = \mathbf{343.4\ mV}$$

$$i_o = \mathbf{24.51\ \mu A}$$



Solution 5.82

A four-bit DAC covers a voltage range of 0 to 10 V. Calculate the resolution of the DAC in volts per discrete binary step.

Solution

The maximum voltage level corresponds to

$$1111 = 2^4 - 1 = 15.$$

Hence, each bit is worth $(10/15) = \mathbf{666.7\ mV}$

Solution 5.83

The result depends on your design. Hence, let R_G = 10 k ohms, R_1 = 10 k ohms, R_2 = 20 k ohms, R_3 = 40 k ohms, R_4 = 80 k ohms, R_5 = 160 k ohms, R_6 = 320 k ohms, then,

$$-v_o = (R_f/R_1)v_1 + \text{---------} + (R_f/R_6)v_6$$

$$= v_1 + 0.5v_2 + 0.25v_3 + 0.125v_4 + 0.0625v_5 + 0.03125v_6$$

(a) $|v_o| = 1.1875 = 1 + 0.125 + 0.0625 = 1 + (1/8) + (1/16)$ which implies,

$[v_1\ v_2\ v_3\ v_4\ v_5\ v_6]$ = **[100110]**

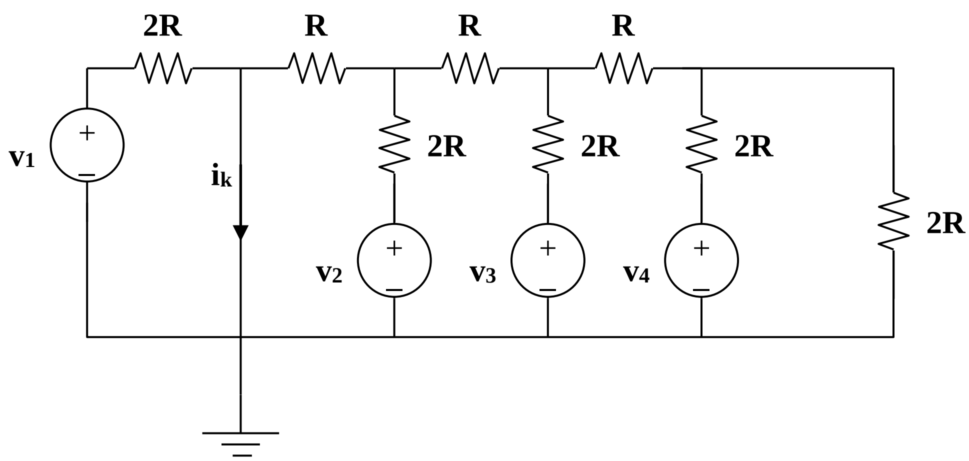
(b) $|v_o| = 0 + (1/2) + (1/4) + 0 + (1/16) + (1/32) = (27/32) =$ **843.75 mV**

(c) This corresponds to [1 1 1 1 1 1].

$|v_o| = 1 + (1/2) + (1/4) + (1/8) + (1/16) + (1/32) = 63/32 =$ **1.96875 V**

Solution 5.84

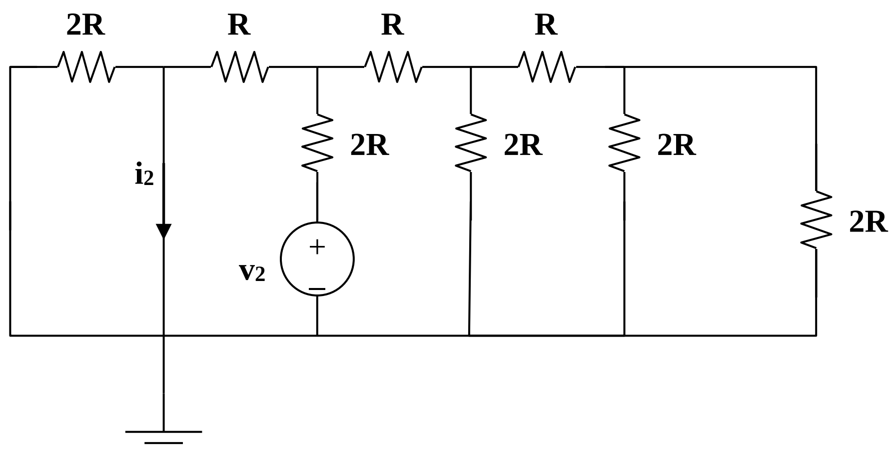
(a) The easiest way to solve this problem is to use superposition and to solve for each term letting all of the corresponding voltages be equal to zero. Also, starting with each current contribution (i_k) equal to one amp and working backwards is easiest.



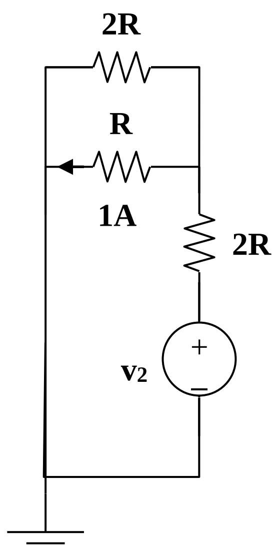
For the first case, let $v_2 = v_3 = v_4 = 0$, and $i_1 = 1$A.

Therefore, $v_1 = 2R$ volts or $i_1 = v_1/(2R)$.

Second case, let $v_1 = v_3 = v_4 = 0$, and $i_2 = 1$A.

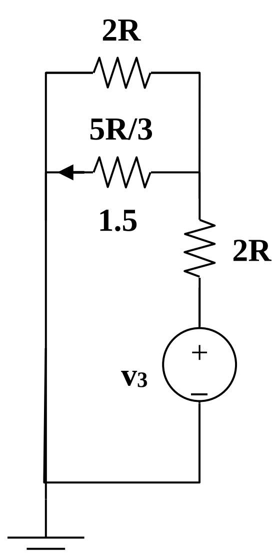


Simplifying, we get,



Therefore, $v_2 = 1\text{x}R + (3/2)(2R) = 4R$ volts or $i_2 = v_2/(4R)$ or $i_2 = 0.25v_2/R$. Clearly this is equal to the desired $1/4^{th}$.

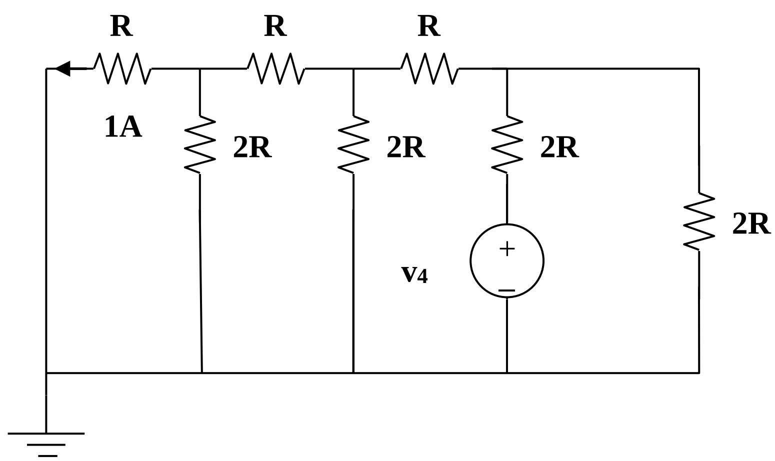
Now for the third case, let $v_1 = v_2 = v_4 = 0$, and $i_3 = 1$A.

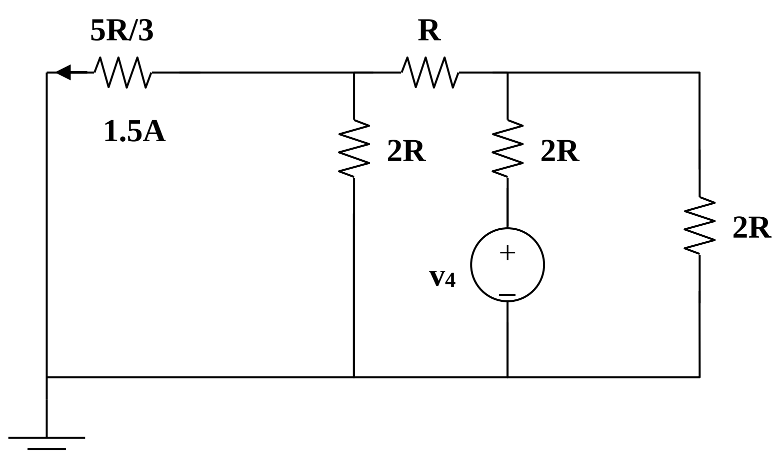


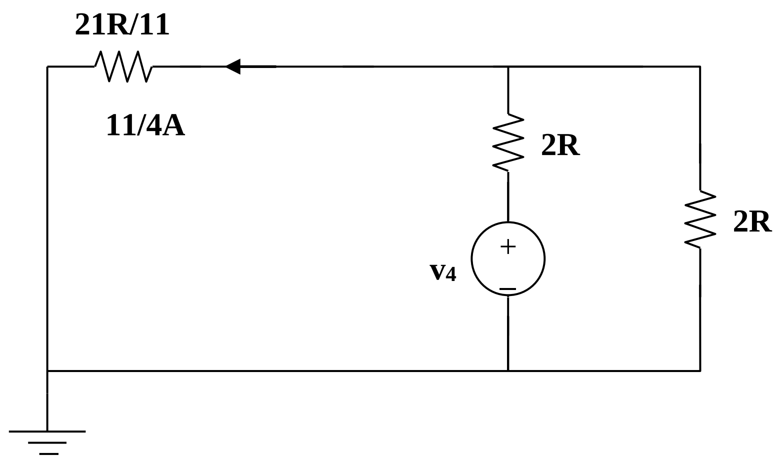
The voltage across the 5R/3-ohm resistor is 5R/2 volts. The current through the 2R resistor at the top is equal to (5/4) A and the current through the 2R-ohm resistor in series with the source is (3/2) + (5/4) = (11/4) A. Thus,

$v_3 = (11/2)R + (5/2)R = (16/2)R = 8R$ volts or $i_3 = v_3/(8R)$ or $0.125v_3/R$. Again, we have the desired result.

For the last case, $v_1 = v_2 = v_3$ and $i_4 = 1A$. Simplifying the circuit we get,







Since the current through the equivalent 21R/11-ohm resistor is (11/4) amps, the voltage across the 2R-ohm resistor on the right is (21/4)R volts. This means the current going through the 2R-ohm resistor is (21/8) A. Finally, the current going through the 2R resistor in series with the source is ((11/4)+(21/8)) = (43/8) A.

Now, $v_4 = (21/4)R + (86/8)R = (128/8)R = 16R$ volts or $i_4 = v_4/(16R)$ or $0.0625v_4/R$. This is just what we wanted.

(b) If $R_f = 12$ k ohms and $R = 10$ k ohms,

$$-v_o = (12/20)[v_1 + (v_2/2) + (v_3/4) + (v_4/8)]$$

$$= 0.6[v_1 + 0.5v_2 + 0.25v_3 + 0.125v_4]$$

For $[v_1\ v_2\ v_3\ v_4] = [1\ 0\ 11]$,

$$|v_o| = 0.6[1 + 0.25 + 0.125] = \mathbf{825\ mV}$$

For $[v_1\ v_2\ v_3\ v_4] = [0\ 1\ 0\ 1]$,

$$|v_o| = 0.6[0.5 + 0.125] = \mathbf{375\ mV}$$

Solution 5.85

In the op amp circuit of Fig. 5.104, find the value of R so that the power absorbed by the 1-kΩ resistor is 10 W. Determine the power gain.

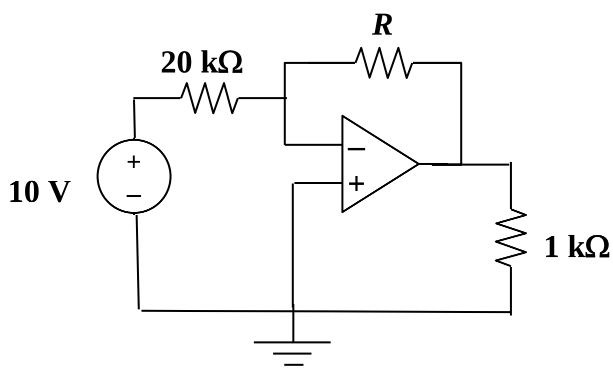


Figure 5.104
For Prob. 5.85.

Solution

This is an inverting amplifier. $v_o = -(R/20k)10 = -0.5R/1{,}000$

The power being delivered to the 1-kΩ give us

$P = 10\ W = (v_o)^2/1k$ or $v_o = \sqrt{10{,}000} = \pm 100V$ in this case we will use the negative value because of the inverting op amp.

Returning to our first equation we get $-100 = -0.5R/1{,}000$.

Thus, R = **200 kΩ**.

The input power is = $(10)^2/20k = 5$ mW which leads to the power gain,

$$10/(0.005) = \mathbf{2{,}000}.$$

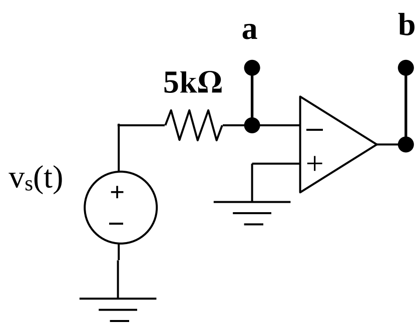
Solution 5.86

Design a voltage controlled ideal current source (within the operating limits of the op amp) where the output current is equal to $200v_s(t)$ μA.

The easiest way to solve this problem is to understand that the op amp creates an output voltage so that the current through the feedback resistor remains equal to the input current.

In the following circuit, the op amp wants to keep the voltage at a equal to zero. So, the input current is $v_s/R = 200v_s(t)$ μA $= v_s(t)/5k$.

Thus, this circuit acts like an ideal voltage controlled current source no matter what (within the operational parameters of the op amp) is connected between a and b. Note, you can change the direction of the current between a and b by sending $v_s(t)$ through an inverting op amp circuit.



Solution 5.87

The output, v_a, of the first op amp is,

$$v_a = (1 + (R_2/R_1))v_1 \qquad (1)$$

Also, $$v_o = (-R_4/R_3)v_a + (1 + (R_4/R_3))v_2 \qquad (2)$$

Substituting (1) into (2),

$$v_o = (-R_4/R_3)\,(1 + (R_2/R_1))v_1 + (1 + (R_4/R_3))v_2$$

Or, $$v_o = \mathbf{(1 + (R_4/R_3))v_2 - (R_4/R_3 + (R_2R_4/R_1R_3))v_1}$$

If $R_4 = R_1$ and $R_3 = R_2$, then,

$$v_o = (1 + (R_4/R_3))(v_2 - v_1)$$

which is a subtractor with a gain of **$(1 + (R_4/R_3))$**.

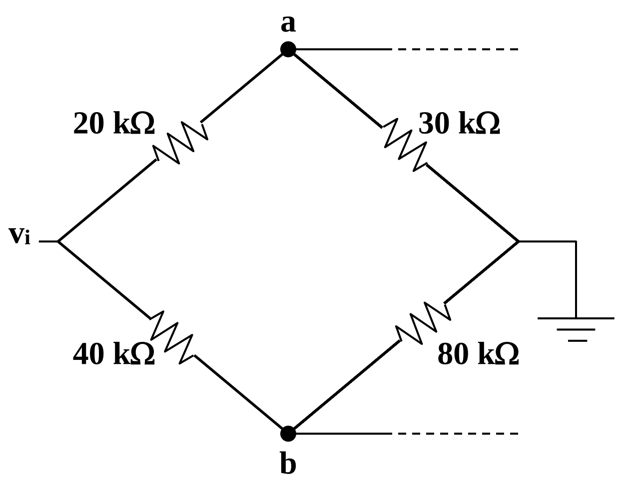
Solution 5.88

We need to find V_{Th} at terminals a – b, from this,

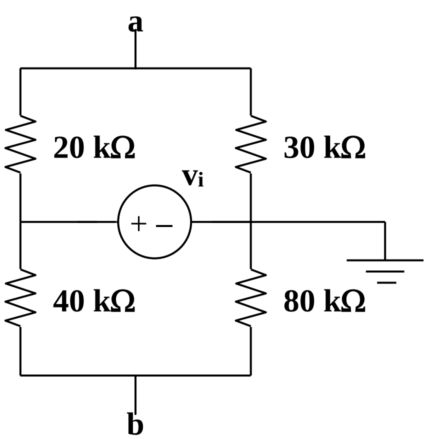
$v_o = (R_2/R_1)(1 + 2(R_3/R_4))V_{Th} = (500/25)(1 + 2(10/2))V_{Th}$

$= 220V_{Th}$

Now we use Fig. (b) to find V_{Th} in terms of v_i.



(a)



(b)

$v_a = (3/5)v_i, \; v_b = (2/3)v_i$

$V_{Th} = v_b – v_a \; (1/15)v_i$

$(v_o/v_i) = A_v = -220/15 = \mathbf{-14.667}$

Solution 5.89

A **summer** with $\mathbf{v_o = -v_1 - (5/3)v_2}$ where v_2 = **6-V battery** and an **inverting amplifier** with $\mathbf{v_1 = -12v_s}$.

Solution 5.90

The op amp circuit in Fig. 5.107 is a *current amplifier*. Find the current gain i_o/i_s of the amplifier.

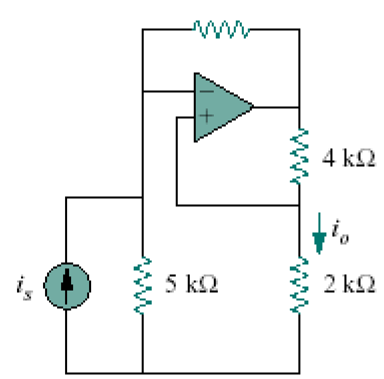
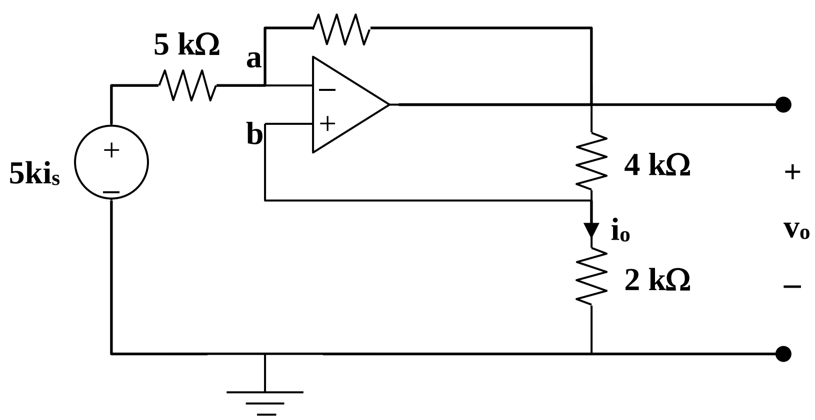


Figure 5.107
For Prob. 5.90.

Solution

Transforming the current source to a voltage source produces the circuit below,

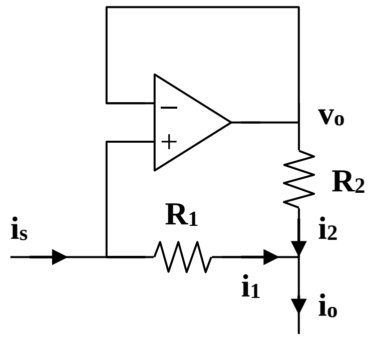
At node b, $v_b = (2/(2 + 4))v_o = v_o/3$



At node a, $[(v_a–5ki_s)/5k] + [(v_a–v_o)/40k] + 0 = 0$ where $v_a = v_b = v_o/3$. This gives us $8v_a – 40ki_s + v_a – v_o = 0 = (9/3)v_o – v_o – 40ki_s = 2v_o – 40ki_s$ or $i_s = v_o/20k$. Finally, $i_o = v_b/2k = v_o/6k$ which leads to,

$i_o/i_s = (v_o/6k)/(v_o/20k) =$ **3.333**.

Solution 5.91



$$i_o = i_1 + i_2 \qquad (1)$$

But $\quad i_1 = i_s \qquad (2)$

R_1 and R_2 have the same voltage, v_o, across them.

$$R_1 i_1 = R_2 i_2, \text{ which leads to } i_2 = (R_1/R_2)i_1 \qquad (3)$$

Substituting (2) and (3) into (1) gives,

$$i_o = i_s(1 + R_1/R_2)$$

$$i_o/i_s = 1 + (R_1/R_2) = 1 + 8/1 = \mathbf{9}$$

Solution 5.92

Refer to the *bridge amplifier* shown in Fig. 5.109. Determine the voltage gain v_o/v_i .

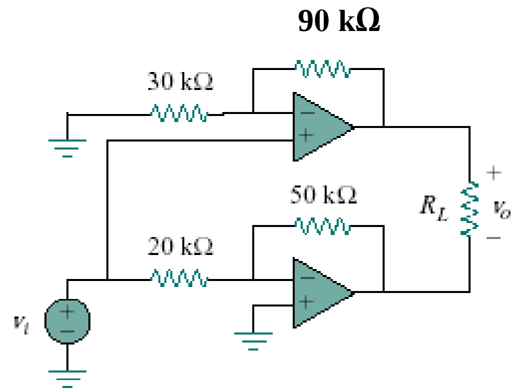


Figure 5.109
For Prob. 5.92.

Solution

The top op amp circuit is a non-inverter, while the lower one is an inverter. The output at the top op amp is

$$v_1 = (1 + 90/30)v_i = 4v_i$$

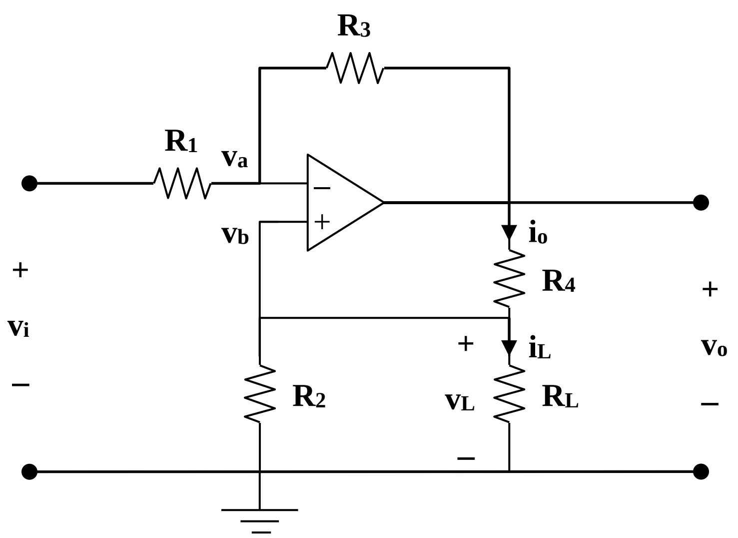
while the output of the lower op amp is

$$v_2 = -(50/20)v_i = -2.5v_i.$$

Hence, $$v_o = v_1 - v_2 = 4v_i + 2.5v_i = 6.5v_i$$

$$v_o/v_i = \mathbf{6.5}$$

Solution 5.93



At node a, $(v_i - v_a)/R_1 = (v_a - v_o)/R_3$

$$v_i - v_a = (R_1/R_2)(v_a - v_o)$$

$$v_i + (R_1/R_3)v_o = (1 + R_1/R_3)v_a \qquad (1)$$

But $v_a = v_b = v_L$. Hence, (1) becomes

$$v_i = (1 + R_1/R_3)v_L - (R_1/R_3)v_o \qquad (2)$$

$i_o = v_o/(R_4 + R_2||R_L)$, $i_L = (R_2/(R_2 + R_L))i_o = (R_2/(R_2 + R_L))(v_o/(R_4 + R_2||R_L))$

Or, $v_o = i_L[(R_2 + R_L)(R_4 + R_2||R_L)/R_2 \qquad (3)$

But, $v_L = i_L R_L \qquad (4)$

Substituting (3) and (4) into (2),

$v_i = (1 + R_1/R_3)\, i_L R_L - R_1[(R_2 + R_L)/(R_2R_3)](R_4 + R_2||R_L)i_L$

$= [((R_3 + R_1)/R_3)R_L - R_1((R_2 + R_L)/(R_2R_3)(R_4 + (R_2R_L/(R_2 + R_L))]i_L$
$= (1/A)i_L$

Thus,

$$A = \frac{\mathbf{1}}{\left(\mathbf{1}+\frac{\mathbf{R_1}}{\mathbf{R_3}}\right)\mathbf{R_L} - \mathbf{R_1}\left(\frac{\mathbf{R_2}+\mathbf{R_L}}{\mathbf{R_2 R_3}}\right)\left(\mathbf{R_4}+\frac{\mathbf{R_2 R_L}}{\mathbf{R_2}+\mathbf{R_L}}\right)}$$

Please note that A has the units of mhos. An easy check is to let every resistor equal 1-ohm and v_i equal to one amp. Going through the circuit produces i_L = 1A. Plugging into the above equation produces the same answer so the answer does check.

Solution 6.1

$i = C\dfrac{dv}{dt} = 7.5\left(2e^{-3t} - 6te^{-3t}\right) = \mathbf{15(1 - 3t)e^{-3t}\ A}$

p = vi = $15(1–3t)e^{-3t} \cdot 2t\ e^{-3t}$ = $\mathbf{30t(1 - 3t)e^{-6t}\ W}$.

$\mathbf{15(1 - 3t)e^{-3t}\ A,\ 30t(1 - 3t)e^{-6t}\ W}$

Solution 6.2

$w(t) = (1/2)C(v(t))^2$ or $(v(t))^2 = 2w(t)/C = (20\cos^2(377t))/(50x10^{-6}) = 0.4x10^6\cos^2(377t)$

or

$v(t) = \pm 632.5\cos(377t)$ V.

Let us assume that $v(t) = 632.5\cos(377t)$ V, which leads to
$i(t) = C(dv/dt) = 50x10^{-6}(632.5)(-377\sin(377t))$

$= \mathbf{-11.923\sin(377t)\ A}$.

Please note that if we had chosen the negative value for v, then i(t) would have been positive.

Solution 6.3

Design a problem to help other students to better understand how capacitors work.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In 5 s, the voltage across a 40-mF capacitor changes from 160 V to 220 V. Calculate the average current through the capacitor.

Solution

$$i = C\frac{dv}{dt} = 40x10^{-3}\frac{220-160}{5} = \mathbf{480\ mA}$$

Solution 6.4

A voltage across a capacitor is equal to [2–2 cos(4t)] V and the current flowing through it is equal to 2sin(4t) μA, determine the value of the capacitance. Calculate the power being stored by the capacitor.

Solution

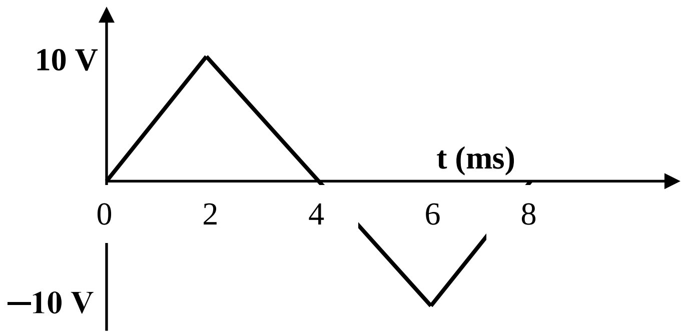
Starting with $i_C = Cdv_C/dt$ and $v_C = [2–2\cos(4t)]$ V and that $i_C = 2\sin(4t)$ μA,

we get $c = 2\sin(4t)x10^{–6}/8\sin(4t)$ = **0.25 μF**.

$P_C = v_C i_C = [2–2\cos(4t)][2\sin(4t)x10^{–6}]$ = **[4sin(4t)–4cos(4t)sin(4t)] μW**.

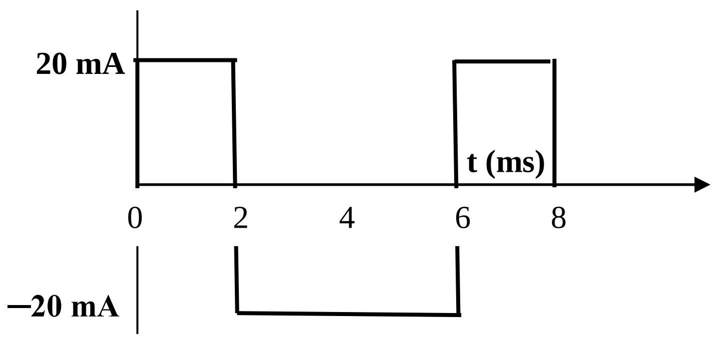
Solution 6.5

The voltage across a 4-μF capacitor is shown below. Find the current waveform.



Step 1. $v = \begin{cases} 5000t, & 0 < t < 2ms \\ 20 - 5000t, & 2 < t < 6ms \\ -40 + 5000t, & 6 < t < 8ms \end{cases}$ and $i_C(t) = Cdv_C(t)/dt$.

Step 2. For 0<t<2ms, $i_C(t) = 4x10^{-6}d(5000t)/dt = 20$ mA;
for 2ms<t<6ms, $i_C(t) = 4x10^{-6}d(20-5000t)/dt = -20$ mA;
and for 6ms<t<8ms, $i_C(t) = 4x10^{-6}d(-40+5000t)/dt = 20$ mA.



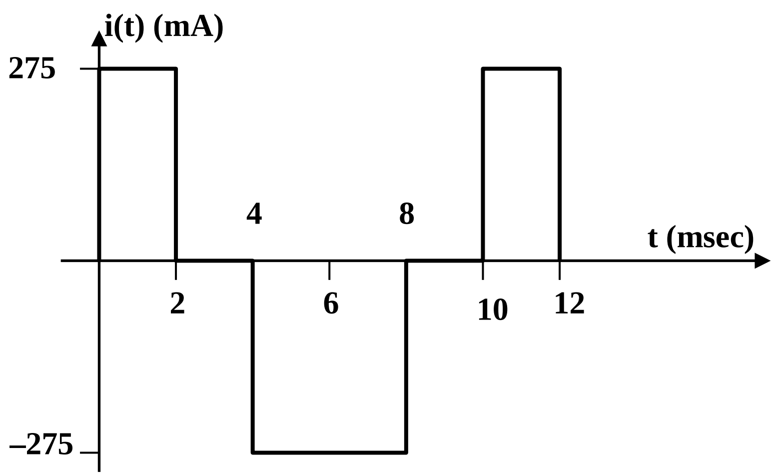
Solution 6.6

$i = C\dfrac{dv}{dt} = 55x10^{-6}$ times the slope of the waveform.
For example, for 0 < t < 2,

$$\frac{\text{dv}}{\text{dt}} = \frac{10}{2\text{x}10^{-3}}$$

i = $C\dfrac{dv}{dt} = (55x10^{-6})\dfrac{10}{2x10^{-3}} = 275mA$

Thus the current i(t) is sketched below.



Solution 6.7

$$v = \frac{1}{C}\int i dt + v(t_o) = \frac{1}{25x10^{-3}}\int_o^t 5tx10^{-3}\, dt + 10$$

$$= \frac{2.5t^2}{25} + 10 = \mathbf{[0.1t^2 + 10]\ V}.$$

Solution 6.8

(a) $i = C\dfrac{dv}{dt} = -100ACe^{-100t} - 600BCe^{-600t}$ (1)

$i(0) = 2 = -100AC - 600BC \quad \longrightarrow \quad 5 = -A - 6B$ (2)

$v(0^+) = v(0^-) \quad \longrightarrow \quad 50 = A + B$ (3)

Solving (2) and (3) leads to

$\underline{A=61,\ B=-11}$

(b) $\text{Energy} = \dfrac{1}{2}Cv^2(0) = \dfrac{1}{2}x4x10^{-3}x2500 = \underline{5\text{ J}}$

(c) From (1),

$$\boldsymbol{i = -100x61x4x10^{-3}e^{-100t} - 600x11x4x10^{-3}e^{-600t} = \underline{-24.4e^{-100t} - 26.4e^{-600t}\ \ \text{A}}}$$

Solution 6.9

$$v(t) = \frac{1}{1/2}\int_o^t 6\left(1 - e^{-t}\right)dt + 0 = 12\left(t + e^{-t}\right)\Big|_0^t \text{ V} \;=\; 12(t + e^{-t}) - 12$$

$$v(2) = 12(2 + e^{-2}) - 12 = \mathbf{13.624\ V}$$

$$p = iv = [12\ (t + e^{-t}) - 12]6(1-e^{-t})$$

$$p(2) = [12\ (2 + e^{-2}) - 12]6(1-e^{-2}) \;=\; \mathbf{70.66\ W}$$

Solution 6.10

$$i = C\frac{dv}{dt} = 5x10^{-3}\frac{dv}{dt}$$

$$v = \begin{cases} 16t, & 0 < t < 1\mu s \\ 16, & 1 < \mathrm{t} < 3\ \mu s \\ 64 - 16\mathrm{t}, & 3 < \mathrm{t} < 4\mu s \end{cases}$$

$$\frac{dv}{dt} = \begin{cases} 16x10^{6}, & 0 < t < 1\mu s \\ 0, & 1 < \mathrm{t} < 3\ \mu s \\ -16\mathrm{x}10^{6}, & 3 < \mathrm{t} < 4\mu s \end{cases}$$

$$\boldsymbol{i(t)} = \begin{cases} \mathbf{80\,kA,} & \mathbf{0 < \boldsymbol{t} < 1\mu s} \\ \mathbf{0,} & \mathbf{1 < t < 3\ \mu s} \\ \mathbf{-80\,kA,} & \mathbf{3 < t < 4\mu s} \end{cases}$$

Solution 6.11

$$v = \frac{1}{C}\int_0^t i dt + v(0) = 10 + \frac{1}{4x10^{-3}}\int_0^t i(t)dt$$

For 0<t <2, i(t)=15mA, V(t)= 10+ $v = 10 + \frac{10^3}{4x10^{-3}}\int_0^t 15dt = 10 + 3.76t$

v(2) = 10+7.5 =17.5

For 2 < t <4, i(t) = –10 mA

$$v(t) = \frac{1}{4x10^{-3}}\int_2^t i(t)dt + v(2) = -\frac{10x10^{-3}}{4x10^{-3}}\int_2^t dt + 17.5 = 22.5 + 2.5t$$

v(4)=22.5-2.5x4 =12.5

For 4<t<6, i(t) = 0, $v(t) = \frac{1}{4x10^{-3}}\int_2^t 0dt + v(4) = 12.5$

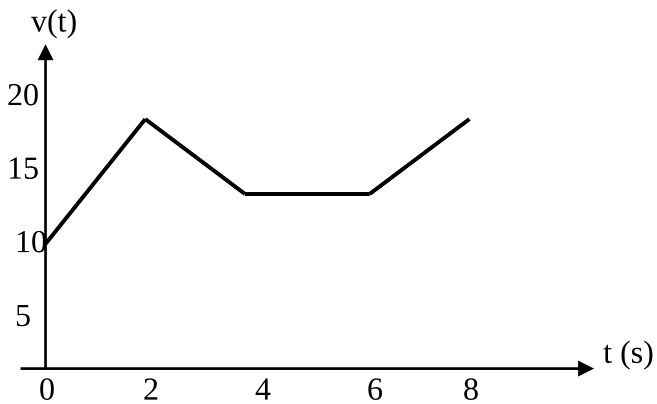
For 6<t<8, i(t) = 10 mA

$$v(t) = \frac{10x10^3}{4x10^{-3}}\int_4^t dt + v(6) = 2.5(t-6) + 12.5 = 2.5t - 2.5$$

Hence,

$$\text{v(t)} = \begin{cases} 10 + 3.75tV, & 0 < t < 2s \\ 22.5 - 2.5tV, & 2 < t < 4s \\ 12.5V, & 4 < t < 6s \\ 2.5t - 2.5V, & 6 < t < 8s \end{cases}$$

which is sketched below.



Solution 6.12

A voltage of $45e^{-2000t}$ V appears across a parallel combination of a 100-mF capacitor and a 12-Ω resistor. Calculate the power absorbed by the parallel combination.

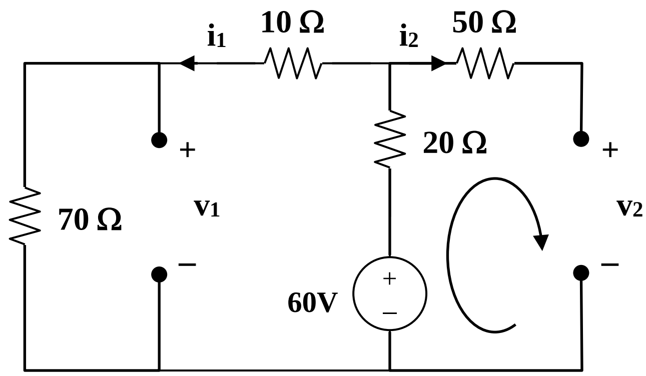
Solution

$i_R = V/R = (45/12)e^{-2000t} = 3.75\ e^{-2000t}$ and
$i_C = C(dv/dt) = 0.1x45(-2000)\ e^{-2000t} = -9000\ e^{-2000t}$ A.

Thus, $i = i_R + i_C = -8{,}996.25e^{-2000t}$. The power is equal to:
$vi =$ **$-40.48\ 179.925\ e^{-4000t}$ kW**.

Solution 6.13

Under dc conditions, the circuit becomes that shown below:



$i_2 = 0$, $i_1 = 60/(70+10+20) = 0.6$ A

$v_1 = 70i_1 = 42$ V, $v_2 = 60–20i_1 = 48$ V

Thus, $v_1 =$ **42 V**, $v_2 =$ **48 V**.

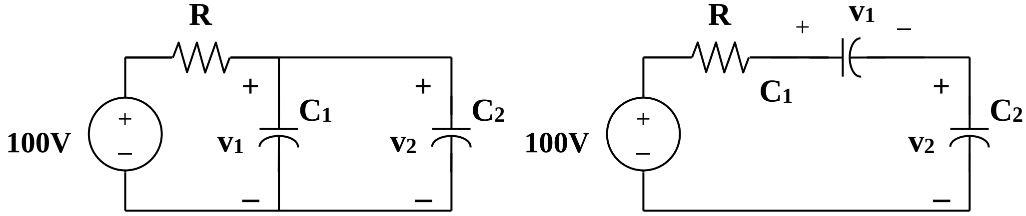
Solution 6.14

20 pF is in series with 60pF = 20*60/80=15 pF
30-pF is in series with 70pF = 30x70/100=21pF
15pF is in parallel with 21pF = 15+21 = **36 pF**

Solution 6.15

Arranging the capacitors in parallel results in circuit shown in Fig. (1) (It should be noted that the resistors are in the circuits only to limit the current surge as the capacitors charge. Once the capacitors are charged the current through the resistors are obviously equal to zero.):

$$v_1 = v_2 = 100$$



$$w_{20} = \frac{1}{2}Cv^2 = \frac{1}{2}x25x10^{-6}x100^2 = \textbf{125 mJ}$$

$$w_{30} = \frac{1}{2}x75x10^{-6}x100^2 = \textbf{375 mJ}$$

(b) Arranging the capacitors in series results in the circuit shown in Fig. (2):

$$v_1 = \frac{C_2}{C_1 + C_2}V = \frac{75}{100}x100 = 75 \text{ V}, \; v_2 = 25 \text{ V}$$

$$w_{25} = \frac{1}{2}x25x10^{-6}x75^2 = \textbf{70.31 mJ}$$

$$w_{75} = \frac{1}{2}x75x10^{-6}x25^2 = \textbf{23.44 mJ}.$$

(a) **125 mJ, 375 mJ (b) 70.31 mJ, 23.44 mJ**

Solution 6.16

The equivalent capacitance at terminals *a-b* in the circuit in Fig. 6.50 is 20 μF. Calculate the value of *C*.

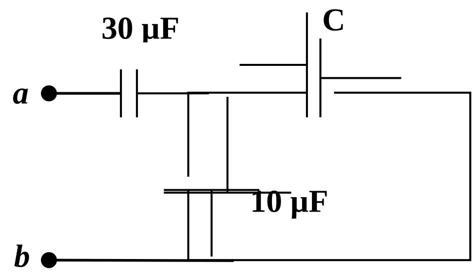


Figure 6.50
For Prob. 6.16.

Solution

The capacitance looking into terminals a and b is equal to,

C_{eq} = $20\text{x}10^{-6} = 30\text{x}10^{-6}(10\text{x}10^{-6}+\text{C})/(\ 30\text{x}10^{-6}+10\text{x}10^{-6}+\text{C})$.

$30\text{x}10^{-6}+10\text{x}10^{-6}+\text{C} = (30/20)(10\text{x}10^{-6}+\text{C})$ or
$(1.5\text{–}1)\text{C} = 30\text{x}10^{-6}+10\text{x}10^{-6}\text{–}15\text{x}10^{-6} = 25\text{x}10^{-6}$ or

$\text{C} = 25\text{x}10^{-6}/0.5 =$ **50 μF**.

Solution 6.17

(a) 4F in series with 12F = 4 x 12/(16) = 3F
3F in parallel with 6F and 3F = 3+6+3 = 12F
4F in series with 12F = 3F
i.e. C_{eq} = **3F**

(b) C_{eq} = 5 + [6x(4 + 2)/(6+4+2)] = 5 + (36/12) = 5 + 3 = **8F**

(c)

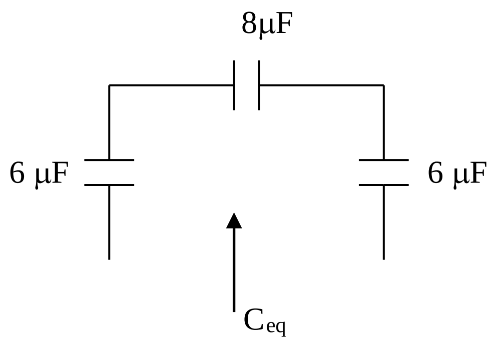
(d) 3F in series with 6F = (3 x 6)/9 = 2F

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{6} + \frac{1}{3} = 1$$

C_{eq} = **1F**

Solution 6.18

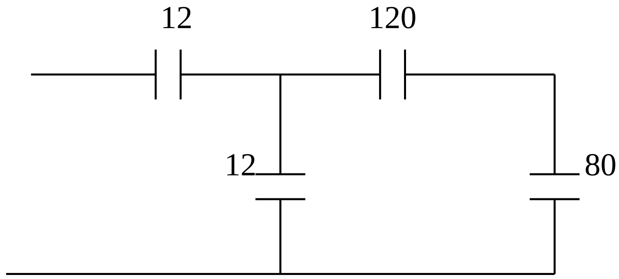
4 μF in parallel with 4 μF = 8μF
4 μF in series with 4 μF = 2 μF
2 μF in parallel with 4 μF = 6 μF
Hence, the circuit is reduced to that shown below.



$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{6} + \frac{1}{8} = 0.4583 \quad \longrightarrow \quad C_{eq} = \underline{2.1818\ \mu F}$$

Solution 6.19

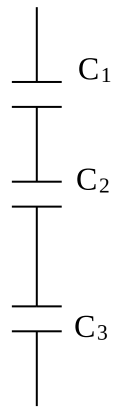
We combine 10-, 20-, and 30- μF capacitors in parallel to get 60 μF. The 60 - μF capacitor in series with another 60- μF capacitor gives 30 μF.
30 + 50 = 80 μF, 80 + 40 = 120 μF
The circuit is reduced to that shown below.



120- μF capacitor in series with 80 μF gives (80x120)/200 = 48
48 + 12 = 60
60- μF capacitor in series with 12 μF gives (60x12)/72 = **10 μF**

Solution 6.20

Consider the circuit shown below.



$C_1 = 1+1 = 2\mu F$
$C_2 = 2+2+2 = 6\mu F$

$C_3 = 4x3 = 12\mu F$

$1/C_{eq} = (1/C_1) + (1/C_2) + (1/C_3) = 0.5 + 0.16667 + 0.08333 = 0.75x10^6$

C_{eq} = **1.3333 μF**.

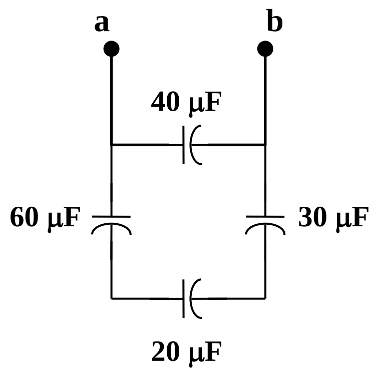
Solution 6.21

$4\mu F$ in series with $12\mu F = (4x12)/16 = 3\mu F$
$3\mu F$ in parallel with $3\mu F = 6\mu F$
$6\mu F$ in series with $6\mu F = 3\mu F$
$3\mu F$ in parallel with $2\mu F = 5\mu F$
$5\mu F$ in series with $5\mu F = 2.5\mu F$

Hence $C_{eq} = \mathbf{2.5\mu F}$

Solution 6.22

Combining the capacitors in parallel, we obtain the equivalent circuit shown below:



Combining the capacitors in series gives C_{eq}^1, where

$$\frac{1}{C_{eq}^1} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10} \longrightarrow C_{eq}^1 = 10\mu F$$

Thus

$$C_{eq} = 10 + 40 = \mathbf{50\ \mu F}$$

Solution 6.23

Using Fig. 6.57, design a problem to help other students better understand how capacitors work together when connected in series and parallel.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 6.57, determine:

(a) the voltage across each capacitor,
(b) the energy stored in each capacitor.

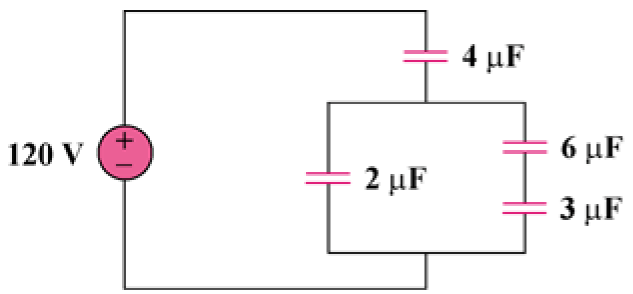


Figure 6.57

Solution

(a) 3μF is in series with 6μF 3x6/(9) = 2μF

$v_{4\mu F}$ = 1/2 x 120 = **60V**

$v_{2\mu F}$ = **60V**

$$v_{6\mu F} = \frac{3}{6+3}(60) = \mathbf{20V}$$

$v_{3\mu F}$ = 60 - 20 = **40V**

(b) Hence w = 1/2 Cv^2

$w_{4\mu F}$ = 1/2 x 4 x 10^{-6} x 3600 = **7.2mJ**

$w_{2\mu F}$ = 1/2 x 2 x 10^{-6} x 3600 = **3.6mJ**

$w_{6\mu F}$ = 1/2 x 6 x 10^{-6} x 400 = **1.2mJ**

$w_{3\mu F}$ = 1/2 x 3 x 10^{-6} x 1600 = **2.4mJ**

Solution 6.24

In the circuit shown in Fig. 6.58 assume that the capacitors were initially uncharged and that the current source has been connected to the circuit long enough for all the capacitors to reach steady-state (no current flowing through the capacitors), determine the voltage across each capacitor and the energy stored in each.

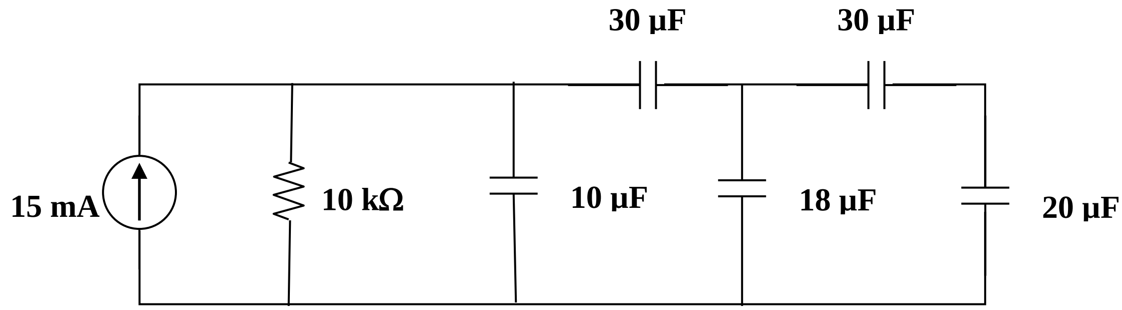


Figure 6.58
For Prob. 6.24.

Solution
Reducing the capacitance starting from right to left. 30µF in series with 20µF we get, 30x20µF/(30+20) = 12µF in parallel with 18µF we get (12+18)µF = 30µF.
Now

v_{10} = 15x10 = **150 V** and w_{10} = $0.5\text{x}10\text{x}150^2\text{x}10^{-6}$ = **112.5 mJ**
v_{30} = **75 V** and w_{30} = $0.5\text{x}30\text{x}75^2\text{x}10^{-6}$ = **84.38 mJ**
v_{18} = **75 V** and w_{18} = $0.5\text{x}18\text{x}75^2\text{x}10^{-6}$ = **50.62 mJ**
v_{30} = [20/(30+20)]75 = **30 V** and w_{30} = $0.5\text{x}30\text{x}30^2\text{x}10^{-6}$ = **13.5 mJ**
v_{20} = [30/(30+20)]75 = **45 V** and w_{20} = $0.5\text{x}20\text{x}45^2\text{x}10^{-6}$ = **20.25 mJ**

Solution 6.25

(a) For the capacitors in series,

$$Q_1 = Q_2 \longrightarrow C_1 v_1 = C_2 v_2 \longrightarrow \frac{v_1}{v_2} = \frac{C_2}{C_1}$$

$$v_s = v_1 + v_2 = \frac{C_2}{C_1} v_2 + v_2 = \frac{C_1 + C_2}{C_1} v_2 \longrightarrow \boldsymbol{v_2 = \frac{C_1}{C_1 + C_2} v_s}$$

Similarly, $\boldsymbol{v_1 = \frac{C_2}{C_1 + C_2} v_s}$

(b) For capacitors in parallel

$$v_1 = v_2 = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_s = Q_1 + Q_2 = \frac{C_1}{C_2} Q_2 + Q_2 = \frac{C_1 + C_2}{C_2} Q_2$$

or

$$Q_2 = \frac{C_2}{C_1 + C_2}$$

$$Q_1 = \frac{C_1}{C_1 + C_2} Q_s$$

$$i = \frac{dQ}{dt} \longrightarrow \boldsymbol{i_1 = \frac{C_1}{C_1 + C_2} i_s}, \quad \boldsymbol{i_2 = \frac{C_2}{C_1 + C_2} i_s}$$

Solution 6.26

Three capacitors, $\boldsymbol{C}_1 = 5\ \mu$F, $\boldsymbol{C}_2 = 10\ \mu$F, and $\boldsymbol{C}_3 = 20\ \mu$F, are connected in parallel across a 200-V source. Determine:
(a) the total capacitance,
(b) the charge on each capacitor,
(c) the total energy stored in the parallel combination.

Solution

(a) $C_{eq} = C_1 + C_2 + C_3 = \mathbf{35\mu F}$

(b) $Q_1 = C_1 v = 5x200\ \mu C = \mathbf{1\ mC}$
$Q_2 = C_2 v = 10x200\ \mu C = \mathbf{2\ mC}$
$Q_3 = C_3 v = 20x200\ \mu C = \mathbf{4\ mC}$

(c) $w = \frac{1}{2} C_{eq} v^2 = \frac{1}{2} x 35 x 200^2\ \mu J = \mathbf{700\ mJ}$

Solution 6.27

Given that four 10-μF capacitors can be connected in series and in parallel, find the minimum and maximum values that can be obtained by such series/parallel combinations.

Solution

If they are all connected in parallel, we get C_{total} = 4x10 μF = 40 μF.

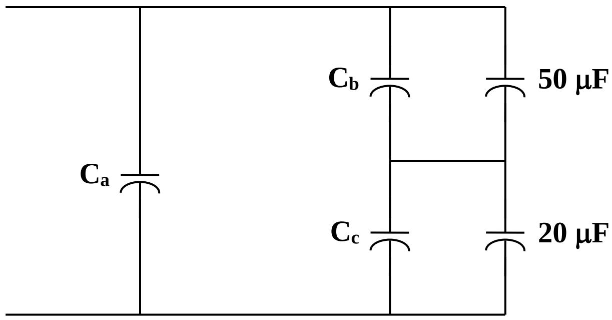
If they are all connected in series, we get C_{total} = 1/(4/10 μF) = 2.5 μF.

Since all other combinations fall within these two extreme cases. Thus,

C_{min} = **2.5 μF**, C_{max} = **40 μF**.

Solution 6.28

We may treat this like a resistive circuit and apply delta-wye transformation, except that R is replaced by 1/C.



$$\frac{1}{C_a} = \frac{\left(\frac{1}{10}\right)\left(\frac{1}{40}\right) + \left(\frac{1}{10}\right)\left(\frac{1}{30}\right) + \left(\frac{1}{30}\right)\left(\frac{1}{40}\right)}{\frac{1}{30}}$$

$$= \frac{3}{40} + \frac{1}{10} + \frac{1}{40} = \frac{2}{10}$$

$C_a = 5\mu F$

$$\frac{1}{C_b} = \frac{\frac{1}{400} + \frac{1}{300} + \frac{1}{1200}}{\frac{1}{10}} = \frac{2}{30}$$

$C_b = 15\mu F$

$$\frac{1}{C_c} = \frac{\frac{1}{400} + \frac{1}{300} + \frac{1}{1200}}{\frac{1}{40}} = \frac{4}{15}$$

$C_c = 3.75\mu F$

C_b in parallel with $50\mu F = 50 + 15 = 65\mu F$

C_c in series with $20\mu F = 23.75\mu F$

$65\mu F$ in series with $23.75\mu F = \dfrac{65x23.75}{88.75} = 17.39\mu F$

$17.39\mu F$ in parallel with $C_a = 17.39 + 5 = 22.39\mu F$

Hence $C_{eq} = \mathbf{22.39\mu F}$

Solution 6.29

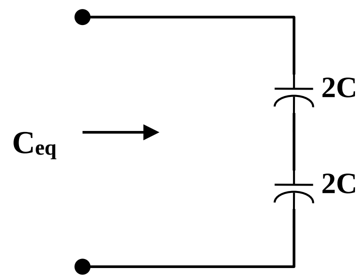
(a) C in series with C = C/(2)

C/2 in parallel with C = 3C/2

$$\frac{3C}{2} \text{ in series with } C = \frac{C\text{x}\,\frac{3C}{2}}{5\frac{C}{2}} = \frac{3C}{5}$$

$$3\frac{C}{5} \text{ in parallel with } C = C + 3\frac{C}{5} = \mathbf{1.6\ C}$$

(b)



$$\frac{1}{C_{eq}} = \frac{1}{2C} + \frac{1}{2C} = \frac{1}{C}$$

C_{eq} = **1 C**

Solution 6.30

$v_o = \frac{1}{C}\int_o^t idt + i(0)$

For $0 < t < 1$, $i = 90t$ mA,

$$v_o = \frac{10^{-3}}{3x10^{-6}}\int_o^t 90tdt + 0 = 15t^2 kV$$

$v_o(1) = 15$ kV

For $1 < t < 2$, $i = (180 – 90t)$ mA,

$$v_o = \frac{10^{-3}}{3x10^{-6}}\int_1^t (180 - 90t)dt + v_o(1)$$

$$= [60t – 15t^2]\Big|_1^t + 15kV$$

$$= [60t – 15t^2 – (60–15) + 15] \text{ kV} = [60t – 15t^2 – 30] \text{ kV}$$

$$\boldsymbol{v_o(t) = \begin{cases} 15t^2 kV, & 0 < t < 1 \\ [60t - 15t^2 - 30]kV, & 1 < t < 2 \end{cases}}$$

Solution 6.31

$$i_s(t) = \begin{bmatrix} 30tmA, & 0 < t < 1 \\ 30mA, & 1 < t < 3 \\ -75 + 15t, & 3 < t < 5 \end{bmatrix}$$

$C_{eq} = 4 + 6 = 10\mu F$

$$v = \frac{1}{C_{eq}} \int_o^t idt + v(0)$$

For 0 < t < 1,

$$v = \frac{10^{-3}}{10x10^{-6}} \int_o^t 30t \text{ dt} + 0 = 1.5t^2 \text{ kV}$$

For 1 < t < 3,

$$v = \frac{10^3}{10} \int_1^t 20dt + v(1) = [3(t-1) + 1.5]kV$$
$$= [3t - 1.5]kV$$

For 3 < t < 5,

$$v = \frac{10^3}{10} \int_3^t 15(t-5)dt + v(3)$$
$$= \left[1.5\frac{t^2}{2} - 7.5t\right]\Big|_3^t + 7.5kV = [0.75t^2 - 7.5t + 23.25]kV$$

$$\boldsymbol{v(t) = \begin{bmatrix} 1.5t^2 kV, & 0 < t < 1s \\ [3t - 1.5]kV, & 1 < t < 3s \\ [0.75t^2 - 7.5t + 23.25]kV, & 3 < t < 5s \end{bmatrix}}$$

$$i_1 = C_1 \frac{dv}{dt} = 6x10^{-6} \frac{dv}{dt}$$

$$\boldsymbol{i_1 = \begin{bmatrix} 18tmA, & 0 < t < 1s \\ 18mA, & 1 < t < 3s \\ [9t - 45]mA, & 3 < t < 5s \end{bmatrix}}$$

$$i_2 = C_2 \frac{dv}{dt} = 4x10^{-6} \frac{dv}{dt}$$

$$i_2 = \begin{cases} 12tmA, & 0 < t < 1s \\ 12mA, & 1 < t < 3s \\ [6t - 30]mA, & 3 < t < 5s \end{cases}$$

Solution 6.32

In the circuit in Fig. 6.64, let $i_s = 4.5e^{-2t}$ mA and the voltage across each capacitor is equal to zero at t = 0. Determine v_1 and v_2 and the energy stored in each capacitor for all t > 0.

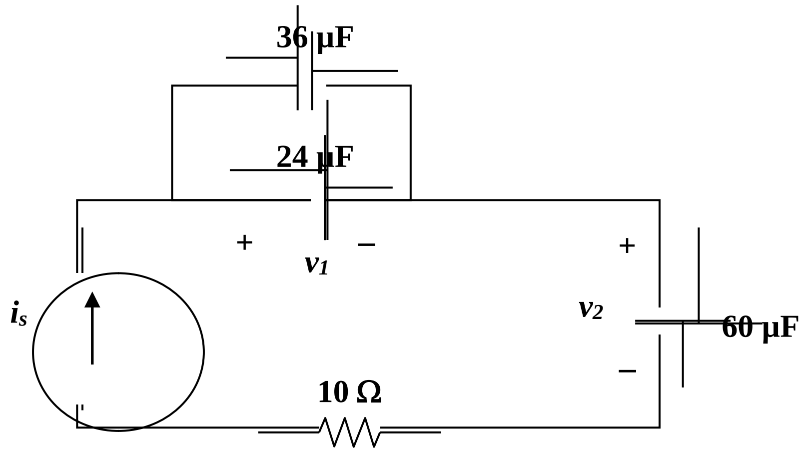


Figure 6.64
For Prob. 6.32.

Solution

Combining the 36 µF with the 24 µF we get 60 µF which leads to $v_1 = \frac{1}{60\mu}\int_0^t 4.5e^{-2\tau} m d\tau$

$=$ **$[37.5 – 37.5e^{-2t}]$ V** $= v_2$.

$(v_1)^2 = [(37.5)^2 – 2(37.5)^2 e^{-2t} + (37.5)^2 e^{-4t}] = 1406.25[1 – 2e^{-2t} + e^{-4t}] = (v_2)^2$

$w_{24} = 0.5\text{x}24\text{x}10^{-6}(v_1)^2 =$ **$16.875[1 – 2e^{-2t} + e^{-4t}]$ mJ**
$w_{36} = 0.5\text{x}36\text{x}10^{-6}(v_1)^2 =$ **$25.31[1 – 2e^{-2t} + e^{-4t}]$ mJ**
$w_{60} = 0.5\text{x}60\text{x}10^{-6}(v_2)^2 =$ **$42.19[1 – 2e^{-2t} + e^{-4t}]$ mJ**

Solution 6.33

Because this is a totally capacitive circuit, we can combine all the capacitors using the property that capacitors in parallel can be combined by just adding their values and we combine capacitors in series by adding their reciprocals. However, for this circuit we only have the three capacitors in parallel.

3 F + 2 F = 5 F (we need this to be able to calculate the voltage)

$C_{Th} = C_{eq} = 5+3+2 = 10$ F

The voltage will divide equally across the two 5 F capacitors. Therefore, we get:

$V_{Th} =$ **15 V**, $C_{Th} =$ **10 F**.

15 V, **10 F**

Solution 6.34

The current through a 25-mH inductor is $10e^{-t/2}$ A. Find the voltage and the power at t = 3 s.

Solution

$i = 10e^{-t/2}$

$$v = L\frac{di}{dt} = 25x10^{-3}(10)\left(\frac{-1}{2}\right)e^{-t/2}$$

$= -125e^{-t/2}$ mV

$v(3) = -125e^{-3/2}$ mV = **–27.89 mV**

$p = vi = -1.25e^{-t}$ W

$p(3) = -1.25e^{-3}$ W = **–62.23 mW**.

Solution 6.35

An inductor has a linear change in current from 100 mA to 200 mA in 2 ms and induces a voltage of 160 mV. Calculate the value of the inductor.

Solution

$v = L(di/dt)$ or $L = v/(di/dt)$

Clearly $di/dt = (0.2–0.1)/0.002 = 50$ amp/sec. Thus,

$L = 0.16/50 =$ **3.2 mH**.

Solution 6.36

Design a problem to help other students to better understand how inductors work.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The current through a 12-mH inductor is $i(t) = 30te^{-2t}$ A, $t \geq 0$. Determine: (a) the voltage across the inductor, (b) the power being delivered to the inductor at t = 1 s, (c) the energy stored in the inductor at t = 1 s.

Solution

(a) $v = L\dfrac{di}{dt} = 12x10^{-3}(30e^{-2t} - 60te^{-2t}) = \underline{(0.36 - 0.72t)e^{-2t}\ \text{V}}$

(b) $p = vi = (0.36 - 0.72x1)e^{-2}x30x1e^{-2} = 0.36x30e^{-4} = \underline{-0.1978\ \text{W}}$

(c) $\text{w} = \dfrac{1}{2}\text{Li}^2 = 0.5\text{x}12\text{x}10^{-3}(30\text{x}1\text{xe}^{-2})^2 =$ **98.9 mJ**.

Solution 6.37

$$v = L\frac{di}{dt} = 12x10^{-3}\,x4(100)\cos 100t$$

$$= \mathbf{4.8\ cos\ (100t)\ V}$$

p = vi = 4.8 x 4 sin 100t cos 100t = 9.6 sin 200t

$$w = \int_o^t p dt = \int_o^{11/200} 9.6\sin 200t$$

$$= -\frac{9.6}{200}\cos 200t\Big|_o^{11/200} J$$

$$= -48(\cos\pi - 1)mJ = \mathbf{96\ mJ}$$

Please note that this problem could have also been done by using (½)Li^2.

Chapter 6.38

$$v = L\frac{di}{dt} = 40x10^{-3}\left(e^{-2t} - 2te^{-2t}\right)dt$$

$$= \boldsymbol{40(1 - 2t)e^{-2t}mV\,, t > 0}$$

Solution 6.39

The voltage across a 50-mH inductor is given by

$v(t) = [5e^{-2t} + 2t + 4]$ V for t > 0.

Determine the current $i(t)$ through the inductor. Assume that $i(0) = 0$ A.

Solution

$$v = L\frac{di}{dt} \longrightarrow i = \frac{1}{L}\int_0^t i\,d\tau + i(0)$$

$$i = \frac{1}{50x10^{-3}}\int_0^t (5e^{-2\tau} + 2\tau + 4)d\tau + 0$$

$$= 20(-2.5e^{-2\tau} + \tau^2 + 4\tau)\Big|_0^t$$

i(t) = $\mathbf{[-50e^{-2t} + 50 + 20t^2 + 80t]}$ **A**.

Solution 6.40

$$i = \begin{cases} 5t, & 0 < t < 2ms \\ 10, & 2 < t < 4ms \\ 30 - 5t, & 4 < t < 6ms \end{cases}$$

$$v = L\frac{di}{dt} = \frac{5x10^{-3}}{10^{-3}} \begin{cases} 5, & 0 < t < 2ms \\ 0, & 2 < t < 4ms \\ -5, & 4 < t < 6ms \end{cases} = \begin{cases} 25, & 0 < t < 2ms \\ 0, & 2 < t < 4ms \\ -25, & 4 < t < 6ms \end{cases}$$

At t = 1ms, v = **25 V**
At t = 3ms, v = **0 V**
At t = 5ms, v= **–25 V**

Solution 6.41

$$i = \frac{1}{L}\int_0^t v dt + C = \left(\frac{1}{2}\right)\int_o^t 20\left(1 - e^{-2t}\right)dt + C$$

$$= 10\left(t + \frac{1}{2}e^{-2t}\right)\Big|_o^t + C = 10t + 5e^{-2t} - 4.7A$$

Note, we get C = –4.7 from the initial condition for i needing to be 0.3 A.

We can check our results be solving for v = Ldi/dt.

$v = 2(10 - 10e^{-2t})V$ which is what we started with.

At t = l s, $i = 10 + 5e^{-2} - 4.7 = 10 + 0.6767 - 4.7 =$ **5.977 A**

$w = \frac{1}{2}L\,i^2 =$ **35.72J**

Solution 6.42

$$i = \frac{1}{L}\int_o^t vdt + i(0) = \frac{1}{5}\int_o^t v(t)dt - 1$$

For $0 < t < 1$, $i = \frac{10}{5}\int_0^t dt - 1 = 2t - 1$ A

For $1 < t < 2$, $i = 0 + i(1) = 1A$

For $2 < t < 3$, $i = \frac{1}{5}\int 10dt + i(2) = 2t\Big|_t^2 + 1$

$= 2t - 3$ A

For $3 < t < 4$, $i = 0 + i(3) = 3$ A

For $4 < t < 5$, $i = \frac{1}{5}\int_4^t 10dt + i(4) = 2t\Big|_4^t + 3$

$= 2t - 5$ A

Thus,

$$\boldsymbol{i(t) = \begin{cases} 2t - 1A, & 0 < t < 1 \\ 1A, & 1 < t < 2 \\ 2t - 3A, & 2 < t < 3 \\ 3A, & 3 < t < 4 \\ 2t - 5, & 4 < t < 5 \end{cases}}$$

Solution 6.43

The current in a 150-mH inductor increases from 0 to 60 mA (steady-state). How much energy is stored in the inductor?

Solution

$w = (1/2)L(i(t))^2 = 0.5 x 0.15 x 0.06^2 =$ **270 μJ**.

Solution 6.44

A 100-mH inductor is connected in parallel with a 2-kΩ resistor. The current through the inductor is $i(t) = 35e^{-400t}$ mA.

(a) Find the voltage $v_L(t)$ across the inductor. (b) Find the voltage $v_R(t)$ across the resistor. (c) Is $v_R(t) + v_L(t) = 0$? (d) Calculate the energy stored in the inductor at t=0.

Solution

(a) $v_L(t) = Ldi/dt = 0.1(-400)0.035e^{-400t} =$ $\mathbf{1.4e^{-400t}}$ **V**.

(b) Since R and L are in parallel, $v_R(t) = v_L(t) =$ $\mathbf{1.4e^{-400t}}$ **V**.

(c) Again, since the two elements are in parallel, $v_R(t) = v_L(t) = 1.4e^{-400t}$ V thus, $v_R(t) + v_L(t) = 2.8e^{-400t}$ and **not equal to zero**!

(d) $w = 0.5Li^2 = 0.5(0.1)(0.035)^2 =$ **61.25 µJ**.

Solution 6.45

If the voltage waveform in Fig. 6.68 is applied to a 25-mH inductor, find the inductor current ***i*(*t*)** for 0 < t < 2 seconds. Assume ***i*(0)** = 0.

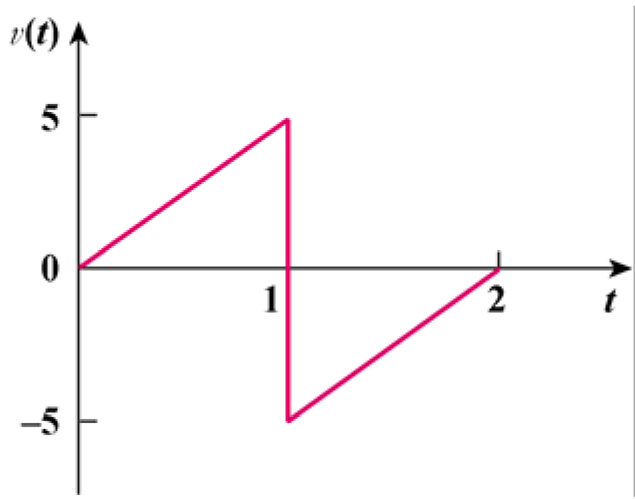


Figure 6.68
For Prob. 6.45.

Solution

i(t) = $\frac{1}{L}\int_o^t v(t) + i(0)$

For 0 < t < 1, v = 5t

$$i = \frac{1}{25x10^{-3}}\int_o^t 5t \text{ dt} + 0$$

= $\mathbf{100t^2}$ **A**.

For 1 < t < 2, v = –10 + 5t

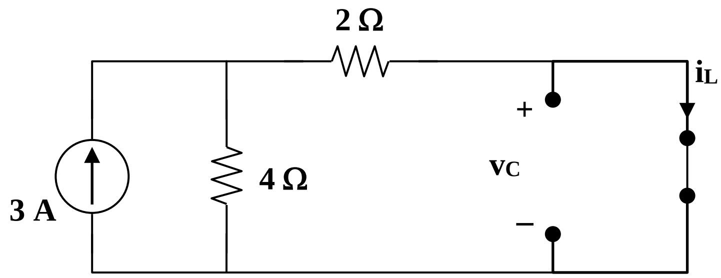
$$i = \frac{1}{25x10^{-3}}\int_1^t (-10+5t)dt + i(1)$$

$$= \left(\int_1^t (0.2\tau - 0.4)d\tau + 0.1\right)kA = \{\left[0.1\tau^2 - 0.4\tau\right]_1^t + 0.1\} \text{ kA}$$

= **[400 – 400t + 100t^2] A**.

Solution 6.46

Under dc conditions, the circuit is as shown below:



By current division,

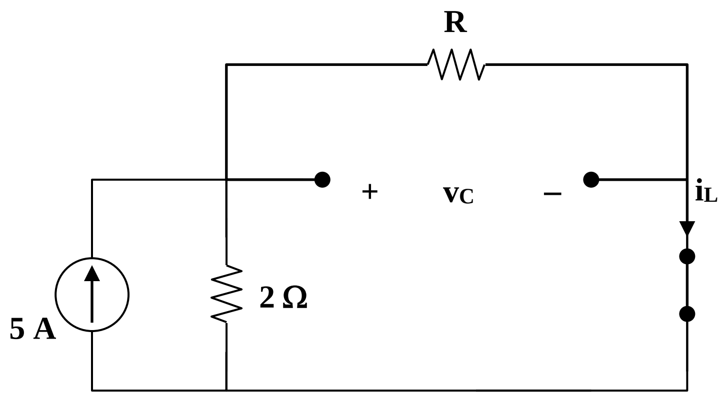
$$i_L = \frac{4}{4+2}(3) = \mathbf{2A}, \quad v_c = \mathbf{0V}$$

$$w_L = \frac{1}{2}L\,i_L^2 = \frac{1}{2}\left(\frac{1}{2}\right)(2)^2 = \mathbf{1J}$$

$$w_c = \frac{1}{2}C\,v_c^2 = \frac{1}{2}(2)(v) = \mathbf{0J}$$

Solution 6.47

Under dc conditions, the circuit is equivalent to that shown below:



$$i_L = \frac{2}{R+2}(5) = \frac{10}{R+2}, \quad v_c = Ri_L = \frac{10R}{R+2}$$

$$w_c = \frac{1}{2}Cv_c^2 = 80x10^{-6}x\frac{100R^2}{(R+2)^2}$$

$$w_L = \frac{1}{2}Li_1^2 = 2x10^{-3}x\frac{100}{(R+2)^2}$$

If $w_c = w_L$,

$$80x10^{-6}x\frac{100R^2}{(R+2)^2} = \frac{2x10^{-3}x100}{(R+2)^2} \longrightarrow 80 \text{ x } 10^{-3}R^2 = 2$$

R = **5Ω**

Solution 6.48

Under steady-state dc conditions, find *i* and *v* in the circuit in Fig. 6.71.

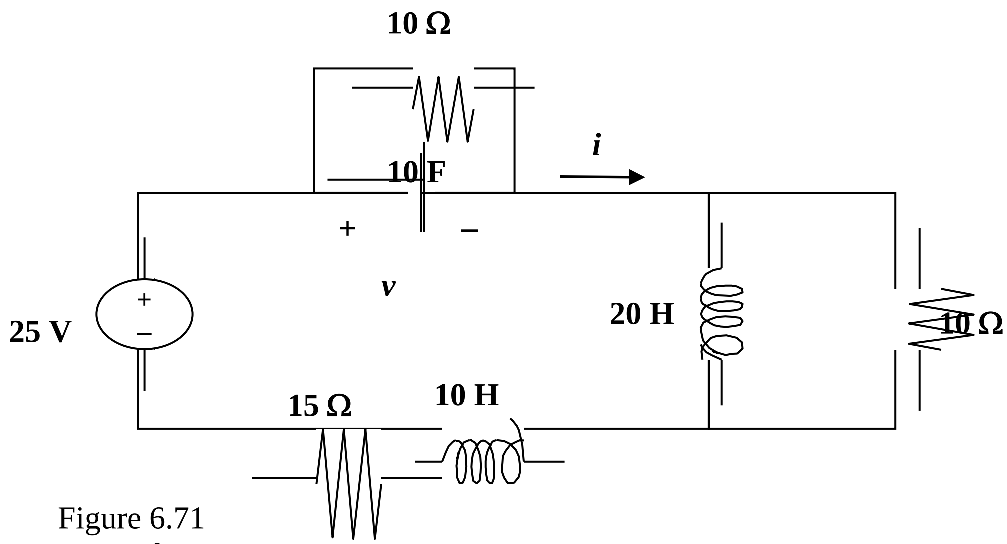


Figure 6.71
For Prob. 6.48.

Solution

Under steady-state, the inductor acts like a short-circuit, while the capacitor acts like an open circuit. Thus the resistor on the right is shorted out and the voltage source only sees the top 10 Ω resistor in series with the 15 Ω resistor for a total of 25 Ω.

Thus $i = 25/25 =$ **1 A** and $v = 10\text{x}1 =$ **10 V**.

Solution 6.49

Find the equivalent inductance of the circuit in Fig. 6.72. Assume all inductors are 40 mH.

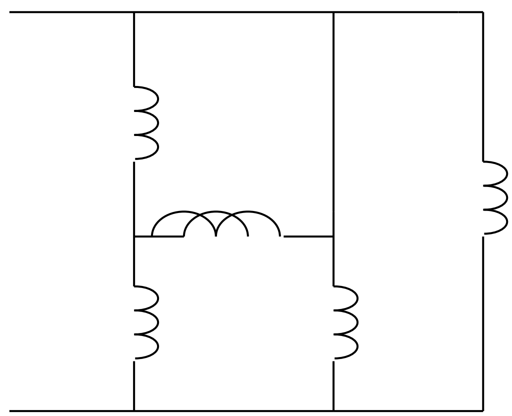
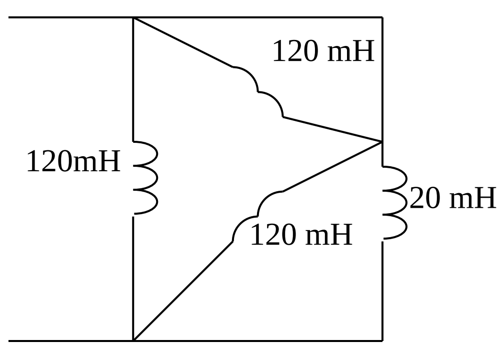


Figure 6.72
For Prob. 6.49.

Solution

Converting the wye-subnetwork to its equivalent delta gives the circuit below.



$$120||0 = 0, \quad 120||20 = 120x20/140 = 17.14286 \text{ mH}.$$

Finally,

L_{eq} = 0.12x0.01714286/(0.12+0.01714286) = **15 mH**.

Solution 6.50

16mH in series with 14 mH = 16+14=30 mH
24 mH in series with 36 mH = 24+36=60 mH
30mH in parallel with 60 mH = 30x60/90 = **20 mH**

Solution 6.51

$$\frac{1}{L} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10} \qquad\qquad L = 10\text{ mH}$$

$$L_{eq} = 10 \| (25 + 10) = \frac{10x35}{45}$$

$$= \mathbf{7.778\ mH}$$

Solution 6.52

Using Fig. 6.74, design a problem to help other students better understand how inductors behave when connected in series and when connected in parallel.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find L_{eq} in the circuit of Fig. 6.74.

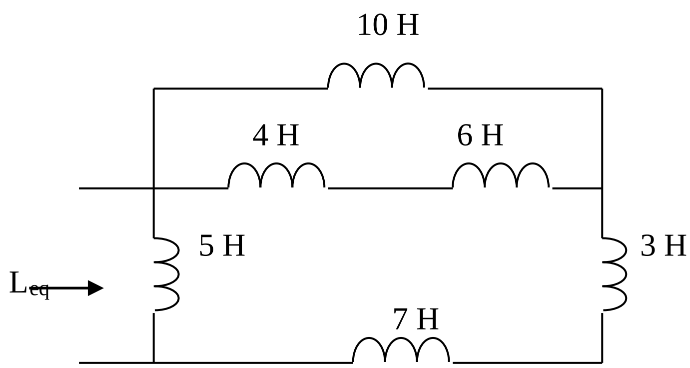


Figure 6.74 For Prob. 6.52.

Solution

$$L_{eq} = 5//(7+3+10//(4+6)) == 5//(7+3+5)) = \frac{5x15}{20} = \underline{3.75\ \text{H}}$$

Solution 6.53

$$L_{eq} = 6 + 10 + 8 \| \left[5 \| (8+12) + 6 \| (8+4)\right]$$

$$= 16 + 8 \| (4+4) = 16 + 4$$

L_{eq} = **20 mH**

Solution 6.54

Find the equivalent inductance looking into the terminals of the circuit in Fig. 6.76.

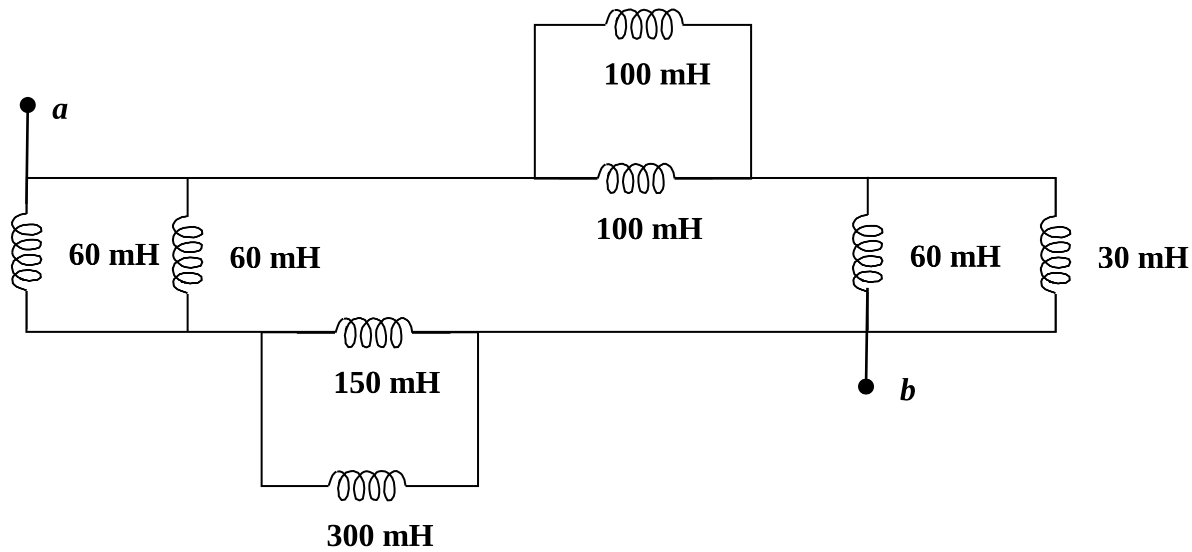


Figure 6.76
For Prob. 6.54.

Solution

The parallel combinations gives us L_{6060} = [60x60/(60+60)]mH = 30 mH; L_{150300} = [150x300/(150+300)]mH = 100 mH; L_{6030} = [60x30/(60+30)]mH = 20 mH; and L_{100100} = [100x100/(100+100)]mH = 50 mH.

We now have inductors in series and then in parallel or L_{30100} = 130 mH and L_{2050} = 70 mH and finally we get L_{ab} = [130x70/(130+70)]mH = **45.5 mH**.

Solution 6.55

(a) L//L = 0.5L, L + L = 2L

$$L_{eq} = L + 2L\,//\,0.5L = L + \frac{2Lx0.5L}{2L+0.5L} = \underline{1.4L} = \mathbf{1.4\ L}.$$

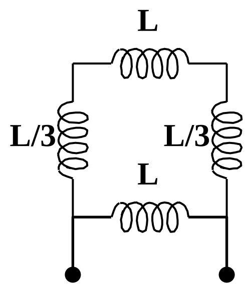
(b) L//L = 0.5L, L//L + L//L = L

$$L_{eq} = L//L = \mathbf{500\ mL}$$

Solution 6.56

$$L\|L\|L = \frac{1}{\frac{3}{L}} = \frac{L}{3}$$

Hence the given circuit is equivalent to that shown below:



$$L_{eq} = L \left\| \left(L + \frac{2}{3}L \right) = \frac{Lx\frac{5}{3}L}{L + \frac{5}{3}L} = \frac{\mathbf{5}}{\mathbf{8}}\boldsymbol{L} \right.$$

Solution 6.57

$$\text{Let } v = L_{eq}\frac{di}{dt} \qquad (1)$$

$$v = v_1 + v_2 = 4\frac{di}{dt} + v_2 \qquad (2)$$

$$i = i_1 + i_2 \longrightarrow i_2 = i - i_1 \qquad (3)$$

$$v_2 = 3\frac{di_1}{dt} \text{ or } \frac{di_1}{dt} = \frac{v_2}{3} \qquad (4)$$

and

$$-v_2 + 2\frac{di}{dt} + 5\frac{di_2}{dt} = 0$$

$$v_2 = 2\frac{di}{dt} + 5\frac{di_2}{dt} \qquad (5)$$

Incorporating (3) and (4) into (5),

$$v_2 = 2\frac{di}{dt} + 5\frac{di}{dt} - 5\frac{di_1}{dt} = 7\frac{di}{dt} - 5\frac{v_2}{3}$$

$$v_2\left(1 + \frac{5}{3}\right) = 7\frac{di}{dt}$$

$$v_2 = \frac{21}{8}\frac{di}{dt}$$

Substituting this into (2) gives

$$v = 4\frac{di}{dt} + \frac{21}{8}\frac{di}{dt}$$

$$= \frac{53}{8}\frac{di}{dt}$$

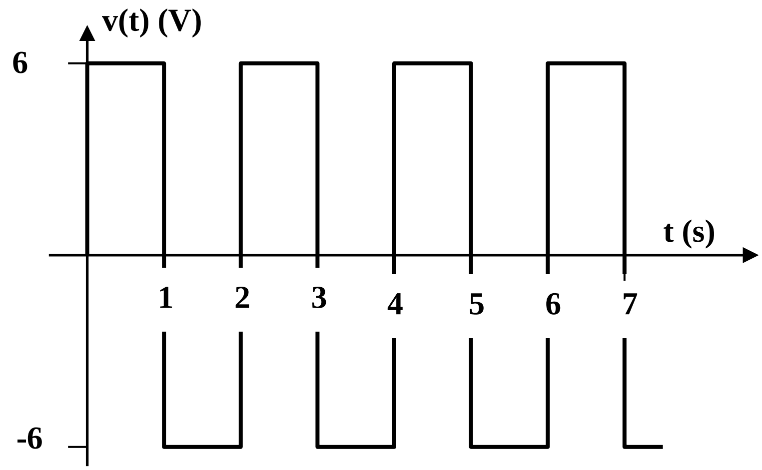
Comparing this with (1),

$$L_{eq} = \frac{53}{8} = \mathbf{6.625\ H}$$

Solution 6.58

$$v = L\frac{di}{dt} = 3\frac{di}{dt} = 3 \text{ x slope of } i(t).$$

Thus v is sketched below:



Solution 6.59

(a) $v_s = (L_1 + L_2)\dfrac{di}{dt}$

$$\frac{di}{dt} = \frac{v_s}{L_1 + L_2}$$

$$v_1 = L_1\frac{di}{dt},\ \ v_2 = L_2\frac{di}{dt}$$

$$\boldsymbol{v_1 = \frac{L_1}{L_1 + L_2}v_s\,,\ v_L = \frac{L_2}{L_1 + L_2}v_s}$$

(b) $v_i = v_2 = L_1\dfrac{di_1}{dt} = L_2\dfrac{di_2}{dt}$

$$i_s = i_1 + i_2$$

$$\frac{di_s}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{v}{L_1} + \frac{v}{L_2} = v\frac{(L_1 + L_2)}{L_1 L_2}$$

$$i_1 = \frac{1}{L_1}\int v\,dt = \frac{1}{L_1}\int \frac{L_1 L_2}{L_1 + L_2}\frac{di_s}{dt}dt = \boldsymbol{\frac{L_2}{L_1 + L_2}i_s}$$

$$i_2 = \frac{1}{L_2}\int v\,dt = \frac{1}{L_2}\int \frac{L_1 L_2}{L_1 + L_2}\frac{di_s}{dt}dt = \boldsymbol{\frac{L_1}{L_1 + L_2}i_s}$$

Solution 6.60

$$L_{eq} = 3//5 = \frac{15}{8}$$

$$v_o = L_{eq}\frac{di}{dt} = \frac{15}{8}\frac{d}{dt}\left(4e^{-2t}\right) = \underline{-15e^{-2t}}$$

$$i_o = \frac{I}{L}\int_0^t v_o(t)dt + i_o(0) = 2 + \frac{1}{5}\int_0^t (-15)e^{-2t}dt = 2 + 1.5e^{-2t}\Big|_0^t$$

i_o = **(0.5 + 1.5e^{-2t}) A**

Solution 6.61

(a) $L_{eq} = 20//(4+6) = 20x10/30 = \underline{6.667 \text{ mH}}$

Using current division,

$$i_1(t) = \frac{10}{10+20} i_s = \underline{e^{-t} \text{ mA}}$$

$$i_2(t) = \underline{2e^{-t} \text{ mA}}$$

(b) $v_o = L_{eq} \dfrac{di_s}{dt} = \dfrac{20}{3} x10^{-3}(-3e^{-t} x10^{-3}) = \underline{-20e^{-t} \ \mu\text{V}}$

(c) $w = \dfrac{1}{2} L i_1^2 = \dfrac{1}{2} x20x10^{-3} xe^{-2} x10^{-6} = \underline{1.3534 \text{ nJ}}$

Solution 6.62

Consider the circuit in Fig. 6.84. Given that $v(t) = 12e^{-3t}$ mV for $t > 0$ and $i_1(0) = -30$ mA, find: (a) $i_2(0)$, (b) $i_1(t)$ and $i_2(t)$.

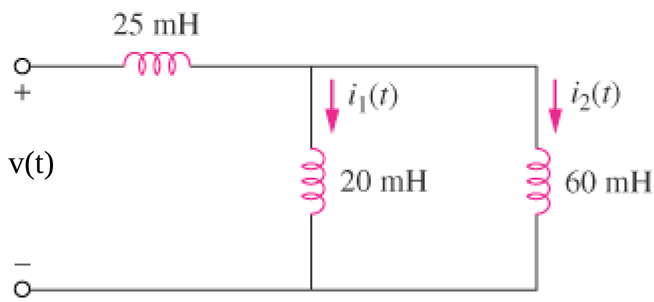


Figure 6.84
For Prob. 6.62.

Solution

(a) $L_{eq} = 25 + 20 \,||\, 60 = 25 + \dfrac{20x60}{80} = 40 \text{ mH}$

$$v = L_{eq}\frac{di}{dt} \quad \longrightarrow \quad i = \frac{1}{L_{eq}}\int v(t)dt + i(0) = \frac{10^{-3}}{40x10^{-3}}\int_0^t 12e^{-3t}dt + i(0) = -0.1(e^{-3t} - 1) + i(0)$$

Using current division and the fact that all the currents were zero when the circuit was put together, we get,

$$i_1 = \frac{60}{80}i = \frac{3}{4}i, \qquad i_2 = \frac{1}{4}i$$

$$i_1(0) = \frac{3}{4}i(0) \quad \longrightarrow \quad 0.75i(0) = -0.03 \quad \longrightarrow \quad i(0) = -0.04$$

$$i_2 = \frac{1}{4}(-0.1e^{-3t} + 0.06) \text{ A} = (-25e^{-3t} + 15) \text{ mA}$$

$i_2(0) = -25+15 =$ **–10 mA**.

(b) $i_1(t) = 0.75(-0.1e^{-3t} + 0.06) =$ **($-75e^{-3t} + 45$) mA** and $i_2(t) =$ **($-25e^{-3t} + 15$) mA**.

Solution 6.63

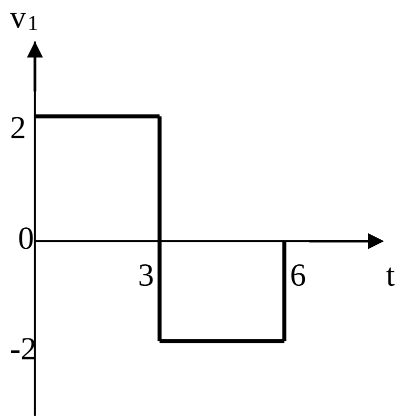
We apply superposition principle and let

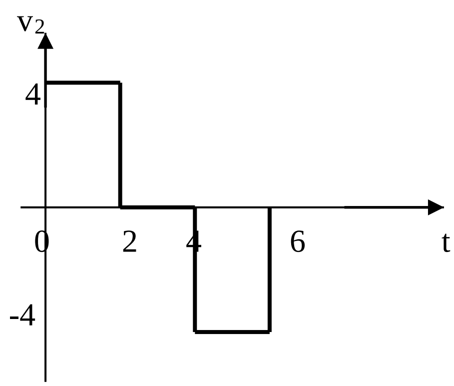
$$v_o = v_1 + v_2$$

where v_1 and v_2 are due to i_1 and i_2 respectively.

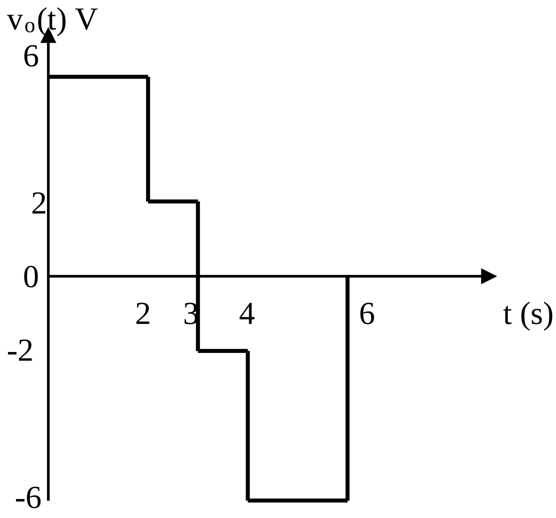
$$v_1 = L\frac{di_1}{dt} = 2\frac{di_1}{dt} = \begin{cases} 2, & 0 < t < 3 \\ -2, & 3 < t < 6 \end{cases}$$

$$v_2 = L\frac{di_2}{dt} = 2\frac{di_2}{dt} = \begin{cases} 4, & 0 < t < 2 \\ 0, & 2 < t < 4 \\ -4, & 4 < t < 6 \end{cases}$$





Adding v_1 and v_2 gives v_o, which is shown below.



Solution 6.64

(a) When the switch is in position A, i= –6 = i(0)

When the switch is in position B, $i(\infty) = 12/4 = 3$, $\tau = L/R = 1/8$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

i(t) = $\mathbf{(3 - 9e^{-8t})}$ **A**

(b) -12 + 4i(0) + v=0, i.e. v = 12 – 4i(0) = **36 V**

(c) At steady state, the inductor becomes a short circuit so that v = **0 V.**

Solution 6.65

(a) $w_5 = \frac{1}{2} L_1 i_1^2 = \frac{1}{2} x5x(4)^2 =$ **40 J**

$w_{20} = \frac{1}{2}(20)(-2)^2 =$ **40 J**

(b) $w = w_5 + w_{20} =$ **80 J**

(c) $i_1 = \frac{1}{L_1}\int_0^t -50e^{-200t}dt + i_1(0) = \frac{1}{5}\left(\frac{1}{200}\right)\left(50e^{-200t}x10^{-3}\right)\Big|_0^t + 4$

= **[5x10^{-5}(e^{-200t} – 1) + 4] A**

$i_2 = \frac{1}{L_2}\int_0^t -50e^{-200t}dt + i_2(0) = \frac{1}{20}\left(\frac{1}{200}\right)\left(50e^{-200t}x10^{-3}\right)\Big|_0^t - 2$

= **[1.25x10^{-5} (e^{-200t} – 1) – 2] A**

(d) $i = i_1 + i_2 =$ **[6.25x10^{-5} (e^{-200t}– 1) + 2] A**

Solution 6.66

If v=i, then

$$i = L\frac{di}{dt} \longrightarrow \frac{dt}{L} = \frac{di}{i}$$

Integrating this gives

$$\frac{t}{L} = \ln(i) - \ln(C_o) = \ln\left(\frac{i}{C_o}\right) \rightarrow i = C_o e^{t/L}$$

$i(0) = 2 = C_o$

$i(t) = 2e^{t/0.02} = \mathbf{2e^{50t}}$ **A**.

Solution 6.67

$$v_o = -\frac{1}{RC}\int vi\,dt, \text{ RC} = 50 \text{ x } 10^3 \text{ x } 0.04 \text{ x } 10^{-6} = 2 \text{ x } 10^{-3}$$

$$v_o = \frac{-10^3}{2}\int 10\sin 50t\ dt$$

v_o = **100cos(50t) mV**

Solution 6.68

A 6-V dc voltage is applied to an integrator with $\boldsymbol{R}$ = 50 kΩ, $\boldsymbol{C}$ = 100 μF at $\boldsymbol{t}$ = 0. How long will it take for the op amp to saturate if the saturation voltages are +12 V and –12 V? Assume that the initial capacitor voltage was zero.

Solution

$v_o = -\frac{1}{RC}\int vi \, dt + v_o(0)$, RC = 50 x 10^3 x 100 x 10^{-6} = 5

$$v_o = -\frac{1}{5}\int_0^t 6d\tau + 0 = -\frac{6t}{5} = -1.2t \; .$$

The op amp will saturate at $v_o = \pm 12$

$-12 = -1.2t \longrightarrow t =$ **10 s**.

Solution 6.69

RC = 4 x 10^6 x 1 x 10^{-6} = 4

$$v_o = -\frac{1}{RC}\int v_i dt = -\frac{1}{4}\int v_i dt$$

For 0 < t < 1, v_i = 20, $v_o = -\frac{1}{4}\int_o^t 20dt =$ -5t mV

For 1 < t < 2, v_i = 10, $v_o = -\frac{1}{4}\int_1^t 10dt + v(1) = -2.5(t-1) - 5$

= -2.5t - 2.5mV

For 2 < t < 4, v_i = - 20, $v_o = +\frac{1}{4}\int_2^t 20dt + v(2) = 5(t-2) - 7.5$

= 5t - 17.5 mV

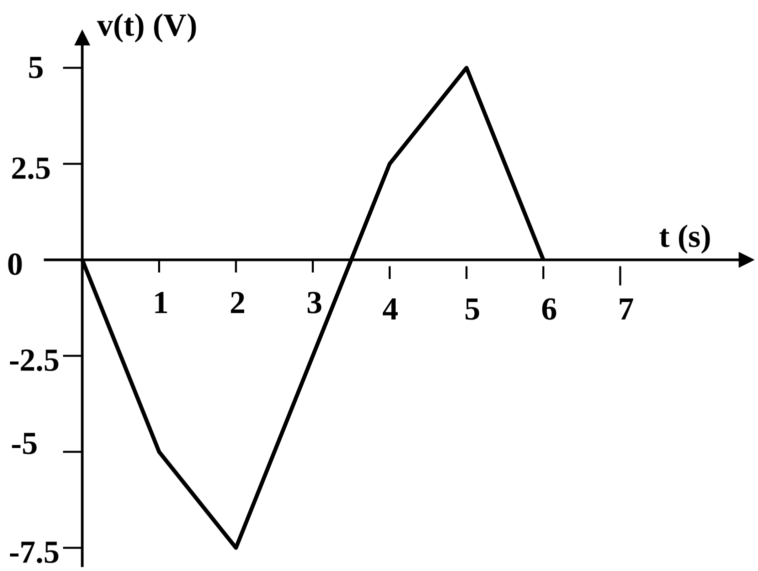
For 4 < t < 5m, v_i = -10, $v_o = \frac{1}{4}\int_4^t 10dt + v(4) = 2.5(t-4) + 2.5$

= 2.5t - 7.5 mV

For 5 < t < 6, v_i = 20, $v_o = -\frac{1}{4}\int_5^t 20dt + v(5) = -5(t-5) + 5$

= - 5t + 30 mV

Thus $v_o(t)$ is as shown below:



Solution 6.70

Using a single op amp, a capacitor, and resistors of 100 kΩ or less, design a circuit to implement

$$v_o = -2\int_0^t \mathrm{vi}(\tau)\,d\tau$$

Assume $\boldsymbol{v_o} = 0$ at $\boldsymbol{t} = 0$.

Solution

One possibility is as follows:

Let RC = 0.5 (which produces (1/RC) = 2.

Next we need to either pick R or C. Let us choose R = **100 kΩ** a practical value. This means that C = $0.5/10^5$ = **5 μF**.

Solution 6.71

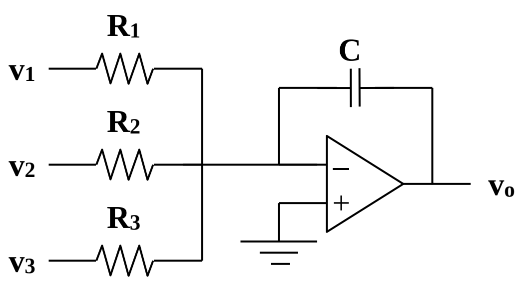
Show how you would use a single op amp to generate

$$v_0 = -\int (v_1 + 4v_2 + 10v_3)\,dt$$

If the integrating capacitor is $\boldsymbol{C}$ = 5 μF, obtain other component values.

Solution

By combining a summer with an integrator, we have the circuit below:



$$v_o = -\frac{1}{R_1C}\int v_1 dt - \frac{1}{R_2C}\int v_2 dt - \frac{1}{R_3C}\int v_2 dt$$

For the given problem, C = 5 μF,

$R_1C = 1$ gives us $R_1 = 1/C = 10^6/5 =$ **200 kΩ**
$R_2C = 1/4$ gives us $R_2 = 0.25/(C) = 0.25x200k\Omega =$ **50 kΩ**
$R_3C = 1/10$ gives us $R_3 = 0.1x200k\Omega =$ 2**0 kΩ**

Solution 6.72

The output of the first op amp is

$$v_1 = -\frac{1}{RC}\int v_i\, dt = -\frac{1}{10x10^3 x2x10^{-6}}\int_o^t v_i dt = -\frac{100t}{2}$$

$$= -50t$$

$$v_o = -\frac{1}{RC}\int v_i\, dt = -\frac{1}{20x10^3 x0.5x10^{-6}}\int_o^t (-50t)dt$$

$$= 2500t^2$$

At t = 1.5ms,

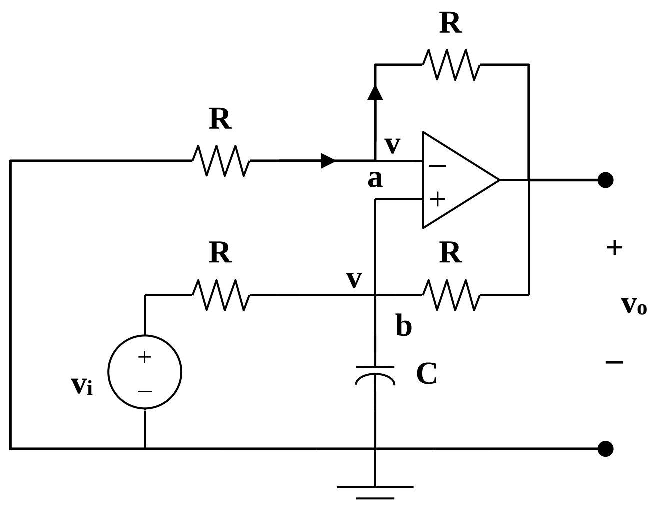
$$v_o = 2500(1.5)^2 x10^{-6} = \mathbf{5.625\ mV}$$

Solution 6.73

Consider the op amp as shown below:

Let $v_a = v_b = v$

At node a, $\frac{0-v}{R} = \frac{v-v_o}{R} \longrightarrow$ **$2v - v_o = 0$** (1)



At node b, $\frac{v_i - v}{R} = \frac{v-v_o}{R} + C\frac{dv}{dt}$

$$\boldsymbol{v_i = 2v - v_o + RC\frac{dv}{dt}} \qquad (2)$$

Combining (1) and (2),

$$\boldsymbol{v_i = v_o - v_o + \frac{RC}{2}\frac{dv_o}{dt}}$$

or

$$\boldsymbol{v_o = \frac{2}{RC}\int v_i\, dt}$$

showing that the circuit is a noninverting integrator.

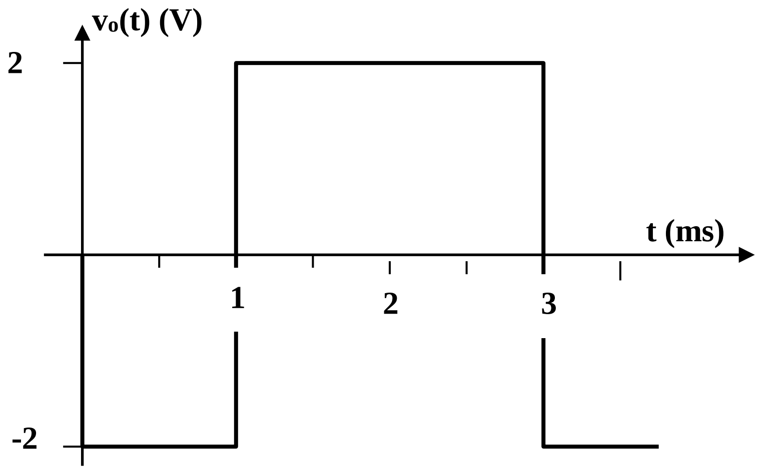
Solution 6.74

RC = 0.01 x 20 x 10^{-3} sec

$$v_o = -RC\frac{dv_i}{dt} = -0.2\frac{dv}{dt}\,m\sec$$

$$v_o = \begin{bmatrix} -2V, & 0 < t < 1 \\ 2V, & 1 < t < 3 \\ -2V, & 3 < t < 4 \end{bmatrix}$$

Thus $v_o(t)$ is as sketched below:



Solution 6.75

An op amp differentiator has **_R_**= 250 kΩ and **_C_** = 10 μF. The input voltage is a ramp **_r_(_t_)** = 7 **_t_** mV. Find the output voltage.

Solution

$$v_0 = -RC\frac{dv_i}{dt}, \quad RC = 250x10^3 x10x10^{-6} = 2.5$$

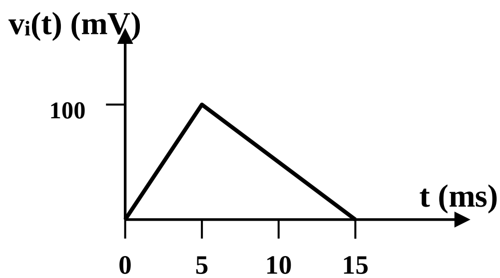
$$v_o = -2.5\frac{d}{dt}(7t) = \mathbf{-17.5\ mV}.$$

Solution 6.76

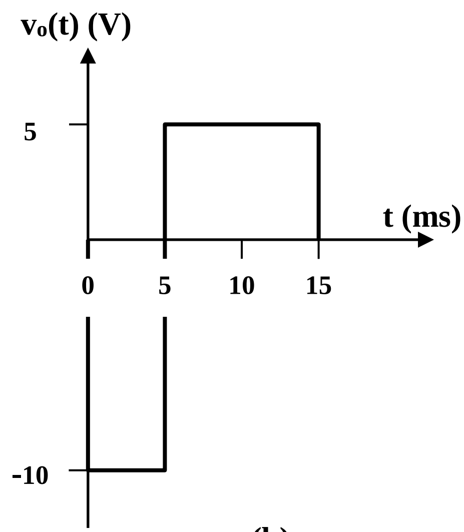
$$v_o = -RC\frac{dv_i}{dt}, \; RC = 50 \text{ x } 10^3 \text{ x } 10 \text{ x } 10^{-6} = 0.5$$

$$v_o = -0.5\frac{dv_i}{dt} = \begin{cases} -10, & 0 < t < 5 \\ 5, & 5 < t < 15 \end{cases}$$

The input is sketched in Fig. (a), while the output is sketched in Fig. (b).



(a)



(b)

Solution 6.77

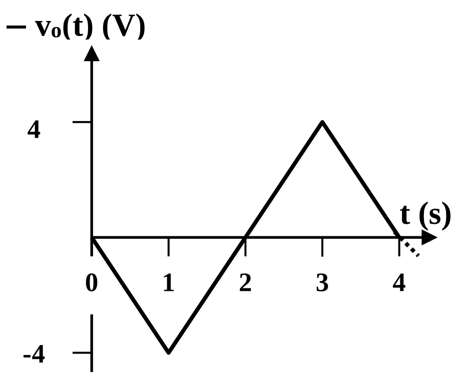
$i = i_R + i_C$

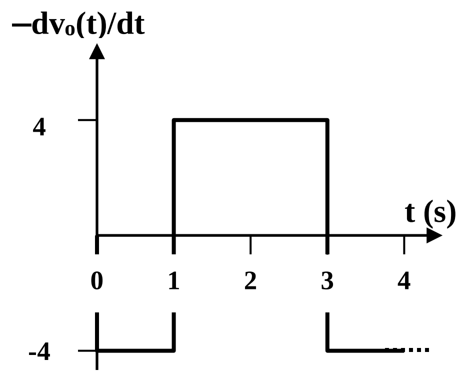
$$\frac{v_i - 0}{R} = \frac{0 - v_0}{R_F} + C\frac{d}{dt}(0 - v_o)$$

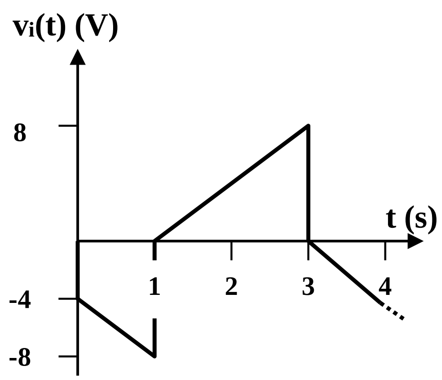
$$R_F C = 10^6 x 10^{-6} = 1$$

Hence $v_i = -\left(v_o + \frac{dv_o}{dt}\right)$

Thus v_i is obtained from v_o as shown below:







Solution 6.78

Design an analog computer to simulate

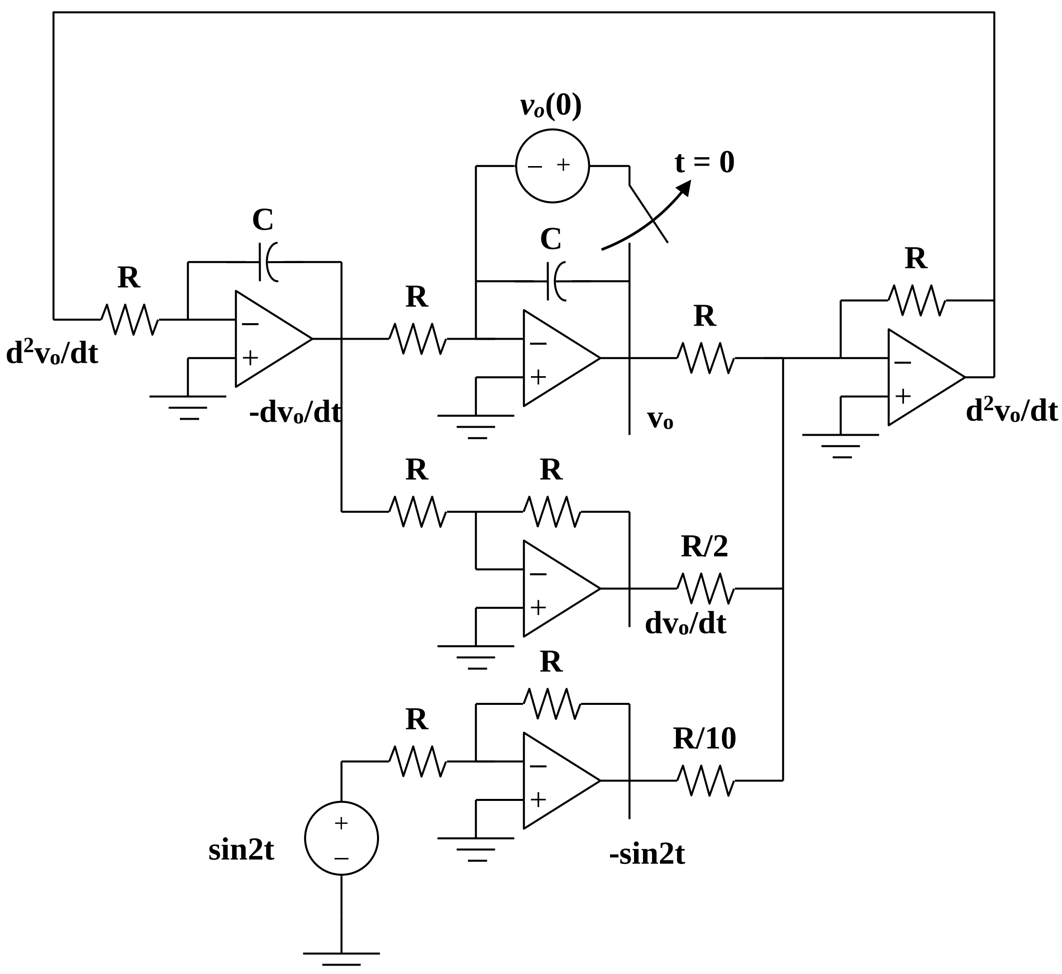
$$\frac{d^2 v_0}{dt^2} + 2\frac{dv_0}{dt} + v_0 = 10\sin 2t$$

where $\boldsymbol{v_0(0)}$ = –6 V and $v'_0\mathbf{(0)}$ = 0.

Solution

$$\frac{d^2 v_o}{dt} = 10\sin 2t - \frac{2dv_o}{dt} - v_o$$

Thus, by combining integrators with a summer, we obtain the appropriate analog computer as shown below, where $v_o(0)$ = –6 V:



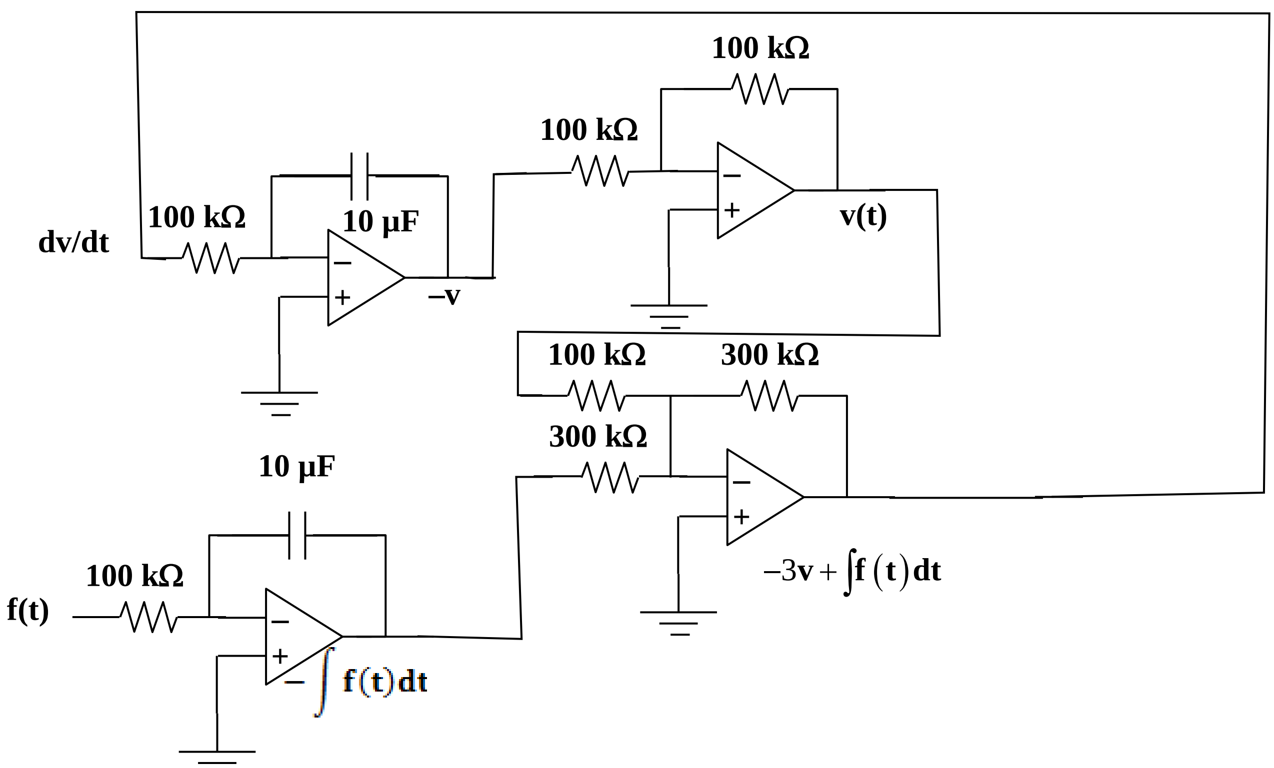
Solution 6.79

Design an analog computer circuit to solve for v(t) given the following equation and a value for f(t) and that v(0) = 0 V.

$$\frac{dv(t)}{dt} + 3v(t) = \int f(t)\,dt$$

Solution

We can write the equation as $\frac{dv(t)}{dt} = \int f(t)\,dt - 3v(t)$. As with any design problem, there are many acceptable solutions, this is just one of them.



Solution 6.80

From the given circuit,

$$\frac{d^2 v_o}{dt^2} = f(t) - \frac{1000\text{k}\Omega}{5000\text{k}\Omega} v_o - \frac{1000\text{k}\Omega}{200\text{k}\Omega} \frac{dv_o}{dt}$$

or

$$\boldsymbol{\frac{d^2 v_o}{dt^2} + 5\frac{dv_o}{dt} + 2v_o = f(t)}$$

Solution 6.81

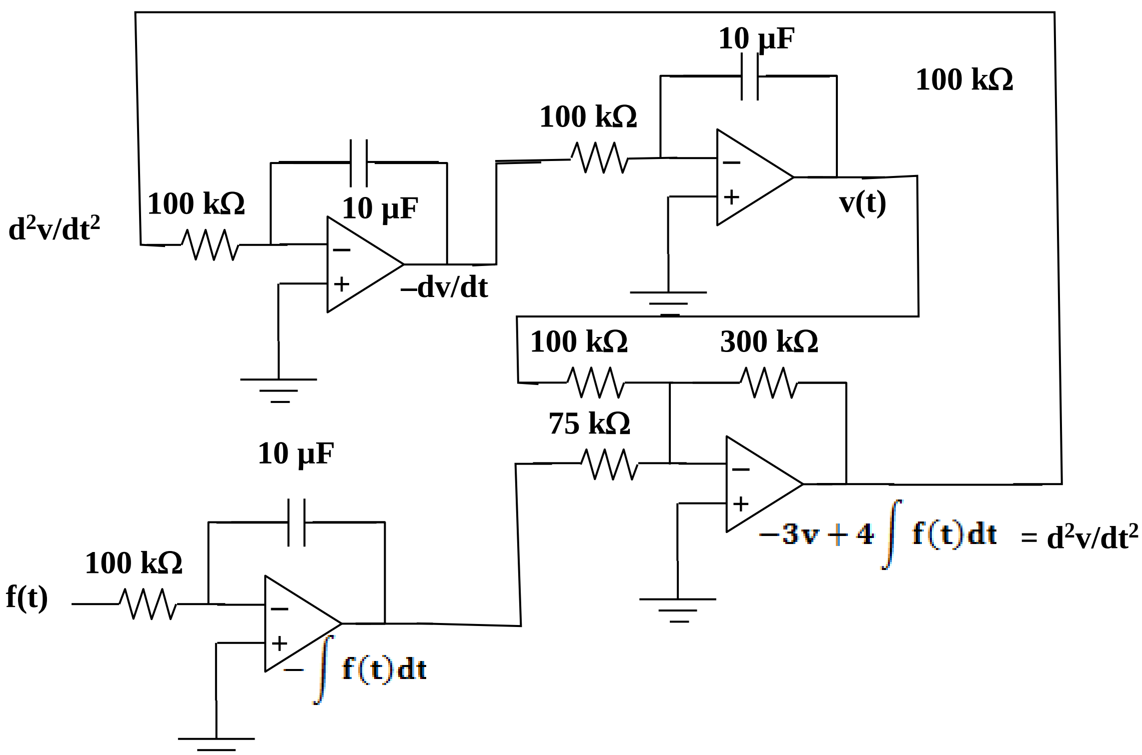
Design an analog computer to simulate the following equation to solve for v(t) (assume the initial conditions are zero):

$$\frac{\mathbf{d^3v(t)}}{\mathbf{dt^3}}+3\frac{\mathbf{dv(t)}}{\mathbf{dt}}=4\mathbf{f(t)}.$$

Solution

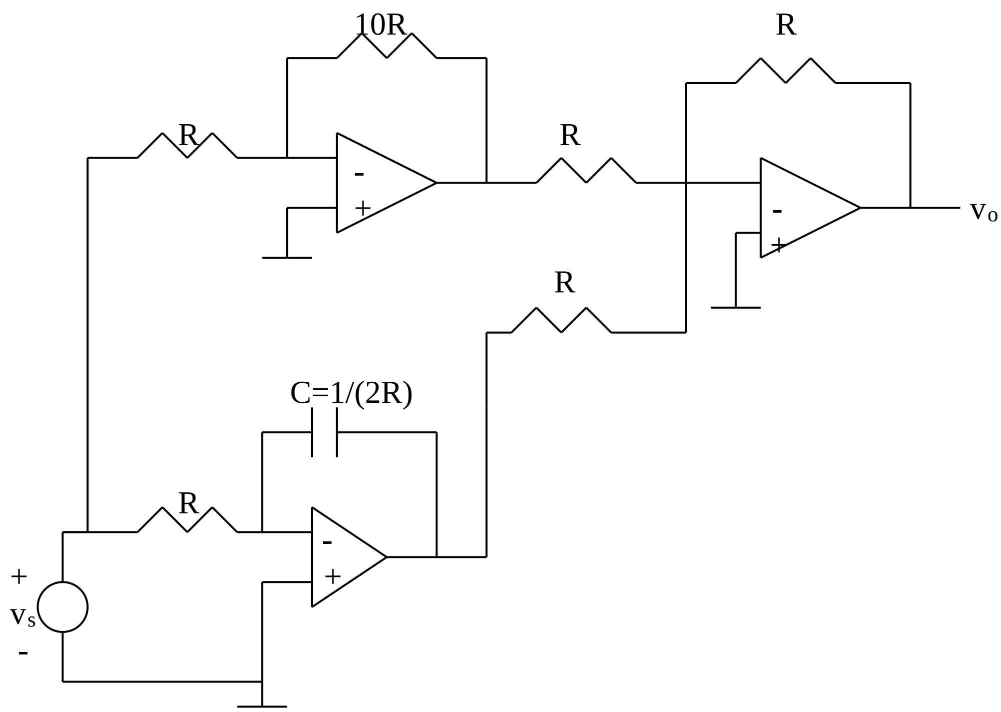
If we integrate both sides we can end up with $\frac{d^2v(t)}{dt^2}=-3v(t)+4\int f(t)dt$.

As with any design problem there are many acceptable solutions, this is one of them. Note, the above can also be solved without the integration step and would also be an accurate solution.



Solution 6.82

The circuit consists of a summer, an inverter, and an integrator. Such circuit is shown below.

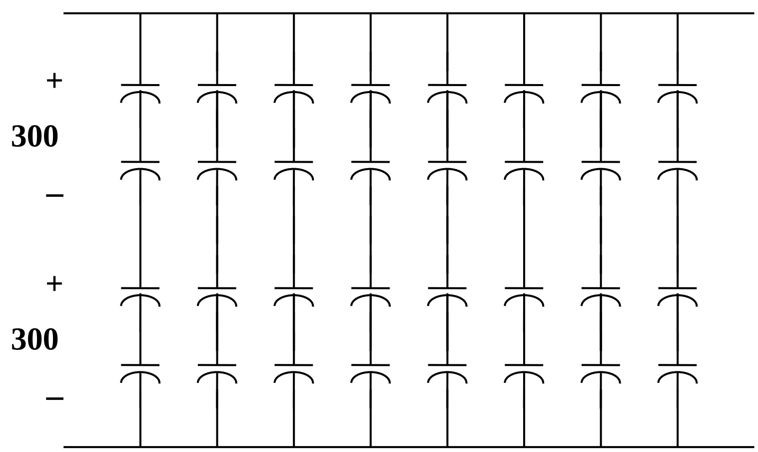


Solution 6.83

Your laboratory has available a large number of 5-μF capacitors rated at 150 V. To design a capacitor bank of 10-μF rated at 600 V, how many 5-μF capacitors are needed and how would you connect them?

Solution

Since four 5 μF capacitors in series gives 1.25 μF, rated at 600V, it requires 8 groups in parallel with each group consisting of four capacitors in series, as shown below:



Answer: **Eight groups in parallel with each group made up of four capacitors in series.**

Solution 6.84

An 8-mH inductor is used in a fusion power experiment. If the current through the inductor is $i(t) = 10\cos^2(\pi t)$ mA, for all $t > 0$ sec, find the power being delivered to the inductor and the energy stored in it at t=0.5s.

Solution

$v = L(di/dt) = 8x10^{-3}x10x(-2\pi)\cos(\pi t)\sin(\pi t)10^{-3} = -80\pi\sin(2\pi t)$ μV.

$p = vi =$ $\mathbf{-80\pi\sin(2\pi t)10\cos^2(\pi t)10^{-9}}$ **W**. At t = 0.5 s, p(0.5) = **0 W**.

At t = 0.5 s, $w(0.5) = (1/2)Li(0.5)^2 = 0.5x8x10^{-3}(10x10^{-3}x0)^2 =$ **0 J**.

Solution 6.85

It is evident that differentiating i will give a waveform similar to v. Hence,

$$v = L\frac{di}{dt}$$

$$i = \begin{bmatrix} 4t, 0 < t < 1ms \\ 8 - 4t, 1 < t < 2ms \end{bmatrix}$$

$$v = L\left[\frac{di}{dt} = \begin{bmatrix} 4000L, 0 < t < 1ms \\ -4000L, 1 < t < 2ms \end{bmatrix}\right]$$

But, $$v = \begin{bmatrix} 5V, 0 < t < 1ms \\ -5V, 1 < t < 2ms \end{bmatrix}$$

Thus, 4000L = 5 $\longrightarrow$ L = 1.25 mH in a **1.25 mH inductor**

Solution 6.86

$$v = v_R + v_L = Ri + L\frac{di}{dt} = 12x2te^{-10t} + 200x10^{-3}x(-20te^{-10t} + 2e^{-10t}) = \underline{(0.4 - 20t)e^{-10t}\ \text{V}}$$

Solution 7.1

(a) τ=RC = 1/200

For the resistor, V=iR= $56e^{-200t} = 8Re^{-200t} x10^{-3} \longrightarrow R = \frac{56}{8} = \underline{7\ k\Omega}$

$$C = \frac{1}{200R} = \frac{1}{200X7X10^3} = \underline{0.7143\mu F}$$

(b) τ =1/200= **5 ms**

(c) If value of the voltage at = 0 is 56 .

$$\frac{1}{2}x56 = 56e^{-200t} \longrightarrow e^{200t} = 2$$

$$200t_o = \ln 2 \longrightarrow t_o = \frac{1}{200}\ln 2 = \underline{3.466\ ms}$$

Solution 7.2

$\tau = R_{th}C$

where R_{th} is the Thevenin equivalent at the capacitor terminals.

$R_{th} = 120 \parallel 80 + 12 = 60\ \Omega$

$\tau = 60x0.05 =$ **3 s**.

Solution 7.3

$R = 6k\ +40kx(25k+35k)/(40k+25k+35k) = 6k+2400k/100 = 30\ k\Omega$.

$\tau = RC = 30kx50x10^{-12} = \mathbf{1.5\ \mu s}$.

Solution 7.4

For $t<0$, $v(0^-)=40$ V.

For $t>0$. we have a source-free RC circuit.

$$\tau = RC = 2x10^3 x10x10^{-6} = 0.02$$

$$v(t) = v(0)e^{-t/\tau} = \underline{40e^{-50t}\ \text{V}}$$

Solution 7.5

Using Fig. 7.85, design a problem to help other students to better understand source-free RC circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit shown in Fig. 7.85, find i(t), t>0.

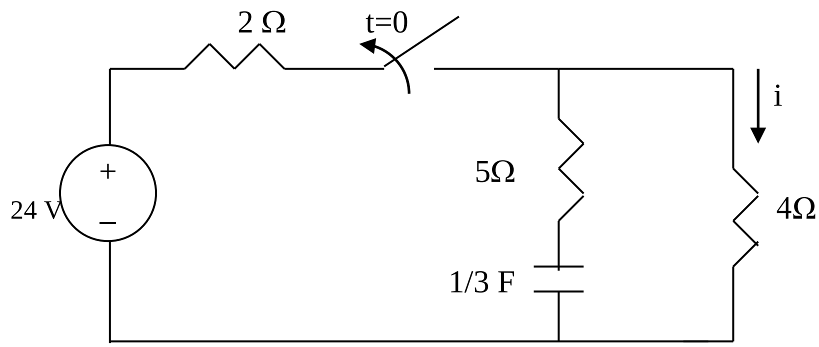


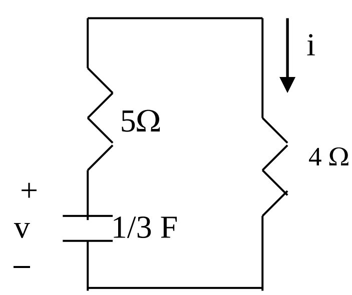
Figure 7.85 For Prob. 7.5.

Solution

Let v be the voltage across the capacitor. For t <0,

$$v(0^-) = \frac{4}{2+4}(24) = 16 \text{ V}$$

For t >0, we have a source-free RC circuit as shown below.



$$\tau = RC = (4+5)\frac{1}{3} = 3s$$

$$v(t) = v(0)e^{-t/\tau} = 16e^{-t/3}\ \mathrm{V}$$

$$i(t) = -C\frac{dv}{dt} = -\frac{1}{3}(-\frac{1}{3})16e^{-t/3} = \underline{1.778e^{-t/3}\ \text{A}}$$

Solution 7.6

The switch in Fig. 7.85 has been closed for a long time, and it opens at $t = 0$. Find $v(t)$ for $t \geq 0$.

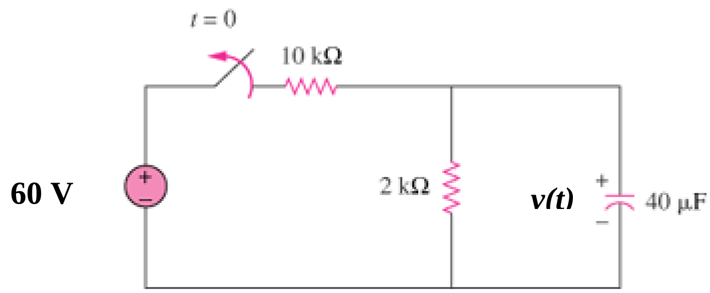


Figure 7.85
For Prob. 7.6.

Solution

v(0) = [2x60/(10+2)] = 10 V and $\tau = RC = 2{,}000 \times 40 \times 10^{-6} = 0.08$.

Note, 1/0.08 = 12.5/s

This then leads to v(t) = **[$10e^{-12.5t}$] V** for all $0 \leq t$.

Solution 7.7

Assuming that the switch in Fig. 7.87 has been in position A for a long time and is moved to position B at t=0. Then at t = 1second, the switch moves from B to C. Find $v_C(t)$ for $t \geq 0$.

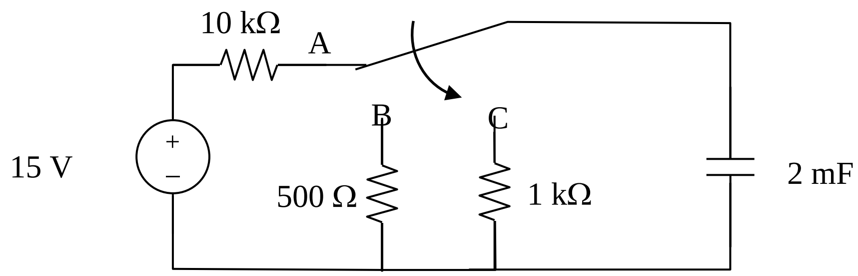


Figure 7.87
For Prob. 7.7

Solution

Step 1. Determine the initial voltage on the capacitor. Clearly it charges to 15 volts when the switch is at position A because the circuit has reached steady state.

This then leaves us with two simple circuits, the first a 500 **Ω** resistor in series with a 2 mF capacitor and an initial charge on the capacitor of 15 volts. The second circuit which exists from t = 1 sec to infinity. The initial condition for the second circuit will be $v_C(1)$ from the first circuit. The time constant for the first circuit is (500)(0.002) = 1 sec and the time constant for the second circuit is (1,000)(0.002) = 2 sec. $v_C(\infty) = 0$ for both circuits.

Step 2.

$v_C(t)$ = [**$15e^{-t}$]u(t) volts** for $0 < t < 1$ sec and $= 15e^{-1}e^{-2(t-1)}$ at t = 1 sec, and

= **[$5.518e^{-2(t-1)}$]u(t–1) volts** for 1 sec $< t < \infty$.

[$15e^{-t}$] volts for $0 < t < 1$ sec, **[$5.518e^{-2(t-1)}$] volts** for 1 sec $< t < \infty$.

Solution 7.8

(a) $\tau = RC = \dfrac{1}{4}$

$$-i = C\frac{dv}{dt}$$

$$-0.2e^{-4t} = C(10)(-4)e^{-4t} \longrightarrow C = \mathbf{5\ mF}$$

$$R = \frac{1}{4C} = \mathbf{50\ \Omega}$$

(b) $\tau = RC = \dfrac{1}{4} = \mathbf{0.25\ s}$

(c) $w_C(0) = \dfrac{1}{2}CV_0^2 = \dfrac{1}{2}(5\times 10^{-3})(100) = \mathbf{250\ mJ}$

(d) $w_R = \dfrac{1}{2}\times\dfrac{1}{2}CV_0^2 = \dfrac{1}{2}CV_0^2\left(1 - e^{-2t_0/\tau}\right)$

$$0.5 = 1 - e^{-8t_0} \longrightarrow e^{-8t_0} = \frac{1}{2}$$

or $e^{8t_0} = 2$

$$t_0 = \frac{1}{8}\ln(2) = \mathbf{86.6\ ms}$$

Solution 7.9

The switch in Fig. 7.89 opens at t=0. Find v_o for $t > 0$.

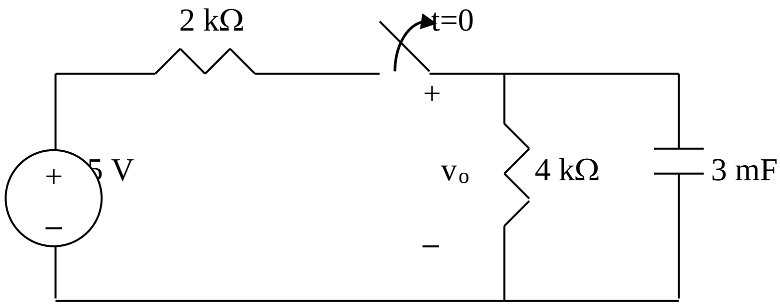


Figure 7.89
For Prob. 7.9.

Solution

For $t < 0$, the switch is closed so that

$v_o(0) = [4/(2+4)]15 = 10$ V.

For $t > 0$, we have a source-free RC circuit where $\tau = 4{,}000 \text{x} 0.003 = 12$ s.

Thus,
$v_o(t) =$ **$[10e^{-t/12}]$ V** for all $t \geq 0$.

Solution 7.10

For t<0, $$v(0^-) = \frac{3}{3+9}(36V) = \underline{9\ V}$$

For t>0, we have a source-free RC circuit

$$\tau = RC = 3x10^3 x20x10^{-6} = 0.06s$$

$v_o(t)$ = $\mathbf{9e^{-16.667t}}$ **V**

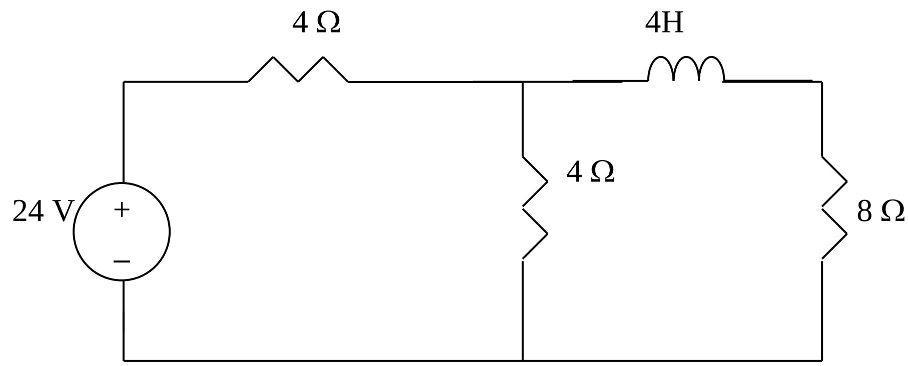
Let the time be t_o.

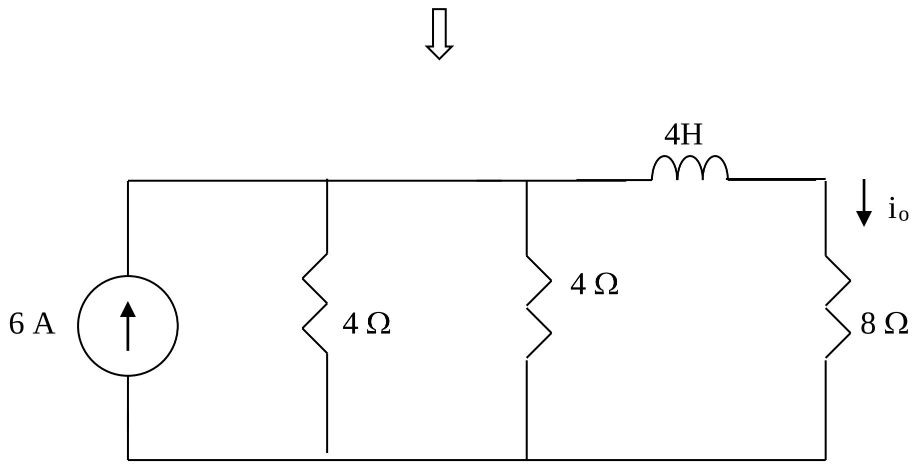
$3 = 9e^{-16.667t_o}$ or $e^{16.667t_o} = 9/3 = 3$

$t_o = \ln(3)/16.667 =$ **65.92 ms**.

Solution 7.11

For t<0, we have the circuit shown below.





4||4= 4x4/8=2
$i_o(0^-) = [2/(2+8)]6 = 1.2$ A

For t >0, we have a source-free RL circuit.

$\tau = \dfrac{L}{R} = \dfrac{4}{4+8} = 1/3$ thus,
$i_o(t) =$ **$1.2e^{-3t}$ A**.

Solution 7.12

Using Fig. 7.92, design a problem to help other students better understand source-free RL circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The switch in the circuit in Fig. 7.90 has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.

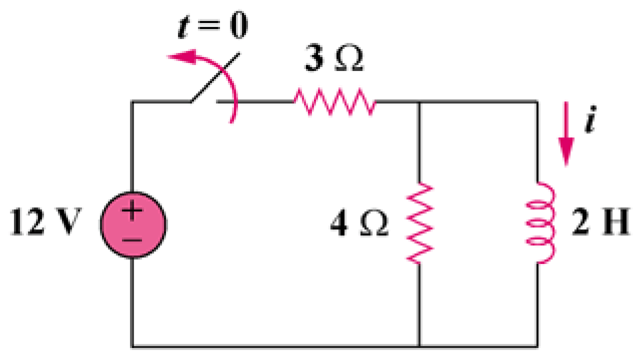
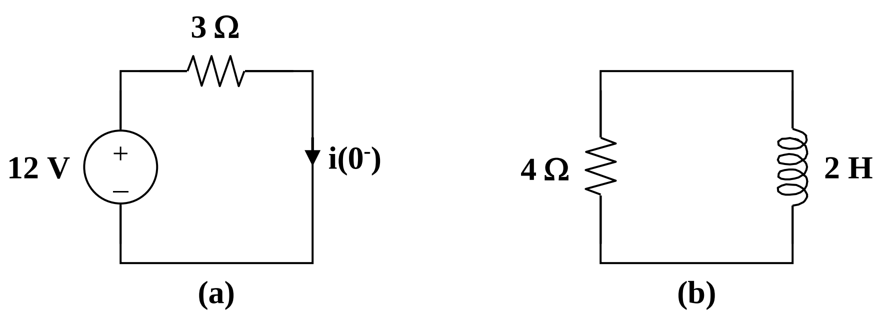


Figure 7.90

Solution

When t < 0, the switch is closed and the inductor acts like a short circuit to dc. The 4 Ω resistor is short-circuited so that the resulting circuit is as shown in Fig. (a).



$$i(0^-) = \frac{12}{3} = 4 \text{ A}$$

Since the current through an inductor cannot change abruptly,

$$i(0) = i(0^-) = i(0^+) = 4 \text{ A}$$

When t > 0, the voltage source is cut off and we have the RL circuit in Fig. (b).

$$\tau = \frac{L}{R} = \frac{2}{4} = 0.5$$

Hence,

$$i(t) = i(0)\,e^{-t/\tau} = \mathbf{4e^{-2t}\ A}$$

Solution 7.13

(a) $\tau = \frac{1}{10^3} = \underline{1ms}$ = **1 ms**.

v(t) = i(t)R = $80e^{-1000t}$ V = $R5e^{-1000t}x10^{-3}$ or R = 80,000/5 = **16 kΩ**.

But $\tau = L/R = 1/10^3$ or $L = 16x10^3/10^3$ = **16 H**.

(b) The energy dissipated in the resistor is

$$w = \int_0^{0.0005} pdt = \int_0^{0.0005} 0.4e^{-2000t}dt = -\frac{0.4}{2000}e^{-2000t}\Big|_0^{0.0005}$$

= $200(1-e^{-1})x10^{-6}$ = **126.42 μJ**.

(a) **16 kΩ, 16 H, 1 ms** **(b) 126.42 μJ**

Solution 7.14

$$R_{Th} = (40 + 20) / / (10 + 30) = \frac{60x40}{100} = 24k\Omega$$

$$\tau = L / R = \frac{5x10^{-3}}{24x10^{3}} = \underline{0.2083\mu s}$$

Solution 7.15

(a) $R_{Th} = 2 + 10 // 40 = 10\Omega, \qquad \tau = \frac{L}{R_{Th}} = 5/10 = \underline{0.5s}$

(b) $R_{Th} = 40//160 + 48 = 40\Omega, \qquad \tau = \frac{L}{R_{Th}} = (20x10^{-3})/80 = \underline{0.25\,ms}$

(a) **10 Ω, 500 ms** **(b) 40 Ω, 250 μs**

Solution 7.16

$$\tau = \frac{L_{eq}}{R_{eq}}$$

(a) $L_{eq} = L$ and $R_{eq} = R_2 + \frac{R_1R_3}{R_1 + R_3} = \frac{R_2(R_1 + R_3) + R_1R_3}{R_1 + R_3}$

$$\tau = \boldsymbol{\frac{L(R_1 + R_3)}{R_2(R_1 + R_3) + R_1R_3}}$$

(b) where $L_{eq} = \frac{L_1L_2}{L_1 + L_2}$ and $R_{eq} = R_3 + \frac{R_1R_2}{R_1 + R_2} = \frac{R_3(R_1 + R_2) + R_1R_2}{R_1 + R_2}$

$$\tau = \boldsymbol{\frac{L_1L_2(R_1 + R_2)}{(L_1 + L_2)(R_3(R_1 + R_2) + R_1R_2)}}$$

Solution 7.17

Consider the circuit of Fig. 7.97. Find $v_0(t)$ if $i(0) = 15$ A and $v(t) = 0$.

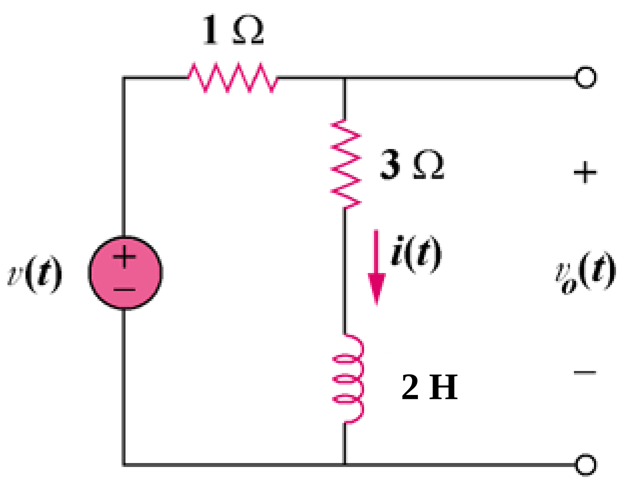


Figure 7.97
For Prob. 7.17.

Solution

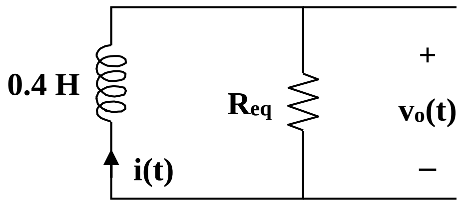
$i(t) = i(0)e^{-t/\tau}$ where $\tau = L/R_{eq}$ $= 2/4 = (½)$ s. Additionally $v_o(t) = 3i(t) + 2di(t)/dt$.

Thus, $i(t) = [15e^{-2t}]u(t)$ A and $v_o(t) = [45e^{-2t}]u(t) - [(2)(2)15e^{-2t}]u(t)$

= $\mathbf{[-15e^{-2t}]}$ **V** for all $t \geq 0$.

Solution 7.18

If $v(t) = 0$, the circuit can be redrawn as shown below.



$$R_{eq} = 2 \| 3 = \frac{6}{5}, \qquad \tau = \frac{L}{R} = \frac{2}{5} \times \frac{5}{6} = \frac{1}{3}$$

$$i(t) = i(0)e^{-t/\tau} = 5e^{-3t}$$

$$v_o(t) = -L\frac{di}{dt} = \frac{-2}{5}(-3)5e^{-3t} = \boldsymbol{6\,e^{-3t}\ V}$$

Solution 7.19

In the circuit of Fig. 7.99, find $i(t)$ for $t > 0$ if $i(0) = 5$ A.

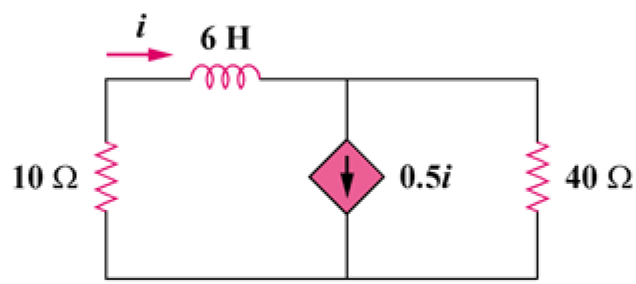
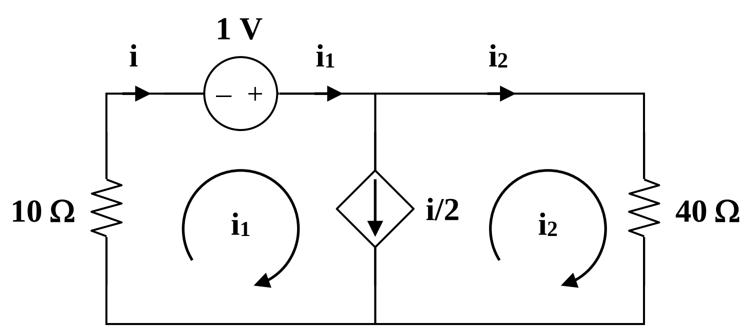


Figure 7.99
For Prob. 7.19.

Solution



To find R_{th} we replace the inductor by a 1-V voltage source as shown above.

$10i_1 - 1 + 40i_2 = 0$

But $\quad i = i_2 + i/2 \quad$ and $\quad i = i_1$

i.e. $\quad i_1 = 2i_2 = i$

$$10i - 1 + 20i = 0 \longrightarrow i = \frac{1}{30}$$

$$R_{th} = \frac{1}{i} = 30\,\Omega$$

$$\tau = \frac{L}{R_{th}} = \frac{6}{30} = 0.2\text{ s}$$

$i(t) =$ **$5e^{-5t}u(t)$ A**.

Solution 7.20

(a) $\tau = \dfrac{L}{R} = \dfrac{1}{50} \longrightarrow R = 50L$

$v = -L\dfrac{di}{dt}$

$90e^{-50t} = -L(30)(-50)e^{-50t} \longrightarrow L = \mathbf{60\ mH}$

$R = 50L = \mathbf{3\ \Omega}$

(b) $\tau = \dfrac{L}{R} = \dfrac{1}{50} = \mathbf{20\ ms}$

(c) $w = \dfrac{1}{2}L\,i^2(0) = \dfrac{1}{2}(0.06)(30)^2 = \mathbf{27\ J}$

The value of the energy remaining at 10 ms is given by:

$w_{10} = 0.03(30e^{-0.5})^2 = 0.03(18.196)^2 = 9.933$ J.

So, the fraction of the energy dissipated in the first 10 ms is given by:

(27–9.933)/27 = 0.6321 or **63.21%**.

Solution 7.21

In the circuit in Fig. 7.101, find the value of *R* for which the steady-state energy stored in the inductor will be 2 J.

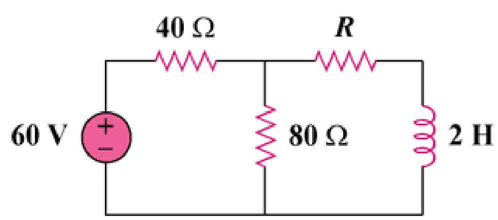
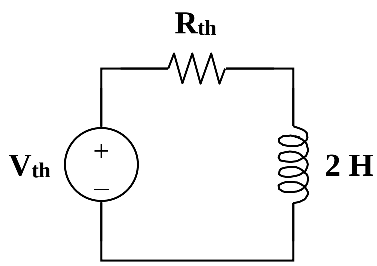


Figure 7.101
For Prob. 7.21.

Solution

The circuit can be replaced by its Thevenin equivalent shown below.



$$V_{th} = \frac{80}{80+40}(60) = 40 \text{ V}$$

$$R_{th} = 40\,||\,80 + R = \frac{80}{3} + R$$

$$I = i(0) = i(\infty) = \frac{V_{th}}{R_{th}} = \frac{40}{80/3 + R}$$

w = $(1/2)LI^2 = 0.5(2)[40/(R+80/3)]^2 = 2$ or $40/(R+80/3) = 1.4142$, thus,

$R+80/3 = 40/1.4142$ or $R = 28.285–26.667 =$ **1.618 Ω**.

Solution 7.23

Since the 2 Ω resistor, 1/3 H inductor, and the (3+1) Ω resistor are in parallel, they always have the same voltage.

$$-\mathrm{i} = \frac{10}{2} + \frac{10}{3+1} = 7.5 \quad \longrightarrow \quad i(0) = -7.5$$

The Thevenin resistance R_{th} at the inductor's terminals is

$$R_{th} = 2 \,\|\, (3+1) = \frac{4}{3}, \qquad \tau = \frac{L}{R_{th}} = \frac{1/3}{4/3} = \frac{1}{4}$$

$$i(t) = i(0)\,e^{-t/\tau} = -7.5\mathrm{e}^{-4t}, \quad t > 0$$

$$v_L = v_o = L\frac{di}{dt} = -7.5(-4)(1/3)\,\mathrm{e}^{-4t}$$

$$\mathrm{v_o} = \mathbf{10e^{-4t}}\ \boldsymbol{V}, \quad \boldsymbol{t > 0}$$

$$\mathrm{v_x} = \frac{1}{3+1}\mathrm{v_L} = \mathbf{2.5\,e^{-4t}\ V}, \quad \boldsymbol{t > 0}$$

Solution 7.24

(a) $v(t) = \mathbf{-5u(t)}$

(b) $i(t) = -10\left[u(t) - u(t-3)\right] + 10\left[u(t-3) - u(t-5)\right]$

$= \mathbf{-10u(t) + 20u(t-3) - 10u(t-5)}$

(c) $x(t) = (t-1)\left[u(t-1) - u(t-2)\right] + \left[u(t-2) - u(t-3)\right]$
$+ (4-t)\left[u(t-3) - u(t-4)\right]$
$= (t-1)u(t-1) - (t-2)u(t-2) - (t-3)u(t-3) + (t-4)u(t-4)$
$= \mathbf{r(t-1) - r(t-2) - r(t-3) + r(t-4)}$

(d) $y(t) = 2u(-t) - 5\left[u(t) - u(t-1)\right]$
$= \mathbf{2u(-t) - 5u(t) + 5u(t-1)}$

Solution 7.25

Design a problem to help other students to better understand singularity functions.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

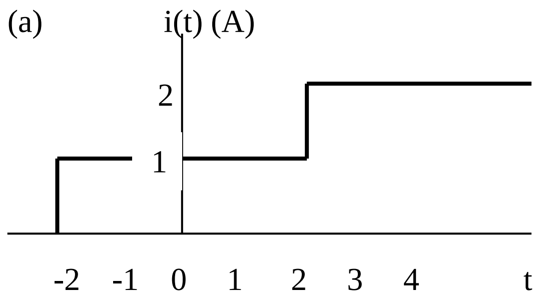
Problem

Sketch each of the following waveforms.
(a) $i(t) = [u(t–2)+u(t+2)]$ A
(b) $v(t) = [r(t) – r(t–3) + 4u(t–5) – 8u(t–8)]$ V

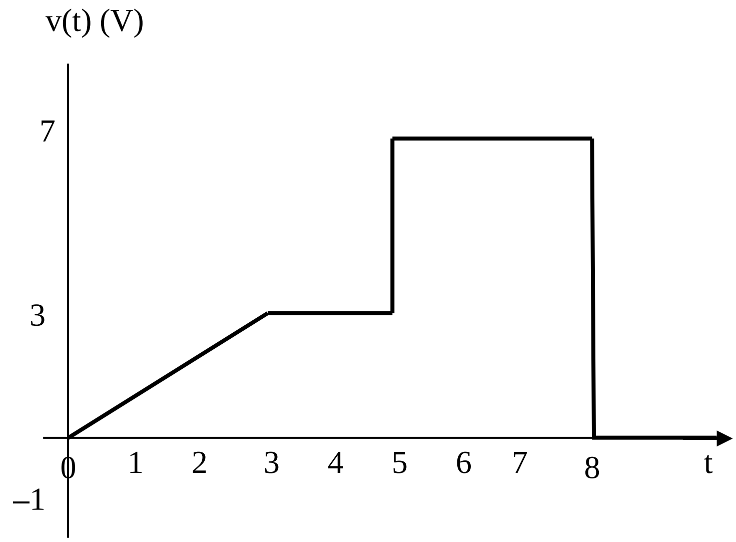
Solution

The waveforms are sketched below.

(a)



(b)



Solution 7.26

(a) $v_1(t) = u(t+1) - u(t) + \left[u(t-1) - u(t) \right]$

$v_1(t) = \mathbf{u(t+1) - 2u(t) + u(t-1)}$

(b) $v_2(t) = (4-t)\left[u(t-2) - u(t-4) \right]$

$v_2(t) = \text{-}(t-4)u(t-2) + (t-4)u(t-4)$

$v_2(t) = \mathbf{2u(t-2) - r(t-2) + r(t-4)}$

(c) $v_3(t) = 2\left[u(t-2) - u(t-4) \right] + 4\left[u(t-4) - u(t-6) \right]$

$v_3(t) = \mathbf{2u(t-2) + 2u(t-4) - 4u(t-6)}$

(d) $v_4(t) = \text{-}t\left[u(t-1) - u(t-2) \right] = \text{-}t\,u(t-1) + t\,u(t-2)$

$v_4(t) = (\text{-}t+1-1)u(t-1) + (t-2+2)u(t-2)$

$v_4(t) = \mathbf{\text{-}\,r(t-1) - u(t-1) + r(t-2) + 2u(t-2)}$

Solution 7.27

v(t)= $[5u(t+1)+10u(t)-25u(t-1)+15u(t-2)]$ V

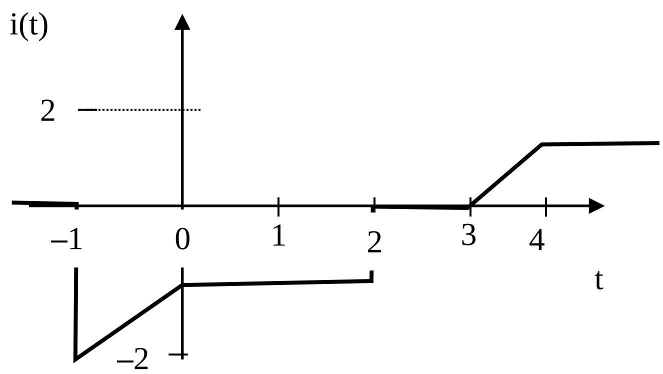
Solution 7.28

Sketch the waveform represented by

$i(t)$ = [r(t+1) – r(t) – 2u(t+1) +u(t–2) + r(t–3) – r(t–4)] A.

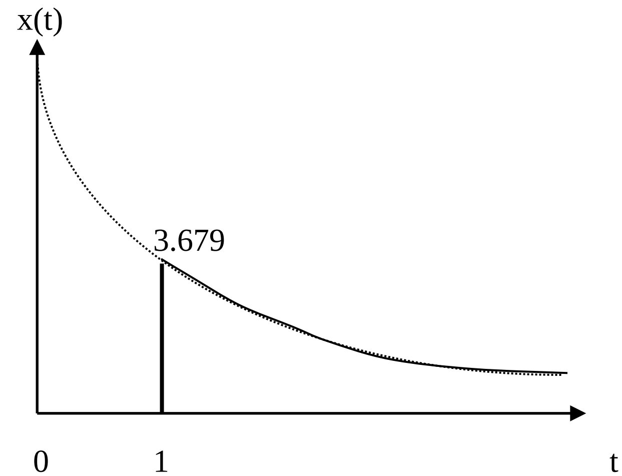
Solution

$i(t)$ is sketched below.

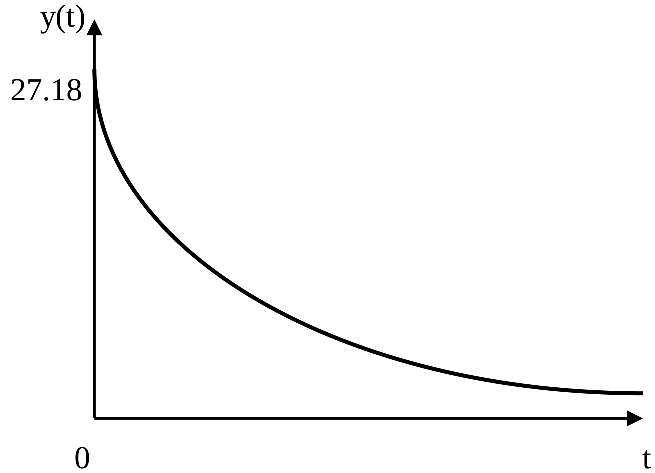


Solution 7.29

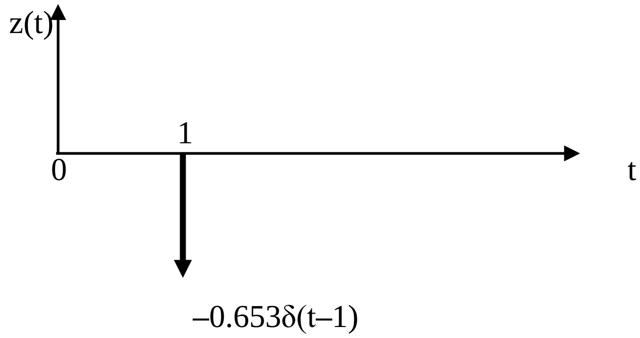
(a)



(b)



(c) $z(t) = \cos 4t\delta(t-1) = \cos 4\delta(t-1) = -0.6536\delta(t-1)$, which is sketched below.



Solution 7.30

(a) $\int_{-\infty}^{\infty} 4t^2\, \delta(t-1)\, dt = 4t^2\big|_{t=1} = \mathbf{4}$

(b) $\int_{-\infty}^{\infty} 4t^2 \cos(2\pi t)\, \delta(t-0.5)\, dt = 4t^2 \cos(2\pi t)\big|_{t=0.5} = \cos\pi = \mathbf{-1}$

Solution 7.31

(a) $$\int_{-\infty}^{\infty}\left[\, e^{-4t^2}\ \delta(t-2)\right] dt = e^{-4t^2}\Big|_{t=2} = e^{-16} = \mathbf{112\times10^{-9}}$$

(b) $$\int_{-\infty}^{\infty}\left[\, 5\delta(t) + e^{-t}\ \delta(t) + \cos 2\pi t\, \delta(t)\right] dt = \left(5 + e^{-t} + \cos(2\pi t)\right)\Big|_{t=0} = 5+1+1 = \mathbf{7}$$

Solution 7.32

(a) $\int_1^t u(\lambda)d\lambda = \int_1^t 1 d\lambda = \lambda \Big|_1^t = \underline{t-1}$

(b) $\int_0^4 r(t-1)dt = \int_0^1 0dt + \int_1^4 (t-1)dt = \frac{t^2}{2} - t\Big|_1^4 = \underline{4.5}$

(c) $\int_1^5 (t-6)^2 \delta(t-2)dt = (t-6)^2\Big|_{t=2} = \underline{16}$

Solution 7.33

The voltage across a 10-mH inductor is 45δ (t – 2) mV. Find the inductor current, assuming that the inductor is initially uncharged.

Solution

$$i(t) = \frac{1}{L}\int_0^t v(\tau)\, d\tau + i(0)$$

$$i(t) = \frac{10^{-3}}{10\times 10^{-3}}\int_0^t 45\delta(\tau - 2)\, d\tau + 0 = 4.5\mathrm{u}(t–2)\ \mathrm{A}.$$

It should be noted that the integration of the impulse function, $\delta(t–t_0)$, produces the unit step, $u(t–t_0)$. Whatever the multiplier (f(t)) of the impulse function at $t = t_0$ ends up multiplying the unit step by the same amount ($f(t_0)$) in this case f(2) = 4.5.

i(t) = **4.5u(t–2) A**.

Solution 7.34

(a) $$\frac{d}{dt}\left[u(t-1)\,u(t+1)\right] = \delta(t-1)u(t+1) + u(t-1)\delta(t+1) = \delta(t-1)1 + 0\delta(t+1) = \underline{\delta(t-1)}$$

(b) $$\frac{d}{dt}\left[r(t-6)\,u(t-2)\right] = u(t-6)u(t-2) + r(t-6)\delta(t-2) = u(t-6)1 + 0\delta(t-2) = \underline{u(t-6)}$$

(c) $$\begin{aligned}\frac{d}{dt}\left[\sin 4t\,u(t-3)\right] &= 4\cos 4t\,u(t-3) + \sin 4t\delta(t-3)\\ &= 4\cos 4t\,u(t-3) + \sin 4x3\delta(t-3)\\ &= \underline{4\cos 4t\,u(t-3) - 0.5366\delta(t-3)}\end{aligned}$$

Solution 7.35

(a)

$$v = Ae^{-2t}, \qquad v(0) = A = -1$$

v(t) = $\mathbf{-e^{-2t}u(t)}$ **V**

(b)

$$i = Ae^{3t/2}, \qquad i(0) = A = 2$$

i(t) = $\mathbf{2e^{-1.5t}u(t)}$ **A**

Solution 7.36

(a) $v(t) = A + Be^{-t}, \quad t > 0$

$A = 1, \ v(0) = 0 = 1 + B \quad \text{or} \qquad B = -1$

$v(t) = \mathbf{1 - e^{-t} \ V, \quad t > 0}$

(b) $v(t) = A + Be^{t/2}, \quad t > 0$

$A = -3, \quad v(0) = -6 = -3 + B \qquad \text{or} \qquad B = -3$

$v(t) = \mathbf{-3\left(1 + e^{t/2}\right) V, \quad t > 0}$

Solution 7.37

Let v = v_h + v_p, v_p =10.

$$\dot{v}_h + \frac{1}{4}v_h = 0 \quad \longrightarrow \quad v_h = Ae^{-t/4}$$

$$v = 10 + Ae^{-0.25t}$$

$$v(0) = 2 = 10 + A \quad \longrightarrow \quad A = -8$$
$$v = 10 - 8e^{-0.25t}$$

(a) $\boldsymbol{\tau = \underline{4s}}$

(b) $\boldsymbol{v(\infty) = 10\ \mathrm{V}}$

(c) $\boldsymbol{\underline{v = \left(10 - 8e^{-0.25t}\right)u(t)\ V}}$

Solution 7.38

Let $i = i_p + i_h$

$$\dot{i}_h + 3i_h = 0 \quad \longrightarrow \quad i_h = Ae^{-3t}u(t)$$

Let $i_p = ku(t), \quad \dot{i}_p = 0, \quad 3ku(t) = 2u(t) \quad \longrightarrow \quad k = \frac{2}{3}$

$$i_p = \frac{2}{3}u(t)$$

$$i = (Ae^{-3t} + \frac{2}{3})u(t)$$

If i(0) =0, then A + 2/3 = 0, i.e. A=-2/3. Thus,

$$\underline{\boldsymbol{i = \frac{2}{3}(1 - e^{-3t})u(t)}}$$

Solution 7.39

(a) Before t = 0,

$$v(t) = \frac{1}{4+1}(20) = \mathbf{4\ V}$$

After t = 0,

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

$$\tau = RC = (4)(2) = 8, \quad v(0) = 4, \qquad v(\infty) = 20$$

$$v(t) = 20 + (4 - 20)\, e^{-t/8}$$

$$v(t) = 20 - 16\, e^{-t/8}\ \mathrm{V}$$

(b) Before t = 0, $v = v_1 + v_2$, where v_1 is due to the 12-V source and v_2 is due to the 2-A source.

$$v_1 = 12\ \mathrm{V}$$

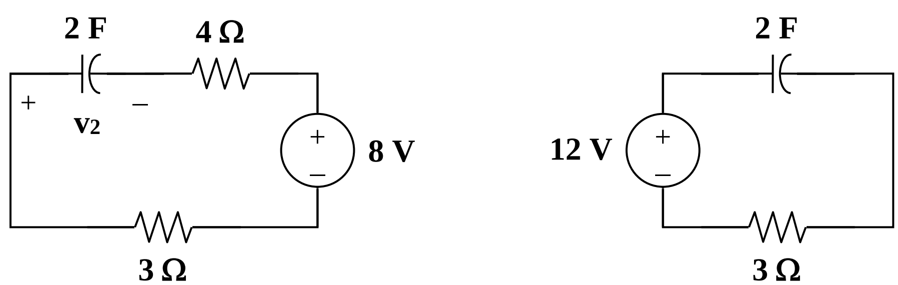
To get v_2, transform the current source as shown in Fig. (a).

$$v_2 = \text{-}8\ \mathrm{V}$$

Thus,

$$v = 12 - 8 = \mathbf{4\ V}$$

After t = 0, the circuit becomes that shown in Fig. (b).



(a) (b)

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

$$v(\infty) = 12, \qquad v(0) = 4, \qquad \tau = RC = (2)(3) = 6$$

$$v(t) = 12 + (4 - 12)\, e^{-t/6}$$

$$v(t) = \mathbf{12 - 8\, e^{-t/6}\ V}$$

Solution 7.40

(a) Before t = 0, v = **12 V** .

After t = 0, $v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$

$v(\infty) = 4$, $\quad v(0) = 12$, $\quad \tau = RC = (2)(3) = 6$

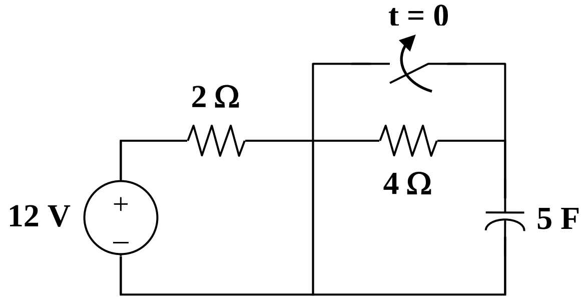
$v(t) = 4 + (12 - 4)\, e^{-t/6}$

$v(t) = \quad \mathbf{4 + 8 e^{-t/6}\ V}$

(b) Before t = 0, v = **12 V** .

After t = 0, $v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$

After transforming the current source, the circuit is shown below.



$v(0) = 12$, $\quad v(\infty) = 12$, $\quad \tau = RC = (2)(5) = 10$

v = **12 V**

Solution 7.41

Using Fig. 7.108, design a problem to help other students to better understand the step response of an RC circuit.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 7.108, find $v(t)$ for $t > 0$.

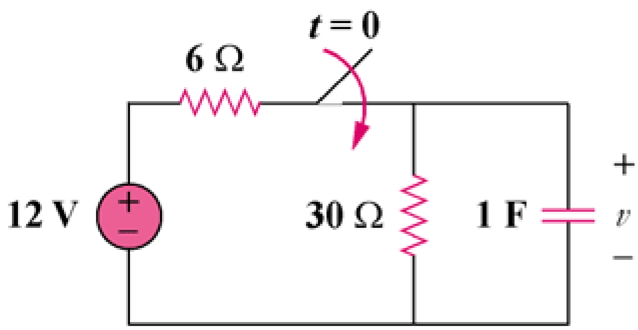


Figure 7.108

Solution

$$v(0) = 0, \qquad v(\infty) = \frac{30}{36}(12) = 10$$

$$R_{eq}C = (6 \,||\, 30)(1) = \frac{(6)(30)}{36} = 5$$

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(t) = 10 + (0 - 10)\, e^{-t/5}$$

$$v(t) = \underline{10(1 - e^{-0.2t})\; u(t)\text{V}}$$

Solution 7.42

(a) If the switch in Fig. 7.109 has been open for a long time and is closed at $t = 0$, find $v_o(t)$.

(b) Suppose that the switch has been closed for a long time and is opened at $t = 0$. Find $v_o(t)$.

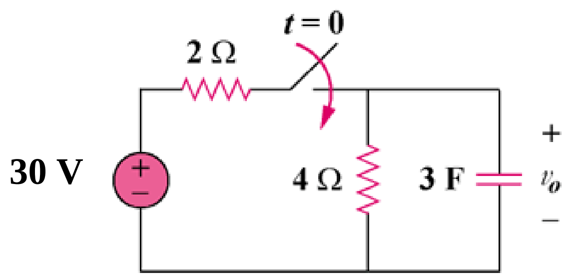


Figure 7.109
For Prob. 7.42.

Solution

(a) $\quad v_o(t) = v_o(\infty) + \left[v_o(0) - v_o(\infty)\right] e^{-t/\tau}$

$v_o(0) = 0, \qquad v_o(\infty) = \dfrac{4}{4+2}(30) = 20V$

$\tau = R_{eq}C_{eq}, \qquad R_{eq} = 2 \,||\, 4 = \dfrac{4}{3}$

$\tau = \dfrac{4}{3}(3) = 4$

$v_o(t) = 20 - 20e^{-t/4}$

$v_o(t) = \mathbf{20(1–e^{-0.25t})u(t)\ V}$.

(b) $\quad$ For this case, $v_o(\infty) = 0$ so that

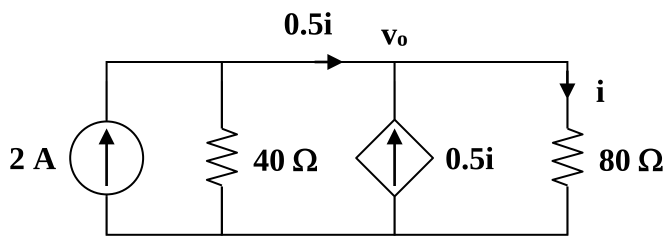
$v_o(t) = v_o(0)e^{-t/\tau}$

$v_o(0) = \dfrac{4}{4+2}(30) = 20, \qquad\qquad \tau = RC = (4)(3) = 12$

$v_o = \mathbf{[20\{1–u(t)\} + (20e^{-t/12})u(t)]\ V}$.

Solution 7.43

Before t = 0, the circuit has reached steady state so that the capacitor acts like an open circuit. The circuit is equivalent to that shown in Fig. (a) after transforming the voltage source.



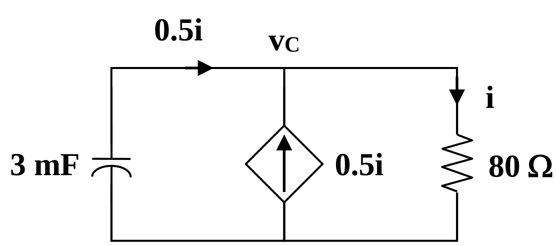
(a)

$$0.5i = 2 - \frac{v_o}{40}, \qquad i = \frac{v_o}{80}$$

Hence, $\frac{1}{2}\frac{v_o}{80} = 2 - \frac{v_o}{40} \longrightarrow v_o = \frac{320}{5} = 64$

$$i = \frac{v_o}{80} = \underline{\mathbf{0.8\ A}}$$

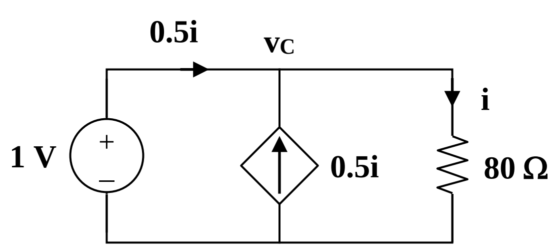
After t = 0, the circuit is as shown in Fig. (b).



(b)

$$v_C(t) = v_C(0)e^{-t/\tau}, \qquad \tau = R_{th}C$$

To find R_{th}, we replace the capacitor with a 1-V voltage source as shown in Fig. (c).



(c)

$$i = \frac{v_C}{80} = \frac{1}{80}, \qquad i_o = 0.5i = \frac{0.5}{80}$$

$$R_{th} = \frac{1}{i_o} = \frac{80}{0.5} = 160\,\Omega, \qquad \tau = R_{th}C = 480$$

$$v_C(0) = 64\text{ V}$$

$$v_C(t) = 64e^{-t/480}$$

$$0.5i = -i_C = -C\frac{dv_C}{dt} = -3\left(\frac{1}{480}\right)64e^{-t/480}$$

$$i(t) = \boldsymbol{800e^{-t/480}u(t)\ mA}$$

Solution 7.44

The switch in Fig. 7.111 has been in position *a* for a long time. At t = 0, it moves to position *b*. Calculate $i(t)$ for all t > 0.

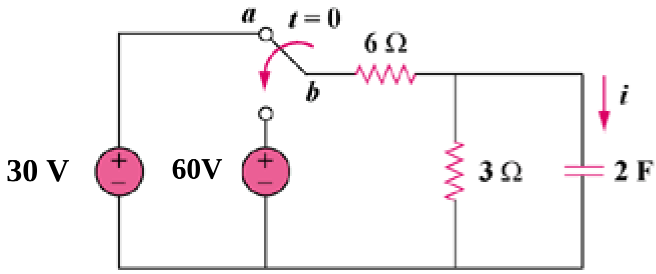


Figure 7.111
For Prob. 7.44.

Solution

Let v(t) be the voltage across the capacitor and $R_{eq} = 6 \,||\, 3 = 2\,\Omega$ and $\tau = RC = 4$, therefore, $v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$

Using voltage division,

$$v(0) = \frac{3}{3+6}(30) = 10\,V\,, \qquad v(\infty) = \frac{3}{3+6}(60) = 20\,V$$

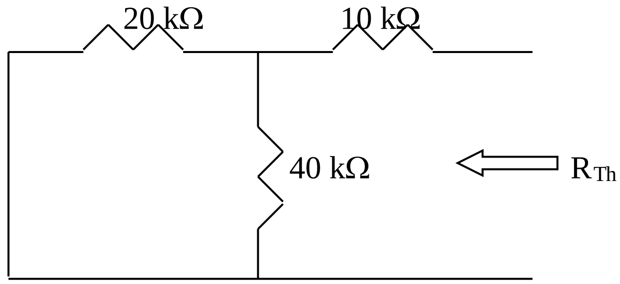
Thus,

$$v(t) = 20 + (10-20)\;e^{-t/4} = 20 - 10\;e^{-t/4}$$

$$i(t) = C\frac{dv}{dt} = (2)(-10)\left(-1/4\right)e^{-t/4} = \mathbf{(5e^{-0.25t})u(t)\ A}.$$

Solution 7.45

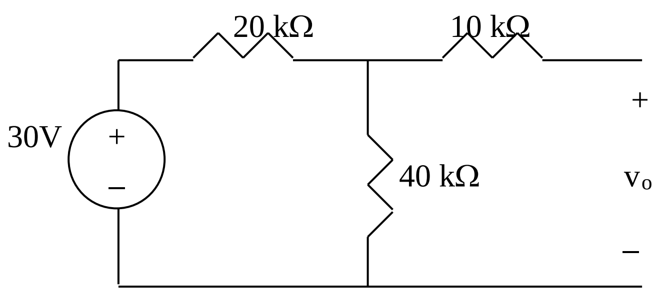
To find R_{Th}, consider the circuit shown below.



$$R_{Th} = 10 + 20 // 40 = 10 + \frac{20x40}{60} = \frac{70}{3} k\Omega$$

$$\tau = R_{Th}C = \frac{70}{3} x10^3 x3x10^{-6} = 0.07$$

To find $v_o(\infty)$, consider the circuit below.



$v_o(\infty) = [40/(40+20)]30 = 20$ V

$v_o(t) = v_o(\infty) + [v_o(0) - v_o(\infty)]e^{-t/0.07}$ = $\mathbf{[20 - 15e^{-14.286t}]u(t)\ V}$.

Solution 7.46

$$\tau = R_{Th}C = (2+6)x0.25 = 2s, \qquad v(0) = 0, \qquad v(\infty) = 6i_s = 6x5 = 30$$

$$\boldsymbol{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} = \underline{30(1 - e^{-t/2})}u(t)\ V}$$

Solution 7.47

Determine $v(t)$ for $t > 0$ in the circuit in Fig. 7.114 if $v(0) = 0$.

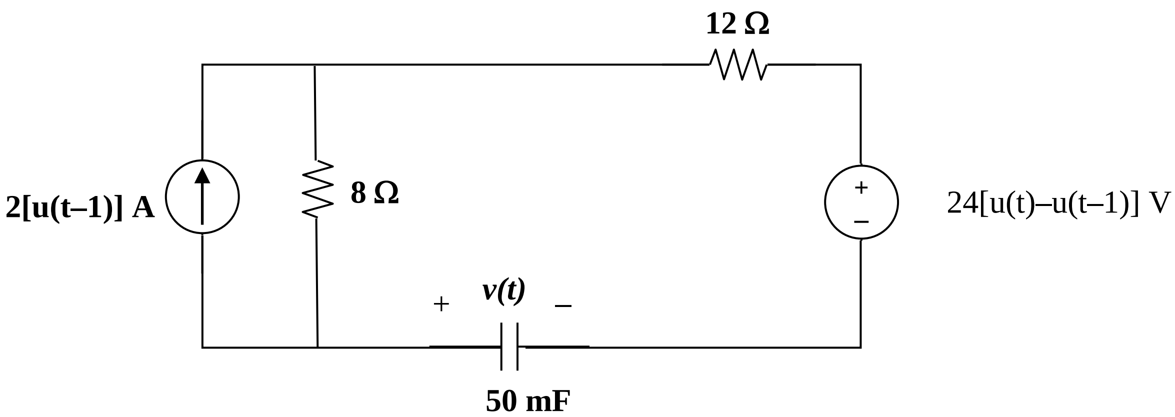


Figure 7.114
For Prob. 7.47.

Solution

For $t < 0$, $u(t) = 0$, $\quad u(t-1) = 0$, $\quad v(0) = 0$

For $0 < t < 1$, $\quad \tau = (8+12)0.05 = 1$ s.
$v(0) = 0$, $v(\infty) = 24$ V.
$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$
$$v(t) = 24\left(1 - e^{-t}\right)$$

For $t > 1$, $\qquad v(1) = 24(1–e^{-1}) = 15.171$ V.
At $t = \infty$ the voltage source is now a short circuit and the capacitor is an open circuit thus, $v(\infty) = –(2)(8) = –16$ V.

Now $v(t) = –16 + [15.171–(–16)]e^{-(t-1)} = –16 + 31.17\, e^{-(t-1)}$

Thus,
$$v(t) = \begin{cases} 24\left(1 - e^{-t}\right)V, & 0 < t < 1 \\ [-16 + 31.17\, e^{-(t-1)}]\, V, & 1 < t \end{cases}$$

Solution 7.48

For t < 0, $u(-t) = 1$,

For t > 0, $u(-t) = 0$, $v(\infty) = 0$

$R_{th} = 20 + 10 = 30$, $\tau = R_{th}C = (30)(0.1) = 3$

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

$$v(t) = \mathbf{10\, e^{-t/3}\ V}$$

$$i(t) = C\frac{dv}{dt} = (0.1)\left(\frac{-1}{3}\right)10\, e^{-t/3}$$

$$i(t) = \mathbf{\frac{-1}{3} e^{-t/3}\ A}$$

Solution 7.49

For $0 < t < 1$, $\quad v(0) = 0$, $\qquad v(\infty) = (2)(4) = 8$

$$R_{eq} = 4 + 6 = 10, \qquad \tau = R_{eq}C = (10)(0.5) = 5$$

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

$$v(t) = 8\left(1 - e^{-t/5}\right) \text{ V}$$

For $t > 1$, $\qquad v(1) = 8\left(1 - e^{-0.2}\right) = 1.45, \qquad v(\infty) = 0$

$$v(t) = v(\infty) + \left[\, v(1) - v(\infty)\right] e^{-(t-1)/\tau}$$

$$v(t) = 1.45e^{-(t-1)/5} \text{ V}$$

Thus,

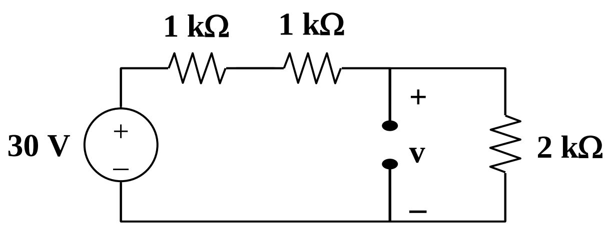
$$v(t) = \begin{cases} \mathbf{8\left(1 - e^{-t/5}\right)V,} & \mathbf{0 < t < 1} \\ \mathbf{1.45e^{-(t-1)/5}\ V,} & \mathbf{t > 1} \end{cases}$$

Solution 7.50

For the capacitor voltage,

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$
$$v(0) = 0$$

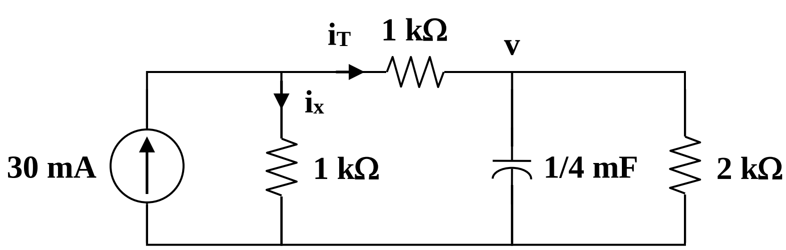
For t > 0, we transform the current source to a voltage source as shown in Fig. (a).



(a)

$$v(\infty) = \frac{2}{2+1+1}(30) = 15 \text{ V}$$
$$R_{th} = (1+1) \,\|\, 2 = 1 \text{ k}\Omega$$
$$\tau = R_{th}C = 10^3 \times \frac{1}{4} \times 10^{-3} = \frac{1}{4}$$
$$v(t) = 15\left(1 - e^{-4t}\right), \quad t > 0$$

We now obtain i_x from v(t). Consider Fig. (b).



(b)

$$i_x = 30 \text{ mA} - i_T$$

But $\quad i_T = \dfrac{v}{R_3} + C\dfrac{dv}{dt}$

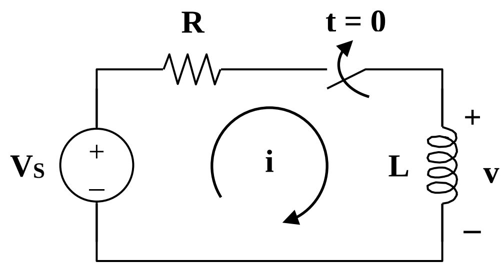
$$i_T(t) = 7.5\left(1 - e^{-4t}\right) \text{ mA} + \frac{1}{4} \times 10^{-3}\,(-15)(-4)e^{-4t} \text{ A}$$
$$i_T(t) = 7.5\left(1 + e^{-4t}\right) \text{ mA}$$

Thus,

$$i_x(t) = 30 - 7.5 - 7.5e^{-4t} \text{ mA}$$
$$i_x(t) = \mathbf{7.5\left(3 - e^{-4t}\right) mA, \quad t > 0}$$

Solution 7.51

Consider the circuit below.



After the switch is closed, applying KVL gives

$$V_S = Ri + L\frac{di}{dt}$$

or $$L\frac{di}{dt} = -R\left(i - \frac{V_S}{R}\right)$$

$$\frac{di}{i - V_S/R} = \frac{-R}{L}dt$$

Integrating both sides,

$$\ln\left(i - \frac{V_S}{R}\right)\bigg|_{I_0}^{i(t)} = \frac{-R}{L}t$$

$$\ln\left(\frac{i - V_S/R}{I_0 - V_S/R}\right) = \frac{-t}{\tau}$$

or $$\frac{i - V_S/R}{I_0 - V_S/R} = e^{-t/\tau}$$

$$\mathbf{i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}}$$

which is the same as Eq. (7.60).

Solution 7.52

Using Fig. 7.118, design a problem to help other students to better understand the step response of an RL circuit.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 7.118, find $i(t)$ for $t > 0$.

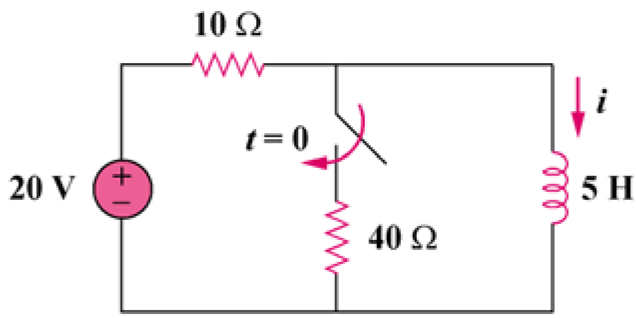


Figure 7.118

Solution

$$i(0) = \frac{20}{10} = 2\text{ A}, \qquad i(\infty) = 2\text{ A}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = \mathbf{2\ A}$$

Solution 7.53

(a) Before t = 0, $i = \frac{25}{3+2} = \mathbf{5\ A}$

After t = 0, $i(t) = i(0)e^{-t/\tau}$

$\tau = \frac{L}{R} = \frac{4}{2} = 2$, $i(0) = 5$

$i(t) = \boldsymbol{5e^{-t/2}\ u(t)A}$

(b) Before t = 0, the inductor acts as a short circuit so that the 2 Ω and 4 Ω resistors are short-circuited.

$i(t) = \mathbf{6\ A}$

After t = 0, we have an RL circuit.

$i(t) = i(0)e^{-t/\tau}$, $\tau = \frac{L}{R} = \frac{3}{2}$

$i(t) = \boldsymbol{6e^{-2t/3}\ u(t)A}$

Solution 7.54

(a) Before t = 0, i is obtained by current division or

$$i(t) = \frac{4}{4+4}(2) = \mathbf{1\ A}$$

After t = 0,

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

$$\tau = \frac{L}{R_{eq}}, \qquad R_{eq} = 4 + (4 \,||\, 12) = 7\ \Omega$$

$$\tau = \frac{3.5}{7} = \frac{1}{2}$$

$$i(0) = 1, \qquad i(\infty) = \frac{(4 \,||\, 12)}{4 + (4 \,||\, 12)}(2) = \frac{3}{4+3}(2) = \frac{6}{7}$$

$$i(t) = \frac{6}{7} + \left(1 - \frac{6}{7} \right) e^{-2t}$$

$$i(t) = \boldsymbol{\frac{1}{7}\left(6 - e^{-2t} \right)\ A}$$

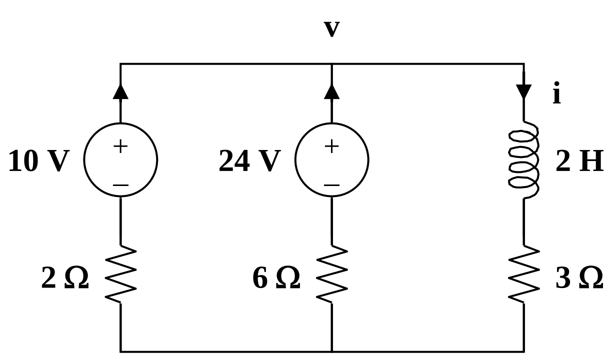
(b) Before t = 0, $i(t) = \frac{10}{2+3} = \mathbf{2\ A}$

After t = 0, $R_{eq} = 3 + (6 \,||\, 2) = 4.5$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{4.5} = \frac{4}{9}$$

$$i(0) = 2$$

To find $i(\infty)$, consider the circuit below, at t = when the inductor becomes a short circuit,



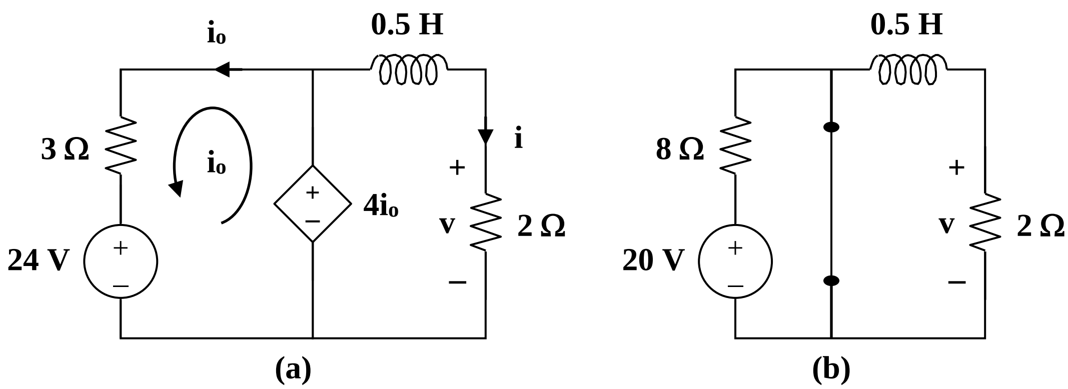
$$\frac{10-v}{2} + \frac{24-v}{6} = \frac{v}{3} \longrightarrow v = 9 \quad i(\infty) = \frac{v}{3} = 3\ \text{A} \ \text{and}$$

$$i(t) = 3 + (2-3)e^{-9t/4}$$

$$i(t) = \mathbf{3 - e^{-9t/4}\ \ A}$$

Solution 7.55

For t < 0, consider the circuit shown in Fig. (a).



$$3i_o + 24 - 4i_o = 0 \longrightarrow i_o = 24$$

$$v(t) = 4i_o = \mathbf{96\ V} \qquad i = \frac{v}{2} = 48\ \text{A}$$

For t > 0, consider the circuit in Fig. (b).

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(0) = 48, \qquad i(\infty) = 0$$

$$R_{th} = 2\ \Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{0.5}{2} = \frac{1}{4}$$

$$i(t) = (48)\,e^{-4t}$$

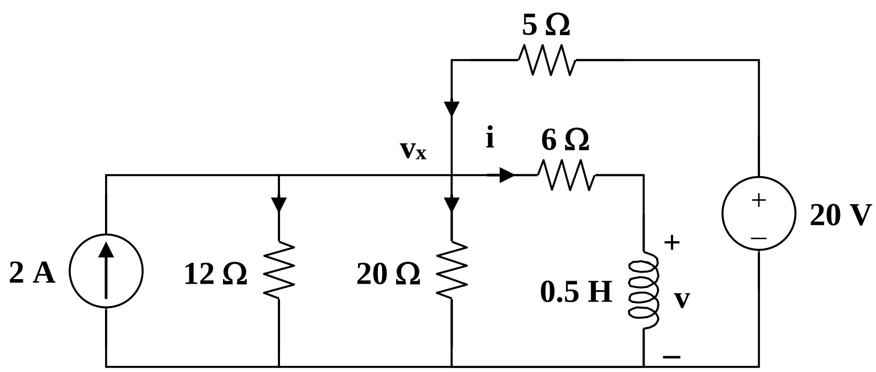
$$v(t) = 2\,i(t) = \mathbf{96}\,\boldsymbol{e}^{\mathbf{-4t}}\ \boldsymbol{u(t)V}$$

Solution 7.56

$$R_{eq} = 6 + 20 \| 5 = 10\ \Omega, \qquad \tau = \frac{L}{R} = 0.05$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

i(0) is found by applying nodal analysis to the following circuit.



$$2 + \frac{20 - v_x}{5} = \frac{v_x}{12} + \frac{v_x}{20} + \frac{v_x}{6} \longrightarrow v_x = 12$$

$$i(0) = \frac{v_x}{6} = 2\ \text{A}$$

Since $20 \| 5 = 4$,

$$i(\infty) = \frac{4}{4+6}(4) = 1.6$$

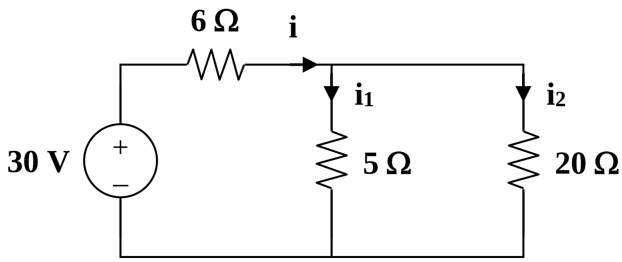
$$i(t) = 1.6 + (2 - 1.6)e^{-t/0.05} = 1.6 + 0.4e^{-20t}$$

$$v(t) = L\frac{di}{dt} = \frac{1}{2}(0.4)(-20)e^{-20t}$$

$$v(t) = \mathbf{-4e^{-20t}\ V}$$

Solution 7.57

At $t = 0^-$, the circuit has reached steady state so that the inductors act like short circuits.



$$i = \frac{30}{6 + (5 \| 20)} = \frac{30}{10} = 3, \qquad i_1 = \frac{20}{25}(3) = 2.4, \qquad i_2 = 0.6$$

$$i_1(0) = 2.4 \text{ A}, \qquad i_2(0) = 0.6 \text{ A}$$

For t > 0, the switch is closed so that the energies in L_1 and L_2 flow through the closed switch and become dissipated in the 5 Ω and 20 Ω resistors.

$$i_1(t) = i_1(0)e^{-t/\tau_1}, \qquad \tau_1 = \frac{L_1}{R_1} = \frac{2.5}{5} = \frac{1}{2}$$

$$i_1(t) = \mathbf{2.4e^{-2t}u(t)\ A}$$

$$i_2(t) = i_2(0)e^{-t/\tau_2}, \qquad \tau_2 = \frac{L_2}{R_2} = \frac{4}{20} = \frac{1}{5}$$

$$i_2(t) = \mathbf{600e^{-5t}u(t)\ mA}$$

Solution 7.58

For $t < 0$, $\quad v_o(t) = 0$

For $t > 0$, $\quad i(0) = 10, \quad i(\infty) = \dfrac{20}{1+3} = 5$

$$R_{th} = 1+3 = 4\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{1/4}{4} = \frac{1}{16}$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = 5\left(1 + e^{-16t}\right) \text{ A}$$

$$v_o(t) = 3i + L\frac{di}{dt} = 15\left(1 + e^{-16t}\right) + \frac{1}{4}(-16)(5)e^{-16t}$$

$$v_o(t) = \mathbf{15 - 5e^{-16t}\ \ V}$$

Solution 7.59

Determine the step response $v_o(t)$ to $i_s = 6u(t)$ A in the circuit of Fig. 7.124.

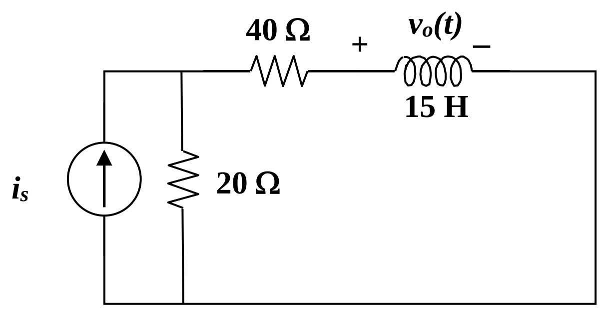


Figure 7.124
For Prob. 7.59.

Solution

Let i(t) be the current through the inductor.
For $t < 0$, $i_s = 0$ A and $i(0) = 0$ A.
For $t > 0$, $R_{eq} = 20+40 = 60\ \Omega$ and $\tau = (L/R_{eq}) = 15/60 = 0.25$ s.

At $t = \infty$, the inductor becomes a short and the current through the 40 Ω can be found by using current division, $i(\infty) = 6x20/(20+40) = 2$ A.

Thus, $i(t) = [2–2e^{–4t}]$ A and $v_o(t) = Ldi/dt = 15[(–4)(–2e^{–4t})]$ or

$v_o(t) = \mathbf{120e^{–4t}u(t)\ V}$.

Solution 7.60

Let I be the inductor current.

For t < 0, $u(t) = 0 \longrightarrow i(0) = 0$

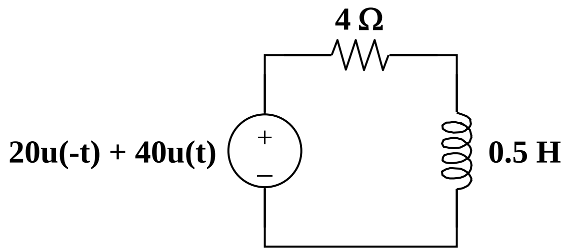
For t > 0, $R_{eq} = 5 \| 20 = 4\,\Omega$, $\tau = \dfrac{L}{R_{eq}} = \dfrac{8}{4} = 2$

$$i(\infty) = 4$$
$$i(t) = i(\infty) + \left[\,i(0) - i(\infty)\right] e^{-t/\tau}$$
$$i(t) = 4\left(1 - e^{-t/2}\right)$$

$$v(t) = L\frac{di}{dt} = (8)(-4)\left(\frac{-1}{2}\right)e^{-t/2}$$
$$v(t) = \mathbf{16\,e^{-0.5t}\ \ V}$$

Solution 7.61

The current source is transformed as shown below.



$$\tau = \frac{L}{R} = \frac{1/2}{4} = \frac{1}{8}, \qquad i(0) = 5, \qquad i(\infty) = 10$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = \mathbf{(10 - 5e^{-8t})u(t)\ A}$$

$$v(t) = L\frac{di}{dt} = \left(\frac{1}{2}\right)(-5)(-8)\, e^{-8t}$$

$$v(t) = \mathbf{20e^{-8t}u(t)\ V}$$

Solution 7.62

$$\tau = \frac{L}{R_{eq}} = \frac{2}{3 \| 6} = 1$$

For 0 < t < 1, $u(t-1) = 0$ so that

$$i(0) = 0\,, \qquad i(\infty) = \frac{1}{6}$$

$$i(t) = \frac{1}{6}\left(1 - e^{-t}\right)$$

For t > 1, $i(1) = \frac{1}{6}\left(1 - e^{-1}\right) = 0.1054$

$$i(\infty) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$i(t) = 0.5 + (0.1054 - 0.5)\,e^{-(t-1)}$$

$$i(t) = 0.5 - 0.3946\,e^{-(t-1)}$$

Thus,

$$i(t) = \begin{cases} \mathbf{\frac{1}{6}\left(1 - e^{-t}\right) A} & \mathbf{0 < t < 1} \\ \mathbf{0.5 - 0.3946\,e^{-(t-1)}\ A} & \mathbf{t > 1} \end{cases}$$

Solution 7.63

For t < 0, $u(-t) = 1$, $i(0) = \frac{10}{5} = 2$

For t > 0, $u(-t) = 0$, $i(\infty) = 0$

$$R_{th} = 5 \,\|\, 20 = 4\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{0.5}{4} = \frac{1}{8}$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = \mathbf{2e^{-8t}u(t)\ A}$$

$$v(t) = L\frac{di}{dt} = \left(\frac{1}{2}\right)(-8)(2)\,e^{-8t}$$

$$v(t) = \mathbf{-8e^{-8t}u(t)\ V}$$

$2e^{-8t}u(t)$ A, $-8e^{-8t}u(t)$ V

Solution 7.64

Determine the value of $i_L(t)$ and the total energy dissipated by the circuit from t = 0 sec to t = ∞ sec. The value of $i_{in}(t)$ is equal to [6 – 6u(t)] A.

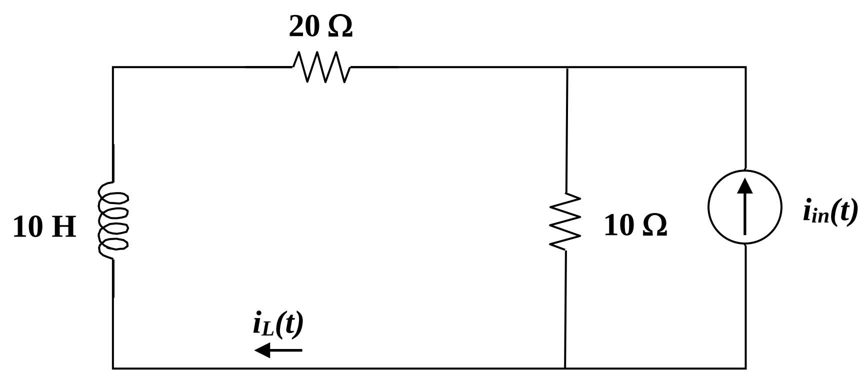


Figure 7.129
For Prob. 7.64.

Solution

For t < 0, the value of i_{in} = 6 A. The value of i_L can be found by using current division, $i_L(0) = -6x10/(20+10) = -2$ A.

For 0 < t i_{in} = 0 A and $i_L(\infty) = -(0)(1/3) = 0$ A and $\tau = L/R = 10/30 = 1/3$. Thus,

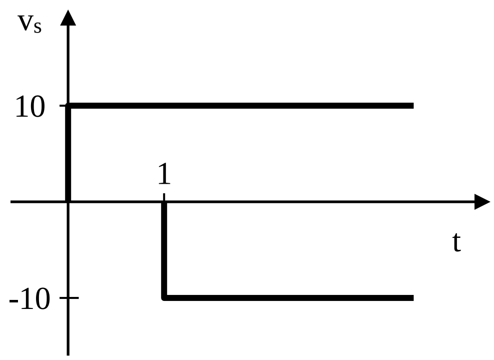
$$i_L(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$
$$= \mathbf{-2e^{-3t}\ A}.$$

To find the total energy that will be dissapaited in the circuit from t = 0 to ∞ we only need to recognize that the inductor is the only divice supplying power to the circuit after t = 0. Thus, the total energy dissapatied by the circuit is equal to the energy stored in the inductor at t = 0.

$$w = (1/2)Li_L(0)^2 = 0.5x10x(-2)^2 = \mathbf{20\ J}.$$

Solution 7.65

Since $v_s = 10\left[u(t) - u(t-1)\right]$, this is the same as saying that a 10 V source is turned on at t = 0 and a -10 V source is turned on later at t = 1. This is shown in the figure below.



For 0 < t < 1, $i(0) = 0$, $\quad i(\infty) = \dfrac{10}{5} = 2$

$$R_{th} = 5 \,\|\, 20 = 4, \qquad \tau = \frac{L}{R_{th}} = \frac{2}{4} = \frac{1}{2}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = 2\left(1 - e^{-2t}\right) \text{ A}$$

$$i(1) = 2\left(1 - e^{-2}\right) = 1.729$$

For t > 1, $i(\infty) = 0$ $\quad$ since $v_s = 0$

$$i(t) = i(1)\, e^{-(t-1)/\tau}$$

$$i(t) = 1.729\, e^{-2(t-1)} \text{ A}$$

Thus,

$$i(t) = \begin{cases} \mathbf{2\left(1 - e^{-2t}\right) A} & \mathbf{0 < t < 1} \\ \mathbf{1.729\, e^{-2(t-1)}\ A} & \mathbf{t > 1} \end{cases}$$

Solution 7.66

Using Fig. 7.131, design a problem to help other students to better understand first-order op amp circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the op-amp circuit of Fig. 7.131, find v_o. Assume that v_s changes abruptly from 0 to 1 V at t=0. Find v_o.

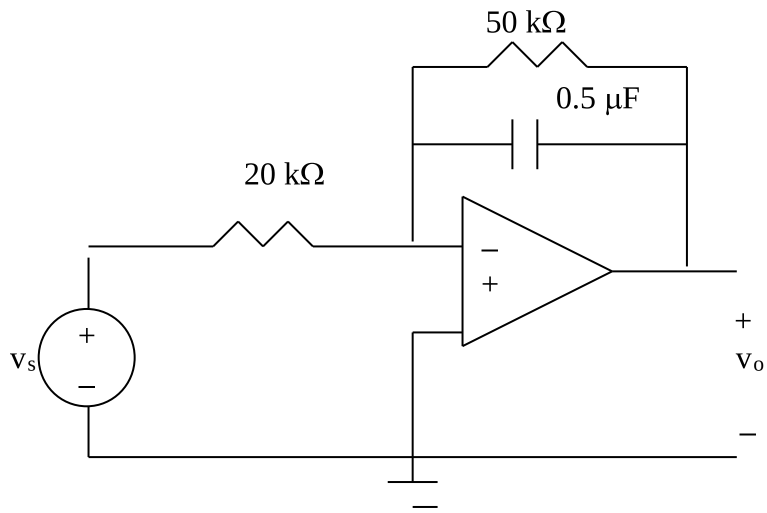


Figure 7.131 For Prob. 7.66.

Solution

For t<0-, v_s =0 so that $v_o(0)=0$
:Let v be the capacitor voltage
For t>0, v_s =1. At steady state, the capacitor acts like an open circuit so that we have an inverting amplifier

$$v_o(\infty) = -(50k/20k)(1V) = -2.5 \text{ V}$$

$$\tau = RC = 50x10^3x0.5x10^{-6} = 25 \text{ ms}$$

$$v_o(t) = v_o(\infty) + (v_o(0) - v_o(\infty))e^{-t/0.025} = \underline{\mathbf{2.5(e^{-40t} - 1)\ V}}.$$

Solution 7.67

If $v(0) = 10$ V, find $v_o(t)$ for $t > 0$ in the op amp circuit in Fig. 7.132.
Let $R = 100$ kΩ and $C = 20$ μF.

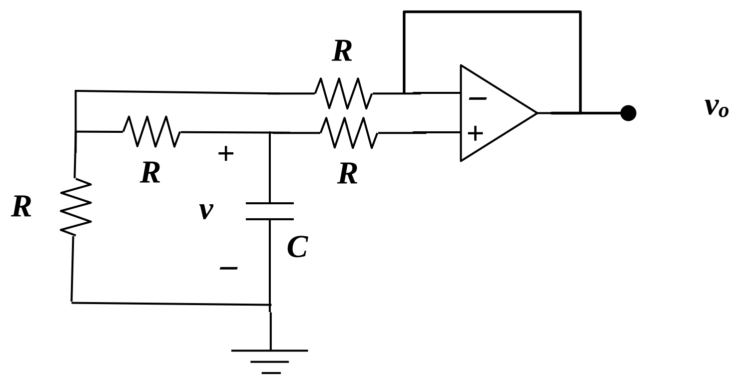
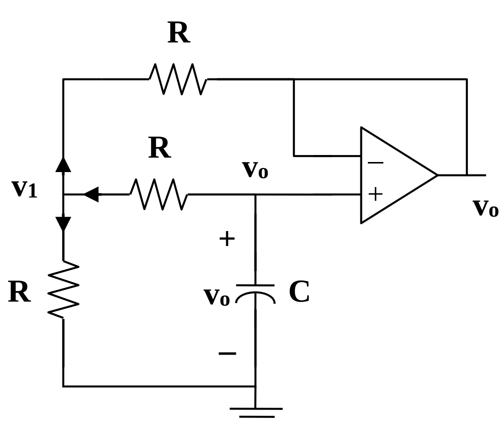


Figure 7.132
For Prob. 7.67.

Solution

In this circuit, the resistor between the capacitor and the positive input terminal of the op amp can be neglected since the current through it has to be equal to zero. This then results in the circuit shown below. Clearly this is a voltage follower circuit with $v_o = v$.



At node 1, $[(v_1–0)/R] + [(v_1–v_o)/R] + [(v_1–v_o)/R] = 0$ or $(3/R)v_1 = (2/R)v_o$ or $v_1 = (2/3)v_o$.

At the noninverting terminal,

$$C\frac{dv_o}{dt} + \frac{v_o - v_1}{R} = 0$$

$$-RC\frac{dv_o}{dt} = v_o - v_1 = v_o - \frac{2}{3}v_o = \frac{1}{3}v_o$$

$$\frac{dv_o}{dt} = -\frac{v_o}{3RC} \text{ or } v_o(t) = V_T\, e^{-t/3RC}$$

$V_T = v_o(0) = 10$ V and $\tau = 3RC = 3x10^5x2x10^{-5} = 6$ s.

Thus,

$v_o(t) =$ **$10e^{-t/6}u(t)$ A**.

Solution 7.68

This is a very interesting problem which has both an ideal solution as well as a realistic solution. Let us look at the ideal solution first. Just before the switch closes, the value of the voltage across the capacitor is zero which means that the voltage at both terminals input of the op amp are each zero. As soon as the switch closes, the output tries to go to a voltage such that both inputs to the op amp go to 4 volts. The ideal op amp puts out whatever current is necessary to reach this condition. An infinite (impulse) current is necessary if the voltage across the capacitor is to go to 8 volts in zero time (8 volts across the capacitor will result in 4 volts appearing at the negative terminal of the op amp). So v_o will be equal to **8 volts** for all t > 0.

What happens in a real circuit? Essentially, the output of the amplifier portion of the op amp goes to whatever its maximum value can be. Then this maximum voltage appears across the output resistance of the op amp and the capacitor that is in series with it. This results in an exponential rise in the capacitor voltage to the steady-state value of 8 volts.

$$vC(t) = V_{\text{op amp max}}(1 - e^{-t/(R_{out}C)}) \text{ volts, for all values of vC less than 8 V,}$$

$$= \mathbf{8\ V} \text{ when t is large enough so that the 8 V is reached.}$$

Solution 7.69

Let v_x be the capacitor voltage.
For t < 0, $v_x(0) = 0$

For t > 0, the 20 kΩ and 100 kΩ resistors are in series and together, they are in parallel with the capacitor since no current enters the op amp terminals. As $t \to \infty$, the capacitor acts like an open circuit so that

$$v_o(\infty) = \frac{-4}{10}(20+100) = -48$$

$$R_{th} = 20 + 100 = 120\ \text{k}\Omega, \qquad \tau = R_{th}C = (120\times 10^3)(25\times 10^{-3}) = 3000$$

$$v_o(t) = v_o(\infty) + \left[v_o(0) - v_o(\infty)\right] e^{-t/\tau}$$

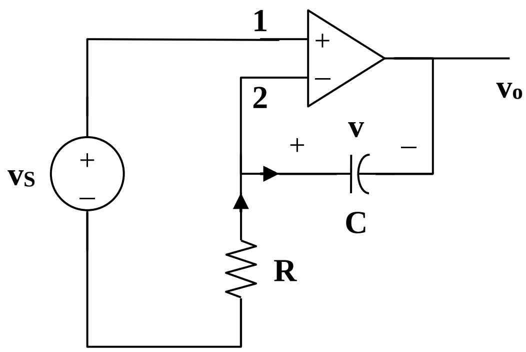
$$v_o(t) = -48\left(1 - e^{-t/3000}\right)V = \mathbf{48(e^{-t/3000}-1)u(t)V}$$

Solution 7.70

Let v = capacitor voltage.

For t < 0, the switch is open and $v(0) = 0$.

For t > 0, the switch is closed and the circuit becomes as shown below.



$$v_1 = v_2 = v_s \qquad (1)$$

$$\frac{0 - v_s}{R} = C\frac{dv}{dt} \qquad (2)$$

where $v = v_s - v_o \quad \longrightarrow \quad v_o = v_s - v \qquad (3)$

From (1),

$$\frac{dv}{dt} = \frac{v_s}{RC} = 0$$

$$v = \frac{-1}{RC}\int v_s \, dt + v(0) = \frac{-t\, v_s}{RC}$$

Since v is constant,

$$RC = (20\times10^3)(5\times10^{-6}) = 0.1$$

$$v = \frac{-20\,t}{0.1}\text{ mV} = -200t \text{ mV}$$

From (3),

$$v_o = v_s - v = 20 + 200\,t$$

$$v_o = \mathbf{20(1 + 10t)\ mV}$$

Solution 7.71

For the op amp circuit in Fig. 7.136, suppose v_s = 10u(t) V. Find *v(t)* for t > 0.

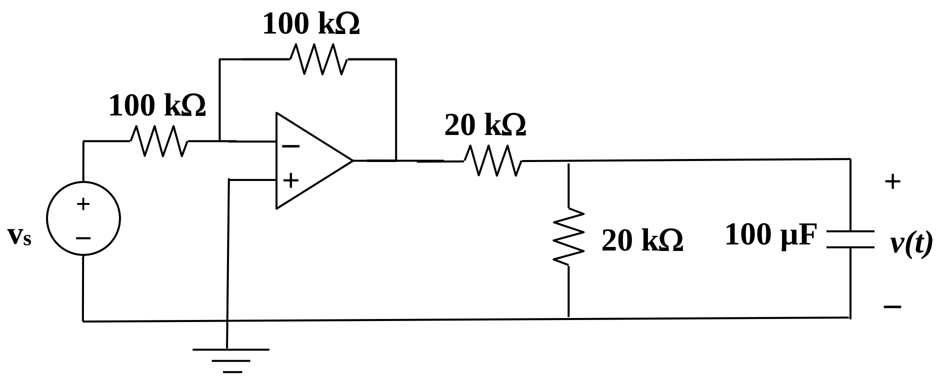


Figure 7.136
For Prob. 7.71.

Solution

We recognize that the op amp operates as an inverting op amp whose output is equal to –v_s = –10u(t) V. Since the output of the op amp acts like an ideal voltage source, we can determine the Thevenin equivalent circuit, as seen by the capacitor, with V_{Thev} = V_{oc} = –10u(t)[20k/(20k+20k)] = –5u(t) V and R_{eq} = 20kx20k/(20k+20k) = 10 kΩ. Next we get $\tau = R_{eq}C = 10^4 x 10^{-4} = 1$ s.

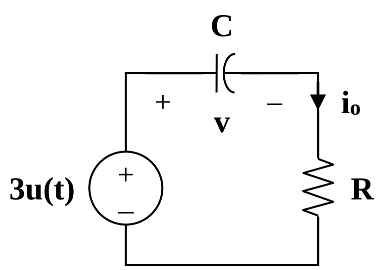
Additionally since the input voltage is equal to zero until t = 0, the value of v(0) = 0.

Finally v(∞) = –5 V which leads to,

v(t) = **[–5 + 5e^{-t}]u(t) V**.

Solution 7.72

The op amp acts as an emitter follower so that the Thevenin equivalent circuit is shown below.



Hence,

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

$$v(0) = -2\text{ V}, \quad v(\infty) = 3\text{ V}, \quad \tau = RC = (10\times10^{3})(10\times10^{-6}) = 0.1$$

$$v(t) = 3 + (-2-3)\,e^{-10t} = 3 - 5\,e^{-10t}$$

$$i_o = C\frac{dv}{dt} = (10\times10^{-6})(-5)(-10)\,e^{-10t}$$

$$i_o = \mathbf{0.5\,e^{-10t}\ mA,\quad t > 0}$$

Solution 7.73

For the op amp circuit of Fig. 7.138, let R_1 = 10 kΩ, R_f = 30 kΩ, $C = 20\ \mu F$, and $v(0)$ = 1 V. Find v_o.

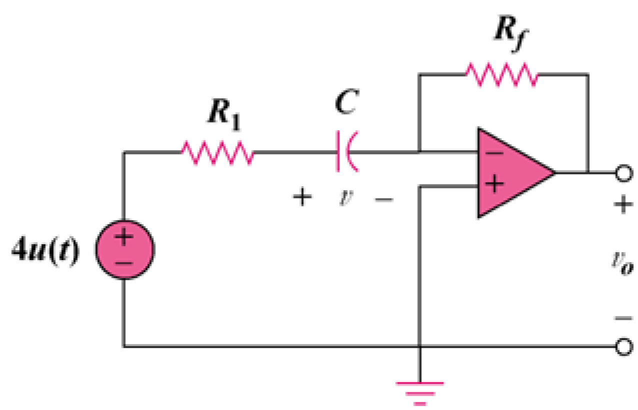
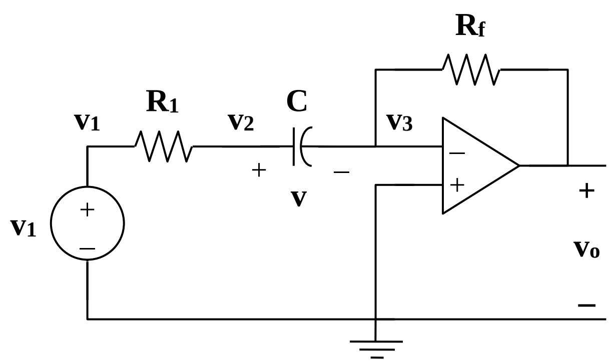


Figure 7.138
For Prob. 7.73.

Solution

Consider the circuit below.



At node 2,

$$\frac{v_1 - v_2}{R_1} = C\frac{dv}{dt} \qquad (1)$$

At node 3,

$$C\frac{dv}{dt} = \frac{v_3 - v_o}{R_f} \qquad (2)$$

But $v_3 = 0$ and $v = v_2 - v_3 = v_2$. Hence, (1) becomes

$$\frac{v_1 - v}{R_1} = C\frac{dv}{dt}$$

$$v_1 - v = R_1 C\frac{dv}{dt}$$

or $$\frac{dv}{dt} + \frac{v}{R_1 C} = \frac{v_1}{R_1 C}$$

which is similar to Eq. (7.42). Hence,

$$v(t) = \begin{cases} v_T & t < 0 \\ v_1 + \left(v_T - v_1\right)e^{-t/\tau} & t > 0 \end{cases}$$

where $v_T = v(0) = 1$ and $v_1 = 4$

$$\tau = R_1 C = (10\times10^3)(20\times10^{-6}) = 0.2$$

$$v(t) = \begin{cases} 1 & t < 0 \\ 4 - 3e^{-5t} & t > 0 \end{cases}$$

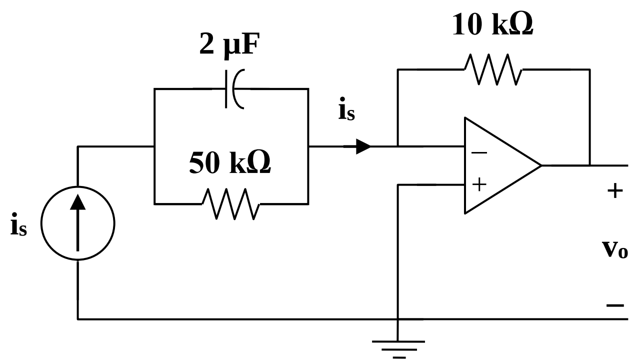
From (2),

$$v_o = -R_f C\frac{dv}{dt} = -(30\times10^3)(20\times10^{-6})(15e^{-5t})$$

$v_o = -9e^{-5t}, \quad t > 0$ or v_o = **$(-9e^{-5t})u(t)$ V**.

Solution 7.74

Let v = capacitor voltage. For t < 0, v(0) = 0



For t > 0, $i_s = 10\ \mu A$.

Since the current through the feedback resistor is i_s, then

$$v_o = -i_s x10^4 \text{ volts} = -10^{-5}x10^4 = \mathbf{-100\ mV}.$$

It is interesting to look at the capacitor voltage.

$$i_s = C\frac{dv}{dt} + \frac{v}{R}$$
$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

It is evident that

$$\tau = RC = (2\times10^{-6})(50\times10^{3}) = 0.1$$

At steady state, the capacitor acts like an open circuit so that i_s passes through R. Hence,

$$v(\infty) = i_s R = (10\times10^{-6})(50\times10^{3}) = 0.5\ V$$

Then the voltage across the capacitor is,

$$v(t) = \mathbf{500(1-e^{-10t})\ mV}.$$

Solution 7.75

In the circuit of Fig. 7.140, find v_o and i_o, given that $v_s = 10[1 - e^{-t}]u(t)$ V.

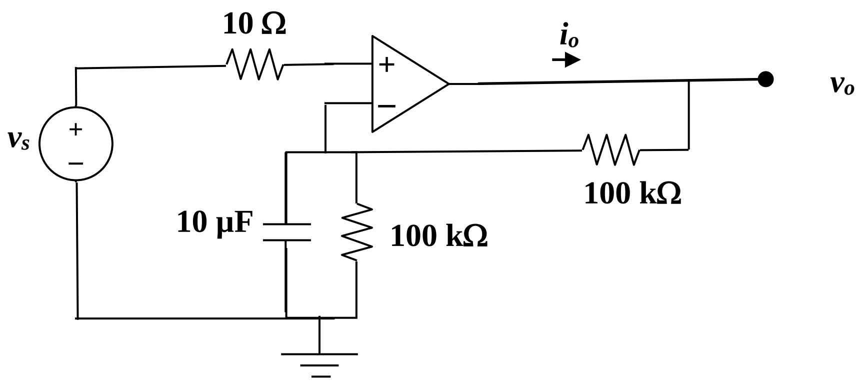


Figure 7.140
For Prob. 7.75.

Solution

Let v_a = voltage at the noninverting terminal and let v_b = voltage at the inverting terminal.

Since $v_s = 0$ for all $t < 0$, all the initial voltages are equal to 0.

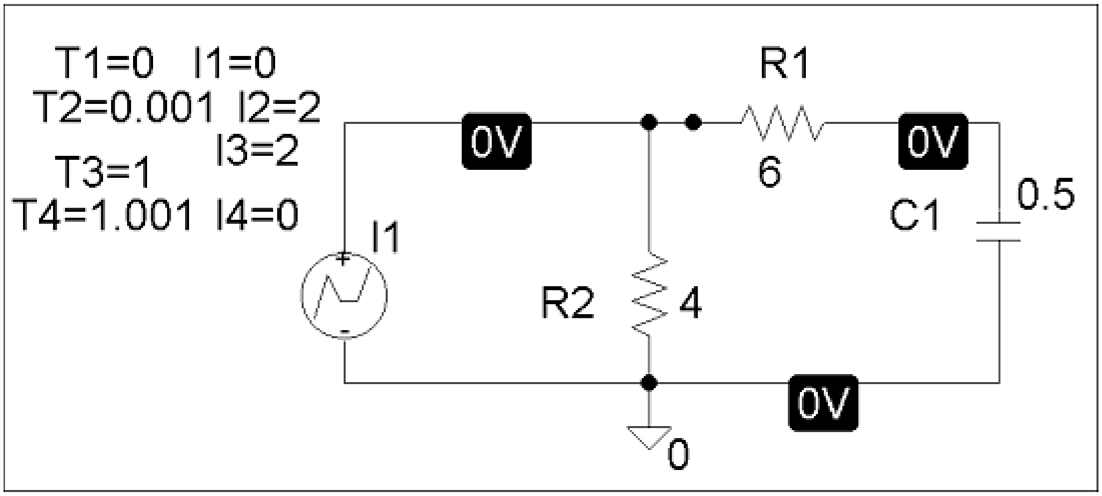
For $t > 0$, $v_a = v_b = v_s = 10[1 - e^{-t}]$.

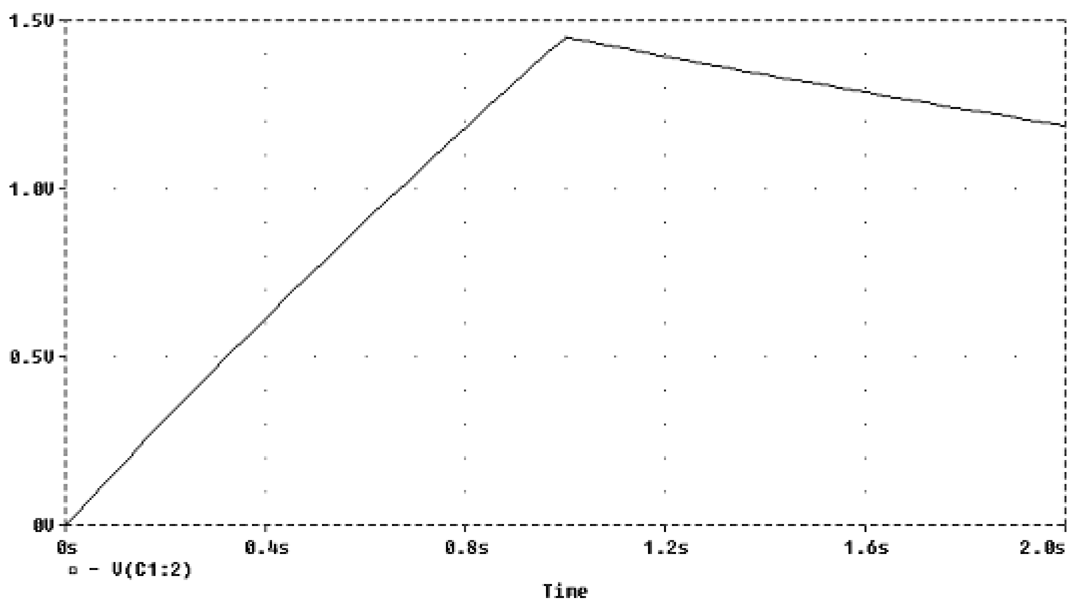
At v_b, $10^{-5}(dv_b/dt) + [(v_b-0)/10^5] + [(v_b-v_o)/10^5] + 0 = 0$. Since $dv_s/dt = 10e^{-t}$ we then get $v_o = 10e^{-t} + 2v_s = 10e^{-t} + 20 - 20e^{-t} =$ $\mathbf{[20 - 10e^{-t}]u(t)\ V}$.

Now, $i_o = [(v_o-v_s)/10^5] = \{[20 - 10e^{-t}] - [10-10e^{-t}]\}/10^5 =$ **100 µA**.

Solution 7.76

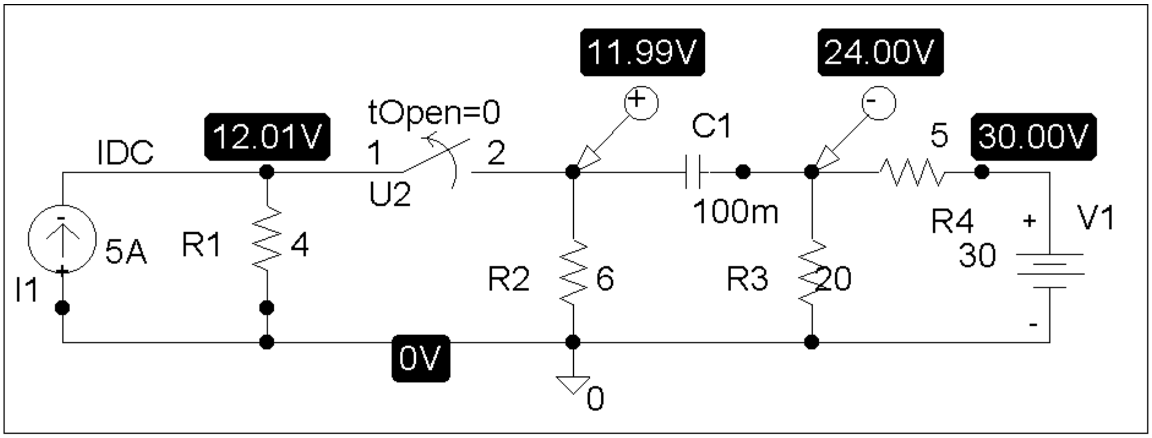
The schematic is shown below. For the pulse, we use IPWL and enter the corresponding values as attributes as shown. By selecting Analysis/Setup/Transient, we let Print Step = 25 ms and Final Step = 2 s since the width of the input pulse is 1 s. After saving and simulating the circuit, we select Trace/Add and display –V(C1:2). The plot of V(t) is shown below.

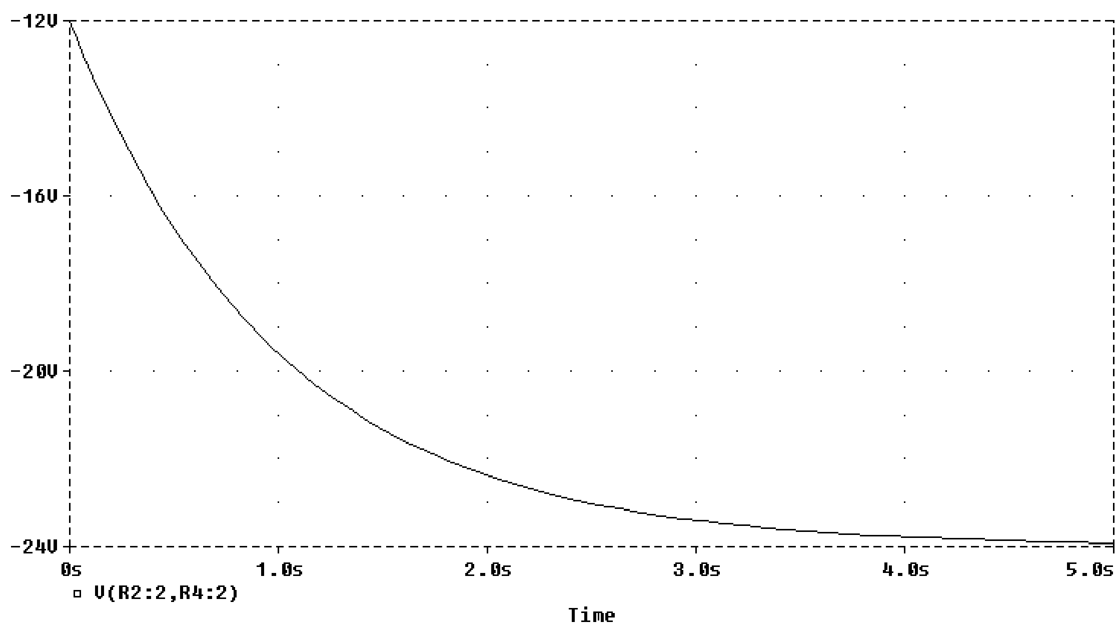




Solution 7.77

The schematic is shown below. We click Marker and insert Mark Voltage Differential at the terminals of the capacitor to display V after simulation. The plot of V is shown below. Note from the plot that V(0) = 12 V and V(∞) = -24 V which are correct.



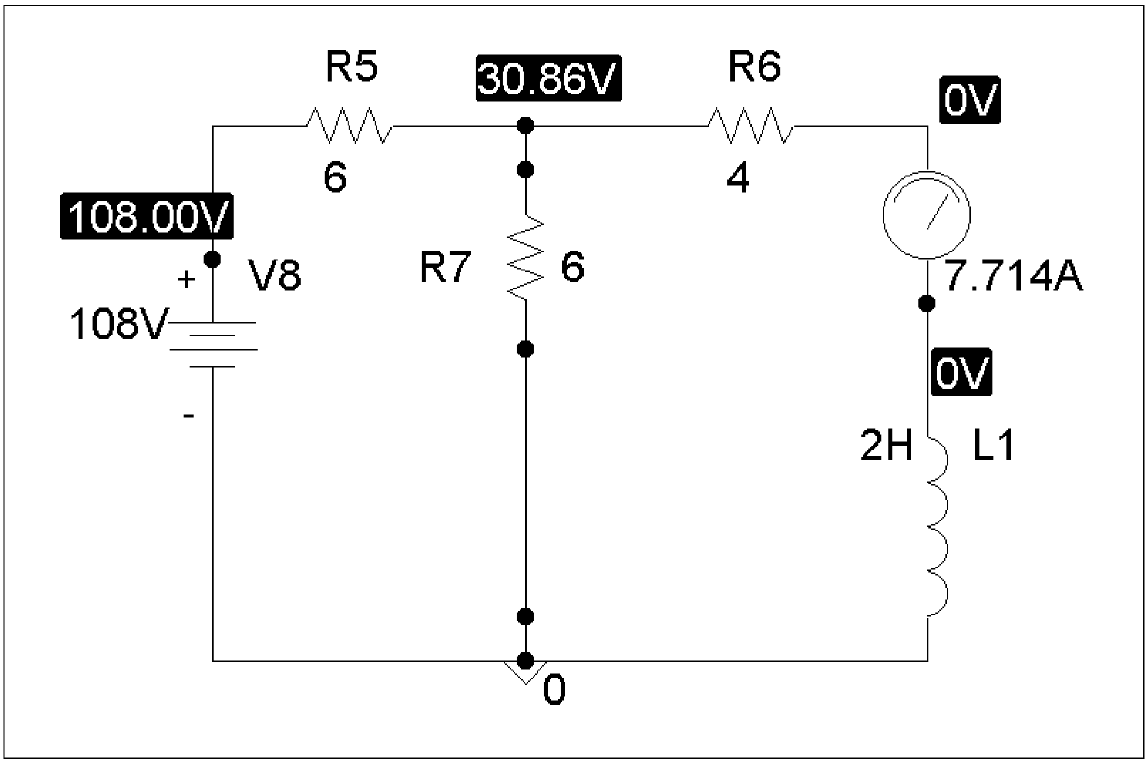


Solution 7.78

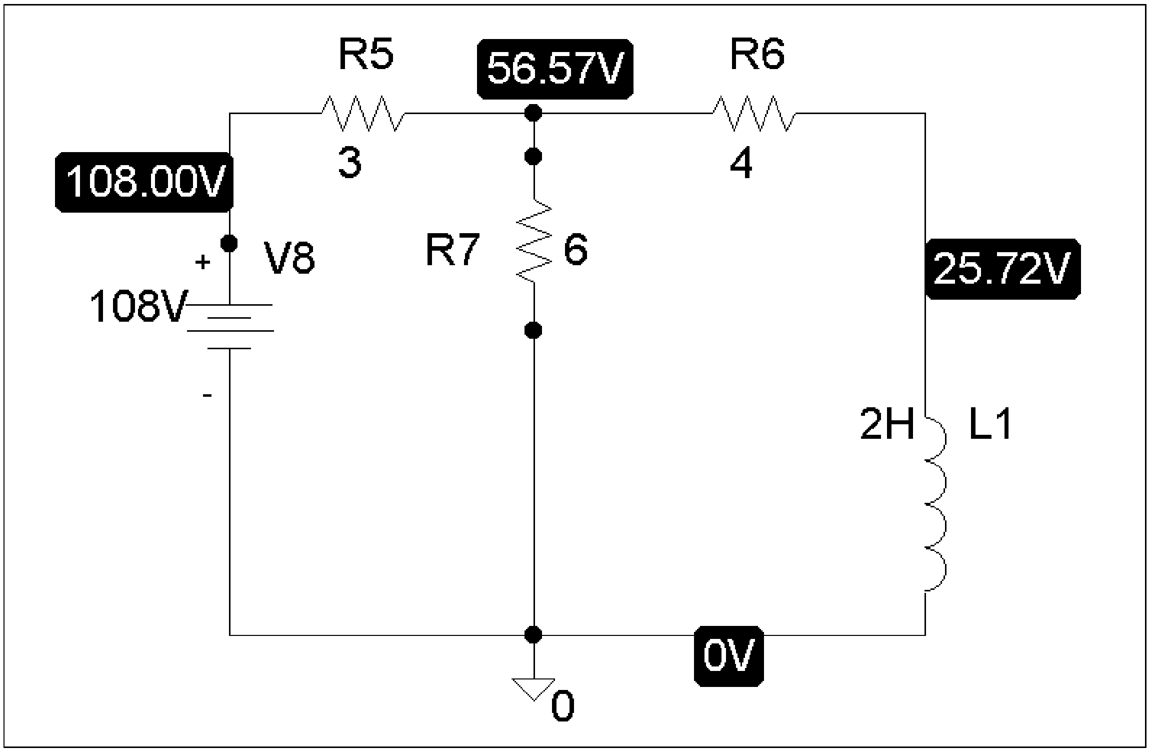
(a) When the switch is in position (a), the schematic is shown below. We insert IPROBE to display i. After simulation, we obtain,

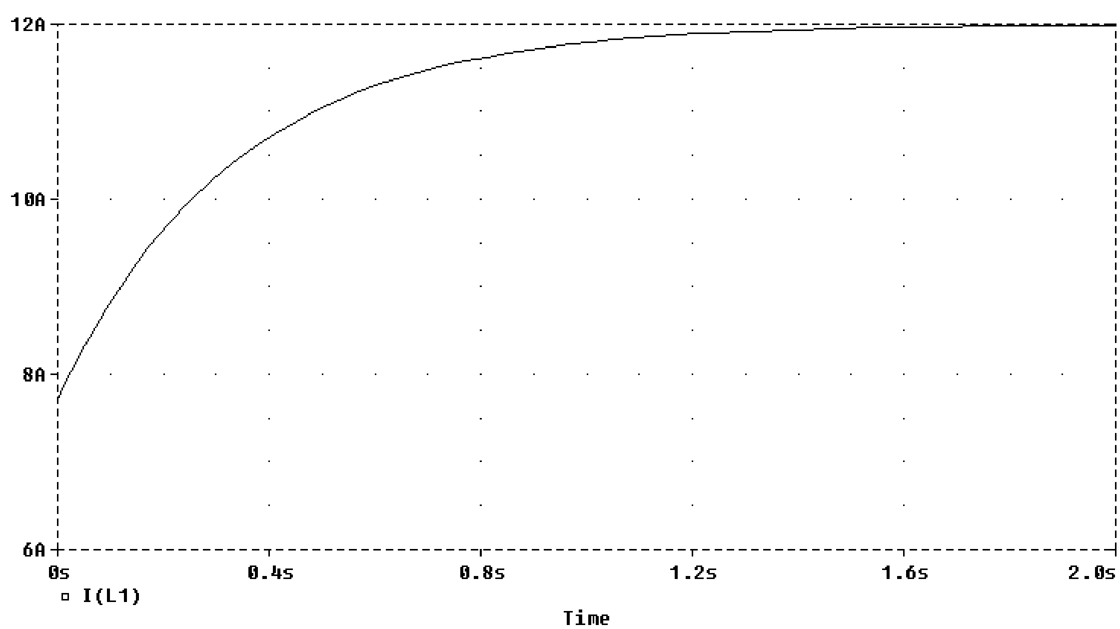
$$i(0) = 7.714 \text{ A}$$

from the display of IPROBE.



(b) When the switch is in position (b), the schematic is as shown below. For inductor I1, we let IC = 7.714. By clicking Analysis/Setup/Transient, we let Print Step = 25 ms and Final Step = 2 s. After Simulation, we click Trace/Add in the probe menu and display I(L1) as shown below. Note that $i(\infty) = 12$A, which is correct.


R5
56.57V
R6
3
4
108.00V
+
V8
R7
6
108V
25.72V
-
2H
L1
0V
0


12A
10A
8A
6A
0s
0.4s
0.8s
1.2s
1.6s
2.0s
I(L1)
Time

Solution 7.79

In the circuit in Fig. 7.143, determine i_o.

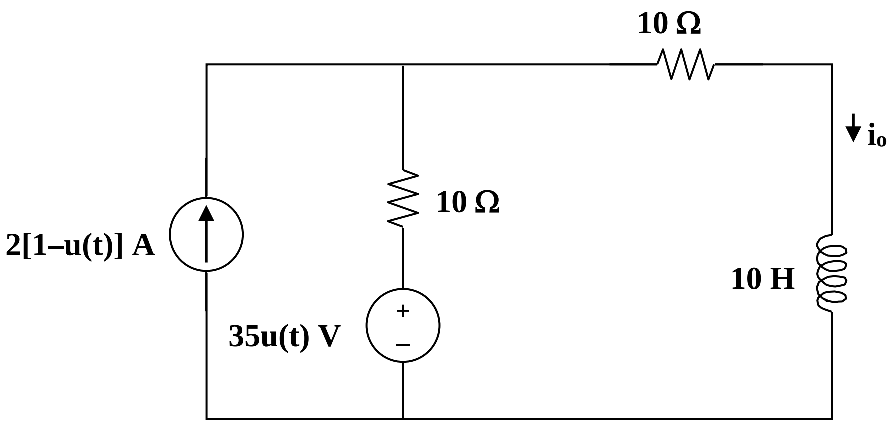


Figure 7.143
For Prob. 7.79.

Solution

For all t < 0, the voltage source is equal to zero (a short) and $i_o = 2x10/(10+10) = 1$ A.

For all 0 < t, the voltage source is equal to 35 V and the current source is equal to zero (an open circuit). At $t = \infty$, $i_o(\infty) = 35/20 = 1.75$ A. Additionally, $R_{eq} = 20\ \Omega$ and $\tau = L/R_{eq} = 10/20 = 0.5$ sec.

Finally,

$i_o(t) = 1.75 + [1–1.75]e^{-2t} = \mathbf{[1.75 - 0.75e^{-2t}]u(t)\ A}$.

Solution 7.80

In the circuit of Fig. 7.144, find the value of i_o for all values of $0 < t$.

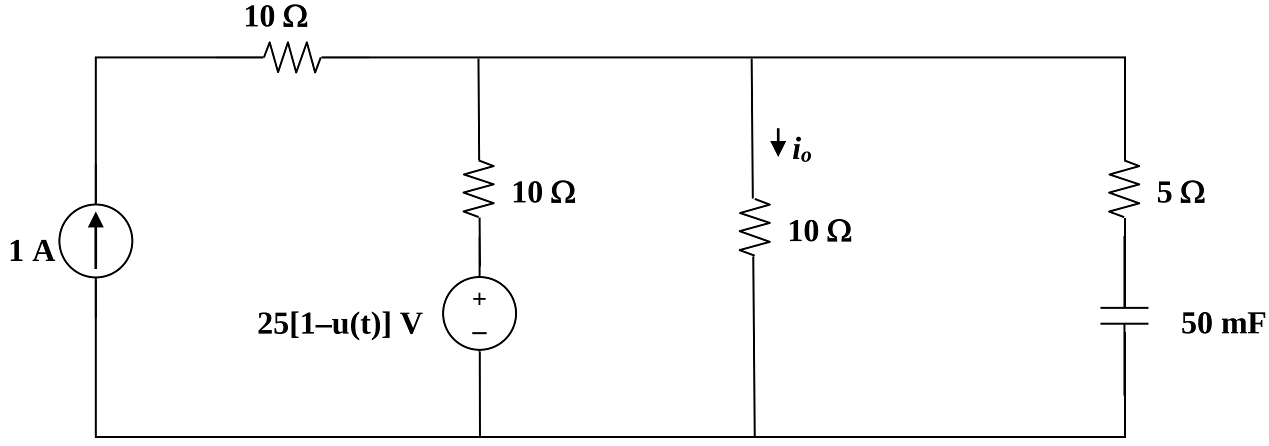


Figure 7.144
For Prob. 7.80.

Solution

For all values of $t < 0$, the current source is equal to 1 A and the voltage source is equal to 25 V. In addition the capacitor is equal to an open circuit. Thus, if we let v_o be the voltage at the top node and taking the bottom node as reference we get,

$-1 + [(v_o-25)/10] + [(v_o-0)/10] = 0$ and $v_o = 35/2 = 17.5$ V and $v_C(0) = 17.5$ V. Note, we can neglect the resistor in series with the current source.

For all values of $0 < t$, the current source is still equal to 1 A but the voltage source is now equal to zero (a short). We can now use the following equations to find i_o.

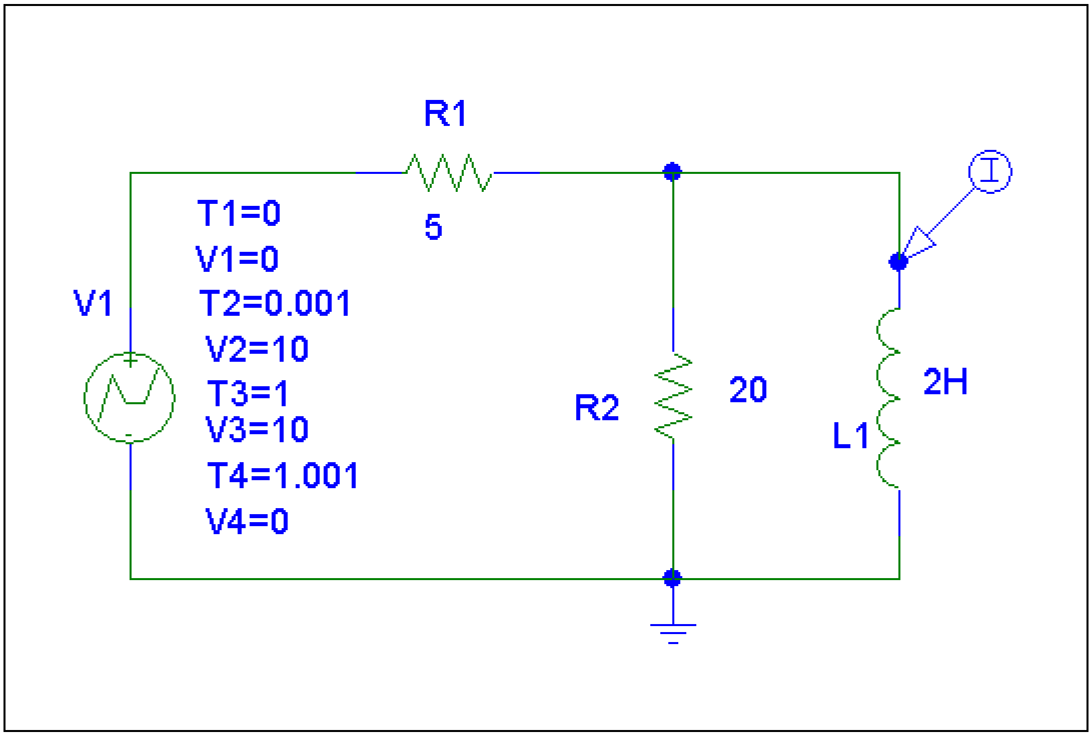
At $t = \infty$, the capacitor is an open circuit and the value of $v_C(\infty) = 5$ V, $R_{eq} = [10x10/(10+10)] + 5 = 10\ \Omega$, and $\tau = 10x0.05 = 0.5$ sec.

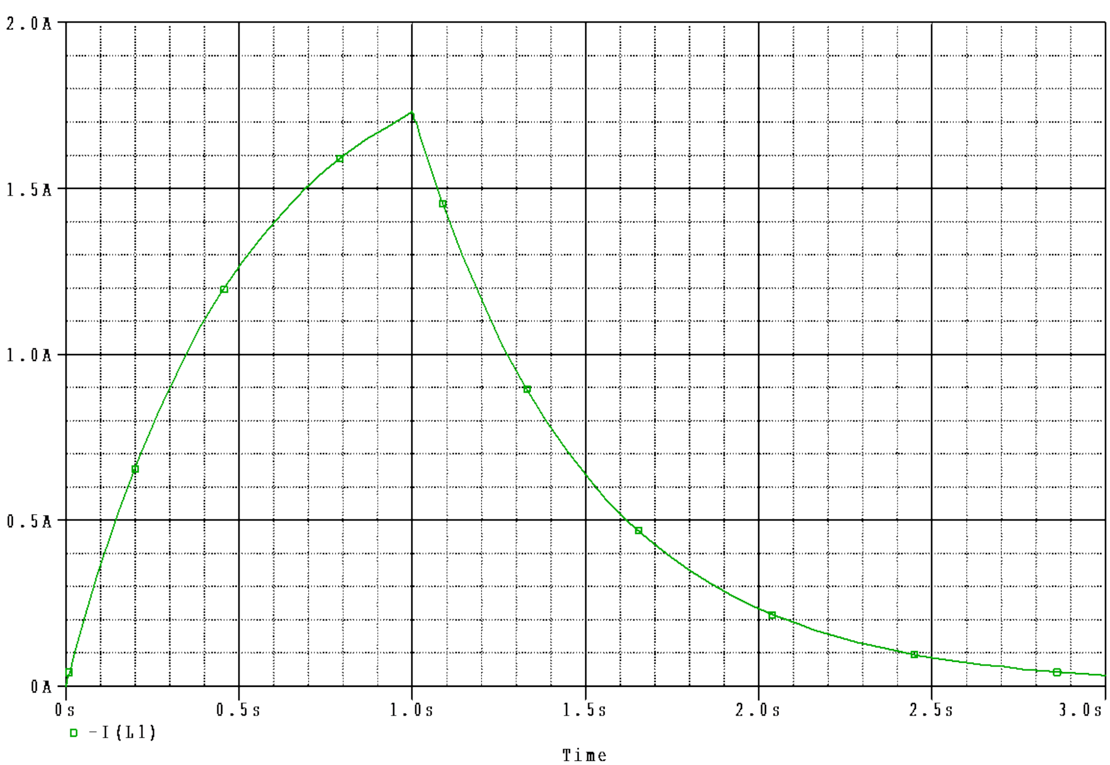
$v_C = [5 + 12.5e^{-2t}]u(t)$ V and $-1 + [(v_o-0)/10] + [(v_o-0)/10] + [(v_o-v_C)/5] = 0$ or $(0.1+0.1+0.2)v_o = 1 + 0.2v_C = 1 + 1 + 2.5e^{-2t} = 2 + 2.5e^{-2t}$ or $v_o = 5 + 6.25e^{-2t}$. Thus,

$i_o = v_o/10 =$ $\mathbf{[500+625e^{-2t}]u(t)\ mA}$.

Solution 7.81

The schematic is shown below. We use VPWL for the pulse and specify the attributes as shown. In the Analysis/Setup/Transient menu, we select Print Step = 25 ms and final Step = 3 S. By inserting a current marker at one terminal of LI, we automatically obtain the plot of i after simulation as shown below.





Solution 7.82

$$\tau = RC \quad \longrightarrow \quad R = \frac{\tau}{C} = \frac{3\times10^{-3}}{100\times10^{-6}} = \mathbf{30\ \Omega}$$

Solution 7.83

$$v(\infty)=120, \qquad v(0)=0, \qquad \tau = RC = 34x10^{6}\, x15x10^{-6} \;= 510s$$

$$v(t)=v(\infty)+[v(0)-v(\infty)]e^{-t/\tau} \qquad \longrightarrow \qquad 85.6=120(1-e^{-t/510})$$

Solving for t gives

$$t = 510\ln 3.488 = 637.16s$$

speed = 4000m/637.16s = **6.278m/s**

Solution 7.84

A capacitor with a value of 10 mF has a leakage resistance of 2 MΩ. How long does it take the voltage across the capacitor to decay to 40% of the initial voltage to which the capacitor is charged? Assume that the capacitor is charged and then set aside by itself.

Solution

The voltage across a charged capacitor is equal to $v_C(t) = v_C(0)e^{-t/\tau}$ where $\tau = R_{leak}C = (2x10^6)(0.01) = 2x10^4$. Thus,

$0.4v_C(0) = v_C(0)e^{-t/20,000}$ or $-t/20,000 = \ln(0.4) = -0.91629$ or $t = 18.326$ ks or

$t =$ **5.091 hours**.

Solution 7.85

(a) The light is on from 75 volts until 30 volts. During that time we essentially have a 120-ohm resistor in parallel with a 6-μF capacitor.

$v(0) = 75$, $v(\infty) = 0$, $\tau = 120x6x10^{-6} = 0.72$ ms

$v(t_1) = 75e^{-t_1/\tau} = 30$ which leads to $t_1 = -0.72\ln(0.4)$ ms = **659.7 μs** of lamp on time.

(b) $\tau = RC = (4\times10^{6})(6\times10^{-6}) = 24$ s

Since $v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$

$$v(t_1) - v(\infty) = [v(0) - v(\infty)]e^{-t_1/\tau} \quad (1)$$

$$v(t_2) - v(\infty) = [v(0) - v(\infty)]e^{-t_2/\tau} \quad (2)$$

Dividing (1) by (2),

$$\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)} = e^{(t_2 - t_1)/\tau}$$

$$t_0 = t_2 - t_1 = \tau\ln\left(\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)}\right)$$

$$t_0 = 24\ln\left(\frac{75-120}{30-120}\right) = 24\ln(2) = \mathbf{16.636\ s}$$

Solution 7.86

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

$$v(\infty) = 12, \qquad v(0) = 0$$

$$v(t) = 12\left(1 - e^{-t/\tau}\right)$$

$$v(t_0) = 8 = 12\left(1 - e^{-t_0/\tau}\right)$$

$$\frac{8}{12} = 1 - e^{-t_0/\tau} \quad \longrightarrow \quad e^{-t_0/\tau} = \frac{1}{3}$$

$$t_0 = \tau \ln(3)$$

For $R = 100\ \text{k}\Omega$,

$$\tau = RC = (100 \times 10^3)(2 \times 10^{-6}) = 0.2\ \text{s}$$

$$t_0 = 0.2 \ln(3) = 0.2197\ \text{s}$$

For $R = 1\ \text{M}\Omega$,

$$\tau = RC = (1 \times 10^6)(2 \times 10^{-6}) = 2\ \text{s}$$

$$t_0 = 2 \ln(3) = 2.197\ \text{s}$$

Thus,

$$\mathbf{0.2197\ s < t_0 < 2.197\ s}$$

Solution 7.87

Let i be the inductor current.

For t < 0, $\quad i(0^-) = \frac{120}{100} = 1.2\text{ A}$

For t > 0, we have an RL circuit

$$\tau = \frac{L}{R} = \frac{50}{100+400} = 0.1, \qquad i(\infty) = 0$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = 1.2\,e^{-10t}$$

At $\quad$ t = 100 ms = 0.1 s,

$$i(0.1) = 1.2\,e^{-1} = \mathbf{441mA}$$

which is the same as the current through the resistor.

Solution 7.88

(a) $\tau = RC = (300 \times 10^3)(200 \times 10^{-12}) = 60\ \mu s$

As a differentiator,

$T > 10\tau = 600\ \mu s = 0.6\ ms$

i.e. $T_{min} = \mathbf{0.6\ ms}$

(b) $\tau = RC = 60\ \mu s$

As an integrator,

$T < 0.1\tau = 6\ \mu s$

i.e. $T_{max} = \mathbf{6\ \mu s}$

Solution 7.89

Since $\tau < 0.1T = 1\ \mu s$

$$\frac{L}{R} < 1\ \mu s$$

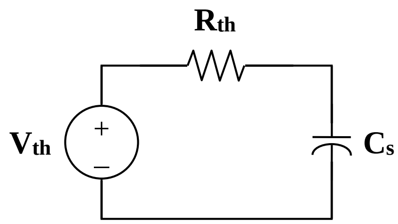
$$L < R \times 10^{-6} = (200 \times 10^{3})(1 \times 10^{-6})$$

$$\mathbf{L < 200\ mH}$$

Solution 7.90

We determine the Thevenin equivalent circuit for the capacitor C_s.

$$v_{th} = \frac{R_s}{R_s + R_p} v_i, \qquad R_{th} = R_s \parallel R_p$$



The Thevenin equivalent is an RC circuit. Since

$$v_{th} = \frac{1}{10} v_i \quad \longrightarrow \quad \frac{1}{10} = \frac{R_s}{R_s + R_p}$$

$$R_s = \frac{1}{9} R_p = \frac{6}{9} = \mathbf{\frac{2}{3}\ M\Omega}$$

Also,

$$\tau = R_{th} C_s = 15\ \mu s$$

where $R_{th} = R_p \parallel R_s = \dfrac{6(2/3)}{6 + 2/3} = 0.6\ M\Omega$

$$C_s = \frac{\tau}{R_{th}} = \frac{15 \times 10^{-6}}{0.6 \times 10^{6}} = \mathbf{25\ pF}$$

Solution 7.91

$$i_o(0) = \frac{12}{50} = 240 \text{ mA}, \qquad i(\infty) = 0$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = 240\, e^{-t/\tau}$$

$$\tau = \frac{L}{R} = \frac{2}{R}$$

$$i(t_0) = 10 = 240\, e^{-t_0/\tau}$$

$$e^{t_0/\tau} = 24 \quad \longrightarrow \quad t_0 = \tau \ln(24)$$

$$\tau = \frac{t_0}{\ln(24)} = \frac{5}{\ln(24)} = 1.573 = \frac{2}{R}$$

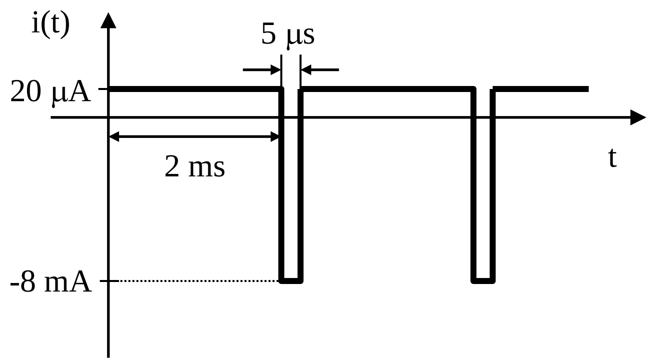
$$R = \frac{2}{1.573} = \mathbf{1.271\,\Omega}$$

Solution 7.92

$$i = C\frac{dv}{dt} = 4\times10^{-9}\cdot\begin{cases}\dfrac{10}{2\times10^{-3}} & 0 < t < t_R \\ \dfrac{-10}{5\times10^{-6}} & t_R < t < t_D\end{cases}$$

$$i(t) = \begin{cases}20\ \mu A & 0 < t < 2\ ms \\ -8\ mA & 2\ ms < t < 2\ ms + 5\ \mu s\end{cases}$$

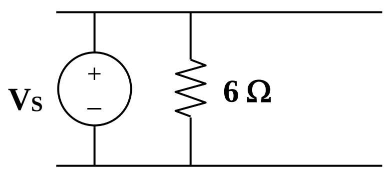
which is sketched below.



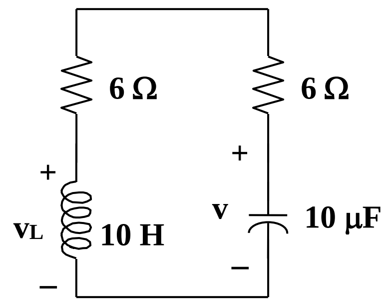
(not to scale)

Solution 8.1

(a) At $t = 0-$, the circuit has reached steady state so that the equivalent circuit is shown in Figure (a).



(a)



(b)

$$i(0-) = 12/6 = 2A, \; v(0-) = 12V$$

$$\text{At } t = 0+, \; i(0+) = i(0-) = \mathbf{2A}, \; v(0+) = v(0-) = \mathbf{12V}$$

(b) For $t > 0$, we have the equivalent circuit shown in Figure (b).

$$v_L = Ldi/dt \text{ or } di/dt = v_L/L$$

Applying KVL at $t = 0+$, we obtain,

$$v_L(0+) – v(0+) + 10i(0+) = 0$$

$$v_L(0+) – 12 + 20 = 0, \text{ or } v_L(0+) = -8$$

Hence, $di(0+)/dt = -8/2 = \mathbf{-4\ A/s}$

Similarly, $i_C = Cdv/dt, \text{ or } dv/dt = i_C/C$

$$i_C(0+) = -i(0+) = -2$$

$$dv(0+)/dt = -2/0.4 = \mathbf{-5\ V/s}$$

(c) As t approaches infinity, the circuit reaches steady state.

$$i(\infty) = \mathbf{0\ A}, \; v(\infty) = \mathbf{0\ V}$$

Solution 8.2

Using Fig. 8.63, design a problem to help other students better understand finding initial and final values.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the circuit of Fig. 8.63, determine:

(a) $i_R(0^+)$, $i_L(0^+)$, and $i_C(0^+)$,

(b) $di_R(0^+)/dt$, $di_L(0^+)/dt$, and $di_C(0^+)/dt$,

(c) $i_R(\infty)$, $i_L(\infty)$, and $i_C(\infty)$.

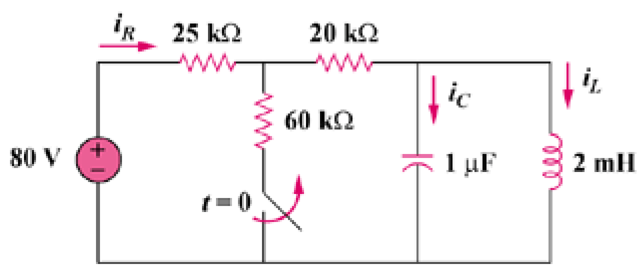
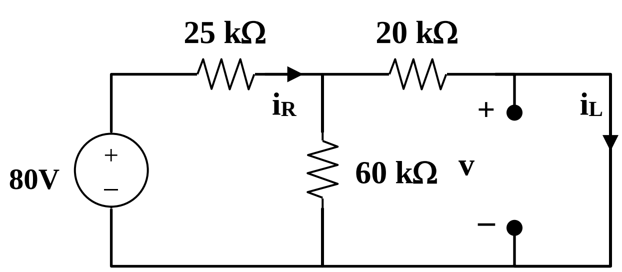


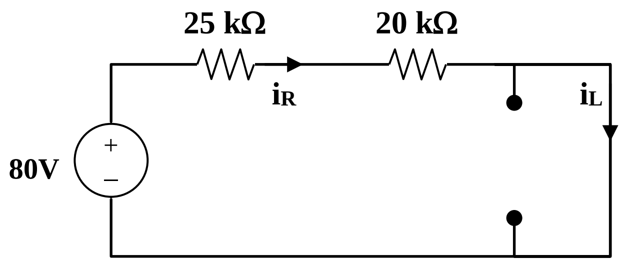
Figure 8.63

Solution

(a) At t = 0-, the equivalent circuit is shown in Figure (a).



(a)



(b)

$$60||20 = 15 \text{ kohms}, \ i_R(0-) = 80/(25 + 15) = 2\text{mA}.$$

By the current division principle,

$$i_L(0-) = 60(2\text{mA})/(60 + 20) = 1.5 \text{ mA}$$

$$v_C(0-) = 0$$

At $t = 0+$,

$$v_C(0+) = v_C(0-) = 0$$

$$i_L(0+) = i_L(0-) = \mathbf{1.5\ mA}$$

$$80 = i_R(0+)(25 + 20) + v_C(0-)$$

$$i_R(0+) = 80/45k = \mathbf{1.778\ mA}$$

But, $$i_R = i_C + i_L$$

$$1.778 = i_C(0+) + 1.5 \ \text{ or } \ i_C(0+) = \mathbf{0.278\ mA}$$

(b) $$v_L(0+) = v_C(0+) = 0$$

But, $v_L = Ldi_L/dt$ and $di_L(0+)/dt = v_L(0+)/L = 0$

$$di_L(0+)/dt = \mathbf{0}$$

Again, $80 = 45i_R + v_C$

$$0 = 45di_R/dt + dv_C/dt$$

But, $dv_C(0+)/dt = i_C(0+)/C = 0.278 \text{ mamps}/1\ \mu\text{F} = 278 \text{ V/s}$

Hence, $di_R(0+)/dt = (-1/45)dv_C(0+)/dt = -278/45$

$di_R(0+)/dt = \mathbf{-6.1778\ A/s}$

Also, $i_R = i_C + i_L$

$di_R(0+)/dt = di_C(0+)/dt + di_L(0+)/dt$

$-6.1788 = di_C(0+)/dt + 0$, or $di_C(0+)/dt = \mathbf{-6.1788\ A/s}$

(c) As t approaches infinity, we have the equivalent circuit in Figure (b).

$i_R(\infty) = i_L(\infty) = 80/45k = \mathbf{1.778\ mA}$

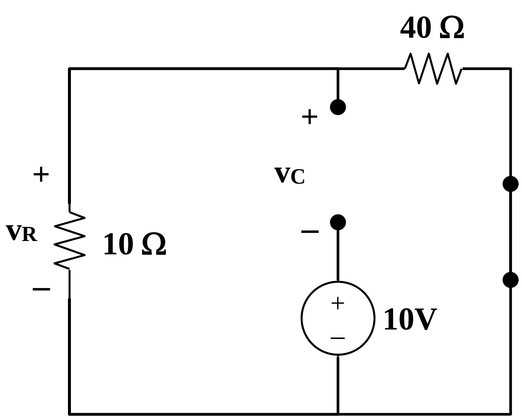
$i_C(\infty) = Cdv(\infty)/dt = \mathbf{0}$.

Solution 8.3

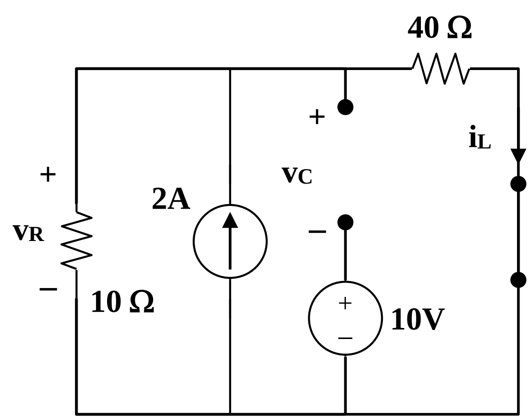
At $t = 0^-$, $u(t) = 0$. Consider the circuit shown in Figure (a). $i_L(0^-) = 0$, and $v_R(0^-) = 0$. But, $-v_R(0^-) + v_C(0^-) + 10 = 0$, or $v_C(0^-) = -10V$.

(a) At $t = 0^+$, since the inductor current and capacitor voltage cannot change abruptly, the inductor current must still be equal to **0A**, the capacitor has a voltage equal to **–10V**. Since it is in series with the +10V source, together they represent a direct short at $t = 0^+$. This means that the entire 2A from the current source flows through the capacitor and not the resistor. Therefore, $v_R(0^+) = \mathbf{0\ V}$.

(b) At $t = 0^+$, $v_L(0+) = 0$, therefore $L di_L(0+)/dt = v_L(0^+) = 0$, thus, $di_L/dt = \mathbf{0A/s}$, $i_C(0^+) = 2$ A, this means that $dv_C(0^+)/dt = 2/C = \mathbf{8\ V/s}$. Now for the value of $dv_R(0^+)/dt$. Since $v_R = v_C + 10$, then $dv_R(0^+)/dt = dv_C(0^+)/dt + 0 = \mathbf{8\ V/s}$.



(a)



(b)

(c) As t approaches infinity, we end up with the equivalent circuit shown in Figure (b).

$$i_L(\infty) = 10(2)/(40 + 10) = \mathbf{400\ mA}$$

$$v_C(\infty) = 2[10||40] - 10 = 16 - 10 = \mathbf{6V}$$

$$v_R(\infty) = 2[10||40] = \mathbf{16\ V}$$

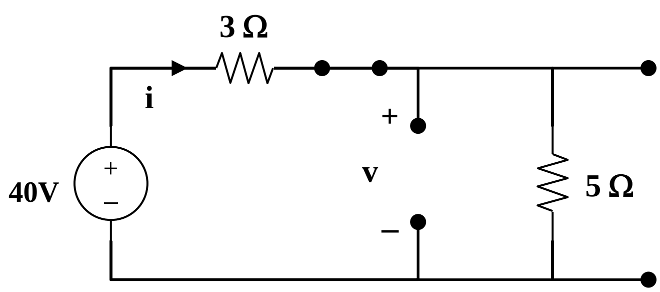
Solution 8.4

(a) At $t = 0^-$, $u(-t) = 1$ and $u(t) = 0$ so that the equivalent circuit is shown in Figure (a).

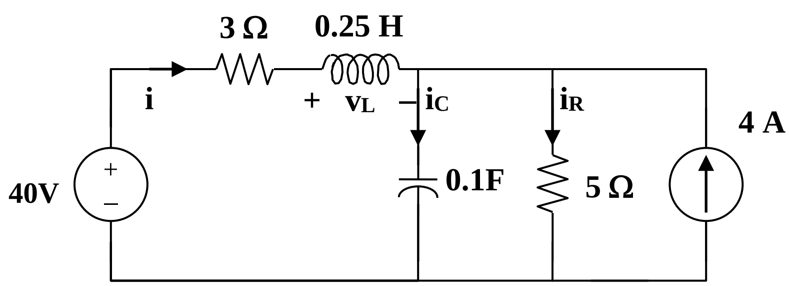
$$i(0^-) = 40/(3 + 5) = 5A, \text{ and } v(0^-) = 5i(0^-) = 25V.$$

Hence, $i(0^+) = i(0^-) = \mathbf{5A}$

$$v(0^+) = v(0^-) = \mathbf{25V}$$



(a)



(b)

(b) $i_C = Cdv/dt$ or $dv(0^+)/dt = i_C(0^+)/C$

For $t = 0^+$, $4u(t) = 4$ and $4u(-t) = 0$. The equivalent circuit is shown in Figure (b). Since i and v cannot change abruptly,

$$i_R = v/5 = 25/5 = 5A, \; i(0^+) + 4 = i_C(0^+) + i_R(0^+)$$

$$5 + 4 = i_C(0^+) + 5 \text{ which leads to } i_C(0^+) = 4$$

$$dv(0^+)/dt = 4/0.1 = \mathbf{40\ V/s}$$

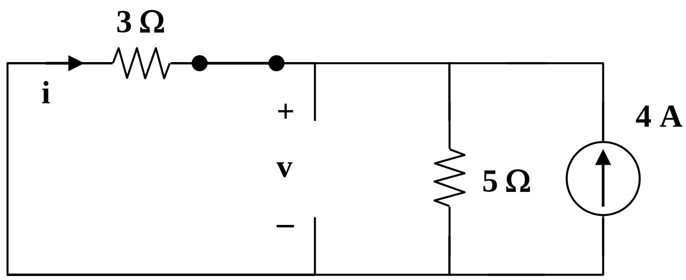
Similarly, $v_L = Ldi/dt$ which leads to $di(0^+)/dt = v_L(0^+)/L$

$$3i(0^+) + v_L(0^+) + v(0^+) = 0$$

$$15 + v_L(0^+) + 25 = 0 \quad \text{or} \quad v_L(0^+) = -40$$

$$di(0^+)/dt = -40/0.25 = \mathbf{-160\ A/s}$$

(c) As t approaches infinity, we have the equivalent circuit in Figure (c).



(c)

$$i(\infty) = -5(4)/(3 + 5) = \mathbf{-2.5\ A}$$

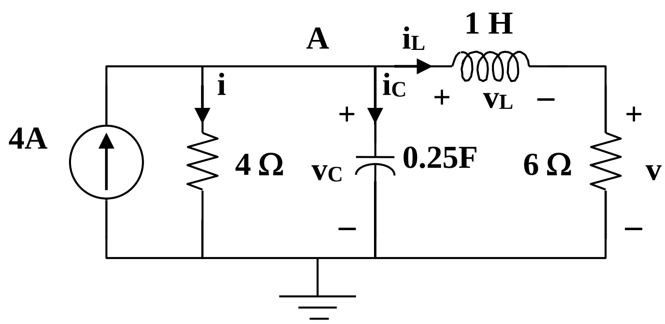
$$v(\infty) = 5(4 - 2.5) = \mathbf{7.5\ V}$$

Solution 8.5

(a) For $t < 0$, $4u(t) = 0$ so that the circuit is not active (all initial conditions = 0).

$$i_L(0-) = 0 \text{ and } v_C(0-) = 0.$$

For $t = 0+$, $4u(t) = 4$. Consider the circuit below.



Since the 4-ohm resistor is in parallel with the capacitor,

$$i(0+) = v_C(0+)/4 = 0/4 = \mathbf{0\ A}$$

Also, since the 6-ohm resistor is in series with the inductor,
$v(0+) = 6i_L(0+) = \mathbf{0V}$.

(b) $di(0+)/dt = d(v_R(0+)/R)/dt = (1/R)dv_R(0+)/dt = (1/R)dv_C(0+)/dt$

$$= (1/4)4/0.25 \text{ A/s} = \mathbf{4\ A/s}$$

$v = 6i_L$ or $dv/dt = 6di_L/dt$ and $dv(0+)/dt = 6di_L(0+)/dt = 6v_L(0+)/L = 0$

$$\text{Therefore } dv(0+)/dt = \mathbf{0\ V/s}$$

(c) As t approaches infinity, the circuit is in steady-state.

$$i(\infty) = 6(4)/10 = \mathbf{2.4\ A}$$

$$v(\infty) = 6(4 - 2.4) = \mathbf{9.6\ V}$$

Solution 8.6

(a) Let i = the inductor current. For $t < 0$, $u(t) = 0$ so that

$$i(0) = 0 \text{ and } v(0) = 0.$$

For $t > 0$, $u(t) = 1$. Since, $v(0+) = v(0-) = 0$, and $i(0+) = i(0-) = 0$.

$$v_R(0+) = Ri(0+) = \mathbf{0\ V}$$

Also, since $v(0+) = v_R(0+) + v_L(0+) = 0 = 0 + v_L(0+)$ or $v_L(0+) = \mathbf{0\ V}$.
(1)

(b) Since $i(0+) = 0$, $i_C(0+) = V_S/R_S$

But, $i_C = Cdv/dt$ which leads to $dv(0+)/dt = V_S/(CR_S)$
(2)

From (1), $dv(0+)/dt = dv_R(0+)/dt + dv_L(0+)/dt$
(3)

$v_R = iR$ or $dv_R/dt = Rdi/dt$
(4)

But, $v_L = Ldi/dt$, $v_L(0+) = 0 = Ldi(0+)/dt$ and $di(0+)/dt = 0$ (5)

From (4) and (5), $dv_R(0+)/dt = \mathbf{0\ V/s}$

From (2) and (3), $dv_L(0+)/dt = dv(0+)/dt = \mathbf{V_s/(CR_s)}$

(c) As t approaches infinity, the capacitor acts like an open circuit, while the inductor acts like a short circuit.

$$v_R(\infty) = \mathbf{[R/(R + R_s)]V_s}$$

$$v_L(\infty) = \mathbf{0\ V}$$

Solution 8.7

$\alpha = [R/(2L)] = 20x10^3/(2x0.2x10^{-3}) = 50x10^6$

$\omega_o = [1/(LC)^{0.5}] = 1/(0.2x10^{-3}x5x10^{-6})^{0.5} = 3.162\ x10^4$

$\alpha > \omega_o \quad \longrightarrow \quad \underline{\text{overdamped}}$

overdamped

Solution 8.8

Design a problem to help other students better understand source-free *RLC* circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The branch current in an *RLC* circuit is described by the differential equation

$$\frac{d^2i}{dt^2} + 6\frac{di}{dt} + 9i = 0$$

and the initial conditions are $i(0) = 0$, $di(0)/dt = 4$. Obtain the characteristic equation and determine $i(t)$ for $t > 0$.

Solution

$$\mathbf{s^2 + 6s + 9 = 0}\text{, thus } s_{1,2} = \frac{-6 \pm \sqrt{6^2 - 36}}{2} = -3,\ \text{repeated roots.}$$

$$i(t) = [(A + Bt)e^{-3t}],\ i(0) = 0 = A$$

$$di/dt = [Be^{-3t}] + [-3(Bt)e^{-3t}]$$

$$di(0)/dt = 4 = B.$$

Therefore, $i(t) = \mathbf{[4te^{-3t}]\ A}$

Solution 8.9

$s^2 + 10s + 25 = 0$, thus $s_{1,2} = \dfrac{-10 \pm \sqrt{10-10}}{2} = -5$, repeated roots.

$$i(t) = [(A + Bt)e^{-5t}],\ \ i(0) = 10 = A$$

$$di/dt = [Be^{-5t}] + [-5(A + Bt)e^{-5t}]$$

$$di(0)/dt = 0 = B - 5A = B - 50 \text{ or } B = 50.$$

Therefore, $i(t) = \mathbf{[(10 + 50t)e^{-5t}]\ A}$

Solution 8.10

The differential equation that describes the current in an *RLC* network is

$$3\frac{di^2}{dt^2}+15\frac{di}{dt}+12i=0$$

Given that $i(0) = 0$, $di(0)/dt = 6$ mA/s, obtain $i(t)$.

Solution

$$s^2 + 5s + 4 = 0, \text{ thus } s_{1,2} = \frac{-5\pm\sqrt{25-16}}{2} = -4, -1.$$

$$i(t) = (Ae^{-4t} + Be^{-t}), \; i(0) = 0 = A + B, \text{ or } B = -A$$

$$di/dt = (-4Ae^{-4t} - Be^{-t})$$

$$di(0)/dt = 0.006 = -4A - B = -3A \text{ or } A = -0.006/3 = -0.002 \text{ and } B = 0.002.$$

Therefore, $i(t)$ = $\mathbf{(-2e^{-4t} + 2e^{-t})}$ **mA.**

Solution 8.11

$$s^2 + 2s + 1 \;=\; 0, \;\text{ thus }\; s_{1,2} \;=\; \frac{-2 \pm \sqrt{4-4}}{2} \;=\; -1, \;\text{ repeated roots.}$$

$$v(t) \;=\; [(A + Bt)e^{-t}], \;\; v(0) \;=\; 10 \;=\; A$$

$$dv/dt \;=\; [Be^{-t}] + [-(A + Bt)e^{-t}]$$

$$dv(0)/dt \;=\; 0 \;=\; B - A \;=\; B - 10 \;\text{ or }\; B \;=\; 10.$$

Therefore, $v(t) \;=\; \mathbf{[(10 + 10t)e^{-t}]\ V}$

Solution 8.12

(a) Overdamped when $C > 4L/(R^2) = 4x1.5/2500 = 2.4x10^{-3}$, or

$C >$ **2.4 mF**

(b) Critically damped when $C =$ **2.4 mF**

(c) Underdamped when $C <$ **2.4 mF**

Solution 8.13

Let $R||60 = R_o$. For a series RLC circuit,

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.01x4}} = 5$$

For critical damping, $\omega_o = \alpha = R_o/(2L) = 5$

or $R_o = 10L = 40 = 60R/(60 + R)$

which leads to $R =$ **120 ohms**

Solution 8.14

When the switch is in position A, $v(0^-)$= 0 and $i_L(0)$ = 80/40 = 2 A. When the switch is in position B, we have a source-free series RCL circuit.

$$\alpha = \frac{R}{2L} = \frac{10}{2x4} = 1.25$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x4}} = 1$$

When the switch is in position A, $v(0^-)$= 0. When the switch is in position B, we have a source-free series RCL circuit.

$$\alpha = \frac{R}{2L} = \frac{10}{2x4} = 1.25$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x4}} = 1$$

Since $\alpha > \omega_o$, we have overdamped case.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1.25 \pm \sqrt{1.56}$$ –0.5 and –2 0.9336

$v(t) = Ae^{-2t} + Be^{-0.5t}$ (1)

$v(0) = 0 = A + B$ (2)

$i_C(0) = C(dv(0)/dt) = -2$ or $dv(0)/dt = -2/C = -8$.

But $$\frac{dv(t)}{dt} = -2Ae^{-2t} - 0.5Be^{-0.5t}$$

$$\frac{dv(0)}{dt} = -2A - 0.5B = -8 \qquad (3)$$

Solving (2) and (3) gives A= 1.3333 and B = –1.3333

$v(t)$ = $\mathbf{5.333e^{-2t} - 5.333e^{-0.5t}}$ **V**.

Solution 8.15

Given that s_1 = -10 and s_2 = -20, we recall that

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -10, -20$$

Clearly, $s_1 + s_2 = -2\alpha = -30$ or $\alpha = 15 = R/(2L)$ or $R = 60L$ (1)

$$s_1 = -15 + \sqrt{15^2 - \omega_o^2} = -10 \text{ which leads to } 15^2 - \omega_o^2 = 25$$

$$\text{or } \omega_o = \sqrt{225 - 25} = \sqrt{200} = 1\big/\sqrt{LC}, \text{ thus } LC = 1/200 \qquad (2)$$

Since we have a series RLC circuit, $i_L = i_C = Cdv_C/dt$ which gives,

$$i_L/C = dv_C/dt = [200e^{-20t} - 300e^{-30t}] \text{ or } i_L = 100C[2e^{-20t} - 3e^{-30t}]$$

But, i is also $= 20\{[2e^{-20t} - 3e^{-30t}]x10^{-3}\} = 100C[2e^{-20t} - 3e^{-30t}]$

Therefore, $C = (0.02/10^2) =$ **200 μF**

$L = 1/(200C) =$ **25 H**

$R = 30L =$ **750 ohms**

Solution 8.16

At $t = 0$, $i(0) = 0$, $v_C(0) = 40x30/50 = 24V$

For $t > 0$, we have a source-free RLC circuit.

$$\alpha = R/(2L) = (40 + 60)/5 = 20 \text{ and } \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}x2.5}} = 20$$

$$\omega_o = \alpha \text{ leads to critical damping}$$

$$i(t) = [(A + Bt)e^{-20t}], \; i(0) = 0 = A$$

$$di/dt = \{[Be^{-20t}] + [-20(Bt)e^{-20t}]\},$$

$$\text{but } di(0)/dt = -(1/L)[Ri(0) + v_C(0)] = -(1/2.5)[0 + 24]$$

Hence, $B = -9.6$ or $i(t) =$ **$[-9.6te^{-20t}]$ A**

Solution 8.17

$$i(0) = I_0 = 0,\ v(0) = V_0 = 4x5 = 20$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -4(0+20) = -80$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}\frac{1}{25}}} = 10$$

$$\alpha = \frac{R}{2L} = \frac{10}{2\frac{1}{4}} = 20,\ which\, is > \omega_o.$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -20 \pm \sqrt{300} = -20 \pm 10\sqrt{3} = -2.679, -37.32$$

$$i(t) = A_1 e^{-2.679t} + A_2 e^{-37.32t}$$

$$i(0) = 0 = A_1 + A_2,\ \frac{di(0)}{dt} = -2.679A_1 - 37.32A_2 = -80$$

$$This\, leads\, to\, A_1 = -2.309 = -A_2$$

$$i(t) = 2.309\left(e^{-37.32t} - e^{-2.679t}\right)$$

$$Since, v(t) = \frac{1}{C}\int_0^t i(t)dt + 20,\ we\, get$$

v(t) = **[$21.55e^{-2.679t} - 1.55e^{-37.32t}$] V**

Solution 8.18

When the switch is off, we have a source-free parallel RLC circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2, \qquad \alpha = \frac{1}{2RC} = 0.5$$

$$\alpha < \omega_o \qquad \longrightarrow \qquad \text{underdamped case} \quad \omega_\mathrm{d} = \sqrt{\omega_o{}^2 - \alpha^2} = \sqrt{4-0.25} = 1.936$$

$I_o(0)$ = i(0) = initial inductor current = 100/5 = 20 A

$V_o(0)$ = v(0) = initial capacitor voltage = 0 V

$$v(t) = e^{-\alpha t}(A_1\cos(\omega_d t) + A_2\sin(\omega_d t)) = e^{-0.5\alpha t}(A_1\cos(1.936t) + A_2\sin(1.936t))$$

v(0) =0 = A_1

$$\frac{dv}{dt} = e^{-0.5\alpha t}(-0.5)(A_1\cos(1.936t) + A_2\sin(1.936t))$$
$$+ e^{-0.5\alpha t}(-1.936A_1\sin(1.936t) + 1.936A_2\cos(1.936t))$$

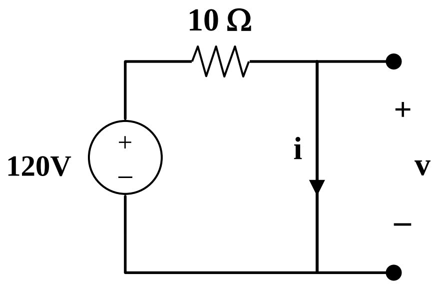
$$\frac{dv(0)}{dt} = -\frac{(V_o + RI_o)}{RC} = -\frac{(0+20)}{1} = -20 = -0.5A_1 + 1.936A_2 \qquad \longrightarrow \qquad A_2 = -10.333$$

Thus,

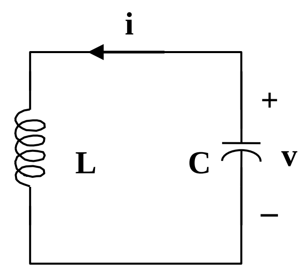
$$\underline{\boldsymbol{v(t) = [-10.333e^{-0.5t}\sin(1.936t)]volts}}$$

Solution 8.19

For $t < 0$, the equivalent circuit is shown in Figure (a).



(a)



(b)

$$i(0) = 120/10 = 12, \; v(0) = 0$$

For $t > 0$, we have a series RLC circuit as shown in Figure (b) with $R = 0 = \alpha$.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4}} = 0.5 = \omega_d$$

$$i(t) = [A\cos 0.5t + B\sin 0.5t], \; i(0) = 12 = A$$

$$v = -Ldi/dt, \text{ and } -v/L = di/dt = 0.5[-12\sin 0.5t + B\cos 0.5t],$$

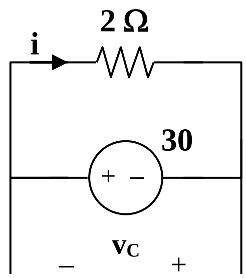
$$\text{which leads to } -v(0)/L = 0 = B$$

Hence, $i(t) = 12\cos 0.5t$ A and $v = 0.5$

However, $v = -Ldi/dt = -4(0.5)[-12\sin 0.5t] =$ **24sin(0.5t) V**

Solution 8.20

For $t < 0$, the equivalent circuit is as shown below.



$$v(0) = -30 \text{ V} \text{ and } i(0) = 30/2 = 15 \text{ A}$$

For $t > 0$, we have a series RLC circuit.

$$\alpha = R/(2L) = 2/(2x0.5) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{0.5x1/4} = 2\sqrt{2}$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{8-4} = 2$$

$$i(t) = (A\cos(2t) + B\sin(2t))e^{-2t}$$

$$i(0) = 15 = A$$

$$di/dt = -2(15\cos(2t) + B\sin(2t))e^{-2t} + (-2x15\sin(2t) + 2B\cos(2t))e^{-\alpha t}$$

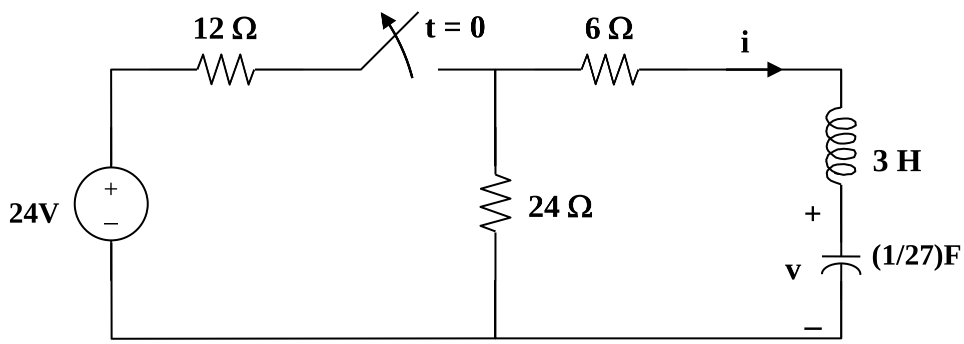
$$di(0)/dt = -30 + 2B = -(1/L)[Ri(0) + v_C(0)] = -2[30 - 30] = 0$$

Thus, $B = 15$ and $i(t) =$ $\mathbf{(15\cos(2t) + 15\sin(2t))e^{-2t}}$ **A**

Solution 8.21

By combining some resistors, the circuit is equivalent to that shown below.

$60\|(15 + 25) = 24$ ohms.



At $t = 0$-, $i(0) = 0$, $v(0) = 24\text{x}24/36 = 16\text{V}$

For $t > 0$, we have a series RLC circuit. $R = 30$ ohms, $L = 3$ H, $C = (1/27)$ F

$$\alpha = R/(2L) = 30/6 = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{3\text{x}1/27} = 3, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm \sqrt{5^2 - 3^2} = -9, -1$$

$$v(t) = [Ae^{-t} + Be^{-9t}], \quad v(0) = 16 = A + B \qquad (1)$$

$$i = Cdv/dt = C[-Ae^{-t} - 9Be^{-9t}]$$

$$i(0) = 0 = C[-A - 9B] \text{ or } A = -9B \qquad (2)$$

From (1) and (2), $B = -2$ and $A = 18$.

Hence, $v(t) = \mathbf{(18e^{-t} - 2e^{-9t})\ V}$

Solution 8.22

Compare the characteristic equation with eq. (8.8), i.e.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

we obtain

$$\frac{R}{L} = 100 \quad \longrightarrow \quad L = \frac{R}{100} = \frac{2000}{100} = \underline{20H}$$

$$\frac{1}{LC} = 10^6 \quad \rightarrow \quad C = \frac{1}{10^6 L} = \frac{10^{-6}}{20} = \underline{50\ \text{nF}}$$

Solution 8.23

Let $C_o = C + 0.01$. For a parallel RLC circuit,

$\alpha = 1/(2RC_o)$, $\omega_o = 1/\sqrt{LC_o}$

$\alpha = 1 = 1/(2RC_o)$, we then have $C_o = 1/(2R) = 1/20 = 50$ mF

$\omega_o = 1/\sqrt{0.02x0.05} = 141.42 > \alpha$ (underdamped)

$C_o = C + 10$ mF $= 50$ mF or C = **40 mF**

Solution 8.24

When the switch is in position A, the inductor acts like a short circuit so

$$i(0^-) = 4$$

When the switch is in position B, we have a source-free parallel RCL circuit

$$\alpha = \frac{1}{2RC} = \frac{1}{2x10x10x10^{-3}} = 5$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x10x10^{-3}}} = 20$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -5 + \sqrt{25-400} = -5 + j19.365$$

$$i(t) = e^{-5t}\left(A_1 \cos 19.365t + A_2 \sin 19.365t\right)$$

$$i(0) = 4 = A_1$$

$$v = L\frac{di}{dt} \longrightarrow \frac{di(0)}{dt} = \frac{v(0)}{L} = 0$$

$$\frac{di}{dt} = eA^{-5t}\left(-5A_1 \cos 19.365t - 5A_2 \sin 19.365t - 19.365A_1 \sin 19.365t + 19.365A_2 \cos 19.365t\right)$$

0 = [di(0)/dt] = –5A_1 + 19.365A_2 or A_2 = 20/19.365 = 1.0328

i(t) = **e^{-5t}[4cos(19.365t) + 1.0328sin(19.365t)] A**

Solution 8.25

Using Fig. 8.78, design a problem to help other students to better understand source-free *RLC* circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the circuit in Fig. 8.78, calculate $i_o(t)$ and $v_o(t)$ for $t > 0$.

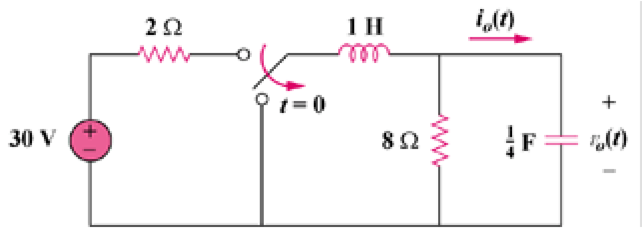


Figure 8.78

Solution

In the circuit in Fig. 8.76, calculate $i_o(t)$ and $v_o(t)$ for $t>0$.

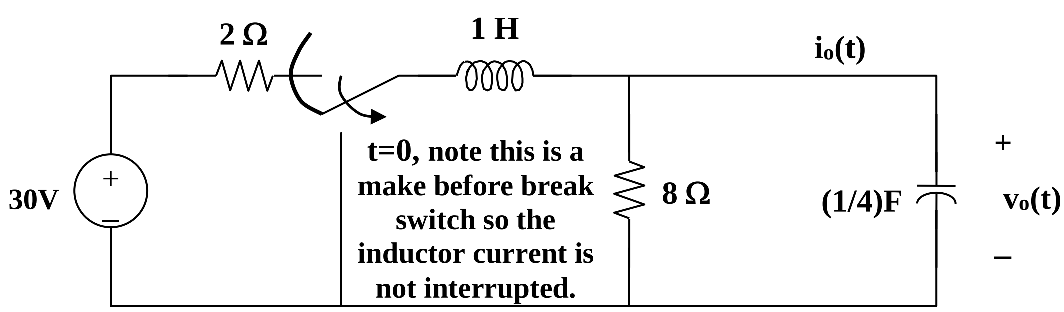


Figure 8.78 For Problem 8.25.

At $t = 0^-$, $v_o(0) = (8/(2 + 8)(30) = 24$

For $t > 0$, we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = ¼$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/4} = 2$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{4-(1/16)} = 1.9843$$

$$v_o(t) = (A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t}$$

$$v_o(0) = 30(8/(2+8)) = 24 = A_1 \text{ and } i_o(t) = C(dv_o/dt) = 0 \text{ when } t = 0.$$

$$dv_o/dt = -\alpha(A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t} + (-\omega_d A_1\sin\omega_d t + \omega_d A_2\cos\omega_d t)e^{-\alpha t}$$

$$\text{at } t = 0, \text{ we get } dv_o(0)/dt = 0 = -\alpha A_1 + \omega_d A_2$$

Thus, $A_2 = (\alpha/\omega_d)A_1 = (1/4)(24)/1.9843 = 3.024$

$v_o(t) =$ **$(24\cos 1.9843t + 3.024\sin 1.9843t)e^{-t/4}$ volts**.

$i_0(t) = Cdv/dt = 0.25[-24(1.9843)\sin 1.9843t + 3.024(1.9843)\cos 1.9843t -$
$0.25(24\cos 1.9843t) - 0.25(3.024\sin 1.9843t)]e^{-t/4}$
$=$ **$[-12.095\sin 1.9843t]e^{-t/4}$ A**.

Solution 8.26

$$s^2 + 2s + 5 \;=\; 0, \;\text{ which leads to }\; s_{1,2} \;=\; \frac{-2 \pm \sqrt{4-20}}{2} \;=\; -1 \pm j4$$

These roots indicate an underdamped circuit which has the generalized solution given as:

$$i(t) \;=\; I_s + [(A_1\cos(4t) + A_2\sin(4t))e^{-t}],$$

At $t = \infty$, $(di(t)/dt) = 0$ and $(d^2i(t)/dt^2) = 0$ so that
$I_s \;=\; 10/5 \;=\; 2$ (from $(d^2i(t)/dt^2)+2(di(t)/dt)+5 = 10$)

$$i(0) \;=\; 2 \;=\; 2 + A_1, \;\text{ or }\; A_1 \;=\; 0$$

$$di/dt \;=\; [(4A_2\cos(4t))e^{-t}] + [(-A_2\sin(4t))e^{-t}] \;=\; 4 \;=\; 4A_2, \;\text{ or }\; A_2 \;=\; 1$$

$i(t) \;=\;$ **$[2 + \sin(4te^{-t})]$ amps**

Solution 8.27

$$s^2 + 4s + 8 = 0 \text{ leads to } s = \frac{-4 \pm \sqrt{16 - 32}}{2} = -2 \pm j2$$

$$v(t) = V_s + (A_1\cos2t + A_2\sin2t)e^{-2t}$$

$$8V_s = 24 \text{ means that } V_s = 3$$

$$v(0) = 0 = 3 + A_1 \text{ leads to } A_1 = -3$$

$$dv/dt = -2(A_1\cos2t + A_2\sin2t)e^{-2t} + (-2A_1\sin2t + 2A_2\cos2t)e^{-2t}$$

$$0 = dv(0)/dt = -2A_1 + 2A_2 \text{ or } A_2 = A_1 = -3$$

v(t) = **[3 – 3(cos(2t) + sin(2t))e^{-2t}] volts**.

Solution 8.28

The characteristic equation is

$$Ls^2 + Rs + \frac{1}{C} = 0 \quad \longrightarrow \quad \frac{1}{2}s^2 + 4s + \frac{1}{0.2} = 0 \quad \longrightarrow \quad s^2 + 8s + 10 = 0$$

$$s_{1,2} = \frac{-8 \pm \sqrt{64 - 40}}{2} = \quad -6.45 \text{ and } -1.5505$$

$$\mathrm{i(t) = i_s + Ae^{-6.45t} + Be^{-1.5505t}}$$

But $[i_s/C] = 10$ or $i_s = 0.2\text{x}10 = 2$

$$i(t) = 2 + Ae^{-6.45t} + Be^{-1.5505t}$$

i(0) = 1 = 2 + A + B or A + B = –1 or A = –1–B (1)

$$\frac{di(t)}{dt} = -6.45Ae^{-6.45t} - 1.5505Be^{-1.5505t}$$

$$but\ \frac{di(0)}{dt} = 0 = -6.45A - 1.5505B \qquad (2)$$

Solving (1) and (2) gives –6.45(–1–B) – 1.5505B = 0 or (6.45–1.5505)B = –6.45
B = –6.45/(4.9) = –1.3163 and A = –1–1.3163 = –2.3163

A= –2.3163, B= –1.3163

Hence,

i(t) = **[2–2.3163e$^{-6.45t}$ –1.3163e$^{-1.5505t}$] A**.

Solution 8.29

(a) $s^2 + 4 = 0$ which leads to $s_{1,2} = \pm j2$ (an undamped circuit)

$$v(t) = V_s + A\cos 2t + B\sin 2t$$

$$4V_s = 12 \text{ or } V_s = 3$$

$$v(0) = 0 = 3 + A \text{ or } A = -3$$

$$dv/dt = -2A\sin 2t + 2B\cos 2t$$

$$dv(0)/dt = 2 = 2B \text{ or } B = 1, \text{ therefore } v(t) = \mathbf{(3 - 3\cos 2t + \sin 2t)\ V}$$

(b) $s^2 + 5s + 4 = 0$ which leads to $s_{1,2} = -1, -4$

$$i(t) = (I_s + Ae^{-t} + Be^{-4t})$$

$$4I_s = 8 \text{ or } I_s = 2$$

$$i(0) = -1 = 2 + A + B, \text{ or } A + B = -3 \qquad (1)$$

$$di/dt = -Ae^{-t} - 4Be^{-4t}$$

$$di(0)/dt = 0 = -A - 4B, \text{ or } B = -A/4 \qquad (2)$$

From (1) and (2) we get $A = -4$ and $B = 1$

$$i(t) = \mathbf{(2 - 4e^{-t} + e^{-4t})\ A}$$

(c) $s^2 + 2s + 1 = 0$, $s_{1,2} = -1, -1$

$$v(t) = [V_s + (A + Bt)e^{-t}], \ V_s = 3.$$

$$v(0) = 5 = 3 + A \text{ or } A = 2$$

$$dv/dt = [-(A + Bt)e^{-t}] + [Be^{-t}]$$

$$dv(0)/dt = -A + B = 1 \text{ or } B = 2 + 1 = 3$$

$$v(t) = \mathbf{[3 + (2 + 3t)e^{-t}]\ V}$$

(d) $s^2 + 2s + 5 = 0$, $s_{1,2} = -1 + j2, -1 - j2$

$$i(t) = [I_s + (A\cos 2t + B\sin 2t)e^{-t}], \text{ where } 5I_s = 10 \text{ or } I_s = 2$$

$$i(0) = 4 = 2 + A \text{ or } A = 2$$

$$di/dt = [-(A\cos 2t + B\sin 2t)e^{-t}] + [(-2A\sin 2t + 2B\cos 2t)e^{-t}]$$

$$di(0)/dt = -2 = -A + 2B \text{ or } B = 0$$

$$i(t) = \mathbf{[2 + (2\cos 2t)e^{-t}]\ A}$$

Solution 8.30

The step responses of a series RLC circuit are

$v_C(t) = [40–10e^{–2000t}–10e^{–4000t}]$ volts, $t > 0$ and
$i_L(t) = [3e^{–2000t}+6e^{–4000t}]$ m A, $t > 0$.

(a) Find C. (b) Determine what type of damping exhibited by the circuit.

Solution

Step 1. For a series RLC circuit, $i_R(t) = i_L(t) = i_C(t)$.

We can determine C from $i_C(t) = i_L(t) = C(dv_C/dt)$ and we can determine that the circuit is **overdamped** since the exponent value are real and negative.

Step 2. $C(dv_C/dt) = C[20,000e^{–2000t}+40,000e^{–4000t}] = 0.003e^{–2000t}+0.006e^{–4000t}$ or

$$C = 0.003/20,000 = \mathbf{150\ \eta F}.$$

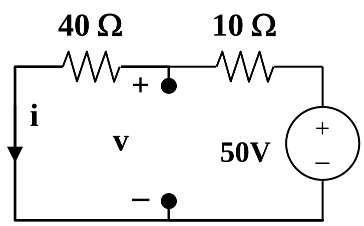
Solution 8.31

For t = 0-, we have the equivalent circuit in Figure (a). For t = 0+, the equivalent circuit is shown in Figure (b). By KVL,

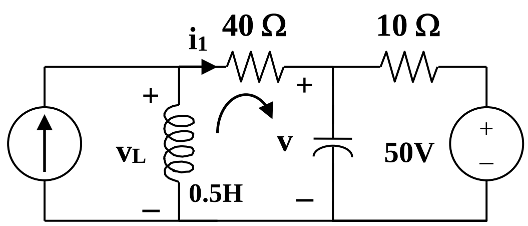
$$v(0+) = v(0-) = 40, \quad i(0+) = i(0-) = 1$$

By KCL, $2 = i(0+) + i_1 = 1 + i_1$ which leads to $i_1 = 1$.
By KVL, $-v_L + 40i_1 + v(0+) = 0$ which leads to $v_L(0+) = 40x1 + 40 = 80$

$$v_L(0+) = \mathbf{80\ V}, \quad v_C(0+) = \mathbf{40\ V}$$



(a)



(b)

Solution 8.32

For the circuit in Fig. 8.80, find $v(t)$ for $t > 0$.

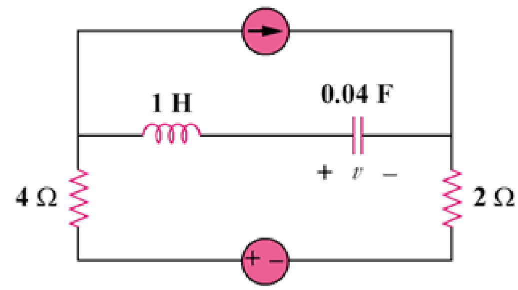
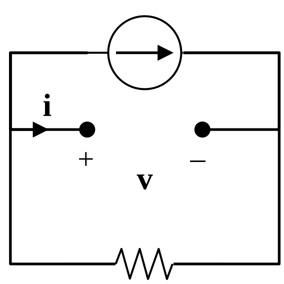


Figure 8.80
For Prob. 8.32.

Solution

For t = 0⁻, the equivalent circuit is shown below.



$$i(0^-) = 0, \ v(0^-) = -3x6 = -18 \text{ V}$$

For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 6/2 = 3, \ \omega_o = 1/\sqrt{LC} = 1/\sqrt{0.04}$$

$$s_{1,2} = -3 \pm \sqrt{9-25} = -3 \pm j4$$

Thus, $v(t) = V_s + [(A\cos 4t + B\sin 4t)e^{-3t}]$

where V_s = final capacitor voltage = 75 V

$v(t) = 75 + [(A\cos 4t + B\sin 4t)e^{-3t}]$

$v(0) = -18 = 75 + A$ which gives $A = -93$.

$i(0) = 0 = Cdv(0)/dt$

$dv/dt = [-3(A\cos 4t + B\sin 4t)e^{-3t}] + [4(-A\sin 4t + B\cos 4t)e^{-3t}]$

$0 = dv(0)/dt = -3A + 4B$ or $B = (3/4)A = -69.75$

$v(t) =$ $\mathbf{\{75 + [(-93\cos 4t - 69.75\sin 4t)e^{-3t}]\}}$ **V for all t > 0.**

Solution 8.33

Find $v(t)$ for $t > 0$ in the circuit in Fig. 8.81.

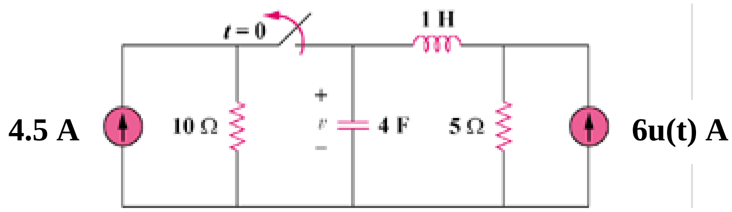
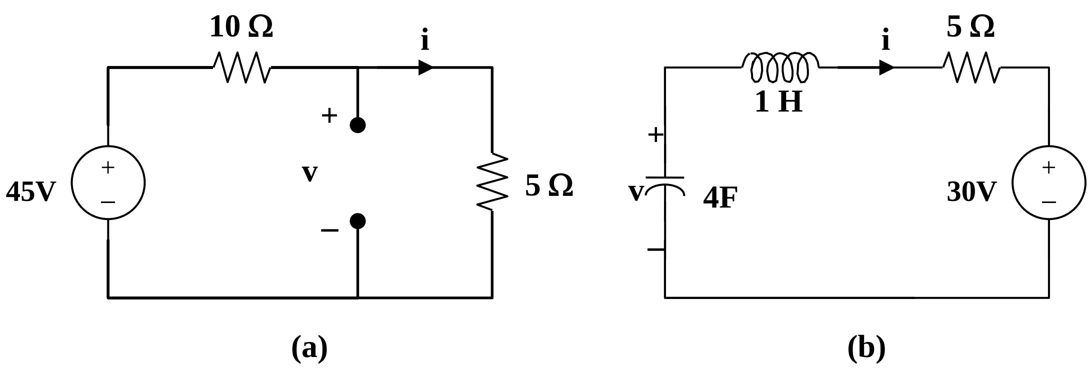


Figure 8.81
For Prob. 8.33.

Solution

We may transform the current sources to voltage sources. For $t = 0^-$, the equivalent circuit is shown in Figure (a).



$$i(0) = 45/15 = 3 \text{ A}, \quad v(0) = 5\text{x}45/15 = 15 \text{ V}$$

For t > 0, we have a series RLC circuit, shown in (b).

$$\alpha = R/(2L) = 5/2 = 2.5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4} = 0.5, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.5 \pm \sqrt{6.25 - 0.25} = -4.949, -0.0505$$

$$v(t) = V_s + [A_1e^{-4.949t} + A_2e^{-0.0505t}], \quad V_s = 30 \text{ V}.$$

$$v(0) = 15 = 30 + A_1 + A_2 \quad \text{or}$$
$$A_2 = -15 - A_1$$
(1)

$$i(0) = -Cdv(0)/dt \text{ or } dv(0)/dt = -3/4 = -0.75$$

Hence, $-0.75 = -4.949A_1 - 0.0505A_2$ (2)

From (1) and (2), $-0.75 = -4.949A_1 + 0.0505(15 + A_1)$ or
$-4.898A_1 = -0.75 - 0.7575 = -1.5075$

$$A_1 = 0.3078, \quad A_2 = -15.308$$

$v(t) = $ **$[30 + 0.3078e^{-4.949t} - 15.308e^{-0.05t}]$ V for all $t > 0$**.

Solution 8.34

Calculate $i(t)$ for $t > 0$ in the circuit in Fig. 8.82.

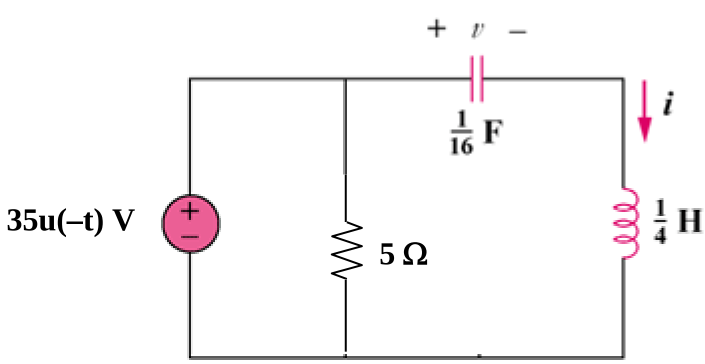


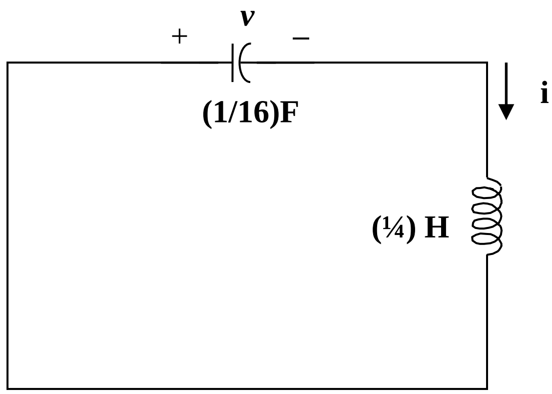
Figure 8.82
For Prob. 8.34.

Solution

Before t = 0, the capacitor acts like an open circuit while the inductor behaves like a short circuit.

$$i(0) = 0, \; v(0) = 35 \text{ V}$$

For t > 0, the LC circuit is disconnected from the voltage source as shown below.



This is a lossless, source-free, series RLC circuit.

$$\alpha = R/(2L) = 0, \; \omega_o = 1/\sqrt{LC} = 1/\sqrt{\frac{1}{16} x \frac{1}{4}} = 8, \; s = \pm j8$$

Since α is equal to zero, we have an undamped response. Therefore,

$$i(t) = A_1\cos(8t) + A_2\sin(8t) \quad \text{where } i(0) = 0 = A_1$$

$$di(0)/dt = (1/L)v_L(0) = -(1/L)v(0) = -4x35 = -140$$

However, $di/dt = 8A_2\cos(8t)$, thus, $di(0)/dt = -140 = 8A_2$ which leads to
$A_2 = -17.5$

Now we have $i(t) = \mathbf{-17.5sin(8t)\ A}$.

Solution 8.35

Using Fig. 8.83, design a problem to help other students to better understand the step response of series *RLC* circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine $v(t)$ for $t > 0$ in the circuit in Fig. 8.83.

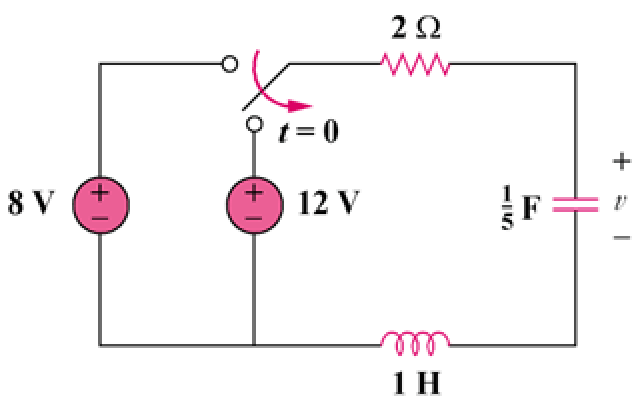


Figure 8.83

Solution

At $t = 0^-$, $i_L(0) = 0$, $v(0) = v_C(0) = 8$ V

For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 2/2 = 1, \quad \omega_o = 1/\sqrt{LC} = 1/\sqrt{1/5} = \sqrt{5}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1 \pm j2$$

$$v(t) = V_s + [(A\cos 2t + B\sin 2t)e^{-t}], \quad V_s = 12.$$

$$v(0) = 8 = 12 + A \text{ or } A = -4, \quad i(0) = C dv(0)/dt = 0.$$

$$\text{But } dv/dt = [-(A\cos 2t + B\sin 2t)e^{-t}] + [2(-A\sin 2t + B\cos 2t)e^{-t}]$$

$$0 = dv(0)/dt = -A + 2B \text{ or } 2B = A = -4 \text{ and } B = -2$$

$$v(t) = \mathbf{\{12 - (4\cos 2t + 2\sin 2t)e^{-t}\ V}.$$

Solution 8.36

Obtain $v(t)$ and $i(t)$ for $t > 0$ in the circuit in Fig. 8.84.

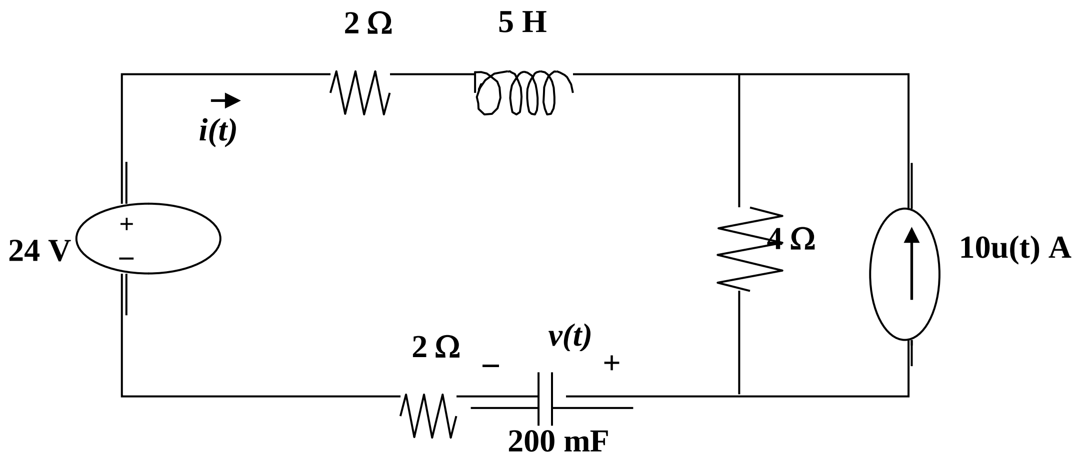
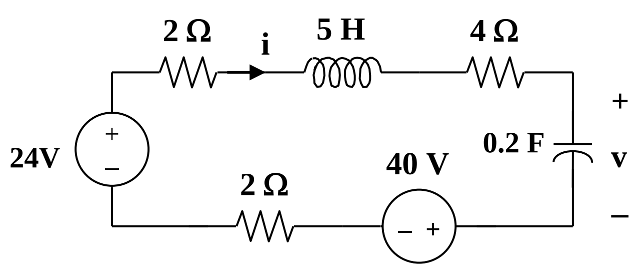


Figure 8.84
For Prob. 8.36.

Solution

For t = 0^-, 10u(t) A = 0. Thus, i(0) = 0, and v(0) = 24 V.

For t > 0, we have the series RLC circuit shown below.



$$\alpha = R/(2L) = (2 + 2 + 4)/(2x5) = 0.8$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.2} = 1$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -0.8 \pm j0.6$$

$$v(t) = V_s + [(A\cos(0.6t) + B\sin(0.6t))e^{-0.8t}]$$

$$V_s = v_{ss} = 24 - 40 = -16 \text{ V} \text{ and } v(0) = 24 = -16 + A \text{ or } A = +40$$

$$i(0) = Cdv(0)/dt = 0$$

But $dv/dt = [-0.8(A\cos(0.6t) + B\sin(0.6t))e^{-0.8t}] + [0.6(-A\sin(0.6t) + B\cos(0.6t))e^{-0.8t}]$

$$0 = dv(0)/dt = -0.8A + 0.6B \text{ which leads to } B = 0.8x(40)/0.6 = 53.33$$

$$v(t) = \mathbf{\{-16 + [(40\cos(0.6t) + 53.33\sin(0.6t))e^{-0.8t}]\}u(t)\ V}$$

$$i = Cdv/dt$$

$$= 0.2\{[-0.8(40\cos(0.6t) + 53.33\sin(0.6)t)e^{-0.8t}] + [0.6(-40\sin(0.6t) + 53.33\cos(0.6t))e^{-0.8t}]\}$$

$$i(t) = \mathbf{[-13.333\sin(0.6t)e^{-0.8t}]u(t)\ A}.$$

Solution 8.37

For the network in Fig. 8.85, solve for $i(t)$ for $t > 0$.

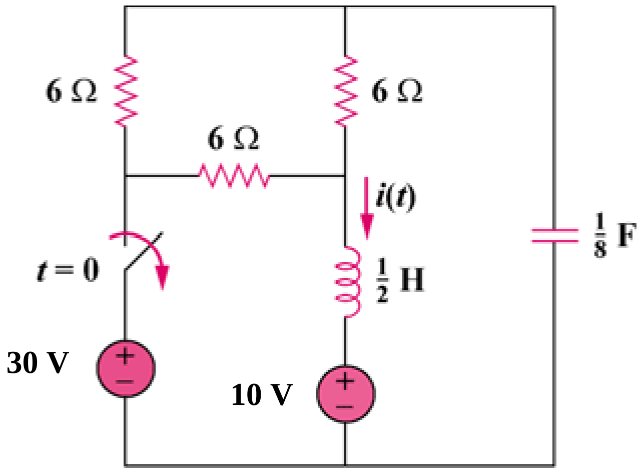
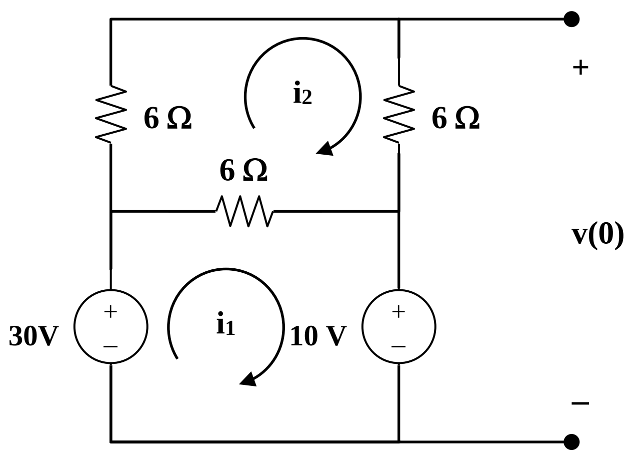


Figure 8.85
For Prob. 8.37.

Solution

For $t = 0^-$, the equivalent circuit is shown below.



$18i_2 - 6i_1 = 0$ or $i_1 = 3i_2$ (1)

$-30 + 6(i_1 - i_2) + 10 = 0$ or $i_1 - i_2 = 20/6 = 10/3$
(2)

From (1) and (2), $(2/3)i_1 = 10/3$ or $i_1 = 5$ and $i_2 = i_1 - 10/3 = 5/3$

$$i(0) = i_1 = 5A$$

$$-10 - 6i_2 + v(0) = 0$$

$$v(0) = 10 + 6x5/3 = 20$$

For $t > 0$, we have a series RLC circuit.

$$R = 6||12 = 4$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/8)} = 4$$

$$\alpha = R/(2L) = (4)/(2x(1/2)) = 4$$

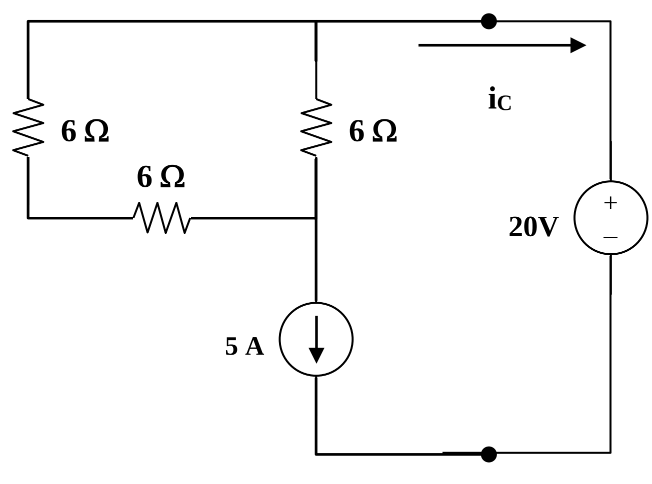
$\alpha = \omega_o$, therefore the circuit is critically damped

$$v(t) = V_s + [(A + Bt)e^{-4t}], \text{ and } V_s = v_{ss} = 10$$

$$v(0) = 20 = 10 + A, \text{ or } A = 10$$

$$i_C = Cdv/dt = C[-4(10 + Bt)e^{-4t}] + C[(B)e^{-4t}]$$

To find $i_C(0)$ we need to look at the circuit right after the switch is opened. At this time, the current through the inductor forces that part of the circuit to act like a current source and the capacitor acts like a voltage source. This produces the circuit shown below. Clearly, $i_C(0+)$ must equal $-i_L(0) = -5A$.



$$i_C(0) = -5 = C(-40 + B) \text{ which leads to } -40 = -40 + B \text{ or } B = 0$$

$$i_C = Cdv/dt = (1/8)[-4(10 + 0t)e^{-4t}] + (1/8)[(0)e^{-4t}]$$

$$i_C(t) = [-(1/2)(10)e^{-4t}]$$

$$i(t) = -i_C(t) = \mathbf{5e^{-4t}\ A\ for\ all\ t > 0}.$$

Solution 8.38

Refer to the circuit in Fig. 8.86. Calculate $i(t)$ for $t > 0$.

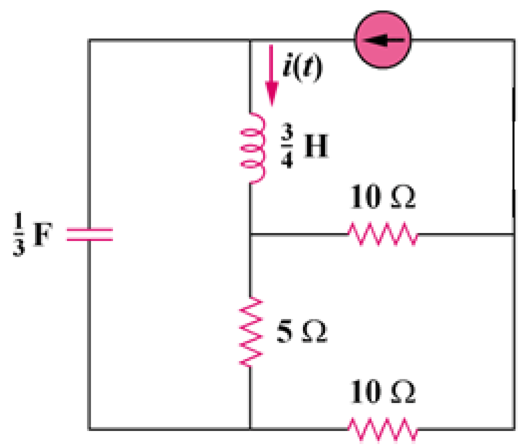
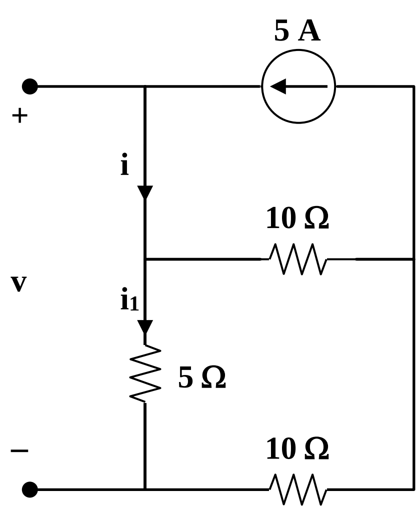


Figure 8.86
For Prob. 8.38.

Solution

At $t = 0^-$, the equivalent circuit is as shown.



$i(0) = 5 \text{ A}, \quad i_1(0) = 10(5)/(10 + 15) = 2 \text{ A}$

$v(0) = 5i_1(0) = 10 \text{ V}$

For $t > 0$, we have a source-free series RLC circuit.

$R = 5||(10 + 10) = 4$ ohms

$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/3)(3/4)} = 2$

$\alpha = R/(2L) = (4)/(2x(3/4)) = 8/3$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.66667 \pm 1.763834 = -4.43\ 05,\ -0.90283$

$i(t) = [Ae^{-4.431t} + Be^{-0.9028t}]$ and $i(0) = A + B = 5$ or $A = 5 - B$

$di(0)/dt = (1/L)[-Ri(0) + v(0)] = (4/3)(-4x5 + 10) = -40/3 = -13.33333$

Hence, $-13.3333 = -4.4305A - 0.90283B = -22.1525 + 4.4305B - 0.90283B$

$3.52767B = 8.8192$ or $B = 2.5$ and $A = 5 - 2.5 = 2.5$. Thus,

$$i(t) = \mathbf{[2.5e^{-4.431t} + 2.5e^{-0.9028t}]\ A}.$$

Solution 8.39

Determine $v(t)$ for $t > 0$ in the circuit in Fig. 8.87.

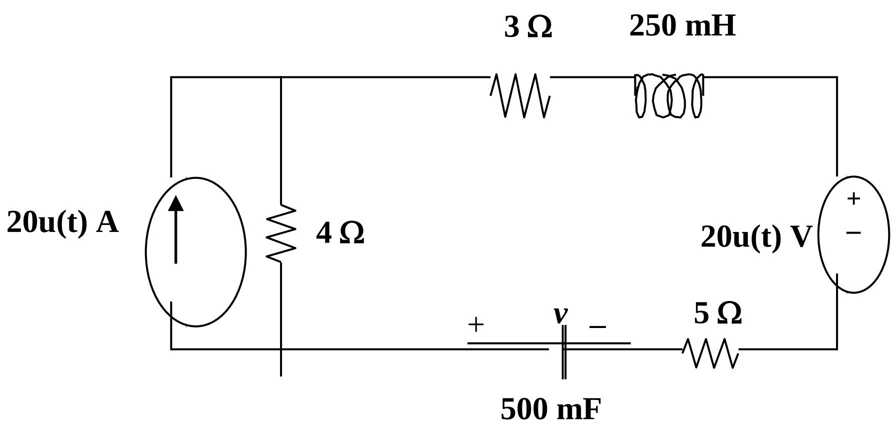


Figure 8.87
For Prob. 8.39.

Solution

For $t = 0^-$, the source voltages are equal to zero thus, the initial conditions are $v(0) = 0$ and $i_L(0) = 0$.

For $t > 0$,

$$R = 3 + 5 + 4 = 12 \text{ ohms}$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/4)} = \sqrt{8}$$

$$\alpha = R/(2L) = (12)/(0.5) = 24$$

Since $\alpha > \omega_o$, we have an overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -24 \pm 23.833 = -47.83, \ -0.167$$

Thus, $v(t) = V_s + [Ae^{-47.83t} + Be^{-0.167t}]$, where

$V_s = v_{ss} = -80 + 20 = -60$ volts.

$$v(0) = 0 = -60 + A + B \text{ or } 60 = A + B \qquad (1)$$

$$i(0) = Cdv(0)/dt = 0$$

But, $dv(0)/dt = -47.83A - 0.167B = 0$ or

$$B = -286.4A \qquad (2)$$

From (1) and (2), A + (–286.4)A = 60 or A = 60/(–285.4) = –0.21023 and
B = –286.4x(–0.21023) = 60.21

$$v(t) = \mathbf{[-60 + (-0.2102e^{-47.83t}+60.21e^{-0.167t})]u(t)\ volts}.$$

Solution 8.40

The switch in the circuit of Fig. 8.88 is moved from position *a* to *b* at $t = 0$. Assume that the voltage across the capacitor is equal to zero at t = 0 and that the switch is a make before break switch. Determine $i(t)$ for $t > 0$.

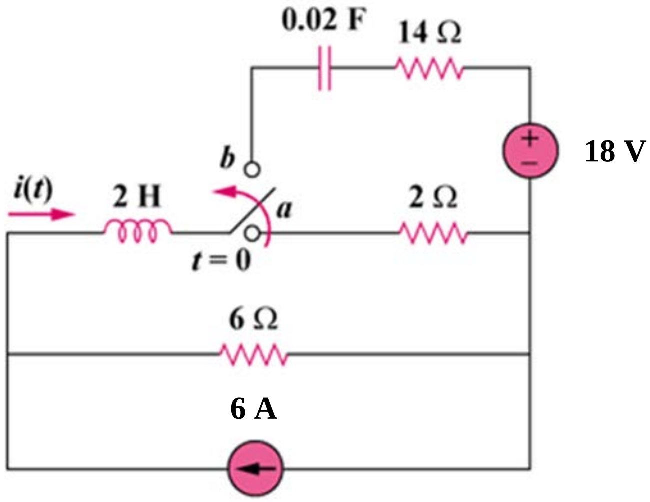
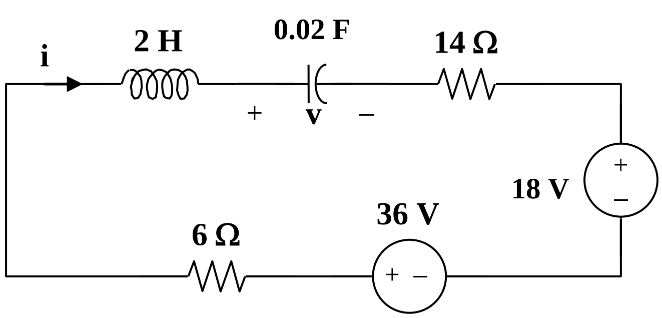


Figure 8.88
For Prob. 8.40.

Solution

At $t = 0^-$, $v_C(0) = 0$ and $i_L(0) = i(0) = (6/(6+2))6 = 4.5$ A.

For $t > 0$, we have a series RLC circuit with a step input as shown below.



$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{2x0.02} = 5$$

$$\alpha = R/(2L) = (6 + 14)/(2x2) = 5$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$v(t) = V_s + [(A + Bt)e^{-5t}], \quad V_s = v_{ss} = 36 - 18 = 18 \text{ V}.$$

$v(0) = 0 = 18 + A$ or $A = -18.$

$i = Cdv/dt = C\{[Be^{-5t}] + [-5(A + Bt)e^{-5t}]\}$

$i(0) = 4.5 = C[-5A + B] = 0.02[90 + B]$ or $B = 135.$

Thus, $i(t) = 0.02\{[135e^{-5t}] + [-5(-18 + 135t)e^{-5t}]\}$

$$i(t) = \mathbf{[(4.5 - 13.5t)e^{-5t}]\ A}.$$

Solution 8.41

For the network in Fig. 8.89, find $i(t)$ for $t > 0$.

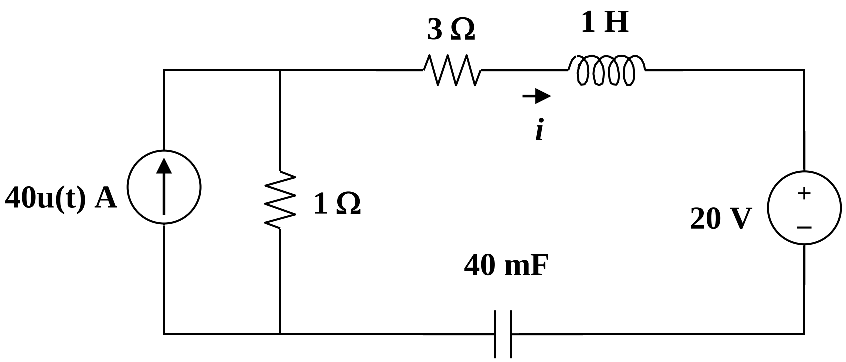


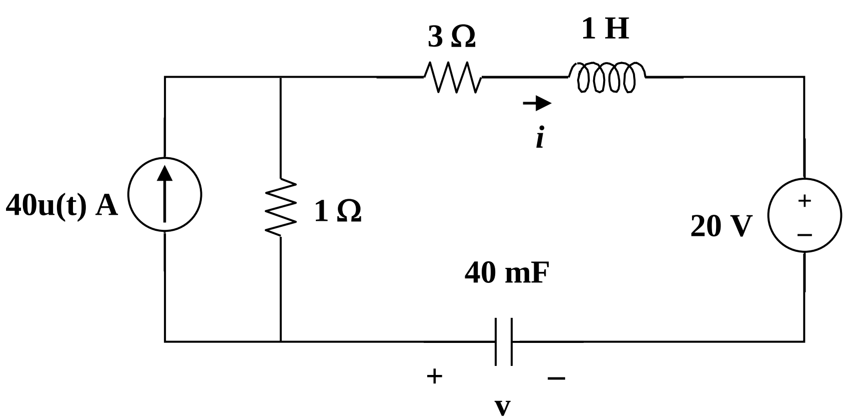
Figure 8.89
For Prob. 8.41.

Solution

At $t = 0^-$, $i(0) = 0$, and

$$v(0) = 20 \text{ V}$$

For $t > 0$, we have a series RLC circuit shown below.



$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5 \text{ rad/sec}$$

$$\alpha = R/(2L) = (4)/(2x1) = 2$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2 \pm j4.583$$

Thus, $v(t) = V_{ss} + [(A\cos(\omega_d t) + B\sin(\omega_d t))e^{-2t}]$,

where $\omega_d = 4.583$ and $V_{ss} = -20$ V

$v(0) = 20 = -20 + A$ or $A = 40$

$i(t) = -Cdv/dt$
$= C(2)\ [(A\cos(\omega_d t) + B\sin(\omega_d t))e^{-2t}] - C\omega_d[(-A\sin(\omega_d t) + B\cos(\omega_d t))e^{-2t}]$

$i(0) = 0 = 2A - \omega_d B$

$B = 2A/\omega_d = 80/(4.583) = 17.456$

$i(t) = C\{[(0\cos(\omega_d t) + (2B + \omega_d A)\sin(\omega_d t))]e^{-2t}\}$

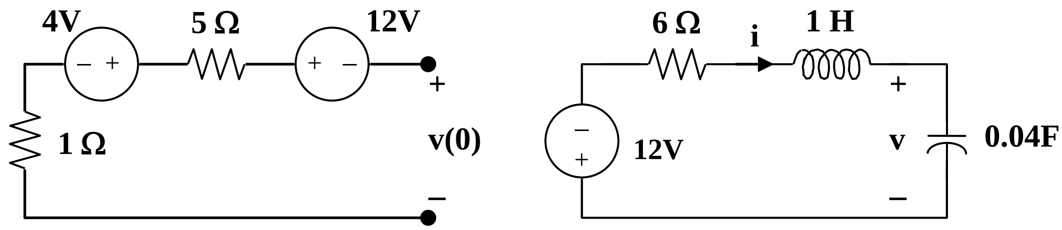
$= (0.04)\{[(34.192 + 183.32)\ \sin(\omega_d t))]e^{-2t}\}$

$i(t) =$ $\mathbf{[8.7\sin(4.583t)e^{-2t}]u(t)\ A}$.

Solution 8.42

For t = 0-, we have the equivalent circuit as shown in Figure (a).

$$i(0) = i(0) = 0, \text{ and } v(0) = 4 - 12 = -8V$$



(a) (b)

For t > 0, the circuit becomes that shown in Figure (b) after source transformation.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$$

$$\alpha = R/(2L) = (6)/(2) = 3$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -3 \pm j4$$

Thus, $v(t) = V_s + [(A\cos 4t + B\sin 4t)e^{-3t}], \quad V_s = -12$

$$v(0) = -8 = -12 + A \text{ or } A = 4$$

$$i = Cdv/dt, \text{ or } i/C = dv/dt = [-3(A\cos 4t + B\sin 4t)e^{-3t}] + [4(-A\sin 4t + B\cos 4t)e^{-3t}]$$

$$i(0) = -3A + 4B \text{ or } B = 3$$

$v(t) =$ **$\{-12 + [(4\cos 4t + 3\sin 4t)e^{-3t}]\}$ A**

Solution 8.43

The switch in Fig. 8.91 is opened at $t = 0$ after the circuit has reached steady state. Choose R and C such that $\alpha = 8$ Np/s and $\omega_d = 30$ rad/s.

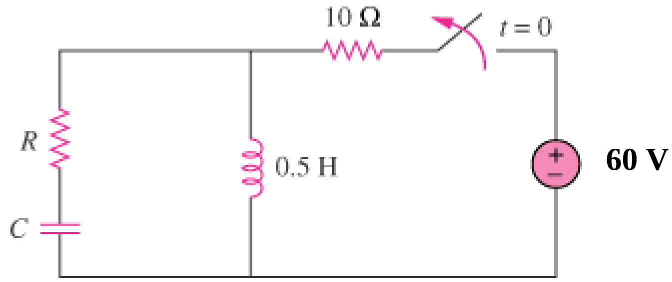


Figure 8.91
For Prob. 8.43.

Solution

For t>0, we have a source-free series RLC circuit.

$$\alpha = \frac{R}{2L} \longrightarrow R = 2\alpha L = 2x8x0.5 = \mathbf{8\Omega}$$

$$\omega_d = \sqrt{\omega_o{}^2 - \alpha^2} = 30 \longrightarrow \omega_o = \sqrt{900 + 64} = \sqrt{964}$$

$$\boldsymbol{\omega_o = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega_o^2 L} = \frac{1}{964x0.5} = 2.075\ \text{mF}}$$

Solution 8.44

$$\alpha = \frac{R}{2L} = \frac{1000}{2x1} = 500, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-9}}} = 10^4$$

$\omega_o > \alpha \quad \longrightarrow \quad$ **underdamped.**

Solution 8.45

In the circuit of Fig. 8.92, find $v(t)$ and $i(t)$ for $t > 0$.

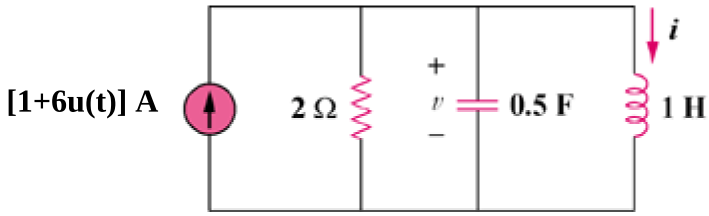


Figure 8.92
For Prob. 8.45.

Solution

$$\omega_o = 1/\sqrt{LC} \;=\; 1/\sqrt{1x0.5} \;=\; \sqrt{2}$$

$$\alpha \;=\; 1/(2RC) \;=\; (1)/(2x2x0.5) \;=\; 0.5$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} \;=\; -\alpha \pm \sqrt{\omega_o^2 - \alpha^2} \;=\; -0.5 \pm j1.3229$$

Thus, $$i(t) \;=\; I_s + [(A\cos 1.3229t + B\sin 1.3229t)e^{-0.5t}], \quad I_s \;=\; 6$$

$$i(0) \;=\; 1 \;=\; 6 + A \;\text{ or }\; A \;=\; -5$$

$$v \;=\; v_C(0) \;=\; v_L(0) \;=\; Ldi(0)/dt \;=\; 0$$

$$di/dt \;=\; [1.3229(-A\sin 1.3229t + B\cos 1.3229t)e^{-0.5t}] + [-0.5(A\cos 1.3229t + B\sin 1.3229t)e^{-0.5t}]$$

$$di(0)/dt \;=\; 0 \;=\; 1.3229B - 0.5A \;\text{ or }\; B \;=\; 0.5(-5)/1.3229 \;=\; -1.8898$$

Thus, $$i(t) \;=\; \mathbf{\{6 - [(5\cos 1.3229t + 1.8898\sin 1.3229t)e^{-t/2}]\}\ A}$$

To find v(t) we use $v(t) = v_L(t) = Ldi(t)/dt$.

From above,

$$di/dt \;=\; [1.3229(-A\sin 1.3229t + B\cos 1.3229t)e^{-0.5t}] + [-0.5(A\cos 1.323t + B\sin 1.323t)e^{-0.5t}]$$

Thus,

$$v(t) = Ldi/dt = [1.323(-A\sin 1.323t + B\cos 1.323t)e^{-0.5t}] + [-0.5(A\cos 1.323t + B\sin 1.323t)e^{-0.5t}]$$

$$= [1.3229(5\sin 1.3229t - 1.8898\cos 1.3229t)e^{-0.5t}] + [(2.5\cos 1.3229t + 0.9449\sin 1.3229t)e^{-0.5t}]$$

$$v(t) = [(-0\cos 1.323t + 4.536\sin 1.323t)e^{-0.5t}] \text{ V}$$

$$= \mathbf{[(7.559\sin 1.3229t)e^{-t/2}] \text{ V}}.$$

Solution 8.46

Using Fig. 8.93, design a problem to help other students to better understand the step response of a parallel *RLC* circuit.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find $i(t)$ for $t > 0$ in the circuit in Fig. 8.93.

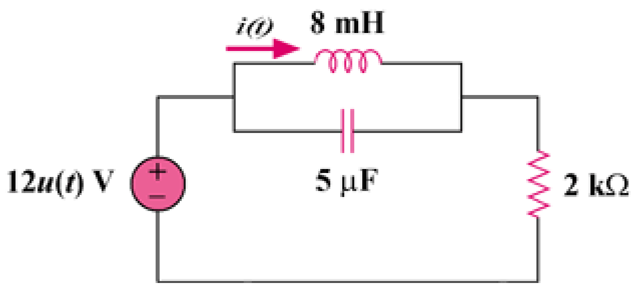
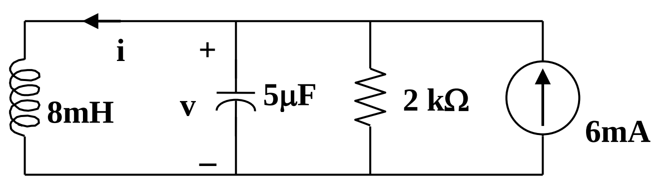


Figure 8.93

Solution

For $t = 0-$, $u(t) = 0$, so that $v(0) = 0$ and $i(0) = 0$.

For $t > 0$, we have a parallel RLC circuit with a step input, as shown below.



$$\alpha = 1/(2RC) = (1)/(2x2x10^3\ x5x10^{-6}) = 50$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{8x10^{-3}x5x10^{-6}} = 5{,}000$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \cong -50 \pm j5{,}000$$

Thus, $\quad i(t) = I_s + [(A\cos5{,}000t + B\sin5{,}000t)e^{-50t}], \quad I_s = 6mA$

$$i(0) = 0 = 6 + A \text{ or } A = -6mA$$

$$v(0) = 0 = Ldi(0)/dt$$

$$di/dt = [5{,}000(-A\sin 5{,}000t + B\cos 5{,}000t)e^{-50t}] + [-50(A\cos 5{,}000t + B\sin 5{,}000t)e^{-50t}]$$

$$di(0)/dt = 0 = 5{,}000B - 50A \text{ or } B = 0.01(-6) = -0.06\text{mA}$$

Thus, $$i(t) = \mathbf{\{6 - [(6\cos 5{,}000t + 0.06\sin 5{,}000t)e^{-50t}]\}\ mA}$$

Solution 8.47

Find the output voltage $v_o(t)$ in the circuit of Fig. 8.94.

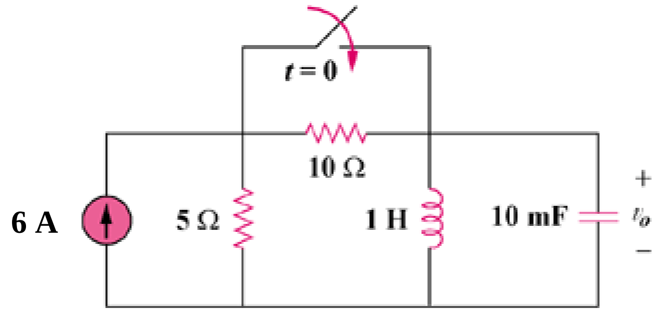


Figure 8.94
For Prob. 8.47.

Solution

At $t = 0^-$, we obtain, $i_L(0) = 6x5/(10 + 5) = 2$ A, and $v_o(0) = 0$.

For $t > 0$, the 10-ohm resistor is short-circuited and we have a parallel RLC circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.01) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.01} = 10$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -10$$

Thus, $i(t) = I_{ss} + [(A + Bt)e^{-10t}]$, $I_{ss} = 6$, $i(0) = 2 = 6 + A$ or $A = -4$

$$v_o = Ldi/dt = [Be^{-10t}] + [-10(A + Bt)e^{-10t}]$$

$$v_o(0) = 0 = B - 10A \text{ or } B = -40$$

Thus, $v_o(t) = \mathbf{(400te^{-10t})\ V}$.

Solution 8.48

For $t = 0^-$, we obtain $i(0) = -6/(1 + 2) = -2$ and $v(0) = 2x1 = 2$.

For $t > 0$, the voltage is short-circuited and we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = (1)/(2x1x0.25) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.25} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $i(t) = [(A + Bt)e^{-2t}]$, $i(0) = -2 = A$

$$v = Ldi/dt = [Be^{-2t}] + [-2(-2 + Bt)e^{-2t}]$$

$$v_o(0) = 2 = B + 4 \text{ or } B = -2$$

Thus,

$i(t) =$ $\mathbf{[(-2 - 2t)e^{-2t}]\ A}$

and $v(t) =$ $\mathbf{[(2 + 4t)e^{-2t}]\ V}$

Solution 8.49

Determine $i(t)$ for $t > 0$ in the circuit of Fig. 8.96.

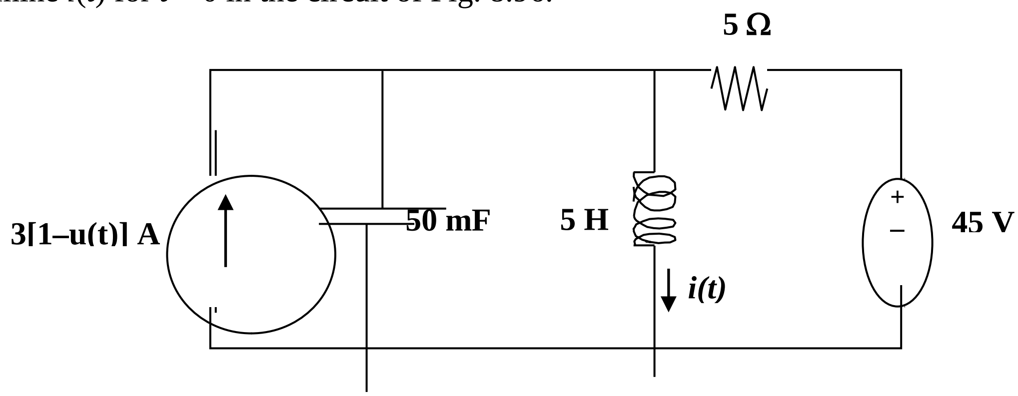


Figure 8.96
For Prob. 8.49.

Solution

For $t = 0^-$, $i(0) = 3 + 45/5 = 12$ A and $v(0) = 0$.

For $t > 0$, we have a parallel *RLC* circuit with a step change in the input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.05) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.05} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $$i(t) = I_{ss} + [(A + Bt)e^{-2t}], \quad I_{ss} = 9$$

$$i(0) = 12 = 9 + A \text{ or } A = 3$$

$$v = Ldi/dt \text{ or } v/L = di/dt = [Be^{-2t}] + [-2(A + Bt)e^{-2t}]$$

$$v(0)/L = 0 = di(0)/dt = B - 2x3 \text{ or } B = 6$$

Thus, $i(t)$ = $\mathbf{\{9 + [(3 + 6t)e^{-2t}]\}u(t)\ A}$

Solution 8.50

For the circuit in Fig. 8.97, find $i(t)$ for $t > 0$.

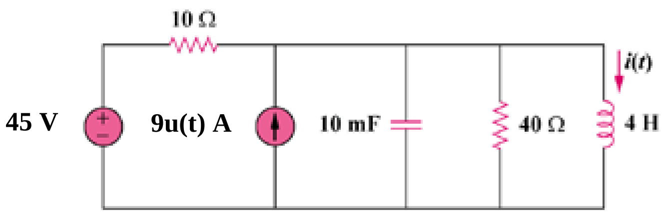
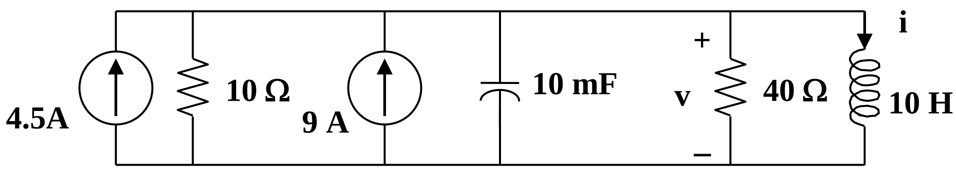


Figure 8.97
For Prob. 8.50.

Solution

For $t = 0^-$, $9u(t) = 0$, $v(0) = 0$, and $i(0) = 45/10 = 4.5$ A.

For $t > 0$, we have a parallel RLC circuit.



$$I_{ss} = 4.5 + 9 = 13.5 \text{ A} \quad \text{and} \quad R = 10||40 = 8 \text{ ohms}$$

$$\alpha = 1/(2RC) = (1)/(2x8x0.01) = 25/4 = 6.25$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4x0.01} = 5$$

Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -10, -2.5$$

Thus, $$i(t) = I_{ss} + [Ae^{-10t}] + [Be^{-2.5t}], \quad I_{ss} = 13.5$$

$$i(0) = 4.5 = 13.5 + A + B \quad \text{or} \quad A + B = -9$$

$$di/dt = [-10Ae^{-10t}] + [-2.5Be^{-2.5t}],$$

$$v(0) = 0 = Ldi(0)/dt \quad \text{or} \quad di(0)/dt = 0 = -10A - 2.5B \quad \text{or} \quad B = -4A$$

Thus, $A - 4A = -9$ or $A = 3$ and $B = -12$.

$$\text{Clearly, } i(t) = \mathbf{\{ 13.5 + [3e^{-10t}] + [-12e^{-2.5t}]\}\ A}.$$

Solution 8.51

Let i = inductor current and v = capacitor voltage.

$$\text{At } t = 0, \; v(0) = 0 \text{ and } i(0) = i_o.$$

For $t > 0$, we have a parallel, source-free LC circuit ($R = \infty$).

$$\alpha = 1/(2RC) = 0 \text{ and } \omega_o = 1/\sqrt{LC} \text{ which leads to } s_{1,2} = \pm j\omega_o$$

$$v = A\cos\omega_o t + B\sin\omega_o t, \; v(0) = 0 \; A$$

$$i_C = Cdv/dt = -i$$

$$dv/dt = \omega_o B\sin\omega_o t = -i/C$$

$$dv(0)/dt = \omega_o B = -i_o/C \text{ therefore } B = i_o/(\omega_o C)$$

$$v(t) = \mathbf{-(i_o/(\omega_o C))\sin\omega_o t \; V \text{ where } \omega_o = 1/\sqrt{\mathit{LC}}}$$

Solution 8.52

The step response of a parallel *RLC* circuit is

$$v = 10 + 20e^{-300t}(\cos 400t - 2\sin 400t)\text{ V},\ t \geq 0$$

when the inductor is 25 mH. Find *R* and *C*.

Solution

$$\alpha = 300 = \frac{1}{2RC} \qquad (1)$$

$$\omega_d = \sqrt{\omega_o{}^2 - \alpha^2} = 400 \quad \longrightarrow \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 160{,}000 + 90{,}000 = \frac{1}{\text{LC}} \quad (2)$$

From (2),

$$C = \frac{1}{250{,}000x25x10^{-3}} = \mathbf{160\ \mu F}$$

From (1),

$$R = \frac{1}{2\alpha C} = \frac{1}{2x300x160x10^{-6}} = \mathbf{10.417\ \Omega}.$$

Solution 8.53

After being open for a day, the switch in the circuit of Fig. 8.99 is closed at t=0. Find the differential equation describing i(t), t >0.

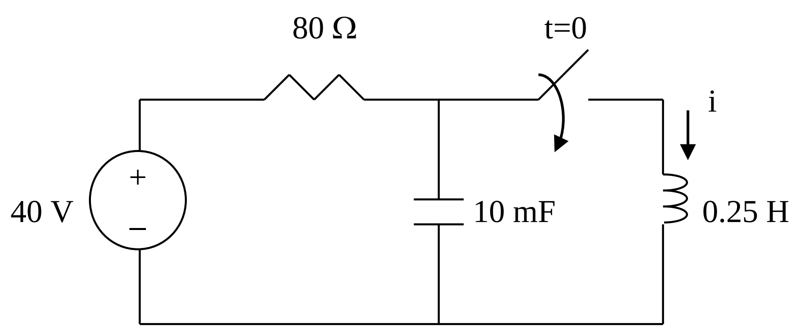
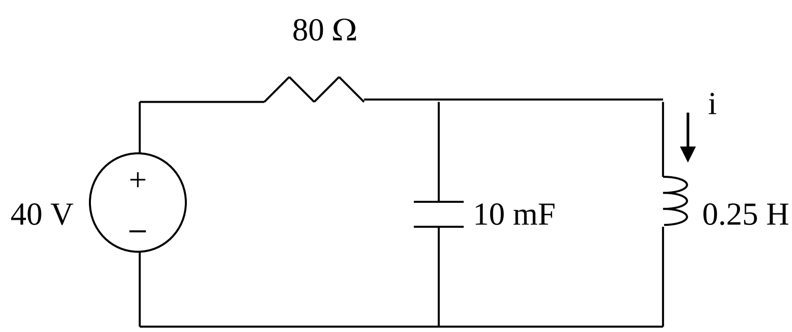


Figure 8.99
For Prob. 8.53.

Solution

For $t < 0$, $i(0) = 0$ and $v_C(0) = 40$.

For $t > 0$, we have the circuit as shown below.



$[(40–v_C)/80] = 0.01[dv_C/dt] + i$ or $40 = v_C + 0.8[dv_C/dt] + 80i$. But v_C is also $= 0.25di/dt$ which leads to $40 = 0.25[di/dt] + (0.8)(0.25)[d^2i/dt^2] + 80i$. Simplifying we get,

$$\mathbf{(d^2i/dt^2) + 1.25(di/dt) + 400i = 200}.$$

Solution 8.54

Using Fig. 8.100, design a problem to help other students better understand general second-order circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 8.100, let I = 9A, R_1 = 40 Ω, R_2 = 20 Ω, C = 10 mF, R_3 = 50 Ω, and L = 20 mH. Determine: (a) $i(0^+)$ and $v(0^+)$, (b) $di(0^+)/dt$ and $dv(0^+)/dt$, (c) $i(\infty)$ and $v(\infty)$.

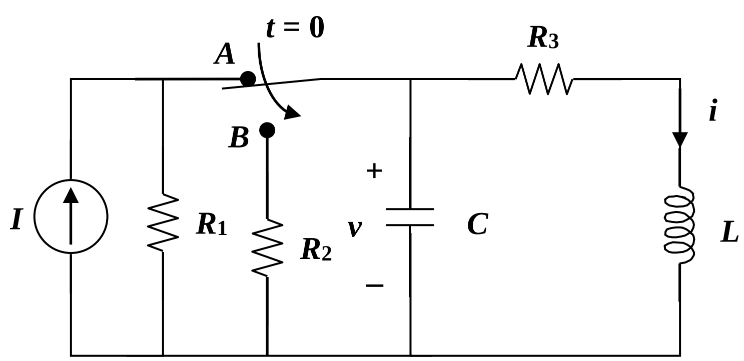
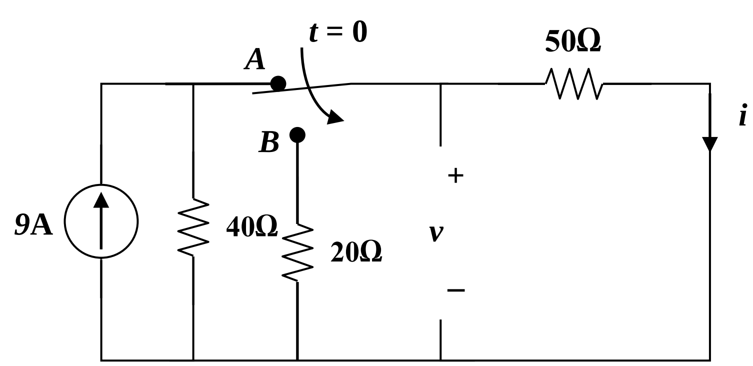


Figure 8.100
For Prob. 8.54.

Solution

(a) When the switch is at A, the circuit has reached steady state. Under this condition, the circuit is as shown below.



(a) When the switch is at A, $i(0^-) = 9[(40x50)/(40+50)]/50 = 4$ A and $v(0^-) = 50i(0^-) = 200$ V. Since the current flowing through the inductor cannot change in zero time, $i(0^+) = i(0^-) =$ **4 A**. Since the voltage across the capacitor cannot change in zero time, $v(0^-) = v(0^-) =$ **200 V**.

(b) For the inductor, $v_L = L(di/dt)$ or $di(0^+)/dt = v_L(0^+)/0.02$.

At $t = 0^+$, the right hand loop becomes,

$$-200 + 50x4 + v_L(0^+) = 0 \text{ or } v_L(0^+) = 0 \text{ and } (di(0^+)/dt) = \underline{\mathbf{0}}.$$

For the capacitor, $i_C = C(dv/dt)$ or $dv(0^+)/dt = i_C(0^+)/0.01$.

At $t = 0^+$, and looking at the current flowing out of the node at the top of the circuit,

$$((200–0)/20) + i_C + 4 = 0 \text{ or } i_C = –14 \text{ A}.$$

Therefore,

$$dv(0^+)/dt = –14/0.01 = \mathbf{–1.4\ kV/s.}$$

(c) When the switch is in position B, the circuit reaches steady state. Since it is source-free, i and v decay to zero with time.

Thus,

$$i(\infty) = \mathbf{0\ A} \text{ and } v(\infty) = \mathbf{0\ V}.$$

Solution 8.55

For the circuit in Fig. 8.101, find $v(t)$ for $t > 0$. Assume that $i(0^+) = 2$ A.

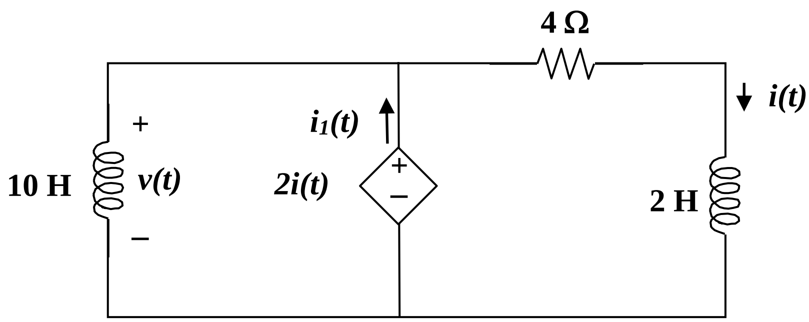
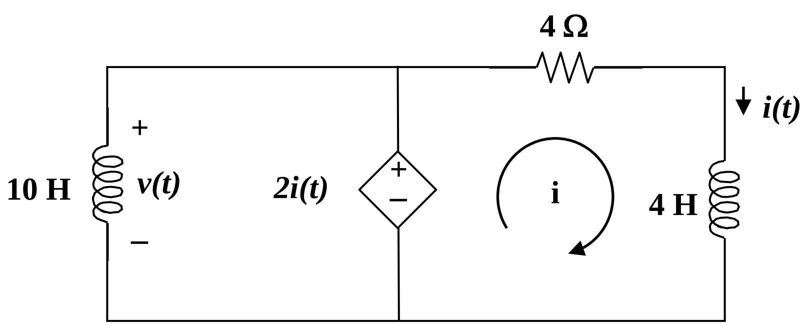


Figure 8.101
For Prob. 8.55.

Solution



We find that $i_1 = v(t) = 2i(t)$.

The inductor on the left does not affect the voltage so it can be neglected. Writing a mesh equation we get $-2i + 4i + 4di/dt = 0$ or $(di/dt) + 0.5i = 0$. This is a first order linear differential equation which has a solution equal to $i(t) =$
$[A + Be^{-t/\tau}]u(t)$ A. We have $\tau = 4/2 = 2$, $A = i(\infty) = 0$ and $A+B = i(0) = 2$ A $= B$. This leads to, $i(t) = 2e^{-t/2}$ A for all $t > 0$. Thus,

$v(t) = 2i(t) =$ **$[4e^{-t/2}]$ V** for all $t > 0$.

Solution 8.56

In the circuit of Fig. 8.102, find $i(t)$ for $t > 0$.

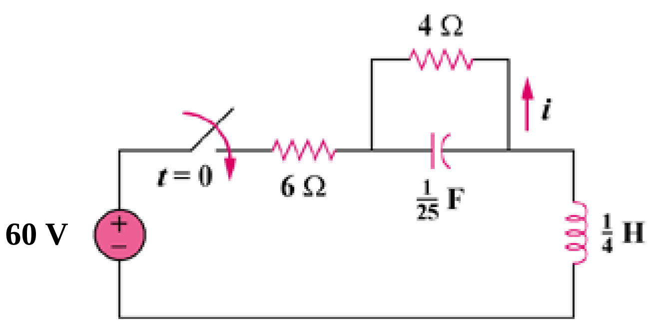
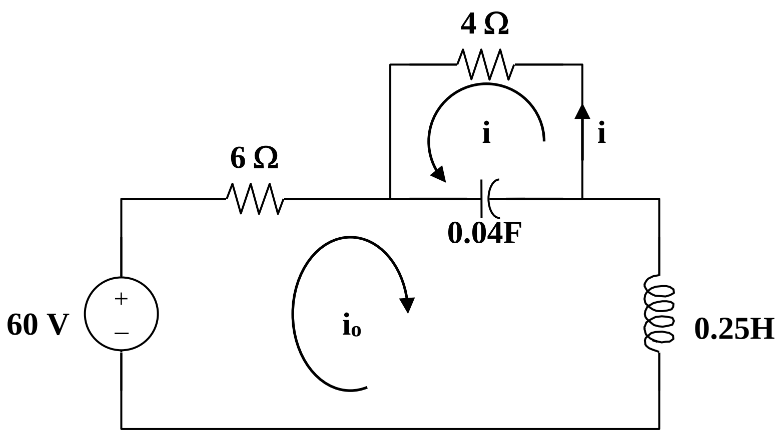


Figure 8.102
For Prob. 8.56.

Solution

For $t < 0$, $i(0) = 0$ and $v(0) = 0$.

For $t > 0$, the circuit is as shown below.



Applying KVL to the larger loop and letting v = the capacitor voltage positive on the left,

$-60 + 6i_o + 0.25di_o/dt + v = 0$ where $i_o = [v/4] + 0.04dv/dt$ which leads to

$-60 + 1.5v + 0.24[dv/dt] + 0.0625[dv/dt] + 0.01[d^2v/dt^2] + v = 0$ or

$[d^2v/dt^2] + 30.25[dv/dt] + 250v = 6{,}000$ which leads to the characteristic equation,

$s^2 + 30.25s + 250 = 0$. $s_{1,2} = [–30.25\pm(915.0625–1{,}000)^{0.5}]/2 = [–30.25\pm j9.21615]/2 = –15.125 \pm j4.608$. Since we have an underdamped system the response v can be expressed as,

$v = V_{ss} + [A_1\cos(4.608t) + A_2\sin(4.608t)]e^{–15.125t}$ where $V_{ss} = 24$ V and $v(0) = 0 = V_{ss} + A_1$ or $A_1 = –24$ V. Since the initial current through the capacitor = 0, then $dv(0)/dt = 0$. $dv(0)/dt = [–(–24)(4.608)\sin(0) + A_2(4.608)\cos(0)]e^{–0} – 15.125[–24\cos(0) + A_2\sin(0)]e^{–0}$ or $0 = 4.608A_2 + 363$ or $A_2 = –78.78$.

Thus, $v = [24 + [–24\cos(4.608t) – 78.78\sin(4.608t)]e^{–15.125t}$ V for all $t > 0$.

Since $i = –v/4 =$ **$[–6 + [6\cos(4.608t) + 19.695\sin(4.608t)]e^{–15.125t}$ A**.

Solution 8.57

Given the circuit shown in Fig. 8.103, determine the characteristic equation of the circuit and the values for *i(t)* and *v(t)* for all t > 0.

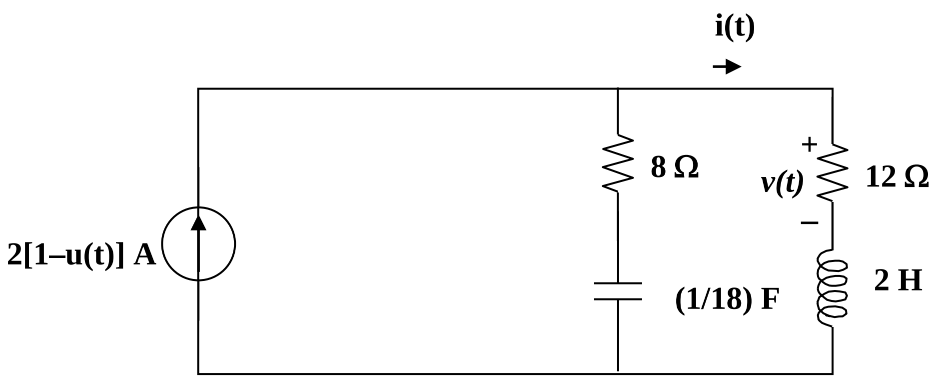


Figure 8.103
For Prob. 8.57.

Solution

Let v_C = capacitor voltage (plus on top and negative on the bottom) and i = inductor current. At t = 0^+, the circuit has reached steady-state and the current source goes to zero.

$$v_C(0^+) = 24 \text{ V} \text{ and } i(0^+) = 2\text{A}$$

We now have a source-free RLC circuit.

$$R = 8 + 12 = 20 \text{ ohms}, \ L = 2 \text{ H}, \ C = (1/18) \text{ F}.$$

$$\alpha = R/(2L) = (20)/(2x2) = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{2x(1/18)} = 3$$

Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1 \text{ and } -9$$

Thus, the characteristic equation is (s + 1)(s + 9) = 0 or $\mathbf{s^2 + 10s + 9 = 0}$.

$$i(t) = [Ae^{-t} + Be^{-9t}] \text{ and } i(0) = 2 = A + B \text{ or } B = 2 - A.$$

To determine A and B will need a second equation or to determine di(0)/dt.

We only need to evaluate the loop equation at $t = 0^+$ or

$-v_C(0) + 20i(0) + Ldi(0^+)/dt = 0$ or $di(0^+)/dt = [24–40]/2 = –8$
$= –A–9B$ or $= –A–9(2–A) = –18+8A = –8$ or $A = 10/8 = 1.25$ and
$B = 2 – (1.25) = 0.75$. Thus,

$$i(t) = \mathbf{[1.25e^{-t} + 0.75e^{-9t}]u(t)\ A}.$$

Finally,

$$v(t) = 12i(t) = \mathbf{[15e^{-t} + 9e^{-9t}]u(t)\ V}.$$

Solution 8.58

In the circuit of Fig. 8.104, the switch has been in position 1 for a long time but moved to position 2 at $t = 0$. Find:

(a) $v(0^+)$, $dv(0^+)/dt$
(b) $v(t)$ for $t \geq 0$.

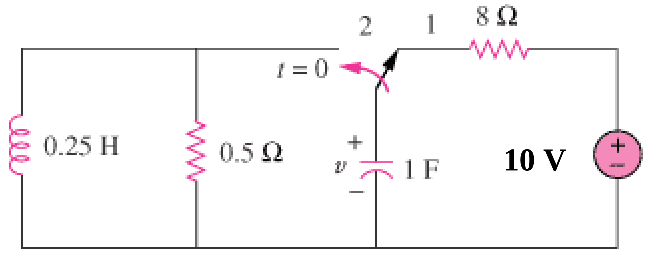


Figure 8.104
For Prob. 8.58.

Solution

(a) Let i =inductor current, v = capacitor voltage i(0) =0, $v(0^+)$ = **10 V**.

$$\frac{dv(0)}{dt} = -\frac{[v(0)+Ri(0)]}{RC} = -\frac{(10+0)}{0.5} = \mathbf{-20\ V/s}.$$

(b) For $t \geq 0$, the circuit is a source-free RLC parallel circuit.

$$\alpha = \frac{1}{2RC} = \frac{1}{2x0.5x1} = 1, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2$$

$$\omega_0 = \sqrt{\omega^2{}_o - \alpha^2} = \sqrt{4-1} = 1.732$$

Thus,

$$v(t) = e^{-t}(A_1 \cos 1.732t + A_2 \sin 1.732t)$$

v(0) = 10 V = A_1

$$\frac{dv}{dt} = -e^{-t}A_1 \cos 1.732t - 1.732e^{-t}A_1 \sin 1.732t - e^{-t}A_2 \sin 1.732t + 1.732e^{-t}A_2 \cos 1.732t$$

$$\frac{dv(0)}{dt} = -20 = -A_1 + 1.732A_2 \quad \longrightarrow \quad A_2 = -5.774$$

v(t) = **[10cos(1.732t) – 5.774sin(1.732t)]e^{-t} V for all t > 0**.

Solution 8.59

The switch in Fig. 8.105 has been in position 1 for $t < 0$. At $t = 0$, it is moved from position 1 to the top of the capacitor at t = 0. Please note that the switch is a make before break switch, it stays in contact with position 1 until it makes contact with the top of the capacitor and then breaks the contact at position 1. Given that the initial voltage across the capacitor is equal to zero, determine $v(t)$.

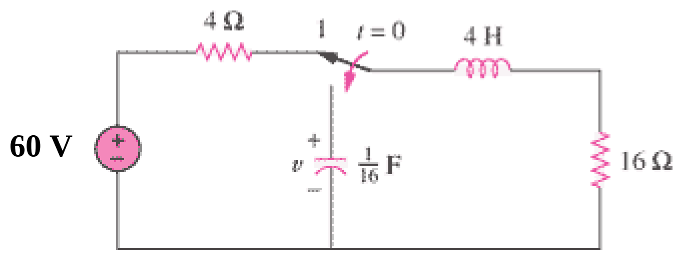


Figure 8.105
For Prob. 8.59.

Solution

Let i = inductor current and v = capacitor voltage
v(0) = 0, i(0) = 60/(4+16) = 3 A
For t>0, the circuit becomes a source-free series RLC with

$$\alpha = \frac{R}{2L} = \frac{16}{2x4} = 2, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4x1/16}} = 2, \qquad \longrightarrow \qquad \alpha = \omega_o = 2$$

$$i(t) = Ae^{-2t} + Bte^{-2t}$$

i(0) = 3 = A

$$\frac{di}{dt} = -2Ae^{-2t} + Be^{-2t} - 2Bte^{-2t}$$

$$\frac{di(0)}{dt} = -2A + B = -\frac{1}{L}[Ri(0) - v(0)] \qquad \longrightarrow \qquad -2A + B = -\frac{1}{4}(48 - 0), \qquad B = -6$$

i(t) = [3e^{-2t} – 6te^{-2t}] and

$$v = \frac{1}{C}\int_0^t -id\tau + v(0) = -48\int_0^t e^{-2\tau}d\tau + 96\int_0^t \tau e^{-2\tau}d\tau = +24e^{-2\tau}\Big|_0^t + \frac{96}{4}e^{-2\tau}(-2\tau - 1)\Big|_0^t$$

v = **–48te^{-2t} V.**

Solution 8.60

Obtain i_1 and i_2 for $t > 0$ in the circuit of Fig. 8.106.

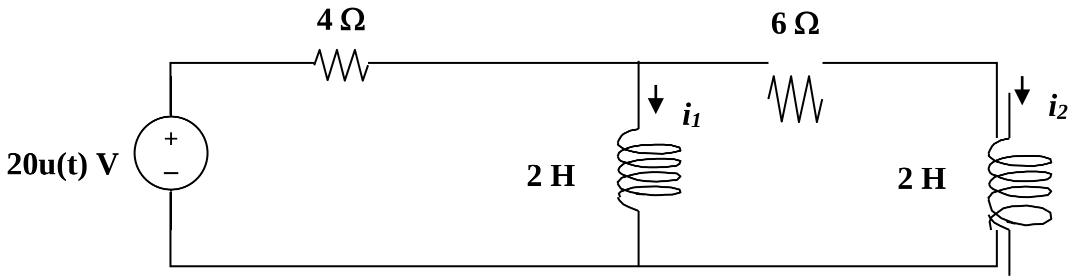


Figure 8.106
For Prob. 8.60.

Solution

Since the independent source is equal to zero until t = 0, $i_1(0) = i_2(0) = 0$.

Applying nodal analysis and letting the voltage at node 1 be v_1 (the voltage across the first inductor) and at node 2 be v_2 (the voltage across the second inductor) we get,

$[(v_1–20)/4] + i_1 + i_2 = 0$ and $[(v_2–v_1)/6] + i_2 = 0$. Also, $v_1 = 2di_1/dt$ and $v_2 = 2di_2/dt$.

The first equation gives us, $[(2di_1/dt)/4] +i_1 + i_2 = 5$ or $i_2 = 5 – 0.5(di_1/dt) – i_1$. We can take the derivative of this and get $(di_2/dt) = –0.5(d^2i_1/dt^2) – di_1/dt$.

The second equation gives us $(di_2/dt) – (di_1/dt) + 3\ i_2 = 0$ or
$[–0.5(d^2i_1/dt^2) – di_1/dt] – (di_1/dt) + 3[5 – 0.5(di_1/dt) – i_1] = 0$ or
$(d^2i_1/dt^2) + 7(di_1/dt) + 6i_1 = 30$.

We have the following $s^2 + 7s + 6 = 0 = (s+1)(s+6)$ and overdamped circuit.

Thus, $i_1(t) = I_s + [Ae^{–t} + Be^{–6t}]$, $I_s = 20/4 = 5$ A. Now let t = 0 we get,
$i_1(0) = 0 = 5 + A + B$ or $A = –5 – B$. Since the two inductors are open at t = 0 we get
$v_1(0^+) = 20\text{ V} = 2di_1(0^+)$ or $(di_1(0^+)/dt) = 10 = –A – 6B = –(–5–B) – 6B = 5–5B$ or B = –
1 and A = –4 which gives us,

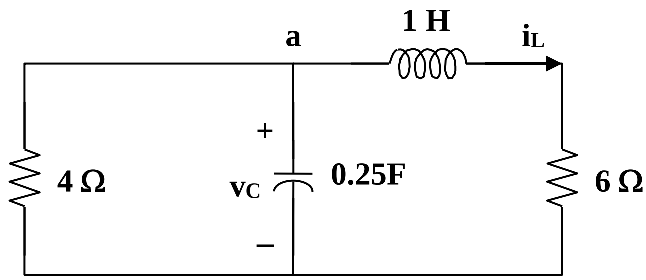
$i_1(t) =$ **$[5 – 4e^{–t} – e^{–6t}]u(t)$ A**.

We can use $i_2 = 5 – 0.5(di_1/dt) – i_1$. to find i_2. $= 5 – 2e^{–t} – 3e^{–6t} – 5 + 4e^{–t} + e^{–6t}$ or

$i_2(t) =$ **$[2e^{–t} – 2e^{–6t}]u(t)$ A**.

Solution 8.61

For $t > 0$, we obtain the natural response by considering the circuit below.



At node a, $v_C/4 + 0.25dv_C/dt + i_L = 0$ (1)

But, $v_C = 1di_L/dt + 6i_L$ (2)

Combining (1) and (2),

$$(1/4)di_L/dt + (6/4)i_L + 0.25d^2i_L/dt^2 + (6/4)di_L/dt + i_L = 0$$

$$d^2i_L/dt^2 + 7di_L/dt + 10i_L = 0$$

$$s^2 + 7s + 10 = 0 = (s + 2)(s + 5) \text{ or } s_{1,2} = -2, -5$$

$$\text{Thus, } i_L(t) = i_L(\infty) + [Ae^{-2t} + Be^{-5t}],$$

where $i_L(\infty)$ represents the final inductor current $= 4(4)/(4 + 6) = 1.6$

$$i_L(t) = 1.6 + [Ae^{-2t} + Be^{-5t}] \text{ and } i_L(0) = 1.6 + [A+B] \text{ or } -1.6 = A+B \quad (3)$$

$$di_L/dt = [-2Ae^{-2t} - 5Be^{-5t}]$$

$$\text{and } di_L(0)/dt = 0 = -2A – 5B \text{ or } A = -2.5B \quad (4)$$

From (3) and (4), $A = -8/3$ and $B = 16/15$

$$i_L(t) = 1.6 + [-(8/3)e^{-2t} + (16/15)e^{-5t}]$$

$$v(t) = 6i_L(t) = \mathbf{\{9.6 + [-16e^{-2t} + 6.4e^{-5t}]\}\ V}$$

$$v_C = 1di_L/dt + 6i_L = [(16/3)e^{-2t} - (16/3)e^{-5t}] + \{9.6 + [-16e^{-2t} + 6.4e^{-5t}]\}$$

$$v_C = \{9.6 + [-(32/3)e^{-2t} + 1.0667e^{-5t}]\}$$

$$i(t) = v_C/4 = \mathbf{\{2.4 + [-2.667e^{-2t} + 0.2667e^{-5t}]\}\ A}$$

Solution 8.62

Find the response $v(t)$ for $t > 0$ in the circuit in Fig. 8.107. Let $R = 8\ \Omega$, $L = 2$ H, and $C = 125$ mF.

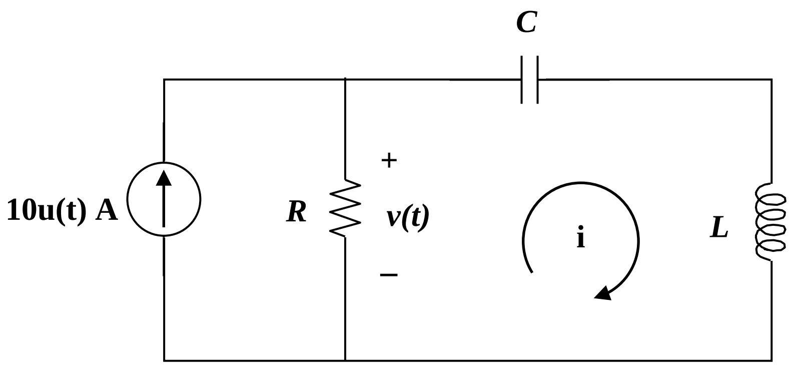


Figure 8.107
For Prob. 8.62.

Solution

This is actually a series RLC circuit where $\alpha = R/(2L) = 2$ and $\omega_0 = 1/\sqrt{LC} = 2$ and $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -2$. Clearly we have a critically damped circuit.

A straight forward way to solve this is to solve for i and then solve for $v(t) = 8[10-i]u(t)$ V.

$i = I_{ss} + [(A + Bt)e^{-2t}]$ where $I_{ss} = 0$. Additionally $i(0) = 0 = 0 + A$ or $A = 0$. Next $di/dt = B[e^{-2t} - 2te^{-2t}]$ or $di(0)/dt = B$. We note that $v_L = L\,di/dt$ or $di/dt = v_L/L$. This leads to $di(0)/dt = 80/2 = 40 = B$. Thus, $i = [40te^{-2t}]u(t)$ A.

Finally,

$$v(t) = \mathbf{[80 - 320te^{-2t}]u(t)\ V}.$$

Solution 8.63

$$\frac{v_s - 0}{R} = C\frac{d(0 - v_o)}{dt} \longrightarrow \frac{v_s}{R} = -C\frac{dv_o}{dt}$$

$$v_o = L\frac{di}{dt} \longrightarrow \frac{dv_o}{dt} = L\frac{d^2 i}{dt^2} = -\frac{v_s}{RC}$$

Thus,

$$\boldsymbol{\frac{d^2 i(t)}{dt^2} = -\frac{v_s}{RCL}}$$

Solution 8.64

Using Fig. 8.109, design a problem to help other students to better understand second-order op amp circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Obtain the differential equation for $v_o(t)$ in the network of Fig. 8.109.

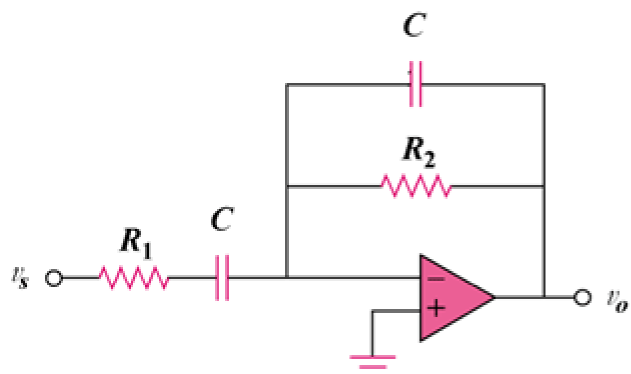
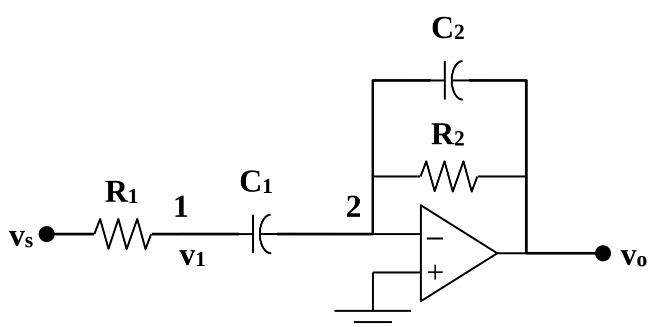


Figure 8.109

Solution



At node 1, $(v_s - v_1)/R_1 = C_1\, d(v_1 - 0)/dt$ or $v_s = v_1 + R_1C_1dv_1/dt$ (1)

At node 2, $C_1dv_1/dt = (0 - v_o)/R_2 + C_2d(0 - v_o)/dt$

or $-R_2C_1dv_1/dt = v_o + R_2C_2dv_o/dt$ (2)

From (1) and (2), $(v_s - v_1)/R_1 = C_1\, dv_1/dt = -(1/R_2)(v_o + R_2C_2dv_o/dt)$

or $v_1 = v_s + (R_1/R_2)(v_o + R_2C_2dv_o/dt)$ (3)

Substituting (3) into (1) produces,

$$v_s = v_s + (R_1/R_2)(v_o + R_2C_2dv_o/dt) + R_1C_1d\{v_s + (R_1/R_2)(v_o + R_2C_2dv_o/dt)\}/dt$$

$$= v_s + (R_1/R_2)(v_o) + (R_1C_2)dv_o/dt + R_1C_1dv_s/dt + (R_1R_1C_1/R_2)dv_o/dt + ((R_1)^2\, C_1C_2)[d^2v_o/dt^2]$$

$$((R_1)^2\, C_1C_2)[d^2v_o/dt^2] + [(R_1C_2) + (R_1R_1C_1/R_2)]dv_o/dt + (R_1/R_2)(v_o) = -R_1C_1dv_s/dt$$

Simplifying we get,

$$[d^2v_o/dt^2] + \{[(R_1C_2) + (R_1R_1C_1/R_2)]/((R_1)^2\, C_1C_2)\}dv_o/dt + \{(R_1/R_2)(v_o)/((R_1)^2\, C_1C_2)\} = -\{R_1C_1/((R_1)^2\, C_1C_2)\}dv_s/dt$$

$$\mathbf{d^2v_o/dt^2 + [(1/R_1C_1) + (1/(R_2C_2))]dv_o/dt + [1/(R_1R_2C_1C_2)](v_o) = -[1/(R_1C_2)]dv_s/dt}$$

Another way to successfully work this problem is to give actual values of the resistors and capacitors and determine the actual differential equation. Alternatively, one could give a differential equations and ask the other students to choose actual value of the differential equation.

Solution 8.65

At the input of the first op amp,

$$(v_o - 0)/R \;=\; Cd(v_1 - 0) \qquad (1)$$

At the input of the second op amp,

$$(-v_1 - 0)/R \;=\; Cdv_2/dt \qquad (2)$$

Let us now examine our constraints. Since the input terminals are essentially at ground, then we have the following,

$$v_o \;=\; -v_2 \;\text{ or }\; v_2 \;=\; -v_o \qquad (3)$$

Combining (1), (2), and (3), eliminating v_1 and v_2 we get,

$$\frac{d^2v_o}{dt^2} - \left(\frac{1}{R^2C^2}\right)v_o = \frac{d^2v_o}{dt^2} - 100v_o = 0$$

Which leads to $s^2 - 100 \;=\; 0$

Clearly this produces roots of –10 and +10.

And, we obtain,

$$v_o(t) \;=\; (Ae^{+10t} + Be^{-10t})V$$

At $t = 0$, $v_o(0+) \;=\; -v_2(0+) \;=\; 0 \;=\; A + B$, thus $B \;=\; -A$

This leads to $v_o(t) \;=\; (Ae^{+10t} - Ae^{-10t})V$. Now we can use $v_1(0+) \;=\; 2V$.

From (2), $v_1 \;=\; -RCdv_2/dt \;=\; 0.1dv_o/dt \;=\; 0.1(10Ae^{+10t} + 10Ae^{-10t})$

$$v_1(0+) \;=\; 2 \;=\; 0.1(20A) \;=\; 2A \;\text{ or }\; A \;=\; 1$$

Thus, $v_o(t) \;=\; \mathbf{(e^{+10t} - e^{-10t})\ V}$

It should be noted that this circuit is unstable (clearly one of the poles lies in the right-half-plane).

Solution 8.66

Obtain the differential equations for $v_o(t)$ in the op amp circuit in Fig. 8.111.

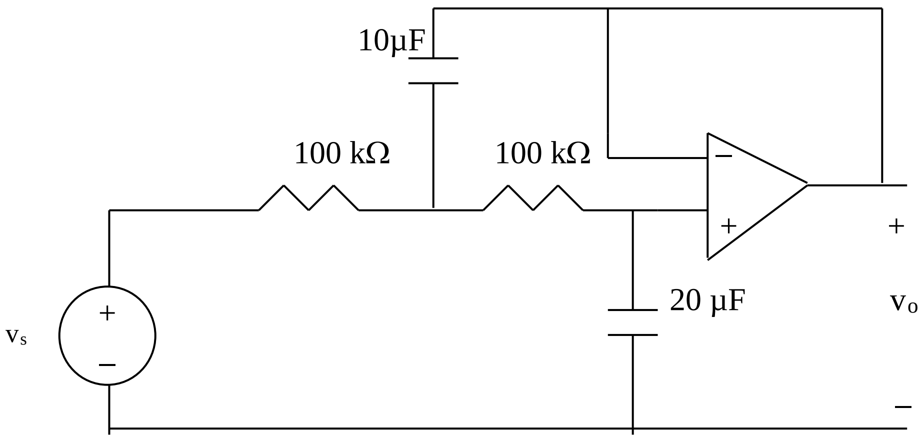
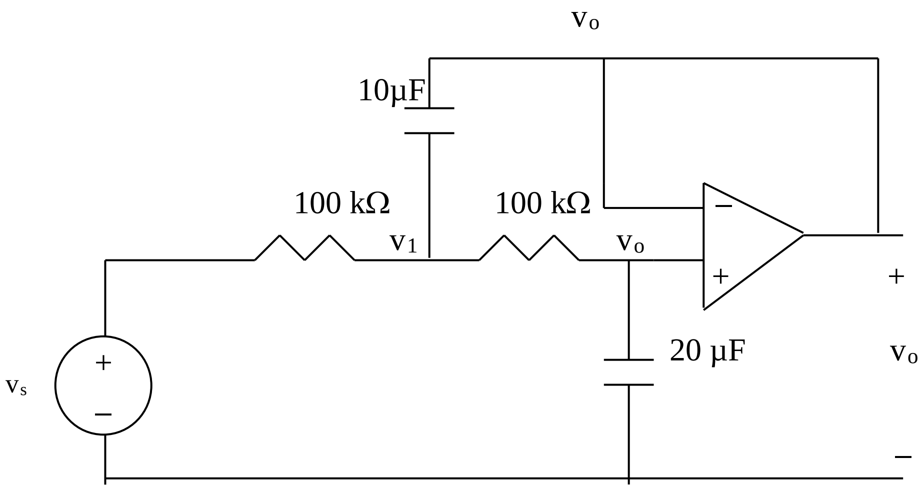


Figure 8.111
For Prob. 8.66.

Solution

We apply nodal analysis to the circuit as shown below.



At node 1,

$$\frac{v_1 - v_s}{10^5} + 10^{-5}\frac{d(v_1 - v_o)}{dt} + \frac{v_1 - v_o}{10^5} = 0 \, or \, v_s = 2v_1 - v_o + \frac{dv_1}{dt} - \frac{dv_o}{dt}$$

At node 2,

$$\frac{v_o - v_1}{10^5} + 2x10^{-5}\frac{d(v_o - 0)}{dt} + 0 = 0\,or\,v_1 = v_o + 2\frac{dv_o}{dt}$$

This leads to $v_s = 2\left(v_o + 2\frac{dv_o}{dt}\right) - v_o + \frac{d\left(v_o + 2\frac{dv_o}{dt}\right)}{dt} - \frac{dv_o}{dt}$ or

$$\mathbf{v_s = [2(d^2v_o/dt^2) + 4(dv_o/dt) + v_o]}$$

Solution 8.67

At node 1,

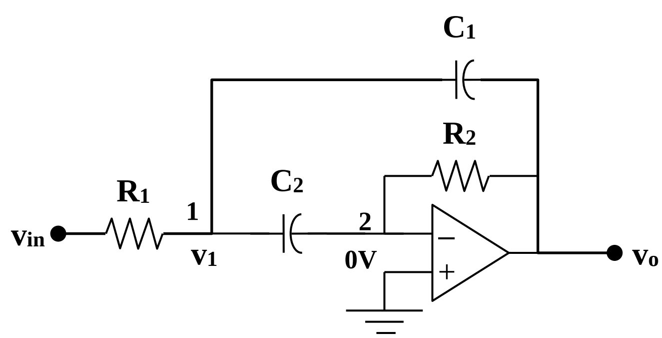
$$\frac{v_{in} - v_1}{R_1} = C_1 \frac{d(v_1 - v_o)}{dt} + C_2 \frac{d(v_1 - 0)}{dt} \qquad (1)$$

At node 2, $$C_2 \frac{d(v_1 - 0)}{dt} = \frac{0 - v_o}{R_2}, \text{ or } \frac{dv_1}{dt} = \frac{-v_o}{C_2 R_2} \qquad (2)$$

From (1) and (2),

$$v_{in} - v_1 = -\frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} - R_1 C_1 \frac{dv_o}{dt} - R_1 \frac{v_o}{R_2}$$

$$v_1 = v_{in} + \frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} + R_1 C_1 \frac{dv_o}{dt} + R_1 \frac{v_o}{R_2} \qquad (3)$$



From (2) and (3),

$$-\frac{v_o}{C_2 R_2} = \frac{dv_1}{dt} = \frac{dv_{in}}{dt} + \frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} + R_1 C_1 \frac{d^2 v_o}{dt^2} + \frac{R_1}{R_2}\frac{dv_o}{dt}$$

$$\frac{d^2 v_o}{dt^2} + \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{dv_o}{dt} + \frac{v_o}{C_1 C_2 R_2 R_1} = -\frac{1}{R_1 C_1}\frac{dv_{in}}{dt}$$

But $C_1 C_2 R_1 R_2 = 10^{-4} \text{x} 10^{-4} \text{x} 10^{4} \text{x} 10^{4} = 1$

$$\frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{2}{R_2 C_1} = \frac{2}{10^4 \text{x} 10^{-4}} = 2$$

$$\frac{d^2v_o}{dt^2}+2\frac{dv_o}{dt}+v_o=-\frac{dv_{in}}{dt}$$

Which leads to $s^2 + 2s + 1 = 0$ or $(s + 1)^2 = 0$ and $s = –1, –1$

Therefore, $v_o(t) = [(A + Bt)e^{-t}] + V_f$

As t approaches infinity, the capacitor acts like an open circuit so that

$$V_f = v_o(\infty) = 0$$

$v_{in} = 10u(t)$ mV and the fact that the initial voltages across each capacitor is 0

means that $v_o(0) = 0$ which leads to $A = 0$.

$$v_o(t) = [Bte^{-t}]$$

$$\frac{dv_o}{dt} = [(B – Bt)e^{-t}] \qquad (4)$$

From (2),
$$\frac{dv_o(0+)}{dt}=-\frac{v_o(0+)}{C_2R_2}=0$$

From (1) at $t = 0+$,

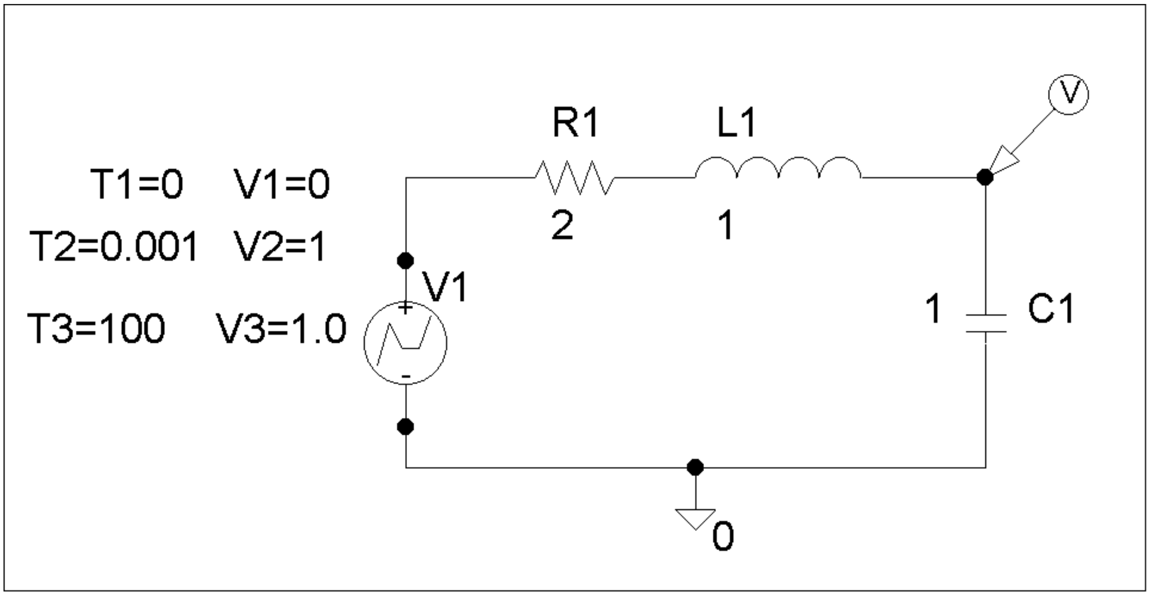
$$\frac{1-0}{R_1}=-C_1\frac{dv_o(0+)}{dt} \text{ which leads to } \frac{dv_o(0+)}{dt}=-\frac{1}{C_1R_1}=-1$$

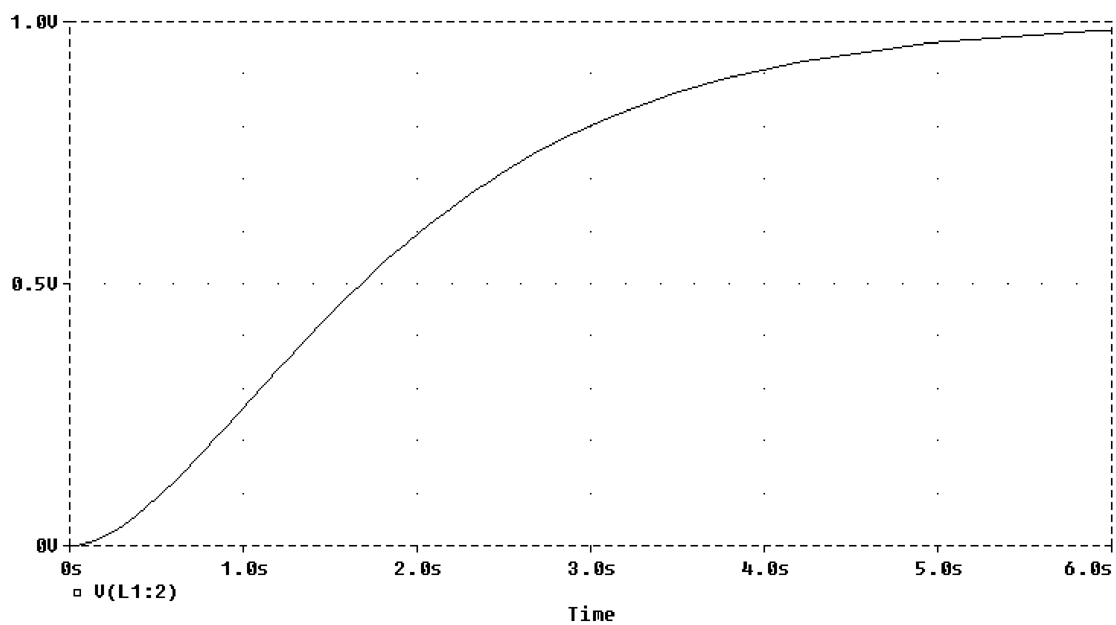
Substituting this into (4) gives $B = –1$

Thus, $v(t) = \mathbf{-te^{-t}u(t)\ V}$

Solution 6.68

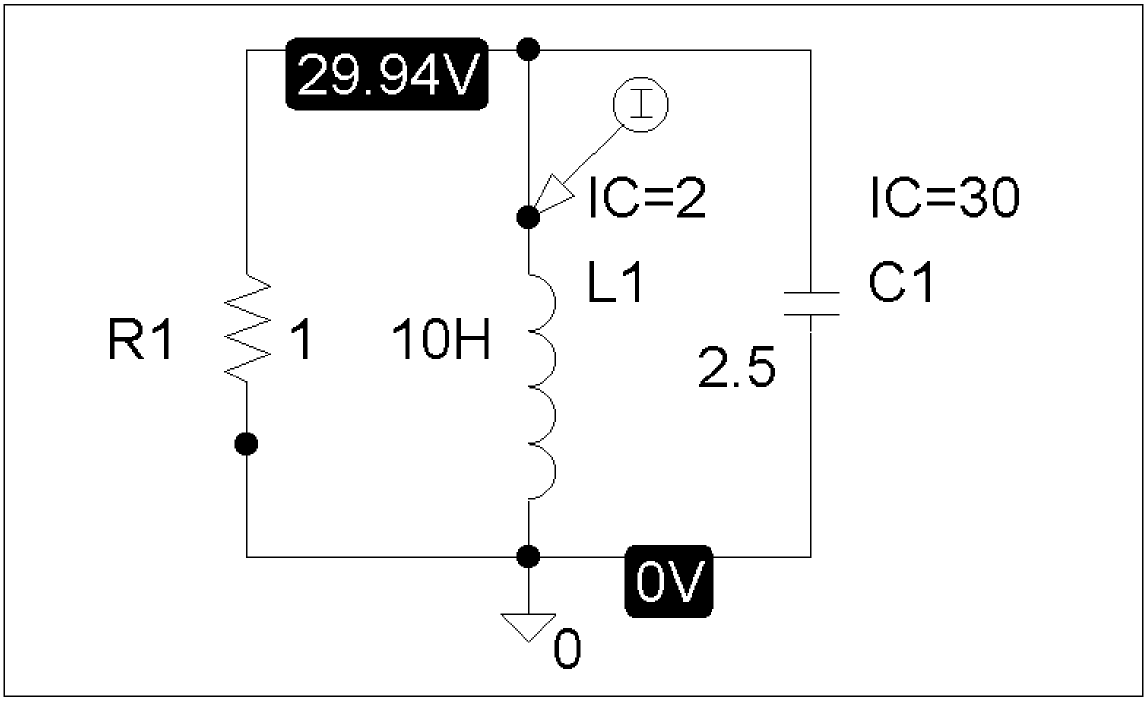
The schematic is as shown below. The unit step is modeled by VPWL as shown. We insert a voltage marker to display V after simulation. We set Print Step = 25 ms and final step = 6s in the transient box. The output plot is shown below.

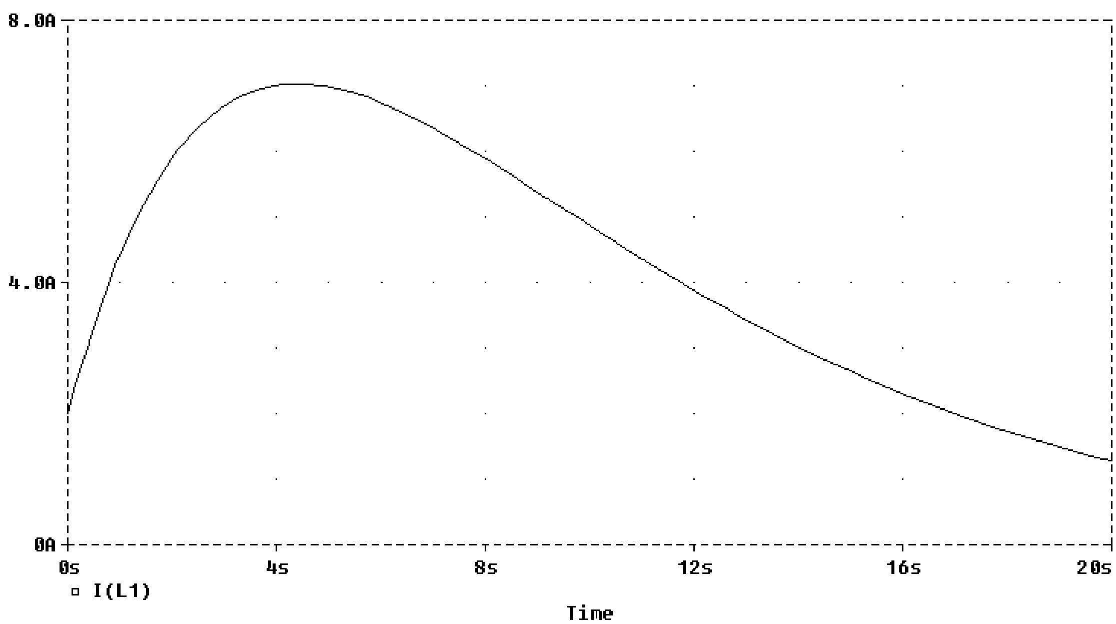




Solution 8.69

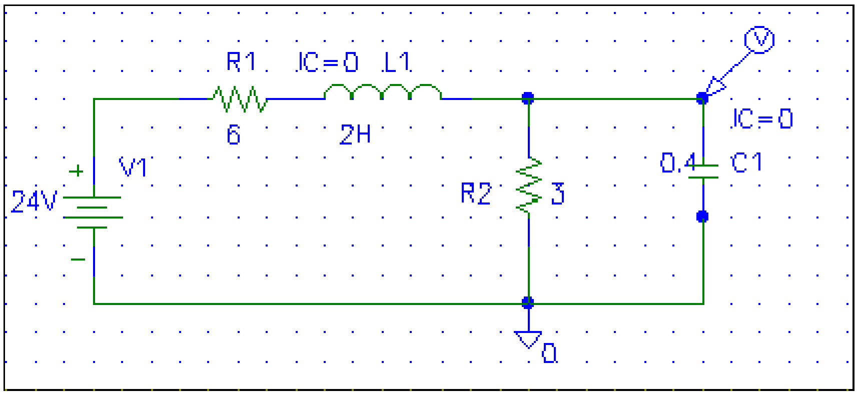
The schematic is shown below. The initial values are set as attributes of L1 and C1. We set Print Step to 25 ms and the Final Time to 20s in the transient box. A current marker is inserted at the terminal of L1 to automatically display i(t) after simulation. The result is shown below.



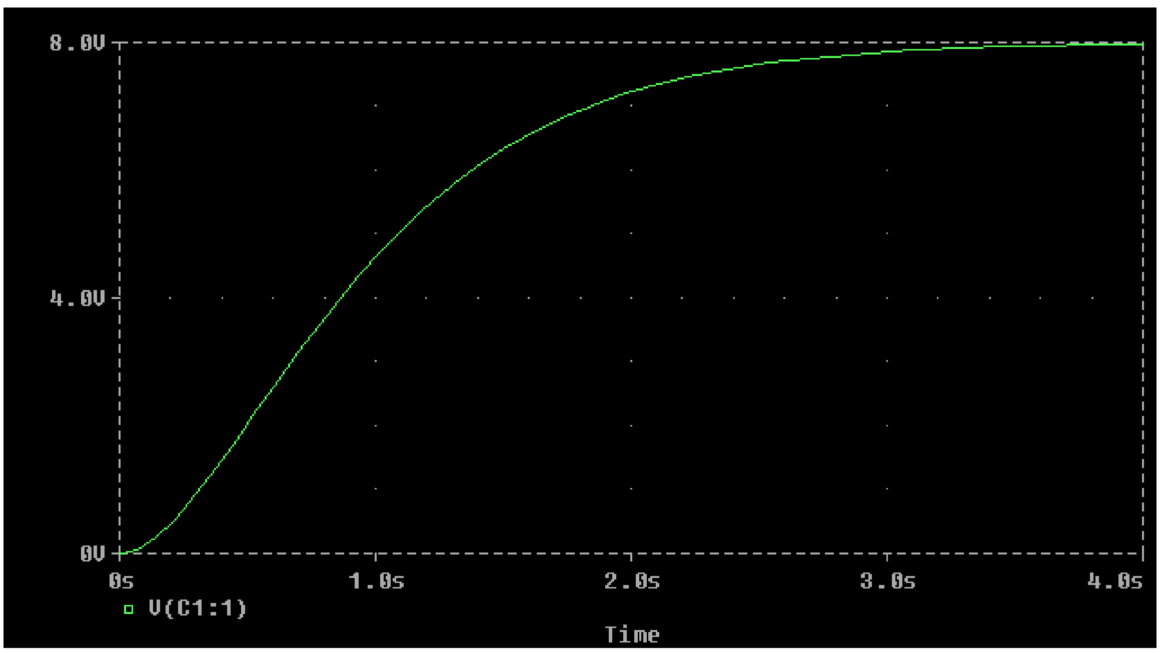


Solution 8.70

The schematic is shown below.

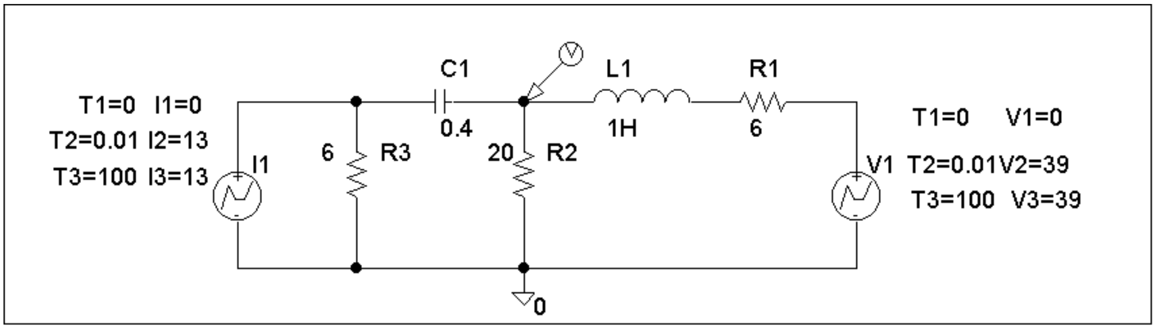


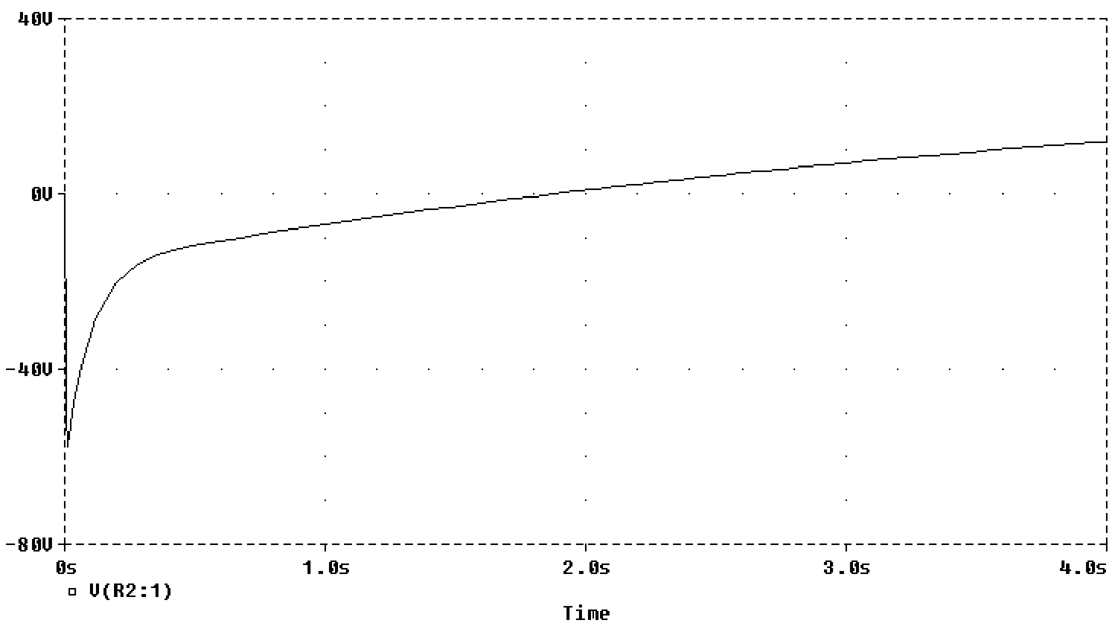
After the circuit is saved and simulated, we obtain the capacitor voltage v(t) as shown below.



Solution 8.71

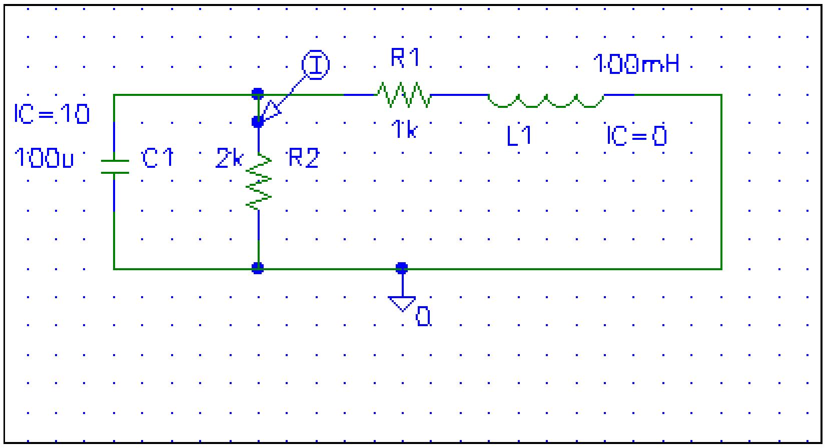
The schematic is shown below. We use VPWL and IPWL to model the 39 u(t) V and 13 u(t) A respectively. We set Print Step to 25 ms and Final Step to 4s in the Transient box. A voltage marker is inserted at the terminal of R2 to automatically produce the plot of v(t) after simulation. The result is shown below.



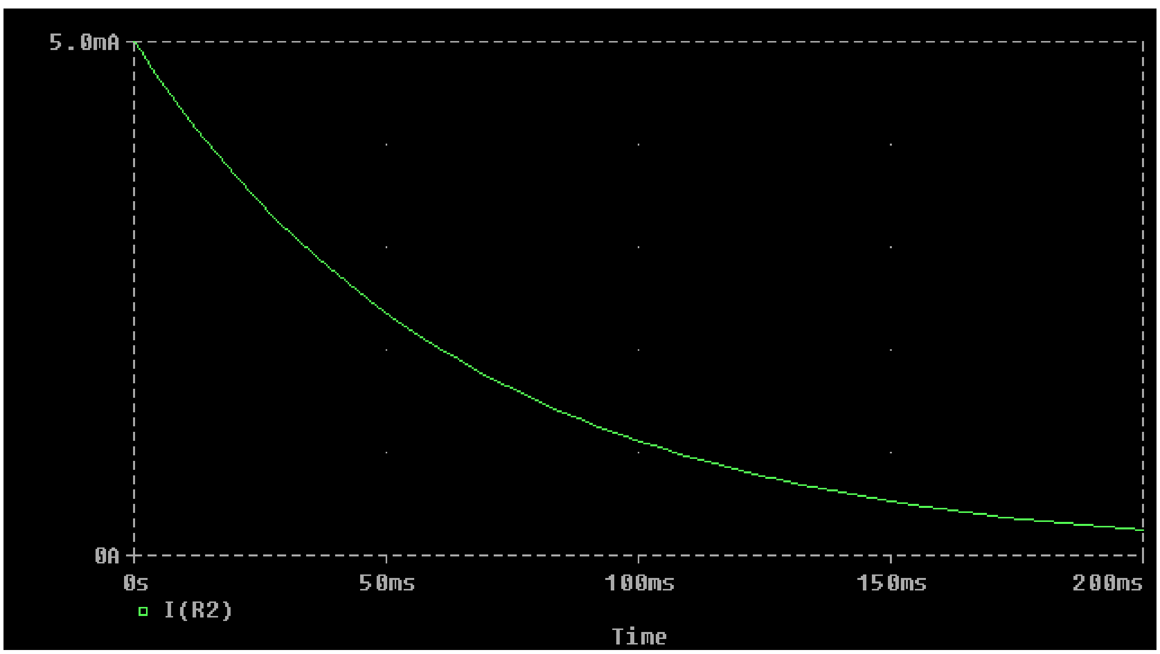


Solution 8.72

When the switch is in position 1, we obtain IC=10 for the capacitor and IC=0 for the inductor. When the switch is in position 2, the schematic of the circuit is shown below.



When the circuit is simulated, we obtain i(t) as shown below.



Solution 8.73

Design a problem, using PSpice, to help other students to better understand source-free *RLC* circuits.

Although there are many ways to work this problem, this is an example based on a somewhat similar problem worked in the third edition.

Problem

The step response of an *RLC* circuit is given by

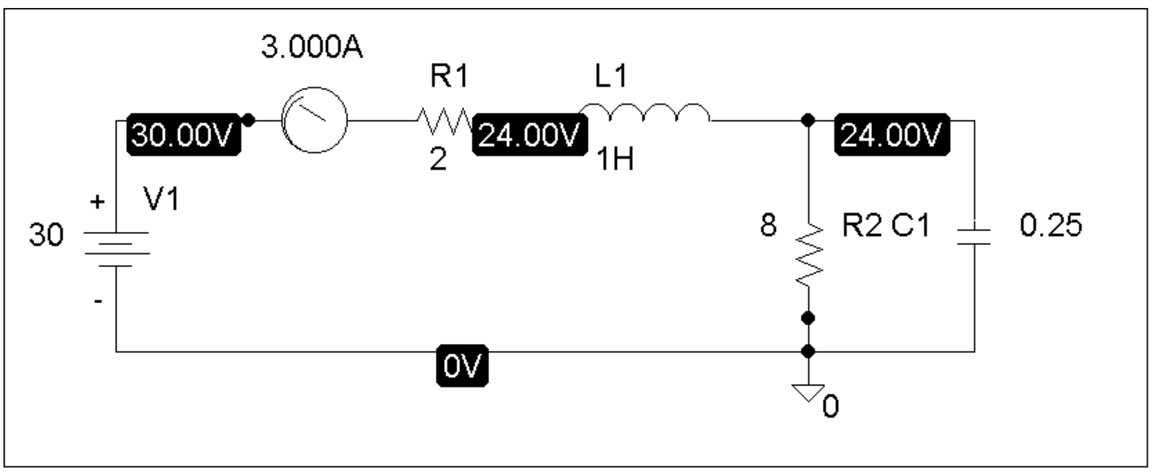
$$\frac{d^2 i_L}{dt^2} + 0.5\frac{di_L}{dt} + 4i_L = 0$$

Given that $i_L(0) = 3$ A and $v_C(0) = 24$ V, solve for v_C(t) and $I_C(t)$.

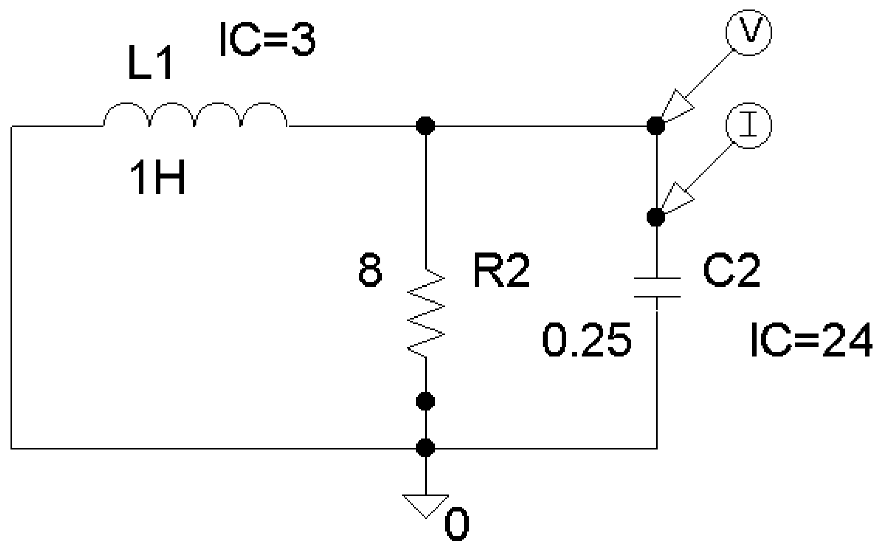
Solution

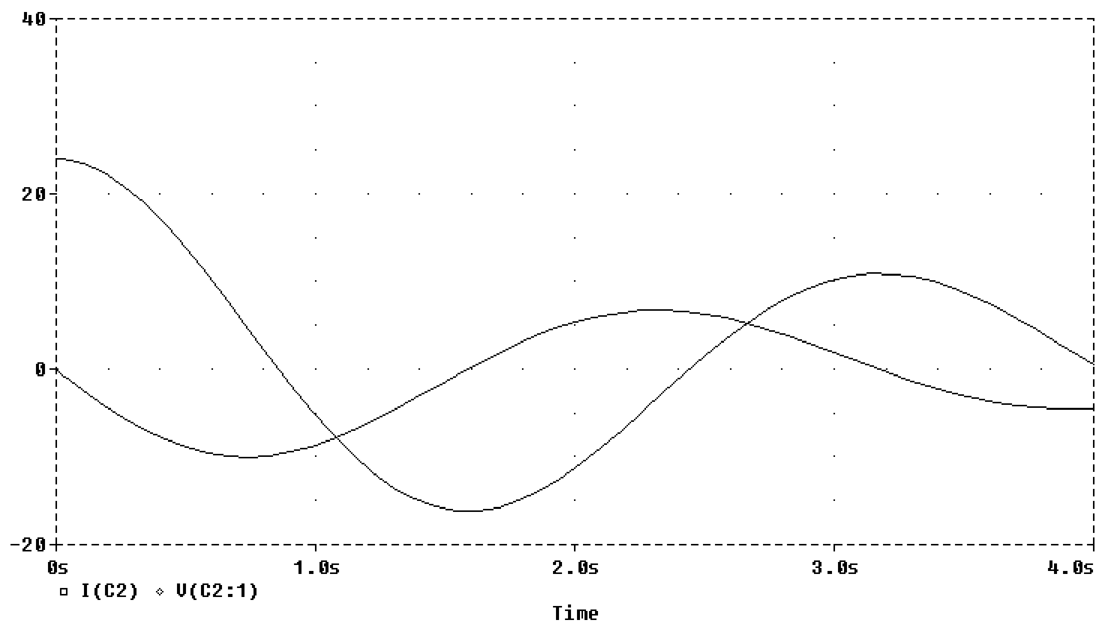
(a) For t < 0, we have the schematic below. When this is saved and simulated, we obtain the initial inductor current and capacitor voltage as

$$i_L(0) = 3 \text{ A} \quad \text{and } v_c(0) = 24 \text{ V}.$$



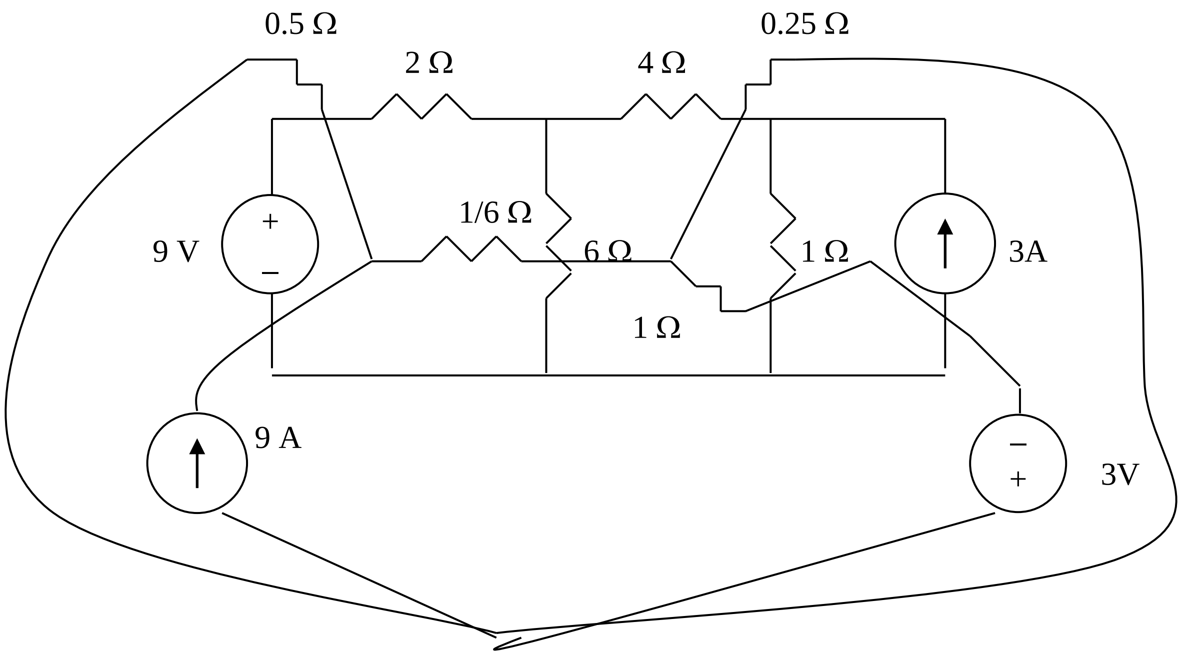
(b) For $t > 0$, we have the schematic shown below. To display i(t) and v(t), we insert current and voltage markers as shown. The initial inductor current and capacitor voltage are also incorporated. In the Transient box, we set Print Step = 25 ms and the Final Time to 4s. After simulation, we automatically have $i_o(t)$ and $v_o(t)$ displayed as shown below.



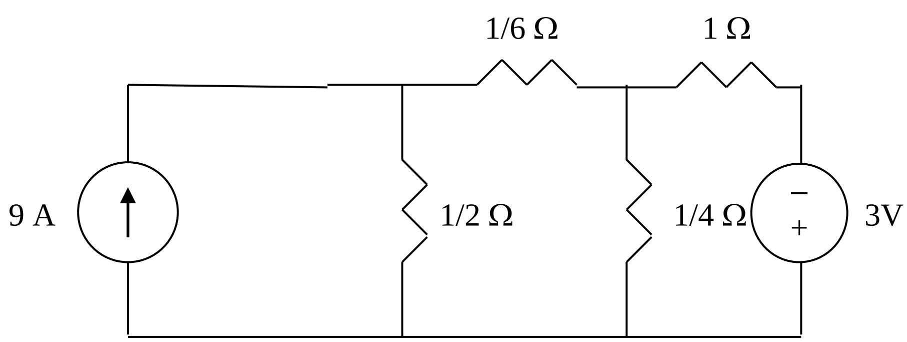


Solution 8.74

The dual is constructed as shown below.

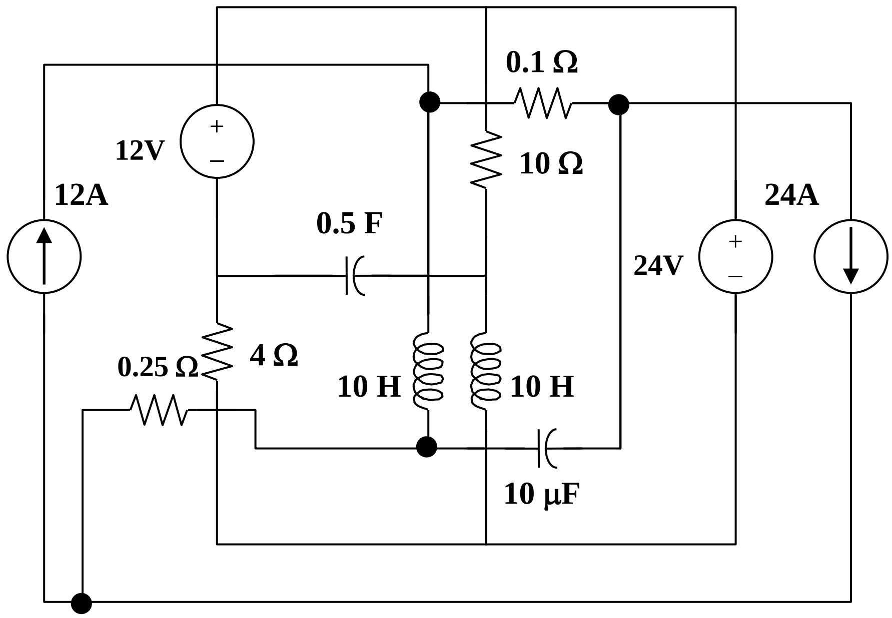


The dual is redrawn as shown below.

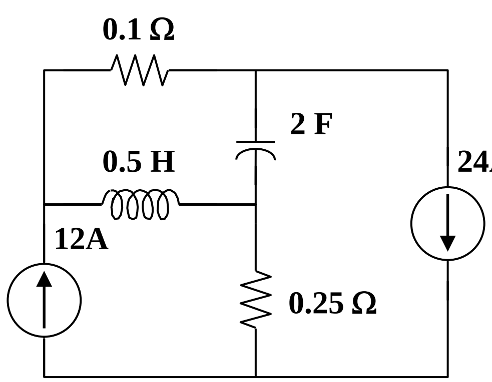


Solution 8.75

The dual circuit is connected as shown in Figure (a). It is redrawn in Figure (b).



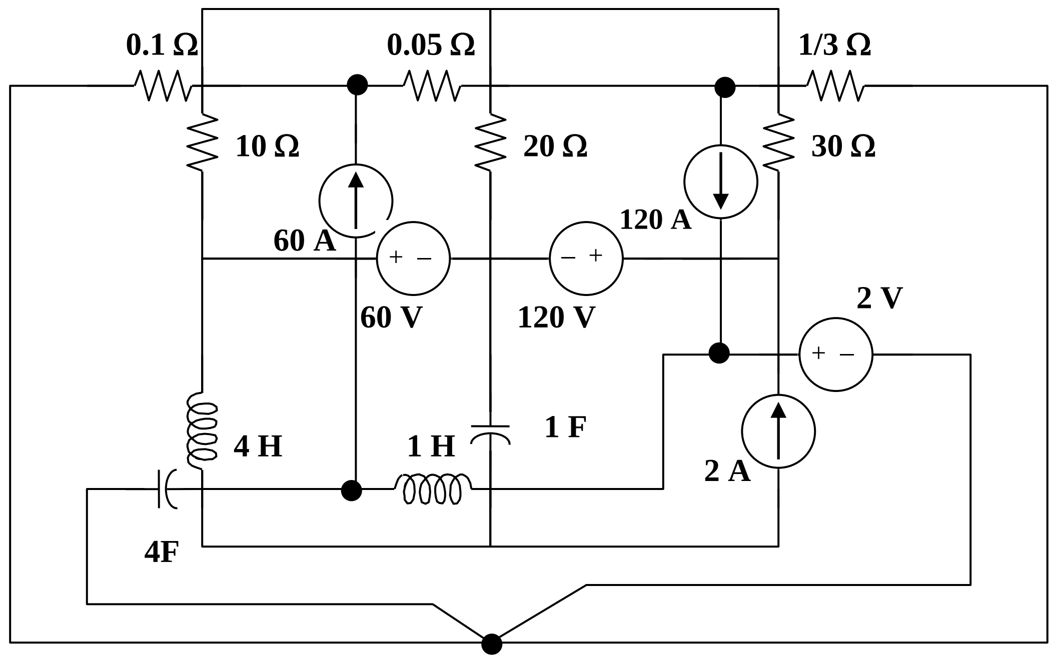
(a)



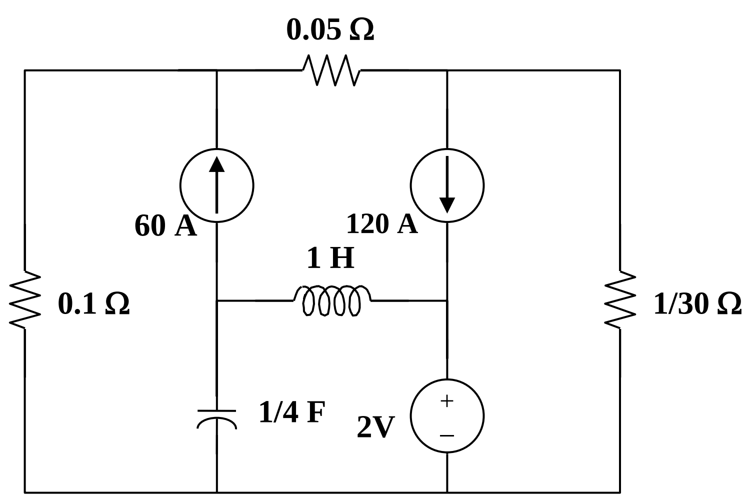
(b)

Solution 8.76

The dual is obtained from the original circuit as shown in Figure (a). It is redrawn in Figure (b).



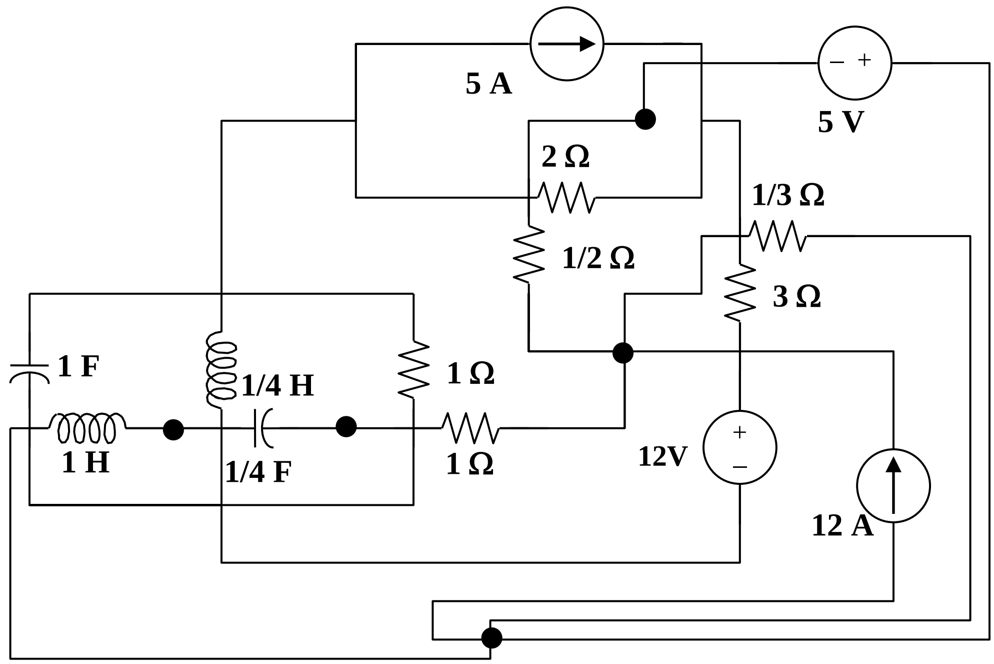
(a)



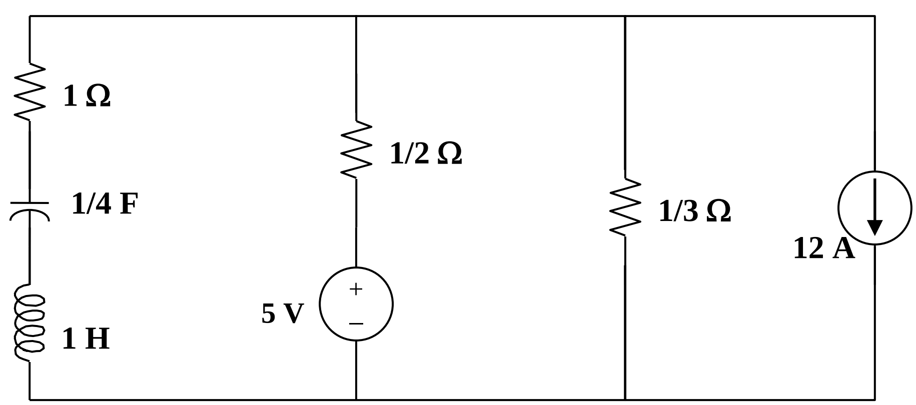
(b)

Solution 8.77

The dual is constructed in Figure (a) and redrawn in Figure (b).



(a)



(b)

Solution 8.78

The voltage across the igniter is $v_R = v_C$ since the circuit is a parallel RLC type.

$$v_C(0) = 12, \text{ and } i_L(0) = 0.$$

$$\alpha = 1/(2RC) = 1/(2x3x1/30) = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{60x10^{-3}x1/30} = 22.36$$

$\alpha < \omega_o$ produces an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm j21.794$$

$$v_C(t) = e^{-5t}(A\cos 21.794t + B\sin 21.794t) \qquad (1)$$

$$v_C(0) = 12 = A$$

$$dv_C/dt = -5[(A\cos 21.794t + B\sin 21.794t)e^{-5t}]$$
$$+ 21.794[(-A\sin 21.794t + B\cos 21.794t)e^{-5t}] \qquad (2)$$

$$dv_C(0)/dt = -5A + 21.794B$$

But, $\quad dv_C(0)/dt = -[v_C(0) + Ri_L(0)]/(RC) = -(12 + 0)/(1/10) = -120$

Hence, $\quad -120 = -5A + 21.794B$, leads to $B\ (5x12 - 120)/21.794 = -2.753$

At the peak value, $\quad dv_C(t_o)/dt = 0$, i.e.,

$$0 = A + B\tan 21.794t_o + (A21.794/5)\tan 21.794t_o - 21.794B/5$$

$$(B + A21.794/5)\tan 21.794t_o = (21.794B/5) - A$$

$$\tan 21.794t_o = [(21.794B/5) - A]/(B + A21.794/5) = -24/49.55 = -0.484$$

Therefore, $\quad 21.7945t_o = |-0.451|$

$$t_o = |-0.451|/21.794 = \mathbf{20.68\ ms}$$

Solution 8.79

A load is modeled as a 100-mH inductor in parallel with a 12-Ω resistor. A capacitor is needed to be connected to the load so that the network is critically damped at 60 Hz. Calculate the size of the capacitor.

Solution

For critical damping of a parallel RLC circuit,

$$\alpha = \omega_o \quad \longrightarrow \quad \frac{1}{2RC} = \frac{1}{\sqrt{LC}}$$

Hence,

$$C = \frac{L}{4R^2} = \frac{0.1}{4x144} = \mathbf{173.61\mu F}.$$

Solution 8.80

$$t_1 = 1/|s_1| = 0.1x10^{-3} \text{ leads to } s_1 = -1000/0.1 = -10{,}000$$

$$t_2 = 1/|s_2| = 0.5x10^{-3} \text{ leads to } s_1 = -2{,}000$$

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}$$

$$s_1 + s_2 = -2\alpha = -12{,}000, \text{ therefore } \alpha = 6{,}000 = R/(2L)$$

$$L = R/12{,}000 = 50{,}000/12{,}000 = \mathbf{4.167H}$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -2{,}000$$

$$\alpha - \sqrt{\alpha^2 - \omega_o^2} = 2{,}000$$

$$6{,}000 - \sqrt{\alpha^2 - \omega_o^2} = 2{,}000$$

$$\sqrt{\alpha^2 - \omega_o^2} = 4{,}000$$

$$\alpha^2 - \omega_o^2 = 16x10^6$$

$$\omega_o^2 = \alpha^2 - 16x10^6 = 36x10^6 - 16x10^6$$

$$\omega_o = 10^3\sqrt{20} = 1/\sqrt{LC}$$

$$C = 1/(20x10^6x4.167) = \mathbf{12\ nF}$$

Solution 8.81

$t = 1/\alpha = 0.25$ leads to $\alpha = 4$

But, α 1/(2RC) or, $C = 1/(2\alpha R) = 1/(2x4x200) =$ **625 μF**

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

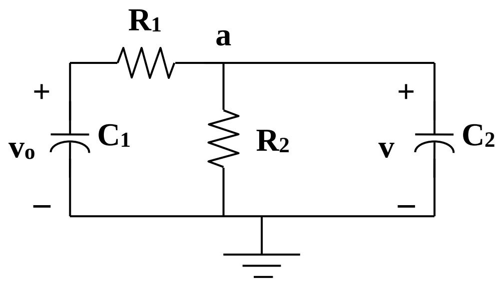
$$\omega_o^2 = \omega_d^2 + \alpha^2 = (2\pi 4x10^3)^2 + 16 \cong (2\pi 4x10^3 0^2 \;\; = \;\; 1/(LC)$$

This results in $L = 1/(64\pi^2 x10^6 x625x10^{-6}) =$ **2.533 μH**

Solution 8.82

For t = 0-, v(0) = 0.

For t > 0, the circuit is as shown below.



At node a,

$$(v_o - v/R_1 = (v/R_2) + C_2 dv/dt$$

$$v_o = v(1 + R_1/R_2) + R_1C_2\ dv/dt$$

$$60 = (1 + 5/2.5) + (5x10^6\ x5x10^{-6})dv/dt$$

$$60 = 3v + 25dv/dt$$

$$v(t) = V_s + [Ae^{-3t/25}]$$

where $3V_s = 60$ yields $V_s = 20$

$$v(0) = 0 = 20 + A \text{ or } A = -20$$

$$v(t) = \mathbf{20(1 - e^{-3t/25})V}$$

Solution 8.83

$$i = i_D + Cdv/dt \qquad (1)$$

$$-v_s + iR + Ldi/dt + v = 0 \qquad (2)$$

Substituting (1) into (2),

$$v_s = Ri_D + RCdv/dt + Ldi_D/dt + LCd^2v/dt^2 + v = 0$$

$$LCd^2v/dt^2 + RCdv/dt + Ri_D + Ldi_D/dt = v_s$$

$$\mathbf{d^2v/dt^2 + (R/L)dv/dt + (R/LC)i_D + (1/C)di_D/dt = v_s/LC}$$

Solution 9.1

(a) V_m = **50 V**.

(b) Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{30} = \underline{0.2094s}$ = **209.4ms**

(c) Frequency f = ω/(2π) = 30/(2π) = **4.775 Hz**.

(d) At t=1ms, v(0.01) = 50cos(30x0.01rad + 10˚) = 50cos(1.72˚ + 10˚) = **44.48 V** and ωt = **0.3 rad**.

Solution 9.2

(a) amplitude = **15 A**

(b) $\omega = 25\pi =$ **78.54 rad/s**

(c) $f = \dfrac{\omega}{2\pi} =$ **12.5Hz**

(d) $I_s = 15\angle 25°$ A

$I_s(2\text{ ms}) = 15\cos((500\pi)(2\times 10^{-3}) + 25°) =$

$15\cos(\pi + 25°) = 15\cos(205°) =$ **–13.595 A**

Solution 9.3

(a) $10\sin(\omega t + 30°) = 10\cos(\omega t + 30° - 90°) = \mathbf{10cos(\omega t - 60°)}$

(b) $-9\sin(8t) = \mathbf{9cos(8t + 90°)}$

(c) $-20\sin(\omega t + 45°) = 20\cos(\omega t + 45° + 90°) = \mathbf{20cos(\omega t + 135°)}$

(a) $10\cos(\omega t - 60°)$, (b) $9\cos(8t + 90°)$, (c) $20\cos(\omega t + 135°)$

Solution 9.4

Design a problem to help other students to better understand sinusoids.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

(a) Express $v = 8\cos(7t + 15°)$ in sine form.

(b) Convert $i = –10\sin(3t - 85°)$ to cosine form.

Solution

(a) $v = 8\cos(7t + 15°) = 8\sin(7t + 15° + 90°) = \mathbf{8\sin(7t + 105°)}$

(b) $i = –10\sin(3t – 85°) = 10\cos(3t – 85° + 90°) = \mathbf{10\cos(3t + 5°)}$

Solution 9.5

$v_1 = 45 \sin(\omega t + 30°)$ V = $45 \cos(\omega t + 30° - 90°)$ = $45 \cos(\omega t - 60°)$ V

$v_2 = 50 \cos(\omega t - 30°)$ V

This indicates that the phase angle between the two signals is **30°** and that **v_1 lags v_2**.

Solution 9.6

(a) $v(t) = 10\cos(4t - 60°)$

$i(t) = 4\sin(4t + 50°) = 4\cos(4t + 50° - 90°) = 4\cos(4t - 40°)$

Thus, **i(t) leads v(t) by 20°**.

(b) $v_1(t) = 4\cos(377t + 10°)$

$v_2(t) = -20\cos(377t) = 20\cos(377t + 180°)$

Thus, **$v_2(t)$ leads $v_1(t)$ by 170°**.

(c) $x(t) = 13\cos(2t) + 5\sin(2t) = 13\cos(2t) + 5\cos(2t - 90°)$

$\mathbf{X} = 13\angle 0° + 5\angle{-90°} = 13 - j5 = 13.928\angle{-21.04°}$

$x(t) = 13.928\cos(2t - 21.04°)$

$y(t) = 15\cos(2t - 11.8°)$

phase difference $= -11.8° + 21.04° = 9.24°$

Thus, **y(t) leads x(t) by 9.24°**.

Solution 9.7

If $f(\phi) = \cos\phi + j\sin\phi$,

$$\frac{df}{d\phi} = -\sin\phi + j\cos\phi = j(\cos\phi + j\sin\phi) = jf(\phi)$$

$$\frac{df}{f} = j\,d\phi$$

Integrating both sides

$\ln f = j\phi + \ln A$

$f = Ae^{j\phi} = \cos\phi + j\sin\phi$

$f(0) = A = 1$

i.e. $\mathbf{f(\phi) = e^{j\phi} = \cos\phi + j\sin\phi}$

Solution 9.8

(a) $\dfrac{60\angle 45^\circ}{7.5 - j10} + j2 = \dfrac{60\angle 45^\circ}{12.5\angle -53.13^\circ} + j2$

$= 4.8\angle 98.13^\circ + j2 = –0.6788+j4.752+j2$

$=$ **$–0.6788 + j6.752$**

(b) $(6–j8)(4+j2) = 24–j32+j12+16 = 40–j20 = 44.72\angle –26.57^\circ$

$$\frac{32\angle -20^\circ}{(6-j8)(4+j2)} + \frac{20}{-10+j24} = \frac{32\angle -20^\circ}{44.72\angle -26.57^\circ} + \frac{20}{26\angle 112.62^\circ}$$

$= 0.7156\angle 6.57^\circ + 0.7692\angle –112.62^\circ = 0.7109+j0.08188–0.2958–j0.71$

$=$ **$0.4151–j0.6281$**

(c) $20 + (16\angle –50^\circ)(13\angle 67.38^\circ) = 20+208\angle 17.38^\circ = 20 + 198.5+j62.13$

$=$ **$218.5+j62.13$**

Solution 9.9

(a) $$(5\angle 30°)(6 - j8 + 1.1197 + j0.7392) = (5\angle 30°)(7.13 - j7.261)$$
$$= (5\angle 30°)(10.176\angle -45.52°) =$$

$$\mathbf{50.88\angle{-15.52}}°.$$

(b) $$\frac{(10\angle 60°)(35\angle -50°)}{(-3 + j5) = (5.83\angle 120.96°)} = \mathbf{60.02\angle{-110.96}}°.$$

Solution 9.10

Design a problem to help other students to better understand phasors.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Given that $z_1 = 6 - j8$, $z_2 = 10\angle{-30^\circ}$, and $z_3 = 8e^{-j120^\circ}$, find:
(a) $z_1 + z_2 + z_3$
(b) $z_1 z_2 / z_3$

Solution

(a) $z_1 = 6 - j8$, $z_2 = 8.66 - j5$, and $z_3 = -4 - j6.9282$

$z_1 + z_2 + z_3 =$ **(10.66 – j19.928)Ω**

(b) $\dfrac{z_1 z_2}{z_3} = [(10\angle{-53.13^\circ})(10\angle{-30^\circ})/(8\angle{-120^\circ})] = 12.5\angle 36.87^\circ\Omega =$ **(10 + j7.5) Ω**

Solution 9.11

(a) $V = \underline{21 < -15^o \text{ V}}$

(b) $i(t) = 8\sin(10t + 70^o + 180^o) = 8\cos(10t + 70^o + 180^o - 90^o) = 8\cos(10t + 160^o)$

I = 8∠160° mA

(c) $v(t) = 120\sin(10^3 t - 50^o) = 120\cos(10^3 t - 50^o - 90^o)$

V = 120∠–140° V

(d) $i(t) = -60\cos(30t + 10^o) = 60\cos(30t + 10^o + 180^o)$

I = 60∠–170° mA

Solution 9.11

Let **X** = 4∠40° and **Y** = 20∠–30°. Evaluate the following quantities and express your results in polar form.

(X + **Y**)/**X***
(X - **Y**)*
(**X** + **Y**)/**X**

X = 3.064+j2.571; **Y** = 17.321–j10

(a) **(X** + **Y**)**X*** = $(20.38 - j7.429)(4\angle -40^\circ)$

$$= (21.69\angle -20.03^\circ)(4\angle -40^\circ) = 86.76\angle -60.03^\circ$$

= **86.76∠–60.03˚**

(b) **(X – Y)*** = (–14.257+j12.571)* = **19.41∠–139.63˚**

(c) (**X** + **Y**)/**X** **=** (21.69∠–20.03˚)/(4∠40˚) = **5.422∠–60.03˚**

Solution 9.13

(a) $\mathbf{(-0.4324 + \mathit{j}0.4054) + (-0.8425 - \mathit{j}0.2534) = \underline{-1.2749 + \mathit{j}0.1520}}$

(b) $\dfrac{50\angle -30^o}{24\angle 150^o} = \underline{-2.0833}$ = **–2.083**

(c) (2+j3)(8-j5) –(-4) = **35 +j14**

Solution 9.14

(a) $\dfrac{3-j14}{-7+j17}=\dfrac{14.318\angle-77.91°}{18.385\angle112.38°}=0.7788\angle169.71°=\underline{-0.7663+j0.13912}$

(b) $\dfrac{(62.116+j231.82+138.56-j80)(60-j80)}{(67+j84)(16.96+j10.5983)}=\dfrac{24186-6944.9}{246.06+j2134.7}=\underline{-1.922-j11.55}$

(c) $\left[\dfrac{10+j20}{3+j4}\right]^2\sqrt{(10+j5)(16-j20)}$

$= [(22.36\angle63.43°)/(5\angle53.13°)]^2[(11.18\angle26.57°)(25.61\angle–51.34°)]^{0.5}$
$= [4.472\angle10.3°]^2[286.3\angle–24.77°]^{0.5} = (19.999\angle20.6°)(16.921\angle–12.38°) = 338.4\angle8.22°$

or **334.9+j48.38**

Solution 9.15

(a) $\begin{vmatrix} 10+j6 & 2-j3 \\ -5 & -1+j \end{vmatrix}$ = -10 – j6 + j10 – 6 + 10 – j15

= **–6 – j11**

(b) $\begin{vmatrix} 20\angle -30^\circ & -4\angle -10^\circ \\ 16\angle 0^\circ & 3\angle 45^\circ \end{vmatrix}$ = $60\angle 15^\circ + 64\angle\text{-}10^\circ$

= 57.96 + j15.529 + 63.03 – j11.114

= **120.99 + j4.415**

(c)

$$\begin{vmatrix} 1-j & -j & 0 \\ j & 1 & -j \\ 1 & j & 1+j \\ 1-j & -j & 0 \\ j & 1 & -j \end{vmatrix}$$

= (1–j)1(1+j) +$j^2$0+1(–j)(–j) – 0(1) – (1–j)(–j)j – j(–j)(1+j)

= 2–1 – 1 + j – 1 – j = **–1**

Solution 9.16

(a) $-20\cos(4t + 135°) = 20\cos(4t + 135° - 180°)$
$= 20\cos(4t - 45°)$
The phasor form is $\mathbf{20\angle{-45°}}$

(b) $8\sin(20t + 30°) = 8\cos(20t + 30° - 90°)$
$= 8\cos(20t - 60°)$
The phasor form is $\mathbf{8\angle{-60°}}$

(c) $20\cos(2t) + 15\sin(2t) = 20\cos(2t) + 15\cos(2t - 90°)$
The phasor form is $20\angle 0° + 15\angle{-90°} = 20 - j15 = \mathbf{25\angle{-36.87°}}$

Solution 9.17

$$V = V_1 + V_2 = 10 < -60^o + 12 < 30^o = 5 - j8.66 + 10.392 + j6 = 15.62 < -9.805^o$$

v(t) = **15.62cos(50t–9.8°) V**

Solution 9.18

(a) $v_1(t) = \mathbf{60\ cos(t + 15°)}$

(b) $\mathbf{V}_2 = 6 + j8 = 10\angle 53.13°$

$v_2(t) = \mathbf{10\ cos(40t + 53.13°)}$

(c) $i_1(t) = \mathbf{2.8\ cos(377t - \pi/3)}$

(d) $\mathbf{I}_2 = -0.5 - j1.2 = 1.3\angle 247.4°$

$i_2(t) = \mathbf{1.3\ cos(10^3 t + 247.4°)}$

Solution 9.19

(a) $3\angle 10° - 5\angle -30° = 2.954 + j0.5209 - 4.33 + j2.5$

$= -1.376 + j3.021$

$= 3.32\angle 114.49°$

Therefore, $3\cos(20t + 10°) - 5\cos(20t - 30°)$

$=$ $\mathbf{3.32\cos(20t + 114.49°)}$

(b) $40\angle -90° + 30\angle -45° = -j40 + 21.21 - j21.21$

$= 21.21 - j61.21$

$= 64.78\angle -70.89°$

Therefore, $40\sin(50t) + 30\cos(50t - 45°) =$ $\mathbf{64.78\cos(50t - 70.89°)}$

(c) Using $\sin\alpha = \cos(\alpha - 90°)$,

$20\angle -90° + 10\angle 60° - 5\angle -110° = -j20 + 5 + j8.66 + 1.7101 + j4.699$

$= 6.7101 - j6.641$

$= 9.44\angle -44.7°$

Therefore, $20\sin(400t) + 10\cos(400t + 60°) - 5\sin(400t - 20°)$

$=$ $\mathbf{9.44\cos(400t - 44.7°)}$

Solution 9.20

7.5cos(10t+30˚) A can be represented by $7.5\angle 30^\circ$ and 120cos(10t+75˚) V can be represented by $120\angle 75^\circ$. Thus,

$\mathbf{Z} = \mathbf{V}/\mathbf{I} = (120\angle 75^\circ)/(7.5\angle 30^\circ) = 16\angle 45^\circ$ or **(11.314+j11.314) Ω**.

Solution 9.21

(a) $F = 5\angle 15^o - 4\angle \underline{-30^o - 90^o} = 6.8296 + j4.758 = 8.3236\angle 34.86^o$

$$\underline{\boldsymbol{f(t) = 8.324\cos(30t + 34.86^o)}}$$

(b) $G = 8\angle -90^o + 4\angle 50^o = 2.571 - j4.9358 = 5.565\angle -62.49^o$

$$\underline{\boldsymbol{g(t) = 5.565\cos(t - 62.49^o)}}$$

(c) $H = \frac{1}{j\omega}\left(10\angle 0^o + 50\angle -90^o\right), \quad \omega = 40$

i.e. $H = 0.25\angle -90^o + 1.25\angle -180^o = -j0.25 - 1.25 = 1.2748\angle -168.69^o$

h(t) **= 1.2748cos(40t – 168.69°)**

Solution 9.22

Let f(t) $=10v(t)+4\dfrac{dv}{dt}-2\displaystyle\int_{-\infty}^{t}v(t)dt$

$$F=10V+j\omega 4V-\frac{2V}{j\omega},\qquad \omega=5,\qquad V=55\angle 45^o$$

$$F=10V+j20V+j0.4V=(10+j20.4)V=22.72\angle 63.89°(55\angle 45°)=1249.6\angle 108.89^o$$

f(t) = **1249.6cos(5t+108.89°)**

Solution 9.23

(a) $v = [110\sin(20t+30^\circ) + 220\cos(20t–90^\circ)]$ V leads to $\mathbf{V} = 110\angle(30^\circ–90^\circ) + 220\angle–90^\circ$ $= 55–j95.26 – j220 = 55–j315.3 = 320.1\angle–80.11^\circ$ or

v = **320.1cos(20t–80.11°) A**.

(b) $i = [30\cos(5t+60^\circ)–20\sin(5t+60^\circ)]$ A leads to $\mathbf{I} = 30\angle60^\circ – 20\angle(60^\circ–90^\circ) =$ $15+j25.98 – (17.321–j10) = –2.321+j35.98 = 36.05\angle93.69^\circ$ or

i = **36.05cos(5t+93.69°) A**.

(a) 320.1cos(20t–80.11°) A, (b) 36.05cos(5t+93.69°) A

Solution 9.24

(a)

$$\mathbf{V} + \frac{\mathbf{V}}{j\omega} = 10\angle 0°, \quad \omega = 1$$

$$\mathbf{V}(1 - j) = 10$$

$$\mathbf{V} = \frac{10}{1 - j} = 5 + j5 = 7.071\angle 45°$$

Therefore,

v(t) = **7.071cos(t + 45°) V**

(b)

$$j\omega\mathbf{V} + 5\mathbf{V} + \frac{4\mathbf{V}}{j\omega} = 20\angle(10° - 90°), \quad \omega = 4$$

$$\mathbf{V}\left(j4 + 5 + \frac{4}{j4}\right) = 20\angle\text{-}80°$$

$$\mathbf{V} = \frac{20\angle\text{-}80°}{5 + j3} = 3.43\angle\text{-}110.96°$$

Therefore,

v(t) = **3.43cos(4t – 110.96°) V**

Solution 9.25

(a)

$$2j\omega\mathbf{I} + 3\mathbf{I} = 4\angle 45°, \quad \omega = 2$$

$$\mathbf{I}(3 + j4) = 4\angle 45°$$

$$I = \frac{4\angle 45°}{3 + j4} = \frac{4\angle 45°}{5\angle 53.13°} = 0.8\angle -8.13°$$

Therefore, i(t) = **800cos(2t – 8.13°) mA**

(b)

$$10\frac{\mathbf{I}}{j\omega} + j\omega\mathbf{I} + 6\mathbf{I} = 5\angle 22°, \quad \omega = 5$$

$$(-j2 + j5 + 6)\mathbf{I} = 5\angle 22°$$

$$\mathbf{I} = \frac{5\angle 22°}{6 + j3} = \frac{5\angle 22°}{6.708\angle 26.56°} = 0.745\angle -4.56°$$

Therefore, i(t) = **745 cos(5t – 4.56°) mA**

Solution 9.26

$$j\omega\mathbf{I} + 2\mathbf{I} + \frac{\mathbf{I}}{j\omega} = 1\angle 0°, \quad \omega = 2$$

$$\mathbf{I}\left(j2 + 2 + \frac{1}{j2}\right) = 1$$

$$\mathbf{I} = \frac{1}{2 + j1.5} = 0.4\angle -36.87°$$

Therefore, i(t) = **0.4 cos(2t – 36.87°)**

Solution 9.27

$$j\omega\mathbf{V} + 50\mathbf{V} + 100\frac{\mathbf{V}}{j\omega} = 110\angle -10°, \quad \omega = 377$$

$$\mathbf{V}\left(j377 + 50 - \frac{j100}{377}\right) = 110\angle -10°$$

$$\mathbf{V}(380.6\angle 82.45°) = 110\angle -10°$$

$$\mathbf{V} = 0.289\angle -92.45°$$

Therefore, v(t) = **289 cos(377t – 92.45°) mV**.

Solution 9.28

Determine the current that flows through a 20-Ω resistor connected in parallel with a voltage source v_s = 120 cos(377t+37˚) V.

Solution

$$i(t) = \frac{v_s(t)}{R} = \frac{120\cos(377t + 37°)}{20} = \mathbf{6cos(377t+37^{\circ})\ A}.$$

Solution 9.29

Given that $v_C(0) = 2\cos(155°)$ V, what is the instantaneous voltage across a 2-μF capacitor when the current through it is $I = 4 \sin(10^6 t + 25°)$ A?

Solution

$$\mathbf{Z} = \frac{1}{j\omega C} = \frac{1}{j(10^6)(2\times 10^{-6})} = -j0.5$$

$$\mathbf{V} = \mathbf{IZ} = (4\angle 25°)(0.5\angle -90°) = 2\angle -65°$$

Therefore $v(t) = \mathbf{2\sin(10^6 t - 65°)\ V}$.

Solution 9.30

Since R and C are in parallel, they have the same voltage across them. For the resistor,

$$V = I_R R \quad \longrightarrow \quad I_R = V/R = \frac{100 < 20^o}{40k} = 2.5 < 20^o \text{ mA}$$

$$i_R = \underline{2.5\cos(60t + 20^o) \text{ mA}}$$

For the capacitor,

$$i_C = C\frac{dv}{dt} = 50x10^{-6}(-60)x100\sin(60t + 20^o) = \underline{-300\sin(60t + 20^o) \text{ mA}}$$

Solution 9.31

A series RLC circuit has R= 80 Ω, L=240 mH, and C = 5mF. If the input voltage is v(t)= 115cos(2t), find the current flowing through the circuit.

Solution

$$L = 240mH \quad \longrightarrow \quad j\omega L = j2x240x10^{-3} = j0.48$$

$$C = 5mF \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j2x5x10^{-3}} = -j100$$

$Z = 80 + j0.48 - j100 = 80 - j99.52 = 127.688\angle{-51.21^\circ}$ Ω.

I = V/Z = $115\angle 0^\circ/(80 - j99.52) = 115/(127.688\angle{-51.21^\circ}) = 0.9006\angle 51.21^\circ$

Thus,

i(t) = **900.6cos(2t+51.21°) mA**

Solution 9.32

Using Fig. 9.40, design a problem to help other students to better understand phasor relationships for circuit elements.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Two elements are connected in series as shown in Fig. 9.40.
If i = 12 cos (2t - 30°) A, find the element values.

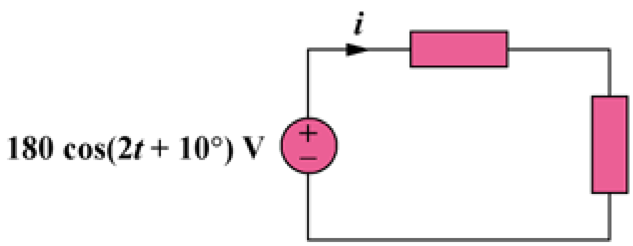


Figure 9.40

Solution

$\mathbf{V} = 180\angle 10°$, $\mathbf{I} = 12\angle -30°$, $\omega = 2$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{180\angle 10°}{12\angle -30°} = 15\angle 40° = 11.49 + j9.642\ \Omega$$

One element is a resistor with R = **11.49 Ω**.
The other element is an inductor with ωL = 9.642 or L = **4.821 H**.

Solution 9.33

A series *RL* circuit is connected to a 220-V ac source. If the voltage across the resistor is 170 V, find the voltage across the inductor.

Solution

$220\angle\theta = v_R + jv_L$ where $220 = \sqrt{v_R^2 + v_L^2}$

$v_L = \sqrt{220^2 - v_R^2}$

$v_L = \sqrt{220^2 - 170^2} =$ **139.64 V**

Solution 9.34

$v_o = 0$ when $jX_L – jX_C = 0$ so $X_L = X_C$ or $\omega L = \dfrac{1}{\omega C} \longrightarrow \omega = \dfrac{1}{\sqrt{LC}}$.

$$\omega = \frac{1}{\sqrt{(5\times10^{-3})(20\times10^{-3})}} = \mathbf{100\ rad/s}$$

Solution 9.35

Find current i in the circuit of Fig. 9.42, when $v_s(t)$ = 115cos(200t) V.

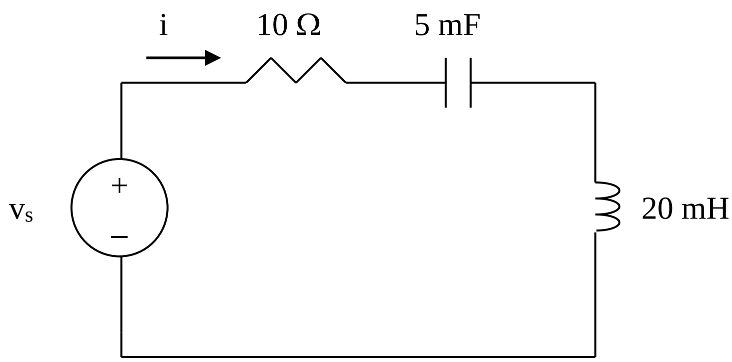


Figure 9.42
For Prob. 9.35.

Solution

$$5mF \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j200x5x10^{-3}} = -j$$

$$20mH \quad \longrightarrow \quad j\omega L = j20x10^{-3}x200 = j4$$

$$Z_{in} = 10 - j + j4 = 10 + j3 = 10.44\angle 16.699°$$

Thus, I = V_s/Z_{in} = 115∠0°/10.44∠16.699° = 11.015∠–16.7°

i(t) = **11.015cos(200t–16.7°) A**

Solution 9.36

Using Fig. 9.43, design a problem to help other students to better understand impedance.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the circuit in Fig. 9.43, determine *i*. Let v_s = 60 cos(200*t* - 10°) V.

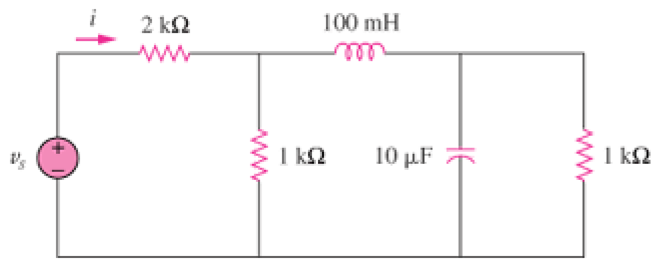


Figure 9.43

Solution

Let Z be the input impedance at the source.

$$100\text{ mH} \longrightarrow j\omega L = j200x100x10^{-3} = j20$$

$$10\mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10x10^{-6}x200} = -j500$$

1000//-j500 = 200 –j400
1000//(j20 + 200 –j400) = 242.62 –j239.84

$$Z = 2242.62 - j239.84 = 2255\angle -6.104^o$$

$$I = \frac{60\angle -10^o}{2255\angle -6.104^o} = 26.61\angle -3.896^o \text{ mA}$$

$$\boldsymbol{i = \underline{266.1\cos(200t - 3.896^o)}\,\text{mA}}$$

Solution 9.37

Determine the admittance **Y** for the circuit in Fig. 9.44.

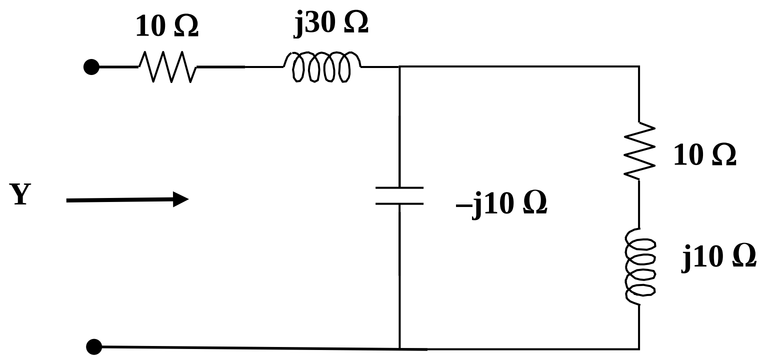


Figure 9.44
For Prob. 9.37.

Solution

Let us start with **Z** = 1/**Y** = 10+j30 + (–j10)(10+j10)/(–j10+10+j10)
= 10 + j30 + (100–j100)/10 = 20+j20 and **Y** = 1/**Z** = 1/28.284∠45°
= 0.035355∠–45° or

Y = 0.025–j0.025 = **(25–j25) mS**.

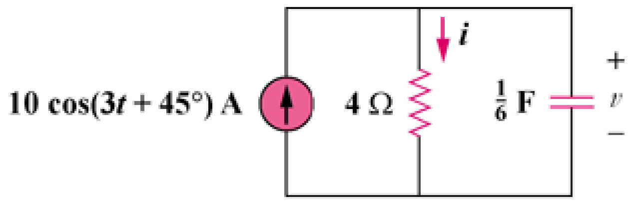
Solution 9.38

Using Fig. 9.45, design a problem to help other students to better understand admittance.

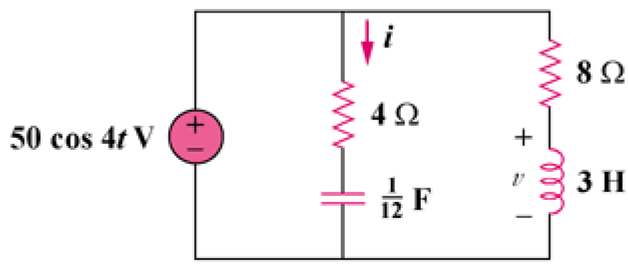
Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find *i*(*t*) and *v*(*t*) in each of the circuits of Fig. 9.45.



(a)



(b)

Figure 9.45

Solution

(a) $\frac{1}{6}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/6)} = \text{-j2}$

$$\mathbf{I} = \frac{\text{- j2}}{4 - \text{j2}}(10\angle 45°) = 4.472\angle\text{-}18.43°$$

Hence, i(t) = **4.472 cos(3t – 18.43°) A**

$$\mathbf{V} = 4\mathbf{I} = (4)(4.472\angle\text{-}18.43°) = 17.89\angle\text{-}18.43°$$

Hence, v(t) = **17.89 cos(3t – 18.43°) V**

(b) $\frac{1}{12}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(1/12)} = \text{-j3}$

$3\text{ H} \longrightarrow j\omega L = j(4)(3) = j12$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\angle 0^\circ}{4 - j3} = 10\angle 36.87^\circ$$

Hence, i(t) = **10 cos(4t + 36.87°) A**

$$\mathbf{V} = \frac{j12}{8 + j12}(50\angle 0^\circ) = 41.6\angle 33.69^\circ$$

Hence, v(t) = **41.6 cos(4t + 33.69°) V**

Solution 9.39

For the circuit shown in Fig. 9.46, find Z_{eq} and use that to find current I. Let ω=10 rad/s.

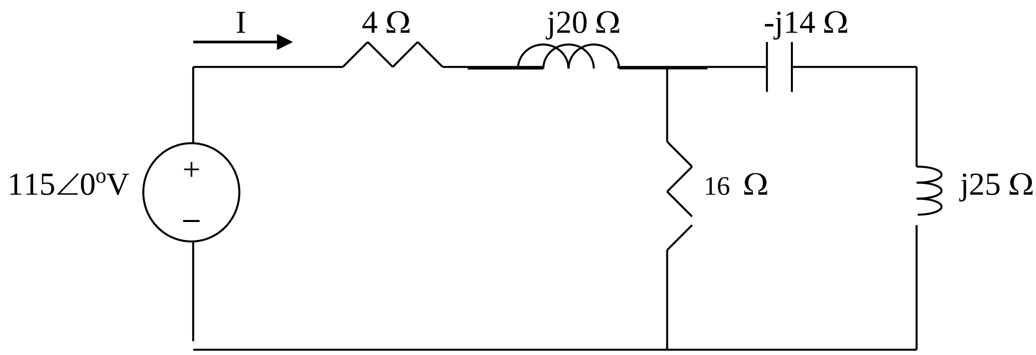


Figure 9.46
For Prob. 9.39.

Solution

Z_{eq} = 4 + j20 + [16(–j14+j25)/(16–j14+j25)] = 4 + j20 + j176(16–j11)/(256+121)
= 4 + j20 + (1,936+j2,816)/377 = (9.135 + j27.47) Ω.

= **(9.135+j27.47) Ω** = **28.95∠71.61° Ω**.

I = V/Z_{eq} = 115/28.95∠71.61° = 3.972∠–71.61°

i(t) = **3.972cos(10t–71.61°) A**

Solution 9.40

In the circuit of Fig. 9.47, find $i_o(t)$ when:

(a) ω = 1 rad/s
(b) ω = 5 rad/s
(c) ω = 10 rad/s

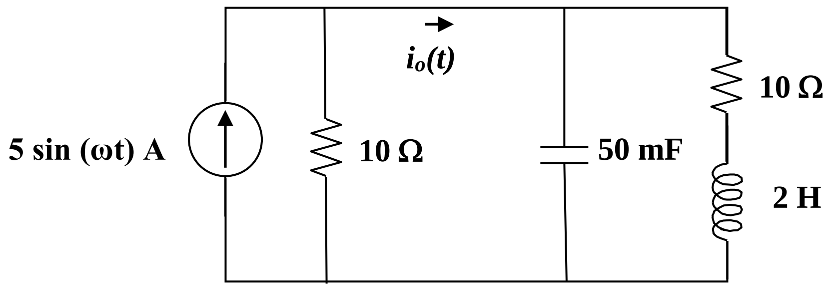


Figure 9.47
For Prob. 9.40.

Solution

It will help to convert the current source resistor into the Thevenin equivalent or 50 V in series with 10 Ω.

(a) For ω = 1 rad/s, the inductor becomes j2 and the capacitor becomes –j20 which leads to Z = 10 + (–j20)(10+j2)/(–j20+10+j2) = 10 + (40–j200)/(10–j18)
= 10 + 203.96$\angle$–78.69°/(20.591$\angle$–60.945°) = 10 + 9.9053$\angle$–17.745°
= 10 + 9.434 – j3.0189 = 19.667$\angle$–8.83°
$I_o = 50/Z_{in}$ = 2.542$\angle$8.83° A or i_o(t) = **2.542sin(t+8.83°) A**.

(b) For ω = 5 rad/s, the inductor becomes j10 and the capacitor becomes –j4 which leads to Z = 10 + (–j4)(10+j10)/(–j4+10+j10) = 10 + (40–j40)/(10+j6)
= 10 + 56.569$\angle$–45°/(11.6619$\angle$30.9638°) = 10 + 4.8508$\angle$–75.964°
= 10 + 1.17647 – j4.706 = 12.12683$\angle$–22.834°
$I_o = 50/Z_{in}$ = 4.1231$\angle$22.83° A or i_o(t) = **4.123sin(5t+22.83°) A**.

(c) For ω = 10 rad/s, the inductor becomes j20 and the capacitor becomes –j2 which leads to Z = 10 + (–j2)(10+j20)/(–j2+10+j20) = 10 + (40–j20)/(10+j18)
= 10 + 44.721$\angle$–26.565°/(20.591$\angle$60.945°) = 10 + 2.1719$\angle$–87.51°
= 10 + 0.0944 – j2.1698 = 10.325$\angle$–12.13°
$\mathbf{I_o} = 50/\mathbf{Z_{in}}$ = 4.843$\angle$12.13° A or i_o(t) = **4.843sin(10t+12.13°) A**.

Solution 9.41

Find $v(t)$ in the *RLC* circuit of Fig. 9.48.

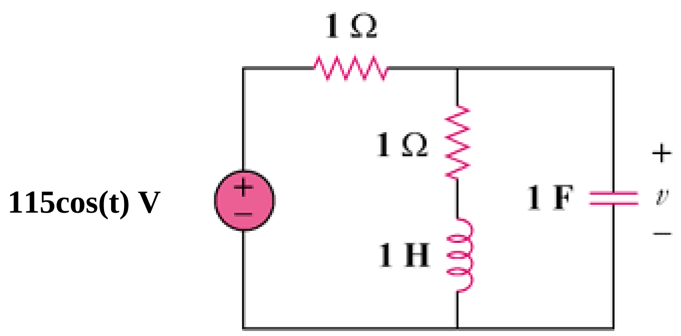


Figure 9.48
For Prob. 9.41.

Solution

$$\omega = 1,$$

$$1\text{ H} \longrightarrow j\omega L = j(1)(1) = j$$

$$1\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1)(1)} = -j$$

$$\mathbf{Z} = 1 + (1+j) \,||\, (-j) = 1 + \frac{-j+1}{1} = 2 - j$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{115}{2-j}, \qquad \mathbf{I}_c = (1+j)\mathbf{I}$$

$$\mathbf{V} = (-j)(1+j)\mathbf{I} = (1-j)\mathbf{I} = \frac{(1-j)(115)}{2-j} = 72.74\angle -18.43°$$

Thus,

$v(t) = $ **72.74cos(t – 18.43°) V**

Solution 9.42

$$\omega = 200$$

$$50\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(50\times 10^{-6})} = -j100$$

$$0.1\ H \longrightarrow j\omega L = j(200)(0.1) = j20$$

$$50 \,||\, -j100 = \frac{(50)(-j100)}{50 - j100} = \frac{-j100}{1 - j2} = 40 - j20$$

$$\mathbf{V}_o = \frac{j20}{j20 + 30 + 40 - j20}(60\angle 0°) = \frac{j20}{70}(60\angle 0°) = 17.14\angle 90°$$

Thus,

$v_o(t) =$ **17.14 sin(200t + 90°) V**

or

$v_o(t) =$ **17.14 cos(200t) V**

Solution 9.43

Find current $\mathbf{I_o}$ in the circuit shown in Fig. 9.50.

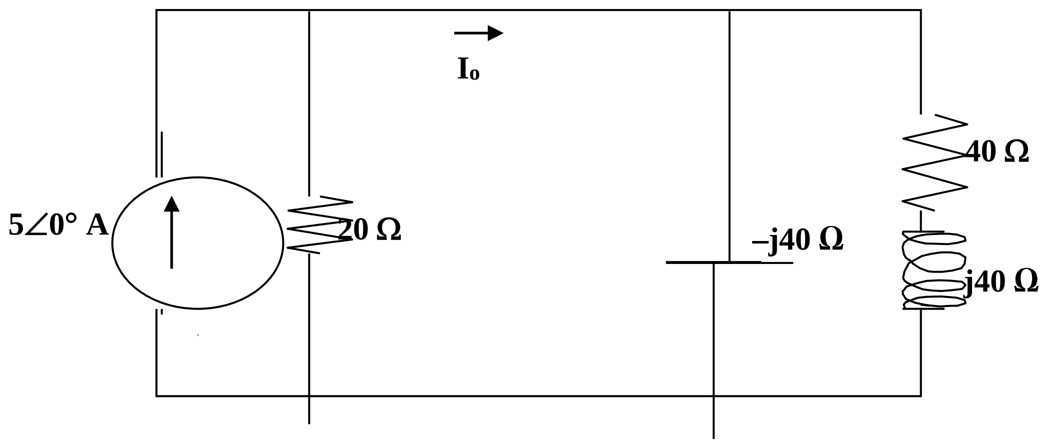


Figure 9.50
For Prob. 9.43.

Solution

First we convert the current source into a voltage source (100∠0° V) in series with 20 Ω. This then gives us an input impedance equal to,

$\mathbf{Z_{in}}$ = 20 + (–j40)(40+j40)/(–j40+40+j40) = 20 + (1,600–j1,600)/40 = 20+40–j40
= 60–j40 = 72.111∠–33.69°.

$\mathbf{I_o}$ = 100/(72.111∠–33.69° = **1.3868∠33.69° A**.

Solution 9.44

Calculate $i(t)$ in the circuit of Fig. 9.51.

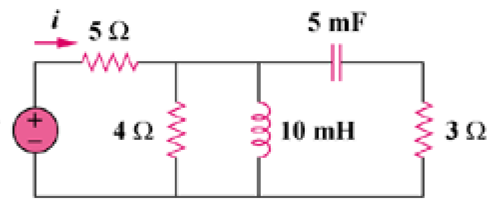


Figure 9.51
For Prob. 9.44.

Solution

$\omega = 200$

$10 \text{ mH} \longrightarrow \quad j\omega L = j(200)(10 \times 10^{-3}) = j2$

$$5 \text{ mF} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j(200)(5 \times 10^{-3})} = -j$$

$$\mathbf{Y} = \frac{1}{4} + \frac{1}{j2} + \frac{1}{3 - j} = 0.25 - j0.5 + \frac{3 + j}{10} = 0.55 - j0.4$$

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{0.55 - j0.4} = 1.1892 + j0.865$$

$$\mathbf{I} = \frac{160\angle 0°}{5 + \mathbf{Z}} = \frac{160\angle 0°}{6.1892 + j0.865} = \frac{160}{6.2494\angle 7.956°} = 25.6\angle -7.956°$$

Thus,
i(t) = **25.6cos(200t – 7.96°) A**

Solution 9.45

Find current $\mathbf{I}_o$ in the network of Fig. 9.52.

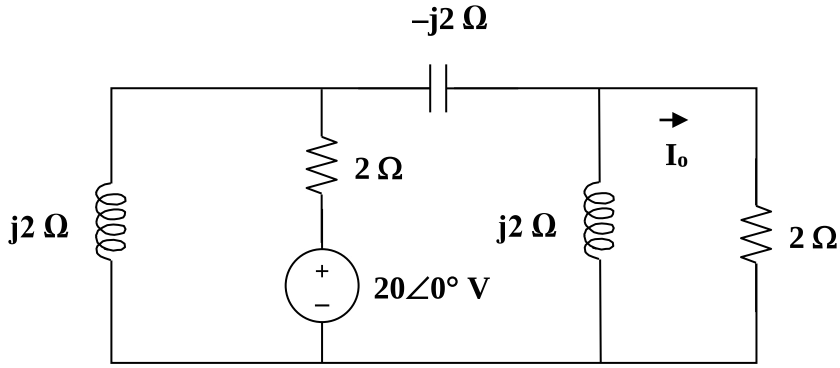


Figure 9.52
For Prob. 9.45.

Solution

There are different ways to solve this problem. We could identify two unknown node voltages and solve for the one that can give us $\mathbf{I_o}$. So, set the bottom as the reference node and let $\mathbf{V_1}$ be the node at the top of the left hand node and $\mathbf{V_2}$ be the node voltage on the right. Thus, $\mathbf{I_o} = \mathbf{V_2}/2$.

The nodal equations are $[(V_1–0)/j2]+[(V_1–20)/2]+[(V_1–V_2)/(–j2)] = 0$ and $[(V_2–V_1)/(–j2)]+[(V_2–0)/j2]+[(V_2–0)/2] = 0$. Simplifying them we get,

$(–j0.5+0.5+j0.5)V_1 – (j0.5)V_2 = 10$ and $–(j0.5)V_1 + (j0.5–j0.5+0.5)V_2 = 0$ or $0.5V_1 – j0.5V_2 = 10$ and $–j0.5V_1 + 0.5V_2 = 0$ or $V_1 = –jV_2$.

Finally, $–j0.5V_2 – j0.5V_2 = 10$ or $V_2 = 10/(–j) = j10$. Thus,

$\mathbf{I_o} = j10/2 = j5 = \mathbf{5\angle 90° \ A}$.

Solution 9.46

If v_s = 100sin(10t+18°) V in the circuit in Fig. 9.53, find i_o.

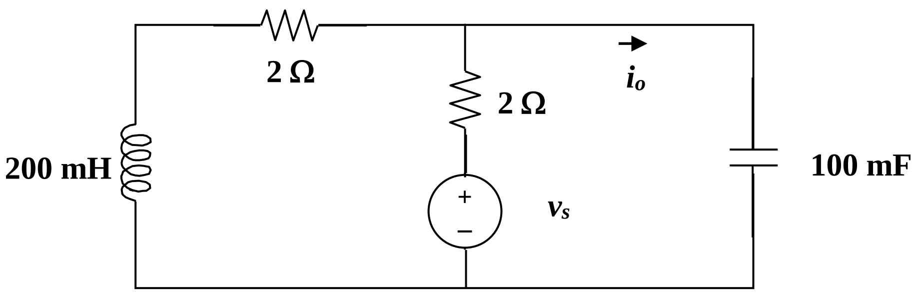


Figure 9.53
For Prob. 9.46.

Solution

Let $V_s = 100\angle 0°$ V (we will account for the 18° when we convert I_o back into the time domain). The inductor becomes j2 Ω and the capacitor becomes –j Ω. We can now write a nodal equation and $I_o = V_C/(-j)$. The nodal equation will give us V_C.

$[(V_C-0)/(2+j2)] + [(V_C-100)/2] + [(V_C-0)/(-j)] = 0$ or
$(0.25-j0.25+0.5+j)V_C = (0.75+j0.75)V_C$
$= (1.06066\angle 45°)V_C = 50$ or $V_C = 47.14\angle -45°$ which leads to

$I_o = 47.14\angle -45°/(-j) = 47.14\angle 45°$. Compensating for the 18° we get,

i_o = **47.14sin(10t+63°) A**.

Solution 9.47

In the circuit shown in Fig. 9.54, determine the value of $i_s(t)$.

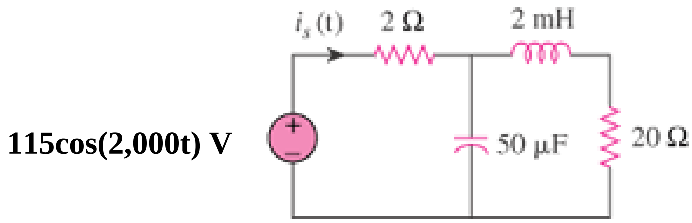
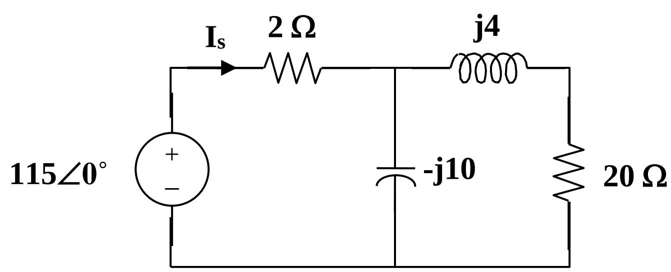


Figure 9.54
For Prob. 9.47.

Solution

First, we convert the circuit into the frequency domain.



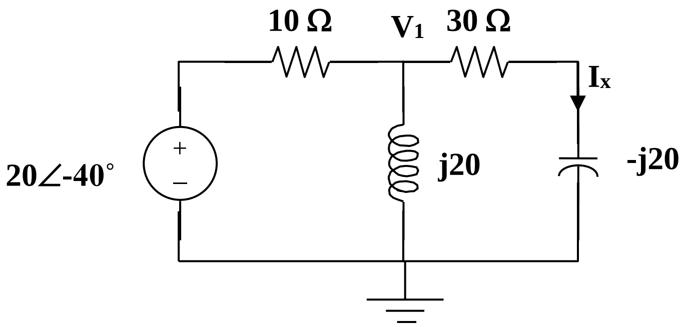
$$I_s = \frac{115}{2+\frac{-j10(20+j4)}{-j10+20+j4}} = \frac{115}{2+\frac{203.961\angle -78.69^\circ}{20.8806\angle -16.699^\circ}} = \frac{115}{2+9.76773\angle -61.991^\circ}$$

$$= \frac{115}{2+4.58703-j8.623673} = \frac{115}{10.8516\angle -52.63^\circ} = 10.598\angle 52.63^\circ$$

$i_s(t)$ = **10.598cos(2000t +52.63°) A**

Solution 9.48

Converting the circuit to the frequency domain, we get:



We can solve this using nodal analysis.

$$\frac{V_1 - 20\angle -40^\circ}{10} + \frac{V_1 - 0}{j20} + \frac{V_1 - 0}{30 - j20} = 0$$

$$V_1(0.1 - j0.05 + 0.02307 + j0.01538) = 2\angle -40^\circ$$

$$V_1 = \frac{2\angle 40^\circ}{0.12307 - j0.03462} = 15.643\angle -24.29^\circ$$

$$I_x = \frac{15.643\angle -24.29^\circ}{30 - j20} = 0.4338\angle 9.4^\circ$$

$$i_x = \underline{0.4338\sin(100t + 9.4^\circ)\ A}$$

Solution 9.49

Find $v_s(t)$ in the circuit of Fig. 9.56 if the current i_x through the 1-Ω resistor is 8 sin 200*t* A.

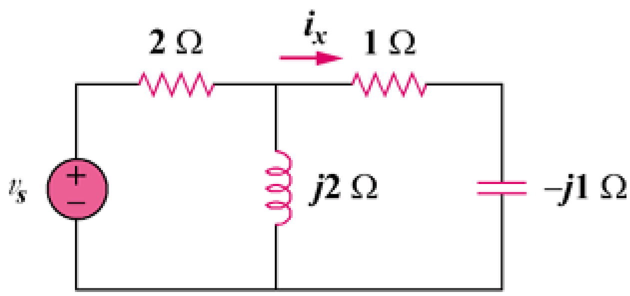
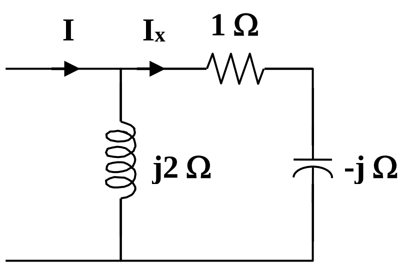


Figure 9.56
For Prob. 9.49.

Solution

$$\mathbf{Z}_T = 2 + j2 \,||\, (1 - j) = 2 + \frac{(j2)(1-j)}{1+j} = 4$$



$$\mathbf{I}_x = \frac{j2}{j2+1-j}\mathbf{I} = \frac{j2}{1+j}\mathbf{I}, \qquad \text{where } \mathbf{I}_x = 8\angle 0° = 8$$

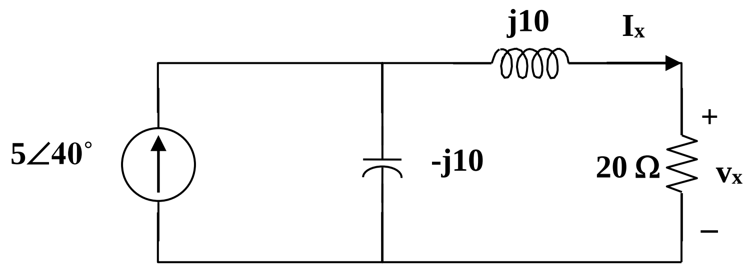
$$\mathbf{I} = \frac{1+j}{j2}\mathbf{I}_x = \frac{8+j8}{j2}$$

$$\mathbf{V}_s = \mathbf{I}\,\mathbf{Z}_T = \frac{8+j8}{j2}(4) = \frac{16(1+j)}{j} = 16(1-j) = 22.627\angle -45°$$

$v_s(t) =$ **22.63 sin(200t – 45°) V**

Solution 9.50

Since ω = 100, the inductor = j100x0.1 = j10 Ω and the capacitor = 1/(j100x10^{-3}) = -j10Ω.



Using the current dividing rule:

$$I_x = \frac{-j10}{-j10+20+j10} 5\angle 40° = -j2.5\angle 40° = 2.5\angle -50°$$
$$V_x = 20I_x = 50\angle -50°$$

$v_x(t)$ = **50cos(100t–50°) V**

Solution 9.51

If the voltage v_o across the 2-Ω resistor in the circuit of Fig. 9.58 is 90cos(2t) V, obtain i_s.

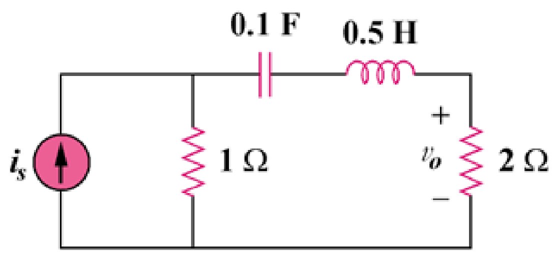


Figure 9.58
For Prob. 9.51.

Solution

$$0.1\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(0.1)} = \text{-j5}$$

$$0.5\text{ H} \longrightarrow j\omega L = j(2)(0.5) = j$$

The current **I** through the 2-Ω resistor is

$$\mathbf{I} = \frac{1}{1 - j5 + j + 2}\mathbf{I}_s = \frac{\mathbf{I}_s}{3 - j4}, \qquad \text{where} \quad \mathbf{I} = \frac{90}{2}\angle 0° = 45$$

$$\mathbf{I}_s = (45)(3 - j4) = 225\angle\text{-}53.13°$$

Therefore,

$$i_s(t) = \mathbf{225cos(2t – 53.13°) A}$$

Solution 9.52

If $\mathbf{V}_o = 8\angle 30°$ V in the circuit of Fig. 9.59, find $\mathbf{V}_s$.

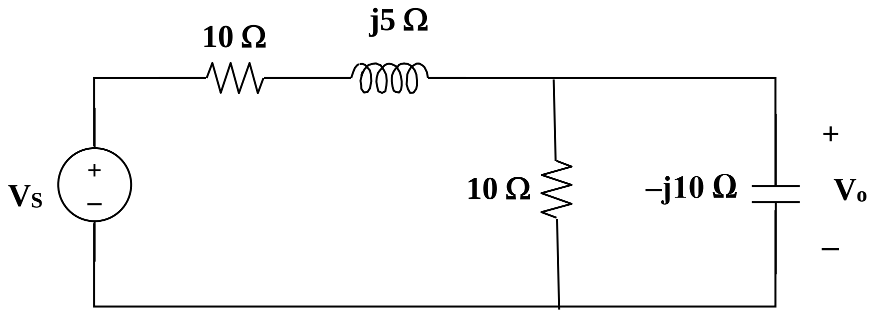


Figure 9.59
For Prob. 9.52

Solution

We can treat $\mathbf{V}_o$ as the node voltage for the circuit and then write the node equations. We get,

$[(\mathbf{V}_o - \mathbf{V}_s)/(10+j5)] + [(\mathbf{V}_o - 0)/10] + [(\mathbf{V}_o - 0)/(-j10)] = 0$ this leads to
$\mathbf{V}_s/(10+j5) = (0.08 - j0.04 + 0.1 + j0.1)\mathbf{V}_o = (0.18+j0.06)8\angle 30°$ or
$\mathbf{V}_s = [(0.189737\angle 18.435°)(8\angle 30°)(11.18034\angle 26.565°$ thus,

$$\mathbf{V}_s = \mathbf{16.971\angle 75°\ V}.$$

Solution 9.53

Find $\mathbf{I}_0$ in the circuit in Fig. 9.60.

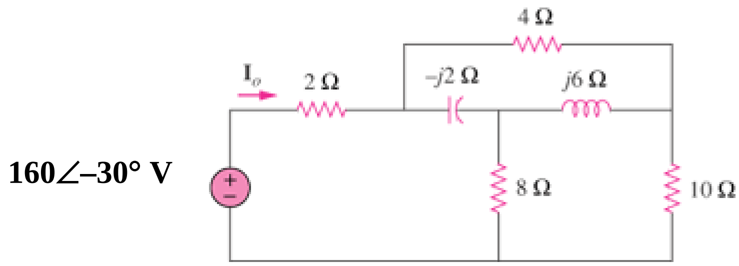
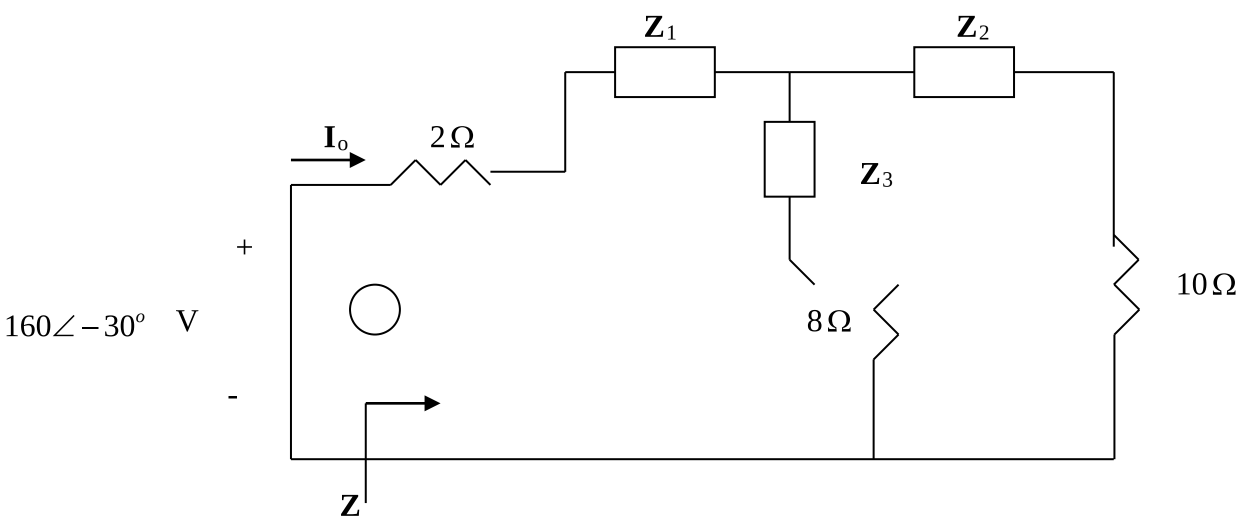


Figure 9.60
For Prob. 9.53.

Solution

Convert the delta to wye subnetwork as shown below.



$$Z_1 = \frac{-j2x4}{4+j4} = \frac{8\angle -90^\circ}{5.6569\angle 45^\circ} = -1 - j1, \qquad Z_2 = \frac{j6x4}{4+j4} = 3 + j3,$$

$$Z_3 = \frac{12}{4+j4} = 1.5 - j1.5$$

$$(Z_3 + 8)//(Z_2 + 10) = (9.5 - j1.5)//(13 + j3) = 5.691\angle 0.21^\circ = 5.691 + j0.02086$$

$$Z = 2 + Z_1 + 5.691 + j0.02086 = 6.691 - j0.9791$$

$$\mathbf{I_o} = \frac{160\angle -30^o}{Z} = \frac{160\angle -30^o}{6.7623\angle -8.33^o} = \mathbf{23.66\angle{-21.67^o}\ A}.$$

Solution 9.54

In the circuit of Fig. 9.61, find $\mathbf{V}_s$ if $\mathbf{I}_o = 30\angle 0°$ A.

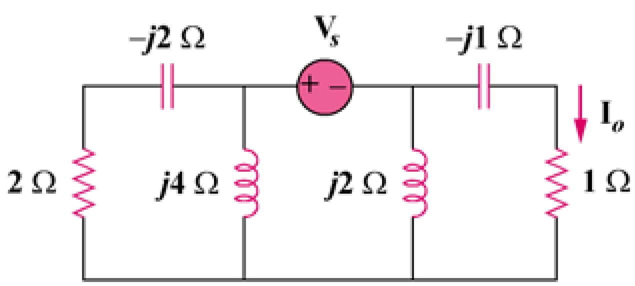
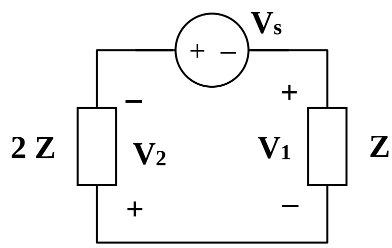


Figure 9.61
For Prob. 9.54.

Solution

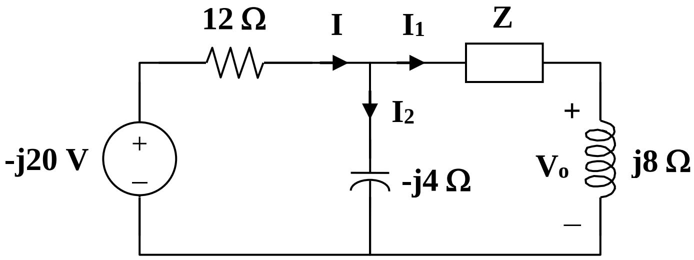
Since the left portion of the circuit is twice as large as the right portion, the equivalent circuit is shown below.



$$\mathbf{V}_1 = \mathbf{I}_o(1-j) = 30(1-j)$$
$$\mathbf{V}_2 = 2\mathbf{V}_1 = 60(1-j)$$
$$\mathbf{V_2} + \mathbf{V_s} + \mathbf{V_1} = 0 \text{ or}$$
$$\mathbf{V}_s = -\mathbf{V}_1 - \mathbf{V}_2 = -90(1-j) = (90\angle 180°)(1.4142\angle -45°)$$

$$\mathbf{V}_s = \mathbf{127.28\angle 135°\ V}$$

Solution 9.55



$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{j8} = \frac{4}{j8} = \text{-j}0.5$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_1(\mathbf{Z} + j8)}{-j4} = \frac{(\text{-j}0.5)(\mathbf{Z} + j8)}{-j4} = \frac{\mathbf{Z}}{8} + j$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \text{-j}0.5 + \frac{\mathbf{Z}}{8} + j = \frac{\mathbf{Z}}{8} + j0.5$$

$$-j20 = 12\mathbf{I} + \mathbf{I}_1(\mathbf{Z} + j8)$$

$$-j20 = 12\left(\frac{\mathbf{Z}}{8} + \frac{j}{2}\right) + \frac{-j}{2}(\mathbf{Z} + j8)$$

$$-4 - j26 = \mathbf{Z}\left(\frac{3}{2} - j\frac{1}{2}\right)$$

$$\mathbf{Z} = \frac{-4 - j26}{\frac{3}{2} - j\frac{1}{2}} = \frac{26.31\angle 261.25^\circ}{1.5811\angle -18.43^\circ} = 16.64\angle 279.68^\circ$$

Z = (2.798 – j16.403) Ω

Solution 9.56

$$50\mu F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j377x50x10^{-6}} = -j53.05$$

$$60mH \quad \longrightarrow \quad j\omega L = j377x60x10^{-3} = j22.62$$

$$Z_{in} = 12 - j53.05 + j22.62 // 40 = \underline{21.692 - j35.91\ \Omega}$$

Solution 9.57

2H $\longrightarrow$ $j\omega L = j2$

1F $\longrightarrow$ $\dfrac{1}{j\omega C} = -j$

$$Z = 1 + j2//(2 - j) = 1 + \frac{j2(2 - j)}{j2 + 2 - j} = 2.6 + j1.2$$

$$\boldsymbol{Y = {}^{1}\!/\!_{Z} = \underline{0.3171 - j0.1463\ \ S}}$$

Solution 9.58

Using Fig. 9.65, design a problem to help other students to better understand impedance combinations.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

At $\omega = 50$ rad/s, determine $\mathbf{Z}_{in}$ for each of the circuits in Fig. 9.65.

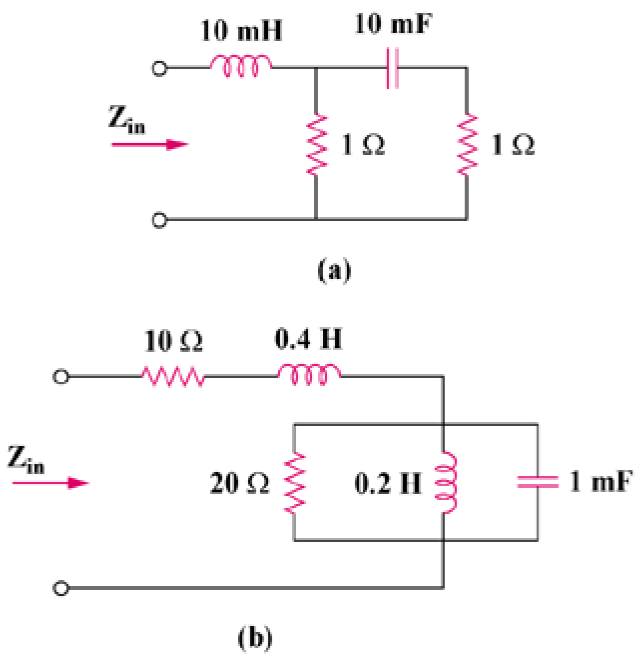


Figure 9.65

Solution

(a) 10 mF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j(50)(10\times10^{-3})} = -j2$

10 mH $\longrightarrow$ $j\omega L = j(50)(10\times10^{-3}) = j0.5$

$$\mathbf{Z}_{in} = j0.5 + 1 \,||\, (1 - j2)$$
$$\mathbf{Z}_{in} = j0.5 + \frac{1 - j2}{2 - j2}$$
$$\mathbf{Z}_{in} = j0.5 + 0.25(3 - j)$$
$$\mathbf{Z}_{in} = \mathbf{0.75 + j0.25\ \Omega}$$

(b) $0.4\text{ H} \longrightarrow j\omega L = j(50)(0.4) = j20$

$0.2\text{ H} \longrightarrow j\omega L = j(50)(0.2) = j10$

$$1\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(50)(1\times 10^{-3})} = -j20$$

For the parallel elements,

$$\frac{1}{\mathbf{Z}_p} = \frac{1}{20} + \frac{1}{j10} + \frac{1}{-j20}$$

$$\mathbf{Z}_p = 10 + j10$$

Then,

$$\mathbf{Z}_{in} = 10 + j20 + \mathbf{Z}_p = \mathbf{20 + j30\ \Omega}$$

Solution 9.59

For the network in Fig. 9.66, find $\mathbf{Z}_{in}$. Let $\omega = 100$ rad/s.

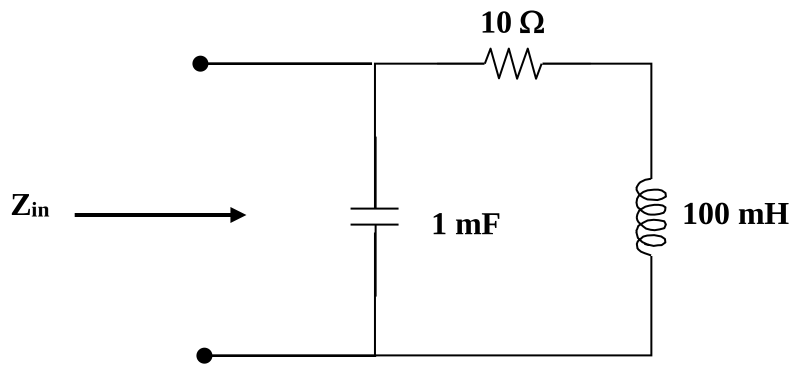


Figure 9.66
For Prob. 9.59.

Solution

At $\omega = 100$ rad/s the capacitor becomes –j10 Ω and the inductor becomes j10 Ω. This then leads to $Z_{in} = (-j10)(10+j10)/(-j10+10+j10) = (100-j100)/10$ or

$$Z_{in} = \mathbf{(10 - j10)\ \Omega = 14.142\angle{-45^\circ}\ \Omega}.$$

Solution 9.60

$$Z = (25 + j15) + (20 - j50)//(30 + j10) = 25 + j15 + 26.097 - j5.122$$

Z = **(51.1+j9.878) Ω**

Solution 9.61

All of the impedances are in parallel.

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{1-j} + \frac{1}{1+j2} + \frac{1}{j5} + \frac{1}{1+j3}$$

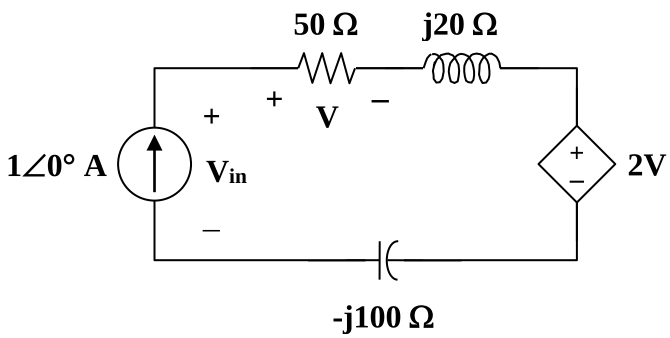
$$\frac{1}{\mathbf{Z}_{eq}} = (0.5 + j0.5) + (0.2 - j0.4) + (-j0.2) + (0.1 - j0.3) = 0.8 - j0.4$$

$$\mathbf{Z}_{eq} = \frac{1}{0.8 - j0.4} = \mathbf{(1 + j0.5)\ \Omega}$$

Solution 9.62

$$2\text{ mH} \longrightarrow \quad j\omega L = j(10\times 10^3)(2\times 10^{-3}) = j20$$

$$1\ \mu\text{F} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j(10\times 10^3)(1\times 10^{-6})} = -j100$$



$$\mathbf{V} = (1\angle 0°)(50) = 50$$

$$\mathbf{V}_{in} = (1\angle 0°)(50 + j20 - j100) + (2)(50)$$
$$\mathbf{V}_{in} = 50 - j80 + 100 = 150 - j80$$

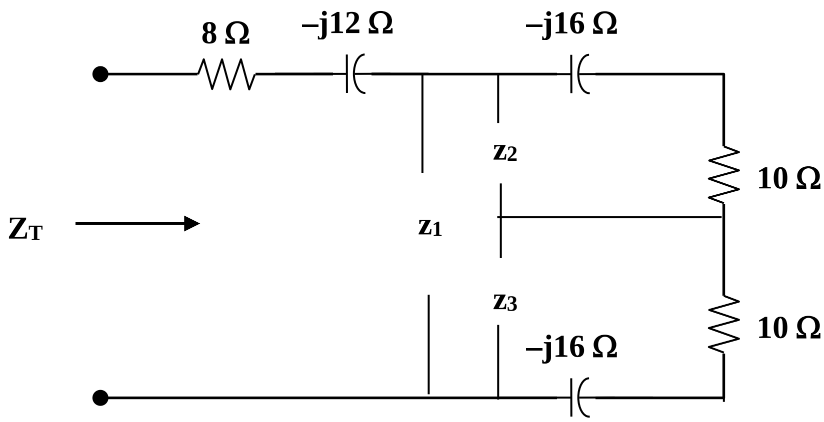
$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{1\angle 0°} = \mathbf{150 - j80\ \Omega}$$

Solution 9.63

First, replace the wye composed of the 20-ohm, 10-ohm, and j15-ohm impedances with the corresponding delta.

$$z_1 = \frac{200 + j150 + j300}{10} = 20 + j45$$

$$z_2 = \frac{200 + j450}{j15} = 30 - j13.333, \; z_3 = \frac{200 + j450}{20} = 10 + j22.5$$



Now all we need to do is to combine impedances.

$$z_2 \| (10 - j16) = \frac{(30 - j13.333)(10 - j16)}{40 - j29.33} = 8.721 - j8.938$$

$$z_3 \| (10 - j16) = 21.70 - j3.821$$

$$Z_T = 8 - j12 + z_1 \| (8.721 - j8.938 + 21.7 - j3.821) = \underline{34.69 - j6.93\Omega}$$

Solution 9.64

Find $\mathbf{Z_T}$ and **V** in the circuit shown in Fig. 9.71. Let the value of the inductance be j20 Ω.

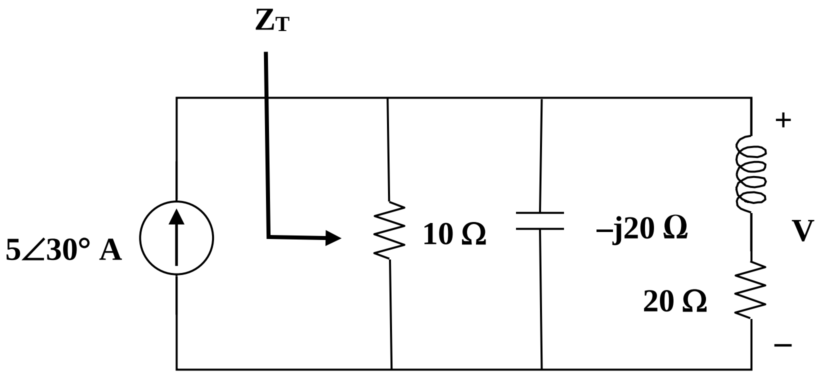


Figure 9.71
For Prob. 9.64.

Solution

$[1/\mathbf{Z_T}] = 0.1 + j0.05 + 0.025 – j0.025 = 0.125 + j0.025 = 0.127475\angle 11.31°$ or
$\mathbf{Z_T} = 7.8447\angle –11.31° = (7.6924 – j1.53848)$.

$\mathbf{Z_T}$ = **(7.692 – j1.5385) Ω**

$\mathbf{V} = \mathbf{I}x\mathbf{Z_T} = (5\angle 30°)(7.8447\angle –11.31°) =$ **39.22∠18.69° V**.

Solution 9.65

$$\mathbf{Z}_{\mathrm{T}} = 2 + (4 - \mathrm{j}6) \,||\, (3 + \mathrm{j}4)$$

$$\mathbf{Z}_{\mathrm{T}} = 2 + \frac{(4 - \mathrm{j}6)(3 + \mathrm{j}4)}{7 - \mathrm{j}2}$$

$$\mathbf{Z}_{\mathrm{T}} = \mathbf{6.83 + j1.094\ \Omega} = 6.917\angle 9.1^\circ\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_{\mathrm{T}}} = \frac{120\angle 10^\circ}{6.917\angle 9.1^\circ} = \mathbf{17.35\angle 0.9^\circ\ A}$$

Solution 9.66

For the circuit in Fig. 9.73, calculate $\mathbf{Z}_T$ and $\mathbf{V}_{ab}$.

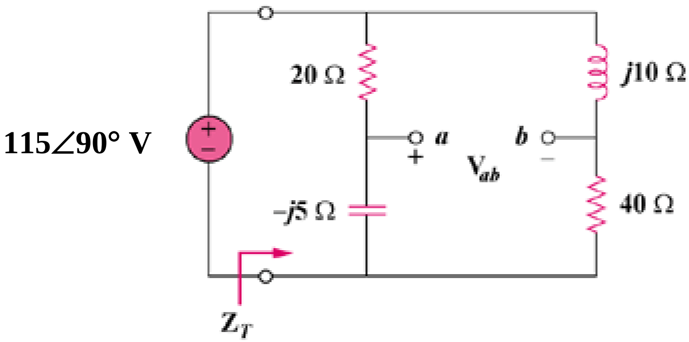


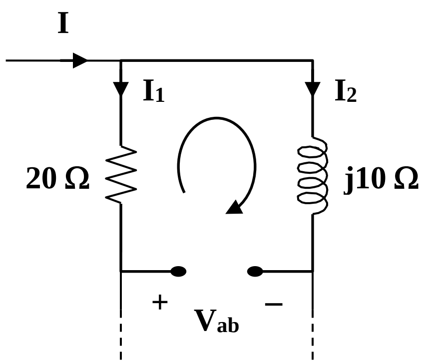
Figure 9.73
For Prob. 9.66.

Solution

$$\mathbf{Z}_T = (20 - j5) \,||\, (40 + j10) = \frac{(20 - j5)(40 + j10)}{60 + j5} = \frac{170}{145}(12 - j)$$

$$\mathbf{Z}_T = \mathbf{14.069 - j1.172\ \Omega} = \mathbf{14.118\angle\text{-}4.76°\ \Omega}$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{115\angle 90°}{14.118\angle -4.76°} = 8.1456\angle 94.76°$$



$$\mathbf{I}_1 = \frac{40 + j10}{60 + j5}\mathbf{I} = \frac{8 + j2}{12 + j}\mathbf{I}$$

$$\mathbf{I}_2 = \frac{20 - j5}{60 + j5}\mathbf{I} = \frac{4 - j}{12 + j}\mathbf{I}$$

$$\mathbf{V}_{ab} = -20\mathbf{I}_1 + j10\mathbf{I}_2$$

$$\mathbf{V}_{ab} = \frac{-(160 + j40)}{12 + j}\mathbf{I} + \frac{10 + j40}{12 + j}\mathbf{I}$$

$$\mathbf{V}_{ab} = \frac{-150}{12 + j}\mathbf{I} = \frac{(-12 + j)(150)}{145}\mathbf{I}$$

$$\mathbf{V}_{ab} = (12.457\angle 175.24°)(8.1456\angle 97.76°)$$

$$\mathbf{V}_{ab} = \mathbf{101.47\angle 273°\ V}$$

Solution 9.67

(a) 20 mH $\longrightarrow$ $j\omega L = j(10^3)(20 \times 10^{-3}) = j20$

12.5 μF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j(10^3)(12.5 \times 10^{-6})} = -j80$

$$\mathbf{Z}_{in} = 60 + j20 \,||\, (60 - j80)$$
$$\mathbf{Z}_{in} = 60 + \frac{(j20)(60 - j80)}{60 - j60}$$
$$\mathbf{Z}_{in} = 63.33 + j23.33 = 67.494\angle 20.22°$$

$$\mathbf{Y}_{in} = \frac{1}{\mathbf{Z}_{in}} = \mathbf{14.8\angle\text{-}20.22° \ mS}$$

(b) 10 mH $\longrightarrow$ $j\omega L = j(10^3)(10 \times 10^{-3}) = j10$

20 μF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j(10^3)(20 \times 10^{-6})} = -j50$

$30 \,||\, 60 = 20$

$$\mathbf{Z}_{in} = -j50 + 20 \,||\, (40 + j10)$$
$$\mathbf{Z}_{in} = -j50 + \frac{(20)(40 + j10)}{60 + j10} = –j50 + 20(41.231\angle 14.036°)/(60.828\angle 9.462°)$$
$= –j50 + (13.5566\angle 4.574° = –j50 + 13.51342 + j1.08109$
$= 13.51342 – j48.9189 = 50.751\angle –74.56°$
$$\mathbf{Z}_{in} = 13.5 - j48.92 = 50.75\angle -74.56°$$

$$\mathbf{Y}_{in} = \frac{1}{\mathbf{Z}_{in}} = \mathbf{19.704\angle 74.56° \ mS} \ = \ 5.246 + j18.993 \text{ mS}$$

Solution 9.68

$$\mathbf{Y}_{eq} = \frac{1}{5 - j2} + \frac{1}{3 + j} + \frac{1}{-j4}$$

$$\mathbf{Y}_{eq} = (0.1724 + j0.069) + (0.3 - j0.1) + (j0.25)$$

$$\mathbf{Y}_{eq} = \mathbf{(472.4 + j219)\ mS}$$

Solution 9.69

$$\frac{1}{\mathbf{Y}_o} = \frac{1}{4} + \frac{1}{-j2} = \frac{1}{4}(1 + j2)$$

$$\mathbf{Y}_o = \frac{4}{1 + j2} = \frac{(4)(1 - j2)}{5} = 0.8 - j1.6$$

$$\mathbf{Y}_o + j = 0.8 - j0.6$$

$$\frac{1}{\mathbf{Y}_o'} = \frac{1}{1} + \frac{1}{-j3} + \frac{1}{0.8 - j0.6} = (1) + (j0.333) + (0.8 + j0.6)$$

$$\frac{1}{\mathbf{Y}_o'} = 1.8 + j0.933 = 2.028\angle 27.41°$$

$$\mathbf{Y}_o' = 0.4932\angle -27.41° = 0.4378 - j0.2271$$

$$\mathbf{Y}_o' + j5 = 0.4378 + j4.773$$

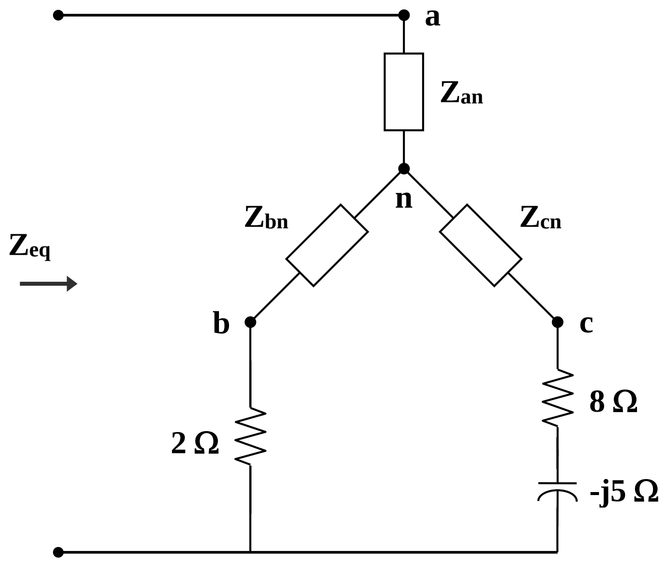
$$\frac{1}{\mathbf{Y}_{eq}} = \frac{1}{2} + \frac{1}{0.4378 + j4.773} = 0.5 + \frac{0.4378 - j4.773}{22.97}$$

$$\frac{1}{\mathbf{Y}_{eq}} = 0.5191 - j0.2078$$

$$\mathbf{Y}_{eq} = \frac{0.5191 - j0.2078}{0.3126} = \mathbf{(1.661 + j0.6647)\ S}$$

Solution 9.70

Make a delta-to-wye transformation as shown in the figure below.



$$\mathbf{Z}_{an} = \frac{(\text{-}j10)(10+j15)}{5-j10+10+j15} = \frac{(10)(15-j10)}{15+j5} = 7-j9$$

$$\mathbf{Z}_{bn} = \frac{(5)(10+j15)}{15+j5} = 4.5+j3.5$$

$$\mathbf{Z}_{cn} = \frac{(5)(\text{-}j10)}{15+j5} = \text{-}1-j3$$

$$\mathbf{Z}_{eq} = \mathbf{Z}_{an} + (\mathbf{Z}_{bn}+2) \,||\, (\mathbf{Z}_{cn}+8-j5)$$

$$\mathbf{Z}_{eq} = 7-j9+(6.5+j3.5) \,||\, (7-j8)$$

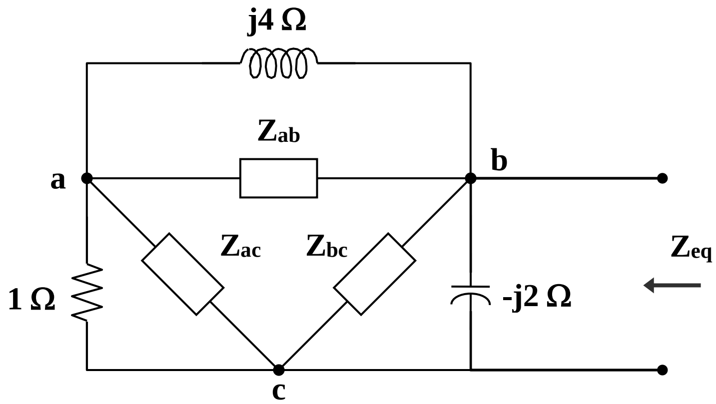
$$\mathbf{Z}_{eq} = 7-j9+\frac{(6.5+j3.5)(7-j8)}{13.5-j4.5}$$

$$\mathbf{Z}_{eq} = 7-j9+5.511-j0.2$$

$$\mathbf{Z}_{eq} = 12.51-j9.2 = \mathbf{15.53\angle\text{-}36.33°\ \Omega}$$

Solution 9.71

We apply a wye-to-delta transformation.



$$\mathbf{Z}_{ab} = \frac{2 - j2 + j4}{j2} = \frac{2 + j2}{j2} = 1 - j$$

$$\mathbf{Z}_{ac} = \frac{2 + j2}{2} = 1 + j$$

$$\mathbf{Z}_{bc} = \frac{2 + j2}{-j} = -2 + j2$$

$$j4 \,||\, \mathbf{Z}_{ab} = j4 \,||\, (1 - j) = \frac{(j4)(1 - j)}{1 + j3} = 1.6 - j0.8$$

$$1 \,||\, \mathbf{Z}_{ac} = 1 \,||\, (1 + j) = \frac{(1)(1 + j)}{2 + j} = 0.6 + j0.2$$

$$j4 \,||\, \mathbf{Z}_{ab} + 1 \,||\, \mathbf{Z}_{ac} = 2.2 - j0.6$$

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{-j2} + \frac{1}{-2 + j2} + \frac{1}{2.2 - j0.6}$$

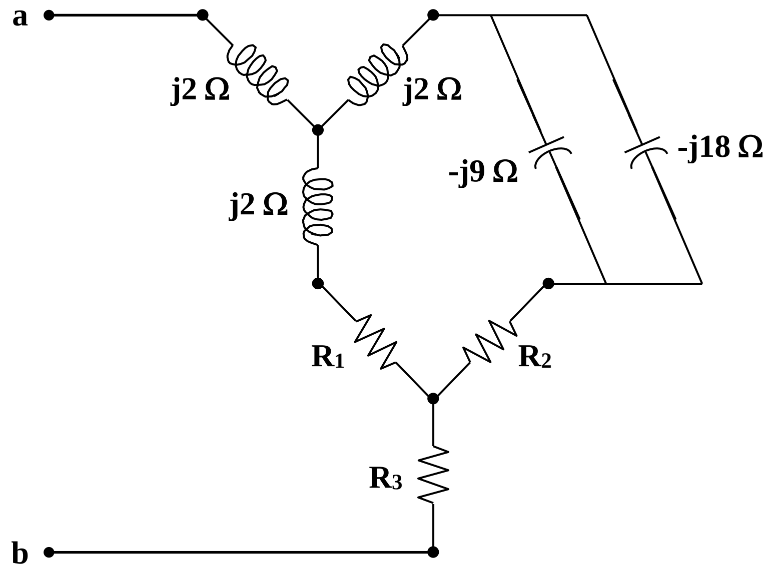
$$= j0.5 - 0.25 - j0.25 + 0.4231 + j0.1154$$

$$= 0.173 + j0.3654 = 0.4043\angle 64.66^\circ$$

$$\mathbf{Z}_{eq} = 2.473\angle\text{-}64.66^\circ\ \Omega = \mathbf{(1.058 - j2.235)\ \Omega}$$

Solution 9.72

Transform the delta connections to wye connections as shown below.



$- j9 \,||\, -j18 = -j6,$

$$R_1 = \frac{(20)(20)}{20+20+10} = 8\,\Omega, \qquad R_2 = \frac{(20)(10)}{50} = 4\,\Omega, \qquad R_3 = \frac{(20)(10)}{50} = 4\,\Omega$$

$$\mathbf{Z}_{ab} = j2 + (j2+8) \,||\, (j2 - j6 + 4) + 4$$

$$\mathbf{Z}_{ab} = 4 + j2 + (8 + j2) \,||\, (4 - j4)$$

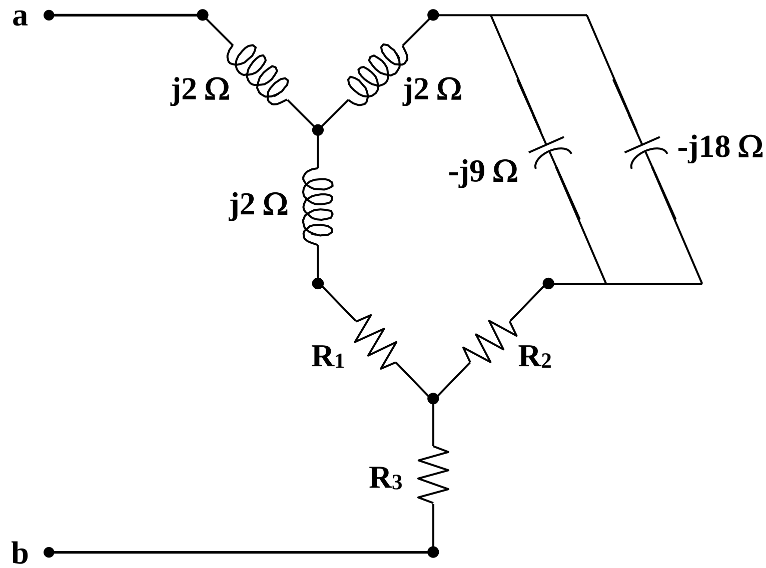
$$\mathbf{Z}_{ab} = 4 + j2 + \frac{(8+j2)(4-j4)}{12 - j2}$$

$$\mathbf{Z}_{ab} = 4 + j2 + 3.567 - j1.4054$$

$$\mathbf{Z}_{ab} = \mathbf{(7.567 + j0.5946)\ \Omega}$$

Solution 9.73

Transform the delta connection to a wye connection as in Fig. (a) and then transform the wye connection to a delta connection as in Fig. (b).



$$\mathbf{Z}_1 = \frac{(j8)(-j6)}{j8 + j8 - j6} = \frac{48}{j10} = -j4.8$$
$$\mathbf{Z}_2 = \mathbf{Z}_1 = -j4.8$$
$$\mathbf{Z}_3 = \frac{(j8)(j8)}{j10} = \frac{-64}{j10} = j6.4$$

$$(2 + \mathbf{Z}_1)(4 + \mathbf{Z}_2) + (4 + \mathbf{Z}_2)(\mathbf{Z}_3) + (2 + \mathbf{Z}_1)(\mathbf{Z}_3) =$$
$$(2 - j4.8)(4 - j4.8) + (4 - j4.8)(j6.4) + (2 - j4.8)(j6.4) = 46.4 + j9.6$$

$$\mathbf{Z}_a = \frac{46.4 + j9.6}{j6.4} = 1.5 - j7.25$$
$$\mathbf{Z}_b = \frac{46.4 + j9.6}{4 - j4.8} = 3.574 + j6.688$$
$$\mathbf{Z}_c = \frac{46.4 + j9.6}{2 - j4.8} = 1.727 + j8.945$$

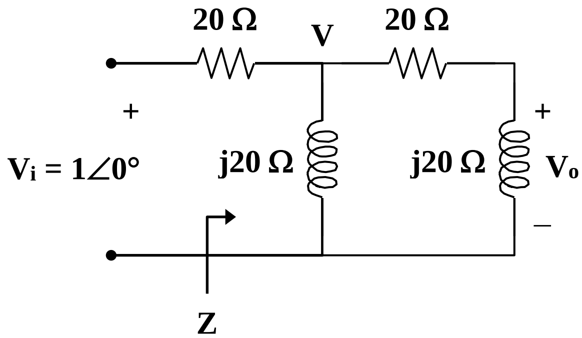
$$j6 \,\|\, \mathbf{Z}_b = \frac{(6\angle 90°)(7.583\angle 61.88°)}{3.574 + j12.688} = 07407 + j3.3716$$
$$-j4 \,\|\, \mathbf{Z}_a = \frac{(-j4)(1.5 - j7.25)}{1.5 - j11.25} = 0.186 - j2.602$$

$$j12 \parallel \mathbf{Z}_c = \frac{(12\angle 90°)(9.11\angle 79.07°)}{1.727 + j20.945} = 0.5634 + j5.1693$$

$$\mathbf{Z}_{eq} = (j6 \parallel \mathbf{Z}_b) \parallel (-j4 \parallel \mathbf{Z}_a + j12 \parallel \mathbf{Z}_c)$$
$$\mathbf{Z}_{eq} = (0.7407 + j3.3716) \parallel (0.7494 + j2.5673)$$
$$\mathbf{Z}_{eq} = 1.508\angle 75.42°\ \Omega = \mathbf{(0.3796 + j1.46)\ \Omega}$$

Solution 9.74

One such RL circuit is shown below.



We now want to show that this circuit will produce a 90° phase shift.

$$\mathbf{Z} = \text{j}20 \,||\, (20 + \text{j}20) = \frac{(\text{j}20)(20 + \text{j}20)}{20 + \text{j}40} = \frac{-20 + \text{j}20}{1 + \text{j}2} = 4(1 + \text{j}3)$$

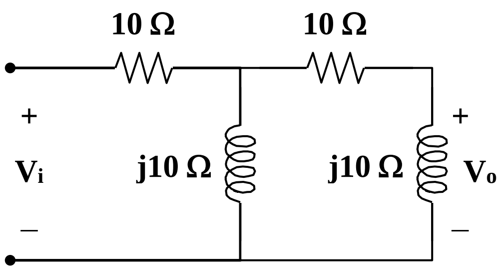
$$\mathbf{V} = \frac{\mathbf{Z}}{\mathbf{Z} + 20}\mathbf{V}_\text{i} = \frac{4 + \text{j}12}{24 + \text{j}12}(1\angle 0°) = \frac{1 + \text{j}3}{6 + \text{j}3} = \frac{1}{3}(1 + \text{j})$$

$$\mathbf{V}_\text{o} = \frac{\text{j}20}{20 + \text{j}20}\mathbf{V} = \left(\frac{\text{j}}{1 + \text{j}}\right)\left(\frac{1}{3}(1 + \text{j})\right) = \frac{\text{j}}{3} = 0.3333\angle 90°$$

This shows that the output leads the input by 90°.

Solution 9.75

Since $\cos(\omega t) = \sin(\omega t + 90°)$, we need a phase shift circuit that will cause the output to lead the input by 90°. **This is achieved by the RL circuit shown below, as explained in the previous problem**.



This can also be obtained by an RC circuit.

Solution 9.76

(a) $v_2 = 8\sin 5t = 8\cos(5t - 90^o)$

v_1 leads v_2 by 70^o.

(b) $v_2 = 6\sin 2t = 6\cos(2t - 90^o)$

v_1 leads v_2 by 180^o.

(c) $v_1 = -4\cos 10t = 4\cos(10t + 180^o)$

$v_2 = 15\sin 10t = 15\cos(10t - 90^o)$

v_1 leads v_2 by 270^o.

Solution 9.77

Refer to the *RC* circuit in Fig. 9.81.
(a) Calculate the phase shift at 2 MHz.
(b) Find the frequency where the phase shift is 45°.

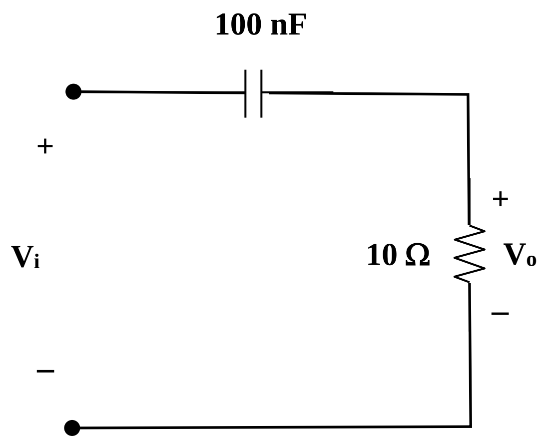


Figure 9.81
For Prob. 9.77.

Solution

In the frequency domain, the capacitance is equal to $-j10^7/\omega$ which leads to,

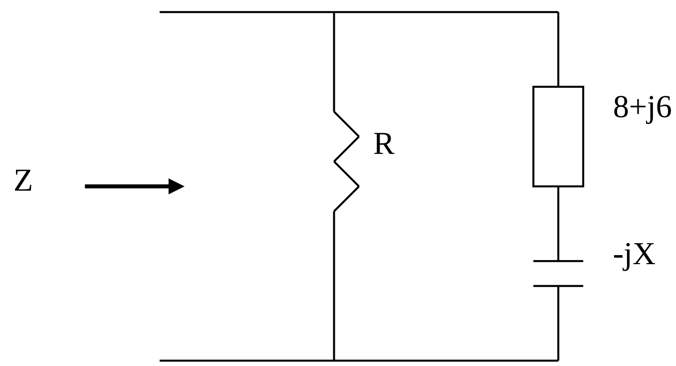
$V_o = V_i(10)/(10-j10^7/\omega)$.

For $\omega = 2x10^6$ $V_o = V_i(10)/(10-j5) = V_i(10)/(11.18034\angle-26.565°) = 0.8944V_i\angle26.57°$ which produces a phase shift = **26.57º (lagging)**.

For a 45° phase shift we need to have $(10^7/\omega) = 10$ or

ω = **1 MHz**.

Solution 9.78



$$Z = R//[8+j(6-X)] = \frac{R[8+j(6-X)]}{R+8+j(6-X)} = 5$$

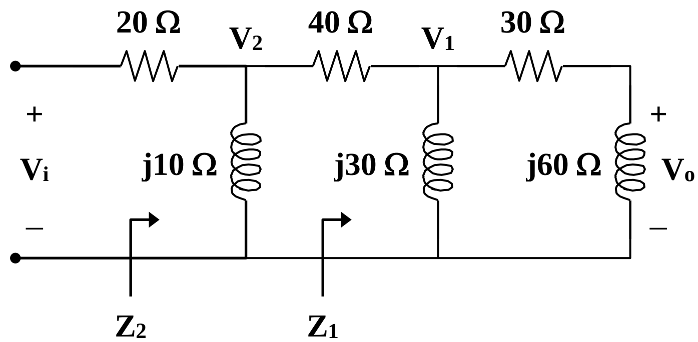
i.e. 8R + j6R – jXR = 5R + 40 + j30 –j5X

Equating real and imaginary parts:

8R = 5R + 40 which leads to **R=13.333Ω**
6R-XR =30-5X which leads to **X= 6 Ω**.

Solution 9.79

(a) Consider the circuit as shown.



$$\mathbf{Z}_1 = j30 \parallel (30 + j60) = \frac{(j30)(30 + j60)}{30 + j90} = 3 + j21$$

$$\mathbf{Z}_2 = j10 \parallel (40 + \mathbf{Z}_1) = \frac{(j10)(43 + j21)}{43 + j31} = 1.535 + j8.896 = 9.028\angle 80.21°$$

Let $\mathbf{V}_i = 1\angle 0°$.

$$\mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + 20}\mathbf{V}_i = \frac{(9.028\angle 80.21°)(1\angle 0°)}{21.535 + j8.896}$$

$$\mathbf{V}_2 = 0.3875\angle 57.77°$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + 40}\mathbf{V}_2 = \frac{3 + j21}{43 + j21}\mathbf{V}_2 = \frac{(21.213\angle 81.87°)(0.3875\angle 57.77°)}{47.85\angle 26.03°}$$

$$\mathbf{V}_1 = 0.1718\angle 113.61°$$

$$\mathbf{V}_o = \frac{j60}{30 + j60}\mathbf{V}_1 = \frac{j2}{1 + j2}\mathbf{V}_1 = \frac{2}{5}(2 + j)\mathbf{V}_1$$

$$\mathbf{V}_o = (0.8944\angle 26.56°)(0.1718\angle 113.6°)$$

$$\mathbf{V}_o = 0.1536\angle 140.2°$$

Therefore, the phase shift is **140.2°**

(b) The phase shift is **leading**.

(c) If $\mathbf{V}_i = 120$ V, then

$$\mathbf{V}_o = (120)(0.1536\angle 140.2°) = 18.43\angle 140.2° \text{ V}$$

and the magnitude is **18.43 V**.

Solution 9.80

$$200\text{ mH} \longrightarrow \quad j\omega L = j(2\pi)(60)(200\times10^{-3}) = j75.4\,\Omega$$

$$\mathbf{V}_o = \frac{j75.4}{R+50+j75.4}\mathbf{V}_i = \frac{j75.4}{R+50+j75.4}\,(120\angle 0°)$$

(a) When $R = 100\,\Omega$,

$$\mathbf{V}_o = \frac{j75.4}{150+j75.4}(120\angle 0°) = \frac{(75.4\angle 90°)(120\angle 0°)}{167.88\angle 26.69°}$$

$\mathbf{V}_o =$ **$53.89\angle 63.31°$ V**

(b) When $R = 0\,\Omega$,

$$\mathbf{V}_o = \frac{j75.4}{50+j75.4}(120\angle 0°) = \frac{(75.4\angle 90°)(120\angle 0°)}{90.47\angle 56.45°}$$

$\mathbf{V}_o =$ **$100\angle 33.55°$ V**

(c) To produce a phase shift of 45°, the phase of $\mathbf{V}_o$ = 90° + 0° – α = 45°.

Hence, α = phase of (R + 50 + j75.4) = 45°.

For α to be 45°, R + 50 = 75.4

Therefore, R = **25.4 Ω**

Solution 9.81

Let $\mathbf{Z}_1 = R_1$, $\mathbf{Z}_2 = R_2 + \dfrac{1}{j\omega C_2}$, $\mathbf{Z}_3 = R_3$, and $\mathbf{Z}_x = R_x + \dfrac{1}{j\omega C_x}$.

$$\mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2$$

$$R_x + \frac{1}{j\omega C_x} = \frac{R_3}{R_1}\left(R_2 + \frac{1}{j\omega C_2}\right)$$

$$R_x = \frac{R_3}{R_1}R_2 = \frac{1200}{400}(600) = \mathbf{1.8\ k\Omega}$$

$$\frac{1}{C_x} = \left(\frac{R_3}{R_1}\right)\left(\frac{1}{C_2}\right) \longrightarrow C_x = \frac{R_1}{R_3}C_2 = \left(\frac{400}{1200}\right)(0.3\times 10^{-6}) = \mathbf{0.1\ \mu F}$$

Solution 9.82

$$C_x = \frac{R_1}{R_2} C_s = \left(\frac{100}{2000}\right)(40 \times 10^{-6}) = \mathbf{2\ \mu F}$$

Solution 9.83

$$L_x = \frac{R_2}{R_1} L_s = \left(\frac{500}{1200}\right)(250 \times 10^{-3}) = \mathbf{104.17\ mH}$$

Solution 9.84

Let $\quad \mathbf{Z}_1 = R_1 \parallel \dfrac{1}{j\omega C_s}, \qquad \mathbf{Z}_2 = R_2, \qquad \mathbf{Z}_3 = R_3$, and $\mathbf{Z}_x = R_x + j\omega L_x$.

$$\mathbf{Z}_1 = \frac{\dfrac{R_1}{j\omega C_s}}{R_1 + \dfrac{1}{j\omega C_s}} = \frac{R_1}{j\omega R_1 C_s + 1}$$

Since $\mathbf{Z}_x = \dfrac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2$,

$$R_x + j\omega L_x = R_2 R_3 \frac{j\omega R_1 C_s + 1}{R_1} = \frac{R_2 R_3}{R_1}(1 + j\omega R_1 C_s)$$

Equating the real and imaginary components,

$$\mathbf{R_x = \frac{R_2 R_3}{R_1}}$$

$\omega L_x = \dfrac{R_2 R_3}{R_1}(\omega R_1 C_s)$ implies that

$$\mathbf{L_x = R_2 R_3 C_s}$$

Given that $R_1 = 40\ \text{k}\Omega$, $R_2 = 1.6\ \text{k}\Omega$, $R_3 = 4\ \text{k}\Omega$, and $C_s = 0.45\ \mu\text{F}$

$$R_x = \frac{R_2 R_3}{R_1} = \frac{(1.6)(4)}{40}\ \text{k}\Omega = 0.16\ \text{k}\Omega = \mathbf{160\ \Omega}$$

$$L_x = R_2 R_3 C_s = (1.6)(4)(0.45) = \mathbf{2.88\ H}$$

Solution 9.85

Let $\mathbf{Z}_1 = R_1$, $\mathbf{Z}_2 = R_2 + \dfrac{1}{j\omega C_2}$, $\mathbf{Z}_3 = R_3$, and $\mathbf{Z}_4 = R_4 \parallel \dfrac{1}{j\omega C_4}$.

$$\mathbf{Z}_4 = \frac{R_4}{j\omega R_4 C_4 + 1} = \frac{-jR_4}{\omega R_4 C_4 - j}$$

Since $\mathbf{Z}_4 = \dfrac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2 \longrightarrow \mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3$,

$$\frac{-jR_4R_1}{\omega R_4 C_4 - j} = R_3\left(R_2 - \frac{j}{\omega C_2}\right)$$

$$\frac{-jR_4R_1(\omega R_4 C_4 + j)}{\omega^2 R_4^2 C_4^2 + 1} = R_3R_2 - \frac{jR_3}{\omega C_2}$$

Equating the real and imaginary components,

$$\frac{R_1R_4}{\omega^2 R_4^2 C_4^2 + 1} = R_2R_3 \tag{1}$$

$$\frac{\omega R_1 R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1} = \frac{R_3}{\omega C_2} \tag{2}$$

Dividing (1) by (2),

$$\frac{1}{\omega R_4 C_4} = \omega R_2 C_2$$

$$\omega^2 = \frac{1}{R_2 C_2 R_4 C_4}$$

$$\omega = 2\pi f = \frac{1}{\sqrt{R_2 C_2 R_4 C_4}}$$

$$\mathbf{f = \frac{1}{2\pi\sqrt{R_2 R_4 C_2 C_4}}}$$

Solution 9.86

$$\mathbf{Y} = \frac{1}{240} + \frac{1}{j95} + \frac{1}{-j84}$$

$$\mathbf{Y} = 4.1667 \times 10^{-3} - j0.01053 + j0.0119$$

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1000}{4.1667 + j1.37} = \frac{1000}{4.3861\angle 18.2^\circ}$$

$\mathbf{Z} = \mathbf{228\angle\text{-}18.2^\circ\ \Omega}$

Solution 9.87

The network in Fig. 9.87 is part of the schematic describing an industrial electronic sensing device. What is the total impedance of the circuit at 4 kHz?

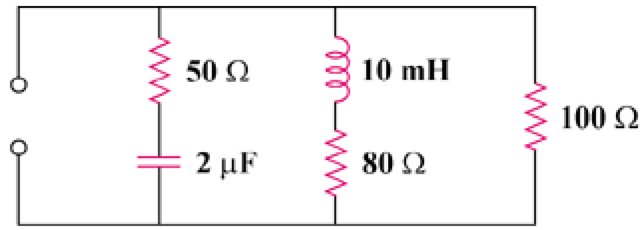


Figure 9.87
For Prob. 9.87.

Solution

$$\mathbf{Z}_1 = 50 + \frac{1}{j\omega C} = 50 + \frac{-j}{(2\pi)(4\times10^3)(2\times10^{-6})}$$
$$\mathbf{Z}_1 = 50 - j19.8944 = 53.813\angle{-21.697°}$$

$$\mathbf{Z}_2 = 80 + j\omega L = 80 + j(2\pi)(4\times10^3)(10\times10^{-3})$$
$$\mathbf{Z}_2 = 80 + j251.327 = 263.752\angle 72.343°$$

$$\mathbf{Z}_3 = 100$$

$$\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}$$
$$\frac{1}{\mathbf{Z}} = (0.0185829)\angle 21.697° + (0.0037914)\angle{-72.343°} + 0.01$$
$$= 0.0172663 + j0.0068701 + 0.00115 - j0.0036128 + 0.01 = 0.0284163 + j0.0032573$$
$$= 0.028602\angle 6.539° \text{ or}$$

Z = 1/0.028602∠6.539° = **34.96∠–6.54° Ω**
= **(34.73 – j3.982) Ω**.

Solution 9.88

(a) $\mathbf{Z} = -j20 + j30 + 120 - j20$

$\mathbf{Z} = \mathbf{(120 - j10)\ \Omega}$

(b) If the frequency were halved, $\frac{1}{\omega C} = \frac{1}{2\pi f\ C}$ would cause the capacitive impedance to double, while $\omega L = 2\pi f\ L$ would cause the inductive impedance to halve. Thus,

$\mathbf{Z} = -j40 + j15 + 120 - j40$

$\mathbf{Z} = \mathbf{(120 - j65)\ \Omega}$

Solution 9.89

An industrial load is modeled as a series combination of an inductor and a resistance as shown in Fig. 9.89. Calculate the value of a capacitor C across the series combination so that the net impedance is resistive at a frequency of 2 kHz.

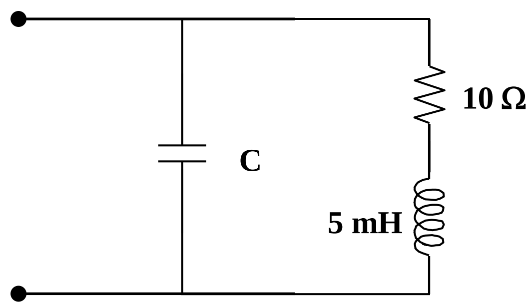


Figure 9.89
For Prob. 9.89.

Solution

Step 1.

There are different ways to solve this problem but perhaps the easiest way is to convert the series R L elements into their parallel equivalents. Then all you need to do is to make the inductance and capacitance cancel each other out to result in a purely resistive circuit.

$X_L = 2x10^3x5x10^{-3} = 10$ which leads to $\mathbf{Y} = 1/(10+j10) = 0.05–j0.05$ or a 20**Ω** resistor in parallel with a j20**Ω** inductor. $X_c = 1/(2x10^3C)$ and the parallel combination of the capacitor and inductor is equal to,

$[(–jX_C)(j20)/(–jX_C+j20)]$.

Step 2.

Now we just need to set $X_C = 20 = 1/(2x10^3C)$ which will create an open circuit.

$$C = 1/(20x2x10^3) = \mathbf{25\ \mu F}.$$

Solution 9.90

Let $\quad \mathbf{V}_s = 145\angle 0°, \qquad X = \omega L = (2\pi)(60)\,L = 377\,L$

$$\mathbf{I} = \frac{\mathbf{V}_s}{80 + R + jX} = \frac{145\angle 0°}{80 + R + jX}$$

$$\mathbf{V}_1 = 80\,\mathbf{I} = \frac{(80)(145)}{80 + R + jX}$$

$$50 = \left| \frac{(80)(145)}{80 + R + jX} \right| \qquad (1)$$

$$\mathbf{V}_o = (R + jX)\,\mathbf{I} = \frac{(R + jX)(145\angle 0°)}{80 + R + jX}$$

$$110 = \left| \frac{(R + jX)(145)}{80 + R + jX} \right| \qquad (2)$$

From (1) and (2),

$$\frac{50}{110} = \frac{80}{\left| R + jX \right|}$$

$$\left| R + jX \right| = (80)\left(\frac{11}{5}\right)$$

$$R^2 + X^2 = 30976 \qquad (3)$$

From (1),

$$\left| 80 + R + jX \right| = \frac{(80)(145)}{50} = 232$$

$$6400 + 160R + R^2 + X^2 = 53824$$

$$160R + R^2 + X^2 = 47424 \qquad (4)$$

Subtracting (3) from (4),

$$160R = 16448 \longrightarrow R = \mathbf{102.8\ \Omega}$$

From (3),

$$X^2 = 30976 - 10568 = 20408$$

$$X = 142.86 = 377L \longrightarrow L = \mathbf{378.9\ mH}$$

Solution 9.91

Figure 9.91 shows a series combination of an inductance and a resistance. If it is desired to connect a capacitor in parallel with the series combination such that the net impedance is resistive at 10 kHz, what is the required value of *C*?

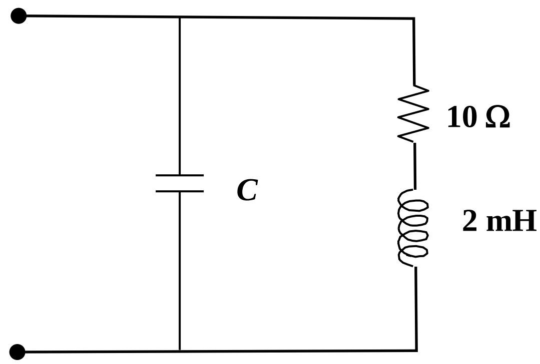


Figure 9.91
For Prob. 9.91.

Solution

At 10 kHz the inductive reactance is equal to j20 Ω and the capacitive reactance is equal to $-j/(10^4C)$. The easiest way to eliminate the effect of the inductor with the capacitor is to convert the resistor and inductor to admittance. Thus,

$1/(10+j20) = (10-j20)/(100+400) = 0.02-j0.04$. This leads to $10^4C = 0.04$ or

$C = 0.04\text{x}10^{-4} =$ **4 μF**.

Note the resultant resistance is equal to $1/0.02 = 50\ \Omega$.

Solution 9.92

(a) $\boldsymbol{Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{100\angle 75^o}{450\angle 48^o\, x10^{-6}}} = \underline{471.4\angle 13.5^o\,\Omega}}$

(b) $\boldsymbol{\gamma = \sqrt{ZY} = \sqrt{100\angle 75^o\, x450\angle 48^o\, x10^{-6}} = \underline{212.1\angle 61.5^o}\; mS}$

Solution 9.93

$$\mathbf{Z} = \mathbf{Z}_s + 2\,\mathbf{Z}_\ell + \mathbf{Z}_L$$
$$\mathbf{Z} = (1 + 0.8 + 23.2) + j(0.5 + 0.6 + 18.9)$$
$$\mathbf{Z} = 25 + j20$$

$$\mathbf{I}_L = \frac{\mathbf{V}_S}{\mathbf{Z}} = \frac{115\angle 0^\circ}{32.02\angle 38.66^\circ}$$

$$\mathbf{I}_L = \mathbf{3.592\angle{-38.66^\circ}\ A}$$

Solution 10.1

We first determine the input impedance.

$$1H \longrightarrow j\omega L = j1x10 = j10$$

$$1F \longrightarrow 1/j\omega C = \frac{1}{j10x1} = -j0.1$$

$$Z_{in} = 1 + \left(\frac{1}{j10} + \frac{1}{-j0.1} + \frac{1}{1}\right)^{-1} = 1.0101 - j0.1 = 1.015 < -5.653^o$$

$$I = \frac{2 < 0^o}{1.015 < -5.653^o} = 1.9704 < 5.653^o$$

i(t) = **1.9704cos(10t+5.65°) A**

Solution 10.2

Using Fig. 10.51, design a problem to help other students better understand nodal analysis.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Solve for V_o in Fig. 10.51, using nodal analysis.

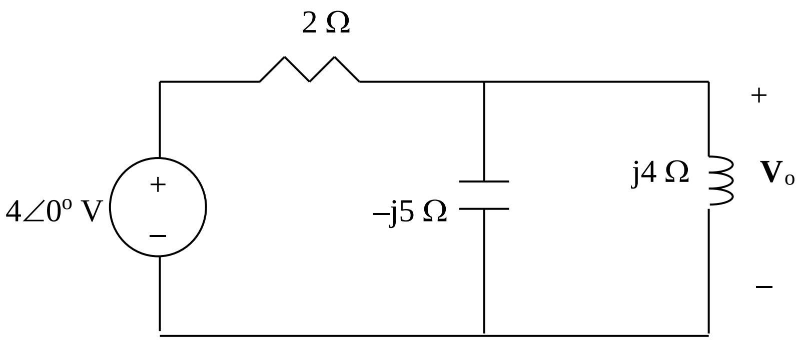
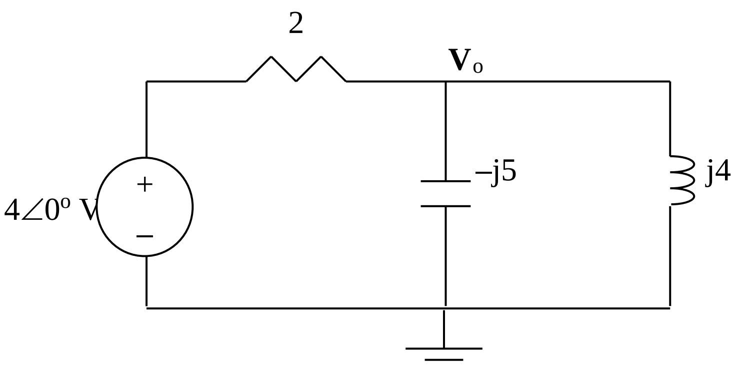


Figure 10.51 For Prob. 10.2.

Solution

Consider the circuit shown below.



At the main node,

$$\frac{4-V_o}{2}=\frac{V_o}{-j5}+\frac{V_o}{j4} \quad \longrightarrow \quad 40=V_o(10+j)$$

$\mathbf{V_o}$ = 40/(10–j) = (40/10.05)∠5.71° = **3.98∠5.71° V**

Solution 10.3

$\omega = 4$

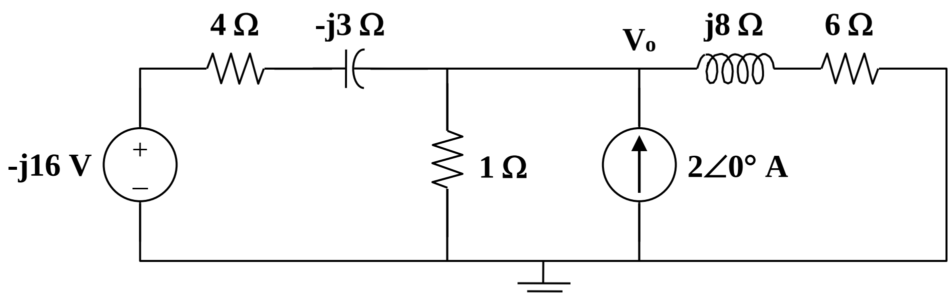
$2\cos(4t) \longrightarrow 2\angle 0°$

$16\sin(4t) \longrightarrow 16\angle -90° = -j16$

$2\text{ H} \longrightarrow j\omega L = j8$

$1/12\text{ F} \longrightarrow \dfrac{1}{j\omega C} = \dfrac{1}{j(4)(1/12)} = -j3$

The circuit is shown below.



Applying nodal analysis,

$$\frac{-j16 - \mathbf{V}_o}{4 - j3} + 2 = \frac{\mathbf{V}_o}{1} + \frac{\mathbf{V}_o}{6 + j8}$$

$$\frac{-j16}{4 - j3} + 2 = \left(1 + \frac{1}{4 - j3} + \frac{1}{6 + j8}\right)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{3.92 - j2.56}{1.22 + j0.04} = \frac{4.682\angle -33.15°}{1.2207\angle 1.88°} = 3.835\angle -35.02°$$

Therefore,

$$v_o(t) = \mathbf{3.835cos(4t – 35.02°) V}$$

Solution 10.4

Compute $v_o(t)$ in the circuit of Fig. 10.53.

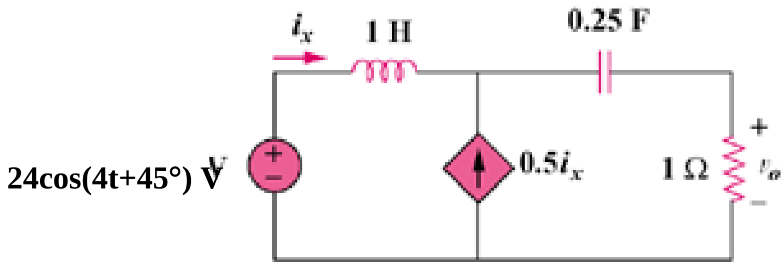
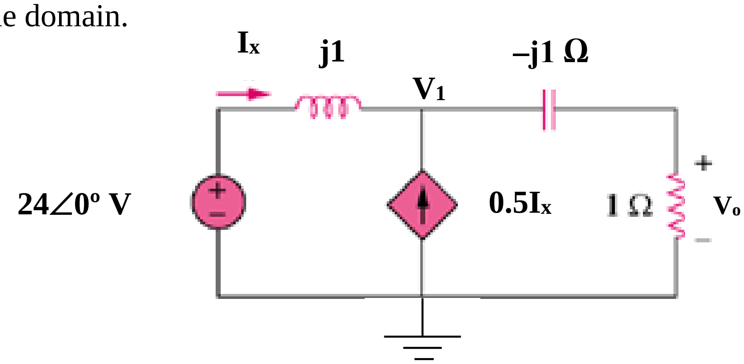


Figure 10.53
For Prob. 10.4.

Solution

Step 1. Convert the circuit into the frequency domain and solve for the node voltage, V_1, using nodal analysis. The find the current $I_C = V_1/[1+(1/(j4x0.25)]$ which then produces $V_o = 1xI_C$. Finally, convert the capacitor voltage back into the time domain.



Note that we represented 24cos(4t+45°) volts by 24∠0° V. That will make our calculations easier and all we have to do is to offset our answer by a 45°.

Our node equation is $[(V_1–24)/j] – (0.5I_x) + [(V_1–0)/(1–j)] = 0$. We have two unknowns, therefore we need a constraint equation. $I_x = [(24–V_1)/j] = j(V_1–24)$. Once we have V_1, we can find $I_o = V_1 /(1–j)$ and $V_o = 1xI_o$.

Step 2. Now all we need to do is to solve our equations.

$[(V_1–24)/j] – [0.5j(V_1–24] + [(V_1–0)/(1–j)] = [–j–j0.5+0.5+j0.5]V_1 +j24+j12 = 0$
or

$[0.5–j]V_1 = –j36$ or $V_1 = j36/(–0.5+j) = (36\angle 90^o)/(1.118\angle 116.57^o)$
$= 32.2\angle –26.57^o$ V.

Finally, $I_x = V_1/(1–j) = (32.2\angle –26.57^o)(0.7071\angle 45^o) = 22.769\angle 18.43^o$ A and V_o $= 1xI_o = 22.77\angle 18.43^o$ V or

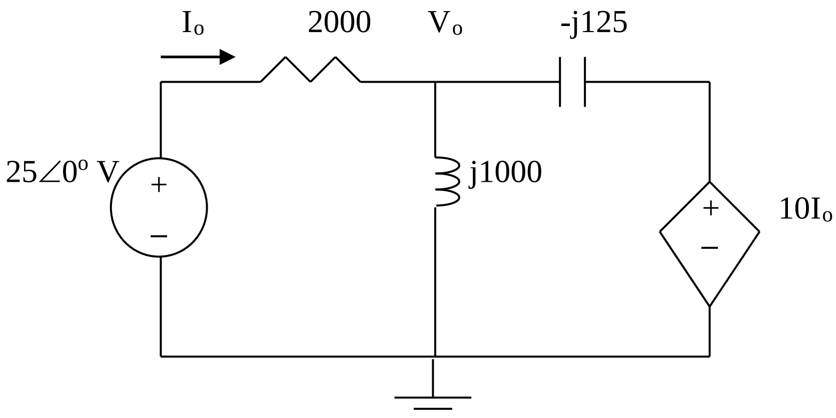
$$v_o(t) = 22.77\cos(4t+45^o+18.43^o) = \mathbf{22.77\cos(4t+63.43^o)\ volts}.$$

Solution 10.5

$$0.25H \longrightarrow j\omega L = j0.25x4x10^3 = j1000$$

$$2\mu F \longrightarrow 1/j\omega C = \frac{1}{j4x10^3 x2x10^{-6}} = -j125$$

Consider the circuit as shown below.



At node V_o,

$$\frac{V_o - 25}{2000} + \frac{V_o - 0}{j1000} + \frac{V_o - 10I_o}{-j125} = 0$$

$$V_o - 25 - j2V_o + j16V_o - j160I_o = 0$$

$$(1 + j14)V_o - j160I_o = 25$$

But $I_o = (25–V_o)/2000$

$$(1 + j14)V_o - j2 + j0.08V_o = 25$$

$$V_o = \frac{25 + j2}{1 + j14.08} = \frac{25.08\angle 4.57°}{14.115\angle 58.94°} 1.7768\angle -81.37°$$

Now to solve for i_o,

$$I_o = \frac{25 - V_o}{2000} = \frac{25 - 0.2666 + j1.7567}{2000} = 12.367 + j0.8784 \text{ mA}$$

$$= 12.398\angle 4.06°$$

i_o = **$12.398\cos(4x10^3 t + 4.06°)$ mA**.

Problem 10.6

Determine $\mathbf{V}_x$ shown in Fig. 10.55

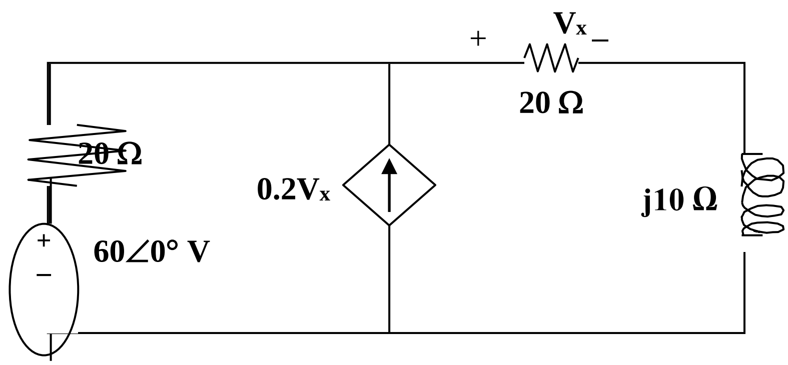


Figure 10.55
For Prob. 10.6.

Solution

Let V_o be the voltage across the dependent current source. Using nodal analysis we get:

$$[(V_o-60)/20] - 0.2V_x + [(V_o-0)/(20+j10)] = 0 \text{ where } V_x = V_o[20/(20+j10)]$$

This leads this to $\{0.05 - [4/(20+j10)] + [1/(20+j10)]\}V_o = 3$ or

$$(1+j0.5-3)V_o = (-2+J0.5)V_o = 3(20+j10) \text{ or } V_o = 3(20+j10)/(-2+j0.5) \text{ or}$$

$$V_x = 3(20)/(-2+j0.5) = 60/(2.06155\angle 165.96°) = \mathbf{29.1\angle -165.96° \ V}.$$

Solution 10.7

At the main node,

$$\frac{120\angle -15^{o} - V}{40 + j20} = 6\angle 30^{o} + \frac{V}{-j30} + \frac{V}{50} \quad \longrightarrow \quad \frac{115.91 - j31.058}{40 + j20} - 5.196 - j3 =$$

$$V\left(\frac{1}{40 + j20} + \frac{j}{30} + \frac{1}{50}\right)$$

$$V = \frac{-3.1885 - j4.7805}{0.04 + j0.0233} = \underline{124.08\angle -154^{o}\ \ V}$$

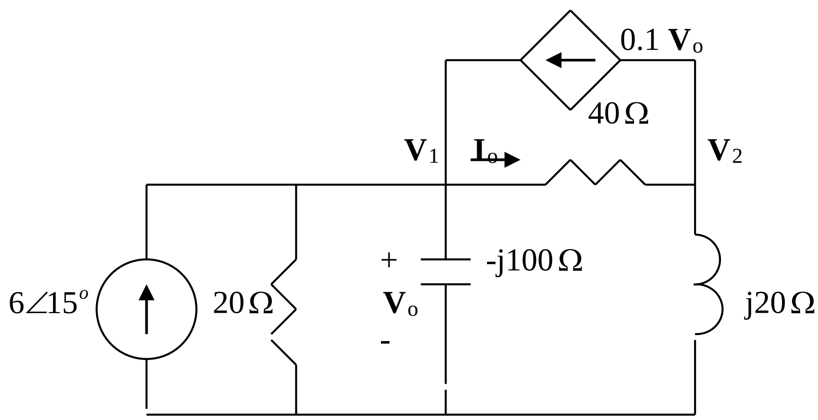
Solution 10.8

$\omega = 200,$

$$100\text{mH} \longrightarrow \quad j\omega L = j200\text{x}0.1 = j20$$

$$50\mu\text{F} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j200\text{x}50\text{x}10^{-6}} = -j100$$

The frequency-domain version of the circuit is shown below.



At node 1,

$$6\angle 15^{o} + 0.1V_1 = \frac{V_1}{20} + \frac{V_1}{-j100} + \frac{V_1 - V_2}{40}$$

or $$5.7955 + j1.5529 = (-0.025 + j0.01)V_1 - 0.025V_2 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{40} = 0.1V_1 + \frac{V_2}{j20} \longrightarrow \quad 0 = 3V_1 + (1 - j2)V_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} (-0.025 + j0.01) & -0.025 \\ 3 & (1 - j2) \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} (5.7955 + j1.5529) \\ 0 \end{pmatrix} \quad \text{or} \quad AV = B$$

Using MATLAB,

$$V = \text{inv}(A)*B$$

leads to $V_1 = -70.63 - j127.23, \qquad V_2 = -110.3 + j161.09$

$$I_o = \frac{V_1 - V_2}{40} = 7.276\angle -82.17^o$$

Thus,

$$\underline{\boldsymbol{i_o(t) = 7.276\cos(200t - 82.17^o)\ \text{A}}}$$

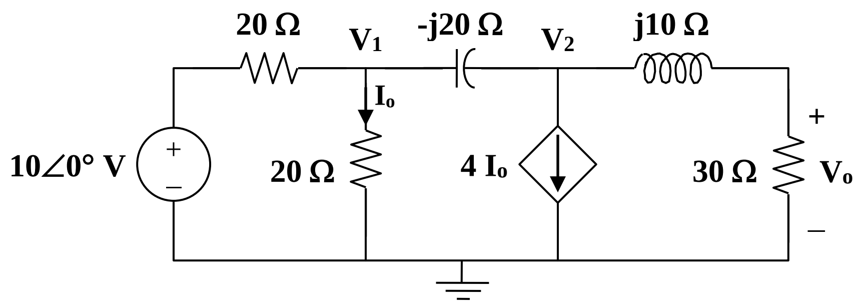
Solution 10.9

$$10\cos(10^3 t) \longrightarrow 10\angle 0°, \quad \omega = 10^3$$

$$10 \text{ mH} \longrightarrow j\omega L = j10$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

Consider the circuit shown below.



At node 1,

$$\frac{10-\mathbf{V}_1}{20} = \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20}$$

$$10 = (2+j)\mathbf{V}_1 - j\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20} = (4)\frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_2}{30+j10}, \text{ where } \mathbf{I}_o = \frac{\mathbf{V}_1}{20} \text{ has been substituted.}$$

$$(-4+j)\mathbf{V}_1 = (0.6+j0.8)\mathbf{V}_2$$

$$\mathbf{V}_1 = \frac{0.6+j0.8}{-4+j}\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1)

$$10 = \frac{(2+j)(0.6+j0.8)}{-4+j}\mathbf{V}_2 - j\mathbf{V}_2$$

or

$$\mathbf{V}_2 = \frac{170}{0.6-j26.2}$$

$$\mathbf{V}_o = \frac{30}{30+j10}\mathbf{V}_2 = \frac{3}{3+j}\cdot\frac{170}{0.6-j26.2} = 6.154\angle 70.26°$$

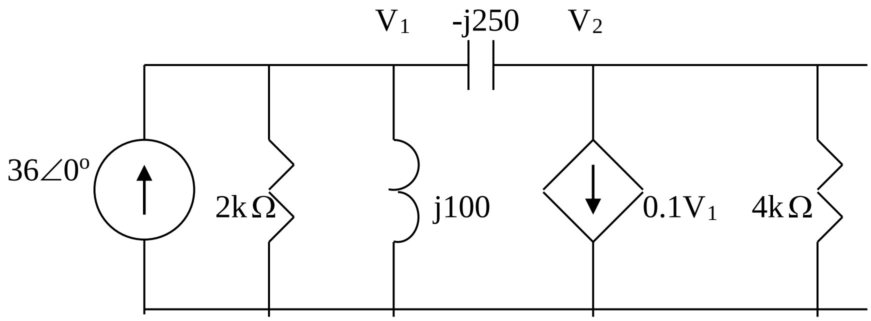
Therefore, $v_o(t) =$ **$6.154\cos(10^3 t + 70.26°)$ V**

Solution 10.10

$$50\,\text{mH} \longrightarrow \quad j\omega L = j2000x50x10^{-3} = j100, \qquad \omega = 2000$$

$$2\mu\text{F} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j2000x2x10^{-6}} = -j250$$

Consider the frequency-domain equivalent circuit below.



At node 1,

$$36 = \frac{V_1}{2000} + \frac{V_1}{j100} + \frac{V_1 - V_2}{-j250} \qquad \longrightarrow \qquad 36 = (0.0005 - j0.006)V_1 - j0.004V_2$$

(1)

At node 2,

$$\frac{V_1 - V_2}{-j250} = 0.1V_1 + \frac{V_2}{4000} \qquad \longrightarrow \qquad 0 = (0.1 - j0.004)V_1 + (0.00025 + j0.004)V_2$$

(2)

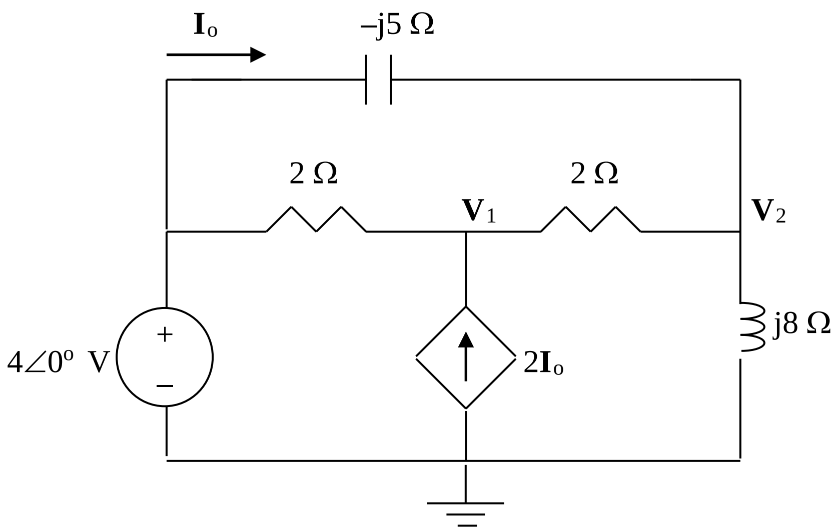
Solving (1) and (2) gives

$$V_o = V_2 = -535.6 + j893.5 = 8951.1\angle 93.43^o$$

v_o (t) = **8.951 sin(2000t +93.43º) kV**

Solution 10.11

Consider the circuit as shown below.



At node 1,

$$\frac{V_1 - 4}{2} - 2I_o + \frac{V_1 - V_2}{2} = 0$$
$$V_1 - 0.5V_2 - 2I_o = 2$$

But, $I_o = (4–V_2)/(–j5) = –j0.2V_2 + j0.8$

Now the first node equation becomes,

$$V_1 – 0.5V_2 + j0.4V_2 – j1.6 = 2 \text{ or}$$
$$V_1 + (–0.5+j0.4)V_2 = 2 + j1.6$$

At node 2,

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 4}{-j5} + \frac{V_2 - 0}{j8} = 0$$
$$–0.5V_1 + (0.5 + j0.075)V_2 = j0.8$$

Using MATLAB to solve this, we get,

```
>> Y=[1,(-0.5+0.4i);-0.5,(0.5+0.075i)]

Y =
```

```
  1.0000             -0.5000 + 0.4000i
 -0.5000              0.5000 + 0.0750i

>> I=[(2+1.6i);0.8i]

I =

  2.0000 + 1.6000i
       0 + 0.8000i

>> V=inv(Y)*I

V =

  4.8597 + 0.0543i
  4.9955 + 0.9050i
```

$I_o = -j0.2V_2 + j0.8 = -j0.9992 + 0.01086 + j0.8 = 0.01086 - j0.1992$

$= \mathbf{199.5\angle 86.89^\circ\ mA}$.

Solution 10.12

Using Fig. 10.61, design a problem to help other students to better understand Nodal analysis.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

By nodal analysis, find i_o in the circuit in Fig. 10.61.

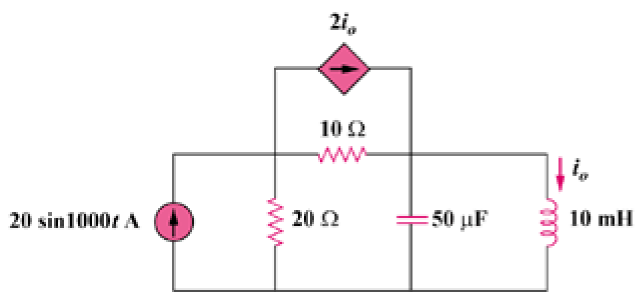


Figure 10.61

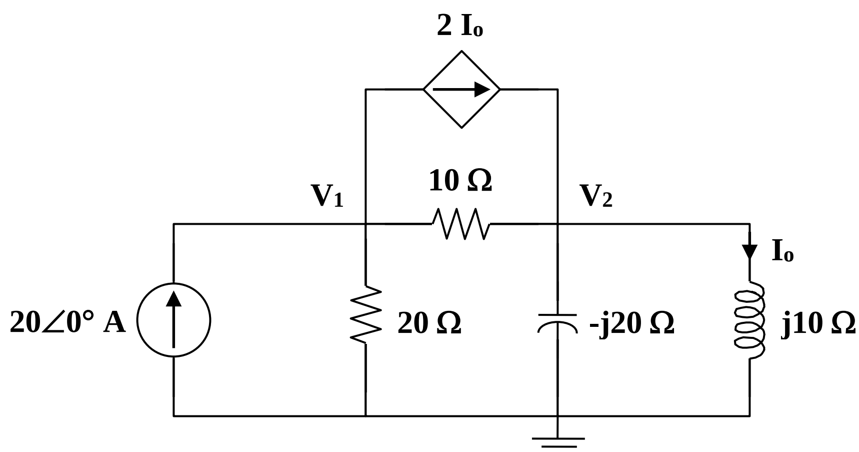
Solution

$$20\sin(1000t) \longrightarrow 20\angle 0°, \quad \omega = 1000$$

$$10 \text{ mH} \longrightarrow j\omega L = j10$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

The frequency-domain equivalent circuit is shown below.



At node 1,

$$20 = 2\mathbf{I}_o + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10},$$ where

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10}$$

$$20 = \frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10}$$

$$400 = 3\mathbf{V}_1 - (2 + j4)\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10} = \frac{\mathbf{V}_2}{-j20} + \frac{\mathbf{V}_2}{j10}$$

$$j2\mathbf{V}_1 = (-3 + j2)\mathbf{V}_2$$

or $$\mathbf{V}_1 = (1 + j1.5)\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$400 = (3 + j4.5)\mathbf{V}_2 - (2 + j4)\mathbf{V}_2 = (1 + j0.5)\mathbf{V}_2$$

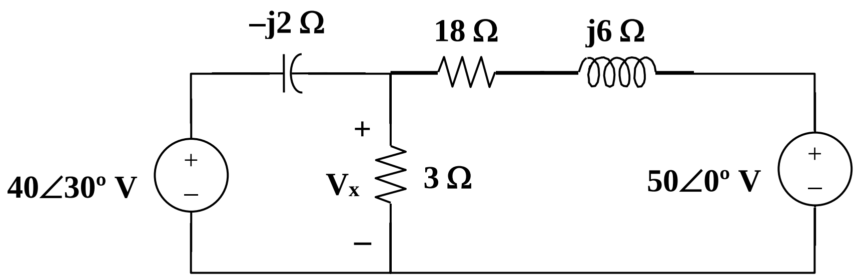
$$\mathbf{V}_2 = \frac{400}{1 + j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10} = \frac{40}{j(1 + j0.5)} = 35.74\angle -116.6°$$

Therefore, $i_o(t) =$ **35.74 sin(1000t – 116.6°) A**

Solution 10.13

Nodal analysis is the best approach to use on this problem. We can make our work easier by doing a source transformation on the right hand side of the circuit.



$$\frac{V_x - 40\angle 30^\circ}{-j2} + \frac{V_x}{3} + \frac{V_x - 50}{18 + j6} = 0$$

which leads to V_x = **29.36∠62.88° A**.

Solution 10.14

At node 1,

$$\frac{0-\mathbf{V}_1}{-j2}+\frac{0-\mathbf{V}_1}{10}+\frac{\mathbf{V}_2-\mathbf{V}_1}{j4}=20\angle 30°$$

$$-(1+j2.5)\mathbf{V}_1 - j2.5\mathbf{V}_2 = 173.2 + j100 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_2}{j2}+\frac{\mathbf{V}_2}{-j5}+\frac{\mathbf{V}_2-\mathbf{V}_1}{j4}=20\angle 30°$$

$$-j5.5\mathbf{V}_2 + j2.5\mathbf{V}_1 = 173.2 + j100 \qquad (2)$$

Equations (1) and (2) can be cast into matrix form as

$$\begin{bmatrix} 1+j2.5 & j2.5 \\ j2.5 & -j5.5 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}=\begin{bmatrix} -200\angle 30° \\ 200\angle 30° \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1+j2.5 & j2.5 \\ j2.5 & -j5.5 \end{vmatrix} = 20 - j5.5 = 20.74\angle -15.38°$$

$$\Delta_1 = \begin{vmatrix} -200\angle 30° & j2.5 \\ 200\angle 30° & -j5.5 \end{vmatrix} = j3(200\angle 30°) = 600\angle 120°$$

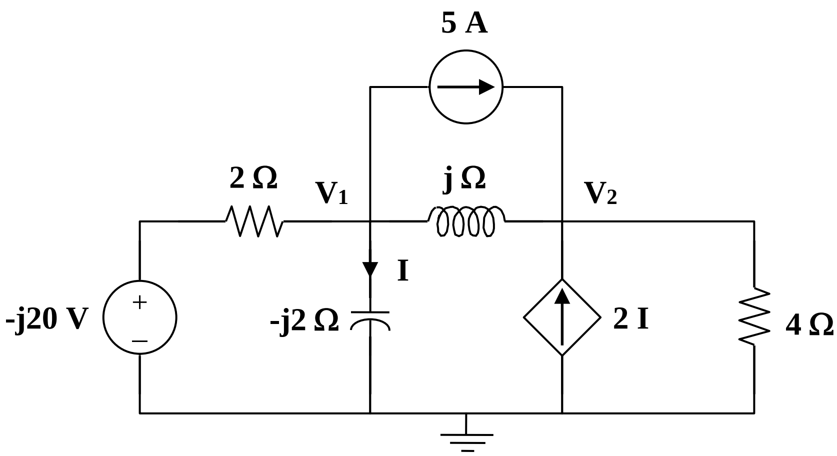
$$\Delta_2 = \begin{vmatrix} 1+j2.5 & -200\angle 30° \\ j2.5 & 200\angle 30° \end{vmatrix} = (200\angle 30°)(1+j5) = 1020\angle 108.7°$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = 28.93\angle 135.38°\ \text{V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 49.18\angle 124.08°\ \text{V}$$

Solution 10.15

We apply nodal analysis to the circuit shown below.



At node 1,

$$\frac{-j20-\mathbf{V}_1}{2}=5+\frac{\mathbf{V}_1}{-j2}+\frac{\mathbf{V}_1-\mathbf{V}_2}{j}$$

$$-5-j10=(0.5-j0.5)\,\mathbf{V}_1+j\,\mathbf{V}_2 \qquad (1)$$

At node 2,

$$5+2\,\mathbf{I}+\frac{\mathbf{V}_1-\mathbf{V}_2}{j}=\frac{\mathbf{V}_2}{4},$$

$$\text{where}\quad \mathbf{I}=\frac{\mathbf{V}_1}{-j2}$$

$$\mathbf{V}_2=\frac{5}{0.25-j}\,\mathbf{V}_1 \qquad (2)$$

Substituting (2) into (1),

$$-5-j10-\frac{j5}{0.25-j}=0.5(1-j)\,\mathbf{V}_1$$

$$(1-j)\,\mathbf{V}_1=-10-j20-\frac{j40}{1-j4}$$

$$(\sqrt{2}\angle -45°)\,\mathbf{V}_1=-10-j20+\frac{160}{17}-\frac{j40}{17}$$

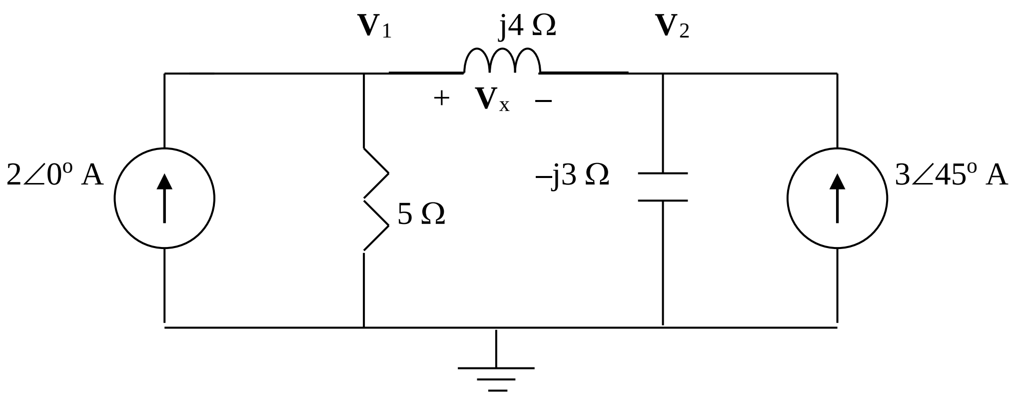
$$\mathbf{V}_1=15.81\angle 313.5°$$

$$\mathbf{I}=\frac{\mathbf{V}_1}{-j2}=(0.5\angle 90°)(15.81\angle 313.5°)$$

$$\mathbf{I}=\mathbf{7.906\angle 43.49°\ A}$$

Solution 10.16

Consider the circuit as shown in the figure below.



At node 1,

$$-2+\frac{V_1-0}{5}+\frac{V_1-V_2}{j4}=0 \qquad (1)$$

$$(0.2-j0.25)V_1+j0.25V_2=2$$

At node 2,

$$\frac{V_2-V_1}{j4}+\frac{V_2-0}{-j3}-3\angle 45^\circ=0 \qquad (2)$$

$$j0.25V_1+j0.08333V_2=2.121+j2.121$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} 0.2-j0.25 & j0.25 \\ j0.25 & j0.08333 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} 2 \\ 2.121+j2.121 \end{bmatrix}$$

Solving this using MATLAB, we get,

```
>> Y=[(0.2-0.25i),0.25i;0.25i,0.08333i]

Y =

   0.2000 - 0.2500i        0 + 0.2500i
        0 + 0.2500i        0 + 0.0833i

>> I=[2;(2.121+2.121i)]
```

I =

   2.0000
   2.1210 + 2.1210i

```
>> V=inv(Y)*I
```

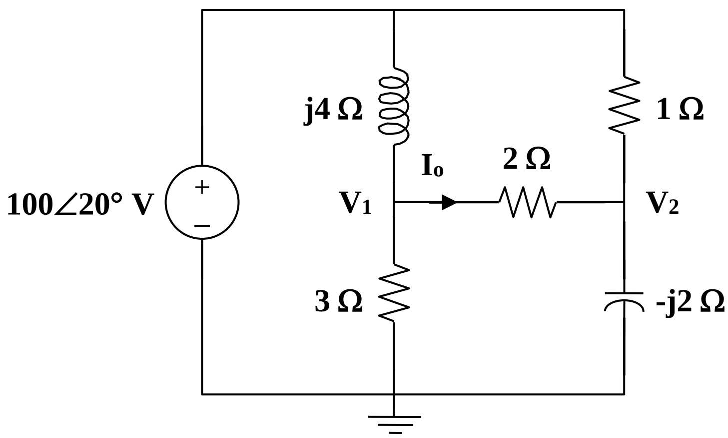
V =

   5.2793 - 5.4190i
   9.6145 - 9.1955i

$$V_s = V_1 - V_2 = -4.335 + j3.776 = \mathbf{5.749\angle 138.94^\circ\ V}.$$

Solution 10.17

Consider the circuit below.



At node 1,

$$\frac{100\angle 20^\circ - \mathbf{V}_1}{j4} = \frac{\mathbf{V}_1}{3} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2}$$

$$100\angle 20^\circ = \frac{\mathbf{V}_1}{3}(3 + j10) - j2\,\mathbf{V}_2$$

(1)

At node 2,

$$\frac{100\angle 20^\circ - \mathbf{V}_2}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} = \frac{\mathbf{V}_2}{-j2}$$

$$100\angle 20^\circ = -0.5\,\mathbf{V}_1 + (1.5 + j0.5)\,\mathbf{V}_2$$

(2)

From (1) and (2),

$$\begin{bmatrix} 100\angle 20^\circ \\ 100\angle 20^\circ \end{bmatrix} = \begin{bmatrix} -0.5 & 0.5(3+j) \\ 1 + j10/3 & -j2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} -0.5 & 1.5 + j0.5 \\ 1 + j10/3 & -j2 \end{vmatrix} = 0.1667 - j4.5$$

$$\Delta_1 = \begin{vmatrix} 100\angle 20^\circ & 1.5 + j0.5 \\ 100\angle 20^\circ & -j2 \end{vmatrix} = -55.45 - j286.2$$

$$\Delta_2 = \begin{vmatrix} -0.5 & 100\angle 20^\circ \\ 1 + j10/3 & 100\angle 20^\circ \end{vmatrix} = -26.95 - j364.5$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = 64.74\angle -13.08^\circ$$

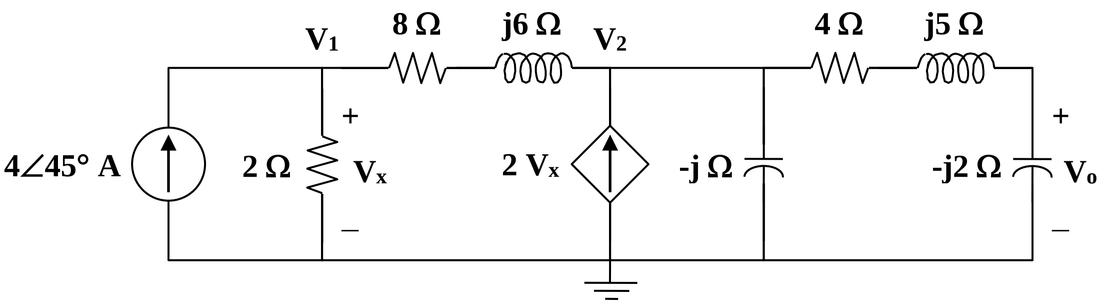
$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 81.17\angle -6.35^\circ$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} = \frac{\Delta_1 - \Delta_2}{2\Delta} = \frac{-28.5 + j78.31}{0.3333 - j9}$$

$$\mathbf{I}_o = \mathbf{9.25\angle\text{-}162.12^\circ\ A}$$

Solution 10.18

Consider the circuit shown below.



At node 1,

$$4\angle 45° = \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j6}$$

$$200\angle 45° = (29 - j3)\,\mathbf{V}_1 - (4 - j3)\,\mathbf{V}_2$$
(1)

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j6} + 2\mathbf{V}_x = \frac{\mathbf{V}_2}{-j} + \frac{\mathbf{V}_2}{4 + j5 - j2},$$ where $\mathbf{V}_x = \mathbf{V}_1$

$$(104 - j3)\,\mathbf{V}_1 = (12 + j41)\,\mathbf{V}_2$$

$$\mathbf{V}_1 = \frac{12 + j41}{104 - j3}\mathbf{V}_2$$
(2)

Substituting (2) into (1),

$$200\angle 45° = (29 - j3)\frac{(12 + j41)}{104 - j3}\mathbf{V}_2 - (4 - j3)\,\mathbf{V}_2$$

$$200\angle 45° = (14.21\angle 89.17°)\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{200\angle 45°}{14.21\angle 89.17°}$$

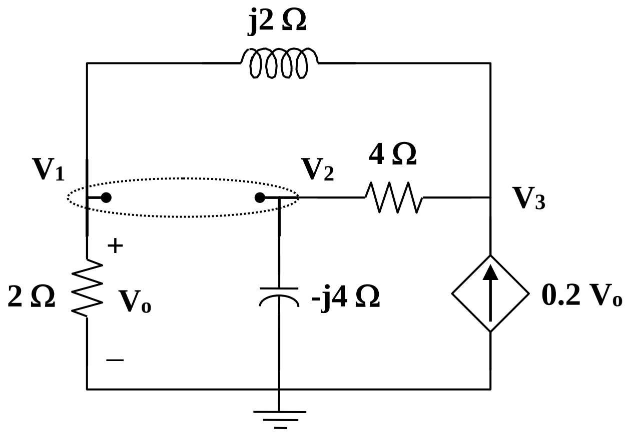
$$\mathbf{V}_o = \frac{-j2}{4 + j5 - j2}\mathbf{V}_2 = \frac{-j2}{4 + j3}\mathbf{V}_2 = \frac{-6 - j8}{25}\mathbf{V}_2$$

$$\mathbf{V}_o = \frac{10\angle 233.13°}{25} \cdot \frac{200\angle 45°}{14.21\angle 89.17°}$$

$$\mathbf{V}_o = \mathbf{5.63\angle 189°\ V}$$

Solution 10.19

We have a supernode as shown in the circuit below.



Notice that $\mathbf{V}_o = \mathbf{V}_1$.

At the supernode,

$$\frac{\mathbf{V}_3 - \mathbf{V}_2}{4} = \frac{\mathbf{V}_2}{-\text{j}4} + \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_3}{\text{j}2}$$

$$0 = (2 - \text{j}2)\,\mathbf{V}_1 + (1 + \text{j})\,\mathbf{V}_2 + (\text{-}1 + \text{j}2)\,\mathbf{V}_3 \qquad (1)$$

At node 3,

$$0.2\mathbf{V}_1 + \frac{\mathbf{V}_1 - \mathbf{V}_3}{\text{j}2} = \frac{\mathbf{V}_3 - \mathbf{V}_2}{4}$$

$$(0.8 - \text{j}2)\,\mathbf{V}_1 + \mathbf{V}_2 + (\text{-}1 + \text{j}2)\,\mathbf{V}_3 = 0 \qquad (2)$$

Subtracting (2) from (1),

$$0 = 1.2\mathbf{V}_1 + \text{j}\,\mathbf{V}_2 \qquad (3)$$

But at the supernode,

$$\mathbf{V}_1 = 12\angle 0° + \mathbf{V}_2$$

or $$\mathbf{V}_2 = \mathbf{V}_1 - 12 \qquad (4)$$

Substituting (4) into (3),

$$0 = 1.2\mathbf{V}_1 + \text{j}\,(\mathbf{V}_1 - 12)$$

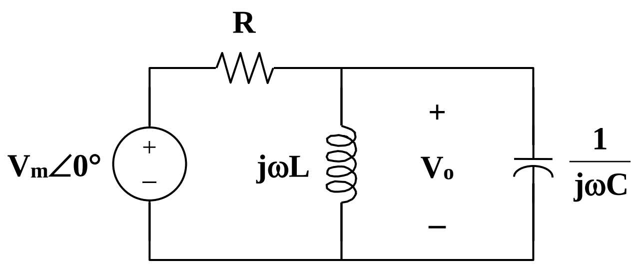
$$\mathbf{V}_1 = \frac{\text{j}12}{1.2 + \text{j}} = \mathbf{V}_o$$

$$\mathbf{V}_o = \frac{12\angle 90°}{1.562\angle 39.81°}$$

$$\mathbf{V}_o = \mathbf{7.682\angle 50.19°\ V}$$

Solution 10.20

The circuit is converted to its frequency-domain equivalent circuit as shown below.



Let $\qquad \mathbf{Z} = j\omega L \parallel \dfrac{1}{j\omega C} = \dfrac{\frac{L}{C}}{j\omega L + \frac{1}{j\omega C}} = \dfrac{j\omega L}{1-\omega^2 LC}$

$$\mathbf{V}_o = \frac{\mathbf{Z}}{R+\mathbf{Z}} V_m = \frac{\frac{j\omega L}{1-\omega^2 LC}}{R + \frac{j\omega L}{1-\omega^2 LC}} V_m = \frac{j\omega L}{R\,(1-\omega^2 LC) + j\omega L} V_m$$

$$\mathbf{V}_o = \frac{\omega L\, V_m}{\sqrt{R^2\,(1-\omega^2 LC)^2 + \omega^2 L^2}} \angle\left(90^\circ - \tan^{-1}\frac{\omega L}{R\,(1-\omega^2 LC)}\right)$$

If $\qquad \mathbf{V}_o = A\angle\phi$, then

$$A = \frac{\boldsymbol{\omega L\, V_m}}{\sqrt{\mathbf{R^2\,(1-\omega^2 LC)^2 + \omega^2 L^2}}}$$

and $\qquad \phi = \mathbf{90^\circ - tan^{-1}\dfrac{\omega L}{R\,(1-\omega^2 LC)}}$

Solution 10.21

(a) $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{1}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{1}{1-\omega^2 LC + j\omega RC}$$

At $\omega = 0$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1} = \mathbf{1}$$

As $\omega \to \infty$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{0}$$

At $\omega = \dfrac{1}{\sqrt{LC}}$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{jRC \cdot \dfrac{1}{\sqrt{LC}}} = \frac{\mathbf{-j}}{\mathbf{R}}\sqrt{\frac{\mathbf{L}}{\mathbf{C}}}$$

(b) $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{-\omega^2 LC}{1-\omega^2 LC + j\omega RC}$$

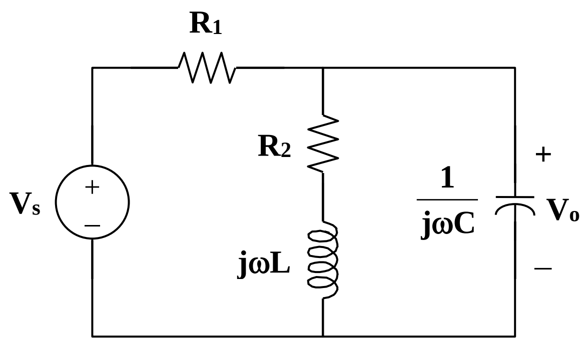
At $\omega = 0$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{0}$$

As $\omega \to \infty$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1} = \mathbf{1}$$

At $\omega = \dfrac{1}{\sqrt{LC}}$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-1}{jRC \cdot \dfrac{1}{\sqrt{LC}}} = \frac{\mathbf{j}}{\mathbf{R}}\sqrt{\frac{\mathbf{L}}{\mathbf{C}}}$$

Solution 10.22

Consider the circuit in the frequency domain as shown below.



Let $\quad \mathbf{Z} = (R_2 + j\omega L) \,||\, \dfrac{1}{j\omega C}$

$$\mathbf{Z} = \frac{\dfrac{1}{j\omega C}(R_2 + j\omega L)}{R_2 + j\omega L + \dfrac{1}{j\omega C}} = \frac{R_2 + j\omega L}{1 + j\omega R_2 - \omega^2 LC}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}}{\mathbf{Z} + R_1} = \frac{\dfrac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C}}{R_1 + \dfrac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C}}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{R_2 + j\omega L}}{\mathbf{R_1 + R_2 - \omega^2 LCR_1 + j\omega(L + R_1 R_2 C)}}$$

Solution 10.23

$$\frac{V - V_s}{R} + \frac{V}{j\omega L + \frac{1}{j\omega C}} + j\omega CV = 0$$

$$V + \frac{j\omega RCV}{-\omega^2 LC + 1} + j\omega RCV = V_s$$

$$\left(\frac{1 - \omega^2 LC + j\omega RC + j\omega RC - j\omega^3 RLC^2}{1 - \omega^2 LC}\right) V = V_s$$

$$\boldsymbol{V = \underline{\frac{(1 - \omega^2 LC)V_s}{1 - \omega^2 LC + j\omega RC(2 - \omega^2 LC)}}}$$

Solution 10.24

Design a problem to help other students to better understand mesh analysis.

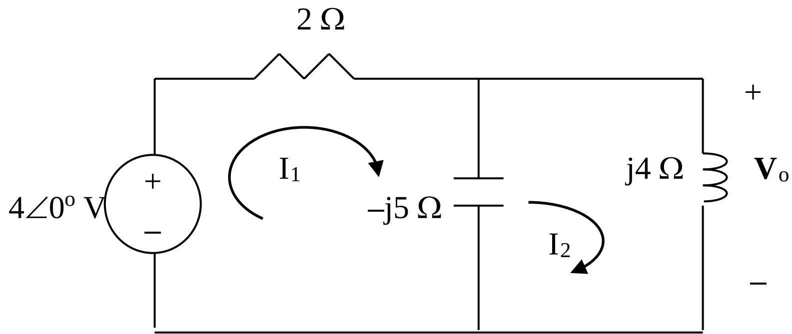
Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use mesh analysis to find $\mathbf{V}_o$ in the circuit in Prob. 10.2.

Solution

Consider the circuit as shown below.



For mesh 1,

$$4 = (2 - j5)I_1 + j5I_1 \qquad (1)$$

For mesh 2,

$$0 = j5I_1 + (j4 - j5)I_2 \quad \longrightarrow \quad I_1 = \frac{1}{5}I_2 \qquad (2)$$

Substituting (2) into (1),

$$4 = (2 - j5)\frac{1}{5}I_2 + j5I_2 \quad \longrightarrow \quad I_2 = \frac{1}{0.1 + j}$$

$\mathbf{V_o}$ = j4$\mathbf{I_2}$ = j4/(0.1+j) = j4/(1.00499∠84.29°) = **3.98∠5.71° V**

Solution 10.25

$\omega = 2$

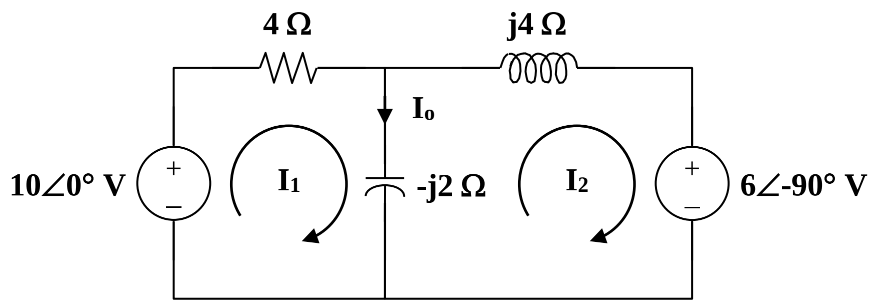
$10\cos(2t) \longrightarrow 10\angle 0°$

$6\sin(2t) \longrightarrow 6\angle -90° = -j6$

$2 \text{ H} \longrightarrow j\omega L = j4$

$0.25 \text{ F} \longrightarrow \dfrac{1}{j\omega C} = \dfrac{1}{j(2)(1/4)} = -j2$

The circuit is shown below.



For loop 1,

$$-10 + (4 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$
$$5 = (2 - j)\mathbf{I}_1 + j\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$j2\mathbf{I}_1 + (j4 - j2)\mathbf{I}_2 + (-j6) = 0$$
$$\mathbf{I}_1 + \mathbf{I}_2 = 3 \qquad (2)$$

In matrix form (1) and (2) become

$$\begin{bmatrix} 2-j & j \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\Delta = 2(1-j), \qquad \Delta_1 = 5 - j3, \qquad \Delta_2 = 1 - j3$$

$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{4}{2(1-j)} = 1 + j = 1.4142\angle 45°$$

Therefore,

$i_o(t) =$ **1.4142cos(2t + 45°) A**

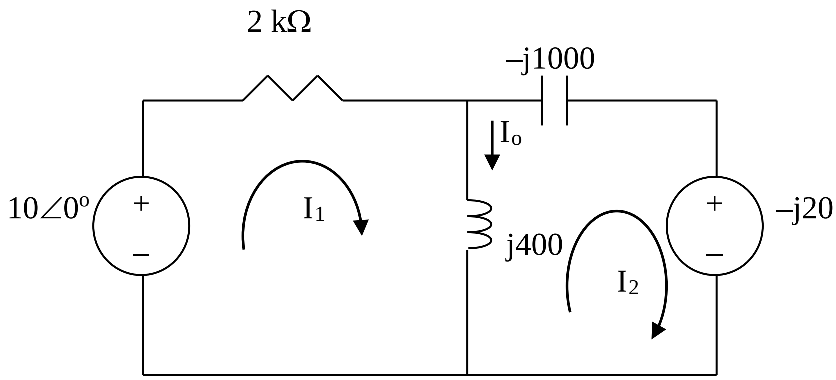
Solution 10.26

$$0.4H \quad \longrightarrow \quad j\omega L = j10^3 x0.4 = j400$$

$$1\mu F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j10^3 x10^{-6}} = -j1000$$

$$20\sin\left(10^3 t\right) = 20\cos\left(10^3 t - 90°\right) which\,leads\,to\,20\angle -90° = -j20$$

The circuit becomes that shown below.



For loop 1,

$$-10 + (12000 + j400)I_1 - j400I_2 = 0 \quad \longrightarrow \quad 1 = (200 + j40)I_1 - j40I_2 \ (1)$$

For loop 2,

$$-j20 + (j400 - j1000)I_2 - j400I_1 = 0 \quad \longrightarrow \quad -12 = 40I_1 + 60I_2$$

(2)

In matrix form, (1) and (2) become

$$\begin{bmatrix} 1 \\ -12 \end{bmatrix} = \begin{bmatrix} 200 + j40 & -j40 \\ 40 & 60 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this leads to

I_1 =0.0025-j0.0075, I_2 = -0.035+j0.005

$I_o = I_1 - I_2$ = 0.0375 – j0.0125 = 39.5∠–18.43° mA

$i_o(t)$ = **39.5cos(10^3t–18.43°) mA**

Solution 10.27

Using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 10.75 as shown in the text.

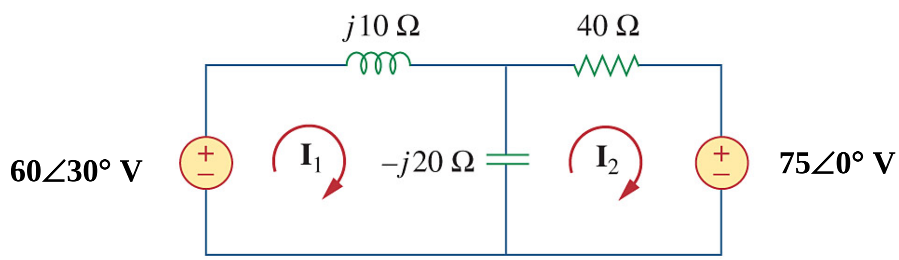


Figure 10.75
For Prob. 10.27.

Solution

For mesh 1,

$$-60\angle 30° + (j10 - j20)\mathbf{I}_1 + j20\mathbf{I}_2 = 0$$

$$6\angle 30° = -j\mathbf{I}_1 + j2\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$75\angle 0° + (40 - j20)\mathbf{I}_2 + j20\mathbf{I}_1 = 0$$

$$7.5 = -j2\mathbf{I}_1 - (4 - j2)\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 6\angle 30° \\ 7.5 \end{bmatrix} = \begin{bmatrix} -j & j2 \\ -j2 & -(4 - j2) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = -2 + 4j = 4.472\angle 116.56°$$

$$\Delta_1 = -(6\angle 30°)(4 - j2) - j15 = 31.515\angle 211.8°$$

$$\Delta_2 = -j7.5 + 12\angle 120° = 6.66\angle 154.27°$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \mathbf{7.047\angle 95.24°\ A}$$

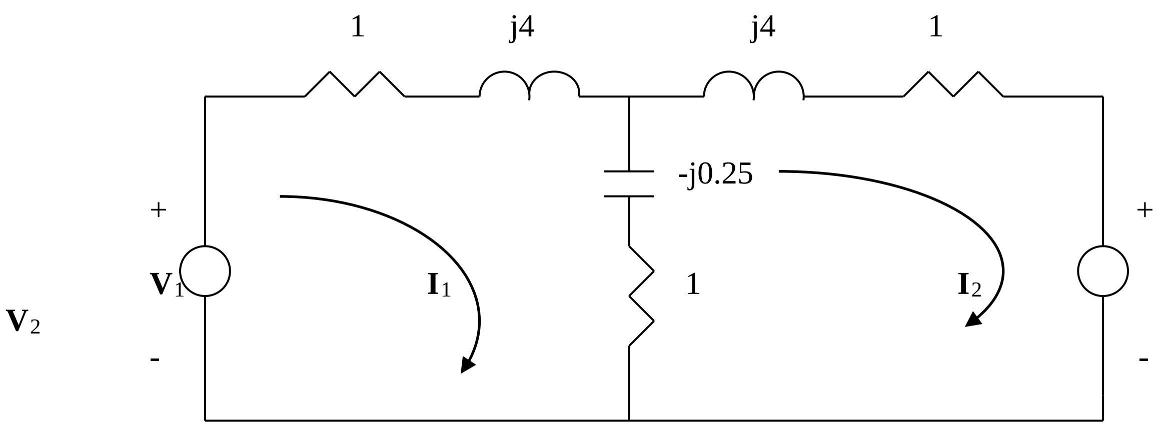
$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \mathbf{1.4892\angle 37.71°\ A}$$

Solution 10.28

$$1\text{H} \longrightarrow \quad j\omega L = j4, \qquad 1\text{F} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j1x4} = -j0.25$$

The frequency-domain version of the circuit is shown below, where

$$V_1 = 10\angle 0^o, \qquad V_2 = 20\angle -30^o \ .$$



$$V_1 = 10\angle 0^o, \qquad V_2 = 20\angle -30^o$$

Applying mesh analysis,

$$10 = (2 + j3.75)I_1 - (1 - j0.25)I_2 \qquad (1)$$

$$-20\angle -30^o = -(1 - j0.25)I_1 + (2 + j3.75)I_2 \qquad (2)$$

From (1) and (2), we obtain

$$\begin{pmatrix} 10 \\ -17.32 + j10 \end{pmatrix} = \begin{pmatrix} 2 + j3.75 & -1 + j0.25 \\ -1 + j0.25 & 2 + j3.75 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

Solving this leads to

$$I_1 = 2.741\angle -41.07^o, \qquad I_2 = 4.114\angle 92^o$$

Hence,

$i_1(t) = \mathbf{2.741cos(4t–41.07^\circ)A}$, $i_2(t) = \mathbf{4.114cos(4t+92^\circ)A}$.

Solution 10.29

Using Fig. 10.77, design a problem to help other students better understand mesh analysis.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

By using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit depicted in Fig. 10.77.

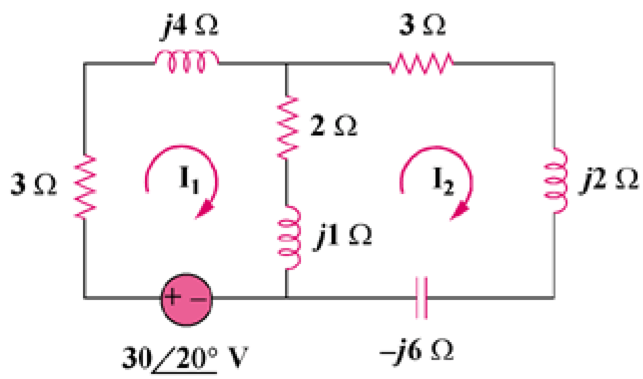


Figure 10.77

Solution

For mesh 1,

$$(5+j5)\mathbf{I}_1-(2+j)\mathbf{I}_2-30\angle 20°=0$$
$$30\angle 20°=(5+j5)\mathbf{I}_1-(2+j)\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$(5+j3-j6)\mathbf{I}_2-(2+j)\mathbf{I}_1=0$$
$$0=-(2+j)\mathbf{I}_1+(5-j3)\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix}30\angle 20°\\0\end{bmatrix}=\begin{bmatrix}5+j5 & -(2+j)\\-(2+j) & 5-j3\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\\mathbf{I}_2\end{bmatrix}$$

$$\Delta=37+j6=37.48\angle 9.21°$$
$$\Delta_1=(30\angle 20°)(5.831\angle -30.96°)=175\angle -10.96°$$

$$\Delta_2 = (30\angle 20°)(2.356\angle 26.56°) = 67.08\angle 46.56°$$

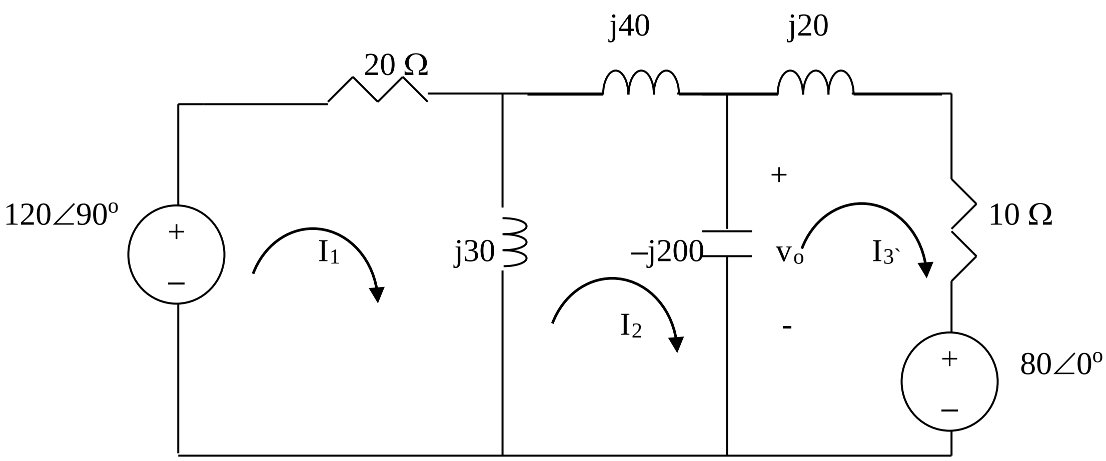
$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \mathbf{4.67\angle{-20.17°}\ A}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \mathbf{1.79\angle 37.35°\ A}$$

Solution 10.30

$$300mH \longrightarrow j\omega L = j100x300x10^{-3} = j30$$
$$200mH \longrightarrow j\omega L = j100x200x10^{-3} = j20$$
$$400mH \longrightarrow j\omega L = j100x400x10^{-3} = j40$$
$$50\mu F \longrightarrow 1/j\omega C = \frac{1}{j100x50x10^{-6}} = -j200$$

The circuit becomes that shown below.



For mesh 1,

$$-120 < 90^o + (20 + j30)I_1 - j30I_2 = 0 \longrightarrow j120 = (20 + j30)I_1 - j30I_2 \qquad (1)$$

For mesh 2,

$$-j30I_1 + (j30 + j40 - j200)I_2 + j200I_3 = 0 \longrightarrow 0 = -3I_1 - 13I_2 + 20I_3 \qquad (2)$$

For mesh 3,

$$80 + \mathrm{j}200\mathrm{I}_2 + (10 - \mathrm{j}180)\mathrm{I}_3 = 0 \rightarrow -8 = \mathrm{j}20\mathrm{I}_2 + (1 - \mathrm{j}18)\mathrm{I}_3 \qquad (3)$$

We put (1) to (3) in matrix form.

$$\begin{bmatrix} 2+\mathrm{j}3 & -\mathrm{j}3 & 0 \\ -3 & -13 & 20 \\ 0 & \mathrm{j}20 & 1-\mathrm{j}18 \end{bmatrix} \begin{bmatrix} \mathrm{I}_1 \\ \mathrm{I}_2 \\ \mathrm{I}_3 \end{bmatrix} = \begin{bmatrix} \mathrm{j}12 \\ 0 \\ -8 \end{bmatrix}$$

This is an excellent candidate for MATLAB.

```
>> Z=[(2+3i),-3i,0;-3,-13,20;0,20i,(1-18i)]

Z =

   2.0000 + 3.0000i        0 - 3.0000i        0
  -3.0000           -13.0000           20.0000
        0                  0 +20.0000i   1.0000 -18.0000i

>> V=[12i;0;-8]

V =

        0 +12.0000i
        0
  -8.0000

>> I=inv(Z)*V

I =

   2.0557 + 3.5651i
   0.4324 + 2.1946i
   0.5894 + 1.9612i
```

$V_o = -j200(I_2 - I_3) = -j200(-0.157+j0.2334) = 46.68 + j31.4 = 56.26\angle 33.93^\circ$

$$v_o = \mathbf{56.26cos(100t + 33.93^\circ)\ V}.$$

Solution 10.31

Use mesh analysis to determine current $\mathbf{I}_o$ in the circuit of Fig. 10.79 below.

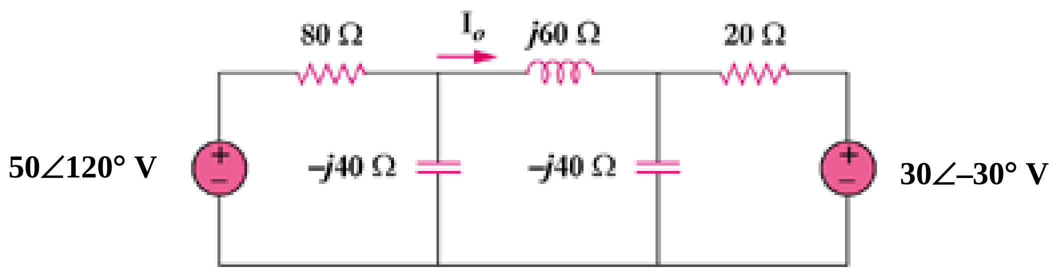
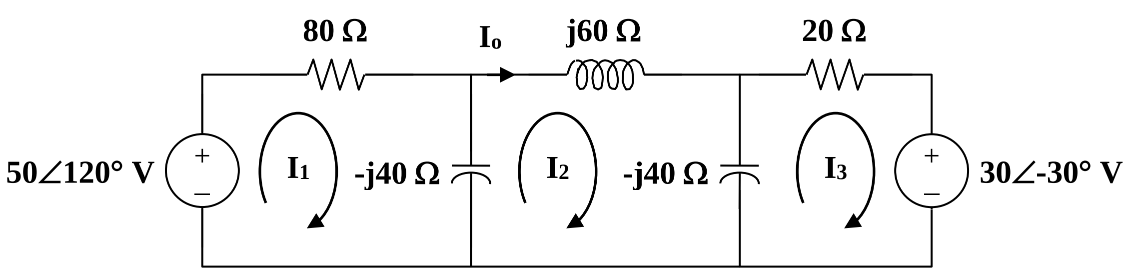


Figure 10.79
For Prob. 10.31.

Solution

Consider the network shown below.



For loop 1,

$$-50\angle 120° + (80 - j40)\mathbf{I}_1 + j40\mathbf{I}_2 = 0$$

$$5\angle 20° = 4(2 - j)\mathbf{I}_1 + j4\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$\text{j}40\mathbf{I}_1 + (\text{j}60 - \text{j}80)\mathbf{I}_2 + \text{j}40\mathbf{I}_3 = 0$$

$$0 = 2\mathbf{I}_1 - \mathbf{I}_2 + 2\mathbf{I}_3 \qquad (2)$$

For loop 3,

$$30\angle -30° + (20 - j40)\mathbf{I}_3 + j40\mathbf{I}_2 = 0$$

$$-3\angle -30° = j4\mathbf{I}_2 + 2(1 - j2)\mathbf{I}_3 \qquad (3)$$

From (2),

$$2\mathbf{I}_3 = \mathbf{I}_2 - 2\mathbf{I}_1$$

Substituting this equation into (3),

$$-3\angle -30° = -2(1 - j2)\mathbf{I}_1 + (1 + j2)\mathbf{I}_2 \qquad (4)$$

From (1) and (4),

$$\begin{bmatrix} 5\angle 120° \\ -3\angle -30° \end{bmatrix} = \begin{bmatrix} 4(2-j) & j4 \\ -2(1-j2) & 1+j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8-\text{j}4 & -\text{j}4 \\ -2+\text{j}4 & 1+\text{j}2 \end{vmatrix} = 32 + \text{j}20 = 37.74\angle 32°$$

$$\Delta_2 = \begin{vmatrix} 8-j4 & 5\angle 120° \\ -2+j4 & -3\angle -30° \end{vmatrix} = -2.464 + j41.06 = 41.125\angle 93.44°$$

$$\mathbf{I}_o = \mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \mathbf{1.0897\angle 61.44°\ A}$$

Solution 10.32

Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the circuit of Fig. 10.80 using mesh analysis.

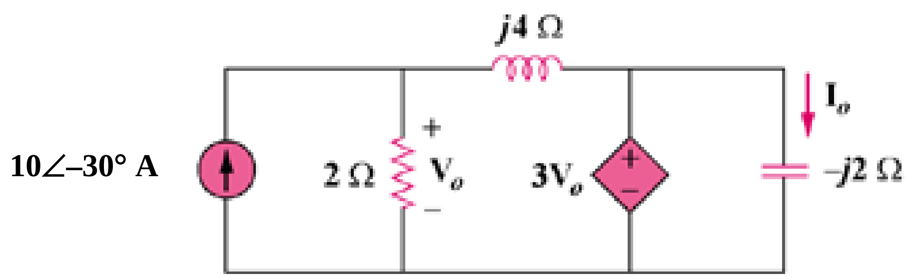
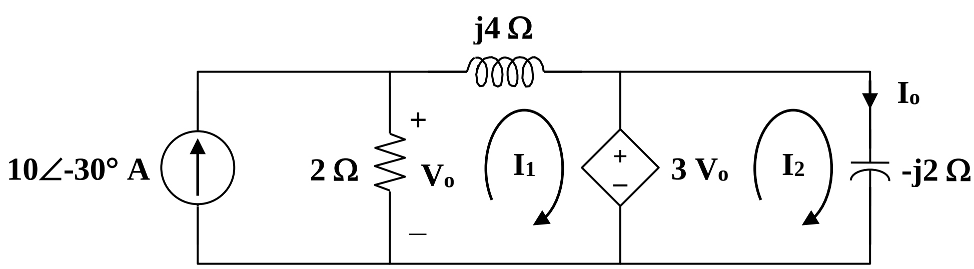


Figure 10.80
For Prob. 10.32.

Solution

Consider the circuit below.



For mesh 1,

$$(2+j4)\mathbf{I}_1 - 2(10\angle\text{-}30°) + 3\mathbf{V}_o = 0$$

where $\quad \mathbf{V}_o = 2(10\angle\text{-}30° - \mathbf{I}_1)$

Hence,

$$(2+j4)\mathbf{I}_1 - 20\angle\text{-}30° + 6(10\angle\text{-}30° - \mathbf{I}_1) = 0$$

$$10\angle\text{-}30° = (1-j)\mathbf{I}_1$$

or $\quad \mathbf{I}_1 = 25\sqrt{2}\angle 15°$

$$\mathbf{I}_o = \frac{3\mathbf{V}_o}{-j2} = \frac{3}{-j2}(2)(10\angle\text{-}30° - \mathbf{I}_1)$$

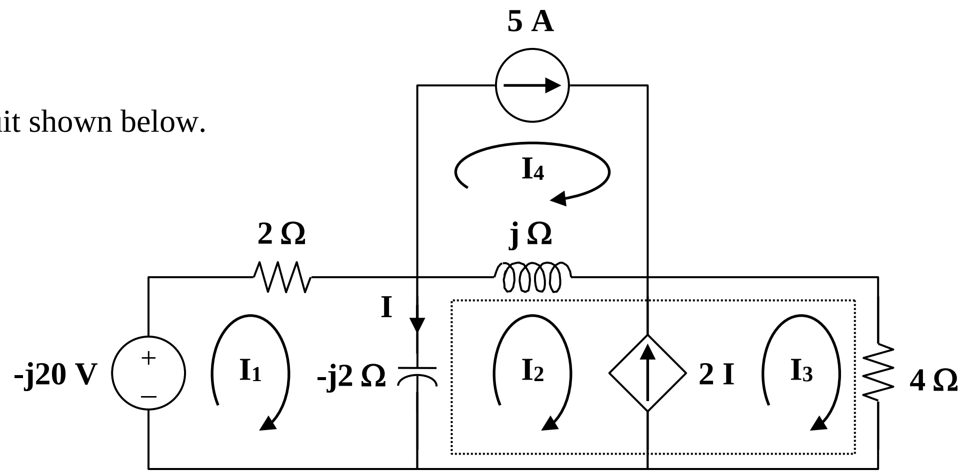
$$\mathbf{I}_o = j3(10\angle\text{-}30° - 5\sqrt{2}\angle 15°)$$

$$\mathbf{I}_o = \mathbf{21.21\angle 15°\ A}$$

$$\mathbf{V}_o = \frac{-j2\mathbf{I}_o}{3} = \mathbf{5.657\angle\text{-}75°\ V}$$

Solution 10.33

Consider the circuit shown below.



For mesh 1,

$$j20 + (2 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$

$$(1 - j)\mathbf{I}_1 + j\mathbf{I}_2 = -j10 \qquad (1)$$

For the supermesh,

$$(j - j2)\mathbf{I}_2 + j2\mathbf{I}_1 + 4\mathbf{I}_3 - j\mathbf{I}_4 = 0 \qquad (2)$$

Also,

$$\mathbf{I}_3 - \mathbf{I}_2 = 2\mathbf{I} = 2(\mathbf{I}_1 - \mathbf{I}_2)$$

$$\mathbf{I}_3 = 2\mathbf{I}_1 - \mathbf{I}_2 \qquad (3)$$

For mesh 4,

$$\mathbf{I}_4 = 5 \qquad (4)$$

Substituting (3) and (4) into (2),

$$(8 + j2)\mathbf{I}_1 - (-4 + j)\mathbf{I}_2 = j5 \qquad (5)$$

Putting (1) and (5) in matrix form,

$$\begin{bmatrix} 1 - j & j \\ 8 + j2 & 4 - j \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} -j10 \\ j5 \end{bmatrix}$$

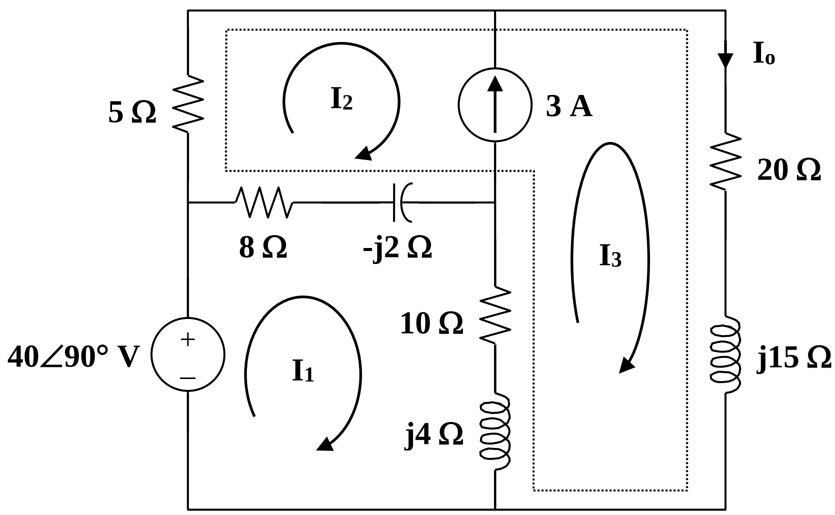
$\Delta = -3 - j5$, $\qquad \Delta_1 = -5 + j40$, $\qquad \Delta_2 = -15 + j85$

$$\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{10 - j45}{-3 - j5} =$$

7.906∠43.49° A

Solution 10.34

The circuit is shown below.



For mesh 1,

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0 \qquad (1)$$

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (30 + j19)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0 \qquad (2)$$

Also,

$$\mathbf{I}_2 = \mathbf{I}_3 - 3 \qquad (3)$$

Adding (1) and (2) and incorporating (3),

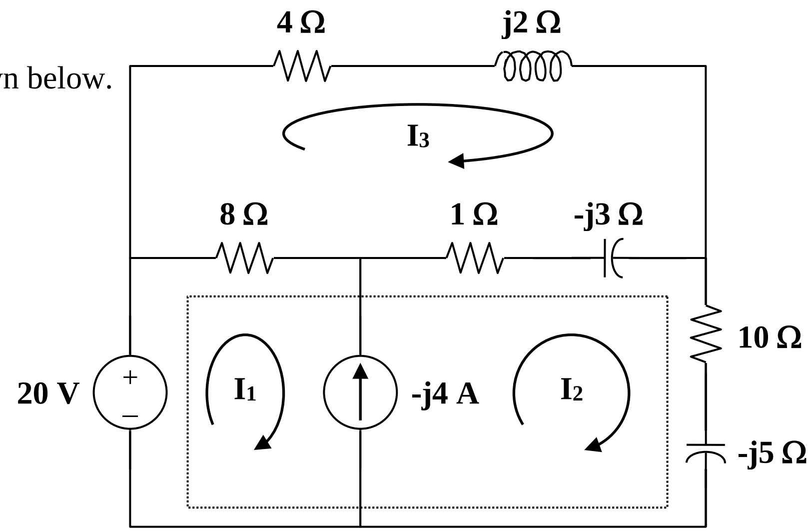
$$-j40 + 5(\mathbf{I}_3 - 3) + (20 + j15)\mathbf{I}_3 = 0$$

$$\mathbf{I}_3 = \frac{3 + j8}{5 + j3} = 1.465\angle 38.48°$$

$$\mathbf{I}_o = \mathbf{I}_3 = \mathbf{1.465\angle 38.48°\ A}$$

Solution 10.35

Consider the circuit shown below.



For the supermesh,

$$-20 + 8\mathbf{I}_1 + (11 - j8)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 0 \qquad (1)$$

Also,

$$\mathbf{I}_1 = \mathbf{I}_2 + j4 \qquad (2)$$

For mesh 3,

$$(13 - j)\mathbf{I}_3 - 8\mathbf{I}_1 - (1 - j3)\mathbf{I}_2 = 0 \qquad (3)$$

Substituting (2) into (1),

$$(19 - j8)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 20 - j32 \qquad (4)$$

Substituting (2) into (3),

$$-(9 - j3)\mathbf{I}_2 + (13 - j)\mathbf{I}_3 = j32 \qquad (5)$$

From (4) and (5),

$$\begin{bmatrix} 19 - j8 & -(9 - j3) \\ -(9 - j3) & 13 - j \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 20 - j32 \\ j32 \end{bmatrix}$$

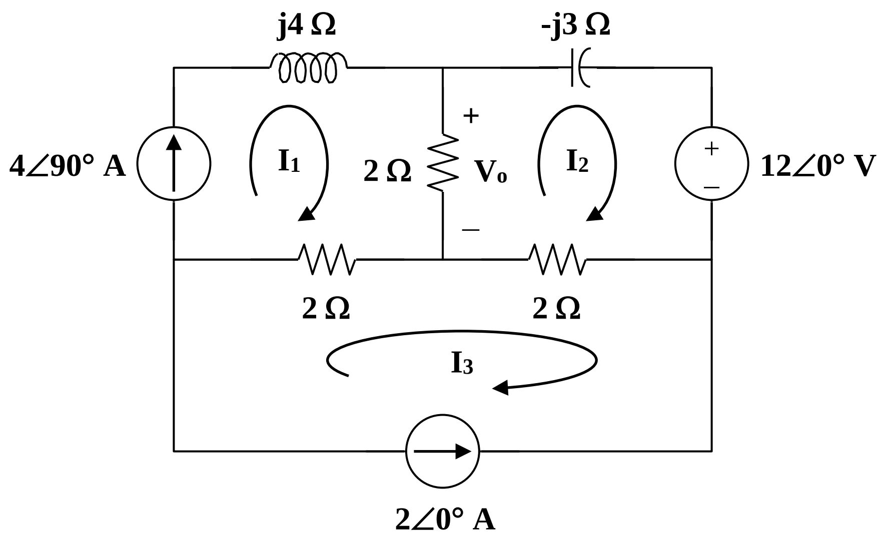
$$\Delta = 167 - j69, \qquad \Delta_2 = 324 - j148$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{324 - j148}{167 - j69} = \frac{356.2\angle -24.55^\circ}{180.69\angle -22.45^\circ}$$

$$\mathbf{I}_2 = \mathbf{1.971\angle{-2.1^\circ}\ A}$$

Solution 10.36

Consider the circuit below.



Clearly,

$\mathbf{I}_1 = 4\angle 90° = \text{j}4 \quad \text{and} \qquad \mathbf{I}_3 = \text{-}2$

For mesh 2,

$$(4 - \text{j}3)\mathbf{I}_2 - 2\mathbf{I}_1 - 2\mathbf{I}_3 + 12 = 0$$

$$(4 - \text{j}3)\mathbf{I}_2 - \text{j}8 + 4 + 12 = 0$$

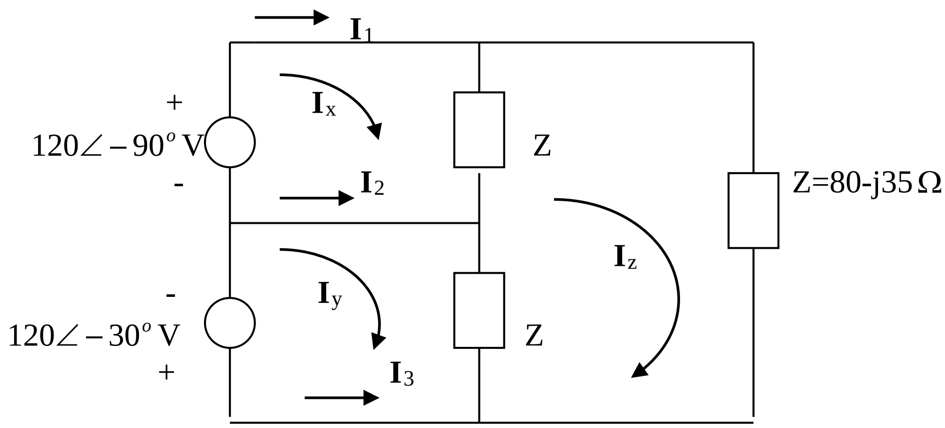
$$\mathbf{I}_2 = \frac{\text{-}16 + \text{j}8}{4 - \text{j}3} = \text{-}3.52 - \text{j}0.64$$

Thus,

$$\mathbf{V}_o = 2(\mathbf{I}_1 - \mathbf{I}_2) = (2)(3.52 + \text{j}4.64) = 7.04 + \text{j}9.28$$

$$\mathbf{V}_o = \mathbf{11.648\angle 52.82°\ V}$$

Solution 10.37



For mesh x,

$$ZI_x - ZI_z = -j120 \qquad (1)$$

For mesh y,

$$ZI_y - ZI_z = -120\angle 30^o = -103.92 + j60 \qquad (2)$$

For mesh z,

$$-ZI_x - ZI_y + 3ZI_z = 0 \qquad (3)$$

Putting (1) to (3) together leads to the following matrix equation:

$$\begin{pmatrix} (80-j35) & 0 & (-80+j35) \\ 0 & (80-j35) & (-80+j35) \\ (-80+j35) & (-80+j35) & (240-j105) \end{pmatrix}\begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} = \begin{pmatrix} -j120 \\ -103.92+j60 \\ 0 \end{pmatrix} \longrightarrow \quad AI = B$$

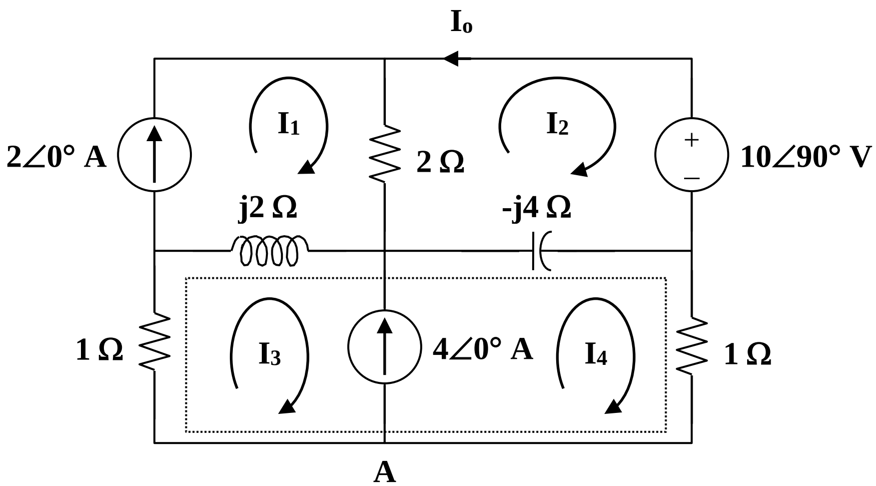
Using MATLAB, we obtain

$$I = inv(A)*B = \begin{pmatrix} -0.2641 - j2.366 \\ -2.181 - j0.954 \\ -0.815 - j1.1066 \end{pmatrix}$$

$$\boldsymbol{I_1 = I_x = -0.2641 - j2.366 = \underline{2.38\angle -96.37^o} \text{ A}}$$
$$\boldsymbol{I_2 = I_y - I_x = -1.9167 + j1.4116 = \underline{2.38\angle 143.63^o} \text{ A}}$$
$$\boldsymbol{I_3 = -I_y = 2.181 + j0.954 = \underline{2.38\angle 23.63^o \text{ A}}}$$

Solution 10.38

Consider the circuit below.



Clearly,

$$\mathbf{I}_1 = 2 \qquad (1)$$

For mesh 2,

$$(2 - j4)\mathbf{I}_2 - 2\mathbf{I}_1 + j4\mathbf{I}_4 + 10\angle 90° = 0 \qquad (2)$$

Substitute (1) into (2) to get

$$(1 - j2)\mathbf{I}_2 + j2\mathbf{I}_4 = 2 - j5$$

For the supermesh,

$$(1 + j2)\mathbf{I}_3 - j2\mathbf{I}_1 + (1 - j4)\mathbf{I}_4 + j4\mathbf{I}_2 = 0$$

$$j4\mathbf{I}_2 + (1 + j2)\mathbf{I}_3 + (1 - j4)\mathbf{I}_4 = j4 \qquad (3)$$

At node A,

$$\mathbf{I}_3 = \mathbf{I}_4 - 4 \qquad (4)$$

Substituting (4) into (3) gives

$$j2\mathbf{I}_2 + (1 - j)\mathbf{I}_4 = 2(1 + j3) \qquad (5)$$

From (2) and (5),

$$\begin{bmatrix} 1 - j2 & j2 \\ j2 & 1 - j \end{bmatrix}\begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_4 \end{bmatrix} = \begin{bmatrix} 2 - j5 \\ 2 + j6 \end{bmatrix}$$

$$\Delta = 3 - j3, \qquad \Delta_1 = 9 - j11$$

$$\mathbf{I}_o = -\mathbf{I}_2 = \frac{-\Delta_1}{\Delta} = \frac{-(9 - j11)}{3 - j3} = \frac{1}{3}(-10 + j)$$

$$\mathbf{I}_o = \mathbf{3.35\angle 174.3° \ A}$$

Solution 10.39

For mesh 1,

$$(28 - j15)I_1 - 8I_2 + j15I_3 = 12\angle 64^o \qquad (1)$$

For mesh 2,

$$-8I_1 + (8 - j9)I_2 - j16I_3 = 0 \qquad (2)$$

For mesh 3,

$$j15I_1 - j16I_2 + (10 + j)I_3 = 0 \qquad (3)$$

In matrix form, (1) to (3) can be cast as

$$\begin{pmatrix} (28 - j15) & -8 & j15 \\ -8 & (8 - j9) & -j16 \\ j15 & -j16 & (10 + j) \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 12\angle 64^o \\ 0 \\ 0 \end{pmatrix} \quad \text{or} \quad AI = B$$

Using MATLAB,

I = inv(A)*B

$$I_1 = -0.128 + j0.3593 = \mathbf{381.4\angle 109.6^\circ\ mA}$$
$$I_2 = -0.1946 + j0.2841 = \mathbf{344.3\angle 124.4^\circ\ mA}$$
$$I_3 = 0.0718 - j0.1265 = \mathbf{145.5\angle{-60.42}^\circ\ mA}$$
$$I_x = I_1 - I_2 = 0.0666 + j0.0752 = \mathbf{100.5\angle 48.5^\circ\ mA}$$

381.4∠109.6° mA, 344.3∠124.4° mA, 145.5∠–60.42° mA, 100.5∠48.5° mA

Solution 10.40

Find i_o in the circuit shown in Fig. 10.85 using superposition.

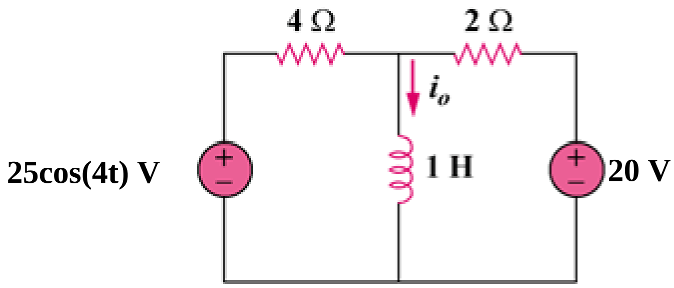
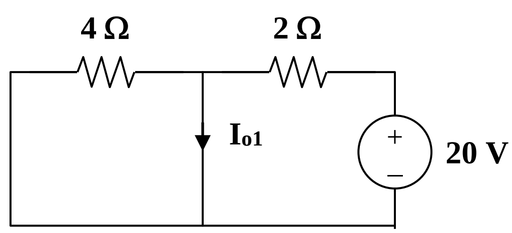


Figure 10.85
For Prob. 10.40.

Solution

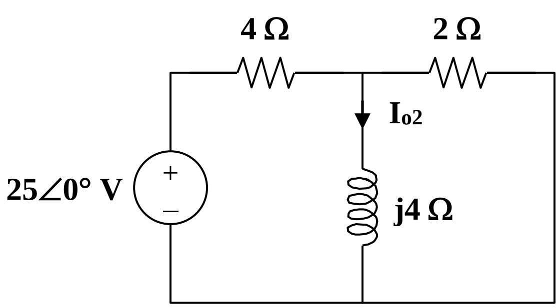
Let $\mathbf{I_o} = \mathbf{I_{o1}} + \mathbf{I_{o2}}$, where I_{o1} is due to the dc source and I_{o2} is due to the ac source. For I_{o1}, consider the circuit in Fig. (a).
Clearly,



(a)

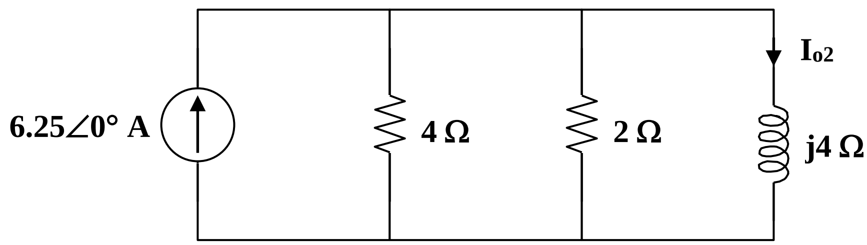
$\mathbf{I_{o1}} = 20/2 = 10$ A

For $\mathbf{I_{o2}}$, consider the circuit in Fig. (b).



(b)

If we transform the voltage source, we have the circuit in Fig. (c), where $4 \,||\, 2 = 4/3\,\Omega$.



(c)

By the current division principle,

$$\mathbf{I}_{o2} = \frac{4/3}{4/3 + j4}(6.25\angle 0°)$$

$$\mathbf{I}_{o2} = 0.625 - j1.875 = 1.9764\angle\text{-}71.56°$$

Thus, $$I_{o2} = 1.9764\cos(4t - 71.56°)\ A$$

Therefore,

$i_o = i_{o1} + i_{o2} =$ **[10 + 1.9764cos(4t–71.56°)] A**

Solution 10.41

Find v_o for the circuit in Fig. 10.86 assuming that $i_s(t)$ = 2sin(2t) + 3cos(4t) A.

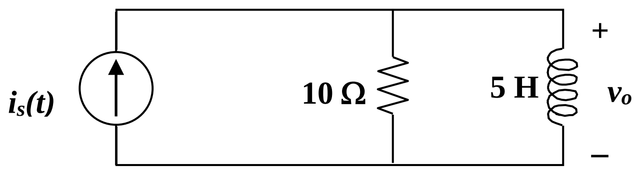


Figure 10.86
For Prob. 10.41.

Solution

This problem is easily solved using superposition. $V_o = I_s[10(j5\omega)]/(10+j5\omega)$.

For ω = 2 rad/s we get $V_o' = 2(j100)/(10+j10) = 14.142\angle 45^\circ$ A and for ω = 4 rad/s we get $V_o'' = 3(j200)/(10+j20) = j600/(22.361\angle 63.43^\circ$
$= 26.83\angle 26.57^\circ$ or

v_o = **[14.142sin(2t+45º) + 26.83cos(4t+26.57º)] V**.

Solution 10.42

Using Fig. 10.87, design a problem to help other students to better understand the superposition theorem.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Solve for I_o in the circuit of Fig. 10.87.

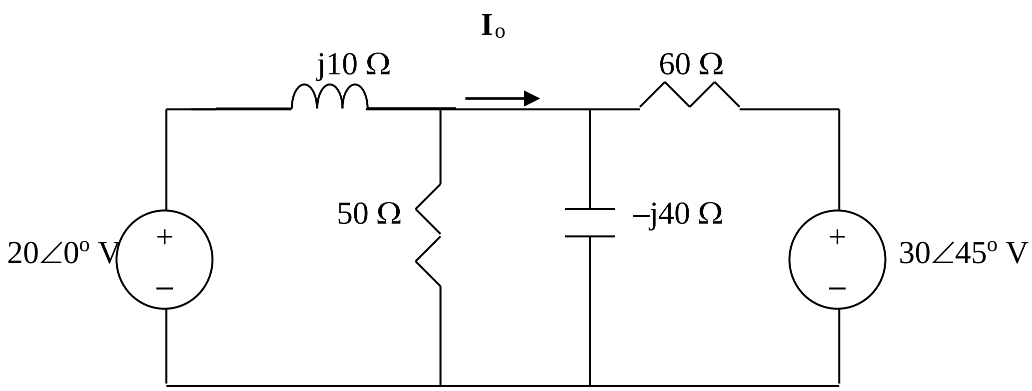
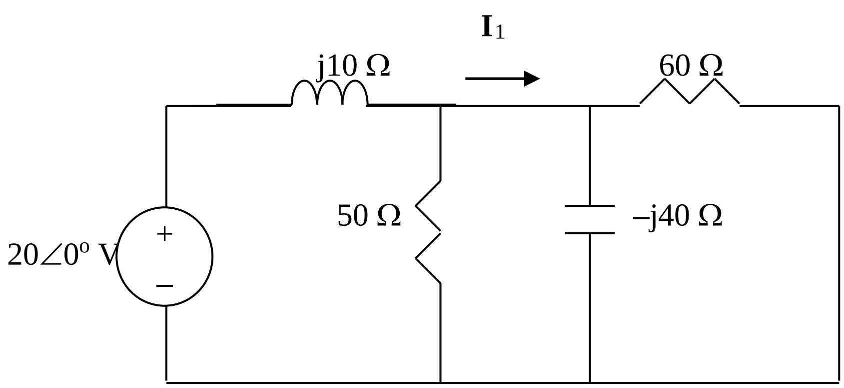


Figure 10.87 For Prob. 10.42.

Solution

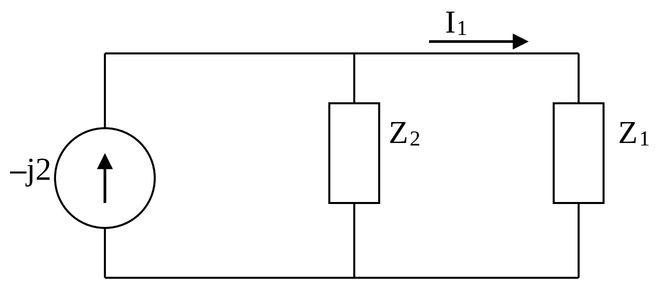
Let $I_o = I_1 + I_2$

where I_1 and I_2 are due to 20<0º and 30<45º sources respectively. To get I_1, we use the circuit below.



Let Z_1 = -j40//60 = 18.4615 –j27.6927, Z_2 = j10//50=1.9231 + j9.615

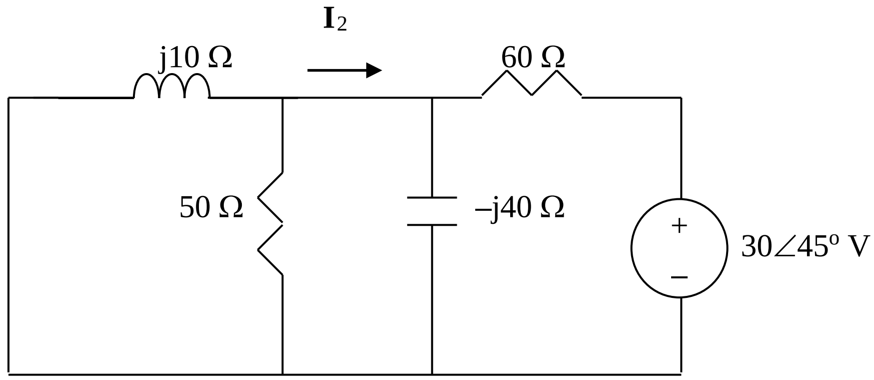
Transforming the voltage source to a current source leads to the circuit below.



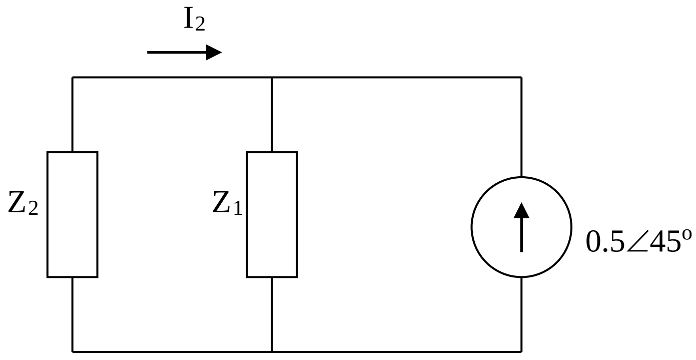
Using current division,

$$I_1 = \frac{Z_2}{Z_1 + Z_2}(-j2) = 0.6217 + j0.3626$$

To get I_2, we use the circuit below.



After transforming the voltage source, we obtain the circuit below.



Using current division,

$$I_2 = \frac{-Z_1}{Z_1 + Z_2}(0.5 < 45^o) = -0.5275 - j0.3077$$

Hence, $\mathbf{I_o} = \mathbf{I_1} + \mathbf{I_2} =$ 0.0942+j0.0509 = **109∠30° mA**.

Solution 10.43

Using the superposition principle, find i_x in the circuit of Fig. 10.88.

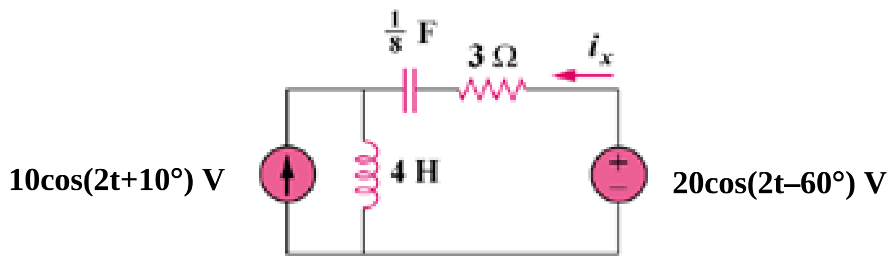


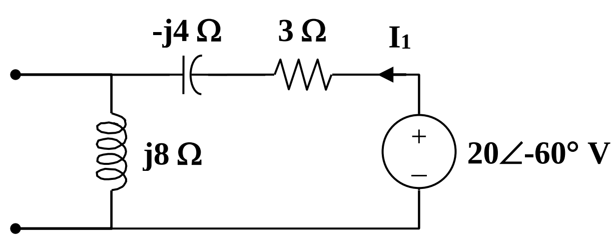
Figure 10.88
For Prob. 10.43.

Solution

Let $\mathbf{I}_x = \mathbf{I}_1 + \mathbf{I}_2$, where $\mathbf{I}_1$ is due to the voltage source and $\mathbf{I}_2$ is due to the current source.

$$\omega = 2$$
$$10\cos(2t+10°) \longrightarrow 10\angle 10°$$
$$20\cos(2t-60°) \longrightarrow 20\angle\text{-}60°$$
$$4\text{ H} \longrightarrow j\omega L = j8$$
$$\frac{1}{8}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/8)} = \text{-}j4$$

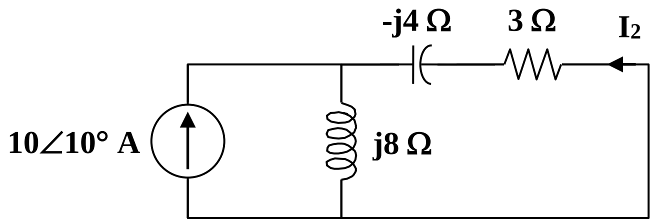
For $\mathbf{I}_1$, consider the circuit in Fig. (a).



(a)

$$\mathbf{I}_1 = \frac{20\angle\text{-}60°}{3+j8-j4} = \frac{20\angle\text{-}60°}{3+j4}$$

For $\mathbf{I}_2$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = \frac{-\mathrm{j}8}{3+\mathrm{j}8-\mathrm{j}4}(10\angle 10°) = \frac{-\mathrm{j}80\angle 10°}{3+j4}$$

$$\mathbf{I}_x = \mathbf{I}_1 + \mathbf{I}_2 = \frac{1}{3+j4}(20\angle -60° - j80\angle 10°)$$

$$\mathbf{I}_x = \frac{99.02\angle -76.04°}{5\angle 53.13°} = 19.804\angle -129.17°$$

Therefore, i_x = **19.804cos(2t – 129.17°) A**

Solution 10.44

Use superposition principle to obtain v_x in the circuit of Fig. 10.89. Let v_s = 50 sin2t V and i_s = 12 cos(6t + 10°) A.

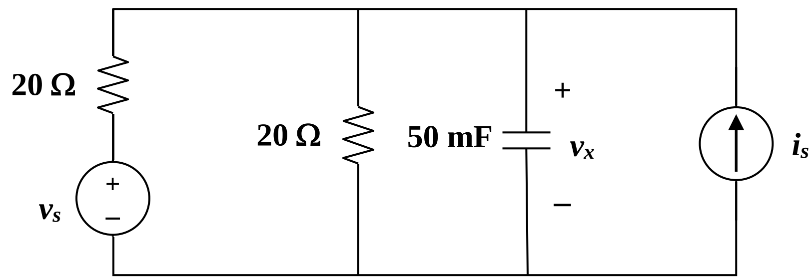


Figure 10.89
For Prob. 10.44.

Solution

Let $v_x = v_1 + v_2$, where v_1 and v_2 are due to the voltage source and current source respectively.

For v_1, ω = 2 rad/s and the capacitive reactance is equal to –j10 Ω and V_s = 50 V. The resulting nodal equation becomes, $[(V_1–50)/20] + [(V_1–0)/20] + [(V_1–0)/(–j10)] + 0 = 0$.

Simplifying we get $(0.05+0.05+j0.1)V_1 = (0.1+j0.1)V_1 = 2.5$ or $V_1 = 17.678\angle–45°$ or $v_1(t) = 17.678\sin(2t–45°)$ V.

For v_2, ω = 6 rad/s and the capacitive reactance is equal to –j(10/3) Ω and I_s = 12 A. Note we will adjust the angle after we calculate the value of V_2 during the conversion back into the time domain. The resulting nodal equation becomes, $[(V_2–0)/20] + [(V_2–0)/20] +$
$[(V_2–0)/(–j10/3)] –12 = 0$.

Simplifying we get $(0.05+0.5+j0.3)V_2 = 12$ or $V_2 = 12/(0.1+j0.3) = 12/(0.31623\angle71.57°)$ or $V_2 = 37.95\angle–71.57°$ V or $v_2(t) = 37.95\cos(6t–61.57°)$ V.

v_x = **[17.678sin(2t–45°)+37.95cos(6t–61.57°)] V**.

Solution 10.45

Use superposition to find *i(t)* in the circuit of Fig. 10.90.

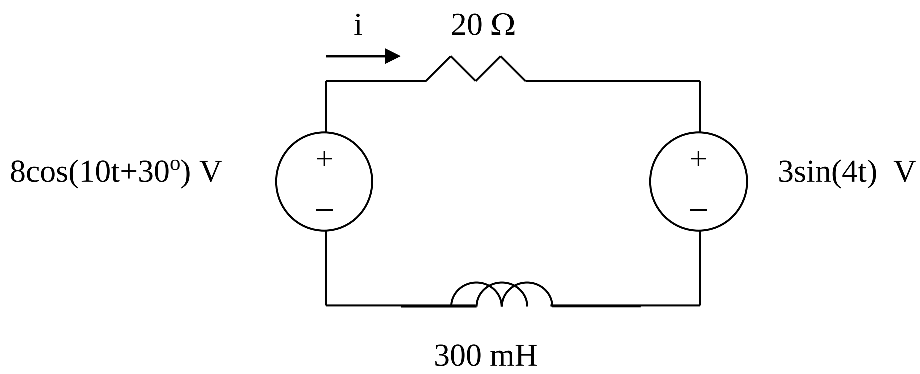
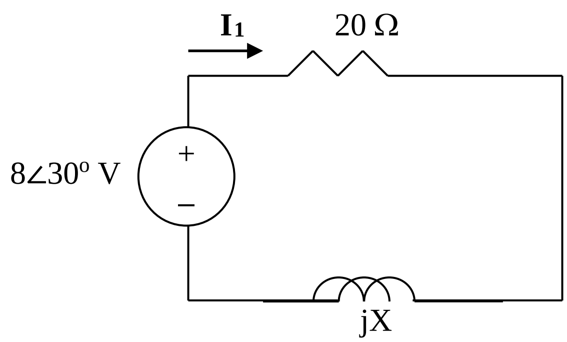


Figure 10.90
For Prob. 10.45.

Solution

Let $i = i_1 + i_2$, where i_1 and i_2 are due to 8cos(10t +30º) and 3sin4t sources respectively.
To find i_1, consider the circuit below.



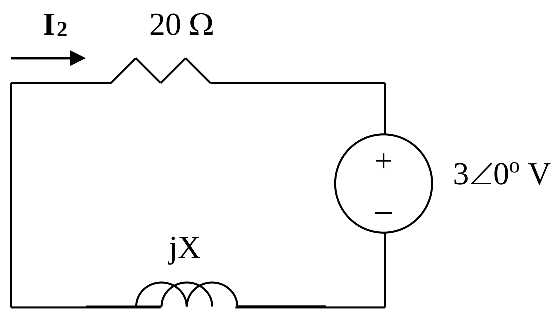
$$X = \omega L = 10x300x10^{-3} = 3$$

Type equation here.

$$\mathbf{I}_1 = \frac{8\angle 30°}{20 + j3} = \frac{8\angle 30°}{20.22\angle 8.53°} = 0.3956\angle 21.47°$$

$i_1(t)$ = 395.6cos(10t+21.47°) mA.

To find $i_2(t)$, consider the circuit below,



$$X = \omega L = 4x300x10^{-3} = 1.2$$

$$\mathbf{I}_2 = -\frac{3\angle 0^\circ}{20 + j1.2} = \frac{3\angle 180^\circ}{20.036\angle 3.43^\circ} = 0.14975\angle 176.57^\circ \text{ or}$$

$i_2(t)$ = 149.75sin(4t+176.57°) mA.

Thus,

i(t) = $i_1(t)$ + $i_2(t)$ = **[395.6cos(10t+21.47°) + 149.75sin(4t+176.57°)] mA**.

Solution 10.46

Solve for $v_o(t)$ in the circuit of Fig. 10.91 using the superposition principle.

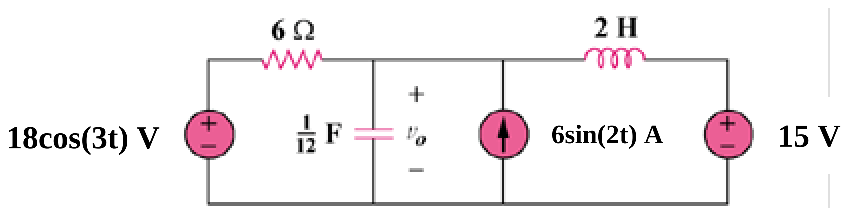
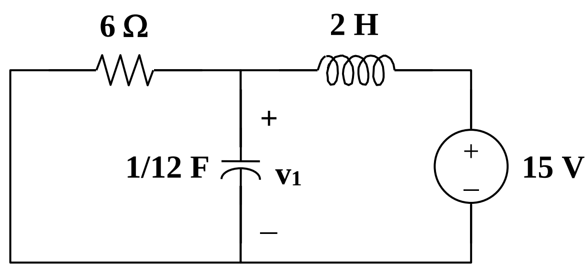


Figure 10.91
For Prob. 10.46.

Solution

Let $v_o = v_1 + v_2 + v_3$, where $\mathbf{V_1}$, $\mathbf{V_2}$, and $\mathbf{V_3}$ are respectively due to the 15-V dc source, the ac current source, and the ac voltage source. For v_1 consider the circuit in Fig. (a).



(a)

The capacitor is open to dc, while the inductor is a short circuit. Hence,

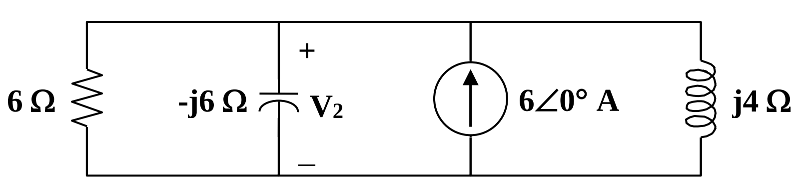
$$v_1 = 15\,V$$

For v_2, consider the circuit in Fig. (b).

$$\omega = 2$$

$$2\text{ H} \longrightarrow j\omega L = j4$$

$$\frac{1}{12}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/12)} = -j6$$



(b)

Applying nodal analysis,

$$6 = \frac{\mathbf{V}_2}{6} + \frac{\mathbf{V}_2}{-j6} + \frac{\mathbf{V}_2}{j4} = \left(\frac{1}{6} + \frac{j}{6} - \frac{j}{4}\right)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{36}{1 - j0.5} = 32.18\angle 26.57°$$

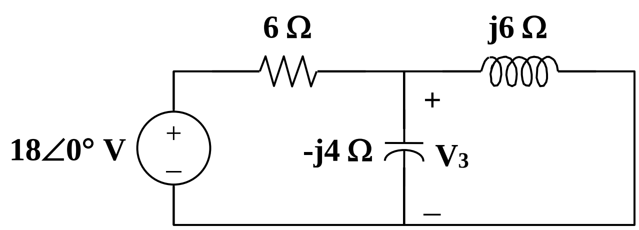
Hence, $v_2 = 32.18\sin(2t + 26.57°)\,V$

For v_3, consider the circuit in Fig. (c).

$$\omega = 3$$

$$2\text{ H} \longrightarrow \mathrm{j}\omega\mathrm{L} = \mathrm{j}6$$

$$\frac{1}{12}\text{ F} \longrightarrow \frac{1}{\mathrm{j}\omega\mathrm{C}} = \frac{1}{\mathrm{j}(3)(1/12)} = -\mathrm{j}4$$



(c)

At the non-reference node,

$$\frac{18 - \mathbf{V}_3}{6} = \frac{\mathbf{V}_3}{-j4} + \frac{\mathbf{V}_3}{j6}$$

$$\mathbf{V}_3 = \frac{18}{1 + j0.5} = 16.1\angle -26.57°$$

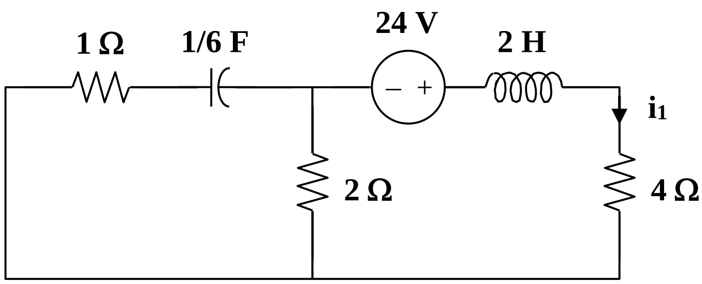
Hence, $v_3 = 16.1\cos(3t - 26.57°)\,V$

Therefore,

$v_o(t)$ = **[15+32.18sin(2t+26.57°)+16.1cos(3t–26.57°)] V**

Solution 10.47

Let $i_o = i_1 + i_2 + i_3$, where i_1, i_2, and i_3 are respectively due to the 24-V dc source, the ac voltage source, and the ac current source. For i_1, consider the circuit in Fig. (a).



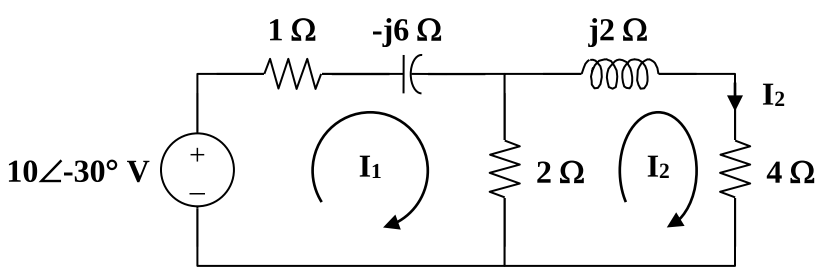
(a)

Since the capacitor is an open circuit to dc,

$$i_1 = \frac{24}{4+2} = 4 \text{ A}$$

For i_2, consider the circuit in Fig. (b).

$$\omega = 1$$
$$2 \text{ H} \longrightarrow j\omega L = j2$$
$$\frac{1}{6} \text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j6$$



(b)

For mesh 1,

$$\text{-}10\angle\text{-}30° + (3 - j6)\mathbf{I}_1 - 2\mathbf{I}_2 = 0$$
$$10\angle\text{-}30° = 3(1 - 2j)\mathbf{I}_1 - 2\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$0 = \text{-}2\mathbf{I}_1 + (6 + j2)\mathbf{I}_2$$
$$\mathbf{I}_1 = (3 + j)\mathbf{I}_2 \qquad (2)$$

Substituting (2) into (1)

$$10\angle -30° = 13 - j15\mathbf{I}_2$$

$$\mathbf{I}_2 = 0.504\angle 19.1°$$

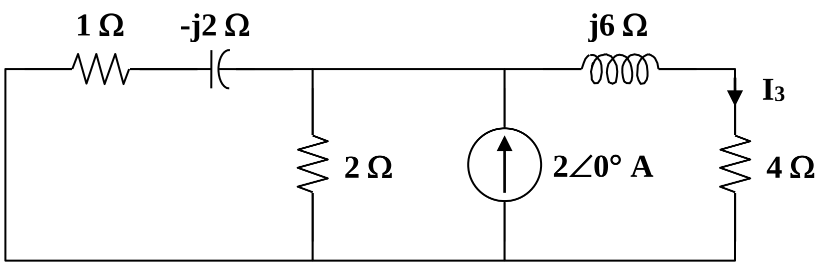
Hence, $\quad i_2 = 0.504\sin(t + 19.1°)$ A

For i_3, consider the circuit in Fig. (c).

$$\omega = 3$$

$$2\text{ H} \longrightarrow j\omega L = j6$$

$$\frac{1}{6}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/6)} = -j2$$



(c)

$$2 \,||\, (1 - j2) = \frac{2(1 - j2)}{3 - j2}$$

Using current division,

$$\mathbf{I}_3 = \frac{\dfrac{2(1-j2)}{3-j2}\cdot(2\angle 0°)}{4 + j6 + \dfrac{2(1-j2)}{3-j2}} = \frac{2(1-j2)}{13 + j3}$$

$$\mathbf{I}_3 = 0.3352\angle -76.43°$$

Hence $\quad i_3 = 0.3352\cos(3t - 76.43°)$ A

Therefore, $\quad i_o =$ **[4 + 0.504 sin(t + 19.1°) + 0.3352 cos(3t – 76.43°)] A**

Solution 10.48

Find i_o in the circuit in Fig. 10.93 using superposition.

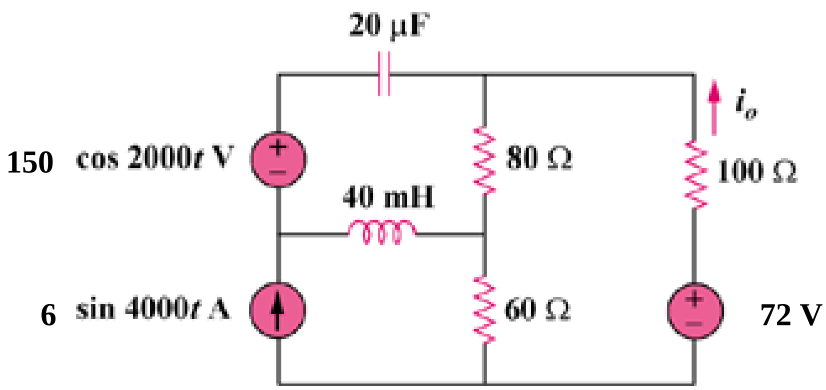


Figure 10.93
For Prob. 10.48.

Solution

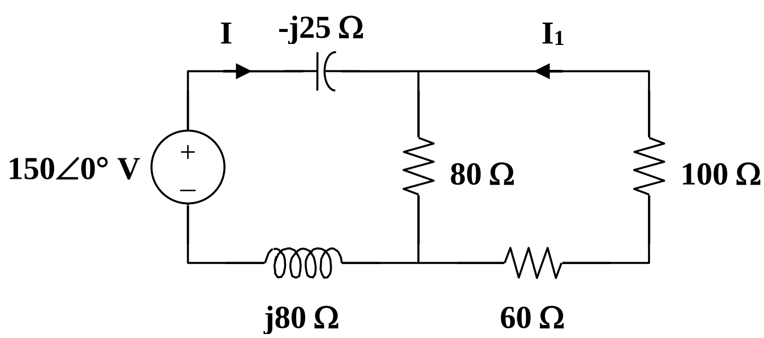
Let $i_o = i_1 + i_2 + i_3$, where I_1 is due to the ac voltage source, I_2 is due to the dc voltage source, and I_3 is due to the ac current source. For I_1, consider the circuit in Fig. (a).

$$\omega = 2000$$

$$50\cos(2000t) \longrightarrow 50\angle 0°$$

$$40 \text{ mH} \longrightarrow j\omega L = j(2000)(40\times 10^{-3}) = j80$$

$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2000)(20\times 10^{-6})} = -j25$$



(a)

$$80 \,||\, (60+100) = 160/3$$

$$\mathbf{I} = \frac{150}{160/3 + j80 - j25} = \frac{90}{32 + j33}$$

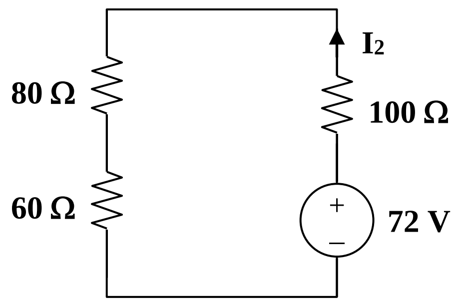
Using current division,

$$\mathbf{I}_1 = \frac{-80\mathbf{I}}{80+160} = \frac{-1}{3}\mathbf{I} = \frac{30\angle 180^\circ}{46\angle 45.9^\circ}$$

$$\mathbf{I}_1 = 0.6522\angle 134.1^\circ$$

Hence, $$i_1 = 0.6522\cos(2000t + 134.1^\circ)\ A$$

For $\mathbf{I_2}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I_2} = \frac{72}{80+60+100} = 0.3\ A$$

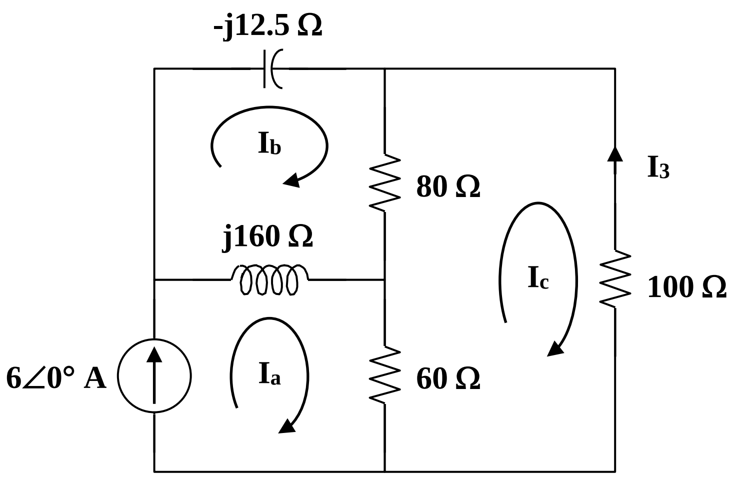
For $\mathbf{I_3}$, consider the circuit in Fig. (c).

$$\omega = 4000$$

$$2\cos(4000t) \longrightarrow 2\angle 0^\circ$$

$$40\text{ mH} \longrightarrow j\omega L = j(4000)(40\times 10^{-3}) = j160$$

$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4000)(20\times 10^{-6})} = -j12.5$$



(c)

`

For mesh 1,

$$\mathbf{I}_a = 6 \text{ A} \qquad (1)$$

For mesh 2,

$$(80 + j160 - j12.5)\mathbf{I_b} - j160\mathbf{I}_a - 80\mathbf{I}_c = 0$$

Simplifying and substituting (1) into this equation yields

$$(8 + j14.75)\mathbf{I}_b - 8\mathbf{I}_c = j96 \qquad (2)$$

For mesh 3,

$$240\mathbf{I}_c - 60\mathbf{I}_a - 80\mathbf{I}_b = 0$$

Simplifying and substituting (1) into this equation yields

$$\mathbf{I_b} = 3\mathbf{I_c} - 4.5 \qquad (3)$$

Substituting (3) into (2) yields

$$(16 + j44.25)\mathbf{I_c} = 36 + j162.375$$

$$\mathbf{I}_c = \frac{36 + j162.375}{16 + j44.25} = 3.5346\angle 7.38^\circ$$

$$\mathbf{I}_3 = \text{-}\mathbf{I_c} = -3.535\angle 7.38^\circ$$

Hence, $\mathbf{i_{O}}_3 = 3.535\sin(4000\mathbf{t} - 172.62^\circ)$ **A**

Therefore,

$i_O =$ **{0.3 + 0.6522cos(2000t + 134.1°) + 3.535sin(4000t – 172.62°)} A**

Solution 10.49

Using source transformation, find *i* in the circuit of Fig. 10.94.

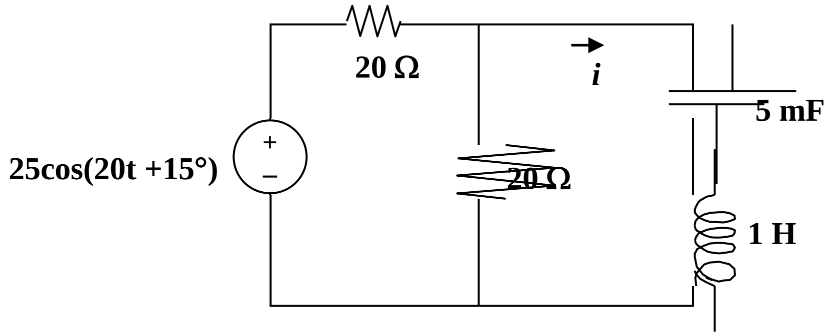


Figure 10.94
For Prob. 10.49.

Solution

First we convert the circuit into the frequency domain and use source transformation to change the voltage source in series with the 20 Ω resistor into a $(25\angle 15°)/20 = 1.25\angle 15°$ A current source in parallel with a 20 Ω resistor. Now the two parallel 20 Ω resistors can be turned into a single $(20)(20)/(20+20) = 10$ Ω resistor. Now we convert the current source in parallel with the 10 Ω resistor into a $(1.25\angle 15°)(10) = 12.5\angle 15°$ V voltage source in series with a 10Ω resistor.

Now we get $I = (12.5\angle 15°)/[10–j(1/((20)(0.005))+j(20)(1)] = (12.5\angle 15°)/(10–j10+j20) = (12.5\angle 15°)/(14.142\angle 45°) = 0.8839\angle –30°$. Thus,

i = **883.9cos(20t–30°) mA**.

Solution 10.50

Using Fig. 10.95, design a problem to help other students to better understand source transformation.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use source transformation to find v_o in the circuit in Fig. 10.95.

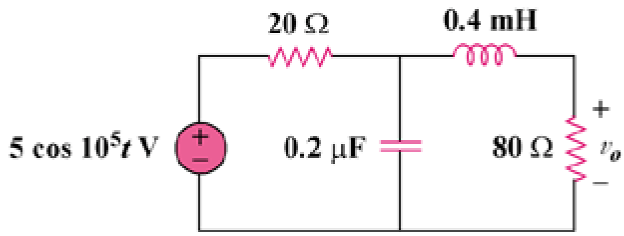


Figure 10.95

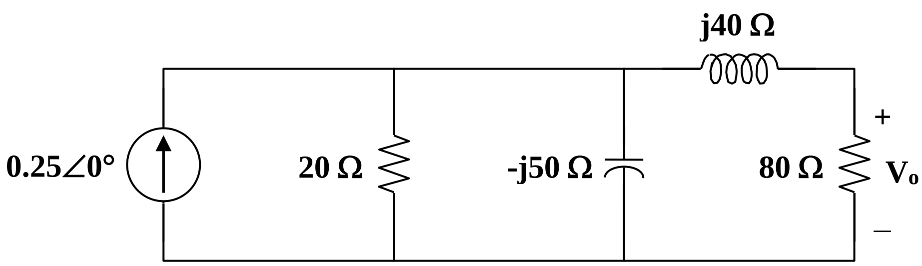
Solution

$$5\cos(10^5 t) \longrightarrow 5\angle 0°, \quad \omega = 10^5$$

$$0.4\text{ mH} \longrightarrow j\omega L = j(10^5)(0.4\times 10^{-3}) = j40$$

$$0.2\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^5)(0.2\times 10^{-6})} = -j50$$

After transforming the voltage source, we get the circuit in Fig. (a).

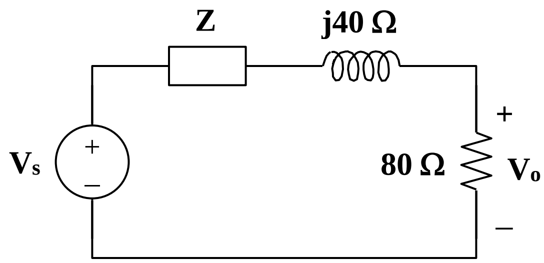


(a)

Let $\quad \mathbf{Z} = 20 \,||\, -j50 = \dfrac{-j100}{2 - j5}$

and $\quad \mathbf{V}_s = (0.25\angle 0°)\,\mathbf{Z} = \dfrac{-j25}{2 - j5}$

With these, the current source is transformed to obtain the circuit in Fig.(b).



(b)

By voltage division,

$$\mathbf{V}_o = \frac{80}{\mathbf{Z} + 80 + j40}\mathbf{V}_s = \frac{80}{\dfrac{-j100}{2 - j5} + 80 + j40} \cdot \frac{-j25}{2 - j5}$$

$$\mathbf{V}_o = \frac{8(-j25)}{36 - j42} = 3.615\angle -40.6°$$

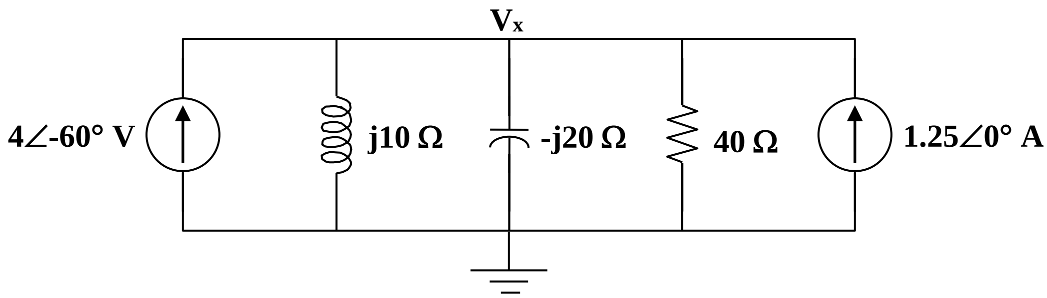
Therefore, $\quad v_o = \mathbf{3.615\ cos(10^5\ t - 40.6°)\ V}$

Solution 10.51

There are many ways to create this problem, here is one possible solution. Let
$V_1 = 40\angle 30°$ V, $X_L = 10\ \Omega$, $X_C = 20\ \Omega$, $R_1 = R_2 = 80\ \Omega$, and $V_2 = 50$ V.

If we let the voltage across the capacitor be equal to V_x, then
$\mathbf{I_o} = [\mathbf{V_x}/(–j20)] + [(\mathbf{V_x}–50)/80] = (0.0125+j0.05)\mathbf{V_x} – 0.625 = (0.051539\angle 75.96°)\mathbf{V_x} – 0.625$.

The following circuit is obtained by transforming the voltage sources.



$\mathbf{V_x} = (4\angle\text{-}60°+1.25)/(–j0.1+j0.05+0.025) = (2–j3.4641+1.25)/(0.025–j0.05)$
$= (3.25–j3.4641)/(0.025–j0.05) = (4.75\angle–46.826°)/(0.055902\angle–63.435°)$
$= 84.97\angle 16.609°$ V.

Therefore,

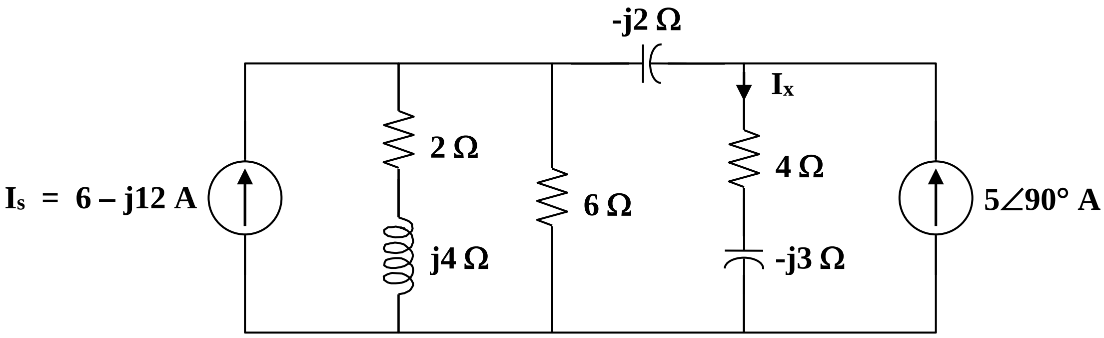
$\mathbf{I_o} = (0.051539\angle 75.96°)(84.97\angle 16.609°) – 0.625 = 4.3793\angle 92.569° – 0.625$
$= –0.196291+j4.3749 – 0.625 = –0.821291+j4.3749 =$ **$4.451\angle 100.63°$ A**.

Solution 10.52

We transform the voltage source to a current source.

$$\mathbf{I}_s = \frac{60\angle 0°}{2 + j4} = 6 - j12$$

The new circuit is shown in Fig. (a).

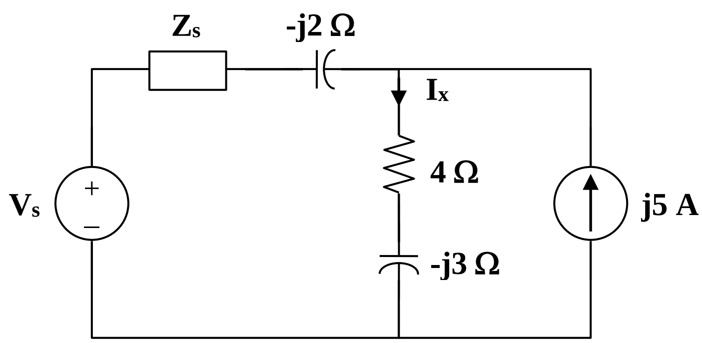


(a)

Let $\quad \mathbf{Z}_s = 6 \,||\, (2 + j4) = \dfrac{6(2 + j4)}{8 + j4} = 2.4 + j1.8$

$$\mathbf{V}_s = \mathbf{I}_s\,\mathbf{Z}_s = (6 - j12)(2.4 + j1.8) = 36 - j18 = 18(2 - j)$$

With these, we transform the current source on the left hand side of the circuit to a voltage source. We obtain the circuit in Fig. (b).

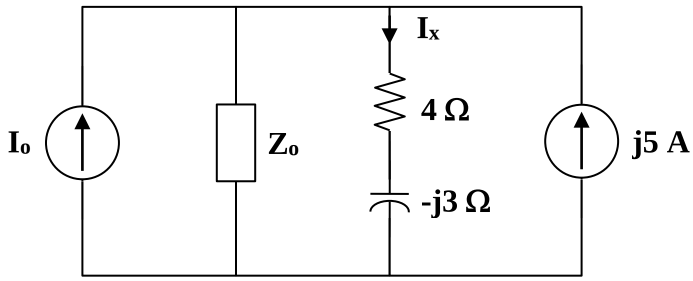


(b)

Let $\quad \mathbf{Z}_o = \mathbf{Z}_s - j2 = 2.4 - j0.2 = 0.2(12 - j)$

$$\mathbf{I}_o = \frac{\mathbf{V}_s}{\mathbf{Z}_o} = \frac{18(2 - j)}{0.2(12 - j)} = 15.517 - j6.207$$

With these, we transform the voltage source in Fig. (b) to a current source. We obtain the circuit in Fig. (c).



(c)

Using current division,

$$\mathbf{I}_x = \frac{\mathbf{Z}_o}{\mathbf{Z}_o + 4 - j3}(\mathbf{I}_o + j5) = \frac{2.4 - j0.2}{6.4 - j3.2}(15.517 - j1.207)$$

$$\mathbf{I}_x = 5 + j1.5625 = \mathbf{5.238\angle 17.35^\circ\ A}$$

Solution 10.53

Use the concept of source transformation to find $\mathbf{V}_o$ in the circuit of Fig. 10.97.

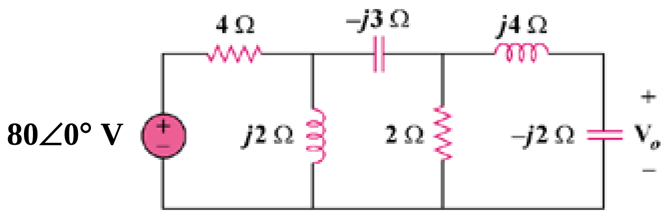
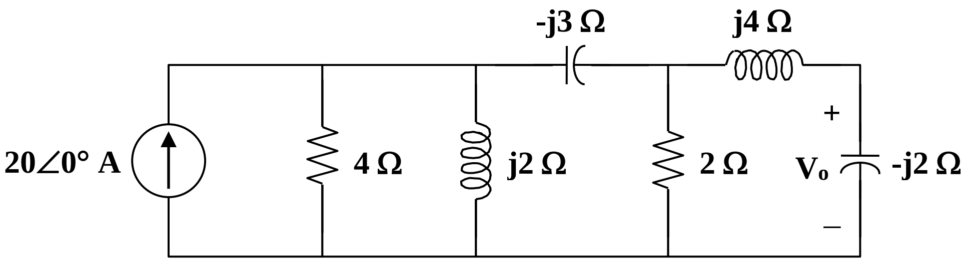


Figure 10.97
For Prob. 10.53.

Solution

We transform the voltage source to a current source to obtain the circuit in Fig. (a).

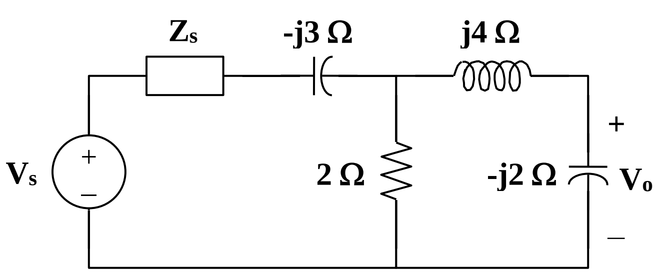


(a)

Let $$\mathbf{Z}_s = 4 \,||\, j2 = \frac{j8}{4 + j2} = 0.8 + j1.6$$

$$\mathbf{V}_s = (20\angle 0°)\,\mathbf{Z}_s = (20)(0.8 + j1.6) = 16 + j32$$

With these, the current source is transformed so that the circuit becomes that shown in Fig. (b).

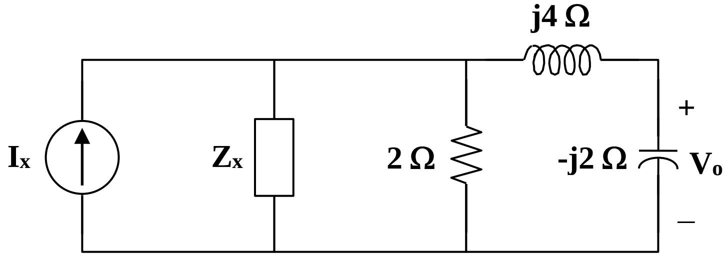


(b)

Let $\quad \mathbf{Z}_x = \mathbf{Z}_s - j3 = 0.8 - j1.4$

$$\mathbf{I_x} = \frac{\mathbf{V_s}}{\mathbf{Zx}} = \frac{16 + \mathbf{j}32}{0.8 - \mathbf{j}1.4} = -12.3076 + \mathbf{j}18.4616$$

With these, we transform the voltage source in Fig. (b) to obtain the circuit in Fig. (c).

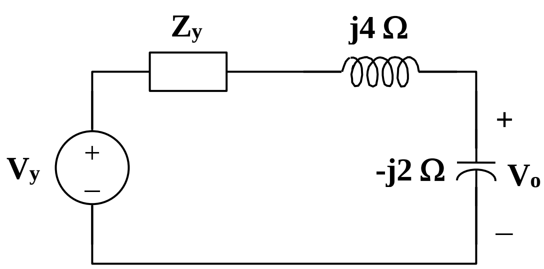


(c)

Let $\quad \mathbf{Z}_y = 2 \,\|\, \mathbf{Z}_x = \dfrac{1.6 - j2.8}{2.8 - j1.4} = 0.8571 - j0.5714$

$$\mathbf{V}_y = \mathbf{I}_x \mathbf{Z}_y = (-12.3076 + j18.4616) \cdot (0.8571 - j0.5714) = j22.8572 \text{ V}.$$

With these, we transform the current source to obtain the circuit in Fig. (d).



(d)

Using current division,

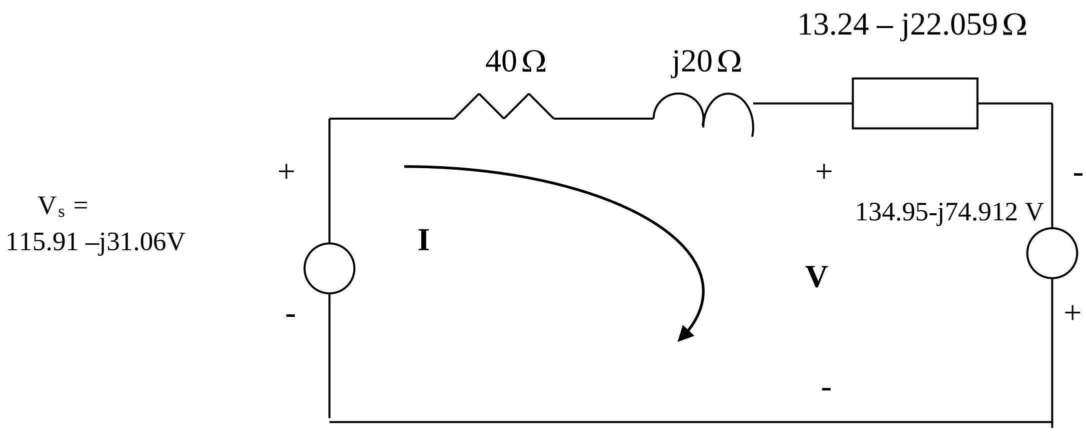
$$\mathbf{V}_o = \frac{-j2}{\mathbf{Z}_y + j4 - j2}\mathbf{V}_y = \frac{-j2(j22.8572)}{0.8571 - j0.5714 + j4 - j2} = \mathbf{(14.116 - j23.532)\ V}.$$

$$\mathbf{V_o} = \mathbf{27.44\angle{-59.04^\circ}\ V}.$$

Solution 10.54

$$50 // (-j30) = \frac{50x(-j30)}{50 - j30} = 13.24 - j22.059$$

We convert the current source to voltage source and obtain the circuit below.



Applying KVL gives

-115.91 + j31.058 + (53.24-j2.059)I -134.95 + j74.912 = 0

or $\quad I = \frac{-250.86 + j105.97}{53.24 - j2.059} = -4.7817 + j1.8055$

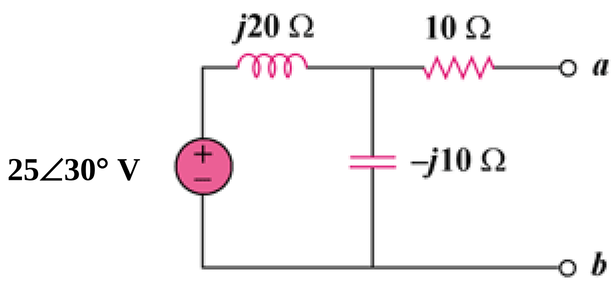
But $\;-V_S + (40 + j20)I + V = 0 \quad \longrightarrow \quad V = V_S - (40 + j20)I$

$\boldsymbol{V = 115.91 - j31.05 - (40 + j20)(-4.7817 + j1.8055) = \underline{124.06\angle -154^o \text{ V}}}$

which agrees with the result in Prob. 10.7.

Solution 10.55

Find the Thevenin and Norton equivalent circuits at terminals *a-b* for each of the circuits in Fig. 10.98.



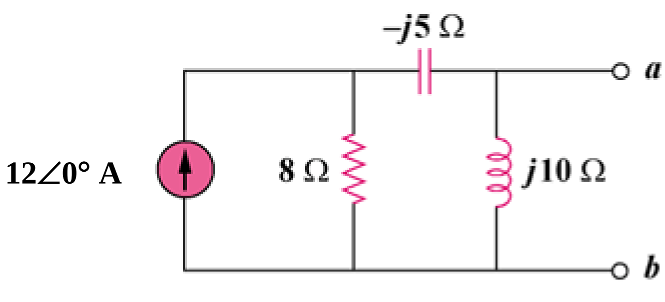
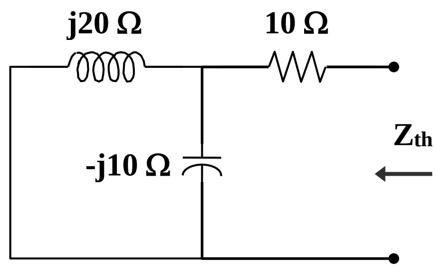


Figure 10.98
For Prob. 10.55.

Solution

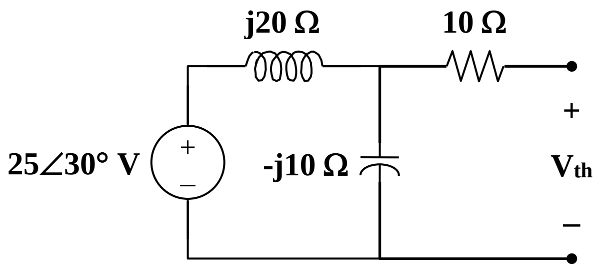
(a) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



$$\mathbf{Z}_N = \mathbf{Z}_{th} = 10 + j20 \,||\, (-j10) = 10 + \frac{(j20)(-j10)}{j20 - j10}$$

$$= 10 - j20 = \mathbf{22.36\angle{-63.43^\circ}\ \Omega}$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).

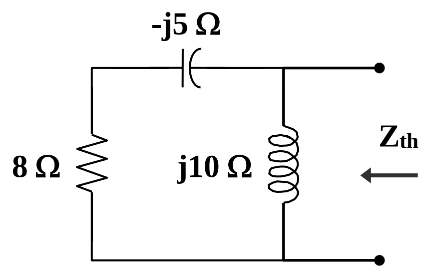


(b)

$$\mathbf{V}_{th} = \frac{-j10}{j20 - j10}(25\angle 30°) = \mathbf{25\angle{-150}°\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V_{th}}}{\mathbf{Z_{th}}} = \frac{25\angle -150°}{22.36\angle -63.43°} = \mathbf{1.1181\angle{-86.57}°\ A}$$

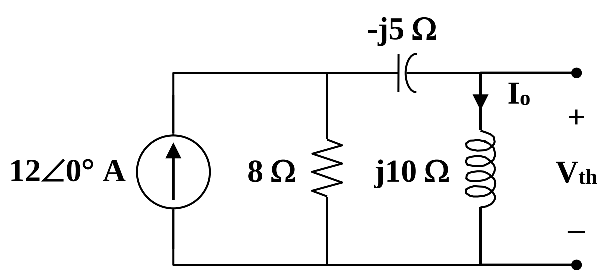
(b) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (c).



(c)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = j10 \,||\, (8 - j5) = \frac{(j10)(8 - j5)}{j10 + 8 - j5} = \mathbf{10\angle 26°\ \Omega}$$

To obtain $\mathbf{V}_{th}$, consider the circuit in Fig. (d).



(d)

By current division,

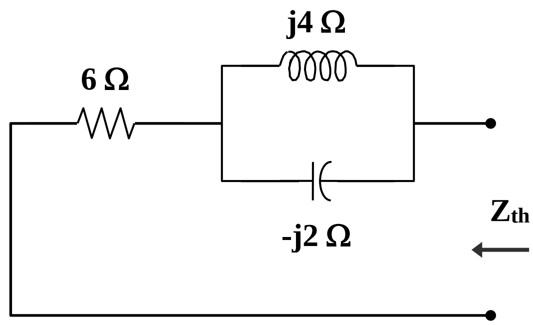
$$\mathbf{I}_o = \frac{8}{8 + j10 - j5}(12\angle 0°) = \frac{96}{8 + j5}$$

$$\mathbf{V}_{th} = j10\mathbf{I}_o = \frac{j960}{8 + j5} = \mathbf{101.76\angle 58°\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{101.76\angle 58°}{10\angle 26°} = \mathbf{10.176\angle 32°\ A}$$

Solution 10.56

(a) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 6 + j4 \,||\, (\text{-}j2) = 6 + \frac{(j4)(\text{-}j2)}{j4 - j2} = 6 - j4$$

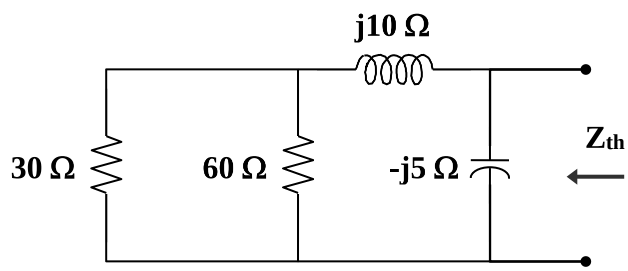
$$= \mathbf{7.211\angle\text{-}33.69^\circ\ \Omega}$$

By placing short circuit at terminals a-b, we obtain,

$\mathbf{I}_N = \mathbf{2\angle 0^\circ\ A}$

$$\mathbf{V}_{th} = \mathbf{Z}_{th}\,\mathbf{I}_{th} = (7.211\angle\text{-}33.69^\circ)(2\angle 0^\circ) = \mathbf{14.422\angle\text{-}33.69^\circ\ V}$$

(b) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (b).



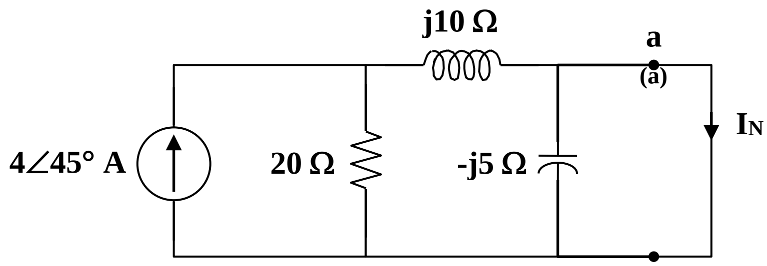
(b)

$$30 \,||\, 60 = 20$$

$$\mathbf{Z}_N = \mathbf{Z}_{th} = \text{-}j5 \,||\, (20 + j10) = \frac{(\text{-}j5)(20 + j10)}{20 + j5}$$

$$= \mathbf{5.423\angle\text{-}77.47^\circ\ \Omega}$$

To find $\mathbf{V}_{th}$ and $\mathbf{I}_N$, we transform the voltage source and combine the 30 Ω and 60 Ω resistors. The result is shown in Fig. (c).



(c)

$$\mathbf{I}_N = \frac{20}{20 + j10}(4\angle 45°) = \frac{2}{5}(2 - j)(4\angle 45°)$$
$$= \mathbf{3.578\angle 18.43°\ A}$$

$$\mathbf{V}_{th} = \mathbf{Z}_{th}\,\mathbf{I}_N = (5.423\angle -77.47°)(3.578\angle 18.43°)$$
$$= \mathbf{19.4\angle -59°\ V}$$

Solution 10.57

Using Fig. 10.100, design a problem to help other students to better understand Thevenin and Norton equivalent circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Thevenin and Norton equivalent circuits for the circuit shown in Fig. 10.100.

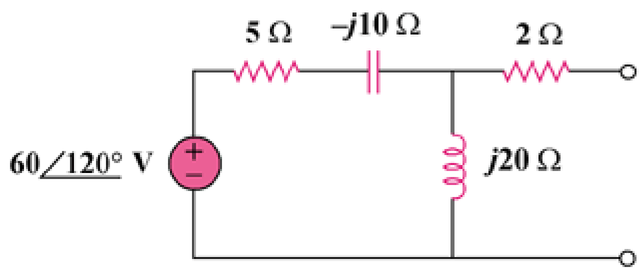
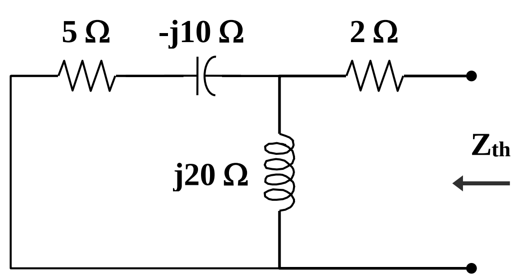


Figure 10.100

Solution

To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).

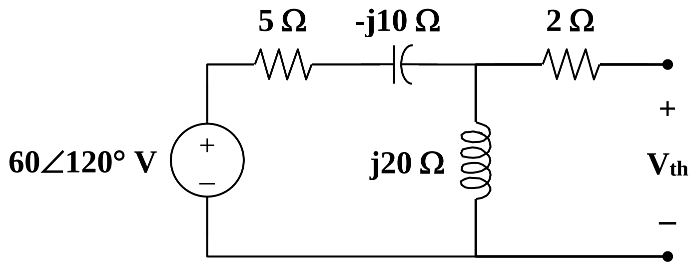


(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 2 + j20 \,\|\, (5 - j10) = 2 + \frac{(j20)(5 - j10)}{5 + j10}$$

$$= 18 - j12 = \mathbf{21.633\angle\text{-}33.7^\circ\ \Omega}$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_{th} = \frac{j20}{5 - j10 + j20}(60\angle 120°) = \frac{j4}{1 + j2}(60\angle 120°)$$
$$= \mathbf{107.3\angle 146.56°\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{107.3\angle 146.56°}{21.633\angle -33.7°} = \mathbf{4.961\angle\text{-}179.7°\ A}$$

Solution 10.58

For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a*-*b*.

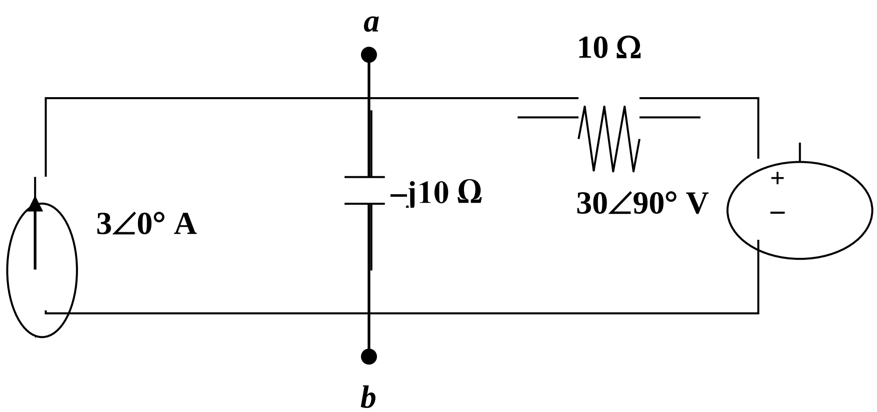


Figure 10.101
For Prob. 10.58.

Solution

The easiest way to do this is to find $\mathbf{V}_{\mathbf{oc}}$ and $\mathbf{I}_{\mathbf{sc}}$. Writing a nodal equation at $\mathbf{V}_{\mathbf{ab}}$ will give us $\mathbf{V}_{\mathbf{oc}} = \mathbf{V}_{\mathbf{ab}}$. $-3 + [(\mathbf{V}_{\mathbf{ab}}-0)/(-j10)] + [(V_{ab}-j30)/10] = 0$ or $(0.1+j0.1)\mathbf{V}_{\mathbf{ab}} = 3+j3$ or

$\mathbf{V}_{\mathbf{oc}} = \mathbf{V}_{\mathbf{Thev}} = 3(1+j)/[0.1(1+j)] = \mathbf{30\ V}$.

I_{sc} is fairly easy in that shorting a to b shorts out the capacitor. Therefore,
$I_{sc} = 3 + [(j30)/10] = 3+j3$. Thus,

$\mathbf{Z}_{\mathbf{eq}} = \mathbf{V}_{\mathbf{Thev}}/\mathbf{I}_{\mathbf{sc}} = 30/[3(1+j)] = \mathbf{(5-j5)\ \Omega}$.

Solution 10.59

Calculate the output impedance of the circuit shown in Fig. 10.102.

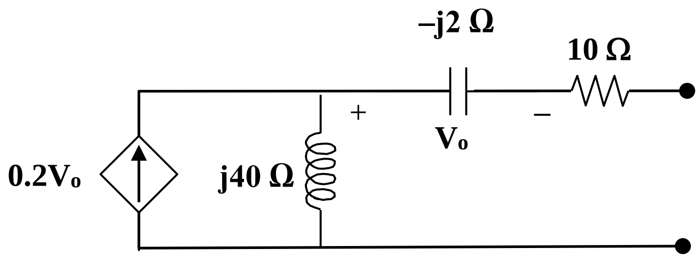
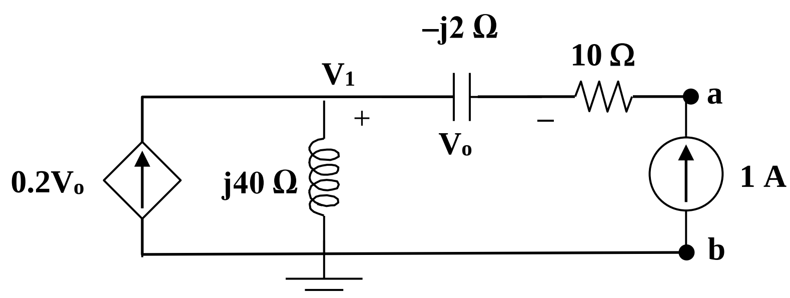


Figure 10.102
For Prob. 10.59.

Solution

Since there are no independent sources, we need to inject a current, best value is to make it 1 amp, into the terminals on the right and then to determine the voltage at the terminals.



Clearly $\mathbf{V_o} = -(-j2) = j2$ and $\mathbf{V_1} = (0.2\mathbf{V_o} + 1)j40 = (1+j0.4)j40 = -16+j40$ V.
Next, $\mathbf{V_{ab}} = 10 - j2 - 16 + j40 = -6+j38 = 38.47\angle 98.97°$ V or

$$\mathbf{Z_{eq}} = \mathbf{(-6+j38)\ \Omega}.$$

Solution 10.60

Find the Thevenin equivalent of the circuit in Fig. 10.103 as seen from:

(a) terminals *a-b*
(b) terminals *c-d*

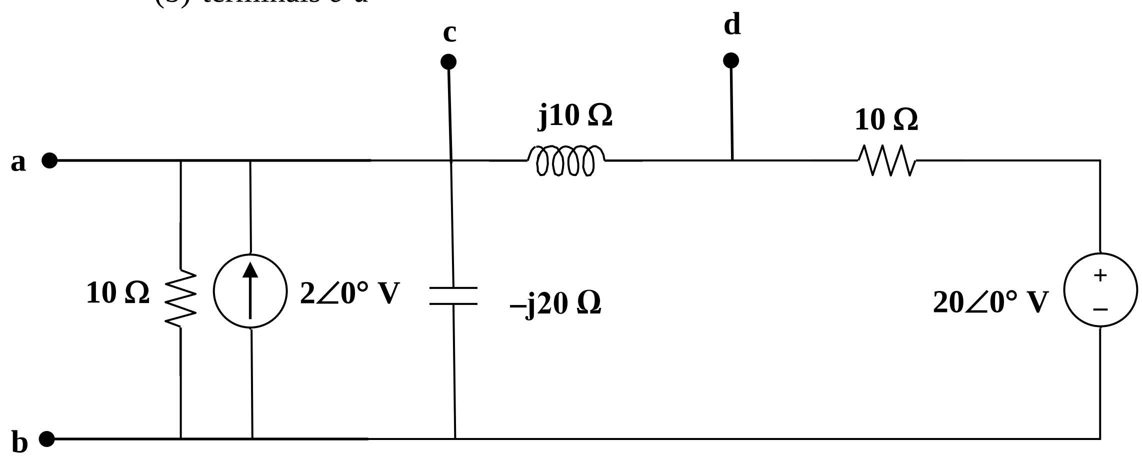


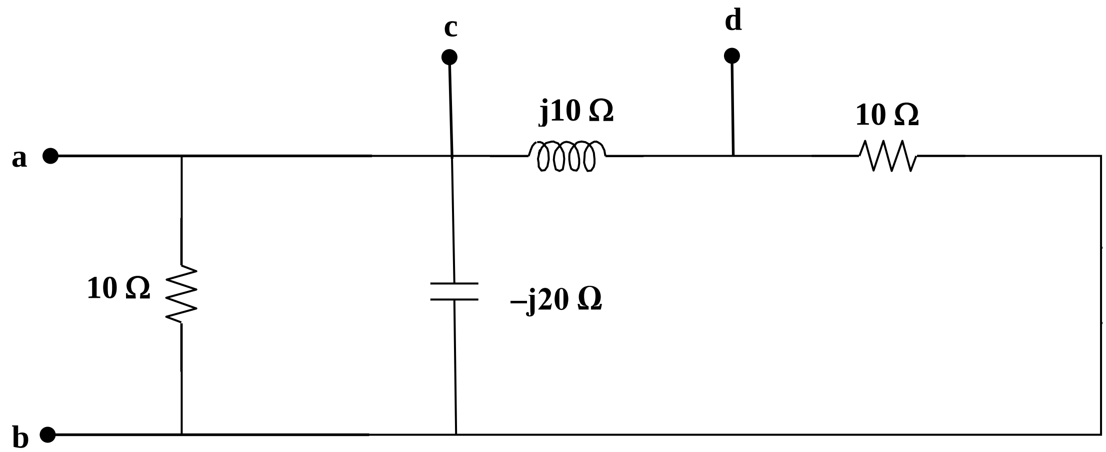
Figure 10.103
For Prob. 10.60.

Solution

Let us find the Thevenin equivalent circuits by finding $\mathbf{V_{ab}}$ and $\mathbf{V_{cd}}$ in the above circuit which gives us the Thevenin voltages. Next we set the independent sources to zero and find $\mathbf{Z_{ab}}$ and $\mathbf{Z_{cd}}$ which are the Thevenin impedances.

We start by observing we only have one unknown node voltage, $\mathbf{V_c}$ which leads to $[(\mathbf{V_c}–0)/10] – 2 + [(\mathbf{V_c}–0)/(–j20)] + [(\mathbf{V_c}–20)/(10+j10)] = 0$. Now we get,

$[0.1+j0.05+(10–j10)/(100+100)]\mathbf{V_c} = [0.1+j0.05+0.05–j0.05]\mathbf{V_c} = 0.15\mathbf{V_c}$ $= 2 + 20(10–j10)/200 = 2 + 1 – j = 3–j$ or $\mathbf{V_c} = 20–j6.6667$. Clearly $\mathbf{V_{ab}} = \mathbf{V_c} = \mathbf{V_{Thevab}} =$ $\mathbf{21.08\angle 18.44° \ V}$. Let I = the current flowing left to right through the inductor. Thus, $\mathbf{I} = [(\mathbf{V_c}–20)/(10+j10)] = –j6.6667(0.05–j0.05) = –0.33333–j0.33333$ which gives us $\mathbf{V_{cd}} = j10(–0.33333–j0.33333) = 3.3333–j3.3333 = \mathbf{4.714\angle –45° \ V}$.



For ab, $1/Z_{eq} = 0.1 + j0.05 + 0.05 - j0.05 = 0.15$ or $Z_{eq} = (20/3)\ \Omega$.

For cd, $1/Z_{eq} = (j10)\{[(10)(-j20)/(10-j20)]+10\}/(j10+\{[(10)(-j20)/(10-j20)]+10\})$.

$[(10)(-j20)/(10-j20)]+10 = [-j200(10+j20)/(100+400)]+10 = 8 - j4 + 10 = 18 - j4$
$= (j10)\{18-j4\}/(j10+\{18-j4\}) = (40+j180)/(18+j6)$
$= 184.391\angle 77.471°/(18.9737\angle 18.435°) = (9.7183\angle 59.036°)\ \Omega = (5 + j25/3)\ \Omega$.

(a) $\mathbf{V_{Thev} = 21.08\angle 18.44°\ V}$ and $\mathbf{Z_{eq} = (20/3)\ \Omega}$,
(b) $\mathbf{V_{Thev} = 4.714\angle -45°\ V}$ and $\mathbf{Z_{eq}} = (\mathbf{9.7183\angle 59.04°)\ \Omega}$ or = $\mathbf{(5 + j25/3)\ \Omega}$.

Solution 10.61

Find the Thevenin equivalent at terminals a-b of the circuit in Fig. 10.104.

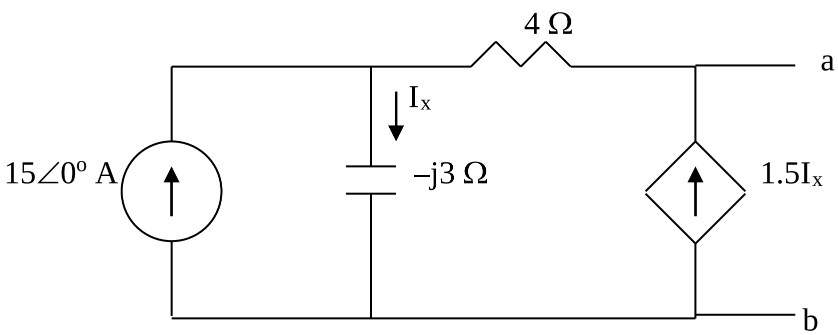
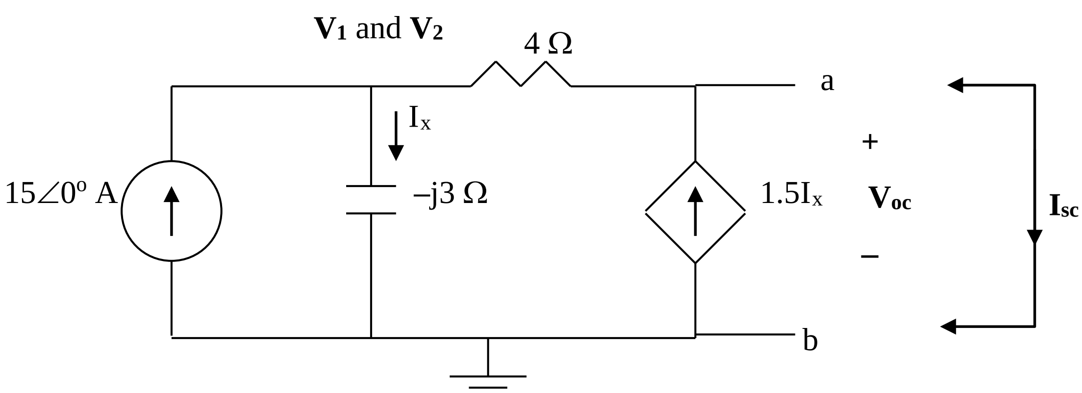


Figure 10.104
For Prob. 10.61.

Solution

Find the Thevenin equivalent at terminals a-b of the circuit in Fig. 10.104.



Step 1. First we solve for the open circuit voltage using the above circuit and writing two node equations. Then we solve for the short circuit current which only needs one node equation. For being able to solve for $\mathbf{V_{oc}}$, we need to solve these three equations,

$$-15 + [(\mathbf{V_1}-0)/(-j3)] + [(\mathbf{V_1}-\mathbf{V_{oc}})/4] = 0 \text{ and}$$

$$[(\mathbf{V_{oc}}-\mathbf{V_1})/4] - 1.5\mathbf{I_x} = 0 \text{ where } \mathbf{I_x} = [(\mathbf{V_1}-0)/(-j3)].$$

To solve for $\mathbf{I_{sc}}$, all we need to do is to solve these three equations,

$$-15 + [(\mathbf{V_2}-0)/(-j3)] + [(\mathbf{V_2}-0)/4] = 0,\ \mathbf{I_{sc}} = [\mathbf{V_2}/4] + 1.5\mathbf{I_x}, \text{ and}$$

$\mathbf{I_x} = [\mathbf{V_2}/–j3]$.

Finally, $\mathbf{V_{Thev}} = \mathbf{V_{oc}}$ and $\mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}}$.

Step 2. Now all we need to do is to solve for the unknowns. For $\mathbf{V_{oc}}$,

$\mathbf{I_x} = j0.33333\mathbf{V_1}$ and $(0.25+(1.5)(j0.33333))\mathbf{V_1} = 0.25\mathbf{V_{oc}}$ or
$(0.25+j0.5)\mathbf{V_1} = (0.55902\angle 63.43°)\mathbf{V_1} = 0.25\mathbf{V_{oc}}$ or
$\mathbf{V_1} = (0.44721\angle{-63.43°})\mathbf{V_{oc}}$ which leads to,

$(0.25+j0.33333)\mathbf{V_1} – 0.25\mathbf{V_{oc}} = 15$
$= (0.41666\angle{+53.13°})(0.44721\angle{-63.43°})V_{oc} – 0.25\mathbf{V_{oc}}$
$= (0.186335\angle{-10.3°})\mathbf{V_{oc}} – 0.25\mathbf{V_{oc}} = (0.183333–0.25–j0.033333)\mathbf{V_{oc}}$
$= (–0.066667–j0.033333)\mathbf{V_{oc}} = (0.074536\angle{-153.435°})\mathbf{V_{oc}} = 15$ or

$$\mathbf{V_{oc}} = \mathbf{V_{Thev}} = \mathbf{201.2\angle 153.44°\ V = (–180+j90)\ V}.$$

Now for $\mathbf{I_{sc}}$,

$\mathbf{I_{sc}} = [\mathbf{V_2}/4] + 1.5\mathbf{I_x} = (0.25+(1.5)(j0.33333))\mathbf{V_2} = (0.25+j0.5)\mathbf{V_2}$.

$[(\mathbf{V_2}–0)/(–j3)] + [(\mathbf{V_2}–0)/4] = 15 = (0.25+j0.3333)\mathbf{V_2}$
$= (0.41667\angle 53.13°)\mathbf{V_2} = 15$ or $\mathbf{V_2} = 4.8\angle{-53.13°}$

$\mathbf{I_{sc}} = (0.25+j0.5)\mathbf{V_2} = (0.55901\angle 63.435°)(36\angle{-53.13°})$
$= 20.124\angle 10.3°$ A

Finally,

$\mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}} = 201.2\angle 153.435°/20.12\angle 10.305°$

$$= \mathbf{10\angle 143.13°\ \Omega} \text{ or } = \mathbf{(–8+j6)\ \Omega}.$$

Solution 10.62

Using Thevenin's theorem, find v_o in the circuit in Fig. 10.105.

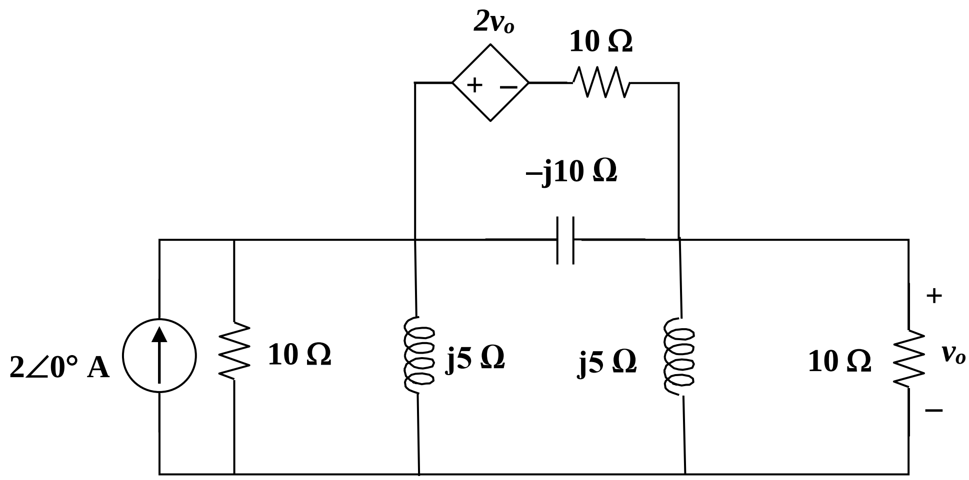
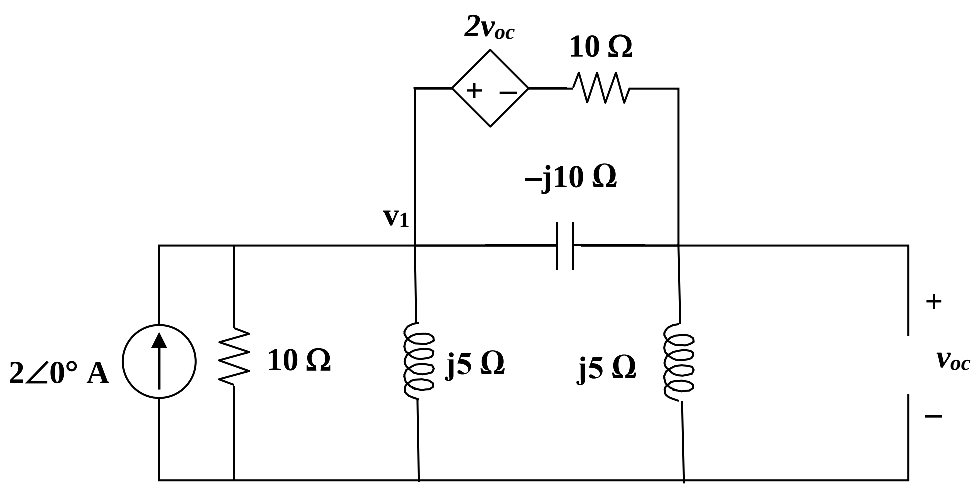


Figure 10.105
For Prob. 10.62.

Solution

We will take out the 10 Ω resistor and determine the Thevenin equivalent looking in from the right. We will calculate V_{oc} and I_{sc}.



We now have two node equations, the first on at v_1 is,

$$-2 + [(v_1-0)/10] + [(v_1-0)/j5] + [(v_1-v_{oc})/(-j10)] + [(v_1-2v_{oc}-v_{oc})/10] = 0 \text{ or}$$

$$[0.1-j0.2+j0.1+0.1]v_1 - [j0.1+0.3]v_{oc} = 2 = (0.22361\angle-26.565°)v_1 - (0.3+j0.1)v_{oc}.$$

The second equation, at v_{oc}, is, $[(v_{oc}–0)/j5] + [(v_{oc}–v_1)/(–j10)] + [(v_{oc}–v_1+2v_{oc})/10] = 0$ or

$$–(0.1+j0.1)v_1 + (–j0.2+j0.1+ 0.3)v_{oc} \text{ or } v_1 = [(0.3–j0.1)/(0.14142\angle 45°)]v_{oc}$$

$$= [(0.316228\angle –18.435°)/(0.14142\angle 45°)]v_{oc} = (2.2361\angle –63.435°)v_{oc}.$$

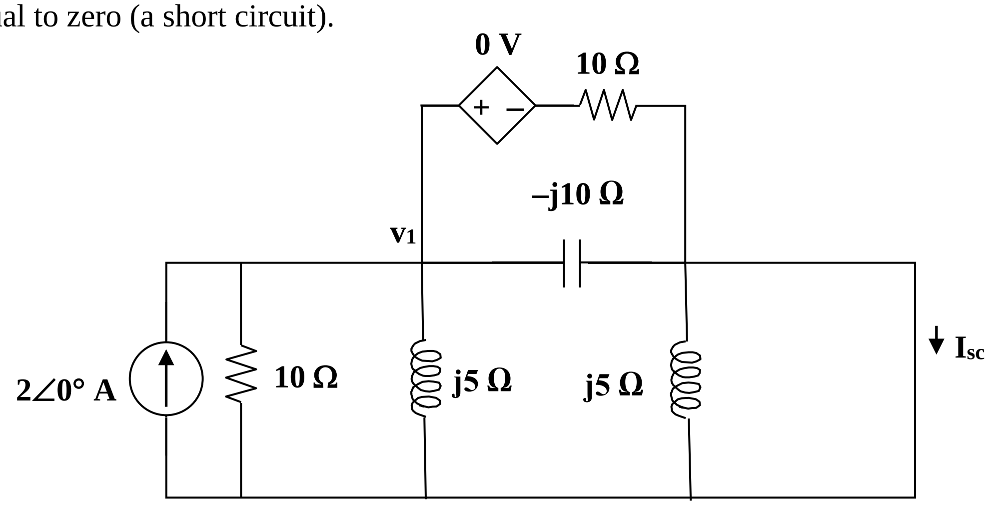
The first equation now becomes, $(0.22361\angle –26.565°)v_1 – (0.3+j0.1)v_{oc} = 2$ or

$$(0.22361\angle –26.565°)(2.2361\angle –63.435°)v_{oc} – (0.3+j0.1)v_{oc} = 2$$

$$= (0.5\angle –90° – 0.3 – j0.1)v_{oc} = (–0.3–j0.6)v_{oc} = 2 \text{ or } v_{oc} =2/(0.67082\angle –116.565°)$$

$$= 2.9814\angle 116.565° \text{ V}.$$

Now for I_{sc}, we have essentially the same equations with voltage across the second inductor equal to zero (a short circuit).



Thus we get, $(0.22361\angle –26.565°)v_1 = 2$ or $v_1 = 8.9441\angle 26.565°$. From the above we get, $I_{sc} = [(v_1–0)/(–j10)] + [(v_1–0)/10] = 0.89441\angle 116.565° + 0.89441\angle 26.565° = –0.4+j0.8 + 0.8+j0.4 = 0.4+j1.2 = 1.2649\angle 71.565°$.
This now leads to, $Z_{eq} = 2.9814\angle 116.565°/1.2649\angle 71.565°$
$= 2.357\angle 45° = (1.66665+j1.66665)\ \Omega$.

$$\mathbf{V_{Thev} = 2.981\angle 116.56° \text{ V}, Z_{eq} = (1.6666 + j1.6666)\ \Omega.}$$

Solution 10.63

Obtain the Norton equivalent of the circuit depicted in Fig. 10.106 at terminals *a-b*.

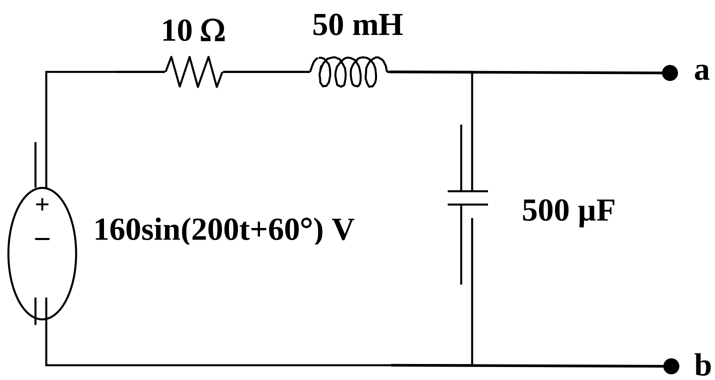
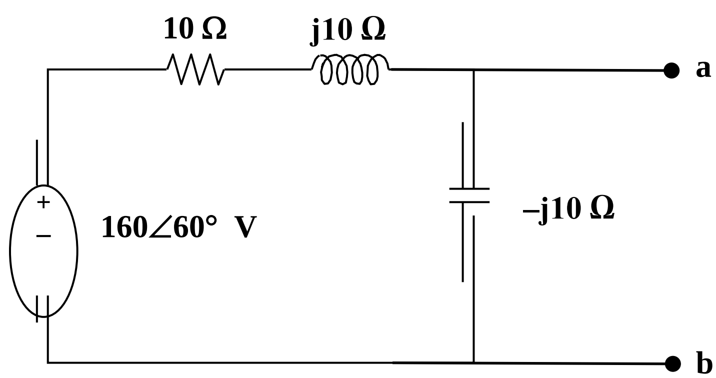


Figure 10.106
For Prob. 10.63.

Solution

First we need to transform this circuit into the frequency domain (where the Norton equivalent circuit exists) and then solve for $\mathbf{V_{oc}}$ and $\mathbf{I_{sc}}$.

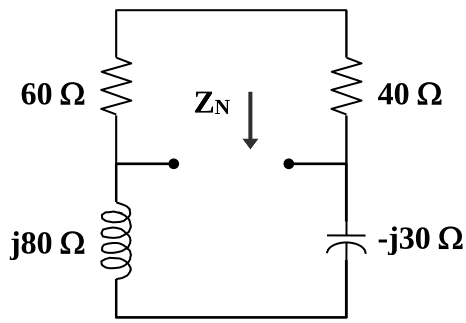


$\mathbf{V_{oc}} = [160\angle 60°/(10+j10-j10)](-j10) = 160\angle -30°$ V.

$I_{sc} = 160\angle 60°/(10+j10) =$ $\mathbf{11.314\angle 15°\ A}$ $= \mathbf{I_N}$ and $Z_{eq} = V_{oc}/I_{sc}$
$= 160\angle -30°/(11.314\angle 15°) =$ $\mathbf{14.142\angle -45°\ \Omega}$ $=$ $\mathbf{(10-j10)\ \Omega}$.

Solution 10.64

$\mathbf{Z}_N$ is obtained from the circuit in Fig. (a).

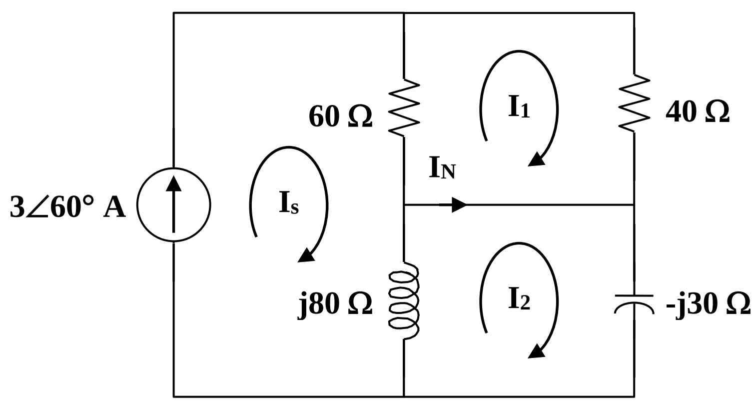


(a)

$$\mathbf{Z}_N = (60+40) \,||\, (j80 - j30) = 100 \,||\, j50 = \frac{(100)(j50)}{100 + j50}$$

$$\mathbf{Z}_N = 20 + j40 = \mathbf{44.72\angle 63.43° \ \Omega}$$

To find $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_s = 3\angle 60°$$

For mesh 1,

$$100\mathbf{I}_1 - 60\mathbf{I}_s = 0$$
$$\mathbf{I}_1 = 1.8\angle 60°$$

For mesh 2,

$$(j80 - j30)\mathbf{I}_2 - j80\mathbf{I}_s = 0$$
$$\mathbf{I}_2 = 4.8\angle 60°$$

$I_N = I_2 - I_1 = \mathbf{3\angle 60° \ A}$

Solution 10.65

Using Fig. 10.108, design a problem to help other students to better understand Norton's theorem.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Compute i_o in Fig. 10.108 using Norton's theorem.

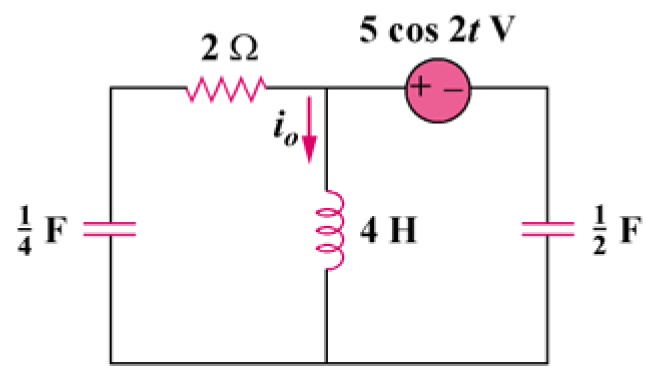


Figure 10.108

Solution

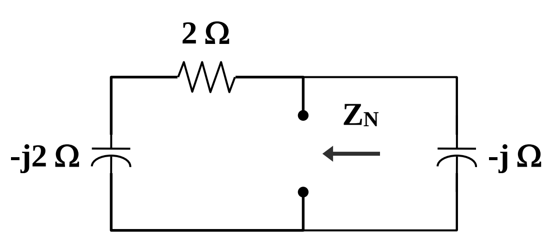
$$5\cos(2t) \longrightarrow 5\angle 0°, \quad \omega = 2$$

$$4\text{ H} \longrightarrow j\omega L = j(2)(4) = j8$$

$$\frac{1}{4}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = -j2$$

$$\frac{1}{2}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/2)} = -j$$

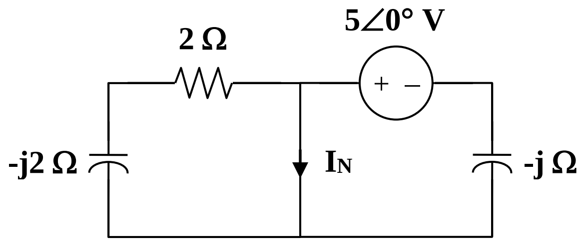
To find $\mathbf{Z}_N$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = \text{-j} \,||\, (2 - j2) = \frac{\text{-j}(2 - j2)}{2 - j3} = \frac{1}{13}(2 - j10)$$

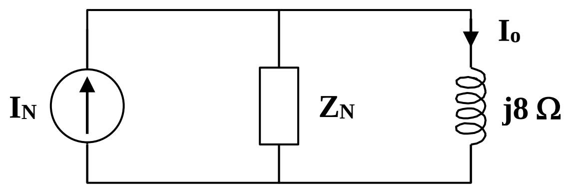
To find $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_N = \frac{5\angle 0°}{\text{-j}} = j5$$

The Norton equivalent of the circuit is shown in Fig. (c).



(c)

Using current division,

$$\mathbf{I}_o = \frac{\mathbf{Z}_N}{\mathbf{Z}_N + j8}\mathbf{I}_N = \frac{(1/13)(2 - j10)(j5)}{(1/13)(2 - j10) + j8} = \frac{50 + j10}{2 + j94}$$

$$\mathbf{I}_o = 0.1176 - j0.5294 = 0542\angle\text{-}77.47°$$

Therefore, $i_o =$ **542 cos(2t – 77.47°) mA**

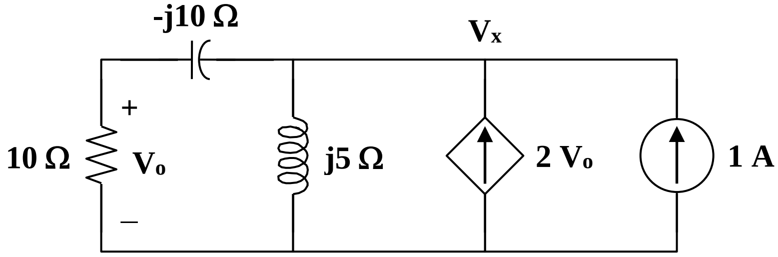
Solution 10.66

$$\omega = 10$$

$$0.5\text{ H} \longrightarrow \quad j\omega L = j(10)(0.5) = j5$$

$$10\text{ mF} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j(10)(10\times 10^{-3})} = -j10$$

To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



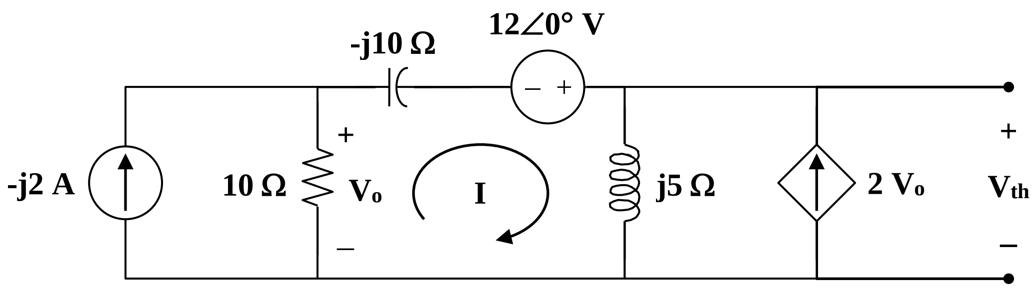
(a)

$$1 + 2\mathbf{V}_o = \frac{\mathbf{V}_x}{j5} + \frac{\mathbf{V}_x}{10 - j10}, \qquad \text{where} \quad \mathbf{V}_o = \frac{10\mathbf{V}_x}{10 - j10}$$

$$1 + \frac{19\mathbf{V}_x}{10 - j10} = \frac{\mathbf{V}_x}{j5} \longrightarrow \quad \mathbf{V}_x = \frac{-10 + j10}{21 + j2}$$

$$\mathbf{Z}_N = \mathbf{Z}_{th} = \frac{\mathbf{V}_x}{1} = \frac{14.142\angle 135°}{21.095\angle 5.44°} = \mathbf{670\angle 129.56°\ m\Omega}$$

To find $\mathbf{V}_{th}$ and $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$(10 - j10 + j5)\mathbf{I} - (10)(-j2) + j5(2\mathbf{V}_o) - 12 = 0$$

where $\quad \mathbf{V}_o = (10)(-j2 - \mathbf{I})$

Thus,

$$(10 - j105)\mathbf{I} = -188 - j20$$

$$\mathbf{I} = \frac{188 + j20}{-10 + j105}$$

$$\mathbf{V}_{th} = j5(\mathbf{I} + 2\mathbf{V}_o) = j5(-19\mathbf{I} - j40) = -j95\mathbf{I} + 200$$

$$\mathbf{V}_{th} = \frac{-j95(188 + j20)}{-10 + j105} + 200 = \frac{(95\angle -90^\circ)(189.06\angle 6.07^\circ)}{105.48\angle 95.44} + 200$$

$$= 170.28\angle -179.37^\circ + 200 = -170.27 - j1.8723 + 200 = 29.73 - j1.8723$$

$$\mathbf{V}_{th} = \mathbf{29.79\angle -3.6^\circ\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{29.79\angle -3.6^\circ}{0.67\angle 129.56^\circ} = \mathbf{44.46\angle -133.16^\circ\ A}$$

Solution 10.67

Find the Thevenin and Norton equivalent circuits at terminals *a*-*b* of the circuit in Fig. 10.110.

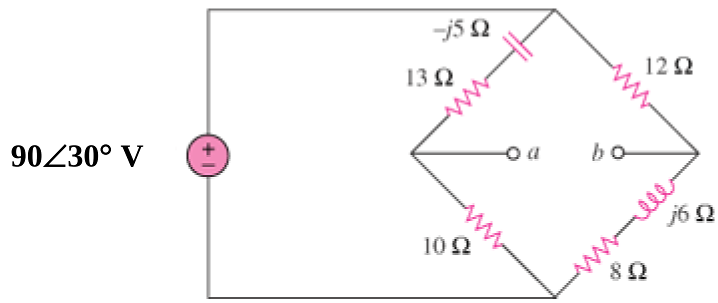


Figure 10.110
For Prob. 10.67.

Solution

$$\mathbf{Z_{eq}} = 10//(13-j5)+12//(8+j6) = \frac{10(13-j5)}{23-j5}+\frac{12(8+j6)}{20+j6} = \mathbf{(11.243 + j1.079)\ \Omega}.$$

$\mathbf{V_a}$ = [10/(23–j5)](90∠30°) = 900/(23.5372∠–12.265°) = 38.237∠42.265°
= 28.297 + j25.717 and $\mathbf{V_b}$ = [(8+j6)/(20+j6)](90∠30°)
= [(10∠36.87°)/(20.881∠16.699°)](90∠30°) = 43.1∠50.17° = (27.61 + j33.1) V.

Thus,
$\mathbf{V_{Thev}} = \mathbf{V_a} - \mathbf{V_b}$ = 0.687 – j7.383 = **7.415∠–84.68° V**.
$\mathbf{I_N} = \mathbf{V_{Thev}}/\mathbf{Z_{eq}}$ = (7.415∠–84.68°)/(11.2947∠5.482°) = **656.5∠–90.16° mA**.

Solution 10.68

For the circuit in Fig. 10.111, obtain the Thèvenin equivalent at terminals *a*-*b*.

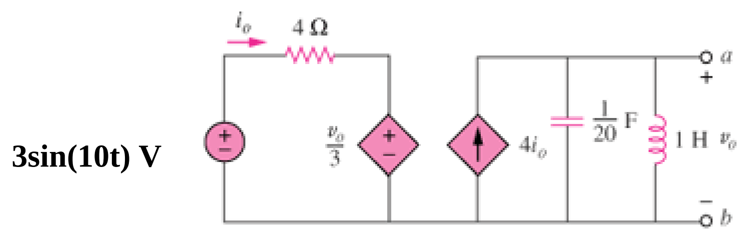


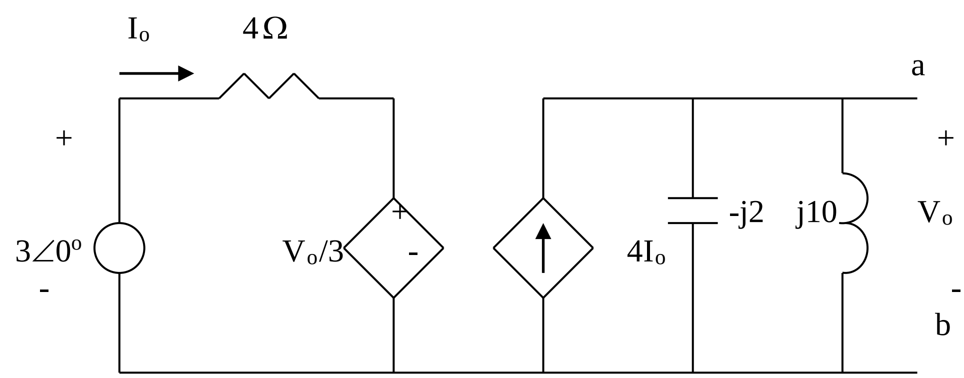
Figure 10.111
For Prob. 10.68.

Solution

$$1\text{H} \longrightarrow j\omega L = j10x1 = j10$$

$$\frac{1}{20}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10x\frac{1}{20}} = -j2$$

We obtain V_{Th} using the circuit below.



$$j10//(-j2) = \frac{j10(-j2)}{j10 - j2} = -j2.5$$

$$V_o = 4I_o x(-j2.5) = -j10I_o \qquad (1)$$

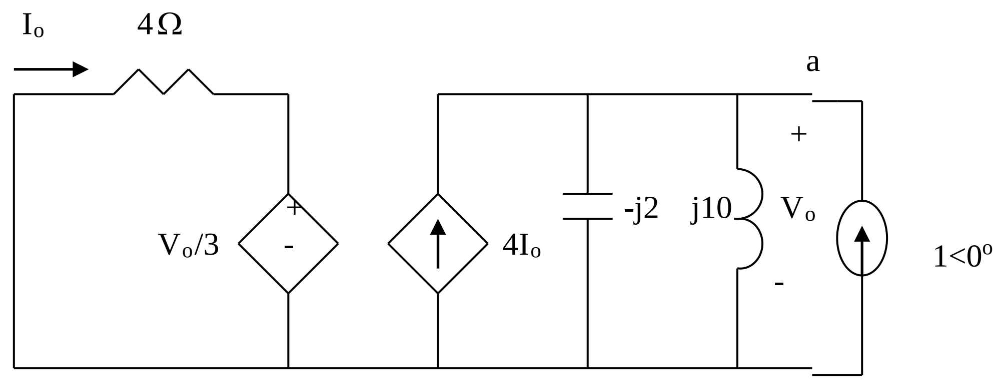
$$-3 + 4I_o + \frac{1}{3}V_o = 0 \qquad (2)$$

Combining (1) and (2) gives

$$I_o = \frac{3}{4 - j10/3}, \qquad V_{Th} = V_o = -j10I_o = \frac{-j30}{4 - j10/3} = 5.7617\angle -50.1945^o$$

$$\mathbf{V_{Thev} = 5.762\angle{-50.19^\circ}\ V}.$$

To find $R_{Th,}$ we insert a 1-A source at terminals a-b, as shown below.



$$4I_o + \frac{1}{3}V_o = 0 \quad \longrightarrow \quad I_o = -\frac{V_o}{12}$$

$1 + 4I_o = \dfrac{V_o}{-j2} + \dfrac{V_o}{j10}$ or $[(1/3)+(j0.5)–(j0.1)]\mathbf{V_o} = 1$ or $\mathbf{V_o} = 1/(0.33333+j0.4)$ or

$\mathbf{V_o}/1 = \mathbf{Z_{eq}} = 1/(0.5206812\angle 50.1947°) = 1.92056\angle –50.1947°$

$\mathbf{Z_{eq}} = \mathbf{(1.2295–j1.4754)\ \Omega}$.

Solution 10.69

For the integrator shown in Fig. 10.112, obtain $\mathbf{V}_o/\mathbf{V}_s$. Find $v_o(t)$ when $v_s(t) = \mathbf{V}_m \sin\omega t$ and $\omega = 1/RC$.

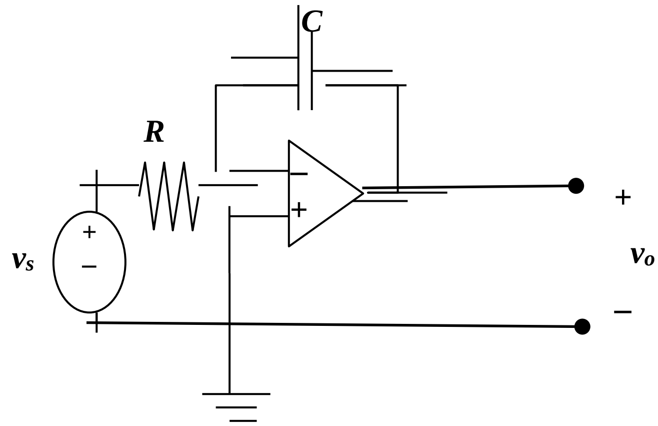


Figure 10.112
For Prob. 10.69.

Solution

This is an inverting op amp so that $\mathbf{V_o}/\mathbf{V_s} = -[1/(j\omega C)]/R = \mathbf{j[1/(\omega RC)]}$.

For $\mathbf{V_s} = V_m\angle 0°$ V and $\omega = 1/(RC)$ we get $\mathbf{V_o} = jV_m$ or

$v_o(t) = \mathbf{V_m sin(\omega t+90°)\ V}$.

Solution 10.70

Using Fig. 10.113, design a problem to help other students to better understand op amps in AC circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The circuit in Fig. 10.113 is an integrator with a feedback resistor. Calculate $v_o(t)$ if $v_s = 2 \cos 4 \times 10^4 t$ V.

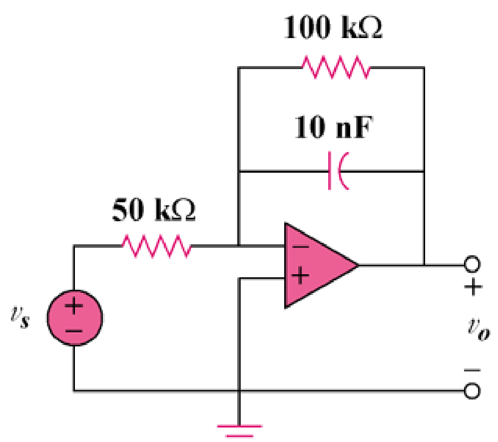


Figure 10.113

Solution

This may also be regarded as an inverting amplifier.

$$2\cos(4\times10^4 t) \longrightarrow 2\angle 0°, \quad \omega = 4\times10^4$$

$$10 \text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4\times10^4)(10\times10^{-9})} = \text{-j}2.5 \text{ k}\Omega$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i}$$

where $\mathbf{Z}_i = 50 \text{ k}\Omega$ and $\mathbf{Z}_f = 100\text{k} \,||\, (\text{-j}2.5\text{k}) = \frac{\text{-j}100}{40 - j} \text{ k}\Omega$.

Thus, $$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-(\text{-j}2)}{40 - j}$$

If $\mathbf{V}_s = 2\angle 0°$,

$$\mathbf{V}_o = \frac{j4}{40 - j} = \frac{4\angle 90°}{40.01\angle -1.43°} = 0.1\angle 91.43°$$

Therefore,

$$v_o(t) = \mathbf{100\ cos(4x10^4\ t + 91.43°)\ mV}$$

Solution 10.71

Find v_o in the op amp circuit shown in Fig. 114.

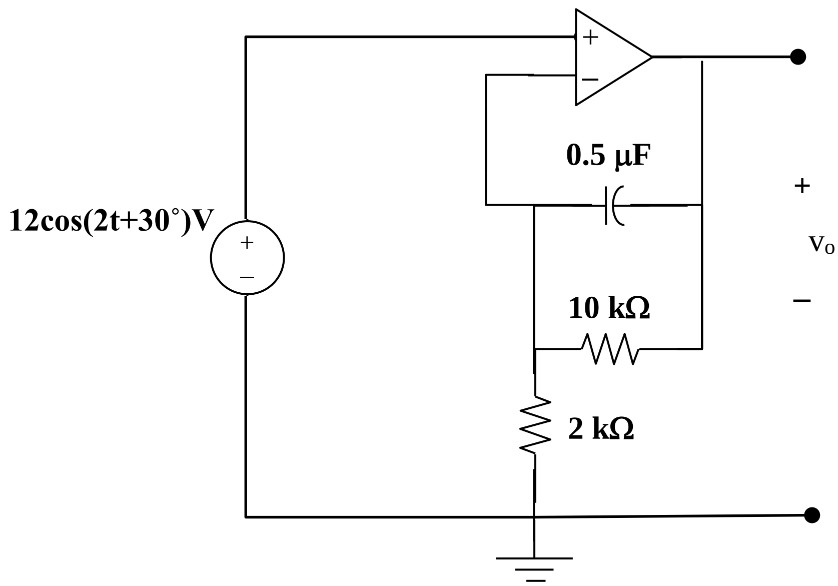


Figure 10.114
For Prob. 10.71.

Solution

$$12\cos(2t+30^o) \quad \longrightarrow \quad 12\angle 30^o$$

$$0.5\mu F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j2x0.5x10^{-6}} = -j1M\Omega$$

At the inverting terminal,

$$\frac{V_o - 12\angle 30^o}{-j1000k} + \frac{V_o - 12\angle 30^o}{10k} = \frac{12\angle 30^o}{2k} \quad \longrightarrow$$

$$V_o(1-j100) = 12\angle 30 + 1200\angle -60° + 6000\angle -60°$$

$$V_o = \frac{10.3923 + j6 + 3600 - j6235.38}{1-j100} = \frac{7200\angle -59.9045°}{100\angle -89.427°} = 72\angle 29.52^o$$

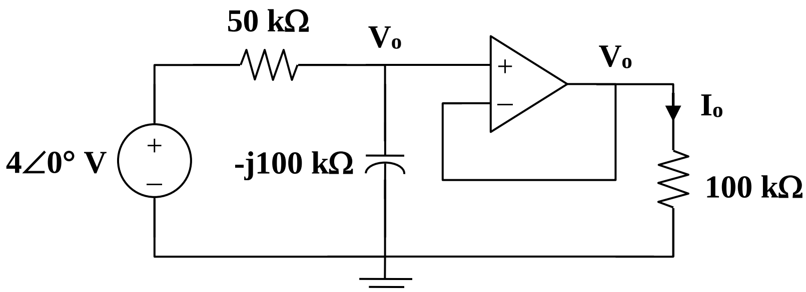
$v_o(t)$ = **72cos(2t+29.52°) V**

Solution 10.72

$$4\cos(10^4 t) \longrightarrow 4\angle 0°, \quad \omega = 10^4$$

$$1\text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^4)(10^{-9})} = -j100\text{ k}\Omega$$

Consider the circuit as shown below.



At the noninverting node,

$$\frac{4-\mathbf{V}_o}{50} = \frac{\mathbf{V}_o}{-j100} \longrightarrow \mathbf{V}_o = \frac{4}{1+j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{100k} = \frac{4}{(100)(1+j0.5)}\text{mA} = 35.78\angle -26.56°\ \mu\text{A}$$

Therefore,

$i_o(t) =$ **35.78cos(10⁴t–26.56°) μA**

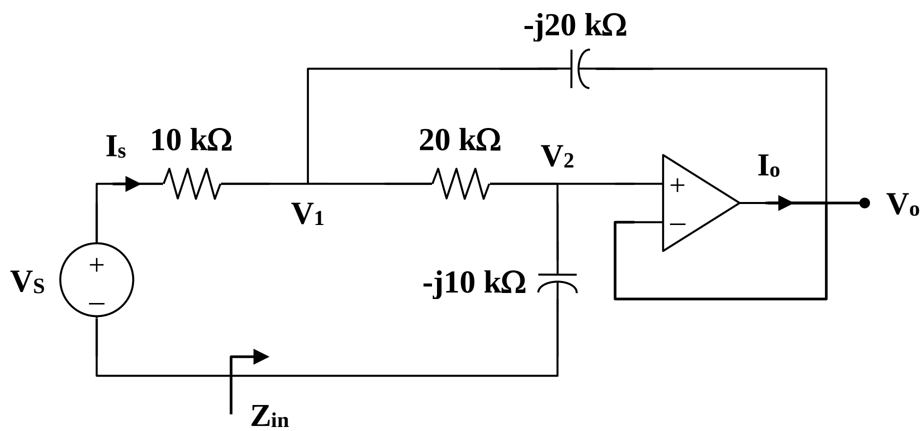
Solution 10.73

As a voltage follower, $\mathbf{V}_2 = \mathbf{V}_o$

$$C_1 = 10 \text{ nF} \longrightarrow \frac{1}{j\omega C_1} = \frac{1}{j(5\times 10^3)(10\times 10^{-9})} = \text{-j}20 \text{ k}\Omega$$

$$C_2 = 20 \text{ nF} \longrightarrow \frac{1}{j\omega C_2} = \frac{1}{j(5\times 10^3)(20\times 10^{-9})} = \text{-j}10 \text{ k}\Omega$$

Consider the circuit in the frequency domain as shown below.



At node 1,

$$\frac{\mathbf{V}_s - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{\text{- j}20} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$2\,\mathbf{V}_s = (3 + j)\mathbf{V}_1 - (1 + j)\mathbf{V}_o$$

(1)

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{20} = \frac{\mathbf{V}_o - 0}{\text{- j}10}$$

$$\mathbf{V}_1 = (1 + j2)\mathbf{V}_o$$

(2)

Substituting (2) into (1) gives

$$2\,\mathbf{V}_s = j6\mathbf{V}_o \qquad \text{or} \qquad \mathbf{V}_o = \text{-j}\frac{1}{3}\mathbf{V}_s$$

$$\mathbf{V}_1 = (1 + j2)\mathbf{V}_o = \left(\frac{2}{3} - j\frac{1}{3}\right)\mathbf{V}_s$$

$$\mathbf{I}_s = \frac{\mathbf{V}_s - \mathbf{V}_1}{10k} = \frac{(1/3)(1+j)}{10k}\mathbf{V}_s$$

$$\frac{\mathbf{I}_s}{\mathbf{V}_s} = \frac{1+j}{30k}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{30k}{1+j} = 15(1-j)\,k$$

$$\mathbf{Z}_{in} = \mathbf{21.21\angle{-45^\circ}\ k\Omega}$$

Solution 10.74

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1}, \qquad \mathbf{Z}_f = R_2 + \frac{1}{j\omega C_2}$$

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = -\frac{R_2 + \frac{1}{j\omega C_2}}{R_1 + \frac{1}{j\omega C_1}} = -\left(\frac{C_1}{C_2}\right)\left(\frac{1 + j\omega R_2 C_2}{1 + j\omega R_1 C_1}\right)$$

At $\omega = 0$, $\qquad \mathbf{A}_v = -\frac{\mathbf{C_1}}{\mathbf{C_2}}$

As $\omega \to \infty$, $\qquad \mathbf{A}_v = -\frac{\mathbf{R_2}}{\mathbf{R_1}}$

At $\omega = \frac{1}{R_1 C_1}$, $\qquad \mathbf{A}_v = -\left(\frac{\mathbf{C_1}}{\mathbf{C_2}}\right)\left(\frac{\mathbf{1 + jR_2C_2/R_1C_1}}{\mathbf{1+j}}\right)$

At $\omega = \frac{1}{R_2 C_2}$, $\qquad \mathbf{A}_v = -\left(\frac{\mathbf{C_1}}{\mathbf{C_2}}\right)\left(\frac{\mathbf{1+j}}{\mathbf{1 + jR_1C_1/R_2C_2}}\right)$

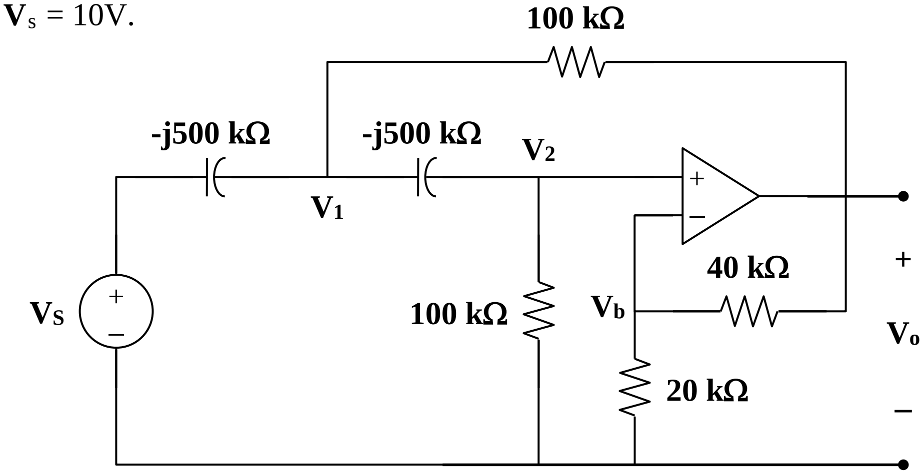
Solution 10.75

$$\omega = 2\times 10^3$$

$$C_1 = C_2 = 1\,\text{nF} \longrightarrow \frac{1}{j\omega C_1} = \frac{1}{j(2\times 10^3)(1\times 10^{-9})} = \text{-j500 k}\Omega$$

Consider the circuit shown below.

Let $\mathbf{V}_s$ = 10V.



At node 1,

$[(\mathbf{V}_1–10)/(–j500k)] + [(\mathbf{V}_1–\mathbf{V}_o)/10^5] + [(\mathbf{V}_1–\mathbf{V}_2)/(–j500k)] = 0$

or $(1+j0.4)\mathbf{V}_1 – j0.2\mathbf{V}_2 – \mathbf{V}_o = j2$ (1)

At node 2,

$[(\mathbf{V}_2–\mathbf{V}_1)/(–j500k)] + [(\mathbf{V}_2–0)/100k] + 0 = 0$ or

$–j0.2\mathbf{V}_1 + (1+j0.2)\mathbf{V}_2 = 0$ or $\mathbf{V}_1 = [–(1+j0.2)/(–j0.2)]V_2$

$= (1–j5)\mathbf{V}_2$ (2)

At node b,

$$\mathbf{V_b} = \frac{R_3}{R_3+R_4}\mathbf{V}_o = \frac{\mathbf{V}_o}{3} = \mathbf{V_2} \qquad (3)$$

From (2) and (3),

$\mathbf{V}_1 = (0.3333–j1.6667)\mathbf{V}_o$ (4)

Substituting (3) and (4) into (1),

$(1+j0.4)(0.3333–j1.6667)\mathbf{V}_o – j0.06667\mathbf{V}_o – \mathbf{V}_o = j2$

$(1+j0.4)(0.3333–j1.6667) = (1.077\angle 21.8^\circ)(1.6997\angle –78.69^\circ)$

$= 1.8306\angle –56.89^\circ = 1–j1.5334$

$$(1–1+j(–1.5334–0.06667))\mathbf{V_o} = (–j1.6001)\mathbf{V_o} = 1.6001\angle–90^\circ$$

Therefore,

$$\mathbf{V}_o = 2\angle 90^\circ/(1.6001\angle–90^\circ) = 1.2499\angle 180^\circ$$

Since $\mathbf{V}_s = 10$,

$$\mathbf{V}_o/\mathbf{V}_s = \mathbf{0.12499\angle 180^\circ}.$$

Solution 10.76

Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the op amp circuit of Fig. 10.119.

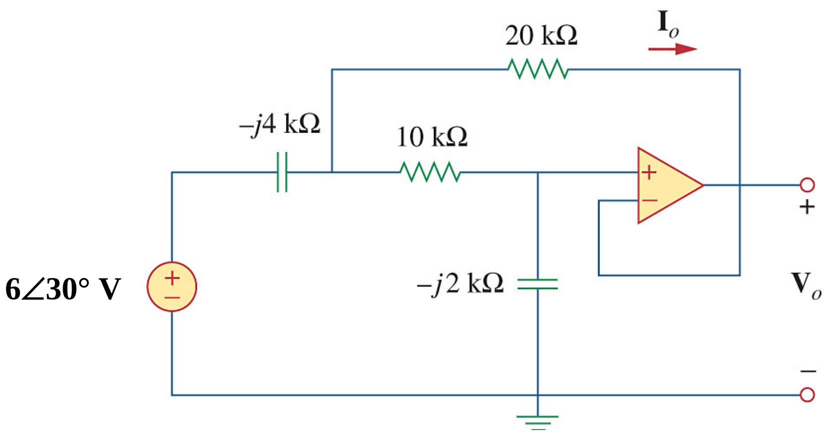


Figure 10.119
For Prob. 10.76.

Solution

Let the voltage between the –j4 kΩ capacitor and the 10 kΩ resistor be $\mathbf{V}_1$.

$$\frac{6\angle 30^o - V_1}{-j4k} = \frac{V_1 - V_o}{10k} + \frac{V_1 - V_o}{20k} \longrightarrow \qquad (1)$$

$$6\angle 30^o = (1 - j0.6)V_1 + j0.6V_o$$
$$= 5.196+j3$$

Also,

$$\frac{V_1 - V_o}{10k} = \frac{V_o}{-j2k} \longrightarrow \quad V_1 = (1 + j5)V_o \qquad (2)$$

Solving (2) into (1) yields

$$6\angle 30° = (1 - j0.6)(1 + j5)V_o + j0.6V_o = (1 + 3 - j0.6 + j5 + j6)V_o$$
$$= (4+j5)\mathbf{V_o}$$

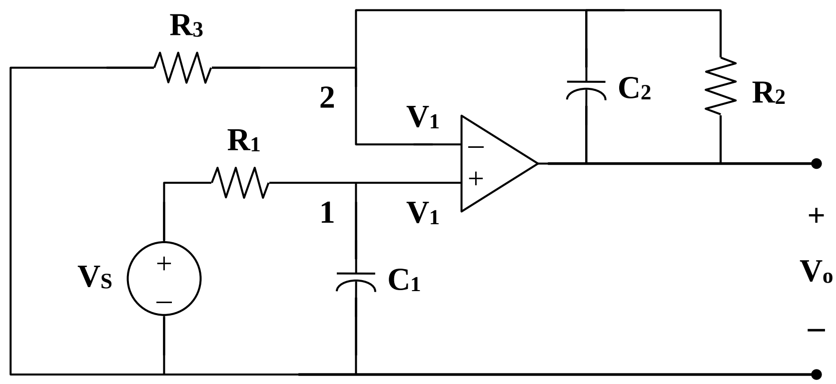
$$V_o = \frac{6\angle 30°}{6.403\angle 51.34°} = \underline{0.9371\angle -21.34^o \;\; V}$$

$$= \mathbf{937.1\angle{-21.34}^\circ\ mV}$$

$\mathbf{I}_o = (\mathbf{V}_1 – \mathbf{V}_o)/20k = \mathbf{V}_o/(–j4k) = (0.9371/4k)\angle(–21.43+90)^\circ$
$= \mathbf{234.3\angle 68.57^\circ\ \mu A}$

Solution 10.77

Consider the circuit below.



At node 1,

$$\frac{\mathbf{V}_s - \mathbf{V}_1}{R_1} = j\omega C\,\mathbf{V}_1$$

$$\mathbf{V}_s = (1 + j\omega R_1 C_1)\,\mathbf{V}_1 \qquad (1)$$

At node 2,

$$\frac{0 - \mathbf{V}_1}{R_3} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{R_2} + j\omega C_2\,(\mathbf{V}_1 - \mathbf{V}_o)$$

$$\mathbf{V}_1 = (\mathbf{V}_o - \mathbf{V}_1)\left(\frac{R_3}{R_2} + j\omega C_2 R_3\right)$$

$$\mathbf{V}_o = \left(1 + \frac{1}{(R_3/R_2) + j\omega C_2 R_3}\right)\mathbf{V}_1 \qquad (2)$$

From (1) and (2),

$$\mathbf{V}_o = \frac{\mathbf{V}_s}{1 + j\omega R_1 C_1}\left(1 + \frac{R_2}{R_3 + j\omega C_2 R_2 R_3}\right)$$

$$\mathbf{\frac{V_o}{V_s} = \frac{R_2 + R_3 + j\omega C_2 R_2 R_3}{(1 + j\omega R_1 C_1)(R_3 + j\omega C_2 R_2 R_3)}}$$

Solution 10.78

Determine $v_o(t)$ in the op amp circuit in Fig. 10.121 below.

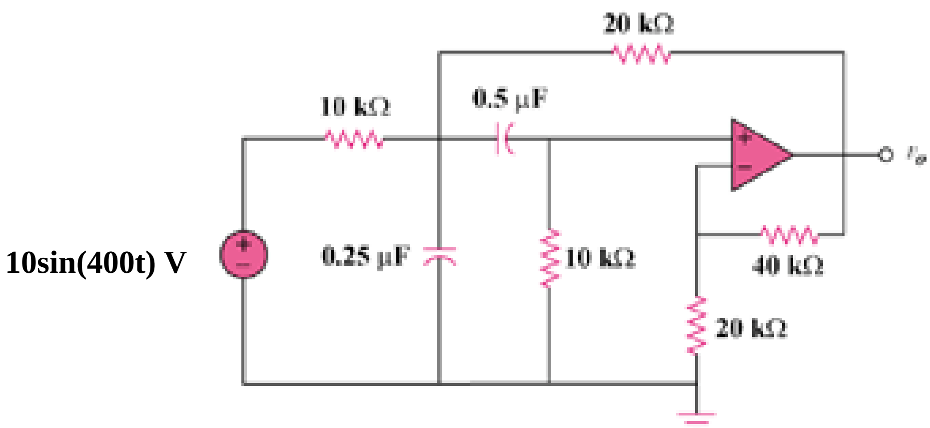


Figure 10.121
For Prob. 10.78.

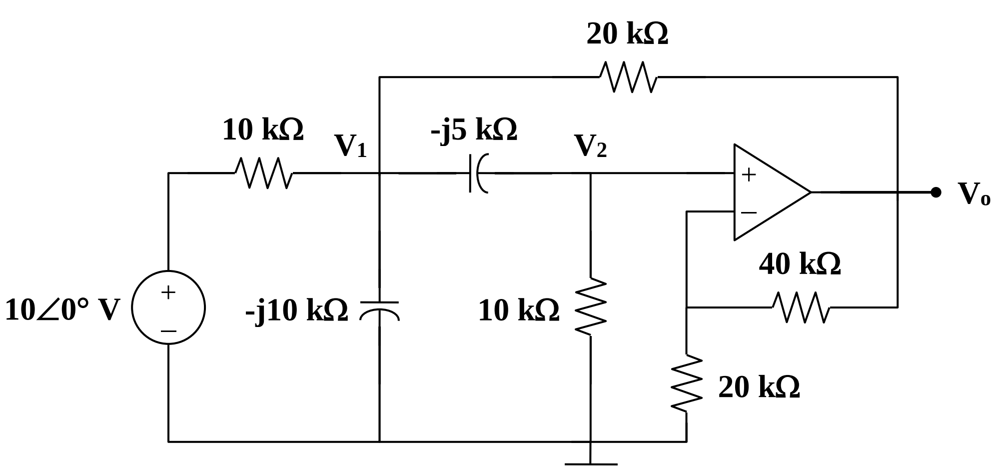
Solution

$$10\sin(400t) \longrightarrow 10\angle 0°, \quad \omega = 400$$

$$0.5\ \mu\text{F} \longrightarrow \frac{1}{\text{j}\omega\text{C}} = \frac{1}{\text{j}(400)(0.5\times 10^{-6})} = \text{-j5 k}\Omega$$

$$0.25\ \mu\text{F} \longrightarrow \frac{1}{\text{j}\omega\text{C}} = \frac{1}{\text{j}(400)(0.25\times 10^{-6})} = \text{-j10 k}\Omega$$

Consider the circuit as shown below.



At node 1,

$$\frac{10-\mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1-\mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1-\mathbf{V}_o}{20}$$

$$20 = (3 + j6)\mathbf{V}_1 - j4\mathbf{V}_2 - \mathbf{V}_o$$

(1)

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-\mathrm{j}5} = \frac{\mathbf{V}_2}{10}$$

$$\mathbf{V}_1 = (1 - \mathrm{j}0.5)\mathbf{V}_2$$

(2)

But

$$\mathbf{V}_2 = \frac{20}{20 + 40}\mathbf{V}_o = \frac{1}{3}\mathbf{V}_o$$

(3)

From (2) and (3),

$$\mathbf{V}_1 = \frac{1}{3} \cdot (1 - \mathrm{j}0.5)\mathbf{V}_o$$

(4)

Substituting (3) and (4) into (1) gives

$$20 = (3 + j6) \cdot \frac{1}{3} \cdot (1 - j0.5)\mathbf{V}_o - j\frac{4}{3}\mathbf{V}_o - \mathbf{V}_o = \left(1 + j\frac{1}{6}\right)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{120}{6 + j} = \frac{120}{6.08276\angle 9.4623^\circ} = 19.728\angle -9.46^\circ$$

Therefore,

$$\mathrm{v}_o(\mathrm{t}) = \mathbf{19.728sin(400t–9.46°)\ V}$$

Solution 10.79

For the op amp circuit in Fig. 10.122, obtain $\mathbf{V_o}$.

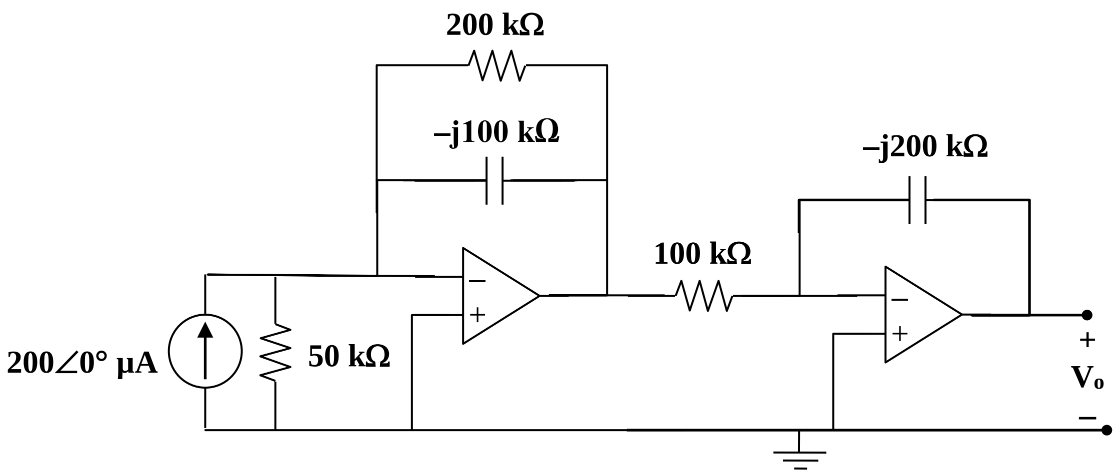
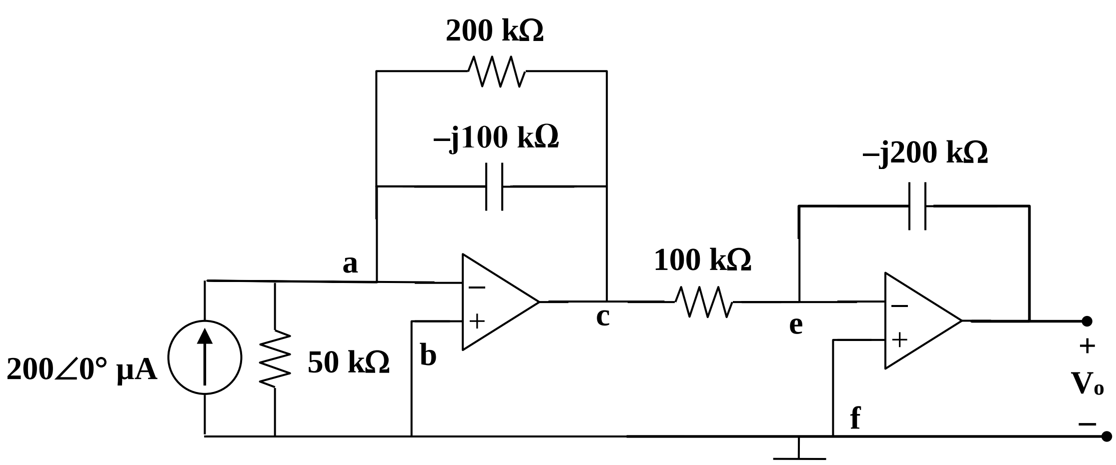


Figure 10.122
For Prob. 10.79.

Solution

First we label all the unknown nodes in the circuit.



At node a we have, $-200\mu + [(\mathbf{V_a}-0)/50k] + [(\mathbf{V_a}-\mathbf{V_c})/200k] + [(\mathbf{V_a}-\mathbf{V_c})/(-j100k)] = 0$. At b we have $\mathbf{V_b} = 0$. At e we have $[(\mathbf{V_e}-\mathbf{V_c})/100k] + [(\mathbf{V_e}-\mathbf{V_o})/(-j200k)] = 0$. At f we have $\mathbf{V_f} = 0$. Now we need to use the constraint equations, $\mathbf{V_a} = \mathbf{V_b} = 0$ and $\mathbf{V_e} = \mathbf{V_f} = 0$.

This leads to the following,

$\{[1/200\text{k}] + [1/(-j100\text{k})]\}\mathbf{V_c} = -200\mu$ or $(0.5+j)\mathbf{V_c} = -20$ or
$\mathbf{V_c} = -20/(1.118034\angle 63.435°) = -17.88854\angle -63.435°$.

Now for the second op amp, $[(-\mathbf{V_c})/100\text{k}] + [(-\mathbf{V_o})/(-j200\text{k})] = 0$ or
$\mathbf{V_o}/(-j2) = -\mathbf{V_c}$ or $\mathbf{V_o} = -(-j2)(-17.88854\angle -63.435°)$
$= 35.777\angle(90°-180°-63.44°) = 35.78\angle -153.44°$ or

$$\mathbf{V_o} = \mathbf{35.78\angle -153.44°\ V}.$$

Solution 10.80

Obtain $v_o(t)$ for the op amp circuit in Fig. 10.123 if v_s = 12cos (1000t – 60°) V.

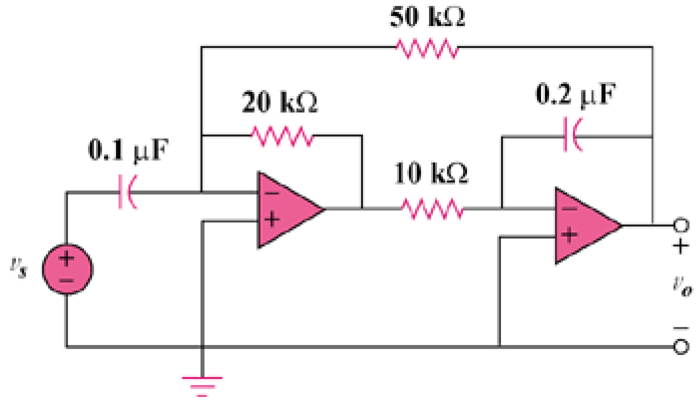


Figure 10.123
For Prob. 10.80.

Solution

$$12\cos(1000t - 60°) \longrightarrow 12\angle\text{-}60°, \quad \omega = 1000$$

$$0.1\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.1\times 10^{-6})} = \text{-j10 k}\Omega$$

$$0.2\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.2\times 10^{-6})} = \text{-j5 k}\Omega$$

Let the input to the inverting terminal of the first op amp be $\mathbf{V_a}$, the output of the first op amp be $\mathbf{V_1}$, and the input to the inverting terminal of the second op amp be $\mathbf{V_b}$. This then gives us the following node equations,

$[(\mathbf{V_a}–\mathbf{V_s})/(–j10k)] + [(\mathbf{V_a}–\mathbf{V_o})/50k] + [(\mathbf{V_a}–\mathbf{V_1})/20k] + 0 = 0$ where $\mathbf{V_a}$ = 0 or
$\mathbf{V_1} = 20k\{[–\mathbf{V_s}/(–j10k)] + [–\mathbf{V_o}/50k]\} = –2j\mathbf{V_s} – 0.4\mathbf{V_o}$.

$[(\mathbf{V_b}–\mathbf{V_1})/10k] + [(\mathbf{V_b}–\mathbf{V_o})/(–j5k)] + 0 = 0$ where $\mathbf{V_b}$ = 0 or
$\mathbf{V_o} = –j5k[–\mathbf{V_1}/10k] = j0.5\mathbf{V_1} = j0.5[–j2\mathbf{V_s} – 0.4\mathbf{V_o}] = \mathbf{V_s} – j0.2\mathbf{V_o}$ or
$(1+j0.2)\mathbf{V_o} = \mathbf{V_s}$ or $\mathbf{V_o} = \mathbf{V_s}/(1.0198\angle 11.31°) = (0.9806\angle –11.31°)\mathbf{V_s}$.

Since $\mathbf{V_s} = 12\angle –60°$ V, which leads to, $\mathbf{V_o} = 11.767\angle –71.31°$ V or

$v_o(t)$ = **11.767cos(1,000t–71.31°) V**.

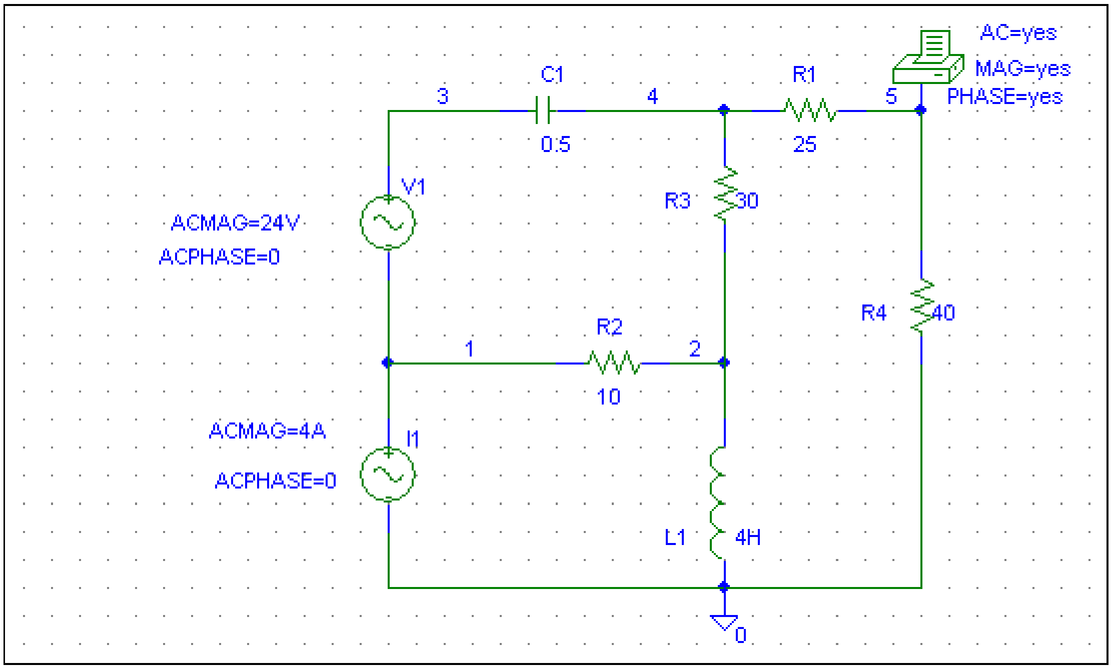
Solution 10.81

We need to get the capacitance and inductance corresponding to –j2 Ω and j4 Ω.

$$-j2 \quad \longrightarrow \quad C = \frac{1}{\omega X_c} = \frac{1}{1x2} = 0.5F$$

$$j4 \quad \longrightarrow \quad L = \frac{X_L}{\omega} = 4H$$

The schematic is shown below.



When the circuit is simulated, we obtain the following from the output file.

FREQ	VM(5)	VP(5)
1.592E-01	1.127E+01	-1.281E+02

From this, we obtain

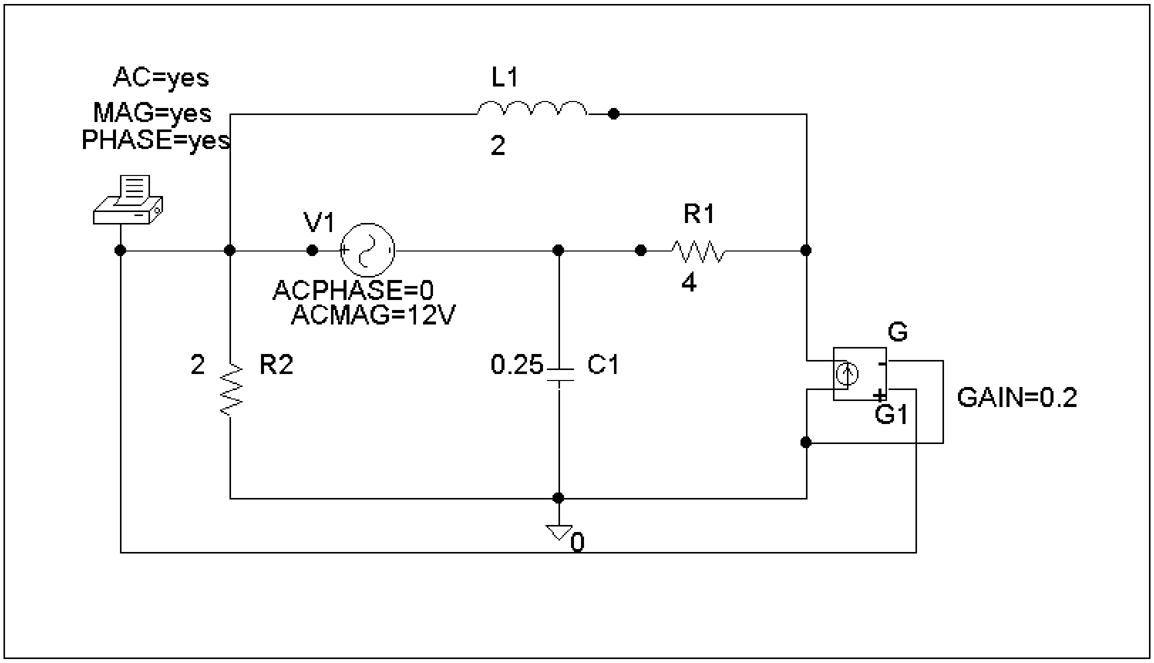
$$V_o = \mathbf{11.27\angle 128.1^o\ \ V}.$$

Solution 10.82

The schematic is shown below. We insert PRINT to print V_o in the output file. For AC Sweep, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we print out the output file which includes:

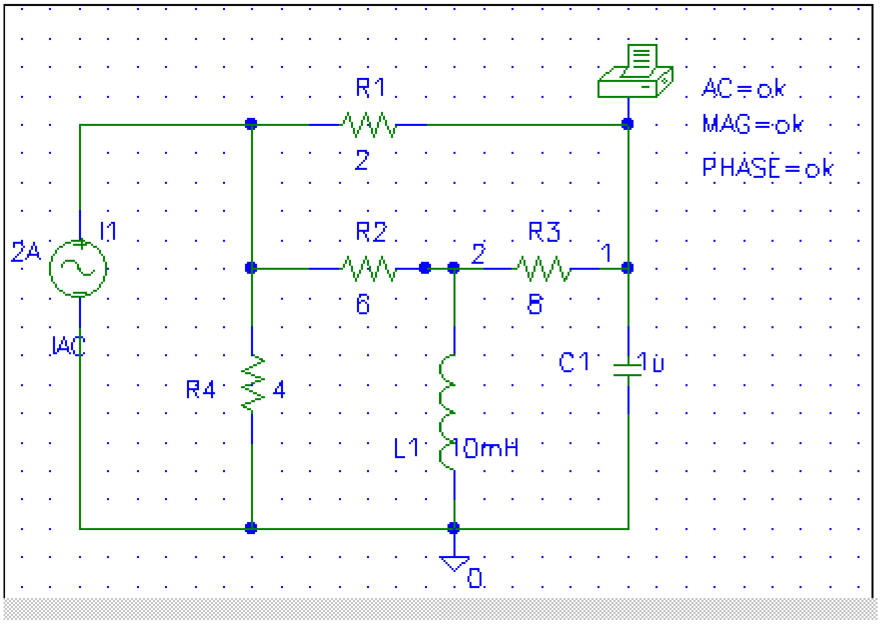
FREQ	VM($N_0001)	VP($N_0001)
1.592 E-01	7.684 E+00	5.019 E+01

which means that $V_o = \mathbf{7.684\angle 50.19^o\ V}$



Solution 10.83

The schematic is shown below. The frequency is $f = \omega / 2\pi = \dfrac{1000}{2\pi} = 159.15$



When the circuit is saved and simulated, we obtain from the output file

FREQ	VM(1)	VP(1)
1.592E+02	6.611E+00	-1.592E+02

Thus,

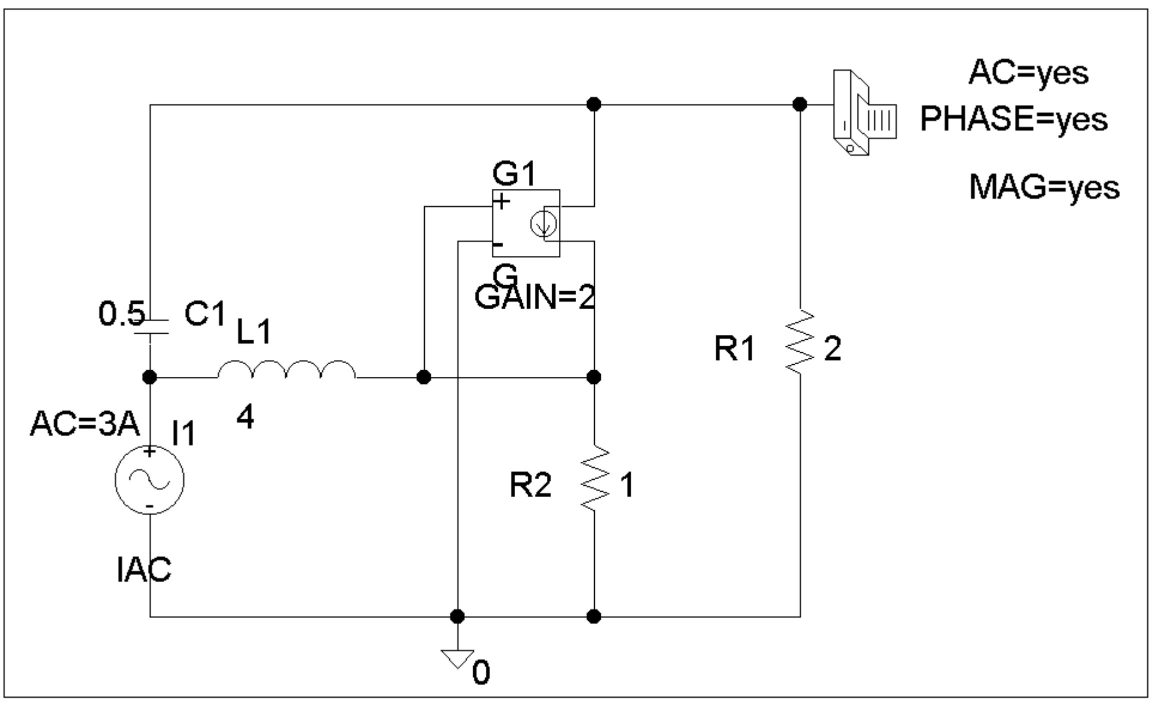
$$v_o = \mathbf{6.611cos(1000t – 159.2^o)\ \ V}$$

Solution 10.84

The schematic is shown below. We set PRINT to print V_o in the output file. In AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain the output file which includes:

FREQ	VM($N_0003)	VP($N_0003)
1.592 E-01	1.664 E+00	–1.646 E+02

Namely, V_o = **1.664∠–146.4º V**



Solution 10.85

Using Fig. 10.127, design a problem to help other students to better understand performing AC analysis with *PSpice*.

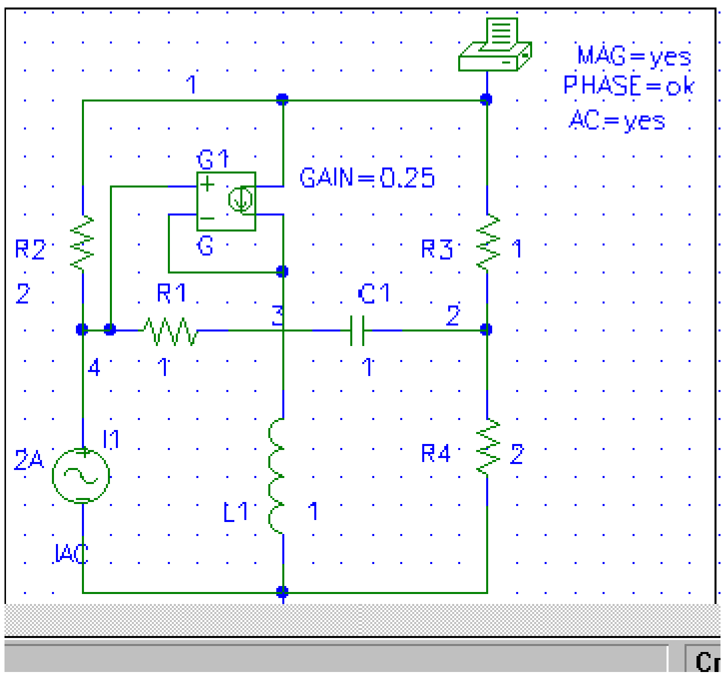
Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use *PSpice* to find V_o in the circuit of Fig. 10.127. Let $R_1 = 2\ \Omega$, $R_2 = 1\ \Omega$, $R_3 = 1\ \Omega$, $R_4 = 2\ \Omega$, $I_s = 2\angle 0^\circ$ A, $X_L = 1\ \Omega$, and $X_C = 1\ \Omega$.

Solution

The schematic is shown below. We let $\omega = 1$ rad/s so that L=1H and C=1F.



When the circuit is saved and simulated, we obtain from the output file

FREQ	VM(1)	VP(1)
1.591E-01	2.228E+00	-1.675E+02

From this, we conclude that

$$V_o = \mathbf{2.228\angle{-167.5^\circ}\ V}.$$

Solution 10.86

The schematic is shown below. We insert three pseudo-component PRINTs at nodes 1, 2, and 3 to print V_1, V_2, and V_3, into the output file. Assume that w = 1, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After saving and simulating the circuit, we obtain the output file which includes:

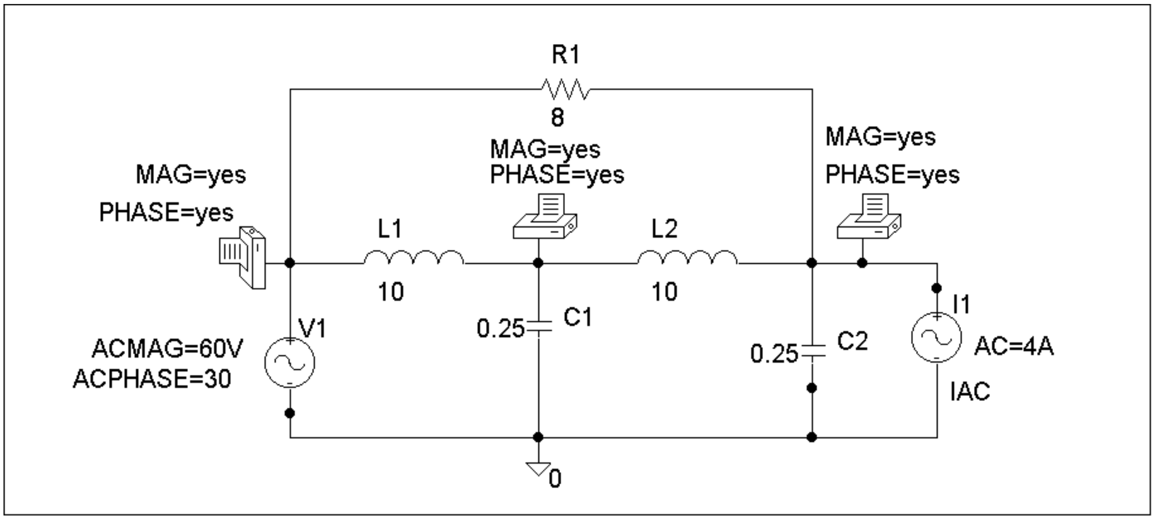
FREQ	VM($N_0002)	VP($N_0002)
1.592 E-01	6.000 E+01	3.000 E+01

FREQ	VM($N_0003)	VP($N_0003)
1.592 E-01	2.367 E+02	-8.483 E+01

FREQ	VM($N_0001)	VP($N_0001)
1.592 E-01	1.082 E+02	1.254 E+02

Therefore,

$$V_1 = \mathbf{60\angle 30^o\ V} \quad V_2 = \mathbf{236.7\angle -84.83^o\ V} \quad V_3 = \mathbf{108.2\angle 125.4^o\ V}$$



Solution 10.87

The schematic is shown below. We insert three PRINTs at nodes 1, 2, and 3. We set Total Pts = 1, Start Freq = 0.1592, End Freq = 0.1592 in the AC Sweep box. After simulation, the output file includes:

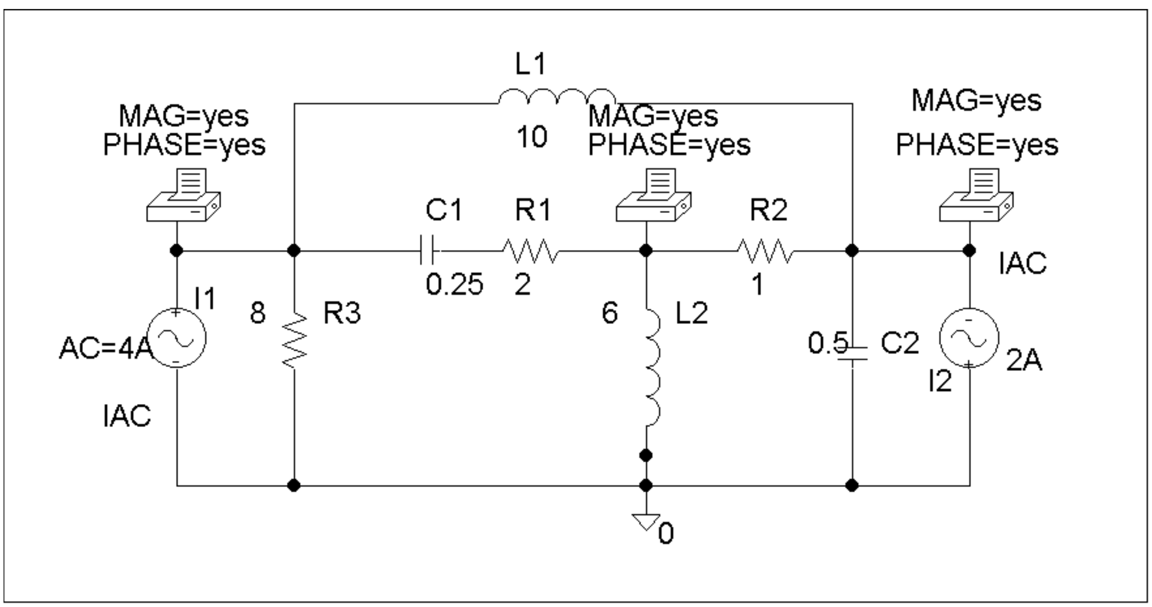
FREQ	VM($N_0004)	VP($N_0004)
1.592 E-01	1.591 E+01	1.696 E+02

FREQ	VM($N_0001)	VP($N_0001)
1.592 E-01	5.172 E+00	-1.386 E+02

FREQ	VM($N_0003)	VP($N_0003)
1.592 E-01	2.270 E+00	-1.524 E+02

Therefore,

V_1 = **$15.91\angle 169.6^o$ V** V_2 = **$5.172\angle -138.6^o$ V** V_3 = **$2.27\angle -152.4^o$ V**



Solution 10.88

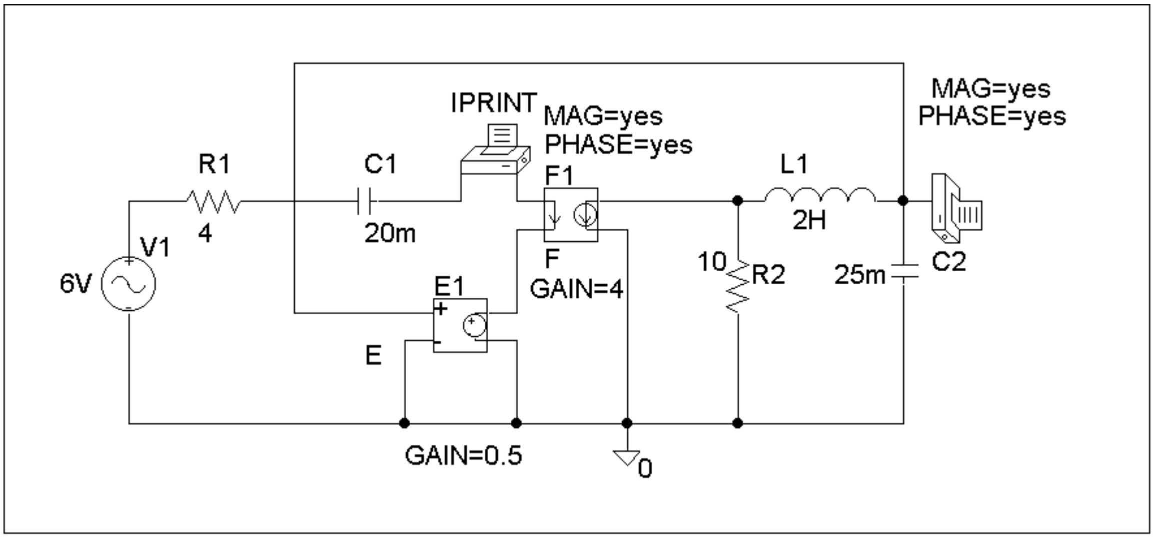
The schematic is shown below. We insert IPRINT and PRINT to print I_o and V_o in the output file. Since w = 4, f = w/2π = 0.6366, we set Total Pts = 1, Start Freq = 0.6366, and End Freq = 0.6366 in the AC Sweep box. After simulation, the output file includes:

FREQ	VM($N_0002)	VP($N_0002)
6.366 E-01	3.496 E+01	1.261 E+01

FREQ	IM(V_PRINT2)	IP (V_PRINT2)
6.366 E-01	8.912 E-01	-8.870 E+01

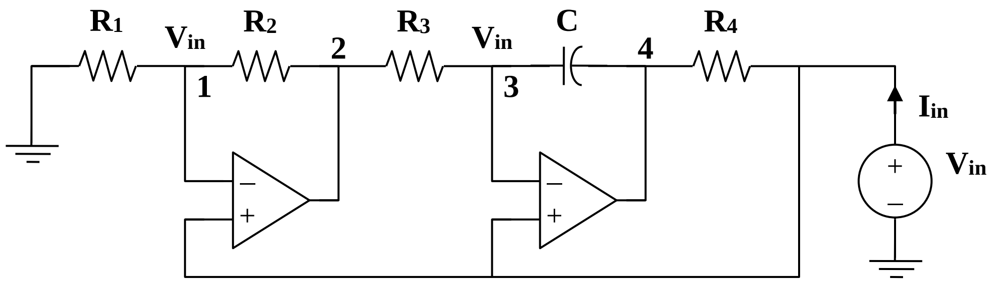
Therefore, $V_o = 34.96\angle 12.6^o$ V, $I_o = 0.8912\angle -88.7^o$ A

v_o = **34.96 cos(4t + 12.6º)V**, i_o = **0.8912cos(4t – 88.7º)A**



Solution 10.89

Consider the circuit below.



At node 1,

$$\frac{0-\mathbf{V}_{in}}{R_1} = \frac{\mathbf{V}_{in}-\mathbf{V}_2}{R_2}$$

$$-\mathbf{V}_{in} + \mathbf{V}_2 = \frac{R_2}{R_1}\mathbf{V}_{in} \qquad (1)$$

At node 3,

$$\frac{\mathbf{V}_2-\mathbf{V}_{in}}{R_3} = \frac{\mathbf{V}_{in}-\mathbf{V}_4}{1/j\omega C}$$

$$-\mathbf{V}_{in} + \mathbf{V}_4 = \frac{\mathbf{V}_{in}-\mathbf{V}_2}{j\omega CR_3} \qquad (2)$$

From (1) and (2),

$$-\mathbf{V}_{in} + \mathbf{V}_4 = \frac{-R_2}{j\omega CR_3R_1}\mathbf{V}_{in}$$

Thus,

$$\mathbf{I}_{in} = \frac{\mathbf{V}_{in}-\mathbf{V}_4}{R_4} = \frac{R_2}{j\omega CR_3R_1R_4}\mathbf{V}_{in}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{\mathbf{I}_{in}} = \frac{j\omega CR_1R_3R_4}{R_2} = j\omega L_{eq}$$

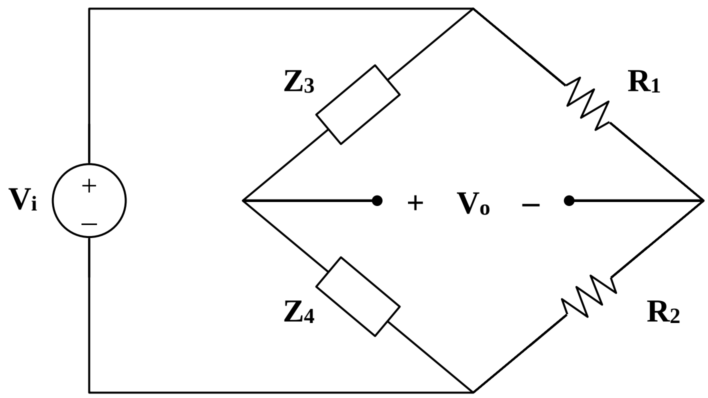
where $\mathbf{L_{eq} = \frac{R_1R_3R_4C}{R_2}}$

Solution 10.90

Let $$\mathbf{Z}_4 = R \parallel \frac{1}{j\omega C} = \frac{R}{1 + j\omega RC}$$

$$\mathbf{Z}_3 = R + \frac{1}{j\omega C} = \frac{1 + j\omega RC}{j\omega C}$$

Consider the circuit shown below.



$$\mathbf{V}_o = \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4}\mathbf{V}_i - \frac{R_2}{R_1 + R_2}\mathbf{V}_i$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{R}{1 + j\omega C}}{\dfrac{R}{1 + j\omega C} + \dfrac{1 + j\omega RC}{j\omega C}} - \frac{R_2}{R_1 + R_2}$$

$$= \frac{j\omega RC}{j\omega RC + (1 + j\omega RC)^2} - \frac{R_2}{R_1 + R_2}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega RC}{1 - \omega^2 R^2 C^2 + j3\omega RC} - \frac{R_2}{R_1 + R_2}$$

For $\mathbf{V}_o$ and $\mathbf{V}_i$ to be in phase, $\dfrac{\mathbf{V}_o}{\mathbf{V}_i}$ must be purely real. This happens when

$$1 - \omega^2 R^2 C^2 = 0$$

$$\omega = \frac{1}{RC} = 2\pi f$$

or $$f = \frac{1}{2\pi RC}$$

At this frequency,

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{3} - \frac{R_2}{R_1 + R_2}$$

Solution 10.91

(a) Let $\mathbf{V}_2 =$ voltage at the noninverting terminal of the op amp

$\mathbf{V}_o =$ output voltage of the op amp

$\mathbf{Z}_p = 10 \text{ k}\Omega = R_o$

$$\mathbf{Z}_s = R + j\omega L + \frac{1}{j\omega C}$$

As in Section 10.9,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} = \frac{R_o}{R + R_o + j\omega L - \dfrac{j}{\omega C}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega C R_o}{\omega C (R + R_o) + j(\omega^2 LC - 1)}$$

For this to be purely real,

$$\omega_o^2 LC - 1 = 0 \longrightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.4\times10^{-3})(2\times10^{-9})}}$$

$$f_o = \mathbf{180\ kHz}$$

(b) At oscillation,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega_o C R_o}{\omega_o C (R + R_o)} = \frac{R_o}{R + R_o}$$

This must be compensated for by

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{80}{20} = 5$$

$$\frac{R_o}{R + R_o} = \frac{1}{5} \longrightarrow R = 4R_o = \mathbf{40\ k\Omega}$$

Solution 10.92

Let $\mathbf{V}_2 =$ voltage at the noninverting terminal of the op amp

$\mathbf{V}_o =$ output voltage of the op amp

$\mathbf{Z}_s = R_o$

$$\mathbf{Z}_p = j\omega L \,||\, \frac{1}{j\omega C} \,||\, R = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}} = \frac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}$$

As in Section 10.9,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} = \frac{\frac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}}{R_o + \frac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega RL}{\omega RL + \omega R_o L + jR_o R\,(\omega^2 LC - 1)}$$

For this to be purely real,

$$\omega_o^2 LC = 1 \longrightarrow f_o = \frac{1}{2\pi\sqrt{LC}}$$

(a) At $\omega = \omega_o$,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega_o RL}{\omega_o RL + \omega_o R_o L} = \frac{R}{R + R_o}$$

This must be compensated for by

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{R_f}{R_o} = 1 + \frac{1000k}{100k} = 11$$

Hence,

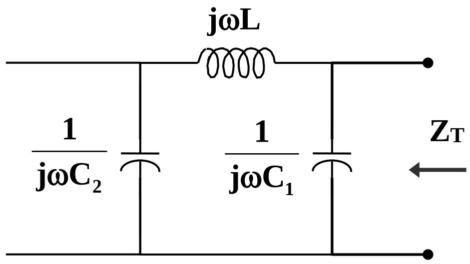
$$\frac{R}{R + R_o} = \frac{1}{11} \longrightarrow R_o = 10R = \mathbf{100\ k\Omega}$$

(b) $$f_o = \frac{1}{2\pi\sqrt{(10\times 10^{-6})(2\times 10^{-9})}}$$

$$f_o = \mathbf{1.125\ MHz}$$

Solution 10.93

As shown below, the impedance of the feedback is



$$\mathbf{Z}_T = \frac{1}{j\omega C_1} \,||\, \left(j\omega L + \frac{1}{j\omega C_2} \right)$$

$$\mathbf{Z}_T = \frac{\dfrac{-j}{\omega C_1}\left(j\omega L + \dfrac{-j}{\omega C_2} \right)}{\dfrac{-j}{\omega C_1} + j\omega L + \dfrac{-j}{\omega C_2}} = \frac{\dfrac{1}{\omega} - \omega L C_2}{j(C_1 + C_2 - \omega^2 L C_1 C_2)}$$

In order for $\mathbf{Z}_T$ to be real, the imaginary term must be zero; i.e.

$$C_1 + C_2 - \omega_o^2 L C_1 C_2 = 0$$

$$\omega_o^2 = \frac{C_1 + C_2}{L C_1 C_2} = \frac{1}{L C_T}$$

$$\mathbf{f_o = \frac{1}{2\pi\sqrt{LC_T}}}$$

Solution 10.94

If we select $C_1 = C_2 = 20$ nF

$$C_T = \frac{C_1C_2}{C_1 + C_2} = \frac{C_1}{2} = 10 \text{ nF}$$

Since $f_o = \dfrac{1}{2\pi\sqrt{LC_T}}$,

$$L = \frac{1}{(2\pi f)^2 C_T} = \frac{1}{(4\pi^2)(2500\times 10^6)(10\times 10^{-9})} = 10.13 \text{ mH}$$

$$X_c = \frac{1}{\omega C_2} = \frac{1}{(2\pi)(50\times 10^3)(20\times 10^{-9})} = 159\ \Omega$$

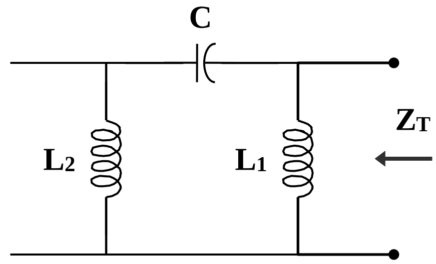
We may select $R_i = 20\ \text{k}\Omega$ and $R_f \geq R_i$, say $R_f = 20\ \text{k}\Omega$.

Thus,

$C_1 = C_2 =$ **20 nF**, $\quad L =$ **10.13 mH** $\quad R_f = R_i =$ **20 kΩ**

Solution 10.95

First, we find the feedback impedance.



$$\mathbf{Z}_T = j\omega L_1 \,\|\, \left(j\omega L_2 + \frac{1}{j\omega C} \right)$$

$$\mathbf{Z}_T = \frac{j\omega L_1 \left(j\omega L_2 - \dfrac{j}{\omega C} \right)}{j\omega L_1 + j\omega L_2 - \dfrac{j}{\omega C}} = \frac{\omega^2 L_1 C (1 - \omega L_2)}{j(\omega^2 C (L_1 + L_2) - 1)}$$

In order for $\mathbf{Z}_T$ to be real, the imaginary term must be zero; i.e.

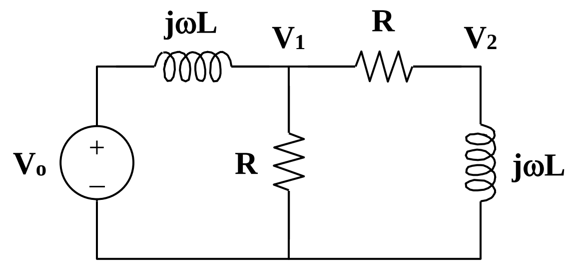
$$\omega_o^2 C (L_1 + L_2) - 1 = 0$$

$$\omega_o = 2\pi f_o = \frac{1}{C(L_1 + L_2)}$$

$$\boldsymbol{f_o = \frac{1}{2\pi \sqrt{C(L_1 + L_2)}}}$$

Solution 10.96

(a) Consider the feedback portion of the circuit, as shown below.



$$\mathbf{V_2} = \frac{j\omega L}{R + j\omega L}\mathbf{V_1} \longrightarrow \mathbf{V}_1 = \frac{R + j\omega L}{j\omega L}\mathbf{V}_2 \qquad (1)$$

Applying KCL at node 1,

$$\frac{\mathbf{V}_o - \mathbf{V}_1}{j\omega L} = \frac{\mathbf{V}_1}{R} + \frac{\mathbf{V}_1}{R + j\omega L}$$

$$\mathbf{V}_o - \mathbf{V}_1 = j\omega L\, \mathbf{V}_1 \left(\frac{1}{R} + \frac{1}{R + j\omega L} \right)$$

$$\mathbf{V}_o = \mathbf{V}_1 \left(1 + \frac{j2\omega RL - \omega^2 L^2}{R\,(R + j\omega L)} \right)$$

(2)

From (1) and (2),

$$\mathbf{V}_o = \left(\frac{R + j\omega L}{j\omega L} \right) \left(1 + \frac{j2\omega RL - \omega^2 L^2}{R\,(R + j\omega L)} \right) \mathbf{V}_2$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{R^2 + j\omega RL + j2\omega RL - \omega^2 L^2}{j\omega RL}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + \dfrac{R^2 - \omega^2 L^2}{j\omega RL}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{1}}{\mathbf{3 + j(\omega L/R - R/\omega L)}}$$

(b) Since the ratio $\frac{\mathbf{V}_2}{\mathbf{V}_o}$ must be real,

$$\frac{\omega_o L}{R} - \frac{R}{\omega_o L} = 0$$

$$\omega_o L = \frac{R^2}{\omega_o L}$$

$$\omega_o = 2\pi f_o = \frac{R}{L}$$

$$\mathbf{f_o = \frac{R}{2\pi L}}$$

(c) When $\omega = \omega_o$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3}$$

This must be compensated for by $\mathbf{A}_v = 3$. But

$$\mathbf{A}_v = 1 + \frac{R_2}{R_1} = 3$$

$$\mathbf{R_2 = 2R_1}$$

Solution 11.1

$v(t) = 160\cos(50t)$
$i(t) = -33\sin(50t-30^\circ) = 33\cos(50t-30^\circ+180^\circ-90^\circ) = 33\cos(50t+60^\circ)$

$p(t) = v(t)i(t) = 160\text{x}33\cos(50t)\cos(50t+60^\circ)$
$= 5280(1/2)[\cos(100t+60^\circ)+\cos(60^\circ)] =$ **[1.320+2.640cos(100t+60˚)] kW**.

$P = [V_m I_m/2]\cos(0-60^\circ) = 0.5\text{x}160\text{x}33\text{x}0.5 =$ **1.320 kW**.

Solution 11.2

Given the circuit in Fig. 11.35, find the average power supplied or absorbed by each element.

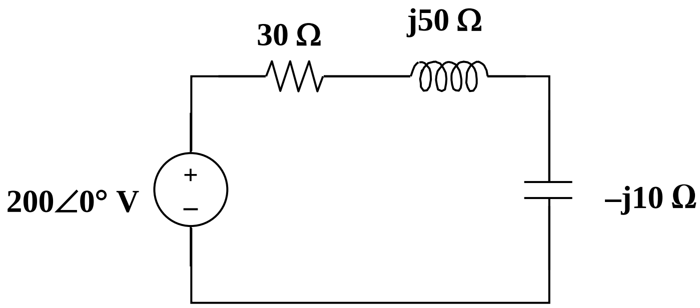
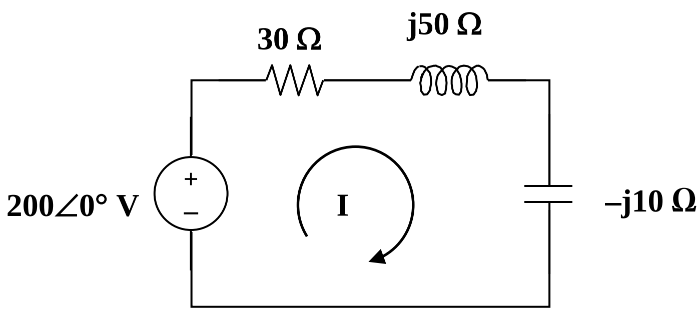


Figure 11.35
For Prob. 11.2.

Solution

Step 1. First we can write one mesh equation and solve for I. Once we have I, we can then find the average power absorbed by each element. Obviously the source will have a negative power absorbed meaning it is supplying power. One last comment, since we still have not covered rms values, we will treat the 200 V as a peak value.

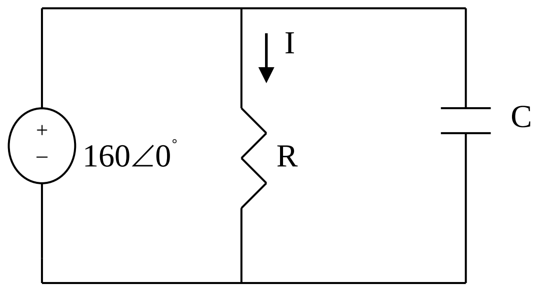


$-200 + 30\mathbf{I} + j50\mathbf{I} + (-j10)\mathbf{I} = 0$ or $\mathbf{I} = 200/(30+j40)$. Finally, $P_{30} = \mathbf{I}(\mathbf{I})^{*}30$, $P_{j50} = \mathbf{0}$, $P_{-j10} = \mathbf{0}$, and $P_{200} = -|\mathbf{V}|\,|\mathbf{I}|\cos(\theta)$

Step 2. $\mathbf{I} = 200/50\angle 53.13° = 4\angle -53.13°$ A. Thus,

P_{30} = **480 W** and P_{200} = **–480 W**.

Solution 11.3



$$90\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j90x10^{-6}x2x10^{3}} = -j5.5556$$

I = 160/60 = 2.667A

The average power delivered to the load is the same as the average power absorbed by the resistor which is

$$P_{avg} = 0.5|I|^2 60 = \mathbf{213.4\ W}.$$

Solution 11.4

Using Fig. 11.36, design a problem to help other students better understand instantaneous and average power.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the average power dissipated by the resistances in the circuit of Fig. 11.36. Additionally, verify the conservation of power. Note, we do not talk about rms values of voltages and currents until Section 11.4, all voltages and currents are peak values.

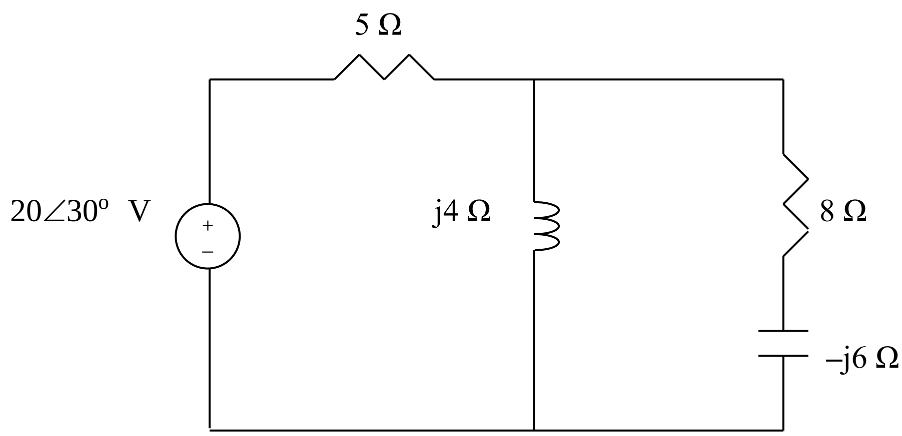
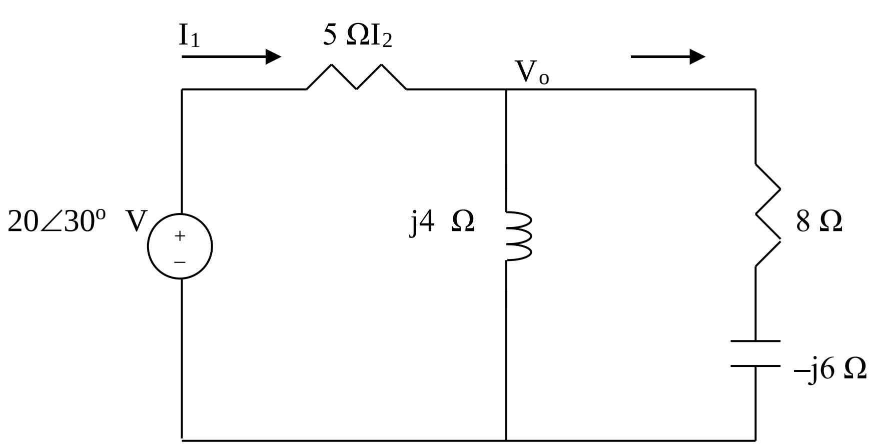


Figure 11.36 For Prob. 11.4.

Solution

We apply nodal analysis. At the main node,



$$\frac{20<30^o - V_o}{5} = \frac{V_o}{j4} + \frac{V_o}{8-j6} \quad \longrightarrow \quad V_o = 5.152 + j10.639 = 11.821\angle 64.16^\circ$$

For the 5-Ω resistor,

$$I_1 = \frac{20<30^o - V_o}{5} = 2.438 < -3.0661^o \quad \text{A}$$

The average power dissipated by the resistor is

$$P_1 = \frac{1}{2}|I_1|^2 R_1 = \frac{1}{2}x2.438^2 x5 = \underline{14.86 \text{ W}}$$

For the 8-Ω resistor,

$\mathbf{I_2} = V_o/(8–j6) = (11.812/10)\angle(64.16+36.87)^\circ = 1.1812\angle 101.03^\circ$ A

The average power dissipated by the resistor is

$P_2 = 0.5|I_2|^2R_2 = 0.5(1.1812)^2 8 =$ **5.581 W**

The complex power supplied is

$\mathbf{S} = 0.5(V_s)(I_1)^* = 0.5(20\angle 30^\circ)(2.438\angle 3.07^\circ) = 24.38\angle 33.07^\circ$

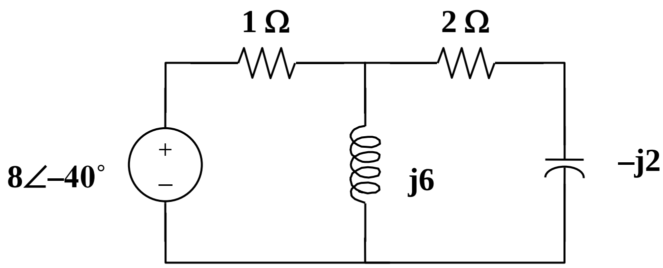
= **(20.43+13.303) VA**

Adding P_1 and P_2 gives the real part of **S**, showing the conservation of power.

P = 14.86+5.581 = **20.44 W** which checks nicely.

Solution 11.5

Converting the circuit into the frequency domain, we get:



$$I_{1\Omega} = \frac{8\angle -40^\circ}{1+\dfrac{j6(2-j2)}{j6+2-j2}} = 1.6828\angle -25.38^\circ$$

$$P_{1\Omega} = \frac{1.6828^2}{2}1 = \underline{1.4159\,\mathrm{W}}$$

$P_{1\Omega}$ = **1.4159 W**

P_{3H} = $P_{0.25F}$ = **0 W**

$$\left|I_{2\Omega}\right| = \left|\frac{j6}{j6+2-j2}1.6828\angle -25.38^\circ\right| = 2.258$$

$$P_{2\Omega} = \frac{2.258^2}{2}2 = \underline{5.097\,\mathrm{W}}$$

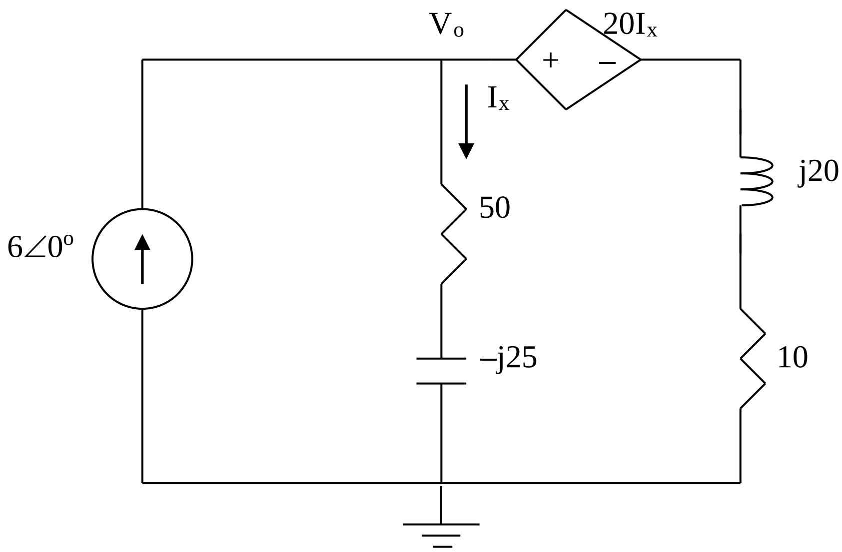
$P_{2\Omega}$ = **5.097 W**

Solution 11.6

$$20\text{ mH} \quad \longrightarrow \quad j\omega L = j10^3 x20x10^{-3} = j20$$

$$40\mu\text{F} \rightarrow \frac{1}{j\omega C} = \frac{1}{j10^3 \text{x}40\text{x}10^{-6}} = -j25$$

We apply nodal analysis to the circuit below.



$$-6 + \frac{V_o - 20I_x}{10 + j20} + \frac{V_o - 0}{50 - j25} = 0$$

But $I_x = \dfrac{V_o}{50 - j25}$. Substituting this and solving for V_o leads

$$\left(\frac{1}{10+j20} - \frac{20}{(10+j20)}\frac{1}{(50-j25)} + \frac{1}{50-j25}\right)V_o = 6$$

$$\left(\frac{1}{22.36\angle 63.43^\circ} - \frac{20}{(22.36\angle 63.43^\circ)(55.9\angle -26.57^\circ)} + \frac{1}{55.9\angle -26.57^\circ}\right)V_o = 6$$

$$(0.02 - j0.04 - 0.012802 + j0.009598 + 0.016 + j0.008)V_o = 6$$

$(0.0232 - j0.0224)V_o = 6$ or $V_o = 6/(0.03225\angle{-43.99^\circ}) = 186.05\angle 43.99^\circ$ volts.

$|I_x| = 186.05/55.9 = 3.328$

We can now calculate the average power absorbed by the 50-Ω resistor.

$P_{avg} = [(3.328)^2/2]x50 =$ **276.8 W**.

Solution 11.7

Given the circuit of Fig. 11.40, find the average power absorbed by the 10-Ω resistor.

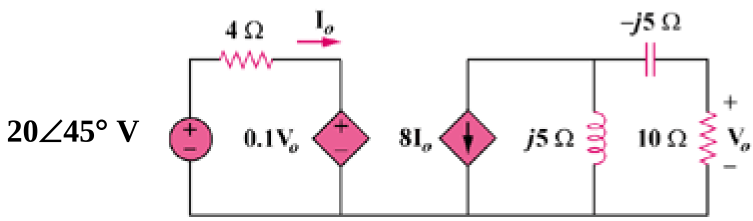


Figure 11.40
For Prob. 11.7.

Solution

Applying KVL to the left-hand side of the circuit,

$$20\angle 45° = 4\mathbf{I}_o + 0.1\mathbf{V}_o \qquad (1)$$

Applying KCL to the right side of the circuit,

$$8\mathbf{I}_o + \frac{\mathbf{V}_1}{j5} + \frac{\mathbf{V}_1}{10 - j5} = 0$$

But, $\mathbf{V}_o = \dfrac{10}{10 - j5}\mathbf{V}_1 \longrightarrow \mathbf{V}_1 = \dfrac{10 - j5}{10}\mathbf{V}_o$

Hence, $8\mathbf{I}_o + \dfrac{10 - j5}{j50}\mathbf{V}_o + \dfrac{\mathbf{V}_o}{10} = 0$

$$\mathbf{I}_o = j0.025\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1),

$$20\angle 45° = 0.1\mathbf{V}_o(1 + j)$$

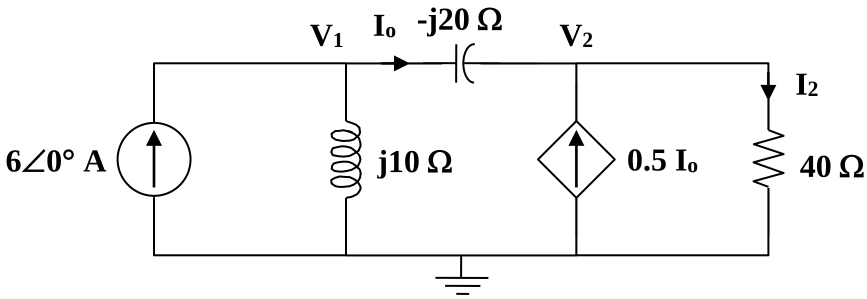
$$\mathbf{V}_o = \frac{200\angle 45°}{1 + j}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{10} = \frac{20}{\sqrt{2}}\angle 0°$$

$$P = \frac{1}{2}|\mathbf{I}_1|^2 R = \left(\frac{1}{2}\right)\left(\frac{400}{2}\right)(10) = \mathbf{1\ kW}$$

Solution 11.8

We apply nodal analysis to the following circuit.



At node 1,

$$6 = \frac{\mathbf{V}_1}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20} \quad \mathbf{V}_1 = j120 - \mathbf{V}_2 \qquad (1)$$

At node 2,

$$0.5\mathbf{I}_o + \mathbf{I}_o = \frac{\mathbf{V}_2}{40}$$

But, $\mathbf{I}_o = \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{-j20}$

Hence, $\dfrac{1.5(\mathbf{V}_1 - \mathbf{V}_2)}{-j20} = \dfrac{\mathbf{V}_2}{40}$

$$3\mathbf{V}_1 = (3 - j)\mathbf{V}_2 \qquad (2)$$

Substituting (1) into (2),

$$j360 - 3\mathbf{V}_2 - 3\mathbf{V}_2 + j\mathbf{V}_2 = 0$$

$$\mathbf{V}_2 = \frac{j360}{6 - j} = \frac{360}{37}(-1 + j6)$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{40} = \frac{9}{37}(-1 + j6)$$

$$P = \frac{1}{2}\left|\mathbf{I}_2\right|^2 R = \frac{1}{2}\left(\frac{9}{\sqrt{37}}\right)^2 (40) = \mathbf{43.78\ W}$$

Solution 11.9

For the op amp circuit in Fig. 11.41, $\mathbf{V}_s = 2\angle 30^o$ V. Find the average power absorbed by the 20-kΩ resistor.

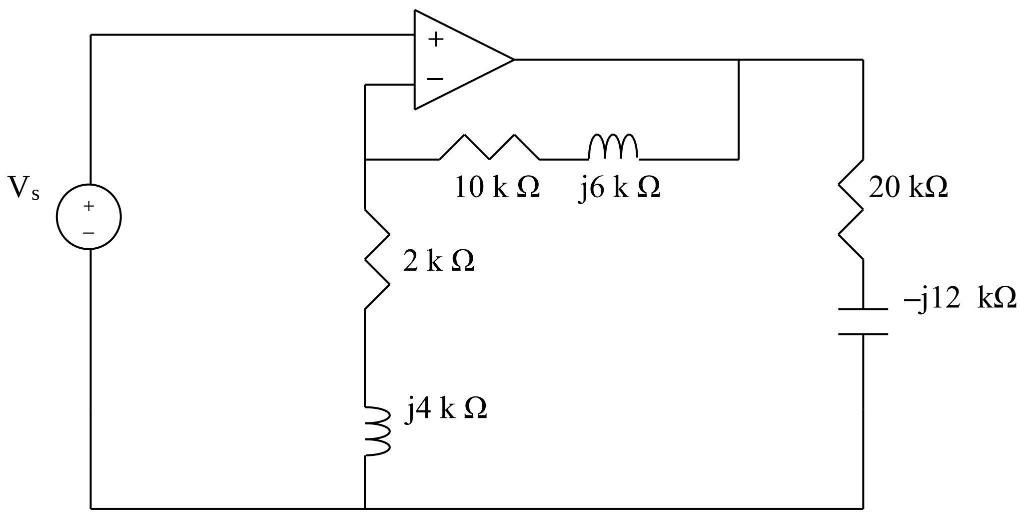


Figure 11.41
For Prob. 11.9.

Solution

This is a non-inverting op amp circuit. At the output of the op amp,

$$\mathbf{V_o} = \left(1+\frac{\mathbf{Z}_2}{\mathbf{Z}_1}\right)\mathbf{V_s} = \left(1+\frac{(10+j6)10^3}{(2+j4)10^3}\right)\mathbf{V_s} = \left(1+\frac{11.6619\angle 30.964^o}{4.4721\angle 63.435^o}\right)2\angle 30^o$$

$= (1+2.6077\angle{-32.471°})2\angle 30° = (1+2.2–j1.4)2\angle 30°$
$= (3.4928\angle{-23.629°})(2\angle 30°) = 6.9856\angle 6.371° = (6.9425+j0.77516)$ V.

The current through the 20-kΩ resistor is

$$\mathbf{I_o} = \frac{\mathbf{V_o}}{20k - j12k} = (6.9856\angle 6.371°)/(23.3238k\angle{-30.964°}) = 0.29951\angle 37.335°$$
mA

or $|\mathbf{I_o}| = 0.2995$ mA.

$P = [|\mathbf{I_o}|^2/2]R = [0.2995^2/2]10^{-6}x20x10^3$

$=$ **897 μW**

Solution 11.10

No current flows through each of the resistors. Hence, for each resistor, $P = \mathbf{0\ W}$. It should be noted that the input voltage will appear at the output of each of the op amps.

Solution 11.11

$\omega = 377, \qquad R = 10^4, \qquad C = 200\times10^{-9}$

$\omega RC = (377)(10^4)(200\times10^{-9}) = 0.754$

$\tan^{-1}(\omega RC) = 37.02°$

$$Z_{ab} = \frac{10k}{\sqrt{1+(0.754)^2}}\angle -37.02° = 7.985\angle -37.02°\ k\Omega$$

$i(t) = 33\sin(377t + 22°) = 33\cos(377t - 68°)\ mA$

$\mathbf{I} = 33\angle{-68°}$ mA

$$S = \frac{I^2 Z_{ab}}{2} = \frac{\left(33x10^{-3}\right)^2(7.985\angle -37.02°)\times10^3}{2}$$

$\mathbf{S} = 4.348\angle{-37.02°}$ VA

$$P = |S|\cos(37.02) = \mathbf{3.472\ W}$$

Solution 11.2

For the circuit shown in Fig. 11.44, determine the load impedance $\boldsymbol{Z_L}$ for maximum power transfer (to $\boldsymbol{Z_L}$). Calculate the maximum power absorbed by the load.

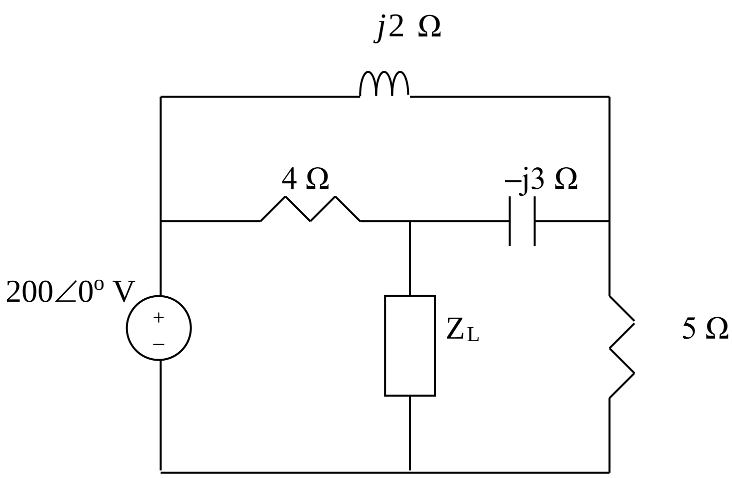
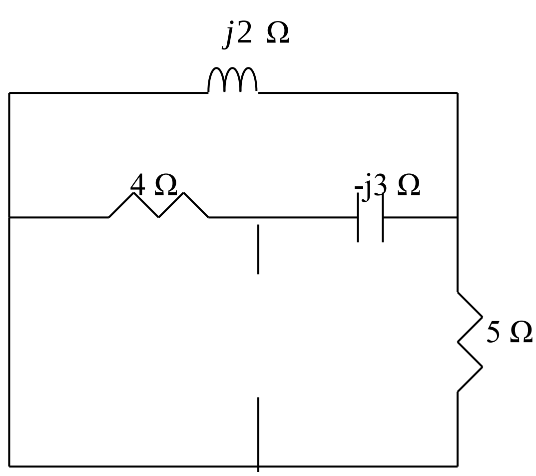


Figure 11.44
For Prob. 11.12.

Solution

We find the Thevenin impedance using the circuit below.

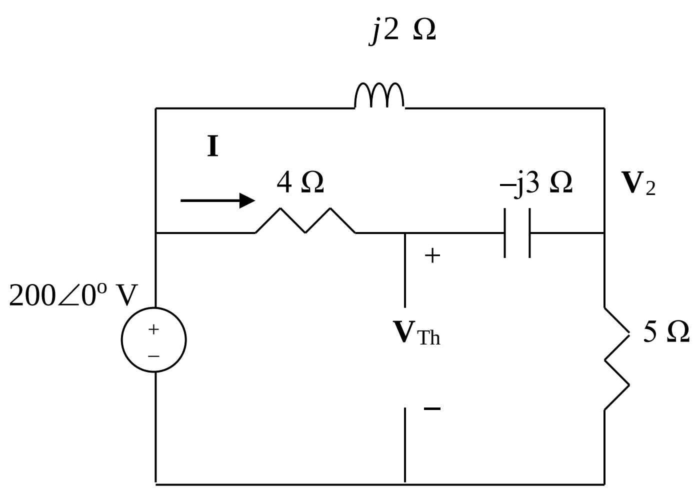


We note that the inductor is in parallel with the 5-Ω resistor and the combination is in series with the capacitor. That whole combination is in parallel with the 4-Ω resistor. Thus,

$$Z_{Thev} = \frac{4\left(-j3+\frac{5xj2}{5+j2}\right)}{4-j3+\frac{5xj2}{5+j2}} = \frac{4(0.6896-j1.2758)}{4.69-j1.2758} = \frac{4(1.4502\angle-61.61°)}{4.86\angle-15.22°}$$
$$= 1.1936\angle-46.39°$$

$\mathbf{Z_{Thev}}$ = 0.8233 – j0.8642 or $\mathbf{Z_L}$ = **[823.3 + j864.2] mΩ**.

We obtain V_{Th} using the circuit below. We apply nodal analysis.



$$\frac{\mathbf{V}_2-200}{4-\mathbf{j}3}+\frac{\mathbf{V}_2-200}{\mathbf{j}2}+\frac{\mathbf{V}_2-0}{5}=0$$
$$(0.16+\mathbf{j}0.12-\mathbf{j}0.5+0.2)\mathbf{V}_2 = (0.16+\mathbf{j}0.12-\mathbf{j}0.5)200$$
$$(0.5235\angle-46.55°)\mathbf{V}_2 = (0.4123\angle-67.17°)200 = 82.46\angle-67.17°$$

Thus, $\mathbf{V_2}$ = 157.52∠–20.62˚V = 147.43–j55.473

I = (200 – V_2)/(4 – j3) = (200 – 147.43 + j55.473)/(4 – j3)
= (52.57+j55.473)/(4–j3) = (76.426∠46.54°)/(5∠–36.87°
= 15.285∠83.41° = 1.7542+j15.184

$\mathbf{V_{Thev}}$ = 200 – 4**I** = 200 – 7.0168 – j60.736 = [192.983 – j60.736] V
= 202.31∠–17.47˚V

To calculate the maximum power to the load, we can use eq. 11.14,
$I_L = V_{THEV}/(2r_l)$, to obtain,

|$\mathbf{I_L}$| = (202.31/(2x0.8233)) = 122.865 A

P_{avg} = [(|$\mathbf{I_L}$|)^{2}0.8233]/2 = **6.214 kW**.

Solution 11.13

For maximum power transfer to the load, $\mathbf{Z_L}$ = **[120 – j60] Ω**.

$I_L = 165/(240) = 0.6875$ A

$$P_{avg} = [|I_L|^2 120]/2 = \mathbf{28.36\ W}.$$

Solution 11.14

Using Fig. 11.45, design a problem to help other students to better understand maximum average power transfer to a load ***Z***.

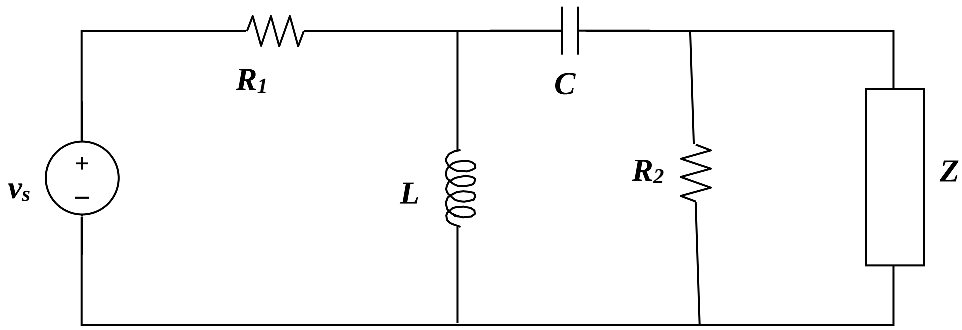
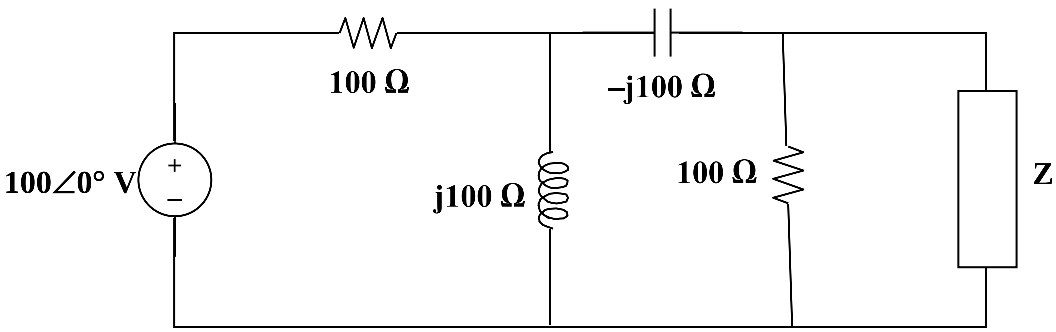


Figure 11.45
For Prob. 11.14.

Although there are many ways to work this problem, this is an example of how a student might pose and solve the problem.

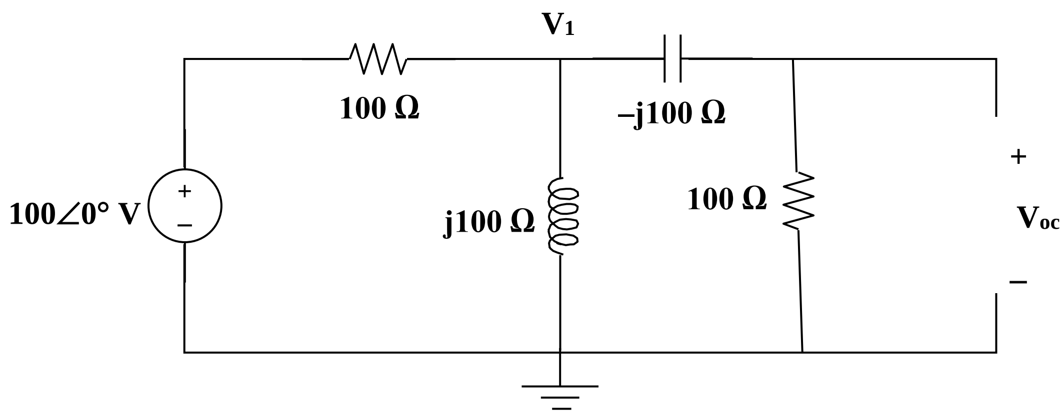
Problem Statement

It is desired to transfer maximum power to the load ***Z*** in the circuit shown below. Find ***Z*** and the maximum average power. Let v_s = 100 sin(100t) V.



Solution

Step 1. We need to find the Thevenin equivalent at the terminals of **Z**. In order to do this we need to find $\mathbf{V_{oc}}$ and $\mathbf{Z_{eq}}$. Finding the open circuit voltage, $\mathbf{V_{oc}}$, we use the following circuit to help us write the nodal equation,
$[(\mathbf{V_1}–100)/100] + [(\mathbf{V_1}–0)/(j100)] + [(\mathbf{V_1}–\mathbf{V_{oc}})/(–j100)] = 0$ and
$[(\mathbf{V_{oc}}–\mathbf{V_1})/(–j100)] + [(\mathbf{V_{oc}}–0)/100] = 0$. Solving this will give us $\mathbf{V_{oc}} = \mathbf{V_{Thev}}$.



To find $\mathbf{Z_{eq}}$ all we need to do is set the voltage source to zero and then find the impedance looking in from the right. $\mathbf{Z_{eq}} = 100||[-j100+j100||100]$.
Once we have Z_{eq} we now have the load that will give us maximum power transfer, $\mathbf{Z} = (\mathbf{Z_{eq}})^*$.

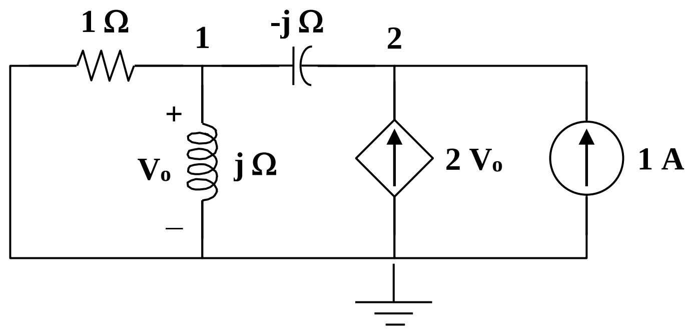
Step 2. $[(\mathbf{V_{oc}}-\mathbf{V_1})/(-j100)] + [(\mathbf{V_{oc}}-0)/100] = 0$ or $j0.01\mathbf{V_1} = (0.01+j0.01)\mathbf{V_{oc}}$ or $\mathbf{V_1} = [(0.01+j0.01)/(j0.01)]\mathbf{V_{oc}} = (1-j)\mathbf{V_{oc}}$. Now we substitute this into the first equation. $[(\mathbf{V_1}-100)/100] + [(\mathbf{V_1}-0)/(j100)] + [(\mathbf{V_1}-\mathbf{V_{oc}})/(-j100)] = 0$ or $(0.01-j0.01+j0.01)\mathbf{V_1} - (j0.01)\mathbf{V_{oc}} = 1$ or $(0.01)\mathbf{V_1} - j0.01\mathbf{V_{oc}} = 1$ which leads to $(0.01)(1-j)\mathbf{V_{oc}} - j0.01\mathbf{V_{oc}} = (0.01-j0.02)\mathbf{V_{oc}} = 1$ or $\mathbf{V_{oc}} = 1/(0.022361\angle-63.43°$ V $= 44.72\angle 63.43°$ V $= \mathbf{V_{Thev}}$.

$100||j100 = 100(j100)/(100+j100) = 100(0.5+j0.5) = 50+j50$.
$100||(-j100+50+j50) = 100(50-j50)/(150-j50)$
$= 100(70.71\angle-45°)/(158.11\angle-18.43°) = 44.72\angle-26.57°\ \Omega$ or
$\mathbf{Zeq} = 44.72\angle-26.57°\ \Omega = (40 - j20)\ \Omega$ or

$$\mathbf{Z} = (\mathbf{Z_{eq}})^* = \mathbf{(40+j20)\ \Omega} \text{ and}$$
$$P_{avg} = [|\mathbf{V_{Thev}}|]^2/[4(\mathbf{Zeq} - \mathbf{Z})] = \mathbf{6.25\ W}.$$

Solution 11.15

To find $\mathbf{Z}_{eq}$, insert a 1-A current source at the load terminals as shown in Fig. (a).



(a)

At node 1,

$$\frac{\mathbf{V}_o}{1}+\frac{\mathbf{V}_o}{j}=\frac{\mathbf{V}_2-\mathbf{V}_o}{-j} \longrightarrow \mathbf{V}_o = j\mathbf{V}_2 \qquad (1)$$

At node 2,

$$1+2\mathbf{V}_o=\frac{\mathbf{V}_2-\mathbf{V}_o}{-j} \longrightarrow 1 = j\mathbf{V}_2-(2+j)\mathbf{V}_o \qquad (2)$$

Substituting (1) into (2),

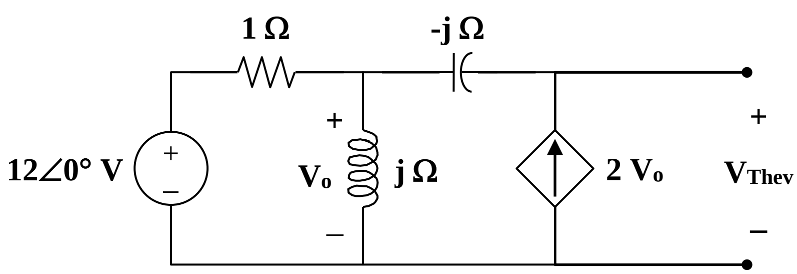
$$1 = j\mathbf{V}_2-(2+j)(j)\mathbf{V}_2=(1-j)\mathbf{V}_2$$

$$\mathbf{V}_2=\frac{1}{1-j}$$

$$\mathbf{Z}_{eq}=\frac{\mathbf{V}_2}{1}=\frac{1+j}{2}=0.5+j0.5$$

$$\mathbf{Z}_L=\mathbf{Z}_{eq}^*=\mathbf{[0.5-\mathit{j}0.5]\,\Omega}$$

We now obtain $\mathbf{V}_{Thev}$ from Fig. (b).



(b)

$$-2\mathbf{V}_o+\frac{\mathbf{V}_o-12}{1}+\frac{\mathbf{V}_o}{j}=0$$

$$\mathbf{V}_o = \frac{-12}{1+j}$$

$$-\mathbf{V}_o - (-j \times 2\mathbf{V}_o) + \mathbf{V}_{Th} = 0$$

$$\mathbf{V}_{Thev} = (1-j2)\mathbf{V}_o = \frac{(-12)(1-j2)}{1+j}$$

$$P_{max} = \frac{\left[\left[\dfrac{V_{Thev}}{0.5+j0.5+0.5-j0.5}\right]\right]^2}{2} 0.5 = \frac{\left(\dfrac{12\sqrt{5}}{\sqrt{2}}\right)^2}{2(2x0.5)^2} 0.5$$

$$= \mathbf{90\ W}$$

Solution 11.16

For the circuit in Fig. 11.47, find the maximum power that can be delivered to the load $\mathbf{Z}_L$.

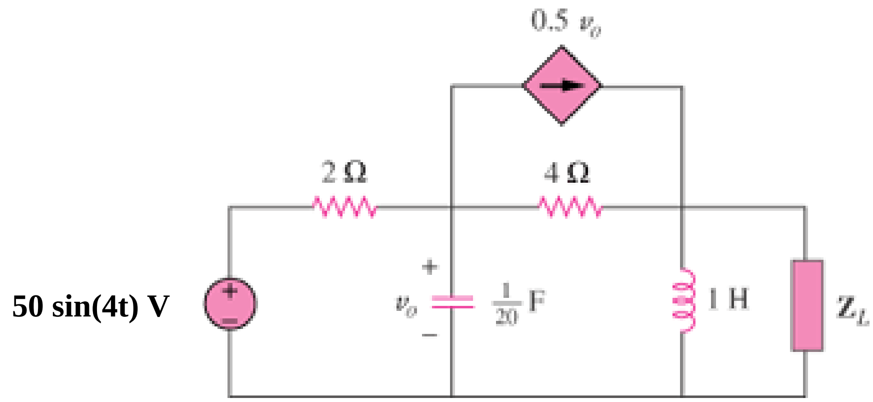
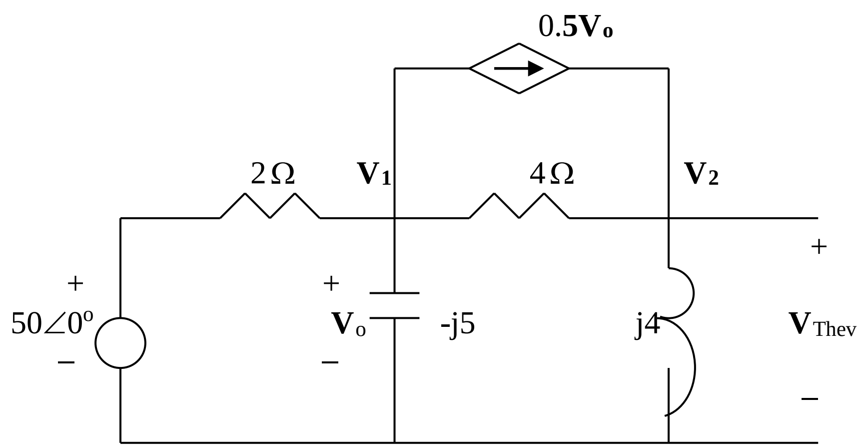


Figure 11.47
For Prob. 11.16.

Solution

$$\omega = 4, \quad 1\text{H} \longrightarrow j\omega L = j4, \quad 1/20\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j4x1/20} = -j5$$

We find the Thevenin equivalent at the terminals of Z_L. To find V_{Thev}, we use the circuit shown below.



At node 1,

$$\frac{50-\mathbf{V}_1}{2} = \frac{\mathbf{V}_1}{-\mathbf{j}5} + 0.5\mathbf{V}_1 + \frac{\mathbf{V}_1-\mathbf{V}_2}{4} \longrightarrow 25 = \mathbf{V}_1(1.25+\mathbf{j}0.2) - 0.25\mathbf{V}_2$$

(1)

At node 2,

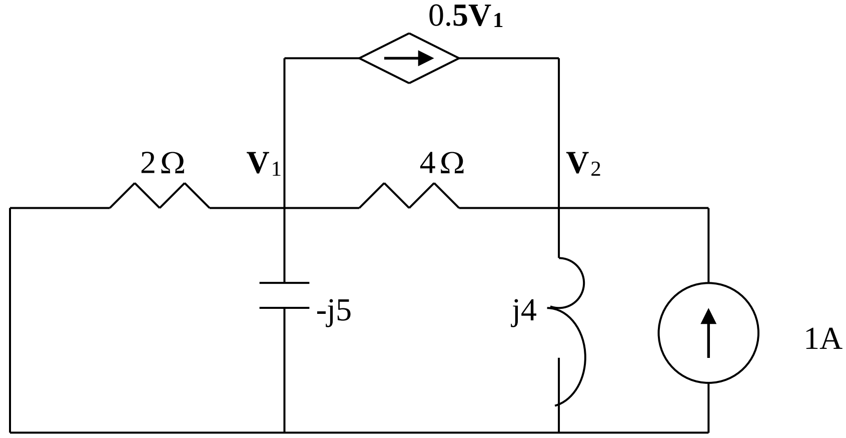
$$\frac{\mathbf{V}_1-\mathbf{V}_2}{4}+0.5\mathbf{V}_1=\frac{\mathbf{V}_2}{\mathbf{j}4} \quad \longrightarrow \quad 0=0.75\mathbf{V}_1+\mathbf{V}_2(-0.25+\mathbf{j}0.25) \quad \text{or}$$

$\mathbf{V_1}$ = (0.33333–j0.33333)$\mathbf{V_2}$ = (0.4714∠–45°)$\mathbf{V_2}$ (2)

Substituting (2) into (1) leads to

(1.25+j0.2)(0.33333–j0.33333)$\mathbf{V_2}$ – 0.25$\mathbf{V_2}$ = 25
= [0.41666+0.066666 – 0.25 + j(0.066666–0.41666)]$\mathbf{V_2}$ = (0.23332–j0.35)$\mathbf{V_2}$
= (0.42064∠–56.311°)$\mathbf{V_2}$ or $\mathbf{V_2}$ = 25/(0.42064∠–56.311°) = 59.433∠56.311° V
= (32.967 + j49.452) V = $\mathbf{V_{Thev}}$.

To obtain R_{eq}, consider the circuit shown below. We replace $\mathbf{Z_L}$ by a 1-A current source.



At node 1,

$$\frac{\mathbf{V}_1}{2}+\frac{\mathbf{V}_1}{-\mathbf{j}5}+0.5\mathbf{V}_1+\frac{\mathbf{V}_1-\mathbf{V}_2}{4}=0 \quad \longrightarrow \quad 0=\mathbf{V}_1(1.25+\mathbf{j}0.2)-0.25\mathbf{V}_2 \text{ or}$$

$\mathbf{V_1}$ = [0.25/(1.2659∠9.09°)]$\mathbf{V_2}$ = (0.197488∠–9.09°)$\mathbf{V_2}$
(3)

At node 2,

$$1+\frac{\mathbf{V}_1-\mathbf{V}_2}{4}+0.5\mathbf{V}_1=\frac{\mathbf{V}_2}{\mathbf{j}4} \quad \longrightarrow \quad -1=0.75\mathbf{V}_1+\mathbf{V}_2(-0.25+\mathbf{j}0.25)$$

(4)

Substituting (3) into (4) gives,

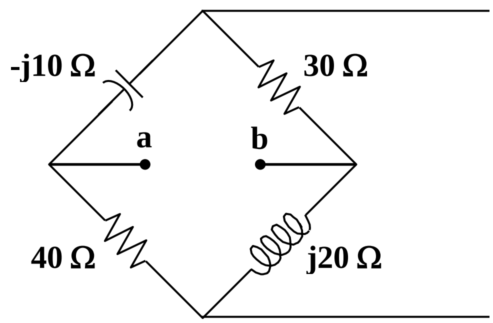
0.75(0.197488∠–9.09°)$\mathbf{V_2}$ + (–0.25+j0.25)$\mathbf{V_2}$ = –1
= (0.146256–j0.0234 – 0.25 + j0.25)$\mathbf{V_2}$ = (–0.103744+j0.2266)$\mathbf{V_2}$
= (0.24922∠114.6°)$\mathbf{V_2}$ or $\mathbf{V_2}$ = 4.0125∠65.4° V = (1.67033+j3.6483) V.

$$\mathbf{Z_{eq}} = \frac{\mathbf{V}_2}{1} = 4.0125\angle 65.4° \ \Omega \text{ and } \mathbf{Z_L} = 4.0125\angle{-65.4°} \ \Omega$$

$$\mathbf{P}_{\max} = \frac{|\mathbf{V_{Th}}|^2}{4[\mathbf{Z_{eq}} + \mathbf{Z_L}]} = \frac{59.433^2}{8\mathbf{x}1.67033} = \mathbf{264.34\ W}.$$

Solution 11.17

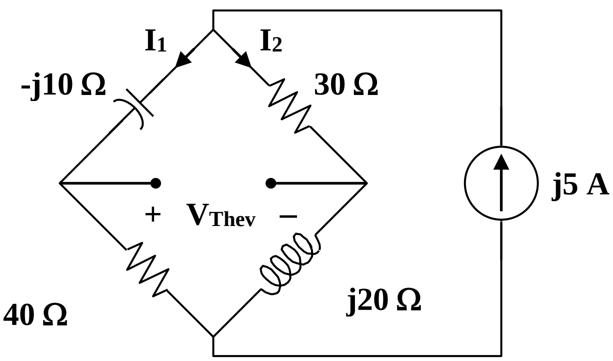
We find Z_{eq} at terminals a-b following Fig. (a).



(a)

$$\mathbf{Z}_{eq} = (-j10+30)\,||\,(j20+40) = \frac{(30-j10)(40+j20)}{70+j10} = \mathbf{20\ \Omega} = \mathbf{Z_L}$$

We obtain $\mathbf{V}_{Thev}$ from Fig. (b).



(b)

Using current division,

$$\mathbf{I}_1 = \frac{30+\mathrm{j}20}{70+\mathrm{j}10}(\mathrm{j}5) = \text{-}1.1+\mathrm{j}2.3$$

$$\mathbf{I}_2 = \frac{40-\mathrm{j}10}{70+\mathrm{j}10}(\mathrm{j}5) = 1.1+\mathrm{j}2.7$$

$$\mathbf{V}_{\mathrm{Th}} = 30\mathbf{I}_2 + \mathrm{j}10\mathbf{I}_1 = 10+\mathrm{j}70$$

$$P_{\max} = \frac{\left|\mathbf{V}_{Th}\right|^2}{2\left(Z_{eq}+Z_L\right)^2} Z_L = \frac{5000}{(2)(2x20)^2} 20 = \mathbf{31.25\ W}$$

Solution 11.18

Find the value of $\mathbf{Z}_L$ in the circuit of Fig. 11.49 for maximum power transfer.

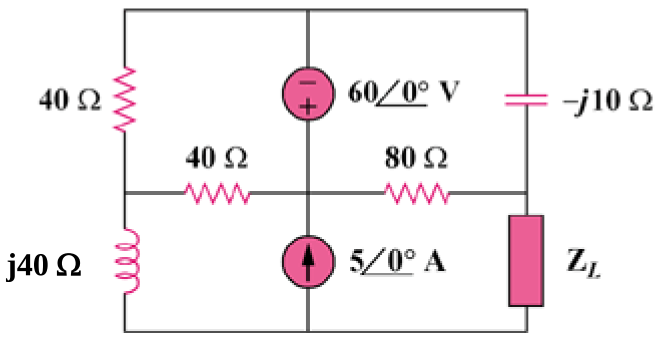
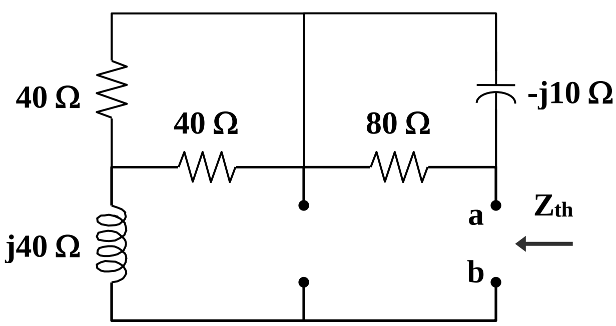


Figure 11.49
For Prob. 11.18.

Solution

We find $\mathbf{Z}_{\mathrm{Th}}$ at terminals a-b as shown in the figure below.



$$\mathbf{Z}_{Th} = j40 + 40 \,||\, 40 + 80 \,||\, (\text{-j10}) = \text{j}40 + 20 + \frac{(80)(\text{-j}10)}{80 - \text{j}10}$$

$$\mathbf{Z}_{Th} = 21.23 + j30.154$$

$$\mathbf{Z}_{\mathrm{L}} = \mathbf{Z}_{\mathrm{Th}}^{*} = \mathbf{[21.23–j30.15]\ \Omega}$$

Solution 11.19

The variable resistor R in the circuit of Fig. 11.50 is adjusted until it absorbs the maximum average power. Find R and the maximum average power absorbed.

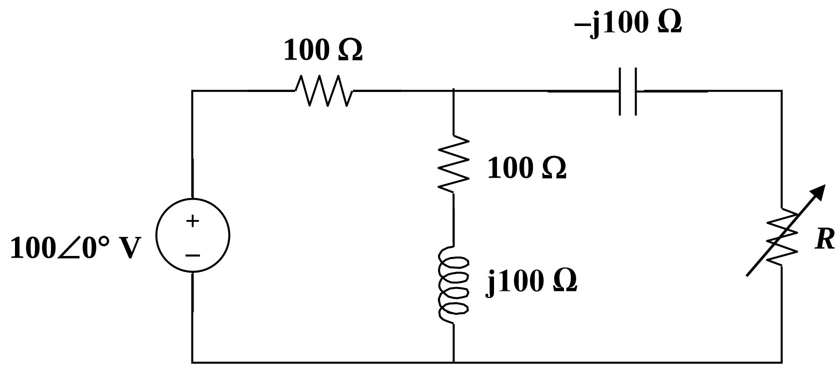
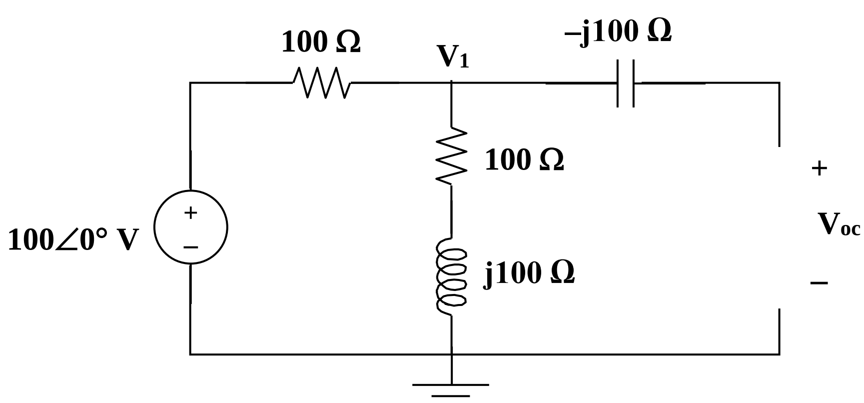


Figure 11.50
For Prob. 11.19.

Solution

Step 1. We first remove R from the circuit and then find the Thevenin equivalent circuit. Once we have $\mathbf{V_{Thev}}$ and $\mathbf{Z_{eq}}$ we then know that for maximum power transfer to the load, R must be equal to $|\mathbf{Z_{eq}}|$ and $P_{avg} = |\mathbf{V_{Thev}}|^2/(8R)$. We now find $\mathbf{V_{Thev}}$ by writing and solving a nodal equation for the circuit shown below. To find $\mathbf{Z_{eq}}$, we just set the source to zero (a short) and determine the impedance looking in from the right. $\mathbf{Z_{eq}} = -j100 + 100(100+j100)/(100+100+j100)$.



$[(V_1-100)/100] + [(V_1-0)/(100+j100)] + 0 = 0$ and $\mathbf{V_{oc}} = \mathbf{V_{Thev}} = \mathbf{V_1}$.

Step 2. $\mathbf{Z_{eq}} = -j100+100(1.4142\angle 45°)/(2.2361\angle 26.57°) = -j100+63.244\angle 18.43°$ $= -j100 + 60 + j20 = (60 - j80)\ \Omega = 100\angle -53.13°\ \Omega$.

The node equation becomes, $(0.01+0.005-j0.005)\mathbf{V_1} = 1 = 0.0158114\angle -18.43°\mathbf{V_1}$ or $\mathbf{V_1} = 63.246\angle 18.43°$. Thus,

$$R = \mathbf{100\ \Omega}$$

and $|\mathbf{I}| = 63.246/|60–j80+100| = 63.246/178.885 = 0.353557$ A and

$$\text{the maximum } P_{avg} = [(0.353557)^2/2]100 = \mathbf{6.25\ W}.$$

Solution 11.20

The load resistance R_L in Fig. 11.51 is adjusted until it absorbs the maximum average power. Calculate the value of R_L and the maximum average power.

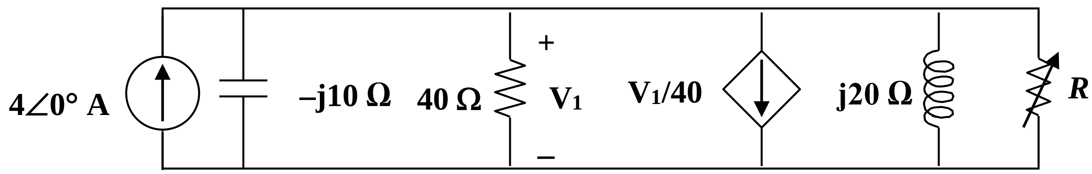
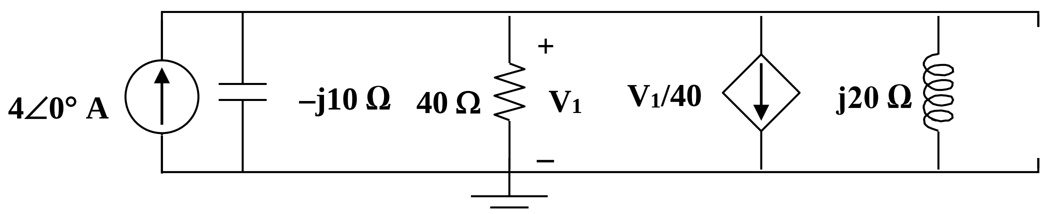


Figure 11.51
For Prob. 11.20.

Solution

Step 1. The easiest way to solve this problem is to find the Thevenin equivalent circuit and then we can now solve for R = $|\mathbf{Z_{eq}}|$ and $P_{avg} = |\mathbf{I}|^2R/2$ where $\mathbf{I} = \mathbf{V_{Thev}}/(\mathbf{Z_{eq}}+R)$. To find the Thevenin equivalent circuit we take R out of the circuit and then find $\mathbf{V_{oc}}$ and $\mathbf{I_{sc}}$ which gives $\mathbf{V_{Thev}} = \mathbf{V_{oc}}$ and $\mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}}$.



We note that $\mathbf{V_1} = \mathbf{V_{oc}} = \mathbf{V_{Thev}}$ and the open circuit nodal equation becomes,
$-4 + [(\mathbf{V_1}-0)/(-j10)] + [(\mathbf{V_1}-0)/(40)] + [\mathbf{V_1}/40] + [(\mathbf{V_1}-0)/(j20)] = 0$.

For I_{sc} we note that $\mathbf{V_1} = 0$ because of the short which means that $\mathbf{I_{sc}} = 4$ A.

Step 2. $(j0.1 + 0.025 - j0.05 + 0.025)\mathbf{V_1} = 4$ or $\mathbf{V_1} = 4/(0.05+j0.05)$ or $\mathbf{V_1} = 4/(0.07071\angle 45°) = 56.569\angle -45°$ V. Therefore, $\mathbf{Z_{eq}} = 56.569\angle -45°/4 = 14.142\angle -45°$ Ω.

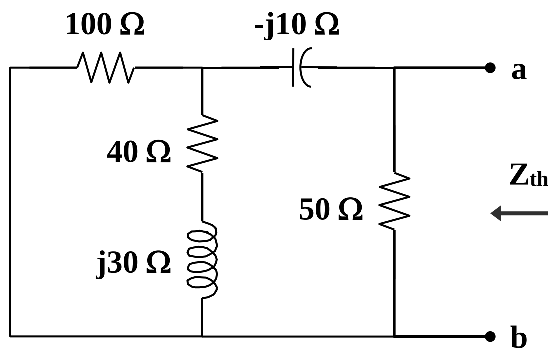
$$R = \mathbf{14.142\ \Omega}$$

and $\mathbf{I} = (56.569\angle -45°)/(7.071-j7.071+14.142)$
$= (56.569\angle -45°)/(21.213-j7.071) = (56.569\angle -45°)/(22.36\angle -18.435°)$
$= 2.5299\angle -26.565°$ A. Therefore,

$$P_{avg} = (2.5299)^2R/2 = \mathbf{45.26\ W}.$$

Solution 11.21

We find $\mathbf{Z}_{Th}$ at terminals a-b, as shown in the figure below.



$$\mathbf{Z}_{Th} = 50 \parallel [\,-j10 + 100 \parallel (40 + j30)\,]$$

where $100 \parallel (40 + j30) = \dfrac{(100)(40 + j30)}{140 + j30} = 31.707 + j14.634$

$$\mathbf{Z}_{Th} = 50 \parallel (31.707 + j4.634) = \frac{(50)(31.707 + j4.634)}{81.707 + j4.634}$$

$$\mathbf{Z}_{Th} = 19.5 + j1.73$$

$$R_L = \left| \mathbf{Z}_{Th} \right| = \mathbf{19.58\ \Omega}$$

Solution 11.22

i(t) = [2–2cos(2t)] amps

$$I_{rms}^2 = \frac{1}{\pi}\left[\int_0^{\pi}\left[2-2\cos\left(2t\right)\right]^2 dt\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} 4dt + \int_0^{\pi}\left[-4\cos(2t)\right]dt + \int_0^{\pi} 4\cos^2(2t)dt\right]$$

$$= \frac{1}{\pi}\left[4\pi + 0 + 4\int_0^{\pi}\left[\frac{1+\cos(4t)}{2}\right]dt\right] = \frac{1}{\pi}\left[4\pi + 4\left(\frac{\pi}{2}\right)\right] = 6$$

$$\mathbf{I_{rms}} = \sqrt{6} = 2.449\,\mathbf{amps}$$

Solution 11.23

Using Fig. 11.54, design a problem to help other students to better understand how to find the rms value of a waveshape.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the rms value of the voltage shown in Fig. 11.54.

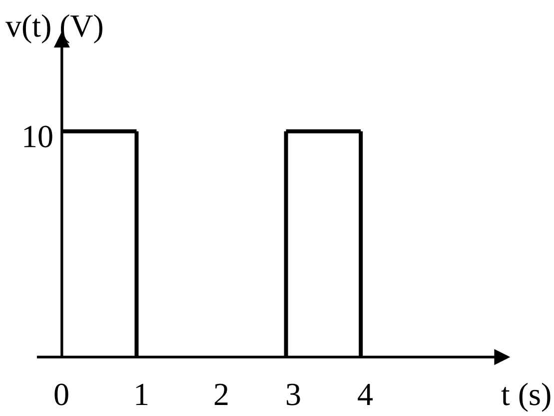


Figure 11.54 For Prob. 11.23.

Solution

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)dt = \frac{1}{3}\int_0^1 10^2 dt = \frac{100}{3}$$

V_{rms} = **5.7735 V**

Solution 11.24

$$T = 2, \qquad v(t) = \begin{cases} 5, & 0 < t < 1 \\ -5, & 1 < t < 2 \end{cases}$$

$$V_{rms}^2 = \frac{1}{2}\left[\int_0^1 5^2 \ dt + \int_1^2 (-5)^2 \ dt\right] = \frac{25}{2}[1+1] = 25$$

$$V_{rms} = \mathbf{5\ V}$$

Solution 11.25

Find the rms value of the signal shown in Fig. 11.56.

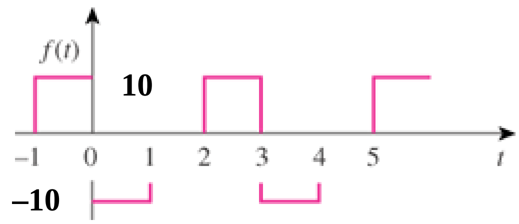


Figure 11.56
For Prob. 11.25.

Solution

$$f_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{3}\left[\int_0^1 (-10)^2 dt + \int_1^2 0dt + \int_2^3 10^2 dt\right]$$

$$= \frac{1}{3}[100 + 0 + 100] = \frac{200}{3}$$

$$f_{rms} = \sqrt{\frac{200}{3}} = 8.165$$

f_{rms} = **8.165**

Solution 11.26

$\mathrm{T} = 4\,,\qquad v(t) = \begin{cases} 5 & 0 < t < 2 \\ 20 & 2 < \mathrm{t} < 4 \end{cases}$

$$V_{rms}^2 = \frac{1}{4}\left[\int_0^2 10^2\ dt + \int_2^4 (20)^2\ dt \right] = \frac{1}{4}[200 + 800] = 250$$

$$\mathrm{V_{rms}} = \mathbf{15.811\ V}.$$

Solution 11.27

$T = 5, \qquad i(t) = t, \quad 0 < t < 5$

$$I_{rms}^2 = \frac{1}{5}\int_0^5 t^2\ dt = \frac{1}{5}\cdot\frac{t^3}{3}\Big|_0^5 = \frac{125}{15} = 8.333$$

$I_{rms} =$ **2.887 A**

Solution 11.28

$$V_{rms}^2 = \frac{1}{5}\left[\int_0^2 (4t)^2\ dt + \int_2^5 0^2\ dt\right]$$

$$V_{rms}^2 = \frac{1}{5}\cdot\frac{16t^3}{3}\Big|_0^2 = \frac{16}{15}(8) = 8.533$$

$$V_{rms} = \mathbf{2.92\ V}$$

$$P = \frac{V_{rms}^2}{R} = \frac{8.533}{2} = \mathbf{4.267\ W}$$

Solution 11.29

$$T = 20, \qquad i(t) = \begin{cases} 60 - 6t & 5 < t < 15 \\ -120 + 6t & 15 < t < 25 \end{cases}$$

$$I_{eff}^2 = \frac{1}{20}\left[\int_5^{15} (60 - 6t)^2 \ dt + \int_{15}^{25} (-120 + 6t)^2 \ dt \ \right]$$

$$I_{eff}^2 = \frac{1}{5}\left[\int_5^{15} (900 - 180t + 9t^2) \ dt + \int_{15}^{25} (9t^2 - 360t + 3600) \ dt \ \right]$$

$$I_{eff}^2 = \frac{1}{5}\left[\left(900t - 90t^2 + 3t^3\right)\Big|_5^{15} + \left(3t^3 - 180t^2 + 3600t\right)\Big|_{15}^{25} \right]$$

$$I_{eff}^2 = \frac{1}{5}[750 + 750] = 300$$

$$I_{eff} = \mathbf{17.321\ A}$$

$$P = I_{eff}^2 R = (17.321)^2 x12 = \mathbf{3.6\ kW}.$$

Solution 11.30

$$v(t) = \begin{cases} t & 0 < t < 2 \\ -1 & 2 < t < 4 \end{cases}$$

$$V_{rms}^2 = \frac{1}{4}\left[\int_0^2 t^2 \ dt + \int_2^4 (-1)^2 \ dt\right] = \frac{1}{4}\left[\frac{8}{3} + 2\right] = 1.1667$$

V_{rms} = **1.08 V**

Solution 11.31

$$V^2{}_{rms} = \frac{1}{2}\int_0^2 v(t)dt = \frac{1}{2}\left[\int_0^1 (2t)^2\, dt + \int_1^2 (-4)^2\, dt\right] = \frac{1}{2}\left[\frac{4}{3} + 16\right] = 8.6667$$

$$\boldsymbol{V_{rms}} = \underline{\mathbf{2.944\ V}}$$

Solution 11.32

$$I_{rms}^2 = \frac{1}{2}\left[\int_0^1 (10t^2)^2\ dt + \int_1^2 0\ dt\right]$$

$$I_{rms}^2 = 50\int_0^1 t^4\ dt = 50 \cdot \frac{t^5}{5}\Big|_0^1 = 10$$

$$I_{rms} = \mathbf{3.162\ A}$$

Solution 11.33

Determine the rms value for the waveform in Fig. 11.64.

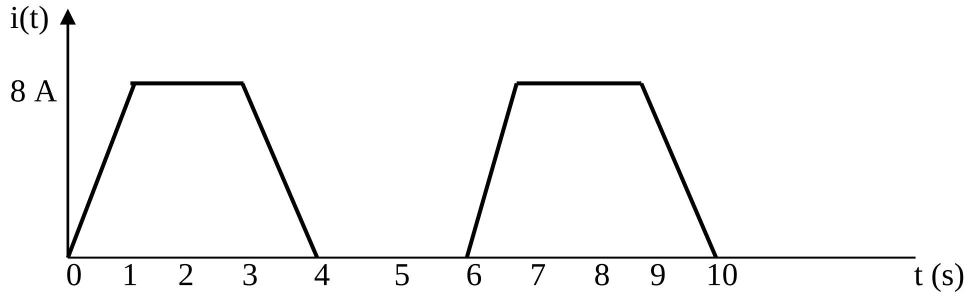


Figure 11.64
For Prob. 11.33.

Solution

$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2(t)\,dt = \frac{1}{6}\left[\int_0^1 64t^2 dt + \int_1^3 64 dt + \int_3^4 (-8t+32)^2 dt\right]$$

$$I_{rms}^2 = \frac{1}{6}\left[64\frac{t^3}{3}\bigg|_0^1 + 64(3-1) + \left(64\frac{t^3}{3} - 256t^2 + 1024t\right)\bigg|_3^4\right]$$
$$= 28.43$$

I_{rms} = **5.332 A**

Solution 11.34

$$f_{rms}^{2} = \frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{3}\left[\int_0^2 (3t)^2\, dt + \int_2^3 6^2\, dt\right]$$

$$= \frac{1}{3}\left[\left.\frac{9t^3}{3}\right|_0^2 + 36\right] = 20$$

$$f_{rms} = \sqrt{20} = 4.472$$

f_{rms} = **4.472**

Solution 11.35

$$V_{rms}^2 = \frac{1}{6}\left[\int_0^1 10^2\ dt + \int_1^2 20^2\ dt + \int_2^4 30^2\ dt + \int_4^5 20^2\ dt + \int_5^6 10^2\ dt\right]$$

$$V_{rms}^2 = \frac{1}{6}[100 + 400 + 1800 + 400 + 100] = 466.67$$

$$V_{rms} = \mathbf{21.6\ V}$$

Solution 11.36

(a) I_{rms} = $\underline{10\ \text{A}}$

(b) $\boldsymbol{V^2_{rms} = 4^2 + \left(\frac{3}{\sqrt{2}}\right)^2} \longrightarrow \boldsymbol{V_{rms} = \sqrt{16 + \frac{9}{2}} = \underline{4.528\ \text{V}}}$ (checked)

(c) $\boldsymbol{I_{rms} = \sqrt{64 + \frac{36}{2}} = \underline{9.055\ \text{A}}}$

(d) $\boldsymbol{V_{rms} = \sqrt{\frac{25}{2} + \frac{16}{2}} = \underline{4.528\ \text{V}}}$

Solution 11.37

Design a problem to help other students to better understand how to determine the rms value of the sum of multiple currents.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Calculate the rms value of the sum of these three currents:

$i_1 = 8$, $i_2 = 4\sin(t + 10°)$, $i_3 = 6\cos(2t + 30°)$ A

Solution

$$i = i_1 + i_2 + i_3 = 8 + 4\sin(t + 10^o) + 6\cos(2t + 30^o)$$

$$\boldsymbol{I_{rms} = \sqrt{I^2{}_{1rms} + I^2{}_{2rms} + I^2{}_{3rms}} = \sqrt{64 + \frac{16}{2} + \frac{36}{2}} = \sqrt{90} = \underline{9.487\ \text{A}}}$$

Solution 11.38

For the power system in Fig. 11.67, find: (a) the average power, (b) the reactive power, (c) the power factor. Note that 440 V is an rms value.

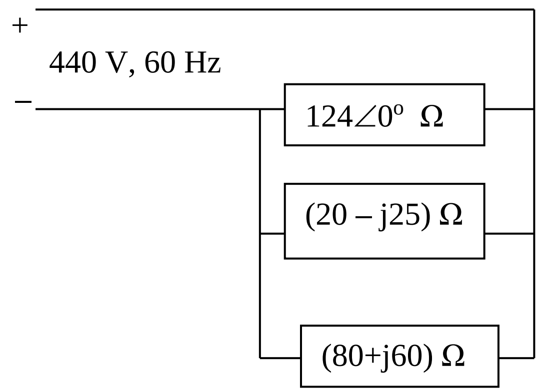


Figure 11.67
For Prob. 11.38.

Solution

$\mathbf{S_1} = V^2/(\mathbf{Z_1})^* = 1.56129$ kW.

$\mathbf{S_2} = V^2/(\mathbf{Z_2})^* = 193{,}600/(32.0156\angle 51.34° = 6{,}047.05\angle{-51.34°}$ VA
$= 3.7776$ kW – j4.7219 kVAR

$\mathbf{S_3} = V^2/(\mathbf{Z_3})^* = 193{,}600/(100\angle{-36.87°} = 1{,}936\angle 36.87°$ VA
$= 1.5488$ kW + j1.1616 lVAR.

$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} + \mathbf{S_3} = (1.56129+3.7776+1.5488)$ kW + j(0–4.7219+1.1616) kVAR
= 6.888 kW – j3.56 kVAR.
Therefore,

(a) P = Re(**S**) = **6.888 kW**

(b) Q = Im (**S**) = **–3.56 kVAR (leading)**

(c) pf = $\cos[\tan^{-1}([-3.56/6.888])] \cos\{\tan^{-1}[-0.51684]\}$
$= \cos(-27.332°) = \mathbf{0.89}$

Solution 11.39

An ac motor with impedance $\mathbf{Z_L} = (2 + j1.2)\ \Omega$ is supplied by a 220-V, 60-Hz source. (a) Find pf, P, and Q. (b) Determine the capacitor required to be connected in parallel with the motor so that the power factor is corrected to unity.

Solution

(a) $\mathbf{Z_L} = 2 + j1.2 = 2.3324\angle 30.964^\circ$

pf = cos(30.964) = **0.8575**

$\mathbf{S} = V^2/(\mathbf{Z_L})^* = 48{,}400/(2.3324\angle -30.964^\circ) = 20{,}751\angle 30.964^\circ$
$= 17.794$ kW + j 10.676 kVAR.

P = **17.794 kW**

Q = **10.676 kVAR (lagging)**

(b) $X_C = V^2/Q_C = 48{,}400/10{,}676 = 4.5335 = 1/(377C)$ or
C = **585.1 μF**.

{It is important to note that this capacitor will see a peak voltage of 220$\sqrt{2}$ = 311.08V, this means that the specifications on the capacitor must be at least this or greater!}

Solution 11.40

Design a problem to help other students to better understand apparent power and power factor.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A load consisting of induction motors is drawing 80 kW from a 220-V, 60 Hz power line at a pf of 0.72 lagging. Find the capacitance of a capacitor required to raise the pf to 0.92.

Solution

$$pf1 = 0.72 = \cos\theta_1 \qquad \longrightarrow \qquad \theta_1 = 43.94^0$$
$$pf2 = 0.92 = \cos\theta_2 \qquad \longrightarrow \qquad \theta_2 = 23.07^0$$

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \frac{80x10^3(0.9637 - 0.4259)}{2\pi x 60 x (220)^2} = \underline{2.4 \text{ mF}},$$

{Again, we need to note that this capacitor will be exposed to a peak voltage of 311.08V and must be rated to at least this level, preferably higher!}

Solution 11.41

(a) $-j2 \parallel (j5 - j2) = -j2 \parallel -j3 = \dfrac{(-j2)(-j3)}{j} = -j6$

$\mathbf{Z}_T = 4 - j6 = 7.211\angle -56.31°$

$\text{pf} = \cos(-56.31°) =$ **0.5547** **(leading)**

(b) $j2 \parallel (4 + j) = \dfrac{(j2)(4+j)}{4 + j3} = 0.64 + j1.52$

$\mathbf{Z} = 1 \parallel (0.64 + j1.52 - j) = \dfrac{0.64 + j0.44}{1.64 + j0.44} = 0.4793\angle 21.5°$

$\text{pf} = \cos(21.5°) =$ **0.9304** **(lagging)**

Solution 11.42

(a) S=120, $pf = 0.707 = \cos\theta \longrightarrow \theta = 45^o$

$$S = S\cos\theta + jS\sin\theta = \underline{84.84 + j84.84 \text{ VA}}$$

(b) $S = V_{rms} I_{rms} \longrightarrow I_{rms} = \dfrac{S}{V_{rms}} = \dfrac{120}{110} = \underline{1.091 \text{ A rms}}$

(c) $S = I_{rms}^2 Z \longrightarrow Z = \dfrac{S}{I_{rms}^2} = \underline{71.278 + j71.278\ \Omega}$

(d) If Z = R + jϖL, then R = **71.278 Ω**

$$\omega L = 2\pi fL = 71.278 \longrightarrow L = \frac{71.278}{2\pi x 60} = \underline{0.1891\ \text{H}} = \mathbf{189.1\ mH}.$$

Solution 11.43

Design a problem to help other students to better understand complex power.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The voltage applied to a 10-ohm resistor is

$v(t) = 5 + 3\cos(t + 10°) + \cos(2t + 30°)$ V

(a) Calculate the rms value of the voltage.

(b) Determine the average power dissipated in the resistor.

Solution

(a) $$V_{rms} = \sqrt{V^2_{1rms} + V^2_{2rms} + V^2_{3rms}} = \sqrt{25 + \frac{9}{2} + \frac{1}{2}} = \sqrt{30} = \underline{5.477\text{ V}}$$

(b) $$P = \frac{V^2_{rms}}{R} = 30/10 = \underline{3\text{ W}}$$

Solution 11.44

$$40\mu F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j2000x40x10^{-6}} = -j12.5$$

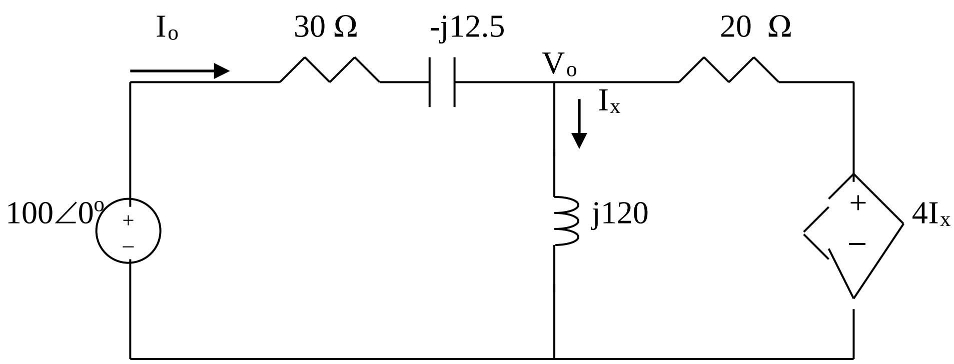
$$60mH \quad \longrightarrow \quad j\omega L = j2000x60x10^{-3} = j120$$

We apply nodal analysis to the circuit shown below.

$$\frac{100-V_o}{30-j12.5} + \frac{4I_x - V_o}{20} = \frac{V_o}{j120}$$

But $\quad I_x = \dfrac{V_o}{j120}$. Solving for V_o leads to

$$V_o = 2.9563 + j1.126$$



$$I_o = \frac{100-V_o}{30-j12.5} = 2.7696 + j1.1165$$

$$S = \frac{1}{2}V_s I_o^* = \frac{1}{2}(100)(2.7696 - j.1165) = \underline{138.48 - j55.825 \ \text{VA}}$$

S = (138.48 – j55.82) VA

Solution 11.45

(a) $\boldsymbol{V^2_{rms} = 20^2 + \dfrac{60^2}{2} = 2200} \quad \longrightarrow \quad \boldsymbol{V_{rms} = \underline{46.9\ \mathrm{V}}}$

$$\boldsymbol{I_{rms} = \sqrt{1^2 + \frac{0.5^2}{2}} = \sqrt{1.125} = \underline{1.061A}}$$

(b) p(t) = v(t)i(t) = 20 + 60cos100t – 10sin100t – 30(sin100t)(cos100t); clearly the average power = **20W**.

Solution 11.46

(a) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (220\angle 30°)(0.5\angle -60°) = 110\angle -30°$
$\mathbf{S} = \mathbf{[95.26 - \mathit{j}55]}\,\mathbf{\mathit{VA}}$

Apparent power = **110 VA**
Real power = **95.26 W**
Reactive power = **55 VAR**
pf is **leading** because current leads voltage

(b) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (250\angle -10°)(6.2\angle 25°) = 1550\angle 15°$
$\mathbf{S} = \mathbf{[497.2 + \mathit{j}401.2]}\,\mathbf{\mathit{VA}}$

Apparent power = **1550 VA**
Real power = **1497.2 W**
Reactive power = **401.2 VAR**
pf is **lagging** because current lags voltage

(c) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (120\angle 0°)(2.4\angle 15°) = 288\angle 15°$
$\mathbf{S} = \mathbf{[278.2 + \mathit{j}74.54]}\,\mathbf{\mathit{VA}}$

Apparent power = **288 VA**
Real power = **278.2 W**
Reactive power = **74.54 VAR**
pf is **lagging** because current lags voltage

(d) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (160\angle 45°)(8.5\angle -90°) = 1360\angle -45°$
$\mathbf{S} = \mathbf{[961.7 - j961.7]\ VA}$

Apparent power = **1360 VA**
Real power = **961.7 W**
Reactive power = **- 961.7 VAR**
pf is **leading** because current leads voltage

Solution 11.47

For each of the following cases, find the complex power, the average power, and the reactive power:

(a) $v(t) = 169.7 \sin(377t + 45°)$ V, $i(t) = 5.657 \sin(377t)$ A

(b) $v(t) = 339.4 \sin(377t + 90°)$ V, $i(t) = 5.657 \sin(377t + 45°)$ A

(c) $\mathbf{V} = 900\angle 90°$ V rms, $\mathbf{Z} = 75\angle 45°\ \Omega$

(d) $\mathbf{I} = 100\angle 60°$ A rms, $\mathbf{Z} = 50\angle 60°\ \Omega$

Solution

Step 1. In the first two cases we need to convert the time varying voltages and currents into complex rms (rms magnitudes are equal to peak values divided by 1.4142) values. $\mathbf{S} = \mathbf{VI}^* = \mathbf{VV}^*/\mathbf{Z}^* = \mathbf{IZI}^* = P + jQ$, where **S** is the complex power, P is the average power, and Q is the reactive power.

Step 2.

(a) $\mathbf{V} = (169.7/1.4142)\angle 45° = 120\angle 45°$ V and
$\mathbf{I} = (5.657/1.4142)\angle 0° = 4\angle 0°$ A.
Now $\mathbf{S} = \mathbf{480\angle 45°\ VA} = \mathbf{339.4\ W} + j\mathbf{339.4\ VAR}$

(b) $\mathbf{V} = 240\angle 90°$ V and $\mathbf{I} = 4\angle 45°$ A which leads to
$\mathbf{S} = (240\angle 90°)(4\angle -45°) = \mathbf{960\angle 45°\ VA} = \mathbf{678.8\ W + j678.8\ VAR}$.

(c) $\mathbf{S} = 900^2/(75\angle -45°) = \mathbf{10.8\angle 45°\ kVA} = \mathbf{7.637\ kW} + j\mathbf{7.637\ kVAR}$.

(d) $\mathbf{S} = 100^2(50\angle 60°) = \mathbf{500\angle 60°\ kVA} = \mathbf{250\ kW} + j\mathbf{433\ kVAR}$.

Solution 11.48

(a) $\mathbf{S} = \text{P} - \text{jQ} = \mathbf{[269 - \mathit{j}150]\,\mathit{VA}}$

(b) $\text{pf} = \cos\theta = 0.9 \quad \longrightarrow \quad \theta = 25.84°$

$$\text{Q} = \text{S}\sin\theta \quad \longrightarrow \quad \text{S} = \frac{\text{Q}}{\sin\theta} = \frac{2000}{\sin(25.84°)} = 4588.31$$

$$\text{P} = \text{S}\cos\theta = 4129.48$$

$\mathbf{S} = \mathbf{[4.129 - \mathit{j}2]\,\mathit{kVA}}$

(c) $\text{Q} = \text{S}\sin\theta \quad \longrightarrow \quad \sin\theta = \dfrac{\text{Q}}{\text{S}} = \dfrac{450}{600} = 0.75$

$\theta = 48.59$, $\quad\quad\quad\quad$ $\text{pf} = 0.6614$

$$\text{P} = \text{S}\cos\theta = (600)(0.6614) = 396.86$$

$\mathbf{S} = \mathbf{[396.9 + \mathit{j}450]\,\mathit{VA}}$

(d) $\text{S} = \dfrac{|\mathbf{V}|^2}{|\mathbf{Z}|} = \dfrac{(220)^2}{40} = 1210$

$\text{P} = \text{S}\cos\theta \quad \longrightarrow \quad \cos\theta = \dfrac{\text{P}}{\text{S}} = \dfrac{1000}{1210} = 0.8264$

$\theta = 34.26°$

$$\text{Q} = \text{S}\sin\theta = 681.25$$

$\mathbf{S} = \mathbf{[1 + \mathit{j}0.6812]\,\mathit{kVA}}$

Solution 11.49

(a) $\mathbf{S} = 4 + j\dfrac{4}{0.86}\sin(\cos^{-1}(0.86))$ kVA

$\mathbf{S} = \mathbf{[4+}\ \boldsymbol{j}\mathbf{2.373]}\ \boldsymbol{kVA}$

(b) $\text{pf} = \dfrac{P}{S} = \dfrac{1.6}{2}0.8 = \cos\theta \quad \longrightarrow \quad \sin\theta = 0.6$

$\mathbf{S} = 1.6 - j2\sin\theta = \mathbf{[1.6-}\ \boldsymbol{j}\mathbf{1.2]}\ \boldsymbol{kVA}$

(c) $\mathbf{S} = \mathbf{V}_{\text{rms}}\,\mathbf{I}^{*}_{\text{rms}} = (208\angle 20°)(6.5\angle 50°)$ VA

$\mathbf{S} = 1.352\angle 70° = \mathbf{[0.4624+}\ \boldsymbol{j}\mathbf{1.2705]}\ \boldsymbol{kVA}$

(d) $\mathbf{S} = \dfrac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \dfrac{(120)^2}{40 - j60} = \dfrac{14400}{72.11\angle -56.31°}$

$\mathbf{S} = 199.7\angle 56.31° = \mathbf{[110.77+}\ \boldsymbol{j}\mathbf{166.16]}\ \boldsymbol{VA}$

Solution 11.50

(a) $\quad \mathbf{S} = \mathrm{P} - \mathrm{jQ} = 1000 - \mathrm{j}\dfrac{1000}{0.8}\sin(\cos^{-1}(0.8))$

$\mathbf{S} = 1000 - \mathrm{j}750$

But, $\quad \mathbf{S} = \dfrac{|\mathbf{V}_{\mathrm{rms}}|^2}{\mathbf{Z}^*}$

$$\mathbf{Z}^* = \frac{|\mathbf{V}_{\mathrm{rms}}|^2}{\mathbf{S}} = \frac{(220)^2}{1000 - \mathrm{j}750} = 30.98 + \mathrm{j}23.23$$

$\mathbf{Z} = \mathbf{[30.98 - \mathit{j}23.23]\,\Omega}$

(b) $\quad \mathbf{S} = |\mathbf{I}_{\mathrm{rms}}|^2\mathbf{Z}$

$$\mathbf{Z} = \frac{\mathbf{S}}{|\mathbf{I}_{\mathrm{rms}}|^2} = \frac{1500 + \mathrm{j}2000}{(12)^2} = \mathbf{[10.42 + \mathit{j}13.89]\,\Omega}$$

(c) $\quad \mathbf{Z}^* = \dfrac{|\mathbf{V}_{\mathrm{rms}}|^2}{\mathbf{S}} = \dfrac{|\mathbf{V}|^2}{2\mathbf{S}} = \dfrac{(120)^2}{(2)(4500\angle 60^\circ)} = 1.6\angle -60^\circ$

$\mathbf{Z} = 1.6\angle 60^\circ = \mathbf{[0.8 + \mathit{j}1.386]\,\Omega}$

Solution 11.51

For the entire circuit in Fig. 11.70, calculate:

(a) the power factor
(b) the average power delivered by the source
(c) the reactive power
(d) the apparent power
(e) the complex power

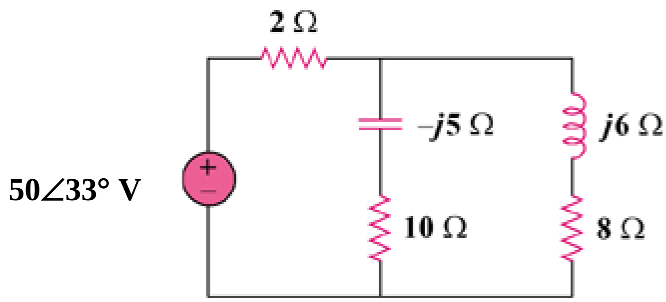


Figure 11.70
For Prob. 11.51.

Solution

(a) $\mathbf{Z}_T = 2 + (10 - j5) \,||\, (8 + j6)$

$$\mathbf{Z}_T = 2 + \frac{(10 - j5)(8 + j6)}{18 + j} = 2 + \frac{110 + j20}{18 + j}$$

$$\mathbf{Z}_T = 8.152 + j0.768 = 8.188\angle 5.382°$$

$\text{pf} = \cos(5.382°) = \mathbf{0.9956}$ **(lagging)**

(b) $\mathbf{S} = \mathbf{V}\mathbf{I}^* = \dfrac{|\mathbf{V}|^2}{(\mathbf{Z}_T)^*} = \dfrac{(50)^2}{(8.188\angle\mathbf{-5.382°})}$

$\mathbf{S} = 305.325\angle 5.382°$

$P = S\cos\theta = \mathbf{304\ W}$

(c) $Q = S\sin\theta = \mathbf{28.64\ VAR}$

(d) $S = |\mathbf{S}| = \mathbf{305.3\ VA}$

(e) $\mathbf{S} = 305.325\angle 5.382° =$ **(304+j28.64) VA**

Solution 11.52

$$S_A = 2000 + j\frac{2000}{0.8}0.6 = 2000 + j1500$$
$$S_B = 3000x0.4 - j3000x0.9165 = 1200 - j2749$$
$$S_C = 1000 + j500$$
$$S = S_A + S_B + S_C = 4200 - j749$$

(a) $pf = \dfrac{4200}{\sqrt{4200^2 + 749^2}} = \mathbf{0.9845\ leading}$

(b) $S = V_{rms}I^*_{rms} \longrightarrow I^*_{rms} = \dfrac{4200 - j749}{120\angle 45°} = 35.55\angle -55.11°$

I_{rms} = $\mathbf{35.55\angle 55.11°\ A}$.

Solution 11.53

$S = S_A + S_B + S_C = 4000(0.8–j0.6) + 2400(0.6+j0.8) + 1000 + j500$

$= 5640 + j20 = 5640\angle 0.2^\circ$

(a) $$\boldsymbol{I_{rms}^* = \frac{S_B}{V_{rms}} + \frac{S_A + S_C}{V_{rms}} = \frac{S}{V_{rms}} = \frac{5640\angle 0.2^\circ}{120\angle 30^\circ} = 47\angle -29.8^\circ}$$

$$\boldsymbol{I = 47\angle 29.8^\circ = \underline{47\angle 29.8^\circ\ A}}$$

(b) pf = cos(0.2˚) **≈ 1.0 lagging**.

Solution 11.54

For the network in Fig. 11.73, find the complex power absorbed by each element.

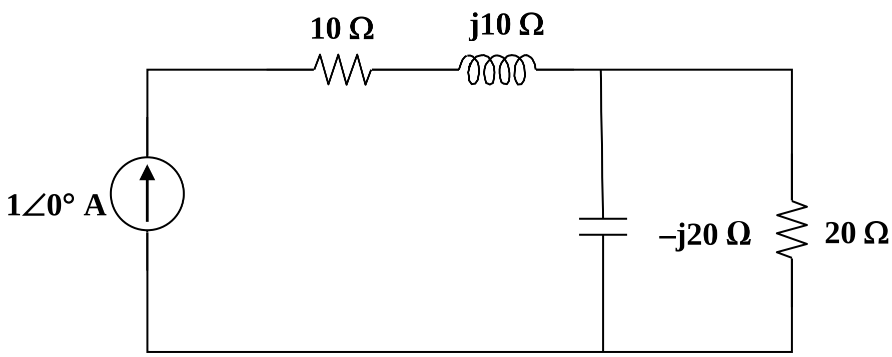


Figure 11.73
For Prob. 11.54.

Solution

Step 1. $P_{10} = (1)^2 10$, $Q_{j10} = (1)^2(10)$, and we need to determine the current division to find the current through the –j20 Ω, $\mathbf{I}_{-j20} = 1(20)/(20–j20)$, and the 20Ω, $\mathbf{I}_{20} = 1(–j20)/(20–j20)$. Finally $Q_{-j20} = |\mathbf{I}_{-j20}|^2(20)$ and $P_{20} = |\mathbf{I}_{20}|^2 20$. Lastly, the power absorbed by the current source can be expressed as
$\mathbf{S}_{\mathbf{absorbed}} = –(P_{10} + P_{20} + jQ_{j10} – jQ_{-j20})$.

Step 2. P_{10} = **10 W**, Q_{j10} = **10 VAR**, $\mathbf{I}_{-j20} = 20/(28.282\angle–45°) = 0.7071\angle45°$ A, and
$\mathbf{I}_{20} = (20\angle–90°)/(28.282\angle–45°) = 0.70711\angle–45°$ A. Thus,

P_{10} = **10 W**, P_{20} = **10 W,**
Q_{j10} = **10 VAR,** Q_{-j20} = **10 VAR** and
S = –(10+10+j10–j10) = **–20 W**.

Solution 11.55

Using Fig. 11.74, design a problem to help other students to better understand the conservation of AC power.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the complex power absorbed by each of the five elements in the circuit of Fig. 11.74.

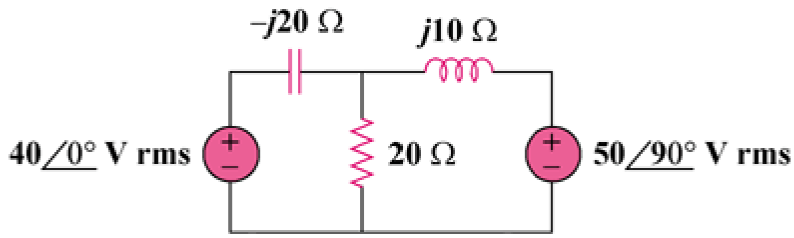
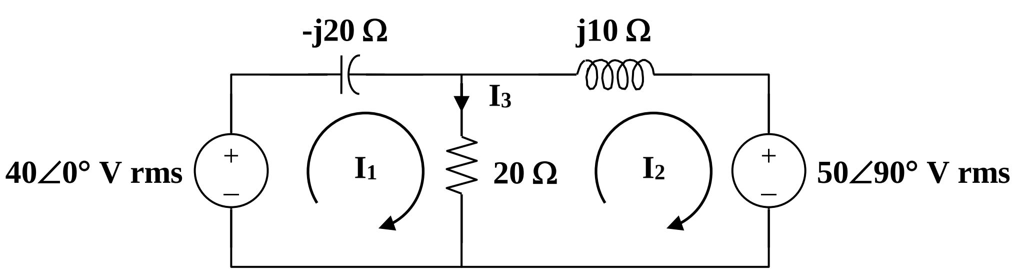


Figure 11.74

Solution

We apply mesh analysis to the following circuit.



For mesh 1,

$$40 = (20 - j20)\,I_1 - 20\,I_2$$

$$2 = (1 - j)\,I_1 - I_2 \qquad (1)$$

For mesh 2,

$$-j50 = (20 + j10)\,I_2 - 20\,I_1$$

$$-j5 = -2\,I_1 + (2 + j)\,I_2 \qquad (2)$$

Putting (1) and (2) in matrix form,

$$\begin{bmatrix} 2 \\ -j5 \end{bmatrix} = \begin{bmatrix} 1-j & -1 \\ -2 & 2+j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 1 - j, \qquad \Delta_1 = 4 - j3, \qquad \Delta_2 = -1 - j5$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{4 - j3}{1 - j} = \frac{1}{2}(7 + j) = 3.535\angle 8.13°$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-1 - j5}{1 - j} = 2 - j3 = 3.605\angle -56.31°$$

$$I_3 = I_1 - I_2 = (3.5 + j0.5) - (2 - j3) = 1.5 + j3.5 = 3.808\angle 66.8°$$

For the 40-V source,

$$\mathbf{S} = -\mathbf{V}\,\mathbf{I}_1^* = -(40)\left(\frac{1}{2}\cdot(7 - j)\right) = \mathbf{[-140 + j20]\ VA}$$

For the capacitor,

$$\mathbf{S} = \left|\mathbf{I}_1\right|^2 \mathbf{Z}_c = \mathbf{-\ j250\ VA}$$

For the resistor,

$$\mathbf{S} = \left|\mathbf{I}_3\right|^2 R = \mathbf{290\ VA}$$

For the inductor,

$$\mathbf{S} = \left|\mathbf{I}_2\right|^2 \mathbf{Z}_L = \mathbf{j130\ VA}$$

For the j50-V source,

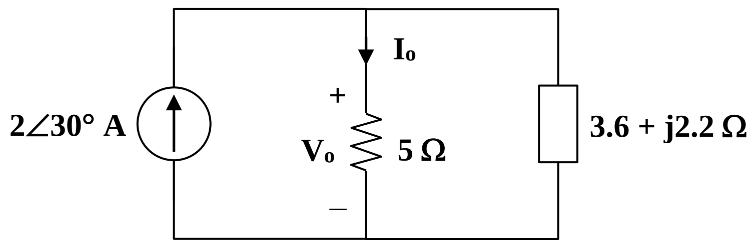
$$\mathbf{S} = \mathbf{V}\,\mathbf{I}_2^* = (j50)(2 + j3) = \mathbf{[-150 + j100]\ VA}$$

Solution 11.56

$$-j2 \parallel 6 = \frac{(6)(-j2)}{6-j2} = \frac{12\angle -90^\circ}{6.32456\angle -18.435^\circ} = 1.897365\angle -71.565^\circ = 0.6 - j1.8$$

$$3 + j4 + [(-j2) \parallel 6] = 3.6 + j2.2$$

The circuit is reduced to that shown below.



$$\mathbf{I}_o = \frac{3.6 + j2.2}{8.6 + j2.2}(2\angle 30^\circ) = \frac{4.219\angle 31.4296^\circ}{8.87694\angle 14.3493^\circ}(2\angle 30^\circ) = 0.95055\angle 47.08^\circ$$

$$\mathbf{V}_o = 5\mathbf{I}_o = 4.75275\angle 47.08^\circ$$

$$\mathbf{S} = \mathbf{V}_o \mathbf{I}_s^* = (4.75275\angle 47.08^\circ)(2\angle -30^\circ)$$

$$\mathbf{S} = 9.5055\angle 17.08^\circ = \mathbf{(9.086+j2.792)\ VA}$$

Solution 11.57

For the circuit in Fig. 11.76, find the average, reactive, and complex power delivered by the dependent voltage source.

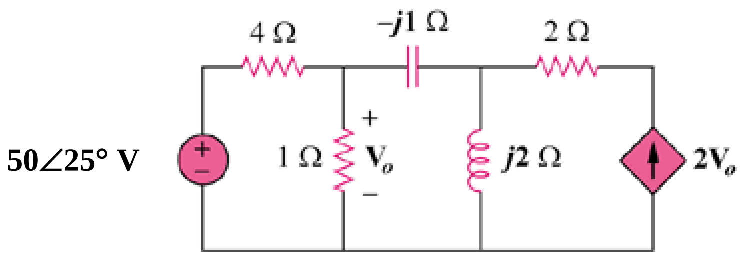
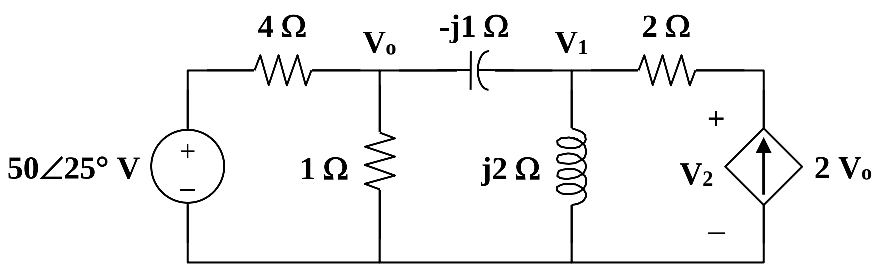


Figure 11.76
For Prob. 11.57.

Solution

Consider the circuit as shown below.



At node o,

$$\frac{50-\mathbf{V}_o}{4}=\frac{\mathbf{V}_o}{1}+\frac{\mathbf{V}_o-\mathbf{V}_1}{-j}$$ (Note, we are neglecting the angle on the source since we are only interested in power from the dependent source.)

$$50=(5+j4)\mathbf{V}_o-j4\mathbf{V}_1 \qquad (1)$$

At node 1,

$$\frac{\mathbf{V}_o-\mathbf{V}_1}{-j}+2\mathbf{V}_o=\frac{\mathbf{V}_1}{j2}$$

$$\mathbf{V}_1=(2-j4)\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1), $50=(5+j4-j8-16)\mathbf{V}_o$

$$\mathbf{V}_o=\frac{-50}{11+j4}, \qquad \mathbf{V}_1=\frac{(-50)(2-j4)}{11+j4}$$

The voltage across the dependent source is

$$\mathbf{V}_2 = \mathbf{V}_1 + (2)(2\,\mathbf{V}_o) = \mathbf{V}_1 + 4\,\mathbf{V}_o$$

$$\mathbf{V}_2 = \frac{-50}{11+\text{j}4}\cdot(2-j4+4) = \frac{(-50)(6-j4)}{11+j4}$$

$$\mathbf{S} = \mathbf{V}_2\,\mathbf{I}^* = \mathbf{V}_2\,(2\,\mathbf{V}_o^*)$$

$$\mathbf{S} = \frac{(-50)(6-\mathbf{j}4)}{11+\mathbf{j}4}\cdot\frac{\mathbf{-100}}{\mathbf{11-j4}} = \left(\frac{5{,}000}{137}\right)(6-\mathbf{j}4)$$

$$\mathbf{S} = \mathbf{(219 - j145.99)\ VA}$$

Solution 11.58

Obtain the complex power delivered to the 10-kΩ resistor in Fig. 11.77 below.

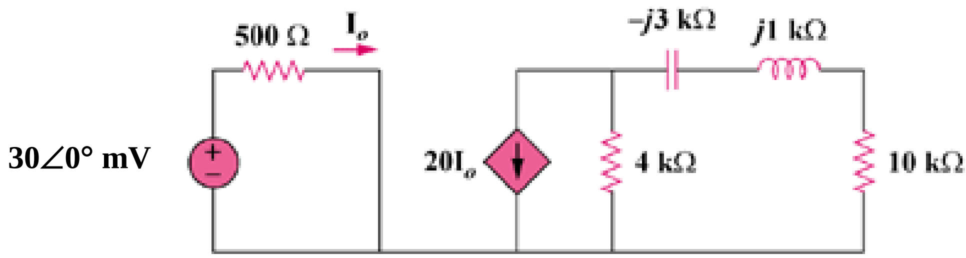


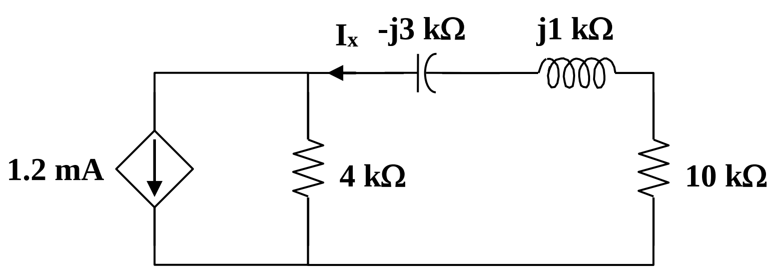
Figure 11.77
For Prob. 11.58.

Solution

From the left portion of the circuit,

$$\mathbf{I}_o = \frac{0.03}{500} = 60\ \mu A$$

$20\mathbf{I}_o = 1.2\ mA$ which then leads to the following circuit,



From the right portion of the circuit,

$$\mathbf{I}_x = \frac{4}{4+10+j-j3}(1.2\ mA) = \frac{2.4}{7-j}\ mA$$

$$\mathbf{S} = |\mathbf{I}_x|^2 R = \frac{(2.4\times10^{-3})^2}{50}\cdot(10\times10^3)\ 1.152$$

$$\mathbf{S = 1.152\ mVA}$$

It should be noted that even though we give the answer in VA, the complex power delivered to a resistor is always in watts so, a value of **S = 1.152 mW** would also be correct.

Solution 11.59

Calculate the reactive power in the inductor and capacitor in the circuit of Fig. 11.78.

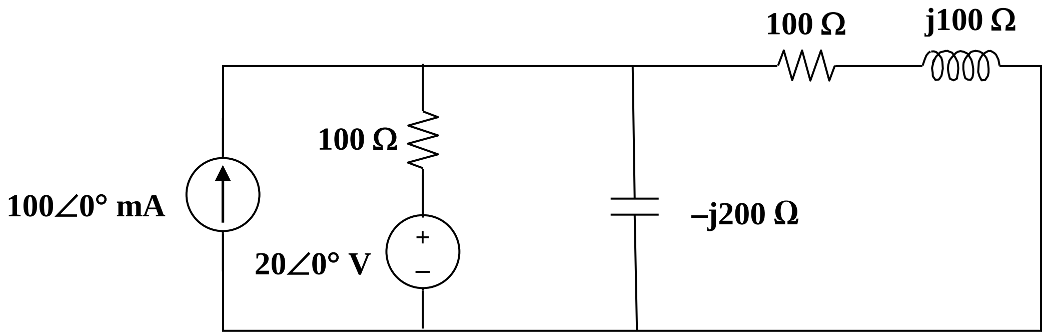
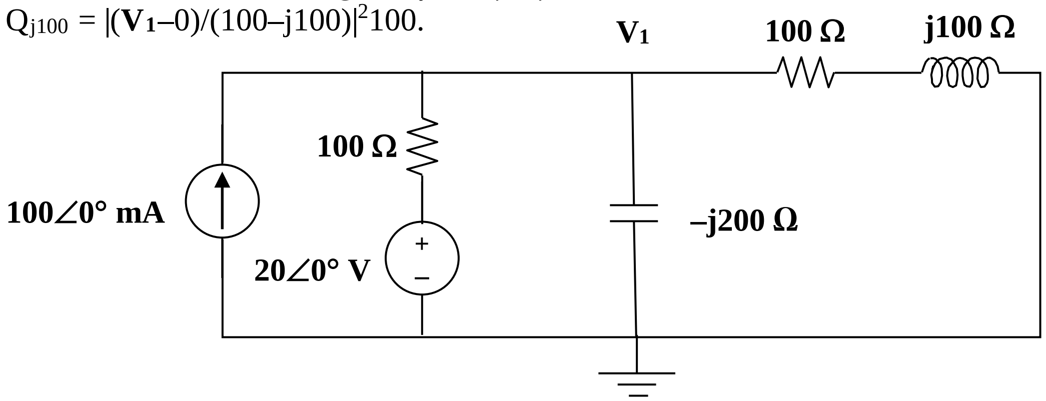


Figure 11.78
For Prob. 11.59.

Solution

Step 1. We write a nodal equation and solve for $\mathbf{V_1}$ in the following circuit. Once we have $\mathbf{V_1}$ we can get $Q_{-j200} = |\mathbf{V_1}|^2/200$ and $Q_{j100} = |(\mathbf{V_1}-0)/(100-j100)|^2 100$.



$-0.1 + [(\mathbf{V_1}-20)/100] + [(\mathbf{V_1}-0)/(-j200)] + [(\mathbf{V_1}-0)/(100+j100)] = 0.$

Step 2. $(0.01+j0.005+0.005-j0.005)\mathbf{V_1} = 0.3$ or $\mathbf{V_1} = 0.3/(0.015) = 20$ V.

$Q_{-j200} = 20^2/200 = 2$ VAR and $Q_{j100} = (20/141.42)^2 100 = 2$ VAR or

$\mathbf{S_{-j200}} = \mathbf{-j2\ VAR}$ and $\mathbf{S_{j100}} = \mathbf{j2\ VAR}$.

Solution 11.60

$$S_1 = 20 + j\frac{20}{0.8}\sin(\cos^{-1}(0.8)) = 20 + j15$$
$$S_2 = 16 + j\frac{16}{0.9}\sin(\cos^{-1}(0.9)) = 16 + j7.749$$

$$S = S_1 + S_2 = 36 + j22.749 = 42.585\angle 32.29°$$

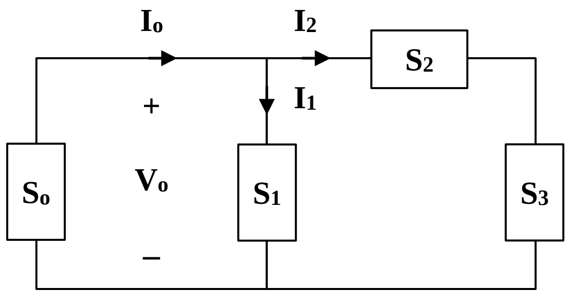
But $\quad S = V_o I^* = 6V_o$

$$V_o = \frac{S}{6} = \mathbf{7.098\angle 32.29°}$$

$$pf = \cos(32.29°) = \mathbf{0.8454} \quad \textbf{(lagging)}$$

Solution 11.61

Consider the network shown below.



$$\mathbf{S}_2 = 1.2 - j0.8 \text{ kVA}$$

$$\mathbf{S}_3 = 4 + j\frac{4}{0.9}\sin(\cos^{-1}(0.9)) = 4 + j1.937 \text{ kVA}$$

Let $\mathbf{S}_4 = \mathbf{S}_2 + \mathbf{S}_3 = 5.2 + j1.137 \text{ kVA}$

But $\mathbf{S}_4 = \mathbf{V}_o \mathbf{I}_2^*$

$$\mathbf{I}_2^* = \frac{\mathbf{S}_4}{\mathbf{V}_o} = \frac{(5.2 + j1.137)\times 10^3}{100\angle 90°} = 11.37 - j52$$

$$\mathbf{I}_2 = 11.37 + j52$$

Similarly, $\mathbf{S}_1 = \sqrt{2} - j\frac{\sqrt{2}}{0.707}\sin(\cos^{-1}(0.707)) = \sqrt{2}(1 - j) \text{ kVA}$

But $\mathbf{S}_1 = \mathbf{V}_o \mathbf{I}_1^*$

$$\mathbf{I}_1^* = \frac{\mathbf{S}_1}{V_o} = \frac{(1.4142 - j1.4142)\times 10^3}{j100} = -14.142 - j14.142$$

$$\mathbf{I}_1 = -14.142 + j14.142$$

$$\mathbf{I}_o = \mathbf{I}_1 + \mathbf{I}_2 = -2.772 + j66.14 = \mathbf{66.2\angle 92.4° \text{ A}}$$

$$\mathbf{S}_o = V_o I_o^*$$

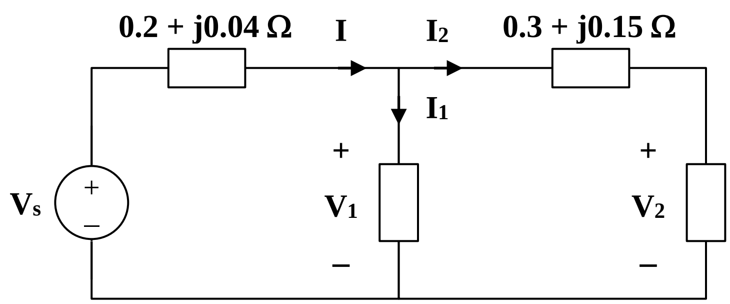
$$\mathbf{S}_o = (100\angle 90°)(66.2\angle -92.4°)\ VA$$

$$\mathbf{S}_o = \mathbf{6.62\angle{-2.4°} \text{ kVA}}$$

66.2∠92.4° A, 6.62∠–2.4° kVA

Solution 11.62

Consider the circuit below.



$$\mathbf{S}_2 = 15 - j\frac{15}{0.8}\sin(\cos^{-1}(0.8)) = 15 - j11.25$$

But $\quad \mathbf{S}_2 = \mathbf{V}_2 \mathbf{I}_2^*$

$$\mathbf{I}_2^* = \frac{\mathbf{S}_2}{\mathbf{V}_2} = \frac{15 - j11.25}{120}$$

$$\mathbf{I}_2 = 0.125 + j0.09375$$

$$\mathbf{V}_1 = \mathbf{V}_2 + \mathbf{I}_2(0.3 + j0.15)$$

$$\mathbf{V}_1 = 120 + (0.125 + j0.09375)(0.3 + j0.15)$$

$$\mathbf{V}_1 = 120.02 + j0.0469$$

$$\mathbf{S}_1 = 10 + j\frac{10}{0.9}\sin(\cos^{-1}(0.9)) = 10 + j4.843$$

But $\quad \mathbf{S}_1 = \mathbf{V}_1 \mathbf{I}_1^*$

$$\mathbf{I}_1^* = \frac{\mathbf{S}_1}{\mathbf{V}_1} = \frac{11.111\angle 25.84°}{120.02\angle 0.02°}$$

$$\mathbf{I}_1 = 0.093\angle -25.82° = 0.0837 - j0.0405$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.2087 + j0.053$$

$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{I}(0.2 + j0.04)$$

$$\mathbf{V}_s = (120.02 + j0.0469) + (0.2087 + j0.053)(0.2 + j0.04)$$

$$\mathbf{V}_s = 120.06 + j0.0658$$

$$\mathbf{V}_s = \mathbf{120.06\angle 0.03°\ V}$$

Solution 11.63

Find $\mathbf{I}_o$ in the circuit of Fig. 11.82.

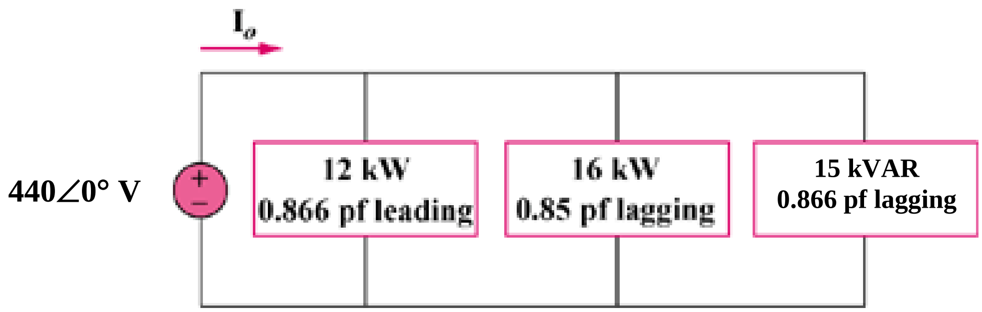


Figure 11.82
For Prob. 11.63.

Solution

Let $\quad \mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$.

$\mathbf{S}_1 = \mathbf{P} + \mathbf{jQ} = 12\mathbf{k} - \mathbf{jQ_C}$ where $\tan\theta = Q_C/12k$ and $\theta = \cos^{-1}(0.866) = 30°$ and $Q_C = 12k(\tan(30°)) = 6.9282k$ so $\mathbf{S_1} = 12k - j6.9282$.

$$\mathbf{S}_2 = 16 + j\frac{16}{0.85}\sin(\cos^{-1}(0.85)) = 16 + j9.916$$

$$\mathbf{S}_3 = \frac{(15)(0.866)}{\sin(\cos^{-1}(0.866))} + j15 = 25.98 + j15$$

$$\mathbf{S} = 53.98 + j17.987 = \mathbf{V}\,\mathbf{I}_o^* = 56.898\angle 18.43° \text{ kVA}$$

$$\mathbf{I}_o^* = \frac{\mathbf{S}}{\mathbf{V}} = \frac{(53.98 + j17.987)x10^3}{440} = (56.898\angle 18.43° \text{ kVA})/(440 \text{ V})$$

$$\mathbf{I}_o = \mathbf{129.31\angle 18.43° \ A}$$

Solution 1l.64

Determine I_s in the circuit shown in Fig. 11.83, if the voltage source supplies 6 kW and 1.2 kVAR (leading).

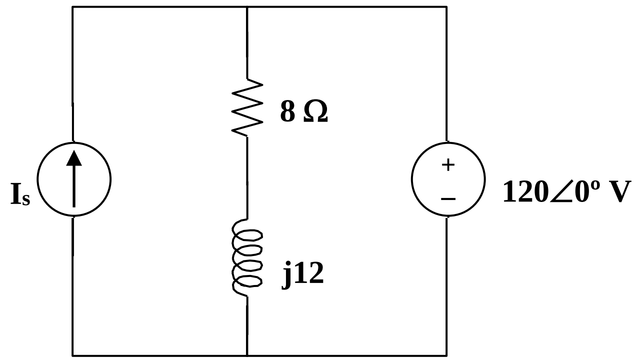
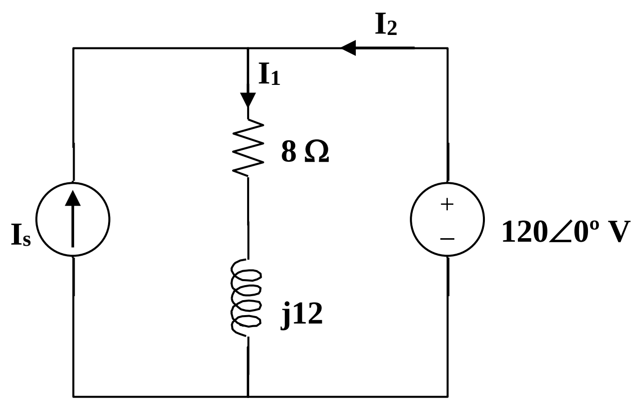


Figure 11.83
For Prob. 11.64.

Solution



$\mathbf{I_s} + \mathbf{I_2} = \mathbf{I_1}$ or $\mathbf{I_s} = \mathbf{I_1} - \mathbf{I_2}$

$$I_1 = \frac{120}{8 + j12} = 4.615 - j6.923$$

But, $$S = VI_2^* \longrightarrow I_2^* = \frac{S}{V} = \frac{6{,}000 - j1{,}200}{120} = 50 - j10$$

$$or\ I_2 = 50 + j10$$

$\mathbf{I_s} = \mathbf{I_1} - \mathbf{I_2} = -45.385 - j16.923 = \mathbf{48.44\angle{-159.55^\circ}\ A}$.

Solution 11.65

$$C = 1\,\text{nF} \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{-j}{10^4 \times 10^{-9}} = -j100\ \text{k}\Omega$$

At the noninverting terminal,

$$\frac{4\angle 0° - \mathbf{V}_o}{100} = \frac{\mathbf{V}_o}{-j100} \quad \longrightarrow \quad \mathbf{V}_o = \frac{4}{1+j}$$

$$\mathbf{V}_o = \frac{4}{\sqrt{2}}\angle -45°$$

$$v_o(t) = \frac{4}{\sqrt{2}}\cos(10^4 t - 45°)$$

$$P = \frac{V_{rms}^2}{R} = \left(\frac{4}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\right)^2 \left(\frac{1}{50 \times 10^3}\right) \text{W}$$

$P =$ **80 μW**

Solution 11.66

Obtain the average power absorbed by the 10-Ω resistor in the op amp circuit in Fig. 11.85.

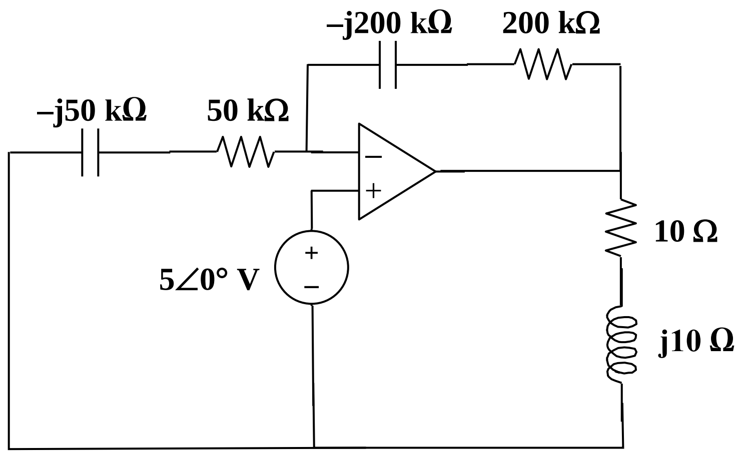
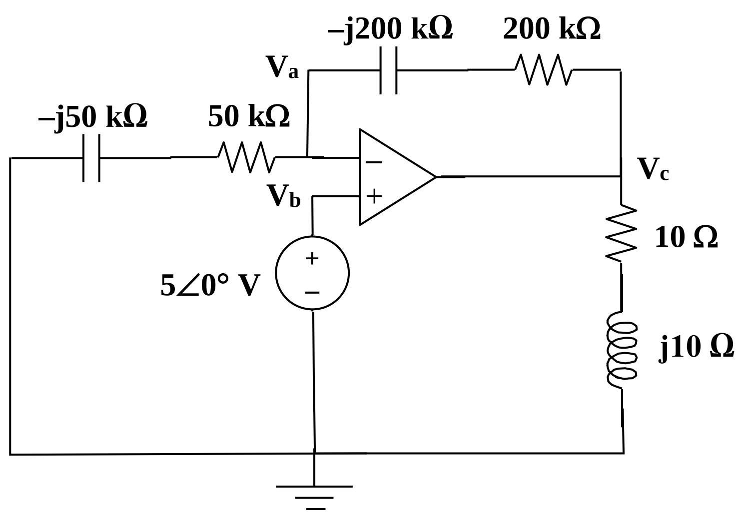


Figure 11.85
For Prob. 11.66.

Solution

Step 1. First we identify a reference node and then label the unknown nodes. Finally we write the node equations and use the constraint equation to solve for the unknown nodes.



$[(\mathbf{V_a}–0)/(50k–j50k)] + [(\mathbf{V_a}–\mathbf{V_c})/(200k–j200k)] + 0 = 0$ and $\mathbf{V_a} = \mathbf{V_b} = 5$ V. This allows us to solve for $\mathbf{V_c}$. The current through the 10 Ω resistor = $\mathbf{V_c}/(10+j10)$.

Step 2. $\mathbf{V_c}/(200k–j200k) = [5/(50k–j50k)] + [5/(200k–j200k)] = 25/(200k–j200k)$
or

$\mathbf{V_c} = 25$ V and $|\mathbf{I_{10}}| = |25/(14.142\angle 45°)| = 1.76778$ and

$$P_{10} = (1.76778)^2 10 = \mathbf{31.25\ W}.$$

Solution 11.67

For the op amp shown in Fig. 11.86, calculate:

(a) the complex power delivered by the voltage source
(b) the average power dissipated by the 10-Ω resistor

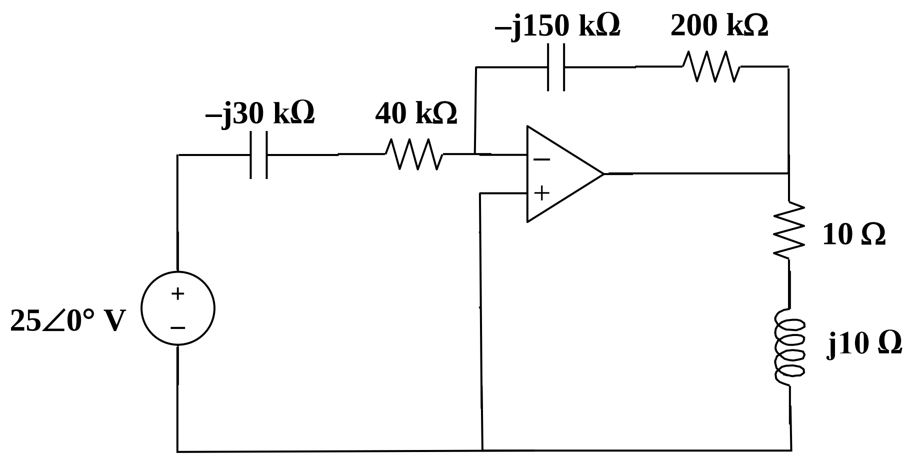
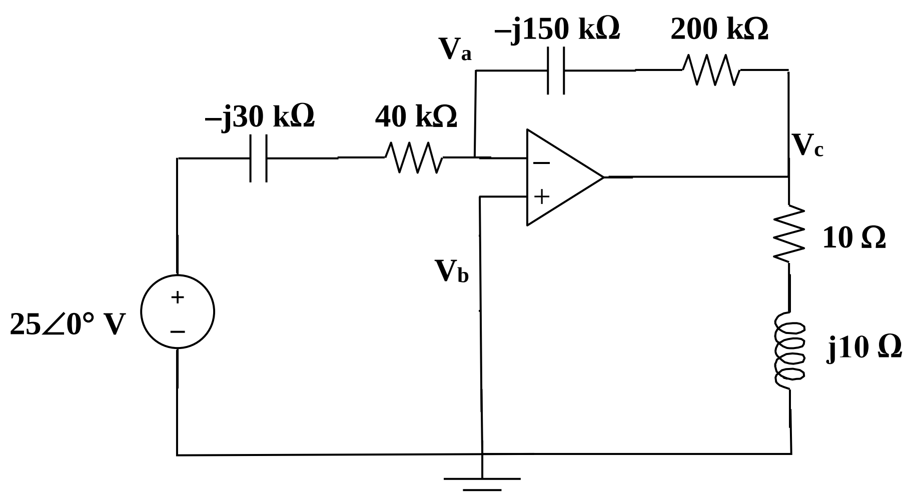


Figure 11.86
For Prob. 11.67.

Solution

Step 1. First we establish a reference node and then identify the unknown nodes. Next we write node equations and apply the constraint equation.



$[(\mathbf{V_a}–25)/(40k–j30k)] + [(\mathbf{V_a}–\mathbf{V_c})/(200k–j150k)] + 0 = 0$ and $\mathbf{V_a} = \mathbf{V_b} = 0$.
Finally,
$\mathbf{I_{10}} = (\mathbf{V_c}–0)/(10+j10)$. For the complex power delivered by the source,

$\mathbf{I_s}$ = 25/(40k–j30k) and $\mathbf{S_{delivered}}$ = 25$(\mathbf{I_s})^*$.

Step 2. $\mathbf{V_c}$/(200k–j150k) = –25/(40k–j30k) or $\mathbf{V_c}$ = –125 V and $\mathbf{I_{10}}$ = –125/(14.142∠45°)
= 8.8389∠135° A and $\mathbf{I_s}$ = 25/(50k∠–36.87°) = 0.5∠36.87° mA.

$\mathbf{S_{delivered}}$ = 25(0.5∠–36.87°) = **12.5∠–36.87° mVA** and
$P_{avg} = (8.8389)^2 10$ = **78.13 W**.

Solution 11.68

Let $\mathbf{S} = \mathbf{S}_R + \mathbf{S}_L + \mathbf{S}_c$

where $\mathbf{S}_R = P_R + jQ_R = \frac{1}{2} I_o^2 R + j0$

$$\mathbf{S}_L = P_L + jQ_L = 0 + j\frac{1}{2} I_o^2 \omega L$$

$$\mathbf{S}_c = P_c + jQ_c = 0 - j\frac{1}{2} I_o^2 \cdot \frac{1}{\omega C}$$

Hence,

$$\mathbf{S} = \frac{\mathbf{1}}{\mathbf{2}} \mathbf{I}_o^2 \left[\mathbf{R} + \mathbf{j}\left(\boldsymbol{\omega} \mathbf{L} - \frac{\mathbf{1}}{\boldsymbol{\omega} \mathbf{C}} \right) \right]$$

Solution 11.69

Refer to the circuit shown in Fig. 11.88.

(a) What is the power factor?
(b) What is the average power dissipated?
(c) What is the value of the capacitance that will give a unity power factor when connected to the load?

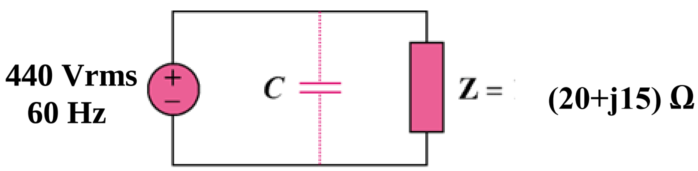


Figure 11.88
For Prob. 11.69.

Solution

(a) Given that $\mathbf{Z} = 20 + j15$

$$\tan\theta = \frac{15}{20} \longrightarrow \theta = 36.87^\circ$$

$$\text{pf} = \cos\theta = \mathbf{0.8}$$

(b)
$$\mathbf{S} = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(440)^2}{25\angle -36.87^\circ} = 6{,}195.2 + j4{,}646.4$$

The average power absorbed $= \text{P} = \text{Re}(\mathbf{S}) = \mathbf{6.195\ kW}$

(c) For unity power factor, $\theta_1 = 0^\circ$, which implies that the reactive power due to the capacitor is Q_C = 4.6464 kVAR

But $$Q_c = \frac{V^2}{X_c} = \omega C V^2$$

$$C = \frac{Q_c}{\omega V^2} = \frac{(4{,}646.4)}{(2\pi)(60)(440)^2} = \mathbf{63.66\ \mu F}$$

Solution 11.70

Design a problem to help other students to better understand power factor correction.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

An 880-VA, 220-V, 50-Hz load has a power factor of 0.8 lagging. What value of parallel capacitance will correct the load power factor to unity?

Solution

$$\text{pf} = \cos\theta = 0.8 \longrightarrow \sin\theta = 0.6$$
$$Q = S\sin\theta = (880)(0.6) = 528$$

If the power factor is to be unity, the reactive power due to the capacitor is

$$Q_c = Q = 528 \text{ VAR}$$

$$\text{But} \qquad Q = \frac{V_{rms}^2}{X_c} = \omega C V^2 \longrightarrow C = \frac{Q_c}{\omega V^2}$$

$$C = \frac{(528)}{(2\pi)(50)(220)^2} = \mathbf{34.72\ \mu F}$$

Solution 11.71

(a) For load 1,

Q_1 = 60 kVAR, pf = 0.85 or $\theta_1 = 31.79^\circ$

$Q_1 = S_1 \sin\theta_1$ = 60k or S_1 = 113.89k and P_1 = 113.89cos(31.79) = 96.8kW

$\mathbf{S}_1$ = 96.8 + j60 kVA

For load 2, $\mathbf{S}_2$ = 90 – j50 kVA

For load 3, $\mathbf{S}_3$ =100 kVA

Hence,

$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} + \mathbf{S_3} = 286.8 + j10\text{kVA} = 287\angle 2^\circ\text{kVA}$

But $\mathbf{S} = (V_{rms})^2/Z^*$ or $Z^* = 120^2/287\angle 2^\circ\text{k} = 0.05017\angle{-2^\circ}$

Thus, $Z = \mathbf{0.05017\angle 2^\circ\,\Omega}$ or **[50.14 + j1.7509] mΩ**.

(b) From above, pf = cos2° = **0.9994**.

(c) $I_{rms} = V_{rms}/Z = 120/0.05017\angle 2^\circ = \mathbf{2.392\angle{-2^\circ}\ kA}$ or **[2.391 – j0.08348] kA**.

Solution 11.72

(a) $P = S\cos\theta_1 \quad \longrightarrow \quad S = \dfrac{P}{\cos\theta_1} = \dfrac{2.4}{0.8} = 3.0 \text{ kVA}$

$pf = 0.8 = \cos\theta_1 \quad \longrightarrow \quad \theta_1 = 36.87^o$

$Q = S\sin\theta_1 = 3.0\sin 36.87^o = 1.8 \text{ kVAR}$

Hence, S = 2.4 + j1.8 kVA

$S_1 = \dfrac{P_1}{\cos\theta} = \dfrac{1.5}{0.707} = 2.122 \text{ kVA}$

$pf = 0.707 = \cos\theta \quad \longrightarrow \quad \theta = 45^o$

$Q_1 = P_1 = 1.5 \text{ kVAR} \quad \longrightarrow \quad \mathrm{S}_1 = 1.5 + j1.5 \text{ kVA}$

Since, $S = S_1 + S_2 \quad \longrightarrow \quad S_2 = S - S_1 = (2.4 + j1.8) - (1.5 + j1.5) = 0.9 + j0.3 \text{ kVA}$

$S_2 = 0.9497 < 18.43^o$

pf = cos 18.43^o = <u>0.9487</u>

(b) $pf = 0.9 = \cos\theta_2 \quad \longrightarrow \quad \theta_2 = 25.84^o$

$C = \dfrac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \dfrac{2400(\tan 36.87 - \tan 25.84)}{2\pi x 60 x (120)^2} = \underline{117.5\ \mu\text{F}}$

Solution 11.73

(a) $\mathbf{S} = 10 - j15 + j22 = 10 + j7 \text{ kVA}$

$S = |\mathbf{S}| = \sqrt{10^2 + 7^2} = \mathbf{12.21\ kVA}$

(b) $\mathbf{S} = \mathbf{V}\mathbf{I}^* \longrightarrow \mathbf{I}^* = \dfrac{\mathbf{S}}{\mathbf{V}} = \dfrac{10{,}000 + j7{,}000}{240}$

$\mathbf{I} = 41.667 - j29.167 = \mathbf{50.86\angle{-35°}\ A}$

(c) $\theta_1 = \tan^{-1}\left(\dfrac{7}{10}\right) = 35°,\qquad \theta_2 = \cos^{-1}(0.96) = 16.26°$

$Q_c = P_1[\tan\theta_1 - \tan\theta_2] = 10[\tan(35°) - \tan(16.26°)]$

$Q_c = \mathbf{4.083\ kVAR}$

$C = \dfrac{Q_c}{\omega V_{rms}^2} = \dfrac{4083}{(2\pi)(60)(240)^2} = \mathbf{188.03\ \mu F}$

(d) $\mathbf{S}_2 = P_2 + jQ_2,\ P_2 = P_1 = 10 \text{ kW}$

$Q_2 = Q_1 - Q_c = 7 - 4.083 = 2.917 \text{ kVAR}$

$\mathbf{S}_2 = 10 + j2.917 \text{ kVA}$

But $\mathbf{S}_2 = \mathbf{V}\mathbf{I}_2^*$

$\mathbf{I}_2^* = \dfrac{\mathbf{S}_2}{\mathbf{V}} = \dfrac{10{,}000 + j2917}{240}$

$\mathbf{I}_2 = 41.667 - j12.154 = \mathbf{43.4\angle{-16.26°}\ A}$

Solution 11.74

(a) $\theta_1 = \cos^{-1}(0.8) = 36.87°$

$$S_1 = \frac{P_1}{\cos\theta_1} = \frac{24}{0.8} = 30 \text{ kVA}$$

$$Q_1 = S_1 \sin\theta_1 = (30)(0.6) = 18 \text{ kVAR}$$

$$\mathbf{S}_1 = 24 + j18 \text{ kVA}$$

$$\theta_2 = \cos^{-1}(0.95) = 18.19°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = \frac{40}{0.95} = 42.105 \text{ kVA}$$

$$Q_2 = S_2 \sin\theta_2 = 13.144 \text{ kVAR}$$

$$\mathbf{S}_2 = 40 + j13.144 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = 64 + j31.144 \text{ kVA}$$

$$\theta = \tan^{-1}\left(\frac{31.144}{64}\right) = 25.95°$$

$$\text{pf} = \cos\theta = \mathbf{0.8992}$$

(b) $\theta_2 = 25.95°$, $\qquad\qquad \theta_1 = 0°$

$$Q_c = P[\tan\theta_2 - \tan\theta_1] = 64[\tan(25.95°) - 0] = 31.144 \text{ kVAR}$$

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{31{,}144}{(2\pi)(60)(120)^2} = \mathbf{5.74\ mF}$$

Solution 11.75

Consider the power system shown in Fig. 11.90. Calculate:

(a) the total complex power
(b) the power factor
(c) the parallel capacitance necessary to establish a unity power factor

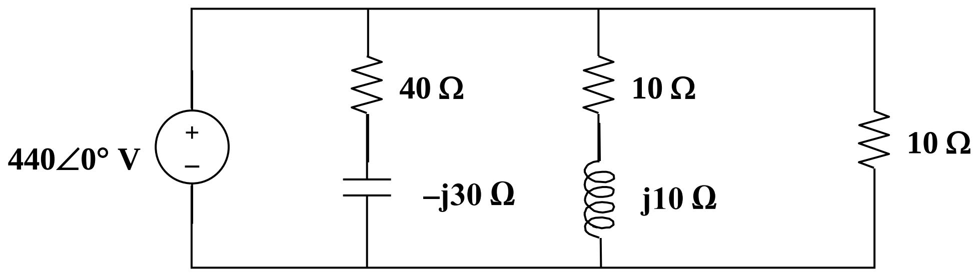


Figure 11.90
For Prob. 11.75.

Solution

Step 1. $S_1 = (440)^2/(40–j30)^*$, $S_2 = (440)^2/(10+j10)^*$, and $S_3 = (440)^2/10$.
$S_{Tot} = S_1 + S_2 + S_3 = P_{Tot} + jQ_{Tot}$. Now pf = $\cos[\mathrm{Tan}^{-1}(Q_{Tot}/P_{Tot})]$.
Finally $Q_{Tot} = Q_C = (440)^2/X_C$ and $X_C = 1/(377C)$ assuming 60 Hz.

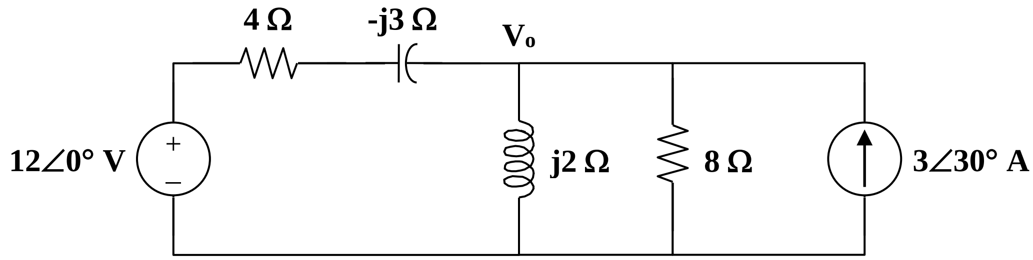
(a) $S_1 = 193{,}600/(50\angle 36.87°) = 3{,}872\angle –36.87° = (3{,}097.6–j2{,}323.2)$ VA, $S_2 = 193{,}600/(14.142\angle –45° = 13{,}689.7\angle 45° = (9{,}680+j9{,}680)$ VA, and $S_3 = 193{,}600/10 = 19{,}3600$. $S_{Tot} = [(3.0976+9.68+19.36)+j(–2.3232+9.68)]$ kVA = **32.14 kW + j7.357 kVAR**.

(b) pf = $\cos[\mathrm{Tan}^{-1}(7.357/32.14)] = \cos(12.893°)$ = **0.9748**

(c) $Q_C = 7{,}357 = 193{,}600/X_C$ and $X_C = 193{,}600/7{,}357 = 26.315 = 1/(377C)$ or C = **100.8 µF**.

Solution 11.76

The wattmeter reads the real power supplied by the current source. Consider the circuit below.



$$3\angle 30° + \frac{12 - \mathbf{V}_o}{4 - j3} = \frac{\mathbf{V}_o}{j2} + \frac{\mathbf{V}_o}{8}$$

$$\mathbf{V}_o = \frac{36.14 + j23.52}{2.28 - j3.04} = 0.7547 + j11.322 = 11.347\angle 86.19°$$

$$\mathbf{S} = \mathbf{V}_o \mathbf{I}_o^* = (11.347\angle 86.19°)(3\angle\text{-}30°)$$

$$\mathbf{S} = 34.04\angle 56.19° \text{ VA}$$

$$\text{P} = \text{Re}(\mathbf{S}) = \mathbf{18.942\ W}$$

Solution 11.77

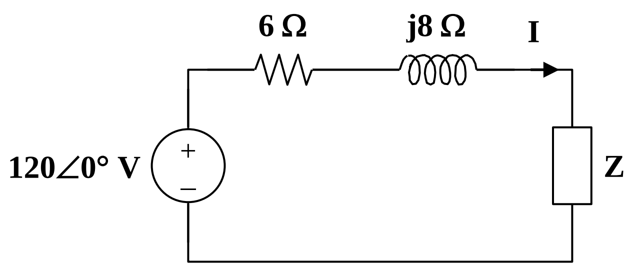
The wattmeter measures the power absorbed by the parallel combination of 0.1 F and 150 Ω.

$$120\cos(2t) \longrightarrow 120\angle 0°, \qquad \omega = 2$$

$$4\text{ H} \longrightarrow j\omega L = j8$$

$$0.1\text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j5$$

Consider the following circuit.



$$\mathbf{Z} = 15 \,||\, (\text{-}j5) = \frac{(15)(\text{-}j5)}{15 - j5} = 1.5 - j4.5$$

$$\mathbf{I} = \frac{120}{(6 + j8) + (1.5 - j4.5)} = 14.5\angle\text{-}25.02°$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\,\mathbf{I}^* = \frac{1}{2}\left|\mathbf{I}\right|^2 \mathbf{Z} = \frac{1}{2}\cdot(14.5)^2(1.5 - j4.5)$$
$$\mathbf{S} = 157.69 - j473.06 \text{ VA}$$

The wattmeter reads

$$P = \text{Re}(\mathbf{S}) = \mathbf{157.69\ W}$$

Solution 11.78

Find the wattmeter reading of the circuit shown in Fig. 11.93 below.

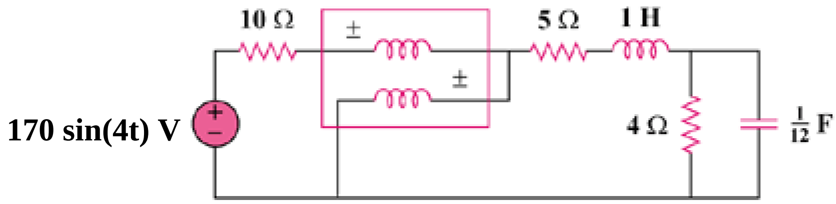


Figure 11.93
For Prob. 11.78.

Solution

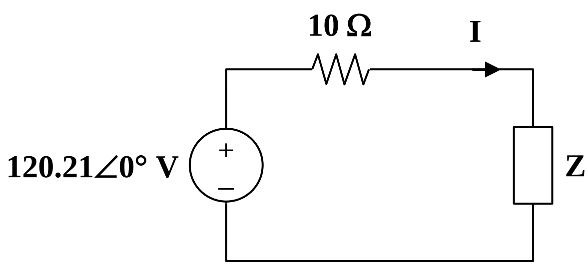
The wattmeter reads the power absorbed by the element to its right side.

$$170\sin(4\mathbf{t}) \longrightarrow 120.21\angle\mathbf{0}°, \qquad \omega = 4$$

$$1\text{ H} \longrightarrow j\omega L = j4$$

$$\frac{1}{12}\text{ F} \longrightarrow \frac{1}{j\omega C} = -j3$$

Consider the following circuit.



$$\mathbf{Z} = 5 + j4 + 4 \,||\, -j3 = 5 + j4 + \frac{(4)(-j3)}{4 - j3}$$

$$\mathbf{Z} = 6.44 + j2.08$$

$$\mathbf{I} = \frac{120.21}{6.7676\angle 17.9°} = 17.7626\angle -17.9°$$

$$\mathbf{S} = |\mathbf{I}|^2\,\mathbf{Z} = (17.7626)^2(6.44 + j2.08) = (2{,}032 + j656.3)\text{VA}$$

$$P = \text{Re}(\mathbf{S}) = \mathbf{2.032\ kW}$$

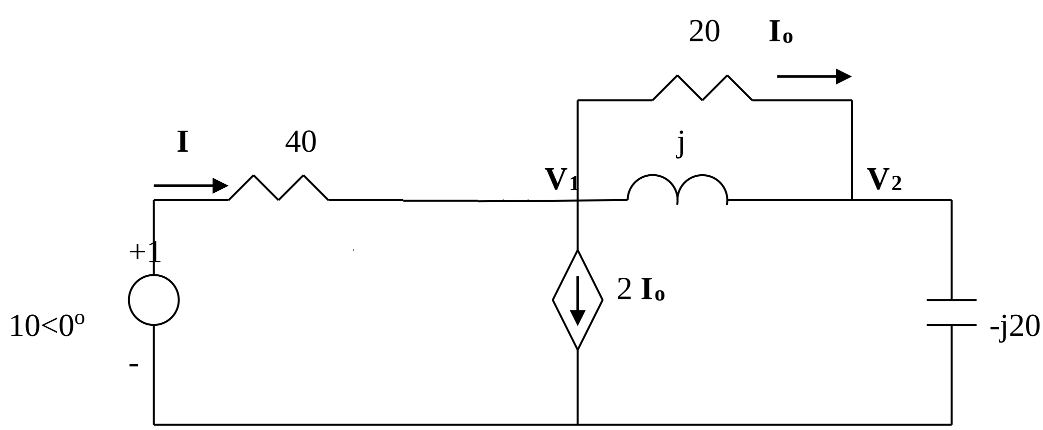
Solution 11.79

The wattmeter reads the power supplied by the source and partly absorbed by the 40-Ω resistor.

$\omega = 100,$

$$10\,\text{mH} \longrightarrow j100x10x10^{-3} = j, \quad 500\mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j100x500x10^{-6}} = -j20$$

The frequency-domain circuit is shown below.



At node 1,

$$\frac{10 - V_1}{40} = 2I_o + \frac{V_1 - V_2}{j} + \frac{V_1 - V_2}{20} = \frac{3(V_1 - V_2)}{20} + \frac{V_1 - V_2}{j} \longrightarrow \tag{1}$$

$$10 = (7 - j40)V_1 + (-6 + j40)V_2$$

At node 2,

$$\frac{V_1 - V_2}{j} + \frac{V_1 - V_2}{20} = \frac{V_2}{-j20} \longrightarrow 0 = (20 + j)V_1 - (19 + j)V_2 \tag{2}$$

Solving (1) and (2) yields V_1 = 1.5568 –j4.1405

$$I = \frac{10 - V_1}{40} = 0.2111 + j0.1035, \quad S = \frac{1}{2}V_1 I^* = -0.04993 - j0.5176$$

P = Re(S) = **50 mW**.

Solution 11.80

The circuit of Fig. 11.95 portrays a wattmeter connected into an ac network.

(a) Find the load current.

(b) Calculate the wattmeter reading.

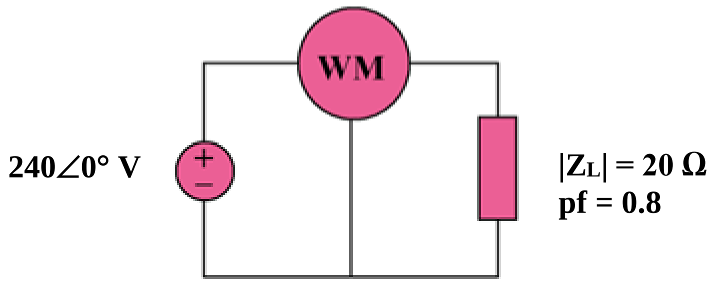


Figure 11.95
For Prob. 11.80.

Solution

(a) $|\mathbf{I}| = \frac{|\mathbf{V}|}{|\mathbf{Z}|} = \frac{240}{20} = \mathbf{12\ A}$

(b) $|\mathbf{S}| = \frac{|\mathbf{V}|^2}{|\mathbf{Z}|} = \frac{(240)^2}{20} = 2{,}880\ \text{VA}$

P = S cos(θ) = 2,880(0.8) = **2.304 kW**.

Solution 11.81

Design a problem to help other students to better understand how to correct power factor to values other than unity.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A 120-V rms, 60-Hz electric hair dryer consumes 600 W at a lagging pf of 0.92. Calculate the rms-valued current drawn by the dryer.

How would you power factor correct this to a value of 0.95?

Solution

P = 600 W, $pf = 0.92 \quad \longrightarrow \quad \theta = 23.074^o$

$$P = S\cos\theta \quad \longrightarrow \quad S = \frac{P}{0.92} = 652.17 \text{ VA}$$

S = P+jQ = 600 + j652.17sin23.09° = 600 +j255.6

But $S = V_{rms} I_{rms}^*$.

$$I_{rms}^* = \frac{S}{V_{rms}} = \frac{600 + j255.6}{120}$$

$$I_{rms} = 5 - j2.13 = \mathbf{5.435\angle{-23.07^\circ}\ A}.$$

To correct this to a pf = 0.95, I would add a capacitor in parallel with the hair dryer (remember, series compensation will increase the power delivered to the load and probably burn out the hair dryer.

pf = 0.95 = 600/S or S = 631.6 VA and θ = 18.19˚ and VARs = 197.17

Thus,

$VARs_{cap}$ = 255.6 – 197.17 = 58.43 = 120xI_C or I_C = 58.43/120 = 0.4869A

Next,

X_C = 120/0.4869 = 246.46 = 1/(377xC) or C = **10.762 μF**

Solution 11.82

(a) $P_1 = 5{,}000, \qquad Q_1 = 0$

$$P_2 = 30{,}000x0.82 = 24{,}600, \qquad Q_2 = 30{,}000\sin(\cos^{-1} 0.82) = 17{,}171$$

$$\bar{S} = \bar{S}_1 + \bar{S}_2 = (P_1 + P_2) + j(Q_1 + Q_2) = 29{,}600 + j17{,}171$$

$$\boldsymbol{S = |\bar{S}| = \underline{34.22\,\text{kVA}}}$$

(b) Q = **17.171 kVAR**

(c) $pf = \dfrac{P}{S} = \dfrac{29{,}600}{34{,}220} = 0.865$

$$\boldsymbol{Q_c = P(\tan\theta_1 - \tan\theta_2)}$$
$$\boldsymbol{= 29{,}600\left[\tan(\cos^{-1} 0.865) - \tan(\cos^{-1} 0.9)\right] = \underline{2833\ \text{VAR}}}$$

(c) $$\boldsymbol{C = \frac{Q_c}{\omega V^2{}_{rms}} = \frac{2833}{2\pi x 60 x 240^2} = \underline{130.46\mu\,\text{F}}}$$

Solution 11.83

(a) $\bar{S} = \frac{1}{2} V I^* = \frac{1}{2}(210\angle 60^o)(8\angle -25^o) = 840\angle 35^o$

$$\boldsymbol{P = S\cos\theta = 840\cos 35^o = \underline{688.1\ \text{W}}}$$

(b) S = **840 VA**

(c) $\boldsymbol{Q = S\sin\theta = 840\sin 35^o = \underline{481.8\ \text{VAR}}}$

(d) $\boldsymbol{pf = P/S = \cos 35^o = \underline{0.8191\ \text{(lagging)}}}$

Solution 11.84

(a) Maximum demand charge $= 2{,}400 \times 30 = \$72{,}000$

Energy cost $= \$0.04 \times 1{,}200 \times 10^3 = \$48{,}000$

Total charge = **$120,000**

(b) To obtain $120,000 from 1,200 MWh will require a flat rate of

$\dfrac{\$120{,}000}{1{,}200 \times 10^3}$ per kWh = **$0.10 per kWh**

Solution 11.85

A regular household system of a single-phase three-wire allows the operation of both 120-V and 240-V, 60-Hz appliances. The household circuit is modeled as shown in Fig. 11.96. Calculate: (a) the currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_n$, (b) the total complex power supplied, (c) the overall power factor of the circuit.

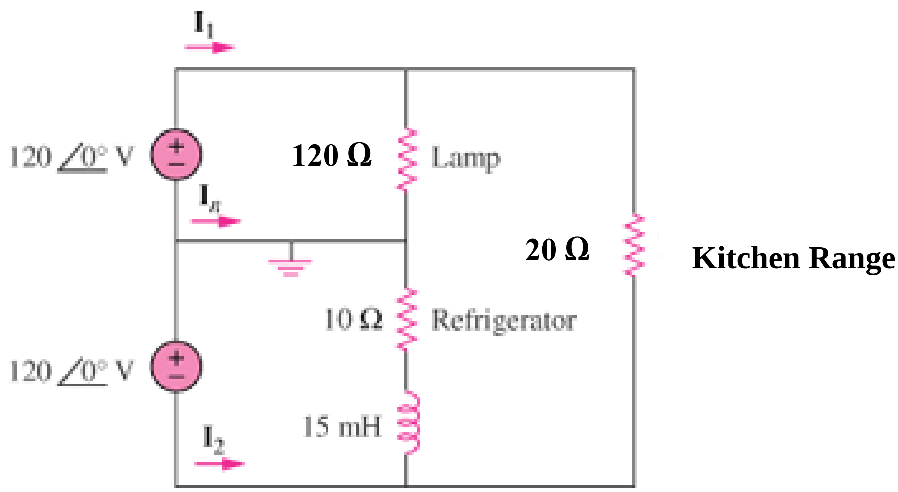


Figure 11.96
For Prob. 11.85.

Solution

(a) 15 mH $\longrightarrow$ $j2\pi x60x15x10^{-3} = j5.655$

Since we know the voltage across each device we can calculate the current through each one. $\mathbf{I}_{Lamp}$ = 120/120 = 1 A, $\mathbf{I}_{Ref}$ = 120/(10+j5.655), and $\mathbf{I}_{range}$ = 240/20 = 12 A. KCL gives us $\mathbf{I}_1 = \mathbf{I}_{Lamp} + \mathbf{I}_{range}$ = **13 A**, $\mathbf{I}_n = \mathbf{I}_{Ref} - \mathbf{I}_{Lamp}$, and $\mathbf{I}_2 = -\mathbf{I}_{Ref} - \mathbf{I}_{range}$. $\mathbf{I}_{Ref}$ = 120/(11.4882∠29.488°)
= (10.4455∠–29.488°) A = (9.0924 – j5.1417) A. Thus,
$\mathbf{I}_n$ = 9.0924 – 1 – j5.1417 = **(8.092 – j5.142) A** = **9.588∠–32.43° A** and
$\mathbf{I}_2$ = –9.0924 – 12 + j5.1417 = **(–21.09 + j5,142) A** = **21.71∠166.3° A**.

(b) The complex power delivered by each source is given by $\mathbf{S}_1 = 120(\mathrm{I}_1)^*$ and $\mathbf{S}_2 = 120(-\mathrm{I}_2)^*$ and $\mathbf{S}_{Tot} = \mathbf{S}_1 + \mathbf{S}_2$. $\mathbf{S}_1$ = 120(13) = 1.56 kW and $\mathbf{S}_2 = 120(21.71\angle{-13.7°})^*$ = 2.6052∠13.7° kVA = 2.531 kW + j617 VAR, thus, **S** = **4.091 kW + j0.617 kVAR**.

(c) Finally, pf = P/|S| = 4.091/4.1373 = **0.9888 (lagging)**.

Solution 11.86

For maximum power transfer

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* \longrightarrow \mathbf{Z}_i = \mathbf{Z}_{Th} = \mathbf{Z}_L^*$$

$$\mathbf{Z}_L = R + j\omega L = 75 + j(2\pi)(4.12\times 10^6)(4\times 10^{-6})$$
$$\mathbf{Z}_L = 75 + j103.55\,\Omega$$

$$\mathbf{Z}_i = \mathbf{[75 - \mathit{j}103.55]\,\Omega}$$

Solution 11.87

$$\mathbf{Z} = R \pm jX$$

$$\mathbf{V}_R = \mathbf{I}R \quad \longrightarrow \quad R = \frac{\mathbf{V}_R}{\mathbf{I}} = \frac{80}{50 \times 10^{-3}} = 1.6\ \text{k}\Omega$$

$$|\mathbf{Z}|^2 = R^2 + X^2 \quad \longrightarrow \quad X^2 = |\mathbf{Z}|^2 - R^2 = (3)^2 - (1.6)^2$$

$$X = 2.5377\ \text{k}\Omega$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{2.5377}{1.6}\right) = 57.77^\circ$$

$$\text{pf} = \cos\theta = \mathbf{0.5333}$$

Solution 11.88

(a) $\mathbf{S} = (110)(2\angle 55°) = 220\angle 55°$

$P = S\cos\theta = 220\cos(55°) = \mathbf{126.2\ W}$

(b) $S = |\mathbf{S}| = \mathbf{220\ VA}$

Solution 11.89

(a) Apparent power $= S =$ **12 kVA**

$$P = S\cos\theta = (12)(0.78) = 9.36 \text{ kW}$$
$$Q = S\sin\theta = 12\sin(\cos^{-1}(0.78)) = 7.51 \text{ kVAR}$$

$$\mathbf{S} = P + jQ = \mathbf{[9.36 + \mathit{j}7.51]}\ \boldsymbol{kVA}$$

(b) $$\mathbf{S} = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} \longrightarrow \mathbf{Z}^* = \frac{|\mathbf{V}|^2}{\mathbf{S}} = \frac{(210)^2}{(9.36 + j7.51)\times 10^3} = 2.866 - j2.3$$

$$\mathbf{Z} = \mathbf{[2.866 + j2.3]\ \Omega}$$

Solution 11.90

Original load :

$$P_1 = 2000 \text{ kW}, \qquad \cos\theta_1 = 0.85 \longrightarrow \theta_1 = 31.79°$$

$$S_1 = \frac{P_1}{\cos\theta_1} = 2352.94 \text{ kVA}$$

$$Q_1 = S_1 \sin\theta_1 = 1239.5 \text{ kVAR}$$

Additional load :

$$P_2 = 300 \text{ kW}, \qquad \cos\theta_2 = 0.8 \longrightarrow \theta_2 = 36.87°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = 375 \text{ kVA}$$

$$Q_2 = S_2 \sin\theta_2 = 225 \text{ kVAR}$$

Total load :

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = (P_1 + P_2) + j(Q_1 + Q_2) = P + jQ$$

$$P = 2000 + 300 = 2300 \text{ kW}$$

$$Q = 1239.5 + 225 = 1464.5 \text{ kVAR}$$

The minimum operating pf for a 2300 kW load and not exceeding the kVA rating of the generator is

$$\cos\theta = \frac{P}{S_1} = \frac{2300}{2352.94} = 0.9775$$

or $\quad \theta = 12.177°$

The maximum load kVAR for this condition is

$$Q_m = S_1 \sin\theta = 2352.94 \sin(12.177°)$$

$$Q_m = 496.313 \text{ kVAR}$$

The capacitor must supply the difference between the total load kVAR (i.e. Q) and the permissible generator kVAR (i.e. Q_m). Thus,

$$Q_c = Q - Q_m = \mathbf{968.2\ kVAR}$$

Solution 11.91

The nameplate of an electric motor has the following information:

Line voltage: 220 V rms
Line current: 15 A rms
Line frequency: 60 Hz
Power: 2700 W

Determine the power factor (lagging) of the motor. Find the value of the capacitance *C* that must be connected across the motor to raise the pf to unity.

Solution

$I = V/Z$ which leads to $Z = [220/15]\angle\theta = 14.6667\angle\theta$, $S = (220)(15)\angle\theta = 3.3\angle\theta$ kVA, where $\cos^{-1}(2700/3300) = \cos^{-1}(0.818182) = 35.097°$, and $X_L = 3300\sin(35.097°) = 1897.38 = X_C$. This leads to $C = 1/[377(1897.38)] =$ **1.398 μF**.

pf = **0.8182** (lagging)

C = **1.398 μF**

0.8182 (lagging), **1.398 μF**

Solution 11.92

As shown in Fig. 11.97, a 550-V feeder line supplies an industrial plant consisting of a motor drawing 90 kW at 0.8 pf (inductive), a capacitor with a rating of 20 kVAR, and lighting drawing 10 kW.

(a) Calculate the total reactive power and apparent power absorbed by the plant.

(b) Determine the overall pf.

(c) Find the magnitude of the current in the feeder line.

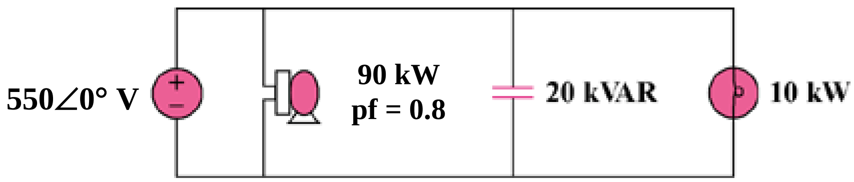


Figure 11.97
For Prob. 11.92.

Solution

(a) Apparent power drawn by the motor is

$$S_m = \frac{P}{\cos\theta} = \frac{90}{0.8} = 112.5\ kVA$$

$$Q_m = \sqrt{S^2 - P^2} = \sqrt{(112.5)^2 - (90)^2} = 67.5\ kVAR$$

Total real power

$$P = P_m + P_c + P_L = 90 + 0 + 10 = 100\ kW$$

Total reactive power

$$Q = Q_m + Q_c + Q_L = 67.5 - 20 + 0 = \mathbf{47.5\ kVAR}$$

Total apparent power

$$\mathrm{S} = \sqrt{\mathrm{P}^2 + \mathrm{Q}^2} = \mathbf{110.71\ kVA}$$

(b) $$pf = \frac{P}{S} = \frac{100}{110.71} = \mathbf{0.9033}$$

(c) |**I**| = S/V = 110,710/550 = **201.3 A**.

Solution 11.93

(a) $P_1 = (5)(0.7457) = 3.7285 \text{ kW}$

$$S_1 = \frac{P_1}{pf} = \frac{3.7285}{0.8} = 4.661 \text{ kVA}$$

$$Q_1 = S_1 \sin(\cos^{-1}(0.8)) = 2.796 \text{ kVAR}$$

$$\mathbf{S}_1 = 3.7285 + j2.796 \text{ kVA}$$

$$P_2 = 1.2 \text{ kW}, \qquad Q_2 = 0 \text{ VAR}$$

$$\mathbf{S}_2 = 1.2 + j0 \text{ kVA}$$

$$P_3 = (10)(120) = 1.2 \text{ kW}, \qquad Q_3 = 0 \text{ VAR}$$

$$\mathbf{S}_3 = 1.2 + j0 \text{ kVA}$$

$$Q_4 = 1.6 \text{ kVAR}, \qquad \cos\theta_4 = 0.6 \longrightarrow \sin\theta_4 = 0.8$$

$$S_4 = \frac{Q_4}{\sin\theta_4} = 2 \text{ kVA}$$

$$P_4 = S_4 \cos\theta_4 = (2)(0.6) = 1.2 \text{ kW}$$

$$\mathbf{S}_4 = 1.2 - j1.6 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4$$

$$\mathbf{S} = 7.3285 + j1.196 \text{ kVA}$$

Total real power = **7.3285 kW**

Total reactive power = **1.196 kVAR**

(b) $$\theta = \tan^{-1}\left(\frac{1.196}{7.3285}\right) = 9.27°$$

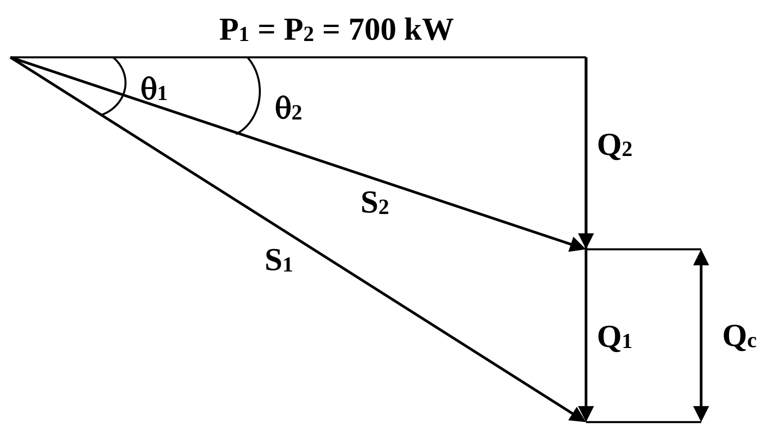
$$pf = \cos\theta = \mathbf{0.987}$$

Solution 11.94

$$\cos\theta_1 = 0.7 \longrightarrow \theta_1 = 45.57°$$
$$S_1 = 1\text{ MVA} = 1000\text{ kVA}$$
$$P_1 = S_1\cos\theta_1 = 700\text{ kW}$$
$$Q_1 = S_1\sin\theta_1 = 714.14\text{ kVAR}$$

For improved pf,

$$\cos\theta_2 = 0.95 \longrightarrow \theta_2 = 18.19°$$
$$P_2 = P_1 = 700\text{ kW}$$
$$S_2 = \frac{P_2}{\cos\theta_2} = \frac{700}{0.95} = 736.84\text{ kVA}$$
$$Q_2 = S_2\sin\theta_2 = 230.08\text{ kVAR}$$



(a) Reactive power across the capacitor

$$Q_c = Q_1 - Q_2 = 714.14 - 230.08 = 484.06\text{ kVAR}$$

Cost of installing capacitors $= \$30 \times 484.06 =$ **$14,521.80**

(b) Substation capacity released $= S_1 - S_2 = 1000 - 736.84 = 263.16$ kVA

Saving in cost of substation and distribution facilities $= \$120 \times 263.16 =$ **$31,579.20**

(c) **Yes**, because (a) is greater than (b). Additional system capacity obtained by using capacitors costs only 46% as much as new substation and distribution facilities.

Solution 11.95

(a) Source impedance $\mathbf{Z}_s = R_s - jX_c$

Load impedance $\mathbf{Z}_L = R_L + jX_2$

For maximum load transfer

$$\mathbf{Z}_L = \mathbf{Z}_s^* \longrightarrow R_s = R_L, \quad X_c = X_L$$

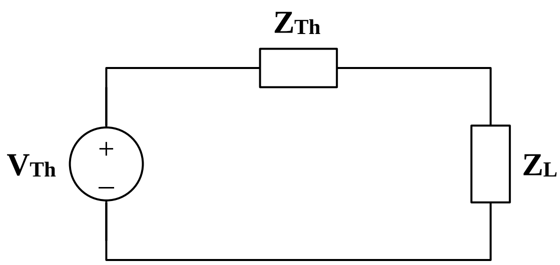
$$X_c = X_L \longrightarrow \frac{1}{\omega C} = \omega L$$

or $\omega = \dfrac{1}{\sqrt{LC}} = 2\pi f$

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(80\times 10^{-3})(40\times 10^{-9})}} = \mathbf{2.814\ kHz}$$

(b) $P = \left(\dfrac{V_S}{(10+4)}\right)^2 4 = \left(\dfrac{4.6}{14}\right)^2 4 = \mathbf{431.8\ mW}$ (since V_s is in rms)

Solution 11.96



(a) $\quad V_{Th} = 146\text{ V}, \quad 300\text{ Hz}$

$Z_{Th} = 40 + j8\,\Omega$

$Z_L = Z_{Th}^* = \mathbf{[40 - \mathit{j}8]\,\Omega}$

(b) $$P = \frac{|V_{Th}|^2}{8R_{Th}} = \frac{(146)^2}{(8)(40)} = \mathbf{66.61\ W}$$

Solution 11.97

A power transmission system is modeled as shown in Fig. 11.99. If $\mathbf{V}_s$ = 440 ∠0° rms, find the average power absorbed by the load.

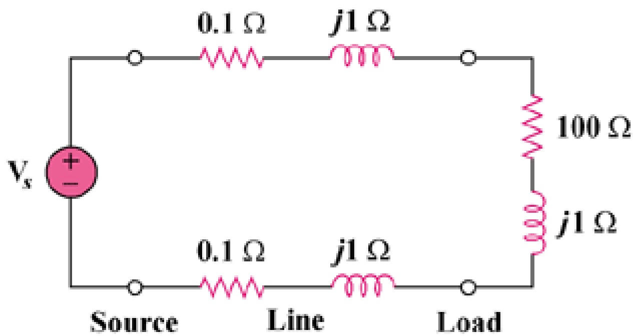


Figure 11.99
For Problem 11.97.

Solution

$$Z_T = (2)(0.1 + j) + (100 + j20) = 100.2 + j22\ \Omega$$

$$I = \frac{V_s}{Z_T} = \frac{440}{100.2 + j22}$$

$$P = |\,I\,|^2 R_L = 100|\,I\,|^2 = \frac{(100)(440)^2}{(100.2)^2 + (22)^2} = \frac{19{,}360{,}000}{10{,}040 + 484}$$

= **1.8396 kW**.

Solution 12.1

(a) If $\mathbf{V}_{ab} = 400$, then

$$\mathbf{V}_{an} = \frac{400}{\sqrt{3}}\angle -30^\circ = \mathbf{231\angle -30^\circ\ V}$$

$$\mathbf{V}_{bn} = \mathbf{231\angle -150^\circ\ V}$$

$$\mathbf{V}_{cn} = \mathbf{231\angle -270^\circ\ V}$$

(b) For the acb sequence,

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0^\circ - V_p\angle 120^\circ$$

$$\mathbf{V}_{ab} = V_p\left(1 + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = V_p\sqrt{3}\angle -30^\circ$$

i.e. in the acb sequence, $\mathbf{V}_{ab}$ lags $\mathbf{V}_{an}$ by 30°.

Hence, if $\mathbf{V}_{ab} = 400$, then

$$\mathbf{V}_{an} = \frac{400}{\sqrt{3}}\angle 30^\circ = \mathbf{231\angle 30^\circ\ V}$$

$$\mathbf{V}_{bn} = \mathbf{231\angle 150^\circ\ V}$$

$$\mathbf{V}_{cn} = \mathbf{231\angle -90^\circ\ V}$$

Solution 12.2

Since phase c lags phase a by 120°, this is an **acb sequence**.

$$\mathbf{V}_{bn} = 120\angle(30° + 120°) = \mathbf{120\angle 150°}\ \boldsymbol{V}$$

Solution 12.3

Given a balanced Y-connected three-phase generator with a line-to-line voltage of $\mathbf{V_{ab}} = 100\angle 45°$ V and $\mathbf{V_{bc}} = 100\angle 165°$ V, determine the phase sequence and the value of $\mathbf{V_{ca}}$.

Solution

Since $\mathbf{V_{bc}}$ leads $\mathbf{V_{ab}}$ by 120° we have a **acb** sequence and $\mathbf{V_{ca}} = \mathbf{100\angle{-75°}\ V}$.

Solution 12.4

Knowing the line-to-line voltages we can calculate the wye voltages and can let the value of V_a be a reference with a phase shift of zero degrees.

$V_L = 440 = \sqrt{3}\, V_p$ or $V_p = 440/1.7321 = 254$ V or $\mathbf{V_{an}} = 254\angle 0°$ V which determines, using abc rotation, both $\mathbf{V_{bn}} = 254\angle{-120°}$ and $\mathbf{V_{cn}} = 254\angle 120°$.

$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y = 254/(40\angle 30°) = \mathbf{6.35\angle{-30}°\ A}$

$\mathbf{I}_b = \mathbf{I}_a\angle{-120}° = \mathbf{6.35\angle{-150}°\ A}$

$\mathbf{I}_c = \mathbf{I}_a\angle{+120}° = \mathbf{6.35\angle 90°\ A}$

Solution 12.5

$\mathbf{V}_{AB} = 1.7321\text{x}\mathbf{V}_{AN}\angle{+30^\circ} = 207.8\angle(32^\circ{+}30^\circ) = 207.8\angle 62^\circ$ V or

v_{AB} = **$207.8\cos(\omega t+62^\circ)$ V**
which also leads to,
v_{BC} = **$207.8\cos(\omega t-58^\circ)$ V**

and
v_{CA} = **$207.8\cos(\omega t+182^\circ)$ V**

$207.8\cos(\omega t+62^\circ)$ V, $207.8\cos(\omega t-58^\circ)$ V, $207.8\cos(\omega t+182^\circ)$ V

Solution 12.6

Using Fig. 12.41, design a problem to help other students to better understand balanced wye-wye connected circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the Y-Y circuit of Fig. 12.41, find the line currents, the line\ voltages, and the load voltages.

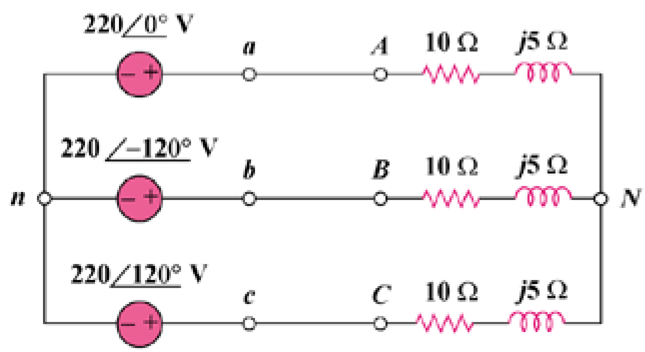


Figure 12.41

Solution

$$\mathbf{Z}_Y = 10 + j5 = 11.18\angle 26.56°$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{220\angle 0°}{11.18\angle 26.56°} = \mathbf{19.68\angle -26.56°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{19.68\angle -146.56°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{19.68\angle 93.44°\ A}$$

The line voltages are

$$\mathbf{V}_{ab} = 220\sqrt{3}\angle 30° = \mathbf{381\angle 30°\ V}$$

$$\mathbf{V}_{bc} = \mathbf{381\angle -90°\ V}$$

$$\mathbf{V}_{ca} = \mathbf{381\angle -210°\ V}$$

The load voltages are

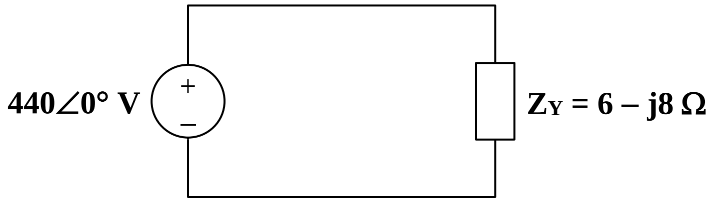
$$\mathbf{V}_{AN} = \mathbf{I}_a\mathbf{Z}_Y = \mathbf{V}_{an} = \mathbf{220\angle 0°\ V}$$

$$\mathbf{V}_{BN} = \mathbf{V}_{bn} = \mathbf{220\angle -120°\ V}$$

$$\mathbf{V}_{CN} = \mathbf{V}_{cn} = \mathbf{220\angle 120°\ V}$$

Solution 12.7

This is a balanced Y-Y system.



Using the per-phase circuit shown above,

$$\mathbf{I}_a = \frac{440\angle 0°}{6 - j8} = \mathbf{44\angle 53.13°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120° = \mathbf{44\angle -66.87°\ A}$$

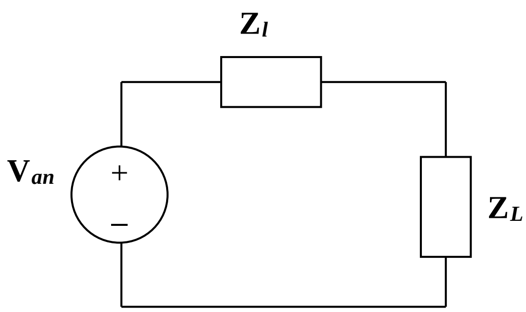
$$\mathbf{I}_c = \mathbf{I}_a \angle 120° = \mathbf{44\angle 173.13°\ A}$$

Solution 12.8

In a balanced three-phase wye-wye system, the source is an acb-sequence of voltages and $\mathbf{V}_{cn} = 120\angle 35^\circ$ V rms. The line impedance per phase is $(1+j2)\Omega$, while the per phase impedance of the load is $(11+j14)\ \Omega$. Calculate the line currents and the load voltages.

Solution

Consider the per phase equivalent circuit shown below.



Since the sequence is acb and $\mathbf{V}_{cn} = 120\angle 35^\circ$ V, then $\mathbf{V}_{an} = 120\angle 155^\circ$ V, and $\mathbf{V}_{bn} = 120\angle -85^\circ$ V.

$\mathbf{I}_a = \mathbf{V}_{an}/(\mathbf{Z}_l + \mathbf{Z}_L) = (120\angle 155^\circ)/(12+j16) = (120\angle 155^\circ)/(20\angle 53.13^\circ)$

$= \mathbf{6\angle 101.87^\circ}$ **amps**.

$\mathbf{I}_b = \mathbf{I}_a \angle 120^\circ = \mathbf{6\angle 221.87^\circ}$ **amps**.

$\mathbf{I}_c = \mathbf{I}_a \angle -120^\circ = \mathbf{6\angle -18.13^\circ}$ **amps**.

$\mathbf{V}_{La} = \mathbf{I}_a\mathbf{Z}_L = (6\angle 101.87^\circ)(11+j14) = (6\angle 101.87^\circ)(17.8045\angle 51.843^\circ)$

$= \mathbf{106.83\angle 153.71^\circ}$ **volts**.

$\mathbf{V}_{Lb} = \mathbf{V}_{La} \angle 120^\circ = \mathbf{106.83\angle -86.29^\circ}$ **volts**.

$\mathbf{V}_{Lc} = \mathbf{V}_{La} \angle -120^\circ = \mathbf{106.83\angle 33.71^\circ}$ **volts**.

Solution 12.9

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}_Y} = \frac{120\angle 0°}{20 + j15} = \mathbf{4.8\angle -36.87°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{4.8\angle -156.87°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{4.8\angle 83.13°\ A}$$

As a balanced system, $\mathbf{I}_n = \mathbf{0\ A}$

Solution 12.10

For the circuit in Fig. 12.43, determine the current in the neutral line.

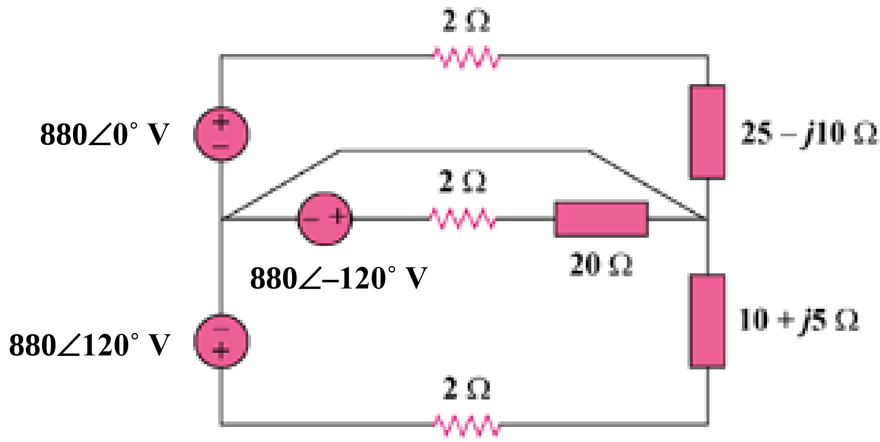


Figure 12.43
For Probs. 12.10 and 12.58.

Solution

Since the neutral line is present, we can solve this problem on a per-phase basis.

For phase a,

$$\mathbf{I_a} = \frac{\mathbf{V_{an}}}{\mathbf{Z_A} + 2} = \frac{880\angle\mathbf{0}°}{27 - \mathbf{j}10} = \frac{880}{28.7924\angle -20.323°} = 30.564\angle\mathbf{20.323}°$$

For phase b,

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_B + 2} = \frac{880\angle\text{-}120°}{22} = 40\angle\text{-}120°$$

For phase c,

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_C + 2} = \frac{880\angle 120°}{12 + j5} = \frac{880\angle 120°}{13\angle 22.62°} = 67.69\angle 97.38°$$

The current in the neutral line is

$$\mathbf{I}_\text{n} = \text{-}(\mathbf{I}_\text{a} + \mathbf{I}_\text{b} + \mathbf{I}_\text{c}) \text{ or } \text{-}\mathbf{I}_\text{n} = \mathbf{I}_\text{a} + \mathbf{I}_\text{b} + \mathbf{I}_\text{c}$$

$$\text{-}\mathbf{I_n} = (28.661 + \mathbf{j}10.6152) + (-20 - \mathbf{j}34.641) + (-8.6947 + \mathbf{j}67.129)$$

$\mathbf{I_n}$ = 0.0337 – j43.103 = **43.1∠–89.96° A**

Solution 12.11

In the wye-delta system shown in Fig. 12.44, the source is a positive sequence with $\mathbf{V}_{an}$ = 440∠0˚ V and phase impedance $\mathbf{Z}_P$ = (2 – j3) Ω. Calculate the line voltage V_L and the line current I_L.

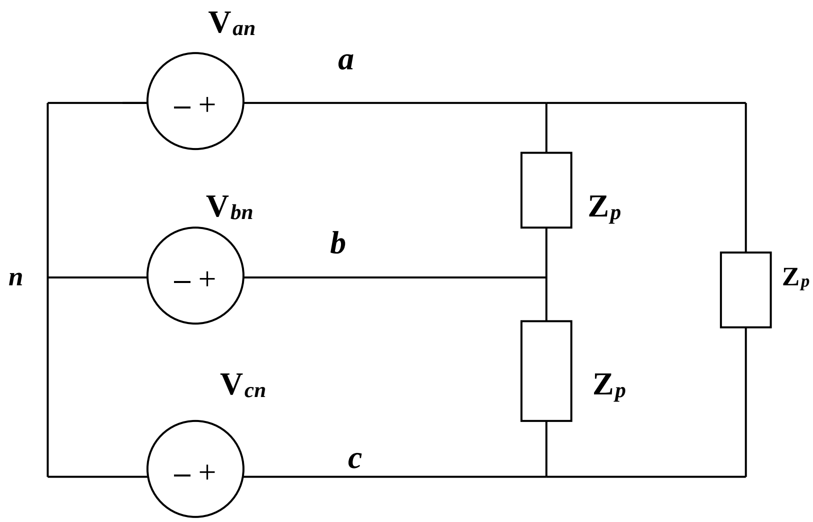


Figure 12.44
For Prob. 12.11.

Solution

Given that V_p = 440 and that the system is balanced, $V_L = 1.7321V_p$ = **762.1 V**.

$I_p = V_L/|2–j3|$ = 762.12/3.6056 = 211.37 A and

I_L = 1.7321x211.37 = **366.1 A**.

Solution 12.12

Using Fig. 12.45, design a problem to help other students to better understand wye-delta connected circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Solve for the line currents in the **Y-Δ** circuit of Fig. 12.45. Take $\mathbf{Z}_\Delta = 60\angle 45°\Omega$.

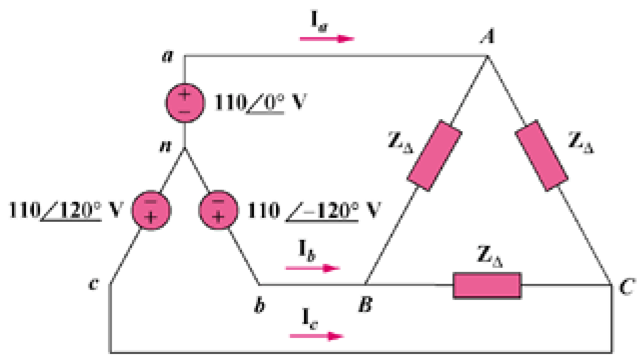
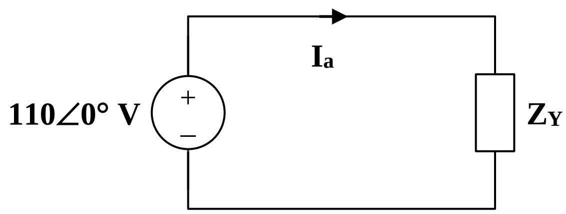


Figure 12.45

Solution

Convert the delta-load to a wye-load and apply per-phase analysis.



$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 45°\,\Omega$$

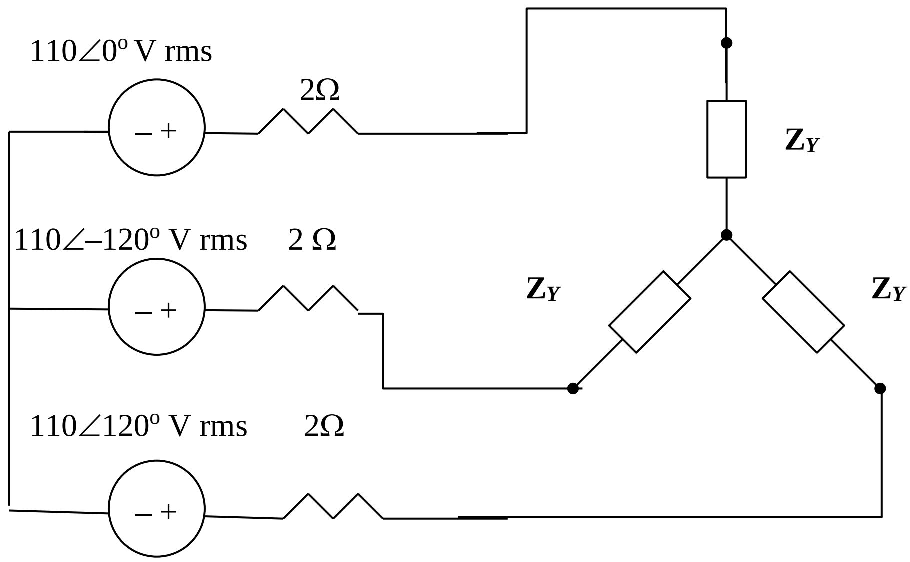
$$\mathbf{I}_a = \frac{110\angle 0°}{20\angle 45°} = \mathbf{5.5\angle -45°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{5.5\angle -165°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{5.5\angle 75°\ A}$$

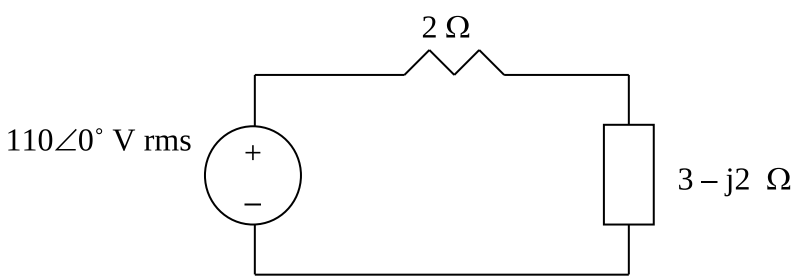
Solution 12.13

Convert the delta load to wye as shown below.



$$Z_Y = \frac{1}{3} Z_\Delta = 3 - j2\ \Omega$$

We consider the single phase equivalent shown below.



$\mathbf{I_a} = 110/(2 + 3 - j2) = 20.43\angle 21.8^\circ$ A

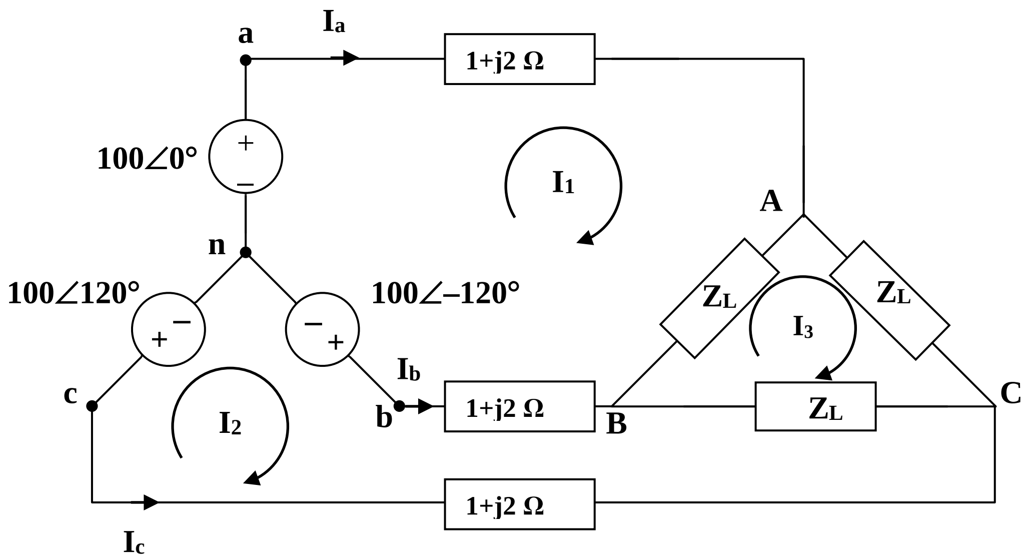
$I_L = |\mathbf{I_a}| = \mathbf{20.43\ A}$

$S = 3|\mathbf{I}_a|^2 Z_Y = 3(20.43)^2(3 - j2) = 4514\angle{-33.96^\circ} = 3744 - j2522$

P = Re(S) = **3.744 kW**.

Solution 12.14

We apply mesh analysis with $Z_L = (12+j12)\ \Omega$.



For mesh 1,

$$-100 + 100\angle -120^o + I_1(14 + j16) - (1 + j2)I_2 - (12 + j12)I_3 = 0 \text{ or}$$

$$(14 + j16)I_1 - (1 + j2)I_2 - (12 + j12)I_3 = 100 + 50 - j86.6 = 150 + j86.6 \qquad (1)$$

For mesh 2,

$$100\angle 120^o - 100\angle -120^o - I_1(1 + j2) - (12 + j12)I_3 + (14 + j16)I_2 = 0 \text{ or}$$

$$-(1 + j2)I_1 + (14 + j16)I_2 - (12 + j12)I_3 = -50 - j86.6 + 50 - j86.6 = -j173.2 \qquad (2)$$

For mesh 3,

$$-(12 + j12)I_1 - (12 + j12)I_2 + (36 + j36)I_3 = 0 \text{ or } \mathbf{I_3 = I_1 + I_2} \qquad (3)$$

Solving for $\mathbf{I_1}$ and $\mathbf{I_2}$ using (1) to (3) gives

$\mathbf{I_1} = 12.804\angle{-50.19°}$ A = (8.198 – j9.836) A and
$\mathbf{I_2} = 12.804\angle{-110.19°}$ A = (–4.419 – j12.018) A

$$\mathbf{I_a = I_1 = 12.804\angle{-50.19°}\ A}$$

$$\mathbf{I_b = I_2 - I_1 = 12.804\angle{-170.19°}\ A}$$

$$\mathbf{I}_c = -\mathbf{I}_2 = 12.804\angle 69.81° \text{ A}$$

As a check we can convert the delta into a wye circuit. Thus,

$\mathbf{Z_Y}$ = (12+j12)/3 = 4+j4 and $\mathbf{I_a}$ = 100/(1+j2+4+j4) = 100/(5+j6)
= 100/(7.8102∠50.19°) =

12.804 ∠–50.19° A.

So, the answer does check.

Solution 12.15

Convert the delta load, $\mathbf{Z}_\Delta$, to its equivalent wye load.

$$\mathbf{Z}_{Ye} = \frac{\mathbf{Z}_\Delta}{3} = 8 - j10$$

$$\mathbf{Z}_p = \mathbf{Z}_Y \parallel \mathbf{Z}_{Ye} = \frac{(12 + j5)(8 - j10)}{20 - j5} = 8.076\angle{-14.68°}$$

$$\mathbf{Z}_p = 7.812 - j2.047$$

$$\mathbf{Z}_T = \mathbf{Z}_p + \mathbf{Z}_L = 8.812 - j1.047$$

$$\mathbf{Z}_T = 8.874\angle{-6.78°}$$

We now use the per-phase equivalent circuit.

$$\mathbf{I}_a = \frac{V_p}{\mathbf{Z}_p + \mathbf{Z}_L}, \qquad \text{where} \quad V_p = \frac{210}{\sqrt{3}}$$

$$\mathbf{I}_a = \frac{210}{\sqrt{3}\,(8.874\angle{-6.78°})} = 13.66\angle 6.78°$$

$$I_L = \left|\mathbf{I}_a\right| = \mathbf{13.66\ A}$$

Solution 12.16

A balanced delta-connected load has a phase current $\mathbf{I}_{AC} = 5\angle{-30°}$ A.

(a) Determine the three line currents assuming that the circuit operates in the positive phase sequence.

(b) Calculate the load impedance if the line voltage is $\mathbf{V}_{AB} = 440\angle 0°$ V.

Solution

(a) $\mathbf{I}_{CA} = -\mathbf{I}_{AC} = 5\angle(-30° + 180°) = 5\angle 150°$

This implies that

$$\mathbf{I}_{AB} = 5\angle 30°$$
$$\mathbf{I}_{BC} = 5\angle -90°$$

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30° = \mathbf{8.66\angle 0° \text{ A}}$$
$$\mathbf{I}_b = \mathbf{8.66\angle{-120}° \text{ A}}$$
$$\mathbf{I}_c = \mathbf{8.66\angle 120° \text{ A}}$$

(b) $\mathbf{Z}_\Delta = \dfrac{\mathbf{V}_{AB}}{\mathbf{I}_{AB}} = \dfrac{440\angle 0°}{5\angle 30°} = \mathbf{88\angle{-30}° \ \Omega}$.

Solution 12.17

A positive sequence wye connected source where $\mathbf{V_{an}} = 120\angle 90°$ V, is connected to a delta connected load where $\mathbf{Z_L} = (60+j45)\ \Omega$. Determine the line currents.

Solution

First the voltages are $\mathbf{V_{an}} = 120\angle 90°$ V, $\mathbf{V_{bn}} = 120\angle -30°$ V, and $\mathbf{V_{cn}} = 120\angle -150°$ V. The phase load is $\mathbf{Z_\Delta} = 75\angle 36.87°\ \Omega$.

$$\mathbf{Z_Y} = \mathbf{Z_\Delta}/3 = 25\angle 36.87°\ \Omega$$

Thus,

$$\mathbf{I_a} = \mathbf{V_{an}}/25\angle 36.87° = 120\angle 90°/25\angle 36.87° = \mathbf{4.8\angle 53.13°\ A}.$$
$$\mathbf{I_b} = 120\angle -30°/25\angle 36.87° = \mathbf{4.8\angle -66.87°\ A}.$$
$$\mathbf{I_c} = 120\angle -150°/25\angle 36.87° = \mathbf{4.8\angle 173.13°\ A}.$$

Solution 12.18

$$\mathbf{V}_{AB} = \mathbf{V}_{an}\sqrt{3}\angle 30° = (220\angle 60°)(\sqrt{3}\angle 30°) = 381.1\angle 90°$$

$$\mathbf{Z}_{\Delta} = 12 + j9 = 15\angle 36.87°$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{381.1\angle 90°}{15\angle 36.87°} = \mathbf{25.4\angle 53.13^\circ\ A}$$

$$\mathbf{I}_{\mathrm{BC}} = \mathbf{I}_{\mathrm{AB}}\angle -120° = \mathbf{25.4\angle{-66.87}^\circ\ A}$$

$$\mathbf{I}_{\mathrm{CA}} = \mathbf{I}_{\mathrm{AB}}\angle 120° = \mathbf{25.4\angle 173.13^\circ\ A}$$

Solution 12.19

For the Δ-Δ circuit of Fig. 12.50, calculate the phase and line currents.

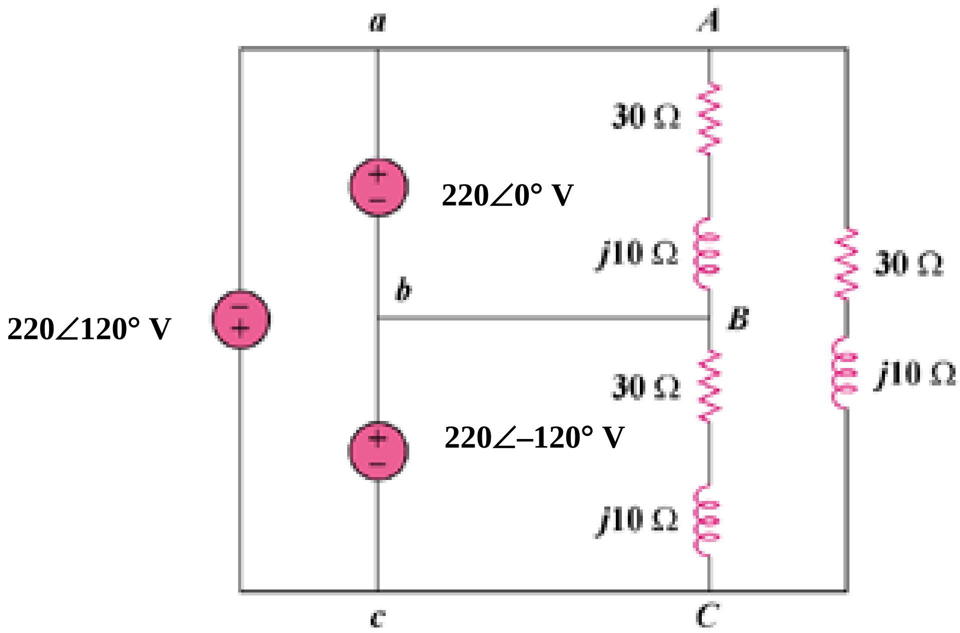


Figure 12.50
For Prob. 12.19.

Solution

$$\mathbf{Z}_\Delta = 30 + j10 = 31.62\angle 18.43°$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{440\angle 0°}{31.62\angle 18.43°} = \mathbf{13.915\angle{-18.43}°\ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle{-120°} = \mathbf{13.915\angle{-138.43}°\ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \mathbf{13.915\angle 101.57°\ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}\sqrt{3}\angle{-30°}$$

$$\mathbf{I}_a = 13.915\sqrt{3}\angle{-48.43°} = \mathbf{24.1\angle{-48.43}°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle{-120°} = \mathbf{24.1\angle{-168.43}°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{24.1\angle 71.57°\ A}$$

Solution 12.20

Using Fig. 12.51, design a problem to help other students to better understand balanced delta-delta connected circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Refer to the Δ-Δ circuit in Fig. 12.51. Find the line and phase currents. Assume that the load impedance is 12 + j9Ω per phase.

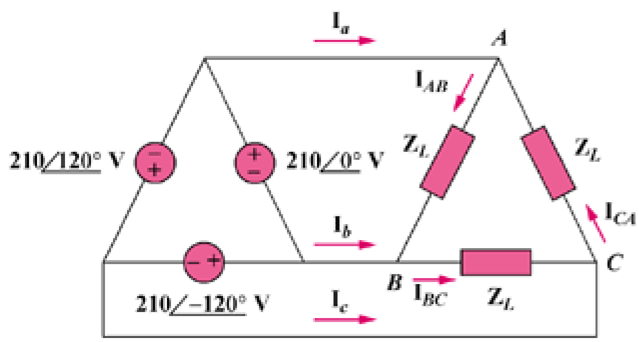


Figure 12.51

Solution

$$\mathbf{Z}_\Delta = 12 + j9 = 15\angle 36.87°$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{210\angle 0°}{15\angle 36.87°} = \mathbf{14\angle -36.87°\ A}$$
$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \mathbf{14\angle -156.87°\ A}$$
$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \mathbf{14\angle 83.13°\ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30° = \mathbf{24.25\angle -66.87°\ A}$$
$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{24.25\angle -186.87°\ A}$$
$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{24.25\angle 53.13°\ A}$$

Solution 12.21

Three 440-volt generators, form a delta connected source which is connected to a balanced delta connected load of $\mathbf{Z}_L$ = (8.66 + j5) Ω per phase as shown in Fig. 12.52. Determine the value of $\mathbf{I}_{BC}$ and $\mathbf{I}_{aA}$. What is the pf of the load?

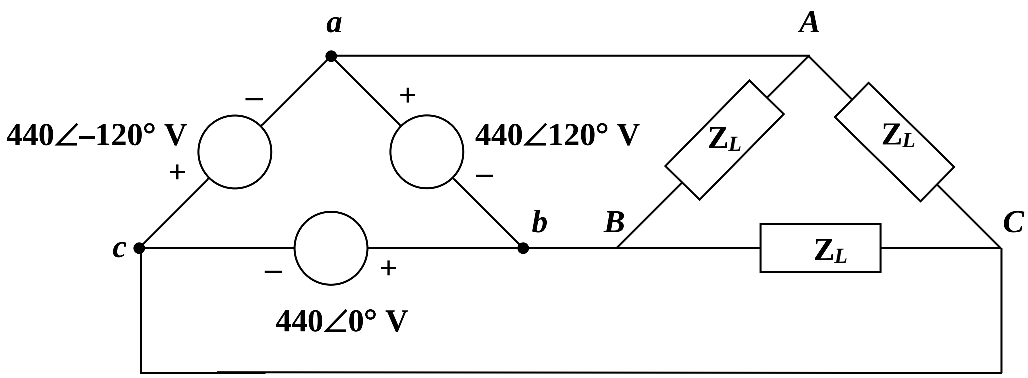


Figure 12.52
For Prob. 12.21.

Solution

$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_L = 440/(10\angle 30°) = \mathbf{44\angle{-30°}\ A}$.

$\mathbf{I}_{aA} = \mathbf{I}_{AC} + \mathbf{I}_{AB} = [440\angle 60°/(10\angle 30°)] + [440\angle 120°/(10\angle 30°)]$
$= [44\angle 30°] + [44\angle 90°] = 38.105 + j22 + j44 = 38.105 + j66 = \mathbf{76.21\angle 60°\ A}$.

pf = 8.66/10 = **0.866**.

Solution 12.22

Find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$ in the three-phase network of Fig. 12.53 below. Take $\mathbf{Z}_L$= (114 + j87) Ω and $\mathbf{Z}_l$ = (2 + j) Ω.

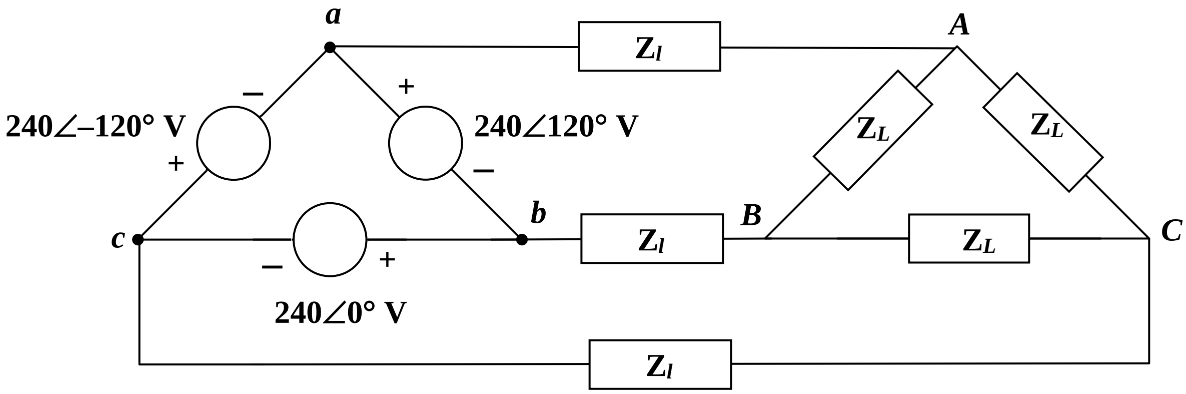
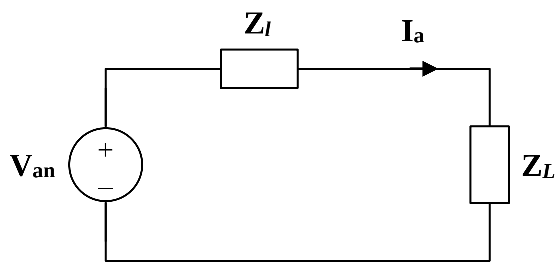


Figure 12.53
For Prob. 12.22.

Solution

Due to the line impedances, converting the Δ-connected source to a Y-connected source will make solving this problem easier.



Therefore,

$\mathbf{V}_{an} = \frac{240}{\sqrt{3}}\angle 90^\circ = 138.564\angle 90^\circ$ V , $\mathbf{V}_{bn}$ = 138.564∠–30° V, and $\mathbf{V}_{cn}$ = 138.564∠–150° V. The angles for the wye connected sources can be seen graphically by noting that the above circuit accurately shows the angles associated with the delta connected source and that the corresponding wye connected sources connect at the center, labeled n, of the delta connected sources. Also, $\mathbf{Z}_p$ = (114+j87)/3 = (38 + j29) Ω.

Finally, $\mathbf{I}_{aA}$ = 138.564∠90°/[38+2+j(29+1)] =138.564∠90°/(50∠36.87°) or

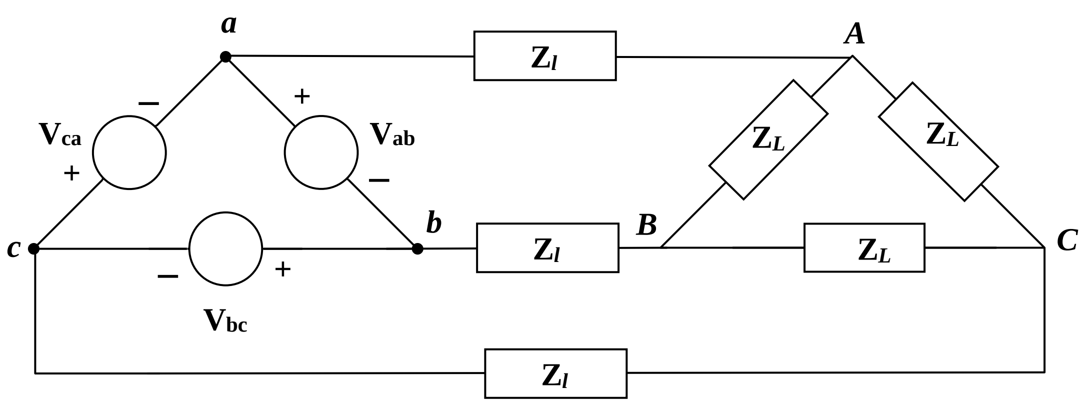
$\mathbf{I}_{aA}$ = 2.772∠53.13° A
$\mathbf{I}_{bB}$ = 2.772∠–66.87° A
$\mathbf{I}_{cC}$ = 2.772∠173.13° A

Solution 12.23*

A balanced delta connected source is connected to a balanced delta connected load where $\mathbf{Z}_L = (80 + j60)\ \Omega$ and $\mathbf{Z}_l = (2 + j)\ \Omega$. Given that the load voltages are $\mathbf{V}_{AB} = 100\angle 0^\circ$ V, $\mathbf{V}_{BC} = 100\angle 120^\circ$ V, and $\mathbf{V}_{CA} = 100\angle -120^\circ$ V. Calculate the source voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$.

Solution

We know that $\mathbf{I}_{aA} = \mathbf{I}_{AB} + \mathbf{I}_{AC} = \mathbf{V}_{AB}/\mathbf{Z}_L + \mathbf{V}_{AC}/\mathbf{Z}_L = [100/(100\angle 37.87^\circ)] + [100\angle 60^\circ/(100\angle 36.87^\circ)] = 1\angle -36.87^\circ + 1\angle 23.13^\circ = 0.8 - j0.6 + 0.91962 + j0.39282 = 1.71962 - j0.20718 = 1.7321\angle -6.87^\circ$ A, $\mathbf{I}_{bB} = \mathbf{I}_{BA} + \mathbf{I}_{BC} = \mathbf{V}_{BA}/\mathbf{Z}_L + \mathbf{V}_{BC}/\mathbf{Z}_L = [100\angle 180^\circ/(100\angle 37.87^\circ)] + [100\angle 120^\circ/(100\angle 36.87^\circ)] = 1\angle 143.13^\circ + 1\angle 83.13^\circ = -0.8 + j0.6 + 0.119617 + j0.99282 = -0.68038 + j1.59282 = 1.73205\angle 113.13^\circ$ A, and $\mathbf{I}_{cC} = \mathbf{I}_{CA} + \mathbf{I}_{CB} = \mathbf{V}_{CA}/\mathbf{Z}_L + \mathbf{V}_{CB}/\mathbf{Z}_L = [100\angle -120^\circ/(100\angle 37.87^\circ)] + [100\angle -60^\circ/(100\angle 36.87^\circ)] = 1\angle -157.87^\circ + 1\angle -96.87^\circ = -0.9263315 - j0.376709 - 0.119617 - j0.99282$
$= -1.0459485 - j1.369529 = 1.7233\angle -127.37^\circ$ A. Finally we need $\mathbf{Z}_l = 2.23607\angle 26.565^\circ$.



Clearly $\mathbf{V}_{ab} = \mathbf{I}_{aA}\mathbf{Z}_l + \mathbf{V}_{AB} - \mathbf{I}_{bB}\mathbf{Z}_l = (1.7321\angle -6.87^\circ)(2.23607\angle 26.56^\circ) + 100 - (1.73205\angle 113.13^\circ)(2.23607\angle 26.565^\circ) = 3.8731\angle 19.69^\circ + 100 - (3.873\angle 139.69^\circ) = 3.6466 + j1.30497 + 100 - (-2.95338 + j2.50553) = 106.6 - j1.20056 =$ **$106.61\angle -0.65^\circ$ V**,
$\mathbf{V}_{bc} = \mathbf{I}_{bB}\mathbf{Z}_l + \mathbf{V}_{BC} - \mathbf{I}_{cC}\mathbf{Z}_l = (1.73205\angle 113.13^\circ)(2.23607\angle 26.56^\circ) + 100\angle 120^\circ$
$- (1.7233\angle -127.37^\circ)(2.23607\angle 26.56^\circ) = 3.8534\angle 139.69^\circ + 100\angle 120^\circ$
$- (3.8534\angle -100.81^\circ) = -2.93843 + j2.4929 - 50 + j86.6 - (-0.72272 - j3.785)$
$= -52.216 + j92.878 =$ **$106.55\angle 119.34^\circ$ V**, and $\mathbf{V}_{ca} = \mathbf{I}_{cC}\mathbf{Z}_l + \mathbf{V}_{CA} - \mathbf{I}_{aA}\mathbf{Z}_l$
$= (1.7233\angle -127.37^\circ)(2.23607\angle 26.56^\circ) + 100\angle -120^\circ$
$- (1.7321\angle -6.87^\circ)(2.23607\angle 26.56^\circ) = 3.8534\angle -100.81^\circ - 50 - j86.6 - (3.8731\angle 19.69^\circ)$
$= -0.72272 - j3.785 - 50 - j86.6 - (3.6466 + j1.305)$
$= -54.369 - j91.69 =$ **$106.6\angle -120.67^\circ$ V**.

$\mathbf{V}_{ab}$ = **$106.61\angle -0.65^\circ$ V** , $\mathbf{V}_{bc}$ = **$106.55\angle 119.34^\circ$ V**, $\mathbf{V}_{ca}$ = **$106.6\angle -120.67^\circ$ V**.

Solution 12.24

A balanced delta-connected source has phase voltage $\mathbf{V}_{ab}$ = 880∠30° V and a positive phase sequence. If this is connected to a balanced delta-connected load, find the line and phase currents. Take the load impedance per phase as 60 ∠30°**Ω** and line impedance per phase as 1 + j1**Ω**.

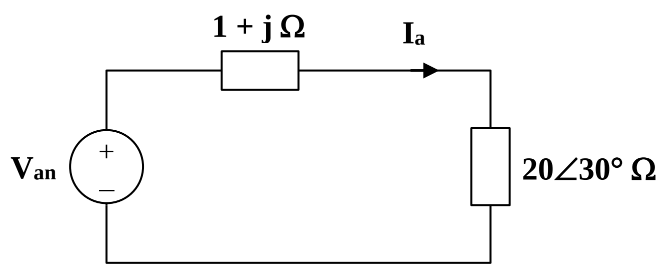
Solution

Convert both the source and the load to their wye equivalents.

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 30° = 17.32 + j10$$

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle\text{-}30° = 508.07\angle 0°$$

We now use per-phase analysis.



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{(1+j)+(17.32+j10)} = \frac{508.07}{21.37\angle 31°} = \mathbf{23.77\angle{-31}°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle\text{-}120° = \mathbf{23.77\angle{-151}°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{23.77\angle 89°\ A}$$

But $\quad \mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle\text{-}30°$

$$\mathbf{I}_{AB} = \frac{23.77\angle\text{-}31°}{\sqrt{3}\angle\text{-}30°} = \mathbf{13.724\angle{-1}°\ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle\text{-}120° = \mathbf{13.724\angle{-121}°\ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \mathbf{13.724\angle 119°\ A}$$

Solution 12.25

Convert the delta-connected source to an equivalent wye-connected source and consider the single-phase equivalent.

$$\mathbf{I}_a = \frac{440\angle(10° - 30°)}{\sqrt{3}\,\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = 3 + j2 + 10 - j8 = 13 - j6 = 14.318\angle -24.78°$

$$\mathbf{I}_a = \frac{440\angle -20°}{\sqrt{3}(14.318\angle -24.78°)} = \mathbf{17.742\angle 4.78°\ amps}.$$

$$\mathbf{I_b = I_a\angle{-120°} = 17.742\angle{-115.22°}\ amps}.$$

$$\mathbf{I_c = I_a\angle{+120°} = 17.742\angle 124.78°\ amps}.$$

Solution 12.26

Using Fig. 12.55, design a problem to help other students to better understand balanced delta connected sources delivering power to balanced wye connected loads.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the balanced circuit in Fig. 12.55, $\mathbf{V}_{ab} = 125\angle 0°$ V. Find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

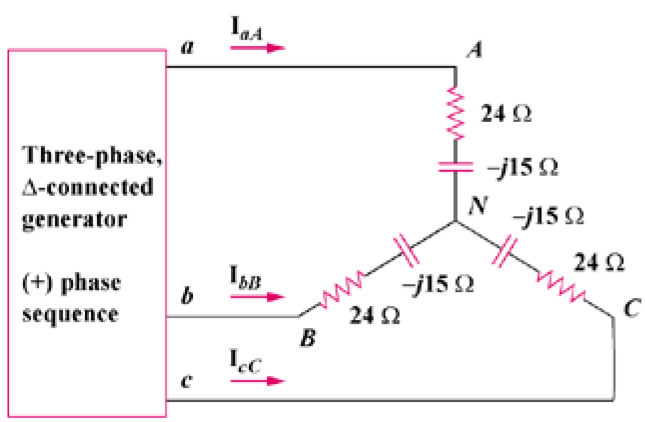


Figure 12.55

Solution

Transform the source to its wye equivalent.

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30° = 72.17\angle -30°$$

Now, use the per-phase equivalent circuit.

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}}, \qquad \mathbf{Z} = 24 - j15 = 28.3\angle -32°$$

$$\mathbf{I}_{aA} = \frac{72.17\angle -30°}{28.3\angle -32°} = \mathbf{2.55\angle 2°\ A}$$

$$\mathbf{I}_{bB} = \mathbf{I}_{aA}\angle -120° = \mathbf{2.55\angle -118°\ A}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{aA}\angle 120° = \mathbf{2.55\angle 122°\ A}$$

Solution 12.27

Since Z_L and Z_ℓ are in series, we can lump them together so that

$$Z_Y = 2 + j + 6 + j4 = 8 + j5$$

$$I_a = \frac{\frac{V_P}{\sqrt{3}} < -30^o}{Z_Y} = \frac{208 < -30^o}{\sqrt{3}(8 + j5)}$$

$$V_L = (6 + j4)I_a = \frac{208(0.866 - j0.5)(6 + j4)}{\sqrt{3}(8 + j5)} = 80.81 - j43.54$$

$|V_L| =$ **91.79 V**

Solution 12.28

The line-to-line voltages in a wye-load have a magnitude of 880 V and are in the positive sequence at 60 Hz. If the loads are balanced with $Z_1 = Z_2 = Z_3 = 25\angle 30^\circ$, find all line currents and phase voltages.

Solution

$V_L = |V_{ab}| = 880 = \sqrt{3}V_P$ or V_P = 880/1.7321 = 508.05

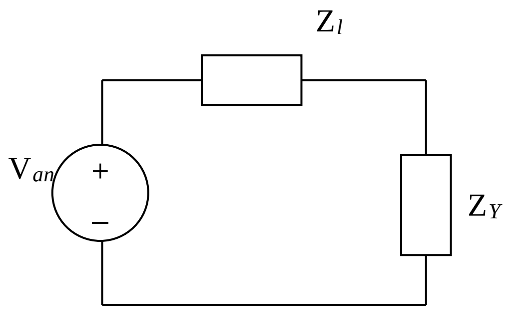
For reference, let V_{AN} = **$508.05\angle 0^\circ$ V** which leads to
V_{BN} = **$508.05\angle{-120^\circ}$ V** and V_{CN} = **$508.05\angle 120^\circ$ V**.

The line currents are found as follows,

$I_a = V_{AN}/Z_Y = 508.05/25\angle 30^\circ$ = **$20.32\angle{-30^\circ}$ A**.
This leads to, I_b = **$20.32\angle{-150^\circ}$ A** and I_c = **$20.32\angle 90^\circ$ A**.

Solution 12.29

We can replace the delta load with a wye load, $Z_Y = Z_\Delta/3 = 17+j15\Omega$. The per-phase equivalent circuit is shown below.



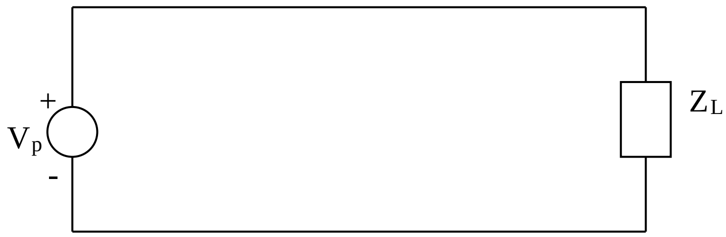
$\mathbf{I_a} = \mathbf{V_{an}}/|\mathbf{Z}_Y + \mathbf{Z}_l| = 240/|17+j15+0.4+j1.2| = 240/|17.4+j16.2| = 240/23.77 = 10.095$

$\mathbf{S} = 3[(I_a)^2(17+j15)] = 3x101.91(17+j15)$

$= \mathbf{[5.197+j4.586]\ kVA}$.

Solution 12.30

Since this a balanced system, we can replace it by a per-phase equivalent, as shown below.



$$\overline{S} = 3\overline{S}_p = \frac{3V^2_p}{Z^*_p}, \qquad V_p = \frac{V_L}{\sqrt{3}}$$

$$\overline{S} = \frac{V^2_L}{Z^*_p} = \frac{(208)^2}{30\angle -45^o} = 1.4421\angle 45^o \text{ kVA}$$

$$\boldsymbol{P = S\cos\theta = \underline{1.02\,\text{kW}}}$$

Solution 12.31

A balanced delta-connected load is supplied by a 60-Hz three-phase source with a line voltage of 480V. Each load phase draws 24 kW at a lagging power factor of 0.8. Find:

(a) the load impedance per phase
(b) the line current
(c) the value of capacitance needed to be connected in parallel with each load phase to minimize the current from the source.

Solution

(a)

$$P_p = 24{,}000, \qquad \cos\theta = 0.8, \qquad S_p = \frac{P_P}{\cos\theta} = 24/0.8 = 30\text{kVA} \text{ and } \theta = 36.87°$$

$$Q_p = S_P \sin\theta = 18\,\text{kVAR}$$

$$\overline{S} = 3\overline{S}_p = 3(24 + j18) = 72 + j54\,\text{kVA}$$

For delta-connected load, $V_p = V_L = 480$ (rms). But

$$\overline{S} = \frac{3V^2{}_p}{Z^*{}_p} \longrightarrow Z^*{}_p = \frac{3V^2{}_p}{S} = \frac{3(480)^2}{(72 + j54)x10^3}, \qquad \underline{Z_P = [6.144 + j4.608]\Omega}$$

(b) $$P_p = \sqrt{3}V_L I_L \cos\theta \longrightarrow I_L = \frac{24{,}000}{\sqrt{3}x480x0.8} = \mathbf{36.08\ A}$$

(c) We find C to bring the power factor to unity

$$Q_c = Q_p = 18\,\text{kVA} \longrightarrow C = \frac{Q_c}{\omega V^2{}_{rms}} = \frac{18{,}000}{2\pi x60x480^2} = \mathbf{207.2\ \mu F}.$$

Solution 12.32

Design a problem to help other students to better understand power in a balanced three-phase system.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A balanced wye load is connected to a 60-Hz three-phase source with $V_{ab} = 240\angle 0^\circ$V. The load has lagging pf =0.5 and each phase draws 5 kW. (a) Determine the load impedance Z_Y. (b) Find I_a, I_b, and I_c.

Solution

(a) $|V_{ab}| = \sqrt{3}V_p = 240 \quad \longrightarrow \quad V_p = \dfrac{240}{\sqrt{3}} = 138.56$

$$V_{an} = V_p < -30^o$$

$$pf = 0.5 = \cos\theta \quad \longrightarrow \quad \theta = 60^o$$

$$P = S\cos\theta \quad \longrightarrow \quad S = \frac{P}{\cos\theta} = \frac{5}{0.5} = 10 \text{ kVA}$$

$$Q = S\sin\theta = 10\sin 60 = 8.66$$

$$S_p = 5 + j8.66 \text{ kVA}$$

But

$$S_P = \frac{V_p^2}{Z_p^*} \quad \longrightarrow \quad Z_p^* = \frac{V_p^2}{S_p} = \frac{138.56^2}{(5 + j8.66)x10^3} = 0.96 - j1.663$$

$\mathbf{Z_P = [0.96 + j1.663]\ \Omega}$

(b) $I_a = \dfrac{V_{an}}{Z_Y} = \dfrac{138.56 < -30^o}{0.96 + j1.6627} = \underline{72.17 < -90^o \text{ A}}$ = **72.17∠–90° A**

$I_b = I_a < -120^o = \underline{72.17 < -210^o \text{ A}}$ = **72.17∠150° A**

$I_c = I_a < +120^o = \underline{72.17 < 30^o \text{ A}}$ = **72.17∠30° A**

Solution 12.33

$\mathbf{S} = \sqrt{3}\,V_L I_L \angle\theta$

$S = \left|\mathbf{S}\right| = \sqrt{3}\,V_L I_L$

For a Y-connected load,

$I_L = I_p, \qquad V_L = \sqrt{3}\,V_p$

$S = 3\,V_p I_p$

$$I_L = I_p = \frac{S}{3V_p} = \frac{4800}{(3)(208)} = \mathbf{7.69\ A}$$

$$V_L = \sqrt{3}\,V_p = \sqrt{3} \times 208 = \mathbf{360.3\ V}$$

Solution 12.34

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}}$$

$$\mathbf{I}_a = \frac{V_p}{\mathbf{Z}_Y} = \frac{220}{\sqrt{3}(10 - j16)} = \frac{127.02}{18.868\angle -58^\circ} = 6.732\angle 58^\circ$$

$I_L = I_p =$ **6.732A**

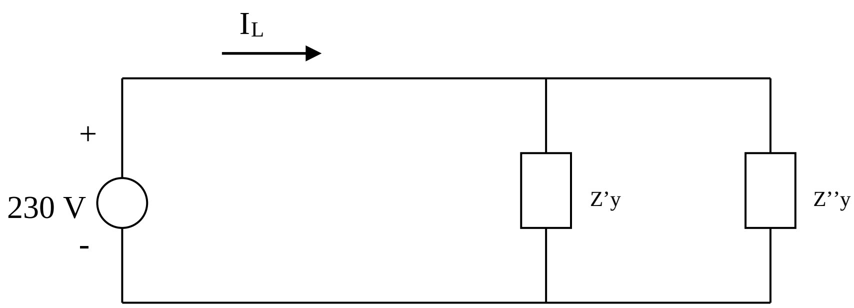
$$\mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta = \sqrt{3} \times 220 \times 6.732\angle -58^\circ = 2565\angle -58^\circ$$

S = [1.3592–j2.175] kVA

Solution 12.35

(a) This is a balanced three-phase system and we can use per phase equivalent circuit. The delta-connected load is converted to its wye-connected equivalent

$$Z''_y = \frac{1}{3} Z_\Delta = (60 + j30)/3 = 20 + j10$$



$$Z_y = Z'_y // Z''_y = (40 + j10)//(20 + j10) = 13.5 + j5.5$$

$$\mathbf{I_L = \frac{230}{13.5 + j5.5} = [14.61 - j5.953]\ A}$$

(b) $\mathbf{S = 3V_s I^*_L = [10.081 + j4.108]\ kVA}$

(c) pf = P/S = **0.9261**

Solution 12.36

(a) S = 1 [0.75 + sin(cos^{-1}0.75)] = **0.75 + j0.6614 MVA**

(b) $\overline{S} = 3V_p I^*{}_p \quad \longrightarrow \quad I^*{}_p = \frac{S}{3V_p} = \frac{(0.75 + j0.6614)x10^6}{3x4200} = 59.52 + j52.49$

$$\boldsymbol{P_L = |I_p|^2 R_l = (79.36)^2 (4) = \underline{25.19\,\mathrm{kW}}}$$

(c) $\boldsymbol{V_s = V_L + I_p(4 + j) = 4.4381 - j0.21\,\mathrm{kV} = \underline{4.443\angle -2.709^o\ \mathrm{kV}}}$

Solution 12.37

The total power measured in a three-phase system feeding a balanced wye-connected load is 12 kW at a power factor of 0.6 leading. If the line voltage is 440 V, calculate the line current $\boldsymbol{I_L}$ and the load impedance $\mathbf{Z}_{\boldsymbol{Y}}$.

Solution

$$S = \frac{P}{pf} = \frac{12}{0.6} = 20\,\text{kVA} \quad \text{also } \theta = -53.13°$$

$\mathbf{S}$ = S∠θ = 20,000∠–53.13° = [12–j16] kVA.

But $$\mathbf{S} = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \angle\theta = \sqrt{3} V_L I_L \angle\theta$$

$$I_L = \frac{20\times10^3}{\sqrt{3}\times440} = \mathbf{26.24\ A}$$

$$\mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p$$

For a Y-connected load, $I_L = I_p$.

$$\mathbf{Z}_p = \frac{\mathbf{S}}{3\left|I_L\right|^2} = \frac{(12-j16)\times10^3}{(3)(26.2432)^2} = \frac{(12-j16)x10^3}{2066.117}$$

$$\mathbf{Z}_p = \mathbf{(5.808 - j7.744)\ \Omega}$$

Solution 12.38

As a balanced three-phase system, we can use the per-phase equivalent shown below.

$$\mathbf{I}_a = \frac{110\angle 0°}{(1+j2)+(9+j12)} = \frac{110\angle 0°}{10+j14}$$

$$\mathbf{S}_p = |\mathbf{I}_a|^2 \mathbf{Z}_Y = \frac{(110)^2}{(10^2+14^2)} \cdot (9+j12)$$

The complex power is

$$\mathbf{S} = 3\mathbf{S}_p = 3\frac{(110)^2}{296} \cdot (9+j12)$$

$$\mathbf{S} = \mathbf{(1.1037+j1.4716)\ kVA}$$

Solution 12.39

Find the real power absorbed by the load in Fig. 12.58.

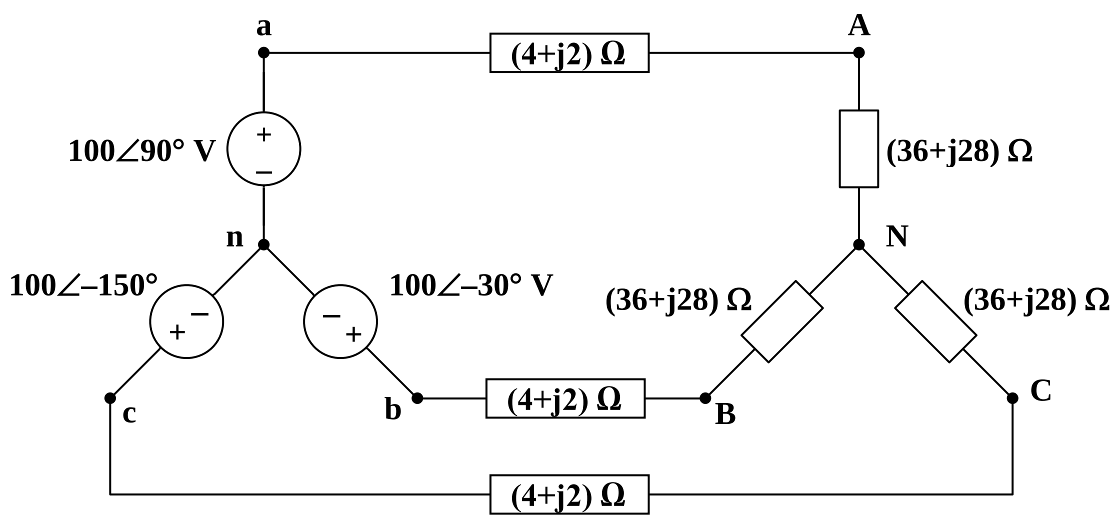


Figure 12.58
For Prob. 12.39.

Solution

To find power delivered to the load, we need to determine the current through the load. Since the load is balanced, the current through the load is equal to

$\mathbf{I_{aA}} = \mathbf{V_{an}}/(\mathbf{Z_l}+\mathbf{Z_L}) = j100/(4+j2 + 36+j28) = j100/(40+j30) = j100/(50\angle 36.87°) = 2\angle 53.13°$ A.

$P = (I_{aA})(36)(I_{aA})^* = (2)^2(36) = 144$ W for a total power absorbed equal to

$P_{Tot} = 3x144 =$ **432 W**.

Solution 12.40

Transform the delta-connected load to its wye equivalent.

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 7 + j8$$

Using the per-phase equivalent circuit above,

$$\mathbf{I}_a = \frac{100\angle 0°}{(1 + j0.5) + (7 + j8)} = 8.567\angle -46.75°$$

For a wye-connected load,

$$I_p = I_a = |\mathbf{I}_a| = 8.567$$

$$\mathbf{S} = 3|\mathbf{I}_p|^2 \mathbf{Z}_p = (3)(8.567)^2 (7 + j8)$$

$$P = \text{Re}(\mathbf{S}) = (3)(8.567)^2 (7) = \mathbf{1.541\ kW}$$

Solution 12.41

$$S = \frac{P}{pf} = \frac{5\ \text{kW}}{0.8} = 6.25\ \text{kVA}$$

But $\quad S = \sqrt{3}\, V_L I_L$

$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{6.25 \times 10^3}{\sqrt{3} \times 400} = \mathbf{9.021\ A}$$

Solution 12.42

The load determines the power factor.

$$\tan\theta = \frac{40}{30} = 1.333 \quad \longrightarrow \quad \theta = -53.13^\circ$$

$$\text{pf} = \cos\theta = 0.6 \quad \text{(leading)}$$

$$\mathbf{S} = 7.2 - j\left(\frac{7.2}{0.6}\right)(0.8) = 7.2 - j9.6 \text{ kVA}$$

But $\quad \mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p$

$$\left|\mathbf{I}_p\right|^2 = \frac{\mathbf{S}}{3\mathbf{Z}_p} = \frac{(7.2 - j9.6)\times 10^3}{(3)(30 - j40)} = 80$$

$$I_p = 8.944 \text{ A}$$

$$I_L = I_p = \mathbf{8.944\ A}$$

$$V_L = \frac{S}{\sqrt{3}\,I_L} = \frac{12\times 10^3}{\sqrt{3}\,(8.944)} = \mathbf{774.6\ V}$$

Solution 12.43

$\mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p$, $I_p = I_L$ for Y-connected loads

$\mathbf{S} = (3)(13.66)^2(7.812 - j2.047)$

$$\mathbf{S} = \mathbf{[4.373 - \mathit{j}1.145]}\ \boldsymbol{kVA}$$

Solution 12.44

For a Δ-connected load,

$$V_p = V_L, \qquad I_L = \sqrt{3}\, I_p$$

$$S = \sqrt{3}\, V_L I_L$$

$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{\sqrt{(12^2 + 5^2)} \times 10^3}{\sqrt{3}\,(240)} = 31.273$$

At the source,

$$\mathbf{V}_L' = \mathbf{V}_L + \mathbf{I}_L \mathbf{Z}_l + \mathbf{I}_L \mathbf{Z}_l$$

$$\mathbf{V}_L' = 240\angle 0° + 2(31.273)(1 + j3) \ = 240+62.546+j187.638$$

$$\mathbf{V}_L' = 302.546+j187.638 = 356\angle 31.81°$$

$$\left|\mathbf{V}_L'\right| = \mathbf{356\ V}$$

Also, at the source,

S' = $3(31.273)^2(1+j3) + (12{,}000+j5{,}000) = 2{,}934+12{,}000+j(8{,}802+5{,}000)$
= $14{,}934+j13{,}802 = 20{,}335\angle 42.744°$ thus, $\theta = 42.744°$.

pf = cos(42.744°) = **0.7344**

Checking, $V_Y = 240/1.73205 = 138.564$, **S** $= 3(138.564)^2/(Z_Y)^* = 12{,}000+l5{,}000$, and $Z_Y = 57{,}600/(12{,}000–j5{,}000) = 57.6/(13\angle –22.62°) = 4.4308\angle 22.62° = 4.09+j1.70416$. The total load seen by the source is $1+j3+4.09+j1.70416 = 5.09+j4.70416 = 6.9309\angle 42.74°$ per phase. This leads to $\theta = \text{Tan}^{-1}(4.70416/5.09) = \text{Tan}^{-1}(0.9242) = 42.744°$. Clearly, the answer checks. $I_l = 138.564/4.4308 = 31.273$ A. Again the answer checks. Finally, $3(31.273)^2(5.09+j4.70416) = 2{,}934(6.9309\angle 42.74°) = 20{,}335\angle 42.74°$, the same as we calculated above.

Solution 12.45

$$\mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta$$

$$I_L = \frac{|\mathbf{S}|\angle -\theta}{\sqrt{3}\, V_L}, \qquad |\mathbf{S}| = \frac{P}{pf} = \frac{450\times 10^3}{0.708} = 635.6 \text{ kVA}$$

$$\mathbf{I}_L = \frac{(635.6)\angle -\theta}{\sqrt{3}\times 440} = 834\angle -45^\circ \text{ A}$$

At the source,

$$\mathbf{V}_L = 440\angle 0^\circ + \mathbf{I}_L (0.5 + j2)$$
$$\mathbf{V}_L = 440 + (834\angle -45^\circ)(2.062\angle 76^\circ)$$
$$\mathbf{V}_L = 440 + 1719.7\angle 31^\circ$$
$$\mathbf{V}_L = 1914.1 + j885.7$$

$$\mathbf{V}_L = \mathbf{2.109\angle 24.83^\circ \text{ V}}$$

Solution 12.46

For the wye-connected load,

$$I_L = I_p, \qquad V_L = \sqrt{3}\,V_p \qquad I_p = V_p / \mathbf{Z}$$

$$\mathbf{S} = 3\mathbf{V}_p\mathbf{I}_p^* = \frac{3\left|\mathbf{V}_p\right|^2}{\mathbf{Z}^*} = \frac{3\left|\mathbf{V}_L/\sqrt{3}\right|^2}{\mathbf{Z}^*}$$

$$\mathbf{S} = \frac{\left|\mathbf{V}_L\right|^2}{\mathbf{Z}^*} = \frac{(110)^2}{100} = 121\ \text{W}$$

For the delta-connected load,

$$V_p = V_L, \qquad I_L = \sqrt{3}\,I_p, \qquad I_p = V_p / \mathbf{Z}$$

$$\mathbf{S} = 3\mathbf{V}_p\mathbf{I}_p^* = \frac{3\left|\mathbf{V}_p\right|^2}{\mathbf{Z}^*} = \frac{3\left|\mathbf{V}_L\right|^2}{\mathbf{Z}^*}$$

$$\mathbf{S} = \frac{(3)(110)^2}{100} = 363\ \text{W}$$

This shows that the **delta-connected load** will absorb three times more average power than the wye-connected load using the same elements.. This is also evident from $\mathbf{Z}_Y = \dfrac{\mathbf{Z}_\Delta}{3}$.

Solution 12.47

$$\text{pf} = 0.8 \quad \text{(lagging)} \quad \longrightarrow \quad \theta = \cos^{-1}(0.8) = 36.87°$$
$$\mathbf{S}_1 = 250\angle 36.87° = 200 + j150 \text{ kVA}$$

$$\text{pf} = 0.95 \quad \text{(leading)} \quad \longrightarrow \quad \theta = \cos^{-1}(0.95) = -18.19°$$
$$\mathbf{S}_2 = 300\angle -18.19° = 285 - j93.65 \text{ kVA}$$

$$\text{pf} = 1.0 \quad \longrightarrow \quad \theta = \cos^{-1}(1) = 0°$$
$$\mathbf{S}_3 = 450 \text{ kVA}$$

$$\mathbf{S}_T = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 935 + j56.35 = 936.7\angle 3.45° \text{ kVA}$$

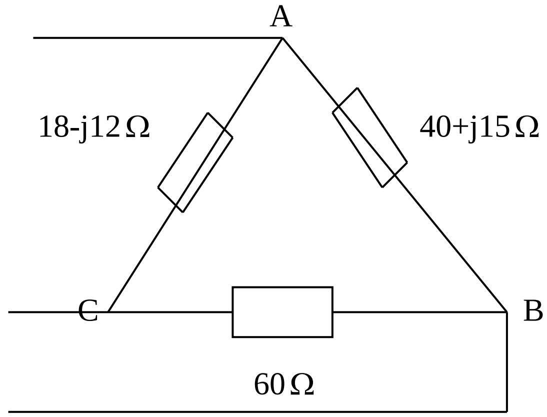
$$\left|\mathbf{S}_T\right| = \sqrt{3}\, V_L I_L$$

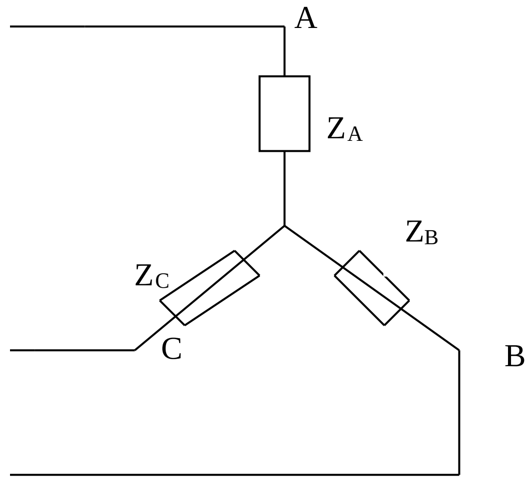
$$I_L = \frac{936.7\times 10^3}{\sqrt{3}\,(13.8\times 10^3)} = \mathbf{39.19 \text{ A rms}}$$

$$\text{pf} = \cos\theta = \cos(3.45°) = \mathbf{0.9982 \quad (lagging)}$$

Solution 12.48

(a) We first convert the delta load to its equivalent wye load, as shown below.



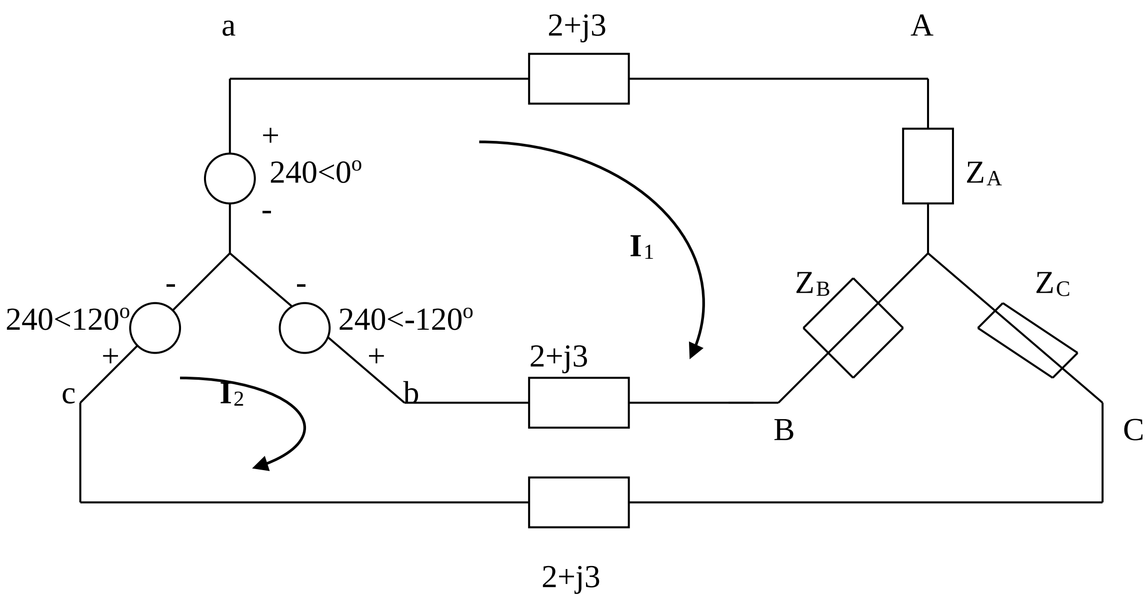


$$Z_A = \frac{(40 + j15)(18 - j12)}{118 + j3} = 7.577 - j1.923$$

$$Z_B = \frac{60(40 + j15).}{118 + j3} = 20.52 + j7.105$$

$$Z_C = \frac{60(18 - j12)}{118 + j3} = 8.992 - j6.3303$$

The system becomes that shown below.



We apply KVL to the loops. For mesh 1,

$$-240+240\angle-120^o+I_1(2Z_l+Z_A+Z_B)-I_2(Z_B+Z_l)=0$$

or

$$(32.097+j11.13)I_1-(22.52+j10.105)I_2=360+j207.85 \qquad (1)$$

For mesh 2,

$$240\angle 120^o-240\angle-120^o-I_1(Z_B+Z_l)+I_2(2Z_l+Z_B+Z_C)=0$$

or

$$-(22.52+j10.105)I_1+(33.51+j6.775)I_2=-j415.69 \qquad (2)$$

Solving (1) and (2) gives

$I_1=23.75-j5.328, \qquad I_2=15.165-j11.89$

$\boldsymbol{I_{aA}=I_1=\underline{24.34\angle-12.64^o\ \text{A}}, \qquad I_{bB}=I_2-I_1=\underline{10.81\angle-142.6^o\ \text{A}}}$

$\boldsymbol{I_{cC}=-I_2=\underline{19.27\angle 141.9^o\ \text{A}}}$

(b) $\mathbf{S_a=(240\angle 0^o)(24.34\angle 12.64^o)=5841.6\angle 12.64^o}$

$\mathbf{S_b=(240\angle-120^o)(10.81\angle 142.6^o)=2594.4\angle 22.6^o}$

$\mathbf{S_c=(240\angle 120^o)(19.27\angle-141.9^o)=4624.8\angle-21.9^o}$

$\mathbf{S=S_a+S_b+S_c=12.386+j0.55\ \ kVA=\underline{12.4\angle 2.54^o\ kVA}}$

Solution 12.49

Each phase load consists of a 20-ohm resistor and a 10-ohm inductive reactance. With a line voltage of 480 V rms, calculate the average power taken by the load if:

(a) the three phase loads are delta-connected,
(b) the loads are wye-connected.

Solution

(a) For the delta-connected load, $Z_p = 20 + j10\Omega, \quad V_p = V_L = 480 \text{ (rms)},$

$$S = \frac{3V^2{}_p}{Z^*{}_p} = \frac{3x480^2}{(20 - j10)} = \frac{(13{,}824 + j6{,}912)k}{500} = (27.648 + j13.824)k$$

P = **27.65 kW**

(b) For the wye-connected load, $Z_p = 20 + j10\Omega, \quad V_p = V_L / \sqrt{3},$

$$S = \frac{3V^2{}_p}{Z^*{}_p} = \frac{3x480^2}{3(20 - j10)} = (9.216 + j4.608)kVA$$

P = **9.216 kW**

Solution 12.50

$\overline{S} = \overline{S}_1 + \overline{S}_2 = 8(0.6 + j0.8) = 4.8 + j6.4\,\text{kVA}, \qquad \overline{S}_1 = 3\,\text{kVA}$
Hence,

$\overline{S}_2 = \overline{S} - \overline{S}_1 = 1.8 + j6.4\,\text{kVA}$

But $\overline{S}_2 = \dfrac{3V^2{}_p}{Z^*{}_p}, \quad V_p = \dfrac{V_L}{\sqrt{3}} \quad \longrightarrow \quad \overline{S}_2 = \dfrac{.V^2{}_L}{Z^*{}_p}$

$$\boldsymbol{Z^*{}_p = \frac{V^*{}_L}{\overline{S}_2} = \frac{240^2}{(1.8 + j6.4)x10^3} \quad \longrightarrow \quad \underline{Z_p = 2.346 + j8.34\Omega}}$$

Solution 12.51

Consider the wye-delta system shown in Fig. 12.60. Let $\mathbf{Z_1} = 100\ \Omega$, $\mathbf{Z_2} = j100\ \Omega$, and $\mathbf{Z_3} = -j100\ \Omega$. Determine the phase currents, $\mathbf{I_{AB}}$, $\mathbf{I_{BC}}$, and $\mathbf{I_{CA}}$, and the line currents, $\mathbf{I_{aA}}$, $\mathbf{I_{bB}}$, and $\mathbf{I_{cC}}$.

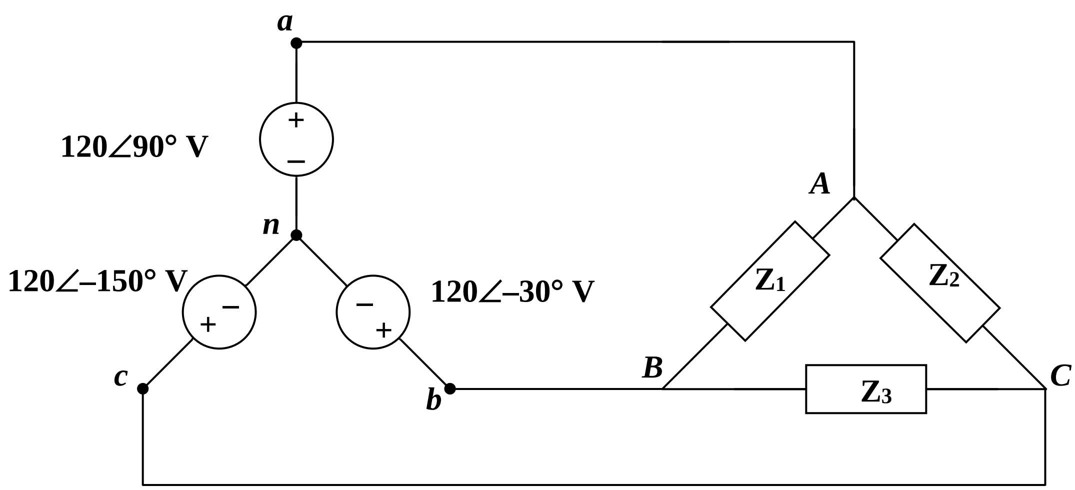


Figure 12.60
For Prob. 12.51.

Solution

Step 1. First we need to determine the Phase voltages, $\mathbf{V_{AB}} = \mathbf{V_{an}} - \mathbf{V_{bn}}$, $\mathbf{V_{BC}} = \mathbf{V_{bn}} - \mathbf{V_{cn}}$, and $\mathbf{V_{CA}} = \mathbf{V_{cn}} - \mathbf{V_{an}}$. Then we can calculate phase currents, $\mathbf{I_{AB}} = \mathbf{V_{AB}}/\mathbf{Z_1}$, $\mathbf{I_{BC}}$, $= \mathbf{V_{BC}}/\mathbf{Z_3}$, and $\mathbf{I_{CA}} = \mathbf{V_{CA}}/\mathbf{Z_2}$. Finally, we can now calculate the line currents, $\mathbf{I_{aA}} = \mathbf{I_{AB}} - \mathbf{I_{CA}}$, $\mathbf{I_{bB}} = \mathbf{I_{BC}} - \mathbf{I_{AB}}$, and $\mathbf{I_{cC}} = \mathbf{I_{CA}} - \mathbf{I_{BC}}$.

Step 2. $\mathbf{V_{AB}} = \mathbf{V_{ab}} - \mathbf{V_{bn}} = 120\angle 90° - 120\angle{-30°} = j120 - 103.923 + j60$ $= -103.923 + j180 = 207.846\angle 120°$ V, $\mathbf{V_{BC}} = \mathbf{V_{bn}} - \mathbf{V_{cn}}$ $= 120\angle{-30°} - 120\angle{-150°} = 103.923 - j60 + 103.923 + j60 = 207.846$ V, and $\mathbf{V_{CA}} = \mathbf{V_{cn}} - \mathbf{V_{an}} = 120\angle{-150°} - j120 = -103.923 - j60 - j120 = -103.923 - j180$ $= 207.846\angle{-120°}$ V.

$\mathbf{I_{AB}} = \mathbf{V_{AB}}/\mathbf{Z_1} = 207.846\angle 120°\ /100 = \mathbf{2.078\angle 120°\ A}$,
$\mathbf{I_{BC}} = \mathbf{V_{BC}}/\mathbf{Z_3} = 207.846\angle 0°\ /(-j100) = \mathbf{2.078\angle 90°\ A}$,
and $\mathbf{I_{CA}} = \mathbf{V_{CA}}/\mathbf{Z_2} = 207.846\angle{-120°}\ /(j100) = \mathbf{2.078\angle 150°\ A}$.

Finally, $\mathbf{I_{aA}} = \mathbf{I_{AB}} - \mathbf{I_{CA}} = 2.07846\angle 120° - 2.07846\angle 30°$
$= -1.03923+j1.8 - 1.8 - j1.03923 = -2.83923 + j0.76077 = \mathbf{2.939\angle 165°\ A}$,
$\mathbf{I_{bB}} = \mathbf{I_{BC}} - \mathbf{I_{AB}} = 2.07846\angle 90° - 2.07846\angle 120° = j2.07846 + 1.03923 - j1.8$
$= 1.03923 + j0.27846 = \mathbf{1.07589\angle 15°\ A}$, and $\mathbf{I_{cC}} = \mathbf{I_{CA}} - \mathbf{I_{BC}} = 2.07846\angle 150°$
$-\ \ 2.07846\angle 90° = -1.8+j1.03923-j2.07846 = -1.8 - j1.03923 = \mathbf{2.078\angle{-150°}\ A}$.

Solution 12.52

A four-wire wye-wye circuit has

$$\mathbf{V}_{an} = 220\ \angle 120°, \qquad \mathbf{V}_{bn} = 220\ \angle 0°$$
$$\mathbf{V}_{cn} = 220\ \angle -120°\ \text{V}$$

If the impedances are

$$\mathbf{Z}_{AN} = 20\ \angle 60°, \qquad \mathbf{Z}_{BN} = 30\ \angle 0°$$
$$\mathbf{Z}_{cn} = 40\ \angle 30°\ \Omega$$

find the current in the neutral line.

Solution

Since the neutral line is present, we can solve this problem on a per-phase basis.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{AN}} = \frac{220\angle 120°}{20\angle 60°} = 11\angle 60°$$
$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_{BN}} = \frac{220\angle 0°}{30\angle 0°} = 7.3333\angle 0°$$
$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_{CN}} = \frac{220\angle -120°}{40\angle 30°} = 5.5\angle -150°$$

Thus,

$$-\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$$
$$-\mathbf{I}_n = 11\angle 60° + 7.3333\angle 0° + 5.5\angle -150°$$
$$-\mathbf{I}_n = (5.5 + j9.5263) + (7.3333) + (-4.7631 - j2.75)$$
$$-\mathbf{I}_n = 8.0702 + j6.7763 = 10.538\angle 40.02°$$

$$\mathbf{I}_n = \mathbf{10.538\angle{-139.98}°\ A}$$

Solution 12.53

Using Fig. 12.61, design a problem that will help other students to better understand unbalanced three-phase systems.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the wye-wye system shown in Fig. 12.61, loads connected to the source are unbalanced. (a) Calculate I_a, I_b, and I_c. (b) Find the total power delivered to the load. Take V_P = 240 V rms.

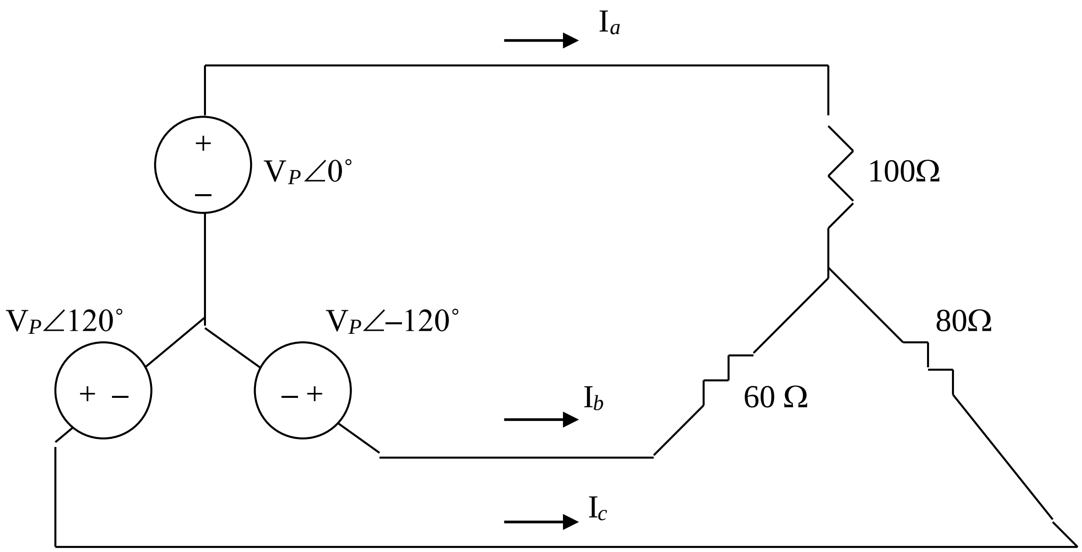
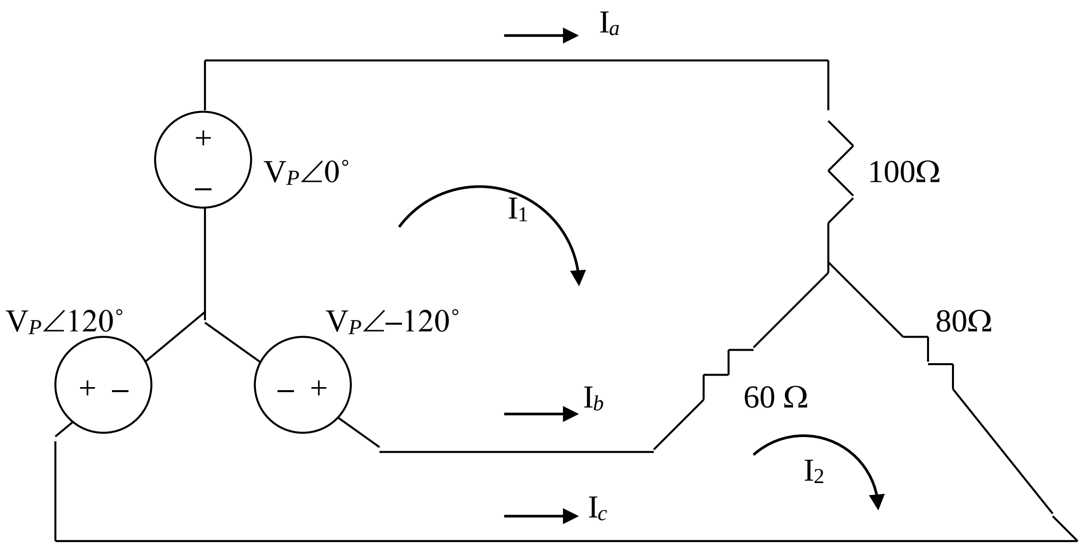


Figure 12.61 For Prob. 12.53.

Solution

Applying mesh analysis as shown below, we get.



$240\angle -120^\circ - 240 + 160\mathbf{I_1} - 60\mathbf{I_2} = 0$ or $160\mathbf{I_1} - 60\mathbf{I_2} = 360+j207.84$ (1)

$240\angle 120^\circ - 240\angle -120^\circ - 60\mathbf{I_1} + 140\mathbf{I_2} = 0$ or $-60\mathbf{I_1} + 140\mathbf{I_2} = -j415.7$ (2)

In matrix form, (1) and (2) become

$$\begin{bmatrix} 160 & -60 \\ -60 & 140 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 360 + j207.84 \\ -j415.7 \end{bmatrix}$$

Using MATLAB, we get,

```
>> Z=[160,-60;-60,140]
Z =
   160   -60
   -60   140
>> V=[(360+207.8i);-415.7i]
V =
  1.0e+002 *
   3.6000 + 2.0780i
        0 - 4.1570i
>> I=inv(Z)*V
I =
   2.6809 + 0.2207i
   1.1489 - 2.8747i
```

$I_1 = 2.681 + j0.2207$ and $I_2 = 1.1489 – j2.875$

$$I_a = I_1 = \mathbf{2.69\angle 4.71^\circ\ A}$$

$$I_b = I_2 - I_1 = -1.5321 - j3.096 = \mathbf{3.454\angle -116.33^\circ\ A}$$

$$I_c = -I_2 = \mathbf{3.096\angle 111.78^\circ\ A}$$

$$S_a = |I_a|^2 Z_a = (2.69)^2 x100 = 723.61$$
$$S_b = |I_b|^2 Z_b = (3.454)^2 x60 = 715.81$$
$$S_c = |I_c|^2 Z_c = (3.0957)^2 x80 = 766.67$$

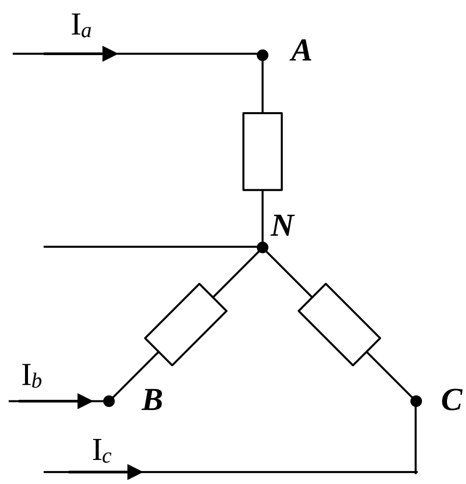
$$S = S_a + S_b + S_c = \underline{2.205\ \ \text{kVA}}$$

Solution 12.54

A balanced three-phase Y-source with V_P = 880 V rms drives a wye-connected three-phase load with phase impedance Z_{AN} = 80 Ω, Z_{BN} = 60+j90 Ω, and Z_{CN} = j80 Ω. Calculate the line currents and total complex power delivered to the load. Assume that the neutrals are connected.

Solution

Consider the load as shown below.



Assume $\mathbf{V_{AN}} = 880\angle0°$ V, $\mathbf{V_{BN}} = 880\angle120°$ V, and $\mathbf{V_{CN}} = 880\angle{-120°}$ V.
$\mathbf{I_a}$ = 880/80 = $\mathbf{11\angle0°}$ **A**, $\mathbf{I_b} = 880\angle120°/(60+j90) = 880\angle120°/(108.17\angle56.13°)$
= $\mathbf{8.135\angle63.87°}$ **A**, and $\mathbf{I_c} = 880\angle{-120°}/(j80)$ = $\mathbf{11\angle150°}$ **A**.

$\mathbf{S_a} = V_{AN}(I_a)^* = 880 \text{x} 11 = 9.68$ kW, $\mathbf{S_b} = 880\angle120°(8.135\angle{-63.87°})$
$= 7.159\angle56.13°$ kVA = 3.99 kW + j5.944 kVAR, and
$\mathbf{S_c} = (880\angle{-120°})(11\angle{-150°}) = 9.68\angle90°$ kVA = j9.68 kVAR.

$\mathbf{S} = \mathbf{S_a} + \mathbf{S_b} + \mathbf{S_c}$ = (9.68+3.99)kW + j(5.944+9.68)kVAR or

$$\mathbf{S = 13.67\ kW + j15.624\ kVAR = 20.76\angle48.82°\ kVA}.$$

Chapter 12, Solution 55.

A three-phase supply, with the line-to-line voltage of 240 V rms, has the unbalanced load as shown in Fig. 12.62. Find the line currents and the total complex power delivered to the load.

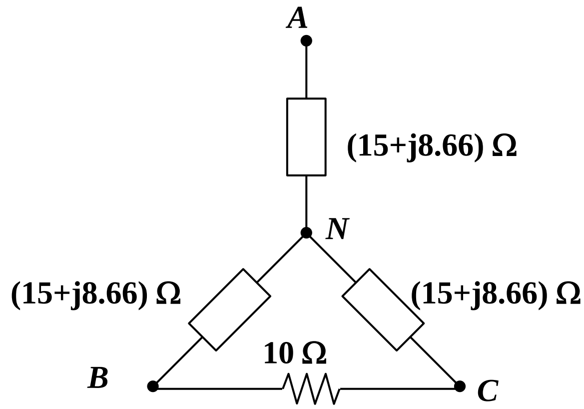


Figure 12.62
For Prob. 12.55.

Solution
To solve this problem we need to arbitrarily select phase angles for the sources which then enables us to find line currents as well as complex power delivered to the load.

Step 1. Let $\mathbf{V_{AB}} = 240\angle 0°$ V, $\mathbf{V_{BC}} = 240\angle 120°$ V, and $\mathbf{V_{CA}} = 240\angle{-120°}$ V. We can treat this as two different circuits and then use superposition to find the line currents and total complex power.

The first circuit consists of a balanced wye with the phase voltages (see Fig. 12.19) of $\mathbf{V_{an}} = 138.564\angle{-30°}$, $\mathbf{V_{bn}} = 138.564\angle{-150°}$, and $\mathbf{V_{cn}} = 138.564\angle 90°$, Therefore, the line currents for this are equal to, $\mathbf{I_{aA}} = \mathbf{V_{an}}/(17.32\angle 30°)$, $\mathbf{I_{bB}} = \mathbf{V_{bn}}/(17.32\angle 30°)$, and $\mathbf{I_{cC}} = \mathbf{V_{cn}}/(17.32\angle 30°)$.

Finally, we note that the current that flows through the 10-Ω resistor impacts the line currents, $\mathbf{I_{bB}}$ and $\mathbf{I_{cC}}$. Let us call the current through the resistor as $\mathbf{I_{BC}}$. $\mathbf{I_{BC}} = \mathbf{V_{BC}}/10$. Thus, $(\mathbf{I_{bB}})' = \mathbf{I_{bB}} + \mathbf{I_{BC}}$ and $(\mathbf{I_{cC}})' = \mathbf{I_{cC}} - \mathbf{I_{BC}}$.

The last thing we need to do is calculate $\mathbf{S_{Tot}} = 3|\mathbf{I_{line}}|^2(15+j8.66) + |\mathbf{I_{AB}}|^2(10)$.

Step 2. $\mathbf{I_{aA}} = (138.564\angle{-30°})/(17.32\angle 30°) = \mathbf{8\angle{-60°}\ A}$,
$\mathbf{I_{bB}} = (138.564\angle{-150°})/(17.32\angle 30°) = 8\angle 180° = -8$, and
$\mathbf{I_{cC}} = (138.564\angle 90°)/(17.32\angle 30°) = 8\angle 60° = 4 + j6.9282$. $\mathbf{I_{BC}} = (240\angle 120°)/10$
$= 24\angle 120° = -12 + j20.785$. Thus, $(\mathbf{I_{bB}})' = -8 - 12 + j20.785 = -20+j20.785$
$= \mathbf{28.84\angle 133.9°\ A}$ and $(\mathbf{I_{cC}})' = \mathbf{I_{cC}} - \mathbf{I_{BC}} = = 4 + j6.9282 +12 - j20.785$
$= 16 - j13.8568 = \mathbf{21.17\angle{-40.89°}\ A}$.
$\mathbf{S_{Tot}} = 3|\mathbf{I_{line}}|^2(15+j86.6) + |\mathbf{I_{BC}}|^2(10) = 3(8)^2(15+j8.66) + (24)^2(10)$
$= 2{,}880 + j1{,}662.72 + 5{,}760 =$ **8.64 kW + j1.6627 kVAR**.

Solution 12.56

Using Fig. 12.63, design a problem to help other students to better understand unbalanced three-phase systems.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Refer to the unbalanced circuit of Fig. 12.63. Calculate:

(a) the line currents

(b) the real power absorbed by the load

(c) the total complex power supplied by the source

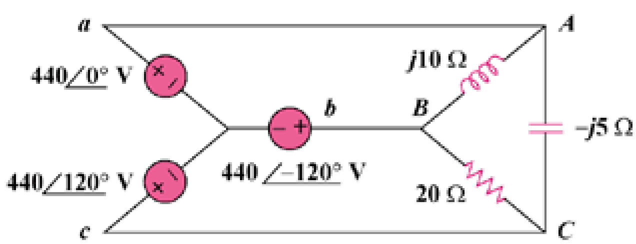
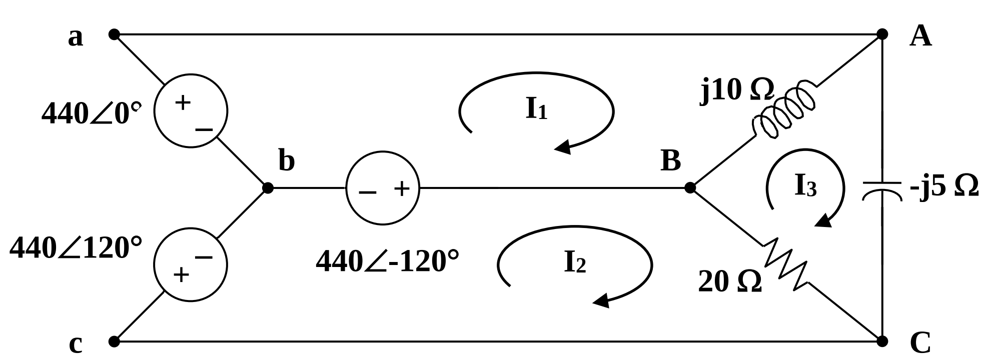


Figure 12.63

Solution

(a) Consider the circuit below.



For mesh 1,

$$440\angle -120° - 440\angle 0° + j10(\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_1 - \mathbf{I}_3 = \frac{(440)(1.5 + j0.866)}{j10} = 76.21\angle -60° \qquad (1)$$

For mesh 2,

$$440\angle 120° - 440\angle -120° + 20(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_3 - \mathbf{I}_2 = \frac{(440)(j1.732)}{20} = j38.1 \qquad (2)$$

For mesh 3,

$$j10(\mathbf{I}_3 - \mathbf{I}_1) + 20(\mathbf{I}_3 - \mathbf{I}_2) - j5\mathbf{I}_3 = 0$$

Substituting (1) and (2) into the equation for mesh 3 gives,

$$\mathbf{I}_3 = \frac{(440)(-1.5 + j0.866)}{j5} = 152.42\angle 60° \qquad (3)$$

From (1),

$$\mathbf{I}_1 = \mathbf{I}_3 + 76.21\angle -60° = 114.315 + j66 = 132\angle 30°$$

From (2),

$$\mathbf{I}_2 = \mathbf{I}_3 - j38.1 = 76.21 + j93.9 = 120.93\angle 50.94°$$

$\mathbf{I}_a = \mathbf{I}_1 =$ **$132\angle 30°$ A**

$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_1 = -38.105 + j27.9 =$ **$47.23\angle 143.8°$ A**

$\mathbf{I}_c = -\mathbf{I}_2 =$ **$120.9\angle 230.9°$ A**

(b) $\mathbf{S}_{AB} = \left|\mathbf{I}_1 - \mathbf{I}_3\right|^2 (j10) = j58.08$ kVA

$\mathbf{S}_{BC} = \left|\mathbf{I}_2 - \mathbf{I}_3\right|^2 (20) = 29.04$ kVA

$\mathbf{S}_{CA} = \left|\mathbf{I}_3\right|^2 (-j5) = (152.42)^2 (-j5) = -j116.16$ kVA

$\mathbf{S} = \mathbf{S}_{AB} + \mathbf{S}_{BC} + \mathbf{S}_{CA} = 29.04 - j58.08$ kVA

Real power absorbed = **29.04 kW**

(c) Total complex supplied by the source is

$\mathbf{S} =$ **$29.04 - j58.08$ kVA**

Solution 12.57

Determine the line currents for the three-phase circuit in Fig. 12.64.
Let $\mathbf{V}_a = 220\angle 0°$, $\mathbf{V}_b = 220\angle{-120°}$, $\mathbf{V}_c = 220\angle 120°$ V.

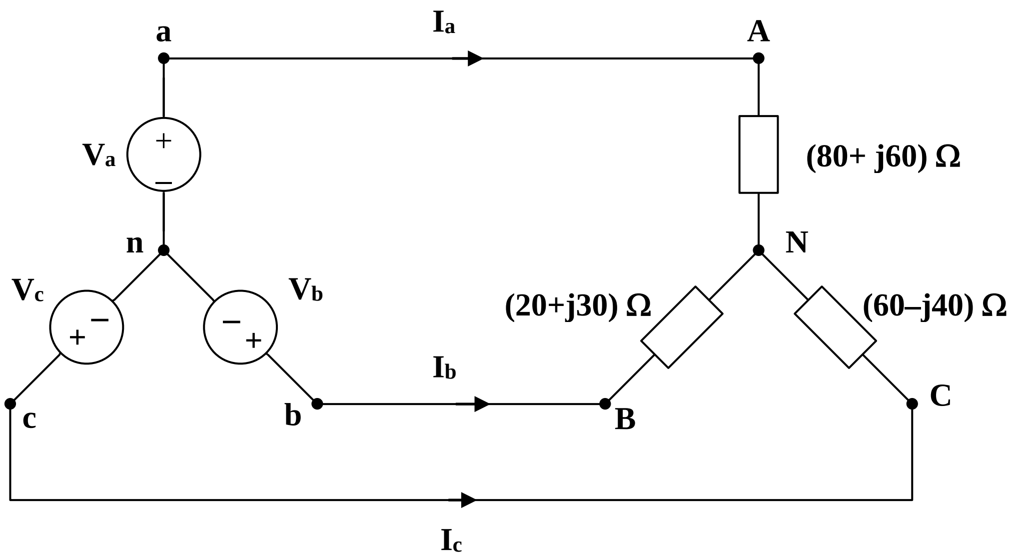
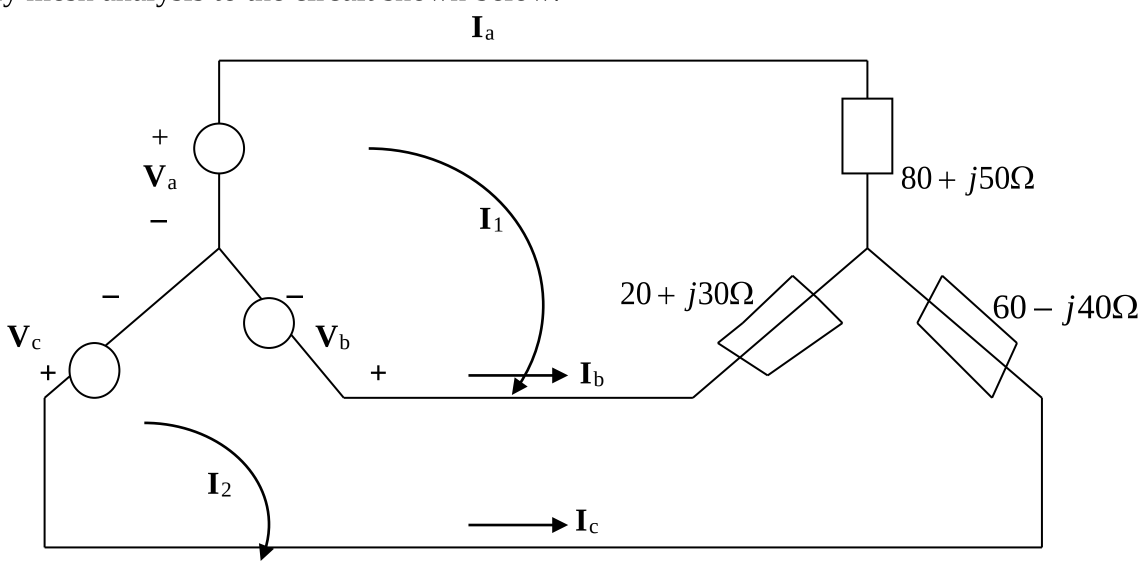


Figure 12.64
For Prob. 12.57.

Solution

We apply mesh analysis to the circuit shown below.



$\mathbf{V_a}$ = 220 V, $\mathbf{V_b}$ = (–110 – j190.53) V, $\mathbf{V_c}$ = (–110 + j190.53) V

$$(100+j80)I_1-(20+j30)I_2=V_a-V_b=330+j190.53 \qquad (1)$$
$$-(20+j30)I_1+(80-j10)I_2=V_b-V_c=-j381.1 \qquad (2)$$

Solving (1) and (2) using MATLAB gives,

```
>> Z=[100+80j,-20-30j;-20-30j,80-10j]

Z =

   1.0e+02 *

   1.0000 + 0.8000i  -0.2000 - 0.3000i
  -0.2000 - 0.3000i   0.8000 - 0.1000i

>> V=[330+190.53j;-381.1j]

V =

   1.0e+02 *

   3.3000 + 1.9053i
   0.0000 - 3.8110i

>> I = inv(Z)*V

I =

   3.7233 - 1.2170i
   1.8178 - 3.4445i or
```

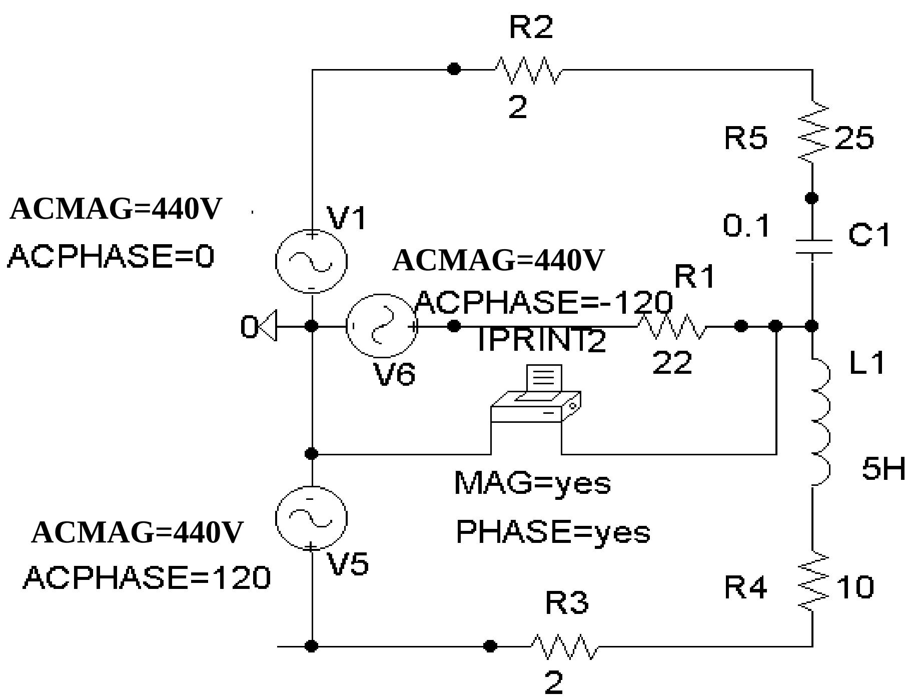
$\mathbf{I_a} = \mathbf{I_1}$ = 3.7233–j1.217 = $\mathbf{3.917\angle{-18.1}^\circ}$ **A**,
$\mathbf{I_b} = -\mathbf{I_1} + \mathbf{I_2}$ = –1.9055–j2.2275 = $\mathbf{2.931\angle{-130.55}}$ **A**,
and $\mathbf{I_c} = -\mathbf{I_2}$ = –1.8178+j3.4445 = $\mathbf{3.895\angle 117.82^\circ}$ **A**.

Solution 12.58

The schematic is shown below. IPRINT is inserted in the neutral line to measure the current through the line. In the AC Sweep box, we select Total Ptss = 1, Start Freq. = 0.1592, and End Freq. = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	2.156 E+01	–8.997 E+01

i.e. I_n = **21.56∠–89.97° A**

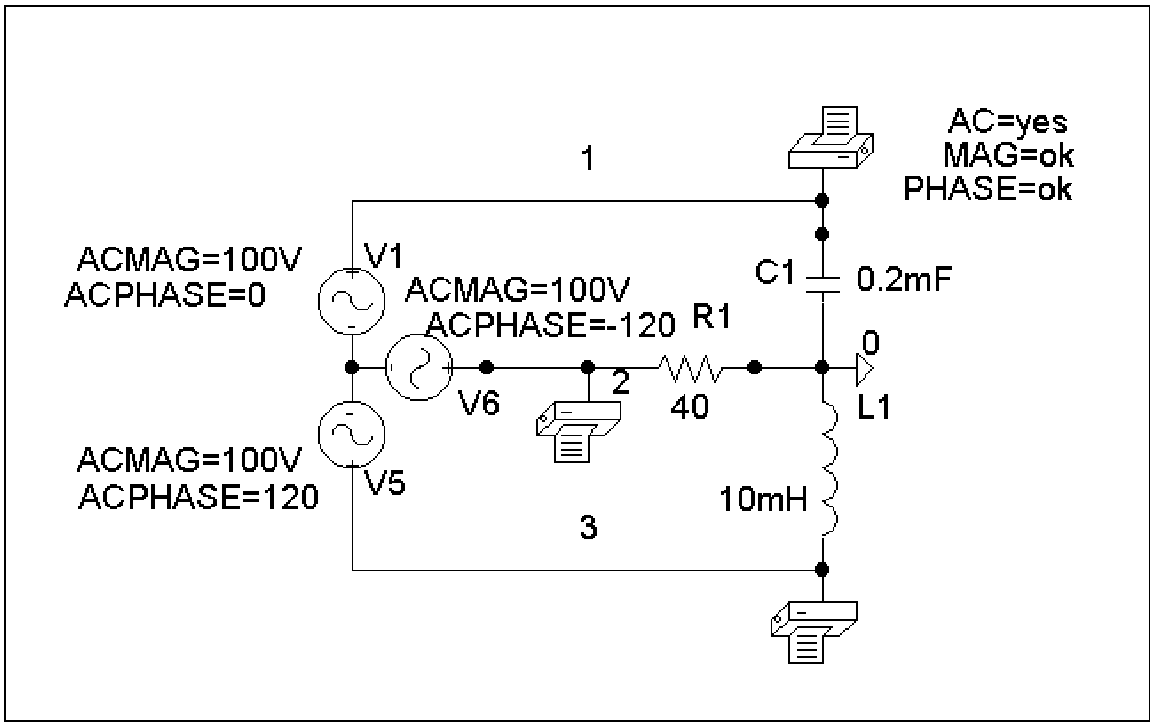


Solution 12.59

The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 60, and End Freq = 60. After simulation, we obtain an output file which includes

FREQ	VM(1)	VP(1)
6.000 E+01	2.206 E+02	–3.456 E+01
FREQ	VM(2)	VP(2)
6.000 E+01	2.141 E+02	–8.149 E+01
FREQ	VM(3)	VP(3)
6.000 E+01	4.991 E+01	–5.059 E+01

i.e. V_{AN} = **220.6∠–34.56°**, V_{BN} = **214.1∠–81.49°**, V_{CN} = **49.91∠–50.59° V**

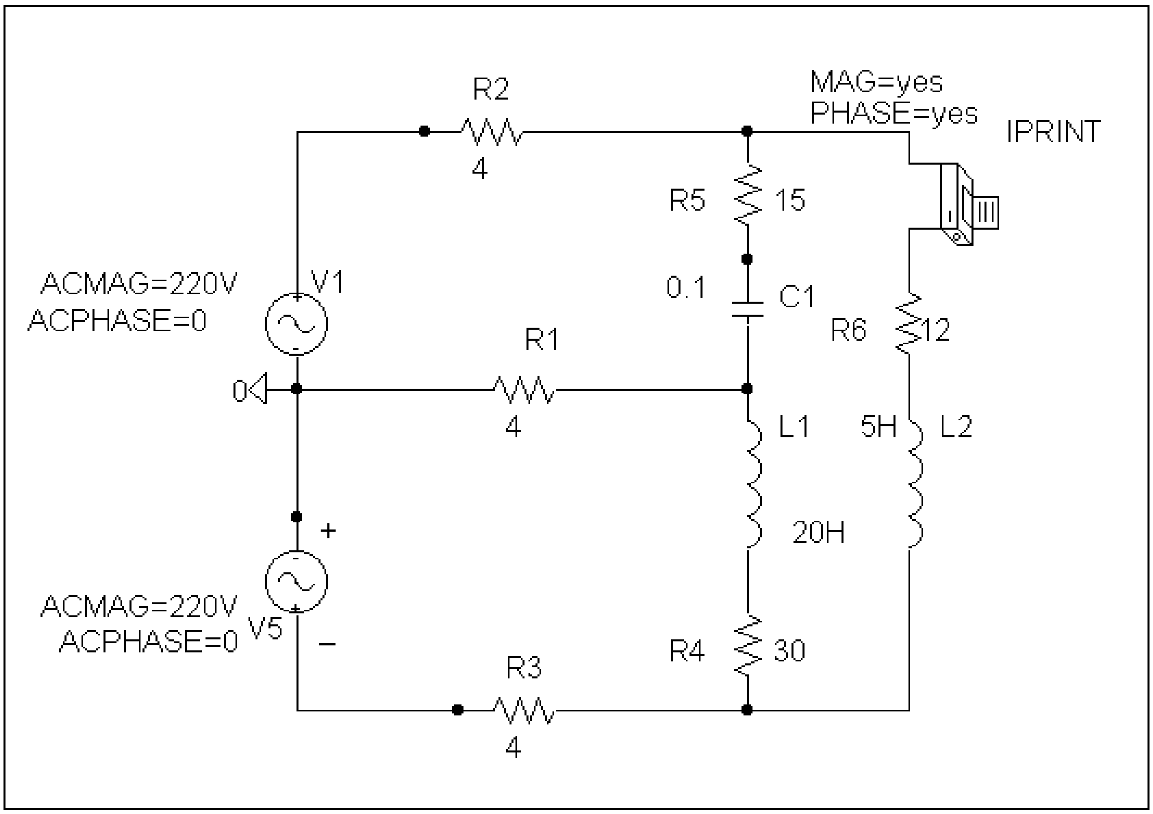


Solution 12.60

The schematic is shown below. IPRINT is inserted to give I_o. We select Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. Upon simulation, the output file includes

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	1.953 E+01	–1.517 E+01

from which, $I_o = \mathbf{19.53\angle{-15.17^\circ}\ A}$



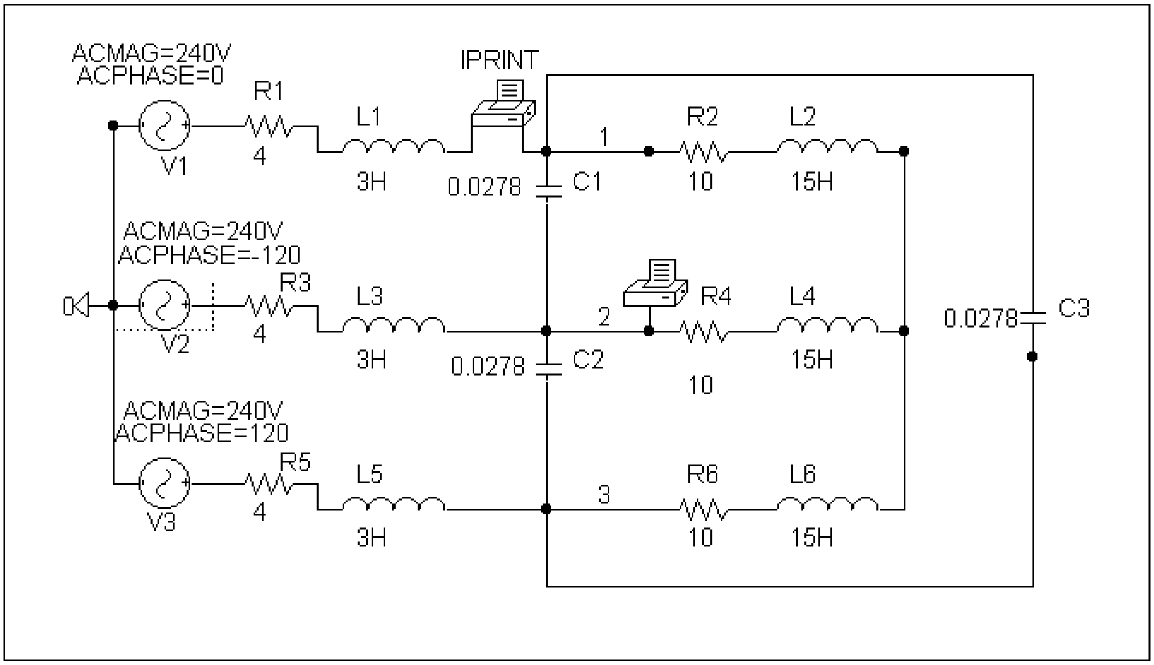
Solution 12.61

The schematic is shown below. Pseudo-components IPRINT and PRINT are inserted to measure I_{aA} and V_{BN}. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. Once the circuit is simulated, we get an output file which includes

FREQ	VM(2)	VP(2)
1.592 E–01	2.308 E+02	–1.334 E+02
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	1.115 E+01	3.699 E+01

from which

$$I_{aA} = \mathbf{11.15\angle 37° \ A}, \quad V_{BN} = \mathbf{230.8\angle{-133.4}° \ V}$$



Solution 12.62

Using Fig. 12.68, design a problem to help other students to better understand how to use *PSpice* to analyze three-phase circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The circuit in Fig. 12.68 operates at 60 Hz. Use *PSpice* to find the source current $\mathbf{I}_{ab}$ and the line current $\mathbf{I}_{bB}$.

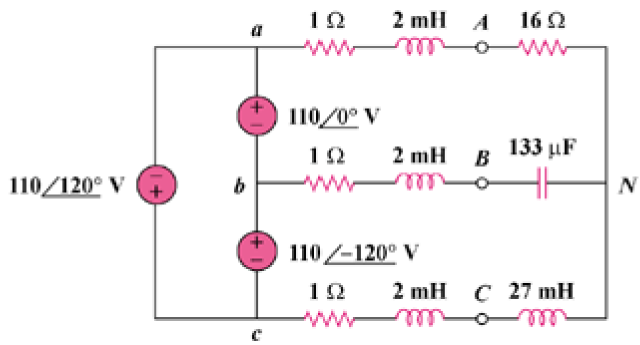


Figure 12.68

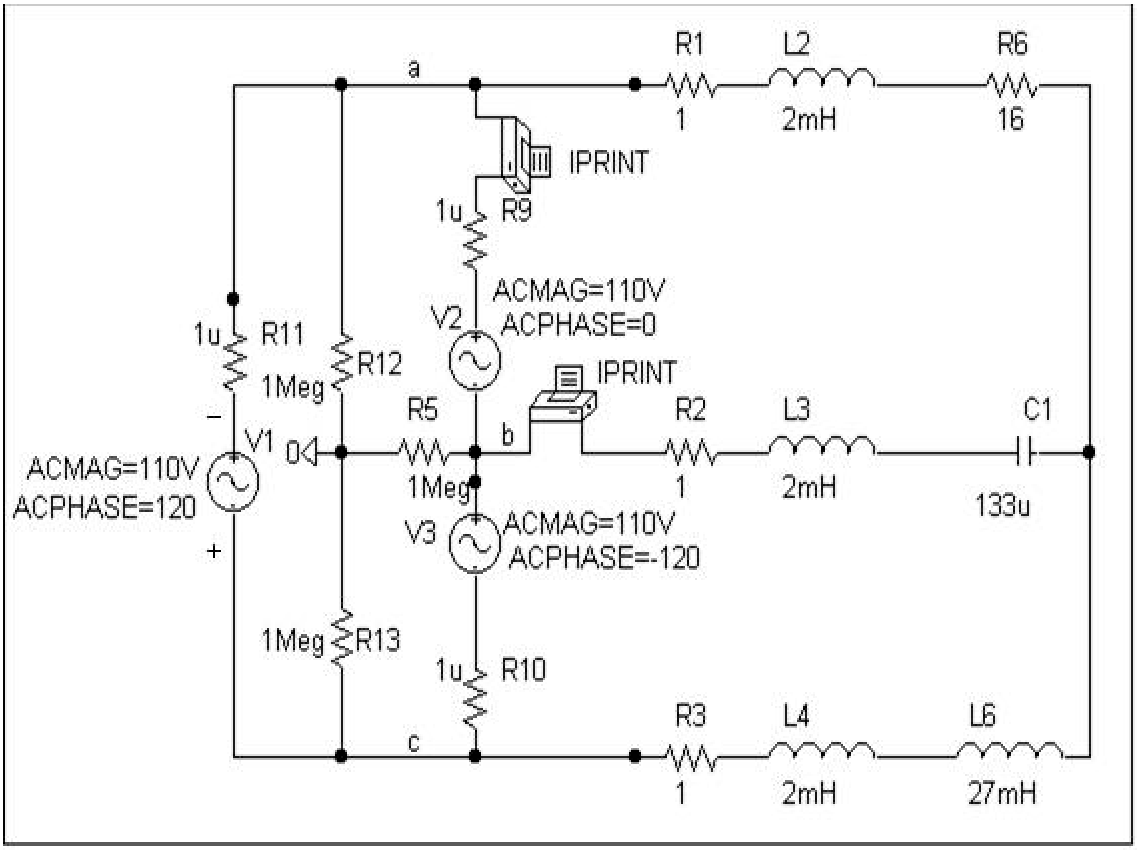
Solution

Because of the delta-connected source involved, we follow Example 12.12. In the AC Sweep box, we type Total Pts = 1, Start Freq = 60, and End Freq = 60. After simulation, the output file includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
6.000 E+01	5.960 E+00	–9.141 E+01
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
6.000 E+01	7.333 E+07	1.200 E+02

From which

$$I_{ab} = \mathbf{3.432\angle{-46.31}^\circ\ A},\quad I_{bB} = \mathbf{10.39\angle{-78.4}^\circ\ A}$$

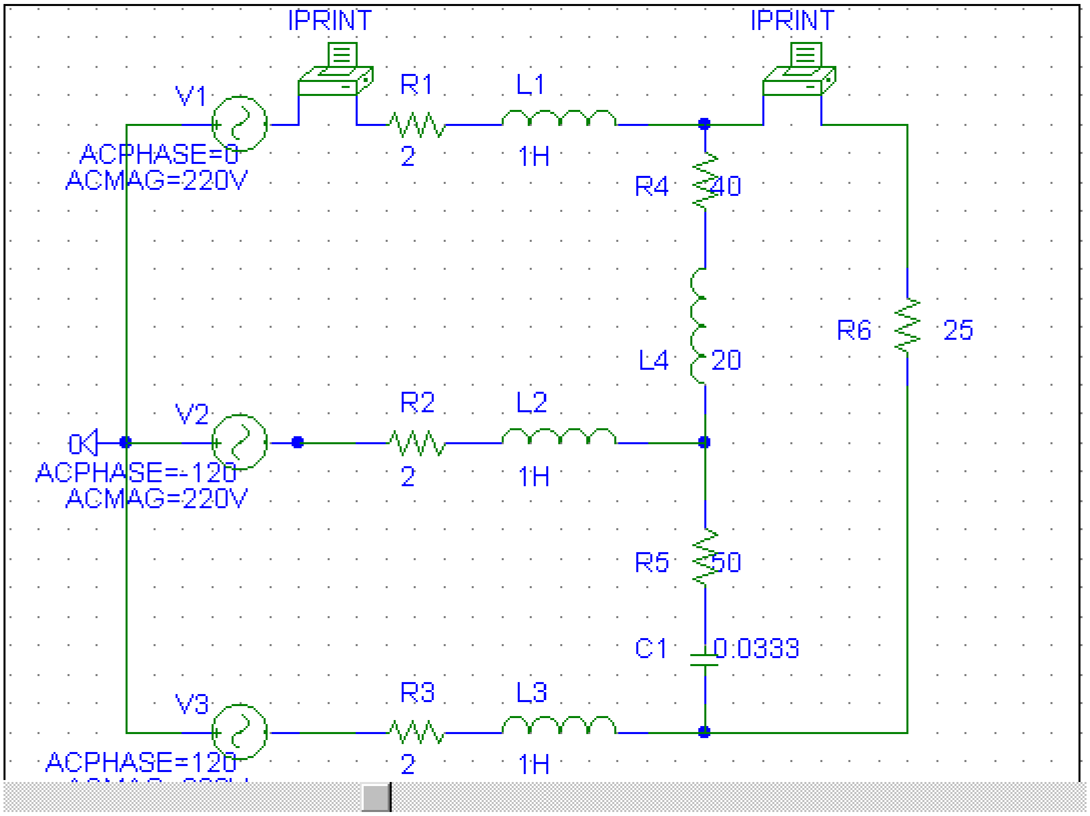


R1
L2
R6
a
1
2mH
16
IPRINT
1u
R9
ACMAG=110V
ACPHASE=0
V2
1u
R11
R12
1Meg
IPRINT
R5
R2
L3
C1
–
V1
0
b
ACMAG=110V
ACPHASE=120
1Meg
1
2mH
133u
V3
ACMAG=110V
ACPHASE=-120
+
1Meg
R13
1u
R10
R3
L4
L6
c
1
2mH
27mH

Solution 12.63

Let $\omega = 1$ so that $L = X/\omega = 20\,\text{H}$, and $C = \dfrac{1}{\omega X} = 0.0333\,\text{F}$

The schematic is shown below..



When the file is saved and run, we obtain an output file which includes the following:

FREQ   IM(V_PRINT1)IP(V_PRINT1)

1.592E-01   1.867E+01   1.589E+02

FREQ   IM(V_PRINT2)IP(V_PRINT2)

1.592E-01   1.238E+01   1.441E+02

From the output file, the required currents are:

$$\underline{\boldsymbol{I_{aA} = 18.67\angle 158.9^o \text{ A}, \quad I_{AC} = 12.38\angle 144.1^o \text{ A}}}$$

Solution 12.64

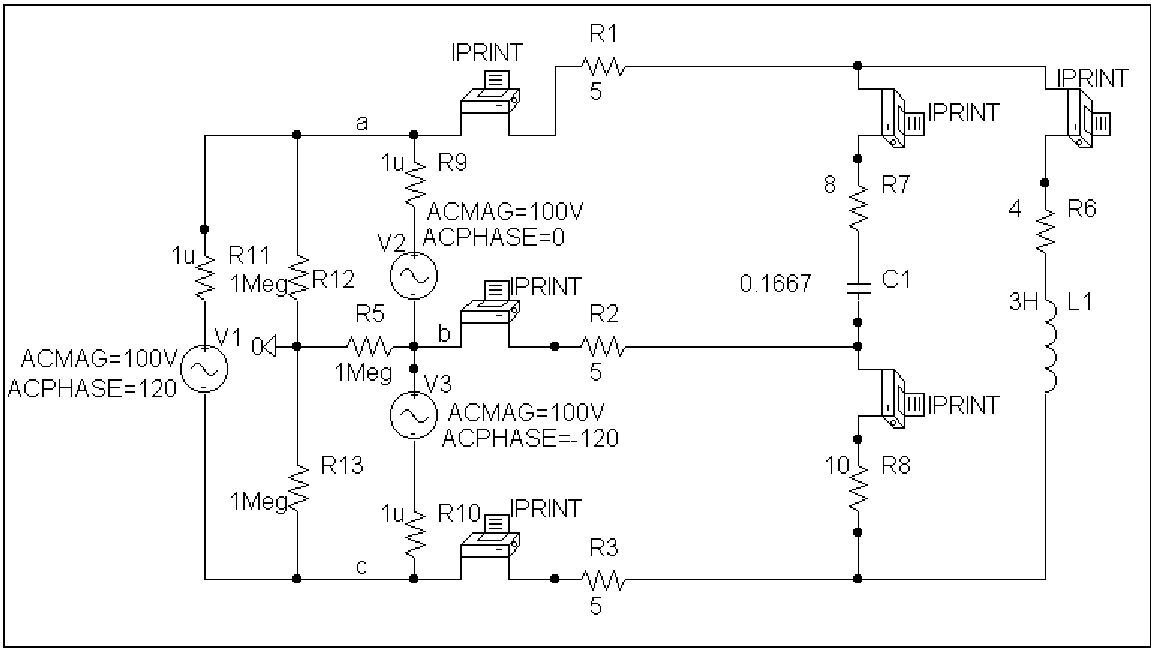
We follow Example 12.12. In the AC Sweep box we type Total Pts = 1,
Start Freq = 0.1592, and End Freq = 0.1592. After simulation the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.710 E+00	7.138 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	6.781 E+07	–1.426 E+02
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	3.898 E+00	–5.076 E+00
FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	3.547 E+00	6.157 E+01
FREQ	IM(V_PRINT5)	IP(V_PRINT5)
1.592 E–01	1.357 E+00	9.781 E+01
FREQ	IM(V_PRINT6)	IP(V_PRINT6)
1.592 E–01	3.831 E+00	–1.649 E+02

from this we obtain

$$I_{aA} = \mathbf{4.71\angle 71.38^\circ\ A},\ I_{bB} = \mathbf{6.781\angle{-142.6^\circ}\ A},\ I_{cC} = \mathbf{3.898\angle{-5.08^\circ}\ A}$$

$$I_{AB} = \mathbf{3.547\angle 61.57^\circ\ A},\ I_{AC} = \mathbf{1.357\angle 97.81^\circ\ A},\ I_{BC} = \mathbf{3.831\angle{-164.9^\circ}\ A}$$

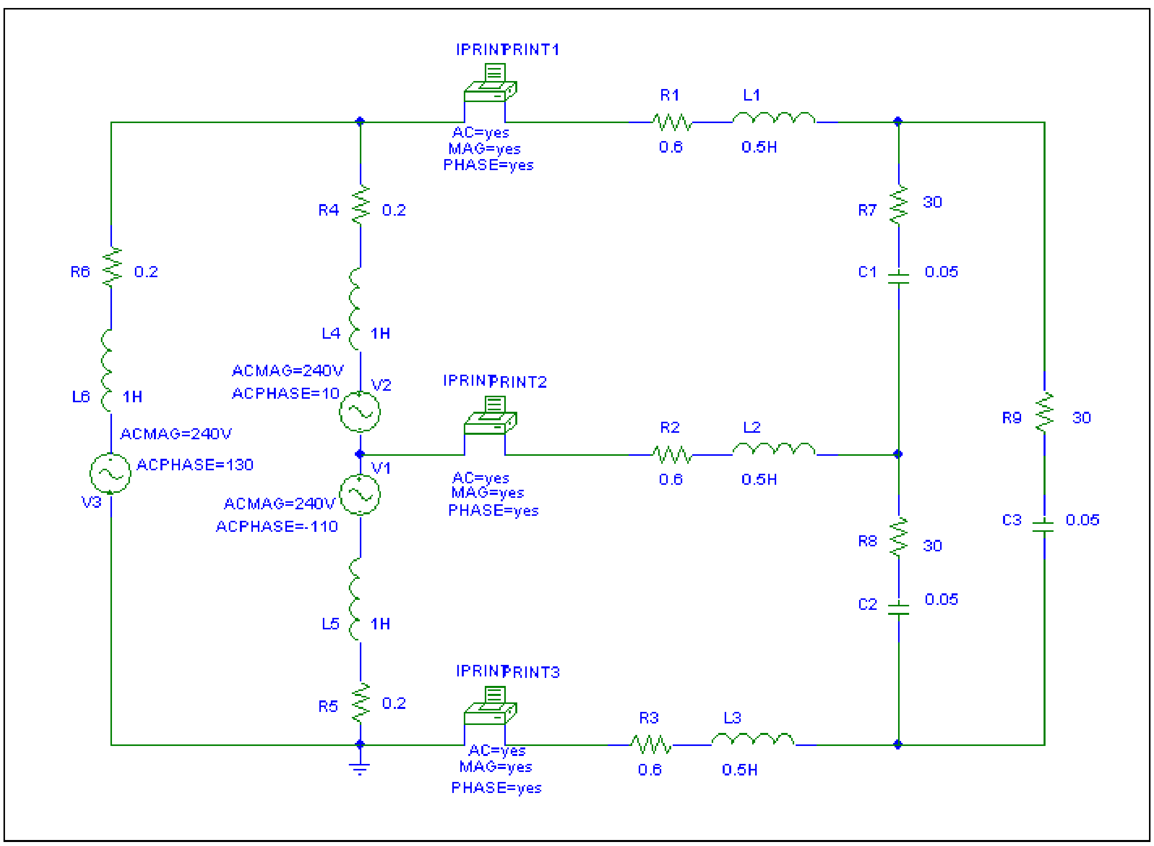

IPRINT
R1
5
IPRINT
IPRINT
a
1u
R9
8
R7
4
R6
ACMAG=100V
ACPHASE=0
V2
1u
R11
1Meg
R12
IPRINT
0.1667
C1
R5
R2
3H
L1
V1
0
b
ACMAG=100V
ACPHASE=120
1Meg
5
V3
IPRINT
ACMAG=100V
ACPHASE=-120
R13
10
R8
1Meg
1u
R10
IPRINT
R3
c
5

Solution 12.65

Due to the delta-connected source, we follow Example 12.12. We type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. The schematic is shown below. After it is saved and simulated, we obtain an output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	1.140E+01	8.664E+00
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E-01	1.140E+01	-1.113E+02
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592E-01	1.140E+01	1.287E+02

Thus, $I_{aA} = \mathbf{11.02\angle 12^\circ\ A}$, $I_{bB} = \mathbf{11.02\angle{-108^\circ}\ A}$, $I_{cC} = \mathbf{11.02\angle 132^\circ\ A}$



Since this is a balanced circuit, we can perform a quick check. The load resistance is large compared to the line and source impedances so we will ignore them (although it would not be difficult to include them).

Converting the sources to a Y configuration we get:

$$V_{an} = 138.56\ \angle{-20^\circ}\ \text{Vrms}$$

and

$$Z_Y = 10 - j6.667 = 12.019\angle{-33.69^\circ}$$

Now we can calculate,

$$I_{aA} = (138.56\ \angle{-20^\circ})/(12.019\angle{-33.69^\circ}) = 11.528\angle 13.69^\circ$$

Clearly, we have a good approximation which is very close to what we really have.

Solution 12.66

A three-phase, four-wire system operating with a 480-V line voltage is shown in Fig. 12.71. The source voltages are balanced. The power absorbed by the resistive wye-connected load is measured by the three-wattmeter method. Calculate:

(a) the voltage to neutral
(b) the currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_n$
(c) the readings of the wattmeters
(d) the total power absorbed by the load

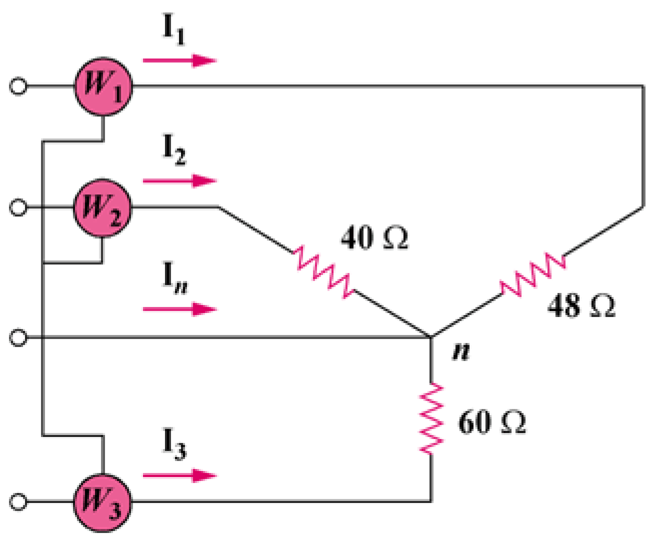


Figure 12.71
For Prob. 12.66.

Solution

(a) $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{480}{\sqrt{3}} = \mathbf{277.1\ V}$

(b) Because the load is unbalanced, we have an unbalanced three-phase system. Assuming an abc sequence,

$$\mathbf{I}_1 = \frac{277.128\angle 0°}{48} = \mathbf{5.774\angle 0°\ A}$$

$$\mathbf{I}_2 = \frac{277.128\angle\text{-}120°}{40} = \mathbf{6.928\angle{-120°}\ A}$$

$$\mathbf{I}_3 = \frac{277.128\angle 120°}{60} = \mathbf{4.619\angle 120°\ A}$$

$$\mathbf{I}_n = -\mathbf{I}_1 - \mathbf{I}_2 - \mathbf{I}_3 = -5.774 - (-3.464 - j6) - (-2.31 + j4)$$

$= (0 + j2)$ A $=$ **$2\angle 90°$ A**.

Hence,

$|\mathbf{I_1}| = 5.774$ A, $|\mathbf{I_2}| = 6.928$ A, $|\mathbf{I_3}| = 4.619$ A.

(c) $P_1 = I_1^2 R_1 = (5.774)^2(48) = \mathbf{1.6003\ kW}$

$P_2 = I_2^2 R_2 = (6.928)^2(40) = \mathbf{1.9199\ kW}$

$P_3 = I_3^2 R_3 = (4.619)^2(60) = \mathbf{1.2801\ kW}$

(d) $\mathrm{P_T} = \mathrm{P_1} + \mathrm{P_2} + \mathrm{P_3} = \mathbf{4.8\ kW}$

Solution 12.67

(a) The power to the motor is

$$P_T = S\cos\theta = (260)(0.85) = 221\text{ kW}$$

The motor power per phase is

$$P_p = \frac{1}{3}P_T = 73.67\text{ kW}$$

Hence, the wattmeter readings are as follows:

$$W_a = 73.67 + 24 = \mathbf{97.67\ kW}$$

$$W_b = 73.67 + 15 = \mathbf{88.67\ kW}$$

$$W_c = 73.67 + 9 = \mathbf{82.67\ kW}$$

(b) The motor load is balanced so that $I_N = 0$.

For the lighting loads,

$$I_a = \frac{24{,}000}{120} = 200\text{ A}$$

$$I_b = \frac{15{,}000}{120} = 125\text{ A}$$

$$I_c = \frac{9{,}000}{120} = 75\text{ A}$$

If we let

$$\mathbf{I}_a = I_a\angle 0° = 200\angle 0°\text{ A}$$

$$\mathbf{I}_b = 125\angle -120°\text{ A}$$

$$\mathbf{I}_c = 75\angle 120°\text{ A}$$

Then,

$$-\mathbf{I}_N = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$$

$$-\mathbf{I}_N = 200 + (125)\left(-0.5 - j\frac{\sqrt{3}}{2}\right) + (75)\left(-0.5 + j\frac{\sqrt{3}}{2}\right)$$

$$-\mathbf{I}_N = 100 - j43.3\text{ A}$$

$$\left|\mathbf{I}_N\right| = \mathbf{108.97\ A}$$

Solution 12.68

(a) $S = \sqrt{3}\,V_L I_L = \sqrt{3}(330)(8.4) = \mathbf{4801\ VA}$

(b) $P = S\cos\theta \longrightarrow \text{pf} = \cos\theta = \dfrac{P}{S}$

$$\text{pf} = \frac{4500}{4801.24} = \mathbf{0.9372}$$

(c) For a wye-connected load,
$I_p = I_L = \mathbf{8.4\ A}$

(d) $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{330}{\sqrt{3}} = \mathbf{190.53\ V}$

Solution 12.69

For load 1,

$$\overline{S}_1 = S_1 \cos\theta_1 + jS_1 \sin\theta_1$$

$$pf = 0.85 = \cos\theta_1 \longrightarrow \theta_1 = 31.79^o$$

$$\overline{S}_1 = 13.6 + j8.43 \quad \text{kVA}$$

For load 2,

$$\overline{S_2} = 12x0.6 + j12x0.8 = 7.2 + j9.6 \ \ \text{kVA}$$

For load 3,

$$\overline{S}_3 = 8 + j0 \quad \text{kVA}$$

Therefore,
S = S_1 + S_2 + S_3 = [28.8+j18.03] kVA

Although we can solve this using a delta load, it will be easier to assume our load is wye connected. We also need the wye voltages and will assume that the phase angle on V_{an} = 208/1.73205 = 120.089 is –30 degrees.

Since
S = 3**VI*** or **I*** = **S**/(3**V**) = (33,978∠32.048°)/[3(120.089)∠–30°] = 94.31∠62.05° A.

I_a = 94.31∠–62.05° A, **I_b = 94.31∠177.95° A**, **I_c = 94.31∠57.95° A**

I = 138.46 – j86.68 = **163.35∠-32° A**.

Solution 12.70

$$P_T = P_1 + P_2 = 1200 - 400 = 800$$

$$Q_T = P_2 - P_1 = -400 - 1200 = -1600$$

$$\tan\theta = \frac{Q_T}{P_T} = \frac{-1600}{800} = -2 \quad \longrightarrow \quad \theta = -63.43°$$

$$\text{pf} = \cos\theta = \mathbf{0.4472 \quad (leading)}$$

$$Z_p = \frac{V_L}{I_L} = \frac{240}{6} = 40$$

$$\mathbf{Z}_p = \mathbf{40\angle -63.43° \ \Omega}$$

Solution 12.71

(a) If $\mathbf{V}_{ab} = 208\angle 0°$, $\mathbf{V}_{bc} = 208\angle -120°$, $\mathbf{V}_{ca} = 208\angle 120°$,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{Ab}} = \frac{208\angle 0°}{20} = 10.4\angle 0°$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{BC}} = \frac{208\angle -120°}{10\sqrt{2}\angle -45°} = 14.708\angle -75°$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{CA}} = \frac{208\angle 120°}{13\angle 22.62°} = 16\angle 97.38°$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 10.4\angle 0° - 16\angle 97.38°$$
$$\mathbf{I}_{aA} = 10.4 + 2.055 - j15.867$$
$$\mathbf{I}_{aA} = 20.171\angle -51.87°$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC} = 16\angle 97.83° - 14.708\angle -75°$$
$$\mathbf{I}_{cC} = 30.64\angle 101.03°$$

$$P_1 = |\mathbf{V}_{ab}||\mathbf{I}_{aA}|\cos(\theta_{V_{ab}} - \theta_{I_{aA}})$$
$$P_1 = (208)(20.171)\cos(0° + 51.87°) = \mathbf{2.590\ \textit{kW}}$$

$$P_2 = |\mathbf{V}_{cb}||\mathbf{I}_{cC}|\cos(\theta_{V_{cb}} - \theta_{I_{cC}})$$

But $\mathbf{V}_{cb} = -\mathbf{V}_{bc} = 208\angle 60°$

$$P_2 = (208)(30.64)\cos(60° - 101.03°) = \mathbf{4.808\ \textit{kW}}$$

(b) $P_T = P_1 + P_2 = 7398.17$ W

$Q_T = \sqrt{3}(P_2 - P_1) = 3840.25$ VAR

$\mathbf{S}_T = P_T + jQ_T = 7398.17 + j3840.25$ VA

$S_T = |\mathbf{S}_T| = \mathbf{8.335\ \textit{kVA}}$

Solution 12.72

From ***Problem 12.11,***

$\mathbf{V}_{AB} = 220\angle 130^\circ \text{ V}$ and $\mathbf{I}_{aA} = 30\angle 180^\circ \text{ A}$

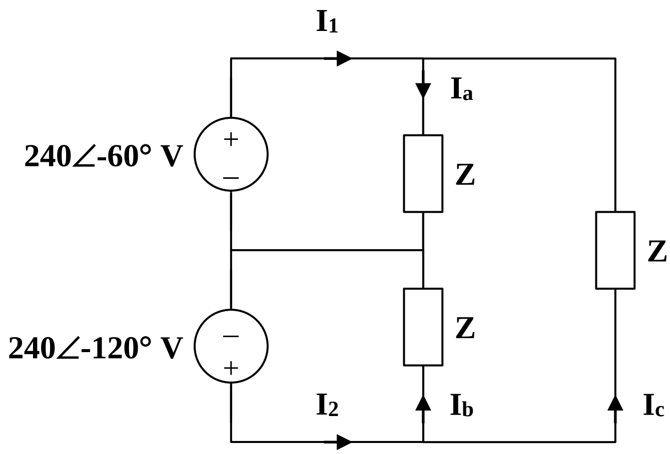
$P_1 = (220)(30)\cos(130^\circ - 180^\circ) = \mathbf{4.242\ \mathit{kW}}$

$\mathbf{V}_{CB} = \mathbf{-V}_{BC} = 220\angle 190^\circ$
$\mathbf{I}_{cC} = 30\angle -60^\circ$

$P_2 = (220)(30)\cos(190^\circ + 60^\circ) = \mathbf{-\ 2.257\ \mathit{k}W}$

Solution 12.73

Consider the circuit as shown below.



$$\mathbf{Z} = 10 + j30 = 31.62\angle 71.57°$$

$$\mathbf{I}_a = \frac{240\angle -60°}{31.62\angle 71.57°} = 7.59\angle -131.57°$$

$$\mathbf{I}_b = \frac{240\angle -120°}{31.62\angle 71.57°} = 7.59\angle -191.57°$$

$$\mathbf{I}_c\,\mathbf{Z} + 240\angle -60° - 240\angle -120° = 0$$

$$\mathbf{I}_c = \frac{-240}{31.62\angle 71.57°} = 7.59\angle 108.43°$$

$$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_c = 13.146\angle -101.57°$$

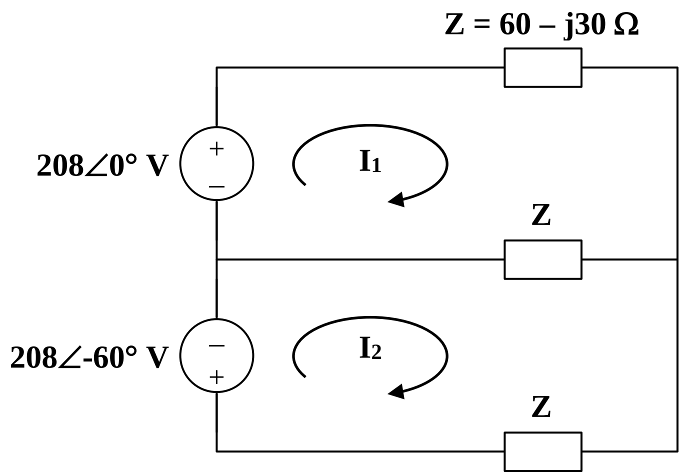
$$\mathbf{I}_2 = \mathbf{I}_b + \mathbf{I}_c = 13.146\angle 138.43°$$

$$P_1 = \text{Re}\left[\mathbf{V}_1\,\mathbf{I}_1^*\right] = \text{Re}\left[(240\angle -60°)(13.146\angle 101.57°)\right] = \mathbf{2.360}\ \boldsymbol{kW}$$

$$P_2 = \text{Re}\left[\mathbf{V}_2\,\mathbf{I}_2^*\right] = \text{Re}\left[(240\angle -120°)(13.146\angle -138.43°)\right] = \mathbf{-632.8\ W}$$

Solution 12.74

Consider the circuit shown below.



For mesh 1,

$$208 = 2\mathbf{Z}\mathbf{I}_1 - \mathbf{Z}\mathbf{I}_2$$

For mesh 2,

$$-208\angle -60° = -\mathbf{Z}\mathbf{I}_1 + 2\mathbf{Z}\mathbf{I}_2$$

In matrix form,

$$\begin{bmatrix} 208 \\ -208\angle -60° \end{bmatrix} = \begin{bmatrix} 2\mathbf{Z} & -\mathbf{Z} \\ -\mathbf{Z} & 2\mathbf{Z} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 3\mathbf{Z}^2, \qquad \Delta_1 = (208)(1.5 + j0.866)\mathbf{Z}, \qquad \Delta_2 = (208)(j1.732)\mathbf{Z}$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{(208)(1.5 + j0.866)}{(3)(60 - j30)} = 1.789\angle 56.56°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{(208)(j1.732)}{(3)(60 - j30)} = 1.79\angle 116.56°$$

$$P_1 = \text{Re}\left[\mathbf{V}_1 \mathbf{I}_1^* \right] = \text{Re}\left[(208)(1.789\angle -56.56°) \right] = \mathbf{208.98\ W}$$

$$P_2 = \text{Re}\left[\mathbf{V}_2 (-\mathbf{I}_2)^* \right] = \text{Re}\left[(208\angle -60°))(1.79\angle 63.44°) \right] = \mathbf{371.65\ W}$$

Solution 12.75

(a) $I = \frac{V}{R} = \frac{12}{600} = \mathbf{20\ mA}$

(b) $I = \frac{V}{R} = \frac{120}{600} = \mathbf{200\ mA}$

Solution 12.76

If both appliances have the same power rating, P,

$$I = \frac{P}{V_s}$$

For the 120-V appliance, $\quad I_1 = \dfrac{P}{120}$.

For the 240-V appliance, $\quad I_2 = \dfrac{P}{240}$.

$$\text{Power loss} = I^2 R = \begin{cases} \dfrac{P^2 R}{120^2} & \text{for the 120-V appliance} \\ \dfrac{P^2 R}{240^2} & \text{for the 240-V appliance} \end{cases}$$

Since $\dfrac{1}{120^2} > \dfrac{1}{240^2}$, **the losses in the 120-V appliance are higher**.

Solution 12.77

A three-phase generator supplied 10 kVA at a power factor of 0.85 lagging. If 7.5 kW are delivered to the load and line losses are 160 W per phase, what are the losses in the generator?

Solution

$P_g = P_T - P_{load} - P_{line}$, $pf = 0.85$

But P_T = 10k cos(θ) = 10k(0.85) = 8.5 kW
P_g = 8.5 kW – 7.5 kW – (3)(160) W = **520 W**

Solution 12.78

$$\cos\theta_1 = \frac{51}{60} = 0.85 \longrightarrow \theta_1 = 31.79°$$
$$Q_1 = S_1 \sin\theta_1 = (60)(0.5268) = 31.61\ \text{kVAR}$$

$$P_2 = P_1 = 51\ \text{kW}$$
$$\cos\theta_2 = 0.95 \longrightarrow \theta_2 = 18.19°$$
$$S_2 = \frac{P_2}{\cos\theta_2} = 53.68\ \text{kVA}$$
$$Q_2 = S_2 \sin\theta_2 = 16.759\ \text{kVAR}$$

$$Q_c = Q_1 - Q_2 = 3.61 - 16.759 = 14.851\ \text{kVAR}$$

For each load,

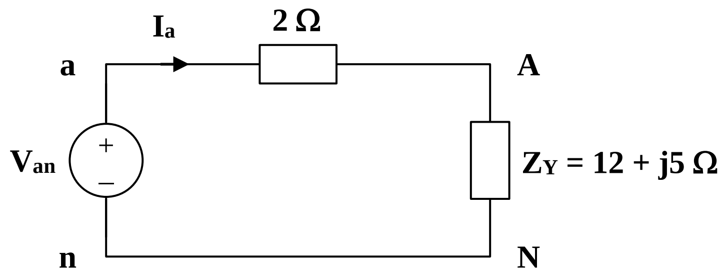
$$Q_{c1} = \frac{Q_c}{3} = 4.95\ \text{kVAR}$$
$$C = \frac{Q_{c1}}{\omega V^2} = \frac{4950}{(2\pi)(60)(440)^2} = \mathbf{67.82\ \mu F}$$

Solution 12.79

A balanced three-phase generator has an *abc* phase sequence with phase voltage $\mathbf{V}_{an}$ = 554.3∠0° V. The generator feeds an induction motor which may be represented by a balanced Y-connected load with an impedance of 12 + j5 Ω per phase. Find the line currents and the load voltages. Assume a line impedance of 2 Ω per phase.

Solution

Consider the per-phase equivalent circuit below.



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y + 2} = \frac{554.3\angle 0°}{14 + j5} = \frac{554.3}{14.866\angle 19.65°} = \mathbf{37.29\angle{-19.65°}\ A}$$

Thus,

$$\mathbf{I}_b = \mathbf{I}_a \angle{-120°} = \mathbf{37.29\angle{-139.65°}\ A}$$
$$\mathbf{I}_c = \mathbf{I}_a \angle 120° = \mathbf{37.29\angle 100.35°\ A}$$

$$\mathbf{V}_{AN} = \mathbf{I}_a\,\mathbf{Z}_Y = (37.286\angle{-19.65°})(13\angle 22.62°) = \mathbf{484.7\angle 2.97°\ V}$$

Thus,

$$\mathbf{V}_{BN} = \mathbf{V}_{AN}\angle{-120°} = \mathbf{484.7\angle{-117.03°}\ V}$$

$$\mathbf{V}_{CN} = \mathbf{V}_{AN}\angle 120° = \mathbf{484.7\angle 122.97°\ V}$$

Solution 12.80

$$S = S_1 + S_2 + S_3 = 6[0.83 + j\sin(\cos^{-1} 0.83)] + S_2 + 8(0.7071 - j0.7071)$$
$$S = 10.6368 - j2.31 + S_2 \text{ kVA} \qquad (1)$$

But
$$S = \sqrt{3} V_L I_L \angle\theta = \sqrt{3}(208)(84.6)(0.8 + j0.6) \text{ VA} = 24.383 + \text{j}18.287 \text{ kVA} \qquad (2)$$

From (1) and (2),
$$S_2 = 13.746 + j20.6 = 24.76\angle 56.28 \text{ kVA}$$
Thus, the unknown load is **24.76 kVA at 0.5551 pf lagging**.

Solution 12.81

$$\text{pf} = 0.8 \quad \text{(leading)} \quad \longrightarrow \quad \theta_1 = -36.87°$$
$$\mathbf{S}_1 = 150\angle -36.87° \text{ kVA}$$

$$\text{pf} = 1.0 \quad \longrightarrow \quad \theta_2 = 0°$$
$$\mathbf{S}_2 = 100\angle 0° \text{ kVA}$$

$$\text{pf} = 0.6 \quad \text{(lagging)} \quad \longrightarrow \quad \theta_3 = 53.13°$$
$$\mathbf{S}_3 = 200\angle 53.13° \text{ kVA}$$

$$\mathbf{S}_4 = 80 + j95 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4$$
$$\mathbf{S} = 420 + j165 = 451.2\angle 21.45° \text{ kVA}$$

$$S = \sqrt{3}\, V_L I_L$$

$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{451.2 \times 10^3}{\sqrt{3} \times 480} = 542.7 \text{ A}$$

For the line,

$$\mathbf{S}_L = 3 I_L^2 \mathbf{Z}_L = (3)(542.7)^2 (0.02 + j0.05)$$
$$\mathbf{S}_L = 17.67 + j44.18 \text{ kVA}$$

At the source,

$$\mathbf{S}_T = \mathbf{S} + \mathbf{S}_L = 437.7 + j209.2$$
$$\mathbf{S}_T = 485.1\angle 25.55° \text{ kVA}$$

$$V_T = \frac{S_T}{\sqrt{3}\, I_L} = \frac{485.1 \times 10^3}{\sqrt{3} \times 542.7} = \mathbf{516\ V}$$

Solution 12.82

$$\overline{S}_1 = 400(0.8 + j0.6) = 320 + j240\,\text{kVA}, \qquad \overline{S}_2 = 3\frac{V^2{}_p}{Z^*{}_p}$$

For the delta-connected load, $V_L = V_p$

$$\overline{S}_2 = 3x\frac{(2400)^2}{10 - j8} = 1053.7 + j842.93\,\text{kVA}$$

$$\overline{S} = \overline{S}_1 + \overline{S}_2 = 1.3737 + j1.0829\,\text{MVA}$$

Let $I = I_1 + I_2$ be the total line current. For I_1,

$$S_1 = 3V_p I^*{}_1, \qquad V_p = \frac{V_L}{\sqrt{3}}$$

$$I^*{}_1 = \frac{S_1}{\sqrt{3}V_L} = \frac{(320 + j240)x10^3}{\sqrt{3}(2400)}, \qquad I_1 = 76.98 - j57.735$$

For I_2, convert the load to wye.

$$I_2 = I_p\sqrt{3}\angle -30^o = \frac{2400}{10 + j8}\sqrt{3}\angle -30^o = 273.1 - j289.76$$

$$I = I_1 + I_2 = 350 - j347.5$$

$$\boldsymbol{V_s = V_L + V_{line} = 2400 + I(3 + j6) = 5.185 + j1.405\,\text{kV} \qquad \longrightarrow \qquad |V_s| = \underline{5.372\,\text{kV}}}$$

Solution 12.83

$$S_1 = 120x746x0.95(0.707 + j0.707) = 60.135 + j60.135\,\text{kVA}, \qquad \text{S}_2 = 80\,\text{kVA}$$

$$S = S_1 + S_2 = 140.135 + j60.135\,\text{kVA}$$

$$\textbf{But}\ \ |\boldsymbol{S}| = \sqrt{3}\boldsymbol{V}_L\boldsymbol{I}_L \quad \longrightarrow \quad \boldsymbol{I}_L = \frac{|\boldsymbol{S}|}{\sqrt{3}\boldsymbol{V}_L} = \frac{\mathbf{152.49}x\mathbf{10^3}}{\sqrt{3}x\mathbf{480}} = \underline{\mathbf{183.42\ A}}$$

Solution 12.84

We first find the magnitude of the various currents.

For the motor,

$$I_L = \frac{S}{\sqrt{3}\,V_L} = \frac{4000}{440\sqrt{3}} = 5.248 \text{ A}$$

For the capacitor,

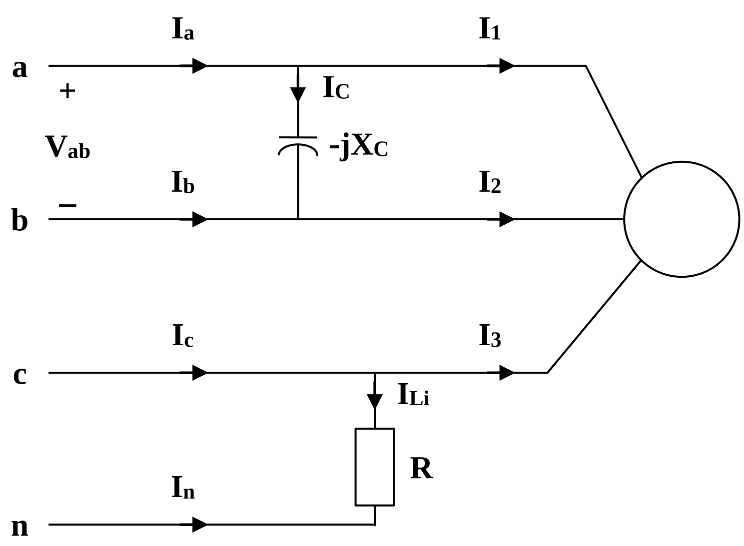
$$I_C = \frac{Q_c}{V_L} = \frac{1800}{440} = 4.091 \text{ A}$$

For the lighting,

$$V_p = \frac{440}{\sqrt{3}} = 254 \text{ V}$$

$$I_{Li} = \frac{P_{Li}}{V_p} = \frac{800}{254} = 3.15 \text{ A}$$

Consider the figure below.



If $\mathbf{V}_{an} = V_p\angle 0°$, $\qquad \mathbf{V}_{ab} = \sqrt{3}\,V_p\angle 30°$

$$\mathbf{V}_{cn} = V_p\angle 120°$$

$$\mathbf{I}_C = \frac{\mathbf{V}_{ab}}{-jX_C} = 4.091\angle 120°$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = 4.091\angle(\theta + 30°)$$

where $\theta = \cos^{-1}(0.72) = 43.95°$

$$\mathbf{I}_1 = 5.249\angle 73.95°$$

$$\mathbf{I}_2 = 5.249\angle -46.05°$$

$$\mathbf{I}_3 = 5.249\angle 193.95°$$

$$\mathbf{I}_{Li} = \frac{\mathbf{V}_{cn}}{R} = 3.15\angle 120°$$

Thus,

$$\mathbf{I}_a = \mathbf{I}_1 + \mathbf{I}_C = 5.249\angle 73.95° + 4.091\angle 120°$$
$$\mathbf{I}_a = \mathbf{8.608\angle 93.96°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_C = 5.249\angle -46.05° - 4.091\angle 120°$$
$$\mathbf{I}_b = \mathbf{9.271\angle -52.16°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_3 + \mathbf{I}_{Li} = 5.249\angle 193.95° + 3.15\angle 120°$$
$$\mathbf{I}_c = \mathbf{6.827\angle 167.6°\ A}$$

$$\mathbf{I}_n = -\mathbf{I}_{Li} = \mathbf{3.15\angle -60°\ A}$$

Solution 12.85

Let $Z_Y = R$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{240}{\sqrt{3}} = 138.56 \text{ V}$$

$$P = V_p I_p = \frac{27}{2} = 9 \text{ kW} = \frac{V_p^2}{R}$$

$$R = \frac{V_p^2}{P} = \frac{(138.56)^2}{9000} = 2.133\,\Omega$$

Thus, $Z_Y = \mathbf{2.133\,\Omega}$

Solution 12.86

For the single-phase three-wire system in Fig. 12.77, find currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{nN}$.

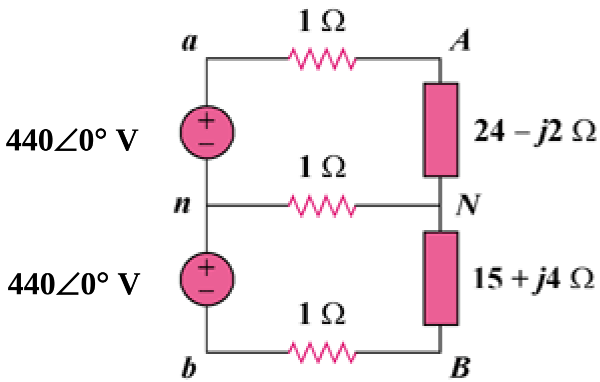
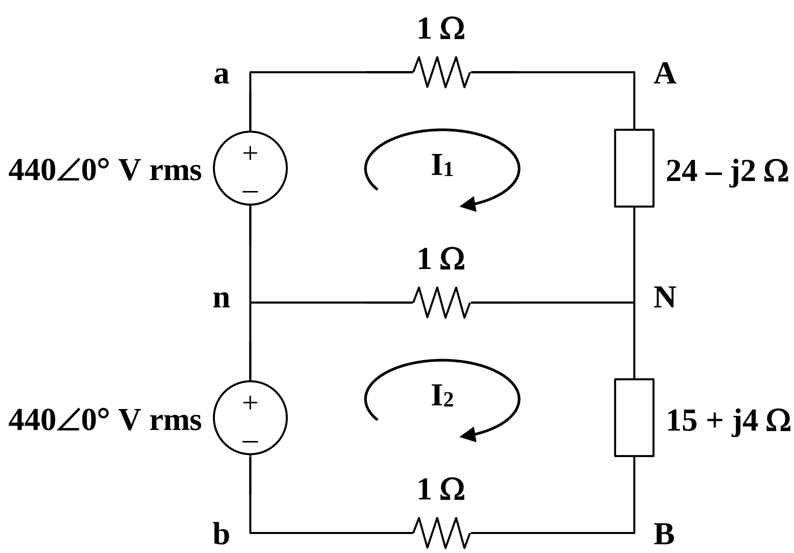


Figure 12.77
For Prob. 12.86.

Solution

Consider the circuit shown below.



For the two meshes,

$$440 = (26 - j2)\mathbf{I}_1 - \mathbf{I}_2 \qquad (1)$$

$$440 = (17 + j4)\mathbf{I}_2 - \mathbf{I}_1 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 440 \\ 440 \end{bmatrix} = \begin{bmatrix} 26 - j2 & -1 \\ -1 & 17 + j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 449 + \text{j}70\,, \qquad \Delta_1 = (440)(18 + j4)\,, \qquad \Delta_2 = (440)(27 - j2)$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{440 \times 18.44\angle 12.53°}{454.42\angle 8.86°} = \mathbf{17.8548\angle 3.67°\ A}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{440 \times 27.07\angle -4.24°}{454.42\angle 8.86°} = \mathbf{26.211\angle -13.1°\ A}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_1 = \mathbf{17.8548\angle 3.67°\ A}$$

$$\mathbf{I}_{\text{bB}} = -\mathbf{I}_2 = \mathbf{26.211\angle 166.9°\ A}$$

$$\mathbf{I}_{nN} = \mathbf{I}_2 - \mathbf{I}_1 = 25.529 - \text{j}5.9408 - 17.8182 - \text{j}1.14288$$

$= 7.711 - \text{j}7.084 =$ **$(10.471\angle -42.57°)$ A**

Solution 12.87

Consider the single-phase three-wire system shown in Fig. 12.78. Find the current in the neutral wire and the complex power supplied by each source. Take $\mathbf{V}_s$ as a $220\angle 0°$–V, 60-Hz source.

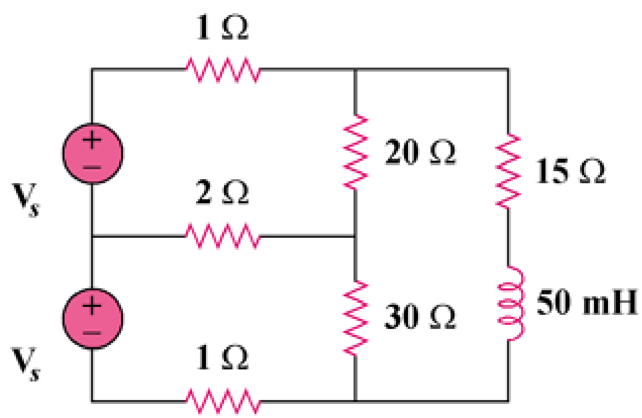
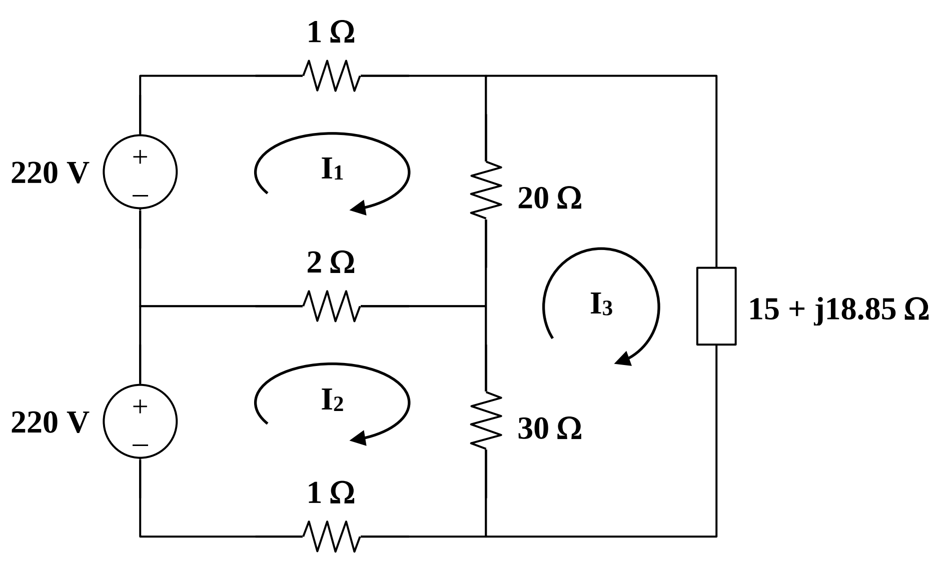


Figure 12.78
For Prob. 12.87.

Solution

$L = 50 \text{ mH} \longrightarrow j\omega L = j(2\pi)(60)(50\text{x}10^{-3}) = j18.85$
Consider the circuit below.



Applying KVl to the three meshes, we obtain

$$23\mathbf{I}_1 - 2\mathbf{I}_2 - 20\mathbf{I}_3 = 220 \quad (1)$$

$$-2\mathbf{I}_1 + 33\mathbf{I}_2 - 30\mathbf{I}_3 = 220 \quad (2)$$

$$-20\mathbf{I}_1 - 30\mathbf{I}_2 + (65 + j18.85)\mathbf{I}_3 = 0 \qquad (3)$$

In matrix form,

$$\begin{bmatrix} 23 & -2 & -20 \\ -2 & 33 & -30 \\ -20 & -30 & 65 + j18.85 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 220 \\ 220 \\ 0 \end{bmatrix}$$

$\Delta = 12{,}775 + j14{,}232$, $\quad \Delta_1 = (220)(1975 + j659.8)$

$\Delta_2 = (220)(1825 + j471.3)$, $\quad \Delta_3 = (220)(1450)$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{220 \times 2082\angle 18.47°}{19214\angle 48.09°} = 23.951\angle{-29.62°}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{220 \times 1884.9\angle 14.48°}{19124\angle 48.09°} = 21.675\angle{-33.61°}$$

$$\mathbf{I}_n = \mathbf{I}_2 - \mathbf{I}_1 = \frac{\Delta_2 - \Delta_1}{\Delta} = \frac{(220)(-150 - j188.5)}{12{,}775 + j14{,}231.75} = \mathbf{2.77\angle{-176.6°}\ A}$$

$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = (220)(23.951\angle 29.62°) =$ **(4.581 + j2.604) kVA**

$\mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^* = (220)(21.675\angle 33.61°) =$ **(3.971 + j2.64) kVA**

Solution 13.1

For coil 1, $L_1 - M_{12} + M_{13} = 12 - 8 + 4 = 8$

For coil 2, $L_2 - M_{21} - M_{23} = 16 - 8 - 10 = -2$

For coil 3, $L_3 + M_{31} - M_{32} = 20 + 4 - 10 = 14$

$$L_T = 8 - 2 + 14 = 20\text{H}$$

or $$L_T = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{13}$$

$$L_T = 12 + 16 + 20 - 2\text{x}8 - 2\text{x}10 + 2\text{x}4 = 48 - 16 - 20 + 8$$

= **20H**

Solution 13.2

Using Fig. 13.73, design a problem to help other students to better understand mutual inductance.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the inductance of the three series-connected inductors of Fig. 13.73.

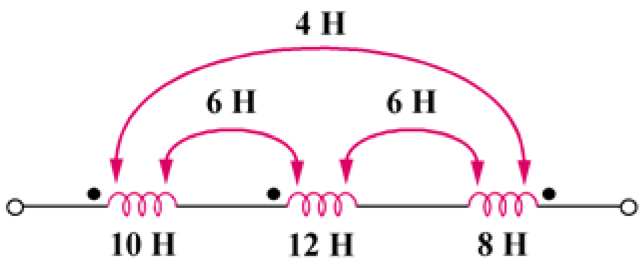


Figure 13.73

Solution

$L = L_1 + L_2 + L_3 + 2M_{12} – 2M_{23} –2M_{31}$
$= 10 + 12 +8 + 2x6 – 2x6 –2x4$

= **22H**

Solution 13.3

$L_1 + L_2 + 2M = 500$ mH (1)

$L_1 + L_2 – 2M = 300$ mH (2)

Adding (1) and (2),

$2L_1 + 2L_2 = 800$ mH

But, $L_1 = 3L_{2,}$, or $8L_2 + 400$, and L_2 = **100 mH**

$L_1 = 3L_2$ = **300 mH**

From (2), 150 + 50 – 2M = 150 leads to M = **50 mH**

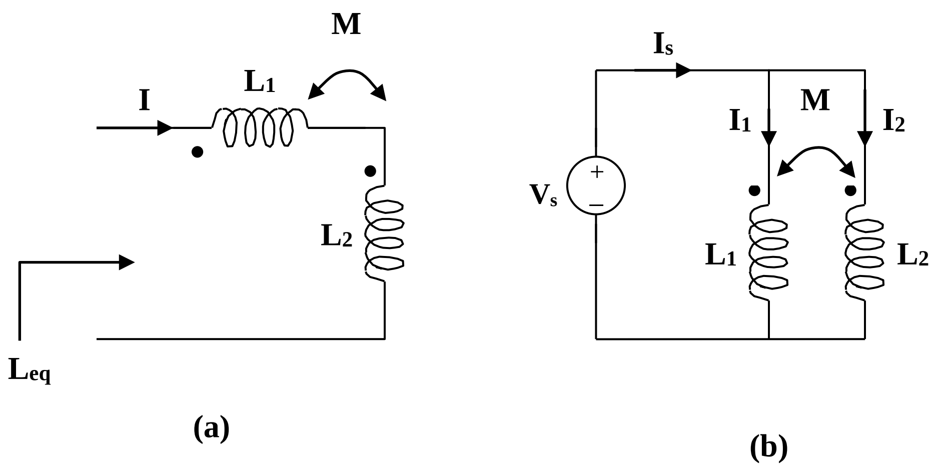
$k = M/\sqrt{L_1L_2} = 50/\sqrt{100x300}$ = **0.2887**

300 mH, 100 mH, 50 mH, 0.2887

Solution 13.4

(a) For the series connection shown in Figure (a), the current I enters each coil from its dotted terminal. Therefore, the mutually induced voltages have the same sign as the self-induced voltages. Thus,

$$L_{eq} = \mathbf{L_1 + L_2 + 2M}$$



(b) For the parallel coil, consider Figure (b).

$$I_s = I_1 + I_2 \quad \text{and} \quad Z_{eq} = V_s/I_s$$

Applying KVL to each branch gives,

$$V_s = j\omega L_1 I_1 + j\omega M I_2 \qquad (1)$$

$$V_s = j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

or

$$\begin{bmatrix} V_s \\ V_s \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = -\omega^2 L_1 L_2 + \omega^2 M^2, \quad \Delta_1 = j\omega V_s(L_2 - M), \quad \Delta_2 = j\omega V_s(L_1 - M)$$

$$I_1 = \Delta_1/\Delta, \quad \text{and} \quad I_2 = \Delta_2/\Delta$$

$$I_s = I_1 + I_2 = (\Delta_1 + \Delta_2)/\Delta = j\omega(L_1 + L_2 - 2M)V_s/(-\omega^2(L_1L_2 - M^2))$$
$$= (L_1 + L_2 - 2M)V_s/(j\omega(L_1L_2 - M^2))$$

$$Z_{eq} = V_s/I_s = j\omega(L_1L_2 - M^2)/(L_1 + L_2 - 2M) = j\omega L_{eq}$$

i.e., $$L_{eq} = \mathbf{(L_1L_2 - M^2)/(L_1 + L_2 - 2M)}$$

Solution 13.5

(a) If the coils are connected in series,

$$L = L_1 + L_2 + 2M = 50 + 120 + 2(0.5)\sqrt{50x120} = \mathbf{247.4\ mH}$$

(b) If they are connected in parallel,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{50x120 - 38.72^2}{50 + 120 - 2x38.72}\,\text{mH} = \mathbf{48.62\ mH}$$

(a) 247.4 mH, (b) 48.62 mH

Problem 13.6

Given the circuit shown in Fig. 13.75. Determine the value of $\mathbf{V_1}$ and $\mathbf{I_2}$.

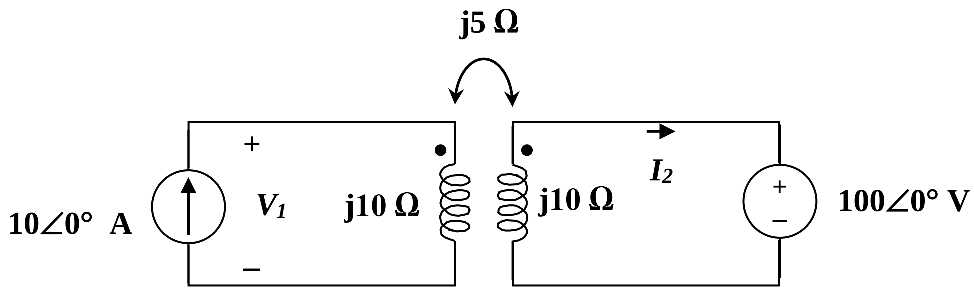
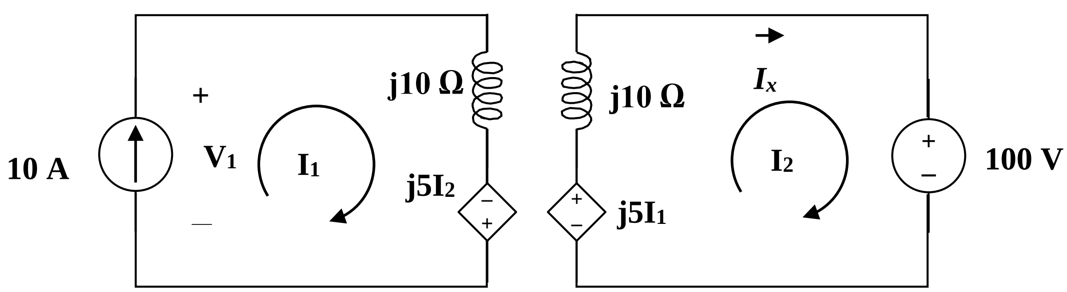


Figure 13.75
For Prob. 13.6.

Solution

Step 1. First we need to replace the coupled inductors with the dependent source model. Next we need to determine the signs on the dependent sources using the dot convention. Now we can write mesh equations around each loop and solve for $\mathbf{I_2}$ and $\mathbf{I_1}$. Once we have $\mathbf{I_1}$ and $\mathbf{I_2}$ we can find $\mathbf{V_1}$.

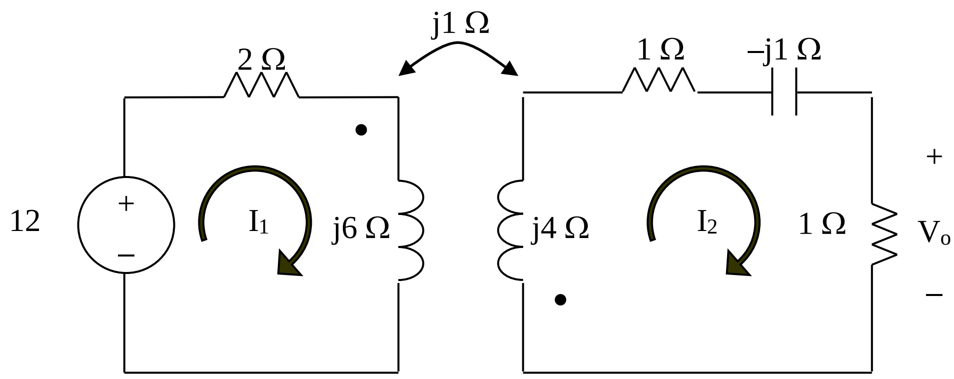


The mesh equations are $-\mathbf{V_1} + j10\mathbf{I_1} - j5\mathbf{I_2}$ where $\mathbf{I_1} = 10$ A and $-j5\mathbf{I_1} + j10\mathbf{I_2} + 100 = 0$. This then gives us two equations with two unknowns, $\mathbf{V_1}$ and $\mathbf{I_2}$.

Step 2. $-\mathbf{V_1} - j5\mathbf{I_2} = -j100$ and $j10\mathbf{I_2} = -100 + j50$ or $\mathbf{I_2} = 5 + j10$ $= \mathbf{11.1803\angle 63.435°}$. Next, $\mathbf{V_1} = -j5\mathbf{I_2} + j100 = 50 - j25 + j100 = 50 + j75$ $= \mathbf{90.139\angle 56.31°\ V}$.

Solution 13.7

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$(2+j6)I_1 + jI_2 = 24$$

For mesh 2,

$$jI_1 + (2–j+j4)I_2 = jI_1 + (2+j3)I_2 = 0 \text{ or } I_1 = (–3+j2)I_2$$

Substituting into the first equation results in $I_2 = (–0.8762+j0.6328)$ A.

$$V_o = I_2 x1 = \mathbf{1.081\angle 144.16^\circ\ V}.$$

Solution 13.8

Find v(t) for the circuit in Fig. 13.77.

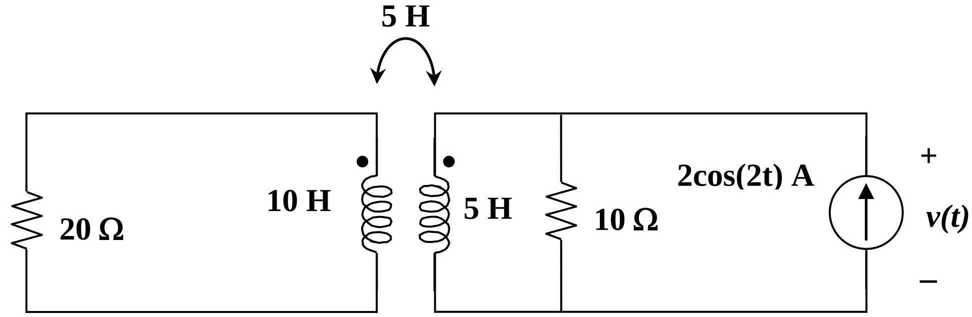
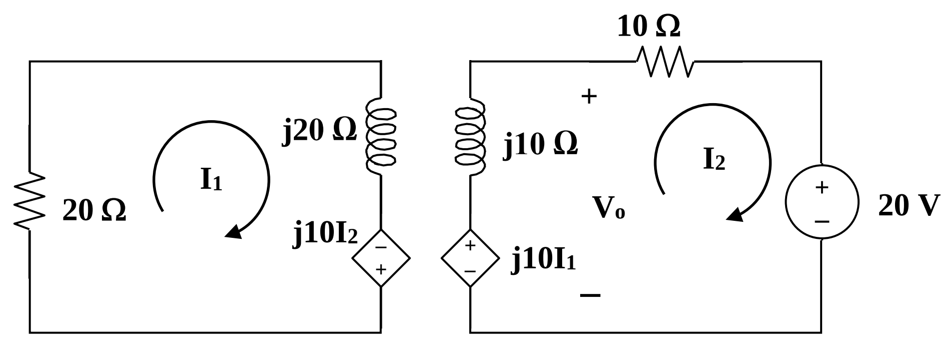


Figure 13.77
For Prob. 13.8.

Solution

Step 1. We need to transform the circuit into the frequency domain and replace the coupled inductors with the dependent source model. In addition, we need to replace the current source in parallel with the resistor with the equivalent voltage source in series with the resistor (source transformation).



The 10 H inductor becomes j2x10 = j20 Ω and the 5 H mutual coupling and 5 H inductor becomes j2x5 = j10 Ω. The source transformation converts the current source resistor combination of 10 Ω in parallel with the 2 A source with a 10 Ω resistor in series with a 20 V source.

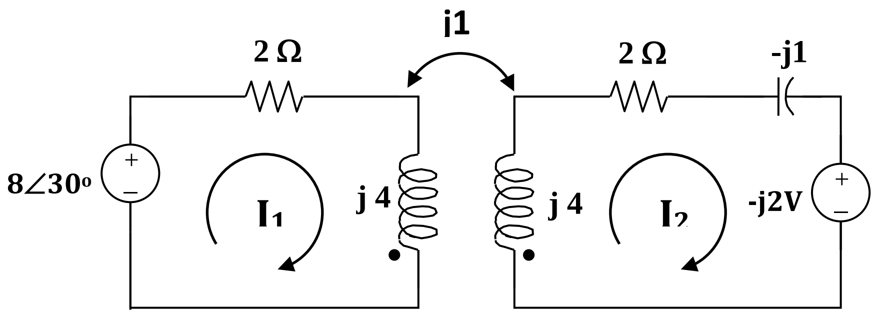
Loop 1, $20\mathbf{I}_1 + j20\mathbf{I}_1 - j10\mathbf{I}_2 = 0$ and loop 2, $-j10\mathbf{I}_1 + j10\mathbf{I}_2 + 10\mathbf{I}_2 + 20 = 0$. Finally $\mathbf{V}_o = -j10\mathbf{I}_2 + j10\mathbf{I}_1$. Note we are representing the frequency domain value of v(t) by $\mathbf{V}_o$.

Step 2. From the first loop equation we get $(20+j20)\mathbf{I}_1 = j10\mathbf{I}_2$ or $\mathbf{I}_1 = (0.25+j0.25)\mathbf{I}_2$.
This leads to $-j10(0.25+j0.25)\mathbf{I}_2 + (10+ j10)\mathbf{I}_2 = -20 = (12.5+j7.5)\mathbf{I}_2$ or
$\mathbf{I}_2 = 20\angle 180°/(14.5774\angle 30.964°) = 1.37199\angle 149.036°$ and
$\mathbf{I}_1 = (0.35355\angle 45°)(1.37199\angle 149.039°) = 0.48507\angle -165.961°$. Finally,
$\mathbf{V}_o = -j10(-1.17647 + j0.70589) + j10(-0.47058-j0.117669)$
$= 8.2356+j7.0589 = 10.847\angle 40.6°$ V or

v(t) = **10.847 cos(10t + 40.6°) A**.

Solution 13.9

Consider the circuit below.



For loop 1,

$$8\angle 30° = (2 + j4)I_1 - jI_2 \qquad (1)$$

For loop 2, $(j4 + 2 - j)I_2 - jI_1 + (-j2) = 0$

or $I_1 = (3 - j2)i_2 - 2 \qquad (2)$

Substituting (2) into (1), $8\angle 30° + (2 + j4)2 = (14 + j7)I_2$

$$I_2 = (10.928 + j12)/(14 + j7) = 1.037\angle 21.12°$$

$$V_x = 2I_2 = \mathbf{2.074\angle 21.12°\ V}$$

Solution 13.10

Find $v_o(t)$ in the circuit in Fig. 13.79.

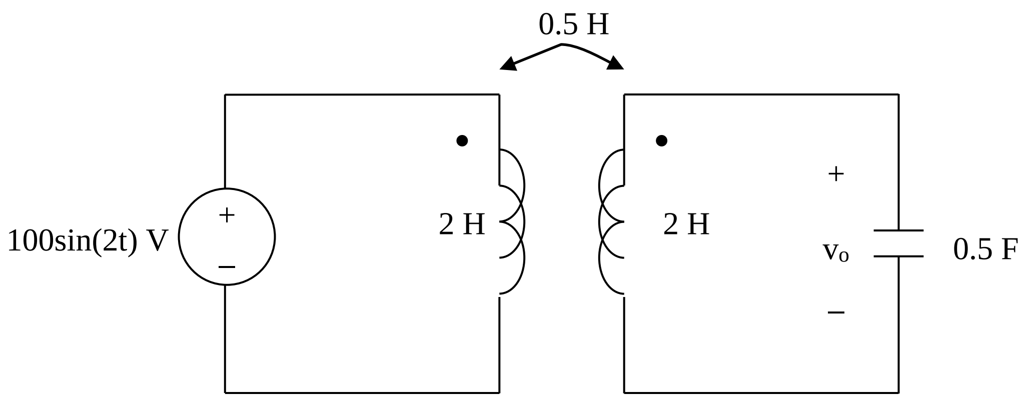
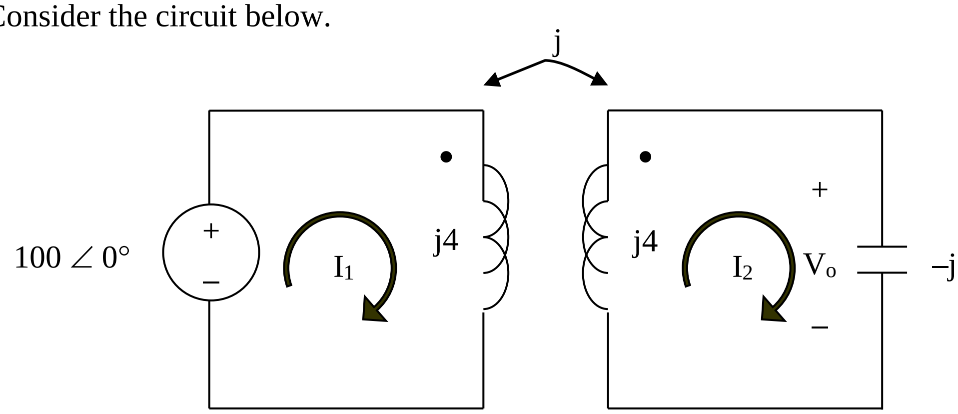


Figure 13.79
For Prob. 13.10.

Solution

$$2H \longrightarrow j\omega L = j2x2 = j4$$
$$0.5H \longrightarrow j\omega L = j2x0.5 = j$$
$$\frac{1}{2}F \longrightarrow 1/j\omega C = \frac{1}{j2x1/2} = -j$$

Consider the circuit below.



$j4I_1 - jI_2 = 100$ (1)
$-jI_1 + (j4-j)I_2 = 0 = -jI_1 + j3I_2$ (2)

In matrix form,

$$\begin{bmatrix} j4 & -j \\ -j & 3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

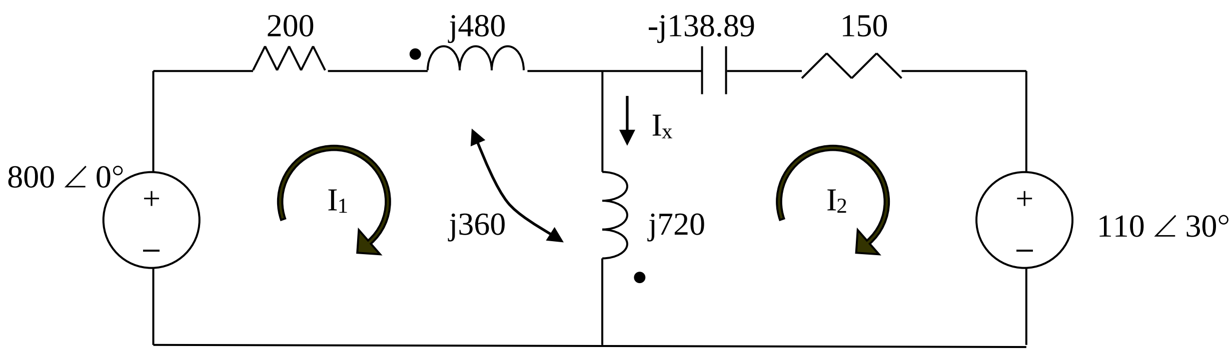
Solving this, $\mathbf{I_2} = -j9.091$ A and $\mathbf{V_o} = -j\mathbf{I_2} = 9.091\angle 180°$ V.

$v_o(t) =$ **9.091sin(2t+180°) V**.

Solution 13.11

$$800mH \longrightarrow j\omega L = j600x800x10^{-3} = j480$$
$$600mH \longrightarrow j\omega L = j600x600x10^{-3} = j360$$
$$1200mH \longrightarrow j\omega L = j600x1200x10^{-3} = j720$$
$$12\mu F \rightarrow \frac{1}{j\omega C} = \frac{-j}{600x12x10^{-6}} = -j138.89$$

After transforming the current source to a voltage source, we get the circuit shown below.



For mesh 1,

$$800 = (200 + j480 + j720)I_1 + j360I_2 - j720I_2 \text{ or}$$
$$800 = (200 + j1200)I_1 - j360I_2 \qquad (1)$$

For mesh 2,

$$110\angle 30^\circ + 150 – j138.89 + j720)I_2 + j360I_1 = 0 \text{ or}$$
$$-95.2628 - j55 = -j360I_1 + (150 + j581.1)I_2 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 800 \\ -95.2628 - j55 \end{bmatrix} = \begin{bmatrix} 200 + j1200 & -j360 \\ -j360 & 150 + j581.1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this using MATLAB leads to:

```
>> Z = [(200+1200i),-360i;-360i,(150+581.1i)]
Z =
  1.0e+003 *
   0.2000 + 1.2000i        0 - 0.3600i
        0 - 0.3600i   0.1500 + 0.5811i
>> V = [800;(-95.26-55i)]
V =
  1.0e+002 *
   8.0000
  -0.9526 - 0.5500i
>> I = inv(Z)*V
I =
   0.1390 - 0.7242i
   0.0609 - 0.2690i
```

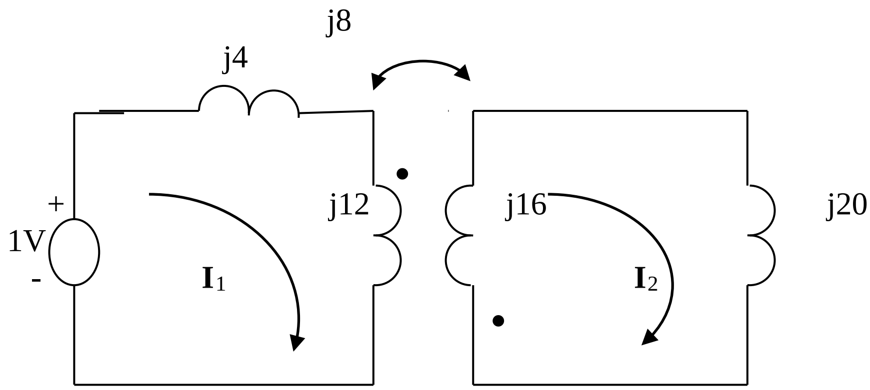
$I_x = I_1 – I_2 = 0.0781 – j0.4552 = 0.4619\angle{–80.26^\circ}$.

Hence,

$$i_x(t) = \mathbf{461.9\cos(600t–80.26^\circ)\ mA}.$$

Solution 13.12

Let $\omega = 1$.



Applying KVL to the loops,

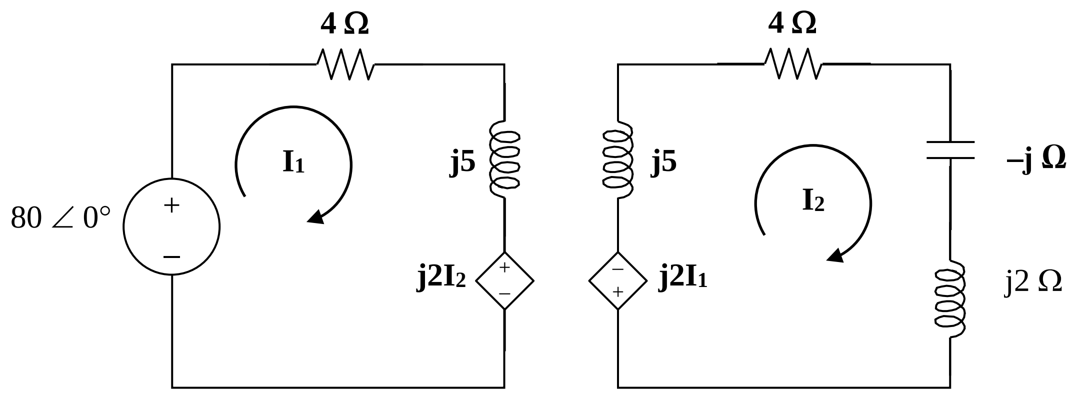
$$1 = j16I_1 + j8I_2 \qquad (1)$$
$$0 = j8I_1 + j36I_2 \qquad (2)$$

Solving (1) and (2) gives $\mathbf{I}_1 = -j0.0703$. Thus

$$Z = \frac{1}{I_1} = jL_{eq} \quad \longrightarrow \quad L_{eq} = \frac{1}{jI_1} = \mathbf{14.225\ H}.$$

We can also use the equivalent T-section for the transform to find the equivalent inductance.

Solution 13.13



$-80 + (4+j5)\mathbf{I_1} + j2\mathbf{I_2} = 0$ or $(4+j5)\mathbf{I_1} + j2\mathbf{I_2} = 80$

$j2\mathbf{I_1} + (4+j6)\mathbf{I_2} = 0$ or $\mathbf{I_2} = [-j2/(7.2111\angle 56.31°)]\mathbf{I_1} = (0.27735\angle -146.31°)\mathbf{J_1}$

$[4+j5 + j2(-0.230769-j0.153846)]I_1 = [4+j5+0.307692-j0.461538]I_1 = 80$

$[4.307692+j4.538462]I_1 = 80$ or $I_1 = 80/(6.2573\angle 46.494°)$
$= 12.78507\angle -46.494°$ A.

$Z_{in} = 80/I_1 = 6.2573\angle 46.494°\ \Omega =$ **$(4.308 + j4.538)\ \Omega$**

An alternate approach would be to use the equation,

$$\mathbf{Z_{in}} = 4 + j(5) + \frac{4}{j5 + 4 - j + j2} = 4 + j5 + \frac{4}{7.2111\angle 56.31°}$$
$= 4+j5+0.5547\angle -56.31° = 4+0.30769+j(5-0.46154)$

$=$ **$[4.308+j4.538]\ \Omega$**.

Solution 13.14

Obtain the Thevenin equivalent circuit for the circuit in Fig. 13.83 at terminals *a*-*b*.

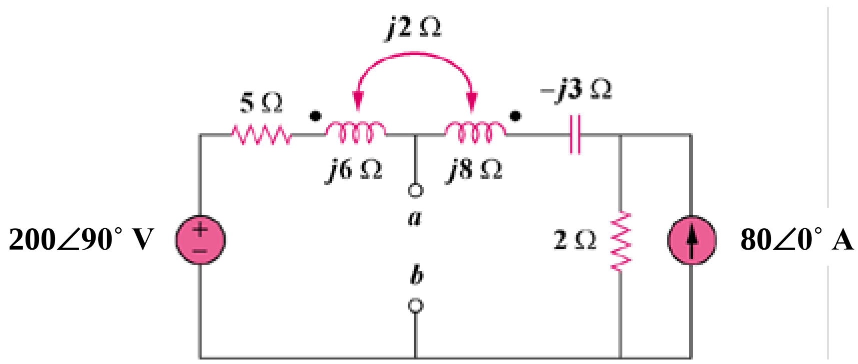
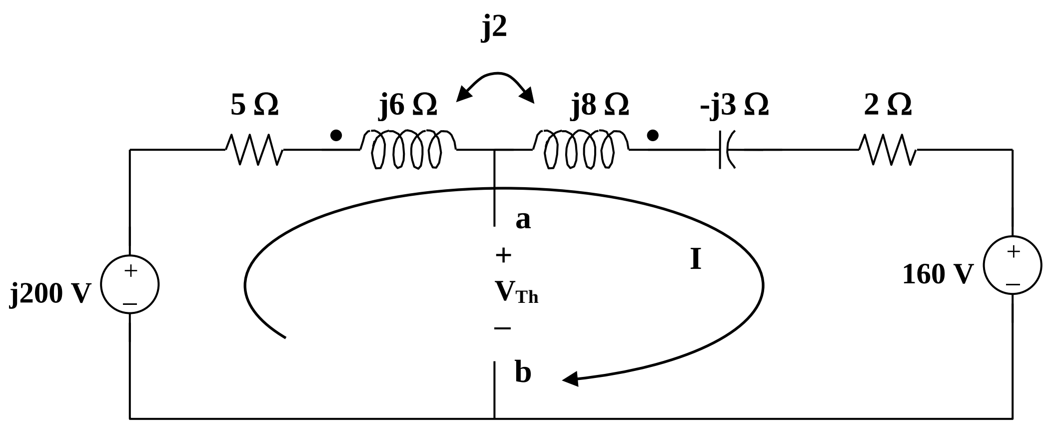


Figure 13.83
For Prob. 13.14.

Solution

To obtain V_{Th}, convert the current source to a voltage source as shown below.



Note that the two coils are connected series aiding.

$$\omega L = \omega L_1 + \omega L_2 - 2\omega M$$

$$j\omega L = j6 + j8 - j4 = j10$$

Thus,

$$-j200 + (5 + j10 - j3 + 2)\mathbf{I} + 160 = 0$$

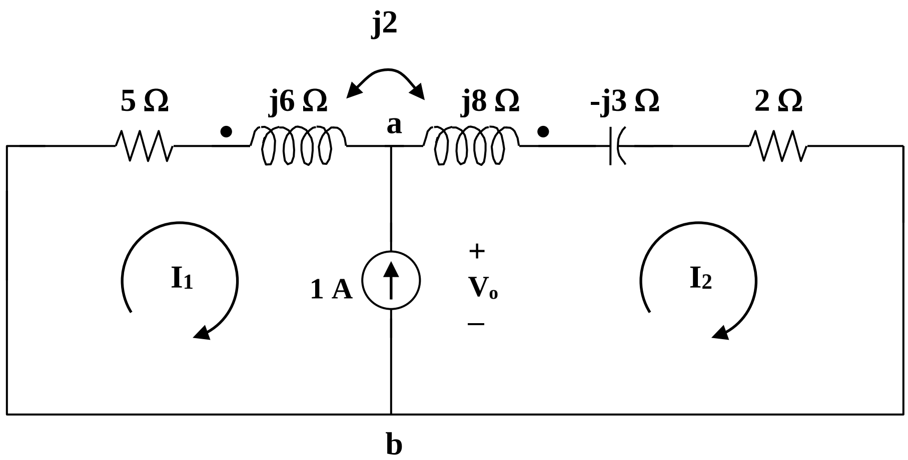
$$\mathbf{I} = (-160 + j200)/(7 + j7)$$

But, $$-j200 + (5 + j6)\mathbf{I} - j2\mathbf{I} + \mathbf{V_{Th}} = 0$$

$$V_{Th} = j200 - (5 + j4)\mathbf{I} = j200 - (5 + j4)(-160 + j200)/(7 + j7)$$
$$= j200 - (6.4031\angle 38.66°)(256.12\angle 128.66°)/(9.8995\angle 45°) = j200 - 165.661\angle 122.32°$$
$$= j200 + 88.57 - j140 = 88.57 + j60 = 106.98\angle 34.115°$$

$$V_{Th} = \mathbf{106.98\angle 34.12° \ V}$$

To obtain Z_{Th}, we set all the sources to zero and insert a 1-A current source at the terminals a–b as shown below.



Clearly, we now have only a super mesh to analyze.

$$(5 + j6)\mathbf{I_1} - j2\mathbf{I_2} + (2 + j8 - j3)\mathbf{I_2} - j2\mathbf{I_1} = 0$$

$$(5 + j4)\mathbf{I_1} + (2 + j3)\mathbf{I_2} = 0 \qquad (1)$$

But, $$\mathbf{I_2} - \mathbf{I_1} = 1 \text{ or } \mathbf{I_2} = \mathbf{I_1} - 1 \qquad (2)$$

Substituting (2) into (1), $$(5 + j4)\mathbf{I_1} + (2 + j3)(1 + \mathbf{I_1}) = 0$$

$$\mathbf{I_1} = -(2 + j3)/(7 + j7)$$

Now, $$(5 + j6)\mathbf{I_1} - j2\mathbf{I_1} + \mathbf{V_o} = 0$$

$$\mathbf{V_o} = -(5 + j4)\mathbf{I_1} = (5 + j4)(2 + j3)/(7 + j7) = (-2 + j23)/(7 + j7) = 2.332\angle 50°$$

$$\mathbf{Z_{eq}} = \mathbf{V_o}/1 = \mathbf{2.332\angle 50° \ \Omega}.$$

Solution 13.15

Find the Norton equivalent for the circuit in Fig. 13.84 at terminals *a-b*.

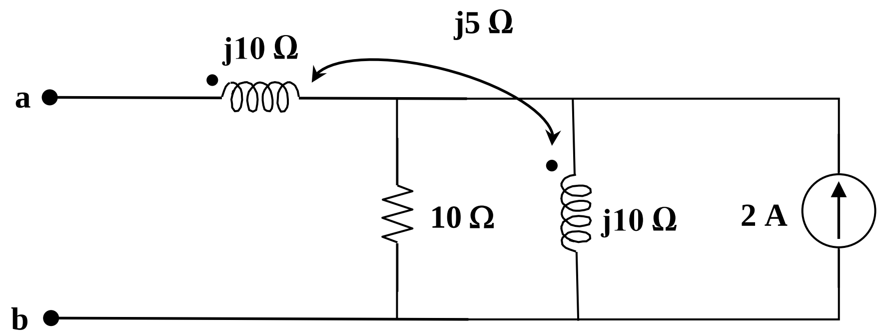
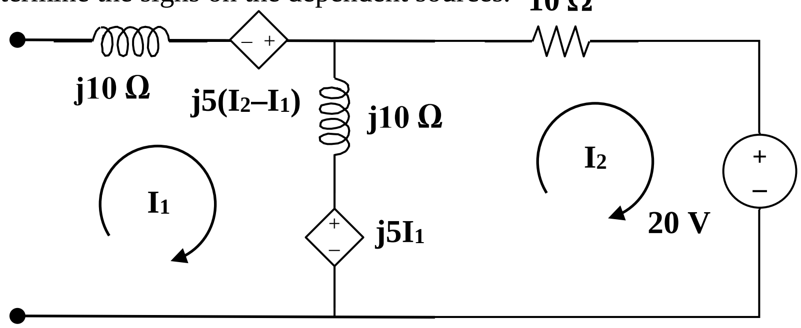


Figure 13.84
For Prob. 13.15.

Solution

Step 1. Since the current source is actually in parallel with the 10 Ω resistor, we can use source transformation to convert them into a resistance of 10Ω in series with a 20 V source. We next replace the mutually coupled inductors with their dependent source equivalent and establish the unknown loop currents. We use the dot convention to determine the signs on the dependent sources.



Clearly we need to find $\mathbf{V}_{oc}$ and $\mathbf{I}_{sc}$ since we have dependent sources. Thus, we have two circuits one with an open circuit at ab and the next is the short circuit at terminals ab.

For $\mathbf{V}_{oc}$ we solve the circuit with $\mathbf{I}_1 = 0$ we get loop 1 equal to $-\mathbf{V}_{oc} - j5\mathbf{I}_2 - j10\mathbf{I}_2 = 0$ or $\mathbf{V}_{oc} = -j15\mathbf{I}_2$ and loop 2 equal to $j10\mathbf{I}_2 + 10\mathbf{I}_2 + 20 = 0$ or $\mathbf{I}_2 = -20/(10+j10) = 1.4142\angle 135°$.

For $\mathbf{I}_{sc}$ we solve
$j10\mathbf{I}_1 - j5(\mathbf{I}_2-\mathbf{I}_1) + j10(\mathbf{I}_1-\mathbf{I}_2) + j5\mathbf{I}_1 = 0$ and $-j5\mathbf{I}_1 + j10(\mathbf{I}_2-\mathbf{I}_1) + 10\mathbf{I}_2 + 20 = 0$.

Now $\mathbf{I}_{sc} = -\mathbf{I}_1$.

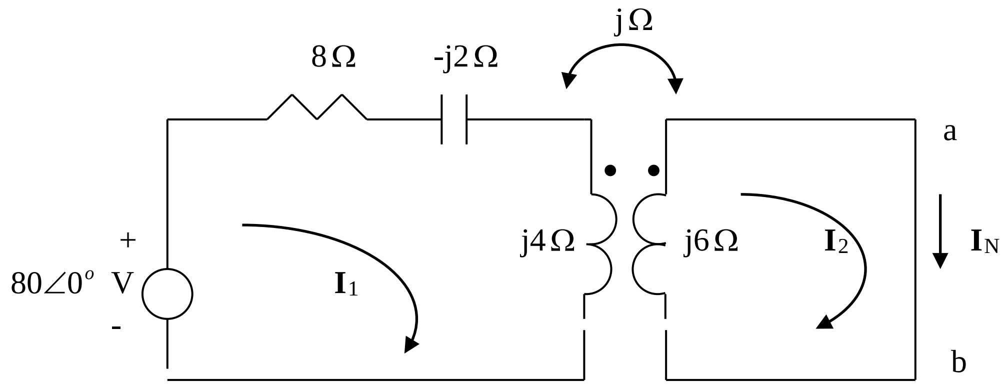
Step 2. $\mathbf{V_{oc}} = -j15(1.4142\angle 135°) = 21.213\angle 45°$ V.

Now for $\mathbf{I_{sc}}$ we solve $(j10+j5+j10+j5)\mathbf{I_1} + (-j5-j10)\mathbf{I_2} = j30\mathbf{I_1} - j15\mathbf{I_2} = 0$ or
$\mathbf{I_2} = 2\mathbf{I_1}$ and then $(-j5-j10)I_1 + (10+j10)I_2 = -20 = (-j15+20+j20)I_1 = (20+j5)I_1$ or
$\mathbf{I_1} = -20/(20+j5) = -20/(20.616\angle 14.036°) = 0.97012\angle 165.964°$ or
$\mathbf{I_{sc}} = \mathbf{I_N} = \mathbf{970.1\angle{-14.04°}}$ **A**.

Finally, $\mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}} = (21.213\angle 45°)/(0.97012\angle{-14.04°}) = \mathbf{21.87\angle 59.04°\ \Omega}$.

Solution 13.16

To find $\mathbf{I}_N$, we short-circuit a-b.

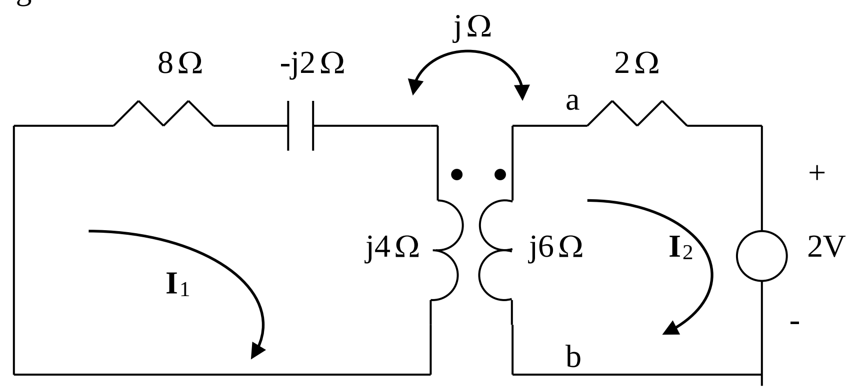


$$-80+(8-j2+j4)I_1-jI_2=0 \longrightarrow (8+j2)I_1-jI_2=80 \quad (1)$$
$$j6I_2-jI_1=0 \longrightarrow I_1=6I_2 \quad (2)$$

Solving (1) and (2) leads to

$$I_N=I_2=\frac{80}{48+j11}=1.584-j0.362=\mathbf{1.6246\angle{-12.91}^\circ\ A}$$

To find Z_N, insert a 1-A current source at terminals a-b. Transforming the current source to voltage source gives the circuit below.



$$0=(8+j2)I_1-jI_2 \longrightarrow I_1=\frac{jI_2}{8+j2}$$
$= [j/(8.24621\angle 14.036°)]\mathbf{I_2} = 0.121268\angle 75.964°\mathbf{I_2}$
$= (0.0294113+j0.117647)\mathbf{I_2}$ (3)

$$2+(2+j6)I_2-jI_1=0 \quad (4)$$

Solving (3) and (4) leads to $(2+j6)\mathbf{I_2} - j(0.0294113+j0.117647)\mathbf{I_2} = -2$ or $(2.117647+j5.882353)\mathbf{I_2} = -2$ or $\mathbf{I_2} = -2/(6.25192\angle 70.201°) = 0.319902\angle 109.8°$.

$V_{ab} = 2(1+\mathbf{I_2}) = 2(1-0.1083629+j0.30099) = (1.78327+j0.601979)$ V $= 1\mathbf{Z_{eq}}$ or

$$\mathbf{Z_{eq}} = (1.78327+j0.601979) = \mathbf{1.8821\angle 18.65°\ \Omega}$$

An alternate approach would be to calculate the open circuit voltage.

$$-80 + (8+j2)\mathbf{I_1} - j\mathbf{I_2} = 0 \text{ or } (8+j2)\mathbf{I_1} - j\mathbf{I_2} = 80 \qquad (5)$$

$$(2+j6)\mathbf{I_2} - j\mathbf{I_1} = 0 \text{ or } \mathbf{I_1} = (2+j6)\mathbf{I_2}/j = (6-j2)\mathbf{I_2} \qquad (6)$$

Substituting (6) into (5) we get,

$$(8.24621\angle 14.036°)(6.32456\angle -18.435°)\mathbf{I_2} - jI_2 = 80 \text{ or}$$

$$[(52.1536\angle -4.399°)-j]I_2 = [52-j5]I_2 = (52.2398\angle -5.492°)I_2 = 80 \text{ or}$$

$I_2 = 1.5314\angle 5.492°$ A and $V_{oc} = 2I_2 = 3.0628\angle 5.492°$ V which leads to,

$$Z_{eq} = V_{oc}/I_{sc} = (3.0628\angle 5.492°)/(1.6246\angle -12.91°) = \mathbf{1.8853\angle 18.4°\ \Omega}$$

This is in good agreement with what we determined before.

Solution 3.17

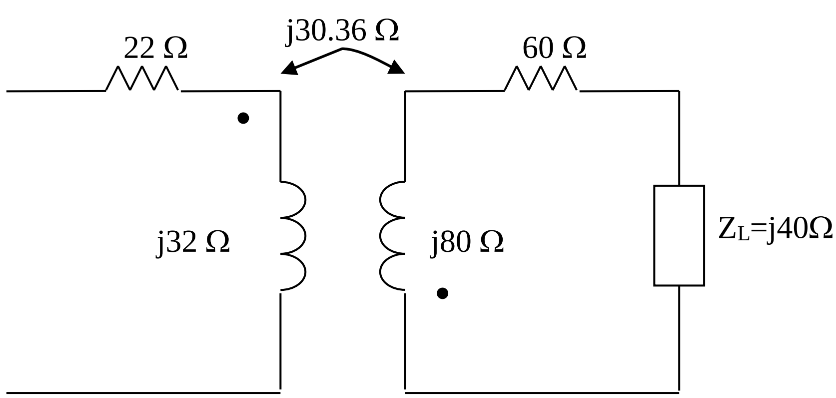
$$j\omega L = j40 \quad \longrightarrow \quad \omega = \frac{40}{L} = \frac{40}{15x10^{-3}} = \ 2667 \text{ rad/s}$$

$$M = k\sqrt{L_1 L_2} = 0.6\sqrt{12x10^{-3}x30x10^{-3}} = \ 11.384 \text{ mH}$$

If 15 mH $\longrightarrow$ 40 Ω

Then 12 mH $\longrightarrow$ 32 Ω
30 mH $\longrightarrow$ 80 Ω
11.384 mH $\longrightarrow$ 30.36 Ω

The circuit becomes that shown below.



$$Z_{in} = 22 + j32 + \frac{\omega^2 M^2}{j80 + 60 + j40} = 22 + j32 + \frac{(30.36)^2}{60 + j120}$$

$$= 22 + j32 + \frac{921.7}{134.16\angle 63.43^\circ} = 22 + j32 + 6.87\angle -63.43^\circ = 22 + j32 + 3.073 - j6.144$$

= **[25.07 + j25.86] Ω**.

Solution 13.18

Find the Thevenin equivalent to the left of the load **Z** in the circuit in Fig. 13.87.

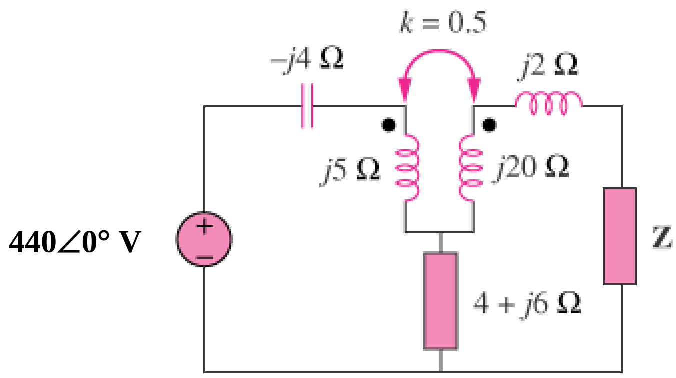
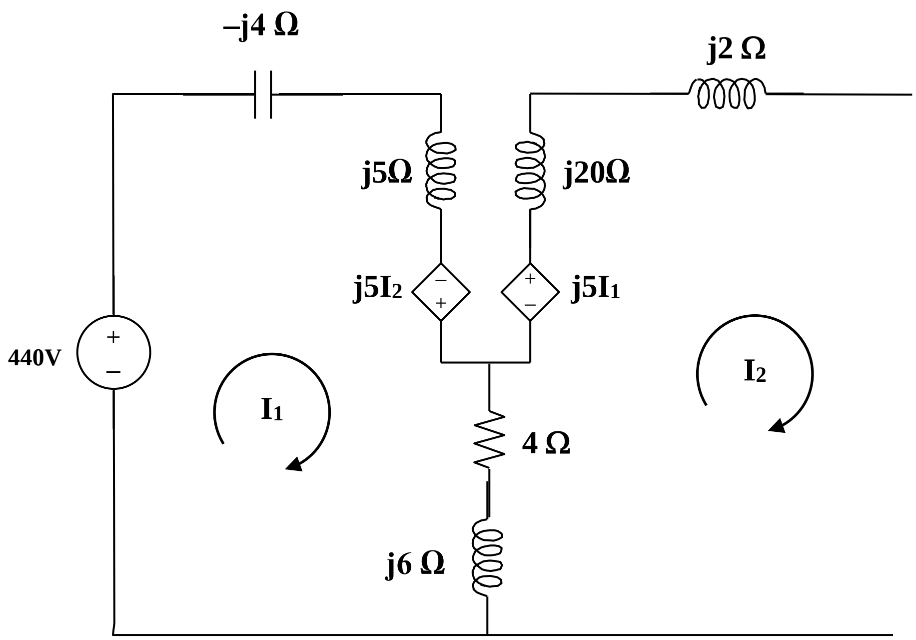


Figure 13.87
For Prob. 1318.

Solution

Replacing the mutually coupled circuit with the dependent source equivalent we get,



Now all we need to do is to find $\mathbf{V_{oc}}$ and $\mathbf{I_{sc}}$. To calculate the open circuit voltage, we note that I_2 is equal to zero. Thus,

$$-440 + (4 + j(-4+5+6))\mathbf{I_1} = 0 \text{ or } \mathbf{I_1} = 440/(4+j7) = 440/(8.06226\angle 60.255°)$$
$$= 54.575\angle -60.255°.$$

$$\mathbf{V_{oc}} = \mathbf{V_{Thev}} = j5\mathbf{I_1} + (4+j6)\mathbf{I_1} = (4+j11)\mathbf{I_1}$$
$$= (11.7047\angle 70.017°)(54.575\angle -60.255°) = \mathbf{638.79\angle 9.76°\ V}$$

To find the short circuit current ($\mathbf{I_{sc} = I_2}$), we need to solve the following mesh equations,

Mesh 1

$$-440 + (-j4+j5)\mathbf{I_1} - j5\mathbf{I_2} + (4+j6)(\mathbf{I_1 - I_2}) = 0 \text{ or}$$
$$(4+j7)\mathbf{I_1} - (4+j11)\mathbf{I_2} = 440 \qquad (1)$$

Mesh 2

$$(4+j6)(\mathbf{I_2 - I_1}) - j5\mathbf{I_1} + j22\mathbf{I_2} = 0 \text{ or } -(4+j11)\mathbf{I_1} + (4+j28)\mathbf{I_2} = 0 \text{ or}$$
$$\mathbf{I_1} = (28.2843\angle 81.87°)\mathbf{I_2}/(11.7047\angle 70.0169°) = (2.4165\angle 11.853°)\mathbf{I_2}$$

Substituting this into equation (1) we get,

$$(8.06226\angle 60.255°)(2.4165\angle 11.853°)\mathbf{I_2} - (4+j11)\mathbf{I_2} = 440 \text{ or}$$
$$[(19.4825\angle 72.108°) - 4 - j11]\mathbf{I_2} = 440 \text{ and}$$
$$[5.9855+j18.5403-4-j11]\mathbf{I_2} = (1.9855+j7.5403)\mathbf{I_2} = 440 \text{ or}$$

$$\mathbf{I_2 = I_{sc}} = 440/(7.79733\angle 75.248°) = 56.43\angle -75.248°\ \text{A}$$

Checking using MATLAB we get,

```
>> Z = [(4+7j) (-4-11j);(-4-11j) (4+28j)]

Z =

   4.0000 + 7.0000i  -4.0000 -11.0000i
  -4.0000 -11.0000i   4.0000 +28.0000i

>> V = [440;0]

V =

   440
     0

>> I = inv(Z)*V

I =
```

61.0687 -121.92583i ($\mathbf{I_1}$)
14.36893 -54.5706i ($\mathbf{I_2} = \mathbf{I_{sc}}$) = $56.431\angle{-75.248°}$ (answer checks)

Finally,

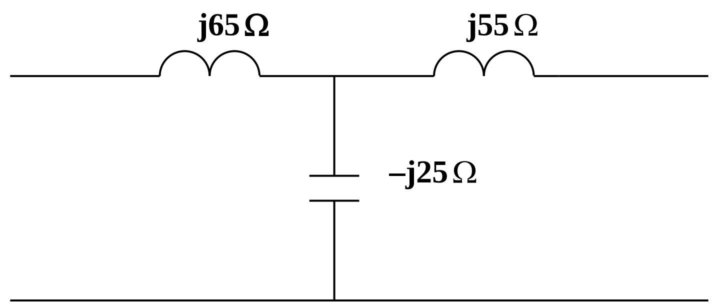
$$\mathbf{Z_{eq}} = \mathbf{V_{Thev}}/\mathbf{I_{sc}} = (638.79\angle 9.76°)/(56.43\angle{-75.248°}) = \mathbf{(11.32\angle 85.01°)\ \Omega}$$

$$\mathbf{V_{Thev}} = \mathbf{638.79\angle 9.76°\ V} \text{ and } \mathbf{Z_{eq}} = \mathbf{11.32\angle 85.01°\ \Omega}$$

Solution 13.19

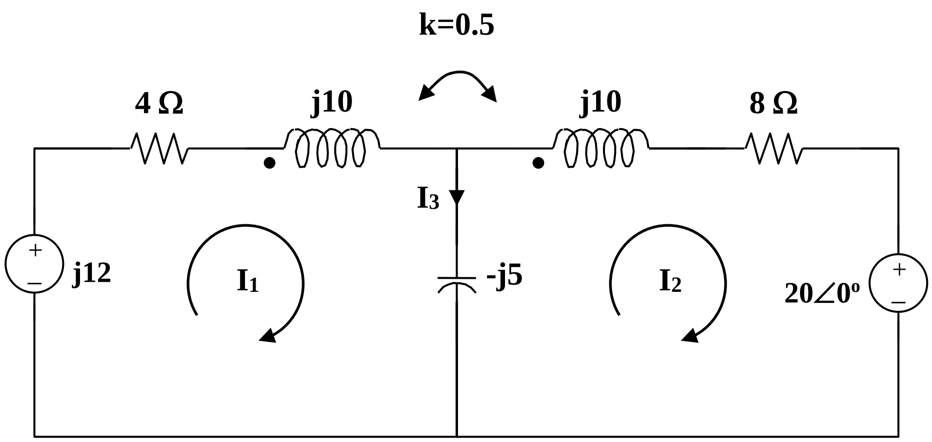
$X_{La} = X_{L1} - (-X_M) = 40 + 25 = 65\ \Omega$ and $X_{Lb} = X_{L2} - (-X_M) = 40 + 25 = 55\ \Omega$.

Finally, $X_C = X_M$ thus, the T-section is as shown below.



Solution 13.20

Transform the current source to a voltage source as shown below.



$$k = M/\sqrt{L_1L_2} \quad \text{or} \quad M = k\sqrt{L_1L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.5(10) = 5$$

For mesh 1, $\quad j12 = (4 + j10 - j5)I_1 + j5I_2 + j5I_2 = (4 + j5)I_1 + j10I_2 \qquad (1)$

For mesh 2, $\quad 0 = 20 + (8 + j10 - j5)I_2 + j5I_1 + j5I_1$

$$-20 = +j10I_1 + (8 + j5)I_2 \qquad (2)$$

From (1) and (2), $\quad \begin{bmatrix} j12 \\ 20 \end{bmatrix} = \begin{bmatrix} 4 + j5 & +j10 \\ +j10 & 8 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 107 + j60, \quad \Delta_1 = -60 - j296, \quad \Delta_2 = 40 - j100$$

$$I_1 = \Delta_1/\Delta = \mathbf{2.462\angle 72.18^\circ\ A}$$

$$I_2 = \Delta_2/\Delta = \mathbf{878\angle{-97.48^\circ}\ mA}$$

$$I_3 = I_1 - I_2 = \mathbf{3.329\angle 74.89^\circ\ A}$$

$$i_1 = 2.462\cos(1000t + 72.18^\circ)\text{ A}$$

$$i_2 = 0.878\cos(1000t - 97.48^\circ)\text{ A}$$

At $t = 2$ ms, $1000t = 2$ rad $= 114.6°$

$$i_1(0.002) = 2.462\cos(114.6° + 72.18°) = -2.445\text{A}$$

$$-2.445$$

$$i_2 = 0.878\cos(114.6° - 97.48°) = -0.8391$$

The total energy stored in the coupled coils is

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2$$

Since $\omega L_1 = 10$ and $\omega = 1000$, $L_1 = L_2 = 10$ mH, $M = 0.5L_1 = 5$mH

$$w = 0.5(0.01)(-2.445)^2 + 0.5(0.01)(-0.8391)^2 + 0.05(-2.445)(-0.8391)$$

$$\mathbf{w = 43.67\ mJ}$$

Solution 13.21

Using Fig. 13.90, design a problem to help other students to better understand energy in a coupled circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.90. Calculate the power absorbed by the 4-Ω resistor.

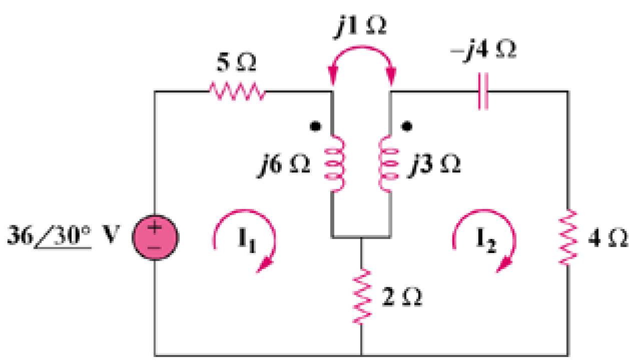


Figure 13.90

Solution

For mesh 1, $36\angle 30° = (7 + j6)I_1 - (2 + j)I_2$ (1)

For mesh 2, $0 = (6 + j3 - j4)I_2 - 2I_1 - jI_1 = -(2 + j)I_1 + (6 - j)I_2$ (2)

Placing (1) and (2) into matrix form, $\begin{bmatrix} 36\angle 30° \\ 0 \end{bmatrix} = \begin{bmatrix} 7 + j6 & -2 - j \\ -2 - j & 6 - j \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 45 + j25 = 51.48\angle 29.05°, \quad \Delta_1 = (6 - j)36\angle 30° = 219\angle 20.54°$$

$\Delta_2 = (2 + j)36\angle 30° = 80.5\angle 56.57°$, $I_1 = \Delta_1/\Delta = \mathbf{4.254\angle{-8.51}° \ A}$, $I_2 = \Delta_2/\Delta = \mathbf{1.5637\angle 27.52° \ A}$

Power absorbed by the 4-ohm resistor,

$$= 0.5(I_2)^2 4 = 2(1.5637)^2 = \mathbf{4.89 \ watts}$$

Solution 13.22

Find current $\mathbf{I}_o$ in the circuit of Fig. 13.91.

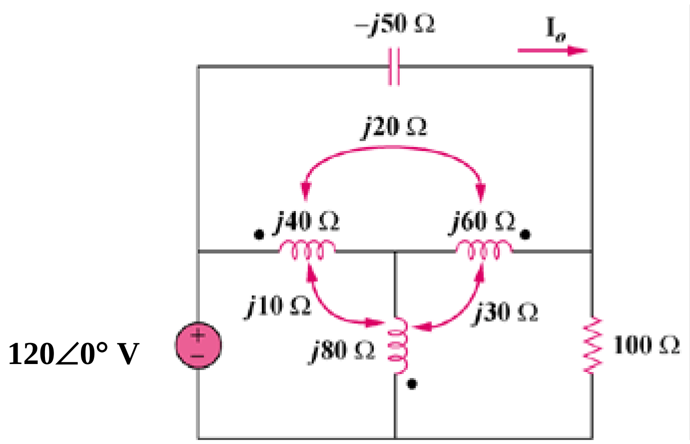
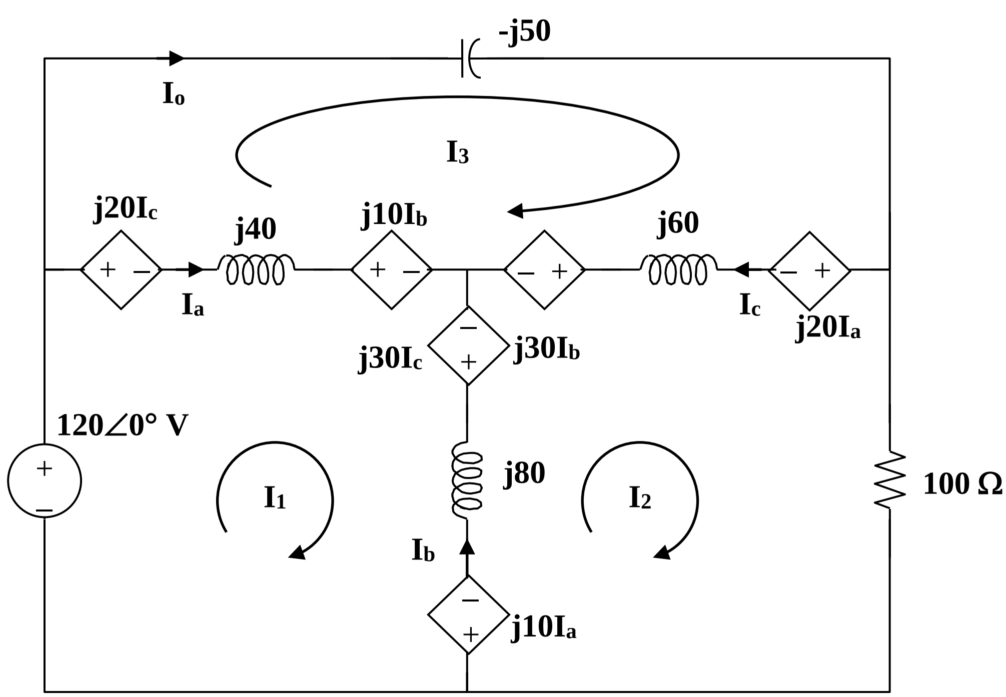


Figure 13.91
For Prob. 13.22.

Solution

With more complex mutually coupled circuits, it may be easier to show the effects of the coupling as sources in terms of currents that enter or leave the dot side of the coil. Figure 13.85 then becomes,



Note the following,

$$I_a = I_1 - I_3$$
$$I_b = I_2 - I_1$$
$$I_c = I_3 - I_2$$

$$\text{and} \quad I_o = I_3$$

Now all we need to do is to write the mesh equations and to solve for I_o.

Loop # 1,

$$-120 + j20(I_3 - I_2) + j40(I_1 - I_3) + j10(I_2 - I_1) - j30(I_3 - I_2)$$
$$+ j80(I_1 - I_2) - j10(I_1 - I_3) = 0$$

$$j100I_1 - j60I_2 - j40I_3 = 120$$

Multiplying everything by (1/j10) yields $\quad 10I_1 - 6I_2 - 4I_3 = -j12 \qquad (1)$

Loop # 2,

$$j10(I_1 - I_3) + j80(I_2 - I_1) + j30(I_3 - I_2) - j30(I_2 - I_1)$$
$$+ j60(I_2 - I_3) - j20(I_1 - I_3) + 100I_2 = 0$$

$$-j60I_1 + (100 + j80)I_2 - j20I_3 = 0 \qquad (2)$$

Loop # 3,

$$-j50I_3 + j20(I_1 - I_3) + j60(I_3 - I_2) + j30(I_2 - I_1)$$
$$-j10(I_2 - I_1) + j40(I_3 - I_1) - j20(I_3 - I_2) = 0$$

$$-j40I_1 - j20I_2 + j10I_3 = 0 \qquad (3)$$

Thus, $\begin{bmatrix} j100 & -j60 & -j40 \\ -j60 & 100+j80 & -j20 \\ -j40 & -j20 & j10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \end{bmatrix}$

Using MATLAB we get,

```
>> Z=[100j,-60j,-40j;-60j,100+80j,-20j;-40j,-20j,10j]

Z =

   1.0e+02 *

   0.0000 + 1.0000i   0.0000 - 0.6000i   0.0000 - 0.4000i
   0.0000 - 0.6000i   1.0000 + 0.8000i   0.0000 - 0.2000i
```

```
   0.0000 - 0.4000i   0.0000 - 0.2000i   0.0000 + 0.1000i

>> V=[120;0;0]

V =

   120
     0
     0

>> I=inv(Z)*V

I =

   0.4523 + 0.3415i
  -0.1938 + 0.7108i
   1.4215 + 2.7877i
```

$I_3 = I_o = 3.129\angle 62.98^\circ$ amp.

Chapter 13, Solution 23.

Let i_s = 5 cos(100t) A. Calculate the voltage across the capacitor, v_C. Also calculate the value of the energy stored in the coupled coils at t = 2.5π ms.

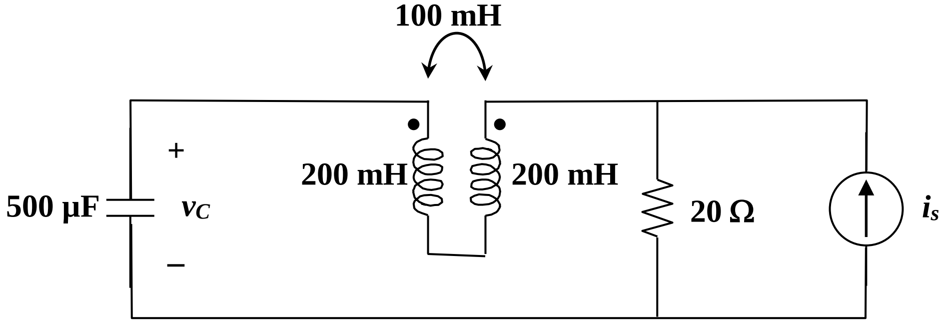
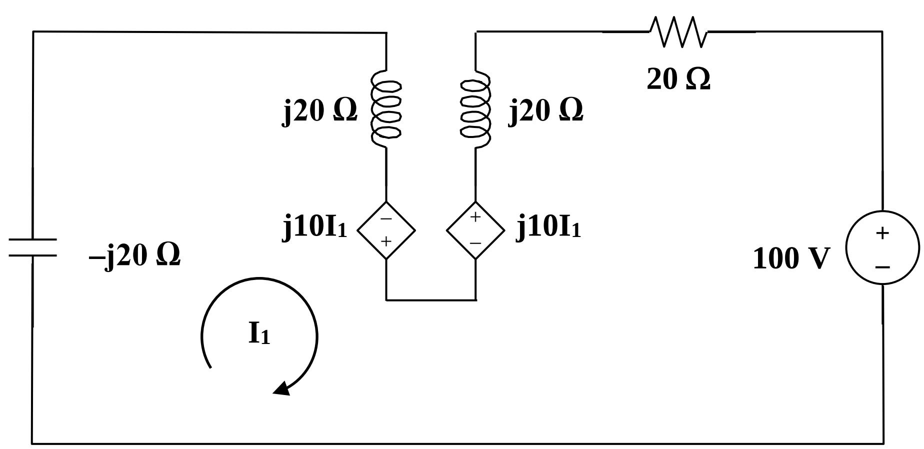


Figure 13.92
For Prob. 13.23.

Solution

Step 1. First we need to convert the circuit into the frequency domain and convert the coupled inductors into their dependent source equivalent. Then we can do source transformation to convert the 20 Ω resistor in parallel with the current source into a 20 Ω resistor in series with a 100 V voltage source. We note that ω = 100 rad/s which leads to the capacitor being equal to –j1/(0.0005x100) = –j20 Ω, the value of the individual inductors will be equal to j(0.2x100) = j20 Ω, and the mutual coupling equal to j(0.1x100) = j10 Ω. Now we have the following circuit,



$-j20I_1 + j20I_1 - j10I_1 - j10I_1 + j20I_1 + 20I_1 + 100 = 0$ and $V_C = j20I_1$. We then convert the value of V_C into the time domain and I_1 into the time domain. Thus, $w(0.25\pi) = 0.5(20-10-10+20)i(0.25\pi)^2$.

Step 2. $(-j20+j20-j10-j10+j20+20)I_1 = -100$ or $I_1 = -100/(20) = -5$ A. Thus, $V_C = -j100 = 100\angle{-90°}$ or $v_C =$ **$100 \cos(100t-90°)$ V** and $i_1 = 5 \cos(100t-180°)$ A.

$$w(0.25\pi) = 0.5(20)[\cos(45°-180°)]^2 = 10(0.5) = \mathbf{5\ J}.$$

Solution 13.24

(a) $k = M/\sqrt{L_1 L_2} = 1/\sqrt{4x2} = \mathbf{0.3535}$

(b) $\omega = 4$

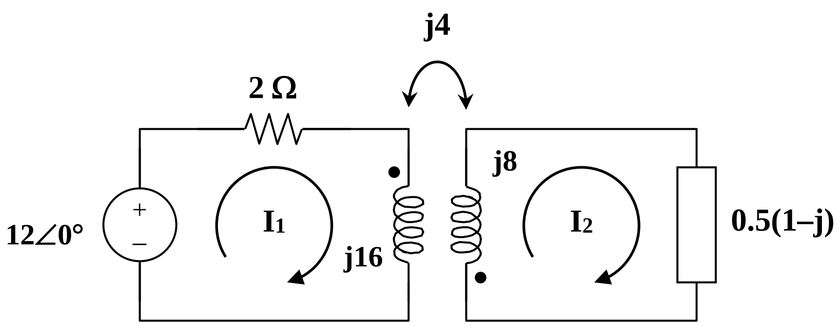
1/4 F leads to $1/(j\omega C) = -j/(4x0.25) = -j$

$1||(-j) = -j/(1-j) = 0.5(1-j)$

1 H produces $j\omega M = j4$

4 H produces j16

2 H becomes j8



$$12 = (2 + j16)I_1 + j4I_2$$

or $\quad 6 = (1 + j8)I_1 + j2I_2 \qquad (1)$

$$0 = (j8 + 0.5 - j0.5)I_2 + j4I_1 \text{ or } I_1 = (0.5 + j7.5)I_2/(-j4) \qquad (2)$$

Substituting (2) into (1),

$$24 = (-11.5 - j51.5)I_2 \text{ or } I_2 = -24/(11.5 + j51.5) = -0.455\angle -77.41^\circ$$

$$V_o = I_2(0.5)(1 - j) = 0.3217\angle 57.59^\circ$$

$$v_o = \mathbf{321.7\cos(4t + 57.6^\circ)\ mV}$$

(c) From (2), $I_1 = (0.5 + j7.5)I_2/(-j4) = 0.855\angle -81.21°$

$$i_1 = 0.885\cos(4t - 81.21°)\text{ A},\ \ i_2 = -0.455\cos(4t - 77.41°)\text{ A}$$

At t = 2s,

$$4t = 8\text{ rad} = 98.37°$$

$$i_1 = 0.885\cos(98.37° - 81.21°) = 0.8169$$

$$i_2 = -0.455\cos(98.37° - 77.41°) = -0.4249$$

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2$$

$$= 0.5(4)(0.8169)^2 + 0.5(2)(-.4249)^2 + (1)(0.1869)(-0.4249) = \mathbf{1.168\ J}$$

Solution 3.25

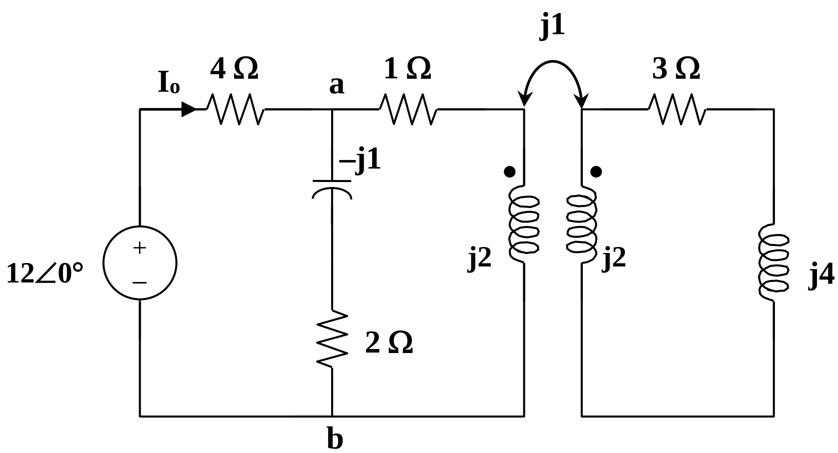
$m = k\sqrt{L_1 L_2} = 0.5\text{ H}$

We transform the circuit to frequency domain as shown below.

$$12\sin 2t \text{ converts to } 12\angle 0°, \ \omega = 2$$

$$0.5\text{ F converts to } 1/(j\omega C) = -j$$

$$2\text{ H becomes } j\omega L = j4$$



Applying the concept of reflected impedance,

$$Z_{ab} = (2 - j)||(1 + j2 + (1)^2/(j2 + 3 + j4))$$

$$= (2 - j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2 - j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2 - j)||(1.0667 + j1.8667)$$

$$=(2 - j)(1.0667 + j1.8667)/(3.0667 + j0.8667) = \mathbf{1.5085\angle 17.9°\ \Omega}$$

$$I_o = 12\angle 0°/(Z_{ab} + 4) = 12/(5.4355 + j0.4636) = 2.2\angle -4.88°$$

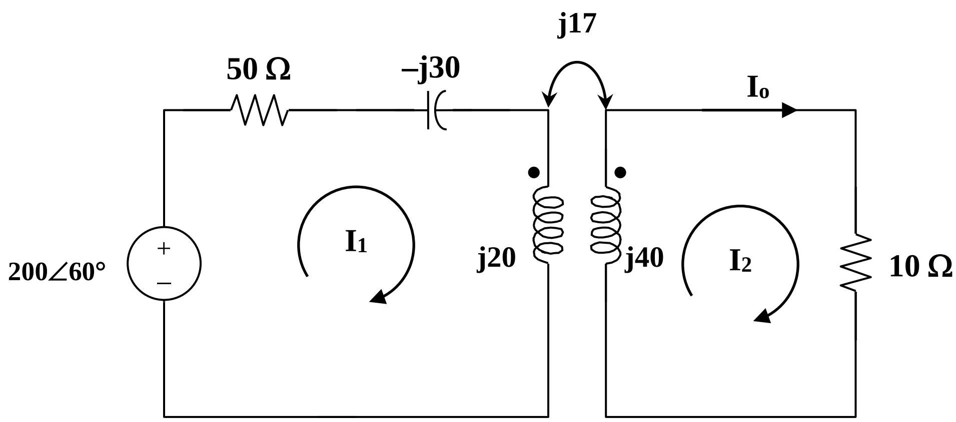
$$i_o = \mathbf{2.2\sin(2t - 4.88°)\ A}$$

Solution 13.26

$$M = k\sqrt{L_1L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.601\sqrt{20x40} = 17$$

The frequency-domain equivalent circuit is shown below.



For mesh 1, $-200\angle 60° + (50-j30+j20)I_1 - j17I_2 = 0$ or

$(50-j10)I_1 - j17I_2 = 200\angle 60°$ (1)

For mesh 2, $(10 + j40)I_2 - j17I_1 = 0$ or $-j17I_1 + (10+j40)I_2 = 0$ (2)

In matrix form,

$$\begin{bmatrix} 200\angle 60° \\ 0 \end{bmatrix} = \begin{bmatrix} 50 - j10 & -j17 \\ -j17 & 10 + j40 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{or}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} 10 + j40 & j17 \\ j17 & 50 - j10 \end{bmatrix}}{500 + 400 + 289 - j100 + j2{,}000} \begin{bmatrix} 200\angle 60° \\ 0 \end{bmatrix}$$

$I_1 = (10+j40)(200\angle 60°)/(1{,}189+j1{,}900)$
$= (41.231\angle 75.964°)(200\angle 60°)/(2{,}241.4\angle 57.962°) = 3.679\angle 78°$ A and
$I_2 = j17(200\angle 60°)/(2{,}241.4\angle 57.962°) = 1.5169\angle 92.04°$ A

$$\mathbf{I_o = I_2 = 1.5169\angle 92.04°\ A}$$

It should be noted that switching the dot on the winding on the right only reverses the direction of **I_o**.

Solution 13.27

Find the average power delivered to the 50-Ω resistor in the circuit of Fig. 13.96.

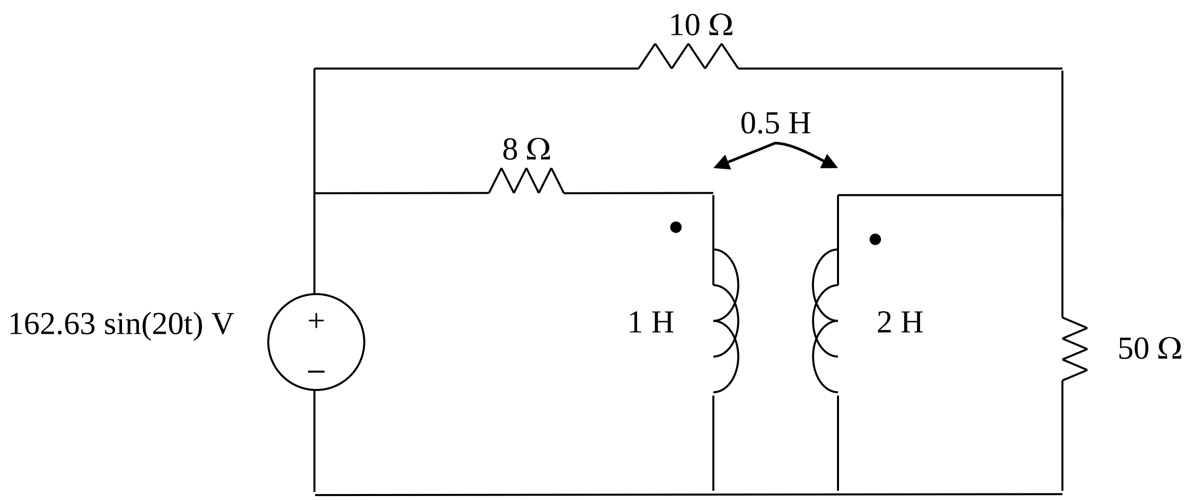


Figure 13.96
For Prob. 13.27.

Solution

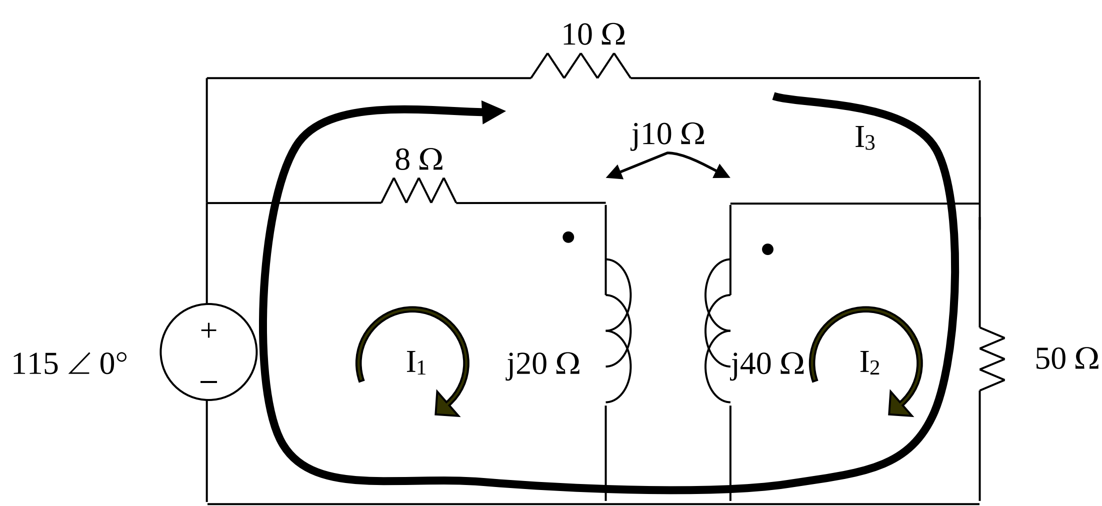
V_s(rms) = 162.63/1.4142 = 115 V

$1H \longrightarrow j\omega L = j20$

$2H \longrightarrow j\omega L = j40$

$0.5H \longrightarrow j\omega L = j10$

We apply mesh analysis to the circuit as shown below.



To make the problem easier to solve, let us have I_3 flow around the outside loop as shown.

For mesh 1,

$$-115 + 8\mathbf{I_1} + j20\mathbf{I_1} - j10\mathbf{I_2} = 0 \text{ or } (8+j20)\mathbf{I_1} - j10\mathbf{I_2} = 40 \qquad (1)$$

For mesh 2,

$$j40\mathbf{I_2} - j10\mathbf{I_1} + 50(\mathbf{I_2} + \mathbf{I_3}) = 0 \text{ or } -j10\mathbf{I_1} + (50+j40)\mathbf{I_2} + 50\mathbf{I_3} = 0 \qquad (2)$$

For mesh 3,

$$-115 + 10\mathbf{I_3} + 50(\mathbf{I_3} + \mathbf{I_2}) = 0 \text{ or } 50\mathbf{I_2} + 60\mathbf{I_3} = 40 \qquad (3)$$

In matrix form, (1) to (3) become

$$\begin{bmatrix} 8+j20 & -j10 & 0 \\ -j10 & 50+j40 & 50 \\ 0 & 50 & 60 \end{bmatrix} I = \begin{bmatrix} 115 \\ 0 \\ 115 \end{bmatrix}$$

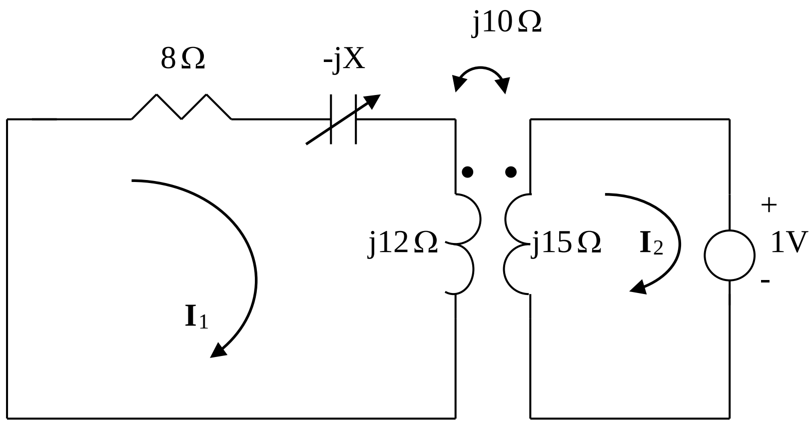
```
>> Z=[(8+20i),-10i,0;-10i,(50+40i),50;0,50,60]
Z =
   8.0000 +20.0000i        0 -10.0000i        0
        0 -10.0000i  50.0000 +40.0000i  50.0000
        0            50.0000            60.0000

>> V=[40;0;40]
V =
    115
     0
     115

>> I=inv(Z)*V
I =
   1.8268 – 4.3464i
   0.1762 + 1.3461i
   1.7698 – 1.1215i
```

Solving this leads to $\mathbf{I_{50}} = \mathbf{I_2} + \mathbf{I_3} = 0.1762 + j1.3461 + 1.7698 - j1.1215$ $= 1.946+j0.2246 = 1.9589\angle 6.58°$ A. This is already an rms value so,

$$P_{50} = (1.9589)^2(50) = \mathbf{191.86\ W}.$$

Solution 13.28

We find Z_{Th} by replacing the 20-ohm load with a unit source as shown below.



For mesh 1, $$0 = (8 - jX + j12)I_1 - j10I_2 \quad (1)$$

For mesh 2,
$$1 + j15I_2 - j10I_1 = 0 \longrightarrow I_1 = 1.5I_2 - 0.1j \quad (2)$$

Substituting (2) into (1) leads to
$$I_2 = \frac{-1.2 + j0.8 + 0.1X}{12 + j8 - j1.5X}$$

$$Z_{Th} = \frac{1}{-I_2} = \frac{12 + j8 - j1.5X}{1.2 - j0.8 - 0.1X}$$

$$|Z_{Th}| = 20 = \frac{\sqrt{12^2 + (8 - 1.5X)^2}}{\sqrt{(1.2 - 0.1X)^2 + 0.8^2}} \longrightarrow 0 = 1.75X^2 + 72X - 624$$

Solving the quadratic equation yields X = **6.425 Ω**

Solution 13.29

In the circuit of Fig. 13.97, find the value of the coupling coefficient *k* that will make the 10-Ω resistor dissipate 1.28 kW. For this value of *k*, find the energy stored in the coupled coils at *t* = 1.5 s.

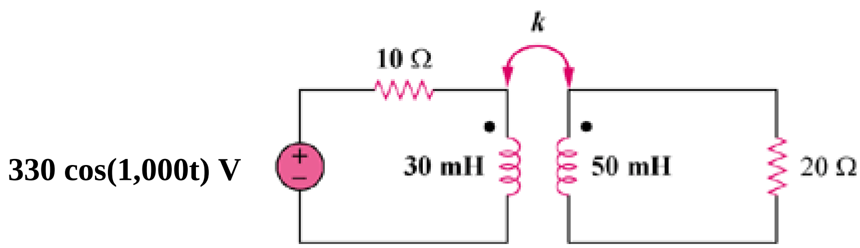


Figure 13.97
For Prob. 13.29.

Solution

30 mH becomes $j\omega L = j30\text{x}10^{-3}\text{x}10^{3} = j30\ \Omega$

50 mH becomes $j50\ \Omega$

Let $X = \omega M = 1{,}000M$

Using the concept of reflected impedance,

$\mathbf{Z_{in}} = 10 + j30 + X^2/(20 + j50)$

$\mathbf{I_1} = \mathbf{V/Z_{in}} = 330/(10 + j30 + X^2/(20 + j50))$

$p = 0.5|\mathbf{I_1}|^2(10) = 1{,}280$ leads to $|\mathbf{I_1}|^2 = 256$ or $|\mathbf{I_1}| = 16$

$16 = |330(20 + j50)/(X^2 + (10 + j30)(20 + j50))|$

$= |330(20 + j50)/(X^2 - 1300 + j1100)|$ or

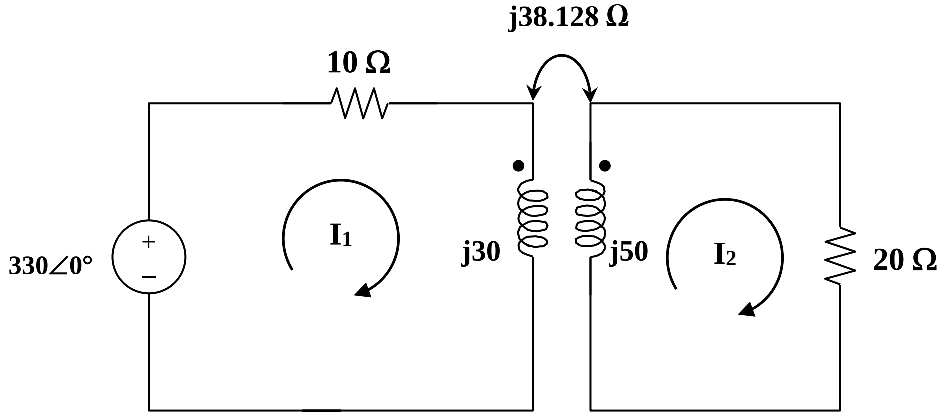
$256 = 108{,}900(400 + 2500)/((X^2 - 1300)^2 + 1{,}210{,}000)$

$(X^2 - 1300)^2 + 1{,}210{,}000 = 1{,}233{,}632.8$ or $(X^2 - 1{,}300)^2 = 23{,}632.8$ or

$X^2 - 1300 = \pm 153.73$ which means that there are two positive values of X which solve this equation. X = 33.857 and 38.128. Let us use the value of X = 38.128.

For X = 38.128 = ωM or M = **38.128 mH**.

$$k = M/\sqrt{L_1 L_2} = 38.128/\sqrt{30x50} = \mathbf{0.9845}$$



$330 = (10 + j30)\mathbf{I_1} - j38.128\mathbf{I_2}$ (1)

$0 = (20 + j50)\mathbf{I_2} - j38.128\mathbf{I_1}$ (2)

In matrix form, $\begin{bmatrix} 330 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 + j30 & -j38.128 \\ -j38.128 & 20 + j50 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$\Delta = 200 - 1500 + j(600+500) + 1453.744 = 153.74 + j1100 = 1,110.69\angle 82.04°$,
$\Delta_1 = 6,600 + j16,500 = 17,771\angle 68.2°$, $\Delta_2 = j12,582$

$I_1 = \Delta_1/\Delta = 16\angle -13.81°$, $I_2 = \Delta_2/\Delta = 11.328\angle 7.97°$

$i_1 = 16\cos(1000t - 13.83°)$, $i_2 = 11.328\cos(1000t + 7.97°)$

At t = 1.5 ms, 1000t = 1.5 rad = 85.94°

$i_1 = 16\cos(85.94° - 13.83°) = 4.915$ A

$i_2 = 11.328\cos(85.94° + 7.97°) = -0.77245$ A

$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2 = 0.5(0.03)(4.915)^2 + 0.5(0.05)(-0.77245)^2$
$- 0.038128(4.915)(-0.77245) = 0.36236 + 0.014917 + 0.144756 = 0.5216$

= **521.6 mJ**

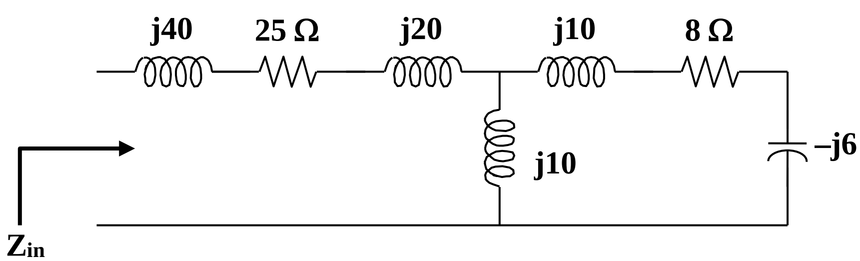
Solution 13.30

(a) $$Z_{in} = j40 + 25 + j30 + (10)^2/(8 + j20 - j6)$$

$$= 25 + j70 + 100/(8 + j14) = \mathbf{(28.08 + j64.62)\ ohms}$$

(b) $$j\omega L_a = j30 - j10 = j20,\ j\omega L_b = j20 - j10 = j10,\ j\omega L_c = j10$$

Thus the Thevenin Equivalent of the linear transformer is shown below.



$$Z_{in} = j40 + 25 + j20 + j10||(8 + j4) = 25 + j60 + j10(8 + j4)/(8 + j14)$$

$$= \mathbf{(28.08 + j64.62)\ ohms}$$

Solution 13.31

Using Fig. 13.100, design a problem to help other students to better understand linear transformers and how to find T-equivalent and Π–equivalent circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 13.99, find:

(a) the *T*-equivalent circuit,

(b) the Π-equivalent circuit.

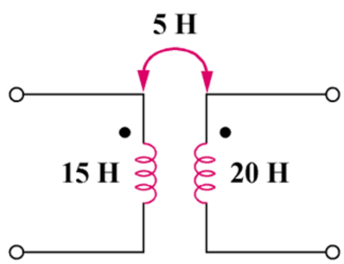


Figure 13.99

Solution

(a) $L_a = L_1 - M = \mathbf{10\ H}$

$L_b = L_2 - M = \mathbf{15\ H}$

$L_c = M = \mathbf{5\ H}$

(b) $L_1L_2 - M^2 = 300 - 25 = 275$

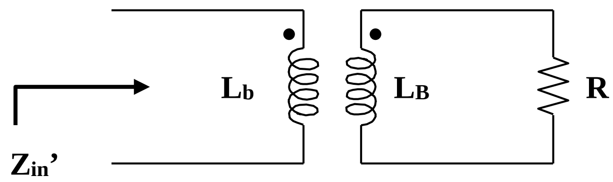
$L_A = (L_1L_2 - M^2)/(L_1 - M) = 275/15 = \mathbf{18.33\ H}$

$L_B = (L_1L_2 - M^2)/(L_1 - M) = 275/10 = \mathbf{27.5\ H}$

$L_C = (L_1L_2 - M^2)/M = 275/5 = \mathbf{55\ H}$

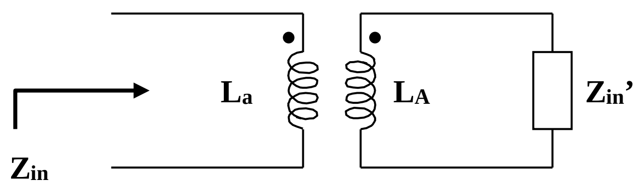
Solution 13.32

We first find Z_{in} for the second stage using the concept of reflected impedance.



$Z_{in}' = j\omega L_b + \omega^2 M_b^2/(R + j\omega L_b) = (j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)/(R + j\omega L_b)$
(1)

For the first stage, we have the circuit below.



$$Z_{in} = j\omega L_a + \omega^2 M_a^2/(j\omega L_a + Z_{in})$$

$$= (-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a Z_{in})/(j\omega L_a + Z_{in}) \qquad (2)$$

Substituting (1) into (2) gives,

$$= \frac{-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a \dfrac{(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{R + j\omega L_b}}{j\omega L_a + \dfrac{j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2}{R + j\omega L_b}}$$

$$= \frac{-R\omega^2 L_a^2 + \omega^2 M_a^2 R - j\omega^3 L_b L_a + j\omega^3 L_b M_a^2 + j\omega L_a(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{j\omega R La - \omega^2 L_a L_b + j\omega L_b R - \omega^2 L_a^2 + \omega^2 M_b^2}$$

$$Z_{in} = \frac{\boldsymbol{\omega^2 R(L_a^2 + L_a L_b - M_a^2) + j\omega^3(L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}}{\boldsymbol{\omega^2(L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}}$$

Solution 13.33

$$
\begin{aligned}
Z_{in} &= 10 + j12 + (15)^2/(20 + j\,40 - j5) = 10 + j12 + 225/(20 + j35) \\
&= 10 + j12 + 225(20 - j35)/(400 + 1225) \\
&= (12.769 + j7.154)\ \Omega
\end{aligned}
$$

Solution 13.34

Using Fig. 13.103, design a problem to help other students to better understand how to find the input impedance of circuits with transformers.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the input impedance of the circuit in Fig. 13.102.

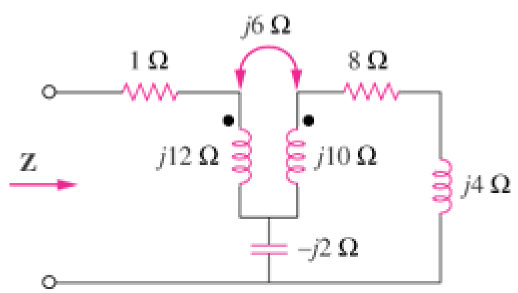
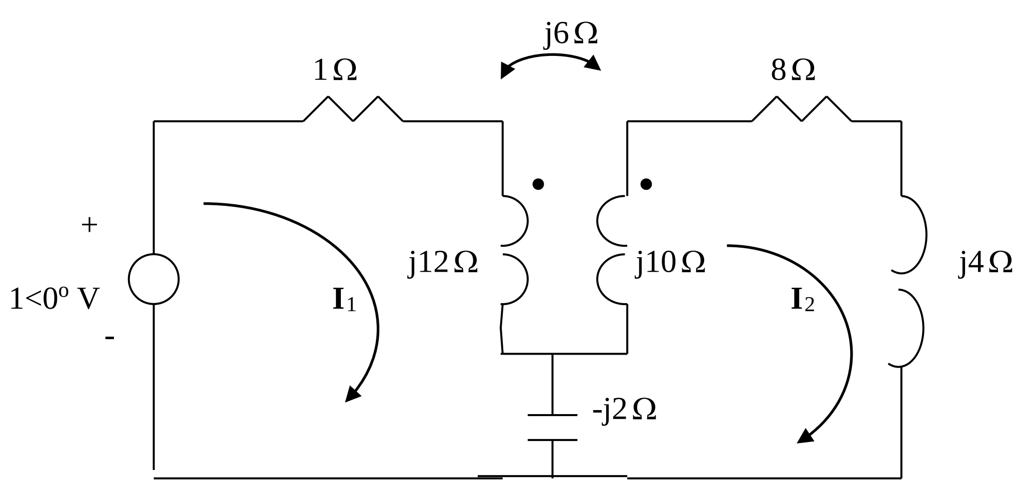


Figure 13.102

Solution

Insert a 1-V voltage source at the input as shown below.



For loop 1,

$$1 = (1 + j10)I_1 - j4I_2 \qquad (1)$$

For loop 2,

$$0 = (8 + j4 + j10 - j2)I_2 + j2I_1 - j6I_1 \quad \longrightarrow \quad 0 = -jI_1 + (2 + j3)I_2 \qquad (2)$$

Solving (1) and (2) leads to $\mathbf{I}_1$=0.019 –j0.1068

$$\boldsymbol{Z} = \frac{\mathbf{1}}{\boldsymbol{I}_1} = \mathbf{1.6154} + \boldsymbol{j}\mathbf{9.077} = \underline{\mathbf{9.219}\angle\mathbf{79.91}^{o}\,\boldsymbol{\Omega}}$$

Alternatively, an easier way to obtain **Z** is to replace the transformer with its equivalent T circuit and use series/parallel impedance combinations. This leads to exactly the same result.

Solution 13.35

For mesh 1,

$$16 = (10 + j4)I_1 + j2I_2 \qquad (1)$$

For mesh 2, $$0 = j2I_1 + (30 + j26)I_2 - j12I_3 \qquad (2)$$

For mesh 3, $$0 = -j12I_2 + (5 + j11)I_3 \qquad (3)$$

We may use MATLAB to solve (1) to (3) and obtain

$$\mathbf{I_1} = 1.3736 - \text{j}0.5385 = \mathbf{1.4754\angle{-21.41}^\circ\ A}$$
$$\mathbf{I_2} = -0.0547 - \text{j}0.0549 = \mathbf{77.5\angle{-134.85}^\circ\ mA}$$
$$\mathbf{I_3} = -0.0268 - \text{j}0.0721 = \mathbf{77\angle{-110.41}^\circ\ mA}$$

1.4754∠–21.41° A, 77.5∠–134.85° mA, 77∠–110.41° mA

Solution 13.36

Following the two rules in section 13.5, we obtain the following:

(a) $V_2/V_1 = \mathbf{-n}$, $I_2/I_1 = \mathbf{-1/n}$ $(n = V_2/V_1)$

(b) $V_2/V_1 = \mathbf{-n}$, $I_2/I_1 = \mathbf{-1/n}$

(c) $V_2/V_1 = \mathbf{n}$, $I_2/I_1 = \mathbf{1/n}$

(d) $V_2/V_1 = \mathbf{n}$, $I_2/I_1 = \mathbf{-1/n}$

Solution 13.37

A 240/2400 V (rms) step-up ideal transformer delivers 50 kW to a resistive load. Calculate: (a) the turns ratio, (b) the primary current, (c) the secondary current.

Solution

(a) $n = \frac{V_2}{V_1} = \frac{2400}{240} = \mathbf{10}$

(b) $\mathbf{S_1} = \mathbf{V_1}(\mathbf{I_1})^* = \mathbf{S_2} = \mathbf{V_2}(\mathbf{I_2})^* = 50{,}000$ which leads to

$\mathbf{I_1} = 50{,}000/240 = \mathbf{208.3\ A}$.

(c) $\mathbf{I_2} = 50{,}000/2{,}400 = \mathbf{20.83\ A}$.

Solution 13.38

Design a problem to help other students to better understand ideal transformers.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A 4-kVA, 2300/230-V rms transformer has an equivalent impedance of $2\angle 10°\Omega$ on the primary side. If the transformer is connected to a load with 0.6 power factor leading, calculate the input impedance.

Solution

$Z_{in} = Z_p + Z_L/n^2, \quad n = v_2/v_1 = 230/2300 = 0.1$

$v_2 = 230 \text{ V}, \quad s_2 = v_2 I_2^*$

$I_2^* = s_2/v_2 = 17.391\angle{-53.13°} \text{ or } I_2 = 17.391\angle 53.13° \text{ A}$

$Z_L = v_2/I_2 = 230\angle 0°/17.391\angle 53.13° = 13.235\angle{-53.13°}$

$Z_{in} = 2\angle 10° + 1323.5\angle{-53.13°}$

$= 1.97 + j0.3473 + 794.1 - j1058.8$

$Z_{in} = \mathbf{1.324\angle{-53.05°}\ k\Omega}$

Solution 13.39

Referred to the high-voltage side,

$Z_L = (1200/240)^2(0.8\angle 10°) = 20\angle 10°$

$Z_{in} = 60\angle -30° + 20\angle 10° = 76.4122\angle -20.31°$

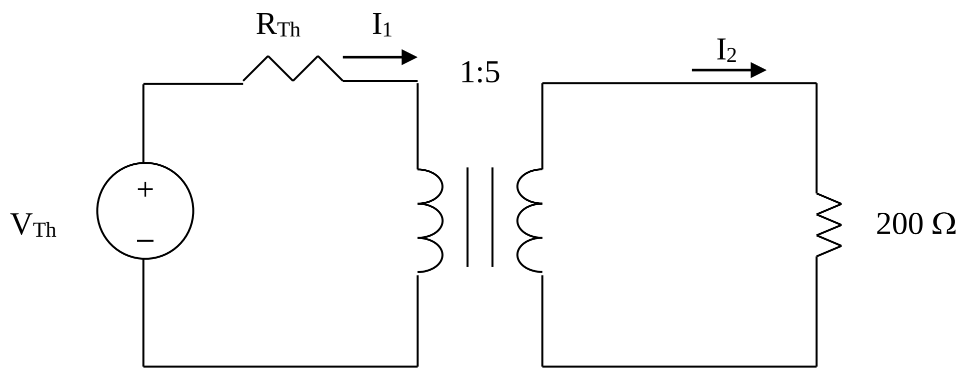
$I_1 = 1200/Z_{in} = 1200/76.4122\angle -20.31° =$ **$15.7\angle 20.31°$ A**

Since $S = I_1 v_1 = I_2 v_2$, $I_2 = I_1 v_1/v_2$

$= (1200/240)(15.7\angle 20.31°) =$ **$78.5\angle 20.31°$ A**

Solution 13.40

Consider the circuit as shown below.



We reflect the 200-Ω load to the primary side.

$$Z_p = 100 + \frac{200}{5^2} = 108$$

$$I_1 = \frac{10}{108}, \qquad I_2 = \frac{I_1}{n} = 2/108$$

$$P = \frac{1}{2}|I_2|^2 R_L = \frac{1}{2}(\frac{2}{108})^2(200) = \underline{34.3 \text{ mW}}$$

Solution 13.41

Given $\mathbf{I}_2$ = 2 A, determine the value of $\mathbf{I}_s$.

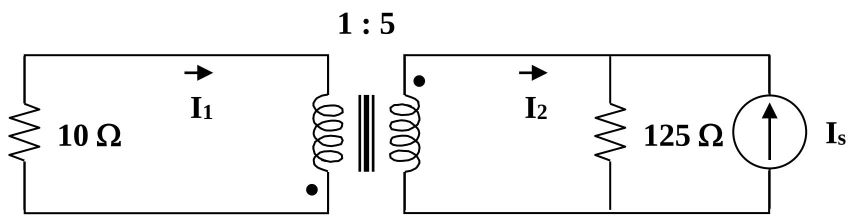


Figure 13.105
For Prob. 13.41.

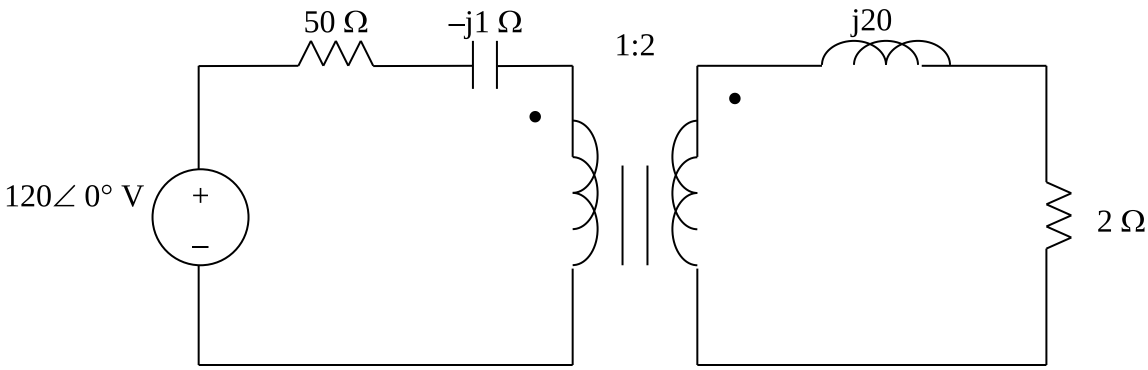
Solution

Step 1. First we note that the dots are not relevant for this problem (the value of I_2 is independent of the location of the dots). Thus, all we need to do is to reflect the 10 Ω to the right hand side of the circuit. The value of the reflected resistance is equal to 25x10 = 250 Ω. Current division gives us $\mathbf{I}_2 = 125(-\mathbf{I}_s)/(250+125) = 2$. Now all we need to do is to solve for $\mathbf{I}_s$.

Step 2. $2\text{x}375 = 125(-\mathbf{I}_s)$ or $\mathbf{I}_s = \mathbf{-6\ A}$.

Solution 13.42

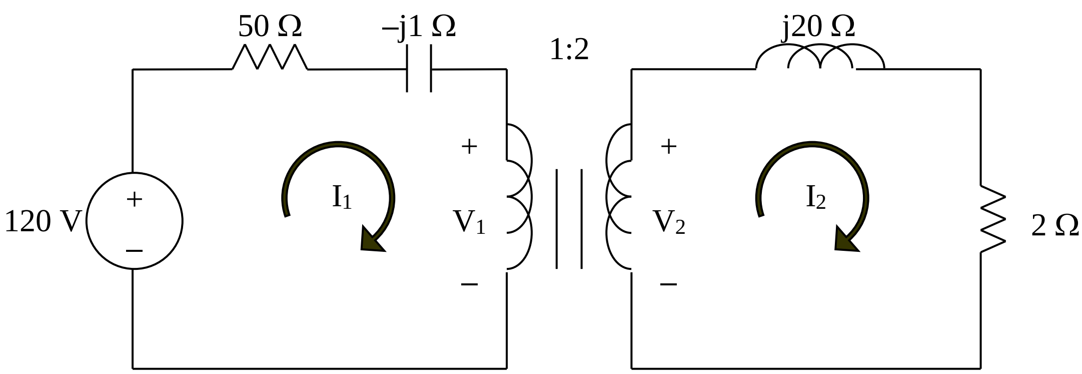
For the circuit in Fig. 13.106, determine the power absorbed by the 2 Ω resistor. Assume the 120 V source is an rms value.



Ideal
Figure 13.106
For Prob. 13.42.

Solution

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$-120 + (50–j)I_1 + V_1 = 0 \qquad (1)$$

For mesh 2,

$$–V_2 + (2+j20)I_2 = 0 \qquad (2)$$

At the transformer terminals,

$$V_2 = 2V_1 \text{ or } 2V_1 – V_2 = 0 \qquad (3)$$

$$I_1 = 2I_2 \text{ or } I_1 – 2I_2 = 0 \qquad (4)$$

From (1) to (4),

$$\begin{bmatrix} 50-j & 0 & 1 & 0 \\ 0 & 2+j20 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 1 & -2 & 0 & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this with MATLAB,

```
>> A = [(50-j) 0 1 0;0 (2+20j) 0 -1;0 0 2 -1;1 -2 0 0]

A =

  Columns 1 through 3

  50.0000 - 1.0000i        0              1.0000
        0              2.0000 +20.0000i        0
        0                   0            2.0000
   1.0000            -2.0000                  0

  Column 4

        0
  -1.0000
  -1.0000
        0

>> B = [120;0;0;0]

B =

    120
     0
     0
     0

>> C = inv(A)*B

C =

   2.3614 – 0.8170i               (I1)
   1.1806 – 0.0934i               (I2)
   2.1159 + 11.7136i              (V1)
   4.2318 + 23.4272i              (V2)
```

$$I_2 = (1.1806 - j0.0934)\text{ A or } 1.1843\angle -4.52^\circ \text{ A}$$

The power absorbed by the 2-Ω resistor is

$$P = |I_2|^2 R = (1.1843)^2 2 = \mathbf{2.805\ W}.$$

Solution 13.43

Obtain $\mathbf{V}_1$ and $\mathbf{V}_2$ in the ideal transformer circuit of Fig. 13.107.

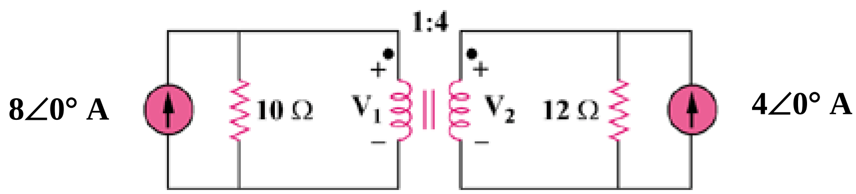
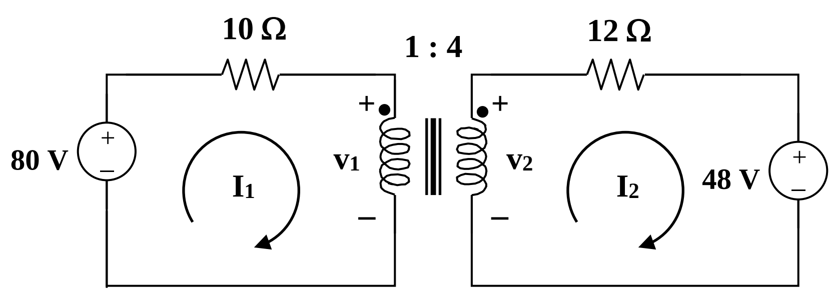


Figure 13.107
For Prob. 13.43.

Solution

Transform the two current sources to voltage sources, as shown below.



Using mesh analysis, $\quad -80 + 10I_1 + v_1 = 0$

$$80 = v_1 + 10I_1 \tag{1}$$

$$48 + 12I_2 - v_2 = 0 \text{ or } 48 = v_2 - 12I_2 \tag{2}$$

At the transformer terminal, $\quad v_2 = nv_1 = 4v_1 \quad (3)$

$$I_1 = nI_2 = 4I_2 \tag{4}$$

Substituting (3) and (4) into (1) and (2), we get,

$$80 = v_1 + 40I_2 \tag{5}$$

$$48 = 4v_1 - 12I_2 \tag{6}$$

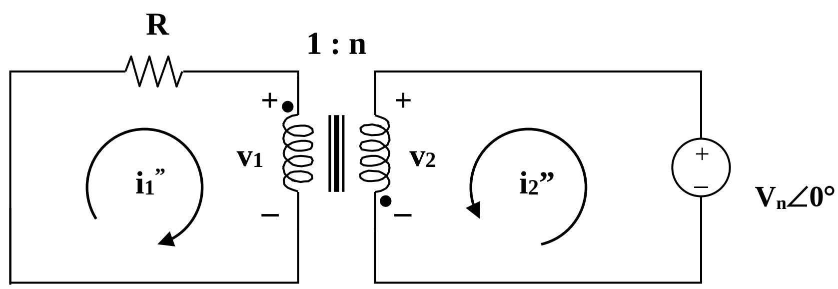
Solving (5) and (6) gives $\quad v_1 = \mathbf{16.744\ V}$ and $v_2 = 4v = \mathbf{66.98\ V}$

Solution 13.44

We can apply the superposition theorem. Let $i_1 = i_1' + i_1''$ and $i_2 = i_2' + i_2''$ where the single prime is due to the DC source and the double prime is due to the AC source. Since we are looking for the steady-state values of i_1 and i_2,

$$i_1' = i_2' = 0.$$

For the AC source, consider the circuit below.



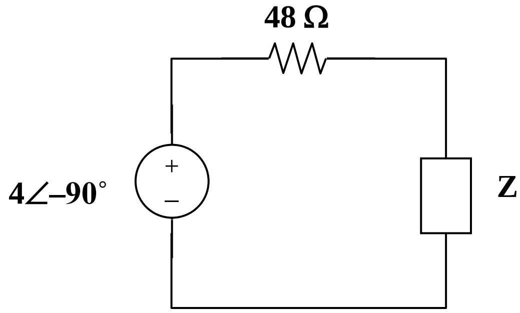
$$v_2/v_1 = -n, \quad I_2''/I_1'' = -1/n$$

But $v_2 = v_m$, $v_1 = -v_m/n$ or $I_1'' = v_m/(Rn)$

$$I_2'' = -I_1''/n = -v_m/(Rn^2)$$

Hence, $i_1(t) = \mathbf{(v_m/Rn)\cos\omega t\ A}$, and $i_2(t) = \mathbf{(-v_m/(n^2R))\cos\omega t\ A}$

Solution 13.45



$$Z_L = 8 - \frac{j}{\omega C} = 8 - j4, \quad n = 1/3$$

$$Z = \frac{Z_L}{n^2} = 9Z_L = 72 - j36$$

$$I = \frac{4\angle -90^\circ}{48 + 72 - j36} = \frac{4\angle -90^\circ}{125.28\angle -16.7^\circ} = 0.03193\angle -73.3^\circ$$

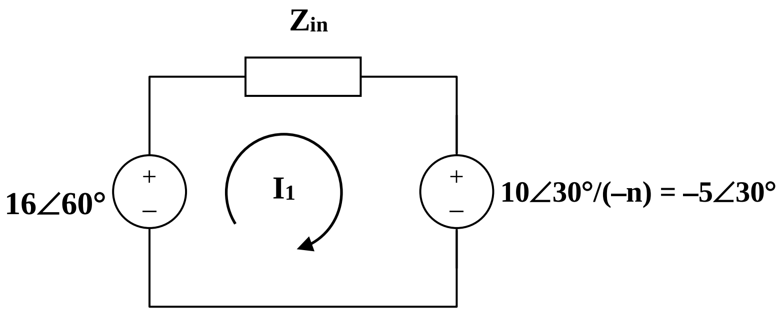
We now have some choices, we can go ahead and calculate the current in the second loop and calculate the power delivered to the 8-ohm resistor directly or we can merely say that the power delivered to the equivalent resistor in the primary side must be the same as the power delivered to the 8-ohm resistor. Therefore,

$$P_{8\Omega} = \left|\frac{I^2}{2}\right| 72 = 0.5098x10^{-3} 72 = \mathbf{36.71\ mW}$$

The student is encouraged to calculate the current in the secondary and calculate the power delivered to the 8-ohm resistor to verify that the above is correct.

Solution 13.46

(a) Reflecting the secondary circuit to the primary, we have the circuit shown below.



$\mathbf{Z_{in}} = 10 + j16 + (1/4)(12 - j8) = 13 + j14$

$-16\angle 60^\circ + \mathbf{Z_{in}}\mathbf{I_1} - 5\angle 30^\circ = 0$ or $\mathbf{I_1} = (16\angle 60^\circ + 5\angle 30^\circ)/(13 + j14)$

Hence, $\mathbf{I_1} = \mathbf{1.072\angle 5.88^\circ\ A}$, and $\mathbf{I_2} = -0.5\mathbf{I_1} = \mathbf{0.536\angle 185.88^\circ\ A}$

(b) Switching a dot will not affect Z_{in} but will affect $\mathbf{I_1}$ and $\mathbf{I_2}$.

$\mathbf{I_1} = (16\angle 60^\circ - 5\angle 30^\circ)/(13 + j14) = \mathbf{0.625\ \angle 25\ A}$

and $\mathbf{I_2} = 0.5\mathbf{I_1} = \mathbf{0.3125\angle 25^\circ\ A}$

Solution 13.47

Find v(t) for the circuit in Fig. 13.111.

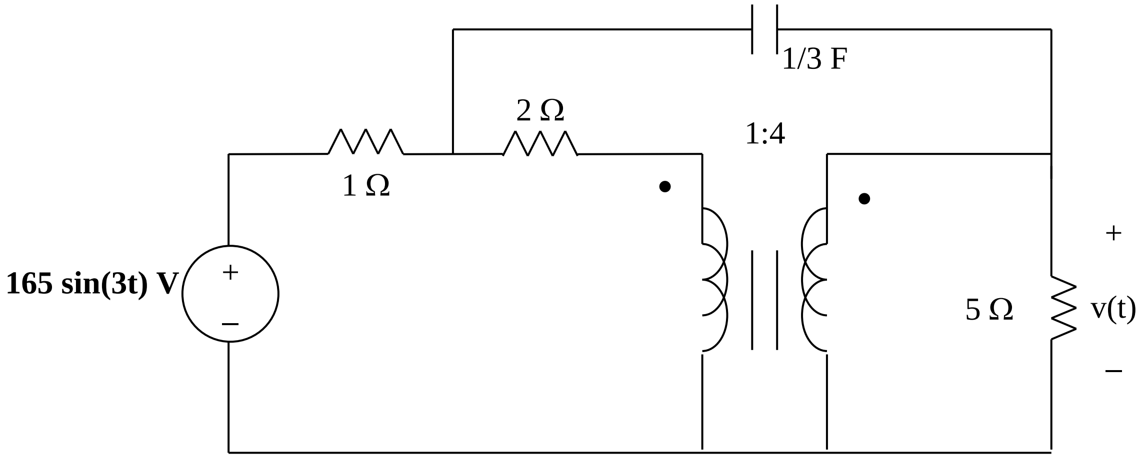
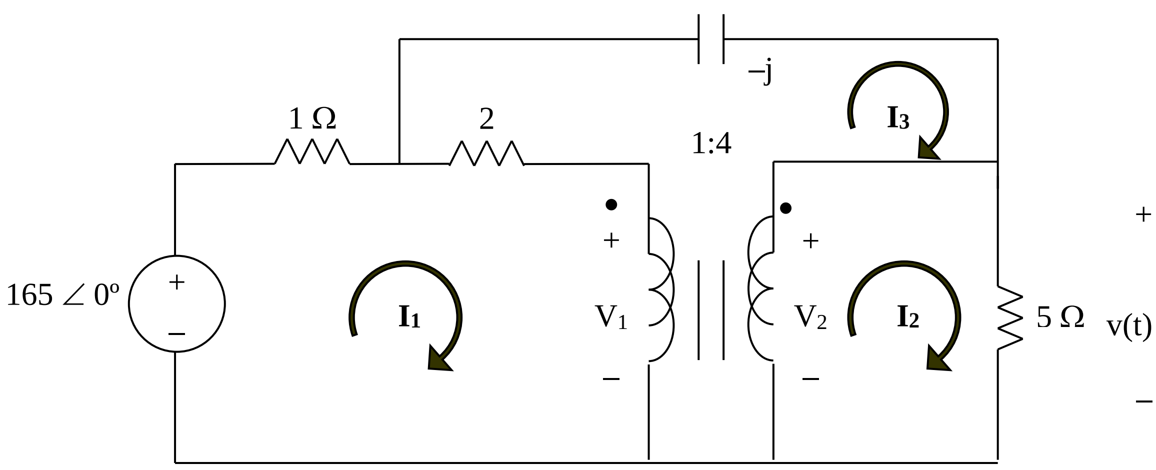


Figure 13.111
For Prob. 13.47.

Solution

$$\text{(1/3) F } 1F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j3x1/3} = -j1$$

Consider the circuit shown below.



For mesh 1,

$$3I_1 - 2I_3 + V_1 = 165 \qquad (1)$$

For mesh 2,

$$5I_2 – V_2 = 0 \qquad (2)$$

For mesh 3,

$$–2I_1 + (2–j)I_3 – V_1 + V_2 =0 \qquad (3)$$

At the terminals of the transformer,

$$V_2 = nV_1 = 4V_1 \qquad (4)$$

$$I_1 – I_3 = 4(I_2 – I_3) \qquad (5)$$

In matrix form,

$$\begin{bmatrix} 3 & 0 & -2 & 1 & 0 \\ 0 & 5 & 0 & 0 & -1 \\ -2 & 0 & 2-j & -1 & 1 \\ 0 & 0 & 0 & -4 & 1 \\ 1 & -4 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 165 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this using MATLAB yields

```
>>A = [3,0,-2,1,0; 0,5,0,0,-1; -2,0,(2-j),-1,1 ;0,0,0,-4,1; 1,-4,3,0,0]


A =

   3.0000         0    -2.0000           1.0000         0
        0    5.0000          0              0    -1.0000
  -2.0000         0    2.0000 - 1.0000i  -1.0000    1.0000
        0         0          0         -4.0000    1.0000
   1.0000   -4.0000     3.0000              0         0

>>U = [165;0;0;0;0]

>>X = inv(A)*U

X =

   53.427 + 0.8085i
   21.809 + 2.0914i
   11.274 + 2.5204i
   27.262 + 2.6152i
   109.053 + 10.465i
```

V = 5**I₂** = **V₂** = 109.053 + j10.465 = 109.55∠5.48° V, therefore,

v(t) = **109.55 sin(3t+5.48°) V**

Solution 13.48

Using Fig. 13.113, design a problem to help other students to better understand how ideal transformers work.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find $\mathbf{I}_x$ in the ideal transformer circuit of Fig. 13.112.

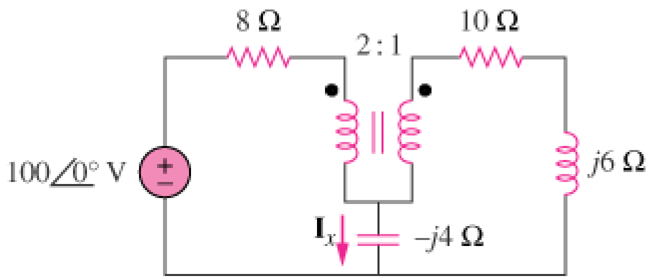
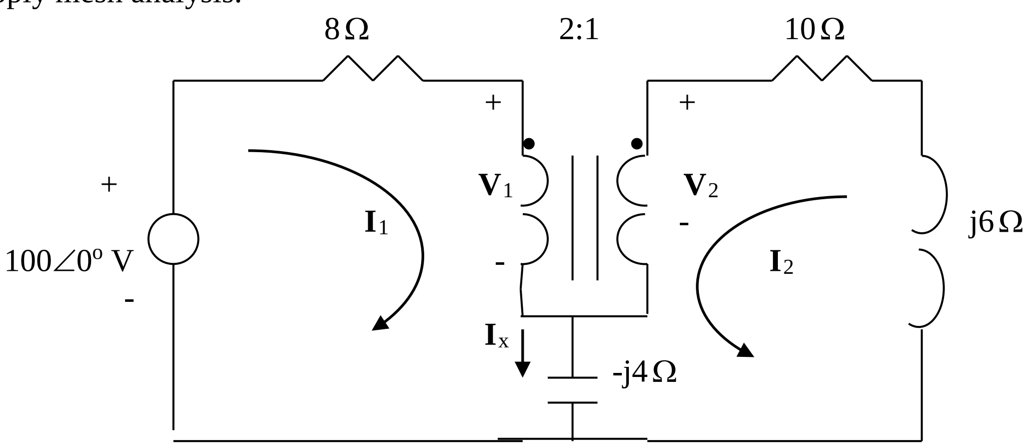


Figure 13.112

Solution

We apply mesh analysis.



$$100 = (8 - j4)I_1 - j4I_2 + V_1 \qquad (1)$$

$$0 = (10 + j2)I_2 - j4I_1 + V_2 \qquad (2)$$

But

$$\frac{V_2}{V_1} = n = \frac{1}{2} \longrightarrow V_1 = 2V_2 \qquad (3)$$

$$\frac{I_2}{I_1} = -\frac{1}{n} = -2 \quad \longrightarrow \quad I_1 = -0.5 I_2 \qquad (4)$$

Substituting (3) and (4) into (1) and (2), we obtain

$$100 = (-4 - j2)I_2 + 2V_2 \qquad (1)a$$

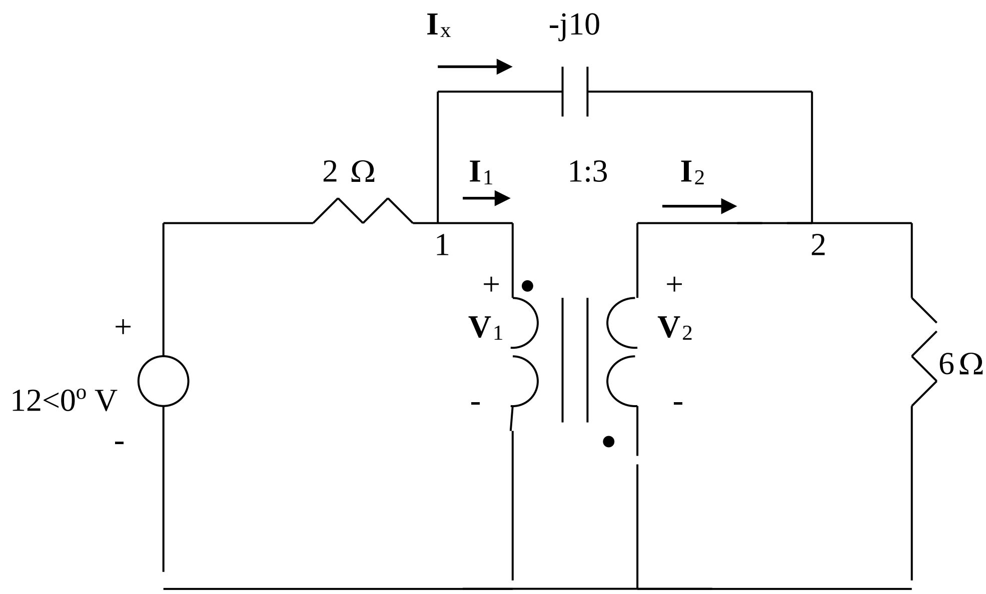
$$0 = (10 + j4)I_2 + V_2 \qquad (2)a$$

Solving (1)a and (2)a leads to $\mathbf{I}_2$ = -3.5503 +j1.4793

$$\boldsymbol{I_x = I_1 + I_2 = 0.5 I_2 = \underline{1.923\angle 157.4^o \text{ A}}}$$

Solution 13.49

$$\omega = 2, \quad \frac{1}{20}\,\text{F} \quad \longrightarrow \quad \frac{1}{j\omega C} = -j10$$



At node 1,

$$\frac{12 - V_1}{2} = \frac{V_1 - V_2}{-j10} + I_1 \quad \longrightarrow \quad 12 = 2I_1 + V_1(1 + j0.2) - j0.2V_2 \qquad (1)$$

At node 2,

$$I_2 + \frac{V_1 - V_2}{-j10} = \frac{V_2}{6} \quad \longrightarrow \quad 0 = 6I_2 + j0.6V_1 - (1 + j0.6)V_2 \qquad (2)$$

At the terminals of the transformer, $V_2 = -3V_1, \quad I_2 = -\frac{1}{3}I_1$

Substituting these in (1) and (2),

$$12 = -6I_2 + V_1(1 + j0.8), \qquad 0 = 6I_2 + V_1(3 + j2.4)$$

Adding these gives **V**$_1$=1.829 –j1.463 and

$$I_x = \frac{V_1 - V_2}{-j10} = \frac{4V_1}{-j10} = 0.937\angle 51.34^o$$

$i_x(t)$ = **937cos(2t+51.34˚) mA**.

Solution 13.50

The value of Z_{in} is not effected by the location of the dots since n^2 is involved.

$$Z_{in}' = (6 - j10)/(n')^2, \quad n' = 1/4$$

$$Z_{in}' = 16(6 - j10) = 96 - j160$$

$$Z_{in} = 8 + j12 + (Z_{in}' + 24)/n^2, \quad n = 5$$

$$Z_{in} = 8 + j12 + (120 - j160)/25 = 8 + j12 + 4.8 - j6.4$$

$$Z_{in} = \mathbf{(12.8 + j5.6)\ \Omega}$$

Solution 13.51

Use the concept of reflected impedance to find the input impedance and current $\mathbf{I}_1$ in Fig. 13.115 below.

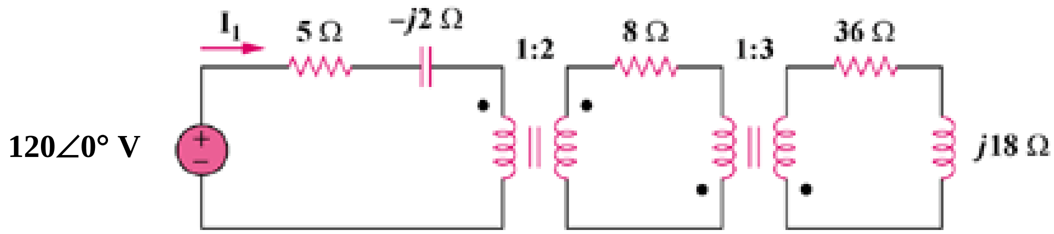


Figure 13.115
For Prob. 13.51.

Solution

Let $\mathbf{Z}_3 = 36 + j18$, where $\mathbf{Z}_3$ is reflected to the middle circuit.

$\mathbf{Z}_R' = \mathbf{Z}_L/n^2 = (12 + j2)/4 = 3 + j0.5$

$$\mathbf{Z}_{in} = 5 - j2 + \mathbf{Z}_R' = \mathbf{[8 - j1.5]\ \Omega}$$

$$\mathbf{I}_1 = 120\angle 0°/\mathbf{Z}_{eq} = 120\angle 0°/(8 - j1.5) = 120\angle 0°/8.1394\angle{-10.62°}$$
$$= \mathbf{14.743\angle 10.62°\ A}$$

$$\mathbf{[8 - j1.5]\ \Omega,\ 14.743\angle 10.62°\ A}$$

Solution 13.52

For maximum power transfer,

$40 = Z_L/n^2 = 10/n^2$ or $n^2 = 10/40$ which yields $n = 1/2 = 0.5$

$I = 120/(40 + 40) = 3/2$

$p = I^2R = (9/4)x40 =$ **90 watts**.

Solution 13.53

Refer to the network in Fig. 13.117.

(a) Find n for maximum power supplied to the 200-Ω load.

(b) Determine the power in the 200-Ω load if n = 10.

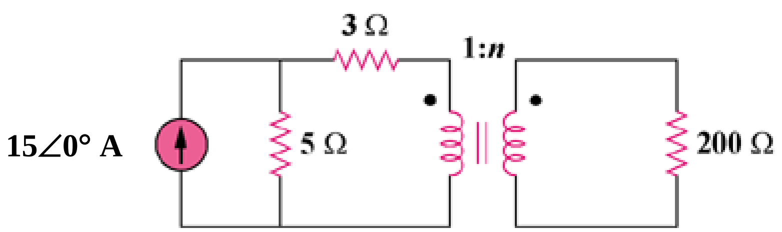
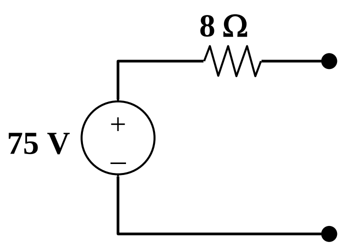


Figure 13.117
For Prob. 13.53.

Solution

(a) The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is $\mathbf{Z_L}' = \mathbf{Z_L}/n^2 = 200/n^2$.

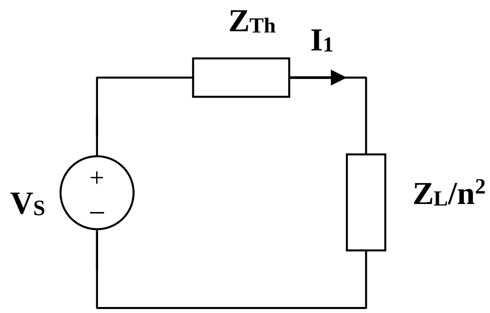
For maximum power transfer, $8 = 200/n^2$ produces $n = \mathbf{5}$.

(b) If $n = 10$, $\mathbf{Z_L}' = 200/100 = 2\ \Omega$ and $I = 75/(8 + 2) = 7.5$ A

$$p = I^2\mathbf{Z_L}' = (7.5)^2(2) = \mathbf{112.5\ W}.$$

Solution 13.54

(a)



For maximum power transfer,

$$Z_{Th} = Z_L/n^2, \text{ or } n^2 = Z_L/Z_{Th} = 8/128$$

$$n = \mathbf{0.25}$$

(b) $I_1 = V_{Th}/(Z_{Th} + Z_L/n^2) = 10/(128 + 128) = \mathbf{39.06\ mA}$

(c) $v_2 = I_2Z_L = 156.24\text{x}8 \text{ mV} = 1.25 \text{ V}$

But $v_2 = nv_1$ therefore $v_1 = v_2/n = 4(1.25) = \mathbf{5\ V}$

Solution 13.55

For the circuit in Fig. 13.119, calculate the equivalent resistance.

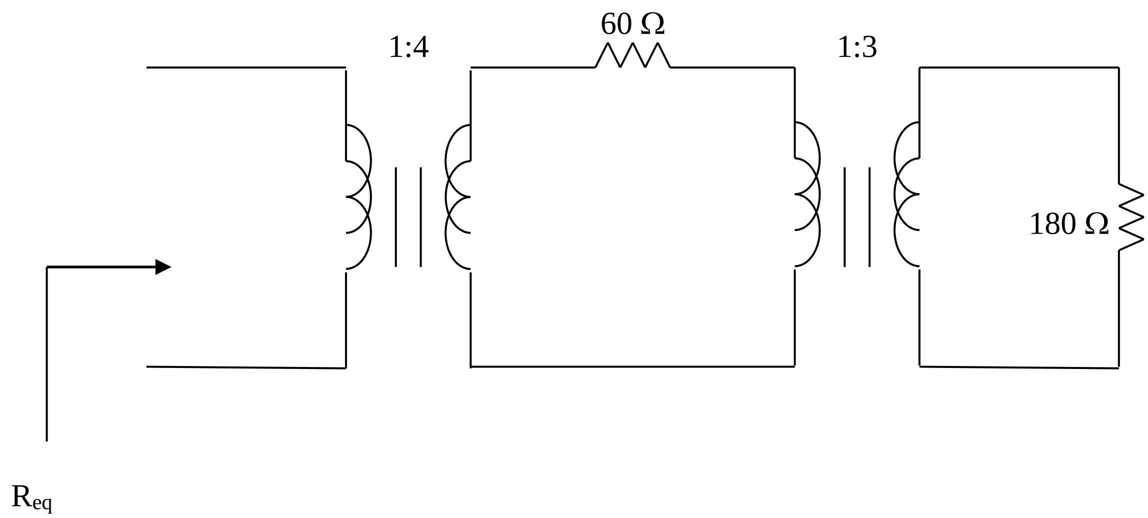


Figure 13.119
For Prob. 13.55.

Solution

We first reflect the 80-Ω resistance to the middle circuit.

$$\mathbf{Z'} = 60 + [180/(3)^2] = 60 + 20 = 80\ \mathbf{\Omega}$$

We now reflect this to the primary side.

$$\mathbf{R_{eq}} = \mathbf{Z'}/(4)^2 = \mathbf{5\ \Omega}.$$

Solution 13.56

Find the power absorbed by the 100-Ω resistor in the ideal transformer circuit of Fig. 13.120.

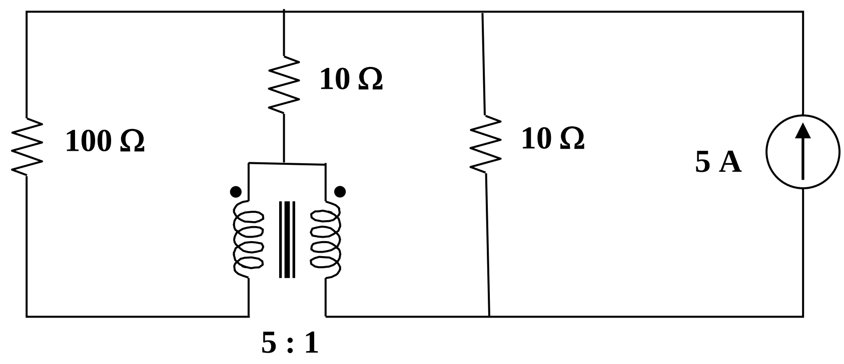
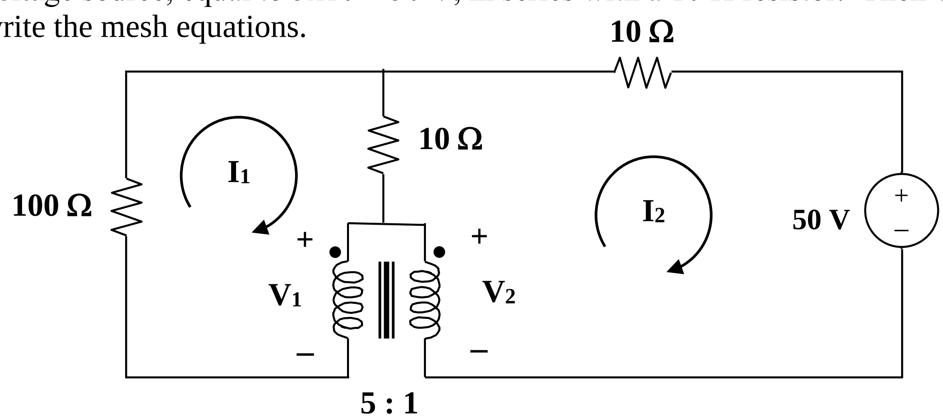


Figure 13.120
For Prob. 13.56.

Solution

Step 1. First we transform the current source in parallel with the 10 Ω into a voltage source, equal to 5x10 = 50 V, in series with a 10 Ω resistor. Then we can write the mesh equations.



$100\mathbf{I_1} + 10(\mathbf{I_1}–\mathbf{I_2}) + \mathbf{V_1} = 0$ and $–\mathbf{V_2} + 10(\mathbf{I_2}–\mathbf{I_1}) + 10\mathbf{I_2} + 50 = 0$. Now for the constraint equations, $\mathbf{V_1} = 5\mathbf{V_2}$ and $\mathbf{I_2} = 5\mathbf{I_1}$. Now all we need to do is to solve for $\mathbf{I_1}$ and then calculate the power. $P_{100} = |\mathbf{I_1}|^2(100)$.

Step 2. Replacing $\mathbf{V_2}$ and $\mathbf{I_2}$ in the above equations gives us, $(100+10–50)\mathbf{I_1} + \mathbf{V_1} = 0$ and $–0.2\mathbf{V_1} + 10(5\mathbf{I_1}–\mathbf{I_1}) + 50\mathbf{I_1} + 50 = 0$ or $0.2\mathbf{V_1} + (40+50)\mathbf{I_1} = –50$. From the first equation we get $\mathbf{V_1} = –60\mathbf{I_1}$ which now can be put into the second equation. $–0.2(–60\mathbf{I_1}) +90\mathbf{I_1} = –50$ or $(12+90)\mathbf{I_1} = –50$ or $\mathbf{I_1} = –0.490196$ A. This then gives us,

$$P_{100} = 100\text{x}(–0.4902)^2 = \mathbf{24.03\ W}.$$

Solution 13.57

(a) $Z_L = j3||(12 – j6) = j3(12 – j6)/(12 – j3) = (12 + j54)/17$

Reflecting this to the primary side gives

$Z_{in} = 2 + Z_L/n^2 = 2 + (3 + j13.5)/17 = 2.3168\angle 20.04°$

$I_1 = v_s/Z_{in} = 60\angle 90°/2.3168\angle 20.04° =$ **$25.9\angle 69.96°$ A(rms)**

$I_2 = I_1/n =$ **$12.95\angle 69.96°$ A(rms)**

(b) $60\angle 90° = 2I_1 + v_1$ or $v_1 = j60 – 2I_1 = j60 – 51.8\angle 69.96°$

$v_1 =$ **$21.06\angle 147.44°$ V(rms)**

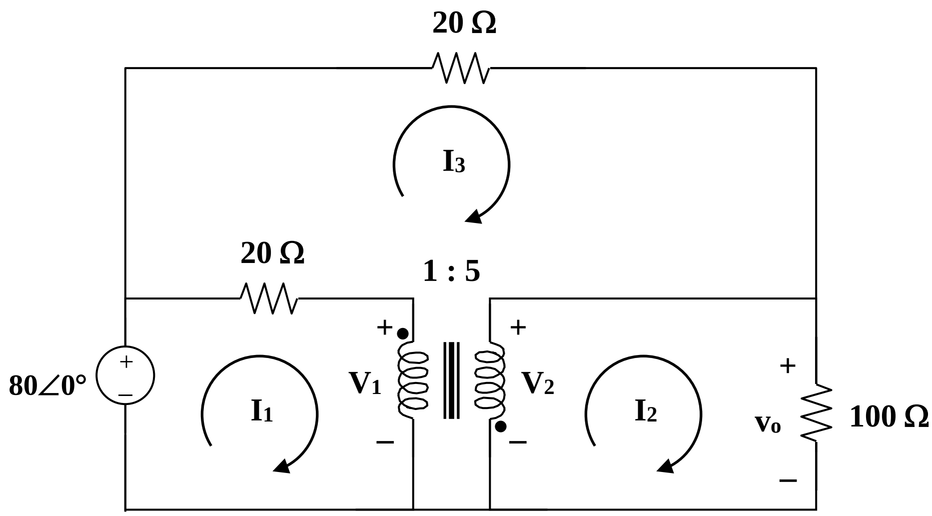
$v_2 = nv_1 =$ **$42.12\angle 147.44°$ V(rms)**

$v_o = v_2 =$ **$42.12\angle 147.44°$ V(rms)**

(c) $S = v_sI_1^* = (60\angle 90°)(25.9\angle –69.96°) =$ **$1.554\angle 20.04°$ kVA**

Solution 13.58

Consider the circuit below.



For mesh1, $-80 + 20I_1 - 20I_3 + V_1 = 0$ or

$$20I_1 - 20I_3 + V_1 = 80 \quad (1)$$

For mesh 2, $V_2 = 100I_2$ or $100I_2 - V_2 = 0 \quad (2)$

For mesh 3, $40I_3 - 20I_1 + V_2 - V_1 = 0$ which leads to

$$-20\,I_1 + 40I_3 - V_1 + V_2 = 0 \quad (3)$$

At the transformer terminals, $V_2 = -nV_1 = -5V_1$ or $5V_1 + V_2 \quad (4)$

$$I_1 - I_3 = -n(I_2 - I_3) = -5(I_2 - I_3) \text{ or}$$
$$I_1 + 5I_2 - 6I_3 = 0 \quad (5)$$

Solving using MATLAB,

```
>>A = [ 20  0  -20  1  0 ; 0  100  0  0  -1; -20  0  40  -1  1; 0  0  0  5  1;  1  5  -6  0
0 ]
```

```
A =

    20     0   -20     1     0
     0   100     0     0    -1
   -20     0    40    -1     1
     0     0     0     5     1
     1     5    -6     0     0

>> B = [ 80  0  0  0  0 ]'

B =

    80
     0
     0
     0
     0

>> Y = inv(A)*B

Y =

    5.9355
    0.5161
    1.4194
  -10.3226
   51.6129
```

P20,1 = 0.5*(I1 – I3)^2*20 = 0.5*(5.9355– 1.4194)^2*20 = 203.95

p_{20}(the one between 1 and 3) $= 0.5(20)(I_1 - I_3)^2 = 10(5.9355-1.4194)^2$

$=$ **203.95 watts**

p_{20}(at the top of the circuit) $= 0.5(20)I_3^2 =$ **20.15 watts**

$p_{100} = 0.5(100)I_2^2 =$ **13.318 watts**

Solution 13.59

In the circuit in Fig. 13.123, let v_s = 165sin(1,000t) V. Find the average power delivered to each resistor.

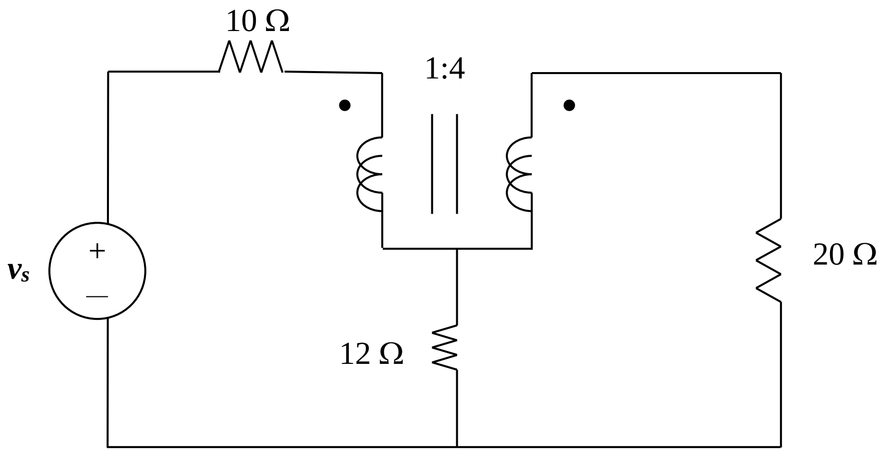
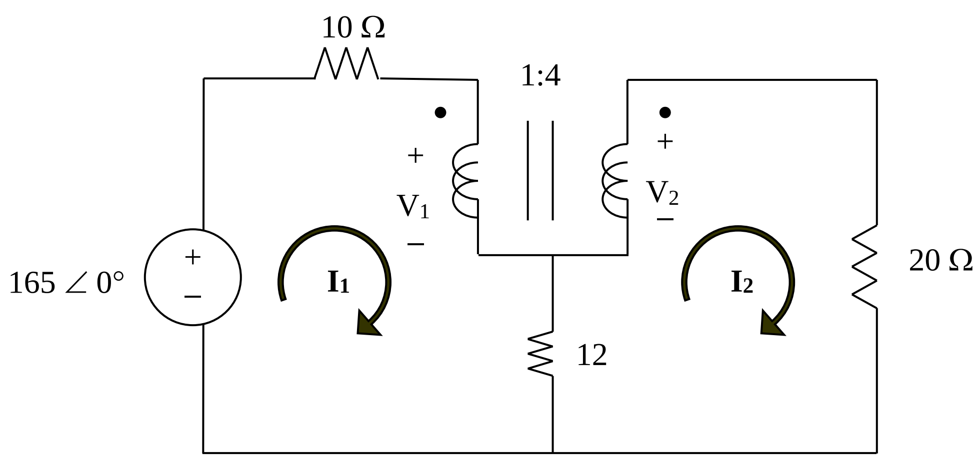


Figure 13.123
For Prob. 13.59.

Solution

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$-165 + 22\mathbf{I_1} - 12\mathbf{I_2} + \mathbf{V_1} = 0 \qquad (1)$$

For mesh 2,

$$-12\mathbf{I_1} + 32\mathbf{I_2} - \mathbf{V_2} = 0 \qquad (2)$$

At the transformer terminals,

$$-4\mathbf{V_1} + \mathbf{V_2} = 0 \qquad (3)$$

$$\mathbf{I_1} - 4\mathbf{I_2} = 0 \qquad (4)$$

Putting (1), (2), (3), and (4) in matrix form, we obtain

$$\begin{bmatrix} 22 & -12 & 1 & 0 \\ -12 & 32 & 0 & -1 \\ 0 & 0 & -4 & 1 \\ 1 & -4 & 0 & 0 \end{bmatrix} I = \begin{bmatrix} 165 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
>> A=[22,-12,1,0;-12,32,0,-1;0,0,-4,1;1,-4,0,0]
A =
   22   -12     1     0
  -12    32     0    -1
    0     0    -4     1
    1    -4     0     0
>> U=[165;0;0;0]
U =
   165
     0
     0
     0
>> X=inv(A)*U
X =
   9.1666
   2.2918
  –9.1666
  –36.6667
```

For 10-Ω resistor,

$$P_{10} = [(9.1666)^2/2]10 = \mathbf{420.1\ W}$$

For 12-Ω resistor,

$$P_{12} = [(9.1666–2.2918)^2/2]12 = \mathbf{283.6\ W}$$

For 20-Ω resistor,

$$P_{20} = [(2.2918)^2/2]20 = \mathbf{52.52\ W}.$$

Solution 13.60

(a) Transferring the 40-ohm load to the middle circuit,

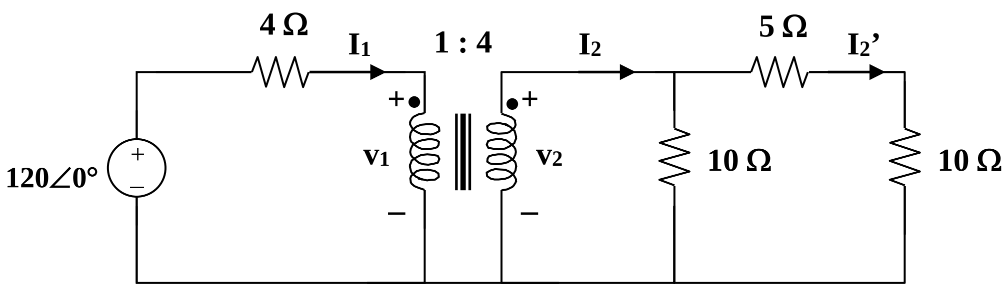
$Z_L' = 40/(n')^2 = 10$ ohms where $n' = 2$

$10||(5 + 10) = 6$ ohms

We transfer this to the primary side.

$Z_{in} = 4 + 6/n^2 = 4 + 0.375 = 4.375$ ohms, where $n = 4$

$I_1 = 120/4.375 = \mathbf{27.43\ A}$ and $I_2 = I_1/n = \mathbf{6.857\ A}$



Using current division, $I_2' = (10/25)I_2 = 2.7429$ and

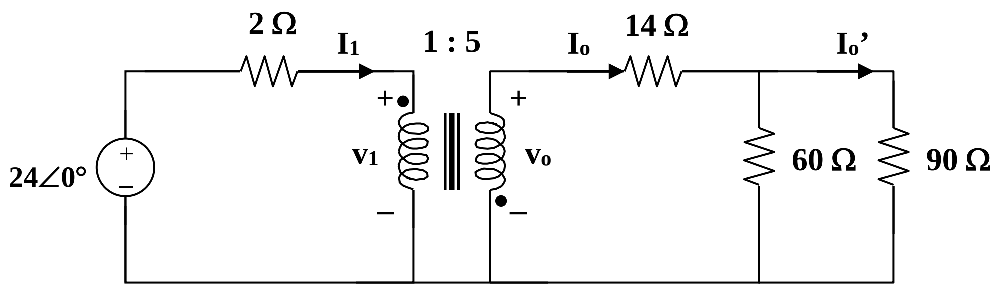
$I_3 = I_2'/n' = \mathbf{1.3714\ A}$

(b) $p = 0.5(I_3)^2(40) = \mathbf{37.62\ watts}$

Solution 13.61

We reflect the 160-ohm load to the middle circuit.

$$Z_R = Z_L/n^2 = 160/(4/3)^2 = 90 \text{ ohms}, \text{ where } n = 4/3$$



$$14 + 60\|90 = 14 + 36 = 50 \text{ ohms}$$

We reflect this to the primary side.

$$Z_R' = Z_L'/(n')^2 = 50/5^2 = 2 \text{ ohms when } n' = 5$$

$I_1 = 24/(2 + 2) = \mathbf{6A}$

$24 = 2I_1 + v_1$ or $v_1 = 24 - 2I_1 = 12$ V

$v_o = -nv_1 = \mathbf{-60\ V}$, $I_o = -I_1/n_1 = -6/5 = -1.2$

$I_o' = [60/(60 + 90)]I_o = -0.48A$

$I_2 = -I_o'/n = 0.48/(4/3) = \mathbf{360\ mA}$

Solution 13.62

(a) Reflect the load to the middle circuit.

$$\mathbf{Z_L}' = 8 - j20 + (18 + j45)/3^2 = 10 - j15$$

We now reflect this to the primary circuit so that

$$\mathbf{Z_{in}} = 6 + j4 + (10 - j15)/n^2 = 7.6 + j1.6 = 7.767\angle 11.89°, \text{ where}$$
$$n = 5/2 = 2.5$$

$$\mathbf{I_1} = 40/\mathbf{Z_{in}} = 40/7.767\angle 11.89° = 5.15\angle -11.89°$$

$$\mathbf{S} = \mathbf{v_s I_1^*} = (40\angle 0°)(5.15\angle 11.89°) = \mathbf{206\angle 11.89°\ VA}$$

(b) $$\mathbf{I_2} = -\mathbf{I_1}/n, \quad n = 2.5$$

$$\mathbf{I_3} = -\mathbf{I_2}/n', \quad n = 3$$

$$\mathbf{I_3} = \mathbf{I_1}/(nn') = 5.15\angle -11.89°/(2.5x3) = 0.6867\angle -11.89°$$

$$p = |\mathbf{I_2}|^2(18) = 18(0.6867)^2 = \mathbf{8.488\ watts}$$

Solution 13.63

Find the mesh currents in the circuit of Fig. 13.128 below.

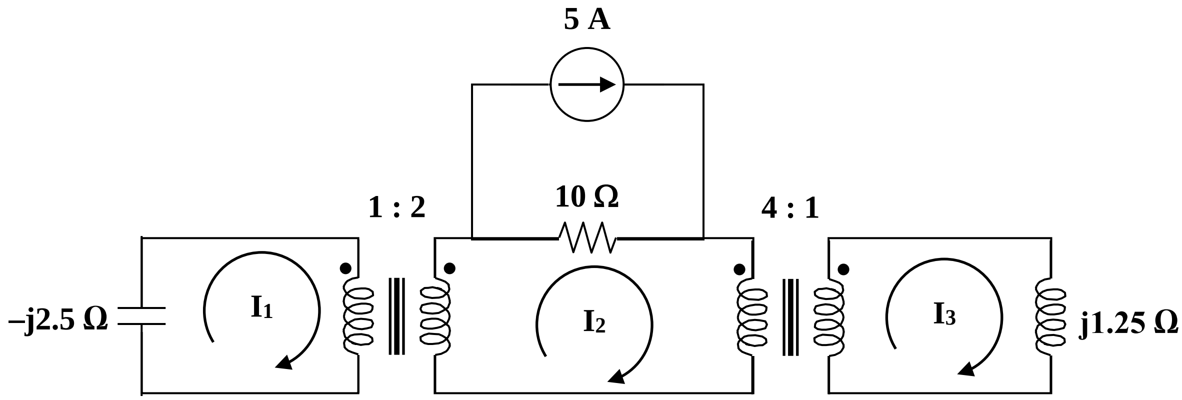


Figure 13.127
For Prob. 13.63.

Solution

Step 1. We can start by reflecting the –j2.5 Ω and the j1.25 Ω impedances into the center circuit and then solve for $\mathbf{I}_2$. The capacitor becomes –j2.5x4 = –j10 Ω and the inductor becomes j1.25x16 = j20 Ω. The mesh equation now becomes $-j10\mathbf{I}_2 + 10(\mathbf{I}_2-5) + j20\mathbf{I}_2 = 0$. Once we solve for $\mathbf{I}_2$ we can find $\mathbf{I}_1 = 2\mathbf{I}_2$ and $\mathbf{I}_3 = 4\mathbf{I}_2$.

Step 2. $(10+j10)\mathbf{I}_2 = 50$ or $\mathbf{I}_2 = 5/(1.4142\angle 45^\circ) = \mathbf{3.536\angle{-45^\circ}\ A}$, $\mathbf{I}_1 = \mathbf{7.071\angle{-45^\circ}\ A}$, and $\mathbf{I}_3 = \mathbf{14.1402\angle{-45^\circ}\ A}$.

Solution 13.64

For the circuit in Fig. 13.128, find the turn ratio so that the maximum power is delivered to the 30-kΩ resistor.

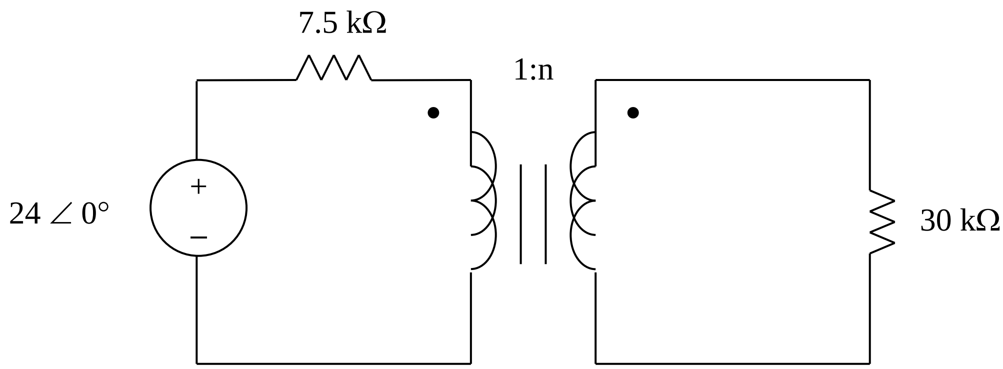
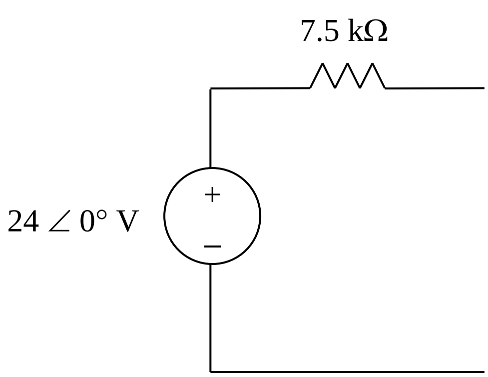


Figure 13.128
For Prob. 13.64.

Solution

The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is

$$\mathbf{Z'} = \mathbf{Z_L}/n^2 = 30k/n^2$$

For maximum power transfer,

$$7.5k = 30k/n^2 \text{ or } n^2 = 30/7.5 = 4 \text{ or } n = \mathbf{2}.$$

Solution 13.65

For the circuit in Fig. 13.128, find the turn ratio so that the maximum power is delivered to the 30-kΩ resistor.

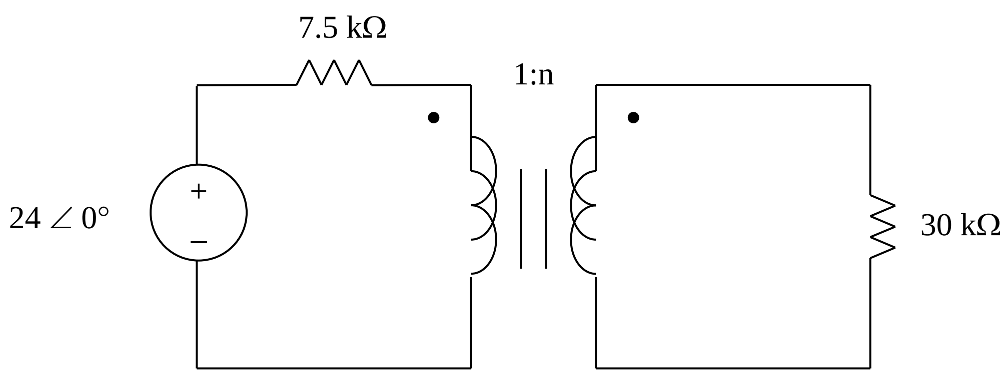
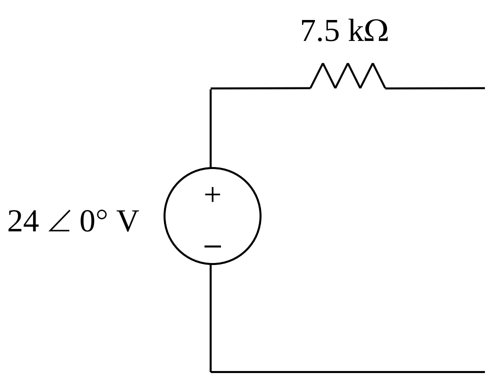


Figure 13.128
For Prob. 13.64.

Solution

The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is

$$\mathbf{Z'} = \mathbf{Z_L}/n^2 = 30k/n^2$$

For maximum power transfer,

$$7.5k = 30k/n^2 \text{ or } n^2 = 30/7.5 = 4 \text{ or } n = \mathbf{2}.$$

Solution 13.66

Design a problem to help other students to better understand how the ideal autotransformer works.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

An ideal autotransformer with a 1:4 step-up turns ratio has its secondary connected to a 120-Ω load and the primary to a 420-V source. Determine the primary current.

Solution

$$v_1 = 420 \text{ V} \quad (1)$$

$$v_2 = 120I_2 \quad (2)$$

$$v_1/v_2 = 1/4 \text{ or } v_2 = 4v_1 \quad (3)$$

$$I_1/I_2 = 4 \text{ or } I_1 = 4 I_2 \quad (4)$$

Combining (2) and (4),

$$v_2 = 120[(1/4)I_1] = 30 I_1$$

$$4v_1 = 30I_1$$

$$4(420) = 1680 = 30I_1 \text{ or } I_1 = \mathbf{56\ A}$$

Solution 13.67

An autotransformer with a 40% tap is supplied by an 880-V, 60-Hz source and is used for step-down operation. A 5-kVA load operating at unity power factor is connected to the secondary terminals. Find: (a) the secondary voltage, (b) the secondary current, (c) the primary current.

Solution

(a) $\frac{V_1}{V_2} = \frac{N_1 + N_2}{N_2} = \frac{1}{0.4} \longrightarrow V_2 = 0.4V_1 = 0.4x880 = \mathbf{352\ V}$.

(b) $\mathbf{S_2} = \mathbf{V_2}(\mathbf{I_2})^* = 5$ kVA or $\mathbf{I_2} = 5{,}000/352 = \mathbf{14.205\ A}$.

(c) $\mathbf{S_1} = \mathbf{V_1}(\mathbf{I_1})^* = \mathbf{S_2} = 5$ kVA or $\mathbf{I_1} = 5{,}000/880 = \mathbf{5.682\ A}$.

Solution 13.68

In the ideal autotransformer of Fig. 13.130, calculate $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_o$. Find the average power delivered to the load.

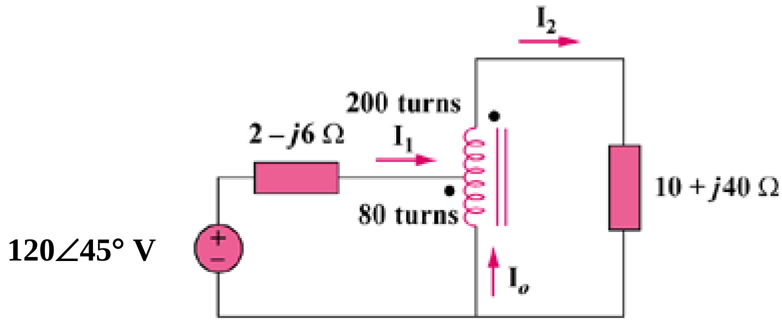
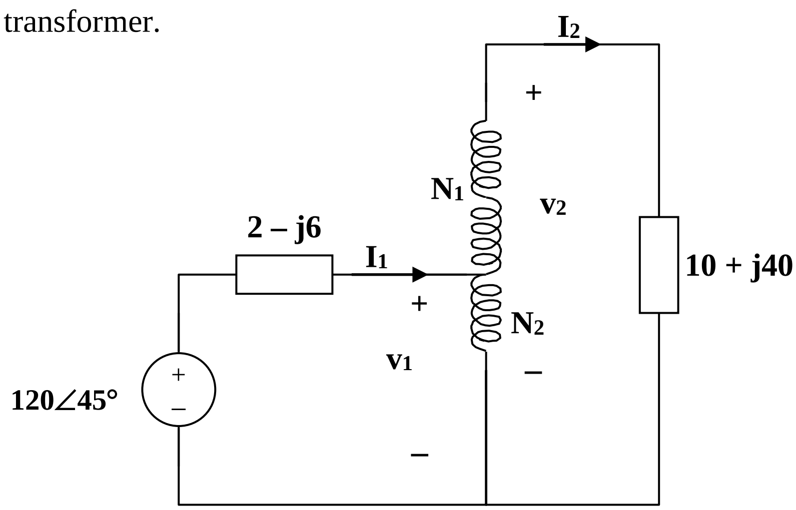


Figure 13.130
For Prob. 13.68.

Solution

This is a step-up transformer.



For the primary circuit, $\quad 120\angle 45° = (2 - j6)\mathbf{I}_1 + \mathbf{v}_1 \quad (1)$

For the secondary circuit, $\quad \mathbf{v}_2 = (10 + j40)\mathbf{I}_2 \quad (2)$

At the autotransformer terminals,

$$\mathbf{v}_1/\mathbf{v}_2 = N_1/(N_1 + N_2) = 200/280 = 5/7,$$

$$\text{thus } \mathbf{v}_2 = 7\mathbf{v}_1/5 \quad (3)$$

Also, $\quad \mathbf{I}_1/\mathbf{I}_2 = 7/5 \text{ or } \mathbf{I}_2 = 5\mathbf{I}_1/7 \quad (4)$

Substituting (3) and (4) into (2), $\mathbf{v_1} = (10 + j40)25\mathbf{I_1}/49$

Substituting that into (1) gives $120\angle 45° = (7.102 + j14.408)\mathbf{I_1}$

$$\mathbf{I_1} = 120\angle 45°/16.063\angle 63.76° = \mathbf{7.4706\angle{-18.76°}\ A}$$

$$\mathbf{I_2} = 5\mathbf{I_1}/7 = \mathbf{5.3361\angle{-18.76°}\ A}$$

$$\mathbf{I_o} = \mathbf{-I_1 + I_2} = [-(5/7) + 1]\mathbf{I_1} = 2\mathbf{I_1}/7 = \mathbf{2.1345\angle{-18.76°}\ A}$$

$$p = |\mathbf{I_2}|^2R = (5.3361)^2(10) = \mathbf{284.7\ W}.$$

Solution 13.69

In the circuit of Fig. 13.131, N_1 = 190 turns and N_2 = 10 turns, determine the Thevenin equivalent circuit looking into terminals a and b. What would be the value of $\mathbf{Z_L}$ that would absorb maximum power from the circuit?

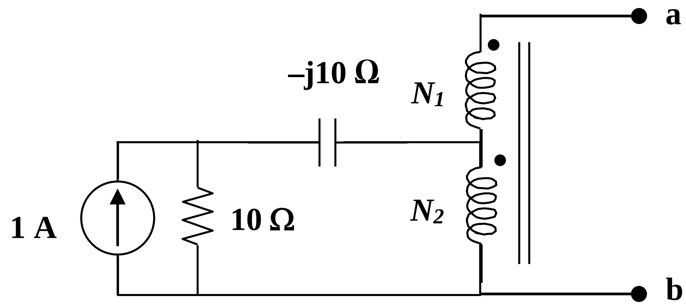
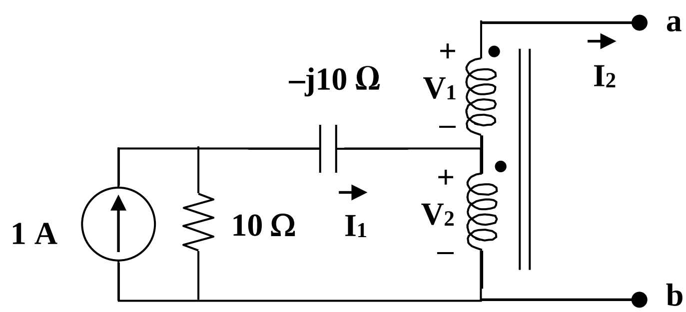


Figure 13.131
For Prob. 13.69.

Solution

Step 1. For the open circuit, $\mathbf{I_1} = 0$ and $\mathbf{I_2} = 0$. Thus, $\mathbf{V_{oc}} = [(190+10)/10](1\text{x}10) = \mathbf{V_{Thev}}$. For the short circuit current, V_1 and V_2 are equal to zero.



$\mathbf{I_1} = 1\text{x}10/(10–j10) = 0.7071\angle 45°$ A. $\mathbf{I_2} = \mathbf{I_{sc}} = [10/200]\mathbf{I_1}$. $\mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}}$.

Step 2. $\mathbf{V_{oc}} = \mathbf{V_{Thev}} = \mathbf{200\ V}$ and $\mathbf{I_{sc}} = 0.035355\angle 45°$ or
$\mathbf{Z_{eq}} = 200/(0.035355\angle 45°) = 5{,}657\angle –45°\ \Omega = \mathbf{(4 – j4)\ k\Omega}$.

For maximum power transfer we need $\mathbf{Z_L} = \mathbf{(4 + j4)\ k\Omega}$.

Solution 13.70

In the ideal transformer circuit shown in Fig. 13.132, determine the average power delivered to the load.

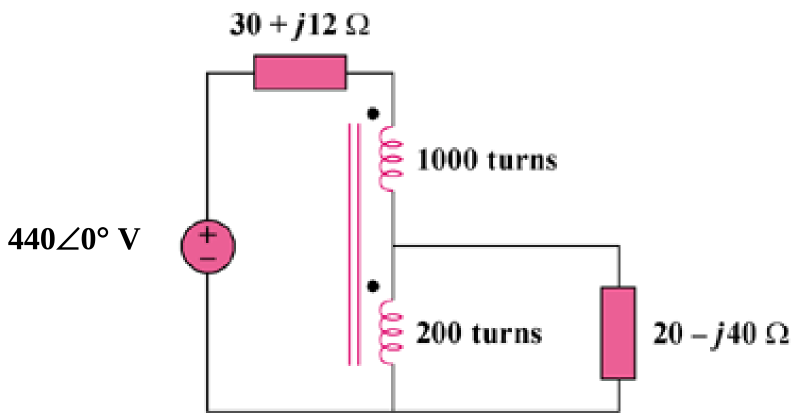
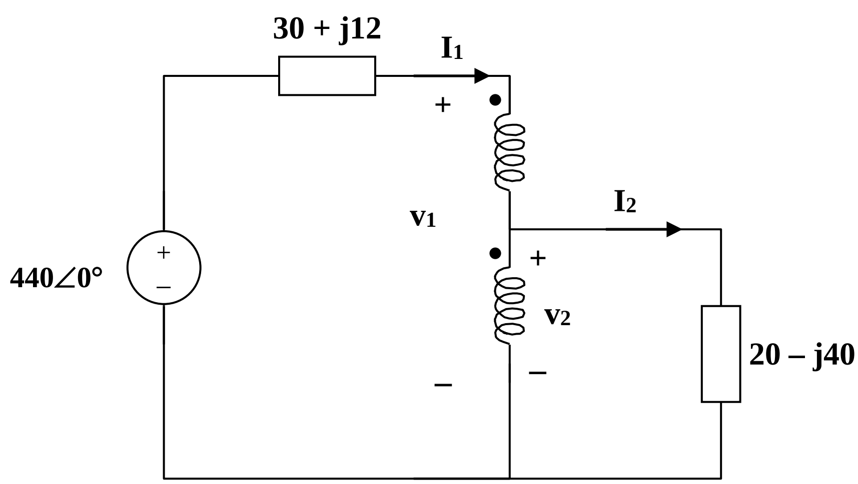


Figure 13.132
For Prob. 13.70.

Solution

This is a step-down transformer.



$$\mathbf{I_1}/\mathbf{I_2} = N_2/(N_1 + N_2) = 200/1200 = 1/6, \text{ or } \mathbf{I_1} = \mathbf{I_2}/6 \qquad (1)$$

$$\mathbf{v_1}/\mathbf{v_2} = (N_2 + N_2)/N_2 = 6, \text{ or } \mathbf{v_1} = 6\mathbf{v_2} \qquad (2)$$

For the primary loop, $440 = (30 + j12)\mathbf{I_1} + \mathbf{v_1}$ (3)

For the secondary loop, $\mathbf{v_2} = (20 - j40)\mathbf{I_2}$ (4)

Substituting (1) and (2) into (3),

$$440 = (30 + j12)(\mathbf{I_2}/6) + 6\mathbf{v_2}$$

and substituting (4) into this yields

$$440 = (5+j2)I_2 + 6(20–j40)I_2 = (125 – j238)\mathbf{I_2} \text{ or}$$
$$\mathbf{I_2} = 440/(268.83\angle–62.291° = 1.63672\angle 62.291° \text{ A.}$$

$$P_{20} = |\mathbf{I_2}|^2(20) = \mathbf{53.58\ W}.$$

Solution 13.71

When individuals travel, their electrical appliances need to have converters to match the voltages required by their appliances to the local voltage available to power their appliances. Today these converters use power electronics to convert voltages. In the past these converters were auto transformers. The auto transformer shown in Fig. 134 is used to convert 115 volts to 220 volts. What is the value of the turns? If the maximum current available from the 115 V source is 15 amps, what will be the maximum current available for the 220 V appliance?

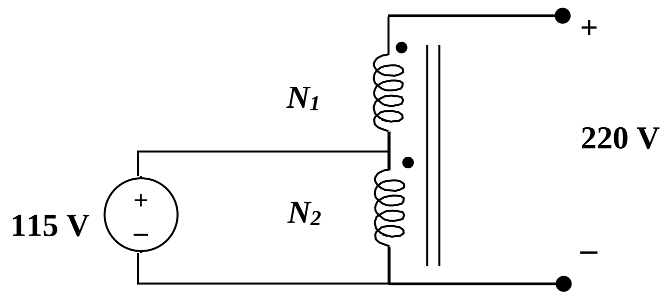


Figure 13.133
For Prob. 13.71.

Solution

Step 1. The relationship between the 115 V and the 220 V is equal to
$115/220 = N_2/(N_1+N_2)$ and $\mathbf{I_{115}} = 15$ A $= [(N_1+N_2)/N_2]\mathbf{I_{220}}$.

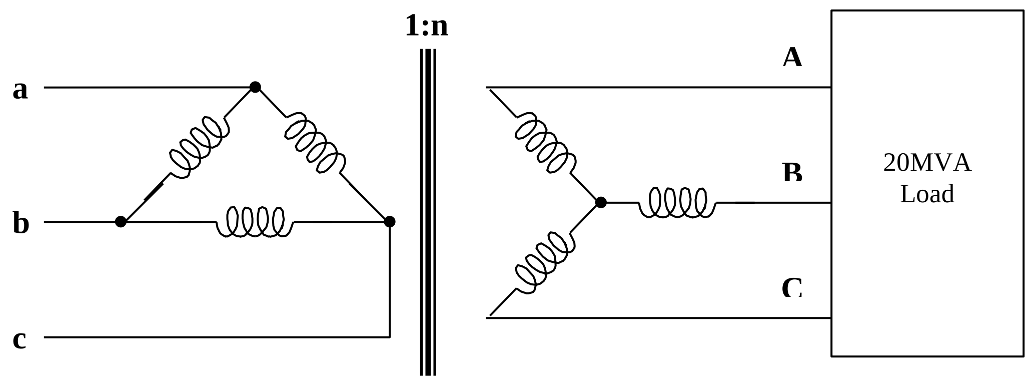
Step 2. $0.52273(N_1+N_2) = N_2$ or $0.52273N_1 = 0.47727N_2$ or

$$N_1/N_2 = \mathbf{0.913}$$

$\mathbf{I_{220}} = 0.52273 \text{x} 15 = \mathbf{7.841\ A}$.

Solution 13.72

(a) Consider just one phase at a time.



$$n = V_L/\sqrt{3}V_{Lp} = 7200/(12470\sqrt{3}) = \mathbf{1/3}$$

(b) The load carried by each transformer is 60/3 = 20 MVA.

Hence I_{Lp} = 20 MVA/12.47 k = **1604 A**

I_{Ls} = 20 MVA/7.2 k = **2778 A**

(c) The current in incoming line a, b, c is

$$\sqrt{3}I_{Lp} = \sqrt{3}x1603.85 = \mathbf{2778\ A}$$

Current in each outgoing line A, B, C is

$$2778/(n\sqrt{3}) = \mathbf{4812\ A}$$

Solution 13.73

(a) This is a **three-phase Δ-Y transformer**.

(b) $V_{Ls} = nv_{Lp}/\sqrt{3} = 450/(3\sqrt{3}) = 86.6$ V, where $n = 1/3$

As a Y-Y system, we can use per phase equivalent circuit.

$I_a = V_{an}/Z_Y = 86.6\angle 0°/(8 - j6) = 8.66\angle 36.87°$

$I_c = I_a\angle 120° =$ $\mathbf{8.66\angle 156.87°\ A}$

$I_{Lp} = n\sqrt{3}\, I_{Ls}$

$I_1 = (1/3)\sqrt{3}\,(8.66\angle 36.87°) = 5\angle 36.87°$

$I_2 = I_1\angle -120° =$ $\mathbf{5\angle -83.13°\ A}$

(c) $p = 3|I_a|^2(8) = 3(8.66)^2(8) =$ **1.8 kw**.

Solution 13.74

(a) This is a **Δ-Δ connection**.

(b) The easy way is to consider just one phase.

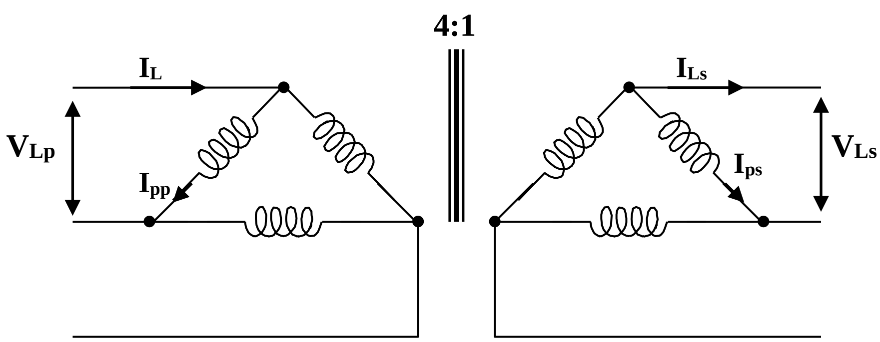
$$1{:}n = 4{:}1 \text{ or } n = 1/4$$

$$n = V_2/V_1 \text{ which leads to } V_2 = nV_1 = 0.25(2400) = 600$$

$$\text{i.e. } V_{Lp} = 2400 \text{ V and } V_{Ls} = 600 \text{ V}$$

$$S = p/\cos\theta = 120/0.8 \text{ kVA} = 150 \text{ kVA}$$

$$p_L = p/3 = 120/3 = 40 \text{ kw}$$



But $p_{Ls} = V_{ps}I_{ps}$

For the Δ-load, $I_L = \sqrt{3}\,I_p$ and $V_L = V_p$

Hence, $I_{ps} = 40{,}000/600 = 66.67$ A

$$I_{Ls} = \sqrt{3}\,I_{ps} = \sqrt{3}\,x66.67 = \mathbf{115.48\ A}$$

(c) Similarly, for the primary side

$$p_{pp} = V_{pp}I_{pp} = p_{ps} \text{ or } I_{pp} = 40{,}000/2400 = \mathbf{16.667\ A}$$

$$\text{and} \quad I_{Lp} = \sqrt{3}\,I_p = \mathbf{28.87\ A}$$

(d) Since $S = 150$ kVA therefore $S_p = S/3 =$ **50 kVA**

Solution 13.75

(a) $n = V_{Ls}/(\sqrt{3}\,V_{Lp}) = 900/(4500\sqrt{3}) = \mathbf{0.11547}$

(b) $S = \sqrt{3}\,V_{Ls}I_{Ls}$ or $I_{Ls} = 120{,}000/(900\sqrt{3}) = \mathbf{76.98\ A}$

$I_{Ls} = I_{Lp}/(n\sqrt{3}) = 76.98/(2.887\sqrt{3}) = \mathbf{15.395\ A}$

Solution 13.76

Using Fig. 13.138, design a problem to help other students to better understand a wye-delta, three-phase transformer and how they work.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A Y-Δ three-phase transformer is connected to a 60-kVA load with 0.85 power factor (leading) through a feeder whose impedance is 0.05 + j0.1Ω per phase, as shown in Fig. 13.137 below. Find the magnitude of:

(a) the line current at the load,

(b) the line voltage at the secondary side of the transformer,

(c) the line current at the primary side of the transformer.

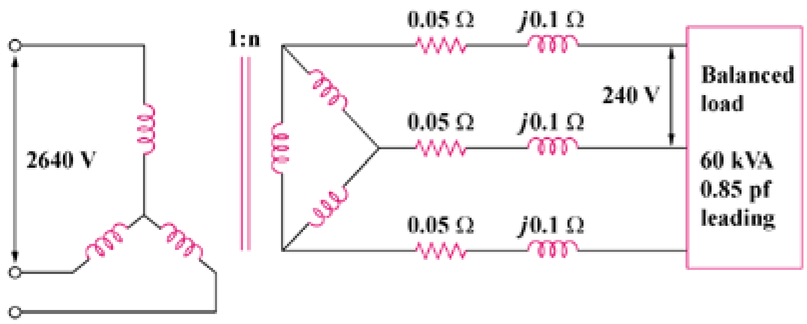


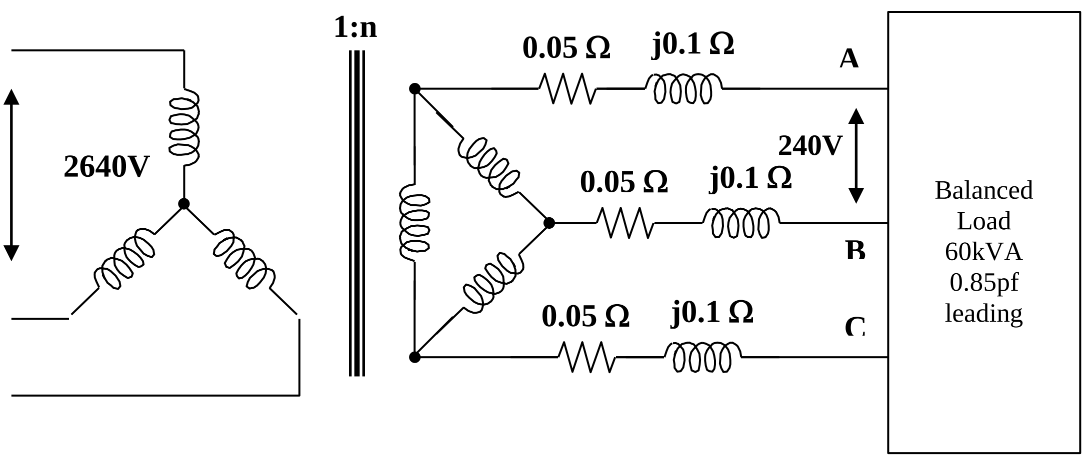
Figure 13.137

Solution

(a) At the load, $V_L = 240\text{ V} = V_{AB}$

$$V_{AN} = V_L/\sqrt{3} = 138.56\text{ V}$$

Since $S = \sqrt{3}V_L I_L$ then $I_L = 60{,}000/(240\sqrt{3}) = \mathbf{144.34\ A}$



(b) Let $V_{AN} = |V_{AN}|\angle 0° = 138.56\angle 0°$

$\cos\theta = pf = 0.85$ or $\theta = 31.79°$

$I_{AA'} = I_L\angle\theta = 144.34\angle 31.79°$

$$V_{A'N'} = ZI_{AA'} + V_{AN}$$

$$= 138.56\angle 0° + (0.05 + j0.1)(144.34\angle 31.79°)$$

$$= 138.03\angle 6.69°$$

$$V_{Ls} = V_{A'N'}\sqrt{3} = 138.03\sqrt{3} = \mathbf{239.1\ V}$$

(c) For Y-Δ connections,

$$n = \sqrt{3}\,V_{Ls}/V_{ps} = \sqrt{3}\,x238.7/2640 = 0.1569$$

$$f_{Lp} = nI_{Ls}/\sqrt{3} = 0.1569x144.34/\sqrt{3} = \mathbf{13.05\ A}$$

Solution 13.77

(a) This is a single phase transformer. $V_1 = 13.2$ kV, $V_2 = 120$ V

$n = V_2/V_1 = 120/13{,}200 = 1/110$, therefore $n = \mathbf{1/110}$

or 110 turns on the primary to every turn on the secondary.

(b) $P = VI$ or $I = P/V = 100/120 = 0.8333$ A

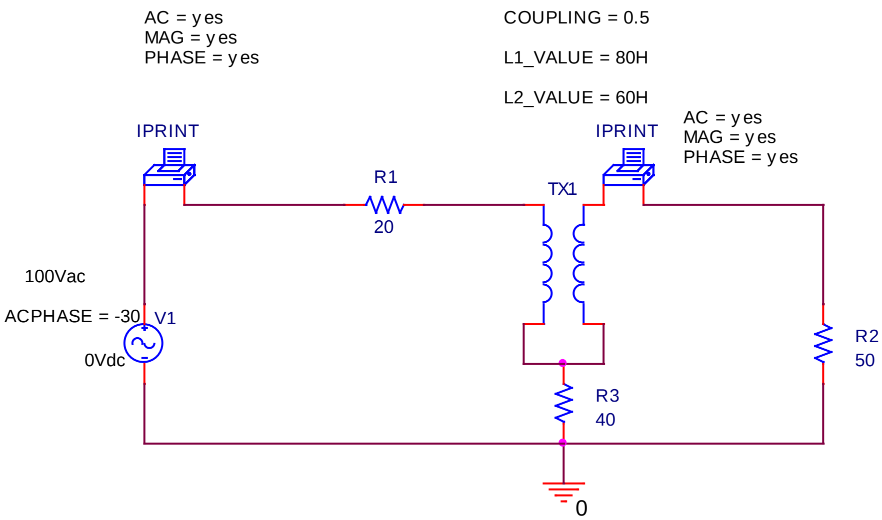
$I_1 = nI_2 = 0.8333/110 = \mathbf{7.576\ mA}$

Solution 13.78

We convert the reactances to their inductive values.

$$X = \omega L \qquad \longrightarrow \qquad L = \frac{X}{\omega}$$

The schematic is as shown below.



FREQ IM(V_PRINT1)IP(V_PRINT1)

1.592E-01 1.347E+00 -8.489E+01

FREQ IM(V_PRINT2)IP(V_PRINT2)

1.592E-01 6.588E-01 -7.769E+01

Thus,

$$\mathbf{I_1 = 1.347\angle{-84.89^\circ}\ amps}\ \text{and}\ \mathbf{I_2 = 658.8\angle{-77.69^\circ}\ mA}$$

Solution 13.79

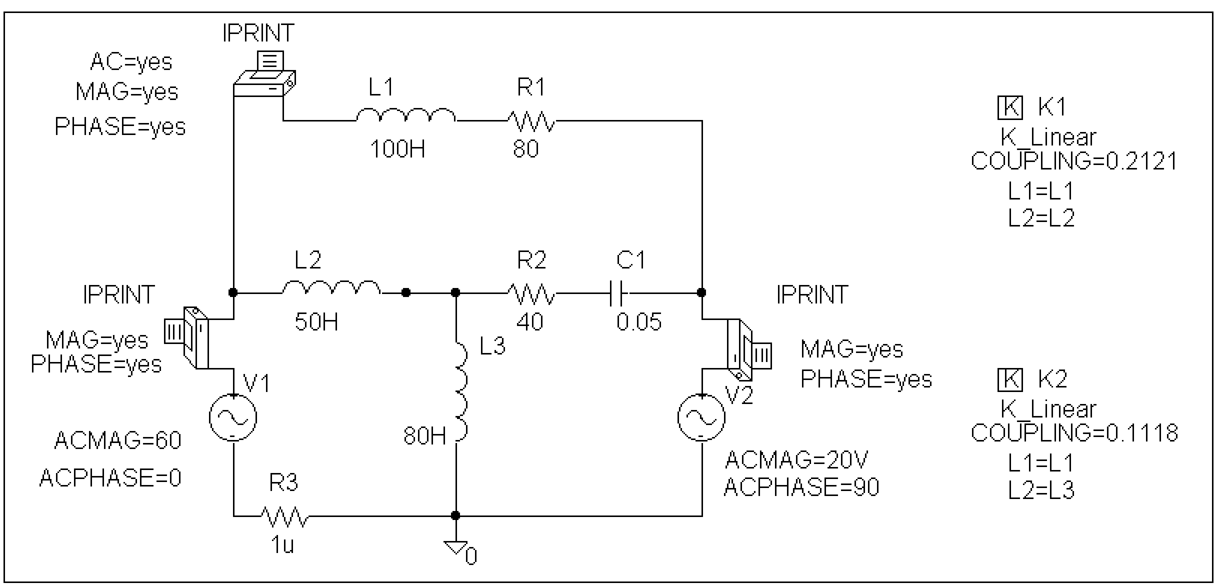
The schematic is shown below.

$$k_1 = 15/\sqrt{5000} = 0.2121, \quad k_2 = 10/\sqrt{8000} = 0.1118$$

In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After the circuit is saved and simulated, the output includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.068 E–01	–7.786 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	1.306 E+00	–6.801 E+01
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	1.336 E+00	–5.492 E+01

Thus, $\mathbf{I_1 = 1.306\angle{-68.01^\circ}\ A}$, $\mathbf{I_2 = 406.8\angle{-77.86^\circ}\ mA}$, $\mathbf{I_3 = 1.336\angle{-54.92^\circ}\ A}$



Solution 13.80

The schematic is shown below.

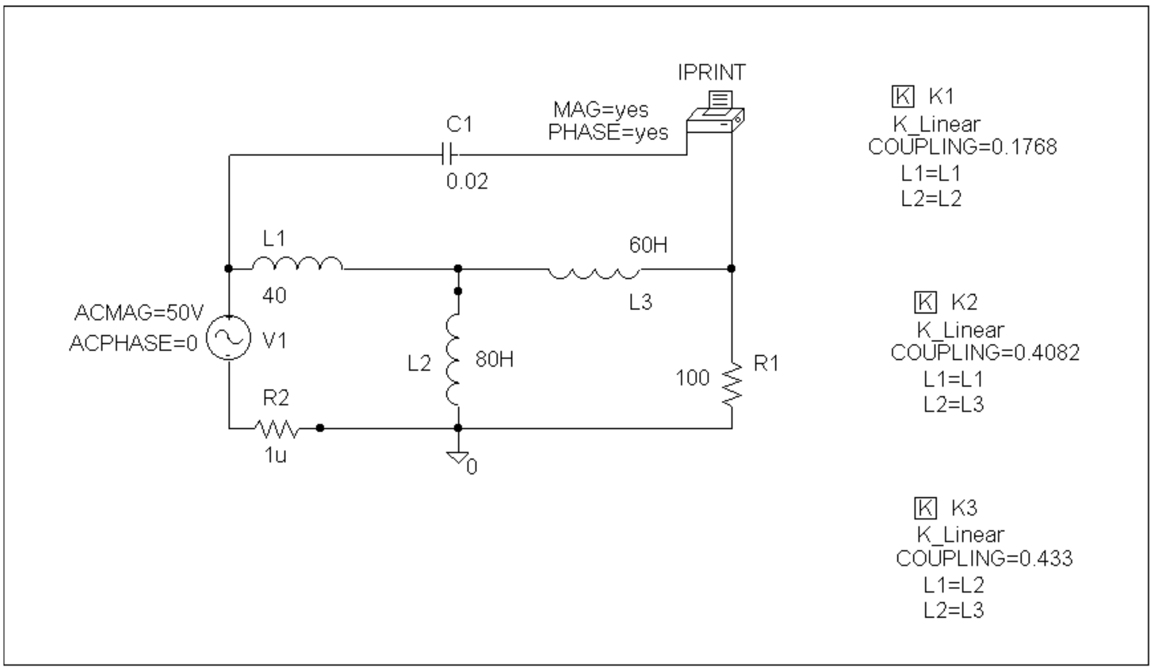
$$k_1 = 10/\sqrt{40x80} = 0.1768, \quad k_2 = 20/\sqrt{40x60} = 0.4082$$

$$k_3 = 30/\sqrt{80x60} = 0.433$$

In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After the simulation, we obtain the output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.304 E+00	6.292 E+01

i.e. I_o = **$1.304\angle 62.92°$ A**



Solution 13.81

The schematic is shown below.

$$k_1 = 2/\sqrt{4x8} = 0.3535, \quad k_2 = 1/\sqrt{2x8} = 0.25$$

In the AC Sweep box, we let Total Pts = 1, Start Freq = 100, and End Freq = 100. After simulation, the output file includes

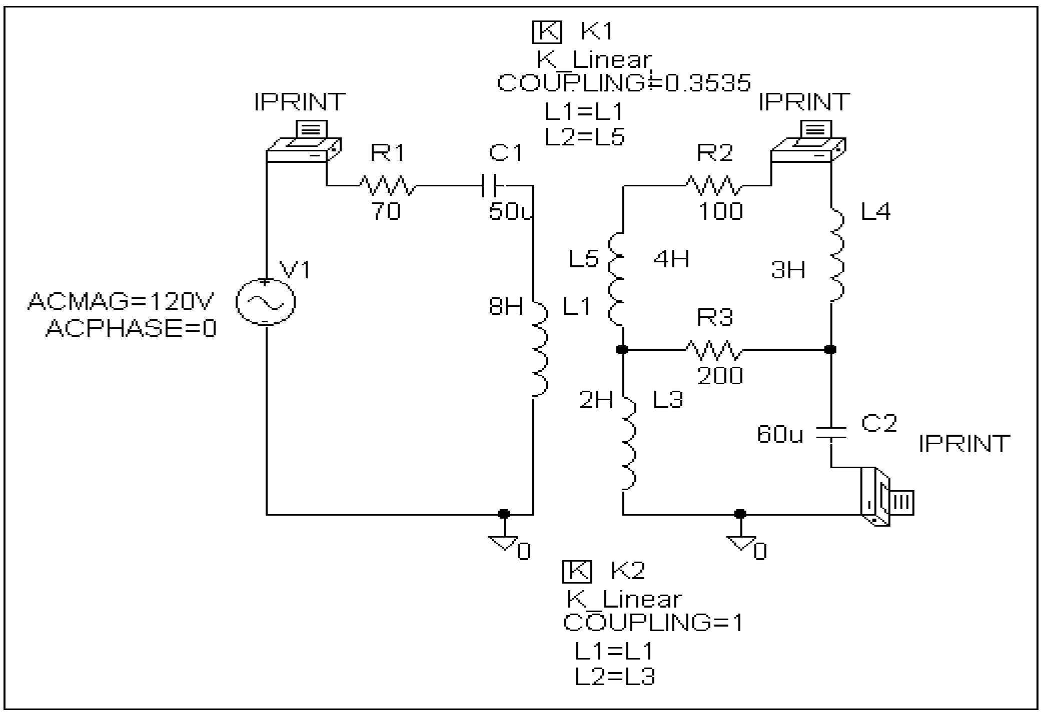
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.000 E+02	1.0448 E–01	1.396 E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.000 E+02	2.954 E–02	–1.438 E+02

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.000 E+02	2.088 E–01	2.440 E+01

i.e. $I_1 = \mathbf{104.5\angle 13.96° \ mA}$, $I_2 = \mathbf{29.54\angle -143.8° \ mA}$,

$I_3 = \mathbf{208.8\angle 24.4° \ mA}$.



Solution 13.82

The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain the output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.955 E+01	8.332 E+01

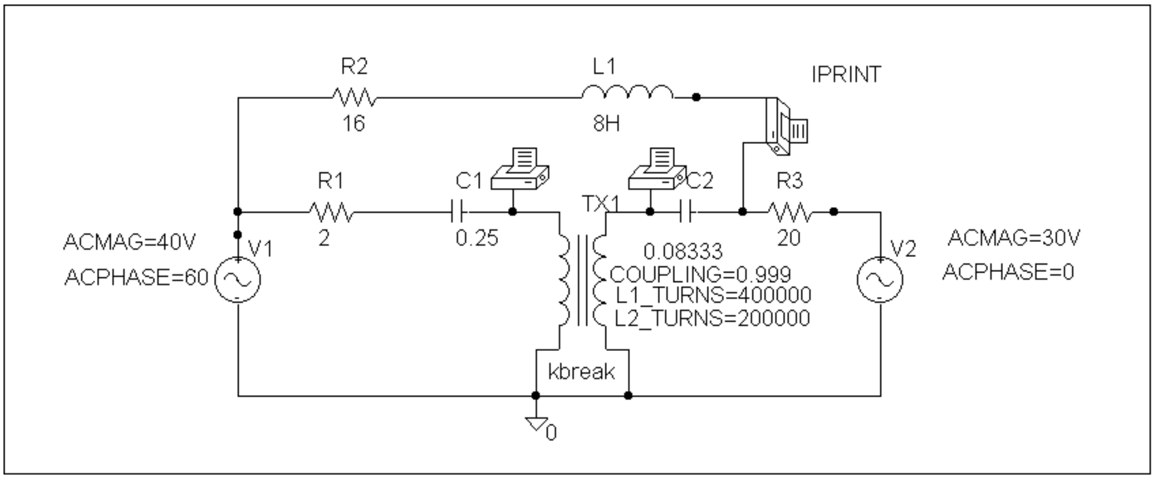
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	6.847 E+01	4.640 E+01

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	4.434 E–01	–9.260 E+01

i.e. V_1 = **19.55∠83.32° V**, V_2 = **68.47∠46.4° V**,

I_o = **443.4∠–92.6° mA**.

These answers are incorrect, we need to adjust the magnitude of the inductances.



Solution 13.83

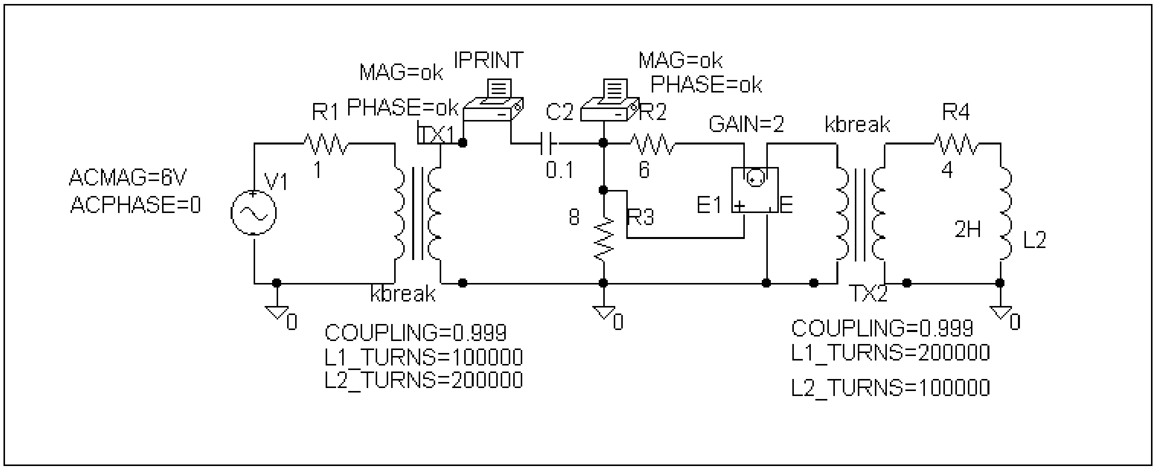
The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.080 E+00	3.391 E+01

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	1.514 E+01	–3.421 E+01

i.e. **ix = 1.08∠33.91° A, V_x = 15.14∠–34.21° V**.

This is most likely incorrect and needs to have the values of turns changed. Clearly the turns ratio makes L2 = 400000 and L1 = 4000000.



Checking with hand calculations.

Loop 1. $-6 + 1I_1 + V_1 = 0$ or $I_1 + V_1 = 6$ (1)

Loop 2. $-V_2 - j10I_2 + 8(I_2 - I_3) = 0$ or $(8 - j10)I_2 - 8I_3 - V_2 = 0$ (2)

Loop 3. $8(I_3 - I_2) + 6I_3 + 2V_x + V_3 = 0$ or $-8I_2 + 14I_3 + V_3 + 2V_x = 0$ but $V_x = 8(I_2 - I_3)$, therefore we get $8I_2 - 2I_3 + V_3 = 0$ (3)

Loop 4. $-V_4 + (4 + j2)I_4 = 0$ or $(4 + j2)I_4 - V_4 = 0$ (4)

We also need the constraint equations, $V_2 = 2V_1$, $I_1 = 2I_2$, $V_3 = 2V_4$, and $I_4 = 2I_3$. Finally,

$$I_x = I_2 \text{ and } V_x = 8(I_2 - I_3).$$

We can eliminate the voltages from the equations (we only need I_2 and I_3 to obtain the required answers) by,

$$(1)+0.5(2) = I_1 + (4–j5)I_2 – 4I_3 = 6 \text{ and}$$

$$0.5(3) + (4) = 4I_2 – I_3 + (4+j2)I_4 = 0.$$

Next we use $I_1 = 2I_2$ and $I_4 = 2I_3$ to end up with the following equations,

$(6–j5)I_2 – 4I_3 = 6$ and $4I_2 + (7+j4)I_3 = 0$ or $I_2 = –[(7+j4)I_3]/4 = (–1.75–j)I_3$
$= (2.01556\angle–150.255°)I_3$

This leads to or $(6–j5)(–1.75–j)I_3 – 4I_3 = (–10.5–5–4+j(8.75–6))I_3 = (–19.5+j2.75)I_3 = 6$

$I_3 = 6/(19.69296\angle171.973°) = 0.304677\angle–171.973°$ amps
$= –0.301692–j0.042545.$
$I_2 = (–1.75–j)(0.304677\angle–171.973°)$
$= (2.01556\angle–150.255°)(0.304677\angle–171.973°)$
$= 614.096\angle37.772°$ mA $= 0.48541+j0.37615$
and $I_2 – I_3 = 0.7871+j0.4187 = 0.89154\angle28.01°.$

Therefore,

$$V_x = 8(0.854876\angle22.97°) = \mathbf{7.132\angle28.01°\ V}$$

$$I_x = I_2 = \mathbf{614.1\angle37.77°\ mA}.$$

Checking with MATLAB we get A and X from equations (1) – (4) and the four constraint equations.

```
>> A = [1 0 0 0 1 0 0 0;0 (8-10j) -8 0 0 -1 0 0;0 8 -2 0 0 0 1 0;0 0 0 (4+2j) 0 0 0 -1;0 0 0 -2 1 0 0;1 -2 0 0 0 0 0 0;0 0 0 0 0 0 1 -2;0 0 -2 1 0 0 0 0]

A =

   1.0000                  0                  0                  0             1.0000                  0                  0
0
        0             8.0000 -10.0000i   -8.0000                  0                  0            -1.0000
0                  0
```

```
       0          8.0000         -2.0000               0               0               0          1.0000
0
       0               0               0          4.0000 + 2.0000i       0               0               0
-1.0000
       0               0               0               0          -2.0000          1.0000               0
0
   1.0000         -2.0000               0               0               0               0               0
0
       0               0               0               0               0               0          1.0000
-2.0000
       0               0          -2.0000          1.0000               0               0               0
0

>> X = [6;0;0;0;0;0;0;0]

X =

     6
     0
     0
     0
     0
     0
     0
     0

>> Y = inv(A)*X

Y =
```

0.9708 + 0.7523i = I_1 = 1.2817∠37.773° amps
0.4854 + 0.3761i = I_2 = 614.056∠37.769° mA = I_x
-0.3017 - 0.0425i = I_3 = 0.30468∠−171.982° amps
-0.6034 - 0.0851i = I_4
5.0292 - 0.7523i = V_1
10.0583 - 1.5046i = V_2
-4.4867 - 3.0943i = V_3
-2.2434 - 1.5471i = V_4

I_x = 614.1∠37.77° mA

Finally, $V_x = 8(I_2 – I_3) = 8(0.7871+j0.4186) = 8(0.891489∠28.01°)$

= **7.132∠28.01° volts**

Solution 13.84

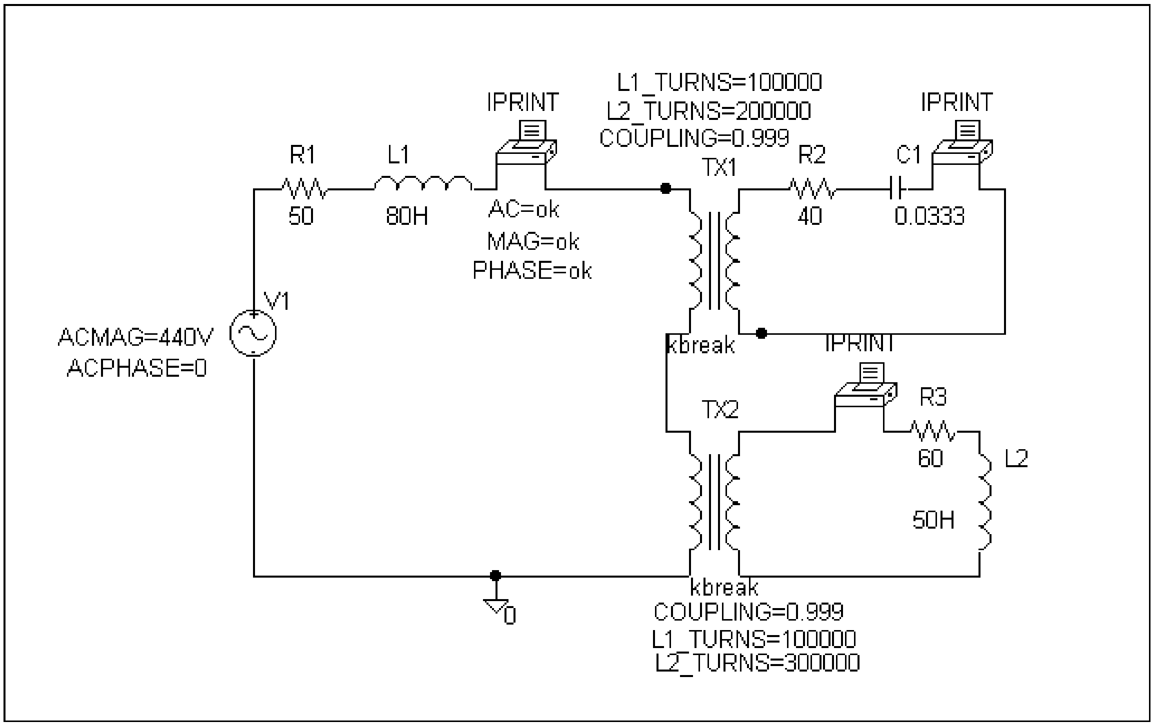
The schematic is shown below. we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.028 E+00	–5.238 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	2.019 E+00	–5.211 E+01
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	1.338 E+00	–5.220 E+01

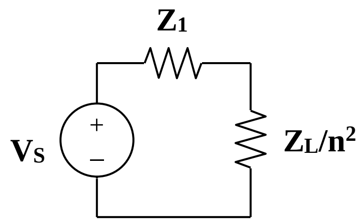
i.e. I_1 = $\mathbf{4.028\angle{-52.38^\circ}}$ **A**, I_2 = $\mathbf{2.019\angle{-52.11^\circ}}$ **A**,

I_3 = $\mathbf{1.338\angle{-52.2^\circ}}$ **A**.

Dot convention is wrong.



Solution 13.85



For maximum power transfer,

$$Z_1 = Z_L/n^2 \text{ or } n^2 = Z_L/Z_1 = 8/7200 = 1/900$$

$$n = 1/30 = N_2/N_1. \text{ Thus } N_2 = N_1/30 = 3000/30 = \mathbf{100\ turns}.$$

Solution 13.86

$n = N_2/N_1 = 48/2400 = 1/50$

$Z_{Th} = Z_L/n^2 = 3/(1/50)^2 = \mathbf{7.5\ k\Omega}$

Solution 13.87

$Z_{Th} = Z_L/n^2$ or $n = \sqrt{Z_L / Z_{Th}} = \sqrt{75/300} = \mathbf{0.5}$

Solution 13.88

$n = V_2/V_1 = I_1/I_2$ or $I_2 = I_1/n = 2.5/0.1 = 25$ A

$p = IV = 25x12.6 =$ **315 watts**

Solution 13.89

$n = V_2/V_1 = 120/240 =$ **0.5**

$S = I_1V_1$ or $I_1 = S/V_1 = 10x10^3/240 =$ **41.67 A**

$S = I_2V_2$ or $I_2 = S/V_2 = 10^4/120 =$ **83.33 A**

Solution 13.90

(a) $n = V_2/V_1 = 240/2400 = \mathbf{0.1}$

(b) $n = N_2/N_1$ or $N_2 = nN_1 = 0.1(250) =$ **25 turns**

(c) $S = I_1V_1$ or $I_1 = S/V_1 = 4\text{x}10^3/2400 =$ **1.6667 A**

$S = I_2V_2$ or $I_2 = S/V_2 = 4\text{x}10^4/240 =$ **16.667 A**

Solution 13.91

(a) The kVA rating is $S = VI = 25{,}000x75 = \mathbf{1.875\ MVA}$

(b) Since $S_1 = S_2 = V_2I_2$ and $I_2 = 1875x10^3/240 = \mathbf{7.812\ kA}$

Solution 13.92

(a) $V_2/V_1 = N_2/N_1 = n, V_2 = (N_2/N_1)V_1 = (28/1200)4800 =$ **112 V**

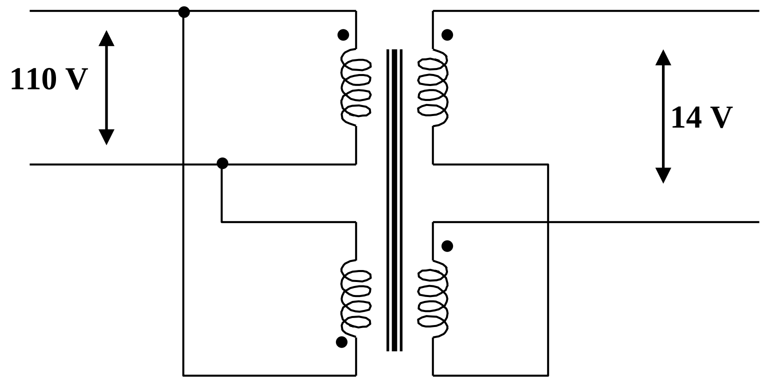
(b) $I_2 = V_2/R = 112/10 =$ **11.2 A** and $I_1 = nI_2,\ n = 28/1200$

$I_1 = (28/1200)11.2 =$ **261.3 mA**

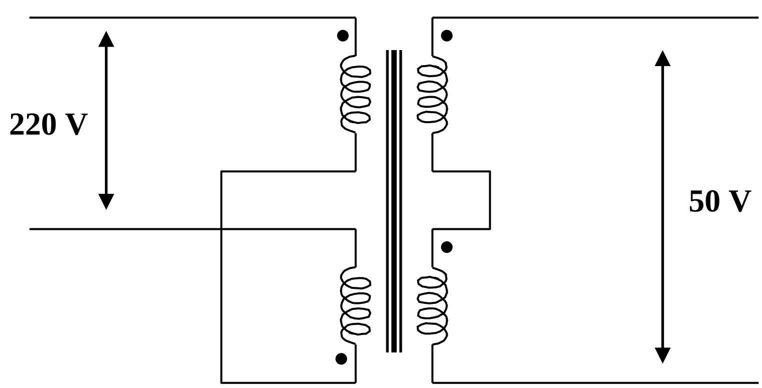
(c) $p = |I_2|^2R = (11.2)^2(10) =$ **1254 watts**.

Solution 13.93

(a) For an input of 110 V, the primary winding must be connected in parallel, with series aiding on the secondary. The coils must be series opposing to give 14 V. Thus, the connections are shown below.



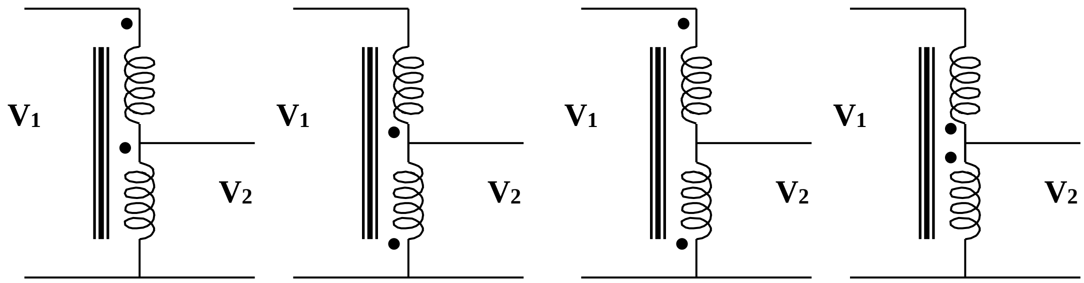
(b) To get 220 V on the primary side, the coils are connected in series, with series aiding on the secondary side. The coils must be connected series aiding to give 50 V. Thus, the connections are shown below.



Solution 13.94

$V_2/V_1 = 110/440 = 1/4 = I_1/I_2$

There are four ways of hooking up the transformer as an auto-transformer. However it is clear that there are only two outcomes.



(1) (2) (3) (4)

(1) and (2) produce the same results and (3) and (4) also produce the same results. Therefore, we will only consider Figure (1) and (3).

(a) For Figure (3), $V_1/V_2 = 550/V_2 = (440 - 110)/440 = 330/440$

Thus, $V_2 = 550\text{x}440/330 =$ **733.4 V (not the desired result)**

(b) For Figure (1), $V_1/V_2 = 550/V_2 = (440 + 110)/440 = 550/440$

Thus, $V_2 = 550\text{x}440/550 =$ **440 V (the desired result)**

Solution 13.95

(a) $n = V_s/V_p = 120/7200 = \mathbf{1/60}$

(b) $I_s = 10x120/144 = 1200/144$

$$S = V_pI_p = V_sI_s$$

$$I_p = V_sI_s/V_p = (1/60)x1200/144 = \mathbf{139\ mA}$$

Solution 13.96*

Problem

Some modern power transmission systems now have major, high voltage DC transmission segments. There are a lot of good reasons for doing this but we will not go into them here. To go from the AC to DC, power electronics are used. We start with three-phase AC and then rectify it (using a full-wave rectifier). It was found that using a delta to wye and delta combination connected secondary would give us a much smaller ripple after the full-wave rectifier. How is this accomplished? Remember that these are real devices and are wound on common cores. Hint, using Figures 13.47 and 13.49, and the fact that each coil of the wye connected secondary and each coil of the delta connected secondary are wound around the same core of each coil of the delta connected primary so the voltage of each of the corresponding coils are in phase. When the output leads of both secondaries are connected through full-wave rectifiers with the same load, you will see that the ripple is now greatly reduced. Please consult the instructor for more help if necessary.

Solution

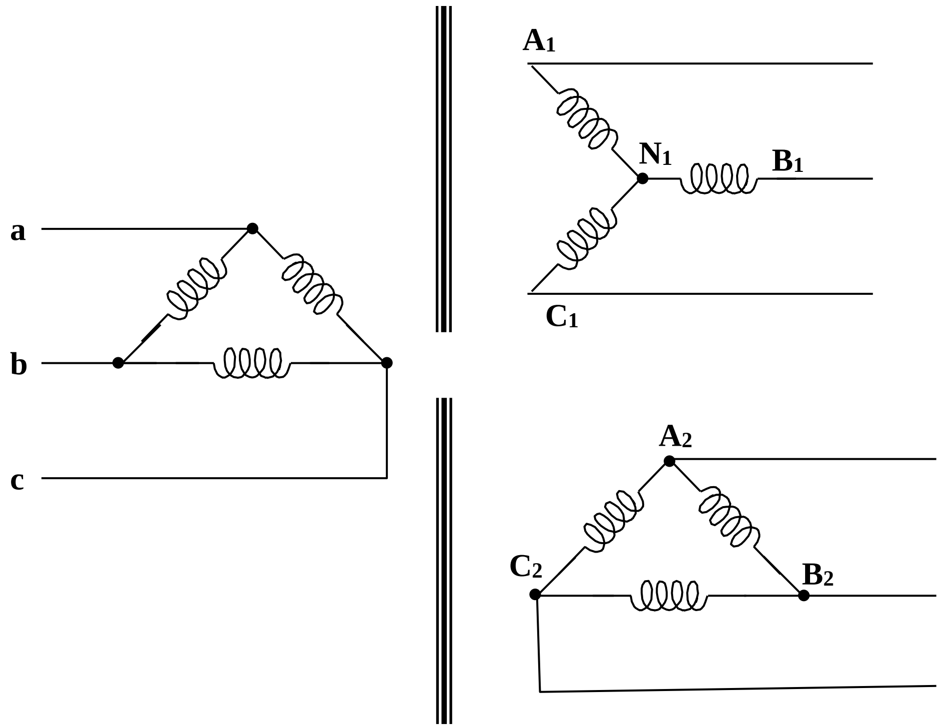
This is a most interesting and very practical problem. The solution is actually quite easy, you are creating a second set of sine waves to send through the full-wave rectifier, 30˚ out of phase with the first set. We will look at this graphically in a minute. We begin by showing the transformer components.

The key to making this work is to wind the secondary coils with each phase of the primary. Thus, a-b is wound around the same core as A_1-N_1 and A_2-B_2. The next thing we need to do is to make sure the voltages come out equal. We need to work the number of turns of each secondary so that the peak of $V_{A1}– V_{B1}$ is equal to $V_{A2}–V_{B2}$. Now, let us look at some of the equations involved.

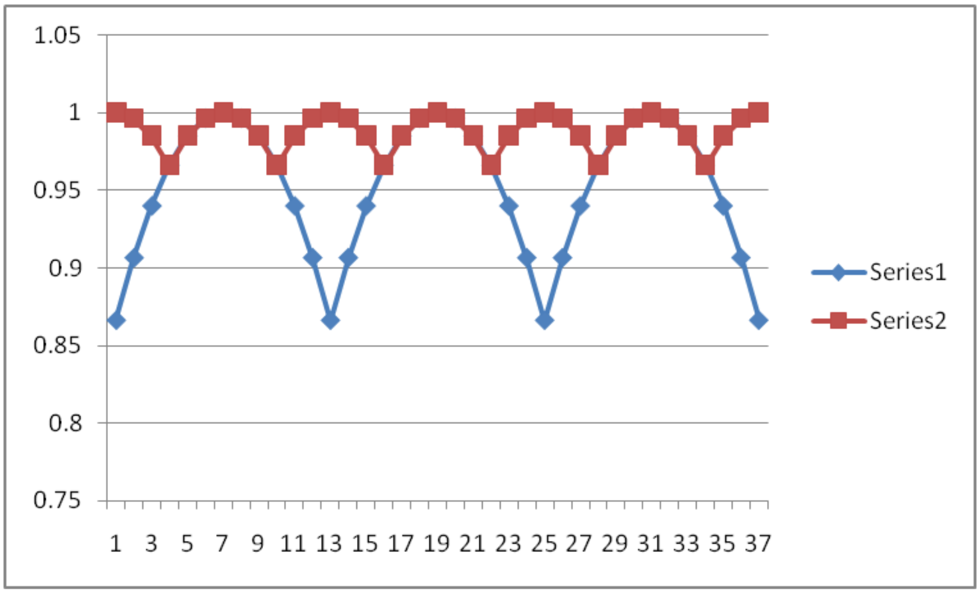
If we let $v_{ab}(t) = 100\sin(t)$ V, assume that we have an ideal transformer, and the turns ratios are such that we get $v_{A1\text{-}N1}(t) = 57.74\sin(t)$ V and $V_{A2\text{-}B2}(t) = 100\sin(t)$ V. Next, let us look at $V_{bc}(t) = 100\sin(t+120^\circ)$ V. This leads to $V_{B1\text{-}N1}(t) = 57.74\sin(t+120^\circ)$ V. We now need to determine $V_{A1\text{-}B1}(t)$.

$$V_{A1\text{-}B1}(t) = 57.74\sin(t) – 57.74\sin(t+120^\circ) = 100\sin(t–30^\circ) \text{ V.}$$

This then leads to the output per phase voltage being equal to $v_{out}(t) = [100\sin(t) + 100\sin(t–30^\circ)]$ V. We can do this for each phase and end up with the output being sent to the full-wave rectifier. This looks like $v_{out}(t) = [|100\sin(t)| + |100\sin(t–30^\circ)| + |\sin(t+120^\circ)|\ |100\sin(t+90^\circ)| + |100\sin(t–120^\circ)| + |100\sin(t–150^\circ)|]$ V. The end result will be more obvious if we look at plots of the rectified output.



In the plot below we see the normalized (1 corresponds to 100 volts) ripple with only one of the secondary sets of windings and then the plot with both. Clearly the ripple is greatly reduced!



Solution 14.1

Find the transfer function $\mathbf{I}_o/\mathbf{I}_i$ of the *RL* circuit in Fig. 14.68. Express the transfer function using $\omega_o = R/L$.

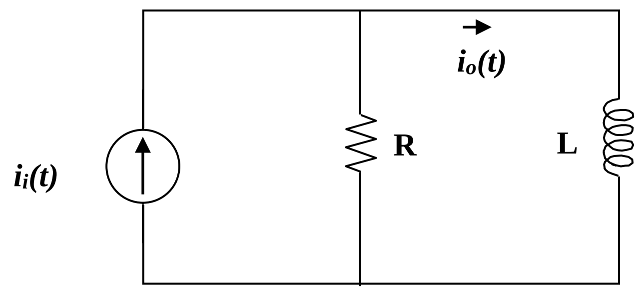


Figure 14.68
For Prob. 14.1.

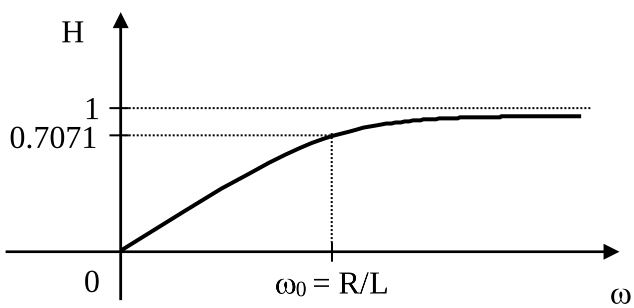
Solution

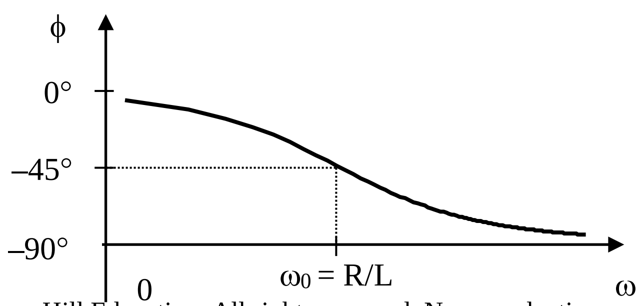
$\mathbf{H}(\omega) = \mathbf{I}_o/\mathbf{I}_i = [Rj\omega L/(R+j\omega L)]/(j\omega L) = 1/(1+j\omega L/R)$

If we let $\omega_o = R/L$ we get $\mathbf{H}(\omega) = 1/(1+j\omega/\omega_o)$.

$$H = |\mathbf{H}(\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_o}\right)^2}} \text{ and } \theta = \angle\mathbf{H}(\omega) = -\tan^{-1}(\omega/\omega_o)$$

This is a highpass filter. The frequency response is the same as that for P.P.14.1 except that $\omega_o = R/L$. Thus, the sketches of H and ϕ are shown below.





Solution 14.2

Using Fig. 14.69, design a problem to help other students to better understand how to determine transfer functions.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Obtain the transfer function **V**$_o$/**V**$_i$ of the circuit in Fig. 14.66.

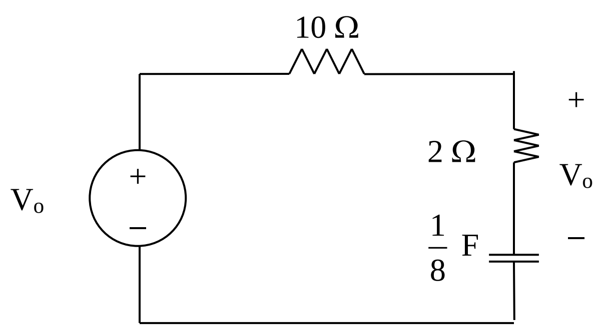


Figure 14.66 For Prob. 14.2.

Solution

$$\boldsymbol{H(s) = \frac{V_o}{V_i} = \frac{2 + \frac{1}{s/8}}{10 + 20 + \frac{1}{s/8}} = \frac{2 + 8/s}{12 + 8/s} = \frac{1}{6}\frac{s+4}{s+0.6667}}$$

Solution 14.3

For the circuit shown in Fig. 14.67, find $\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{I}_i(s)$.

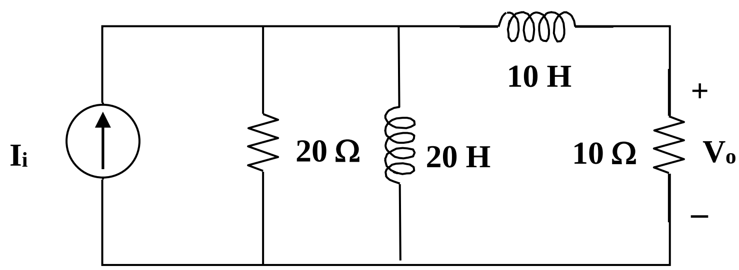
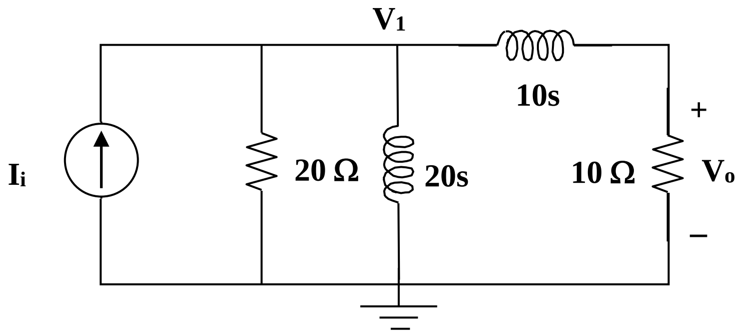


Figure 14.67
For Prob. 14.3.

Solution

Step 1. We can transform this circuit into the s-domain where 10 H becomes 10s and 20 H becomes 20s. This leads to the following circuit,



We can use nodal analysis to solve this problem. Clearly we can write the following nodal equation.

$-\mathbf{I_i} + [(\mathbf{V_1}-0)/20] + [(\mathbf{V_1}-0)/(20s)] + [(\mathbf{V_1}-0)/(10s+10)] = 0$. Finally,
$\mathbf{V_o} = [(\mathbf{V_1}-0)/(10s+10)]10$.

Step 2. $[0.05+(0.05/s)+0.1/(s+1)]\mathbf{V_1} = 0.05[(s^2+s+s+1+2s)/(s(s+1))]\mathbf{V_1}$ $= \mathbf{I_i}$ and $\mathbf{V_1} = 20[s(s+1)/(s^2+4s+1)]\mathbf{I_i}$. Finally,

$\mathbf{V_o} = \mathbf{V_1}/(s+1) = [20s/(s^2+4s+1)]\mathbf{I_i}$ or

$$\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{I}_i(s) = \mathbf{20s/(s^2+4s+1)}.$$

Solution 14.4

Find the transfer function **H(s)** = $\mathbf{V}_o/\mathbf{V}_i$ of the circuit shown in Fig. 14.71.

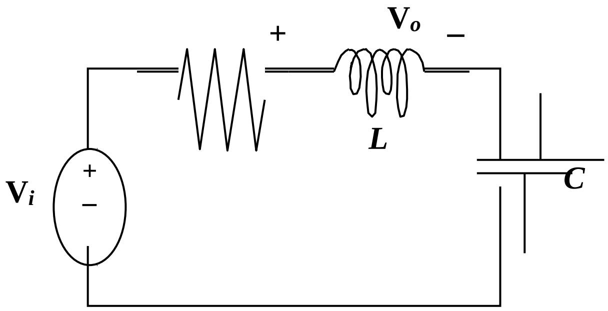
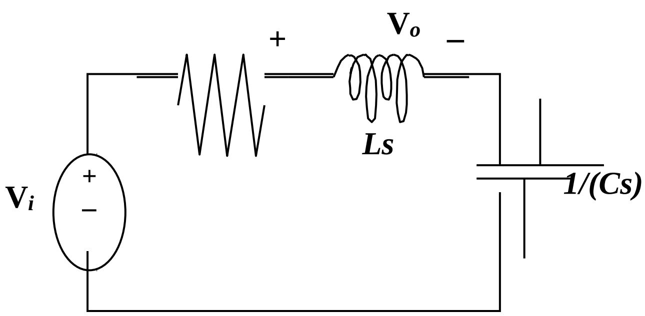


Figure 14.71
For Prob. 14.4.

Solution

Step 1. First we convert the circuit into the s-domain where the capacitor becomes 1/(Cs) and the inductor becomes Ls. Now we redraw the circuit as follows,



We can write a mesh equation, $-\mathbf{V}_i + R\mathbf{I} + Ls\mathbf{I} + [1/(Cs)]\mathbf{I} = 0$ and note that $\mathbf{V}_o = Ls\mathbf{I}$. This now leads to $\mathbf{V}_o/\mathbf{V}_i$.

Step 2. $[R+Ls+1/(Cs)]\mathbf{I} = \mathbf{V}_i$ or $\mathbf{I} = [Cs/(CLs^2+CRs+1]\mathbf{V}_i$. Thus, $\mathbf{V}_o = Ls\mathbf{I}$ or

$$\mathbf{H(s)} = \mathbf{V}_o/\mathbf{V}_i = \mathbf{LCs^2/(CLs^2+CRs+1)}.$$

Solution 14.5

For the circuit shown in Fig. 14.72, find $\mathbf{H}(s) = \mathbf{V}_o/\mathbf{I}_s$.

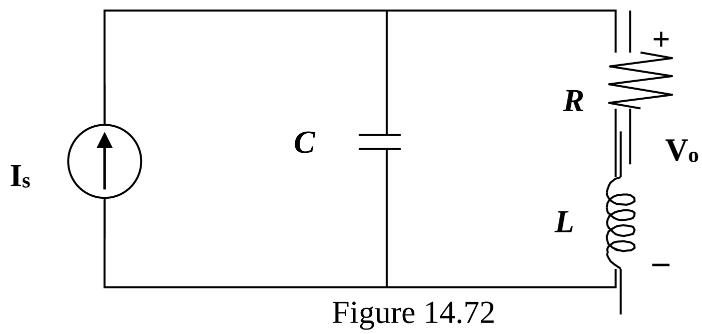
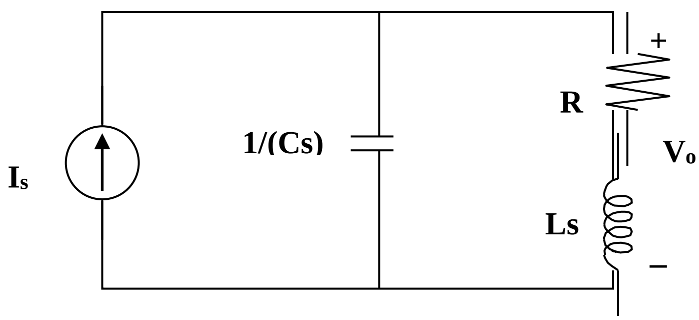


Figure 14.72
For Prob. 14.5.

Solution

Step 1. Let the capacitor be represented by 1/(Cs) and the inductor by Ls. Then convert the circuit into the s-domain.



Very simply by current division we can represent $\mathbf{V_o}$ as,
$\mathbf{I_o} = [1/(Cs)][\mathbf{I_s}/((1/(Cs))+R+Ls)]$ which leads to $\mathbf{V_o} = (Ls+R)\mathbf{I_o}$.

Step 2. $\mathbf{I_o} = \mathbf{I_s}/[1+RCs+LCs^2]$ or $\mathbf{V_o} = (Ls+R)\mathbf{I_s}/(LCs^2+RCs+1)$ or

$$\mathbf{H}(s) = \mathbf{V}_o/\mathbf{I}_s = (Ls+R)/(LCs^2+RCs+1).$$

Solution 14.6

For the circuit in Fig. 14.73, find $\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{V}_s(s)$.

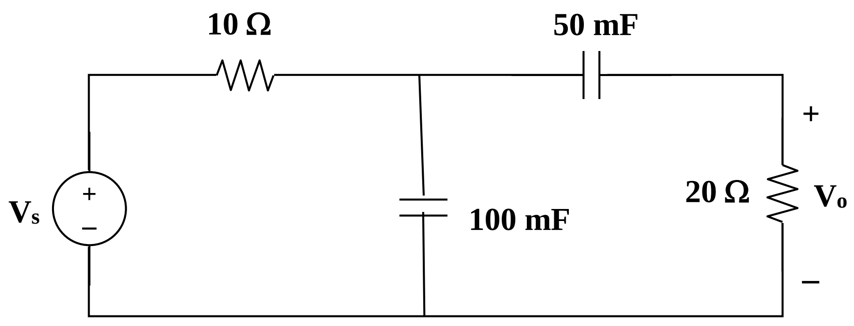
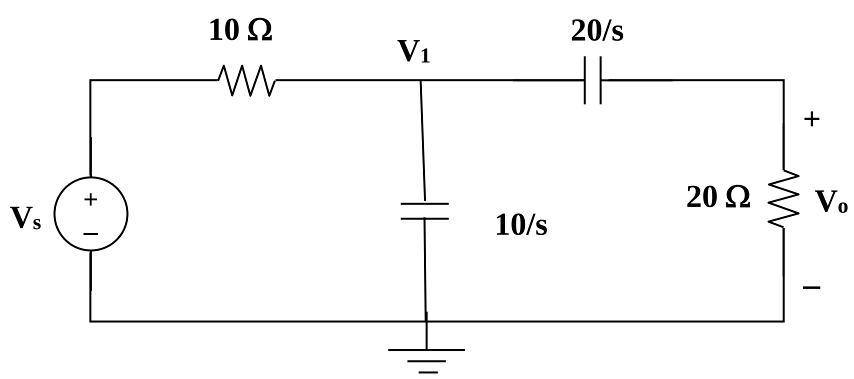


Figure 14.73
For Prob. 14.6.

Solution

Step 1. The 50 mF capacitor becomes 20/s and the 100 mF capacitor becomes 10/s. Now we convert the circuit into the s-domain and using nodal analysis we can solve for V_o.



$[(\mathbf{V}_1 - \mathbf{V}_s)/10] + [(\mathbf{V}_1 - 0)/(10/s)] + [(\mathbf{V}_1 - 0)/(20+20/s)] = 0$ and
$\mathbf{V}_o = \mathbf{V}_1/(20+20/s)$.

Step 2. $[(0.1+0.1s+0.05s/((s+1)]\mathbf{V}_1 = 0.1\mathbf{V}_s$ or$[(s+1+s^2+s+0.5s)/(s+1)]\mathbf{V}_1 = \mathbf{V}_s$ or $\mathbf{V}_1 = [(s+1)/(s^2+2.5s+1)]V_s$ which then gives us
$\mathbf{V}_o = [(s+1)/(s^2+2.5s+1)][0.05s/(s+1)]\mathbf{V}_s$.

$$\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{V}_s(s) = \mathbf{0.05s/(s^2+2.5s+1)}.$$

Solution 14.7

Calculate $|\mathbf{H}|$ if H_{dB} equals

(a) 0.1 dB
(b) –5 dB
(c) 215 dB

Solution

(a) $0.1 \text{ dB} = 20\log_{10}|\mathbf{H}|$, thus, $|\mathbf{H}| = \mathbf{1.0116}$

(b) $-5 \text{ dB} = 20\log_{10}|\mathbf{H}|$, thus, $|\mathbf{H}| = \mathbf{0.5623}$

(c) $215 \text{ dB} = 20\log_{10}|\mathbf{H}|$, thus, $|\mathbf{H}| = \mathbf{5.623x10^{10}}$

Solution 14.8

Design a problem to help other students to better calculate the magnitude in dB and phase in degrees of a variety of transfer functions at a single value of ω.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the magnitude (in dB) and the phase (in degrees) of $\mathbf{H}(\omega)$ at $\omega = 1$ if $\mathbf{H}(\omega)$ equals

(a) 0.05
(b) 125
(c) $\dfrac{10j\omega}{2+j\omega}$
(d) $\dfrac{3}{1+j\omega}+\dfrac{6}{2+j\omega}$

Solution

(a) $H = 0.05$

$H_{dB} = 20\log_{10} 0.05 = \mathbf{-26.02}$, $\qquad \boldsymbol{\varphi = 0°}$

(b) $H = 125$

$H_{dB} = 20\log_{10} 125 = \mathbf{41.94}$, $\qquad \boldsymbol{\varphi = 0°}$

(c) $H(1) = \dfrac{j10}{2+j} = 4.472\angle 63.43°$

$H_{dB} = 20\log_{10} 4.472 = \mathbf{13.01}$, $\qquad \boldsymbol{\varphi = 63.43°}$

(d) $H(1) = \dfrac{3}{1+j}+\dfrac{6}{2+j} = 3.9 - j2.7 = 4.743\angle -34.7°$

$H_{dB} = 20\log_{10} 4.743 = \mathbf{13.521}$, $\qquad \boldsymbol{\varphi = -34.7°}$

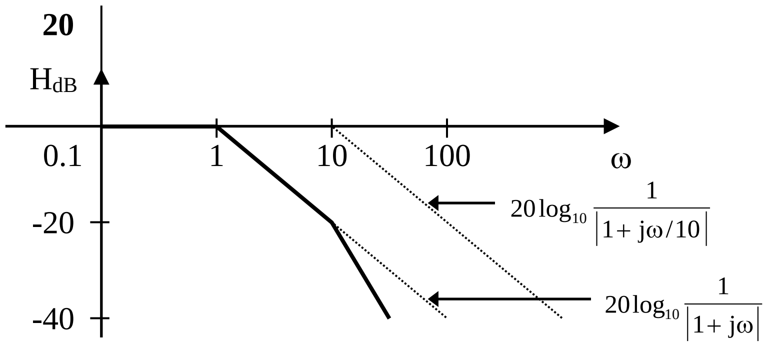
Solution 14.9

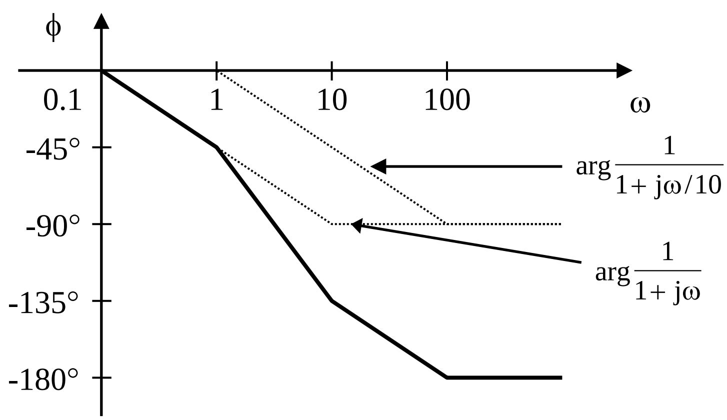
$$\mathbf{H}(\omega) = \frac{10}{10(1 + j\omega)(1 + j\omega/10)}$$

$$H_{dB} = 20\log_{10}|1| - 20\log_{10}|1 + j\omega| - 20\log_{10}|1 + j\omega/10|$$

$$\phi = -\tan^{-1}(\omega) - \tan^{-1}(\omega/10)$$

The magnitude and phase plots are shown below.





Solution 14.10

Design a problem to help other students to better understand how to determine the Bode magnitude and phase plots of a given transfer function in terms of jω.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

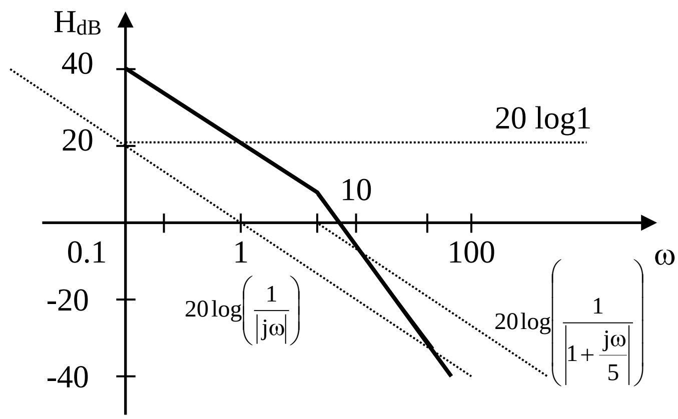
Problem

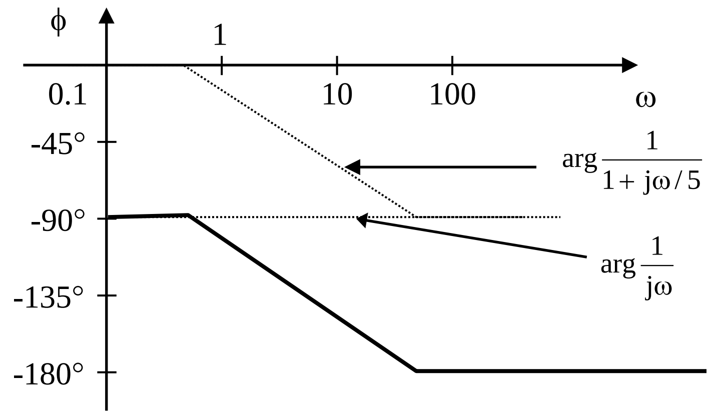
Sketch the Bode magnitude and phase plots of:

$$\mathrm{H}(j\omega)=\frac{50}{j\omega(5+j\omega)}$$

Solution

$$\mathrm{H(j\omega)}=\frac{50}{\mathrm{j\omega(5+j\omega)}}=\frac{10}{1\mathrm{j\omega}\left(1+\dfrac{\mathrm{j\omega}}{5}\right)}$$





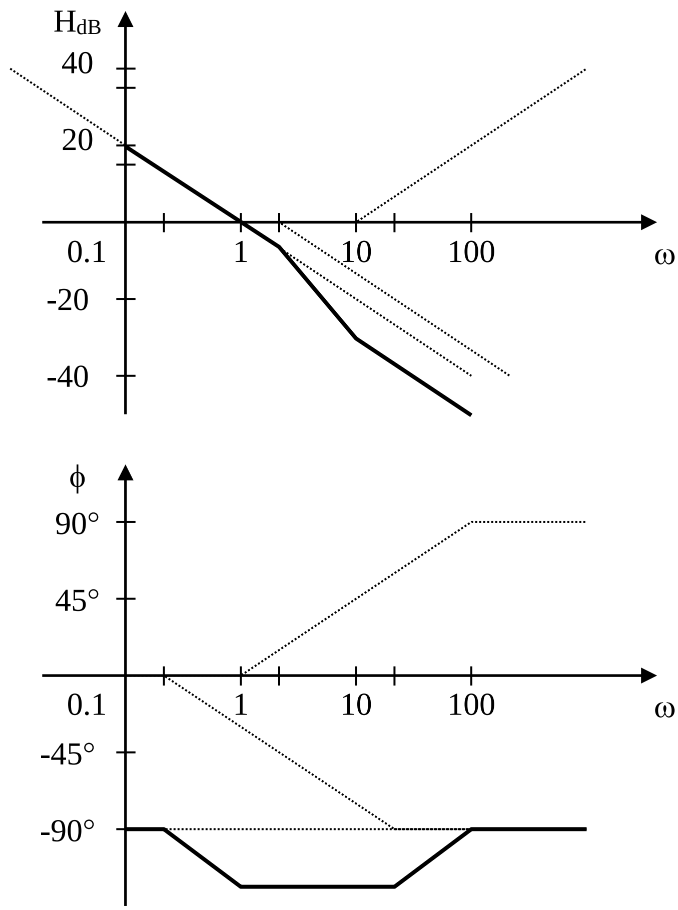
Solution 14.11

$$\mathbf{H}(\omega) = \frac{0.2x10(1 + j\omega/10)}{2[j\omega(1 + j\omega/2)]}$$

$$H_{dB} = 20\log_{10} 1 + 20\log_{10}\left|1 + j\omega/10\right| - 20\log_{10}\left|j\omega\right| - 20\log_{10}\left|1 + j\omega/2\right|$$

$$\phi = \text{-}90° + \tan^{-1}\omega/10 - \tan^{-1}\omega/2$$

The magnitude and phase plots are shown below.

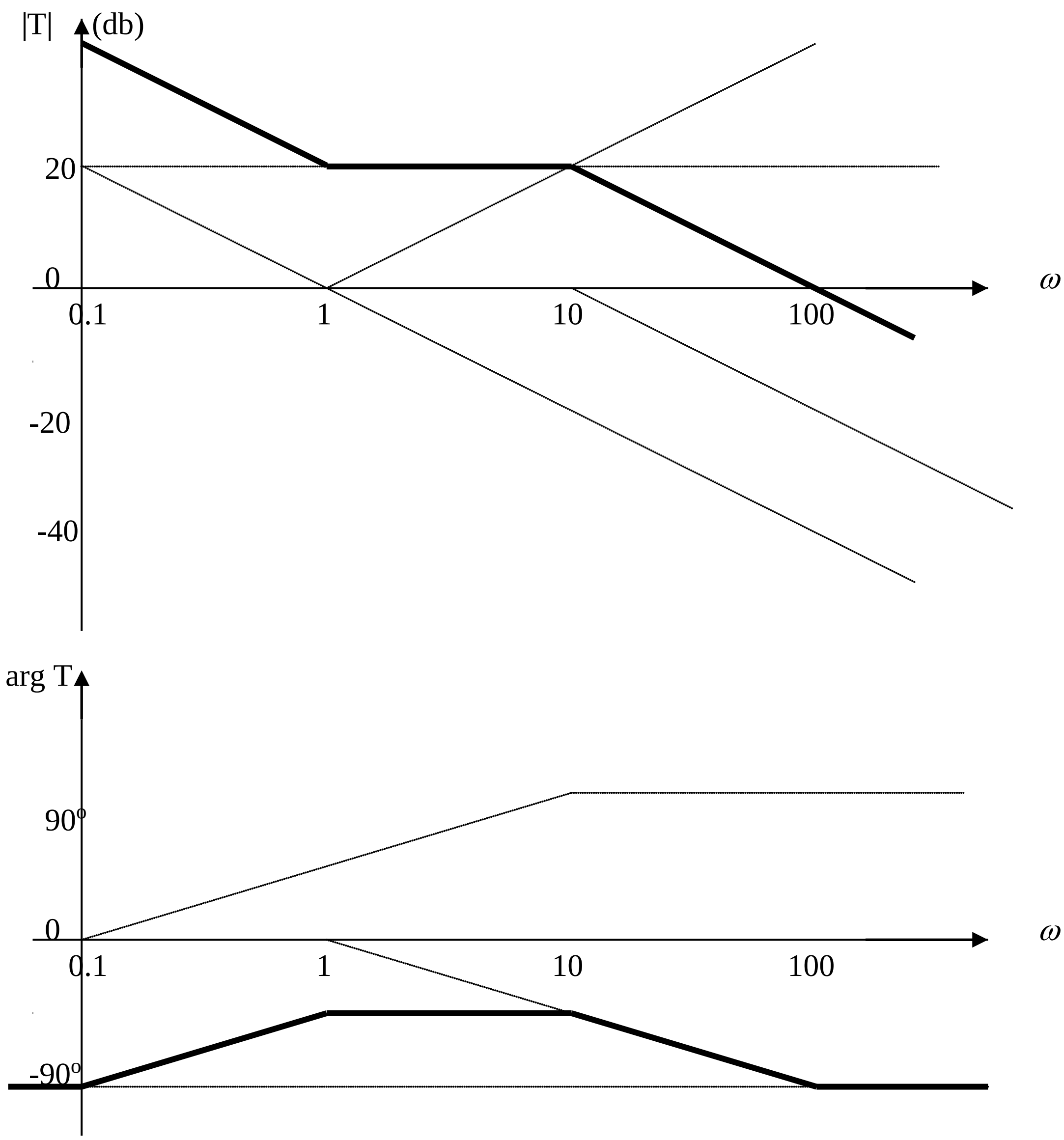


Solution 14.12

$$T(\omega) = \frac{10(1+j\omega)}{j\omega(1+j\omega/10)}$$

To sketch this we need $20\log_{10}|T(\omega)| = 20\log_{10}|10| + 20\log_{10}|1+j\omega| - 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega/10|$ and the phase is equal to $\tan^{-1}(\omega) - 90° - \tan^{-1}(\omega/10)$.

The plots are shown below.



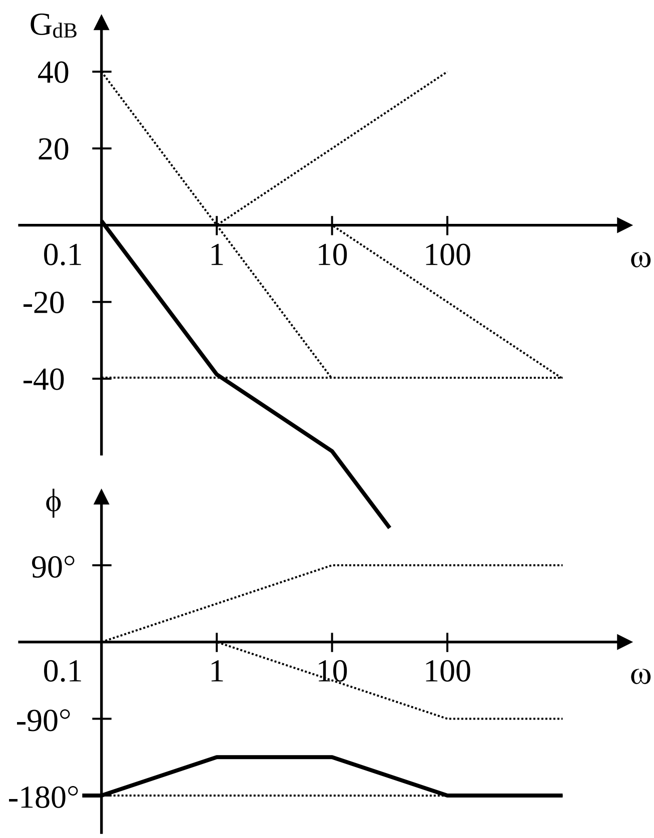
Solution 14.13

$$\mathbf{G}(\omega) = \frac{0.1(1+j\omega)}{(j\omega)^2(10+j\omega)} = \frac{(1/100)(1+j\omega)}{(j\omega)^2(1+j\omega/10)}$$

$$G_{dB} = -40 + 20\log_{10}|1+j\omega| - 40\log_{10}|j\omega| - 20\log_{10}|1+j\omega/10|$$

$$\phi = -180° + \tan^{-1}\omega - \tan^{-1}\omega/10$$

The magnitude and phase plots are shown below.



Solution 14.14

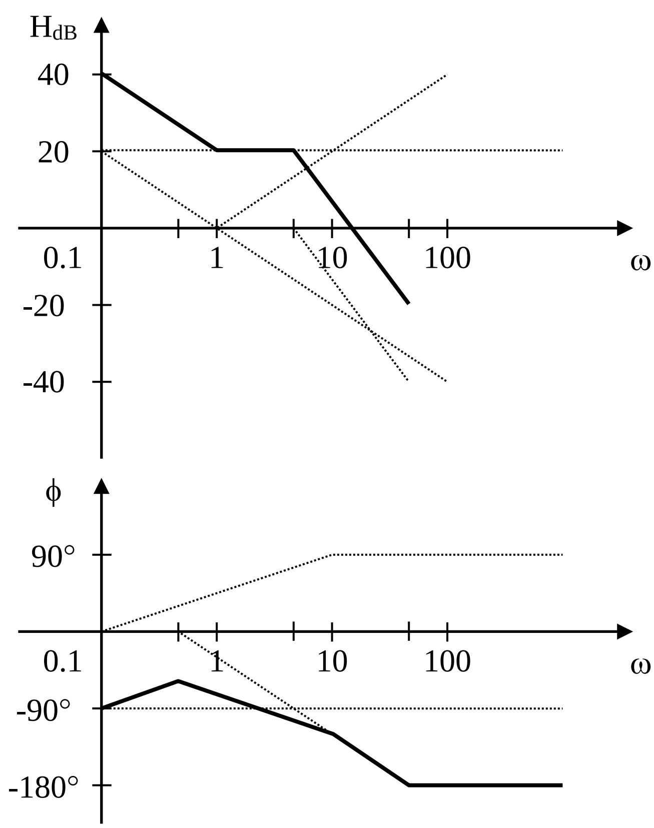
$$\mathbf{H}(\omega) = \frac{250}{25}\frac{1+j\omega}{j\omega\left(1+\frac{j\omega 10}{25}+\left(\frac{j\omega}{5}\right)^2\right)}$$

$$H_{dB} = 20\log_{10}10 + 20\log_{10}\left|1+j\omega\right| - 20\log_{10}\left|j\omega\right|$$

$$-20\log_{10}\left|1+j\omega 2/5+(j\omega/5)^2\right|$$

$$\phi = \text{-}90° + \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega 10/25}{1-\omega^2/5}\right)$$

The magnitude and phase plots are shown below.

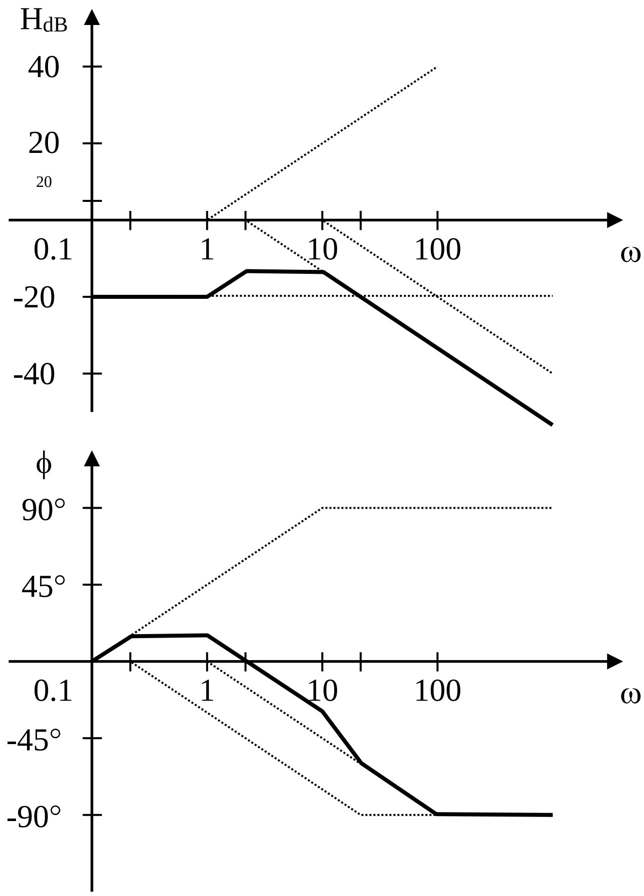


Solution 14.15

$$\mathbf{H}(\omega)=\frac{2(1+j\omega)}{(2+j\omega)(10+j\omega)}=\frac{0.1(1+j\omega)}{(1+j\omega/2)(1+j\omega/10)}$$

$$H_{dB}=20\log_{10}0.1+20\log_{10}\left|1+j\omega\right|-20\log_{10}\left|1+j\omega/2\right|-20\log_{10}\left|1+j\omega/10\right|$$
$$\phi=\tan^{-1}\omega-\tan^{-1}\omega/2-\tan^{-1}\omega/10$$

The magnitude and phase plots are shown below.

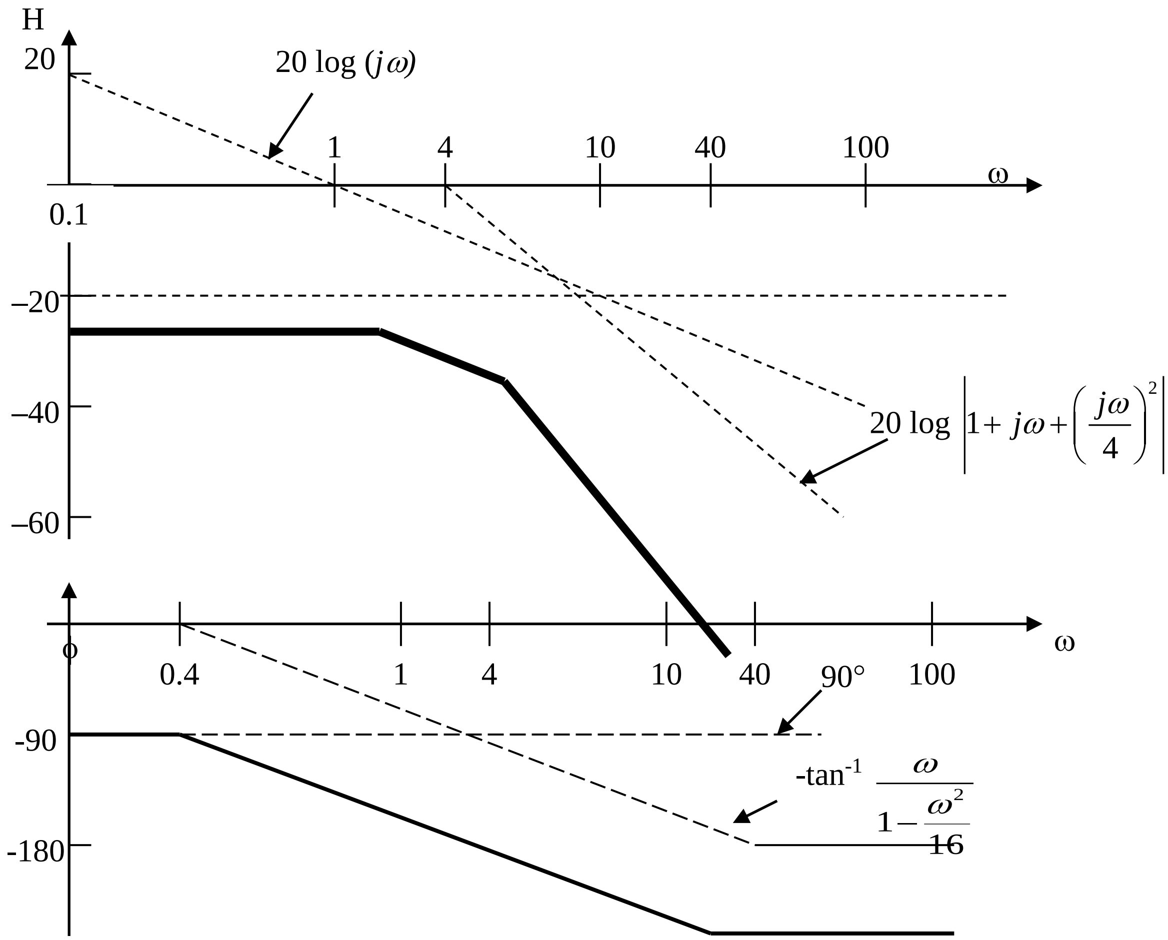


Solution 14.16

$$H(\omega) = \frac{\frac{1.6}{16}}{j\omega\left[1 + j\omega + \left(\frac{j\omega}{4}\right)^2\right]} = \frac{0.1}{j\omega\left[1 + j\omega + \left(\frac{j\omega}{4}\right)^2\right]}$$

$H_{db} = 20\log_{10}|0.1| - 20\text{olg}_{10}|j\omega| - 20\log_{10}|1+j\omega+(j\omega/4)^2|$

The magnitude and phase plots are shown below.



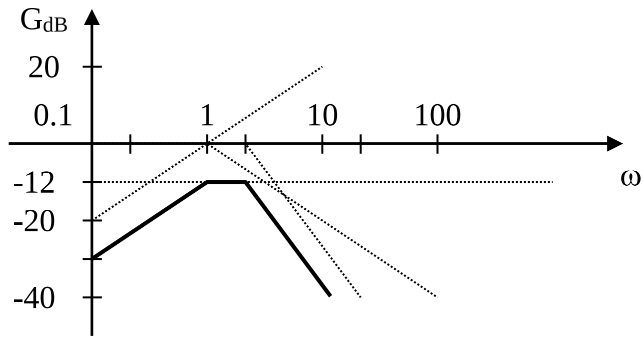
Solution 14.17

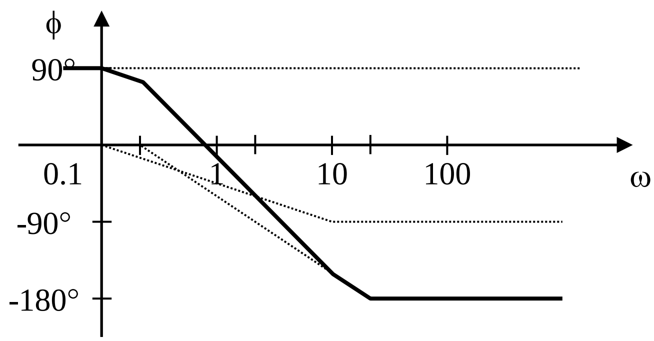
$$\mathbf{G}(\omega) = \frac{(1/4)\,j\omega}{(1+j\omega)(1+j\omega/2)^2}$$

$$G_{dB} = -20\log_{10}4 + 20\log_{10}\left|\,j\omega\right| - 20\log_{10}\left|1+j\omega\right| - 40\log_{10}\left|1+j\omega/2\right|$$

$$\phi = -90° - \tan^{-1}\omega - 2\tan^{-1}\omega/2$$

The magnitude and phase plots are shown below.

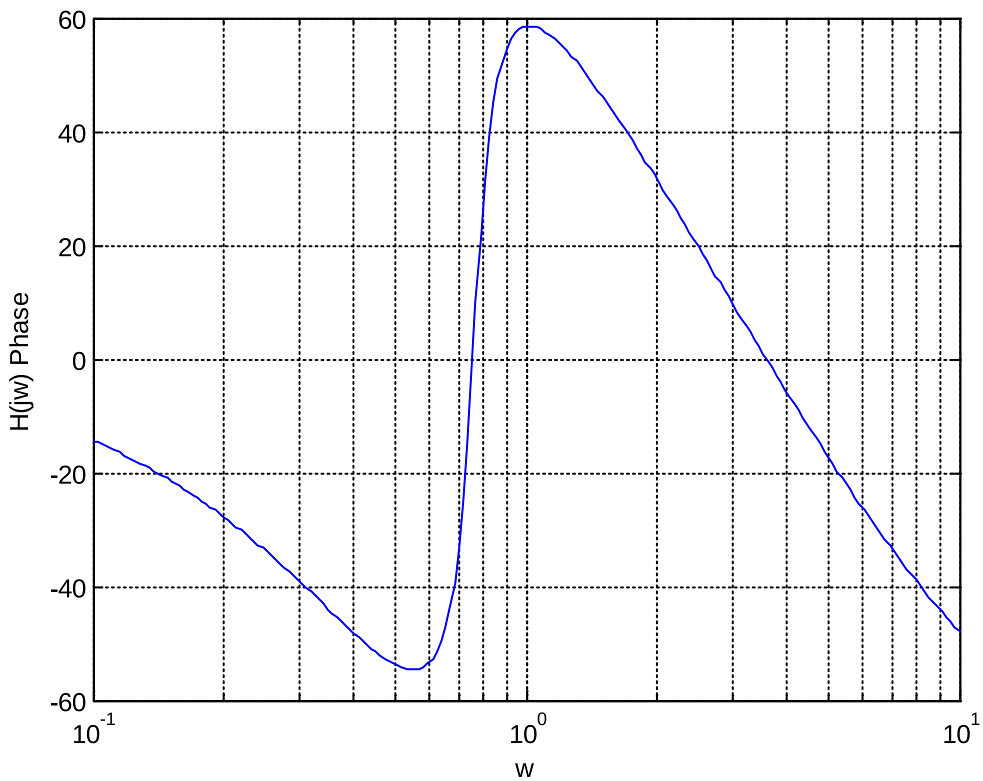




Solution 14.18

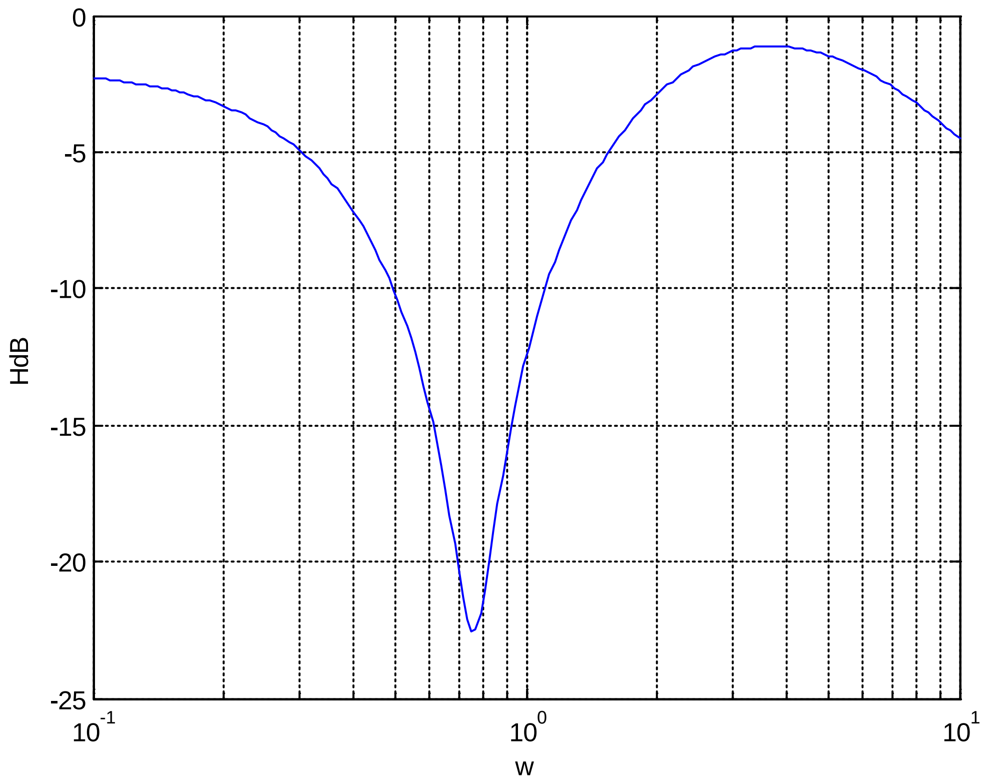
The MATLAB code is shown below.

```
>> w=logspace(-1,1,200);
>> s=i*w;
>> h=(7*s.^2+s+4)./(s.^3+8*s.^2+14*s+5);
>> Phase=unwrap(angle(h))*57.23;
>> semilogx(w,Phase)
>> grid on
```



Now for the magnitude, we need to add the following to the above,

```
>> H=abs(h);
>> HdB=20*log10(H);
>> semilogx(w,HdB);
>> grid on
```



0
-5
-10
-15
-20
-25
HdB
10^-1
10^0
10^1
w

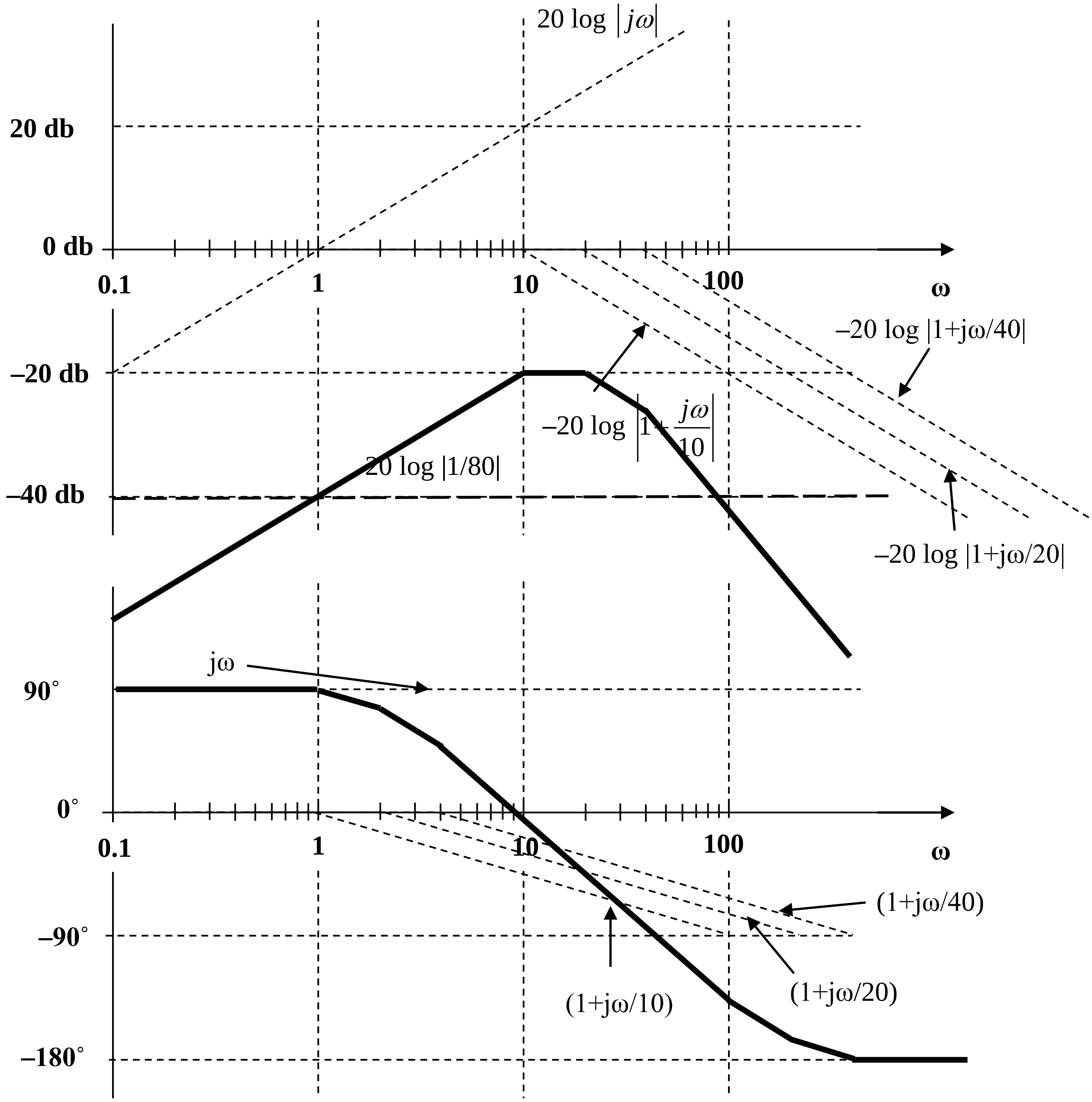
Solution 14.19

$H(\omega) = 80j\omega/[(10+j\omega)(20+j\omega)(40+j\omega)]$

$= [80/(10x20x40)](j\omega)/[(1+j\omega/10)(1+j\omega/20)(1+j\omega/40)]$

$H_{db} = 20\log_{10}|0.01| + 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega/10| - 20\log_{10}|1+j\omega/20| - 20\log_{10}|1+j\omega/40|$

The magnitude and phase plots are shown below.



Solution 14.20

Design a more complex problem than given in Prob. 14.10, to help other students to better understand how to determine the Bode magnitude and phase plots of a given transfer function in terms of jω. Include at least a second order repeated root.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Sketch the magnitude phase Bode plot for the transfer function

$$H(\omega) = \frac{25j\omega}{(j\omega + 1)(j\omega + 5)^2(j\omega + 10)}$$

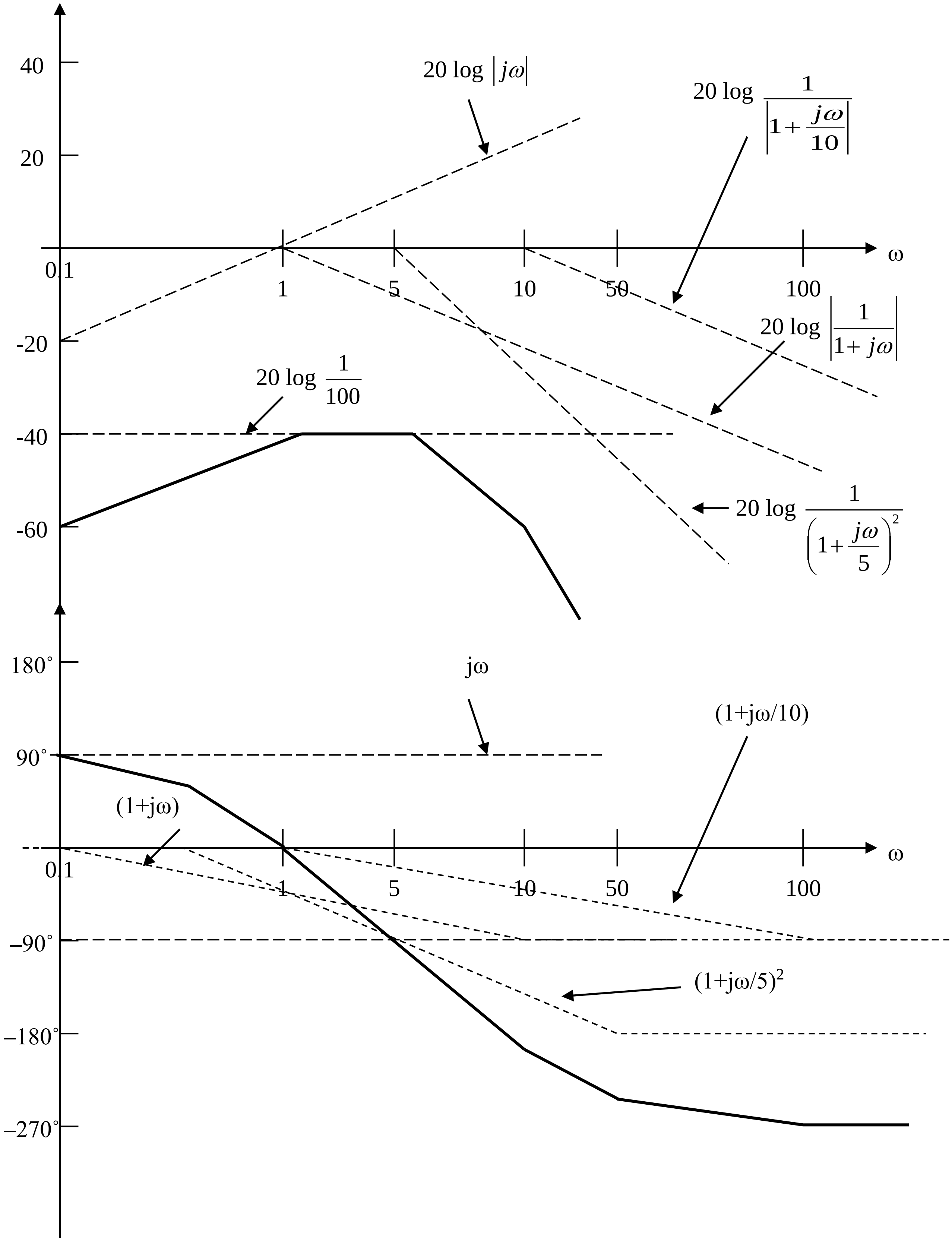
Solution

$$H(\omega) = \frac{\left(\frac{1}{100}\right)j\omega}{(1 + j\omega)\left(1 + \frac{j\omega}{5}\right)^2\left(1 + \frac{j\omega}{10}\right)}$$

20log(1/100) = -40

For the plots, see the next page.

The magnitude and phase plots are shown below.



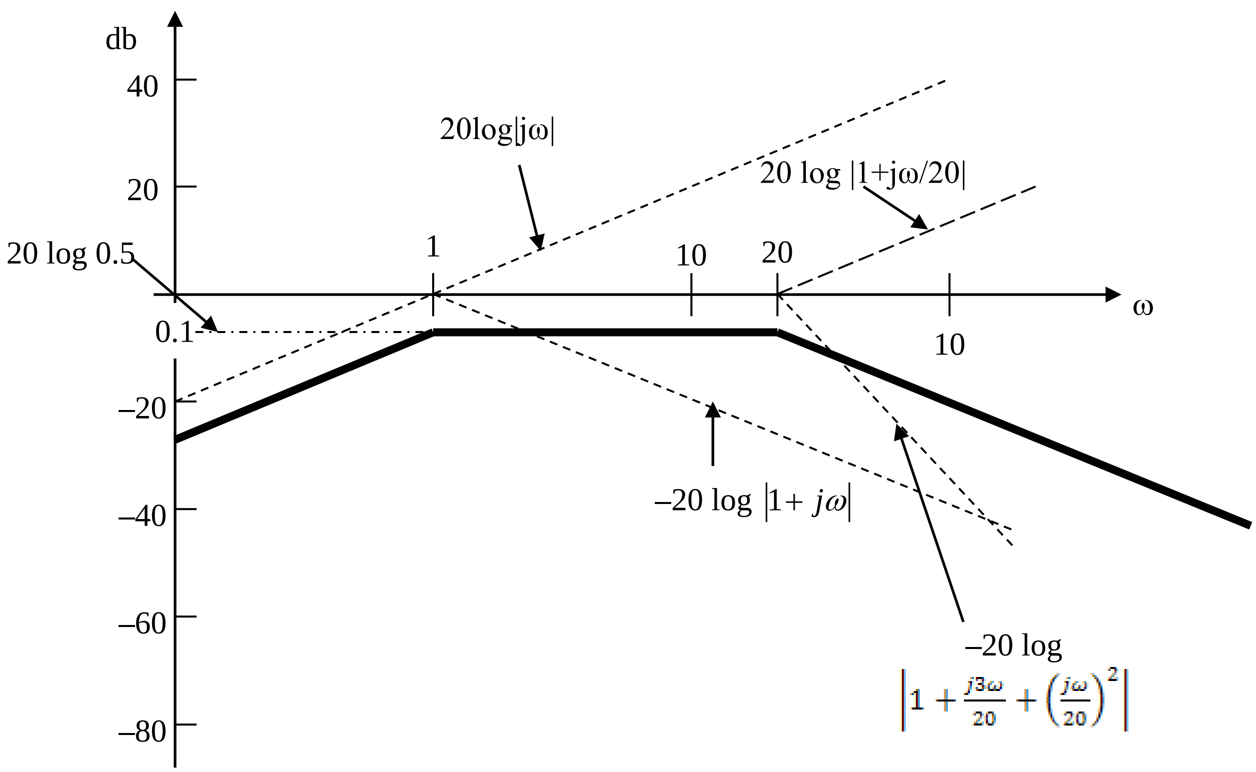
Solution 14.21

H(ω) = 10(jω)(20+jω)/[(1+jω)(400+60jω–$ω^2$)]

= [10x20/400](jω)(1+jω/20)/[(1+jω)(1+(3jω/20)+$(jω/20)^2$)]

$$H_{dB} = 20\log(0.5) + 20\log|j\omega| + 20\log\left|1+\frac{j\omega}{20}\right| - 20\log|1+j\omega| - 20\log\left|1+\frac{j3\omega}{20}+\left(\frac{j\omega}{20}\right)^2\right|$$

The magnitude plot is as sketched below. $20\log_{10}|0.5| = –6$ db



Solution 14.22

Find the transfer function $\mathbf{H}(\omega)$ with the Bode magnitude plot shown in Fig. 14.74.

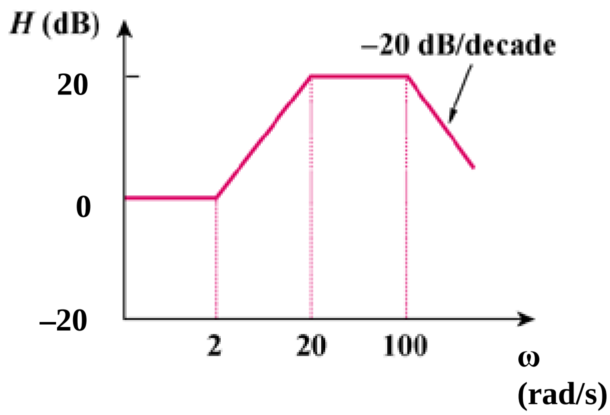


Figure 14.74
For Prob. 14.22.

Solution

$$0 = 20\log_{10} k \quad \longrightarrow \quad k = 1$$

A zero of slope $+20$ dB/dec at $\omega = 2 \longrightarrow 1 + j\omega/2$

A pole of slope -20 dB/dec at $\omega = 20 \longrightarrow \dfrac{1}{1 + j\omega/20}$

A pole of slope -20 dB/dec at $\omega = 100 \longrightarrow \dfrac{1}{1 + j\omega/100}$

Hence,

$$\mathbf{H}(\omega) = \frac{1(1 + j\omega/2)}{(1 + j\omega/20)(1 + j\omega/100)} = \frac{1{,}000(2 + j\omega)}{(20 + j\omega)(100 + j\omega)}$$

Solution 14.23

The Bode magnitude plot of **H**(ω) is shown in Fig. 14.75. Find **H**(ω).

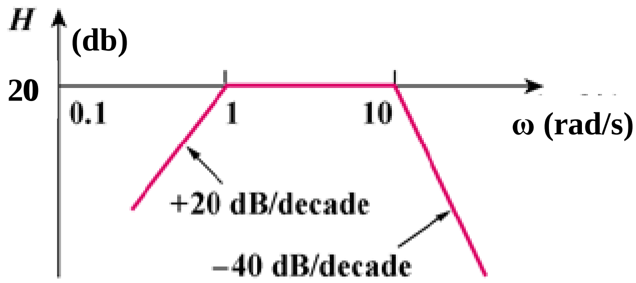


Figure 14.75
For Prob. 14.23.

Solution

The initial slope indicates we have jω in the numerator. Our approach to plotting requires the plot of jω to cross 0db at ω = 1 rad/s. Since it crosses at 20db, that indicates that the overall gain is 20db or,

$20 = 20\log_{10}|\text{gain}|$ the gain has to be 10.

A zero of slope $+20\ \text{dB/dec}$ at the origin $\longrightarrow$ $j\omega$

A pole of slope $-20\ \text{dB/dec}$ at $\omega = 1$ $\longrightarrow$ $\dfrac{1}{1 + j\omega/1}$

A pole of slope $-40\ \text{dB/dec}$ at $\omega = 10$ $\longrightarrow$ $\dfrac{1}{(1 + j\omega/10)^2}$

Hence,

$$\mathbf{H}(\omega) = \frac{10\,j\omega}{(1 + j\omega)(1 + j\omega/10)^2}$$

$$\mathbf{H}(\omega) = \frac{1{,}000\,j\omega}{(1 + j\omega)(10 + j\omega)^2}$$

(It should be noted that this function could also have a minus sign out in front and still be correct. The magnitude plot does not contain this information. It can only be obtained from the phase plot.)

Solution 14.24

The magnitude plot in Fig. 14.76 represents the transfer function of a preamplifier. Find H(s).

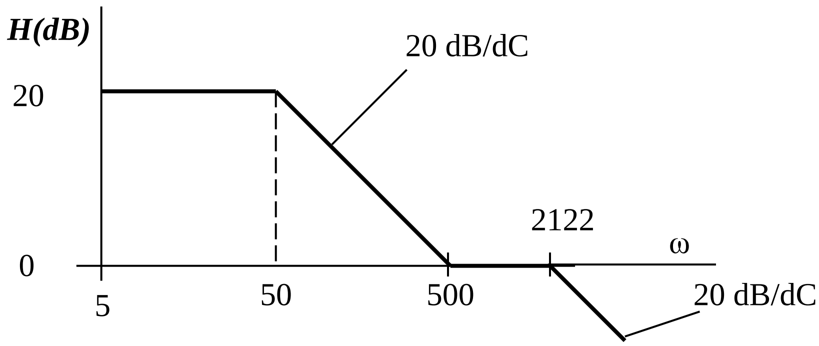


Figure 14.76
For Prob. 14.24.

Solution

$20 = 20\log_{10}|\text{gain}|$ or gain = 10.

There is a pole at ω=50 giving 1/(1+jω/50)
There is a zero at ω=500 giving (1 + jω/500).
There is another pole at ω=2122 giving 1/(1 + jω/2122).
Thus,

$$H(j\omega) = 10(1+j\omega/500)/[(1+j\omega/50)(1+j\omega/2122)]$$

$$= [10(50\text{x}2122)/500](j\omega+500)/[(j\omega+50)(j\omega+2122)]$$

or

$$H(s) = \mathbf{2{,}122(s+500)/[(s+50)(s+2122)]}.$$

Solution 14.25

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(40\times10^{-3})(1\times10^{-6})}} = 5\text{ krad/s}$$

$$\mathbf{Z}(\omega_0) = R = \mathbf{2\ k\Omega}$$

$$\mathbf{Z}(\omega_0/4) = R + j\left(\frac{\omega_0}{4}L - \frac{4}{\omega_0 C}\right)$$

$$\mathbf{Z}(\omega_0/4) = 2000 + j\left(\frac{5\times10^3}{4}\cdot 40\times10^{-3} - \frac{4}{(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(\omega_0/4) = 2000 + j(50 - 4000/5)$$

$$\mathbf{Z}(\omega_0/4) = \mathbf{2 - j0.75\ k\Omega}$$

$$\mathbf{Z}(\omega_0/2) = R + j\left(\frac{\omega_0}{2}L - \frac{2}{\omega_0 C}\right)$$

$$\mathbf{Z}(\omega_0/2) = 2000 + j\left(\frac{(5\times10^3)}{2}(40\times10^{-3}) - \frac{2}{(5\times10^3)(1\times10^{-6})}\right)$$

$\mathbf{Z}(\omega_0/2) = 200+j(100-2000/5)$

$$\mathbf{Z}(\omega_0/2) = \mathbf{2 - j0.3\ k\Omega}$$

$$\mathbf{Z}(2\omega_0) = R + j\left(2\omega_0 L - \frac{1}{2\omega_0 C}\right)$$

$$\mathbf{Z}(2\omega_0) = 2000 + j\left((2)(5\times10^3)(40\times10^{-3}) - \frac{1}{(2)(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(2\omega_0) = \mathbf{2 + j0.3\ k\Omega}$$

$$\mathbf{Z}(4\omega_0) = R + j\left(4\omega_0 L - \frac{1}{4\omega_0 C}\right)$$

$$\mathbf{Z}(4\omega_0) = 2000 + j\left((4)(5\times10^3)(40\times10^{-3}) - \frac{1}{(4)(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(4\omega_0) = \mathbf{2 + j0.75\ k\Omega}$$

Solution 14.26

Design a problem to help other students to better understand ω_o, Q, and B at resonance in series *RLC* circuits.

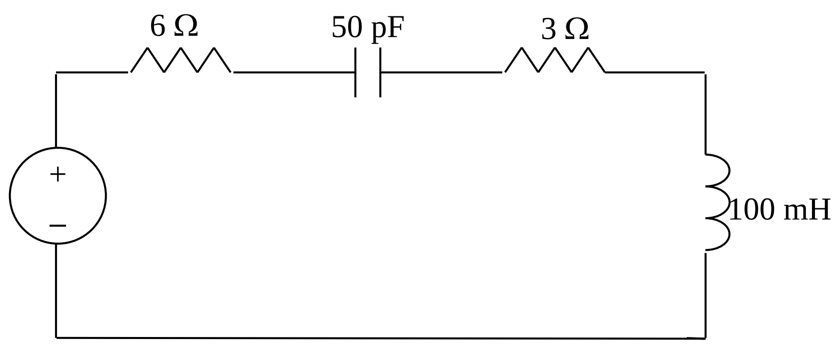
Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A coil with resistance 3 Ω and inductance 100 mH is connected in series with a capacitor of 50 pF, a resistor of 6 Ω, and a signal generator that gives 110V-rms at all frequencies. Calculate ω_o, Q, and B at resonance of the resultant series RLC circuit.

Solution

Consider the circuit as shown below. This is a series RLC resonant circuit.



R = 6 + 3 = 9 Ω

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-3}x50x10^{-12}}} = \underline{447.21\text{ krad/s}}$$

$$Q = \frac{\omega_o L}{R} = \frac{447.21x10^3 x100x10^3}{9} = \underline{4969}$$

$$B = \frac{\omega_o}{Q} = \frac{447.21x10^3}{4969} = \underline{90\text{ rad/s}}$$

Solution 14.27

$$\omega_o = \frac{1}{\sqrt{LC}} = 40 \quad \longrightarrow \quad LC = \frac{1}{40^2}$$

$$B = \frac{R}{L} = 10 \quad \longrightarrow \quad R = 10L$$

If we select R =**1 Ω**, then L = R/10 = **100 mH** and

$$C = \frac{1}{40^2 L} = \frac{1}{40^2 x 0.1} = \underline{6.25 \text{ mF}}$$

Solution 14.28

Design a series *RLC* circuit with $\boldsymbol{B}$ = 20 rad/s and ω_0 = 1,000 rad/s. Find the circuit's Q. Let R = 10 Ω.

Solution

$$R = 10\,\Omega.$$

$$L = \frac{R}{B} = \frac{10}{20} = 0.5\text{ H}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(1000)^2(0.5)} = 2\ \mu\text{F}$$

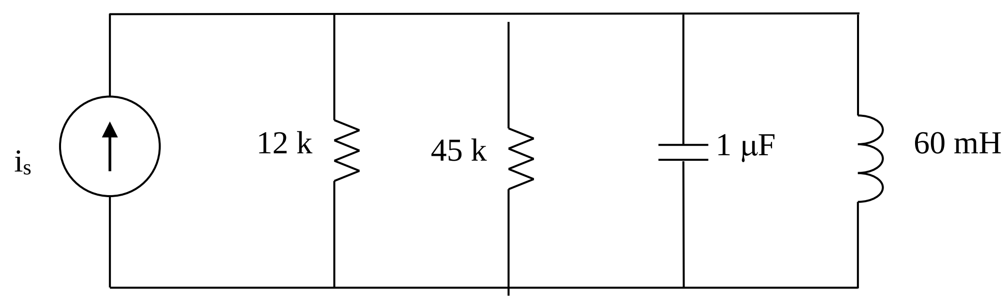
$$Q = \frac{\omega_0}{B} = \frac{1000}{20} = 50$$

Therefore, if $R = 10\,\Omega$ then

$L = \mathbf{500\ mH}$, $C = \mathbf{2\ \mu F}$, $Q = \mathbf{50}$

Solution 14.29

We convert the voltage source to a current source as shown below.



$$i_s = \frac{20}{12}\cos\omega t, \quad R = 12//45 = 12x45/57 = 9.4737 \text{ k}\Omega$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{60x10^{-3}x1x10^{-6}}} = \underline{4.082 \text{ krad/s}} = \mathbf{4.082\ krad/s}$$

$$B = \frac{1}{RC} = \frac{1}{9.4737x10^{3}x10^{-6}} = \underline{105.55 \text{ rad/s}} = \mathbf{105.55\ rad/s}$$

$$Q = \frac{\omega_o}{B} = \frac{4082}{105.55} = \underline{38.674} = \mathbf{38.67}$$

4.082 krads/s, 105.55 rad/s, 38.67

Solution 14.30

(a) $f_o = 15{,}000$ Hz leads to $\omega_o = 2\pi f_o = 94.25$ krad/s $= 1/(LC)^{0.5}$ or

$LC = 1/8.883\text{x}10^9$ or $C = 1/(8.883\text{x}10^9\text{x}10^{-2}) = 11.257\text{x}10^{-9}$ F = **11.257 pF**.

(b) since the capacitive reactance cancels out the inductive reactance at resonance, the current through the series circuit is given by

$I = 120/20 =$ **6 A**.

(c) $Q = \omega_o L/R = 94.25\text{x}10^3(0.01)/20 =$ **47.12**.

Solution 14.31

Design a parallel resonant *RLC* circuit with ω_0 = 100 krad/s and a bandwidth of 10 krad/s.
Additionally what is the value of Q?

Solution

Step 1. We note that,

$$\omega_o = \frac{1}{\sqrt{LC}}$$
$$Q = R/(\omega_o L) = \omega_o RC$$
$$B = \omega_o/Q.$$

Since this is a design problem, we need to find out where to start. Let us pick a value of L = **10 mH**. Now all we need to do is to solve for Q, R, and C and make sure we meet the design criterion.

Step 2. Q = 100/10 = **10**. Next

$$R/L = \omega_0 Q = 10^5 x10 = 10^6 \text{ and } RC = Q/\omega_0 = 10/10^5 = 10^{-4}$$

Since L = 10 mH we get $R = \omega_0 QL = 10^6 x0.01 =$ **10 kΩ**. Next we get,

$$C = 10^{-4}/R = 10^{-4}/10^4 = 10^{-8} = \mathbf{10\ nF}.$$

Solution 14.32

Design a problem to help other students to better understand the quality factor, the resonant frequency, and bandwidth of a parallel *RLC* circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A parallel RLC circuit has the following values:

R = 60 Ω,   L = 1 mH, and C = 50 μF

Find the quality factor, the resonant frequency, and the bandwidth of the RLC circuit.

Solution

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}x50x10^{-6}}} = \underline{4.472\ \text{krad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{60x50x10^{-6}} = \underline{333.33\ \text{rad/s}}$$

$$Q = \frac{\omega_o}{B} = \frac{4472}{333.33} = \underline{13.42}$$

Solution 14.33

A parallel resonant circuit has a bandwidth of 40 krad/s and the half power frequencies are ω_1 = 4.98 Mrad/s and ω_2 = 5.02 Mrad/s, calculate the quality factor and resonant frequency.

Solution

Since $\omega_1 = \omega_o - B/2$ then ω_o = (4.98+0.04/2) M = **5 Mrad/s**. Since $B = \omega_o/Q$ then Q = 5 M/0.04 M = **125**.

Solution 14.34

A parallel RLC circuit has an R = 100 kΩ, L = 100 mH, and a C = 10 μF, determine the value of Q, the resonant frequency, and the bandwidth. If R = 200 kΩ, how does that effect the values of Q, resonant frequency, and the bandwidth?

Solution

Since,

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$Q = R/(\omega_o L) = \omega_o RC$

$B = \omega_o/Q.$

$\omega_o = 1/\sqrt{0.1x10^{-5}}$ = **1,000 rad/s** and $Q = \omega_o RC = 10^3 x 10^5 x 10^{-5}$ = **1,000**. Finally,

$B = \omega_o/Q = 1{,}000/1{,}000$ = **1 rad/s**.

When R = 200 kΩ the value of **ω_o does not change** since it is only dependent on L and C.

$Q = \omega_o RC = 1{,}000x2x10^5 x10^{-5} = 2{,}000$ and $B = 1{,}000/2{,}000$ = **0.5**.

Solution 15.35

A parallel RLC circuit has an R = 10 kΩ, an L = 100 mH, and a resonant frequency of 200 krad/s, calculate the value of C, the value of the quality factor, and the bandwidth.

Solution

Since,

$$\omega_o = \frac{1}{\sqrt{LC}} \text{ or } LC = 1/(\omega_o)^2$$
$$Q = R/(\omega_o L) = \omega_o RC$$
$$B = \omega_o/Q$$

we get $C = 1/(4x10^{10}x0.1) = 0.25x10^{-9}$ = **0.25 nF** and $Q = 2x10^5x10^4x0.25x10^{-9}$ = **0.5**.

Finally, B = 200k/0.5 = **400 krad/s**. Note, since the bandwidth is equal to 400 krad/s, the lower frequency must be equal to 0 Hz! Clearly the bandwidth goes from DC to 400 krad/s.

Solution 14.36

It is expected that a parallel *RLC* resonant circuit has a midband admittance of 25×10^{-3} S, quality factor of 120, and a resonant frequency of 200 krad/s. Calculate the values of *R*, *L*, and *C*. Find the bandwidth and the half-power frequencies.

Solution

At resonance,

$$\mathbf{Y} = \frac{1}{\mathrm{R}} \quad \longrightarrow \quad \mathrm{R} = \frac{1}{\mathrm{Y}} = \frac{1}{25 \times 10^{-3}} = \mathbf{40\ \Omega}$$

$$Q = \omega_0 RC \quad \longrightarrow \quad C = \frac{Q}{\omega_0 R} = \frac{120}{(200 \times 10^3)(40)} = \mathbf{15\ \mu F}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \longrightarrow \quad L = \frac{1}{\omega_0^2 C} = \frac{1}{(4 \times 10^{10})(15 \times 10^{-6})} = \mathbf{1.6667\ \mu H}$$

$$B = \frac{\omega_0}{Q} = \frac{200 \times 10^3}{120} = \mathbf{1.6667\ krad/s}$$

$$\omega_1 = \omega_0 - \frac{B}{2} = 200 - 0.8333 = \mathbf{199.167\ krad/s}$$

$$\omega_2 = \omega_0 + \frac{\mathrm{B}}{2} = 200 + 0.8333 = \mathbf{200.833\ krad/s}$$

Solution 4.37

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5000\ \text{rad/s}$$

$$\mathbf{Y}(\omega_0) = \frac{1}{R} \quad \longrightarrow \quad \mathbf{Z}(\omega_0) = R = \mathbf{2\ k\Omega}$$

$$\mathbf{Y}(\omega_0/4) = \frac{1}{R} + j\left(\frac{\omega_0}{4}C - \frac{4}{\omega_0 L}\right) = 0.5 - j18.75\ \text{mS}$$

$$\mathbf{Z}(\omega_0/4) = \frac{1}{0.0005 - j0.01875} = \mathbf{(1.4212 + j53.3)\ \Omega}$$

$$\mathbf{Y}(\omega_0/2) = \frac{1}{R} + j\left(\frac{\omega_0}{2}C - \frac{2}{\omega_0 L}\right) = 0.5 - j7.5\ \text{mS}$$

$$\mathbf{Z}(\omega_0/2) = \frac{1}{0.0005 - j0.0075} = \mathbf{(8.85 + j132.74)\ \Omega}$$

$$\mathbf{Y}(2\omega_0) = \frac{1}{R} + j\left(2\omega_0 L - \frac{1}{2\omega_0 C}\right) = 0.5 + j7.5\ \text{mS}$$

$$\mathbf{Z}(2\omega_0) = \mathbf{(8.85 - j132.74)\ \Omega}$$

$$\mathbf{Y}(4\omega_0) = \frac{1}{R} + j\left(4\omega_0 L - \frac{1}{4\omega_0 C}\right) = 0.5 + j18.75\ \text{mS}$$

$$\mathbf{Z}(4\omega_0) = \mathbf{(1.4212 - j53.3)\ \Omega}$$

Solution 14.38

Find the resonant frequency of the circuit in Fig. 14.78.

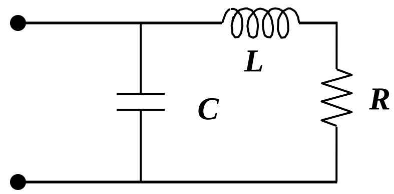


Figure 14.78
For Prob. 14.38.

Solution

$$Z = (1/(j\omega C))(R+j\omega L)/[(1/(j\omega C))+R+j\omega L] = (R+j\omega L)/(1+j\omega RC-\omega^2LC)$$

$$= (R+j\omega L)(1-\omega^2LC-j\omega RC)/[(1-\omega^2LC)^2+(\omega RC)^2]$$

$$=[R-\omega^2RLC+\omega^2RLC+j(\omega L-\omega^3L^2C-\omega R^2C)]/[(1-\omega^2LC)^2+(\omega RC)^2]$$

To find the resonant frequency all we need to do is to set the imaginary part to zero.

Thus, $(\omega L-\omega^3L^2C-\omega R^2C) = 0 = (L-\omega^2L^2C-R^2C)$ gives us $(\omega_o)^2 = (L-R^2C)/(L^2C)$ or

$$\omega_o = \sqrt{\frac{L-R^2C}{L^2C}} \text{ rad/s}$$

Solution 4.39

$$Y = \frac{1}{R + j\omega L} + j\omega C = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

At resonance, $\mathrm{Im}(\mathbf{Y}) = 0$, i.e.

$$\omega_0 C - \frac{\omega_0 L}{R^2 + \omega_0^2 L^2} = 0$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{(40\times10^{-3})(1\times10^{-6})} - \left(\frac{50}{40\times10^{-3}}\right)^2}$$

$$\omega_0 = \mathbf{4.841\ krad/s}$$

Solution 14.40

(a) $B = \omega_2 - \omega_1 = 2\pi(f_2 - f_1) = 2\pi(90-86)x10^3 = 8\pi krad/s$

$$\omega_o = \frac{1}{2}(\omega_1 + \omega_2) = 2\pi(88)x10^3 = 176\pi X10^3$$

$$B = \frac{1}{RC} \longrightarrow C = \frac{1}{BR} = \frac{1}{8\pi x10^3 x2x10^3} = \underline{19.89nF}$$

(b) $\omega_o = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega^2{}_o C} = \frac{1}{(176\pi X10^3)^2 x19.89x10^{-9}} = \mathbf{164.45\ \mu H}$

(c) $\omega_o = 176\pi = \underline{552.9krad/s}$

(d) $B = 8\pi = \underline{25.13krad/s}$

(e) $Q = \frac{\omega_o}{B} = \frac{176\pi}{8\pi} = \underline{22}$

Solution 14.41

Using Fig. 14.80, design a problem to help other students to better understand the quality factor, the resonant frequency, and bandwidth of an *RLC* circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in Example 14.9.

Problem

For the circuits in Fig. 14.80, find the resonant frequency ω_0, the quality factor Q, and the bandwidth B. Let C = 0.1 F, R_1 = 10 Ω, R_2 = 2 Ω, and L = 2 H.

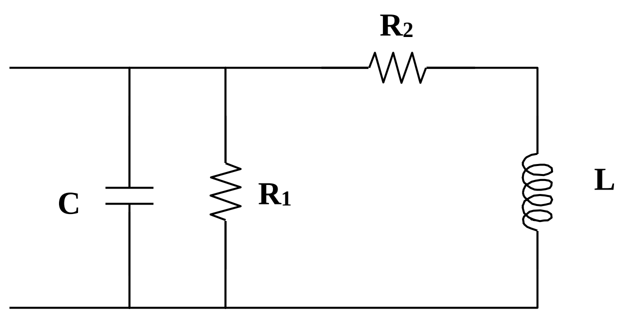


Figure 14.80
For Prob. 14.41.

Solution

To find ω_o, we need to find the input impedance or input admittance and set imaginary component equal to zero. Finding the input admittance seems to be the easiest approach.

$$\mathbf{Y} = j\omega 0.1 + 0.1 + 1/(2+j\omega 2) = j\omega 0.1 + 0.1 + [2/(4+4\omega^2)] - [j\omega 2/(4+4\omega^2)]$$

At resonance,

$$0.1\omega = 2\omega/(4+4\omega^2) \text{ or } 4\omega^2 + 4 = 20 \text{ or } \omega^2 = 4 \text{ or } \omega_o = \mathbf{2\ rad/s}$$

and,

$$\mathbf{Y} = 0.1 + 2/(4+16) = 0.1 + 0.1 = 0.2 \text{ S}$$

The bandwidth is define as the two values of ω such that $|\mathbf{Y}|$ = 1.4142(0.2) = 0.28284 S.

I do not know about you, but I sure would not want to solve this analytically. So how about using MATLAB or excel to solve for the two values of ω?

Using Excel, we get $\omega_1 = 1.414$ rad/s and $\omega_2 = 3.741$ rad/s or B = **2.327 rad/s**

We can now use the relationship between ω_o and the bandwidth.

$$Q = \omega_o/B = 2/2.327 = \mathbf{0.8595}$$

Solution 14.42

(a) This is a series RLC circuit.

$$R = 2 + 6 = 8\,\Omega, \qquad L = 1\,H, \qquad C = 0.4\,F$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.4}} = \mathbf{1.5811\,rad/s}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1.5811}{8} = \mathbf{0.1976}$$

$$B = \frac{R}{L} = \mathbf{8\,rad/s}$$

(b) This is a parallel RLC circuit.

$$3\,\mu F \text{ and } 6\,\mu F \longrightarrow \frac{(3)(6)}{3+6} = 2\,\mu F$$

$$C = 2\,\mu F, \qquad R = 2\,k\Omega, \qquad L = 20\,mH$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-6})(20\times10^{-3})}} = \mathbf{5\,krad/s}$$

$$Q = \frac{R}{\omega_0 L} = \frac{2\times10^{3}}{(5\times10^{3})(20\times10^{-3})} = \mathbf{20}$$

$$B = \frac{1}{RC} = \frac{1}{(2\times10^{3})(2\times10^{-6})} = \mathbf{250\,rad/s}$$

Solution 14.43

(a) $\quad \mathbf{Z}_{in} = (1/j\omega C) \,||\, (R + j\omega L)$

$$\mathbf{Z}_{in} = \frac{\dfrac{R + j\omega L}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{(R + j\omega L)(1 - \omega^2 LC - j\omega RC)}{(1 - \omega^2 LC)^2 + \omega^2 R^2 C^2}$$

At resonance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$, i.e.

$$0 = \omega_0 L(1 - \omega_0^2 LC) - \omega_0 R^2 C$$

$$\omega_0^2 L^2 C = L - R^2 C$$

$$\omega_0 = \sqrt{\frac{L - R^2 C}{L^2 C}} = \sqrt{\frac{\mathbf{1}}{\mathbf{LC}} - \frac{\mathbf{R^2}}{\mathbf{L^2}}}$$

(b) $\quad \mathbf{Z}_{in} = R \,||\, (j\omega L + 1/j\omega C)$

$$\mathbf{Z}_{in} = \frac{R\,(j\omega L + 1/j\omega C)}{R + j\omega L + 1/j\omega C} = \frac{R\,(1 - \omega^2 LC)}{(1 - \omega^2 LC) + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{R\,(1 - \omega^2 LC)[(1 - \omega^2 LC) - j\omega RC]}{(1 - \omega^2 LC)^2 + \omega^2 R^2 C^2}$$

At resonance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$, i.e.

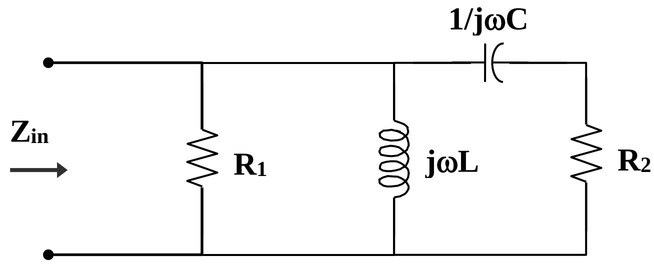
$$0 = R\,(1 - \omega^2 LC)\,\omega RC$$

$$1 - \omega^2 LC = 0$$

$$\omega_0 = \frac{\mathbf{1}}{\sqrt{\mathbf{LC}}}$$

Solution 14.44

Consider the circuit below.



(a) $\quad \mathbf{Z}_{in} = (R_1 \parallel j\omega L) \parallel (R_2 + 1/j\omega C)$

$$\mathbf{Z}_{in} = \left(\frac{R_1 j\omega L}{R_1 + j\omega L}\right) \parallel \left(R_2 + \frac{1}{j\omega C}\right)$$

$$\mathbf{Z}_{in} = \frac{\dfrac{j\omega R_1 L}{R_1 + j\omega L} \cdot \left(R_2 + \dfrac{1}{j\omega C}\right)}{R_2 + \dfrac{1}{j\omega C} + \dfrac{jR_1\omega L}{R_1 + j\omega L}}$$

$$\mathbf{Z}_{in} = \frac{j\omega R_1 L\,(1 + j\omega R_2 C)}{(R_1 + j\omega L)(1 + j\omega R_2 C) - \omega^2 LCR_1}$$

$$\mathbf{Z}_{in} = \frac{-\omega^2 R_1 R_2 LC + j\omega R_1 L}{R_1 - \omega^2 LCR_1 - \omega^2 LCR_2 + j\omega(L + R_1 R_2 C)}$$

$$\mathbf{Z}_{in} = \frac{(-\omega^2 R_1 R_2 LC + j\omega R_1 L)[R_1 - \omega^2 LCR_1 - \omega^2 LCR_2 - j\omega(L + R_1 R_2 C)]}{(R_1 - \omega^2 LCR_1 - \omega^2 LCR_2)^2 + \omega^2 (L + R_1 R_2 C)^2}$$

At resonance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$, i.e.

$$0 = \omega^3 R_1 R_2 LC\,(L + R_1 R_2 C) + \omega R_1 L\,(R_1 - \omega^2 LCR_1 - \omega^2 LCR_2)$$

$$0 = \omega^3 R_1^2 R_2^2 LC^2 + R_1^2 \omega L - \omega^3 R_1^2 L^2 C$$

$$0 = \omega^2 R_2^2 C^2 + 1 - \omega^2 LC$$

$$\omega^2 (LC - R_2^2 C^2) = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC - R_2^2 C^2}}$$

$$\omega_0 = \frac{1}{\sqrt{(0.02)(9 \times 10^{-6}) - (0.1)^2 (9 \times 10^{-6})^2}}$$

$$\omega_0 = \mathbf{2.357\ krad/s}$$

(b) At $\omega = \omega_0 = 2.357 \text{ krad/s}$,

$$j\omega L = j(2.357\times10^{3})(20\times10^{-3}) = j47.14$$

$$R_1 \parallel j\omega L = \frac{j47.14}{1+j47.14} = 0.9996 + j0.0212$$

$$R_2 + \frac{1}{j\omega C} = 0.1 + \frac{1}{j(2.357\times10^{3})(9\times10^{-6})} = 0.1 - j47.14$$

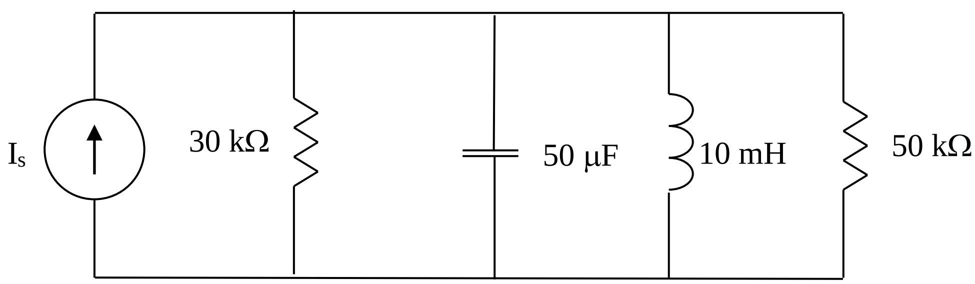
$$\mathbf{Z}_{in}(\omega_0) = (R_1 \parallel j\omega L) \parallel (R_2 + 1/j\omega C)$$

$$\mathbf{Z}_{in}(\omega_0) = \frac{(0.9996 + j0.0212)(0.1 - j47.14)}{(0.9996 + j0.0212) + (0.1 - j47.14)}$$

$$\mathbf{Z}_{in}(\omega_0) = \mathbf{1\,\Omega}$$

Solution 14.45

Convert the voltage source to a current source as shown below.



R = 30//50 = 30x50/80 = 18.75 kΩ

This is a parallel resonant circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10x10^{-3}x50x10^{-6}}} = \underline{447.21 \text{ rad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{18.75x10^{3}x50x10^{-6}} = \underline{1.067 \text{ rad/s}}$$

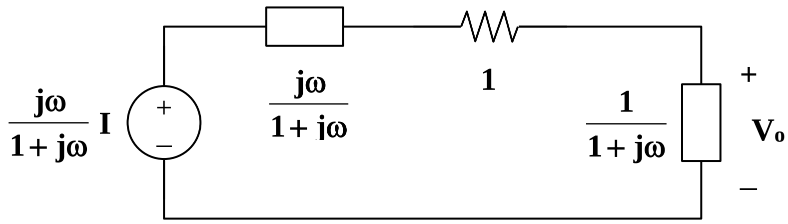
$$Q = \frac{\omega_o}{B} = \frac{447.21}{1.067} = \underline{419.13}$$

447.2 rad/s, 1.067 rad/s, 419.1

Solution 14.46

(a) $$1 \| j\omega = \frac{j\omega}{1+j\omega}, \qquad 1 \| \frac{1}{j\omega} = \frac{1/j\omega}{1+1/j\omega} = \frac{1}{1+j\omega}$$

Transform the current source gives the circuit below.



$$\mathbf{V}_o = \frac{\dfrac{1}{1+j\omega}}{1+\dfrac{1}{1+j\omega}+\dfrac{j\omega}{1+j\omega}} \cdot \frac{j\omega}{1+j\omega}\,\mathbf{I}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{I}} = \frac{\mathbf{j\omega}}{\mathbf{2\,(1+j\omega)^2}}$$

(b) $$\mathbf{H}(1) = \frac{1}{2\,(1+j)^2}$$

$$\left|\mathbf{H}(1)\right| = \frac{1}{2\,(\sqrt{2})^2} = \mathbf{0.25}$$

Solution 14.47

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L/R}$$

$H(0) = 1$ and $H(\infty) = 0$ showing that this circuit is a lowpass filter.

At the corner frequency, $\left|H(\omega_c)\right| = \dfrac{1}{\sqrt{2}}$, i.e.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\left(\dfrac{\omega_c L}{R}\right)^2}} \longrightarrow 1 = \frac{\omega_c L}{R} \qquad \text{or} \qquad \omega_c = \frac{R}{L}$$

Hence,

$$\omega_c = \frac{R}{L} = 2\pi f_c$$

$$f_c = \frac{1}{2\pi} \cdot \frac{R}{L} = \frac{1}{2\pi} \cdot \frac{10\times10^3}{2\times10^{-3}} = \mathbf{796\ kHz}$$

Solution 14.48

Find the transfer function $\mathbf{V}_o/\mathbf{V}_s$ of the circuit in Fig. 14.86. Show that the circuit is a lowpass filter.

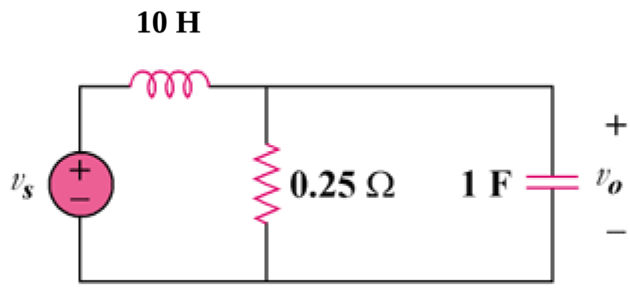


Figure 14.86
For Prob. 14.48.

Solution

$$\mathbf{H}(\omega) = \frac{R \parallel \dfrac{1}{j\omega C}}{j\omega L + R \parallel \dfrac{1}{j\omega C}}$$

$$\mathbf{H}(\omega) = \frac{\dfrac{R/j\omega C}{R + 1/j\omega C}}{j\omega L + \dfrac{R/j\omega C}{R + 1/j\omega C}} = \frac{\dfrac{R}{1 + j\omega RC}}{j\omega L + \dfrac{R}{1 + j\omega RC}} = \frac{R}{R + j\omega L + \dfrac{R/j\omega C}{R + 1/j\omega C}}$$

$$\mathbf{H}(\omega) = \frac{0.25}{(0.25 - \omega^2 2.5) + j\omega 10}$$

$\mathrm{H}(0) = 1$ and $\mathrm{H}(\infty) = 0$ showing that **this circuit is a lowpass filter**.

Solution 14.49

Design a problem to help other students to better understand lowpass filters described by transfer functions.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the cutoff frequency of the lowpass filter described by

$$H(\omega) = \frac{4}{2 + j\omega 10}$$

Find the gain in dB and phase of $\mathbf{H}(\omega)$ at $\omega = 2$ rad/s.

Solution

At dc, $H(0) = \frac{4}{2} = 2$.

Hence, $\left|H(\omega)\right| = \frac{1}{\sqrt{2}} H(0) = \frac{2}{\sqrt{2}}$

$$\frac{2}{\sqrt{2}} = \frac{4}{\sqrt{4 + 100\omega_c^2}}$$

$$4 + 100\omega_c^2 = 8 \quad \longrightarrow \quad \omega_c = 0.2$$

$$H(2) = \frac{4}{2 + j20} = \frac{2}{1 + j10}$$

$$\left|H(2)\right| = \frac{2}{\sqrt{101}} = 0.199$$

In dB, $20\log_{10}\left|H(2)\right| = \mathbf{-14.023}$

$\arg H(2) = -\tan^{-1}10 = -84.3°$ or ω_c = **1.4713 rad/sec**.

Solution 14.50

Determine what type of filter is in Fig. 14.87. Calculate the corner frequency f_c.

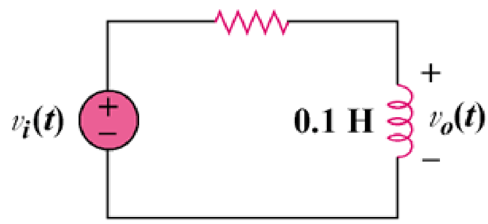


Figure 14.87
For Prob. 14.50.

Solution

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L}$$

$H(0) = 0$ and $H(\infty) = 1$ showing that **this circuit is a highpass filter**.

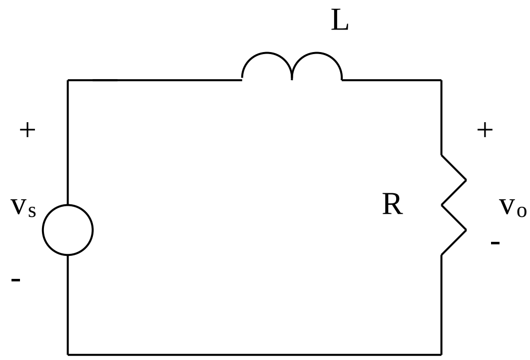
$$\left|\mathbf{H}(\omega_c)\right| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\left(\frac{R}{\omega_c L}\right)^2}} \longrightarrow 1 = \frac{R}{\omega_c L}$$

or $\qquad \omega_c = \frac{R}{L} = 2\pi f_c$

$$f_c = \frac{1}{2\pi}\cdot\frac{R}{L} = \frac{1}{2\pi}\cdot\frac{1{,}000}{0.1} = \mathbf{1.5915\ kHz}.$$

Solution 14.51

The lowpass RL filter is shown below.



$$H = \frac{V_o}{V_s} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L / R}$$

$$\boldsymbol{\omega_c = \frac{R}{L} = 2\pi f_c \quad \longrightarrow \quad R = 2\pi f_c L = 2\pi x 5x10^3 x 40x10^{-3} = \underline{1.256k\Omega}}$$

Solution 14.52

Design a problem to help other students to better understand passive highpass filters.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In a highpass *RL* filter with a cutoff frequency of 100 kHz, L = 40 mH. Find R.

Solution

$$\omega_c = \frac{R}{L} = 2\pi f_c$$

$$R = 2\pi f_c L = (2\pi)(10^5)(40\times10^{-3}) = \mathbf{25.13\ k\Omega}$$

Solution 14.53

$$\omega_1 = 2\pi f_1 = 20\pi \times 10^3$$
$$\omega_2 = 2\pi f_2 = 22\pi \times 10^3$$

$$B = \omega_2 - \omega_1 = 2\pi \times 10^3$$
$$\omega_0 = \frac{\omega_2 + \omega_1}{2} = 21\pi \times 10^3$$
$$Q = \frac{\omega_0}{B} = \frac{21\pi}{2\pi} = \mathbf{10.5}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega_0^2 C}$$
$$L = \frac{1}{(21\pi \times 10^3)^2 (80 \times 10^{-12})} = \mathbf{2.872\ H}$$

$$B = \frac{R}{L} \longrightarrow R = BL$$
$$R = (2\pi \times 10^3)(2.872) = \mathbf{18.045\ k\Omega}$$

Solution 14.54

We start with a series RLC circuit and the use the equations related to the circuit and the values for a bandstop filter.

$Q = \omega_o L/R = 1/(\omega_o CR) = 20$; $B = R/L = \omega_o/Q = 10/20 = 0.5$; $\omega_o = 1/(LC)^{0.5} = 10$

$(LC)^{0.5} = 0.1$ or $LC = 0.01$. Pick L = **10 H** then C = **1 mF**.

$Q = 20 = \omega_o L/R = 10\text{x}10/R$ or R = 100/20 = **5 Ω**.

Solution 14.55

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(25\times10^{-3})(0.4\times10^{-6})}} = 10\ \text{krad/s}$$

$$B = \frac{R}{L} = \frac{10}{25\times10^{-3}} = 0.4\ \text{krad/s}$$

$$Q = \frac{10}{0.4} = \mathbf{25}$$

$$\omega_1 = \omega_o - B/2 = 10 - 0.2 = 9.8\ \text{krad/s} \qquad \text{or} \qquad f_1 = \frac{9.8}{2\pi} = 1.56\ \text{kHz}$$

$$\omega_2 = \omega_o + B/2 = 10 + 0.2 = 10.2\ \text{krad/s} \qquad \text{or} \qquad f_2 = \frac{10.2}{2\pi} = 1.62\ \text{kHz}$$

Therefore,

$$\mathbf{1.56\ kHz < f < 1.62\ kHz}$$

Solution 14.56

(a) From Eq. 14.54,

$$\mathbf{H}(s) = \frac{R}{R + sL + \dfrac{1}{sC}} = \frac{sRC}{1 + sRC + s^2LC} = \frac{s\dfrac{R}{L}}{s^2 + s\dfrac{R}{L} + \dfrac{1}{LC}}$$

Since $B = \dfrac{R}{L}$ and $\omega_0 = \dfrac{1}{\sqrt{LC}}$,

$$\mathbf{H}(s) = \frac{\mathbf{sB}}{\mathbf{s^2 + sB + \omega_0^2}}$$

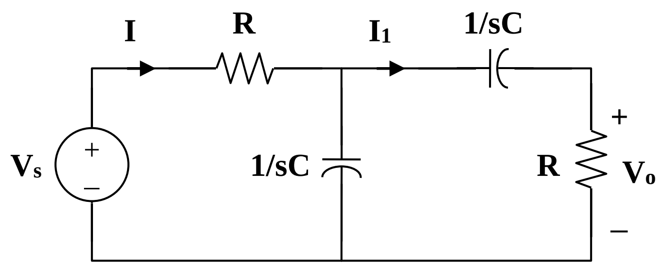
(b) From Eq. 14.56,

$$\mathbf{H}(s) = \frac{sL + \dfrac{1}{sC}}{R + sL + \dfrac{1}{sC}} = \frac{s^2 + \dfrac{1}{LC}}{s^2 + s\dfrac{R}{L} + \dfrac{1}{LC}}$$

$$\mathbf{H}(s) = \frac{\mathbf{s^2 + \omega_0^2}}{\mathbf{s^2 + sB + \omega_0^2}}$$

Solution 14.57

(a) Consider the circuit below.



$$\mathbf{Z}(s) = R + \frac{1}{sC} \,\|\, \left(R + \frac{1}{sC}\right) = R + \frac{\frac{1}{sC}\left(R + \frac{1}{sC}\right)}{R + \frac{2}{sC}}$$

$$\mathbf{Z}(s) = R + \frac{1 + sRC}{sC\,(2 + sRC)}$$

$$\mathbf{Z}(s) = \frac{1 + 3sRC + s^2R^2C^2}{sC\,(2 + sRC)}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}}$$

$$\mathbf{I}_1 = \frac{1/sC}{2/sC + R}\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}\,(2 + sRC)}$$

$$\mathbf{V}_o = \mathbf{I}_1 R = \frac{R\,\mathbf{V}_s}{2 + sRC} \cdot \frac{sC\,(2 + sRC)}{1 + 3sRC + s^2R^2C^2}$$

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{sRC}{1 + 3sRC + s^2R^2C^2}$$

$$\mathbf{H}(s) = \frac{1}{3}\left[\frac{\frac{3}{RC}s}{s^2 + \frac{3}{RC}s + \frac{1}{R^2C^2}}\right]$$

Thus, $\omega_0^2 = \dfrac{1}{R^2C^2}$ or $\omega_0 = \dfrac{1}{RC} = \mathbf{1\,rad/s}$

$$B = \frac{3}{RC} = \mathbf{3\,rad/s}$$

(b) Similarly,

$$\mathbf{Z}(s) = sL + R \,||\, (R + sL) = sL + \frac{R\,(R + sL)}{2R + sL}$$

$$\mathbf{Z}(s) = \frac{R^2 + 3sRL + s^2L^2}{2R + sL}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}}, \qquad \mathbf{I}_1 = \frac{R}{2R + sL}\mathbf{I} = \frac{R\,\mathbf{V}_s}{\mathbf{Z}\,(2R + sL)}$$

$$\mathbf{V}_o = \mathbf{I}_1 \cdot sL = \frac{sLR\,\mathbf{V}_s}{2R + sL} \cdot \frac{2R + sL}{R^2 + 3sRL + s^2L^2}$$

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{sRL}{R^2 + 3sRL + s^2L^2} = \frac{\frac{1}{3}\left(\frac{3R}{L}s\right)}{s^2 + \frac{3R}{L}s + \frac{R^2}{L^2}}$$

Thus, $\omega_0 = \dfrac{R}{L} = \mathbf{1\,rad/s}$

$$B = \frac{3R}{L} = \mathbf{3\,rad/s}$$

Solution 14.58

The circuit parameters for a series *RLC* bandstop filter are $R = 250\ \Omega$, $L = 1$ mH, $C = 40$ pF. Calculate:

(a) the center frequency
(b) the half-power frequencies
(c) the quality factor.

Solution

(a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.001)(40\times 10^{-12})}} = \mathbf{5\ Mrad/s}$$

(b) $$B = \frac{R}{L} = \frac{250}{0.001} = 0.25\times 10^6 \text{ rad/s}$$

$$Q = \frac{\omega_0}{B} = \frac{5\times 10^6}{0.25\times 10^6} = 20$$

As a high Q (Q greater than 10) circuit,

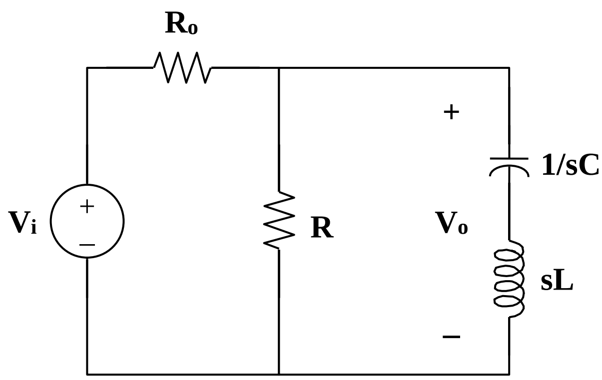
$$\omega_1 = \omega_0 - \frac{B}{2} = 10^6(5 - 0.125) = \mathbf{4.875\ Mrad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 10^6(5 + 0.125) = \mathbf{5.125\ Mrad/s}$$

(c) As seen in part (b), $\mathrm{Q} = \mathbf{20}$

Solution 14.59

Consider the circuit below.



where L = 1 mH, C = 4 μF, R_o = 6 Ω, and R = 4 Ω.

$$\mathbf{Z}(s) = R \,\|\, \left(sL + \frac{1}{sC}\right) = \frac{R\,(sL + 1/sC)}{R + sL + 1/sC}$$

$$\mathbf{Z}(s) = \frac{R\,(1 + s^2LC)}{1 + sRC + s^2LC}$$

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{Z}}{\mathbf{Z} + R_o} = \frac{R\,(1 + s^2LC)}{R_o + sRR_oC + s^2LCR_o + R + s^2LCR}$$

$$\mathbf{Z}_{in} = R_o + \mathbf{Z} = R_o + \frac{R\,(1 + s^2LC)}{1 + sRC + s^2LC}$$

$$\mathbf{Z}_{in} = \frac{R_o + sRR_oC + s^2LCR_o + R + s^2LCR}{1 + sRC + s^2LC}$$

$s = j\omega$

$$\mathbf{Z}_{in} = \frac{R_o + j\omega RR_oC - \omega^2LCR_o + R - \omega^2LCR}{1 - \omega^2LC + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{(R_o + R - \omega^2LCR_o - \omega^2LCR + j\omega RR_oC)(1 - \omega^2LC - j\omega RC)}{(1 - \omega^2LC)^2 + (\omega RC)^2}$$

$\mathrm{Im}(\mathbf{Z}_{in}) = 0$ implies that

$$-\omega RC[R_o + R - \omega^2LCR_o - \omega^2LCR] + \omega RR_oC(1 - \omega^2LC) = 0$$

$$R_o + R - \omega^2 LCR_o - \omega^2 LCR - R_o + \omega^2 LCR_o = 0$$
$$\omega^2 LCR = R$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1\times10^{-3})(4\times10^{-6})}} = \mathbf{15.811\ krad/s}$$

$$\mathbf{H} = \frac{R\,(1-\omega^2 LC)}{R_o + j\omega RR_o C + R - \omega^2 LCR_o - \omega^2 LCR}$$

$$H_{max} = H(0) = \frac{R}{R_o + R}$$

or $\quad H_{max} = H(\infty) = \lim_{\omega\to\infty} \dfrac{R\left(\dfrac{1}{\omega^2} - LC\right)}{\dfrac{R_o + R}{\omega^2} + j\dfrac{RR_o C}{\omega} - LC(R + R_o)} = \dfrac{R}{R + R_o}$

At ω_1 and ω_2, $\left|\mathbf{H}\right| = \dfrac{1}{\sqrt{2}} H_{mzx}$

$$\frac{R}{\sqrt{2}(R_o + R)} = \left|\frac{R\,(1-\omega^2 LC)}{R_o + R - \omega^2 LC(R_o + R) + j\omega RR_o C}\right|$$

$$\frac{1}{\sqrt{2}} = \frac{(R_o + R)(1-\omega^2 LC)}{\sqrt{(\omega RR_o C)^2 + (R_o + R - \omega^2 LC(R_o + R))^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{10\,(1-\omega^2\cdot 4\times10^{-9})}{\sqrt{(96\times10^{-6}\omega)^2 + (10-\omega^2\cdot 4\times10^{-8})^2}}$$

$$0 = \frac{10\,(1-\omega^2\cdot 4\times10^{-9})}{\sqrt{(96\times10^{-6}\omega)^2 + (10-\omega^2\cdot 4\times10^{-8})^2}} - \frac{1}{\sqrt{2}}$$

$$(10-\omega^2\cdot 4\times10^{-8})(\sqrt{2}) - \sqrt{(96\times10^{-6}\omega)^2 + (10-\omega^2\cdot 4\times10^{-8})^2} = 0$$

$$(2)(10-\omega^2\cdot 4\times10^{-8})^2 = (96\times10^{-6}\omega)^2 + (10-\omega^2\cdot 4\times10^{-8})^2$$

$$(96\times10^{-6}\omega)^2 - (10-\omega^2\cdot 4\times10^{-8})^2 = 0$$

$$1.6\times10^{-15}\omega^4 - 8.092\times10^{-7}\omega^2 + 100 = 0$$

$$\omega^4 - 5.058\times10^8 + 6.25\times10^{16} = 0$$

$$\omega^2 = \begin{cases} 2.9109\times10^8 \\ 2.1471\times10^8 \end{cases}$$

Hence,

$$\omega_1 = 14.653\ \text{krad/s}$$

$$\omega_2 = 17.061\ \text{krad/s}$$

$$B = \omega_2 - \omega_1 = 17.061 - 14.653 = \mathbf{2.408\ krad/s}$$

Solution 14.60

Obtain the transfer function of a highpass filter with a passband gain of 100 and a cutoff frequency of 40 rad/s.

Solution

$$\mathbf{H}'(\omega) = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega}{j\omega + 1/RC} \qquad \text{(from Eq. 14.52)}$$

This has a unity passband gain, i.e. $H(\infty) = 1$.

$$\frac{1}{RC} = \omega_c = 40$$

$$\mathbf{H}^{\wedge}(\omega) = 100\,\mathbf{H}'(\omega) = \frac{j100\omega}{40 + j\omega}$$

$$\mathbf{H}(\omega) = \mathbf{j100\omega/(40+j\omega)}$$

Solution 14.61

(a) $\mathbf{V}_+ = \dfrac{1/j\omega C}{R + 1/j\omega C}\mathbf{V}_i$, $\mathbf{V}_- = \mathbf{V}_o$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{1}}{\mathbf{1 + j\omega RC}}$$

(b) $\mathbf{V}_+ = \dfrac{R}{R + 1/j\omega C}\mathbf{V}_i$, $\mathbf{V}_- = \mathbf{V}_o$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i = \mathbf{V}_o$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{j\omega RC}}{\mathbf{1 + j\omega RC}}$$

Solution 14.62

This is a highpass filter.

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC} = \frac{1}{1 - j/\omega RC}$$

$$\mathbf{H}(\omega) = \frac{1}{1 - j\omega_c/\omega}, \qquad \omega_c = \frac{1}{RC} = 2\pi(1000)$$

$$\mathbf{H}(\omega) = \frac{1}{1 - jf_c/f} = \frac{1}{1 - j1000/f}$$

(a) $$\mathbf{H}(f = 200\text{ Hz}) = \frac{1}{1 - j5} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$\left|\mathbf{V}_o\right| = \frac{120\text{ mV}}{\left|1 - j5\right|} = \mathbf{23.53\ mV}$$

(b) $$\mathbf{H}(f = 2\text{ kHz}) = \frac{1}{1 - j0.5} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$\left|\mathbf{V}_o\right| = \frac{120\text{ mV}}{\left|1 - j0.5\right|} = \mathbf{107.3\ mV}$$

(c) $$\mathbf{H}(f = 10\text{ kHz}) = \frac{1}{1 - j0.1} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$\left|\mathbf{V}_o\right| = \frac{120\text{ mV}}{\left|1 - j0.1\right|} = \mathbf{119.4\ mV}$$

Solution 14.63

For an active highpass filter,

$$H(s) = -\frac{sC_iR_f}{1+sC_iR_i} \qquad (1)$$

But

$$H(s) = -\frac{10s}{1+s/10} \qquad (2)$$

Comparing (1) and (2) leads to:

$$\mathbf{C_iR_f = 10 \quad \longrightarrow \quad R_f = \frac{10}{C_i} = \underline{10M\Omega}}$$

$$\mathbf{C_iR_i = 0.1 \quad \longrightarrow \quad R_i = \frac{0.1}{C_i} = \underline{100k\Omega}}$$

Solution 14.64

$$Z_f = R_f \parallel \frac{1}{j\omega C_f} = \frac{R_f}{1 + j\omega R_f C_f}$$

$$Z_i = R_i + \frac{1}{j\omega C_i} = \frac{1 + j\omega R_i C_i}{j\omega C_i}$$

Hence,

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = \boldsymbol{\frac{-j\omega R_f C_i}{(1 + j\omega R_f C_f)(1 + j\omega R_i C_i)}}$$

This is a bandpass filter. $\mathbf{H}(\omega)$ is similar to the product of the transfer function of a lowpass filter and a highpass filter.

Solution 14.65

$$\mathbf{V}_+ = \frac{R}{R + 1/j\omega C}\mathbf{V}_i = \frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i$$

$$\mathbf{V}_- = \frac{R_i}{R_i + R_f}\mathbf{V}_o$$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\frac{R_i}{R_i + R_f}\mathbf{V}_o = \frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left(\mathbf{1} + \frac{\mathbf{R_f}}{\mathbf{R_i}}\right)\left(\frac{\mathbf{j\omega RC}}{\mathbf{1 + j\omega RC}}\right)$$

It is evident that as $\omega \to \infty$, the gain is $\mathbf{1} + \frac{\mathbf{R_f}}{\mathbf{R_i}}$ and that the corner frequency is $\frac{\mathbf{1}}{\mathbf{RC}}$.

Solution 14.66

(a) **Proof**

(b) When $\mathbf{R_1R_4 = R_2R_3}$,

$$\mathbf{H}(s) = \frac{R_4}{R_3 + R_4} \cdot \frac{s}{s + 1/R_2C}$$

(c) When $\mathbf{R_3} \to \infty$,

$$\mathbf{H}(s) = \frac{-1/R_1C}{s + 1/R_2C}$$

Solution 14.67

$$\text{DC gain} = \frac{R_f}{R_i} = \frac{1}{4} \quad \longrightarrow \quad R_i = 4R_f$$

$$\text{Corner frequency} = \omega_c = \frac{1}{R_f C_f} = 2\pi(500)\ \text{rad/s}$$

If we select $R_f = 20\ \text{k}\Omega$, then $R_i = 80\ \text{k}\Omega$ and

$$C = \frac{1}{(2\pi)(500)(20\times10^3)} = 15.915\ \text{nF}$$

Therefore, if R_f = **$20\ \text{k}\Omega$**, then R_i = **$80\ \text{k}\Omega$** and C = **15.915 nF**

Solution 14.68

Design a problem to help other students to better understand the design of active highpass filters when specifying a high-frequency gain and a corner frequency.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Design an active highpass filter with a high-frequency gain of 5 and a corner frequency of 200 Hz.

Solution

$$\text{High frequency gain} = 5 = \frac{R_f}{R_i} \longrightarrow R_f = 5R_i$$

$$\text{Corner frequency} = \omega_c = \frac{1}{R_i C_i} = 2\pi(200)\ \text{rad/s}$$

If we select $R_i = 20\ \text{k}\Omega$, then $R_f = 100\ \text{k}\Omega$ and

$$C = \frac{1}{(2\pi)(200)(20\times10^3)} = 39.8\ \text{nF}$$

Therefore, if $R_i =$ $\mathbf{20\ k\Omega}$, then $R_f =$ $\mathbf{100\ k\Omega}$ and $C =$ $\mathbf{39.8\ nF}$

Solution 14.69

This is a highpass filter with $f_c = 2$ kHz.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$RC = \frac{1}{2\pi f_c} = \frac{1}{4\pi \times 10^3}$$

10^8 Hz may be regarded as high frequency. Hence the high-frequency gain is

$$\frac{-R_f}{R} = \frac{-10}{4} \qquad \text{or} \qquad R_f = 2.5R$$

If we let $R = \mathbf{10\ k\Omega}$, then $R_f = \mathbf{25\ k\Omega}$, and $C = \dfrac{1}{4000\pi \times 10^4} = \mathbf{7.96\ nF}$.

Solution 14.70

(a) $$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)} = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_4(Y_1 + Y_2 + Y_3)}$$

where $Y_1 = \dfrac{1}{R_1} = G_1$, $Y_2 = \dfrac{1}{R_2} = G_2$, $Y_3 = sC_1$, $Y_4 = sC_2$.

$$\mathbf{H}(s) = \frac{\mathbf{G_1 G_2}}{\mathbf{G_1 G_2 + sC_2 (G_1 + G_2 + sC_1)}}$$

(b) $$H(0) = \frac{G_1 G_2}{G_1 G_2} = 1, \qquad H(\infty) = 0$$

showing that **this circuit is a lowpass filter**.

Solution 14.71

$R = 50\,\Omega,\ L = 40\text{ mH},\ C = 1\,\mu\text{F}$

$$L' = \frac{K_m}{K_f}L \quad \longrightarrow \quad 1 = \frac{K_m}{K_f}\cdot(40\times10^{-3})$$

$$25K_f = K_m \qquad (1)$$

$$C' = \frac{C}{K_m K_f} \quad \longrightarrow \quad 1 = \frac{10^{-6}}{K_m K_f}$$

$$10^6 K_f = \frac{1}{K_m} \qquad (2)$$

Substituting (1) into (2),

$$10^6 K_f = \frac{1}{25K_f}$$

$$K_f = \mathbf{2x10^{-4}}$$

$$K_m = 25K_f = \mathbf{5\times10^{-3}}$$

Solution 14.72

Design a problem to help other students to better understand magnitude and frequency scaling.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

What values of K_m and K_f will scale a 4-mH inductor and a 20-μF capacitor to 1 H and 2 F respectively?

Solution

$$L'C' = \frac{LC}{K_f^2} \longrightarrow K_f^2 = \frac{LC}{L'C'}$$

$$K_f^2 = \frac{(4\times 10^{-3})(20\times 10^{-6})}{(1)(2)} = 4\times 10^{-8}$$

$$K_f = \mathbf{2\times 10^{-4}}$$

$$\frac{L'}{C'} = \frac{L}{C}K_m^2 \longrightarrow K_m^2 = \frac{L'}{C'}\cdot\frac{C}{L}$$

$$K_m^2 = \frac{(1)(20\times 10^{-6})}{(2)(4\times 10^{-3})} = 2.5\times 10^{-3}$$

$$K_m = \mathbf{5\times 10^{-2}}$$

Solution 14.73

$$R' = K_m R = (12)(800\times10^3) = \mathbf{9.6\ M\Omega}$$

$$L' = \frac{K_m}{K_f} L = \frac{800}{1000}(40\times10^{-6}) = \mathbf{32\ \mu F}$$

$$C' = \frac{C}{K_m K_f} = \frac{300\times10^{-9}}{(800)(1000)} = \mathbf{0.375\ pF}$$

Solution 14.74

$$\mathbf{R'_1 = K_m R_1 = 3x100 = \underline{300\Omega}}$$

$$\mathbf{R'_2 = K_m R_2 = 10x100 = \underline{1\,k\Omega}}$$

$$\mathbf{L' = \frac{K_m}{K_f} L = \frac{10^2}{10^6}(2) = \underline{200\,\mu H}}$$

$$\mathbf{C' = \frac{C}{K_m K_f} = \frac{\frac{1}{10}}{10^8} = \underline{1\,nF}}$$

Solution 14.75

$$R' = K_m R = 20x10 = \underline{200\,\Omega}$$

$$L' = \frac{K_m}{K_f} L = \frac{10}{10^5}(4) = \underline{400\,\mu H}$$

$$C' = \frac{C}{K_m K_f} = \frac{1}{10x10^5} = \underline{1\,\mu F}$$

Solution 14.76

$$R' = K_m R = 500x5x10^3 = \underline{25\ \text{M}\Omega}$$

$$L' = \frac{K_m}{K_f} L = \frac{500}{10^5}(10mH) = \underline{50\ \mu\text{H}}$$

$$C' = \frac{C}{K_m K_f} = \frac{20x10^{-6}}{500x10^5} = \underline{0.4\ \text{pF}}$$

Solution 14.77

L and C are needed before scaling.

$$B = \frac{R}{L} \longrightarrow L = \frac{R}{B} = \frac{10}{5} = 2 \text{ H}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega_0^2 L} = \frac{1}{(1600)(2)} = 312.5\ \mu\text{F}$$

(a) $$L' = K_m L = (600)(2) = \mathbf{1.200\ kH}$$

$$C' = \frac{C}{K_m} = \frac{3.125\times10^{-4}}{600} = \mathbf{0.5208\ \mu F}$$

(b) $$L' = \frac{L}{K_f} = \frac{2}{10^3} = \mathbf{2\ mH}$$

$$C' = \frac{C}{K_f} = \frac{3.125\times10^{-4}}{10^3} = \mathbf{312.5\ nF}$$

(c) $$L' = \frac{K_m}{K_f} L = \frac{(400)(2)}{10^5} = \mathbf{8\ mH}$$

$$C' = \frac{C}{K_m K_f} = \frac{3.125\times10^{-4}}{(400)(10^5)} = \mathbf{7.81\ pF}$$

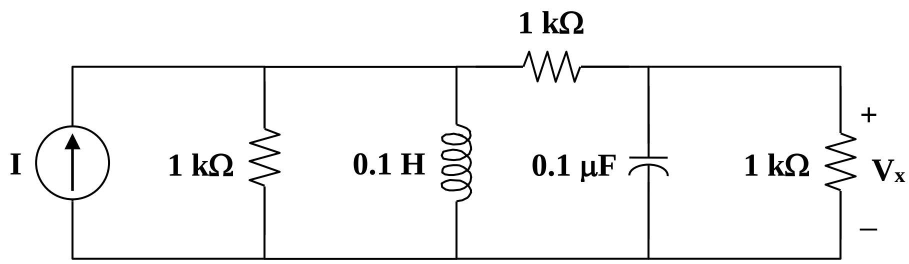
Solution 14.78

$$R' = K_m R = (1000)(1) = 1\,\text{k}\Omega$$

$$L' = \frac{K_m}{K_f} L = \frac{10^3}{10^4}(1) = 0.1\,\text{H}$$

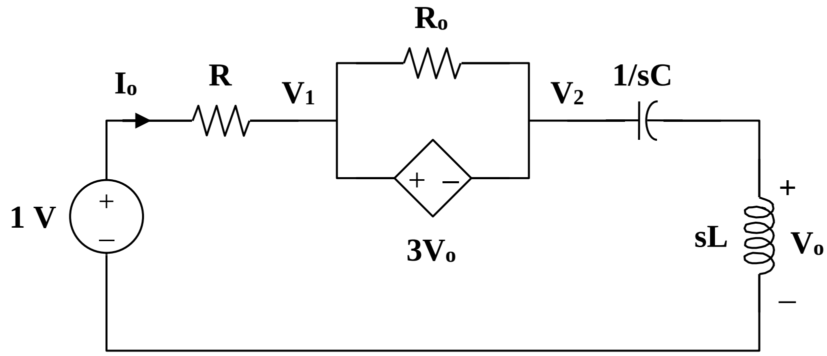
$$C' = \frac{C}{K_m K_f} = \frac{1}{(10^3)(10^4)} = 0.1\,\mu\text{F}$$

The new circuit is shown below.



Solution 14.79

(a) Insert a 1-V source at the input terminals.



There is a supernode.

$$\frac{1-\mathbf{V}_1}{R} = \frac{\mathbf{V}_2}{sL + 1/sC} \tag{1}$$

But $\mathbf{V}_1 = \mathbf{V}_2 + 3\mathbf{V}_o \longrightarrow \mathbf{V}_2 = \mathbf{V}_1 - 3\mathbf{V}_o$ (2)

Also, $\mathbf{V}_o = \dfrac{sL}{sL + 1/sC}\mathbf{V}_2 \longrightarrow \dfrac{\mathbf{V}_o}{sL} = \dfrac{\mathbf{V}_2}{sL + 1/sC}$ (3)

Combining (2) and (3)

$$\mathbf{V}_2 = \mathbf{V}_1 - 3\mathbf{V}_o = \frac{sL + 1/sC}{sL}\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{s^2LC}{1 + 4s^2LC}\mathbf{V}_1 \tag{4}$$

Substituting (3) and (4) into (1) gives

$$\frac{1-\mathbf{V}_1}{R} = \frac{\mathbf{V}_o}{sL} = \frac{sC}{1+4s^2LC}\mathbf{V}_1$$

$$1 = \mathbf{V}_1 + \frac{sRC}{1+4s^2LC}\mathbf{V}_1 = \frac{1+4s^2LC+sRC}{1+4s^2LC}\mathbf{V}_1$$

$$\mathbf{V}_1 = \frac{1+4s^2LC}{1+4s^2LC+sRC}$$

$$\mathbf{I}_o = \frac{1-\mathbf{V}_1}{R} = \frac{sRC}{R\,(1+4s^2LC+sRC)}$$

$$\mathbf{Z}_{in} = \frac{1}{\mathbf{I}_o} = \frac{1+sRC+4s^2LC}{sC}$$

$$\mathbf{Z}_{in} = 4sL + R + \frac{1}{sC} \tag{5}$$

When $R = 5$, $L = 2$, $C = 0.1$,

$$\mathbf{Z}_{in}(s) = \mathbf{8s + 5 + \frac{10}{s}}$$

At resonance,

$$\mathrm{Im}(\mathbf{Z}_{in}) = 0 = 4\omega L - \frac{1}{\omega C}$$

or $$\omega_0 = \frac{1}{2\sqrt{LC}} = \frac{1}{2\sqrt{(0.1)(2)}} = \mathbf{1.118\ rad/s}$$

(b) After scaling,

$$R' \longrightarrow K_m R$$

$$4\,\Omega \longrightarrow 40\,\Omega$$

$$5\,\Omega \longrightarrow 50\,\Omega$$

$$L' = \frac{K_m}{K_f} L = \frac{10}{100}(2) = 0.2\ \text{H}$$

$$C' = \frac{C}{K_m K_f} = \frac{0.1}{(10)(100)} = 10^{-4}$$

From (5),

$$\mathbf{Z}_{in}(s) = \mathbf{0.8s + 50 + \frac{10^4}{s}}$$

$$\omega_0 = \frac{1}{2\sqrt{LC}} = \frac{1}{2\sqrt{(0.2)(10^{-4})}} = \mathbf{111.8\ rad/s}$$

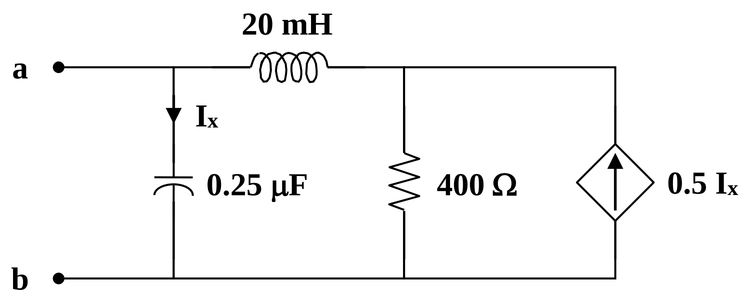
Solution 14.80

(a) $R' = K_m R = (200)(2) = 400\ \Omega$

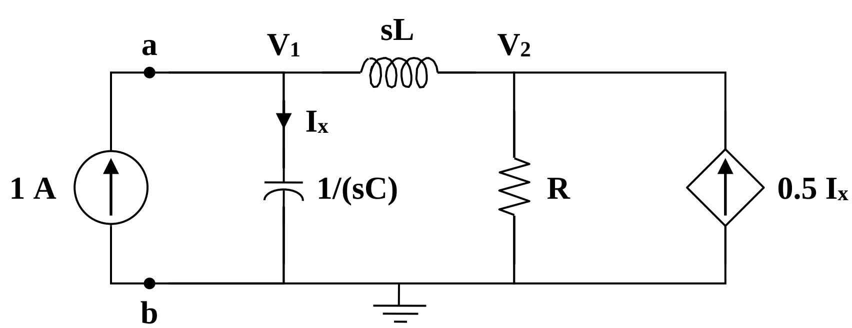
$$L' = \frac{K_m L}{K_f} = \frac{(200)(1)}{10^4} = 20\text{ mH}$$

$$C' = \frac{C}{K_m K_f} = \frac{0.5}{(200)(10^4)} = 0.25\ \mu\text{F}$$

The new circuit is shown below.



(b) Insert a 1-A source at the terminals a-b.



At node 1,

$$1 = sC\mathbf{V}_1 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} + 0.5\mathbf{I}_x = \frac{\mathbf{V}_2}{R}$$

But, $\mathbf{I}_x = sC\mathbf{V}_1$.

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} + 0.5sC\mathbf{V}_1 = \frac{\mathbf{V}_2}{R} \qquad (2)$$

Solving (1) and (2),

$$\mathbf{V}_1 = \frac{sL + R}{s^2LC + 0.5sCR + 1}$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_1}{1} = \frac{sL + R}{s^2LC + 0.5sCR + 1}$$

At $\omega = 10^4$,

$$\mathbf{Z}_{Th} = \frac{(j10^4)(20\times10^{-3}) + 400}{(j10^4)^2(20\times10^{-3})(0.25\times10^{-6}) + 0.5(j10^4)(0.25\times10^{-6})(400) + 1}$$

$$\mathbf{Z}_{Th} = \frac{400 + j200}{0.5 + j0.5} = 600 - j200$$

$$\mathbf{Z}_{Th} = \mathbf{632.5\angle -18.435^\circ\ ohms}$$

Solution 14.81

(a)

$$\frac{1}{Z} = G + j\omega C + \frac{1}{R + j\omega L} = \frac{(G + j\omega C)(R + j\omega L) + 1}{R + j\omega L}$$

which leads to $\quad Z = \dfrac{j\omega L + R}{-\omega^2 LC + j\omega(RC + LG) + GR + 1}$

$$Z(\omega) = \frac{j\frac{\omega}{C} + \frac{R}{LC}}{-\omega^2 + j\omega\left(\frac{R}{L} + \frac{G}{C}\right) + \frac{GR + 1}{LC}} \qquad (1)$$

We compare this with the given impedance:

$$Z(\omega) = \frac{1000(j\omega + 1)}{-\omega^2 + 2j\omega + 1 + 2500} \qquad (2)$$

Comparing (1) and (2) shows that

$$\frac{1}{C} = 1000 \quad \longrightarrow \quad C = 1\,\text{mF}, \quad R/L = 1 \quad \longrightarrow \quad R = L$$

$$\frac{R}{L} + \frac{G}{C} = 2 \quad \longrightarrow \quad G = C = 1\ \text{mS}$$

$$2501 = \frac{GR + 1}{LC} = \frac{10^{-3}R + 1}{10^{-3}R} \quad \longrightarrow \quad R = 0.4 = L$$

Thus,

R = **0.4Ω**, L = **0.4 H**, C = **1 mF**, G = **1 mS**

(b) By frequency-scaling, K_f =1000.

R’ = **0.4 Ω**, G’ = **1 mS**

$$\mathbf{L' = \frac{L}{K_f} = \frac{0.4}{10^3} = \underline{0.4mH}}, \qquad \mathbf{C' = \frac{C}{K_f} = \frac{10^{-3}}{10^{-3}} = \underline{1\mu F}}$$

Solution 14.82

$$C' = \frac{C}{K_m K_f}$$

$$K_f = \frac{\omega'_c}{\omega} = \frac{200}{1} = 200$$

$$K_m = \frac{C}{C'} \cdot \frac{1}{K_f} = \frac{1}{10^{-6}} \cdot \frac{1}{200} = 5000$$

$R' = K_m R = \mathbf{5\ k\Omega}$, thus, $R'_f = 2R_i = \mathbf{10\ k\Omega}$

Solution 14.83

$$1\mu F \quad \longrightarrow \quad C' = \frac{1}{K_m K_f} C = \frac{10^{-6}}{100x10^5} = \underline{0.1\,pF}$$

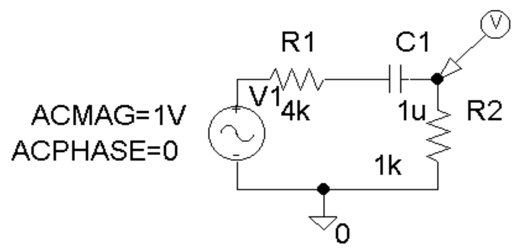
$$5\mu F \quad \longrightarrow \quad C' = \underline{0.5\,pF}$$

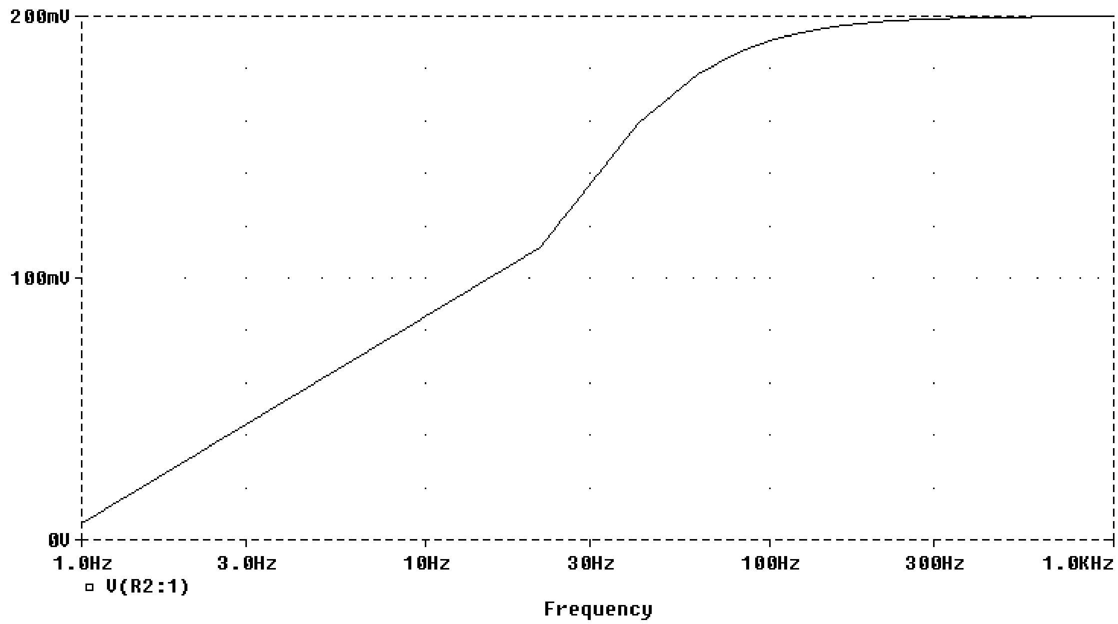
$$10\,k\Omega \quad \longrightarrow \quad R' = K_m R = 100x10\,k\Omega = \underline{1\,M\Omega}$$

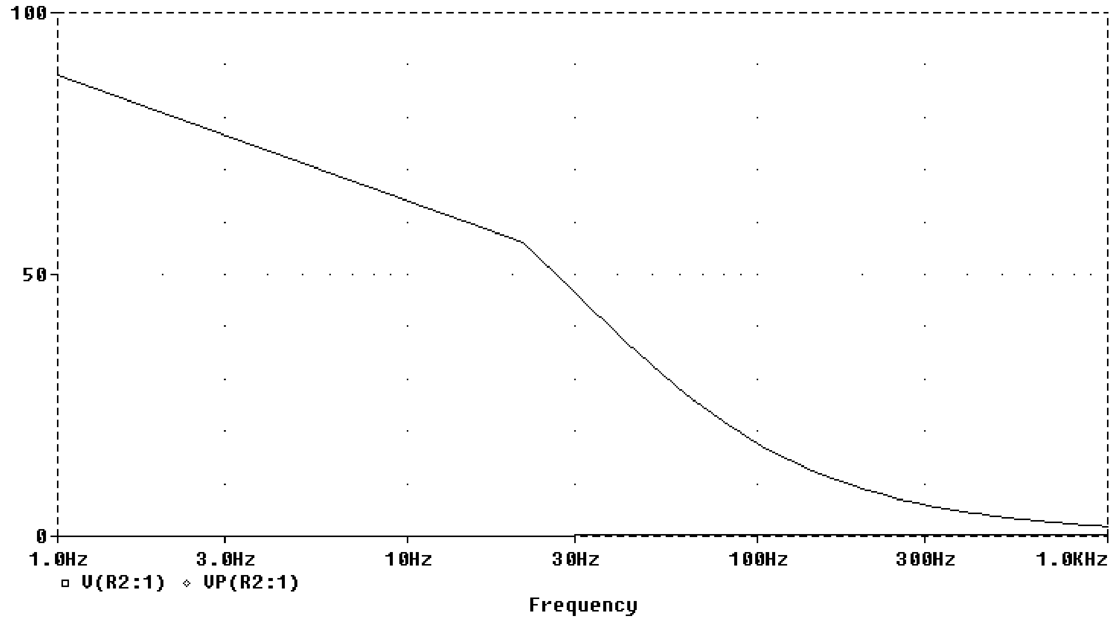
$$20\,k\Omega \quad \longrightarrow \quad R' = \underline{2\,M\Omega}$$

Solution 14.84

The schematic is shown below. A voltage marker is inserted to measure v_o. In the AC sweep box, we select Total Points = 50, Start Frequency = 1, and End Frequency = 1000. After saving and simulation, we obtain the magnitude and phase plots in the probe menu as shown below.

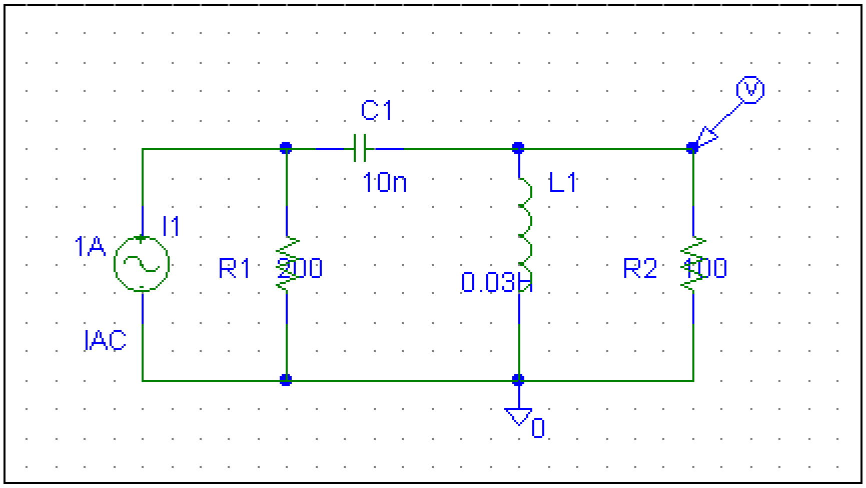


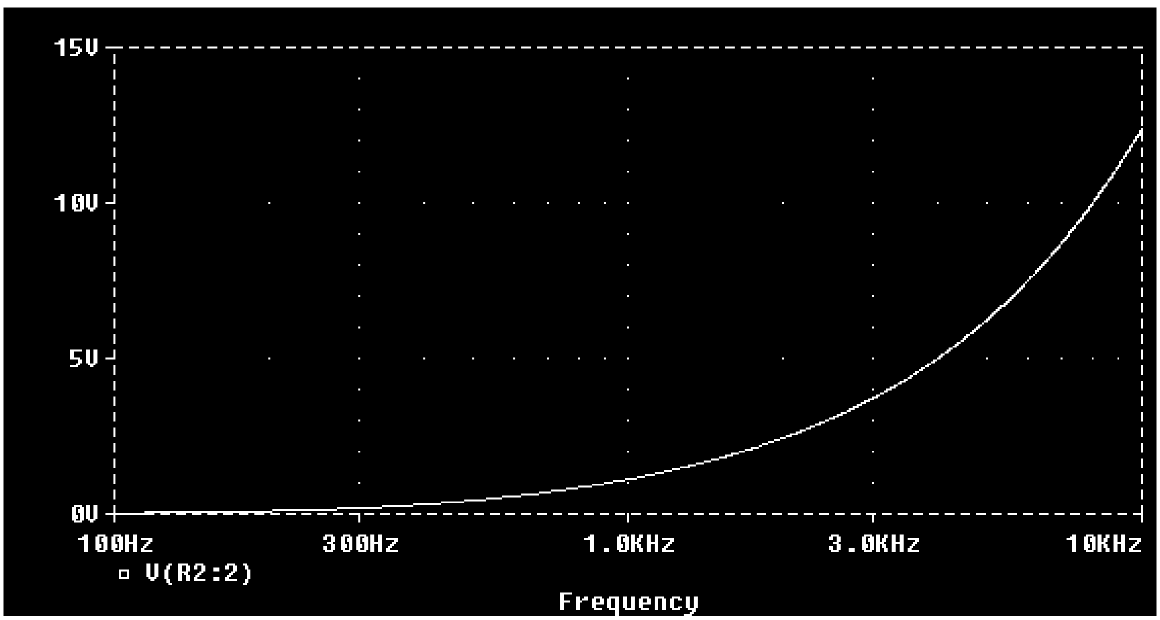


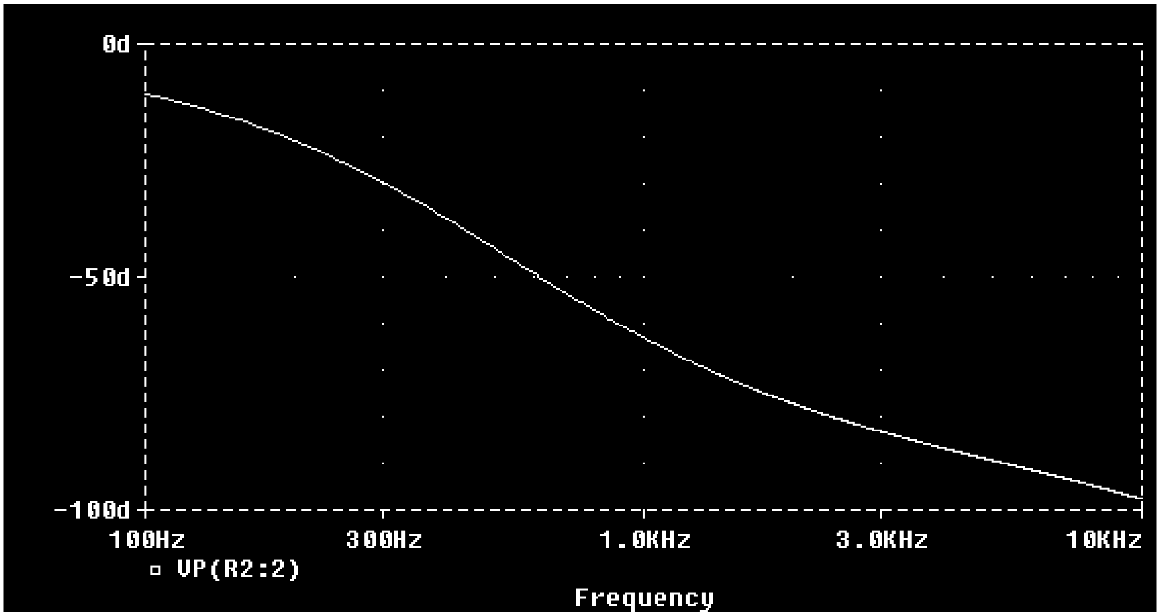

100
50
0
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
▫ V(R2:1) ◇ VP(R2:1)
Frequency

Solution 14.85

We let $I_s = 1\angle 0^o$ A so that $V_o / I_s = V_o$. The schematic is shown below. The circuit is simulated for $100 < f < 10$ kHz.






0d
-50d
-100d
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
VP(R2:2)
Frequency

Solution 14.86

Using Fig. 14.103, design a problem to help other students to better understand how to use PSpice to obtain the frequency response (magnitude and phase of I) in electrical circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use *PSpice* to provide the frequency response (magnitude and phase of *i*) of the circuit in Fig. 14.103. Use linear frequency sweep from 1 to 10,000 Hz.

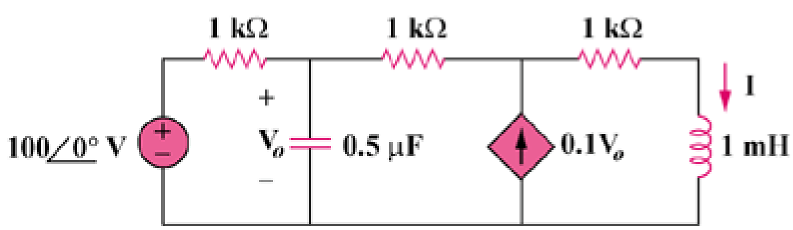
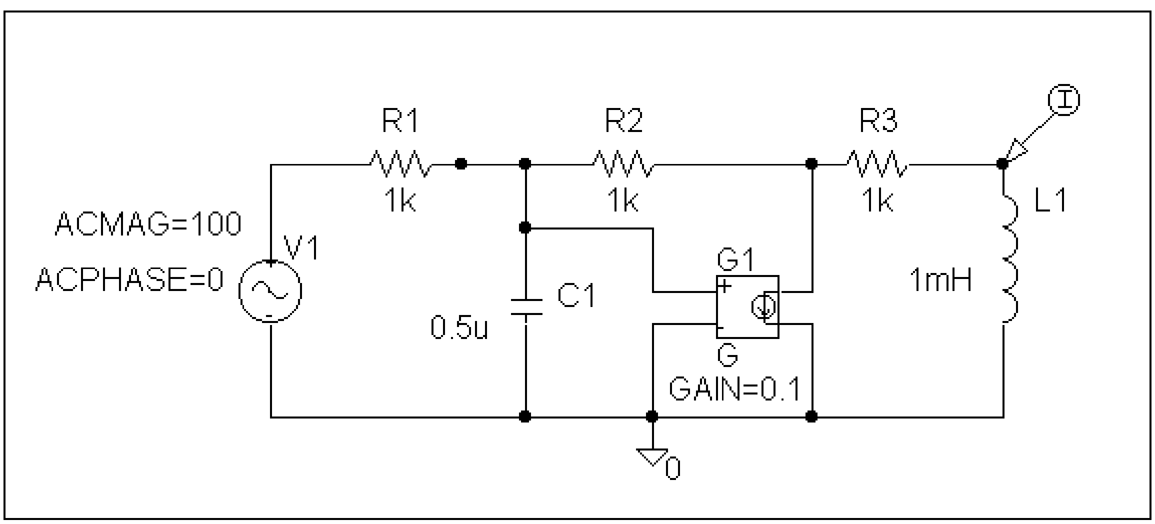
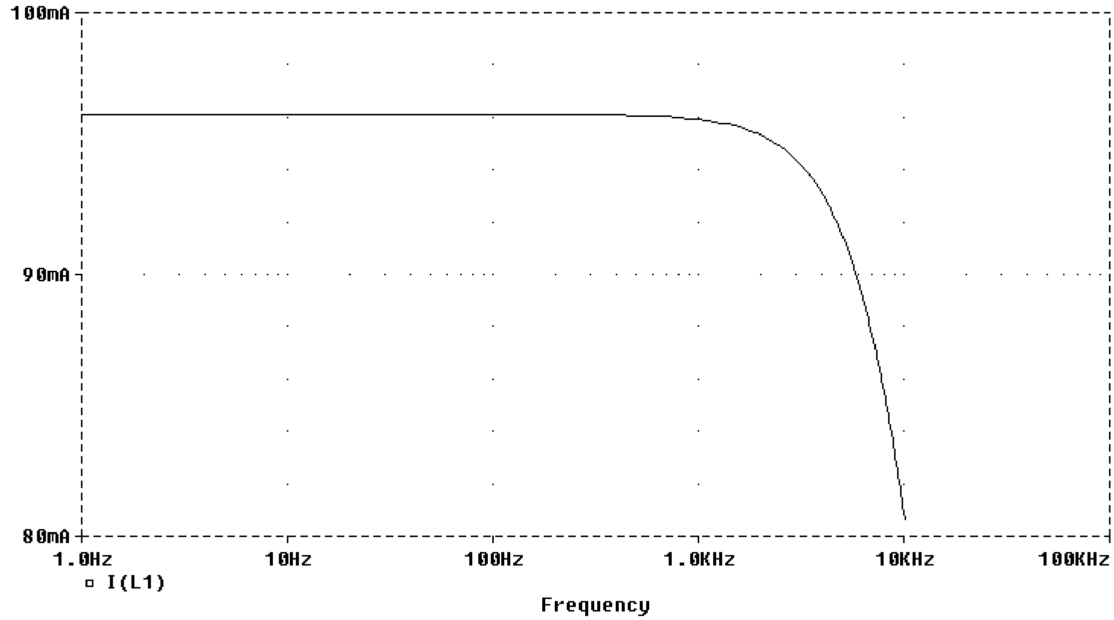


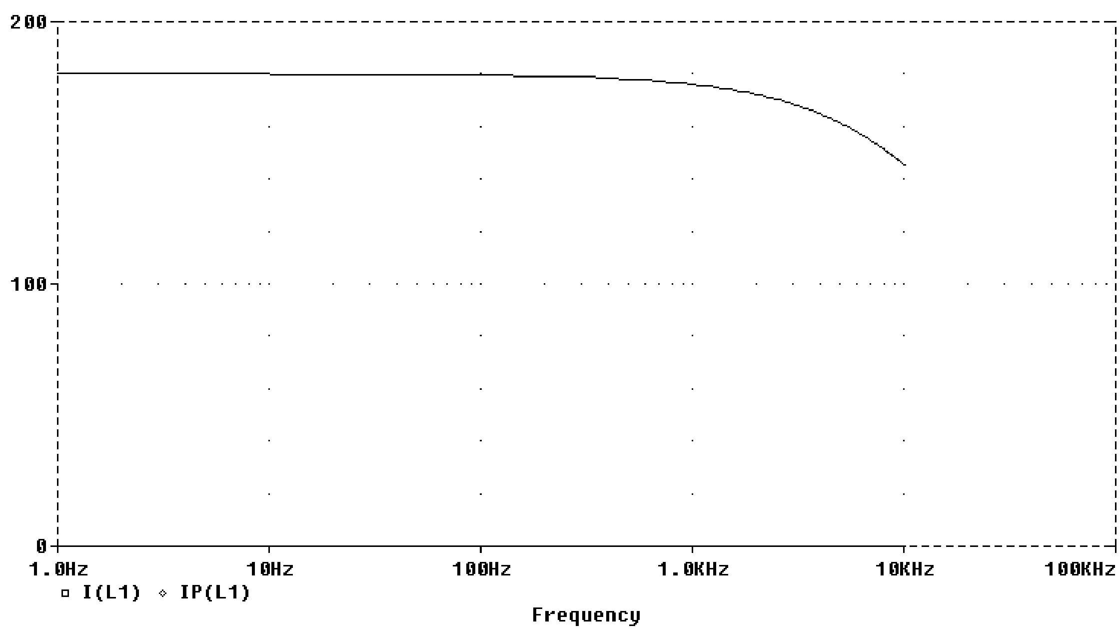
Figure 14.103

Solution

The schematic is shown below. A current marker is inserted to measure **I**. We set Total Points = 101, start Frequency = 1, and End Frequency = 10 kHz in the AC sweep box. After simulation, the magnitude and phase plots are obtained in the Probe menu as shown below.

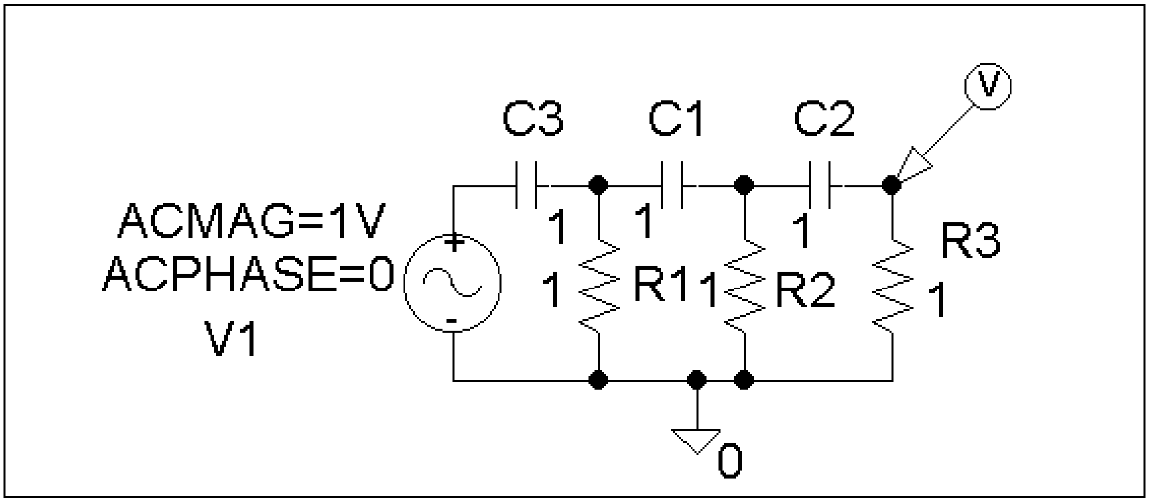


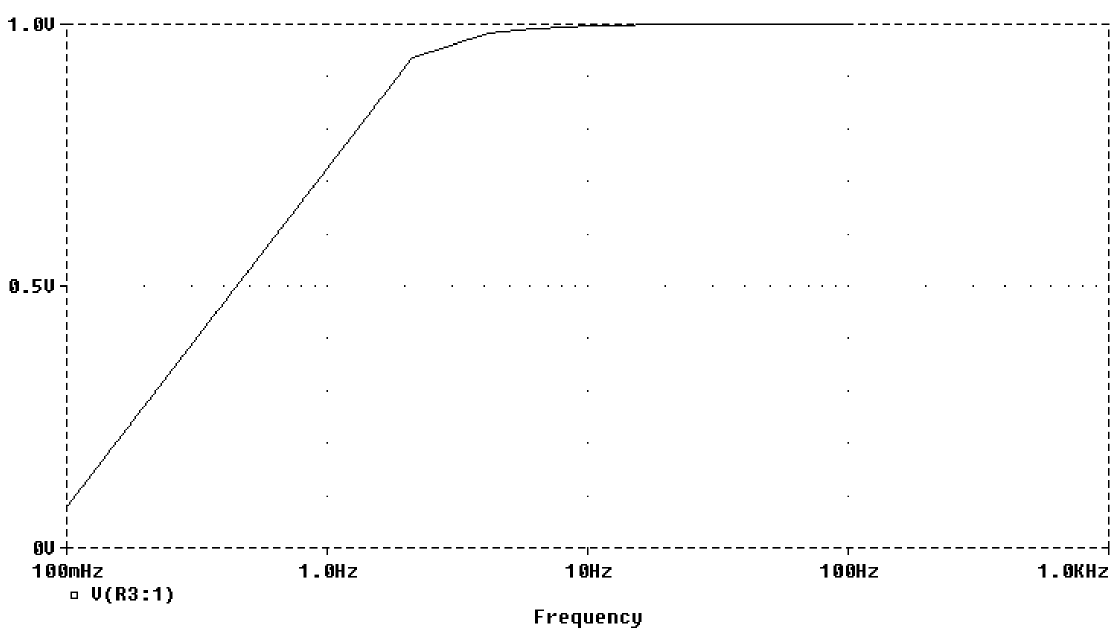

100mA
90mA
80mA
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
100KHz
I(L1)
Frequency


200
100
0
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
100KHz
I(L1) IP(L1)
Frequency

Solution 14.87

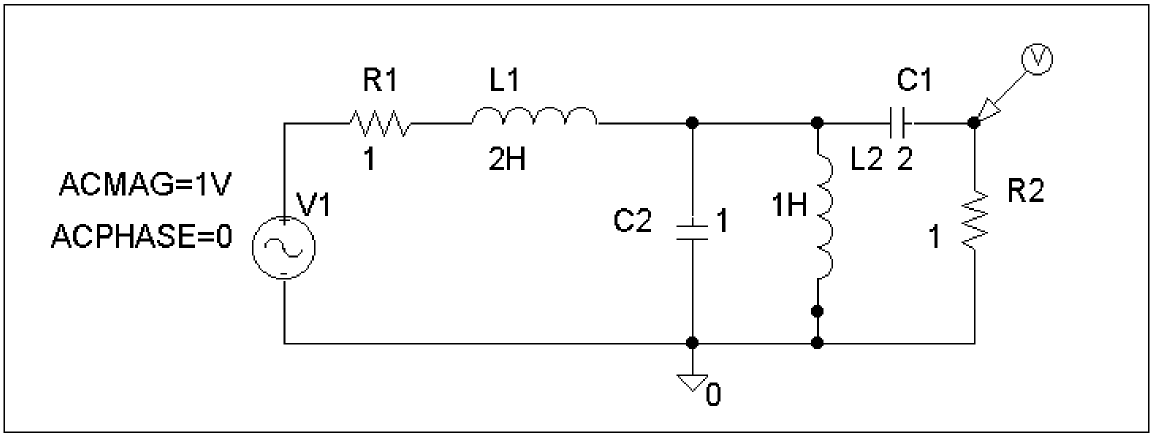
The schematic is shown below. In the AC Sweep box, we set Total Points = 50, Start Frequency = 1, and End Frequency = 100. After simulation, we obtain the magnitude response as shown below. It is evident from the response that the circuit represents a high-pass filter.

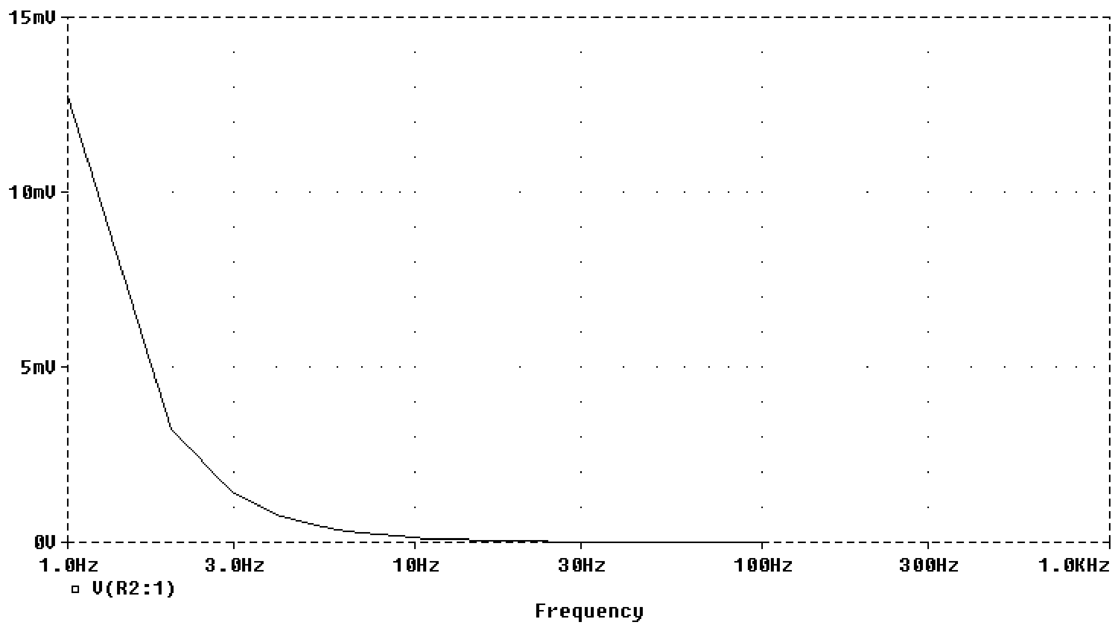


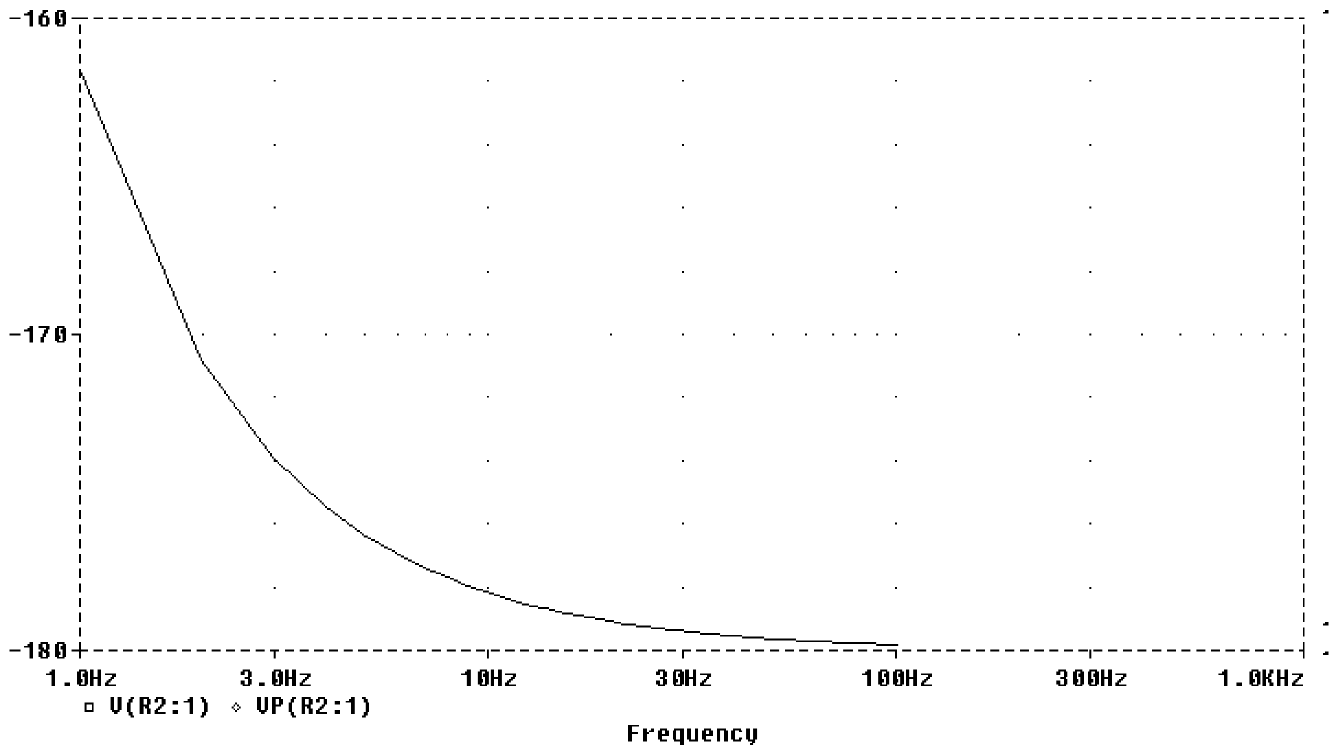


Solution 14.88

The schematic is shown below. We insert a voltage marker to measure V_o. In the AC Sweep box, we set Total Points = 101, Start Frequency = 1, and End Frequency = 100. After simulation, we obtain the magnitude and phase plots of V_o as shown below.

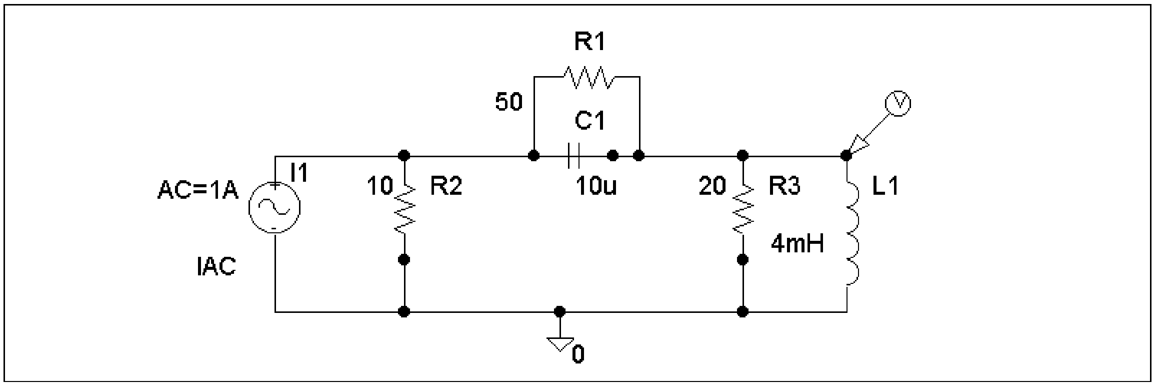


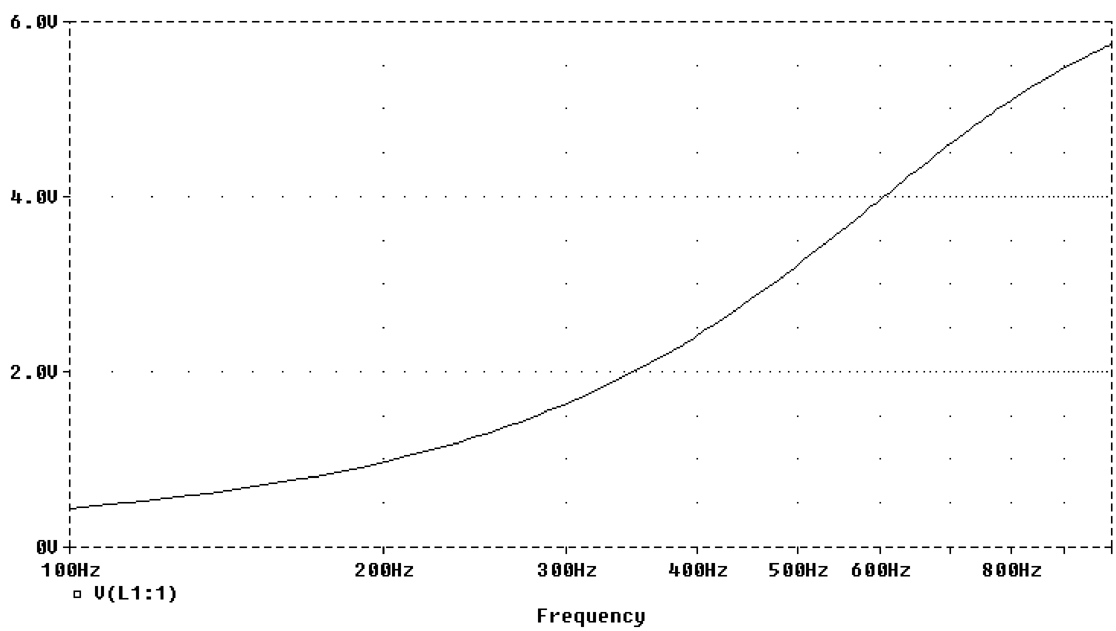



-160
-170
-180
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
V(R2:1) VP(R2:1)
Frequency

Solution 14.89

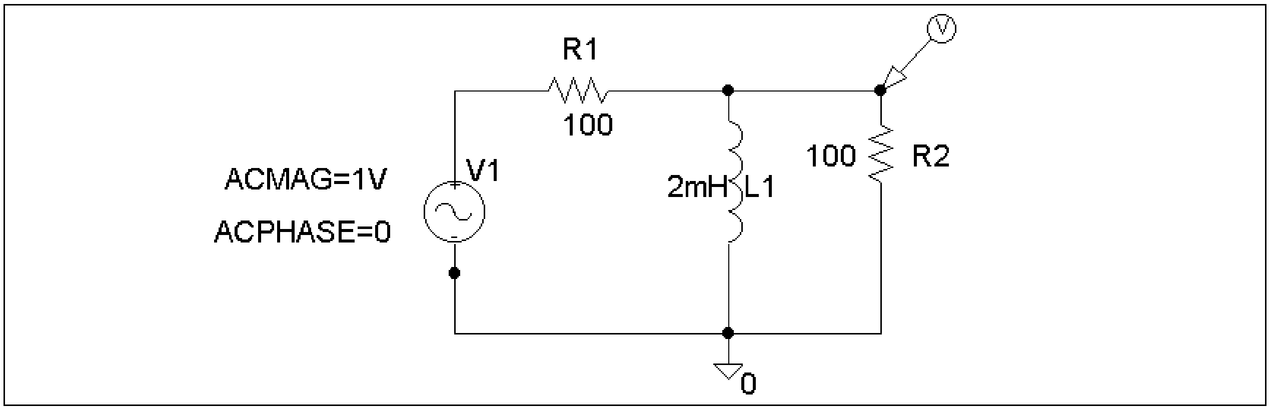
The schematic is shown below. In the AC Sweep box, we type Total Points = 101, Start Frequency = 100, and End Frequency = 1 k. After simulation, the magnitude plot of the response V_o is obtained as shown below.

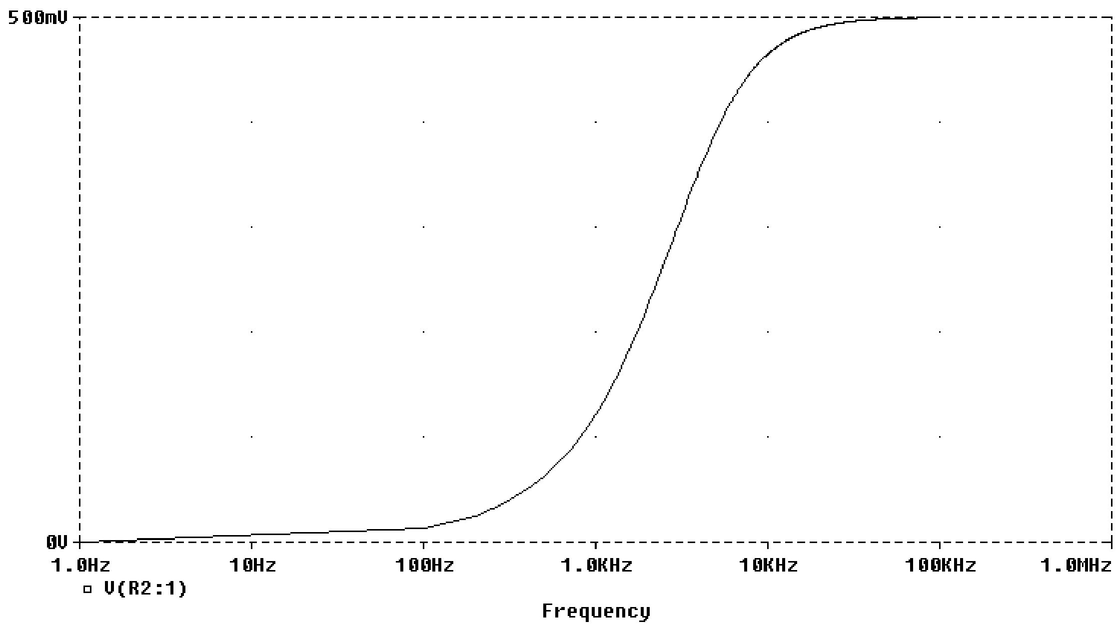




Solution 14.90

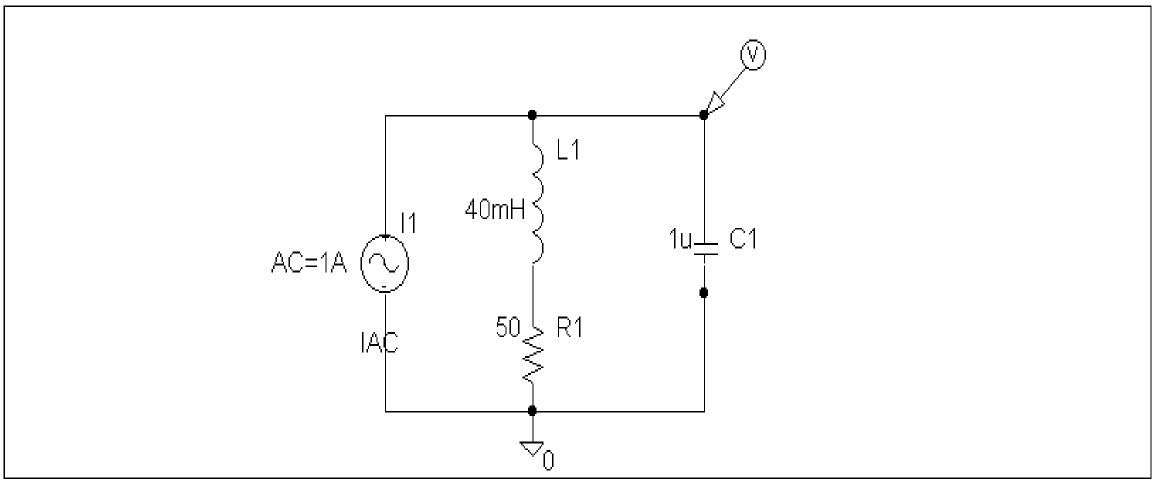
The schematic is shown below. In the AC Sweep box, we set
Total Points = 1001, Start Frequency = 1, and End Frequency = 100k. After simulation, we obtain the magnitude plot of the response as shown below. The response shows that the circuit is a high-pass filter.

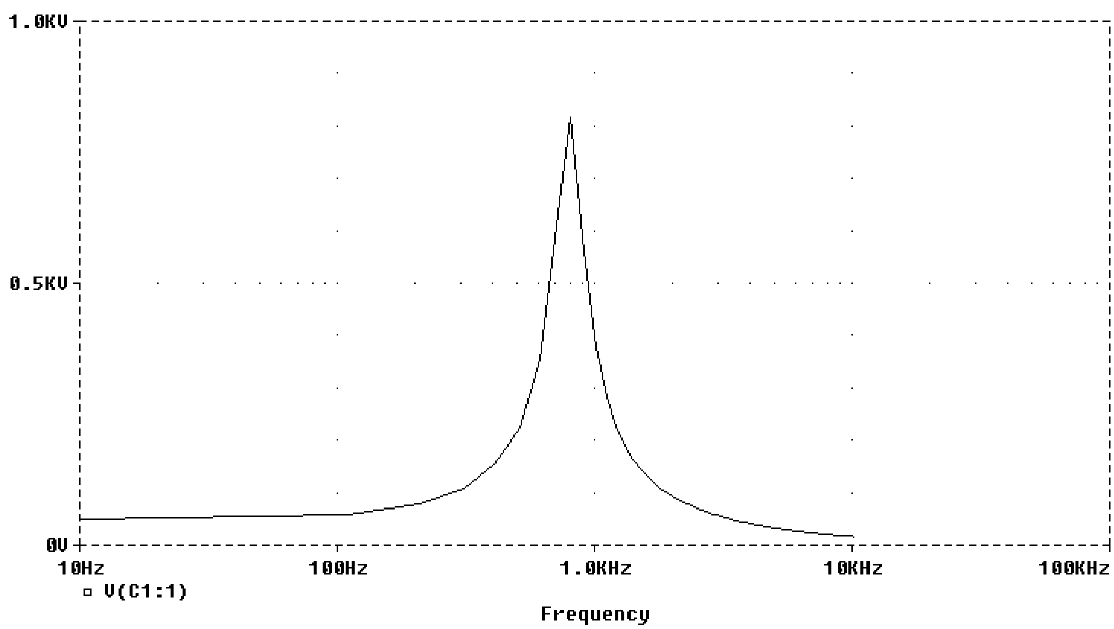




Solution 14.91

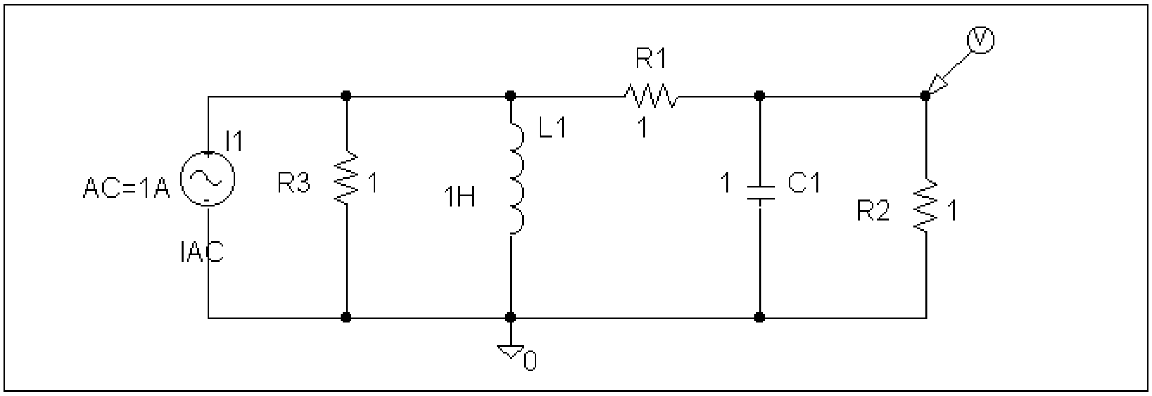
The schematic is shown below. In the AC Sweep box, we select Total Points = 101, Start Frequency = 10, and End Frequency = 10 k. After simulation, the magnitude plot of the frequency response is obtained. From the plot, we obtain the resonant frequency f_o is approximately equal to **800 Hz** so that $\omega_o = 2\pi f_o = 5026$ rad/s.

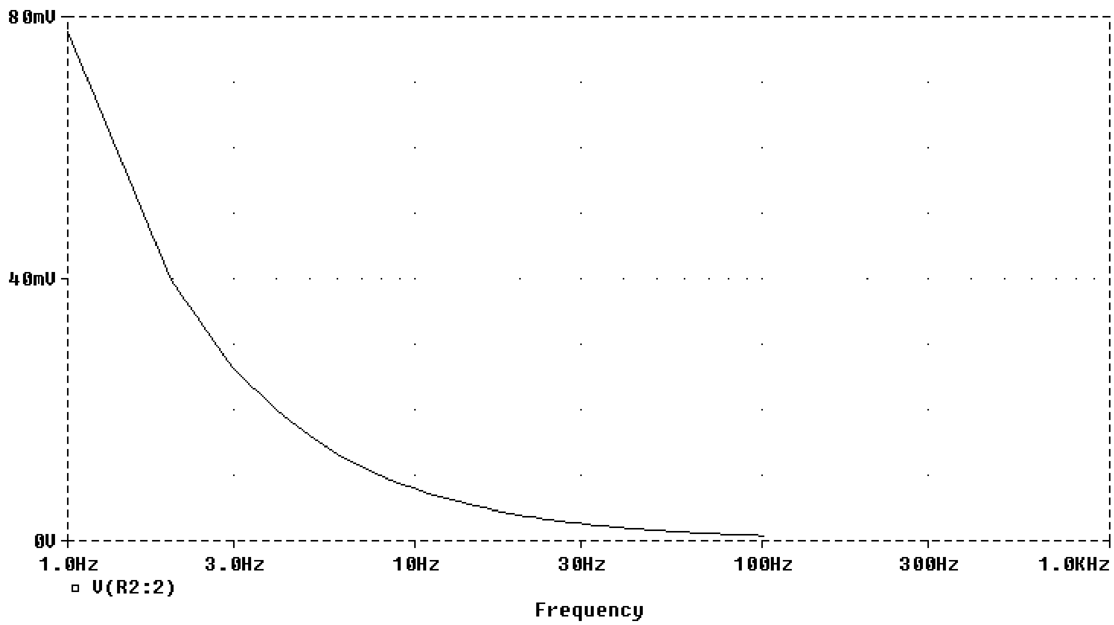




Solution 14.92

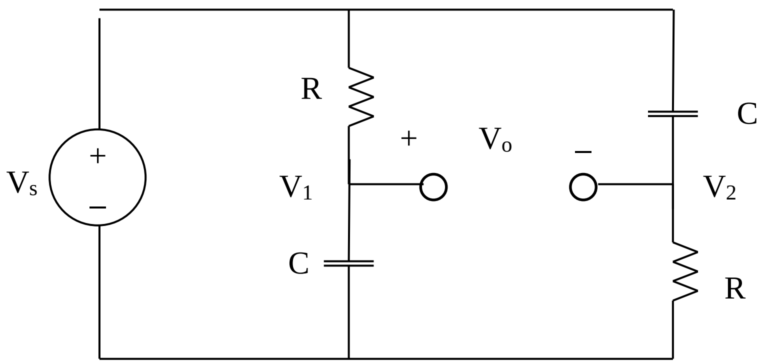
The schematic is shown below. We type Total Points = 101, Start Frequency = 1, and End Frequency = 100 in the AC Sweep box. After simulating the circuit, the magnitude plot of the frequency response is shown below.





Solution 14.93

Consider the circuit as shown below.



$$V_1 = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_s = \frac{V}{1 + sRC}$$

$$V_2 = \frac{R}{R + sC} V_s = \frac{sRC}{1 + sRC} V_s$$

$$V_o = V_1 - V_2 = \frac{1 - sRC}{1 + sRC} V_s$$

Hence,

$$H(s) = \frac{V_o}{V_s} = \underline{\frac{1 - sRC}{1 + sRC}}$$

Solution 14.94

$$\omega_C = \frac{1}{RC}$$

We make R and C as small as possible. To achieve this, we connect 1.8 $k\Omega$ and 3.3 $k\Omega$ in parallel so that

$$R = \frac{1.8x3.3}{1.8 + 3.3} = 1.164\,k\Omega$$

We place the 10-pF and 30-pF capacitors in series so that

C = (10x30)/40 = 7.5 pF

Hence,

$$\boldsymbol{\omega_c = \frac{1}{RC} = \frac{1}{1.164x10^3 x7.5x10^{-12}} = \underline{114.55x10^6 \text{ rad/s}}}$$

Solution 14.95

(a) $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

When $C = 360\text{ pF}$,

$$f_0 = \frac{1}{2\pi\sqrt{(240\times10^{-6})(360\times10^{-12})}} = 0.541\text{ MHz}$$

When $C = 40\text{ pF}$,

$$f_0 = \frac{1}{2\pi\sqrt{(240\times10^{-6})(40\times10^{-12})}} = 1.624\text{ MHz}$$

Therefore, the frequency range is

$$\mathbf{0.541\text{ MHz} < f_0 < 1.624\text{ MHz}}$$

(b) $$Q = \frac{2\pi fL}{R}$$

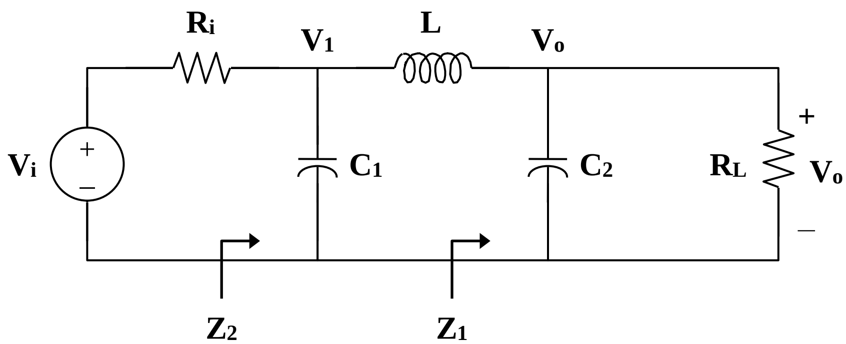
At $f_0 = 0.541\text{ MHz}$,

$$Q = \frac{(2\pi)(0.541\times10^{6})(240\times10^{-6})}{12} = \mathbf{67.98}$$

At $f_0 = 1.624\text{ MHz}$,

$$Q = \frac{(2\pi)(1.624\times10^{6})(240\times10^{-6})}{12} = \mathbf{204.1}$$

Solution 14.96



$$\mathbf{Z}_1 = R_L \parallel \frac{1}{sC_2} = \frac{R_L}{1 + sR_2C_2}$$

$$\mathbf{Z}_2 = \frac{1}{sC_1} \parallel (sL + \mathbf{Z}_1) = \frac{1}{sC_1} \parallel \left(\frac{sL + R_L + s^2R_LC_2L}{1 + sR_LC_2} \right)$$

$$\mathbf{Z}_2 = \frac{\dfrac{1}{sC_1} \cdot \dfrac{sL + R_L + s^2R_LC_2L}{1 + sR_LC_2}}{\dfrac{1}{sC_1} + \dfrac{sL + R_L + s^2R_LC_2L}{1 + sR_LC_2}}$$

$$\mathbf{Z}_2 = \frac{sL + R_L + s^2R_LLC_2}{1 + sR_LC_2 + s^2LC_1 + sR_LC_1 + s^3R_LLC_1C_2}$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_i}\mathbf{V}_i$$

$$\mathbf{V}_o = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL}\mathbf{V}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_2} \cdot \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL}\mathbf{V}_i$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_2} \cdot \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL}$$

where

$$\frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_2} =$$

$$\frac{sL + R_L + s^2R_LLC_2}{sL + R_L + s^2R_LLC_2 + R_i + sR_iR_LC_2 + s^2R_iLC_1 + sR_iR_LC_1 + s^3R_iR_LLC_1C_2}$$

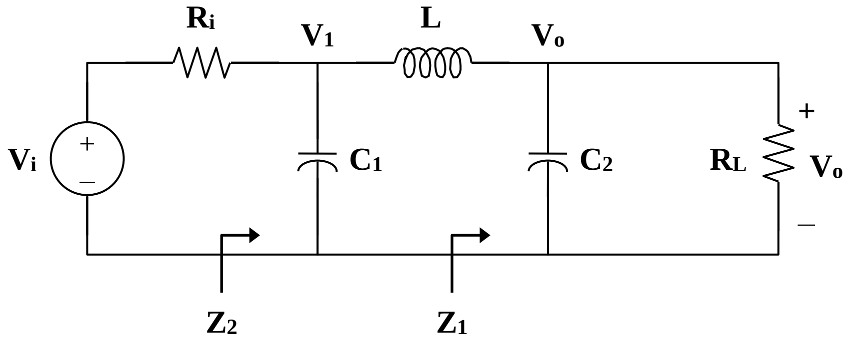
and $$\frac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL} = \frac{R_L}{R_L + sL + s^2R_LLC_2}$$

Therefore,

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{R_L(sL + R_L + s^2R_LLC_2)}}{\mathbf{(sL + R_L + s^2R_LLC_2 + R_i + sR_iR_LC_2 + s^2R_iLC_1 + sR_iR_LC_1 + s^3R_iR_LLC_1C_2)(R_L + sL + s^2R_LLC_2)}}$$

where $s = j\omega$.

Solution 14.97



$$\mathbf{Z} = sL \parallel \left(R_L + \frac{1}{sC_2} \right) = \frac{sL\,(R_L + 1/sC_2)}{R_L + sL + 1/sC_2}, \qquad s = j\omega$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}}{\mathbf{Z} + R_i + 1/sC_1}\mathbf{V}_i$$

$$\mathbf{V}_o = \frac{R_L}{R_L + 1/sC_2}\mathbf{V}_1 = \frac{R_L}{R_L + 1/sC_2} \cdot \frac{\mathbf{Z}}{\mathbf{Z} + R_i + 1/sC_1}\mathbf{V}_i$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_L}{R_L + 1/sC_2} \cdot \frac{sL\,(R_L + 1/sC_2)}{sL\,(R_L + 1/sC_2) + (R_i + 1/sC_1)(R_L + sL + 1/sC_2)}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{s^3 L R_L C_1 C_2}}{\mathbf{(sR_i C_1 + 1)(s^2 L C_2 + sR_L C_2 + 1) + s^2 L C_1 (sR_L C_2 + 1)}}$$

where $s = j\omega$.

Solution 14.98

$$B = \omega_2 - \omega_1 = 2\pi(f_2 - f_1) = 2\pi(454 - 432) = 44\pi$$

$$\omega_0 = 2\pi f_0 = QB = (20)(44\pi)$$

$$f_0 = \frac{(20)(44\pi)}{2\pi} = (20)(22) = \mathbf{440\ Hz}$$

Solution 14.99

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f\, C}$$

$$C = \frac{1}{2\pi f\, X_c} = \frac{1}{(2\pi)(2\times 10^6)(5\times 10^3)} = \frac{10^{-9}}{20\pi}$$

$$X_L = \omega L = 2\pi f\, L$$

$$L = \frac{X_L}{2\pi f} = \frac{300}{(2\pi)(2\times 10^6)} = \frac{3\times 10^{-4}}{4\pi}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\dfrac{3\times 10^{-4}}{4\pi}\cdot\dfrac{10^{-9}}{20\pi}}} = \mathbf{8.165\ MHz}$$

$$B = \frac{R}{L} = (100)\left(\frac{4\pi}{3\times 10^{-4}}\right) = \mathbf{4.188\times 10^{6}\ rad/s}$$

Solution 14.100

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$R = \frac{1}{2\pi f_c C} = \frac{1}{(2\pi)(20\times10^3)(0.5\times10^{-6})} = \mathbf{15.91\ \Omega}$$

Solution 14.101

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$R = \frac{1}{2\pi f_c C} = \frac{1}{(2\pi)(15)(10\times 10^{-6})} = \mathbf{1.061\ k\Omega}$$

Solution 14.102

(a) When $R_s = 0$ and $R_L = \infty$, we have a low-pass filter.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(4\times 10^3)(40\times 10^{-9})} = \mathbf{994.7\ Hz}$$

(b) We obtain R_{Th} across the capacitor.

$$R_{Th} = R_L \,\|\, (R + R_s)$$

$$R_{Th} = 5 \,\|\, (4+1) = 2.5\ \text{k}\Omega$$

$$f_c = \frac{1}{2\pi R_{Th} C} = \frac{1}{(2\pi)(2.5\times 10^3)(40\times 10^{-9})}$$

$$f_c = \mathbf{1.59\ kHz}$$

Solution 14.103

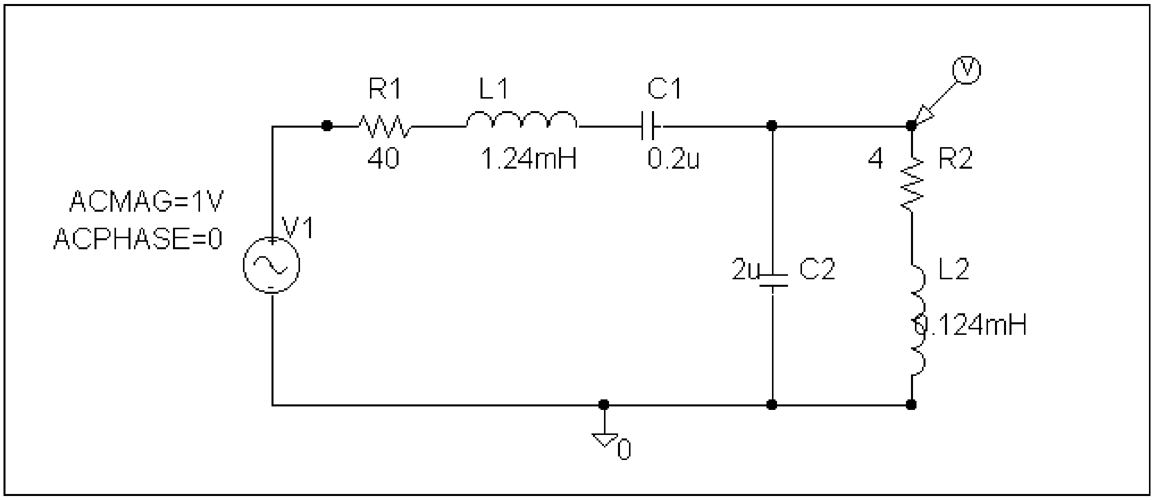
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_2}{R_2 + R_1 \,\|\, 1/j\omega C}, \qquad\qquad s = j\omega$$

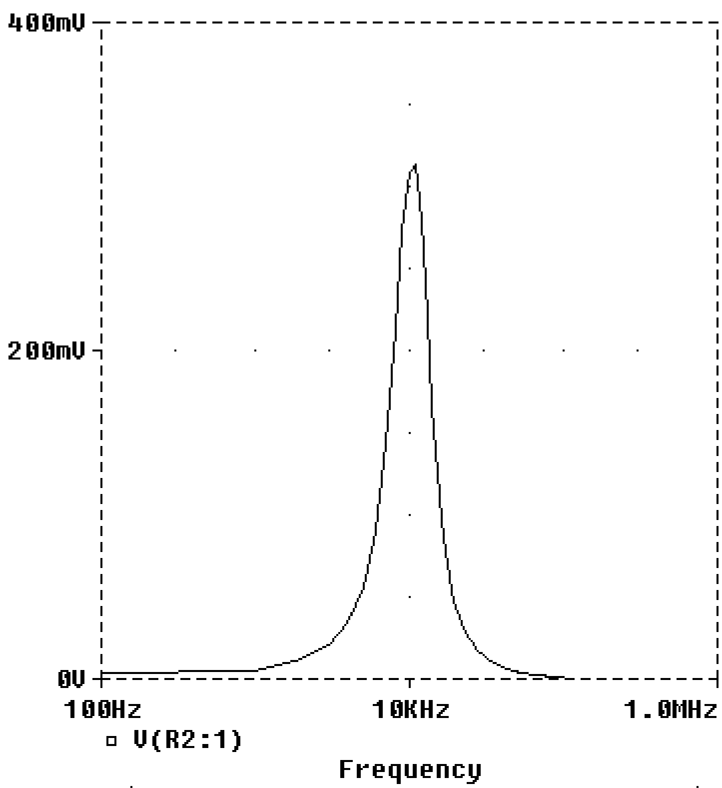
$$\mathbf{H}(s) = \frac{R_2}{R_2 + \dfrac{R_1(1/sC)}{R_1 + 1/sC}} = \frac{R_2\,(R_1 + 1/sC)}{R_1 R_2 + (R_1 + R_2)(1/sC)}$$

$$\mathbf{H}(s) = \frac{\mathbf{R_2(1 + sCR_1)}}{\mathbf{R_1 + R_2 + sCR_1R_2}}$$

Solution 14.104

The schematic is shown below. We click Analysis/Setup/AC Sweep and enter Total Points = 1001, Start Frequency = 100, and End Frequency = 100 k. After simulation, we obtain the magnitude plot of the response as shown.





Solution 15.1

Find the Laplace transform of 5 sin(at)cos(bt). [Hint: using the exponential representation for both functions may make this problem easier.]

Solution.

Step 1. Although we could work with trigonometric identities and the Laplace transforms for sin and cos, it is easier to follow using the exponential forms for sin and cos.

We note that $\cos(bt) = 0.5(e^{jbt} + e^{-jbt})$ and $\sin(at) = \cos(at–90°) = 0.5[e^{jat-90°}+e^{-jat+90°}] = j0.5[-e^{jat}+e^{-jat}]$. Now all we need to do is to multiply these together and take the Laplace transforms of the individual terms.

Step 2. First let us just do the sin and cos terms, thus, sin(at)cos(bt) $= 0.25\{j[-e^{jat}+e^{-jat}](e^{jbt} + e^{-jbt})\} = j0.25\{-e^{j(a+b)t}-e^{j(a-b)t}+e^{j(-a+b)t}+e^{j(-a-b)t}\}$ or $£(5\sin(at)\cos(bt)) = j1.25\{[-1/(s-j(a+b))] + [-1/(s-j(a-b))] + [1/(s+j(a-b))] + [1/(s+j(a+b))]\}$. This is our answer, however, let us simplify it.

Let us take the expressions two at a time, $[-1/(s-j(a+b))] + [1/(s+j(a+b))] = [-(s+j(a+b))+(s-j(a+b))]/[s^2+(a+b)^2] = [-j2(a+b)]/[s^2+(a+b)^2]$ and $[-1/(s-j(a-b))] + [1/(s+j(a-b))] = [-(s+j(a-b))+(s-j(a-b))]/[s^2+(a-b)^2] = [-j2(a-b)]/[s^2+(a-b)^2]$. Combining them and multiplying through by j1.25 we get,

$$£(5\sin(at)\cos(bt)) = \mathbf{[2.5(a+b)]/[s^2+(a+b)^2] + [2.5(a-b)]/[s^2+(a-b)^2]}.$$

Solution 15.2

Determine the Laplace transform of 3.5cos(5t–45°).

Solution

Step 1. We could use the tables to find this but let us go with the exponential form of the cosine function. Namely, $3.5\cos(5t–45°) = f(t) = 3.5[e^{–j5t+45°}+e^{j5t–45°}]/2 = 1.75[e^{45°}e^{–j5t}+e^{–45°}e^{j5t}]$. This is now easy to convert into the s-domain.

Step 2. $F(s) = 1.75\{[1\angle 45°/(s+j5)] + [1\angle –45°/(s–j5)]\}$. This is an answer, however, let us simplify it.

$$F(s) = 1.75\{[(0.7071+j0.7071)(s–j5) + (0.7071–j0.7071)(s+j5)]/(s^2+25)\}$$

$$= 1.75\{[1.4142s+7.071] /(s^2+25)\} = \mathbf{2.475(s+5) /(s^2+25)}.$$

Solution 15.3

(a) $$\mathcal{L}\left[\, e^{-2t}\cos(3t)\, u(t) \right] = \mathbf{\frac{s+2}{(s+2)^2+9}}$$

(b) $$\mathcal{L}\left[\, e^{-2t}\sin(4t)\, u(t) \right] = \mathbf{\frac{4}{(s+2)^2+16}}$$

(c) Since $\mathcal{L}\left[\cosh(at)\right] = \dfrac{s}{s^2-a^2}$

$$\mathcal{L}\left[\, e^{-3t}\cosh(2t)\, u(t) \right] = \mathbf{\frac{s+3}{(s+3)^2-4}}$$

(d) Since $\mathcal{L}\left[\sinh(at)\right] = \dfrac{a}{s^2-a^2}$

$$\mathcal{L}\left[\, e^{-4t}\sinh(t)\, u(t) \right] = \mathbf{\frac{1}{(s+4)^2-1}}$$

(e) $$\mathcal{L}\left[\, e^{-t}\sin(2t) \right] = \frac{2}{(s+1)^2+4}$$

If $\quad f(t) \longleftrightarrow F(s)$

$$t\, f(t) \longleftrightarrow \frac{-d}{ds}F(s)$$

Thus, $$\mathcal{L}\left[\, t\, e^{-t}\sin(2t) \right] = \frac{-d}{ds}\left[\, 2\left((s+1)^2+4\right)^{-1} \right]$$

$$= \frac{2}{((s+1)^2+4)^2}\cdot 2(s+1)$$

$$\mathcal{L}\left[\, t\, e^{-t}\sin(2t) \right] = \mathbf{\frac{4(s+1)}{((s+1)^2+4)^2}}$$

Solution 15.4

Design a problem to help other students better understand how to find the Laplace transform of different time varying functions.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Laplace transforms of the following:

(a) $g(t) = 6\cos(4t - 1)$
(b) $f(t) = 2tu(t) + 5e^{-3(t-2)}u(t-2)$

Solution

(a) $$\mathbf{G(s) = 6\frac{s}{s^2+4^2}e^{-s} = \underline{\frac{6se^{-s}}{s^2+16}}}$$

(b) $$\mathbf{F(s) = \underline{\frac{2}{s^2} + 5\frac{e^{-2s}}{s+3}}}$$

Solution 15.5

(a) $$\mathcal{L}\left[\cos(2t+30°)\right] = \frac{s\cos(30°) - 2\sin(30°)}{s^2+4}$$

$$\mathcal{L}\left[t^2\cos(2t+30°)\right] = \frac{d^2}{ds^2}\left[\frac{s\cos(30°)-1}{s^2+4}\right]$$

$$= \frac{d}{ds}\frac{d}{ds}\left[\left(\frac{\sqrt{3}}{2}s-1\right)\left(s^2+4\right)^{-1}\right]$$

$$= \frac{d}{ds}\left[\frac{\sqrt{3}}{2}\left(s^2+4\right)^{-1} - 2s\left(\frac{\sqrt{3}}{2}s-1\right)\left(s^2+4\right)^{-2}\right]$$

$$= \frac{\frac{\sqrt{3}}{2}(-2s)}{\left(s^2+4\right)^2} - \frac{2\left(\frac{\sqrt{3}}{2}s-1\right)}{\left(s^2+4\right)^2} - \frac{2s\left(\frac{\sqrt{3}}{2}\right)}{\left(s^2+4\right)^2} + \frac{(8s^2)\left(\frac{\sqrt{3}}{2}s-1\right)}{\left(s^2+4\right)^3}$$

$$= \frac{-\sqrt{3}s - \sqrt{3}s + 2 - \sqrt{3}s}{\left(s^2+4\right)^2} + \frac{(8s^2)\left(\frac{\sqrt{3}}{2}s-1\right)}{\left(s^2+4\right)^3}$$

$$= \frac{(-3\sqrt{3}s+2)(s^2+4)}{\left(s^2+4\right)^3} + \frac{4\sqrt{3}s^3 - 8s^2}{\left(s^2+4\right)^3}$$

$$\mathcal{L}\left[t^2\cos(2t+30°)\right] = \mathbf{\frac{8 - 12\sqrt{3}s - 6s^2 + \sqrt{3}s^3}{\left(s^2+4\right)^3}}$$

(b) $$\mathcal{L}\left[3t^4 e^{-2t}\right] = 3\cdot\frac{4!}{(s+2)^5} = \mathbf{\frac{72}{(s+2)^5}}$$

(c) $$\mathcal{L}\left[2t\,u(t) - 4\frac{d}{dt}\delta(t)\right] = \frac{2}{s^2} - 4(s\cdot 1 - 0) = \mathbf{\frac{2}{s^2} - 4s}$$

(d) $$2e^{-(t-1)}\,u(t) = 2e^{-t}\,u(t)$$

$$\mathcal{L}\left[2e^{-(t-1)}\,u(t)\right] = \mathbf{\frac{2e}{s+1}}$$

(e) Using the scaling property,

$$\mathcal{L}\left[5u(t/2)\right] = 5\cdot\frac{1}{1/2}\cdot\frac{1}{s/(1/2)} = 5\cdot 2\cdot\frac{1}{2s} = \mathbf{\frac{5}{s}}$$

(f) $$\mathbf{L}\left[6e^{-t/3}\,u(t)\right] = \frac{6}{s+1/3} = \mathbf{\frac{18}{3s+1}}$$

(g) Let $f(t) = \delta(t)$. Then, $F(s) = 1$.

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \mathcal{L}\left[\frac{d^n}{dt^n}f(t)\right] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \cdots$$

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \mathcal{L}\left[\frac{d^n}{dt^n}f(t)\right] = s^n \cdot 1 - s^{n-1} \cdot 0 - s^{n-2} \cdot 0 - \cdots$$

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \mathbf{s^n}$$

Solution 15.6

Find G(s) given that g(t) = 2r(t) – 2r(t–2).

Solution

Step 1. To make this easier to solve we just use the relationship that r(t) = tu(t).

Thus, r(t–2) = (t–2)u(t–2). Now all we need to find the Laplace transform.

Step 2. $G(s) = (2/s^2) - 2e^{-2s}(1/s^2) = \mathbf{2(1-e^{-2s})(1/s^2)}$.

Solution 15.7

(a) $F(s)=\dfrac{2}{s^2}+\dfrac{4}{s}$

(b) $G(s)=\dfrac{4}{s}+\dfrac{3}{s+2}$

(c) $\mathbf{H(s)=6\dfrac{3}{s^2+9}+8\dfrac{s}{s^2+9}=\dfrac{8s+18}{s^2+9}}$

(d) From Problem 15.1,

$$L\{\cosh at\}=\frac{s}{s^2-a^2}$$

$$X(s)=\frac{s+2}{(s+2)^2-4^2}=\frac{s+2}{s^2+4s-12}$$

$$\mathbf{(a)\ \frac{2}{s^2}+\frac{4}{s},(b)\frac{4}{s}+\frac{3}{s+2},(c)\frac{8s+18}{s^2+9},(d)\frac{s+2}{s^2+4s-12}}$$

Solution 15.8

(a) 2t=2(t-4) + 8

f(t) = 2tu(t-4) = 2(t-4)u(t-4) + 8u(t-4)

$$F(s) = \frac{2}{s^2}e^{-4s} + \frac{8}{s}e^{-4s} = \underline{\left(\frac{2}{s^2} + \frac{8}{s}\right)e^{-4s}}$$

(b) $F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^\infty 5\cos t\delta(t-2)e^{-st}dt = 5\cos te^{-st}\Big|_{t=2} = \underline{5\cos 2e}$ $\underline{\mathbf{5cos(2)e^{-2s}}}$

(c)

$$e^{-t} = e^{-(t-\tau)}e^{-\tau}$$

$$f(t) = e^{-\tau}e^{-(t-\tau)}u(t-\tau)$$

$$F(s) = e^{-\tau}e^{-\tau s}\frac{1}{s+1} = \underline{\frac{e^{-\tau(s+1)}}{s+1}}$$

(d) $\sin 2t = \sin[2(t-\tau)+2\tau] = \sin 2(t-\tau)\cos 2\tau + \cos 2(t-\tau)\sin 2\tau$

$$f(t) = \cos 2\tau \sin 2(t-\tau)u(t-\tau) + \sin 2\tau \cos 2(t-\tau)u(t-\tau)$$

$$F(s) = \underline{\cos 2\tau e^{-\tau s}\frac{2}{s^2+4} + \sin 2\tau e^{-\tau s}\frac{s}{s^2+4}}$$

Solution 15.9

(a) $f(t) = (t-4)\,u(t-2) = (t-2)\,u(t-2) - 2\,u(t-2)$

$$F(s) = \boldsymbol{\frac{e^{-2s}}{s^2} - \frac{2e^{-2s}}{s}}$$

(b) $g(t) = 2e^{-4t}\,u(t-1) = 2e^{-4}\,e^{-4(t-1)}\,u(t-1)$

$$G(s) = \mathbf{\frac{2e^{-s}}{e^4\,(s+4)}}$$

(c) $h(t) = 5\cos(2t-1)\,u(t)$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\cos(2t-1) = \cos(2t)\cos(1) + \sin(2t)\sin(1)$$

$$h(t) = 5\cos(1)\cos(2t)\,u(t) + 5\sin(1)\sin(2t)\,u(t)$$

$$H(s) = 5\cos(1)\cdot\frac{s}{s^2+4} + 5\sin(1)\cdot\frac{2}{s^2+4}$$
$$H(s) = \mathbf{\frac{2.702\,s}{s^2+4} + \frac{8.415}{s^2+4}}$$

(d) $p(t) = 6u(t-2) - 6u(t-4)$

$$P(s) = \mathbf{\frac{6}{s}e^{-2s} - \frac{6}{s}e^{-4s}}$$

Solution 15.10

(a) By taking the derivative in the time domain,

$$g(t) = (-t\,e^{-t} + e^{-t})\cos(t) - t\,e^{-t}\sin(t)$$
$$g(t) = e^{-t}\cos(t) - t\,e^{-t}\cos(t) - t\,e^{-t}\sin(t)$$

$$G(s) = \frac{s+1}{(s+1)^2+1} + \frac{d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right] + \frac{d}{ds}\left[\frac{1}{(s+1)^2+1}\right]$$

$$G(s) = \frac{s+1}{s^2+2s+2} - \frac{s^2+2s}{(s^2+2s+2)^2} - \frac{2s+2}{(s^2+2s+2)^2} =$$

$$\mathbf{\frac{s^2(s+2)}{(s^2+2s+2)^2}}$$

(b) By applying the time differentiation property,

$$G(s) = sF(s) - f(0)$$

where $f(t) = t\,e^{-t}\cos(t)$, $f(0) = 0$

$$G(s) = (s)\cdot\frac{-d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right] = \frac{(s)(s^2+2s)}{(s^2+2s+2)^2} =$$

$$\mathbf{\frac{s^2(s+2)}{(s^2+2s+2)^2}}$$

Solution 15.11

(a) Since $\mathcal{L}\left[\cosh(at)\right] = \dfrac{s}{s^2 - a^2}$

$$F(s) = \frac{6(s+1)}{(s+1)^2 - 4} = \mathbf{\frac{6(s+1)}{s^2 + 2s - 3}}$$

(b) Since $\mathcal{L}\left[\sinh(at)\right] = \dfrac{a}{s^2 - a^2}$

$$\mathcal{L}\left[3e^{-2t}\sinh(4t)\right] = \frac{(3)(4)}{(s+2)^2 - 16} = \frac{12}{s^2 + 4s - 12}$$

$$F(s) = \mathcal{L}\left[t \cdot 3e^{-2t}\sinh(4t)\right] = \frac{-d}{ds}\left[12(s^2 + 4s - 12)^{-1}\right]$$

$$F(s) = (12)(2s+4)(s^2 + 4s - 12)^{-2} = \mathbf{\frac{24(s+2)}{(s^2 + 4s - 12)^2}}$$

(c) $\cosh(t) = \dfrac{1}{2} \cdot (e^t + e^{-t})$

$$f(t) = 8e^{-3t} \cdot \frac{1}{2} \cdot (e^t + e^{-t})\,u(t-2)$$

$$= 4e^{-2t}\,u(t-2) + 4e^{-4t}\,u(t-2)$$

$$= 4e^{-4}\,e^{-2(t-2)}\,u(t-2) + 4e^{-8}\,e^{-4(t-2)}\,u(t-2)$$

$$\mathcal{L}\left[4e^{-4}\,e^{-2(t-2)}\,u(t-2)\right] = 4e^{-4}\,e^{-2s} \cdot \mathcal{L}\left[e^{-2}\,u(t)\right]$$

$$\mathcal{L}\left[4e^{-4}\,e^{-2(t-2)}\,u(t-2)\right] = \frac{4e^{-(2s+4)}}{s+2}$$

Similarly, $\mathcal{L}\left[4e^{-8}\,e^{-4(t-2)}\,u(t-2)\right] = \dfrac{4e^{-(2s+8)}}{s+4}$

Therefore,

$$F(s) = \frac{4e^{-(2s+4)}}{s+2} + \frac{4e^{-(2s+8)}}{s+4} = \mathbf{\frac{e^{-(2s+6)}\left[(4e^2 + 4e^{-2})s + (16e^2 + 8e^{-2})\right]}{s^2 + 6s + 8}}$$

Solution 15.12

If $g(t) = 4e^{-2t}\cos(4t)$, find G(s).

Solution

From the table 15.1, G(s) = $\dfrac{4(s+2)}{(s+2)^2+4^2} = \dfrac{4(\mathbf{s}+2)}{\mathbf{s}^2+4\mathbf{s}+20}$.

Solution 15.13

(a) $tf(t) \quad \longleftrightarrow \quad -\frac{d}{ds}F(s)$

If f(t) = cost, then $F(s) = \frac{s}{s^2+1}$ and $-\frac{d}{ds}F(s) = -\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}$

$$\underline{\mathbf{L(t\cos t) = \frac{s^2-1}{(s^2+1)^2}}}$$

(b) Let f(t) = e^{-t} sin t.

$$F(s) = \frac{1}{(s+1)^2+1} = \frac{1}{s^2+2s+2}$$

$$\frac{dF}{ds} = \frac{(s^2+2s+2)(0) - (1)(2s+2)}{(s^2+2s+2)^2}$$

$$\underline{\mathbf{L(e^{-t}t\sin t) = -\frac{dF}{ds} = \frac{2(s+1)}{(s^2+2s+2)^2}}}$$

(c) $\frac{f(t)}{t} \quad \longleftrightarrow \quad \int_s^\infty F(s)ds$

Let $f(t) = \sin\beta t$, then $F(s) = \frac{\beta}{s^2+\beta^2}$

$$\mathbf{L\left[\frac{\sin\beta t}{t}\right] = \int_s^\infty \frac{\beta}{s^2+\beta^2}ds = \beta\frac{1}{\beta}\tan^{-1}\frac{s}{\beta}\Big|_s^\infty = \frac{\pi}{2} - \tan^{-1}\frac{s}{\beta} = \underline{\tan^{-1}\frac{\beta}{s}}}$$

Solution 15.14

Find the Laplace transform of the signal in Fig. 15.26.

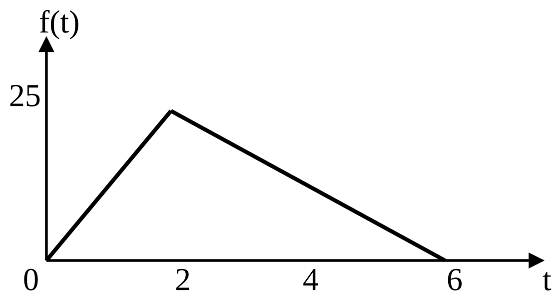
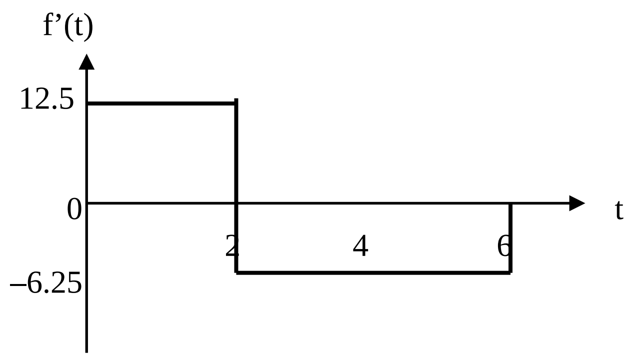


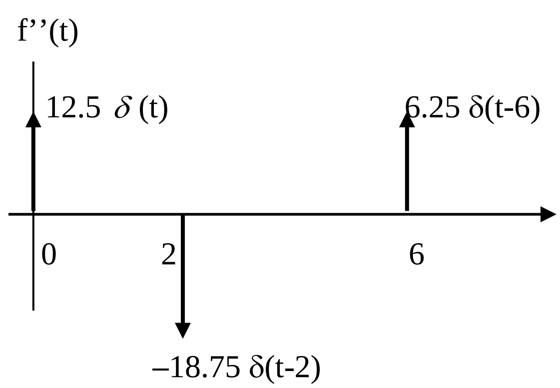
Figure 15.26
For Prob. 15.14.

Solution

Taking the derivative of f(t) twice, we obtain the figures below.



f' = 12.5u(t) – 18.75u(t–2) + 6.25u(t–6)



f" = 12.5δ(t) – 18.75δ(t–2) + 6.25δ(t–6)

Taking the Laplace transform of each term,

$$s^2F(s) = 12.5 - 18.75e^{-2s} + 6.25e^{-6s} \text{ or}$$

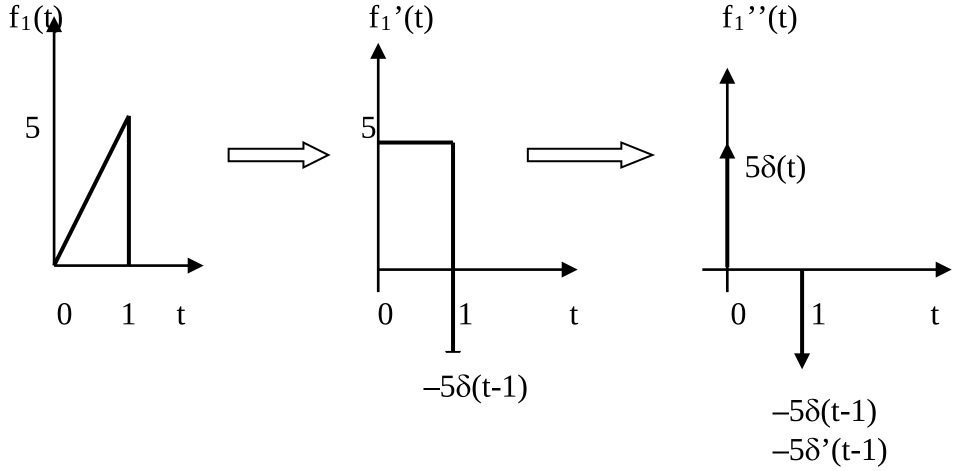
$$F(s) = \mathbf{[12.5 - 18.75e^{-2s} + 6.25e^{-6s}]/s^2}.$$

Solution 15.15

This is a periodic function with T=3.

$$F(s) = \frac{F_1(s)}{1 - e^{-3s}}$$

To get $F_1(s)$, we consider f(t) over one period.



$$f_1'' = 5\delta(t) - 5\delta(t-1) - 5\delta'(t-1)$$

Taking the Laplace transform of each term,

$$s^2 F_1(s) = 5 - 5e^{-s} - 5se^{-s} \quad \text{or} \quad F_1(s) = 5(1 - e^{-s} - se^{-s})/s^2$$

Hence,

$$F(s) = \mathbf{5\frac{1 - e^{-s} - se^{-s}}{s^2(1 - e^{-3s})}}$$

Alternatively, we can obtain the same answer by noting that
$f_1(t) = 5tu(t) - 5tu(t-1) - 5u(t-1)$.

Solution 15.16

Obtain the Laplace transform of $f(t)$ in Fig. 15.28.

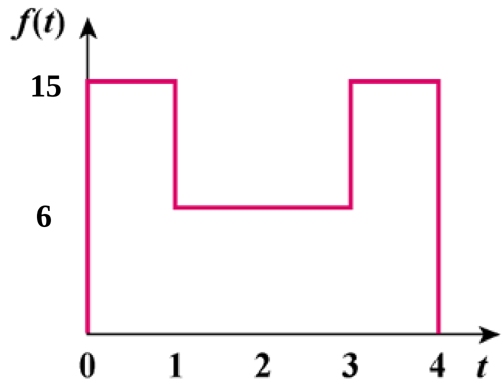


Figure 15.28
For Prob. 15.16.

Solution

$$f(t) = 15u(t) - 9u(t-1) + 9u(t-3) - 15u(t-4)$$

$$F(s) = \mathbf{[15 - 9e^{-s} + 9e^{-3s} - 15e^{-4s}]/s}.$$

Solution 15.17

Using Fig. 15.29, design a problem to help other students to better understand the Laplace transform of a simple, non-periodic waveshape.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Laplace transform of f(t) shown in Fig. 15.29.

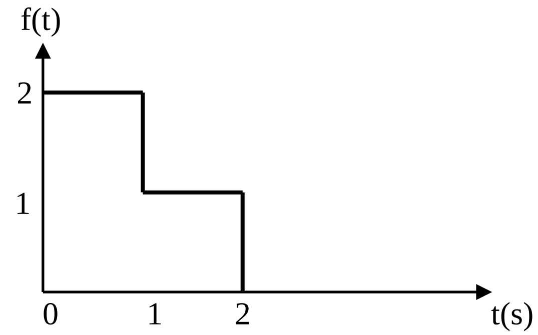
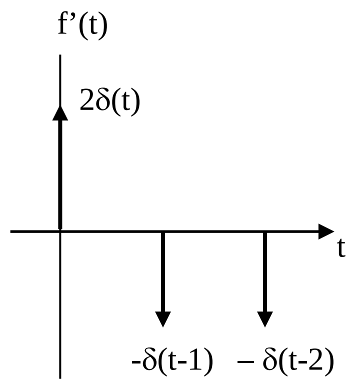


Figure 15.29 For Prob. 15.17.

Solution

Taking the derivative of f(t) gives f'(t) as shown below.



$$f'(t) = 2\delta(t) - \delta(t-1) - \delta(t-2)$$

Taking the Laplace transform of each term,

$$sF(s) = 2 - e^{-s} - e^{-2s}$$ which leads to

$$F(s) = \mathbf{[2 - e^{-s} - e^{-2s}]/s}$$

We can also obtain the same answer noting that $f(t) = 2u(t) - u(t-1) - u(t-2)$.

Solution 15.18

(a) $$g(t) = u(t) - u(t-1) + 2\left[\, u(t-1) - u(t-2)\right] + 3\left[\, u(t-2) - u(t-3)\right]$$
$$= u(t) + u(t-1) + u(t-2) - 3u(t-3)$$

$$G(s) = \mathbf{\frac{1}{s}(1 + e^{-s} + e^{-2s} - 3e^{-3s})}$$

(b) $$h(t) = 2t\left[\, u(t) - u(t-1)\right] + 2\left[\, u(t-1) - u(t-3)\right]$$
$$+ (8-2t)\left[\, u(t-3) - u(t-4)\right]$$

$$= 2t\,u(t) - 2(t-1)u(t-1) - 2u(t-1) + 2u(t-1) - 2u(t-3)$$
$$- 2(t-3)u(t-3) + 2u(t-3) + 2(t-4)u(t-4)$$

$$= 2t\,u(t) - 2(t-1)u(t-1) - 2(t-3)u(t-3) + 2(t-4)u(t-4)$$

$$H(s) = \frac{2}{s^2}(1 - e^{-s}) - \frac{2}{s^2}e^{-3s} + \frac{2}{s^2}e^{-4s} = \mathbf{\frac{2}{s^2}(1 - e^{-s} - e^{-3s} + e^{-4s})}$$

Solution 15.19

Since $L\left[\delta(t)\right] = 1$ and $T = 2$, $F(s) = \dfrac{\mathbf{1}}{\mathbf{1 - e^{-2s}}}$

Solution 15.20

Using Fig. 15.32, design a problem to help other students to better understand the Laplace transform of a simple, periodic waveshape.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The periodic function shown in Fig. 15.32 is defined over its period as

$$g(t) = \begin{cases} \sin \pi t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

Find $G(s)$.

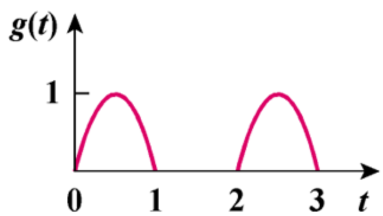


Figure 15.32

Solution

Let $\quad g_1(t) = \sin(\pi t), \quad 0 < t < 1$

$$= \sin(\pi t)\left[u(t) - u(t-1)\right] \; 0 < t < 2$$

$$= \sin(\pi t)\,u(t) - \sin(\pi t)\,u(t-1)$$

Note that $\sin(\pi(t-1)) = \sin(\pi t - \pi) = -\sin(\pi t)$.

So, $\quad g_1(t) = \sin(\pi t)\,u(t) + \sin(\pi(t-1))\,u(t-1)$

$$G_1(s) = \frac{\pi}{s^2 + \pi^2}(1 + e^{-s})$$

$$G(s) = \frac{G_1(s)}{1 - e^{-2s}} = \boldsymbol{\frac{\pi(1 + e^{-s})}{(s^2 + \pi^2)(1 - e^{-2s})}}$$

Solution 15.21

$$T = 2\pi$$

Let $$f_1(t) = \left(1 - \frac{t}{2\pi}\right)\left[u(t) - u(t-2\pi)\right]$$

$$f_1(t) = u(t) - \frac{t}{2\pi}u(t) + \frac{1}{2\pi}(t-2\pi)\,u(t-2\pi)$$

$$F_1(s) = \frac{1}{s} - \frac{1}{2\pi s^2} + \frac{e^{-2\pi s}}{2\pi s^2} = \frac{2\pi\, s + \left[-1 + e^{-2\pi s}\right]}{2\pi s^2}$$

$$F(s) = \frac{F_1(s)}{1 - e^{-Ts}} = \mathbf{\frac{2\pi s - 1 + e^{-2\pi s}}{2\pi s^2\,(1 - e^{-2\pi s})}}$$

Solution 15.22

(a) Let $$\begin{aligned} g_1(t) &= 2t, \quad 0 < t < 1 \\ &= 2t\left[u(t) - u(t-1)\right] \\ &= 2t\,u(t) - 2(t-1)u(t-1) + 2u(t-1) \end{aligned}$$

$$G_1(s) = \frac{2}{s^2} - \frac{2e^{-s}}{s^2} + \frac{2}{s}e^{-s}$$

$$G(s) = \frac{G_1(s)}{1 - e^{-sT}}, \quad T = 1$$

$$\mathbf{G(s) = \frac{2(1 - e^{-s} + s\,e^{-s})}{s^2(1 - e^{-s})}}$$

(b) Let $h = h_0 + u(t)$, where h_0 is the periodic triangular wave.

Let h_1 be h_0 within its first period, i.e.

$$h_1(t) = \begin{cases} 2t & 0 < t < 1 \\ 4 - 2t & 1 < t < 2 \end{cases}$$

$$h_1(t) = 2t\,u(t) - 2t\,u(t-1) + 4u(t-1) - 2t\,u(t-1) - 2(t-2)u(t-2)$$
$$h_1(t) = 2t\,u(t) - 4(t-1)u(t-1) - 2(t-2)u(t-2)$$

$$H_1(s) = \frac{2}{s^2} - \frac{4}{s^2}e^{-s} - \frac{2e^{-2s}}{s^2} = \frac{2}{s^2}(1 - e^{-s})^2$$

$$H_0(s) = \frac{2}{s^2}\frac{(1 - e^{-s})^2}{(1 - e^{-2s})}$$

$$\mathbf{H(s) = \frac{1}{s} + \frac{2}{s^2}\frac{(1 - e^{-s})^2}{(1 - e^{-2s})}}$$

Solution 15.23

(a) Let $f_1(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \end{cases}$

$$f_1(t) = [\,u(t) - u(t-1)] - [\,u(t-1) - u(t-2)]$$
$$f_1(t) = u(t) - 2\,u(t-1) + u(t-2)$$

$$F_1(s) = \frac{1}{s}(1 - 2\,e^{-s} + e^{-2s}) = \frac{1}{s}(1 - e^{-s})^2$$

$$F(s) = \frac{F_1(s)}{(1 - e^{-sT})}, \quad T = 2$$
$$F(s) = \frac{\mathbf{(1 - e^{-s})^2}}{\mathbf{s\,(1 - e^{-2s})}}$$

(b) Let $h_1(t) = t^2\,[\,u(t) - u(t-2)] = t^2\,u(t) - t^2\,u(t-2)$

$$h_1(t) = t^2\,u(t) - (t-2)^2\,u(t-2) - 4\,(t-2)\,u(t-2) - 4\,u(t-2)$$

$$H_1(s) = \frac{2}{s^3}(1 - e^{-2s}) - \frac{4}{s^2}e^{-2s} - \frac{4}{s}e^{-2s}$$

$$H(s) = \frac{H_1(s)}{(1 - e^{-Ts})}, \quad T = 2$$
$$H(s) = \frac{\mathbf{2\,(1 - e^{-2s}) - 4s\,e^{-2s}\,(s + s^2)}}{\mathbf{s^3\,(1 - e^{-2s})}}$$

Solution 15.24

Design a problem to help other students to better understand how to find the initial and final values of a transfer function.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Given that

$$F(s) = \frac{s^2 + 10s + 6}{s(s+1)^2(s+2)}$$

Evaluate f(0) and f(∞) if they exist.

Solution

$$f(0) = \left[\lim_{n \to \infty} \left(\frac{s(s^2 + 10s + 6)}{s(s+1)^2 (s+2)} \right)\right] = \mathbf{0}$$

$$f(\infty) = \lim_{s \to 0} sF(s) = \lim_{s \to 0} \frac{s^2 + 10s + 6}{(s+1)^2(s+2)} = \frac{6}{(1)(2)} = \underline{3} = \mathbf{3}$$

Solution 15.25

Let $F(s) = \dfrac{18(s+1)}{(s+2)(s+3)}$.

(a) Use the initial and final value theorems to find *f(0)* and *f(∞)*.
(b) Verify your answer in part (a) by finding *f(t)* using partial fractions.

Solution

(a) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{18s(s+1)}{(s+2)(s+3)} = \mathbf{18}.$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{18s(s+1)}{(s+2)(s+3)} = \mathbf{0}.$$

(*b*) $f(s) = \dfrac{18(s+1)}{(s+2)(s+3)} = \dfrac{A}{s+2} + \dfrac{B}{s+3}$ where A = 18(–2+1)/(–2+3) = –18 and B = 18(–3+1)/(–3+2) = 36 therefore f(t) = $[-18e^{-2t} + 36e^{-3t}]u(t)$ which gives us f(0) = 18 and f(∞) = 0 and our answers check!

Solution 15.26

(a) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{5s^3 + 3s}{s^3 + 4s^2 + 6} = \mathbf{5}$$

Two poles are not in the left-half plane.
$f(\infty)$ **does not exist**

(b) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^3 - 2s^2 + s}{4(s-2)(s^2 + 2s + 4)}$$

$$= \lim_{s\to\infty} \frac{1 - \frac{2}{s} + \frac{1}{s^2}}{\left(1 - \frac{2}{s}\right)\left(1 + \frac{2}{s} + \frac{4}{s^2}\right)} = \mathbf{0.25}$$

One pole is not in the left-half plane.
$f(\infty)$ **does not exist**

Solution 15.27

(a) $f(t) = \mathbf{u(t) + 2e^{-t}u(t)}$

(b) $G(s) = \dfrac{3(s+4)-11}{s+4} = 3 - \dfrac{11}{s+4}$

$g(t) = \mathbf{3\delta(t) - 11e^{-4t}u(t)}$

(c) $H(s) = \dfrac{4}{(s+1)(s+3)} = \dfrac{A}{s+1} + \dfrac{B}{s+3}$

$A = 2, \qquad B = -2$

$H(s) = \dfrac{2}{s+1} - \dfrac{2}{s+3}$

$h(t) = \mathbf{[2e^{-t} - 2e^{-3t}]u(t)}$

(d) $J(s) = \dfrac{12}{(s+2)^2(s+4)} = \dfrac{A}{s+2} + \dfrac{B}{(s+2)^2} + \dfrac{C}{s+4}$

$B = \dfrac{12}{2} = 6, \qquad C = \dfrac{12}{(-2)^2} = 3$

$12 = A(s+2)(s+4) + B(s+4) + C(s+2)^2$

Equating coefficients :

$s^2: \quad 0 = A + C \longrightarrow A = -C = -3$

$s^1: \quad 0 = 6A + B + 4C = 2A + B \longrightarrow B = -2A = 6$

$s^0: \quad 12 = 8A + 4B + 4C = -24 + 24 + 12 = 12$

$J(s) = \dfrac{-3}{s+2} + \dfrac{6}{(s+2)^2} + \dfrac{3}{s+4}$

$j(t) = \mathbf{[3e^{-4t} - 3e^{-2t} + 6te^{-2t}]u(t)}$

Solution 15.28

Design a problem to help other students to better understand how to find the inverse Laplace transform.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the inverse Laplace transform of the following functions:

(a) $F(s) = \dfrac{20(s+2)}{s(s^2+6s+25)}$

(b) $P(s) = \dfrac{6s^2+36s+20}{(s+1)(s+2)(s+3)}$

Solution

(a) $F(s) = \dfrac{20(s+2)}{s(s^2+6s+25)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+6s+25}$

$20(s+2) = A(s^2+6s+25s) + Bs^2 + Cs$

Equating components,

s^2 : 0 = A + B or B= - A

s: 20 = 6A + C

constant: 40 – 25 A or A = 8/5, B = -8/5, C= 20 – 6A= 52/5

$$F(s) = \frac{8}{5s} + \frac{-\frac{8}{5}s + \frac{52}{5}}{(s+3)^2+4^2} = \frac{8}{5s} + \frac{-\frac{8}{5}(s+3) + \frac{24}{5} + \frac{52}{5}}{(s+3)^2+4^2}$$

$$\underline{f(t) = \frac{8}{5}u(t) - \frac{8}{5}e^{-3t}\cos 4t + \frac{19}{5}e^{-3t}\sin 4t}$$

(b) $P(s) = \dfrac{6s^2+36s+20}{(s+1)(s+2)(s+3)} = \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+3}$

$$A = \frac{6-36+20}{(-1+2)(-1+3)} = -5$$

$$B = \frac{24-72+20}{(-1)(1)} = 28$$

$$C = \frac{54-108+20}{(-2)(-1)} = -17$$

$$P(s) = \frac{-5}{s+1} + \frac{28}{s+2} - \frac{17}{s+3}$$

$$p(t) = \underline{(-5e^{-t} + 28e^{-2t} - 17e^{-3t})u(t)}$$

Solution 15.29

Find the inverse Laplace transform of:

$$F(s) = \frac{s^2 + 2}{s^3 + 2s^2 + 2s}.$$

Solution

Step 1. First we simplify and find the roots of the denominator. Then we perform a partial fraction expansion and then solve for f(t).

$F(s) = (s^2+2)/[s(s^2+2s+2)]$ and $s_{1,2} = \dfrac{-2 \pm \sqrt{4-8}}{2} = -1\pm j$. Now,
$F(s) = (s^2+2)/[s(s+1+j)(s+1-j)] = [A/s] + [B/(s+1+j)] + [C/(s+1-j)]$ where
$A = (s^2+2)/[(s+1+j)(s+1-j)]|_{s=0}$, $B = (s^2+2)/[s(s+1-j)]|_{s=-1-j}$, and
$C = (s^2+2)/[s(s+1+j)]|_{s=-1+j}$.

Step 2. $A = 2/2 = 1$, $B = (j2+2)/[(-1-j)(-j2)] = -j$, and
$C = (-j2+2)/[(-1+j)(j2)] = j$. Therefore, $f(t) = [1+e^{-(1+j)t-90^\circ}+e^{-(1-j)t+90^\circ}]u(t)$
$= [1+e^{-t}(e^{-jt-90^\circ}+e^{jt+90^\circ}]u(t)$ or

$$f(t) = \mathbf{[1+2e^{-t}\cos(t+90^\circ)]}.$$

Solution 15.30

(a) $F_1(s) = \dfrac{6s^2 + 8s + 3}{s(s^2 + 2s + 5)} = \dfrac{A}{s} + \dfrac{Bs + C}{s^2 + 2s + 5}$

$$6s^2 + 8s + 3 = A(s^2 + 2s + 5) + Bs^2 + Cs$$

We equate coefficients.

s^2 : 6 = A + B

s: 8= 2A + C

constant: 3=5A or A=3/5

B=6-A = 27/5, C=8-2A = 34/5

$$F_1(s) = \frac{3/5}{s} + \frac{27s/5 + 34/5}{s^2 + 2s + 5} = \frac{3/5}{s} + \frac{27(s+1)/5 + 7/5}{(s+1)^2 + 2^2}$$

$$\underline{f_1(t) = \left[\frac{3}{5} + \frac{27}{5}e^{-t}\cos 2t + \frac{7}{10}e^{-t}\sin 2t\right]u(t)}$$

(b) $F_2(s) = \dfrac{s^2 + 5s + 6}{(s+1)^2(s+4)} = \dfrac{A}{s+1} + \dfrac{B}{(s+1)^2} + \dfrac{C}{s+4}$

$$s^2 + 5s + 6 = A(s+1)(s+4) + B(s+4) + C(s+1)^2$$

Equating coefficients,

s^2 : 1=A+C

s: 5=5A+B+2C

constant: 6=4A+4B+C

Solving these gives

A=7/9, B= 2/3, C=2/9

$$F_2(s) = \frac{7/9}{s+1} + \frac{2/3}{(s+1)^2} + \frac{2/9}{s+4}$$

$$\underline{f_2(t) = \left[\frac{7}{9}e^{-t} + \frac{2}{3}te^{-t} + \frac{2}{9}e^{-4t}\right]u(t)}$$

(c) $F_3(s) = \dfrac{10}{(s+1)(s^2 + 4s + 8)} = \dfrac{A}{s+1} + \dfrac{Bs + C}{s^2 + 4s + 8}$

$$10 = A(s^2 + 4s + 8) + B(s^2 + s) + C(s+1)$$

s^2 : 0 = A + B or B = -A

s: 0=4A+ B + C

constant: 10=8A+C

Solving these yields

A=2, B= -2, C= -6

$$F_3(s) = \frac{2}{s+1} + \frac{-2s-6}{s^2+4s+8} = \frac{2}{s+1} - \frac{2(s+1)}{(s+1)^2+2^2} - \frac{4}{(s+1)^2+2^2}$$

$f_3(t) =$ **$(2e^{-t} - 2e^{-t}\cos(2t) - 2e^{-t}\sin(2t))u(t)$**.

Solution 15.31

(a) $$F(s)=\frac{10s}{(s+1)(s+2)(s+3)}=\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{s+3}$$

$$A=F(s)\,(s+1)\Big|_{s=-1}=\frac{-10}{2}=-5$$
$$B=F(s)\,(s+2)\Big|_{s=-2}=\frac{-20}{-1}=20$$
$$C=F(s)\,(s+3)\Big|_{s=-3}=\frac{-30}{2}=-15$$

$$F(s)=\frac{-5}{s+1}+\frac{20}{s+2}-\frac{15}{s+3}$$

$$f(t)=\mathbf{(-5e^{-t}+20e^{-2t}-15e^{-3t})u(t)}$$

(b) $$F(s)=\frac{2s^2+4s+1}{(s+1)(s+2)^3}=\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{(s+2)^2}+\frac{D}{(s+2)^3}$$

$$A=F(s)\,(s+1)\Big|_{s=-1}=-1$$
$$D=F(s)\,(s+2)^3\Big|_{s=-2}=-1$$
$$2s^2+4s+1=A(s+2)(s^2+4s+4)+B(s+1)(s^2+4s+4)$$
$$+C(s+1)(s+2)+D(s+1)$$

Equating coefficients :

s^3: $\quad 0=A+B \longrightarrow B=-A=1$

s^2: $\quad 2=6A+5B+C=A+C \longrightarrow C=2-A=3$

s^1: $\quad 4=12A+8B+3C+D=4A+3C+D$

$\quad 4=6+A+D \longrightarrow D=-2-A=-1$

s^0: $\quad 1=8A+4B+2C+D=4A+2C+D=-4+6-1=1$

$$F(s)=\frac{-1}{s+1}+\frac{1}{s+2}+\frac{3}{(s+2)^2}-\frac{1}{(s+2)^3}$$

$$f(t)=-e^{-t}+e^{-2t}+3te^{-2t}-\frac{t^2}{2}e^{-2t}$$
$$f(t)=\mathbf{\left(-e^{-t}+\left(1+3t-\frac{t^2}{2}\right)e^{-2t}\right)u(t)}$$

(c) $$F(s) = \frac{s+1}{(s+2)(s^2+2s+5)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+2s+5}$$

$$A = F(s)\,(s+2)\Big|_{s=-2} = \frac{-1}{5}$$

$$s+1 = A\,(s^2+2s+5) + B\,(s^2+2s) + C\,(s+2)$$

Equating coefficients :

$$s^2: \quad 0 = A + B \longrightarrow B = -A = \frac{1}{5}$$

$$s^1: \quad 1 = 2A + 2B + C = 0 + C \longrightarrow C = 1$$

$$s^0: \quad 1 = 5A + 2C = -1 + 2 = 1$$

$$F(s) = \frac{-1/5}{s+2} + \frac{1/5 \cdot s + 1}{(s+1)^2 + 2^2} = \frac{-1/5}{s+2} + \frac{1/5\,(s+1)}{(s+1)^2 + 2^2} + \frac{4/5}{(s+1)^2 + 2^2}$$

$$f(t) = \mathbf{(-0.2\,e^{-2t} + 0.2\,e^{-t}\cos(2t) + 0.4\,e^{-t}\sin(2t))u(t)}$$

Solution 15.32

(a) $$F(s) = \frac{8(s+1)(s+3)}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$A = F(s)\,s\big|_{s=0} = \frac{(8)(3)}{(2)(4)} = 3$$

$$B = F(s)\,(s+2)\big|_{s=-2} = \frac{(8)(-1)}{(-4)} = 2$$

$$C = F(s)\,(s+4)\big|_{s=-4} = \frac{(8)(-1)(-3)}{(-4)(-2)} = 3$$

$$F(s) = \frac{3}{s} + \frac{2}{s+2} + \frac{3}{s+4}$$

$$f(t) = \mathbf{3u(t) + 2e^{-2t} + 3e^{-4t}}$$

(b) $$F(s) = \frac{s^2 - 2s + 4}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$s^2 - 2s + 4 = A(s^2 + 4s + 4) + B(s^2 + 3s + 2) + C(s+1)$$

Equating coefficients :

s^2: $1 = A + B \longrightarrow B = 1 - A$

s^1: $-2 = 4A + 3B + C = 3 + A + C$

s^0: $4 = 4A + 2B + C = -B - 2 \longrightarrow B = -6$

$A = 1 - B = 7$ $C = -5 - A = -12$

$$F(s) = \frac{7}{s+1} - \frac{6}{s+2} - \frac{12}{(s+2)^2}$$

$$f(t) = \mathbf{7e^{-t} - 6(1+2t)e^{-2t}}$$

(c) $$F(s) = \frac{s^2+1}{(s+3)(s^2+4s+5)} = \frac{A}{s+3} + \frac{Bs+C}{s^2+4s+5}$$

$$s^2+1 = A(s^2+4s+5) + B(s^2+3s) + C(s+3)$$

Equating coefficients :

s^2: $\quad 1 = A + B \longrightarrow B = 1 - A$

s^1: $\quad 0 = 4A + 3B + C = 3 + A + C \longrightarrow A + C = -3$

s^0: $\quad 1 = 5A + 3C = -9 + 2A \longrightarrow A = 5$

$B = 1 - A = -4 \qquad C = -A - 3 = -8$

$$F(s) = \frac{5}{s+3} - \frac{4s+8}{(s+2)^2+1} = \frac{5}{s+3} - \frac{4(s+2)}{(s+2)^2+1}$$

$$f(t) = \mathbf{5e^{-3t} - 4e^{-2t}\cos(t)}$$

Solution 15.33

(a) $$F(s)=\frac{6(s-1)}{s^4-1}=\frac{6}{(s^2+1)(s+1)}=\frac{As+B}{s^2+1}+\frac{C}{s+1}$$

$$6=A(s^2+s)+B(s+1)+C(s^2+1)$$

Equating coefficients :

s^2: $\quad 0=A+C \longrightarrow A=\text{-}C$

s^1: $\quad 0=A+B \longrightarrow B=\text{-}A=C$

s^0: $\quad 6=B+C=2B \longrightarrow B=3$

$A=\text{-}3,\qquad B=3,\qquad C=3$

$$F(s)=\frac{3}{s+1}+\frac{-3s+3}{s^2+1}=\frac{3}{s+1}+\frac{-3s}{s^2+1}+\frac{3}{s^2+1}$$

$$f(t)=\mathbf{(3e^{-t}+3\sin(t)-3\cos(t))u(t)}$$

(b) $$F(s)=\frac{s\,e^{-\pi s}}{s^2+1}$$

$$f(t)=\mathbf{\cos(t-\pi)u(t-\pi)}$$

(c) $$F(s)=\frac{8}{s(s+1)^3}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{(s+1)^2}+\frac{D}{(s+1)^3}$$

$A=8,\qquad D=\text{-}8$

$$8=A(s^3+3s^2+3s+1)+B(s^3+2s^2+s)+C(s^2+s)+Ds$$

Equating coefficients :

s^3: $\quad 0=A+B \longrightarrow B=\text{-}A$

s^2: $\quad 0=3A+2B+C=A+C \longrightarrow C=\text{-}A=B$

s^1: $\quad 0=3A+B+C+D=A+D \longrightarrow D=\text{-}A$

s^0: $\quad A=8,\quad B=-8,\quad C=-8,\quad D=-8$

$$F(s)=\frac{8}{s}-\frac{8}{s+1}-\frac{8}{(s+1)^2}-\frac{8}{(s+1)^3}$$

$$f(t)=\mathbf{8\left[1-e^{-t}-t\,e^{-t}-0.5t^2e^{-t}\right]u(t)}$$

(a) $(3e^{-t}+3\sin(t)-3\cos(t))u(t)$, (b) $\cos(t-\pi)u(t-\pi)$, (c) $8\left[1-e^{-t}-t\,e^{-t}-0.5t^2e^{-t}\right]u(t)$

Solution 15.34

(a) $$F(s) = 10 + \frac{s^2+4-3}{s^2+4} = 11 - \frac{3}{s^2+4}$$

$$f(t) = \mathbf{11\delta(t) - 1.5\sin(2t)}$$

(b) $$G(s) = \frac{e^{-s} + 4e^{-2s}}{(s+2)(s+4)}$$

Let $$\frac{1}{(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4}$$

$A = 1/2 \qquad B = 1/2$

$$G(s) = \frac{e^{-s}}{2}\left(\frac{1}{s+2} + \frac{1}{s+4}\right) + 2e^{-2s}\left(\frac{1}{s+2} + \frac{1}{s+4}\right)$$

$$g(t) = \mathbf{0.5\left[e^{-2(t-1)} - e^{-4(t-1)}\right]u(t-1) + 2\left[e^{-2(t-2)} - e^{-4(t-2)}\right]u(t-2)}$$

(c) Let $$\frac{s+1}{s(s+3)(s+4)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4}$$

$A = 1/12, \qquad B = 2/3, \qquad C = -3/4$

$$H(s) = \left(\frac{1}{12}\cdot\frac{1}{s} + \frac{2/3}{s+3} - \frac{3/4}{s+4}\right)e^{-2s}$$

$$h(t) = \mathbf{\left(\frac{1}{12} + \frac{2}{3}e^{-3(t-2)} - \frac{3}{4}e^{-4(t-2)}\right)u(t-2)}$$

Solution 15.35

(a) Let $G(s) = \dfrac{s+3}{(s+1)(s+2)} = \dfrac{A}{s+1} + \dfrac{B}{s+2}$

$A = 2, \qquad B = -1$

$G(s) = \dfrac{2}{s+1} - \dfrac{1}{s+2} \longrightarrow g(t) = 2e^{-t} - e^{-2t}$

$F(s) = e^{-6s}\,G(s) \longrightarrow f(t) = g(t-6)\,u(t-6)$

$f(t) = \left[\mathbf{2e^{-(t-6)} - e^{-2(t-6)}}\right]\mathbf{u(t-6)}$

(b) Let $G(s) = \dfrac{1}{(s+1)(s+4)} = \dfrac{A}{s+1} + \dfrac{B}{s+4}$

$A = 1/3, \qquad B = -1/3$

$G(s) = \dfrac{1}{3(s+1)} - \dfrac{1}{3(s+4)}$

$g(t) = \dfrac{1}{3}\left[e^{-t} - e^{-4t}\right]$

$F(s) = 4G(s) - e^{-2t}\,G(s)$

$f(t) = 4g(t)\,u(t) - g(t-2)\,u(t-2)$

$f(t) = \mathbf{\dfrac{4}{3}\left[e^{-t} - e^{-4t}\right]u(t) - \dfrac{1}{3}\left[e^{-(t-2)} - e^{-4(t-2)}\right]u(t-2)}$

(c) Let $G(s) = \dfrac{s}{(s+3)(s^2+4)} = \dfrac{A}{s+3} + \dfrac{Bs+C}{s^2+4}$

$A = -3/13$

$s = A(s^2+4) + B(s^2+3s) + C(s+3)$

Equating coefficients :

$s^2: \quad 0 = A + B \longrightarrow B = -A$

$s^1: \quad 1 = 3B + C$

$$s^0: \qquad 0 = 4A + 3C$$

$$A = -3/13, \qquad B = 3/13, \qquad C = 4/13$$

$$13G(s) = \frac{-3}{s+3} + \frac{3s+4}{s^2+4}$$

$$13g(t) = -3e^{-3t} + 3\cos(2t) + 2\sin(2t)$$

$$F(s) = e^{-s}\,G(s)$$

$$f(t) = g(t-1)\,u(t-1)$$

$$f(t) = \mathbf{\frac{1}{13}\left[-3e^{-3(t-1)} + 3\cos(2(t-1)) + 2\sin(2(t-1))\right]u(t-1)}$$

Solution 15.36

(a) $$X(s) = 3\frac{1}{s^2(s+2)(s+3)} = 3\left\{\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{D}{s+3}\right\}$$

$\mathrm{B} = 1/6$, $\quad \mathrm{C} = 1/4$, $\quad \mathrm{D} = -1/9$

$$1 = \mathrm{A}(s^3 + 5s^2 + 6s) + \mathrm{B}(s^2 + 5s + 6) + \mathrm{C}(s^3 + 3s^2) + \mathrm{D}(s^3 + 2s^2)$$

Equating coefficients :

s^3: $\quad 0 = \mathrm{A} + \mathrm{C} + \mathrm{D}$

s^2: $\quad 0 = 5\mathrm{A} + \mathrm{B} + 3\mathrm{C} + 2\mathrm{D} = 3\mathrm{A} + \mathrm{B} + \mathrm{C}$

s^1: $\quad 0 = 6\mathrm{A} + 5\mathrm{B}$

s^0: $\quad 1 = 6\mathrm{B} \longrightarrow \mathrm{B} = 1/6$

$\mathrm{A} = -5/6\,\mathrm{B} = -5/36$

$$X(s) = 3\left(\frac{-5/36}{s} + \frac{1/6}{s^2} + \frac{1/4}{s+2} - \frac{1/9}{s+3}\right)$$

$$\mathrm{x(t)} = \boldsymbol{\left(\frac{-5}{12}\mathrm{u(t)} + \frac{1}{2}\mathrm{t} + \frac{3}{4}\mathrm{e}^{-2t} - \frac{1}{3}\mathrm{e}^{-3t}\right)\mathrm{u(t)}}$$

(b) $$Y(s) = 2\frac{1}{s(s+1)^2} = 2\left(\frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}\right)$$

$\mathrm{A} = 1$, $\quad \mathrm{C} = -1$

$$1 = \mathrm{A}(s^2 + 2s + 1) + \mathrm{B}(s^2 + s) + \mathrm{C}s$$

Equating coefficients :

s^2: $\quad 0 = \mathrm{A} + \mathrm{B} \longrightarrow \mathrm{B} = -\mathrm{A}$

s^1: $\quad 0 = 2\mathrm{A} + \mathrm{B} + \mathrm{C} = \mathrm{A} + \mathrm{C} \longrightarrow \mathrm{C} = -\mathrm{A}$

s^0: $\quad 1 = \mathrm{A}, \quad \mathrm{B} = -1, \quad \mathrm{C} = -1$

$$Y(s) = 2\left(\frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}\right)$$

$$\mathrm{y(t)} = \boldsymbol{\left(2 - 2\mathrm{e}^{-t} - 2\mathrm{t}\,\mathrm{e}^{-t}\right)\mathrm{u(t)}}$$

(c) $$Z(s)=5\left(\frac{A}{s}+\frac{B}{s+1}+\frac{Cs+D}{s^2+6s+10}\right)$$

$$\mathrm{A}=1/10, \qquad \mathrm{B}=-1/5$$

$$1=\mathrm{A}(s^3+7s^2+16s+10)+\mathrm{B}(s^3+6s^2+10s)+\mathrm{C}(s^3+s^2)+\mathrm{D}(s^2+s)$$

Equating coefficients :

s^3: $\quad 0=\mathrm{A}+\mathrm{B}+\mathrm{C}$

s^2: $\quad 0=7\mathrm{A}+6\mathrm{B}+\mathrm{C}+\mathrm{D}=6\mathrm{A}+5\mathrm{B}+\mathrm{D}$

s^1: $\quad 0=16\mathrm{A}+10\mathrm{B}+\mathrm{D}=10\mathrm{A}+5\mathrm{B} \longrightarrow \mathrm{B}=-2\mathrm{A}$

s^0: $\quad 1=10\mathrm{A} \longrightarrow \mathrm{A}=1/10$

$$\mathrm{A}=1/10, \qquad \mathrm{B}=-2\mathrm{A}=-1/5, \qquad \mathrm{C}=\mathrm{A}=1/10, \qquad \mathrm{D}=4\mathrm{A}=\frac{4}{10}$$

$$\frac{10}{5}Z(s)=\frac{1}{s}-\frac{2}{s+1}+\frac{s+4}{s^2+6s+10}$$

$$2Z(s)=\frac{1}{s}-\frac{2}{s+1}+\frac{s+3}{(s+3)^2+1}+\frac{1}{(s+3)^2+1}$$

$$z(t)=\mathbf{0.5\left[1-2e^{-t}+e^{-3t}\cos(t)+e^{-3t}\sin(t)\right]u(t)}$$

Solution 15.37

(a) $H(s)=\dfrac{s+4}{s(s+2)}=\dfrac{A}{s}+\dfrac{B}{s+2}$

s+4 =A(s+2) + Bs
Equating coefficients,
s: 1 = A + B
constant: 4= 2 A $\longrightarrow$ A =2, B=1-A = -1

$$H(s)=\frac{2}{s}-\frac{1}{s+2}$$
$$h(t)=2u(t)-e^{-2t}u(t)=\underline{(2-e^{-2t})u(t)}$$

(b) $G(s)=\dfrac{A}{s+3}+\dfrac{Bs+C}{s^2+2s+2}$

$$s^2+4s+5=(Bs+C)(s+3)+A(s^2+2s+2)$$

Equating coefficients,
s^2: 1= B + A (1)
s: 4 = 3B + C + 2A (2)
Constant: 5 =3C + 2A (3)

Solving (1) to (3) gives

$$A=\frac{2}{5},\qquad B=\frac{3}{5},\qquad C=\frac{7}{5}$$
$$G(s)=\frac{0.4}{s+3}+\frac{0.6s+1.4}{s^2+2s+2}=\frac{0.4}{s+3}+\frac{0.6(s+1)+0.8}{(s+1)^2+1}$$
$$g(t)=\underline{0.4e^{-3t}+0.6e^{-t}\cos t+0.8e^{-t}\sin t}\ \mathrm{u(t)}$$

(c) $f(t)=\underline{e^{-2(t-4)}u(t-4)}$

(d) $D(s)=\dfrac{10s}{(s^2+1)(s^2+4)}=\dfrac{As+B}{s^2+1}+\dfrac{Cs+D}{s^2+4}$

$$10s=(s^2+4)(As+B)+(s^2+1)(Cs+D)$$

Equating coefficients,
s^3 : 0 = A + C
s^2 : 0 = B + D
s: 10 = 4A + C
constant: 0 = 4B+D

Solving these leads to

A = -10/3, B = 0, C = -10/3, D = 0

$$D(s) == \frac{10s/3}{s^2+1} - \frac{10s/3}{s^2+4}$$

$$d(t) = \underline{\frac{10}{3}\cos t - \frac{10}{3}\cos 2t}\,\mathrm{u(t)}$$

Solution 15.38

(a) $$F(s) = \frac{s^2 + 4s}{s^2 + 10s + 26} = \frac{s^2 + 10s + 26 - 6s - 26}{s^2 + 10s + 26}$$

$$F(s) = 1 - \frac{6s + 26}{s^2 + 10s + 26}$$

$$F(s) = 1 - \frac{6(s+5)}{(s+5)^2 + 1^2} + \frac{4}{(s+5)^2 + 1^2}$$

$$f(t) = \mathbf{\delta(t) - 6e^{-t}\cos(5t) + 4e^{-t}\sin(5t)}$$

(b) $$F(s) = \frac{5s^2 + 7s + 29}{s(s^2 + 4s + 29)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4s + 29}$$

$$5s^2 + 7s + 29 = A(s^2 + 4s + 29) + Bs^2 + Cs$$

Equating coefficients :

s^0: $\quad 29 = 29A \longrightarrow A = 1$

s^1: $\quad 7 = 4A + C \longrightarrow C = 7 - 4A = 3$

s^2: $\quad 5 = A + B \longrightarrow B = 5 - A = 4$

$A = 1, \qquad B = 4, \qquad C = 3$

$$F(s) = \frac{1}{s} + \frac{4s + 3}{s^2 + 4s + 29} = \frac{1}{s} + \frac{4(s+2)}{(s+2)^2 + 5^2} - \frac{5}{(s+2)^2 + 5^2}$$

$$f(t) = \mathbf{u(t) + 4e^{-2t}\cos(5t) - e^{-2t}\sin(5t)}$$

Solution 15.39

(a) $$F(s)=\frac{2s^3+4s^2+1}{(s^2+2s+17)(s^2+4s+20)}=\frac{As+B}{s^2+2s+17}+\frac{Cs+D}{s^2+4s+20}$$

$$s^3+4s^2+1=A(s^3+4s^2+20s)+B(s^2+4s+20)$$
$$+C(s^3+2s^2+17s)+D(s^2+2s+17)$$

Equating coefficients :

s^3: $2=A+C$

s^2: $4=4A+B+2C+D$

s^1: $0=20A+4B+17C+2D$

s^0: $1=20B+17D$

Solving these equations (Matlab works well with 4 unknowns),

$A=-1.6$, $B=-17.8$, $C=3.6$, $D=21$

$$F(s)=\frac{-1.6s-17.8}{s^2+2s+17}+\frac{3.6s+21}{s^2+4s+20}$$
$$F(s)=\frac{(-1.6)(s+1)}{(s+1)^2+4^2}+\frac{(-4.05)(4)}{(s+1)^2+4^2}+\frac{(3.6)(s+2)}{(s+2)^2+4^2}+\frac{(3.45)(4)}{(s+2)^2+4^2}$$

$f(t)=$

$$\mathbf{[-1.6e^{-t}\cos(4t)-4.05e^{-t}\sin(4t)+3.6e^{-2t}\cos(4t)+3.45e^{-2t}\sin(4t)]u(t)}$$

(b) $$F(s)=\frac{s^2+4}{(s^2+9)(s^2+6s+3)}=\frac{As+B}{s^2+9}+\frac{Cs+D}{s^2+6s+3}$$

$$s^2+4=A(s^3+6s^2+3s)+B(s^2+6s+3)+C(s^3+9s)+D(s^2+9)$$

Equating coefficients :

s^3: $0=A+C \longrightarrow C=-A$

s^2: $1=6A+B+D$

s^1: $0=3A+6B+9C=6B+6C \longrightarrow B=-C=A$

s^0: $4=3B+9D$

Solving these equations,

$A=1/12$, $B=1/12$, $C=-1/12$, $D=5/12$

$$12F(s)=\frac{s+1}{s^2+9}+\frac{-s+5}{s^2+6s+3}$$

$$s^2 + 6s + 3 = 0 \quad \longrightarrow \quad \frac{-6 \pm \sqrt{36 - 12}}{2} = -0.551, -5.449$$

Let $\quad G(s) = \dfrac{-s+5}{s^2 + 6s + 3} = \dfrac{E}{s + 0.551} + \dfrac{F}{s + 5.449}$

$$E = \frac{-s+5}{s+5.449}\Big|_{s=-0.551} = 1.133$$

$$F = \frac{-s+5}{s+0.551}\Big|_{s=-5.449} = -2.133$$

$$G(s) = \frac{1.133}{s+0.551} - \frac{2.133}{s+5.449}$$

$$12F(s) = \frac{s}{s^2 + 3^2} + \frac{1}{3} \cdot \frac{3}{s^2 + 3^2} + \frac{1.133}{s + 0.551} - \frac{2.133}{s + 5.449}$$

$f(t) =$

$$\mathbf{[\,0.08333\cos(3t) + 0.02778\sin(3t) + 0.0944\,e^{-0.551t} - 0.1778\,e^{-5.449t}\,]u(t)}$$

Solution 15.40

Let $H(s) = \left[\dfrac{4s^2 + 7s + 13}{(s+2)(s^2+2s+5)}\right] = \dfrac{A}{s+2} + \dfrac{Bs + C}{s^2 + 2s + 5}$

$$4s^2 + 7s + 13 = A(s^2 + 2s + 5) + B(s^2 + 2s) + C(s+2)$$

Equating coefficients gives:

$s^2:\quad 4 = A + B$

$s:\quad 7 = 2A + 2B + C \longrightarrow C = -1$

$\text{constant}:\quad 13 = 5A + 2C \longrightarrow 5A = 15 \text{ or } A = 3,\ B = 1$

$$H(s) = \frac{3}{s+2} + \frac{s-1}{s^2+2s+5} = \frac{3}{s+2} + \frac{(s+1)-2}{(s+1)^2 + 2^2}$$

Hence,

$$h(t) = 3e^{-2t} + e^{-t}\cos 2t - e^{-t}\sin 2t = 3e^{-2t} + e^{-t}(A\cos\alpha\cos 2t - A\sin\alpha\sin 2t)$$

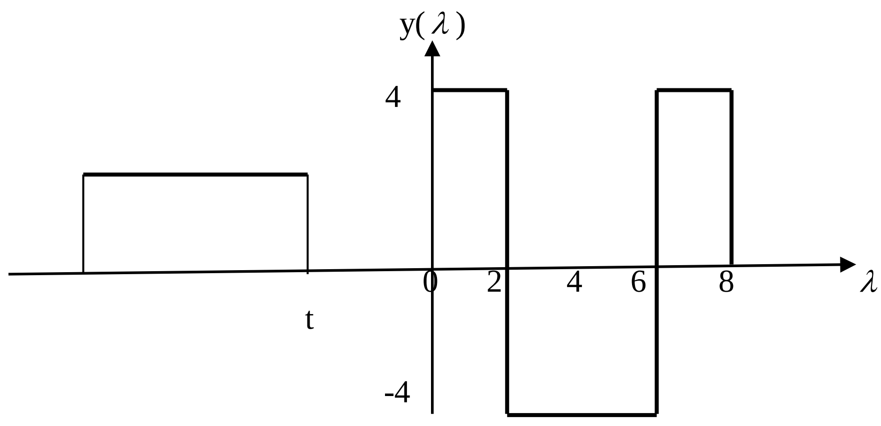
where $A\cos\alpha = 1,\quad A\sin\alpha = 1 \longrightarrow A = \sqrt{2},\quad \alpha = 45^\circ$

Thus,

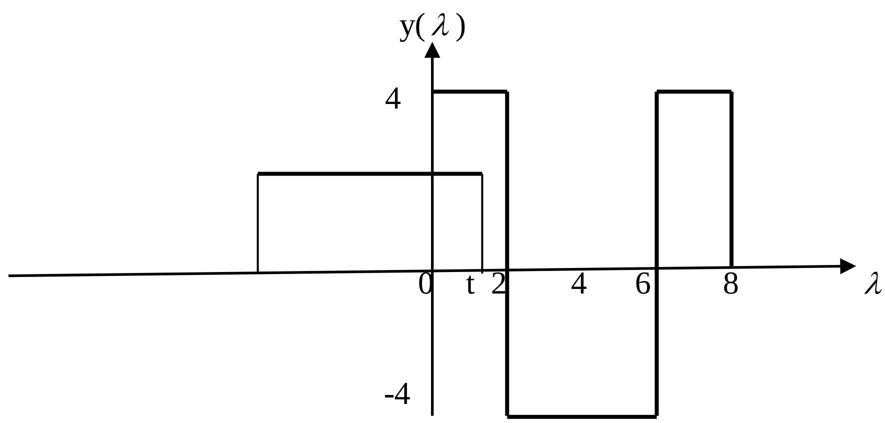
$$\mathbf{h(t)} = \underline{\left[\sqrt{2}e^{-t}\cos(2t + 45^\circ) + 3e^{-2t}\right]u(t)}$$

Solution 15.41

We fold x(t) and slide on y(t). For t<0, no overlapping as shown below. x(t) =0.

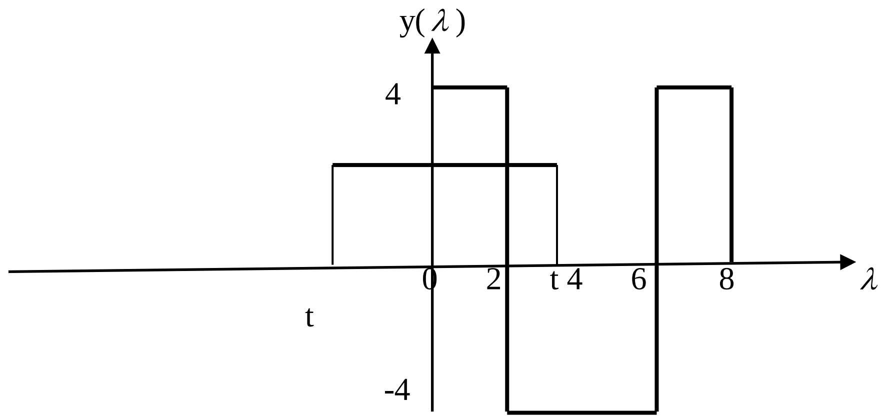


For 0 < t < 2, there is overlapping, as shown below.



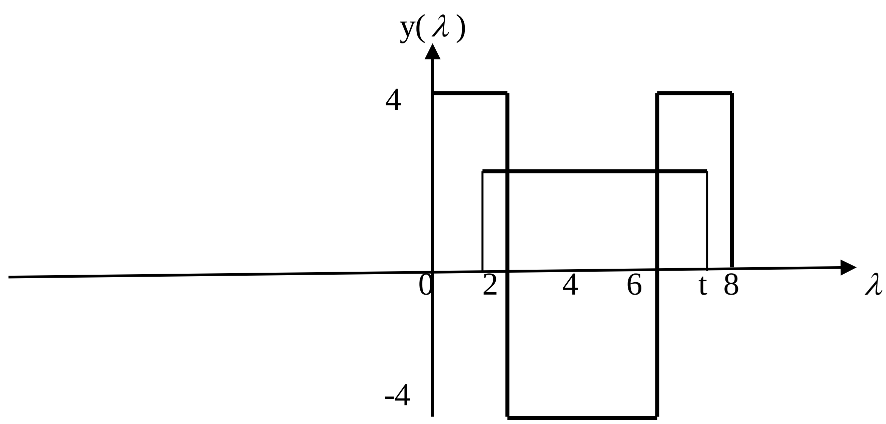
$$z(t) = \int_0^t (2)(4)dt = 8t$$

For 2 < t < 6, the two functions overlap, as shown below.



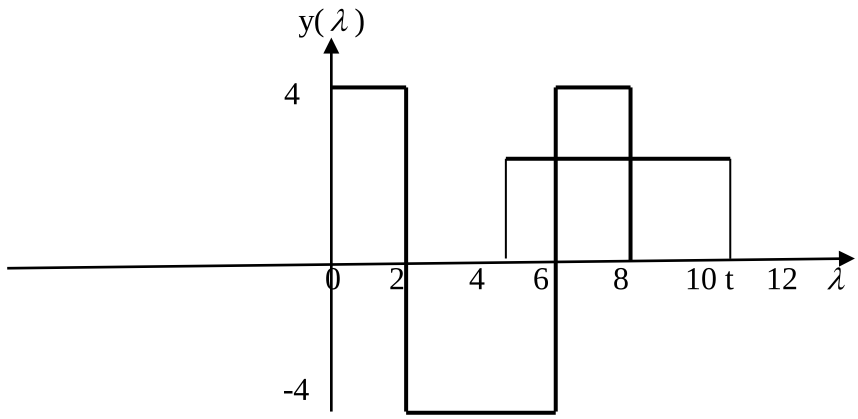
$$z(t) = \int_0^2 (2)(4)d\lambda + \int_0^t (2)(-4)d\lambda = 16 - 8t$$

For 6<t<8, they overlap as shown below.



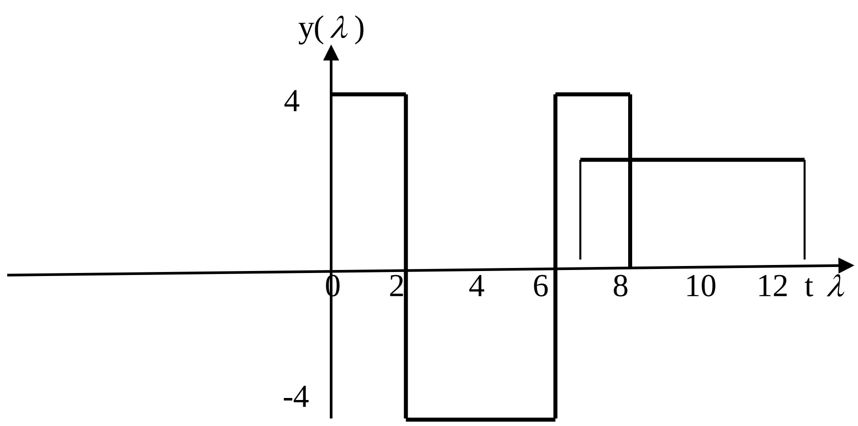
$$z(t) = \int_{t-6}^{2} (2)(4)d\lambda + \int_2^6 (2)(-4)d\lambda + \int_6^t (2)(4)d\lambda = 8\lambda\Big|_{t-6}^{2} - 8\lambda\Big|_{2}^{6} + 8\lambda\Big|_{6}^{t} = -16$$

For 8< t <12, they overlap as shown below.



$$z(t) = \int_{t-6}^{6} (2)(-4)d\lambda + \int_{6}^{8} (2)(4)d\lambda = -8\lambda \Big|_{t-6}^{6} + 8\lambda \Big|_{6}^{8} = 8t - 80$$

For 12 < t < 14, they overlap as shown below.



$$z(t) = \int_{t-6}^{8} (2)(4)d\lambda = 8\lambda \Big|_{t-6}^{8} = 112 - 8t$$

Hence,

z(t) =	**8t,**	**0<t<2**
	16–8t,	**2<t<6**
	–16,	**6<t<8**
	8t–80,	**8<t<12**
	112–8t,	**12<t<14**
	0,	**otherwise**.

Solution 15.42

Design a problem to help other students to better understand how to convolve two functions together.

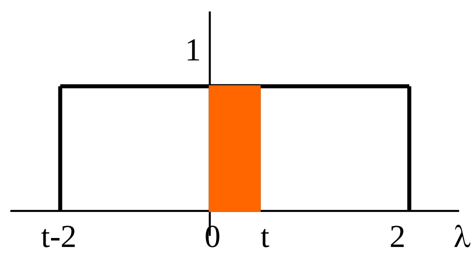
Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Suppose that f(t) = u(t) – u(t-2). Determine f(t)*f(t).

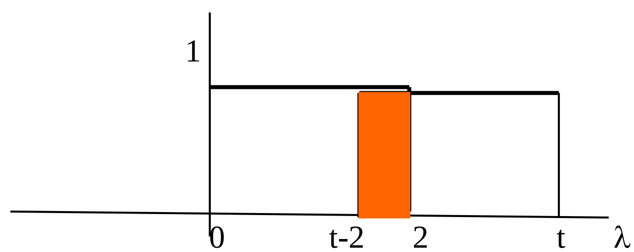
Solution

For 0<t<2, the signals overlap as shown below.



$$y(t) = f(t) * f(t) = \int_0^t (1)(1)d\lambda = t$$

For 2 < t< 4, they overlap as shown below.



$$y(t) = \int_{t-2}^{2} (1)(1)d\lambda = t\Big|_{t-2}^{2} = 4 - t$$

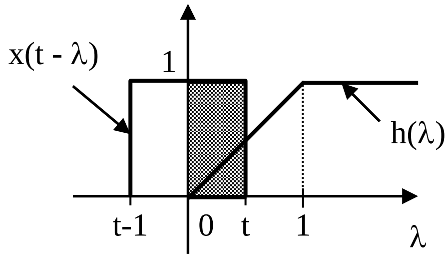
Thus,

$$y(t)=\begin{cases} t, & 0<t<2 \\ 4-t, & 2<t<4 \\ 0, & \text{otherwise} \end{cases}$$

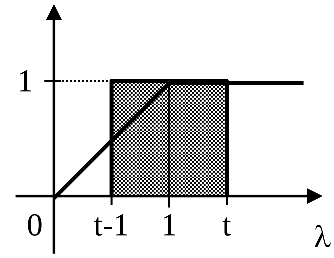
Solution 15.43

(a) For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (a).

$$y(t) = x(t) * h(t) = \int_0^t (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_0^t = \frac{t^2}{2}$$



(a)



(b)

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (b).

$$y(t) = \int_{t-1}^1 (1)(\lambda)\, d\lambda + \int_1^t (1)(1)\, d\lambda = \frac{\lambda^2}{2}\Big|_{t-1}^1 + \lambda\Big|_1^t = \frac{-1}{2}t^2 + 2t - 1$$

For $t > 2$, there is a complete overlap so that

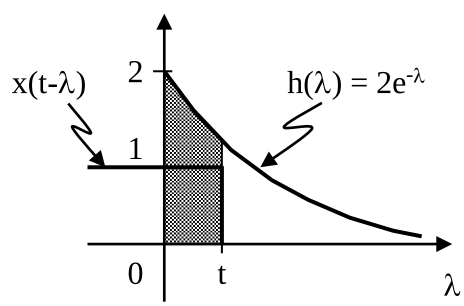
$$y(t) = \int_{t-1}^t (1)(1)\, d\lambda = \lambda\Big|_{t-1}^t = t - (t-1) = 1$$

Therefore,

$$y(t) = \begin{cases} \mathbf{t^2/2,} & \mathbf{0 < t < 1} \\ \mathbf{-(t^2/2) + 2t - 1,} & \mathbf{1 < t < 2} \\ \mathbf{1,} & \mathbf{t > 2} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

(b) For $t > 0$, the two functions overlap as shown in Fig. (c).

$$y(t) = x(t) * h(t) = \int_0^t (1)\, 2e^{-\lambda}\, d\lambda = -2e^{-\lambda}\Big|_0^t$$



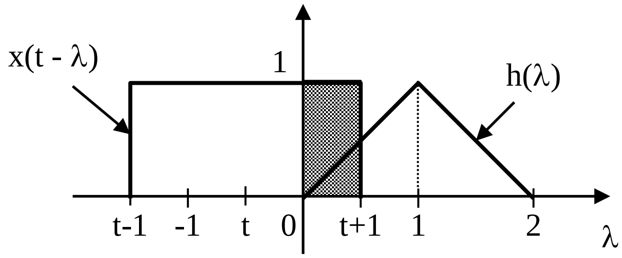
(c)

Therefore,

$$y(t) = \mathbf{2(1 - e^{-t}), \quad t > 0}$$

(c) For $-1 < t < 0$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (d).

$$y(t) = x(t) * h(t) = \int_0^{t+1} (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_0^{t+1} = \frac{1}{2}(t+1)^2$$

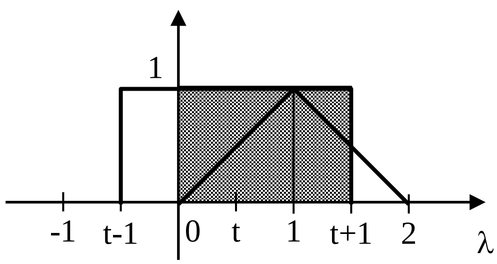


(d)

For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = \int_0^1 (1)(\lambda)\, d\lambda + \int_1^{t+1} (1)(2-\lambda)\, d\lambda$$

$$y(t) = \frac{\lambda^2}{2}\Big|_0^1 + \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_1^{t+1} = \frac{-1}{2}t^2 + t + \frac{1}{2}$$

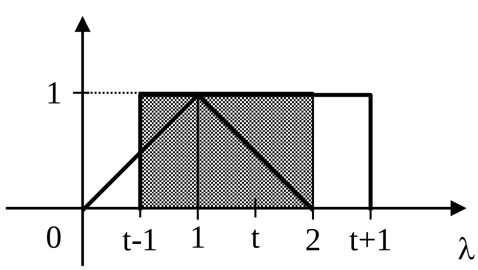


(e)

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (f).

$$y(t) = \int_{t-1}^1 (1)(\lambda)\, d\lambda + \int_1^2 (1)(2-\lambda)\, d\lambda$$

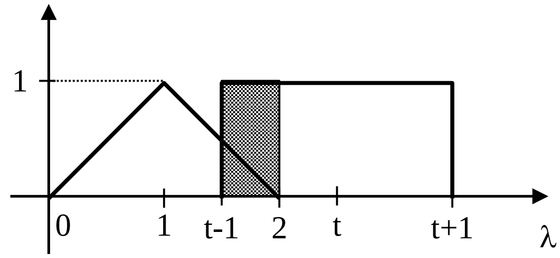
$$y(t) = \frac{\lambda^2}{2}\Big|_{t-1}^1 + \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_1^2 = \frac{-1}{2}t^2 + t + \frac{1}{2}$$



(f)

For $2 < t < 3$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (g).

$$y(t) = \int_{t-1}^{2} (1)(2-\lambda)\, d\lambda = \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_{t-1}^{2} = \frac{9}{2} - 3t + \frac{1}{2}t^2$$



(g)

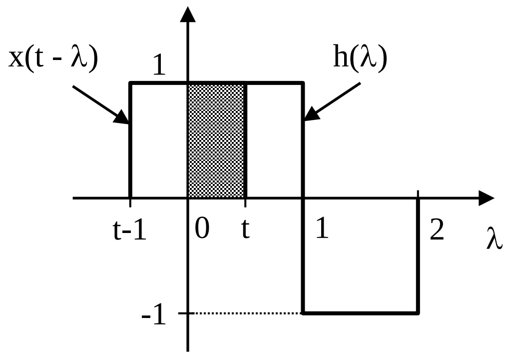
Therefore,

$$y(t) = \begin{cases} \mathbf{(t^2/2) + t + 1/2,} & \mathbf{-1 < t < 0} \\ \mathbf{-(t^2/2) + t + 1/2,} & \mathbf{0 < t < 2} \\ \mathbf{(t^2/2) - 3t + 9/2,} & \mathbf{2 < t < 3} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

Solution 15.44

(a) For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (a).

$$y(t) = x(t) * h(t) = \int_0^t (1)(1)\, d\lambda = t$$



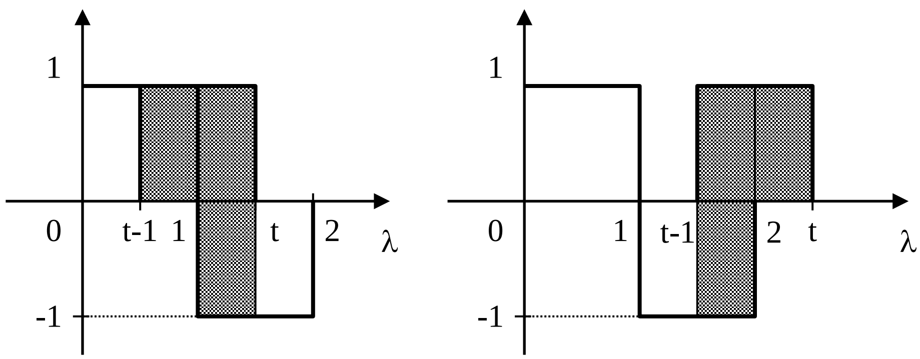
(a)

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (b).

$$y(t) = \int_{t-1}^{1} (1)(1)\, d\lambda + \int_1^t (-1)(1)\, d\lambda = \lambda\Big|_{t-1}^{1} - \lambda\Big|_1^t = 3 - 2t$$

For $2 < t < 3$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (c).

$$y(t) = \int_{t-1}^{2} (1)(-1)\, d\lambda = -\lambda\Big|_{t-1}^{2} = t - 3$$



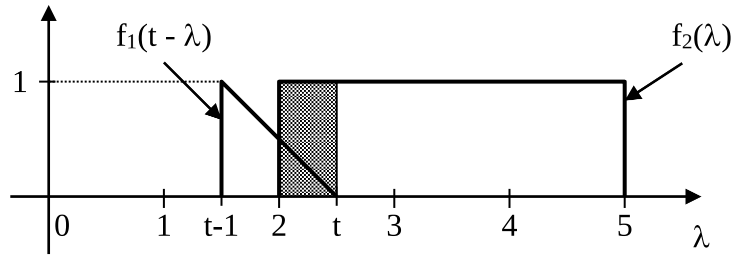
(b) **(c)**

Therefore,

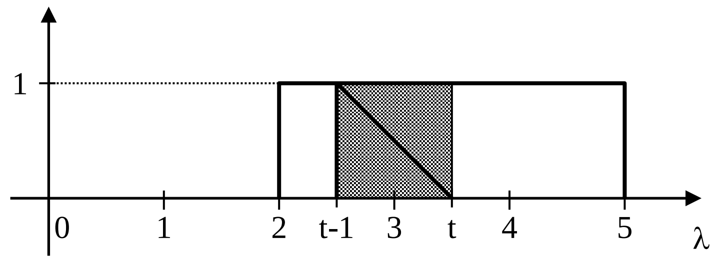
$$y(t) = \begin{cases} \mathbf{t}, & \mathbf{0 < t < 1} \\ \mathbf{3 - 2t}, & \mathbf{1 < t < 2} \\ \mathbf{t - 3}, & \mathbf{2 < t < 3} \\ \mathbf{0}, & \textbf{otherwise} \end{cases}$$

(b) For $t<2$, there is no overlap. For $2<t<3$, $f_1(t-\lambda)$ and $f_2(\lambda)$ overlap, as shown in Fig. (d).

$$y(t) = f_1(t) * f_2(t) = \int_2^t (1)(t-\lambda)\,d\lambda$$
$$= \left(\lambda t - \frac{\lambda^2}{2}\right)\Big|_2^t = \frac{t^2}{2} - 2t + 2$$



(d)



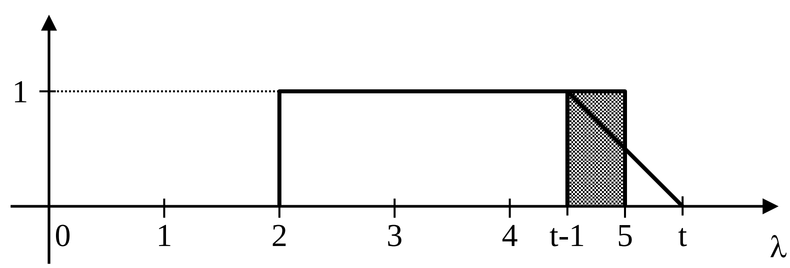
(e)

For $3<t<5$, $f_1(t-\lambda)$ and $f_2(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = \int_{t-1}^t (1)(t-\lambda)\,d\lambda = \left(\lambda t - \frac{\lambda^2}{2}\right)\Big|_{t-1}^t = \frac{1}{2}$$

For $5<t<6$, the functions overlap as shown in Fig. (f).

$$y(t) = \int_{t-1}^5 (1)(t-\lambda)\,d\lambda = \left(\lambda t - \frac{\lambda^2}{2}\right)\Big|_{t-1}^5 = \frac{-1}{2}t^2 + 5t - 12$$



(f)

Therefore,

$$y(t) = \begin{cases} \mathbf{(t^2/2) - 2t + 2,} & \mathbf{2 < t < 3} \\ \mathbf{1/2,} & \mathbf{3 < t < 5} \\ \mathbf{-(t^2/2) + 5t - 12,} & \mathbf{5 < t < 6} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

Solution 15.45

$$y(t) = h(t) * x(t) = \left[4e^{-2t}u(t)\right] * \left[\delta(t) - 2e^{-2t}u(t)\right]$$

$$= 4e^{-2t}u(t) * \delta(t) - 4e^{-2t}u(t) * 2e^{-2t}u(t) = 4e^{-2t}u(t) - 8e^{-2t}\int_0^t e^{o}\, d\lambda$$

$$= \underline{4e^{-2t}u(t) - 8te^{-2t}u(t)}$$

Solution 15.46

(a) $x(t) * y(t) = 2\delta(t) * 4u(t) = \underline{8u(t)}$

(b) $x(t) * z(t) = 2\delta(t) * e^{-2t}u(t) = \underline{2e^{-2t}u(t)}$

(c) $y(t) * z(t) = 4u(t) * e^{-2t}u(t) = 4\int_0^t e^{-2\lambda}d\lambda = \left.\dfrac{4e^{-2\lambda}}{-2}\right|_0^t = \underline{2(1-e^{-2t})}$

(d) $y(t) * [y(t) + z(t)] = 4u(t) * [4u(t) + e^{-2t}u(t)] = 4\int [4u(\lambda) + e^{-2\lambda}u(\lambda)]d\lambda$

$$= 4\int_0^t [4 + e^{-2\lambda}]d\lambda = 4[4t + \frac{e^{-2\lambda}}{-2}]\Bigg|_0^t = \underline{16t - 2e^{-2t} + 2}$$

Solution 15.47

A system has the transfer function

$$H(s) = \frac{6s}{(s+1)(s+2)}$$

(a) Find the impulse response of the system.
(b) Determine the output y(t) given that the input is $x(t) = u(t)$.

Solution

$(a)\ H(s) = \frac{6s}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$ where A = 6(–1)/(–1+2) = –6 and B = 6(–2)/(–2+1) = 12 or h(t) = **[–6e^{-t} + 12e^{-2t}]u(t)**.

(b) H(s) = Y(s)/X(s) or Y(s) = H(s)X(s) = {6s/[(s+1)(s+2)]}(1/s) = [[A/(s+1)] + [B/(s+2)] where A = 6/(–1+2) = 6 and B = 6/(–2+1) = –6. Therefore,

y(t) = **[6e^{-t} – 6e^{-2t}]u(t)**.

Solution 15.48

(a) Let $G(s) = \dfrac{2}{s^2 + 2s + 5} = \dfrac{2}{(s+1)^2 + 2^2}$

$$g(t) = e^{-t}\sin(2t)$$

$$F(s) = G(s)\,G(s)$$

$$f(t) = \mathsf{L}^{-1}\left[\,G(s)\,G(s)\right] = \int_0^t g(\lambda)\,g(t-\lambda)\,d\lambda$$

$$f(t) = \int_0^t e^{-\lambda}\sin(2\lambda)\,e^{-(t-\lambda)}\sin(2(t-\lambda))\,d\lambda$$

$$\sin(A)\sin(B) = \frac{1}{2}\left[\,\cos(A-B) - \cos(A+B)\right]$$

$$f(t) = \frac{1}{2}e^{-t}\int_0^t e^{-\lambda}\left[\,\cos(2t) - \cos(2(t-2\lambda))\right]d\lambda$$

$$f(t) = \frac{e^{-t}}{2}\cos(2t)\int_0^t e^{-2\lambda}\,d\lambda - \frac{e^{-t}}{2}\int_0^t e^{-2\lambda}\cos(2t-4\lambda)\,d\lambda$$

$$f(t) = \frac{e^{-t}}{2}\cos(2t)\cdot\frac{e^{-2\lambda}}{-2}\Big|_0^t - \frac{e^{-t}}{2}\int_0^t e^{-2\lambda}\left[\,\cos(2t)\cos(4\lambda) + \sin(2t)\sin(4\lambda)\right]d\lambda$$

$$f(t) = \frac{1}{4}e^{-t}\cos(2t)\,(-e^{-2t}+1) - \frac{e^{-t}}{2}\cos(2t)\int_0^t e^{-2\lambda}\cos(4\lambda)\,d\lambda$$
$$- \frac{e^{-t}}{2}\sin(2t)\int_0^t e^{-2\lambda}\sin(4\lambda)\,d\lambda$$

$$f(t) = \frac{1}{4}e^{-t}\cos(2t)\,(1-e^{-2t})$$
$$- \frac{e^{-t}}{2}\cos(2t)\left[\,\frac{e^{-2\lambda}}{4+16}\Big(-2\cos(4\lambda) - 4\sin(4\lambda)\Big)\right]\Big|_0^t$$
$$- \frac{e^{-t}}{2}\sin(2t)\left[\,\frac{e^{-2\lambda}}{4+16}\Big(-2\sin(4\lambda) + 4\cos(4\lambda)\Big)\right]\Big|_0^t$$

$$f(t) = \frac{e^{-t}}{2}\cos(2t) - \frac{e^{-3t}}{4}\cos(2t) - \frac{e^{-t}}{20}\cos(2t) + \frac{e^{-3t}}{20}\cos(2t)\cos(4t)$$
$$+ \frac{e^{-3t}}{10}\cos(2t)\sin(4t) + \frac{e^{-t}}{10}\sin(2t)$$
$$+ \frac{e^{-t}}{20}\sin(2t)\sin(4t) - \frac{e^{-t}}{10}\sin(2t)\cos(4t)$$

(b) Let $X(s) = \dfrac{2}{s+1}$, $Y(s) = \dfrac{s}{s+4}$

$x(t) = 2\,e^{-t}\,u(t)$, $y(t) = \cos(2t)\,u(t)$

$F(s) = X(s)\,Y(s)$

$$f(t) = \mathcal{L}^{-1}\left[\,X(s)\,Y(s)\right] = \int_0^{\infty} y(\lambda)\,x(t-\lambda)\,d\lambda$$
$$f(t) = \int_0^{t} \cos(2\lambda)\cdot 2\,e^{-(t-\lambda)}\,d\lambda$$
$$f(t) = 2\,e^{-t}\cdot\frac{e^{\lambda}}{1+4}\left(\cos(2\lambda) + 2\sin(2\lambda)\right)\Big|_0^t$$
$$f(t) = \frac{2}{5}e^{-t}\left[\,e^{t}\left(\cos(2t) + 2\sin(2t) - 1\right)\right]$$
$$f(t) = \frac{2}{5}\cos(2t) + \frac{4}{5}\sin(2t) - \frac{2}{5}e^{-t}$$

Solution 15.49

(a) $t*e^{\alpha t}u(t) =$

$$\int_0^t \mathbf{e^{a\lambda}(t-\lambda)d\lambda = t\frac{e^{a\lambda}}{a}\Big|_0^t - \frac{e^{a\lambda}}{a^2}(a\lambda-1)\Big|_0^t = \underline{\frac{t}{a}(e^{at}-1) - \frac{1}{a^2} - \frac{e^{at}}{a^2}(at-1)}\,u(t)}$$

(b) $\cos t * \cos t u(t) = \int_0^t \cos\lambda\cos(t-\lambda)d\lambda = \int_0^t \{\cos t\cos\lambda\cos\lambda + \sin t\sin\lambda\cos\lambda\}d\lambda$

$$= \left[\cos t\int_0^t \frac{1}{2}[1+\cos 2\lambda]d\lambda + \sin t\int_0^t \cos\lambda d(-\cos\lambda)\right] = \left[\frac{1}{2}\cos t[\lambda + \frac{\sin 2\lambda}{2}]\Big|_0^t - \sin t\frac{\cos\lambda}{2}\Big|_0^t\right]$$

=**[0.5cos(t)(t+0.5sin(2t)) – 0.5sin(t)(cos(t) – 1)]u(t)**.

Solution 15.50

Take the Laplace transform of each term.

$$\left[s^2\,V(s) - s\,v(0) - v'(0)\right] + 2\left[s\,V(s) - v(0)\right] + 10\,V(s) = \frac{3s}{s^2+4}$$

$$s^2\,V(s) - s + 2 + 2s\,V(s) - 2 + 10\,V(s) = \frac{3s}{s^2+4}$$

$$(s^2 + 2s + 10)\,V(s) = s + \frac{3s}{s^2+4} = \frac{s^3 + 7s}{s^2+4}$$

$$V(s) = \frac{s^3 + 7s}{(s^2+4)(s^2+2s+10)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+2s+10}$$

$$s^3 + 7s = A\,(s^3 + 2s^2 + 10s) + B\,(s^2 + 2s + 10) + C\,(s^3 + 4s) + D\,(s^2 + 4)$$

Equating coefficients :

s^3: $\quad 1 = A + C \longrightarrow C = 1 - A$

s^2: $\quad 0 = 2A + B + D$

s^1: $\quad 7 = 10A + 2B + 4C = 6A + 2B + 4$

s^0: $\quad 0 = 10B + 4D \longrightarrow D = \text{-}2.5\,B$

Solving these equations yields

$$A = \frac{9}{26}, \qquad B = \frac{12}{26}, \qquad C = \frac{17}{26}, \qquad D = \frac{-30}{26}$$

$$V(s) = \frac{1}{26}\left[\frac{9s+12}{s^2+4} + \frac{17s-30}{s^2+2s+10}\right]$$

$$V(s) = \frac{1}{26}\left[\frac{9s}{s^2+4} + 6\cdot\frac{2}{s^2+4} + 17\cdot\frac{s+1}{(s+1)^2+3^2} - \frac{47}{(s+1)^2+3^2}\right]$$

$$v(t) = \mathbf{\frac{9}{26}\cos(2t) + \frac{6}{26}\sin(2t) + \frac{17}{26}e^{-t}\cos(3t) - \frac{47}{78}e^{-t}\sin(3t)}$$

Solution 15.51

Given that $v(0) = 5$ and $dv(0)/dt = 10$, solve

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 25e^{-t}u\left(t\right).$$

Solution

Taking the Laplace transform of the differential equation yields

$$\left[s^2V(s) - sv(0) - v'(0)\right] + 5\left[sV(s) - v(0)]\right] + 6V(s) = \frac{25}{s+1}$$

or $\left(s^2 + 5s + 6\right)V(s) - 5s - 10 - 25 = \dfrac{25}{s+1} \quad \longrightarrow \quad V(s) = \dfrac{5s^2 + 40s + 60}{(s+1)(s+2)(s+3)}$

Let $V(s) = \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+3}, \quad A = 12.5, \quad B = 0, \quad C = -7.5$

Hence,

$$v(t) = \mathbf{[12.5e^{-t} - 7.5e^{-3t}]u(t)}.$$

Solution 15.52

Take the Laplace transform of each term.

$$\left[\, s^2\, I(s) - s\, i(0) - i'(0) \right] + 3\left[\, s\, I(s) - i(0) \right] + 2\, I(s) + 1 = 0$$

$$(s^2 + 3s + 2)\, I(s) - s - 3 - 3 + 1 = 0$$

$$I(s) = \frac{s+5}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$A = 4$, $B = -3$

$$I(s) = \frac{4}{s+1} - \frac{3}{s+2}$$

$$i(t) = \mathbf{(4e^{-t} - 3e^{-2t})u(t)}$$

Solution 15.53

Transform each term.

We begin by noting that the integral term can be rewritten as,

$\int_0^t x(\lambda)e^{-(t-\lambda)}d\lambda$ which is convolution and can be written as $e^{-t}*x(t)$.

Now, transforming each term produces,

$$X(s) = \frac{s}{s^2+1} + \frac{1}{s+1}X(s) \rightarrow \left(\frac{s+1-1}{s+1}\right)X(s) = \frac{s}{s^2+1}$$

$$X(s) = \frac{s+1}{s^2+1} = \frac{s}{s^2+1} + \frac{1}{s^2+1}$$

x(t) = **cos(t) + sin(t)**.

If partial fraction expansion is used we obtain,

x(t) = **1.4142cos(t–45°)**.

This is the same answer and can be proven by using trigonometric identities.

Solution 15.54

Design a problem to help other students to better understand solving second order differential equations with a time varying input.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Using Laplace transform, solve the following differential equation for t >0

$$\frac{d^2i}{dt^2}+4\frac{di}{dt}+5i=2e^{-2t}$$

subject to i(0)=0, i'(0)=2.

Solution

Taking the Laplace transform of each term gives

$$\left[s^2I(s)-si(0)-i'(0)\right]+4\left[sI(s)-i(0)\right]+5I(s)=\frac{2}{s+2}$$
$$\left[s^2I(s)-0-2\right]+4\left[sI(s)-0\right]+5I(s)=\frac{2}{s+2}$$

$$I(s)(s^2+4s+5)=\frac{2}{s+2}+2=\frac{2s+6}{s+2}$$
$$I(s)=\frac{2s+6}{(s+2)(s^2+4s+5)}=\frac{A}{s+2}+\frac{Bs+C}{s^2+4s+5}$$
$$2s+6=A(s^2+4s+5)+B(s^2+2s)+C(s+2)$$

We equate the coefficients.

s^2 : 0 = A+ B
s: 2= 4A + 2B + C
constant: 6 = 5A + 2C

Solving these gives
A = 2, B= -2, C = -2

$$I(s)=\frac{2}{s+2}-\frac{2s+2}{s^2+4s+5}=\frac{2}{s+2}-\frac{2(s+2)}{(s+2)^2+1}+\frac{2}{(s+2)^2+1}$$

Taking the inverse Laplace transform leads to:

$$i(t) = \left(2e^{-2t} - 2e^{-2t}\cos t + 2e^{-2t}\sin t\right)u(t) = \underline{2e^{-2t}(1-\cos t + \sin t)u(t)}$$

Solution 15.55

Take the Laplace transform of each term.

$$\left[s^3\,Y(s) - s^2\,y(0) - s\,y'(0) - y''(0)\right] + 6\left[s^2\,Y(s) - s\,y(0) - y'(0)\right]$$
$$+8\left[s\,Y(s) - y(0)\right] = \frac{s+1}{(s+1)^2 + 2^2}$$

Setting the initial conditions to zero gives

$$(s^3 + 6s^2 + 8s)\,Y(s) = \frac{s+1}{s^2 + 2s + 5}$$

$$Y(s) = \frac{(s+1)}{s(s+2)(s+4)(s^2+2s+5)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4} + \frac{Ds+E}{s^2+2s+5}$$

$$A = \frac{1}{40}, \qquad B = \frac{1}{20}, \qquad C = \frac{-3}{104}, \qquad D = \frac{-3}{65}, \qquad E = \frac{-7}{65}$$

$$Y(s) = \frac{1}{40}\cdot\frac{1}{s} + \frac{1}{20}\cdot\frac{1}{s+2} - \frac{3}{104}\cdot\frac{1}{s+4} - \frac{1}{65}\cdot\frac{3s+7}{(s+1)^2+2^2}$$

$$Y(s) = \frac{1}{40}\cdot\frac{1}{s} + \frac{1}{20}\cdot\frac{1}{s+2} - \frac{3}{104}\cdot\frac{1}{s+4} - \frac{1}{65}\cdot\frac{3(s+1)}{(s+1)^2+2^2} - \frac{1}{65}\cdot\frac{4}{(s+1)^2+2^2}$$

$$y(t) = \boldsymbol{\left(\frac{1}{40} + \frac{1}{20}e^{-2t} - \frac{3}{104}e^{-4t} - \frac{3}{65}e^{-t}\cos(2t) - \frac{2}{65}e^{-t}\sin(2t)\right)u(t)}$$

Solution 15.56

Solve for $v(t)$ in the integrodifferential equation

$$12\frac{dv}{dt}+36\int_0^{\tau} v\,d\tau = 0$$

given that $v(0) = 2$.

Solution

Taking the Laplace transform of each term we get:

$$12\left[\,sV(s)-v(0)\right]+\frac{36}{s}V(s)=0$$

$$\left[\,12\,s+\frac{36}{s}\right]V(s)=24$$

$$V(s)=\frac{24s}{12s^2+36}=\frac{2s}{s^2+3}$$

$$v(t) = 2\cos\left(\sqrt{3}t\right)$$

Solution 15.57

Although there is no correct way to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Solve the following integrodifferential equation using the Laplace transform method:

$$\frac{dy(t)}{dt} + 9\int_0^t y(t)dt = \cos 2t, \qquad y(0) = 1$$

Solution

Take the Laplace transform of each term.

$$\left[\mathrm{s\,Y(s) - y(0)} \right] + \frac{9}{\mathrm{s}}\mathrm{Y(s)} = \frac{\mathrm{s}}{\mathrm{s^2 + 4}}$$

$$\left(\frac{\mathrm{s^2 + 9}}{\mathrm{s}} \right)\mathrm{Y(s)} = 1 + \frac{\mathrm{s}}{\mathrm{s^2 + 4}} = \frac{\mathrm{s^2 + s + 4}}{\mathrm{s^2 + 4}}$$

$$\mathrm{Y(s)} = \frac{\mathrm{s^3 + s^2 + 4s}}{\mathrm{(s^2 + 4)(s^2 + 9)}} = \frac{\mathrm{As + B}}{\mathrm{s^2 + 4}} + \frac{\mathrm{Cs + D}}{\mathrm{s^2 + 9}}$$

$$\mathrm{s^3 + s^2 + 4s = A\,(s^3 + 9s) + B\,(s^2 + 9) + C\,(s^3 + 4s) + D\,(s^2 + 4)}$$

Equating coefficients :

$$\begin{aligned} &\mathrm{s^0:} && \mathrm{0 = 9B + 4D} \\ &\mathrm{s^1:} && \mathrm{4 = 9A + 4C} \\ &\mathrm{s^2:} && \mathrm{1 = B + D} \\ &\mathrm{s^3:} && \mathrm{1 = A + C} \end{aligned}$$

Solving these equations gives

$$\mathrm{A = 0}, \qquad \mathrm{B = -4/5}, \qquad \mathrm{C = 1}, \qquad \mathrm{D = 9/5}$$

$$\mathrm{Y(s)} = \frac{-4/5}{\mathrm{s^2 + 4}} + \frac{\mathrm{s + 9/5}}{\mathrm{s^2 + 9}} = \frac{-4/5}{\mathrm{s^2 + 4}} + \frac{\mathrm{s}}{\mathrm{s^2 + 9}} + \frac{9/5}{\mathrm{s^2 + 9}}$$

$$\mathrm{y(t)} = \mathbf{[\,-0.4\sin(2t) + \cos(3t) + 0.6\sin(3t)\,]u(t)}$$

Solution 15.58

We take the Laplace transform of each term.

$$[sV(s) - v(0)] + 2V(s) + \frac{5}{s}V(s) = \frac{4}{s}$$

$$[sV(s) + 1] + 2V(s) + \frac{5}{s}V(s) = \frac{4}{s} \qquad \longrightarrow \qquad V(s) = \frac{4 - s}{s^2 + 2s + 5}$$

$$V(s) = \frac{-(s+1) + 5}{(s+1)^2 + 2^2} = \frac{-(s+1)}{(s+1)^2 + 2^2} + \tfrac{5}{2}\frac{2}{(s+1)^2 + 2^2}$$

$$v(t) = \underline{(-e^{-t}\cos 2t + 2.5e^{-t}\sin 2t)u(t)}$$

Solution 15.59

Solve the following integrodifferential equation

$$\frac{dy}{dt} + 4y + 3\int_0^t y\,d\tau = 18e^{-2t}u(t)\text{ , } y(0) = -3.$$

Solution

Take the Laplace transform of each term of the integrodifferential equation.

$$\left[\,sY(s) - y(0)\right] + 4Y(s) + \frac{3}{s}Y(s) = \frac{18}{s+2}$$

$$(s^2 + 4s + 3)Y(s) = s\left(\frac{18}{s+2} - 3\right)$$

$$Y(s) = \frac{s(12-3s)}{(s+2)(s^2+4s+3)} = \frac{(12-3s)\,s}{(s+1)(s+2)(s+3)}$$

$$Y(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

A = –7.5, B = 36, and C = –31.5

$$Y(s) = \frac{-7.5}{s+1} + \frac{36}{s+2} - \frac{31.5}{s+3}$$

$$y(t) = \mathbf{[-7.5e^{-t} + 36e^{-2t} - 31.5e^{-3t}]u(t)}.$$

Solution 15.60

Take the Laplace transform of each term of the integrodifferential equation.

$$2\left[\, s\,X(s) - x(0)\right] + 5\,X(s) + \frac{3}{s}X(s) + \frac{4}{s} = \frac{4}{s^2+16}$$

$$(2s^2+5s+3)\,X(s) = 2s - 4 + \frac{4s}{s^2+16} = \frac{2s^3-4s^2+36s-64}{s^2+16}$$

$$X(s) = \frac{2s^3-4s^2+36s-64}{(2s^2+5s+3)(s^2+16)} = \frac{s^3-2s^2+18s-32}{(s+1)(s+1.5)(s^2+16)}$$

$$X(s) = \frac{A}{s+1} + \frac{B}{s+1.5} + \frac{Cs+D}{s^2+16}$$

$$A = (s+1)\,X(s)\Big|_{s=-1} = -6.235$$

$$B = (s+1.5)\,X(s)\Big|_{s=-1.5} = 7.329$$

When $s = 0$,

$$\frac{-32}{(1.5)(16)} = A + \frac{B}{1.5} + \frac{D}{16} \quad \longrightarrow \quad D = 0.2579$$

$$s^3 - 2s^2 + 18s - 32 = A\,(s^3 + 1.5s^2 + 16s + 24) + B\,(s^3 + s^2 + 16s + 16)$$
$$+\, C\,(s^3 + 2.5s^2 + 1.5s) + D\,(s^2 + 2.5s + 1.5)$$

Equating coefficients of the s^3 terms,

$$1 = A + B + C \quad \longrightarrow \quad C = -0.0935$$

$$X(s) = \frac{-6.235}{s+1} + \frac{7.329}{s+1.5} + \frac{-0.0935s + 0.2579}{s^2+16}$$

$$x(t) = \mathbf{-6.235\,e^{-t} + 7.329\,e^{-1.5t} - 0.0935\cos(4t) + 0.0645\sin(4t)}$$

Solution 15.61

Solve the following differential equations subject to the specified initial conditions

(a) $d^2v/dt^2 + 4v = 12$, $v(0) = 0$, $dv(0)/dt = 2$

(b) $d^2i/dt^2 + 5di/dt + 4i = 8$, $i(0) = -1$, $di(0)/dt = 0$

(c) $d^2v/dt^2 + 2dv/dt + v = 3$, $v(0) = 5$, $dv(0)/dt = 1$

(d) $d^2i/dt^2 + 2di/dt + 5i = 10$, $i(0) = 4$, $di(0)/dt = -2$

8.29

Solution

(a) Converting into the s-domain we get

$s^2V(s)–sv(0^–)–v'(0^–)+4V(s) = 12/s = s^2V(s)–s0–2+4V(s)$ or
$(s^2+4)V(s) = 2+12/s = 2(s+6)/s$ or $V(s) = 2(s+6)/[s(s+j2)(s–j2)]$
$= [A/s] + [B/(s+j2)] + [C/(s–j2)]$ where $A = 12/4 = 3$, $B = 2(–j2+6)/[–j2(–j4)]$
$= 2(6.325\angle–18.43^\circ)/(8\angle180^\circ) = 1.5812\angle161.57^\circ$ and C
$= s(j2+6)/[j2(j4)] = 2(6.325\angle18.43^\circ)/(8\angle180^\circ) = 1.5812\angle–161.57^\circ$

$v(t) = [3+1.5812e^{161.57^\circ}e^{–j2t}+1.\ 5812e^{–161.57^\circ}e^{j2t}]u(t)$ volts
$=$ **$[3+3.162\cos(2t–161.12^\circ)]u(t)$ volts**.

(b) Converting into the s-domain we get

$s^2I(s)–si(0^–)–i'(0^–)+5sI(s)–5i(0^–)+4I(s) = 8/s$
$= s^2I(s)–s(–1)–0+5sI(s)–5(–1)+4I(s)$ or
$(s^2+5s+4)I(s) = (–s–5) + 8/s = –(s^2+5s–8)/s$
$I(s) = –(s^2+5s–8)/[s(s+1)(s+4)] = [A/s] + [B/(s+1)] + [C/(s+4)]$ where
$A = 8/[(1)(4)] = 2$; $B = –[(–1)^2+5(–1)–8]/[(–1)(–1+4)] = 12/(–3) = –4$;
$C = –[(–4)^2+5(–4)–8]/[(–4)(–4+1)] = 12/(12) = 1$ therefore

$i(t) =$ **$[2–4e^{–t}+e^{–4t}]u(t)$ amps**.

(c) $s^2V(s)–sv(0^–)–v'(0^–)+2sV(s)–2v(0^–)+V(s) = 3/s$
$= s^2V(s)–s5–1+2sV(s)–2x5+V(s) = (s^2+2s+1)V(s) – (5s+11)$ or
$(s^2+2s+1)V(s) = (5s+11) + 3/s = (5s^2+11s+3)/s$ or
$V(s) = (5s^2+11s+3)/[s(s+1)^2] = [A/s] + [B/(s+1)] + [C/(s+1)^2]$ where
$A = 3$; $C = [5(–1)^2+11(–1)+3]/(–1) = (–3)/(–1) = 3$; going back to the original and eliminating the denominators we get $5s^2+11s+3 = 3(s^2+2s+1)+Bs^2+Bs+3s$ or
$B = 2$, thus,
$v(t) =$ **$[3+2e^{–t}+3te^{–t}]u(t)$ volts**.

(d) $s^2I(s)–si(0^–)–i'(0^–)+2sI(s)–2i(0^–)+5I(s) = 10/s$

$= s^2I(s)–s(4)–(–2)+2sI(s)–2(4)+5I(s)$ or
$(s^2+2s+5)I(s) – (4s–2+8) = 10/s$ or $(s^2+2s+5)I(s) = (4s^2+6s+10)/s$ or

$I(s) = (4s^2+6s+10)/[s(s+1+j2)(s+1–j2)] = [A/s] + [B/(s+1+j2)] + [C/(s+1–j2)]$
where $A = 10/5 = 2$; $B = [4(–1–j2)^2+6(–1–j2)+10]/[(–1–j2)(–j4)]$
$= [4(1+j4–4)–6–j12+10]/[–8+j4] = [–12+j16–6–j12+10]/(8.944\angle 153.43^\circ)$
$= [–8+j4]/(8.944\angle 153.43^\circ) = 1$; $C = [4(–1+j2)^2+6(–1+j2)+10]/[(–1+j2)(j4)]$
$= [4(1–j4–4)–6+j12+10]/[–8–j4] = [–12–j16–6+j12+10]/(8.944\angle –153.43^\circ)$
$= [–8–j4]/(8.944\angle 153.43^\circ) = 1$ thus

$i(t) = [2+e^{–t}e^{–j2t}+ e^{–t}e^{j2t}]u(t)$ amps = **$[2+2e^{–t}\cos(2t)]u(t)$ amps**.

(a) **$[3+3.162\cos(2t–161.12^\circ)]u(t)$ volts**, (b) **$[2–4e^{–t}+e^{–4t}]u(t)$ amps**,
(c) **$[3+2e^{–t}+3te^{–t}]u(t)$ volts**, (d) **$[2+2e^{–t}\cos(2t)]u(t)$ amps**

Solution 16.1

The current in an *RLC* circuit is described by

$$\frac{d^2i}{dt^2}+10\frac{di}{dt}+25i=0$$

If $i(0)$ = 7 A and $di(0)/dt$ = 0, find $i(t)$ for $t > 0$.

Solution

Step 1. Transform the equation into the s-domain and solve for I(s).

$s^2I(s) – (di(0^–)/dt) – si(0^–) + 10sI(s) – 10i(0^–) +25\ I(s) = 0$

$(s^2+10s+25)I(s) + [–(di(0^–)/dt)–si(0^–)–10i(0^–)] = 0$

$(s^2+10s+25)I(s) + [–7s–70] = 0$ or $(s^2+10s+25)I(s) = 7(s+10)$ or

$I(s) = 7(s+10)/ (s^2+10s+25)$

Step 2. Perform a partial fraction expansion and then solve for i(t) in the time domain.

$s^2 + 10s + 25 = 0$, thus $s_{1,2} = \dfrac{-10\pm\sqrt{10-10}}{2} = –5$, repeated roots.

$I(s) = 7(s+10)/(s+5)^2 = A/(s+5) + B/(s+5)^2 = (As+A5+B)/(s+5)^2$ or

$7s + 70 = As + 5A + B$

$A = 7$ and $5A+B = 70$ or $B = 70 – 35 = 35$ or

$I(s) = 7/(s+5) + 35/(s+5)^2$ or

$$i(t) = \mathbf{[(7+35t)e^{–5t}]u(t)\ A}$$

Solution 16.2

The differential equation that describes the current in an *RLC* network is

$$3\frac{di^2}{dt^2}+15\frac{di}{dt}+12i=0$$

Given that $i(0) = 0$, $di(0)/dt = 6$ mA/s, obtain $i(t)$.

Solution

Step 1. Convert the equation into the s-domain and then solve for I(s) and then perform a partial fraction expansion and then solve for i(t).

$3[s^2I–si(0)–i'(0)] +15sI–15i(0)+12I = 0$ or $(s^2+5s+4)I = sx0+6+5x0 = 6$ or

$i = 6/[s^2+5s+4]$ where $s_{1,2} = \dfrac{-5\pm\sqrt{25-16}}{2} = -1, -4$.

Step 2. We now have, $I = 6/[(s+1)(s+4)] = [A/(s+1)] + [B/(s+4)]$ where $A = 6/(–1+4) = 2$ mA and $B = 6/(–4+1) = –2$ mA. Finally,

$$i(t) = \mathbf{[2e^{-t} - 2e^{-4t}]u(t)\ mA}.$$

Solution 16.3

The natural response of an *RLC* circuit is described by the differential equation

$$\frac{d^2v}{dt^2} + 2\frac{dv}{dt} + v = 0$$

for which the initial conditions are v(0) = 350 V and dv(0)/dt = 0. Solve for v(t).

Solution

Step 1. Transform the equation into the s-domain and solve for v(t).

$s^2V(s) – (dv(0^–)/dt) – sv(0^–) + 2sV(s) – 2v(0^–) + V(s) = 0$ or

$(s^2+2s+1)V(s) – 350s – 700 = 0$ or $V(s) = 350(s+2)/(s^2+2s+1)$

Step 2. Perform a partial fraction expansion and solve for V(s). Inverse transform into the time-domain and solve for v(t).

$s^2 + 2s + 1 = 0$, thus $s_{1,2} = \dfrac{-2 \pm \sqrt{4-4}}{2} = –1$, repeated roots.

$V(s) = 350(s+2)/(s^2+2s+1) = 350(s+2)/(s+1)^2 = A/(s+1) + B/(s+1)^2$

As+A+B = 350s+700 or A = 350 and A+B = 700 = 350 + B or
B = 700 – 350 = 350

Thus,

$$v(t) = \mathbf{[(350 + 350t)e^{-t}]u(t)\ V}.$$

Solution 16.4

If $R = 20\ \Omega$, $L = 0.6$ H, what value of C will make an RLC series circuit:

(a) overdamped,
(b) critically damped,
(c) underdamped?

Solution

Step 1. Since we are working with a series RLC circuit, we can express our values in terms of I(s) and the s equation that multiplies it in the s-domain. From here we can easily find the values that produce over damped, critically damped, and underdamped conditions.

Equating the mesh equation we get, $RI(s) + LsI(s) + (1/C)I(s)/s – V(s) = 0$ or

$(0.6s + 20 + 1/(Cs))I(s) = V(s)$ or $[s^2+(20/0.6)+(1/(0.6Cs)]I(s) = V(s)/0.6$ or

$[s^s+(20/0.6)s+1/(0.6C)]I(s) = sV(s)/0.6$

The roots for the denominator are $s_{1,2} = \dfrac{-(20/0.6) \pm \sqrt{(400/0.36) - 4/(0.6C)}}{2}$.

Step 2. To find the values of our roots that produces overdamped, critically damped, and underdamped conditions, we note that when s_1 and s_2 values that produces these values,

overdamped is when s_1 and s_2 are real with no complex values
critically damped is when $s_1 = s_2$
underdamped is when both s_1 and s_2 have complex roots and $s_1 = s_2^*$

Now all we need to do is to solve for these conditions.

(a) Overdamped is when [4/(0.6C)] is less than 400/0.36 or
$C > 4x0.36/(400x0.6) = 6x10^{-3}$, or $C >$ **6 mF**

(b) Critically damped is when [4/(0.6C)] is equal to 400/0.36 or
$C = 4x0.36/(400x0.6) = 6x10^{-3} =$ **6 mF**

(c) Underdamped is when [4/(0.6C)] is greater than 400/0.36 or
$C < 4x0.36/(400x0.6) = 6x10^{-3}$ or $C <$ **6mF**

Solution 16.5

The responses of a series *RLC* circuit are

$v_C(t) = [30 - 10e^{-20t} + 30e^{-10t}]u(t)$ V

$i_L(t) = [40e^{-20t} - 60e^{-10t}]u(t)$ mA

where $v_C(t)$ and $i_L(t)$ are the capacitor voltage and inductor current, respectively. Determine the values of *R*, *L*, and *C*.

Solution

Step 1. We can start with the generalized mesh equation for a series RLC network. We can lump the initial conditions ($v_C(0) = 30–10+30 = 50$ volts and $i_L(0) = 40–60 = 20$ amps) with the source in the loop since all we are currently after are the values of R, L, and C.

RI(s) + Ls(I(s)+20/s) + (1/C)I(s)/s – 50/s – V(s) = 0 or [s^2 + (R/L)s + 1/(LC)]I(s) = (V(s)/L) – 20 + 50/(Ls)

Step 2. The values of R, L, and C will come from the roots of the denominator s equation. We already know that they are equal to –10 and –20. We note however, that this will give us only two equations. Obviously we need a third, and that will come from knowing the current through the capacitor and the voltage across it.

$$s_{1,2} = \frac{-(R/L) \pm \sqrt{(R/L)^2 - 4/(LC)}}{2} = -10, -20$$

We can simplify our effort by noting that $s_1 + s_2 = -R/L[(1/2)+(1/2)] = -30$ or R = 30L.

Next, $s_1 - s_2 = \dfrac{2\sqrt{(R/L)^2 - 4/(LC)}}{2} = 10$ or $(R/L)^2 - 4/(LC) = 100$. Since (R/L) = 30, we then get 900 – 100 = 4/(LC) or LC = 4/800 = 1/200.

Now we work with $i_C(t) = Cdv_C(t)/dt$ or $40e^{-20t} - 60e^{-10t}$ mA = $C[200e^{-20t} - 300e^{-10t}]$ V or $C = 0.2x10^{-3}$ = **200 μF**. Since LC = 1/200 then $L = 1/(200x200x10^{-6}) = 1/0.04$ = **25 H**. Finally R = 30L = 30x25 = **750 Ω**.

750 Ω, 25 H, 200 μF

Solution 16.6

Design a parallel RLC circuit that has the characteristic equation

$$.s^2+100s+10^6 = 0.$$

Solution

Step 1. Develop a general equation for a parallel RLC circuit with initial conditions lumped into a parallel current source i(t).

$$[Cs + (1/R) + (1/(Ls))]V(s) – I(s) = 0 \text{ or } [s^2+(1/(RC))s+1/(LC)]V(s) = sI(s)/C$$

Step 2. The next step is to equate the unknowns to the parameters in the characteristic equation. This does become a design problem in that we have two equations with three unknowns. We need to pick one of the values so that the other values are realistic.

$1/(RC) = 100$ and $1/(LC) = 10^6$ or $RC = 0.01$ and $LC = 10^{-6}$. We can start with some values of R and see what happens to the values of L and C.

R	**L**	**C**
1 Ω	100 μH	10 mF
10 Ω	1 mH	1 mF
100 Ω	10 mH	100 μF
1 k Ω	100 mH	10 μF
10 k Ω	1 H	1 μF
100 k Ω	10 H	0.1 μF

We now need to pick reasonable values, R = **10 kΩ**, L = **1 H**, and C = **1 μF** represents an acceptable set since their values are relatively common and inexpensive.

Solution 16.7

The step response of an *RLC* circuit is given by

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 5i = 30\,\mathrm{u(t)}$$

Given that $i(0)$ = 18 amps and $di(0)/dt$ = 36 amps/sec, solve for $i(t)$.

Solution

Step 1. We start by transforming the equation into the s-domain. We then solve for I(s).

$$s^2I(s) – (di(0^–)/dt) – si(0^–) + 2sI(s) – 2i(0^–) + 5\ I(s) = 30/s \text{ or}$$

$$s^2I(s) – (36) – 18s + 2sI(s) – 2x18 + 5I(s) = 30/s = (s^2+2s+5)I(s) – 18(s+4) \text{ or}$$

$$(s^2+2s+5)I(s) = [18(s^2+4s)+30]/s \text{ or } I(s) = [18(s^2+4s)+30]/[s(s^2+2s+5)]$$

Step 2. We need to find the roots of (s^2+2s+5) and then perform a partial fraction expansion and then transform back into the time domain and realize i(t).

$s^2 + 2s + 5$, has the roots $s_{1,2} = \dfrac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j2$

$I(s) = [18(s^2+4s)+30]/[s(s+1+j2)(s+1–j2)] = [A/s] + [B/(s+1+j2)] + [C/(s+1–j2)]$

$A = 30/5 = 6$; $B = [18(1+j4–4–4–j8)+30]/[(–1–j2)(–j4)] = [18(–7–j4)+30]/[–8+j4] =$

$(–96–j72)/[4(–2+j)] = 12(–8–j6)/[4(–2+j)] = 6(–4–j3)/(–2+j) =$

$[6(–4–j3)(–2–j)]/[(–2+j)(–2–j)] = 6(8–3+j6+j4)/5 = 6(1+j2)$;

$C = [18(1–j4–4–4+j8)+30]/[(–1+j2)(j4)] = [18(–7+j4)+30]/[–8–j4] =$

$(–96+j72)/[4(–2–j)] = 12(–8+j6)/[4(–2–j)] = 6(–4+j3)/(–2–j) =$

$[6(–4+j3)(–2+j)]/[(–2–j)(–2+j)] = 6(8–3–j6–j4)/5 = 6(1–j2)$.

$I(s) = [6/s] + [(6+j12)/(s+1+j2)] + [(6–j12)/(s+1–j2)]$ or

$$i(t) = \mathbf{[6+12e^{–t}(\cos(2t)+2\sin(2t))]u(t)\ A}$$

Solution 16.8

A branch voltage in an *RLC* circuit is described by

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 8v = 120$$

If the initial conditions are $v(0) = 0 = dv(0)/dt$, find $v(t)$.

Solution

Step 1. First we transform the equation into the s-domain. Then we solve for V(s).

$$s^2V(s) - (dv(0^-)/dt) - sv(0^-) + 4sV(s) - 4v(0^-) + 8V(s) = 120/s \text{ or}$$

$$s^2V(s) + 4sV(s) + 8V(s) = 120/s = (s^2+4s+8)V(s) \text{ or}$$

$$V(s) = 120/[s(s^2+4s+8)]$$

Step 2. Now we need to solve for the roots of the denominator and perform a partial fraction expansion. Then we can inverse transform the answer back into the time domain.

$s^2 + 4s + 8$ has the roots $s_{1,2} = \dfrac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$ thus,

$V(s) = 120/[s(s+2+j2)(s+2-j2)] = [A/s] + [B/(s+2+j2)] + [C/(s+2-j2)]$

where $A = 120/8 = 15$; $B = 120/[(-2-j2)(-j4)] = 120/(-8+j8) = 120(-1-j)/[8(-1+j)(-1-j)] = 15(-1-j)/2 = 7.5(-1-j)$; and
$C = 120/[(-2+j2)(j4)] = 120/(-8-j8) = 120(-1+j)/[8(-1-j)(-1+j)]$
$= 15(-1+j)/2 = 7.5(-1+j)$.

Therefore, $V(s) = [15/s] + [7.5(-1-j)/(s+2+j2)] + [7.5(-1+j)/(s+2-j2)]$

$v(t) =$ $\mathbf{[15 - 15e^{-2t}(\cos 2t + \sin 2t)]u(t)}$ **volts**.

Solution 16.9

A series RLC circuit is described by

$$L\frac{d^2i(t)}{dt}+R\frac{di(t)}{dt}+\frac{i(t)}{C}=15$$

Find the response when L = 0.5 H, R = 4 Ω, and C = 0.2 F. Let $i(0^-)$ = 7.5 A and $[di(0^-)/dt] = 0$.

Solution

Step 1. First transform the equation into the s-domain. Then solve for I(s).

$0.5s^2I(s) – 0.5(di(0^–)/dt) – 0.5si(0^–) + 4sI(s) – 4i(0^–) + 5I(s) = 15/s$ or
$s^2I(s) – 7.5s + 8sI(s) – 60 + 10I(s) = 30/s$ or
$(s^2+8s+10)I(s) = 7.5s+60+30/s = 7.5(s^2+8s+4)/s$ or
$I(s) = 7.5(s^2+8s+4)/[s(s^2+8s+10)]$

Step 2. Next we need to find the roots of $(s^2+8s+10)$ and then perform a partial fraction expansion and then inverse transform back into the time domain.

$s_{1,2} = –1.5505$ and $–6.45$

$7.5(s^2+8s+2)/[s(s^2+8s+10)] = [A/s] + [B/(s+1.5505)] + [C/(s+6.45)]$

A = 3; B = 5.924; and C = –1.4235 thus,

$I(s) = [3/s] + [5.924/(s+1.5505)] + [–1.4235/(s+6.45)]$ and

$i(t) =$ $\mathbf{[3+5.924e^{–1.5505t}–1.4235e^{–6.45t}]\ A}$.

Solution 16.10

The step responses of a series RLC circuit are

$$v_c = 40 - 10e^{-2000t} - 10e^{-4000t} \text{ V, t>0}$$
$$i_L(t) = 3e^{-2000t} + 6e^{-4000t} \text{ mA, t>0}$$

(a) Find C. (b) Determine what type of damping exhibited by the circuit.

Solution

(a) $i_L(t) = i_C(t) = C\dfrac{dv_o}{dt}$ (1)

$$\frac{dv}{dt} = 2000x10e^{-2000t} + 4000x10e^{-4000t} = 2x10^4(e^{-2000t} + 2e^{-4000t}) \quad (2)$$

But $i_L(t) = 3[e^{-2000t} + 2e^{-4000t}]\text{x}10^{-3}$ (3)

Substituting (2) and (3) into (1), we get

$$2x10^4 xC = 3x10^{-3} \longrightarrow C = 1.5x10^{-7} = \underline{150 \text{ nF}}$$

(b) Since s_1 = - 2000 and s_2 = - 4000 are real and negative, it is an **overdamped** case.

Solution 16.11

The step response of a parallel RLC circuit is

$$v = 10 + 20e^{-300t} (\cos 400t - 2 \sin 400t) \text{ V}, \ \ t \geq 0$$

when the inductor is 50 mH. Find R and C.

Solution

Step 1. There are different ways to approach this problem so, we will convert everything into the s-domain and then solve for the unknowns. We should also note that the steady-state voltage is 10 volts, then the circuit is a step input voltage across a parallel combination of a capacitor and an inductor all in series with an output resistor.

The nodal equation for this circuit is given by,

[(V–10/s)/R] + [(V–0)/(0.05s)] + [(V–0)/(1/sC)] + = 0 or

[(1/R)+(1/(0.05s))+sC]V = 10/(Rs) = [(20R+RCs^2+s)/(Rs)]V or

V = [10/(Rs)][Rs/(RCs^2+s+20R)] = 10/[(RCs)(s^2+(1/(RC))s+(20/C))]

Step 2. From the value of v(t) we can determine the value of the roots of the polynomial (s^2+(1/(RC))s+(20/C)) = (s+300+j400)(s+300–j400) thus, 20/C = 300^2+400^2 = 90,000 + 160,000 = 250,000 or

C = 20/250,000 = **80 μF**

and 1/(RC) = 600 or

R = 1/(600x80x10^{-6}) = **20.83 Ω**.

Solution 16.12

Determine $i(t)$ in the circuit of Fig. 16.35 by means of the Laplace transform.

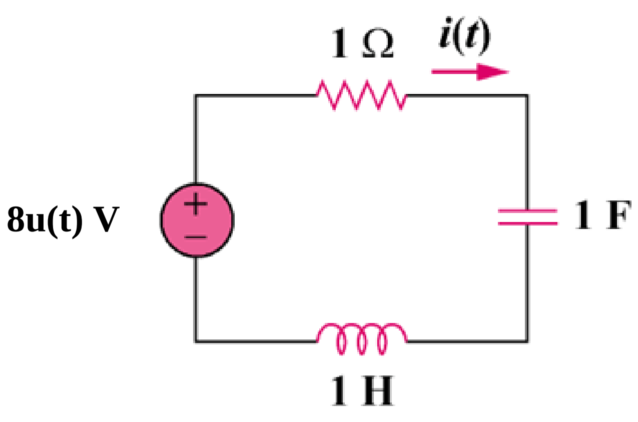
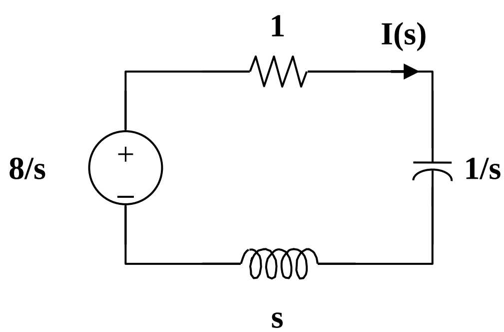


Figure 16.35
For Prob. 16.12.

Solution

Consider the s-domain form of the circuit which is shown below.



$$I(s) = \frac{8/s}{1+s+1/s} = \frac{8}{s^2+s+1} = \frac{8}{(s+1/2)^2+(\sqrt{3}/2)^2}$$

$$i(t) = \frac{16}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}}{2}t\right) \text{u(t) A}$$

$$\text{i(t)} = \mathbf{9.238e^{-t/2}\,sin(0.866t)u(t)\ A}.$$

Solution 16.13

Using Fig. 16.36, design a problem to help other students to better understand circuit analysis using Laplace transforms.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find v_x in the circuit shown in Fig. 16.36 given $v_s = 4u(t)$ V.

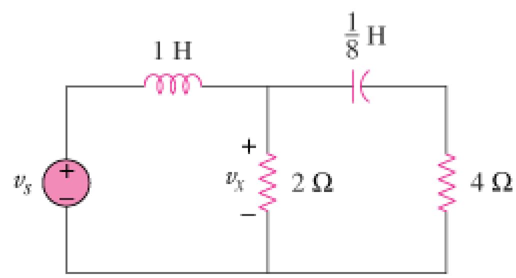
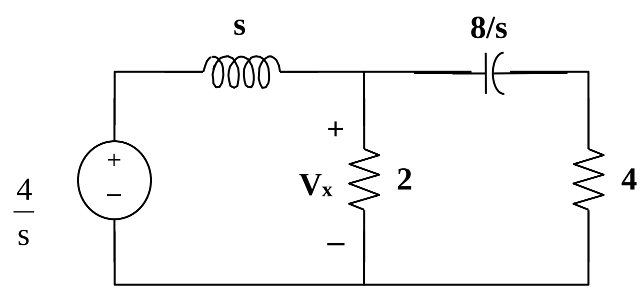


Figure 16.36
For Prob. 16.13.

Solution



$$\frac{V_x - \frac{4}{s}}{s} + \frac{V_x - 0}{2} + \frac{V_x - 0}{4 + \frac{8}{s}} = V_x(4s+8) - \frac{(16s+32)}{s} + (2s^2 + 4s)V_x + s^2 V_x = 0$$

$$V_x(3s^2 + 8s + 8) = \frac{16s + 32}{s}$$

$$V_x = 16\frac{s+2}{s(3s^2+8s+8)} = 16\left(\frac{0.25}{s} + \frac{-0.125}{s + \frac{4}{3} + j\frac{\sqrt{8}}{3}} + \frac{-0.125}{s + \frac{4}{3} - j\frac{\sqrt{8}}{3}}\right)$$

$$v_x = \underline{(4 - 2e^{-(1.3333 + j0.9428)t} - 2e^{-(1.3333 - j0.9428)t})u(t)\,V}$$

$$v_x = \left[\mathbf{4 - 4e^{-4t/3}\cos\left(\frac{2\sqrt{2}}{3}t\right)}\right]\mathbf{u(t)V}$$

Solution 16.14

Find i(t) for t>0 for the circuit in Fig. 16.37. Assume $i_s(t) = [6u(t) + 3\delta(t)]$ mA.

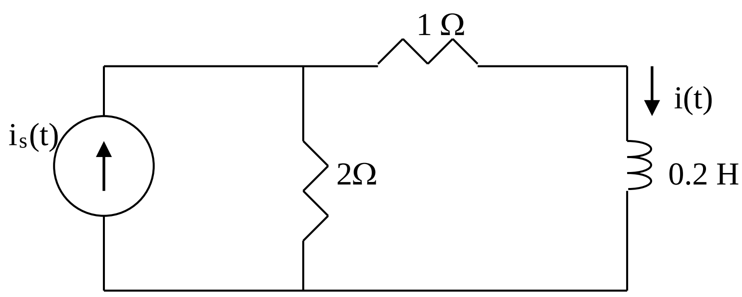
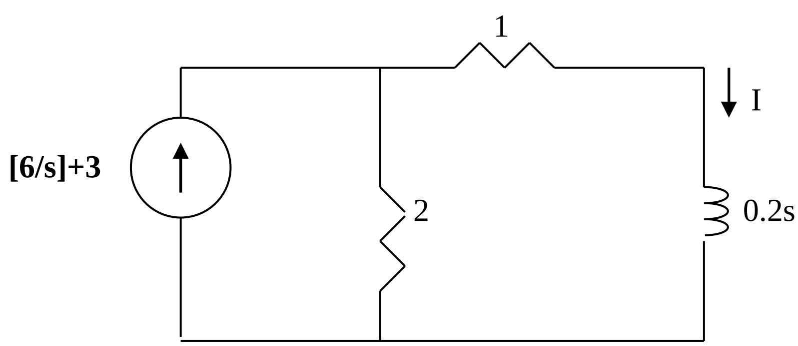


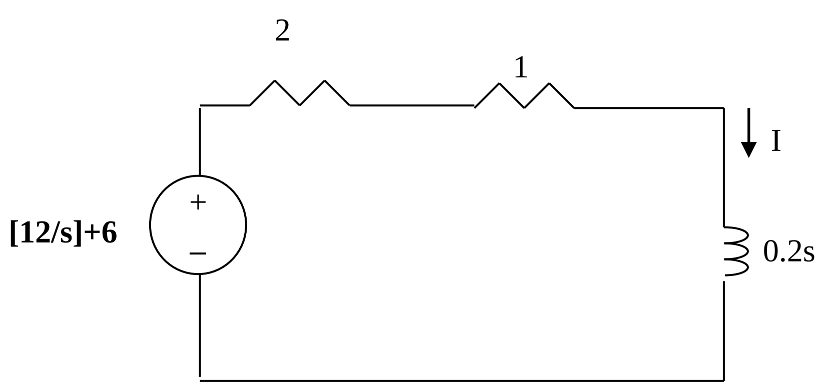
Figure 16.37
For Prob. 16.14.

Solution

In the s-domain, the circuit becomes that shown below.



We transform the current source to a voltage source and obtain the circuit shown below.



$I = [(12/s)+6]/(0.2s+3) = (30s+60)/[s(s+15)] = [A/s] + [B/(s+15)]$ where

$A = 60/15 = 4$ and $B = (30(-15)+60)/(-15) = 390/15 = 26$

$I = [4/s] + [26/(s+15)]$ or

$$i(t) = \mathbf{[4 + 26e^{-15t}]u(t)\ A}.$$

Solution 16.15

For the circuit in Fig. 16.38, calculate the value of *R* needed to have a critically damped response.

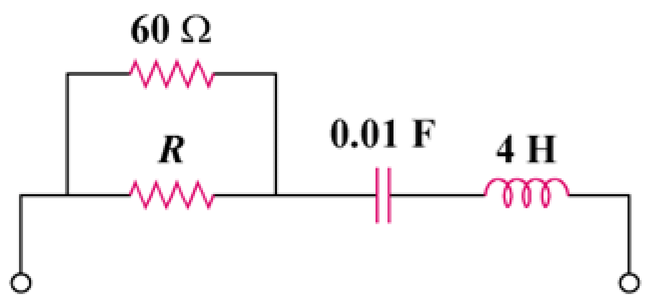


Figure 16.38
For Prob. 16.15.

Solution

Step 1. Let $R||60 = R_o$. Next, convert the circuit into the s-domain and solve for $T(s) = R_o + [1/(0.01s)] + 4s = R_o + (100/s) + 4s = [(4s^2+R_os+100)/s]$. Now to solve for the roots that represent a critically damped system.

$s_{1,2} = \{-R_o \pm [(R_o)^2-4(400)]^{0.5}\}/2.$

The system is critically damped when $[(R_o)^2-4(400)] = 0$.

Step 2. $(R_o)^2 = 1600$ or $R_o = 40$. Since $R_o = [Rx60/(R+60)] = 40$ or $60R = 40R+2400$ or $20R = 2400$ or $R =$ **120 Ω**.

Solution 16.16

The capacitor in the circuit of Fig. 16.39 is initially uncharged. Find $v_o(t)$ for $t>0$.

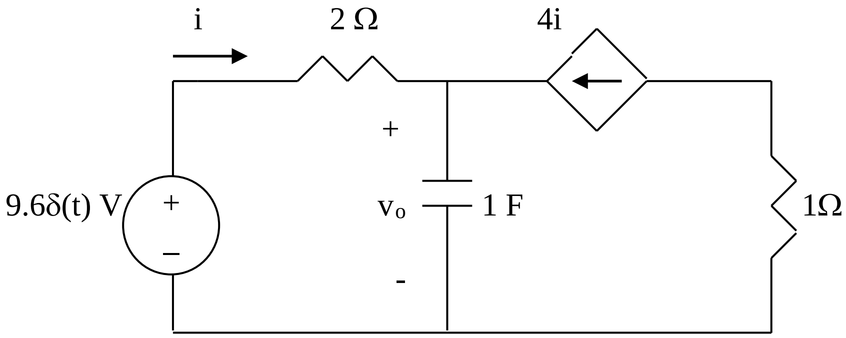
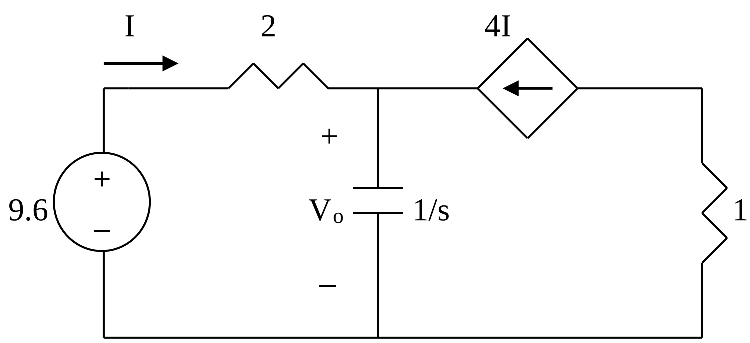


Figure 16.39
For Prob. 16.16.

Solution

The circuit in the s-domain is shown below.



$$I + 4I = \frac{V_o}{1/s} \quad \longrightarrow \quad 5I = sV_o$$

But $I = (9.6–V_o)/2$ which gives us $5[(9.6–V_o)/2] = sV_o$ or

$V_o(s+2.5) = 24$ or $V_o = 24/(s+2.5)$ therefore,

$v_o(t) = \mathbf{24e^{-2.5t}u(t)\ V}$.

Solution 16.17

If $i_s(t) = 7.5e^{-2t}u(t)$ A in the circuit shown in Fig. 16.40, find the value of $i_o(t)$.

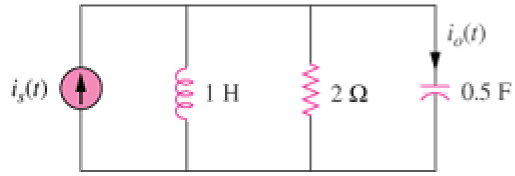
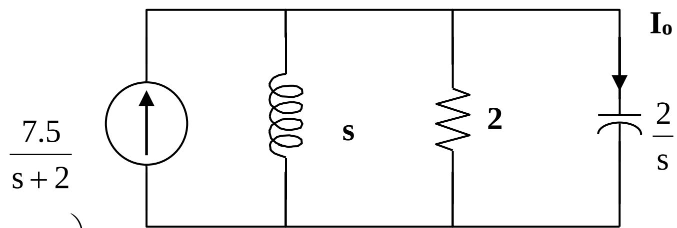


Figure 16.40 For Prob. 16.17.

Solution

We need to determine the initial conditions which in this case are all equal to zero since there are no sources before t = 0. Next we convert the circuit into the s-domain. We can write a nodal equation and then calculate $I_o = V/(2/s)$. We then perform a partial fraction expansion and convert back into the time domain.



$$V = \frac{7.5}{s+2}\left(\frac{1}{\frac{1}{s}+\frac{1}{2}+\frac{s}{2}}\right) = \frac{7.5}{s+2}\left(\frac{2s}{s^2+s+2}\right) = \frac{15s}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$I_o = \frac{Vs}{2} = \frac{7.5s^2}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$= \frac{7.5}{s+2} + \frac{\frac{7.5(-0.5-j1.3229)^2}{(1.5-j1.3229)(-j2.646)}}{s+0.5+j1.3229} + \frac{\frac{7.5(-0.5+j1.3229)^2}{(1.5+j1.3229)(+j2.646)}}{s+0.5-j1.3229}$$

$$i_o(t) = \underline{7.5\left(e^{-2t} + 0.3779e^{-90^\circ}e^{-t/2}e^{-j1.3229t} + 0.3779e^{90^\circ}e^{-t/2}e^{j1.3229t}\right)u(t)\,A}$$

or

$$\underline{= 7.5\left(e^{-2t} - 0.7559e^{-0.5t}\sin 1.3229t\right)u(t)\,A}$$

$$\text{or } i_o(t) = 7.5\left(e^{-2t} - \frac{2}{\sqrt{7}}e^{-0.5t}\sin\left(\frac{\sqrt{7}}{2}t\right)\right)u(t)A$$

Solution 16.18

Find v(t), t>0 in the circuit of Fig. 16.41. Let v_s = 12 V.

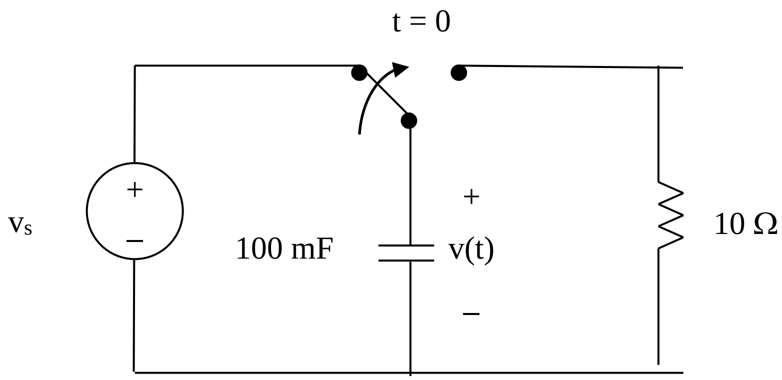
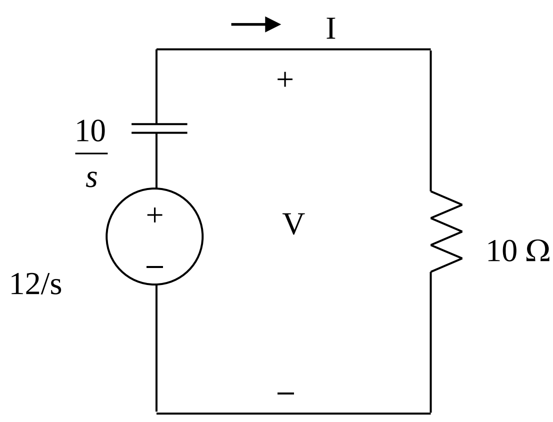


Figure 16.41
For Prob. 16.18.

Solution

For t<0, v(0) = v_s = 12 V.
For t>0, the circuit in the s-domain is as shown below.



$$100mF = 0.1F \quad \longrightarrow \quad \frac{1}{sC} = \frac{10}{s} \quad \text{and}$$

I = (12/s)/[10+(10/s)] = 1.2/(s+1) and V = 10I which gives us

v(t) = **[12e^{-t}]u(t) V**.

Solution 16.19

The switch in Fig. 16.42 moves from position A to position B at t=0 (please note that the switch must connect to point B before it breaks the connection at A, a make before break switch). Find v(t) for t >0.

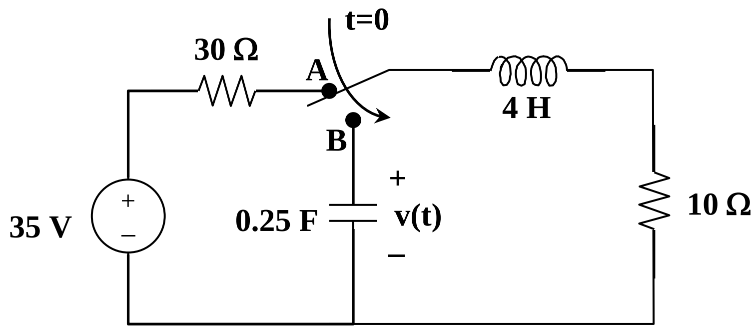
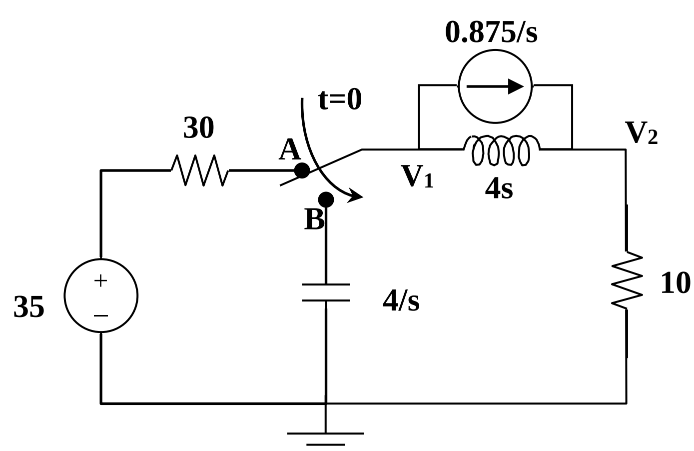


Figure 16.42 For Prob. 16.19.

Solution

Step 1. First find all the initial conditions and then transform into the s-domain. Since the capacitor is not connected to a circuit, we do not know its initial condition so we can assume it is zero ($v(0) = 0$). We can find $i_L(0)$ by letting the inductor be a short and $i_L(0) = 35/40 = 0.875$ amp.



$[(V_1–0)/(4/s)] + [(V_1–V_2)/(4s)] + (0.875/s) = 0$ and $[(V_2–V_1)/(4s)] + (–0.875/s) + [(V_2–0)/10] = 0$ where $V = V_1$. Next, add these together, $[sV_1/4] + [V_2/10] = 0$ or $V_2 = –2.5sV_1$. Now we can solve for V_1 and V_2.

Step 2. $[(s/4)+(1/(4s))+(2.5s/(4s))]V_1 = –0.875/s$
$= [(s^2+2.5s+1)/(4s)]V_1$ or $V_1 = –0.875(4)/(s^2+2.5s+1) = –3.5/[(s+0.5)(s+2)]$
$= [–2.3333/(s+0.5)]+[2.3333/(s+2)]$ or

$v(t) =$ **$[–2.333e^{–t/2}+2.333e^{–2t}]u(t)$ volts**.

Solution 16.20

Find $i(t)$ for $t > 0$ in the circuit of Fig. 8.43.

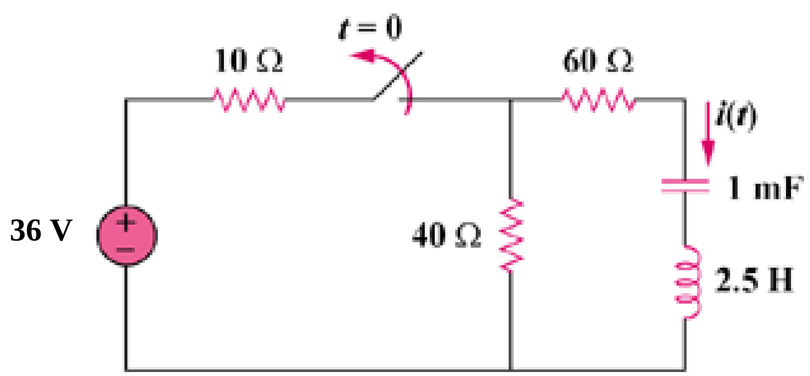
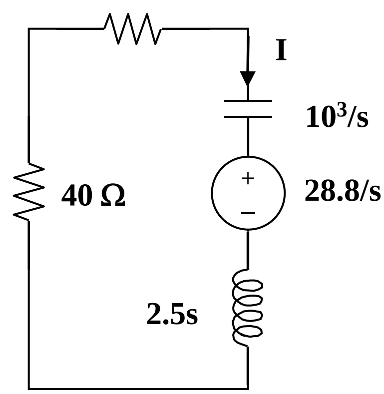


Figure 16.43
For Prob. 16.20.

Step 1. Convert the circuit into the s-domain and write one loop equation noting that $i(0) = 0$ and $v_c(0) = 28.8$ volts.



$[(1000/s)]I + [28.8/s] + [2.5s]I + [40+60]I = 0$ or
$[(2.5s^2+100s+1000)/s]I = -28.8/s$ or $I = -28.8/[2.5(s^2+40s+400)] = -11.52/(s+20)^2$

Step 2. $I = [A/(s+20)] + [B/(s+20)^2]$ where $B = -11.52$ so $A = 0$. Thus,

$i(t) = \mathbf{-11.52te^{-20t}u(t)}$ **A**.

Solution 16.21

In the circuit of Fig. 16.44, the switch moves (make before break switch) from position *A* to *B* at $t = 0$. Find $v(t)$ for all $t \geq 0$.

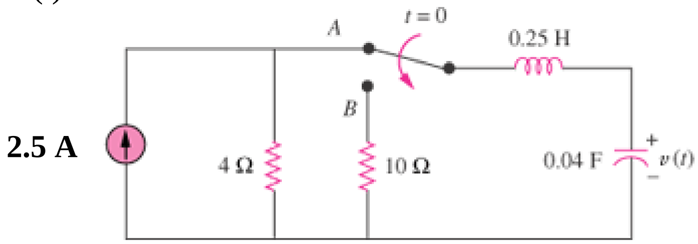
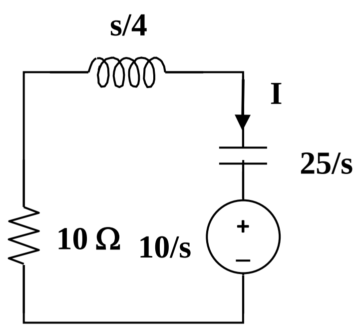


Figure 16.44 For Prob. 16.21.

Solution

Step 1. First we need to find our initial conditions, clearly i(0) = 0 and v(0) = 4x2.5 = 10 volts. Next we convert the circuit into the s-domain. We can then write a mesh equation and solve for v(t).



$[10+(s/4)+(25/s)]I + 10/s = 0$ and $V = (25/s)I + 10/s$

Step 2. $I = -(10/s)/[\ 10+(s/4)+(25/s)] = -(10/s)\{4s/[s^2+40s+100]$
$= -40/[(s+2.679)(s+37.32)] = [A/(s+2.679)]+[B/(s+37.32)]$ where
$A = -40/(-2.679+37.32) = -1.1547$ and $B = -40/(-37.32+2.679) = 1.1547$.

This now leads to $V = (25/s)I+10/s$
$= \{(25)(-1.1547)/[s(s+2.679)]\}+\{(25)(1.1547)/[s(s+37.32)]\}+10/s$
$= \{-28.868/[s(s+2.679)]\}+\{28.868/[s(s+37.32)]\}+10/s$
$= [a/s]+[b/(s+2.679)]+[c/(s+37.32)]$ where
$a = [-28.868/2.679]+[28.868/37.32]+10 = -10.7757+0.773531+10 = -0.0022$ (In practice and theoretically, this term must be equal to be zero since there will be no energy in the circuit at $t = \infty$!);
$b = [-28.868/(-2.679)] = 10.776$; and $c = 173.2/(-37.32) = -0.7735$ Thus,

$$v(t) = \mathbf{[10.776e^{-2.679t}-0.774e^{-37.32t}]u(t)\ volts}.$$

Note that at t = 0 we get 10.002 volts and not 10, this is due to rounding error.

Solution 16.22

Find the voltage across the capacitor as a function of time for $t > 0$ for the circuit in Fig. 16.45. Assume steady-state conditions exist at $t = 0^-$.

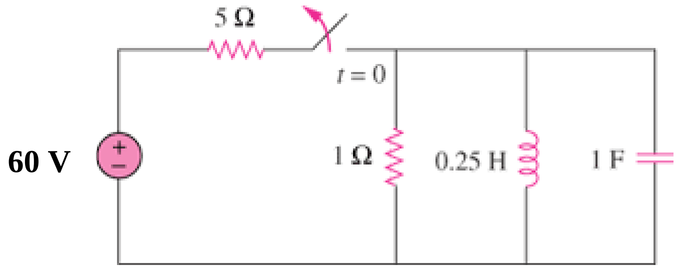
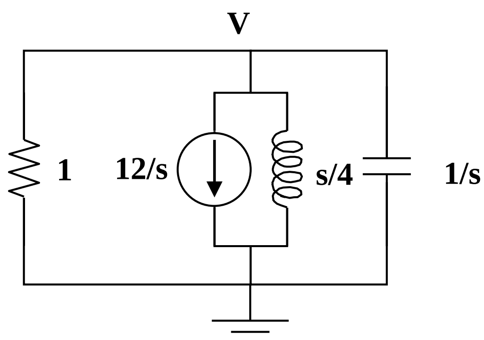


Figure 16.45
For Prob. 16.22.

Solution

Step 1. First we need to calculate the initial conditions, $v_C(0) = 0$ and $i_L(0) = 60/5 = 12$ amps. Next we need to convert the circuit into the s-domain and solve for the node voltage $V = V_C$. Convert this back into the time domain and obtain $v_C(t)$.



$[(V–0)/1]+[12/s]+[(V–0)/(s/4)]+[(V–0)/(1/s)] = 0$ then solve for V, next complete a partial fraction expansion, and then convert back into the time domain.

Step 2. $[1+(4/s)+s]V = [(s^2+s+4)/s]V = –12/s$ or
$V = –12/[(s+0.5+j1.9365)(s+0.5–j1.9365)]$
$= [A/(s+0.5+j1.9365)] + [B/(s+0.5–j1.9365)]$ where $A = –12/(–j3.873)$
$= 3.098\angle–90^\circ$ and $B = –12/(j3.873) = 3.098\angle 90^\circ$. Thus,

$$v_C(t) = 3.098e^{–t/2}[e^{–(j1.9365t+90^\circ)}+e^{(j1.9365t+90^\circ)}]u(t)$$

$$= \mathbf{6.197e^{–t/2}\cos(1.9365t+90^\circ)u(t)\ volts}.$$

Solution 16.23

Obtain $v(t)$ for $t > 0$ in the circuit of Fig. 16.46.

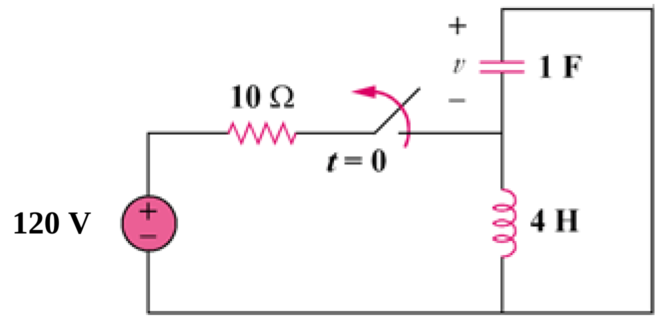
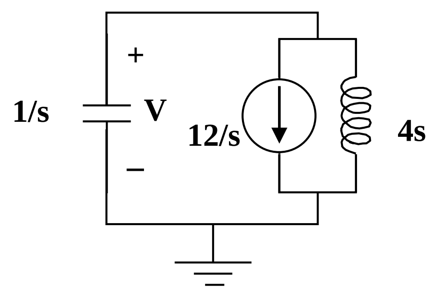


Figure 16.46
For Prob. 16.23.

Solution

Step 1. First we need to calculate the initial conditions. Clearly since the inductor looks like a short, $v(0) = 0$ and $i_L(0) = 120/10 = 12$ amps. Next we convert the circuit into the s-domain and solve for V and then obtain the partial fraction expansion and convert back into the time domain.



$[(V–0)/(1/s)]+(12/s)+[(V–0)/4s] = 0$

Step 2.

$[s+(1/(4s))]V = –12/s = [(s^2+0.25)/(4s)]V$ or $V = –12/[(s+j0.5)(s–j0.5)]$
$= [A/(s+j0.5)] + [B/(s–j0.5)]$ where $A = –12/(–j) = 12\angle–90^\circ$ and
$B = –12/(j) = 12\angle 90^\circ$. Thus,
$v(t) = 12[e^{–(j0.5t+90^\circ)}+e^{(j0.5t+90^\circ)}]u(t)$

$=$ **$24\cos(0.5t+90^\circ)u(t)$ volts**.

Solution 16.24

The switch in the circuit of Fig. 16.47 has been closed for a long time but is opened at $t = 0$. Determine $i(t)$ for $t > 0$.

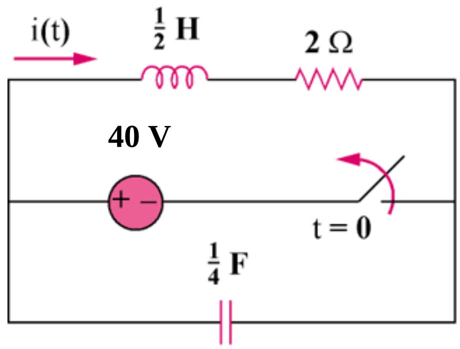
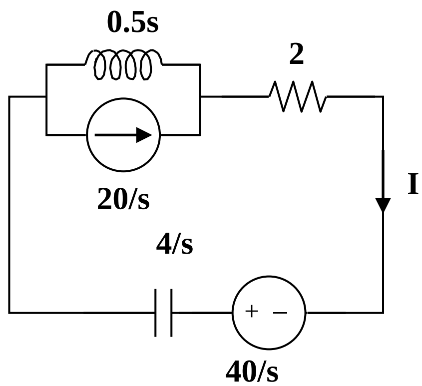


Figure 16.47
For Prob. 16.24.

Solution

Step 1. First we solve for the initial conditions and then convert the circuit into the s-domain and then solve for I, perform a partial fraction expansion, then convert back into the time domain. We recognize that the capacitor becomes an open circuit and the inductor becomes a short circuit at $t = 0^-$.
Therefore, $v(0) = 40$ volts and $i(0) = 40/2 = 20$ amps.



We can use mesh analysis, $-(40/s) + (4/s)I + (0.5s)(I-20/s) + 2I = 0$.

Step 2. $[(4/s)+0.5s+2]I = (40/s)+(10) = (10s+40)/s = [(s^2+4s+8)/(2s)]I$ or
$I = (20s+80)/[(s+2+j2)(s+2-j2)] = [A/(s+2+j2)] + [B/(s+2-j2)]$ where
$A = (-40-j40+80)/(-j4) = 56.568\angle -45^\circ/4\angle -90^\circ = 14.142\angle 45^\circ$ and
$B = (-40+j40+80)/(j4) = 14.142\angle -45^\circ$. Thus,
$i(t) = 14.142e^{-2t}[e^{-(j2t-45^\circ)}+e^{(j2t-45^\circ)}]u(t)$

$= \mathbf{28.28e^{-2t}\cos(2t-45^\circ)}$ **amps**.

Solution 16.25

Calculate $v(t)$ for $t > 0$ in the circuit of Fig. 16.48.

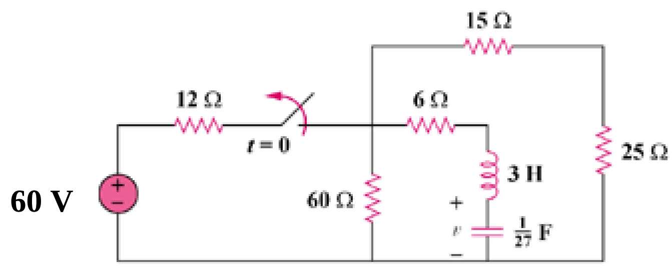
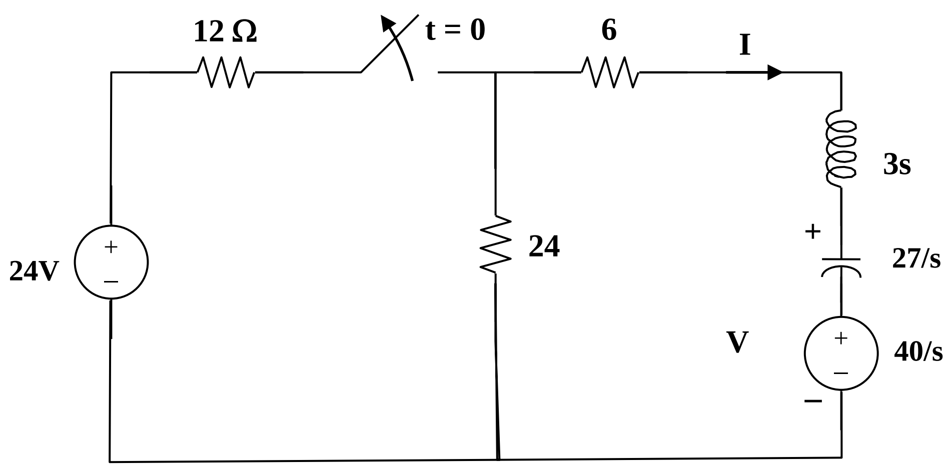


Figure 16.48
For Prob. 16.25.

Step 1. First solve for the initial conditions. Then simplify the circuit and then convert it into the s-domain and then solve for v(t). Since the capacitor becomes an open circuit, $i_L(0) = 0$ and $v(0) = (60)24/36 = 40$ volts.



We can now use mesh analysis to solve for V(s). $(30+3s+27/s)I + 40/s = 0$ or $[3(s^2+10s+9)/s]I = -40/s$ or $I = -(40/3)/[(s+1)(s+9)]$ and $v = (27/s)I + 40/s$.

Step 2. $V = -(40/3)(27)/[s(s+1)(s+9)] + 40/s = -360/[s(s+1)(s+9)] + 40/s$. Thus,

$V = [A/s]+[B/(s+1)]+[C/(s+9)]$ where $A = -(360/9)+40 = 0$ (as expected) and $B = -360/[(-1)(-1+9)] = 360/8 = 45$ and $C = -360/[(-9)(-9+1)] = -360/72 = -5$.

$$v(t) = \mathbf{[45e^{-t}-5e^{-9t}]u(t)\ volts}.$$

Solution 16.26

The switch in Fig. 16.49 moves from position A to position B at t=0 (please note that the switch must connect to point B before it breaks the connection at A, a make before break switch). Determine i(t) for t >0. Also assume that the initial voltage on the capacitor is zero.

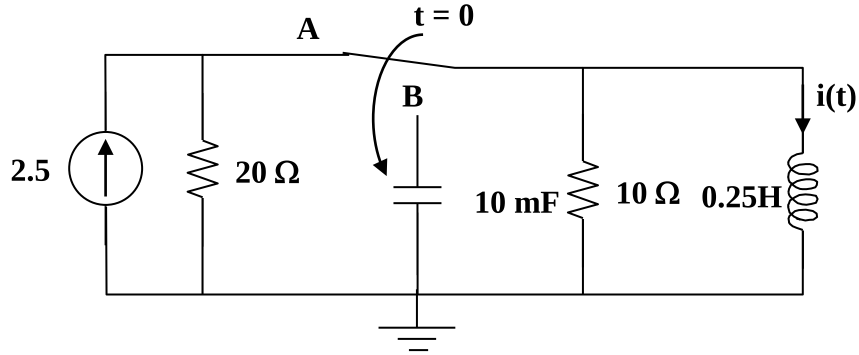
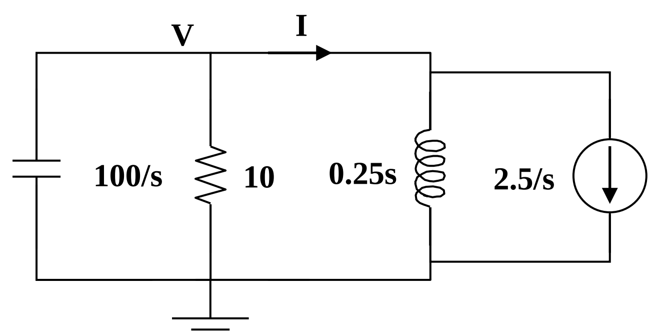


Figure 16.49 For Prob. 16.26.

Solution

Step 1. Determine the initial conditions and then convert the circuit into the s-domain. Then solve for V and then find I. Convert it into the time domain. It is clear from the circuit that $i_L(0) = 2.5$ A.



Applying nodal analysis we get,
$[(V–0)/(100/s)]+[(V–0)/10]+[(V–0)/(0.25s)]+2.5/s = 0$ where
$I = [(V–0)/(0.25s)]+2.5/s$.

Step 2. $[(s^2+10s+400)/(100s)]V = –2.5/s$ or
$V = –250/[(s+5+j19.365)(s+5–j19.365)]$ or
$I = –\{1{,}000/[s(s+5+j19.365)(s+5–j19.365)]\}+2.5/s$
$= [A/s]+[B/(s+5+j19.365)]+[C/(s+5–j19.365)]$ where $A = –2.5+2.5 = 0$, as to be expected; $B = –1{,}000/[(–5–j19.365)(–j38.73)]$
$= 1{,}000\angle 180^\circ/[(20\angle –104.48)(38.73\angle –90^\circ)] = 1.291\angle 14.48^\circ$; and
$C = –1{,}000/[(–5+j19.365)(j38.73)] = 1{,}000\angle 180^\circ/[(20\angle 104.48)(38.73\angle 90^\circ)]$
$= 1.291\angle –14.48^\circ$.

$i(t) =$ **$[2.582e^{–5t}\cos(19.365t+14.48^\circ)]u(t)$ amps**.

Solution 16.27

Find $v(t)$ for $t > 0$ in the circuit in Fig. 16.50.

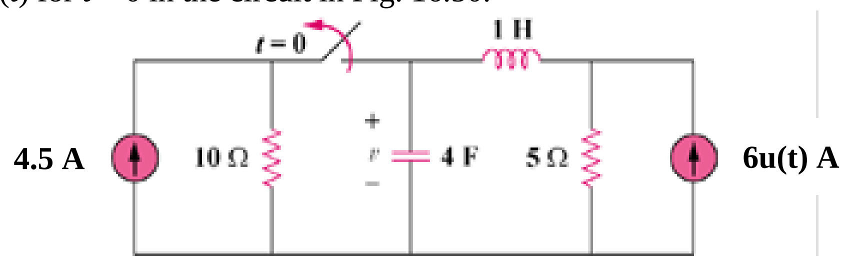


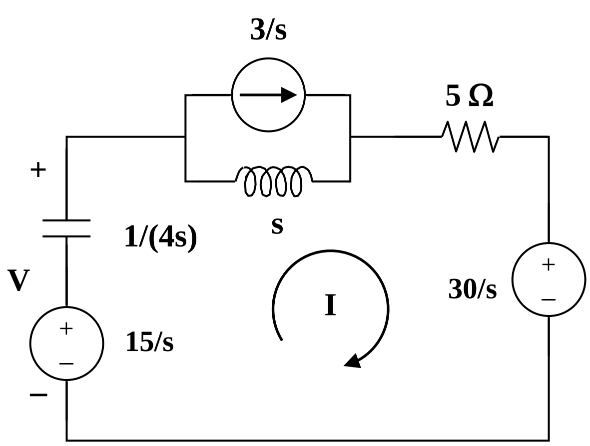
Figure 16.50
For Problem 16.27.

Solution

Step 1. First we need to determine the initial conditions. We note that the source on the right is equal to zero until the switch opens. So, all initial conditions come from the 4.5-amp source on the left. Since the capacitor looks like an open and the inductor looks like a short we get,

$v(0) = 4.5[5\text{x}10/(5+10)] = 15$ volts and $i_L(0) = 15/5 = 3$ amps.

Next we convert the circuit ($t > 0$) into the s-domain with initial conditions. Then we can solve for V, perform a partial fraction expansion and solve for v(t).



$-[15/s]+[1/(4s)]I+[s(I-(3/s))]+5I+[30/s] = 0$ and $V = [1/(4s)](-I) + [15/s]$

Step 2. $\{[1/(4s)]+s+5\}I = \{[s^2+5s+0.25]/(s)\}I = 3-15/s = 3(s-5)/s$ or
$I = 3(s-5)/[(s+0.05051)(s+4.949)]$ and
$V = \{-3(s-5)/[(4s)(s+0.05051)(s+4.949)]\}+15/s$
$= \{-0.75(s-5)/[s(s+0.05051)(s+4.949)]\}+15/s$

$V = [A/s] + [B/(s+0.05051)] + [C/(s+4.949)]$ where

$A = [3.75/[(0.05051)(4.949)]]+15 = 30;$
$B = –0.75(–0.05051–5)/[(–0.05051)(–0.05051+4.949)]$
$= 3.7879/(–0.24742) = –15.309;$ and
$C = –0.75(–4.949–5)/[(–4.949)(–4.949+0.05051)] = 7.4618/24.243 = 0.30779.$

$$v(t) = \mathbf{[30–15.309e^{–0.05051t}+0.3078e^{–4.949t}]u(t)\ volts}.$$

$dv/dt = –15.309(–0.05051)+0.30779(–4.949) = 0.77326–1.52325 = 0.75$ or
$Cdv/dt = –4x0.75 = –3$ amps, the answer checks!

Solution 16.28

For the circuit in Fig. 16.51, find $v(t)$ for $t > 0$.

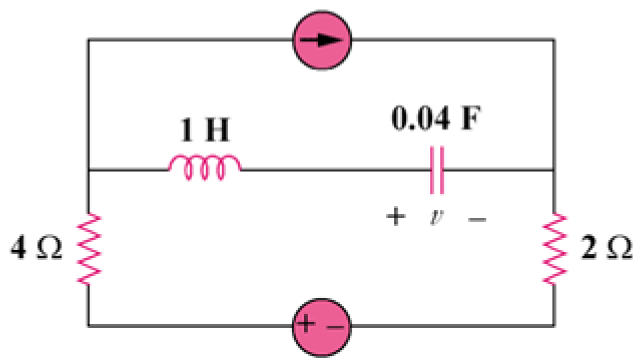
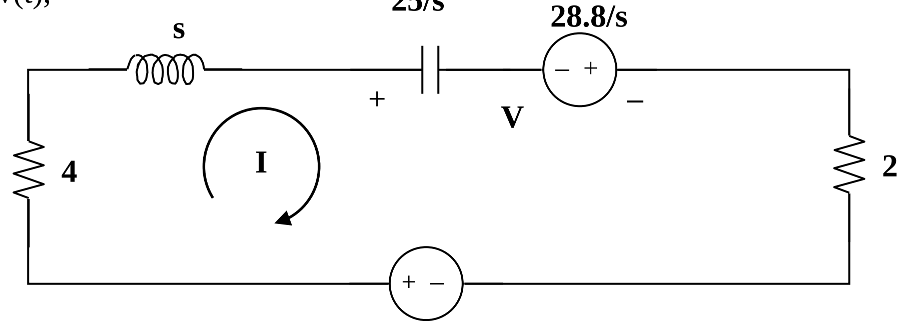


Figure 16.51
For Prob. 16.28.

Solution

Step 1. Determine the initial conditions (at t = 0, the 4.8 amp current source turns off and the 120 volt voltage source becomes active). Since the capacitor becomes an open circuit, $i_L(0) = 0$ and $v(0) = –4.8x6 = –28.8$ volts. Now convert the circuit into the s-domain and solve for V and then convert it into the time domain to obtain v(t),



Now for the mesh equation, $[4+s+(25/s)+2]I–(28.8/s)–(120/s) = 0$. $V = (25/s)I–28.8/s$.

Step 2.
$[(s^2+6s+25)/s]I = 148.8/s$ or $I = 148.8/(s^2+6s+25) = 148.8/[(s+3+j4)(s+3–j4)]$ thus,

$V = \{3,720/[s(s+3+j4)(s+3–j4)]\}–28.8/s = [A/s]+[B/(s+3+j4)]+[C/(s+3–j4)]$
where

$A = (3{,}720/25)–28.8 = 148.8–28.8 = 120$; $B = 3{,}720/[(–3–j4)(–j8)]$
$= 3{,}720/[(5\angle–126.87^\circ)(8\angle–90^\circ)] = 93\angle–143.13^\circ$; and
$C = 3{,}720/[(j8)(–3+j4)] = 3{,}720/[(5\angle126.87^\circ)(8\angle90^\circ)] = 93\angle143.13^\circ$.

$v(t) = \mathbf{[120+186e^{–3t}\cos(4t+143.13^\circ)]u(t)}$ **volts**.

Solution 16.29

Calculate $i(t)$ for $t > 0$ in the circuit in Fig. 16.52.

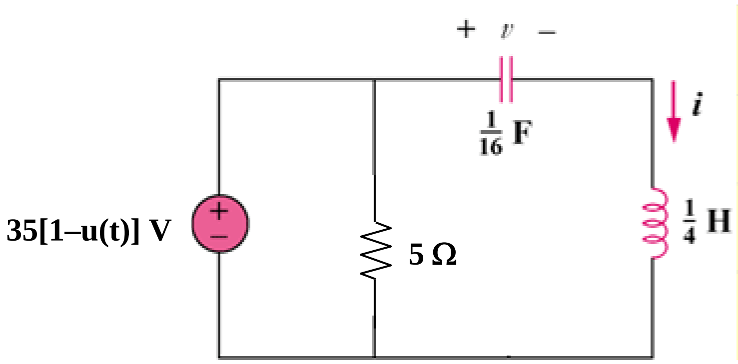
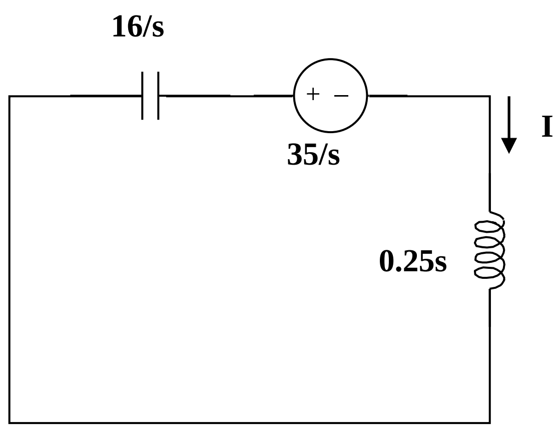


Figure 16.52
For Prob. 16.29.

Solution

Step 1. Calculate the initial conditions and then convert the above circuit into the s-domain. Then solve for I, perform a partial fraction expansion, and convert into the time domain. v(0) = 35 volts and i(0) = 0.



$[16/s]I + [35/s] + 0.25sI = 0.$

Step 2. $\{[16/s]+0.25s\}I = -35/s = \{[s^2+64]/(4s)\}I$ or $I = -140/[(s+j8)(s-j8)]$ or

$I = [A/(s+j8)] + [B/(s-j8)]$ where $A = -140/(-j16) = 8.75\angle{-90^\circ}$ and
$B = -140/(j16) = 8.75\angle 90^\circ$ thus,

$i(t) = [8.75e^{-j8t-90^\circ}+8.75e^{j8t+90^\circ}]u(t) =$ $\mathbf{17.5\cos(8t+90^\circ)u(t)}$ **amps**.

Solution 16.30

Find $v_o(t)$, for all $t>0$, in the circuit of Fig. 16.53.

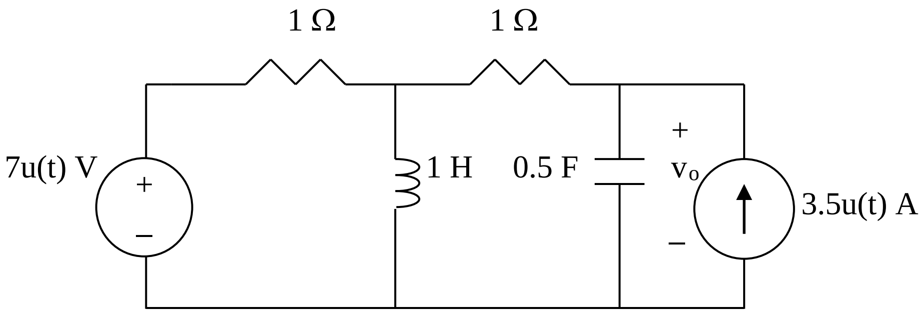
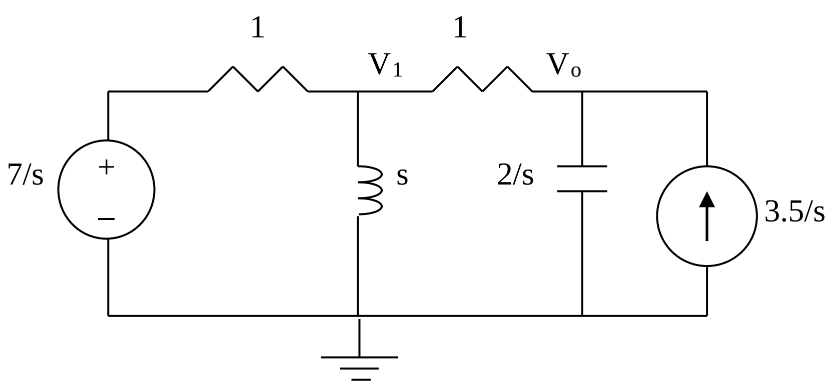


Figure 16.53
For Prob. 16.30.

Solution

The circuit in the s-domain is shown below. Please note, $i_L(0) = 0$ and $v_o(0) = 0$ because both sources were equal to zero for all $t<0$.



At node 1

$[(V_1–7/s)/1] + [(V_1–0)/s] + [(V_1–V_2)/1] = 0$ or $[1+(1/s)+1]V_1–V_2 = 7/s$ or
$[(2s+1)/s]V_1–V_2 = 7/s$

At node o

$[(V_o–V_1)/1] + [(V_o–0)/(2/s)] – (3.5/s) = 0$ or
$–V_1+[(s+2)/2]V_o = 3.5/s$

In matrix form we get,

$$\begin{bmatrix} \frac{2s+1}{s} & -1 \\ -1 & \frac{s+2}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_o \end{bmatrix} = \begin{bmatrix} \frac{7}{s} \\ \frac{3.5}{s} \end{bmatrix} \text{ or}$$

$$\begin{bmatrix} V_1 \\ V_o \end{bmatrix} = \frac{\begin{bmatrix} \frac{s+2}{2} & 1 \\ 1 & \frac{2s+1}{s} \end{bmatrix}}{\frac{(2s+1)(s+2)}{2s} - (-1)(-1)} \begin{bmatrix} \frac{7}{s} \\ \frac{3.5}{s} \end{bmatrix} = \frac{\begin{bmatrix} \frac{s+2}{2} & 1 \\ 1 & \frac{2s+1}{s} \end{bmatrix}}{\frac{s^2+1.5s+1}{s}} \begin{bmatrix} \frac{7}{s} \\ \frac{3.5}{s} \end{bmatrix}$$

$s^2+1.5s+1 = (s+0.75+j0.6614)(s+0.75–j0.6614)$

$V_o = s[(7/s)+[(2s+1)3.5]/s^2]/[(s+0.75+j0.6614)(s+0.75–j0.6614)]$
$= (14s+3.5)/[s(s+0.75+j0.6614)(s+0.75–j0.6614)]$
$= [A/s] + [B/(s+0.75+j0.6614)] + [C/(s+0.75–j0.6614]$ where $A = 3.5$;
$B = [14(–0.75–j0.6614)+3.5]/[(–0.75–j0.6614)(–j1.3228)]$
$= [–11.6078–j9.2596+3.5]/[(1\angle–138.59^\circ)(1.3228\angle–90^\circ)]$
$= (11.6078\angle–127.09^\circ)/[(1\angle–138.59^\circ)(1.3228\angle–90^\circ)] = 8.7751\angle101.5^\circ$
$C = [14(–0.75+j0.6614)+3.5]/[(–0.75+j0.6614)(j1.3228)]$
$= [–10.5+j9.2596+3.5]/[(1\angle138.59^\circ)(1.3228\angle90^\circ)]$
$= (11.6078\angle127.09^\circ)/[(1\angle138.59^\circ)(1.3228\angle90^\circ)] = 8.7751\angle–101.5^\circ$

Therefore,

$v_o(t) = [3.5+8.7751e^{–0.75t}e^{–(j0.6614t–101.5^\circ)}+8.7751e^{–0.75t}e^{(j0.6614t–101.5^\circ)}]u(t)$ volts or
$=$ **$[3.5+17.55e^{–0.75t}\cos(0.6614t–101.5^\circ)]u(t)$ volts**.

Solution 16.31

Obtain $v(t)$ and $i(t)$ for $t > 0$ in the circuit in Fig. 16.54.

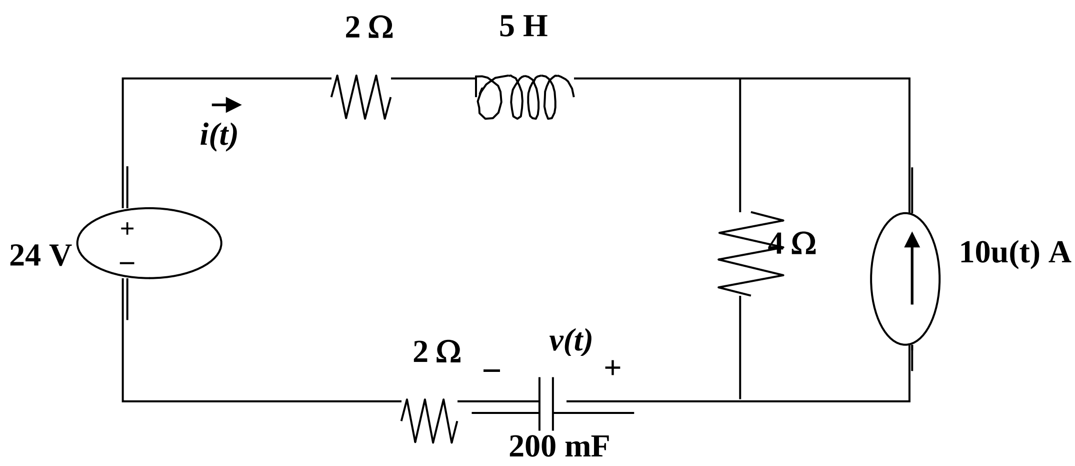


Figure 16.54
For Prob. 16.31.

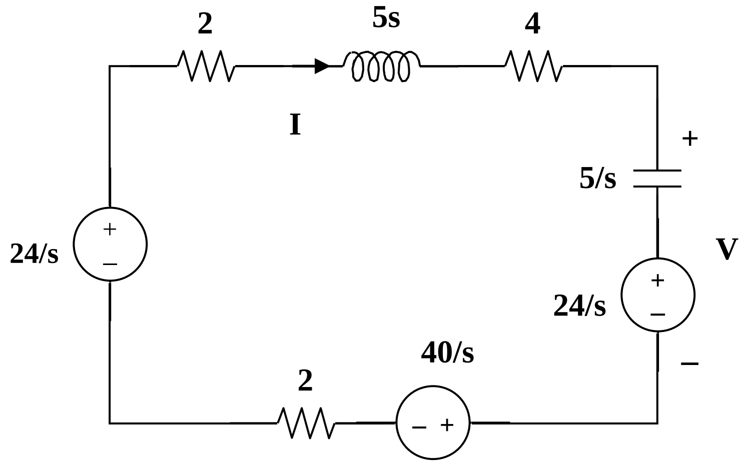
Solution

Step 1. First we need to determine the initial conditions. Then, we need to convert the circuit into the s-domain. We then can solve for I using a mesh equation. Once we have I we can solve for V = (1/(Cs))I + v(0)/s. Finally we perform a partial fraction expansion and solve for I and then do the same for V.

Finally we can solve for i(t) and v(t).

For t = 0^-, 10u(t) A = 0. Thus, i(0) = 0, and v(0) = 24 V.

For t > 0, we have the series RLC circuit shown below.



$-(24/s) + 2I + 5sI + 4I + (5/s)I + (24/s) + (40/s) + 2I = 0.$

Step 2. $[5s+2+4+2+(5/s)]I = -40/s = [(5s^2+8s+5)/s]I$ or

$I = -40/[5(s^2+1.6s+1)]$ where $s_{1,2} = \dfrac{-1.6 \pm \sqrt{2.56-4}}{2} = -0.8 - j0.6, -0.8 + j0.6.$

Now, $I = [A/(s+0.8+j0.6)] + [B/(s+0.8-j0.6)]$ where $A = -8/(-0.8-j0.6+0.8-j0.6)$ $= -8/(-j1.2) = 6.6667\angle-90°$ and $B = 6.6667\angle 90°$. We can find V by solving $V = I(5/s) + 24/s = -40/[s(s+0.8+j0.6)(s+0.8-j0.6)] = [C/s] + [D/(s+0.8+j0.6)] +$ $[E/(s+0.8+j0.6)] + 24/s$ where $C = -40/(0.64+0.36) = -40$, $D = -40/[(-0.8-j0.6)(-0.8-j0.6+0.8-j0.6)] = -40/[(1\angle-143.13°)(1.2\angle-90°)]$ $= 33.333\angle 53.13°$, and $E = 33.333\angle-53.13°$. Finally, we get

$$i(t) = \mathbf{6.667e^{-0.8t}[e^{-j0.6t-90°} + e^{j0.6t+90°}]u(t)\ A} \text{ and}$$
$$v(t) = \mathbf{[-16 + 33.33e^{-0.8t}(e^{-j0.6t+53.13°} + e^{j0.6t-53.13°})]u(t)\ V}.$$

Solution 16.32

For the network in Fig. 16.55, solve for $i(t)$ for $t > 0$.

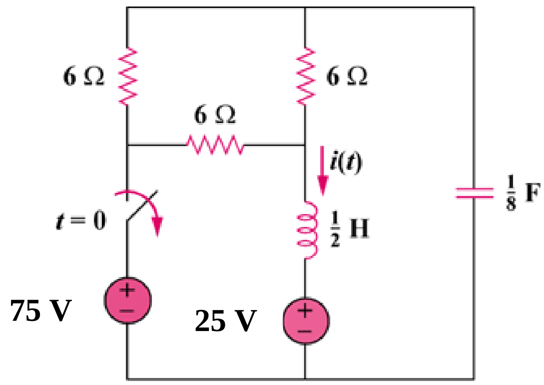
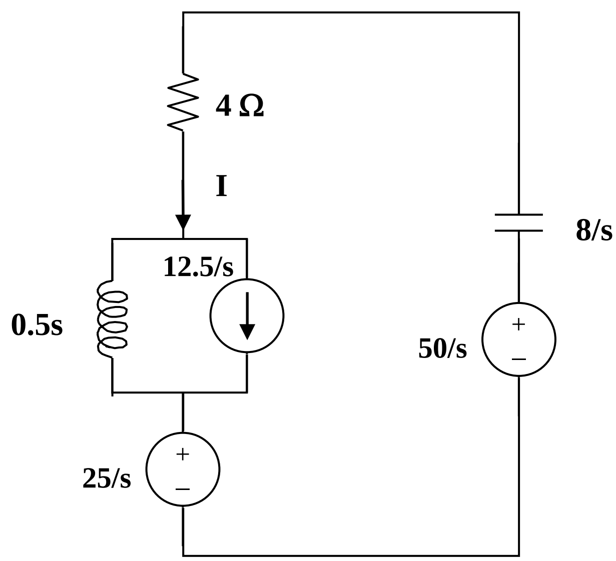


Figure 16.55 For Prob. 16.32.

Solution

Step 1. First we need to find all the initial conditions. Then we need to transform the circuit into the s-domain and solve for I. We then perform a partial fraction expansion and convert the results into the time domain. The inductor becomes a short and the capacitor becomes an open circuit. Thus, i(0) = [50/6] + [50/12] = 12.5 amps and $v_C(0)$ = 25+25 = 50 volts.



Loop equation, –[25/s]–0.5s(I–12.5/s)–4I–[8/s]I+[50/s] = 0.

Step 2. [0.5s+4+8/s]I = [(s^2+8s+16)/(2s)]I = –[25/s]+6.25+50/s = (s+4)/(0.16s) or

I = 12.5(s+4)/[$(s+4)^2$] = [A/(s+4)]+[B/$(s+4)^2$] where A(s+4)+B = 12.5(s+4) or A = 12.5 and B = 0. Therefore,

i(t) = **[12.5e^{-4t}]u(t) amps**.

Solution 16.33

Using Fig. 16.56, design a problem to help other students to better understand how to use Thevenin's theorem (in the s-domain) to aid in circuit analysis.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use Thevenin's theorem to determine $v_o(t)$, $t>0$ in the circuit of Fig. 16.56.

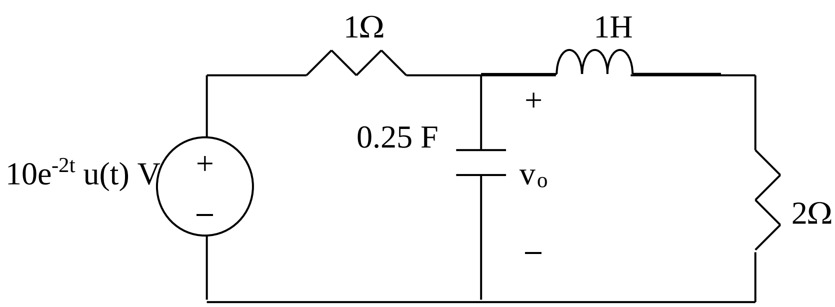


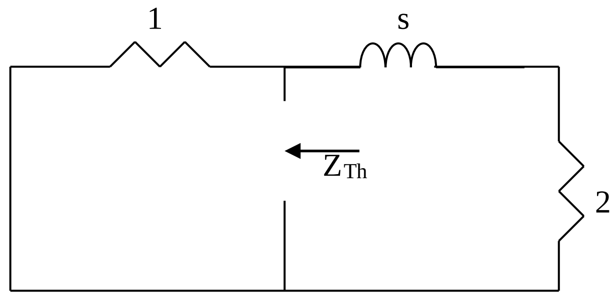
Figure 16.56 For Prob. 16.33.

Solution

$1H \longrightarrow 1s$ and $i_L(0) = 0$ (the sources is zero for all t<0).

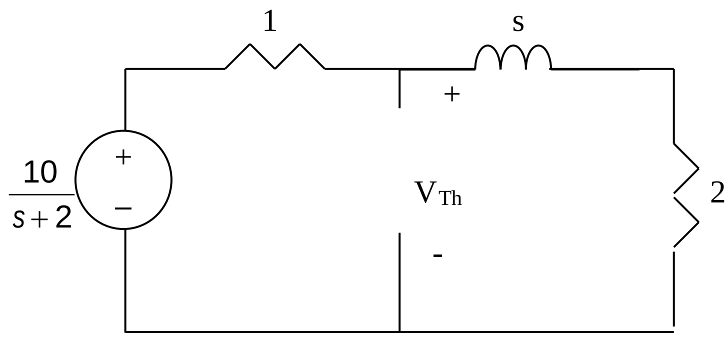
$\frac{1}{4}F \longrightarrow \frac{1}{sC} = \frac{4}{s}$ and $v_C(0) = 0$ (again, there are no source contributions for all t<0).

To find Z_{Th}, consider the circuit below.



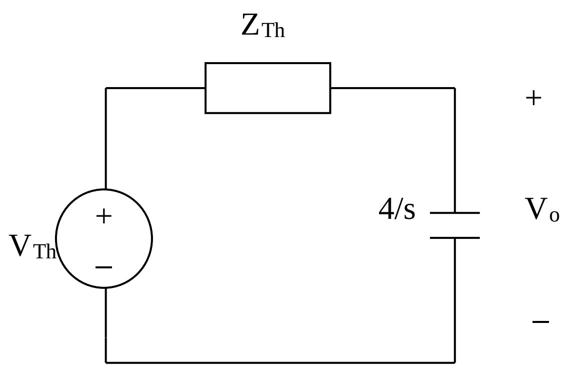
$$Z_{Th} = 1//(s+2) = \frac{s+2}{s+3}$$

To find V_{Th}, consider the circuit below.



$$V_{Th} = \frac{s+2}{s+3} \cdot \frac{10}{s+2} = \frac{10}{s+3}$$

The Thevenin equivalent circuit is shown below



$$V_o = \frac{\frac{4}{s}}{\frac{4}{s} + Z_{Th}} V_{Th} = \frac{\frac{4}{s}}{\frac{4}{s} + \frac{s+2}{s+3}} \cdot \frac{10}{s+3} = \frac{40}{s^2 + 6s + 12} = \frac{\frac{40}{\sqrt{3}}\sqrt{3}}{(s+3)^3 + (\sqrt{3})^2}.$$

$$v_o(t) = \underline{23.094 e^{-3t} \sin \sqrt{3} t} \text{ V}$$

Solution 16.34

Solve for the mesh currents in the circuit of Fig. 16.57. You may leave your results in the s-domain.

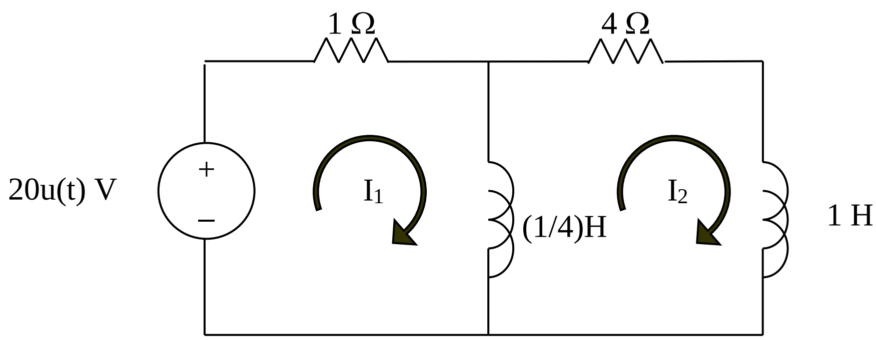
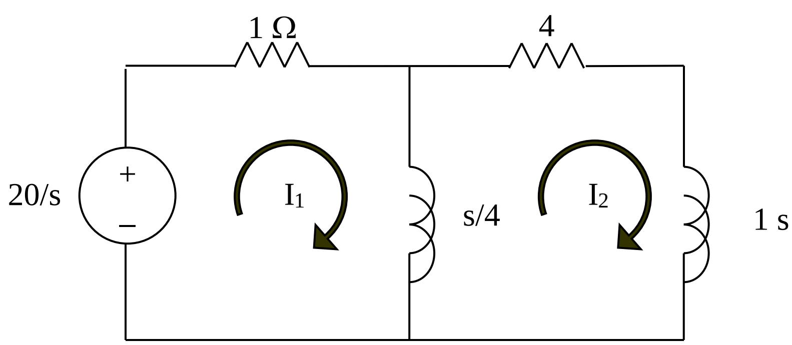


Figure 16.57 For Prob. 16.34.

Solution

In the s-domain, the circuit is as shown below.



$[20/s] = [1+(s/4)]I_1 - 0.25sI_2$ (1)

$-0.25sI_1 + [4+(5s/4)]I_2 = 0$ (2)

In matrix form,

$$\begin{bmatrix} 1+0.25s & -0.25s \\ -0.25s & 4+1.25s \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20/s \\ 0 \end{bmatrix} \text{ where } \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} 4+1.25s & 0.25s \\ 0.25s & 1+0.25s \end{bmatrix}}{0.25s^2+2.25s+4}\begin{bmatrix} 20/s \\ 0 \end{bmatrix}$$

$$I_1 = \frac{\frac{80}{s}+25}{0.25s^2+2.25s+4} = \underline{\frac{100s+320}{s(s^2+9s+16)}}$$

$$I_2 = \frac{5}{0.25s^2+2.25s+4} = \underline{\frac{20}{s^2+9s+16}}$$

Solution 16.35

Find $v_o(t)$ in the circuit in Fig. 16.58.

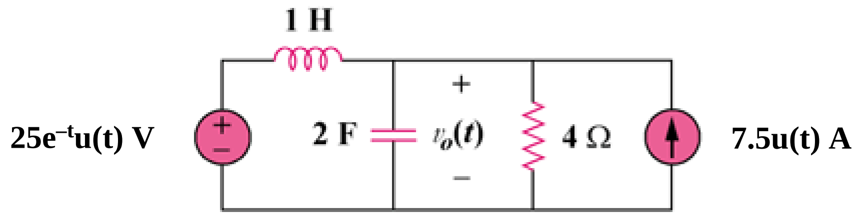
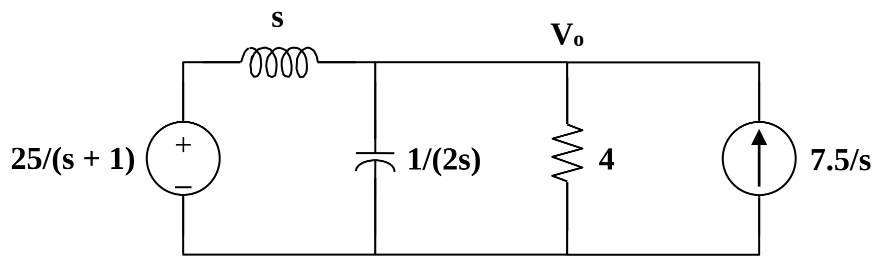


Figure 16.58
For Prob. 16.35.

Solution

Step 1. First we note that the initial condition on the capacitor and inductor must be equal to zero since the circuit is unexcited until t = 0. Next we transform the circuit into the s-domain.



We then can solve for V_o using nodal analysis.

$$\frac{V_o - \frac{25}{s+1}}{s} + \frac{2s(V_o - 0)}{1} + \frac{V_o - 0}{4} - \frac{7.5}{s} = 0$$

$$\left(\frac{1}{s} + 2s + 0.25\right)V_o = \frac{25}{s(s+1)} + \frac{7.5}{s}$$

Finally we solve for V_o, perform a partial fraction expansion and then convert into the time-domain.

Step 2. $$2\left(\frac{s^2 + 0.125s + 0.5}{s}\right)V_o = \frac{7.5s + 32.5}{s(s+1)} \text{ or } V_o = \frac{3.75s + 16.25}{(s^2 + 0.125s + 0.5)(s+1)}$$

Next

$$s_{1,2} = \frac{-0.125 \pm \sqrt{0.015625 - 2}}{2} = -0.0625 \pm \frac{\sqrt{-1.984375}}{2} = -0.0625 \pm j0.70435$$

$$V_o = \frac{3.75s+16.25}{(s+1)(s+0.0625+j0.70435)(s+0.0625-j0.70435)}$$
$$= \frac{A}{s+1} + \frac{B}{s+0.0625+j0.70435} + \frac{C}{s+0.0625-j0.70435}$$

where A = $\frac{-3.75+16.25}{1-0.125+0.5} = \frac{12.5}{1.375} = 9.091$

B =
$$\frac{3.75(-0.0625-j0.70435)+16.25}{(-0.0625-j0.70435+1)(-j1.4087)} = \frac{16.0156-j2.6413}{(0.9375-j0.70435)(-j1.4087)}$$

$$= \frac{16.232\angle -9.36497°)}{(1.17261\angle -36.9178°)(1.4087\angle -90°)} = 9.8265\angle 117.553°$$

C =
$$\frac{3.75(-0.0625+j0.70435)+16.25}{(-0.0625+j0.70435+1)(j1.4087)} = \frac{16.0156+j2.6413}{(0.9375+j0.70435)(j1.4087)}$$

$$= \frac{16.232\angle 9.36497°}{(1.17261\angle 36.9178°)(1.4087\angle 90°)} = 9.8265\angle -117.553°$$

Thus,

$v_o(t)$ =

$$\left[9.091\mathbf{e}^{-\mathbf{t}} + 9.8265\mathbf{e}^{-0.0625}\left(\mathbf{e}^{117.55°}\mathbf{e}^{-\mathbf{j}0.7044\mathbf{t}} + \mathbf{e}^{-117.55°}\mathbf{e}^{\mathbf{j}0.7044\mathbf{t}}\right)\right]\mathbf{u}(\mathbf{t})\,\textbf{volts}$$

or

$$\left[9.091\mathbf{e}^{-\mathbf{t}} + 19.653\mathbf{e}^{-0.0625\mathbf{t}}\mathbf{cos}(0.7044\mathbf{t} - 117.55°)\right]\mathbf{u}(\mathbf{t})\,\textbf{volts}.$$

$[9.091e^{-t} + 19.653e^{-0.0625t}\cos(0.7044t-117.55^o]u(t)$ V.

Solution 16.36

Refer to the circuit in Fig. 16.59. Calculate $i(t)$ for $t > 0$.

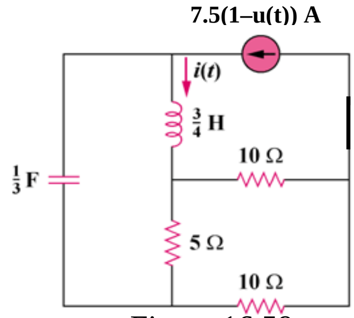
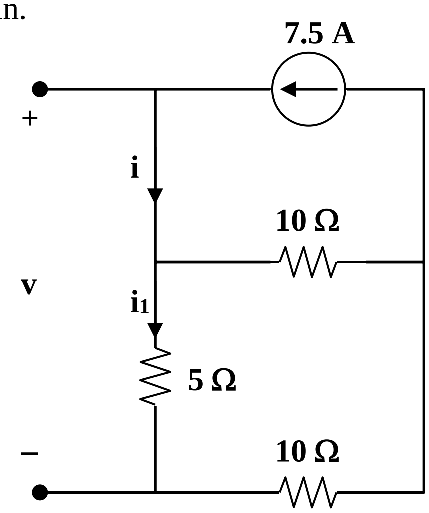


Figure 16.59
For Prob. 16.36.

Solution

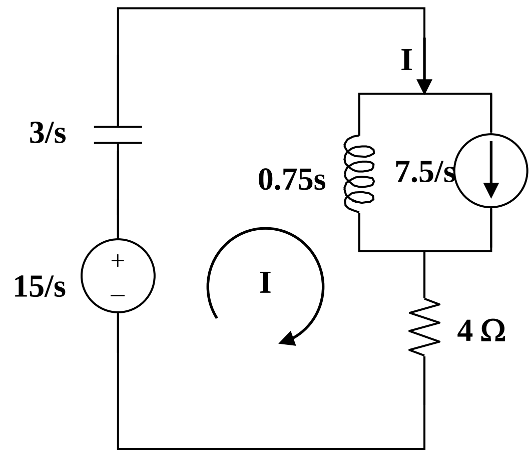
Step 1. First we need to determine the initial conditions and then transform the circuit into the s-domain.



Clearly i = 7.5 A. The current then travels through the parallel combination of the 10 ohm resistor and the combined 15 ohm resistance. i_1 = 7.5[(15)(10)/(15+10)]/(15) = 3 A. Therefore, v(0) = 5x3 = 15 V and i(0) = 7.5 A. We also note that the two 10 ohm resistors are in series and the combination is in parallel with the 5 ohm resistor resulting in a 100/25 = 4 ohm resistor.

The circuit in the s-domain is shown below.

$-[15/s] + [3/s]I + [0.75s](I-7.5/s) + 4I = 0.$



Step 2. $[(s^2+5.333s+4)/(4s/3)]I = 5.625+15/s = 5.625(s+2.667)/s$ or

$I = 7.5(s+2.667)/[(s+0.903)(s+4.43)] = [A/(s+0.903)]+[B/(s+4.43)]$ where

$A = 7.5(-0.903+2.667)/(-0.903+4.43) = 7.5x1.764/3.527 = 3.75$ and
$B = 7.5(-4.43+2.667)/(-4.43+0.903) = 7.5x(-1.764)/(-3.527) = 3.75$ or

$i(t) = \mathbf{[3.75e^{-0.903t}+3.75e^{-4.43t}]u(t)}$ **amps**.

Solution 16.37

Determine v for $t > 0$ in the circuit in Fig. 16.60.

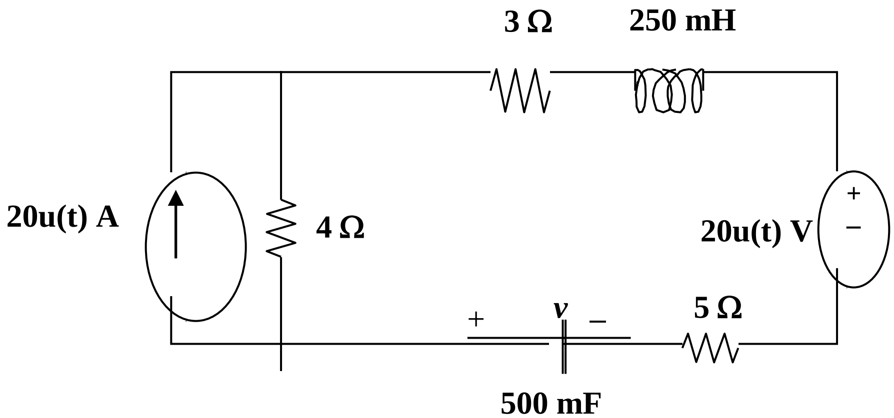
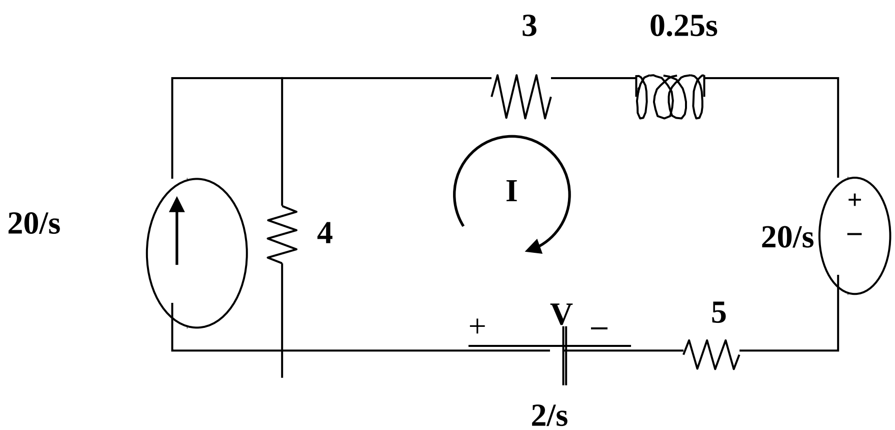


Figure 16.60
For Prob. 16.37.

Solution

Step 1. First we establish the initial conditions and then transform the circuit into the s- domain. Now we write a mesh equation and solve for I. We note that V = –(2/s)I. We now preform a partial fraction expansion and then transform V into the time domain and solve for v.

For t = 0^-, the source voltages are equal to zero thus, the initial conditions are v(0) = 0 and $i_L(0) = 0$. Our circuit looks like this,



4(I–20/s) + 3I + 0.25sI + (20/s) + 5I + (2/s)I = 0. Again, V = –(2/s)I.

Step 2. $[4+3+0.25s+5+(2/s)]I = 60/s = [(0.25s^2+12s+2)/s]I$ or
$I = 240/[s^2+48s+8]$. Thus, $V = -480/[s(s^2+48s+8)]$ where $s_{1,2} = \dfrac{-48 \pm \sqrt{2304-32}}{2}$

$= -24+23.8328, -24-23.8328 = -0.16725, -47.8328$. This leads to,
$V = -480/[s(s+0.16725)(s+47.8328)] = [A/s] + [B/(s+0.16725)] + [C/(s+47.8328)]$ where $A = -480/8 = -60$, $B = -480/[-0.16725(-0.16725+47.8328)] = 60.21$, and
$C = -480/[-47.8328(0.16725-47.8328)] = -0.21053$. Finally we get,

$$v = \mathbf{[-60 + 60.21e^{-0.16725t} - 0.21e^{-47.83t}]u(t)\ V}.$$

Solution 16.38

The switch in the circuit of Fig. 16.61 is moved from position *a* to *b* (a make before break switch) at $t = 0$. Determine $i(t)$ for $t > 0$.

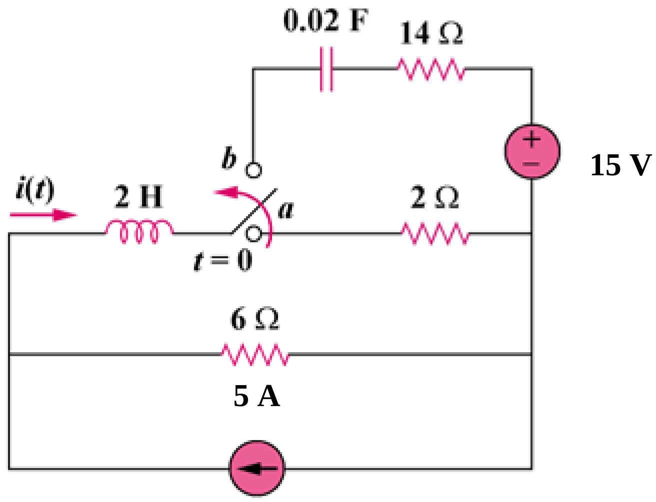
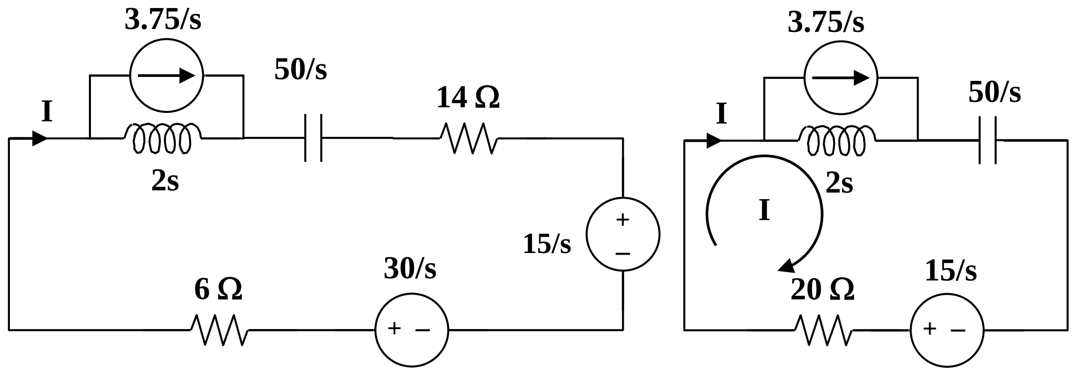


Figure 16.61 For Prob. 16.38.

Solution

Step 1. We first determine the initial conditions. We assume that $v_C(0) = 0$ since we are not given otherwise. $i(0) = [5(2x6)/(2+6)]/2 = 3.75$ amps. Next we need to convert the circuit for $t > 0$ into the s-domain converting the current source in parallel with the 6Ω into a voltage source in series with 6Ω.



Using the simplified circuit on the right, $2s(I–3.75/s) + [50/s]I –(15/s) +20I = 0$. Now we solve for I, perform a partial fraction expansion, and then convert into the time domain.

Step 2. $[2s+(50/s)+20]I = 7.5+15/s = [(s^2+10s+25)/(0.5s)]I = 7.5(s+2)/s$ or $I = [3.75(s+2)/(s+5)^2] = [A/(s+5)] + [B/(s+5)^2]$ where $As+5A+B = 3.75s+7.5$ or $A = 3.75$ and $B = –5A+7.5 = –18.75+7.5 = –11.25$. Thus,

$$i(t) = \mathbf{[(3.75–11.25t)e^{–5t}]u(t)\ amps}.$$

Solution 16.39

For the network in Fig. 16.62, find $i(t)$ for $t > 0$.

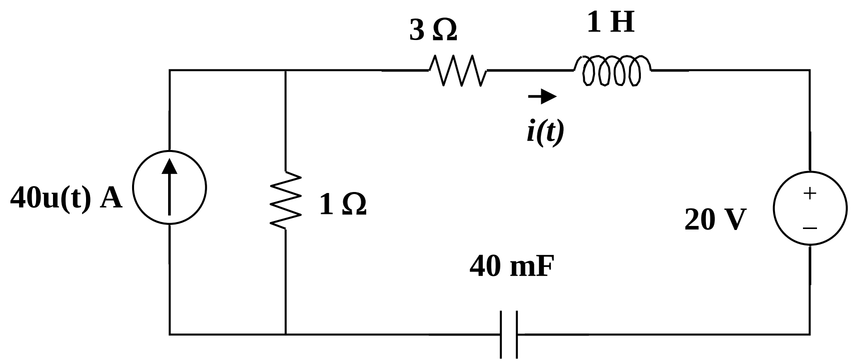
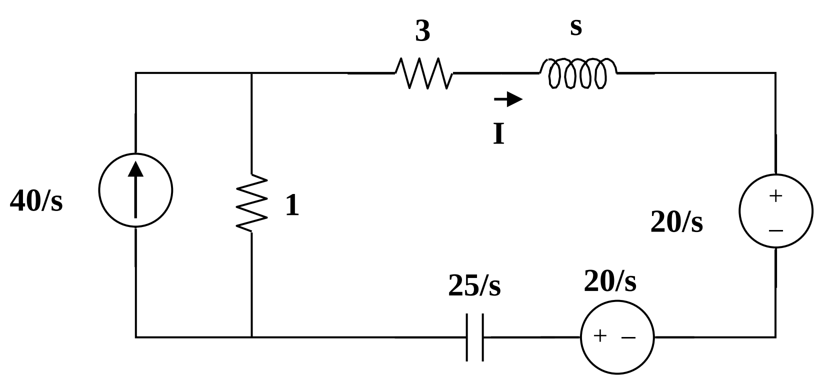


Figure 16.62 For Prob. 16.39.

Solution

Step 1. First we need to determine the initial conditions and then transform the circuit into the s-domain. Then we solve for I, preform a partial fraction expansion a d then convert the answer back into the time domain.

The dc source on the right is active for t < 0, so that i(0) = 0 and the voltage across the capacitor left to right is 20 volts. This leads to,



Our mesh equation, $1(I–40/s) + 3I + sI + (20/s) – (20/s) + (25/s)I = 0$.

Step 2. Now we get $[1+3+s+25/s]I = 40/s = [(s^2+4s+25)/s]I$ or

$I = 40/(s^2+4s+25)$ where $s_{1,2} = \dfrac{-4 \pm \sqrt{16-100}}{2} = –2\pm j4.5826$ or

$I = 40/[(s+2+j4.5826)(s+2–j4.5826)] = [A/(s+2+j4.5826)] + [B/(s+2–j4.5826)]$ where

$A = 40/(–2–j4.5826+2–j4.5826) = 4.364\angle 90° A$ and $B = 4.364\angle –90° A$. Thus,

$$i(t) = \mathbf{[4.364e^{-2t}(e^{-j4.583t+90°} + e^{j4.583t-90°})]u(t)\ A}.$$

Solution 16.40

In the circuit of Fig. 16.63, find $v(t)$ and $i(t)$ for $t > 0$. Assume $v(0) = 0$ V and $i(0) = 1.25$ A.

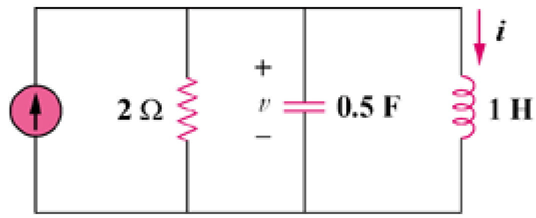
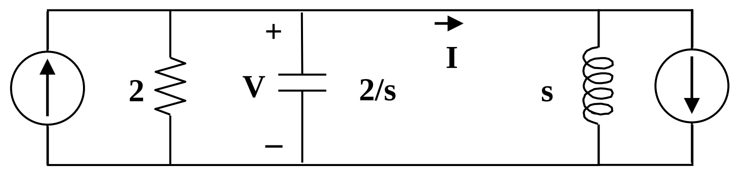


Figure 16.63 For Prob. 16.40.

Solution

Step 1. First we note that the initial conditions are given so that they were generated before the current source turned on. We next transform the circuit into the s-domain.



Now all we need to do is to write a nodal equation and solve for V and I. Once we have them, we perform partial fraction expansions and convert them back into the time domain.

$-(5/s) + [(V-0)/2] + [(V-0)/(2/s)] + [(V-0)/(s)] + 1.25/s = 0$ and
$I = [V/(s)] + (1.25/s)$

Step 2. $[0.5+0.5s+(1/s)]V = 3.75/s = [(s^2+s+2)/(2s)]V$ or $V = 7.5/(s^2+s+2)$

where $s_{1,2} = \dfrac{-1\pm\sqrt{1-8}}{2} = -0.5 \pm j1.3229$. This leads to
$V = [A/(s+0.5+j1.3229)] + [B/(s+0.5-j1.3229)]$ where $A = 7.5/(-j2.6458)$
$= j2.8347$ and $B = 7.5/(j2.6458) = -j2.8347$. Now for $I = (V/s) +1.25/s$
$= [7.5/(s(s+0.5+j1.3229)(s+0.5-j1.3229)] + 1.25/s$. We only need to perform the partial fraction expansion on $[7.5/(s(s+0.5+j1.3229)(s+0.5-j1.3229)]$
$= [C/s] + [D/(s+0.5+j1.3229)] + [E/(s+0.5-j1.3229)]$ where $C = 3.75$,
$D = 7.5/[(-0.5-j1.3229)(-j2.6458) = 7.5/[(1.41424\angle -110.7)(2.6458\angle -90°)]$
$= 2.0044\angle 200.7°$ and $E = 2.0044\angle -200.7°$. Now we get,

$$v(t) = \mathbf{[2.835e^{-(0.5+j1.3229)t+90°} + 2.835e^{-(0.5-j1.3229)t-90°}]u(t)\ V}$$
$$= \mathbf{[5.669e^{-t/2}\cos(1.3229t-90^o)]u(t)\ V} \text{ and}$$
$$i(t) = \mathbf{[5 + 2.004e^{-(0.5+j1.3229)t+200.7°} + 2.004e^{-(0.5-j1.3229)t-200.7°}]u(t)\ A}$$
$$= \mathbf{[5 + 4.009e^{-t/2}\cos(1.3229t-200.7^o)]\ A}.$$

Solution 16.41

Find the output voltage $v_o(t)$ in the circuit of Fig. 16.64.

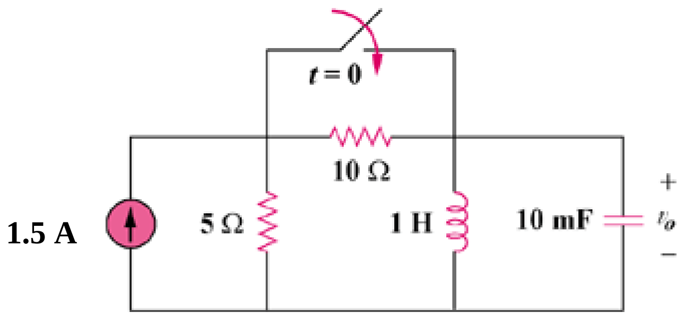
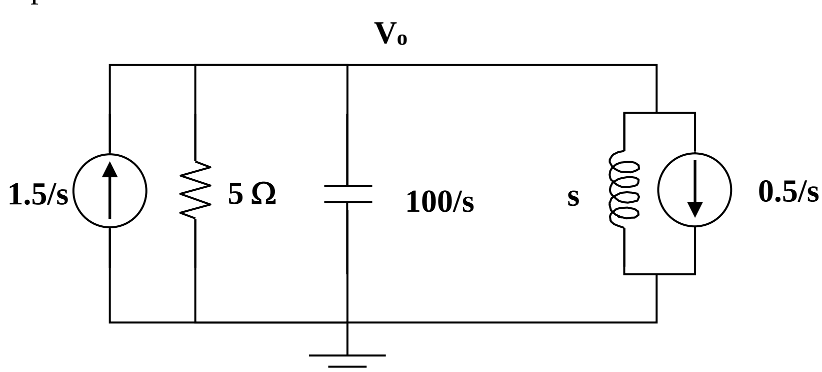


Figure 16.64
For Prob. 16.41.

Solution

Step 1. First we need to determine the initial conditions. We see that $v_o(0) = 0$ since the inductor becomes a short. We also note that the initial current through the inductor is the same as the current through the 10 Ω resistor or $i_L(0) = [1.5(5x10)/(5+10)]/10 = 0.5$ amp. Then we simplify the circuit and convert it into the s-domain and solve for V_o. We then perform a partial fraction expansion and convert into the time domain.



$$-[1.5/s] + [(V_o-0)/5] + [(V_o-0)/(100/s)] + [(V_o-0)/s] + [0.5/s] = 0.$$

Step 2. $[0.2+(s/100)+(1/s)]V_o = 1/s = [(s^2+20s+100)/(100s)]V_o$ or

$V_o = 100/[(s+10)^2]$ and

$$v_o(t) = \mathbf{[100te^{-10t}]u(t)\ volts}.$$

Solution 16.42

Given the circuit in Fig. 16.65, find $i(t)$ and $v(t)$ for $t > 0$.

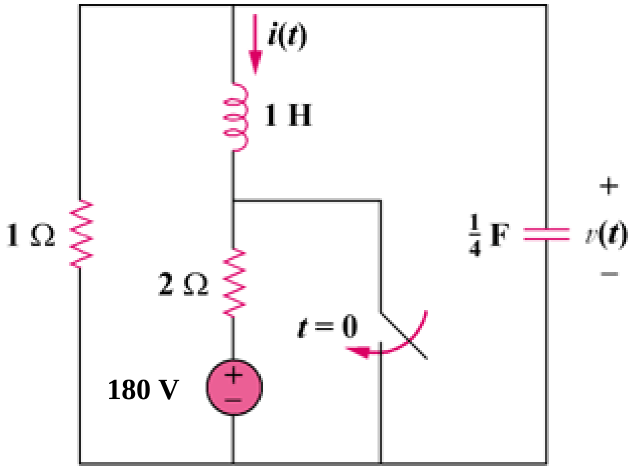
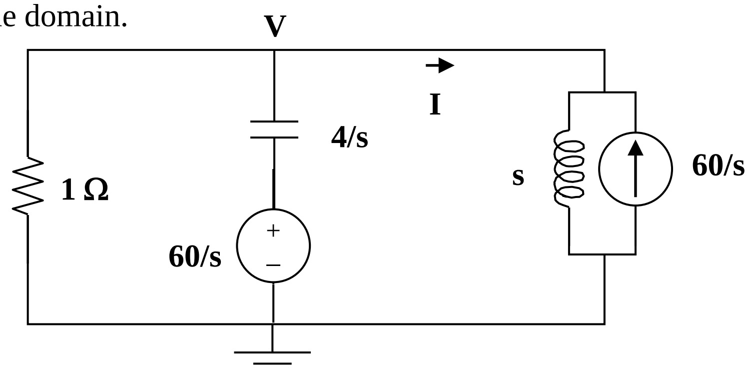


Figure 16.65
For Prob. 16.42.

Solution

Step 1. First we need to find the initial conditions. Since the inductor becomes a short and the capacitor becomes an open circuit, all the current flows through the 1 Ω and 2 Ω resistors or i(0) = –180/3 = –60 amps and v(0) = 60x1 = 60 volts. Next we need to convert the circuit into the s-domain and solve for V and I. Once we have done that, we can perform partial fraction expansions and convert back into the time domain.



$[(V–0)/1] + [(V–60/s)/(4/s)] + [(V–0)/s] – [60/s] = 0$ and $I = [(V–0)/s] – [60/s]$.

Step 2. $[(1+(s/4)+(1/s)]V = [(s^2+4s+4)/(4s)]V = 15+60/s = 15(s+4)/s$ or
$V = 15(s+4)/[(s+2)^2] = [A/(s+2)]+[B/(s+2)^2]$ where $As+2A + B = 60s+240$ and
$A = 60$ and $B = 240–2A = 120$. $I = [60/(s(s+2))]+[120/(s(s+2)^2)]–60/s$

The partial fraction expansion is straight forward for the first and third terms, but the second term takes a little work. $120/(s(s+2)^2 = [a/s]+[b/(s+2)]+[c/(s+2)^2]$ or $as^2+a4s+a4+bs^2+b2s+cs = 120$ or $a = 30$, $b = –30$, and $c = –60$.

Thus, $I = [30/s]+[–30/(s+2)]+[30/s]+[–30/(s+2)]+[–60/(s+2)^2]–60/s = –[60/(s+2)] – [60/(s+2)^2]$ and we finally get,

v(t) = **[60e^–2t^ + 120te^–2t^]u(t) volts** and

$$v(t) = [60e^{-2t} + 120te^{-2t}]u(t) \text{ volts}$$

i(t) = **[–60e^–2t^ – 60te^–2t^]u(t) amps**.

Solution 16.43

Determine $i(t)$ for $t > 0$ in the circuit of Fig. 16.66.

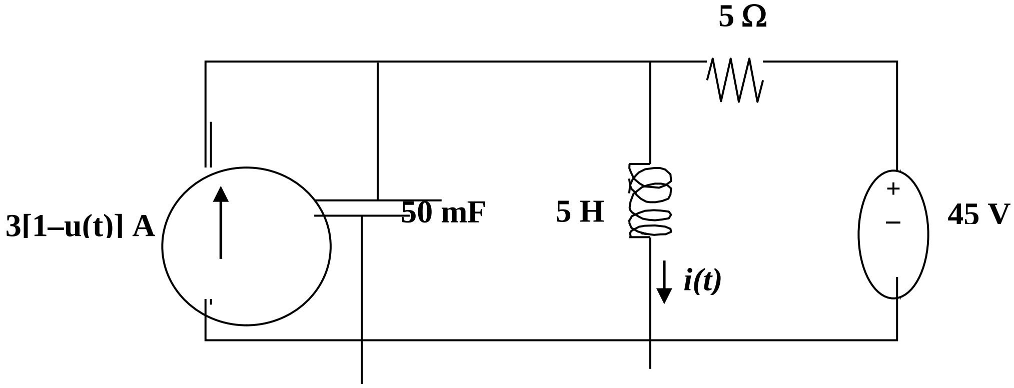
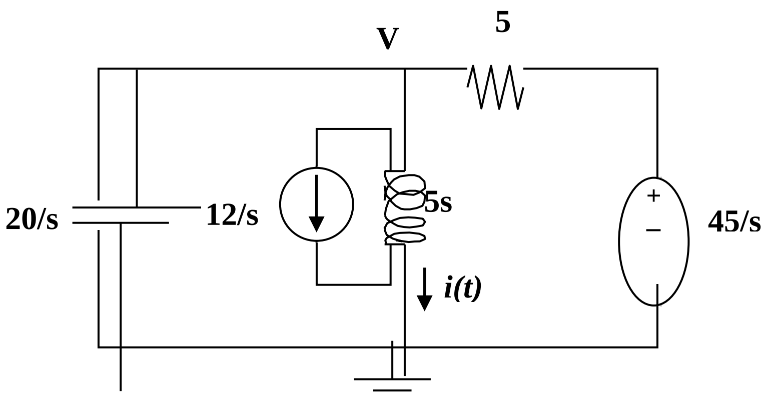


Figure 16.66 For Prob. 16.43.

Solution

Step 1. First we determine the initial conditions and then transform the circuit into the s-domain. We can then write a node equation and solve for V. Once we have V we can find I = [V/(5s)] + i(0)/s. Next we perform a partial fraction expansion and then convert back into the time domain to solve for i(t). At steady-state the capacitor voltage is equal to zero and i(0) = 3 + 45/5 = 12 A.



$$[(V–0)/(20/s)] + (12/s) + [(V–0)/(5s)] + [(V–45/s)/5] = 0$$

Step 2. Now we get $[(s/20)+(1/(5s))+1/5]V = [(s^2+4s+4)/(20s)]V = –3/s$ or $V = –60/[(s+2)^2]$. Finally we get $I = –\{12/[s(s+2)^2]\} + 12/s$ or just for the first term, $–\{12/[s(s+2)^2]\} = [A/s] + [B/(s+2)] + [C/(s+2)^2]$ where A = –12/4 = –3 V and C = –12/(–2) = 6 A. Now we can calculate B by multiplying both sides by s(s+2) which gives us $–12 = –3(s+2)^2 + Bs(s+2) + 6s$ $= –3s^2 – 12s –12 + Bs^2 + 2Bs + 6s = (B–3)s^2 + (–12+2B+6)s – 12$. Therefore B = 3 A. $I = [(–3+12)/s] + [3/(s+2)] + [6/(s+2)^2]$ or

$$i(t) = \mathbf{[9 + 3e^{–2t} + 6te^{–2t}]u(t)\ A}.$$

Solution 16.44

For the circuit in Fig. 16.67, find $i(t)$ for $t > 0$.

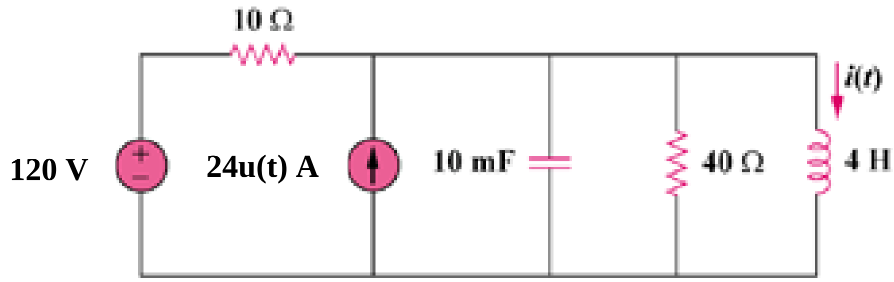
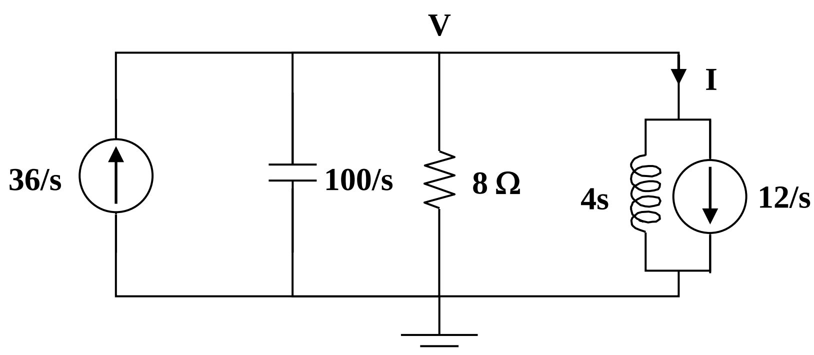


Figure 16.67
For Prob. 16.44.

Solution

Step 1. First we identify the initial conditions. Then we simplify the circuit (for $t > 0$) and then transform it into the s-domain. We simplify the circuit by performing a source transformation and combining the 10 Ω resistor with the 40 Ω resistor (10x40/(10+40) = 8 Ω) and adding the two current sources together 12+24 = 36 A. We then solve for the node voltage, V, and then find I. Finally we perform a partial fraction expansion and convert the answer into the time domain. For $t < 0$, the inductor looks like a short circuit producing $v_C(0) = 0$ and $i(0) = 120/10 = 12$ amps.



$-[36/s] + [(V-0)/(100/s)] + [(V-0)/8] + [(V-0)/(4s)] + [12/s] = 0$ and
$I = [(V-0)/(4s)] + [12/s]$.

Step 2. $[(s/100)+(1/8)+1/(4s)]V = [(s^2+12.5s+25)/(100s)]V = 24/s$ or
$V = 2{,}400/[(s+2.5)(s+10)]$ and $I = 600/[s(s+2.5)(s+10)]+[12/s]$
$= [A/s] + [B/(s+2.5)] + [C/(s+10)]$ where $A = 24+12 = 36$;
$B = 600/[-2.5(-2.5+10)] = -32$; and $C = 600/[-10(-10+2.5)] = 8$.

$$i(t) = \mathbf{[36-32e^{-2.5t}+8e^{-10t}]u(t)\ amps}.$$

Solution 16.45

Find $v(t)$ for $t > 0$ in the circuit in Fig. 16.68.

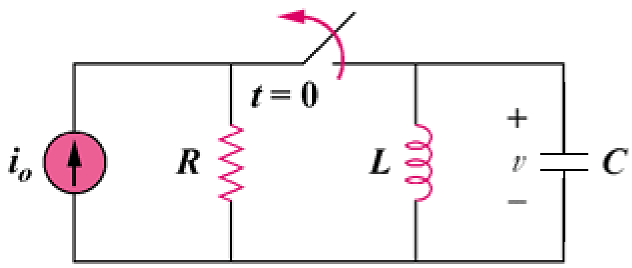
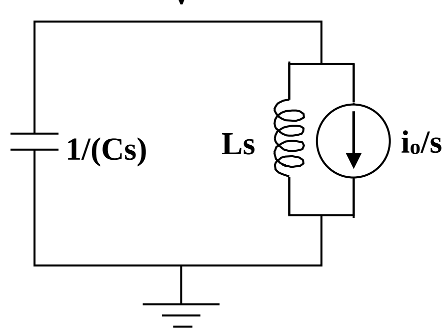


Figure 16.68
For Prob. 16.45.

Solution

Step 1. First, determine the initial conditions. Next convert the circuit into the s-domain and solve for V. Perform a partial fraction expansion and convert back into the time domain. For t < 0, the inductor looks like a short circuit so that v(0) = 0 and $i_L(0) = i_o$.



$[(V–0)/(1/(Cs))] + [(V–0)/(Ls)] + [i_o/s] = 0$.

Step 2. $[Cs+(1/(Ls))]V = [C\{s^2+(1/(LC))\}/s]V = –i_o/s$ or
$V = –(i_o/C)/[(s^2+1/(LC))]$. If we let $\boldsymbol{\omega^2 = 1/(LC)}$ then we get,
$V = –(i_o/C)/[(s^2+\omega^2)] = –(i_o/C)/[(s+j\omega)(s–j\omega)] = [A/(s+j\omega)]+[B/(s–j\omega)]$ where
$A = –(i_o/C)/(–j2\omega) = [i_o/(2\omega C)]\angle–90^\circ$ and $B = –(i_o/C)/(j2\omega) = [i_o/(2\omega C)]\angle–90^\circ$.
Thus,

$v(t) = [i_o/(2\omega C)][e^{–j\omega t–90^\circ}+e^{j\omega t+90^\circ}] = \mathbf{[i_o/(\omega C)]\cos(\omega t+90^\circ)u(t)}$ **volts**.

Solution 16.46

Determine $i_o(t)$ in the circuit in Fig. 16.69.

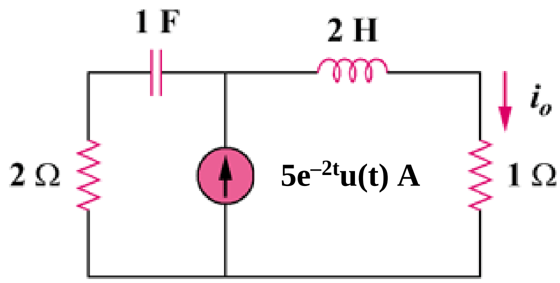
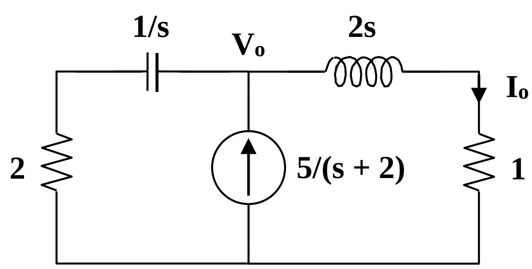


Figure 16.69
For Prob. 16.46.

Solution

Step 1. First we determine the initial conditions which in this case are equal to zero since there is no source present until t = 0. We then transform the circuit into the s-domain and solve for V_o using a nodal equation. Once we have V_o we can find I = V_o/(2s+1). Finally we perform a partial fraction expansion and the transform this into the time domain.



$[(V_o–0)/(2+(1/s))] – [5/(s+2)] + [(V_o–0)/(2s+1)] = 0.$

Step 2. This leads to $[(s+1)/(2s+1)]V_o = 5/(s+2)$

$V_o = \frac{5(2s+1)}{(s+1)(s+2)}$ and $I_o = \frac{V_o}{2s+1} = \frac{5}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$ where A = 5 and B = –5. Thus,

$$i_o(t) = \mathbf{5[e^{-t} - e^{-2t}]u(t)\ A}.$$

Solution 16.47

Determine $i_o(t)$ in the network shown in Fig. 16.70.

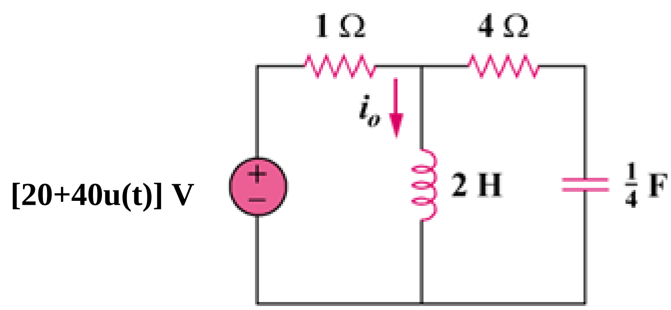
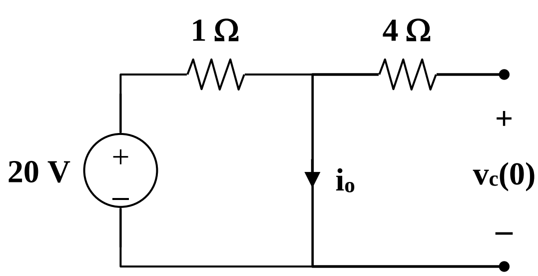


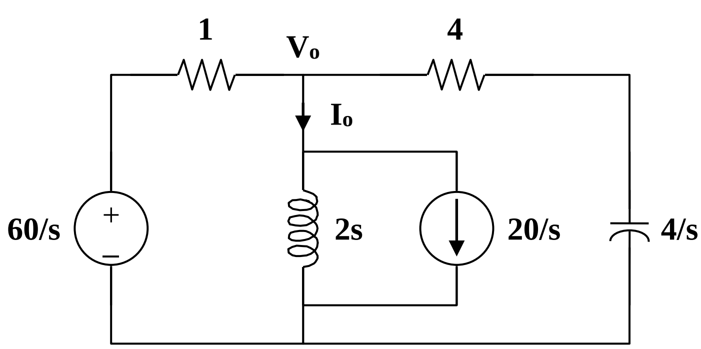
Figure 16.70
For Prob. 16.47.

Solution

Step 1. We first find the initial conditions from the circuit in Fig. (a),



$i_o(0^-) = 20\ A,\ v_c(0^-) = 0\ V$. Next we transform the circuit into the s-domain.



At node V_o we get,

$\frac{V_o - 60/s}{1} + \frac{V_o}{2s} + \frac{20}{s} + \frac{V_o - 0}{4+4/s} = 0$ and note that $I_o = [V_o/(2s)] + 20/s$. We now can solve for V_o and then for I_o. We then perform a partial fraction expansion and transform back into the time domain.

Step 2. $\frac{60}{s} - \frac{20}{s} = \left(1 + \frac{1}{2s} + \frac{s}{4(s+1)}\right)V_o$ and

$\frac{40}{s} = \frac{4s^2 + 4s + 2s + 2 + s^2}{4s(s+1)}V_o = \frac{5s^2+6s+2}{4s(s+1)}V_o$ which leads to $V_o = \frac{160(s+1)}{5s^2+6s+2}$ and

$$I_o = \frac{V_o}{2s} + \frac{20}{s} = \frac{16(s+1)}{s(s^2+1.2s+0.4)} + \frac{20}{s} = \frac{20}{s} + \frac{A}{s} + \frac{Bs+C}{s^2+1.2s+0.4}$$

$$16(s+1) = A(s^2+1.2s+0.4) + Bs^s + Cs$$

Equating coefficients :

$$\begin{aligned}
&s^0: \quad 16 = 0.4A \longrightarrow A = 40\\
&s^1: \quad 16 = 1.2A + C \longrightarrow C = \text{-}1.2\text{A} + 16 = \text{-}32\\
&s^2: \quad 0 = A + B \longrightarrow B = \text{-A} = \text{-}40
\end{aligned}$$

$$I_o = \frac{20}{s} + \frac{40}{s} - \frac{40s+32}{s^2+1.2s+0.4}$$

$$I_o = \frac{60}{s} - \frac{40(s+0.6)}{(s+0.6)^2+0.2^2} - \frac{40(0.2)}{(s+0.6)^2+0.2^2}$$

$i_o(t) =$ **[60–40e$^{-0.6t}$(cos(0.2t)–sin(0.2t))]u(t) A**.

Solution 16.48

Find $v_x(s)$ in the circuit shown in Fig. 16.71.

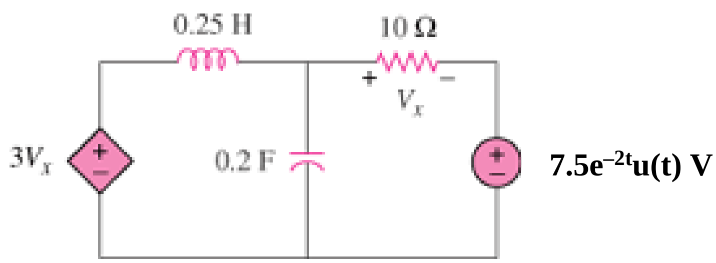
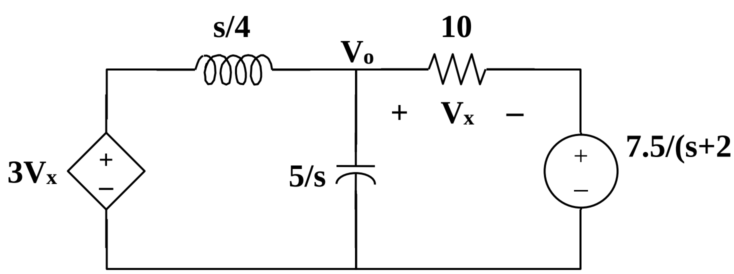


Figure 16.71
For Prob. 16.48.

Solution

Step 1. First we need to determine the initial conditions which in this case are equal to zero since there are no independent sources active until t = 0. Next we transform the circuit into the s-domain where we can write a nodal equation and solve for V_o. Once we have V_o, we can find $V_x = V_o - 7.5/(s+2)$. We then perform a partial fraction and transform back into the time domain.



$$\frac{V_o - 3V_x}{s/4} + \frac{V_o - 0}{5/s} + \frac{V_o - \frac{7.5}{s+2}}{10} = 0$$

$$40V_o - 120V_x + 2s^2V_o + sV_o - \frac{7.5s}{s+2} = 0 = (2s^2 + s + 40)V_o - 120V_x - \frac{7.5s}{s+2}$$

$$But, V_x = V_o - \frac{7.5}{s+2} \rightarrow V_o = V_x + \frac{7.5}{s+2}$$

Step 2. We can now solve for V_x.

$$(2s^2+s+40)\left(V_x+\frac{7.5}{s+2}\right)-120V_x-\frac{7.5s}{s+2}=0$$

$$2(s^2+0.5s-40)V_x=-15\frac{(s^2+20)}{s+2}$$

$$\underline{V_x=-7.5\frac{(s^2+20)}{(s+2)(s^2+0.5s-40)}}$$

We should note that this represents an unstable system because at least one root of the denominator has to be positive resulting in an exponential term that has a positive exponent. This is clearly the result of the dependent source.

Solution 16.49

Find $i_o(t)$ for $t > 0$ in the circuit in Fig. 16.72.

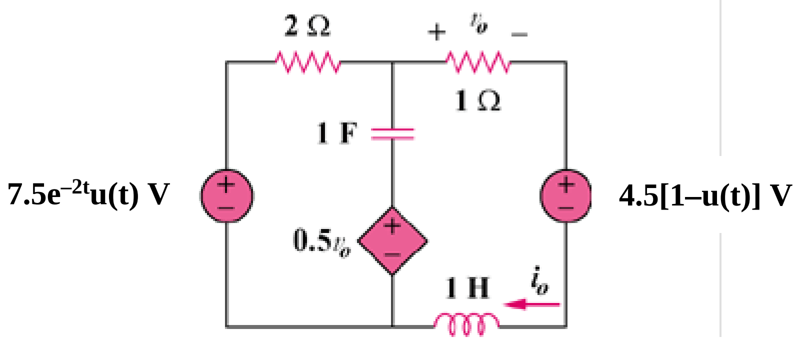
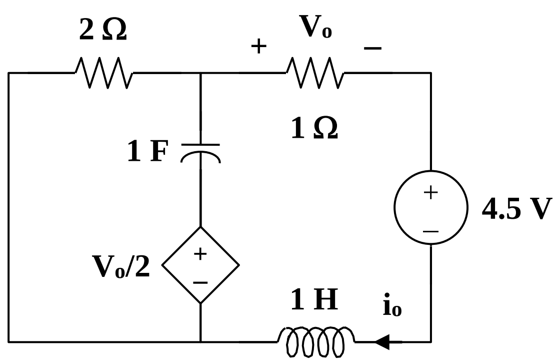


Figure 16.72
For Prob. 16.49.

Solution

We first need to find the initial conditions. For $t < 0$, the circuit is shown in Fig. (a).



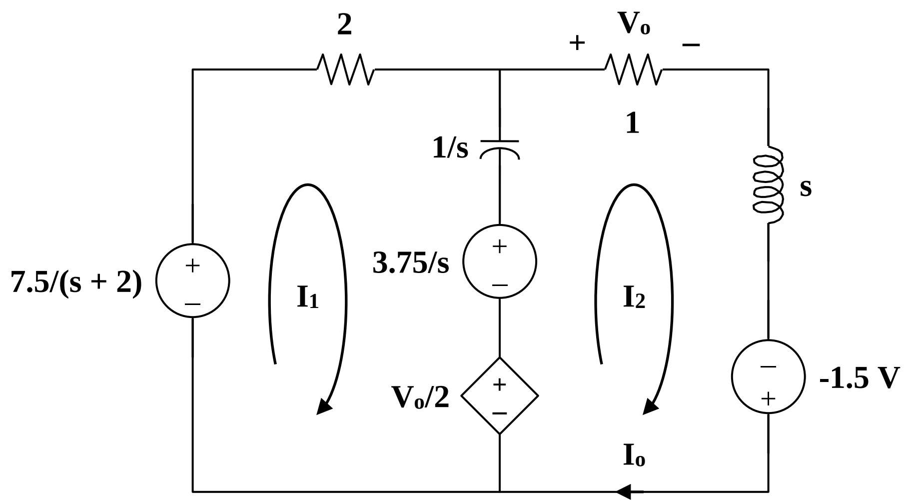
(a)

To dc, the capacitor acts like an open circuit and the inductor acts like a short circuit. Hence,

$$i_L(0) = i_o = \frac{-4.5}{3} = -1.5\text{ A}, \qquad v_o = -1.5\,V$$

$$v_c(0) = -(2)(-1.5) - \left(\frac{-1.5}{2}\right) = 3.75\,V$$

We now incorporate the initial conditions and transform the circuit into the s-domain.



(b)

For mesh 1,

$$\frac{-7.5}{s+2}+\left(2+\frac{1}{s}\right)I_1-\frac{1}{s}I_2+\frac{3.75}{s}+\frac{V_o}{2}=0$$

But, $\quad V_o = I_o = I_2$

$$\left(2+\frac{1}{s}\right)I_1+\left(\frac{1}{2}-\frac{1}{s}\right)I_2=\frac{7.5}{s+2}-\frac{3.75}{s} \qquad (1)$$

For mesh 2,

$$\left(1+s+\frac{1}{s}\right)I_2-\frac{1}{s}I_1+1.5-\frac{V_o}{2}-\frac{3.75}{s}=0$$

$$-\frac{1}{s}I_1+\left(\frac{1}{2}+s+\frac{1}{s}\right)I_2=\frac{3.75}{s}-1.5 \qquad (2)$$

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 2+\frac{1}{s} & \frac{1}{2}-\frac{1}{s} \\ \frac{-1}{s} & \frac{1}{2}+s+\frac{1}{s} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} \frac{7.5}{s+2}-\frac{3.75}{s} \\ \frac{3.75}{s}-1.5 \end{bmatrix}$$

$$\Delta = 2s+2+\frac{3}{s}, \qquad \Delta_2 = -3+\frac{6}{s}+\frac{7.5}{s(s+2)}$$

$$I_o = I_2 = \frac{\Delta_2}{\Delta} = \frac{-3s^2 + 19.5}{(s+2)(2s^2+2s+3)} = \frac{A}{s+2} + \frac{Bs+C}{2s^2+2s+3}$$

$$-3s^2 + 19.5 = A(2s^2+2s+3) + B(s^2+2s) + C(s+2)$$

Equating coefficients :

$$\begin{aligned}
s^2 &: \quad -3 = 2A + B \\
s^1 &: \quad 0 = 2A + 2B + C \\
s^0 &: \quad 19.5 = 3A + 2C
\end{aligned}$$

Solving these equations leads to A = 1.0714, B = –5.143, and C = 8.143.

$$I_o = \frac{1.0714}{s+2} - \frac{5.143s - 8.143}{2s^2+2s+3} = \frac{1.0714}{s+2} - \frac{2.572\,s - 4.072}{s^2+s+1.5}$$

$$I_o = \frac{1.0714}{s+2} - \frac{2.572\,(s+0.5)}{(s+0.5)^2+1.25} + \frac{(4.791)(\sqrt{1.25})}{(s+0.5)^2+1.25}$$

$i_o(t) =$ **$[1.0714e^{-2t} - 2.572e^{-t/2}\cos(1.118t) + 4.791e^{-t/2}\sin(1.118t)]u(t)$ A**.

Solution 16.50

For the circuit in Fig. 16.73, find $v(t)$ for $t > 0$. Assume that $i(0) = 2$ A.

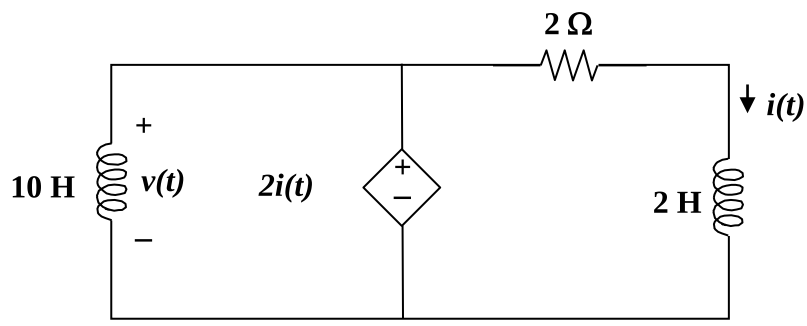
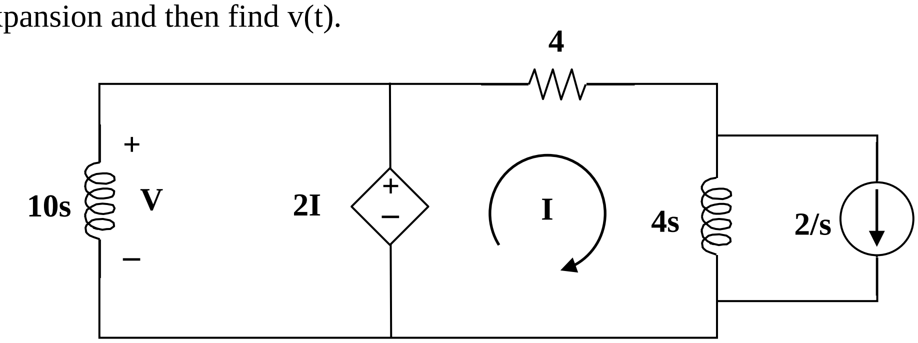


Figure 16.73
For Prob. 16.50.

Solution

Step 1. This is an interesting problem in that we can neglect the left hand side of the circuit in that it is in parallel with an ideal voltage source even though it is a dependent source. The first thing to do is to transform the circuit into the s-domain. Then we can solve for I by writing a mesh equation for the circuit on the right. Once we have I we can solve for V = 2I and then perform a partial fraction expansion and then find v(t).



$-2I + 4I + 4sI - 4sx2/s = 0.$

Step 2. $4(s+0.5)I = 8$ or $I = 2/[(s+0.5)]$ and $V = 4/(s+0.5)$. Thus,

$$v(t) = \mathbf{[4e^{-0.5t}]u(t)\ V}.$$

Solution 16.51

In the circuit of Fig. 16.74, find $i(t)$ for $t > 0$.

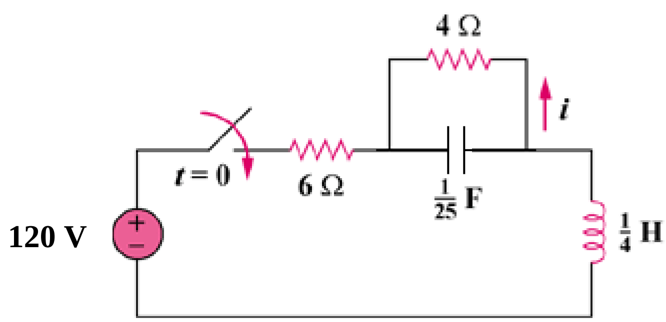
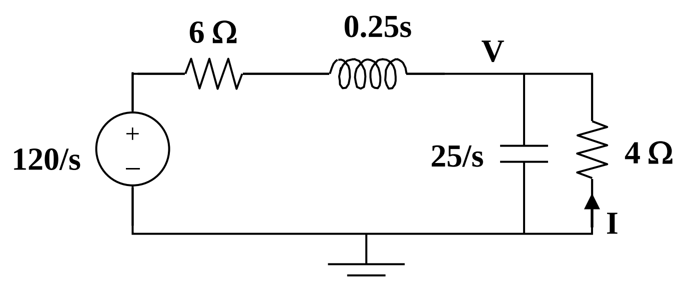


Figure 16.74
For Prob. 16.51.

Solution

Step 1. First we note that the initial conditions for the capacitor and inductor have to be equal to zero. Next we simplify the circuit and then convert the circuit into the s-domain and solve for V. Then we can solve for I and then perform a partial fraction expansion and convert I back into the time domain.



$[(V–120/s)/(0.25(s+24))]+[s(V–0)/25]+[(V–0)/4] = 0$ and $I = [(0–V)/4] = –V/4$.

Step 2. $[(4/(s+24))+(s/25)+0.25]V = [(s^2+24s+6.25s+100+150)/(25(s+24))]V$
$= [(s^2+30.25s+250)/(25(s+24))]V$
$= [\{(s+15.125+j4.608)(s+15.125–j4.608)\}/(25(s+24))]V = [480/(s(s+24))]$ or
$V = 12{,}000/[s(s+15.125+j4.608)(s+15.125–j4.608)]$ and
$I = –3{,}000/[s(s+15.125+j4.608)(s+15.125–j4.608)] =$
$[A/s]+[B/(s+15.125+j4.608)]$
$+[C/(s+15.125–j4.608)]$ where $A = –3{,}000/250 = –12$;
$B = –3{,}000/[(–15.125–j4.608)(–j9.216)] = 3{,}000\angle 180^\circ/[(15.811\angle –163.06^\circ)(9.216\angle –90^\circ)]$
$= 20.587\angle 73.06^\circ$; and $C = 3{,}000\angle 180^\circ/[(15.811\angle 163.06^\circ)(9.216\angle 90^\circ)] = 20.587\angle –73.06^\circ$.
Thus, $i(t) = [–12+20.587e^{–15.125t}(e^{–j4.608t+73.06^\circ}+e^{j4.608–73.06^\circ})]u(t)$ amps

$i(t) =$ $\mathbf{[–12+41.17e^{–15.125t}\cos(4.608t–73.06^\circ)]u(t)}$ **amps**.

Solution 16.52

Given the circuit shown in Fig. 16.75, determine the values for *i(t)* and *v(t)* for all t > 0.

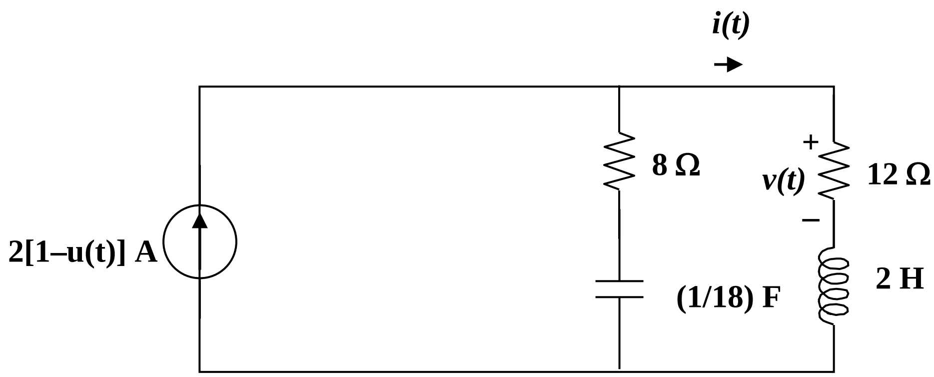
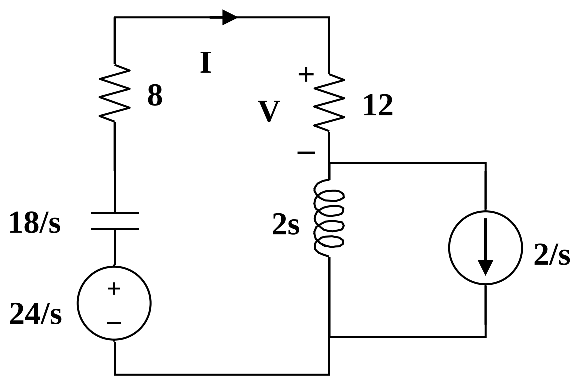


Figure 16.75
For Prob. 16.52.

Solution

Step 1. First we determine the initial conditions and then transform the circuit into the s-domain. We can solve for I by writing a mesh equation and then solve for I. We then perform a partial fraction expansion to solve for I and then transform this back into the time domain. Finally we note that v(t) = 12I which then gives us v(t).



$-(24/s) + (18/s)I + 8I + 12I + 2s(I-2/s) = 0$

Step 2. $[(18/s)+8+12+2s]I = 2[s+10+(9/s)]I = 2[(s^2+10s+9)/s]I = 4+24/s = 4(s+6)/s$ or $I = 2(s+6)/[(s+1)(s+9)] = [A/(s+1)] + [B/(s+9)]$ where $A = 2(-1+6)/(-1+9) = 10/8 = 1.25$ and $B = 0.75$ which leads to,

i(t) = $\mathbf{[1.25e^{-t} + 0.75e^{-9t}]u(t)}$ **A** and
v(t) = $\mathbf{[15e^{-t} + 9e^{-9t}]u(t)}$ **V**.

Solution 16.53

In the circuit of Fig. 16.76, the switch has been in position 1 for a long time but moved to position 2 at $t = 0$. Find:

(a) $v(0^+)$, $dv(0^+)/dt$
(b) $v(t)$ for $t \geq 0$.

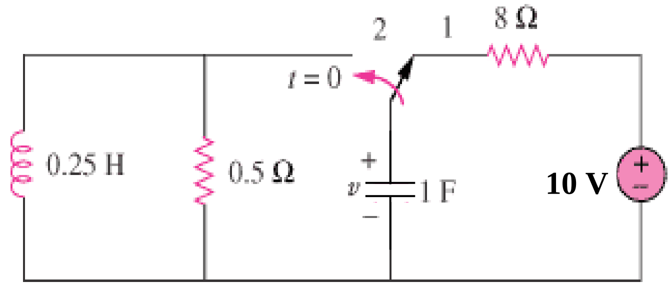
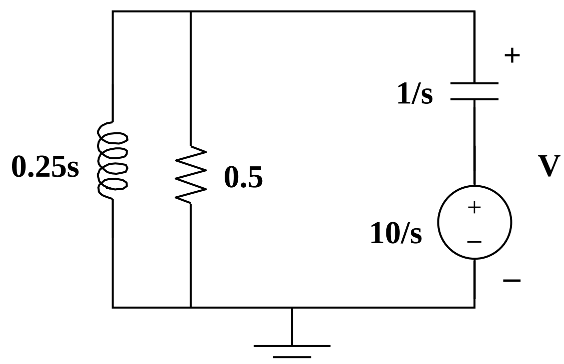


Figure 16.76
For Prob. 16.53.

Solution

Step 1. Clearly $i_L(0) = 0$ and $v(0) =$ **10 volts**. When the switch moves to 2, $i_C(0^+) = Cdv(0)/dt = -10/0.5 =$ **–20 volts/second** $= 1dv(0)/dt$. Next we convert the circuit into the s-domain and solve for V. Then we perform a partial fraction expansion and then convert back into the time domain.



$[(V-0)/(0.25s)]+[(V-0)/0.5]+[(V-10/s)s/1] = 0.$

Step 2. $[(4/s)+2+s]V = [(s^2+2s+4)/s]V = 10$ or
$V = 10s/[(s+1+j1.7321)(s+1-j1.7321)] = [A/(s+1+j1.7321)] + [B/(s+1-j1.7321)]$
where $A = 10(-1-j1.7321)/(3.464\angle-90^\circ) = 10(2\angle-120^\circ)/(3.464\angle-90^\circ)$
$= 5.7737\angle-30^\circ$ and $B = 10(2\angle120^\circ)/(3.464\angle90^\circ) = 5.7737\angle30^\circ$ or
$v(t) = 5.7737e^{-t}[e^{-j1.7321t-30^\circ}+e^{j1.7321t+30^\circ}]u(t)$ volts or

$v(t) =$ **$[11.547e^{-t}\cos(1.7321t+30^\circ)]u(t)$ volts**.

Solution 16.54

The switch in Fig. 16.77 has been in position 1 for $t < 0$. At $t = 0$, it is moved from position 1 to the top of the capacitor at t = 0. Please note that the switch is a make before break switch, it stays in contact with position 1 until it makes contact with the top of the capacitor and then breaks the contact at position 1. Determine $v(t)$.

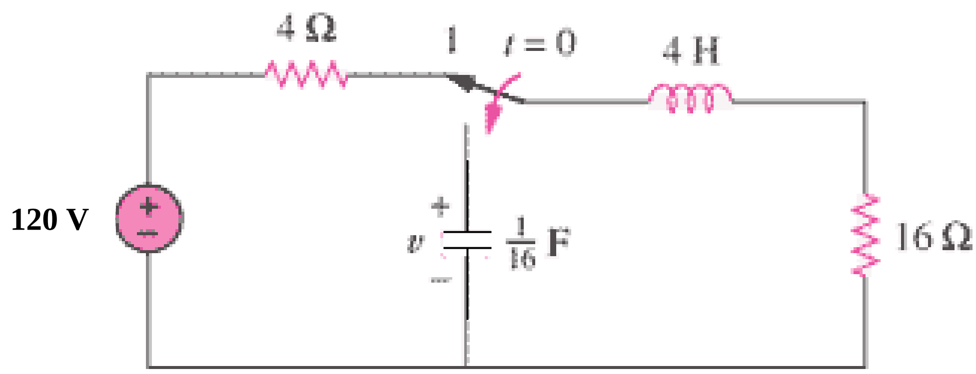
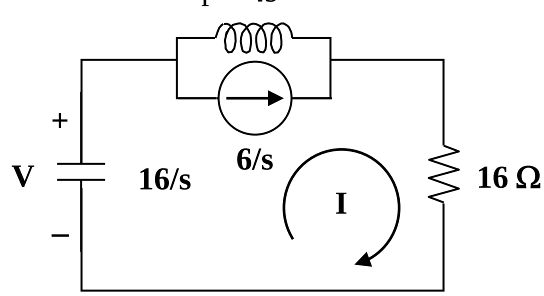


Figure 16.77
For Prob. 16.54.

Solution

Step 1. First determine the initial conditions and then transform the circuit into the s-domain and solve for V. Then perform a partial fraction expansion and then find v(t). We will assume that the value of v(0) = 0. Now we can calculate $i_L(0) = 120/20 = 6$ amps.



$[16/s]I + [4s](I–6/s) + 16I = 0$ and $V = [16/s](–I)$.

Step 2. $[(16/s)+4s+16]I = [4(s^2+4s+4)/s]I = 24$ or
$I = 24s/[4(s+2)^2] = 6s/[(s+2)^2]$ and $V = –96/[(s+2)^2]$

$v(t) = $ **$[–96te^{–2t}]u(t)$ volts**.

Solution 16.55

Obtain i_1 and i_2 for $t > 0$ in the circuit of Fig. 16.78.

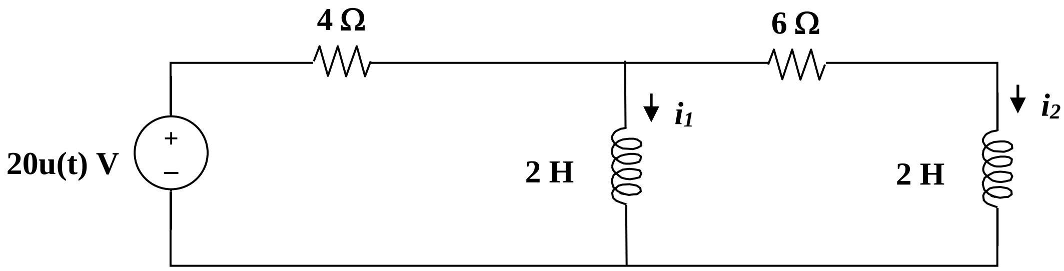
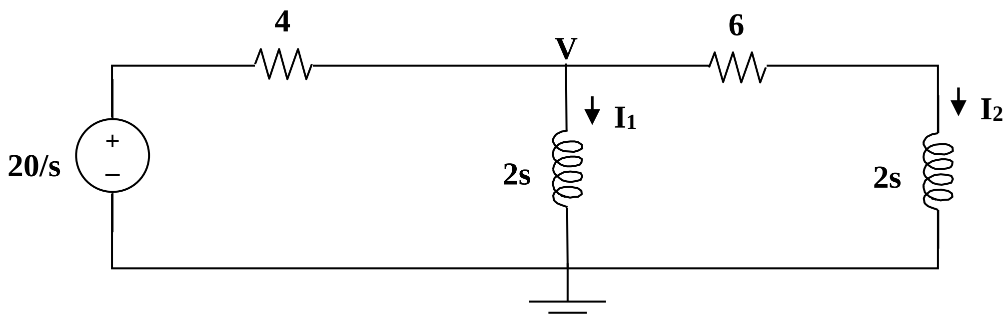


Figure 16.78
For Prob. 16.55.

Solution

Step 1. Since the independent source is equal to zero for all t < 0, there will not be any initial conditions. We then transform the circuit into the s-domain and set up a nodal equation and solve for V. We can then calculate $I_1 = V/2s$ and $I_2 = V/[2(s+3)]$. We finally perform a partial fraction expansion for both terms and then convert back into the time domain.



$[(V–20/s)/4] + [(V–0)/(2s)] + [(V–0)/(2s+6)] = 0.$

Step 2. $[0.25+(0.5/s)+0.5/(s+3)]V = 5/s$ or
$[(0.25s^2+0.75s+0.5s+1.5+0.5s)/(s(s+3))]V = 5/s = 0.25[(s^2+7s+6)/(s(s+3))]V$ or
$V = 5(s+3)/[0.25(s^2+7s+6)] = 20(s+3)/(s^2+7s+6)$ which leads to
$I_1 = 10(s+3)/[s(s+1)(s+6)]$ and $I_2 = 10/[(s+1)(s+6)]$. Partial fraction expansion,
$I_1 = [A/s] + [B/(s+1)] + [C/(s+6)]$ and $I_2 = [D/(s+1)] + [E/(s+6)]$ where
$A = 30/6 = 5$; $B = 20/(–5) = –4$, $C = –30/30 = –1$, $D = 2$, and $E = –2$.

$i_1(t) = \mathbf{[5 – 4e^{–t} – e^{–6t}]u(t)\ A}$ and
$i_2(t) = \mathbf{[2e^{–t} – 2e^{–6t}]u(t)\ A}$.

Solution 16.56

Calculate $i_o(t)$ for $t > 0$ in the network of Fig. 16.79.

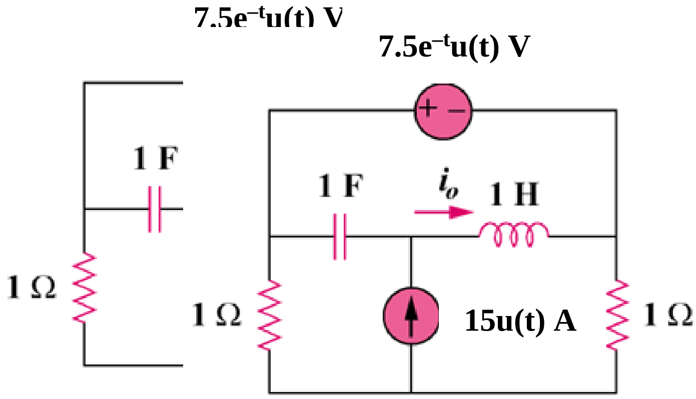
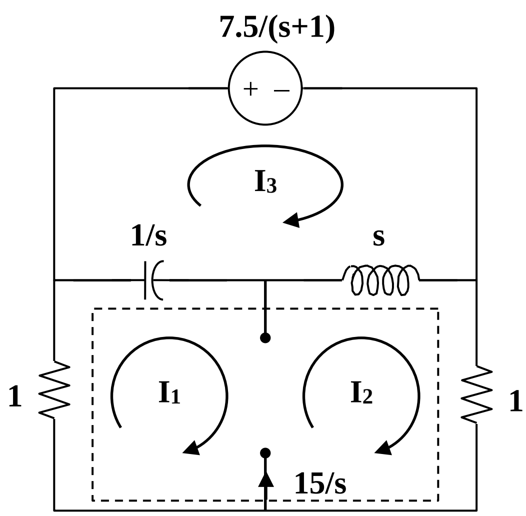


Figure 16.79
For Prob. 16.56.

Solution

Step 1. First we need to determine the initial conditions which in this case are equal to zero since the sources are equal to zero for all time less than zero. Next we apply mesh analysis to the s-domain form of the circuit as shown below and solve for I_3 and I_2 which gives $I_o = I_2 - I_3$.



For mesh 3,

$$\frac{7.5}{s+1}+\left(s+\frac{1}{s}\right)I_3-\frac{1}{s}I_1-sI_2=0 \tag{1}$$

For the supermesh,

$$\left(1+\frac{1}{s}\right)I_1 + (1+s)\,I_2 - \left(\frac{1}{s}+s\right)I_3 = 0 \qquad (2)$$

Step 2. Adding (1) and (2) we get, $I_1 + I_2 = -7.5/(s+1)$ (3)

and $-I_1 + I_2 = 15/s$
(4)

Adding (3) and (4) we get, $I_2 = (7.5/s) - 3.75/(s+1)$ (5)

Substituting (5) into (4) yields, $I_1 = -(7.5/s) - (3.75/(s+1))$ (6)

Substituting (5) and (6) into (1) we get,

$$\frac{7.5}{s^2} + \frac{3.75}{s(s+1)} - 7.5 + \frac{3.75s}{s+1} + \left(\frac{s^2+1}{s}\right)I_3 = -\frac{7.5}{s+1}$$

$$I_3 = -\frac{7.5}{s} + \frac{5.625-1.875j}{s+j} + \frac{5.625+1.875j}{s-j}$$

Substituting (3) into (1) and (2) leads to

$$-\left(s+\frac{1}{s}\right)I_2 + \left(s+\frac{1}{s}\right)I_3 = \frac{7.5(-s^2+2s+2)}{s^2(s+1)} \qquad (4)$$

$$\left(2+s+\frac{1}{s}\right)I_2 - \left(s+\frac{1}{s}\right)I_3 = -\frac{15(s+1)}{s^2} \qquad (5)$$

We can now solve for I_o.

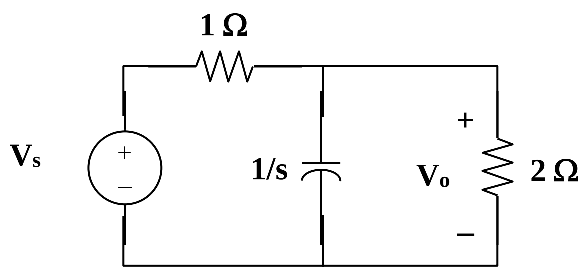
$$I_o = I_2 - I_3 = (15/s) - (3.75/(s+1)) + ((-5.625+1.875j)/(s+j)) + ((-5.625-1.875)/(s-j))$$

or

$i_o(t) =$ **$[15 - 3.75e^{-t} + 5.929e^{-jt+161.57^\circ} + 5.929e^{jt-161.57^\circ}]u(t)$A**.

Solution 16.57

$v_s(t) = 3u(t) - 3u(t-1)$ or $V_s = \frac{3}{s} - \frac{e^{-s}}{s} = \frac{3}{s}(1 - e^{-s})$



$$\frac{V_o - V_s}{1} + sV_o + \frac{V_o}{2} = 0 \rightarrow (s + 1.5)V_o = V_s$$

$$V_o = \frac{3}{s(s+1.5)}(1 - e^{-s}) = \left(\frac{2}{s} - \frac{2}{s+1.5}\right)(1 - e^{-s})$$

$$v_o(t) = \underline{[(2 - 2e^{-1.5t})u(t) - (2 - 2e^{-1.5(t-1)})u(t-1)]\,V}$$

(a) $(3/s)[1-e^{-s}]$, (b) $[(2-2e^{-1.5t})u(t) - (2-2e^{-1.5(t-1)})u(t-1)]$ V

Solution 16.58

Using Fig. 16.81, design a problem to help other students to better understand circuit analysis in the s-domain with circuits that have dependent sources.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the circuit of Fig. 16.81, let $i(0) = 1$ A, $v_o(0) = 2$ V, and $v_s = 4\,e^{-2t}\,u(t)$ V. Find $v_o(t)$ for $t > 0$.

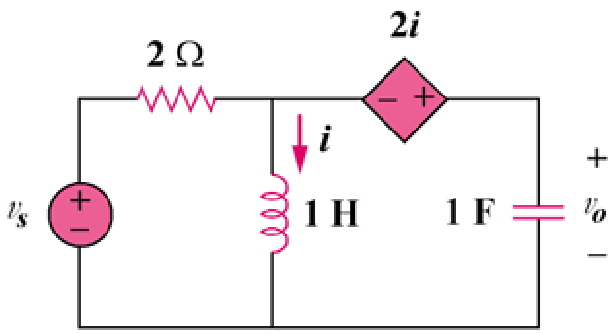
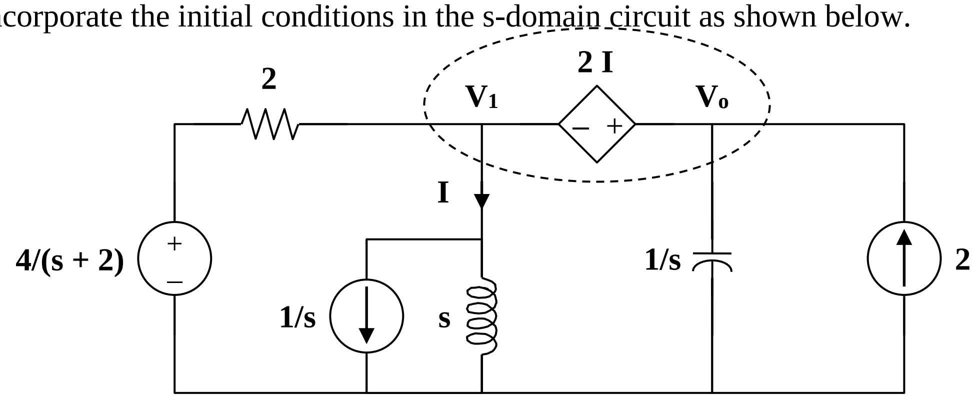


Figure 16.81
For Prob. 16.58.

Solution

We incorporate the initial conditions in the s-domain circuit as shown below.



At the supernode,

$$\frac{(4/(s+2)) - V_1}{2} + 2 = \frac{V_1}{s} + \frac{1}{s} + sV_o$$

$$\frac{2}{s+2} + 2 = \left(\frac{1}{2} + \frac{1}{s}\right)V_1 + \frac{1}{s} + s\,V_o \qquad (1)$$

But $\quad V_o = V_1 + 2I$ and $\quad I = \dfrac{V_1 + 1}{s}$

$$V_o = V_1 + \frac{2(V_1 + 1)}{s} \quad \longrightarrow \quad V_1 = \frac{V_o - 2/s}{(s+2)/s} = \frac{sV_o - 2}{s+2} \qquad (2)$$

Substituting (2) into (1)

$$\frac{2}{s+2} + 2 - \frac{1}{s} = \left(\frac{s+2}{2s}\right)\left[\left(\frac{s}{s+2}\right)V_o - \frac{2}{s+2}\right] + sV_o$$

$$\frac{2}{s+2} + 2 - \frac{1}{s} + \frac{1}{s} = \left[\left(\frac{1}{2}\right) + s\right]V_o$$

$$\frac{2s+4+2}{(s+2)} = \frac{2s+6}{s+2} = (s+1/2)V_o$$

$$V_o = \frac{2s+6}{(s+2)(s+1/2)} = \frac{A}{s+1/2} + \frac{B}{s+2}$$

$$A = (-1+6)/(-0.5+2) = 3.333, \quad B = (-4+6)/(-2+1/2) = -1.3333$$

$$V_o = \frac{3.333}{s+1/2} - \frac{1.3333}{s+2}$$

Therefore,

$$v_o(t) = \underline{(3.333e^{-t/2} - 1.3333e^{-2t})u(t)\ \mathbf{V}}$$

Solution 16.59

Find $v_o(t)$ in the circuit in Fig. 16.82 if $v_x(0) = 10$ V and $i(0) = 5$ A.

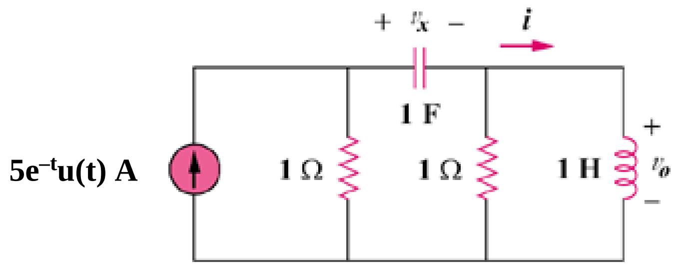
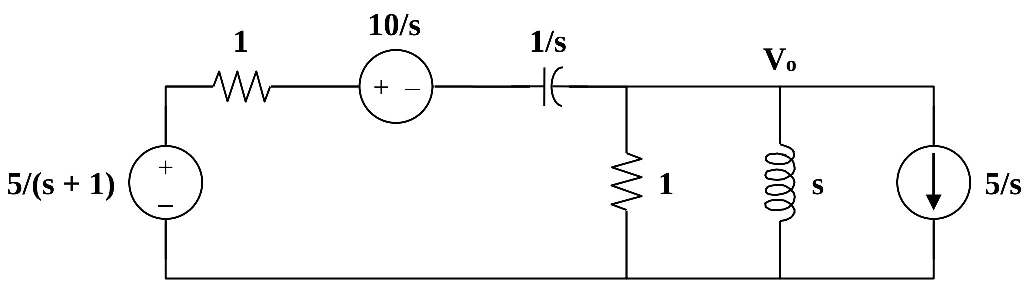


Figure 16.82
For Prob. 16.59.

Solution

Step 1. We incorporate the initial conditions and transform the current source to a voltage source as shown and then convert the circuit into the s-domain.



At the main non-reference node, KCL gives

$$\frac{[5/(s+1)]-[10/s]-V_o}{1+1/s} = \frac{V_o}{1} + \frac{V_o}{s} + \frac{5}{s}$$

Now all we need to do is to solve for V_o, perform a partial fraction expansion, and convert the answer back into the time domain.

Step 2.

$$\frac{5s}{s+1} - 10 - sV_o = (s+1)(1+(1/s))V_o + \frac{5(s+1)}{s}$$

$$\frac{5s}{s+1} - \frac{5(s+1)}{s} - 10 = (2s+2+1/s)V_o$$

$$V_o = \frac{-10s^2 - 20s - 5}{(s+1)(2s^2+2s+1)}$$

$$V_o = \frac{-5s^2 - 10s - 2.5}{(s+1)(s^2+s+0.5)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+s+0.5}$$

$$A = (s+1)V_o\Big|_{s=-1} = 5$$

$$-5s^2 - 10s - 2.5 = A(s^2 + s + 0.5) + B(s^2 + s) + C(s+1)$$

Equating coefficients :

$$s^2: \quad -5 = A + B \longrightarrow B = -10$$

$$s^1: \quad -10 = A + B + C \longrightarrow C = -5$$

$$s^0: \quad -2.5 = 0.5A + C = 2.5 - 5 = -2.5$$

$$V_o = \frac{5}{s+1} - \frac{10s+5}{s^2+s+0.5} = \frac{5}{s+1} - \frac{10(s+0.5)}{(s+0.5)^2 + (0.5)^2}$$

$v_o(t) =$ **[5e^–t^ – 10e^–t/2^cos(t/2)]u(t) V**.

Solution 16.60

Find the response $v(t)$ for $t > 0$ in the circuit in Fig. 16.83. Let $R = 8\ \Omega$, $L = 2$ H, and $C = 125$ mF.

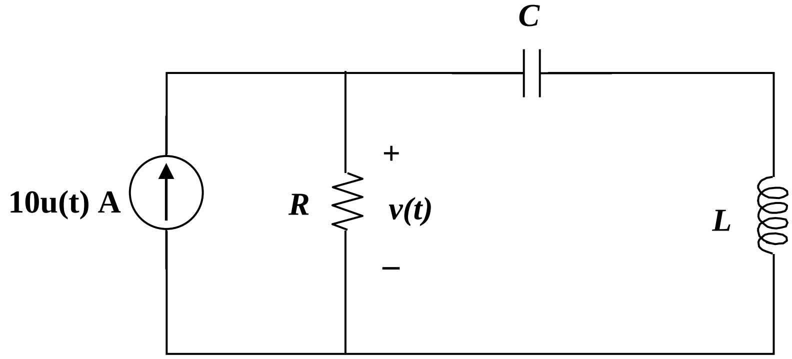
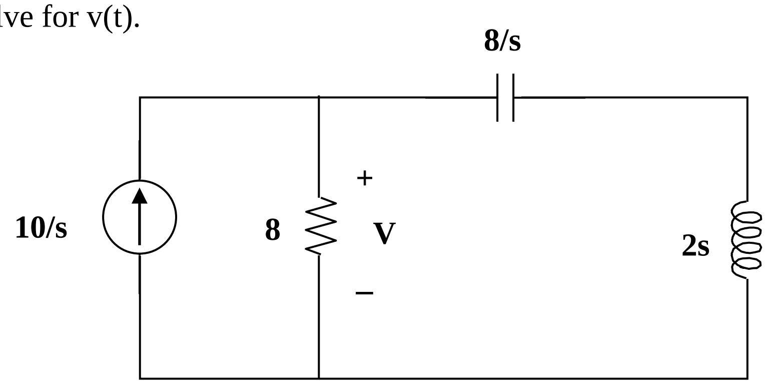


Figure 16.83
For Prob. 16.60.

Solution

Step 1. The first thing is to note that there are no initial conditions since the source = 0 for all t < 0. Next we transform the circuit into the s-domain and write one nodal equation and solve for V. The we perform a partial fraction expansion and solve for v(t).



$-[10/s] + [(V-0)/8] + [(V-0)/\{2(s+4/s)\}] = 0.$

Step 2. $[0.125+0.5s/(s^2+4)]V = 10/s = [(0.125s^2+0.5+0.5s)/(s^2+4)]V$ or
$V = 80(s^2+4)/[s(s^2+4s+4)]$ where $s_{1,2} = -2, -2$, note a repeated root.
$V = [A/s] + [B/(s+2)] + [C/(s+2)^2]$ where A = 80 and C = 640/(-2) = –320.
Now we can find B. $80(s^2+4) = 80(s+2)^2 + B(s^2+4s) - 320s$ or
B = 0. This now gives us,

$$v(t) = \mathbf{[80 - 320te^{-2t}]u(t)\ V}.$$

Solution 16.61

Find the voltage $v_o(t)$ in the circuit of Fig. 16.84 by means of the Laplace transform.

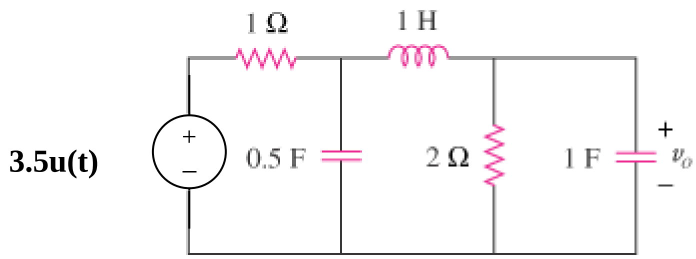
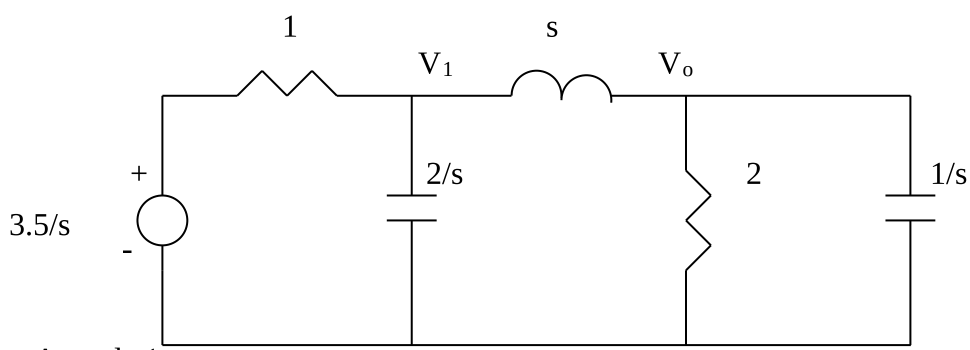


Figure 16.84
For Prob. 16.61.

Solution

Step 1. We first need to determine the initial conditions which in this case are equal to zero since there are no sources before t = 0. Next we transform the circuit into the s-domain. Next we solve for V_o using nodal analysis. Then we perform a partial fraction expansion and then transform this back into the time domain.



At node 1,

$$\frac{V_1-\frac{3.5}{s}}{1}+\frac{V_1-V_o}{s}+\frac{s}{2}(V_1-0)=0 \ or \qquad \left(\frac{s^2}{2}+s+1\right)V_1+(-1)V_o=3.5 \qquad (1)$$

At node 2,

$$\frac{V_o-V_1}{s}+\frac{V_o-0}{2}+s(V_o-0)=0 \ or \qquad V_1=(s^2+0.5s+1)V_o \qquad (2)$$

Step 2. Substituting (2) into (1) gives

$$3.5 = [0.5(s^2 + 2s + 2)(s^2 + 0.5s + 1)V_o - V_o = 0.5(s^4 + 2.5s^3 + 4s^2 + 3s + 2 - 2)V_o$$

$$V_o = \frac{7}{s(s^3 + 2.5s^2 + 4s + 3)}$$

Use MATLAB to find the roots.

```
>> p=[1 2.5 4 3]

p =

    1.0000    2.5000    4.0000    3.0000

>> r=roots(p)

r =

  -0.6347 + 1.4265i
  -0.6347 - 1.4265i
  -1.2306
```

Thus,

$$V_o = \frac{7}{s(s+1.2306)(s+0.6347+j1.4265)(s+0.6347-j1.4265)}$$

$$= \frac{A}{s} + \frac{B}{(s+1.2306)} + \frac{C}{(s+0.6347+j1.4265)} + \frac{D}{(s+0.6347-j1.4265)}$$

Where A = 7/3 = 2.3333; B =

$$\frac{7}{(-1.2306)(-1.2306+0.6347+j1.4265)(-1.2306+0.6347-j1.4265)}$$

$$= \frac{-5.6882}{(0.3551+2.035)} = -2.38$$

$$C = \frac{7}{(-0.6347-j1.4265)(-0.6347-j1.4265+1.2306)(-j2.853)}$$

$$= \frac{7}{(1.5613\angle -113.99°)(1.546\angle -67.33°)(2.853\angle -90°)} = \frac{7}{6.886\angle 88.68°} = 1.0166\angle -88.68°$$

$$D = \frac{7}{(-0.6347 + j1.4265)(-0.6347 + j1.4265 + 1.2306)(j2.853)}$$

$$= \frac{7}{(1.5613\angle 113.99°)(1.546\angle 67.33°)(2.853\angle 90°)} = \frac{7}{6.886\angle -88.68°} = 1.0166\angle 88.68°$$

$$V_o = \frac{2.333}{s} + \frac{-2.38}{(s+1.2306)} + \frac{1.0166\angle -88.68°}{(s+0.6347+j1.4265)} + \frac{1.0166\angle 88.68°}{(s+0.6347-j1.4265)} \text{ or}$$

$v_o(t) = [2.333–2.38e^{–1.2306t}+1.0166e^{–0.6347t}(e^{–(1.4265t+88.68^\circ)}+e^{(1.4265t+88.68^\circ)})]u(t)$ volts or

$$= \mathbf{[2.333–2.38e^{–1.2306t}+2.033e^{–0.6347t}\cos(1.4265t+88.68^\circ)]u(t)\ V}.$$

Answer does check for initial values and final values.

Solution 16.62

Using Fig. 16.85, design a problem to help other students better understand solving for node voltages by working in the s-domain.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the node voltages v_1 and v_2 in the circuit of Fig. 16.85 using Laplace transform technique. Assume that $i_s = 12e^{-t}\, u(t)$ A and that all initial conditions are zero.

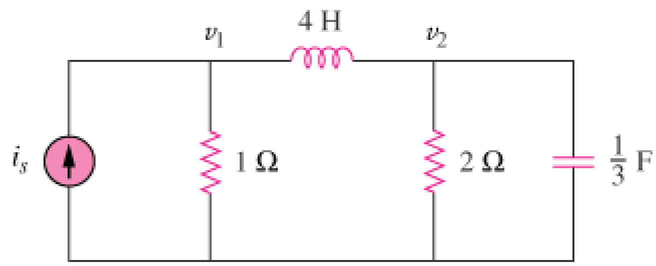
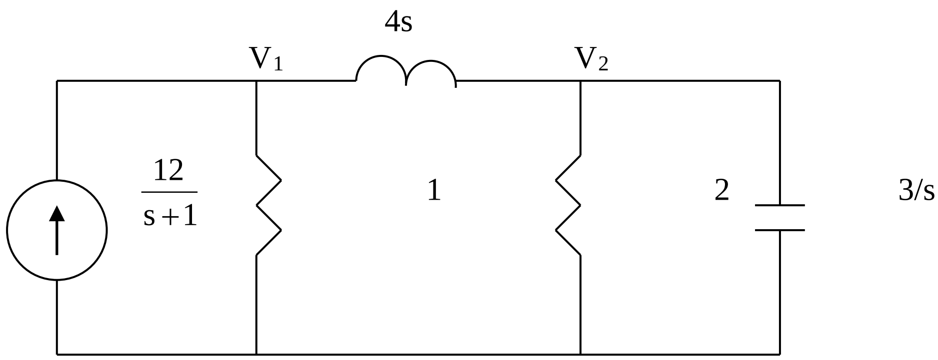


Figure 16.85
For Prob. 16.62.

Solution

The s-domain version of the circuit is shown below.



At node 1,

$$\frac{12}{s+1} = \frac{V_1}{1} + \frac{V_1 - V_2}{4s} \longrightarrow \frac{12}{s+1} = V_1\left(1 + \frac{1}{4s}\right) - \frac{V_2}{4s} \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{4s} = \frac{V_2}{2} + \frac{s}{3}V_2 \longrightarrow V_1 = V_2\left(\frac{4}{3}s^2 + 2s + 1\right) \qquad (2)$$

Substituting (2) into (1),

$$\frac{12}{s+1} = V_2\left[\left(\frac{4}{3}s^2 + 2s + 1\right)\left(1 + \frac{1}{4s}\right) - \frac{1}{4s}\right] = \left(\frac{4}{3}s^2 + \frac{7}{3}s + \frac{3}{2}\right)V_2$$

$$V_2 = \frac{9}{(s+1)(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{A}{(s+1)} + \frac{Bs + C}{(s^2 + \frac{7}{4}s + \frac{9}{8})}$$

$$9 = A(s^2 + \frac{7}{4}s + \frac{9}{8}) + B(s^2 + s) + C(s+1)$$

Equating coefficients:

$$s^2: \qquad 0 = A + B$$

$$s: \qquad 0 = \frac{7}{4}A + B + C = \frac{3}{4}A + C \longrightarrow C = -\frac{3}{4}A$$

$$\text{constant}: \qquad 9 = \frac{9}{8}A + C = \frac{3}{8}A \longrightarrow A = 24,\ B = -24,\ C = -18$$

$$V_2 = \frac{24}{(s+1)} - \frac{24s + 18}{(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{24}{(s+1)} - \frac{24(s + 7/8)}{(s + \frac{7}{8})^2 + \frac{23}{64}} + \frac{3}{(s + \frac{7}{8})^2 + \frac{23}{64}}$$

Taking the inverse of this produces:

$$\underline{\mathbf{v_2(t) = \left[24e^{-t} - 24e^{-0.875t}\cos(0.5995t) + 5.004e^{-0.875t}\sin(0.5995t)\right]u(t)V}}$$

Similarly,

$$V_1 = \frac{9\left(\frac{4}{3}s^2 + 2s + 1\right)}{(s+1)(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{D}{(s+1)} + \frac{Es + F}{(s^2 + \frac{7}{4}s + \frac{9}{8})}$$

$$9\left(\frac{4}{3}s^2 + 2s + 1\right) = D(s^2 + \frac{7}{4}s + \frac{9}{8}) + E(s^2 + s) + F(s+1)$$

Equating coefficients:

$$s^2: \qquad 12 = D + E$$

$$s: \qquad 18 = \frac{7}{4}D + E + F \text{ or } 6 = \frac{3}{4}D + F \quad \longrightarrow \quad F = 6 - \frac{3}{4}D$$

$$\text{constant}: \qquad 9 = \frac{9}{8}D + F \text{ or } 3 = \frac{3}{8}D \quad \longrightarrow \quad D = 8,\ E = 4,\ F = 0$$

$$V_1 = \frac{8}{(s+1)} + \frac{4s}{(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{8}{(s+1)} + \frac{4(s+7/8)}{(s+\frac{7}{8})^2 + \frac{23}{64}} - \frac{7/2}{(s+\frac{7}{8})^2 + \frac{23}{64}}$$

Thus,

$$\underline{\mathbf{v_1(t) = \left[8e^{-t} + 4e^{-0.875t}\cos(0.5995t) - 5.838e^{-0.875t}\sin(0.5995t)\right]u(t)V}}$$

Solution 16.63

Consider the parallel *RLC* circuit of Fig. 16.86. Find *v*(*t*) and *i*(*t*) given that *v*(0) = 7.5 and *i*(0) = –3 A.

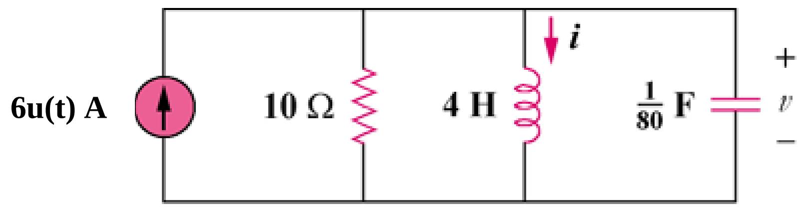
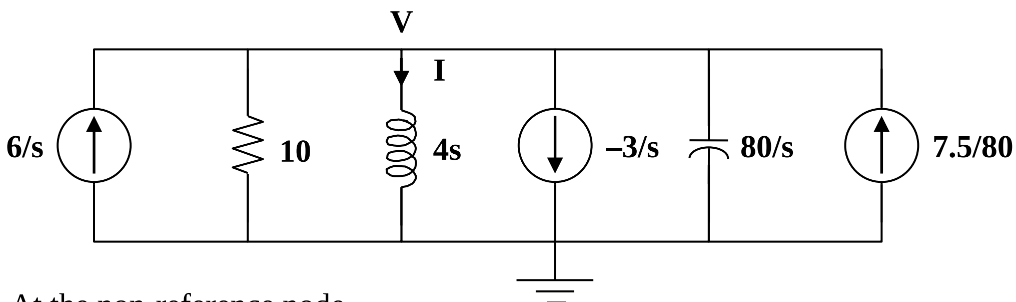


Figure 16.86
For Prob. 16.63.

Solution

Step 1. We now need to transform the circuit into the s-domain and solve for V using nodal analysis. Once we have V we can find I = [V/(4s)] – 3/s. Both terms can now have a partial fraction expansion and transformed back into the time domain.



At the non-reference node,

$$\frac{6}{s}+\frac{3}{s}+0.09375=\frac{V}{10}+\frac{V}{4s}+\frac{sV}{80} \text{ or}$$

$$\frac{9+0.09375s}{s}=\frac{V}{80s}\left(s^2+8s+20\right)$$

Step 2.
$$V=\frac{7.5s+720}{s^2+8s+20}=\frac{7.5(s+4)}{(s+4)^2+2^2}+\frac{(345)(2)}{(s+4)^2+2^2}$$

v(t) = **[7.5e^{-4t}cos(2t) + 345e^{-4t}sin(2t)]u(t) V**.

$$I=\frac{V}{4s}-\frac{3}{s}=\frac{7.5s+720}{4s(s^2+8s+20)}-\frac{3}{s}=\frac{1.875s+180}{s(s^2+8s+20)}-\frac{3}{s}=\frac{A}{s}+\frac{Bs+C}{s^2+8s+20}-\frac{3}{s}$$

A = 9, B = –9, and C = –70.125

$$I=\frac{6}{s}-\frac{6s+46.75}{s^2+8s+20}=\frac{6}{s}-\frac{9(s+4)}{(s+4)^2+2^2}-\frac{(17.0625)(2)}{(s+4)^2+2^2}$$

$\mathrm{i(t)} =$ **[6 – 9e$^{–4t}$cos(2t) – 17.062e$^{–4t}$sin(2t)]u(t) A**.

Checking, Ldi/dt = 4{36e$^{–4t}$cos(2t) + 18e$^{–4t}$sin(2t) + 68.25e$^{–4t}$sin(2t) – 34.12e$^{–4t}$cos(2t)}u(t) = [7.52e$^{–4t}$cos(2t) + 345e$^{–4t}$sin(2t)]u(t) and i(0) = –3 A. The answer checks.

Solution 16.64

The switch in Fig. 16.87 moves from position 1 to position 2 at $t = 0$. Find $v(t)$, for all $t > 0$.

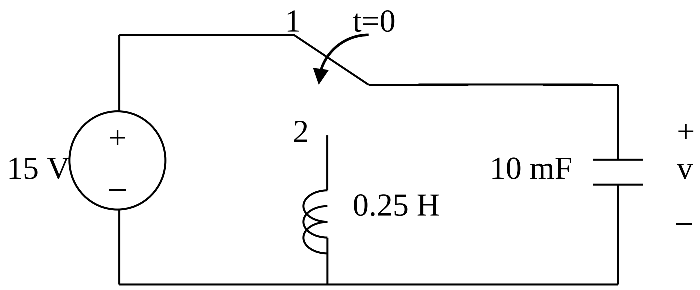
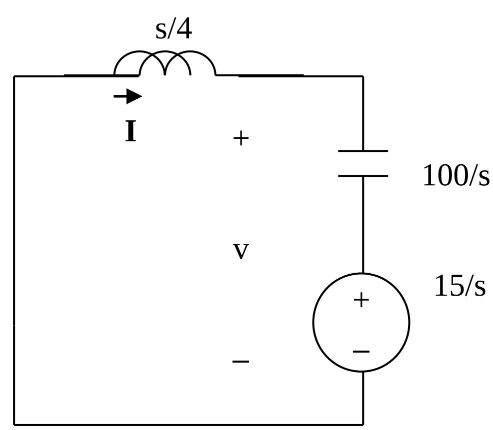


Figure 16.87
For Prob. 16.64.

Solution

When the switch is position 1, $v(0)=15$ V, and $i_L(0) = 0$. When the switch is in position 2, we have the circuit as shown below.



$I = -(15/s)/[0.25s+(100/s)] = -60/(s^2+400)$ and $V = -0.25sI$ (or $(100/s)I + 15/s$).

$I = 15s/(s^2+400)$ which leads to,

$$v(t) = \mathbf{[15\cos(20t)]u(t)\ V}$$

Solution 16.65

For the *RLC* circuit shown in Fig. 16.88, find the complete response if *v*(0) = 100 V when the switch is closed.

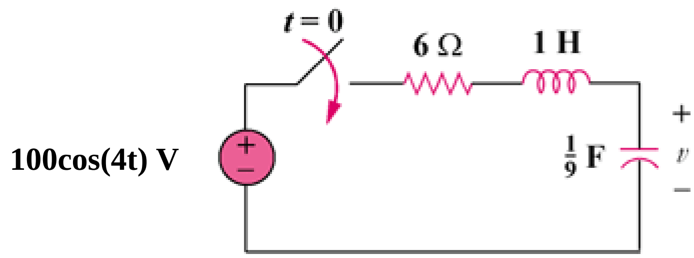
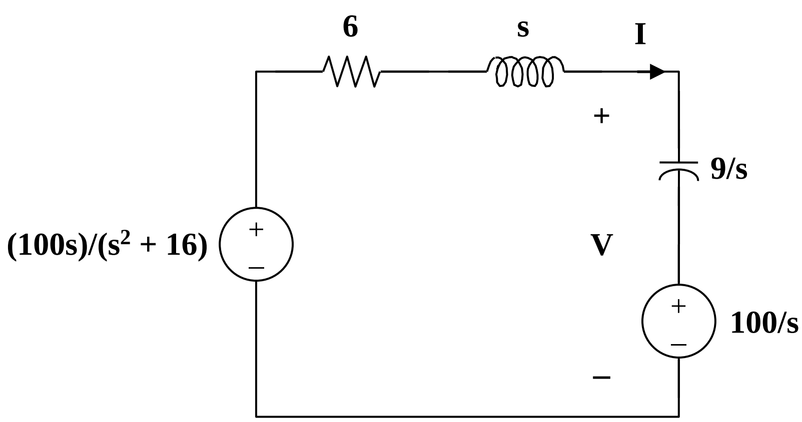


Figure 16.88
For Prob. 16.65.

Solution

For $t > 0$, the circuit in the s-domain is shown below.



Applying KVL,

$$\frac{-100s}{s^2+16} + \left(6 + s + \frac{9}{s}\right) I + \frac{100}{s} = 0$$

$$I = \frac{-1{,}600}{(s^2+6s+9)(s^2+16)}$$

$$V = \frac{9}{s} I + \frac{100}{s} = \frac{100}{s} + \frac{-14{,}400}{s(s+3)^2(s^2+16)}$$

$$= \frac{100}{s} + \frac{A}{s} + \frac{B}{s+3} + \frac{C}{(s+3)^2} + \frac{Ds+E}{s^2+16}$$

$$-14{,}400 = A(s^4+6s^3+25s^2+96s+144) + B(s^4+3s^3+16s^2+48s)$$
$$+ C(s^3+16s) + D(s^4+6s^3+9s^2) + E(s^3+6s^2+9s)$$

Equating coefficients :

$$\begin{aligned}
&s^0: && -14{,}400 = 144A && (1)\\
&s^1: && 0 = 96A + 48B + 16C + 9E && (2)\\
&s^2: && 0 = 25A + 16B + 9D + 6E && (3)\\
&s^3: && 0 = 6A + 3B + C + 6D + E && (4)\\
&s^4: && 0 = A + B + D && (5)
\end{aligned}$$

Solving equations (1), (2), (3), (4) and (5) gives

A = –100, B = 110.1, C = 192, D = 10.1, and E = 138.3.

$$V(s) = \frac{110.1}{s+3} + \frac{192}{(s+3)^2} - \frac{10.1s}{s^2+16} + \frac{(34.58)(4)}{s^2+16}$$

$v(t) =$ $\mathbf{\{110.1e^{-3t} + 192te^{-3t} - 10.1\cos(4t) + 34.58\sin(4t)\}u(t)\ V}$

Solution 16.66

For the op amp circuit in Fig. 16.89, find $v_o(t)$ for t > 0. Take $v_s = 12\, e^{-5t}\, u(t)$ V.

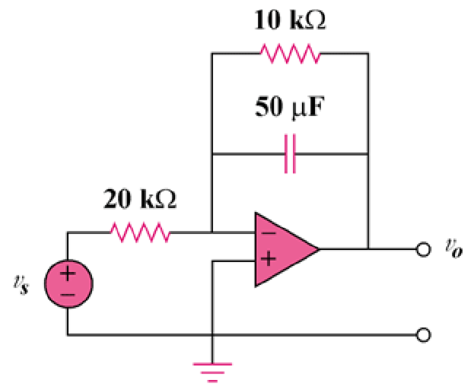
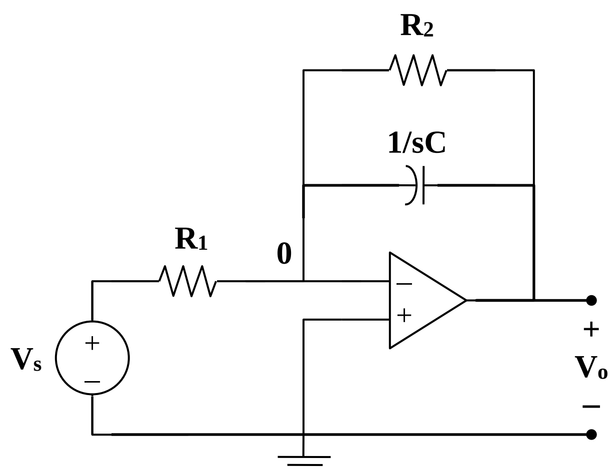


Figure 16.89
For Prob. 16.66.

Solution

Consider the op-amp circuit below where R_1 = 20 kΩ, R_2 = 10 kΩ, C = 50 μF, and $v_s(t) = [12e^{-5t}]u(t)$ V.



At node 0,

$$\frac{V_s - 0}{R_1} = \frac{0 - V_o}{R_2} + (0 - V_o)sC$$

$$V_s = R_1\left(\frac{1}{R_2} + sC\right)(-V_o)$$

$$\frac{V_o}{V_s} = \frac{-1}{sR_1C + R_1/R_2}$$

But $\quad \dfrac{R_1}{R_2} = \dfrac{20}{10} = 2, \qquad R_1C = (20\times10^3)(50\times10^{-6}) = 1$

So, $\quad \dfrac{V_o}{V_s} = \dfrac{-1}{s+2}$

$$v_s(t) = 12e^{-5t} \quad \longrightarrow \quad V_s = 12/(s+5)$$

$$V_o = \frac{-12}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5}$$ where A = –4 and B = 4.

$$V_o = \frac{4}{s+5} - \frac{4}{s+2}$$ which leads to,

$$v_o(t) = \mathbf{4[\left(e^{-5t} - e^{-2t}\right)u(t)\ V}.$$

Solution 16.67

Given the op amp circuit in Fig. 16.90. If $v_1(0^+) = 2$ V and $v_2(0^+) = 0$ V, find v_o for $t > 0$. Let $R = 100$ kΩ and C = 1 μF.

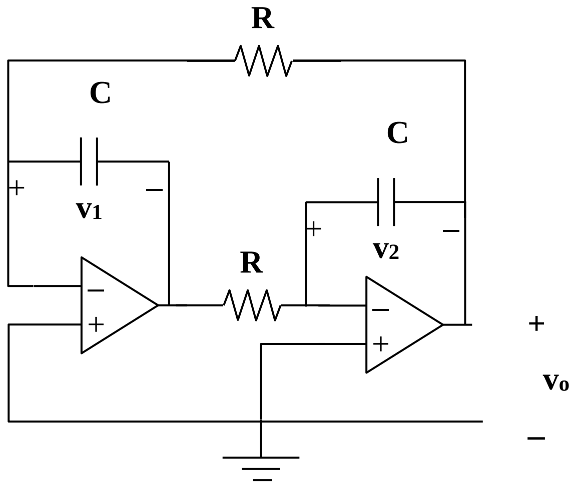
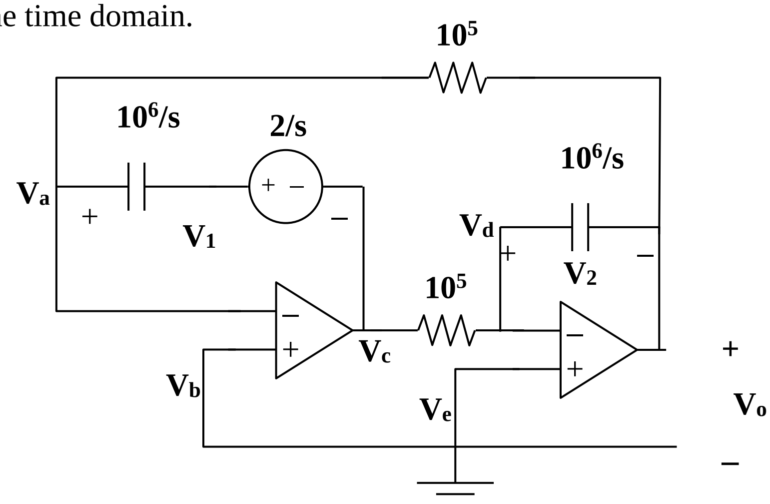


Figure 16.90For Prob. 16.67.

Solution

Step 1. Convert the circuit into the s-domain and insert initial conditions. Next, solve for $V_o(s)$, then obtain the partial fraction expansion and convert back into the time domain.



$[(V_a–(V_c+2/s))/(10^6/s)]+[(V_a–V_o)/10^5]+0 = 0$; $V_a = V_b = 0$ and
$[(V_d–V_c)/10^5]+[(V_d–V_o)/(10^6/s)]+0 = 0$; $V_d = V_e = 0$.

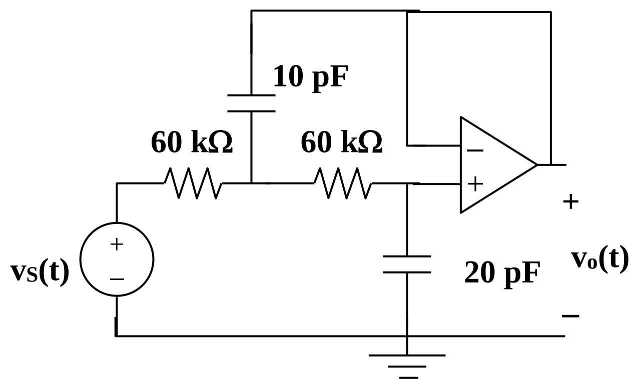
Step 2. $sV_c + 10V_o = –2$ and $10V_c + sV_o = 0$ or $V_c = –0.1sV_o$ thus,

$(–0.1s^2+10)V_o = –2$ or $V_o = 20/(s^2–100) = [A/(s–10)]+[B/(s+10)]$ where
$A = 20/(10+10) = 1$ and $B = 20/(–10–10) = –1$. This now leads to

$v_o(t) =$ **$[e^{10t}–e^{–10t}]u(t)$ volts**. It should be noted that this is an unstable circuit!

Solution 16.68

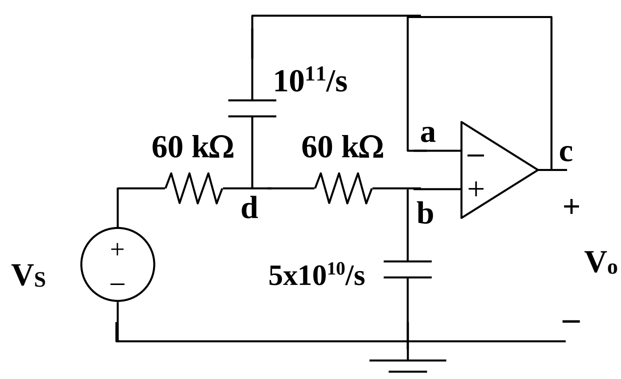
Obtain V_o/V_s in the op amp circuit in Fig. 8.91.



I)1
For Prob. 8.68.

Solution

Step 1. Convert the circuit into the s-domain and then solve for $V_o(s)$ in terms of $V_s(s)$. Then solve for $V_o/V_s = T(s)$.



At a, $V_a = V_b = V_c = V_o$. At b, $[(V_b–V_d)/60k]+[(V_b–0)/(5x10^{10}/s)]+0 = 0$ or $[(V_o–V_d)/60k]+[(V_o–0)/(5x10^{10}/s)] = 0$ or $[(1/60k)V_d = [(1/60k)+(s/(5x10^{10}))]V_o$ or
$V_d = [(1.2x10^{–6})s+1]V_o$.

At d, $[(V_d–V_s)/60k]+[(V_d–V_c)/(10^{11}/s)]+(V_d–V_b)/60k] = 0$ or
$[(2/60k)+(s/10^{11})]V_d–(s/10^{11})V_o–(1/60k)V_o = (1/60k)V_s$ or
$[(2/60k)+(s/10^{11})]\ [(1.2x10^{–6})s+1]V_o–(s/10^{11})V_o–(1/60k)V_o = (1/60k)V_s$ or
$[2+(6x10^{–7})s]\ [(1.2x10^{–6})s+1]V_o–(6x10^{–7})sV_o–V_o = V_s$ or
$[7.2x10^{–13}s^2+(2.4x10^{–6}+0.6x10^{–6}–0.6x10^{–6})s+(2–1)]V_o = V_s$ or

$$T(s) = V_o/V_s = \mathbf{1/[(7.2x10^{–}13)s^2+(2.4x10^{–6})s+1]}.$$

Solution 16.69

Find $I_1(s)$ and $I_2(s)$ in the circuit of Fig. 16.92.

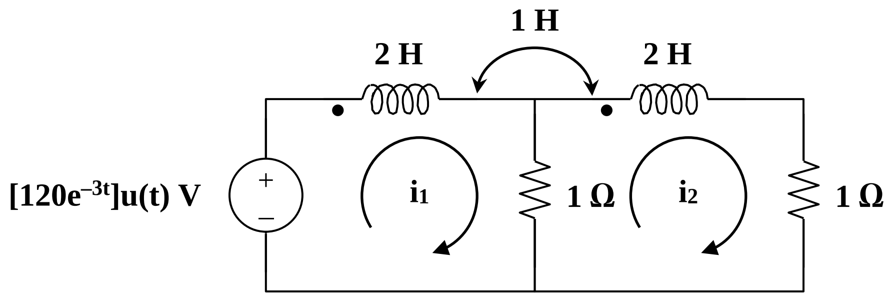
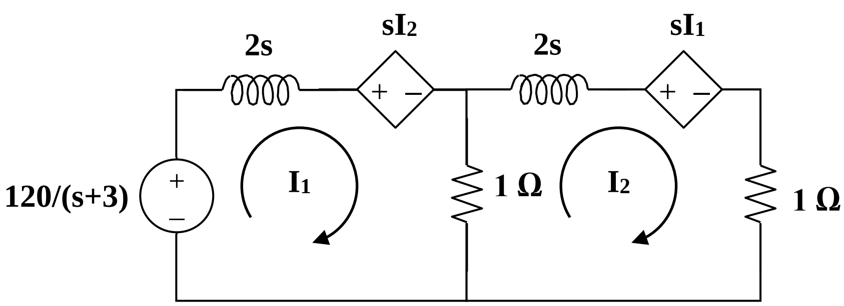


Figure 16.92 For Prob. 16.69.

Solution

Step 1. We note that the initial conditions in this case are equal to zero. Next, we need to convert the circuit into the s-domain and use the model for mutually coupled circuits. Then we can write the mesh equations and solve for I_1 and I_2.



Step 2. $-[120/(s+3)] + 2sI_1 + sI_2 + 1(I_1-I_2) = 0$ and $1(I_2-I_1) + 2sI_2 + sI_1 + 1I_2 = 0$. Simplifying we get,

$(2s+1)I_1 + (s-1)I_2 = 120/(s+3)$ and $(s-1)I_1 + (2s+2)I_2 = 0$.

We can solve this directly using substitution or use matrices. Let us use matrices.

$$\begin{bmatrix} 2s+1 & s-1 \\ s-1 & 2s+2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{120}{(s+3)} \\ 0 \end{bmatrix}$$ The matrix inverse

$$\begin{bmatrix} 2s+1 & s-1 \\ s-1 & 2s+2 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} 2s+2 & -s+1 \\ -s+1 & 2s+1 \end{bmatrix}}{4s^2+4s+2s+2-s^2+2s-1} = \frac{\begin{bmatrix} 2s+2 & -s+1 \\ -s+1 & 2s+1 \end{bmatrix}}{3s^2+8s+1}$$

Therefore,

I_1 = $\mathbf{240(s+1)/[(s+3)(3s^2+8s+1)]}$ and
I_2 = $\mathbf{-120(s-1)/[(s+3)(3s^2+8s+1)]}$.

Solution 16.70

Using Fig. 16.93, design a problem to help other students better understand how to do circuit analysis with circuits that have mutually coupled elements by working in the s-domain.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 16.93, find $v_o(t)$ for $t > 0$.

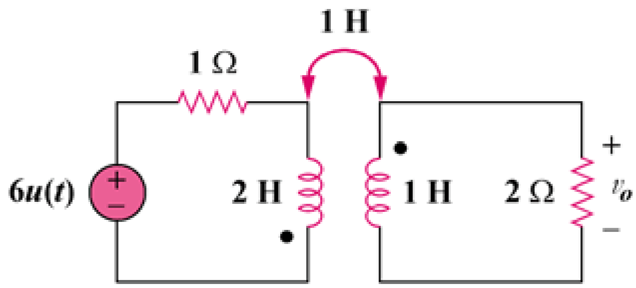
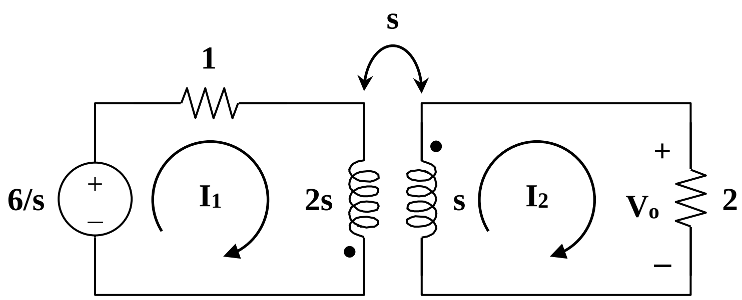


Figure 16.93
For Prob. 16.70.

Solution

Consider the circuit shown below.



For mesh 1,

$$\frac{6}{s} = (1+2s)\,I_1 + s\,I_2 \tag{1}$$

For mesh 2,

$$0 = s\,I_1 + (2+s)\,I_2$$

$$I_1 = -\left(1+\frac{2}{s}\right)I_2 \tag{2}$$

Substituting (2) into (1) gives

$$\frac{6}{s} = -(1+2s)\left(1+\frac{2}{s}\right)I_2 + s\,I_2 = \frac{-(s^2+5s+2)}{s}I_2$$

or $\quad I_2 = \dfrac{-6}{s^2+5s+2}$

$$V_o = 2\,I_2 = \frac{-12}{s^2+5s+2} = \frac{-12}{(s+0.438)(s+4.561)}$$

Since the roots of $s^2+5s+2=0$ are -0.438 and -4.561,

$$V_o = \frac{A}{s+0.438} + \frac{B}{s+4.561}$$

$$A = \frac{-12}{4.123} = -2.91, \qquad B = \frac{-12}{-4.123} = 2.91$$

$$V_o(s) = \frac{-2.91}{s+0.438} + \frac{2.91}{s+4.561}$$

$$v_o(t) = \underline{\mathbf{2.91\left[e^{-4.561t} - e^{0.438t}\right]u(t)\ V}}$$

Solution 16.71

For the ideal transformer circuit in Fig. 16.94, determine $i_o(t)$.

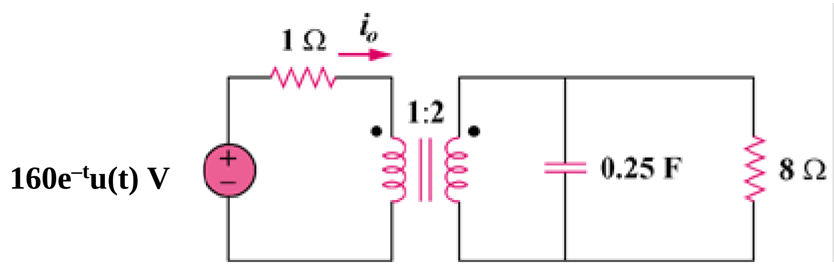
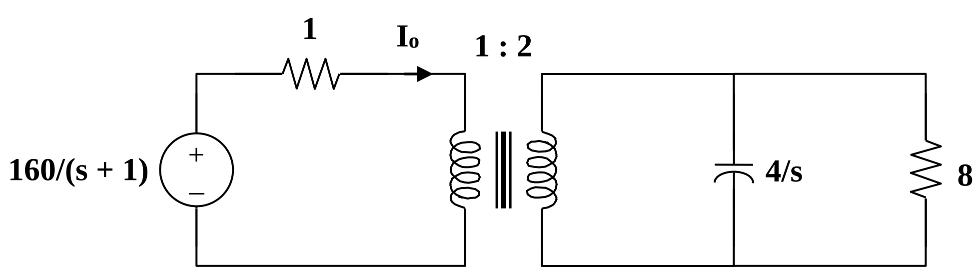


Figure 16.94
For Prob. 16.71.

Solution

Consider the following circuit.



Let $\quad Z_L = 8 \| \frac{4}{s} = \frac{(8)(4/s)}{8+4/s} = \frac{8}{2s+1}$

When this is reflected to the primary side,

$$Z_{in} = 1 + \frac{Z_L}{n^2}, \quad n = 2$$

$$Z_{in} = 1 + \frac{2}{2s+1} = \frac{2s+3}{2s+1}$$

$$I_o = \frac{160}{s+1} \cdot \frac{1}{Z_{in}} = \frac{160}{s+1} \cdot \frac{2s+1}{2s+3}$$

$$I_o = \frac{160s+80}{(s+1)(s+1.5)} = \frac{A}{s+1} + \frac{B}{s+1.5}$$

A = –160 and B = 320.

$$I_o(s) = \frac{-160}{s+1} + \frac{320}{s+1.5}$$

$$i_o(t) = \mathbf{160[2e^{-1.5t} - e^{-t}]u(t)\ A}.$$

Solution 16.72

The transfer function of a system is

$$H(s) = \frac{s^2}{3s+1}$$

Find the output when the system has an input of $14e^{-t/3}\,u(t)$.

Solution

$$Y(s) = H(s)\,X(s)\,, \qquad X(s) = \frac{14}{s+1/3} = \frac{42}{3s+1}$$

$$Y(s) = \frac{42\,s^2}{(3s+1)^2} = \frac{14}{3} - \frac{28s+14/3}{(3s+1)^2}$$

$$Y(s) = \frac{14}{3} - \frac{28}{9}\cdot\frac{s}{(s+1/3)^2} - \frac{14}{27}\cdot\frac{1}{(s+1/3)^2}$$

Let $G(s) = \frac{-8}{9}\cdot\frac{s}{(s+1/3)^2}$

Using the time differentiation property,

$$g(t) = \frac{-8}{9}\cdot\frac{d}{dt}(t\,\mathrm{e}^{-t/3}) = \frac{-8}{9}\left(\frac{-1}{3}t\,e^{-t/3} + e^{-t/3}\right)$$

$$g(t) = \frac{28}{27}t\,e^{-t/3} - \frac{28}{9}e^{-t/3}$$

Hence,

$$\text{y(t)} = \frac{14}{3}\delta(t) + \left[\frac{28}{27}te^{-\frac{t}{3}} - \frac{28}{9}e^{-\frac{t}{3}} - \frac{14}{27}te^{-\frac{t}{3}}\right]u(t)$$

$y(t) =$ **(14/3)δ(t) + [– (28/9)$e^{-t/3}$ + (14/27)t$e^{-t/3}$]u(t)** or
= **4.667δ(t) + [– 3.111$e^{-t/3}$ + 0.5185t$e^{-t/3}$]u(t)**.

Solution 16.73

When the input to a system is a unit step function, the response is 120 cos(2*t*). Obtain the transfer function of the system.

Solution

$$\mathrm{x(t) = u(t)} \quad \longrightarrow \quad \mathrm{X(s)} = \frac{1}{\mathrm{s}}$$

$$y(t) = 120\cos(2t) \quad \longrightarrow \quad Y(s) = \frac{120s}{s^2+4}$$

H(s) = Y(S)/X(s) = $\mathbf{120s^2/(s^2+4)}$.

Solution 16.74

Design a problem to help other students to better understand how to find outputs when given a transfer function and an input.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A circuit is known to have its transfer function as

$$H(s) = \frac{s+3}{s^2+4s+5}$$

Find its output when:

(a) the input is a unit step function

(b) the input is $6te^{-2t}\,u(t)$.

Solution

(a) $\quad \mathrm{Y(s) = H(s)\,X(s)}$

$$= \frac{s+3}{s^2+4s+5} \cdot \frac{1}{s}$$

$$= \frac{s+3}{s(s^2+4s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+5}$$

$$s+3 = A(s^2+4s+5) + Bs^2 + Cs$$

Equating coefficients :

$s^0: \quad 3 = 5A \longrightarrow A = 3/5$

$s^1: \quad 1 = 4A + C \longrightarrow C = 1 - 4A = -7/5$

$s^2: \quad 0 = A + B \longrightarrow B = -A = -3/5$

$$Y(s) = \frac{3/5}{s} - \frac{1}{5} \cdot \frac{3s+7}{s^2+4s+5}$$

$$Y(s) = \frac{0.6}{s} - \frac{1}{5} \cdot \frac{3(s+2)+1}{(s+2)^2+1}$$

$$y(t) = \left[\mathbf{0.6 - 0.6\,e^{-2t}\cos(t) - 0.2\,e^{-2t}\sin(t)}\right]\mathbf{u(t)}$$

(b) $\quad x(t) = 6\,t\,e^{-2t} \quad \longrightarrow \quad X(s) = \dfrac{6}{(s+2)^2}$

$$Y(s) = H(s)\,X(s) = \frac{s+3}{s^2+4s+5}\cdot\frac{6}{(s+2)^2}$$

$$Y(s) = \frac{6\,(s+3)}{(s+2)^2(s^2+4s+5)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{Cs+D}{s^2+4s+5}$$

Equating coefficients :

$s^3: \quad 0 = A + C \quad \longrightarrow \quad C = -A \qquad (1)$

$s^2: \quad 0 = 6A + B + 4C + D = 2A + B + D \qquad (2)$

$s^1: \quad 6 = 13A + 4B + 4C + 4D = 9A + 4B + 4D \qquad (3)$

$s^0: \quad 18 = 10A + 5B + 4D = 2A + B \qquad (4)$

Solving (1), (2), (3), and (4) gives

$A = 6, \qquad B = 6, \qquad C = -6, \qquad D = -18$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6s+18}{(s+2)^2+1}$$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6\,(s+2)}{(s+2)^2+1} - \frac{6}{(s+2)^2+1}$$

$$y(t) = \left[\mathbf{6\,e^{-2t} + 6\,t\,e^{-2t} - 6\,e^{-2t}\cos(t) - 6\,e^{-2t}\sin(t)}\right]\mathbf{u(t)}$$

Solution 16.75

When a unit step is applied to a system at t = 0, its response is

$$y(t) = \left[6+0.75e^{-3t} - e^{-2t}\left(3\cos\left(4t\right)+4.5\sin\left(4t\right)\right)\right]u(t).$$

What is the transfer function of the system?

Solution

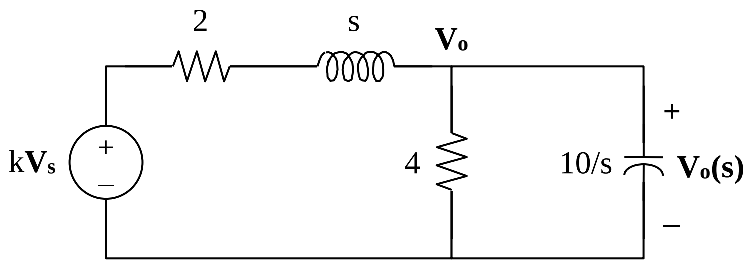
$$H(s)=\frac{Y(s)}{X(s)}, \qquad X(s)=\frac{1}{s}$$

$$Y(s)=\frac{6}{s}+\frac{1.5}{2(s+3)}-\frac{3s}{(s+2)^2+16}-\frac{(4.5)(4)}{(s+2)^2+16}$$

$$H(s)=sY(s)=6+\frac{1.5s}{2(s+3)}-\frac{3s(s+2)}{s^2+4s+20}-\frac{18s}{s^2+4s+20}$$

Solution 16.76

Consider the following circuit.



Using nodal analysis,

$$\frac{kV_s - V_o}{s+2} = \frac{V_o}{4} + \frac{V_o}{10/s}$$

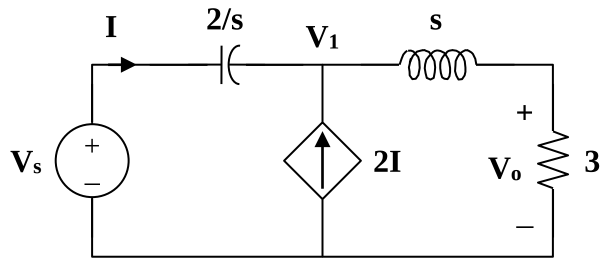
$$V_s = (1/k)(s+2)\left(\frac{1}{s+2} + \frac{1}{4} + \frac{s}{10}\right)V_o = (1/k)\left(1 + \frac{1}{4}(s+2) + \frac{1}{10}(s^2+2s)\right)V_o$$

$$V_s = \frac{1}{20k}\left(2s^2 + 9s + 30\right)V_o$$

$$\frac{V_o}{V_s} = \mathbf{10k/(s^2+4.5s+15)}$$

Solution 16.77

Consider the following circuit.



At node 1,

$$2I + I = \frac{V_1}{s+3}, \qquad \text{where} \quad I = \frac{V_s - V_1}{2/s}$$

$$3 \cdot \frac{V_s - V_1}{2/s} = \frac{V_1}{s+3}$$

$$\frac{V_1}{s+3} = \frac{3s}{2}V_s - \frac{3s}{2}V_1$$

$$\left(\frac{1}{s+3} + \frac{3s}{2}\right)V_1 = \frac{3s}{2}V_s$$

$$V_1 = \frac{3s\,(s+3)}{3s^2 + 9s + 2}V_s$$

$$V_o = \frac{3}{s+3}V_1 = \frac{9s}{3s^2 + 9s + 2}V_s$$

$$H(s) = \frac{V_o}{V_s} = \boldsymbol{\frac{9s}{3s^2 + 9s + 2}}$$

Solution 16.78

The transfer function of a certain circuit is

$$H(s) = \frac{10}{s+1} - \frac{6}{s+2} + \frac{12}{s+4}$$

Find the impulse response of the circuit.

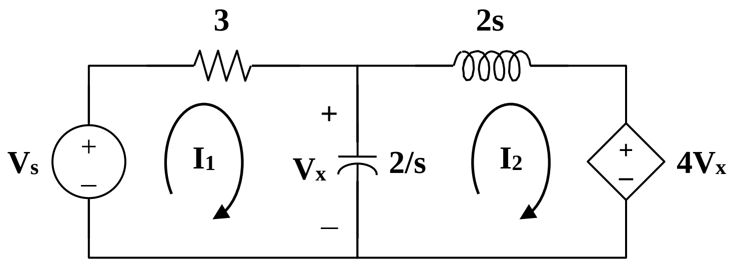
Solution

Taking the inverse Laplace transform of each term gives,

$$h(t) = \mathbf{[10e^{-t} - 6e^{-2t} + 12e^{-4t}]u(t)}.$$

Solution 16.79

(a) Consider the circuit shown below.



For loop 1,

$$V_s = \left(3+\frac{2}{s}\right)I_1 - \frac{2}{s}I_2 \qquad (1)$$

For loop 2,

$$4V_x + \left(2s+\frac{2}{s}\right)I_2 - \frac{2}{s}I_1 = 0$$

But, $V_x = (I_1 - I_2)\left(\frac{2}{s}\right)$

So, $\frac{8}{s}(I_1 - I_2) + \left(2s+\frac{2}{s}\right)I_2 - \frac{2}{s}I_1 = 0$

$$0 = \frac{-6}{s}I_1 + \left(\frac{6}{s} - 2s\right)I_2 \qquad (2)$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} 3+2/s & -2/s \\ -6/s & 6/s-2s \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \left(3+\frac{2}{s}\right)\left(\frac{6}{s}-2s\right) - \left(\frac{6}{s}\right)\left(\frac{2}{s}\right)$$

$$\Delta = \frac{18}{s} - 6s - 4$$

$$\Delta_1 = \left(\frac{6}{s} - 2s\right)V_s, \qquad \Delta_2 = \frac{6}{s}V_s$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{(6/s - 2s)}{18/s - 4 - 6s}V_s$$

$$\frac{I_1}{V_s} = \frac{3/s - s}{9/s - 2 - 3} = \frac{\mathbf{s^2 - 3}}{\mathbf{3s^2 + 2s - 9}}$$

(b) $I_2 = \frac{\Delta_2}{\Delta}$

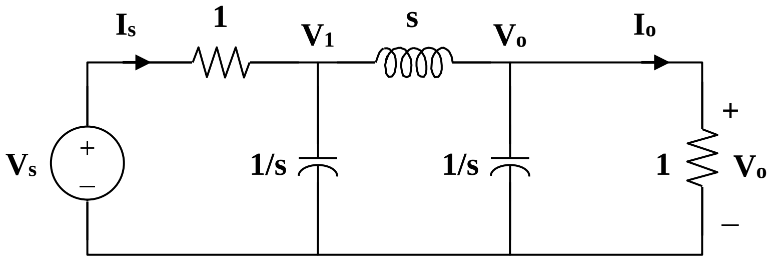
$$V_x = \frac{2}{s}(I_1 - I_2) = \frac{2}{s}\left(\frac{\Delta_1 - \Delta_2}{\Delta}\right)$$

$$V_x = \frac{2/s\, V_s\, (6/s - 2s - 6/s)}{\Delta} = \frac{-4V_s}{\Delta}$$

$$\frac{I_2}{V_x} = \frac{6/s\, V_s}{-4V_s} = \frac{\mathbf{-3}}{\mathbf{2s}}$$

Solution 16.80

(a) Consider the following circuit.



At node 1,

$$\frac{V_s - V_1}{1} = s V_1 + \frac{V_1 - V_o}{s}$$

$$V_s = \left(1 + s + \frac{1}{s}\right) V_1 - \frac{1}{s} V_o \qquad (1)$$

At node o,

$$\frac{V_1 - V_o}{s} = s V_o + V_o = (s+1) V_o$$

$$V_1 = (s^2 + s + 1) V_o \qquad (2)$$

Substituting (2) into (1)

$$V_s = (s + 1 + 1/s)(s^2 + s + 1) V_o - 1/s\, V_o$$

$$V_s = (s^3 + 2s^2 + 3s + 2) V_o$$

$$H_1(s) = \frac{V_o}{V_s} = \underline{\mathbf{\frac{1}{s^3 + 2s^2 + 3s + 2}}}$$

(b) $I_s = V_s - V_1 = (s^3 + 2s^2 + 3s + 2) V_o - (s^2 + s + 1) V_o$

$I_s = (s^3 + s^2 + 2s + 1) V_o$

$$H_2(s) = \frac{V_o}{I_s} = \underline{\mathbf{\frac{1}{s^3 + s^2 + 2s + 1}}}$$

(c) $I_o = \dfrac{V_o}{1}$

$$H_3(s) = \frac{I_o}{I_s} = \frac{V_o}{I_s} = H_2(s) = \underline{\mathbf{\frac{1}{s^3 + s^2 + 2s + 1}}}$$

(d) $$H_4(s) = \frac{I_o}{V_s} = \frac{V_o}{V_s} = H_1(s) = \underline{\boldsymbol{\frac{1}{s^3 + 2s^2 + 3s + 2}}}$$

Solution 16.81

For the op-amp circuit in Fig. 16.99, find the transfer function, $T(s) = I_o(s)/V_s(s)$. Assume all initial conditions are zero.

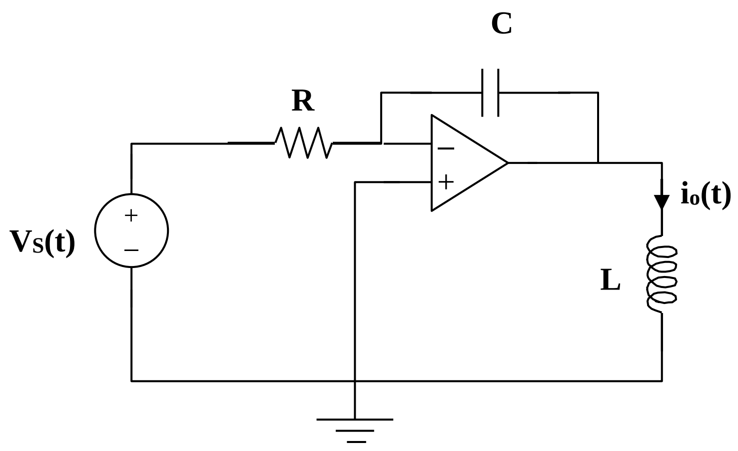
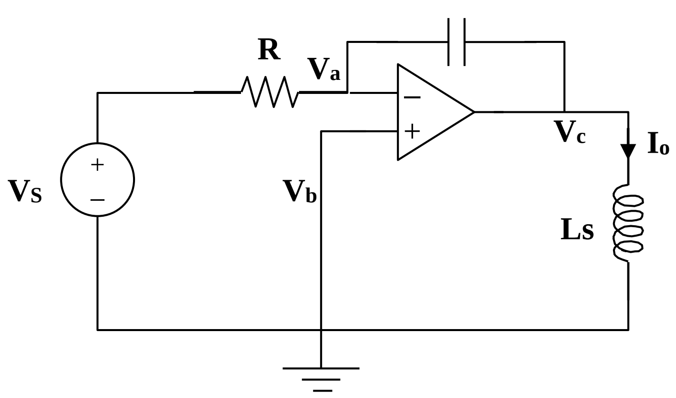


Figure 16.99
For Prob. 16.81.

Solution

Step 1. Convert the circuit into the s-domain. The write the node equations at the input to the op amp and solve for T(s).



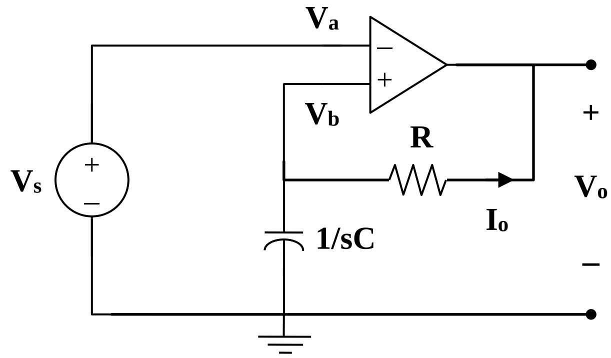
$$[(V_a–V_s)/R]+[(V_a–V_c)/(1/(Cs))]+0 = 0;\ V_a = V_b = 0 \text{ and } I_o = (V_c–0)/(Ls).$$

Step 2. $CsV_c = –V_s/R$ or $V_c = –V_s/(RCs)$ and $I_o = –V_s/(RLCs^2)$ or

$$T(s) = \mathbf{–1/(RLCs^2)}.$$

Solution 16.82

Consider the circuit below.



Since no current enters the op amp, I_o flows through both R and C.

$$V_o = -I_o\left(R + \frac{1}{sC}\right)$$

$$V_a = V_b = V_s = \frac{-I_o}{sC}$$

$$H(s) = \frac{V_o}{V_s} = \frac{R + 1/sC}{1/sC} = \mathbf{sRC + 1}$$

Solution 16.83

(a) $H(s) = \frac{V_o}{V_s} = \frac{R}{R + sL} = \frac{R/L}{s + R/L}$

$h(t) = \mathbf{\frac{R}{L} e^{-Rt/L}\ u(t)}$

(b) $v_s(t) = u(t) \longrightarrow V_s(s) = 1/s$

$V_o = \frac{R/L}{s + R/L} V_s = \frac{R/L}{s(s + R/L)} = \frac{A}{s} + \frac{B}{s + R/L}$

$A = 1, \qquad B = -1$

$V_o = \frac{1}{s} - \frac{1}{s + R/L}$

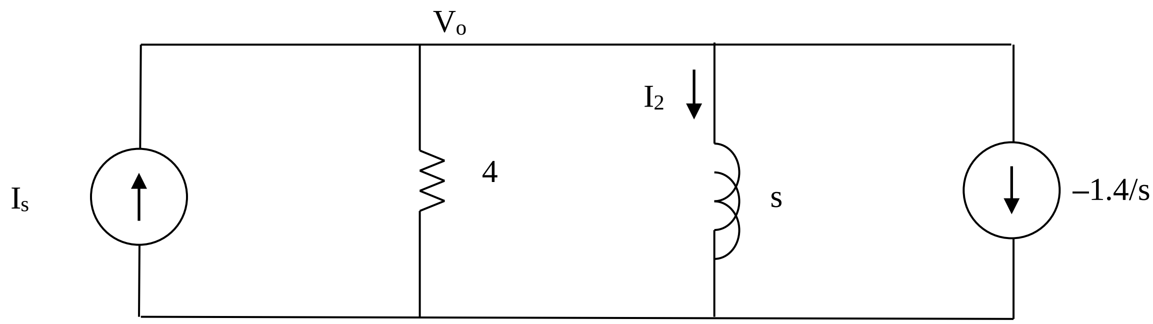
$v_o(t) = u(t) - e^{-Rt/L}\ u(t) = \mathbf{(1 - e^{-Rt/L})u(t)}$

Solution 16.84

A parallel RL circuit has R = 4 Ω and L = 1H. The input to the circuit is $i_s(t) = 1.4e^{-t}u(t)$ A. Find the inductor current $i_L(t)$ for all t>0 and assume that $i_L(0) = -1.4$ A.

Solution

Consider the circuit as shown below.



We start by writing the nodal equation, $-I_s + [(V_o-0)/4] + [(V_o-0)/s] - [1.4/s] = 0$. We also note that $I_L = [(V_o)/s] - 1.4/s$ and $I_s = 1.4/(s+1)$.

This leads to $[0.25+(1/s)]V_o = [(s+4)/(4s)]V_o = [1.4/(s+1)] + 1.4/s$ $= 1.4[(s+s+1)/(s(s+1))] = 2.8[(s+0.5)/(s(s+1))]$ or $V_o = 11.2(s+0.5)/[(s+1)(s+4)]$.

Thus, $I_L = [V_o/s] - 1.4/s = [11.2(s+0.5)/[s(s+1)(s+4)]] - 1.4/s$ $= [A/s] + [B/(s+1)] + [C/(s+4)] - 1.4/s$ where $A = 11.2(0.5)/4 = 1.4$, $B = 11.2(-0.5)/(-3) = 1.8667$, and $C = 11.2(-3.5)/(12) = -3.267$. Therefore,

$$i_L(t) = \mathbf{[1.8667e^{-t} - 3.267e^{-4t}]u(t)\ A}.$$

Solution 16.85

A circuit has a transfer function

$$H(s) = \frac{3(s+4)}{(s+1)(s+2)^2}$$

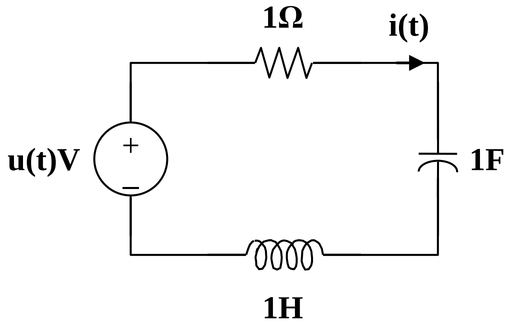
Find the impulse response.

Solution

$H(s) = \frac{3(s+4)}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$ where A = 9 and C = –6. So all we need to do is to find B. We note that $3(s+4) = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$ so,
$0 = As^2 + Bs^2$ or B = –A = –9. Therefore,

h(t) = $\mathbf{[9e^{-t} - 9e^{-2t} - 6te^{-2t}]u(t)}$.

Solution 16.86



First select the inductor current i_L and the capacitor voltage v_C to be the state variables.

Applying KVL we get:

$$-u(t)+i+v_C+i'=0;\ \ i=v_C'$$

Thus,

$$\dot{v}_C = i$$
$$\dot{i} = -v_C - i + u(t)$$

Finally we get,

$$\begin{bmatrix}\dot{\mathbf{v}}_C\\ \dot{\mathbf{i}}\end{bmatrix}=\begin{bmatrix}\mathbf{0} & \mathbf{1}\\ \mathbf{-1} & \mathbf{-1}\end{bmatrix}\begin{bmatrix}\mathbf{v}_C\\ \mathbf{i}\end{bmatrix}+\begin{bmatrix}\mathbf{0}\\ \mathbf{1}\end{bmatrix}\mathbf{u(t)};\ \mathbf{i(t)}=\begin{bmatrix}\mathbf{0} & \mathbf{1}\end{bmatrix}\begin{bmatrix}\mathbf{v}_C\\ \mathbf{i}\end{bmatrix}+\begin{bmatrix}\mathbf{0}\end{bmatrix}\mathbf{u(t)}$$

Solution 16.87

Develop the state equations for the problem you designed in Prob. 16.13.

Although there is no correct way to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Develop the state equations for Problem 16.13.

Problem 16.13

Find v_x in the circuit shown in Fig. 16.36 given $v_s = 4u(t)$ V.

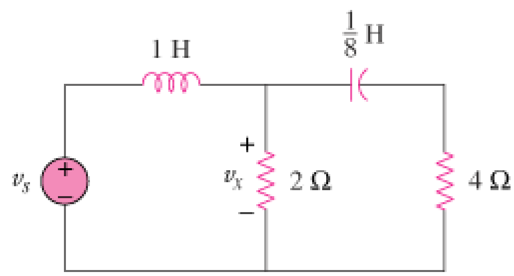
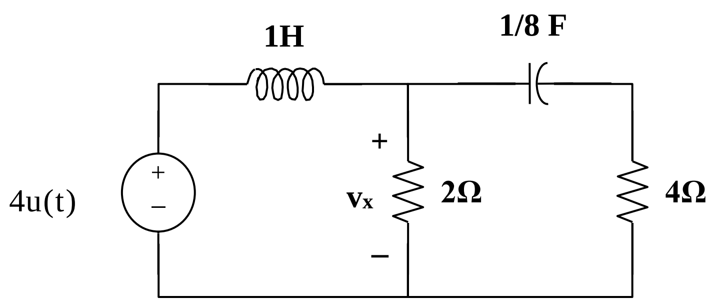


Figure 16.36

Solution



First select the inductor current i_L and the capacitor voltage v_C to be the state variables.

Applying KCL we get:

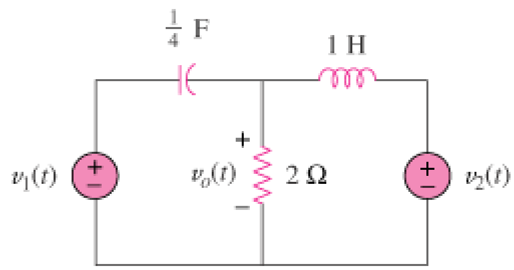
$$-i_L + \frac{v_x}{2} + \frac{\dot{v}_C}{8} = 0; \quad or \;\; \dot{v}_C = 8i_L - 4v_x$$
$$\dot{i}_L = 4u(t) - v_x$$
$$v_x = v_C + 4\frac{\dot{v}_C}{8} = v_C + \frac{\dot{v}_C}{2} = v_C + 4i_L - 2v_x; \; or \;\; v_x = 0.3333v_C + 1.3333i_L$$

$$\dot{v}_C = 8i_L - 1.3333v_C - 5.333i_L = -1.3333v_C + 2.666i_L$$
$$\dot{i}_L = 4u(t) - 0.3333v_C - 1.3333i_L$$

Now we can write the state equations.

$$\begin{bmatrix} \dot{\mathbf{v}}_{\mathbf{C}} \\ \dot{\mathbf{i}}_{\mathbf{L}} \end{bmatrix} = \underline{\begin{bmatrix} -\mathbf{1.3333} & \mathbf{2.666} \\ -\mathbf{0.3333} & -\mathbf{1.3333} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{\mathbf{C}} \\ \mathbf{i}_{\mathbf{L}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{4} \end{bmatrix} \mathbf{u(t)}}; \;\; \mathbf{v}_{\mathbf{x}} = \underline{\begin{bmatrix} \mathbf{0.3333} & \mathbf{1.3333} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{\mathbf{C}} \\ \mathbf{i}_{\mathbf{L}} \end{bmatrix}}$$

Solution 16.88



First select the inductor current i_L (current flowing left to right) and the capacitor voltage v_C (voltage positive on the left and negative on the right) to be the state variables.

Applying KCL we get:

$$-\frac{\dot{v}_C}{4}+\frac{v_o}{2}+i_L=0 \ or \ \dot{v}_C=4i_L+2v_o$$

$$\dot{i}_L=v_o-v_2$$

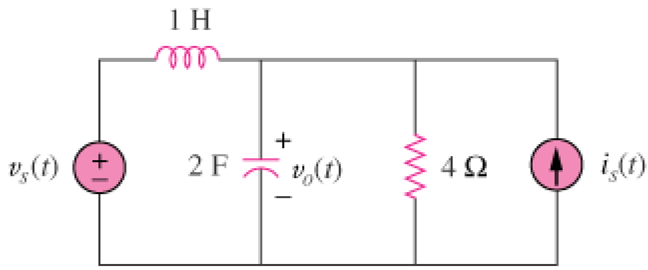
$$v_o=-v_C+v_1$$

$$\dot{v}_C=4i_L-2v_C+2v_1$$

$$\dot{i}_L=-v_C+v_1-v_2$$

$$\begin{bmatrix}\dot{\mathbf{i}}_L\\ \dot{\mathbf{v}}_C\end{bmatrix}=\underline{\begin{bmatrix}0 & -1\\ 4 & -2\end{bmatrix}\begin{bmatrix}\mathbf{i}_L\\ \mathbf{v}_C\end{bmatrix}+\begin{bmatrix}1 & -1\\ 2 & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_1(\mathbf{t})\\ \mathbf{v}_2(\mathbf{t})\end{bmatrix}};\ \mathbf{v}_o(\mathbf{t})=\underline{\begin{bmatrix}0 & -1\end{bmatrix}\begin{bmatrix}\mathbf{i}_L\\ \mathbf{v}_C\end{bmatrix}+\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_1(\mathbf{t})\\ \mathbf{v}_2(\mathbf{t})\end{bmatrix}}$$

Solution 16.89



First select the inductor current i_L (left to right) and the capacitor voltage v_C to be the state variables.

Letting $v_o = v_C$ and applying KCL we get:

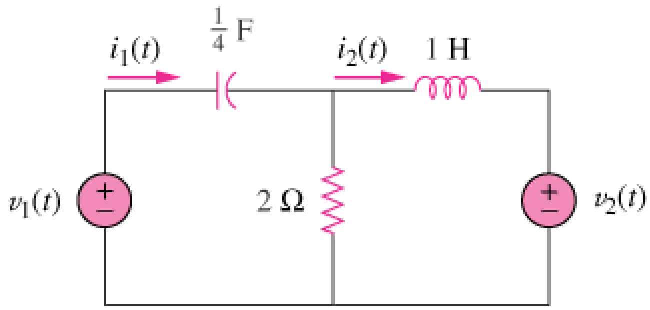
$$-i_L + \dot{v}_C + \frac{v_C}{4} - i_s = 0 \ \ or \ \ \dot{v}_C = -0.25v_C + i_L + i_s$$

$$\dot{i}_L = -v_C + v_s$$

Thus,

$$\begin{bmatrix} \dot{\mathbf{v}}_C \\ \dot{\mathbf{i}}_L \end{bmatrix} = \underline{\begin{bmatrix} -0.25 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{v}_C \\ \mathbf{i}_L \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{v}_s \\ \mathbf{i}_s \end{bmatrix}}; \ \mathbf{v}_o(\mathbf{t}) = \underline{\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{v}_C \\ \mathbf{i}_L \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{v}_s \\ \mathbf{i}_s \end{bmatrix}}$$

Solution 16.90



First select the inductor current i_L (left to right) and the capacitor voltage v_C (+ on the left) to be the state variables.

Letting $i_1 = \dfrac{\dot{v}_C}{4}$ and $i_2 = i_L$ and applying KVL we get:

Loop 1:

$$-v_1 + v_C + 2\left(\frac{\dot{v}_C}{4} - i_L\right) = 0 \ or\ \dot{v}_C = 4i_L - 2v_C + 2v_1$$

Loop 2:

$$2\left(i_L - \frac{\dot{v}_C}{4}\right) + \dot{i}_L + v_2 = 0 \ or$$

$$\dot{i}_L = -2i_L + \frac{4i_L - 2v_C + 2v_1}{2} - v_2 = -v_C + v_1 - v_2$$

$$i_1 = \frac{4i_L - 2v_C + 2v_1}{4} = i_L - 0.5v_C + 0.5v_1$$

$$\begin{bmatrix} \dot{\mathbf{i}}_L \\ \dot{\mathbf{v}}_C \end{bmatrix} = \underline{\begin{bmatrix} 0 & -1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} \mathbf{i}_L \\ \mathbf{v}_C \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{v}_1(\mathbf{t}) \\ \mathbf{v}_2(\mathbf{t}) \end{bmatrix}};\ \begin{bmatrix} \mathbf{i}_1(\mathbf{t}) \\ \mathbf{i}_2(\mathbf{t}) \end{bmatrix} = \underline{\begin{bmatrix} 1 & -0.5 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{i}_L \\ \mathbf{v}_C \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{v}_1(\mathbf{t}) \\ \mathbf{v}_2(\mathbf{t}) \end{bmatrix}}$$

Solution 16.91

Let x_1 = y(t). Thus, $\dot{x}_1 = \dot{y} = x_2 \; and \; \dot{x}_2 = \ddot{y} = -3x_1 - 4x_2 + z(t)$

This gives our state equations.

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-3} & \mathbf{-4} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix} \mathbf{z(t)};} \quad \mathbf{y(t)} = \underline{\begin{bmatrix} \mathbf{1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \end{bmatrix} \mathbf{z(t)}}$$

Solution 16.92

$Let\ x_1 = y(t)\ and\ \ x_2 = \dot{x}_1 - z = \dot{y} - z\ or\ \ \dot{y} = x_2 + z$

Thus,

$$\dot{x}_2 = \ddot{y} - \dot{z} = -9x_1 - 7(x_2 + z) + \dot{z} + 2z - \dot{z} = -9x_1 - 7x_2 - 5z$$

This now leads to our state equations,

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -\mathbf{9} & -\mathbf{7} \end{bmatrix}\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{1} \\ -\mathbf{5} \end{bmatrix}\mathbf{z(t)};}\ \ \mathbf{y(t)} = \underline{\begin{bmatrix} \mathbf{1} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \end{bmatrix}\mathbf{z(t)}}$$

Solution 16.93

Let $x_1 = y(t)$, $x_2 = \dot{x}_1$, *and* $x_3 = \dot{x}_2$.

Thus,

$$\ddot{x}_3 = -6x_1 - 11x_2 - 6x_3 + z(t)$$

We can now write our state equations.

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \\ \dot{\mathbf{x}}_3 \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{-6} & \mathbf{-11} & \mathbf{-6} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \end{bmatrix} \mathbf{z(t)}}; \ \mathbf{y(t)} = \underline{\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \end{bmatrix} \mathbf{z(t)}}$$

Solution 16.94

We transform the state equations into the s-domain and solve using Laplace transforms.

$$sX(s) - x(0) = AX(s) + B\left(\frac{1}{s}\right)$$

Assume the initial conditions are zero.

$$(sI - A)X(s) = B\left(\frac{1}{s}\right)$$

$$X(s) = \begin{bmatrix} s+4 & -4 \\ 2 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 2 \end{bmatrix} \left(\frac{1}{s}\right) = \frac{1}{s^2 + 4s + 8} \begin{bmatrix} s & 4 \\ 2 & s+4 \end{bmatrix} \begin{bmatrix} 0 \\ (2/s) \end{bmatrix}$$

$$Y(s) = X_1(s) = \frac{8}{s(s^2 + 4s + 8)} = \frac{1}{s} + \frac{-s-4}{s^2 + 4s + 8}$$

$$= \frac{1}{s} + \frac{-s-4}{(s+2)^2 + 2^2} = \frac{1}{s} + \frac{-(s+2)}{(s+2)^2 + 2^2} + \frac{-2}{(s+2)^2 + 2^2}$$

$$\mathbf{y(t) = \left(1 - e^{-2t}\left(\cos 2t + \sin 2t\right)\right)u(t)}$$

Solution 16.95

Assume that the initial conditions are zero. Using Laplace transforms we get,

$$X(s) = \begin{bmatrix} s+2 & 1 \\ -2 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1/s \\ 2/s \end{bmatrix} = \frac{1}{s^2+6s+10} \begin{bmatrix} s+4 & -1 \\ 2 & s+2 \end{bmatrix} \begin{bmatrix} 3/s \\ 4/s \end{bmatrix}$$

$$X_1 = \frac{3s+8}{s((s+3)^2+1^2)} = \frac{0.8}{s} + \frac{-0.8s-1.8}{(s+3)^2+1^2}$$

$$= \frac{0.8}{s} - 0.8\frac{s+3}{(s+3)^2+1^2} + .6\frac{1}{(s+3)^2+1^2}$$

$$x_1(t) = (0.8 - 0.8e^{-3t}\cos t + 0.6e^{-3t}\sin t)u(t)$$

$$X_2 = \frac{4s+14}{s((s+3)^2+1^2} = \frac{1.4}{s} + \frac{-1.4s-4.4}{(s+3)^2+1^2}$$

$$= \frac{1.4}{s} - 1.4\frac{s+3}{(s+3)^2+1^2} - 0.2\frac{1}{(s+3)^2+1^2}$$

$$x_2(t) = (1.4 - 1.4e^{-3t}\cos t - 0.2e^{-3t}\sin t)u(t)$$

$$\mathbf{y_1(t) = -2x_1(t) - 2x_2(t) + 2u(t)}$$
$$\mathbf{= \underline{(-2.4 + 4.4e^{-3t}\cos t - 0.8e^{-3t}\sin t)u(t)}}$$

$$\mathbf{y_2(t) = x_1(t) - 2u(t) = \underline{(-1.2 - 0.8e^{-3t}\cos t + 0.6e^{-3t}\sin t)u(t)}}$$

[–2.4 + 4.4e^{-3t}cos(t) – 0.8e^{-3t}sin(t)]u(t), [–1.2 – 0.8e^{-3t}cos(t) + 0.6e^{-3t}sin(t)]u(t)

Solution 16.96

If V_o is the voltage across R, applying KCL at the non-reference node gives

$$I_s = \frac{V_o}{R} + sC\,V_o + \frac{V_o}{sL} = \left(\frac{1}{R} + sC + \frac{1}{sL}\right)V_o$$

$$V_o = \frac{I_s}{\frac{1}{R} + sC + \frac{1}{sL}} = \frac{sRL\,I_s}{sL + R + s^2RLC}$$

$$I_o = \frac{V_o}{R} = \frac{sL\,I_s}{s^2RLC + sL + R}$$

$$H(s) = \frac{I_o}{I_s} = \frac{sL}{s^2RLC + sL + R} = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

The roots

$$s_{1,\,2} = \frac{-1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}$$

both lie in the left half plane since R, L, and C are positive quantities.

Thus, **the circuit is stable**.

Solution 16.97

A system is formed by cascading two systems as shown in Fig. 16.106. Given that the impulse responses of the systems are,

$h_1(t) = 21e^{-t}\, u(t)$, $\quad h_2(t) = e^{-4t}\, u(t)$

(a) Obtain the impulse response of the overall system.
(b) Check if the overall system is stable.

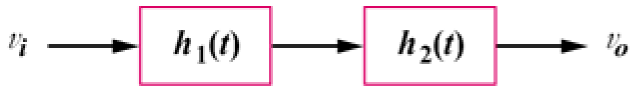


Figure 16.106
For Prob. 16.97.

Solution

(a) $H_1(s) = \dfrac{21}{s+1}$, $\qquad \mathrm{H}_2(\mathrm{s}) = \dfrac{1}{\mathrm{s}+4}$

$H(s) = H_1(s)H_2(s) = \dfrac{21}{(s+1)(s+4)}$ = [A/(s+1)] + [B/(s+4)] where
A = 21/3 = 7 and B = 21/(–3) therefore,

h(t) = **7[e^{-t} – e^{-4t}]u(t)**.

(b) Since the poles of H(s) all lie in the left half s-plane, **the system is stable**.

Solution 16.98

Let V_{o1} be the voltage at the output of the first op amp.

$$\frac{V_{o1}}{V_s} = \frac{-1/sC}{R} = \frac{-1}{sRC}, \qquad \frac{V_o}{V_{o1}} = \frac{-1}{sRC}$$

$$H(s) = \frac{V_o}{V_s} = \frac{1}{s^2R^2C^2}$$

$$h(t) = \frac{t}{R^2C^2}$$

$\lim\limits_{t\to\infty} h(t) = \infty$, i.e. the output is unbounded.

Hence, **the circuit is unstable**.

Solution 16.99

$$sL \parallel \frac{1}{sC} = \frac{sL \cdot \frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{sL}{1+s^2LC}$$

$$\frac{V_2}{V_1} = \frac{\frac{sL}{1+s^2LC}}{R + \frac{sL}{1+s^2LC}} = \frac{sL}{s^2RLC + sL + R}$$

$$\frac{V_2}{V_1} = \frac{s \cdot \frac{1}{RC}}{s^2 + s \cdot \frac{1}{RC} + \frac{1}{LC}}$$

Comparing this with the given transfer function,

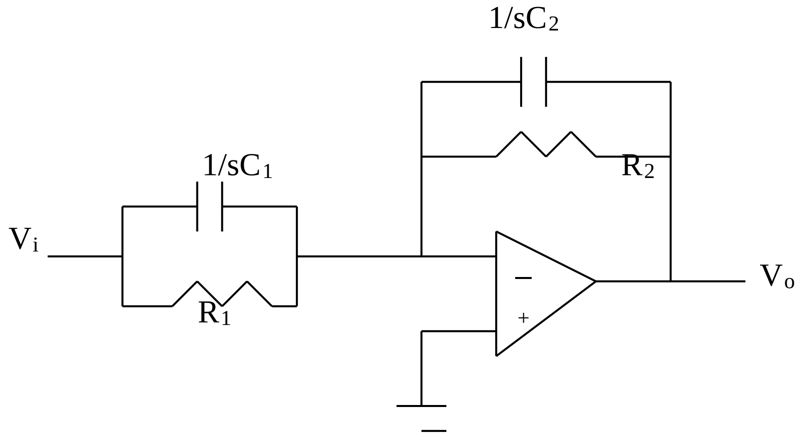
$$2 = \frac{1}{RC}, \qquad 6 = \frac{1}{LC}$$

If $R = 1\,k\Omega$, $\quad C = \frac{1}{2R} =$ **500 μF**

$\quad L = \frac{1}{6C} =$ **333.3 H**

Solution 16.100

The circuit is transformed in the s-domain as shown below.



Let $Z_1 = R_1 // \frac{1}{sC_1} = \frac{R_1 \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{1 + sR_1C_1}$

$$Z_2 = R_2 // \frac{1}{sC_2} = \frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{1 + sR_2C_2}$$

This is an inverting amplifier.

$$H(s) = V_o/V_i = -\frac{Z_2}{Z_1} = \frac{-\frac{R_2}{1 + sR_2C_2}}{\frac{R_1}{1 + sR_1C_1}} = -\frac{R_2}{R_1}\frac{R_1C_1}{R_2C_2}\left[\frac{s + \frac{1}{R_1C_1}}{s + \frac{1}{R_2C_2}}\right] = \frac{-C_1}{C_2}\left[\frac{s + \frac{1}{R_1C_1}}{s + \frac{1}{R_2C_2}}\right]$$

Comparing this with

$$H(s) = -\frac{(s + 1000)}{2(s + 4000)}$$

we obtain:

$$\frac{C_1}{C_2} = 1/2 \quad \longrightarrow \quad C_2 = 2C_1 = \underline{20\mu F}$$

$$\frac{1}{R_1C_1} = 1000 \quad \longrightarrow \quad R_1 = \frac{1}{1000C_1} = \frac{1}{10^3 x 10 x 10^{-6}} = \underline{100\Omega}$$

$$\frac{1}{R_2C_2} = 4000 \quad \longrightarrow \quad R_2 = \frac{1}{4000C_2} = \frac{1}{4x10^3 x 20 x 10^{-6}} = \underline{\mathbf{12.5\Omega}}$$

Solution 16.101

We apply KCL at the noninverting terminal at the op amp.

$$(V_s - 0)\,Y_3 = (0 - V_o)(Y_1 - Y_2)$$

$$Y_3\,V_s = -(Y_1 + Y_2)V_o$$

$$\frac{V_o}{V_s} = \frac{-Y_3}{Y_1 + Y_2}$$

Let $\quad Y_1 = sC_1, \qquad Y_2 = 1/R_1, \qquad Y_3 = sC_2$

$$\frac{V_o}{V_s} = \frac{-sC_2}{sC_1 + 1/R_1} = \frac{-sC_2/C_1}{s + 1/R_1C_1}$$

Comparing this with the given transfer function,

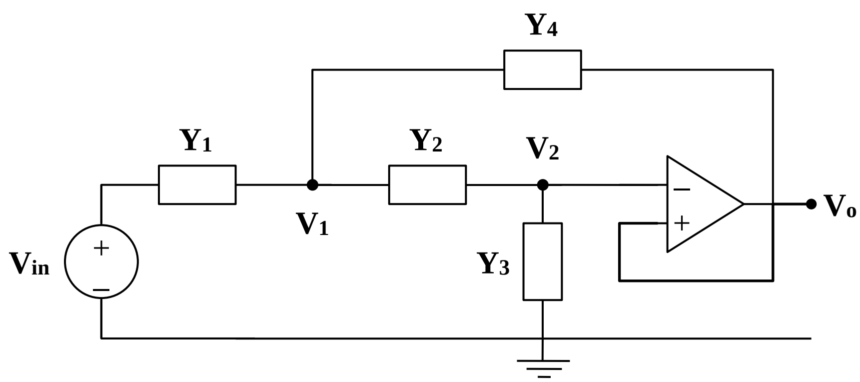
$$\frac{C_2}{C_1} = 1, \qquad \frac{1}{R_1C_1} = 10$$

If $R_1 = 1\,\text{k}\Omega$,

$$C_1 = C_2 = \frac{1}{10^4} = \mathbf{100\ \mu F}$$

Solution 16.102

Consider the circuit shown below. We notice that $V_3 = V_o$ and $V_2 = V_3 = V_o$.



At node 1,

$$(V_{in} - V_1)Y_1 = (V_1 - V_o)Y_2 + (V_1 - V_o)Y_4$$

$$V_{in}\,Y_1 = V_1(Y_1 + Y_2 + Y_4) - V_o(Y_2 + Y_4) \qquad (1)$$

At node 2,

$$(V_1 - V_o)Y_2 = (V_o - 0)Y_3$$

$$V_1 Y_2 = (Y_2 + Y_3)V_o$$

$$V_1 = \frac{Y_2 + Y_3}{Y_2} V_o \qquad (2)$$

Substituting (2) into (1),

$$V_{in}\,Y_1 = \frac{Y_2 + Y_3}{Y_2} \cdot (Y_1 + Y_2 + Y_4)V_o - V_o(Y_2 + Y_4)$$

$$V_{in} Y_1 Y_2 = V_o(Y_1 Y_2 + Y_2^2 + Y_2 Y_4 + Y_1 Y_3 + Y_2 Y_3 + Y_3 Y_4 - Y_2^2 - Y_2 Y_4)$$

$$\frac{V_o}{V_{in}} = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_1 Y_3 + Y_2 Y_3 + Y_3 Y_4}$$

Y_1 and Y_2 must be resistive, while Y_3 and Y_4 must be capacitive.

Let $\quad Y_1 = \dfrac{1}{R_1}, \qquad Y_2 = \dfrac{1}{R_2}, \qquad Y_3 = sC_1, \qquad Y_4 = sC_2$

$$\frac{V_o}{V_{in}} = \frac{\dfrac{1}{R_1 R_2}}{\dfrac{1}{R_1 R_2} + \dfrac{sC_1}{R_1} + \dfrac{sC_1}{R_2} + s^2 C_1 C_2}$$

$$\frac{V_o}{V_{in}} = \frac{\dfrac{1}{R_1 R_2 C_1 C_2}}{s^2 + s \cdot \left(\dfrac{R_1 + R_2}{R_1 R_2 C_2} \right) + \dfrac{1}{R_1 R_2 C_1 C_2}}$$

Choose $R_1 = 1\,\mathrm{k\Omega}$, then

$$\frac{1}{R_1 R_2 C_1 C_2} = 10^6 \qquad \text{and} \qquad \frac{R_1 + R_2}{R_1 R_2 C_2} = 100$$

We have three equations and four unknowns. Thus, there is a family of solutions. One such solution is

$$R_2 = \mathbf{1\ k\Omega}, \quad C_1 = \mathbf{50\,nF}, \quad C_2 = \mathbf{20\ \mu F}$$

Solution 16.103

Using the result of Practice Problem 16.14,

$$\frac{V_o}{V_i} = \frac{-Y_1Y_2}{Y_2Y_3 + Y_4(Y_1 + Y_2 + Y_3)}$$

When $Y_1 = sC_1, \qquad C_1 = 0.5\ \mu F$

$$Y_2 = \frac{1}{R_1}, \qquad R_1 = 10\ k\Omega$$

$$Y_3 = Y_2, \qquad Y_4 = sC_2, \qquad C_2 = 1\ \mu F$$

$$\frac{V_o}{V_i} = \frac{-sC_1/R_1}{1/R_1^2 + sC_2(sC_1 + 2/R_1)} = \frac{-sC_1R_1}{1 + sC_2R_1(2 + sC_1R_1)}$$

$$\frac{V_o}{V_i} = \frac{-sC_1R_1}{s^2C_1C_2R_1^2 + s\cdot 2C_2R_1 + 1}$$

$$\frac{V_o}{V_i} = \frac{-s(0.5\times10^{-6})(10\times10^3)}{s^2(0.5\times10^{-6})(1\times10^{-6})(10\times10^3)^2 + s(2)(1\times10^{-6})(10\times10^3) + 1}$$

$$\frac{V_o}{V_i} = \frac{-100s}{s^2 + 400s + 2\times10^4}$$

Therefore,

$$a = \mathbf{-100}, \qquad b = \mathbf{400}, \qquad c = \mathbf{2\times10^4}$$

Solution 16.104

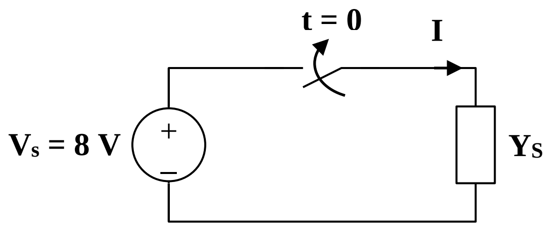
(a) Let $Y(s) = \dfrac{K(s+1)}{s+3}$

$$Y(\infty) = \lim_{s\to\infty} \frac{K(s+1)}{s+3} = \lim_{s\to\infty} \frac{K(1+1/s)}{1+3/s} = K$$

i.e. $0.25 = K$.

Hence, $Y(s) = \dfrac{\mathbf{s+1}}{\mathbf{4(s+3)}}$

(b) Consider the circuit shown below.



$$V_s = 8u(t) \longrightarrow V_s = 8/s$$

$$I = \frac{V_s}{Z} = Y(s)V_s(s) = \frac{8}{4s}\cdot\frac{s+1}{s+3} = \frac{2(s+1)}{s(s+3)}$$

$$I = \frac{A}{s} + \frac{B}{s+3}$$

$A = 2/3$, $\quad B = 2(–3+1)/(–3) = 4/3$

$$i(t) = \frac{\mathbf{1}}{\mathbf{3}}\left[\mathbf{2 + 4e^{-3t}}\right]\mathbf{u(t)\ A}$$

Solution 16.105

The gyrator is equivalent to two cascaded inverting amplifiers. Let V_1 be the voltage at the output of the first op amp.

$$V_1 = \frac{-R}{R} V_i = -V_i$$

$$V_o = \frac{-1/sC}{R} V_1 = \frac{1}{sCR} V_i$$

$$I_o = \frac{V_o}{R} = \frac{V_o}{sR^2C}$$

$$\frac{V_o}{I_o} = sR^2C$$

$\boldsymbol{\frac{V_o}{I_o} = sL}$**, when** $\boldsymbol{L = R^2C}$, so **if you let** $\boldsymbol{L = R^2C}$ **then** $\boldsymbol{V_o/I_o = sL}$.

Solution 17.1

(a) This is **periodic** with $\omega = \pi$ which leads to $T = 2\pi/\omega =$ **2**.

(b) y(t) is **not periodic** although sin t and 4 cos 2πt are independently periodic.

(c) Since $\sin A \cos B = 0.5[\sin(A + B) + \sin(A - B)]$,

$g(t) = \sin 3t \cos 4t = 0.5[\sin 7t + \sin(-t)] = -0.5 \sin t + 0.5 \sin 7t$

which is harmonic or **periodic** with the fundamental frequency

$$\omega = 1 \text{ or } T = 2\pi/\omega = \mathbf{2\pi}.$$

(d) $h(t) = \cos^2 t = 0.5(1 + \cos 2t)$. Since the sum of a periodic function and a constant is also **periodic**, h(t) is periodic. $\omega = 2$ or $T = 2\pi/\omega = \boldsymbol{\pi}$.

(e) The frequency ratio $0.6|0.4 = 1.5$ makes z(t) **periodic**.

$$\omega = 0.2\pi \text{ or } T = 2\pi/\omega = \mathbf{10}.$$

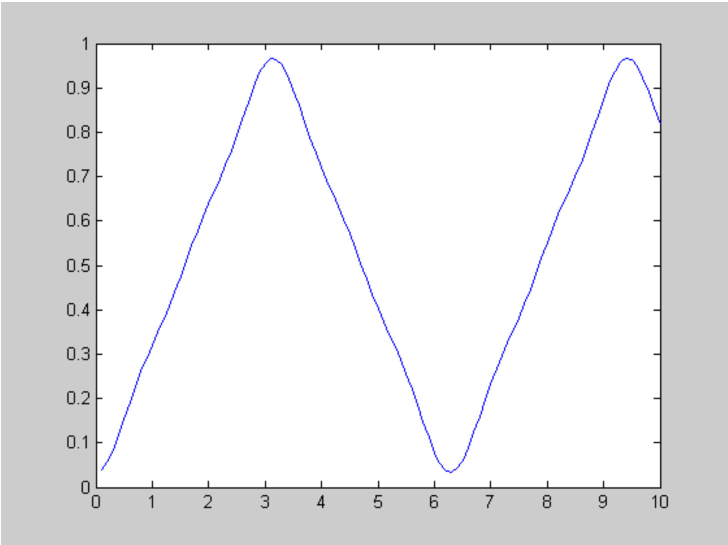
(f) $p(t) = 10$ is **not periodic**.

(g) g(t) is **not periodic**.

Solution 17.2

The function f(t) has a DC offset and is even. We use the following MATLAB code to plot f(t). The plot is shown below. If more terms are taken, the curve is clearly indicating a triangular wave shape which is easily represented with just the DC component and three, cosinusoidal terms of the expansion.

```
for n=1:100
   tn(n)=n/10;
   t=n/10;
   y1=cos(t);
   y2=(1/9)*cos(3*t);
   y3=(1/25)*cos(5*t);
   factor=4/(pi*pi);
   y(n)=0.5- factor*(y1+y2+y3);
end
plot(tn,y)
```



Solution 17.3

Give the Fourier coefficients a_0, a_n, and b_n of the waveform in Fig. 17.47. Plot the amplitude and phase spectra.

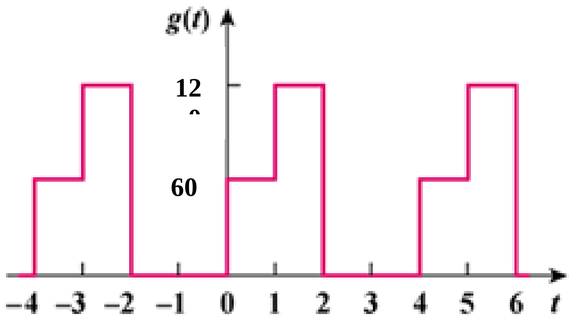


Figure 17.47
For Prob. 17.3.

Solution

$T = 4,\ \omega_o = 2\pi/T = \pi/2$

$$g(t) = \begin{cases} 60, & 0 < t < 1 \\ 120, & 1 < t < 2 \\ 0, & 2 < t < 4 \end{cases}$$

$$a_o = (1/T)\int_0^T g(t)dt = 0.25[\int_0^1 60dt + \int_1^2 120dt] = \mathbf{45}$$

$$a_n = (2/T)\int_0^T g(t)\cos(n\omega_o t)dt = (2/4)[\int_0^1 60\cos(\frac{n\pi}{2}t)dt + \int_1^2 120\cos(\frac{n\pi}{2}t)dt]$$

$$= 0.5[60\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_0^1 + 120\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_1^2] = (-1/(n\pi))60\sin(n\pi/2)$$

$$a_n = \begin{cases} \mathbf{(60/(n\pi))(-1)^{(n+1)/2},} & \mathbf{n = odd} \\ \mathbf{0,} & \mathbf{n = even} \end{cases}$$

$$b_n = (2/T)\int_0^T g(t)\sin(n\omega_o t)dt = (2/4)[\int_0^1 60\sin(\frac{n\pi}{2}t)dt + \int_1^2 120\sin(\frac{n\pi}{2}t)dt]$$

$$= 0.5[\frac{-2x60}{n\pi}\cos\frac{n\pi}{2}t\Big|_0^1 - \frac{2x120}{n\pi}\cos\frac{n\pi}{2}t\Big|_1^2] = \mathbf{(60/(n\pi))[1 - 2\cos n\pi + \cos(n\pi/2)]}$$

n	a_n	b_n	A_n	phase
1	–19.08	57.29	97.32	–101.31
2	0	–19.099	0	0
3	6.36	19.099	32.4	–78.69
4	0	0	9.6	–90
5	–3.84	11.459	19.44	–101.31
6	0	–6.366	0	0
7	2.76	8.185	14.04	–78.69
8	0	0	4.80	–90

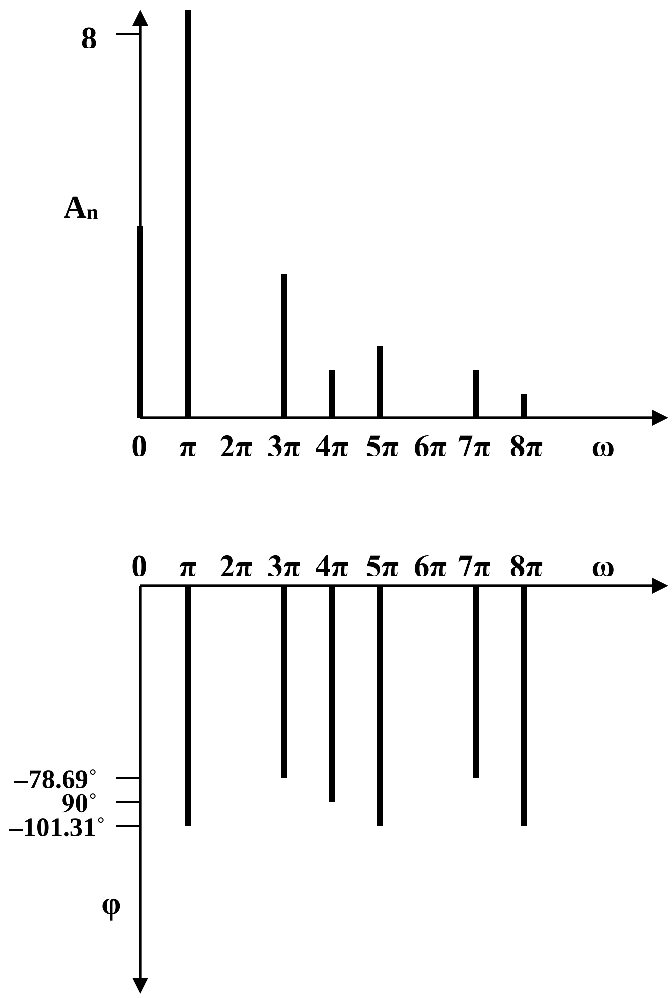


Figure D. 35
For Prob. 17.3.

Solution 17.4

$f(t) = 10 - 5t, \;\; 0 < t < 2, \;\; T = 2, \;\; \omega_o = 2\pi/T = \pi$

$$a_o = (1/T)\int_0^T f(t)dt = (1/2)\int_0^2 (10 - 5t)dt = 0.5[10t - (5t^2/2)]\Big|_0^2 = \mathbf{5}$$

$$a_n = (2/T)\int_0^T f(t)\cos(n\omega_o t)dt = (2/2)\int_0^2 (10 - 5t)\cos(n\pi t)dt$$

$$= \int_0^2 (10)\cos(n\pi t)dt - \int_0^2 (5t)\cos(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\cos n\pi t\Big|_0^2 + \frac{5t}{n\pi}\sin n\pi t\Big|_0^2 = [-5/(n^2\pi^2)](\cos 2n\pi - 1) = \mathbf{0}$$

$$b_n = (2/2)\int_0^2 (10 - 5t)\sin(n\pi t)dt$$

$$= \int_0^2 (10)\sin(n\pi t)dt - \int_0^2 (5t)\sin(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\sin n\pi t\Big|_0^2 + \frac{5t}{n\pi}\cos n\pi t\Big|_0^2 = 0 + [10/(n\pi)](\cos 2n\pi) = \mathbf{10/(n\pi)}$$

Hence

$$f(t) = \mathbf{5 + \frac{10}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi t)}.$$

Solution 17.5

Obtain the Fourier series expansion for the waveform shown in Fig. 17.49.

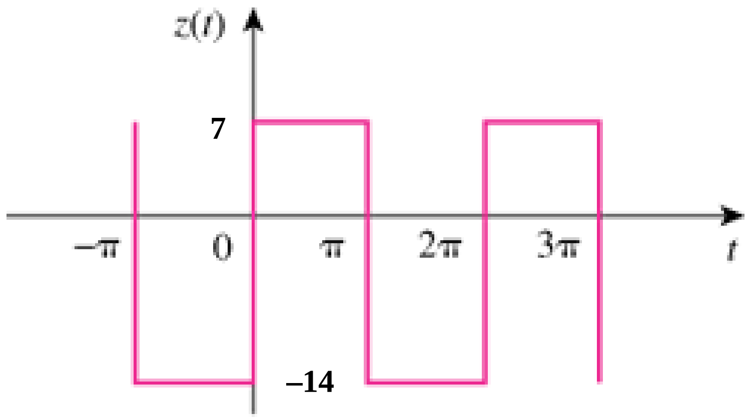


Figure 17.49
For Prob. 17.5.

Solution

$$T = 2\pi, \qquad \omega = 2\pi / T = 1$$

$$a_o = \frac{1}{T}\int_0^T z(t)dt = \frac{1}{2\pi}[7x\pi - 14x\pi] = -3.5$$

$$a_n = \frac{2}{T}\int_0^T z(t)\cos(n\omega_o)dt = \frac{1}{\pi}\int_0^{\pi} 7\cos(nt)dt - \frac{1}{\pi}\int_{\pi}^{2\pi} 14\cos(nt)dt = \frac{7}{n\pi}\sin(nt)\ \Big|_0^{\pi} - \frac{14}{n\pi}\sin(nt)\ \Big|_{\pi}^{2\pi} = 0$$

$$b_n = \frac{2}{T}\int_0^T z(t)\sin(n\omega_o)dt = \frac{1}{\pi}\int_0^{\pi} 7\sin(nt)dt - \frac{1}{\pi}\int_{\pi}^{2\pi} 14\sin(nt)dt = -\frac{7}{n\pi}\cos(nt)\ \Big|_0^{\pi} + \frac{14}{n\pi}\cos(nt)\ \Big|_{\pi}^{2\pi}$$

$$= \begin{cases} \dfrac{42}{n\pi}, n = odd \\ 0,\ \ n = even \end{cases}$$

Thus,

$$z(t) = -3.5 + \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{42}{n\pi}\sin(nt)$$

Solution 17.6

Find the trigonometric Fourier series for

$$f(t) = \begin{cases} 7.5, & 0 < t < \pi \\ 15, & \pi < t < 2\pi \end{cases} \quad \text{and } f(t+2\pi)=f(t) \text{ for all t.}$$

Solution

$T=2\pi$, $\omega_o=2\pi/T = 1$

$$a_o = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{2\pi}\left[\int_0^\pi 7.5dt + \int_\pi^{2\pi} 15dt\right] = \frac{1}{2\pi}(7.5\pi + 15\pi) = 11.25$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_o t)dt = \frac{2}{2\pi}\left[\int_0^\pi 7.5\cos(nt)\,dt + \int_\pi^{2\pi} 15\cos(nt)\,dt\right] = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_o t)\,dt = \frac{2}{2\pi}\left[\int_0^\pi 7.5\sin(nt)\,dt + \int_\pi^{2\pi} 15\sin(nt)\,dt\right]$$

$= \frac{1}{\pi}\left[-\frac{7.5}{n}\cos(nt)\Big|_0^\pi - \frac{15}{n}\cos(nt)\Big|_\pi^{2\pi}\right]$ = [7.5/(nπ)][–(cos(nπ)–1) – 2(cos(2nπ)–cos(nπ)] = [7.5/(nπ)][(cos(nπ)+1) – 2] = [7.5/(nπ)][(cos(nπ) – 1].

So for n = even, $b_n = 0$ and for n = odd, $b_n = -15/(n\pi)$.

Thus,

$$f(t) = 11.25 - \sum_{\mathbf{n=odd}}^{\infty} \frac{15}{\mathbf{n\pi}}\mathbf{sin(nt)}$$

Solution 17.7

$T = 3, \quad \omega_o = 2\pi / T = 2\pi / 3$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{3}\left[\int_0^2 2dt + \int_2^3 (-1)dt\right] = \frac{1}{3}(4-1) = 1$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\frac{2n\pi t}{3}dt = \frac{2}{3}\left[\int_0^2 2\cos\frac{2n\pi t}{3}dt + \int_2^3 (-1)\cos\frac{2n\pi t}{3}dt\right]$$

$$= \frac{2}{3}\left[2\frac{3}{2n\pi}\sin\frac{2n\pi t}{3}\bigg|_0^2 - 1\frac{3}{2n\pi}\sin\frac{2n\pi t}{3}\bigg|_2^3\right] = \frac{3}{n\pi}\sin\frac{4n\pi}{3}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\frac{2n\pi t}{3}dt = \frac{2}{3}\left[\int_0^2 2\sin\frac{2n\pi t}{3}dt + \int_2^3 (-1)sin\frac{2n\pi t}{3}dt\right]$$

$$= \frac{2}{3}\left[-2x\frac{3}{2n\pi}\cos\frac{2n\pi t}{3}\bigg|_0^2 + \frac{3}{2n\pi}\cos\frac{2n\pi t}{3}\bigg|_2^3\right] = \frac{3}{n\pi}(1 - 2\cos\frac{4n\pi}{3})$$

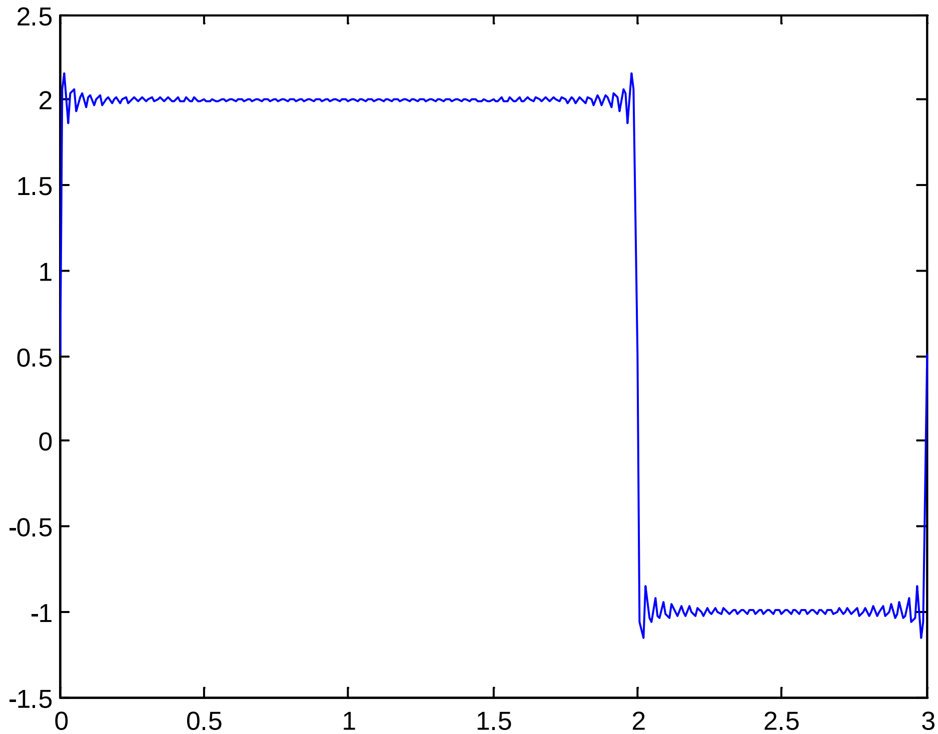
$$= \frac{1}{n\pi}\left(2 - 3\cos\frac{4n\pi}{3} + 1\right) = \frac{3}{n\pi}\left(1 - \cos\frac{4n\pi}{3}\right)$$

Hence,

$$f(t) = \underline{\mathbf{1 + \sum_{n=0}^{\infty}\left[\frac{3}{n\pi}\sin\frac{4n\pi}{3}\cos\frac{2n\pi t}{3} + \frac{3}{n\pi}\left(1 - \cos\frac{4n\pi}{3}\right)\sin\frac{2n\pi t}{3}\right]}}$$

We can now use MATLAB to check our answer,

```
>> t=0:.01:3;
>> f=1*ones(size(t));
>> for n=1:1:99,
f=f+(3/(n*pi))*sin(4*n*pi/3)*cos(2*n*pi*t/3)+(3/(n*pi))*(1-
cos(4*n*pi/3))*sin(2*n*pi*t/3);
end
>> plot(t,f)
```



Clearly we have nearly the same figure we started with!!

Solution 17.8

Using Fig. 17.51, design a problem to help other students to better understand how to determine the exponential Fourier Series from a periodic wave shape.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Obtain the exponential Fourier series of the function in Fig. 17.51.

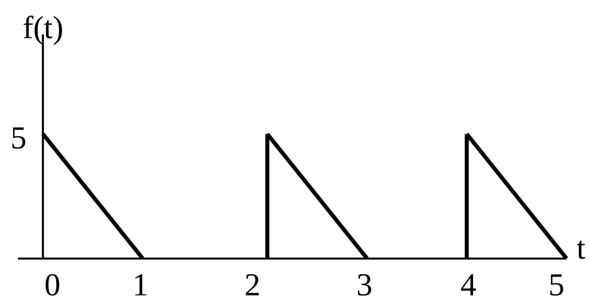


Figure 17.51 For Prob. 17.8.

Solution

$T = 2, \qquad \omega_o = 2\pi / T = \pi$

$$f(t) = \begin{cases} 5(1-t), & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

$$c_n = \frac{1}{T}\int_0^T f(t) e^{-jn\omega_o t} dt = \frac{1}{2}\int_0^1 5(1-t) e^{-jn\pi t} dt$$

$$= \frac{5}{2}\int_0^1 e^{-jn\pi t} dt - \frac{5}{2}\int_0^1 t e^{-jn\pi t} dt = \frac{5}{2}\frac{e^{-jn\pi t}}{-jn\pi}\Bigg|_0^1 - \frac{5}{2}\frac{e^{-jn\pi t}}{(-jn\pi)^2}(-jn\pi t - 1)\Bigg|_0^1$$

$$= \frac{5}{2}\frac{\left[e^{-jn\pi} - 1\right]}{-jn\pi} - \frac{5}{2}\frac{e^{-jn\pi}}{-n^2\pi^2}(-jn\pi - 1) + \frac{5}{2}\frac{(-1)}{-n^2\pi^2}$$

But $e^{-jn\pi} = \cos \pi n - j \sin n\pi = \cos n\pi + 0 = (-1)^n$

$$c_n = \frac{2.5[1-(-1)^n]}{jn\pi} - \frac{2.5(-1)^n[1 + jn\pi]}{n^2\pi^2} + \frac{2.5}{n^2\pi^2}$$

Solution 17.9

Determine the Fourier coefficients a_n and b_n of the first three harmonic terms of the rectified cosine wave in Fig. 17.52.

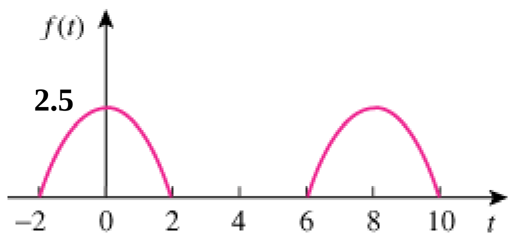


Figure 17.52
For Prob. 17.9.

Solution

f(t) is an even function, $b_n=0$.

$$T = 8, \qquad \omega = 2\pi / T = \pi / 4$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{2}{8}\left[\int_0^2 2.5\cos \pi t/4dt + 0\right] = \frac{2.5}{4}(\frac{4}{\pi})\sin \pi t/4 \Big|_0^2 = \frac{2.5}{\pi} = 0.7958$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_o dt = \frac{4}{8}\left[\int_0^2 2.5\cos(\pi t/4)\cos(n\pi t/4)dt + 0\right] = 0.625\int_0^2 [\cos(\pi t(n+1)/4) + \cos(\pi t(n-1)/4)]$$

For n = 1,

$$a_1 = 0.625\int_0^2 [\cos \pi t/2 + 1]dt = 0.625\left[\frac{2}{\pi}\sin \pi t/2dt + t\right]_0^2 = 1.25$$

For n>1,

$$a_n = \left[\frac{2.5}{\pi(n+1)}\sin\left(\frac{\pi(n+1)t}{4}\right) + \frac{2.5}{\pi(n-1)}\sin\left(\frac{\pi(n-1)t}{4}\right)\right]_0^2 = \frac{2.5}{\pi(n+1)}\sin\left(\frac{\pi(n+1)}{2}\right) + \frac{2.5}{\pi(n-1)}\sin\left(\frac{\pi(n-1)}{2}\right)$$

$$a_2 = \frac{2.5}{3\pi}\sin(1.5\pi) + \frac{2.5}{\pi}\sin(\pi/2) = 0.26528\sin(270°) + 0.795774\sin(90°)$$

$$= -0.265258 + 0.795774 = 0.530516, \qquad a_3 = \frac{5}{4\pi}\sin 2\pi + \frac{2.5}{\pi}\sin \pi = 0$$

Thus,

a_0 = **0.7958**, a_1 = **1.25**, a_2 = **0.5305**, a_3 = **0**, b_1 = **0** = b_2 = b_3.

Solution 17.10

Find the exponential Fourier series for the waveform in Fig. 17.53.

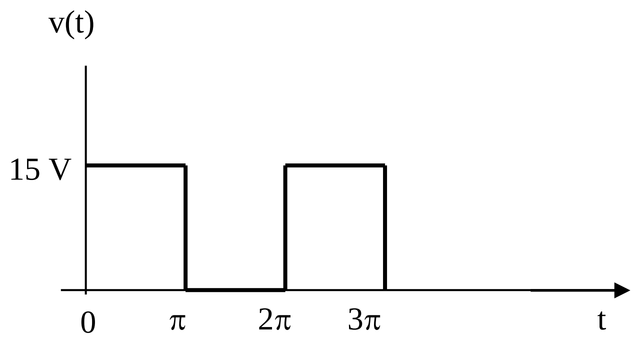


Figure 17.53
For Prob. 17.10.

Solution

$T = 2\pi, \quad \omega_o = 2\pi / T = 1$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{15}{2\pi}\int_0^{\pi} e^{-jnt}dt = \frac{7.5}{\pi}\frac{e^{-jnt}}{(-jn)}\bigg|_0^{\pi}$$

$$= [7.5/(n\pi)][je^{-jn\pi} - j] = [j7.5/(n\pi)][\cos(n\pi)-1]$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{j7.5}{n\pi}(\cos(n\pi)-1)e^{jnt}$$

Solution 17.11

Obtain the exponential Fourier series for the signal in Fig. 17.54.

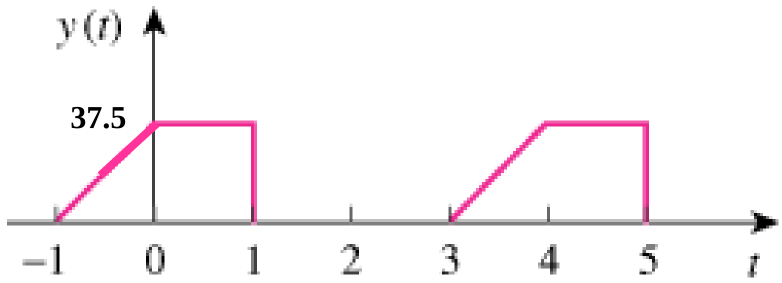


Figure 17.54
For Prob. 17.11.

Solution

$$T = 4, \qquad \omega_o = 2\pi / T = \pi / 2$$

$$c_n = \frac{1}{T}\int_0^T y(t)e^{-jn\omega_o t}dt = \frac{1}{4}\left[\int_{-1}^{0} 37.5(t+1)e^{-jn\pi t/2}dt + \int_0^1 (37.5)e^{-jn\pi t/2}dt\right]$$

$$c_n = \frac{37.5}{4}\left[\left(\frac{e^{-jn\pi t/2}}{-n^2\pi^2/4}(-jn\pi t/2 - 1) - \frac{2}{jn\pi}e^{-jn\pi t/2}\right)\Big|_{-1}^{0} - \left(\frac{2}{jn\pi}e^{-jn\pi t/2}\right)\Big|_0^1\right]$$

$$= \frac{37.5}{4}\left[\frac{4}{n^2\pi^2} - \frac{2}{jn\pi} + \frac{4}{n^2\pi^2}e^{jn\pi/2}((jn\pi/2) - 1) + \frac{2}{jn\pi}e^{jn\pi/2} - \frac{2}{jn\pi}e^{-jn\pi/2} + \frac{2}{jn\pi}\right]$$

But

$$e^{jn\pi/2} = \cos(n\pi/2) + j\sin(n\pi/2),$$

$$e^{-jn\pi/2} = \cos(n\pi/2) - j\sin(n\pi/2)$$

Thus, c_n

$$= \frac{37.5}{4}\left[\begin{array}{l} \frac{4}{n^2\pi^2} - \frac{2}{jn\pi} + \frac{4}{n^2\pi^2}(\cos(n\pi/2) + j\sin(n\pi/2))((jn\pi/2) - 1) \\ + \frac{2}{jn\pi}(\cos(n\pi/2) + j\sin(n\pi/2)) \\ - \frac{2}{jn\pi}(\cos(n\pi/2) - j\sin(n\pi/2)) + \frac{2}{jn\pi} \end{array}\right]$$

$$= \frac{37.5}{4}\left[\frac{4}{n^2\pi^2} + \frac{4}{n^2\pi^2}(\cos(n\pi/2) + j\sin(n\pi/2))((jn\pi/2) - 1) + \frac{4}{jn\pi}(j\sin(n\pi/2))\right]$$

$$= \frac{37.5}{n^2\pi^2}\left[1 + (\cos(n\pi/2) + j\sin(n\pi/2))((jn\pi/2) - 1) + \frac{n\pi}{1}(\sin(n\pi/2))\right]$$

$$= \frac{37.5}{n^2\pi^2}\left[\begin{array}{l} 1 - \cos(n\pi/2) - j\sin(n\pi/2) + (\cos(n\pi/2) \\ + j\sin(n\pi/2))((jn\pi/2) + \frac{n\pi}{1}(\sin(n\pi/2)) \end{array}\right]$$

$$= \frac{37.5}{n^2\pi^2}\left[\begin{array}{l} 1 - \cos(n\pi/2) - j\sin(n\pi/2) + \frac{jn\pi}{2}\cos(n\pi/2) - \frac{n\pi}{2}\sin(n\pi/2) \\ + \frac{n\pi}{1}(\sin(n\pi/2)) \end{array}\right]$$

$$c_n = \frac{75}{4n^2\pi^2}\left[2 - 2\cos(n\pi/2) - 2j\sin(n\pi/2) + \frac{jn\pi}{1}\cos(n\pi/2) + \frac{n\pi}{1}(\sin(n\pi/2))\right]$$

y(t) =

$$\sum_{n=-\infty}^{\infty} \frac{75}{4n^2\pi^2}\left[2 - 2\cos(n\pi/2) - 2j\sin(n\pi/2) + \frac{jn\pi}{1}\cos(n\pi/2) + \frac{n\pi}{1}(\sin(n\pi/2))\right]e^{jn\pi t/2}$$

Solution 17.12

A voltage source has a periodic waveform defined over its period as

$$v(t) = 120t(2\pi - t)\text{V}, \qquad 0 < t < 2\pi$$

Find the Fourier series for this voltage.

Solution

A voltage source has a periodic waveform defined over its period as
$v(t) = 120t(2\pi - t)$ V, for all $0 < t < 2\pi$

Find the Fourier series for this voltage.

$v(t) = 120(2\pi t - t^2)$, $0 < t < 2\pi$, $T = 2\pi$, $\omega_o = 2\pi/T = 1$

a_o =

$$(1/T)\int_0^T f(t)dt = \frac{1}{2\pi}\int_0^{2\pi} 120(2\pi t - t^2)dt$$

$$= \frac{120}{2\pi}(\pi t^2 - t^3/3)\Big|_0^{2\pi} = \frac{480\pi^3}{2\pi}(1 - 2/3) = \frac{240\pi^2}{3} = 80\pi^2$$

$$a_n = \frac{2}{T}\int_0^T 120(2\pi t - t^2)\cos(nt)dt = \frac{120}{\pi}\left[\frac{2\pi}{n^2}\cos(nt) + \frac{2\pi t}{n}\sin(nt)\right]\Bigg|_0^{2\pi}$$

$$-\frac{120}{\pi n^3}\left[2nt\cos(nt) - 2\sin(nt) + n^2t^2\sin(nt)\right]_0^{2\pi}$$

$$= \frac{240}{n^2}(1-1) - \frac{120}{\pi n^3}4n\pi\cos(2\pi n) = \frac{-480}{n^2}$$

$$b_n = \frac{240}{T}\int_0^T (2nt - t^2)\sin(nt)dt = \frac{120}{\pi}\int_0^T (2nt - t^2)\sin(nt)dt$$

$$= \frac{2n}{\pi}\frac{120}{n^2}(\sin(nt) - nt\cos(nt))\Big|_0^{\pi} - \frac{120}{\pi n^3}(2nt\sin(nt) + 2\cos(nt) - 1n^2t^2\cos(nt))\Big|_0^{2\pi}$$

$$= \frac{-480\pi}{n} + \frac{480\pi}{n} = 0$$

Hence, $\qquad f(t) = \dfrac{80\pi^2}{1} - \displaystyle\sum_{n=1}^{\infty}\frac{480}{n^2}\cos(nt)$

Solution 17.13

Design a problem to help other students to better understand obtaining the Fourier series from a periodic function.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A periodic function is defined over its period as

$$h(t) = \begin{cases} 10\sin t, & 0 < t < \pi \\ 20\sin(t-\pi), & \pi < t < 2\pi \end{cases}$$

Find the Fourier series of h(t).

Solution

$T = 2\pi,\ \omega_o = 1$

$$a_o = (1/T)\int_0^T h(t)dt = \frac{1}{2\pi}[\int_0^\pi 10\sin t\,dt + \int_\pi^{2\pi} 20\sin(t-\pi)\,dt]$$

$$= \frac{1}{2\pi}\left[-10\cos t\Big|_0^\pi - 20\cos(t-\pi)\Big|_\pi^{2\pi}\right] = \frac{30}{\pi}$$

$$a_n = (2/T)\int_0^T h(t)\cos(n\omega_o t)dt$$

$$= [2/(2\pi)]\left[\int_0^\pi 10\sin t\cos(nt)dt + \int_\pi^{2\pi} 20\sin(t-\pi)\cos(nt)dt\right]$$

Since $\sin A\cos B = 0.5[\sin(A+B) + \sin(A-B)]$

$\sin t\cos nt = 0.5[\sin((n+1)t) + \sin((1-n))t]$

$\sin(t-\pi) = \sin t\cos\pi - \cos t\sin\pi = -\sin t$

$\sin(t-\pi)\cos(nt) = -\sin(t)\cos(nt)$

$$a_n = \frac{1}{2\pi}\left[10\int_0^\pi [\sin([1+n]t) + \sin([1-n]t)]dt - 20\int_\pi^{2\pi}[\sin([1+n]t) + \sin([1-n]t)]dt\right]$$

$$= \frac{5}{\pi}\left[\left(-\frac{\cos([1+n]t)}{1+n} - \frac{\cos([1-n]t)}{1-n}\right)\Bigg|_0^\pi + \left(\frac{2\cos([1+n]t)}{1+n} + \frac{2\cos([1-n]t)}{1-n}\right)\Bigg|_\pi^{2\pi}\right]$$

$$a_n = \frac{5}{\pi}\left[\frac{3}{1+n} + \frac{3}{1-n} - \frac{3\cos([1+n]\pi)}{1+n} - \frac{3\cos([1-n]\pi)}{1-n}\right]$$

But, $[1/(1+n)] + [1/(1-n)] = 2/(1–n^2)$

$$\cos([n–1]\pi) = \cos([n+1]\pi) = \cos \pi \cos n\pi – \sin \pi \sin n\pi = –\cos n\pi$$

$$a_n = (5/\pi)[(6/(1–n^2)) + (6 \cos(n\pi)/(1–n^2))]$$

$$= [30/(\pi(1–n^2))](1 + \cos n\pi) = \mathbf{[–60/(\pi(n^2–1))], n = even}$$
$$= \mathbf{0, \quad n = odd}$$

$$b_n = (2/T)\int_0^T h(t)\sin n\omega_o t\, dt$$

$$= [2/(2\pi)][\int_0^{\pi} 10\sin t \sin nt\, dt + \int_{\pi}^{2\pi} 20(-\sin t)\sin nt\, dt$$

This is an interesting function which will have a value for b_1 but not for any of the other b_n terms (they will be zero).

$$b_1 = [2/(2\pi)][\left(\int_0^{\pi} 10\sin t \sin t\, dt = 10\int_0^{\pi}\frac{1-\cos(2t)}{2}dt = 5\pi\right.$$
$$\left. + \int_{\pi}^{2\pi} 20(-\sin t)\sin t\, dt = -20\int_{\pi}^{2\pi}(\sin t)^2 dt = -10\pi\right) = -5$$

Now we can calculate the rest of the b_n for values of n = 2 and greater than 2. We note that,

$$\sin A \sin B = 0.5[\cos(A–B) – \cos(A+B)]$$

$$\sin t \sin nt = 0.5[\cos([1–n]t) – \cos([1+n]t)]$$

$$b_n = (5/\pi)\{[(\sin([1–n]t)/(1–n)) – (\sin([1+n]t)/(1+n)]\Big|_0^{\pi}$$

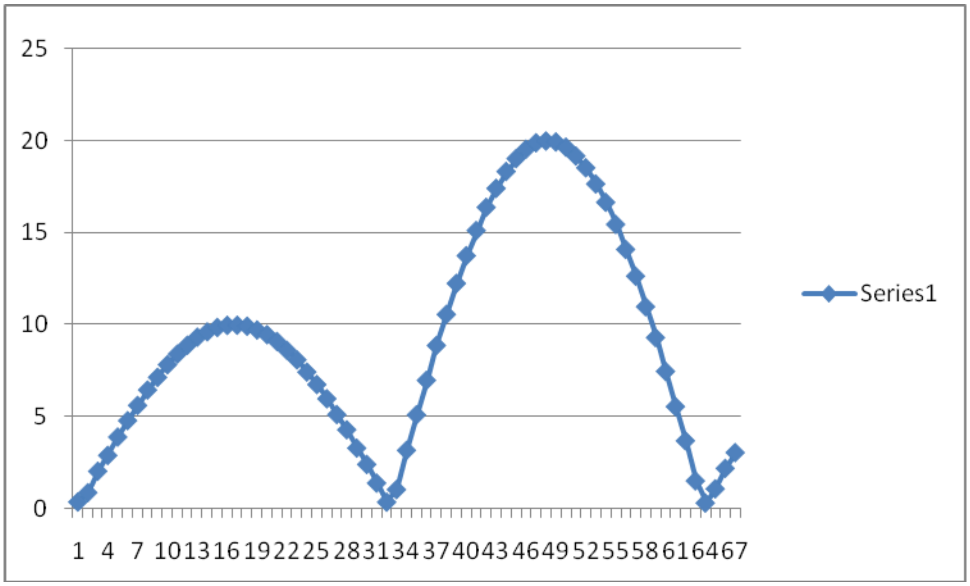
$$+ [(2\sin([1-n]t)/(1-n)) – (2\sin([1+n]t)/(1+n)]\Big|_{\pi}^{2\pi}\}$$

$$= \frac{5}{\pi}\left[-\frac{\sin([1-n]\pi)}{1-n} + \frac{\sin([1+n]\pi)}{1+n}\right] = 0$$

{Note, that if we substitute 1 for n, the first term is undefined!}

Thus, $\qquad h(t) = \dfrac{30}{\pi} - 5\sin(t) - \dfrac{60}{\pi}\sum_{k=1}^{\infty}\dfrac{\cos(2kt)}{(4k^2-1)}$

This does make a very good approximation!



Solution 17.14

Find the quadrature (cosine and sine) form of the Fourier series

$$f\left(t\right)=7.5+\sum_{n=1}^{\infty}\frac{37.5}{n^3+1}cos\left(2nt+\frac{n\pi}{4}\right)$$

Solution

Since cos(A + B) = cos A cos B – sin A sin B.

$$\text{f(t)} \;=\; 7.5+\sum_{n=1}^{\infty}\left(\frac{37.5}{n^3+1}\cos(n\pi/4)\cos(2nt)-\frac{37.5}{n^3+1}\sin(n\pi/4)\sin(2nt)\right)$$

Solution 17.15

(a) $$\text{D}\cos\omega t + \text{E}\sin\omega t = \text{A}\cos(\omega t - \theta)$$

where $$\text{A} = \sqrt{\text{D}^2 + \text{E}^2}, \quad \theta = \tan^{-1}(\text{E/D})$$

$$\text{A} = \sqrt{\frac{16}{(n^2+1)^2} + \frac{1}{n^6}}, \quad \theta = \tan^{-1}((n^2+1)/(4n^3))$$

$$f(t) = \mathbf{10 + \sum_{n=1}^{\infty}\sqrt{\frac{16}{(n^2+1)^2} + \frac{1}{n^6}}\cos\left(10nt - \tan^{-1}\frac{n^2+1}{4n^3}\right)}$$

(b) $$\text{D}\cos\omega t + \text{E}\sin\omega t = \text{A}\sin(\omega t + \theta)$$

where $$\text{A} = \sqrt{\text{D}^2 + \text{E}^2}, \quad \theta = \tan^{-1}(\text{D/E})$$

$$f(t) = \mathbf{10 + \sum_{n=1}^{\infty}\sqrt{\frac{16}{(n^2+1)^2} + \frac{1}{n^6}}\sin\left(10nt + \tan^{-1}\frac{4n^3}{n^2+1}\right)}$$

Solution 17.16

The waveform in Fig. 17.55(a) has the following Fourier series:

$$v(t)=\frac{1}{2}-\frac{4}{\pi^2}\left(\cos\pi t+\frac{1}{9}\cos 3\pi t+\frac{1}{25}\cos 5\pi t+\cdots\right)V$$

Obtain the Fourier series of $v_2(t)$ in Fig. 17.55(b).

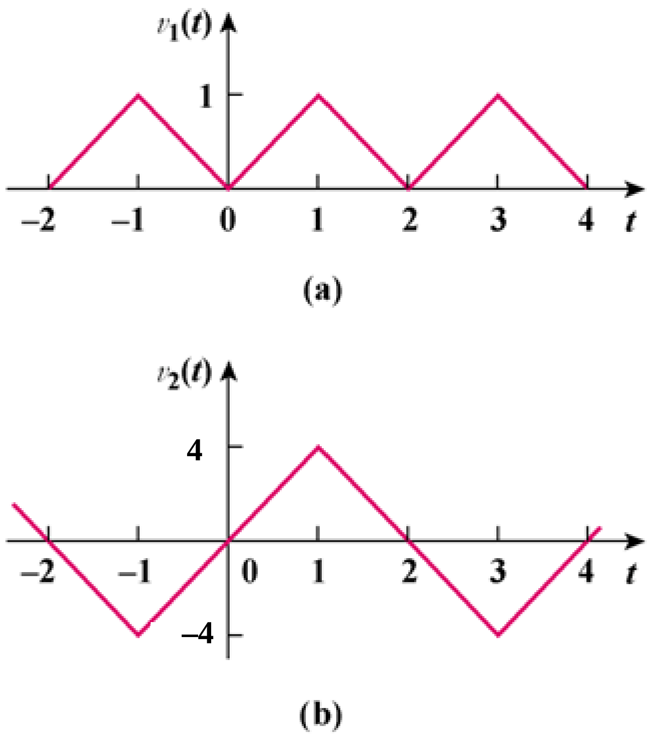
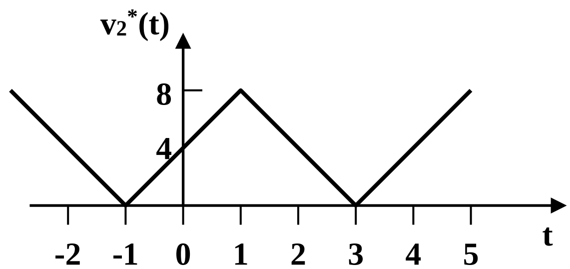


Figure 17.55
For Prob. 17.16.

Solution

If $v_2(t)$ is shifted by 4 along the vertical axis, we obtain $v_2^*(t)$ shown below, i.e. $v_2^*(t) = v_2(t) + 4$.



Comparing $v_2^*(t)$ with $v_1(t)$ shows that

$$v_2^*(t) = 8v_1((t + t_o)/2)$$

where $(t + t_o)/2 = 0$ at $t = -1$ or $t_o = 1$

Hence $$v_2^*(t) = 8v_1((t + 1)/2)$$

But $$v_2^*(t) = v_2(t) + 4$$

$$v_2(t) + 4 = 8v_1((t+1)/2)$$

$$v_2(t) = -4 + 8v_1((t+1)/2)$$

$$= -4 + 4 - \frac{32}{\pi^2}\left[\cos\pi\left(\frac{t+1}{2}\right) + \frac{1}{9}\cos 3\pi\left(\frac{t+1}{2}\right) + \frac{1}{25}\cos 5\pi\left(\frac{t+1}{2}\right) + \cdots\right]$$

$$v_2(t) = -\frac{32}{\pi^2}\left[\cos\left(\frac{\pi t}{2} + \frac{\pi}{2}\right) + \frac{1}{9}\cos\left(\frac{3\pi t}{2} + \frac{3\pi}{2}\right) + \frac{1}{25}\cos\left(\frac{5\pi t}{2} + \frac{5\pi}{2}\right) + \cdots\right]$$

$$v_2(t) = -\frac{32}{\pi^2}\left[\sin\left(\frac{\pi t}{2}\right) + \frac{1}{9}\sin\left(\frac{3\pi t}{2}\right) + \frac{1}{25}\sin\left(\frac{5\pi t}{2}\right) + \cdots\right]$$

Solution 17.17

We replace t by –t in each case and see if the function remains unchanged.

(a) $1 - t$, **neither odd nor even.**

(b) $t^2 - 1$, **even**

(c) $\cos n\pi(-t) \sin n\pi(-t) = -\cos n\pi t \sin n\pi t$, **odd**

(d) $\sin^2 n(-t) = (-\sin \pi t)^2 = \sin^2 \pi t$, **even**

(e) e^t, **neither odd nor even**.

Solution 17.18

(a) $T = 2$ leads to $\omega_o = 2\pi/T = \boldsymbol{\pi}$

$f_1(-t) = -f_1(t)$, showing that $f_1(t)$ is **odd and half-wave symmetric**.

(b) $T = 3$ leads to $\omega_o = 2\pi/3$

$f_2(t) = f_2(-t)$, showing that $f_2(t)$ is **even**.

(c) $T = 4$ leads to $\omega_o = \pi/2$

$f_3(t)$ is **even and half-wave symmetric**.

Solution 17.19

Obtain the Fourier series for the periodic waveform in Fig. 17.57.

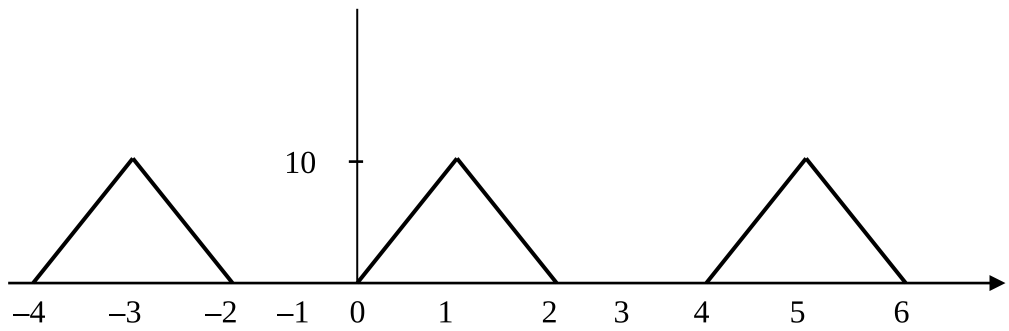


Figure 17.57
For Prob. 17.19.

Solution

This is an odd function.

$a_n = 0$, $T = 4$, $\omega_o = 2\pi/T = \pi/2$.

$$f(t) = \begin{cases} 10t, & 0 < t < 1 \\ 20 - 10t, & 1 < t < 2 \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{4}\int_0^1 10t\,dt + \frac{1}{4}\int_1^2 10(2-t)dt = \frac{1}{4}5t^2\Big|_0^1 + \frac{10}{4}\left(2t - \frac{t^2}{2}\right)\Big|_1^2 = 2.5$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t\,dt = \frac{2}{4}\int_0^1 10t\cos n\omega_o t\,dt + \frac{2}{4}\int_1^2 10(2-t)\cos n\omega_o t\,dt$$

$$= \frac{20}{n\omega_o}\cos n\omega_o t + \frac{t}{n\omega_o}\sin n\omega_o t\Big|_0^1 + \frac{10}{n\omega_o}\sin n\omega_o t\Big|_1^2 + \frac{5}{n^2\omega_o^2}\cos n\omega_o t + \frac{5t}{n\omega_o}\sin n\omega_o t\Big|_1^2$$

$$= \frac{20}{n\omega_o}(\cos n\pi/2 - 1) + \frac{1}{n\omega_o}\sin n\pi/2 + \frac{10}{n\omega_o}(\sin n\pi - \sin n\pi/2) + \frac{5}{n^2\pi^2/4}\cos n\pi$$

$$-\frac{5}{n^2\pi^2/4}\cos n\pi/2 + \frac{10}{n\omega_o}\sin n\pi - \frac{5}{n\pi/2}\sin n\pi/2$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t\,dt = \frac{2}{4}\int_0^1 10t\sin n\omega_o t\,dt + \frac{2}{4}\int_1^2 10(2-t)\sin n\omega_o t\,dt$$

$$= \frac{5}{n\omega_o}\sin n\omega_o t\Big|_0^1 - \frac{10}{n\omega_o}\cos n\omega_o t\Big|_0^1 - \frac{5}{n^2\omega_o^2}\sin n\omega_o t\Big|_1^2 + \frac{t}{n\omega_o}\cos n\omega_o t\Big|_1^2$$

$$= \frac{5}{n^2\omega_o^2}\sin n\pi/2 - \frac{10}{n\omega_o}(\cos\pi n - \cos n\pi/2) - \frac{5}{n^2\omega_o^2}(\sin\pi n - \sin n\pi/2)$$

$$-\frac{2}{n\omega_o}\cos n\pi - \frac{\cos\pi n/2}{n\omega_o}$$

Solution 17.20

Find the Fourier series for the signal in Fig. 17.58. Evaluate $f(t)$ at $t = 2$ using the first three nonzero harmonics.

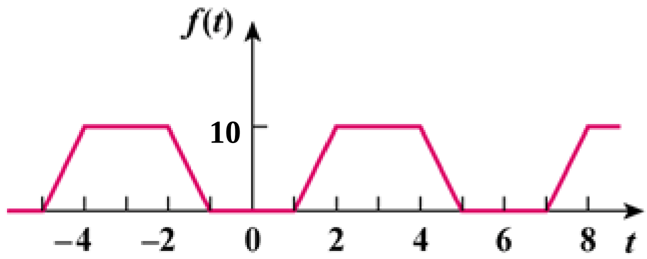


Figure 17.58
For Prob. 17.20.

Solution

This is an even function.

$$b_n = 0, \; T = 6, \; \omega = 2\pi/6 = \pi/3$$

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{6}\left[\int_1^2 (10t-10)dt + \int_2^3 10\,dt\right]$$

$$= \frac{1}{3}\left[(5t^2-10t)\Big|_1^2 + 10(3-2)\right] = (1/3)[20–5–20+10+30–20] = 5$$

$$a_n = \frac{4}{T}\int_0^{T/4} f(t)\cos(n\pi t/3)dt$$

$$= (4/6)[\int_1^2 (10t-10)\cos(n\pi t/3)dt \quad + \quad \int_2^3 10\cos(n\pi t/3)dt\,]$$

$$= \frac{40}{6}\left[\frac{9}{n^2\pi^2}cos\left(\frac{n\pi t}{3}\right)+\frac{3t}{n\pi}sin\left(\frac{n\pi t}{3}\right)-\frac{3}{n\pi}sin\left(\frac{n\pi t}{3}\right)\right]_1^2+\frac{40}{6}\left[\frac{3}{n\pi}sin\left(\frac{n\pi t}{3}\right)\right]_2^3$$

$$= [60/(n^2\pi^2)][\cos(2n\pi/3) - \cos(n\pi/3)]$$

Thus $$f(t) = 5+\frac{60}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\cos\left(\frac{2\pi n}{3}\right)-\cos\left(\frac{\pi n}{3}\right)\right]\cos\left(\frac{n\pi t}{3}\right)$$

At t = 2,

$$f(2) = 5 + (60/\pi^2)\{[(\cos(2\pi/3) - \cos(\pi/3))\cos(2\pi/3)]$$

$$+ [(1/4)(\cos(4\pi/3) - \cos(2\pi/3))\cos(4\pi/3)]$$

$$+ [(1/9)(\cos(2\pi) - \cos(\pi))\cos(2\pi)]$$

$$+ [(1/16)(\cos(8\pi/3) - \cos(4\pi/3))\cos(8\pi/3)]$$

$$+ [(1/25)(\cos(10\pi/3) - \cos(5\pi/3))\cos(10\pi/3)]$$
$$-----\}$$

$$= 5 + 6.07927(0.5 + 0 + 0.2222 + 0 + 0.02 + -----)$$

Taking just the first 3 non zero harmonics we get,

$$f(2) = \mathbf{9.512}.$$

Solution 17.21

Determine the trigonometric Fourier series of the signal in Fig. 17.59.

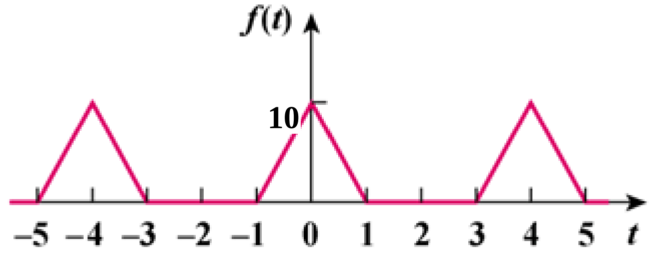


Figure 17.59
For Prob. 17.21.

Solution

This is an even function.

$b_n = 0, \; T = 4, \; \omega_o = 2\pi/T = \pi/2.$

$$f(t) = \begin{cases} 10 - 10t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

$$a_o = \frac{2}{4}\int_0^1 10(1-t)dt = 5\left[t - \frac{t^2}{2}\right]_0^1 = 2.5$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{4}\int_0^1 10(1-t)\cos\left(\frac{n\pi t}{2}\right)dt$$

$$= [40/(\pi^2 n^2)][1 - \cos(n\pi/2)]$$

$$f(t) = 2.5 + \sum_{n=1}^{\infty} \frac{40}{n^2\pi^2}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right]\cos\left(\frac{n\pi t}{2}\right)$$

Solution 17.22

Calculate the Fourier coefficients for the function in Fig. 17.60.

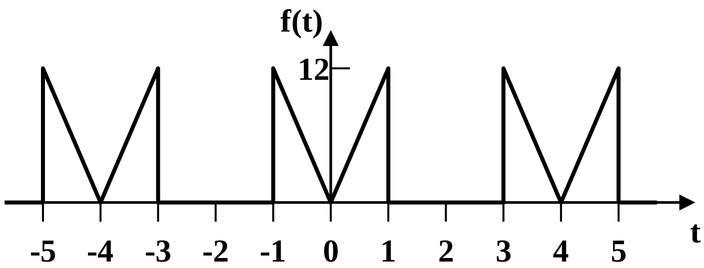


Figure 17.60
For Prob. 17.60

Solution

This is an even function, therefore $b_n = 0$. In addition, T=4 and $\omega_o = \pi/2$.

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{4}\int_0^1 12tdt = 3t^2\Big|_0^1 = \mathbf{3}$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(\omega_o nt)dt = \frac{4}{4}\int_0^1 12t\cos(n\pi t/2)dt$$

$$= 12\left[\frac{4}{n^2\pi^2}\cos(n\pi t/2) + \frac{2t}{n\pi}\sin(n\pi t/2)\right]\Bigg|_0^1$$

$$a_n = \frac{48}{n^2\pi^2}(\cos(n\pi/2) - 1) + \frac{24}{n\pi}\sin(n\pi/2)$$

Solution 17.23

Using Fig. 17.61, design a problem to help other students to better understand finding the Fourier series of a periodic wave shape.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Fourier series of the function shown in Fig. 17.61.

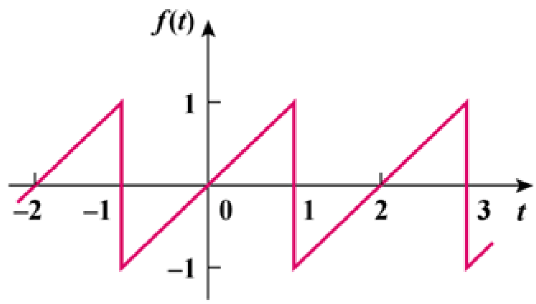


Figure 17.61

Solution

f(t) is an odd function.

$f(t) = t, \quad -1 < t < 1$

$$a_o = 0 = a_n, \quad T = 2, \quad \omega_o = 2\pi/T = \pi$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_o t)dt = \frac{4}{2}\int_0^1 t\sin(n\pi t)dt$$

$$= \frac{2}{n^2\pi^2}\left[\sin(n\pi t) - n\pi t\cos(n\pi t)\right]_0^1$$

$$= -[2/(n\pi)]\cos(n\pi) = 2(-1)^{n+1}/(n\pi)$$

$$\mathbf{f(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(n\pi t)}$$

Solution 17.24

(a) This is an odd function.

$$a_o = 0 = a_n,\ T = 2\pi,\ \omega_o = 2\pi/T = 1$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(\omega_o nt)dt$$

$$f(t) = 1 + t/\pi, \qquad 0 < t < \pi$$

$$b_n = \frac{4}{2\pi}\int_0^{\pi}(1 + t/\pi)\sin(nt)dt$$

$$= \frac{2}{\pi}\left[-\frac{1}{n}\cos(nt) + \frac{1}{n^2\pi}\sin(nt) - \frac{t}{n\pi}\cos(nt)\right]_0^{\pi}$$

$$= [2/(n\pi)][1 - 2\cos(n\pi)] = [2/(n\pi)][1 + 2(-1)^{n+1}]$$

$a_2 = \underline{\mathbf{0}}$, $b_2 = [2/(2\pi)][1 + 2(-1)] = -1/\pi = \mathbf{-0.3183}$

(b) $\omega n = n\omega_o = 10$ or $n = 10$

$$a_{10} = 0,\ b_{10} = [2/(10\pi)][1 - \cos(10\pi)] = -1/(5\pi)$$

Thus the magnitude is $A_{10} = \sqrt{a_{10}^2 + b_{10}^2} = 1/(5\pi) = \mathbf{0.06366}$

and the phase is $\phi_{10} = \tan^{-1}(b_n/a_n) = \mathbf{-90°}$

(c) $$f(t) = \sum_{n=1}^{\infty}\frac{2}{n\pi}[1 - 2\cos(n\pi)]\sin(nt)\,\pi$$

$$f(\pi/2) = \sum_{n=1}^{\infty}\frac{2}{n\pi}[1 - 2\cos(n\pi)]\sin(n\pi/2)\,\pi$$

For $n = 1$, $f_1 = (2/\pi)(1 + 2) = 6/\pi$

For $n = 2$, $f_2 = 0$

For $n = 3$, $f_3 = [2/(3\pi)][1 - 2\cos(3\pi)]\sin(3\pi/2) = -6/(3\pi)$

For $n = 4$, $f_4 = 0$

For n = 5, $f_5 = 6/(5\pi)$, ----

Thus, $f(\pi/2) = 6/\pi - 6/(3\pi) + 6/(5\pi) - 6/(7\pi)$ ---------

$= (6/\pi)[1 - 1/3 + 1/5 - 1/7 +$ --------]

$f(\pi/2) \cong$ **1.3824**

which is within 8% of the exact value of 1.5.

(d) From part (c)

$f(\pi/2) = 1.5 = (6/\pi)[1 - 1/3 + 1/5 - 1/7 +$ - - -]

$(3/2)(\pi/6) = [1 - 1/3 + 1/5 - 1/7 +$ - - -]

or $\pi/4 =$ **$1 - 1/3 + 1/5 - 1/7 +$ - - -**

Solution 17.25

Determine the Fourier series representation of the function in Fig. 17.63.

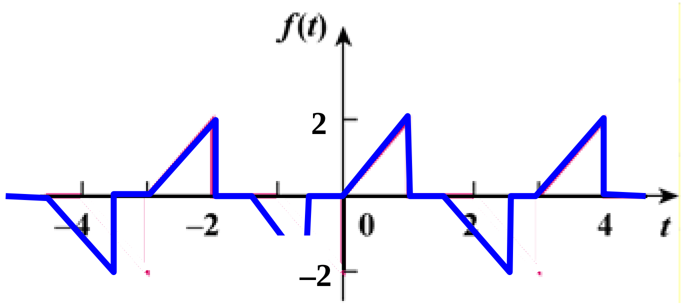


Figure 17.63
For Prob. 17.25.

Solution

This is a half-wave (odd) function since f(t–T/2) = –f(t).

$a_o = 0$, $a_n = b_n = 0$ for n = even, T = 3, $\omega_o = 2\pi/3$.

For n = odd,

$$a_n = \frac{4}{3}\int_0^{1.5} f(t)\cos n\omega_0 t dt = \frac{4}{3}\int_0^{1} 2t\cos n\omega_0 t dt$$

$$= \frac{8}{3}\left[\frac{9}{4\pi^2 n^2}\cos\left(\frac{2\pi nt}{3}\right) + \frac{3t}{2\pi n}\sin\left(\frac{2\pi nt}{3}\right)\right]_0^1$$

$$= \left[\frac{6}{\pi^2 n^2}\left(\cos\left(\frac{2\pi n}{3}\right) - 1\right) + \frac{2}{\pi n}\sin\left(\frac{2\pi n}{3}\right)\right]$$

$$b_n = \frac{4}{3}\int_0^{1.5} f(t)\sin(n\omega_o t)dt = \frac{4}{3}\int_0^{1} 2t\sin(2\pi nt/3)dt$$

$$= \frac{8}{3}\left[\frac{9}{4\pi^2 n^2}\sin\left(\frac{2\pi nt}{3}\right) - \frac{3t}{2n\pi}\cos\left(\frac{2\pi nt}{3}\right)\right]_0^1$$

$$= \left[\frac{6}{\pi^2 n^2}\sin\left(\frac{2\pi n}{3}\right) - \frac{4}{\pi n}\cos\left(\frac{2\pi n}{3}\right)\right]$$

$$f(t) = \sum_{\substack{n=1 \\ n=odd}}^{\infty} \left\{ \begin{array}{l} \left[\frac{6}{\pi^2 n^2} \left(\cos\left(\frac{2\pi n}{3} \right) - 1 \right) + \frac{4}{\pi n} \sin\left(\frac{2\pi n}{3} \right) \right] \cos\left(\frac{2\pi n t}{3} \right) \\ + \left[\frac{6}{\pi^2 n^2} \sin\left(\frac{2\pi n}{3} \right) - \frac{4}{n\pi} \cos\left(\frac{2\pi n}{3} \right) \right] \sin\left(\frac{2\pi n t}{3} \right) \end{array} \right\}$$

Solution 17.26

Find the Fourier series representation of the signal shown in Fig. 17.64.

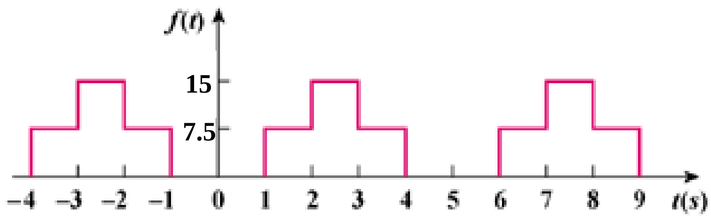


Figure 17.64
For Prob. 17.26.

Solution

$T = 5, \ \omega_o = 2\pi/T = 2\pi/5$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{5}\left[\int_1^2 7.5\,dt + \int_2^3 15\,dt + \int_3^4 7.5\,dt\right] = 0.2(7.5+15+7.5) = 6$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_o t)dt$$

$$a_n = \frac{2}{5}\left[\int_1^2 7.5\cos(2n\pi t/5)dt + \int_2^3 15\cos(2n\pi t/5)dt + \int_3^4 7.5\cos(2n\pi t/5)dt\right]$$

$$= 3\left[\frac{5}{2n\pi}\sin\left(\frac{2n\pi t}{5}\right)\bigg|_1^2 + \frac{10}{2n\pi}\sin\left(\frac{2n\pi t}{5}\right)\bigg|_2^3 + \frac{5}{2n\pi}\sin\left(\frac{2n\pi t}{5}\right)\bigg|_3^4\right]$$

$$= \frac{15}{2n\pi}\left[\sin\left(\frac{2n\pi t}{5}\right)\bigg|_1^2 + 2\sin\left(\frac{2n\pi t}{5}\right)\bigg|_2^3 + \sin\left(\frac{2n\pi t}{5}\right)\bigg|_3^4\right]$$

$$= \frac{15}{2n\pi}\left[\sin\left(\frac{4n\pi}{5}\right) - \sin\left(\frac{2n\pi}{5}\right) + 2\sin\left(\frac{6n\pi}{5}\right) - 2\sin\left(\frac{4n\pi}{5}\right) + \sin\left(\frac{8n\pi}{5}\right) - \sin\left(\frac{6n\pi}{5}\right)\right]$$

$$a_n = \frac{15}{2n\pi}\left[-\sin\left(\frac{2n\pi}{5}\right) + \sin\left(\frac{6n\pi}{5}\right) - \sin\left(\frac{4n\pi}{5}\right) + \sin\left(\frac{8n\pi}{5}\right)\right]$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_o t)dt$$

$$= \frac{2}{5}\left[\int_1^2 7.5\sin\left(\frac{2n\pi t}{5}\right)dt + \int_2^3 15\sin\left(\frac{2n\pi t}{5}\right)dt + \int_3^4 7.5\sin\left(\frac{2n\pi t}{5}\right)dt\right]$$

$$= 3\left[-\frac{5}{2n\pi}\cos\left(\frac{2n\pi t}{5}\right)\Big|_1^2 - \frac{10}{2n\pi}\cos\left(\frac{2n\pi t}{5}\right)\Big|_2^3 - \frac{5}{2n\pi}\cos\left(\frac{2n\pi t}{5}\right)\Big|_3^4\right]$$

$$= \frac{15}{2n\pi}\left[-\cos\left(\frac{2n\pi t}{5}\right)\Big|_1^2 - 2\cos\left(\frac{2n\pi t}{5}\right)\Big|_2^3 - \cos\left(\frac{2n\pi t}{5}\right)\Big|_3^4\right]$$

$$= \frac{15}{2n\pi}\left[-\cos\left(\frac{2n\pi t}{5}\right)\Big|_1^2 - 2\cos\left(\frac{2n\pi t}{5}\right)\Big|_2^3 - \cos\left(\frac{2n\pi t}{5}\right)\Big|_3^4\right]$$

$$= \frac{15}{2n\pi}\left[-\cos\left(\frac{4n\pi}{5}\right) + \cos\left(\frac{2n\pi}{5}\right) - 2\cos\left(\frac{6n\pi}{5}\right) + 2\cos\left(\frac{4n\pi}{5}\right) - \cos\left(\frac{8n\pi}{5}\right) + \cos\left(\frac{6n\pi}{5}\right)\right]$$

$$b_n = \frac{15}{2n\pi}\left[\cos\left(\frac{2n\pi}{5}\right) - \cos\left(\frac{6n\pi}{5}\right) + \cos\left(\frac{4n\pi}{5}\right) - \cos\left(\frac{8n\pi}{5}\right)\right]$$

Hence,

$$f(t) = 6 + \frac{6}{\mathbf{n\pi}}\sum_{\mathbf{n}=1}^{\infty}\left[\mathbf{a}_n\mathbf{cos}\left(\frac{2\mathbf{n\pi t}}{5}\right) + \mathbf{b}_n\mathbf{sin}\left(\frac{2\mathbf{n\pi t}}{5}\right)\right]$$

Solution 17.27

For the waveform shown in Fig. 17.65 below,

(a) specify the type of symmetry it has,
(b) calculate a_3 and b_3,
(c) find the rms value using the first five nonzero harmonics.

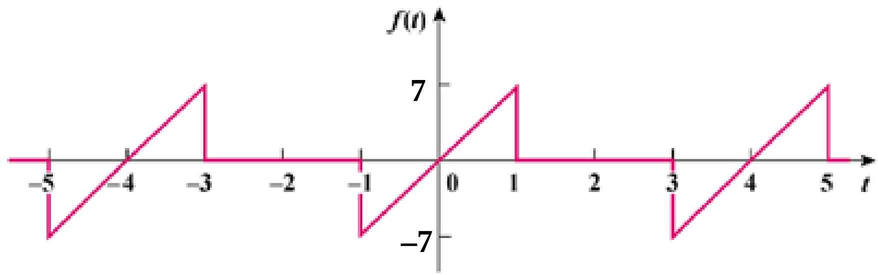


Figure 17.65
For Prob. 17.27.

Solution

(a) **odd** symmetry.

(b) $a_o = 0 = a_n, \; T = 4, \; \omega_o = 2\pi/T = \pi/2$

$$f(t) = 7t, \quad 0 < t < 1$$
$$= 0, \quad 1 < t < 2$$

$$b_n = \frac{4}{4}\int_0^1 7t\sin\frac{n\pi t}{2}dt = \left[\frac{28}{n^2\pi^2}\sin\frac{n\pi t}{2} - \frac{14t}{n\pi}\cos\frac{n\pi t}{2}\right]_0^1$$

$$= \frac{28}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{14}{n\pi}\cos\frac{n\pi}{2} - 0$$

$$= 28(-1)^{(n-1)/2}/(n^2\pi^2), \quad n = \text{odd}$$

$$-14(-1)^{n/2}/(n\pi), \quad n = \text{even}$$

$$a_3 = \mathbf{0}, \; b_3 = 28(-1)/(9\pi^2) = \mathbf{-0.3152}$$

(c) $b_1 = 28/\pi^2, \; b_2 = 7/\pi, \; b_3 = -28/(9\pi^2), \; b_4 = -7/(2\pi), \; b_5 = 28/(25\pi^2)$

$$F_{rms} = \sqrt{a_o^2 + \frac{1}{2}\sum\left(a_n^2 + b_n^2\right)}$$

$F_{rms}^2 = 0.5\Sigma b_n^2$

$= [1/(2\pi^2)][(784/\pi^2) + 49 + (784/(81\pi^2)) + (49/4) + (784/(625\pi^2))]$

$= (1/19.729)(128.433 + 13.23 + 0.127) = 7.1869$

$F_{rms} = \sqrt{7.1869} = \mathbf{2.681}$

Compare this with the exact value of $F_{rms} = \sqrt{\frac{98}{T}\int_0^1 t^2 dt} = \sqrt{49/6} = 2.858$ or

$(2.681/2.858)x100 = 93.81\%$, quite close.

Solution 17.28

Obtain the trigonometric Fourier series for the voltage waveform shown in Fig. 17.66.

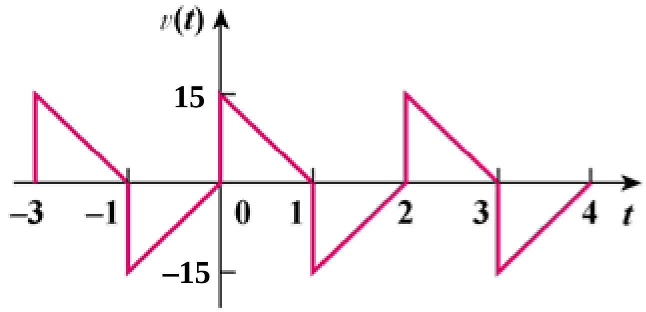


Figure 17.66
For Prob. 17.28.

Solution

This is half-wave symmetric since f(t – T/2) = –f(t).

$$a_o = 0, \quad T = 2, \quad \omega_o = 2\pi/2 = \pi$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{2}\int_0^1 (15-15t)\cos(n\pi t)dt$$

$$= 30\left[\frac{1}{n\pi}\sin(n\pi t) - \frac{1}{n^2\pi^2}\cos(n\pi t) - \frac{t}{n\pi}\sin(n\pi t)\right]\Bigg|_0^1$$

$$= [30/(n^2\pi^2)][1 - \cos(n\pi)] = \begin{cases} 60/(n^2\pi^2), & n = \text{odd} \\ 0, & n = \text{even} \end{cases}$$

$$b_n = 30\int_0^1 (1-t)\sin(n\pi t)dt$$

$$= 30\left[-\frac{1}{n\pi}\cos(n\pi t) - \frac{1}{n^2\pi^2}\sin(n\pi t) + \frac{t}{n\pi}\cos(n\pi t)\right]\Bigg|_0^1$$

$= 30/(n\pi)$, n = odd

$$f(t) = \sum_{k=1}^{\infty}\left(\frac{60}{n^2\pi^2}\cos(n\pi t) + \frac{30}{n\pi}\sin(n\pi t)\right), \quad \mathbf{n = 2k - 1}$$

Solution 17.29

This function is half-wave symmetric.

$$T = 2\pi, \quad \omega_o = 2\pi/T = 1, \quad f(t) = -t, \quad 0 < t < \pi$$

For odd n, $$a_n = \frac{2}{T}\int_0^{\pi} (-t)\cos(nt)dt = -\frac{2}{n^2\pi}\big[\cos(nt) + nt\sin(nt)\big]_0^{\pi} = 4/(n^2\pi)$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} (-t)\sin(nt)dt = -\frac{2}{n^2\pi}\big[\sin(nt) - nt\cos(nt)\big]_0^{\pi} = -2/n$$

Thus,

$$f(t) = \mathbf{2\sum_{k=1}^{\infty}\left[\frac{2}{n^2\pi}\cos(nt) - \frac{1}{n}\sin(nt)\right]}, \qquad \mathbf{n = 2k - 1}$$

Solution 17.30

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_o t}dt = \frac{1}{T}\left[\int_{-T/2}^{T/2} f(t)\cos n\omega_o t dt - j\int_{-T/2}^{T/2} f(t)\sin n\omega_o t dt\right] \qquad (1)$$

(a) The second term on the right hand side vanishes if f(t) is even. Hence

$$c_n = \frac{2}{T}\int_{0}^{T/2} f(t)\cos n\omega_o t dt$$

(b) The first term on the right hand side of (1) vanishes if f(t) is odd. Hence,

$$c_n = -\frac{j2}{T}\int_{0}^{T/2} f(t)\sin n\omega_o t dt$$

Solution 17.31

If $\mathbf{h(t) = f(\alpha t)}$, $\quad \mathbf{T' = T/\alpha} \quad \longrightarrow \quad \mathbf{\omega_o' = \frac{2\pi}{T'} = \frac{2\pi}{T/\alpha} = \underline{\alpha\omega_o}}$

$$\mathbf{a_n' = \frac{2}{T'}\int_0^{T'} h(t)\cos n\omega_o' t dt = \frac{2}{T'}\int_0^{T'} f(\alpha t)\cos n\omega_o' t dt}$$

Let $\mathbf{\alpha t = \lambda,,} \quad \mathbf{dt = d\lambda/\alpha,} \quad \mathbf{\alpha T' = T}$

$$\mathbf{a_n' = \frac{2\alpha}{T}\int_0^{T} f(\lambda)\cos n\omega_o \lambda d\lambda/\alpha = a_n}$$

Similarly, $\quad \mathbf{\underline{b_n' = b_n}}$

Solution 17.32

Find *i*(*t*) in the circuit of Fig. 17.68 given that

$$i_s(t) = \left[3.5 + \sum_{n=1}^{\infty} \frac{4}{n^2}\cos(3nt)\right]\text{A}$$

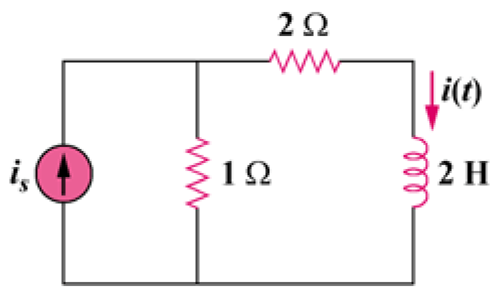


Figure 17.68
For Prob. 17.32.

Solution

When i_s = 3.5 A, (DC component)

$$i = 3.5/(1 + 2) = 3.5/3$$

For n ≥ 1, $\omega_n = 3n, \; I_s = 4/n^2\angle 0°$ A

$$I = [1/(1 + 2 + j2\omega_n^2)]I_s = I_s/(3 + j6n)$$

$$= \frac{\frac{4}{n^2}\angle 0°}{3\sqrt{1+4n^2}\angle\tan^{-1}(6n/3)} = \frac{4}{3n^2\sqrt{1+4n^2}}\angle -\tan(2n)$$

Thus,

$$i(t) = \left[1.1667 + \sum_{n=1}^{\infty}\frac{4}{3n^2\sqrt{1+4n^2}}\cos(3n - \tan^{-1}(2n))\right]\mathbf{A}$$

Solution 17.33

In the circuit shown in Fig. 17.69, the Fourier series expansion of $v_s(t)$ is

$$v_s(t) = \left[10 + \frac{5}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi t)\right]\text{V}$$

Find $v_o(t)$.

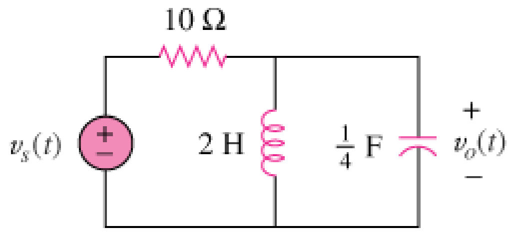


Figure 17.69 For Prob. 17.33.

Solution

For the DC case, the inductor acts like a short, $V_o = 0$.
For the AC case, we obtain the following:

$$\frac{V_o - V_s}{10} + \frac{V_o}{j2n\pi} + \frac{jn\pi V_o}{4} = 0$$

$$\left(1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)\right)V_o = V_s$$

$$V_o = \frac{V_s}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)}$$

$$A_n\angle\Theta_n = \frac{5}{n\pi}\frac{1}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)} = \frac{5}{n\pi + j(2.5n^2\pi^2 - 5)}$$

$$A_n = \frac{5}{\sqrt{n^2\pi^2 + (2.5n^2\pi^2 - 5)^2}}; \quad \Theta_n = -\tan^{-1}\left(\frac{2.5n^2\pi^2 - 5}{n\pi}\right)$$

$$\underline{v_o(t) = \sum_{n=1}^{\infty} A_n \sin(n\pi t + \Theta_n)V}$$

Solution 17.34

Using Fig. 17.70, design a problem to help other students to better understand circuit responses to a Fourier series.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Obtain $v_o(t)$ in the network of Fig. 17.70 if

$$v(t) = \sum_{n=1}^{\infty} \frac{10}{n^2} \cos\left(nt + \frac{n\pi}{4} \right) V$$

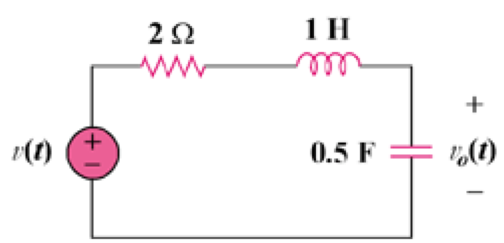
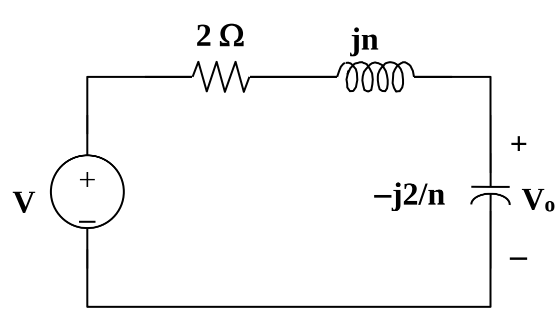


Figure 17.70

Solution

For any n, V = [10/n^2]∠(nπ/4), ω = n.

1 H becomes jω_nL = jn and 0.5 F becomes 1/(jω_nC) = –j2/n



V_o = {–j(2/n)/[2 + jn – j(2/n)]}V = {–j2/[2n + j(n^2 – 2)]}[(10/n^2)∠(nπ/4)]

$$= \frac{20\angle((n\pi/4) - \pi/2)}{n^2\sqrt{4n^2 + (n^2 - 2)^2}\angle\tan^{-1}((n^2 - 2)/2n)}$$

$$= \frac{20}{n^2\sqrt{n^4 + 4}}\angle[(n\pi/4) - (\pi/2) - \tan^{-1}((n^2 - 2)/2n)]$$

$$v_o(t) = \sum_{n=1}^{\infty} \frac{20}{n^2\sqrt{n^4 + 4}} \cos\left(nt + \frac{n\pi}{4} - \frac{\pi}{2} - \tan^{-1}\frac{n^2 - 2}{2n}\right)$$

Solution 17.35

If v_s in the circuit of Fig. 17.72 is the same as function $f_2(t)$ in Fig. 17.57(b), determine the dc component and the first three nonzero harmonics of $v_o(t)$.

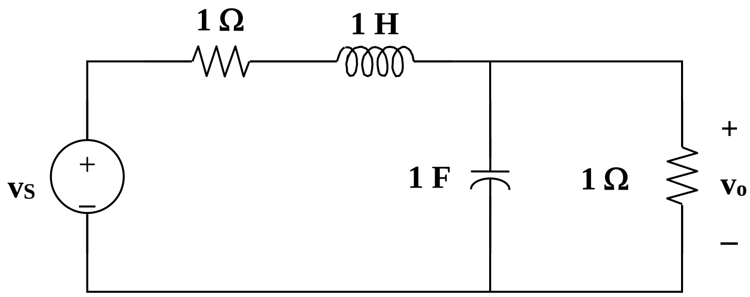


Figure 16.64 For Prob. 16.25

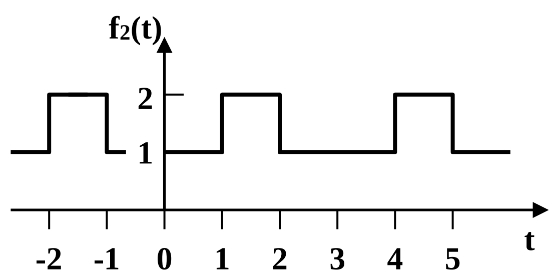


Figure 16.50(b) For Prob. 16.25

The signal is even, hence, $b_n = 0$. In addition, $T = 3$, $\omega_o = 2\pi/3$.

$$v_s(t) \;=\; 1 \text{ for all } 0 < t < 1$$
$$\;=\; 2 \text{ for all } 1 < t < 1.5$$

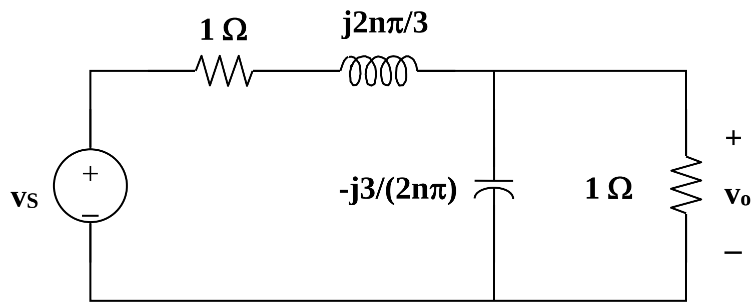
$$a_o = \frac{2}{3}\left[\int_0^1 1dt + \int_1^{1.5} 2dt\right] = \frac{4}{3}$$

$$a_n = \frac{4}{3}\left[\int_0^1 \cos(2n\pi t/3)dt + \int_1^{1.5} 2\cos(2n\pi t/3)dt\right]$$

$$= \frac{4}{3}\left[\frac{3}{2n\pi}\sin(2n\pi t/3)\Big|_0^1 + \frac{6}{2n\pi}\sin(2n\pi t/3)\Big|_1^{1.5}\right] = -\frac{2}{n\pi}\sin(2n\pi/3)$$

$$v_s(t) = \frac{4}{3} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2n\pi/3)\cos(2n\pi t/3)$$

Now consider this circuit,



Let Z = [-j3/(2nπ)](1)/(1 – j3/(2nπ)) = -j3/(2nπ - j3)

Therefore, v_o = $Zv_s/(Z + 1 + j2n\pi/3)$. Simplifying, we get

$$v_o = \frac{-j9v_s}{12n\pi + j(4n^2\pi^2 - 18)}$$

For the dc case, n = 0 and v_s = ¾ V and $v_o = v_s/2$ = 3/8 V.

We can now solve for $v_o(t)$

$$v_o(t) = \left[\frac{3}{8} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2n\pi t}{3} + \Theta_n\right)\right] \text{volts}$$

where $$A_n = \frac{\frac{6}{n\pi}\sin(2n\pi/3)}{\sqrt{16n^2\pi^2 + \left(\frac{4n^2\pi^2}{3} - 6\right)^2}} \text{ and } \Theta_n = 90^o - \tan^{-1}\left(\frac{n\pi}{3} - \frac{3}{2n\pi}\right)$$

where we can further simplify A_n to this, $$A_n = \frac{9\sin(2n\pi/3)}{n\pi\sqrt{4n^4\pi^4 + 81}}$$

Solution 17.36

We first find the Fourier series expansion of v_s. $T = 1, \quad \omega_o = 2\pi / T = 2\pi$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\int_0^1 10(1-t)tdt = 10(t - \frac{t^2}{2})\Big|_1^0 = 5$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o tdt = 2\int_0^1 10(1-t)\cos 2n\pi tdt$$

$$= 20\left[\frac{1}{2\pi n}\sin 2n\pi t - \frac{1}{4n^2\pi^2}\cos 2n\pi t - \frac{t}{2n\pi}\sin 2n\pi t\right]\Big|_o^1 = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o tdt = \frac{2}{2}\int_0^1 10(1-t)t\sin n\omega_o tdt$$

$$= 20\left[-\frac{1}{2n\pi}\cos 2n\pi t - \frac{1}{4n^2\pi^2}\sin 2n\pi t + \frac{1}{2n\pi}\cos 2n\pi t\right]\Big|_0^1 = \frac{10}{n\pi}$$

$$v_s(t) = 5 + \sum_{n=1}^{\infty}\frac{10}{n\pi}\sin 2n\pi t$$

$$1H \quad \longrightarrow \quad j\omega_n L = j\omega_n$$

$$10mF \quad \longrightarrow \quad \frac{1}{j\omega_n C} = \frac{1}{j\omega_n 0.01} = \frac{-j100}{\omega_n}$$

$$I_o = \frac{V_s}{5 + j\omega_n - \dfrac{j100}{\omega_n}}$$

For dc component, $\omega_0 = 0$ which leads to $I_0 = 0$.

For the nth harmonic,

$$\mathbf{I_n} = \frac{\frac{\mathbf{10}}{\mathbf{n\pi}}\angle\mathbf{0°}}{\mathbf{5 + j2n\pi - \frac{j100}{2n\pi}}} = \frac{\mathbf{10}}{\mathbf{5n\pi + j(2n^2\pi^2 - 50)}} = \mathbf{A_n\angle\phi_n}$$

where

$$A_n = \frac{10}{\sqrt{25n^2\pi^2 + (2n^2\pi^2 - 50)^2}}, \qquad \phi_n = -\tan^{-1}\frac{2n^2\pi^2 - 50}{5n\pi}$$

$$i_o(t) = \underline{\sum_{n=1}^{\infty} A_n \sin(2n\pi t + \phi_n)}$$

Solution 17.37

We first need to express i_s in Fourier series. $T = 2, \quad \omega_o = 2\pi / T = \pi$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\left[\int_0^1 3dt + \int_1^2 1dt\right] = \frac{1}{2}(3+1) = 2$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t dt = \frac{2}{2}\left[\int_0^1 3\cos n\pi t dt + \int_1^2 \cos n\pi t dt\right] = \frac{3}{n\pi}\sin n\pi t\Big|_0^1 + \frac{1}{n\pi}\sin n\pi t\Big|_1^2 = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t dt = \frac{2}{2}\left[\int_0^1 3\sin n\pi t dt + \int_1^2 \sin n\pi t dt\right] =$$

$$\frac{-3}{n\pi}\cos n\pi t\Big|_0^1 + \frac{-1}{n\pi}\cos n\pi t\Big|_1^2 = \frac{2}{n\pi}(1-\cos n\pi)$$

$$i_s(t) = 2 + \sum_{n=1}^{\infty}\frac{2}{n\pi}(1-\cos n\pi)\sin n\pi t$$

By current division,

$$I_o = \frac{1}{1+2+j\omega_n L}I_s = \frac{I_s}{3+j3\omega_n}$$

$$V_o = j\omega_n L I_o = \frac{j\omega_n 3 I_s}{3+j3\omega_n} = \frac{j\omega_n I_s}{1+j\omega_n}$$

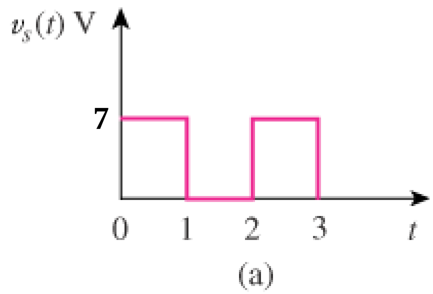
For dc component (n=0), $V_o = 0$.
For the nth harmonic,

$$V_o = \frac{jn\pi}{1+jn\pi}\frac{2}{n\pi}(1-\cos n\pi)\angle -90^\circ = \frac{2(1-\cos n\pi)}{\sqrt{1+n^2\pi^2}}\angle(90^\circ - \tan^{-1} n\pi - 90^\circ)$$

$$\underline{v_o(t) = \sum_{n=1}^{\infty}\frac{2(1-\cos \pi n)}{\sqrt{1+n^2\pi^2}}\cos(n\pi t - \tan^{-1} n\pi)}$$

Solution 17.38

If the square wave shown in Fig. 17.74(a) is applied to the circuit in Fig. 17.74(b), find the Fourier series for $v_o(t)$.



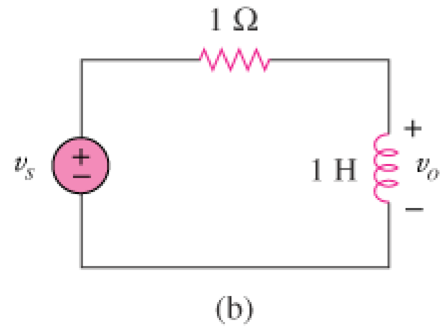


Figure 17.74 For Prob. 17.38.

Solution

$$v_s(t) = 3.5 + \frac{14}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin n\pi t, \qquad n = 2k+1$$

$$V_{on} = \frac{j\omega_n}{1+j\omega_n}V_s, \qquad \omega_n = n\pi$$

For dc, $\omega_n = 0, \quad V_s = 3.5, \quad V_o = 0$

For nth harmonic, $V_s = \frac{14}{n\pi}\angle -90^o$

$$V_{on} = \frac{n\pi\angle 90^o}{\sqrt{1+n^2\pi^2}\angle \tan^{-1} n\pi} \bullet \frac{14}{n\pi}\angle -90^o = \frac{14\angle -\tan^{-1} n\pi}{\sqrt{1+n^2\pi^2}}$$

$$\underline{v_o(t) = \sum_{k=1}^{\infty}\frac{14}{\sqrt{1+n^2\pi^2}}\cos(n\pi t - \tan^{-1} n\pi), \qquad n = 2k-1}$$

Solution 17.39

If the periodic voltage in Fig. 17.75(a) is applied to the circuit in Fig. 17.75(b), find $i_o(t)$.

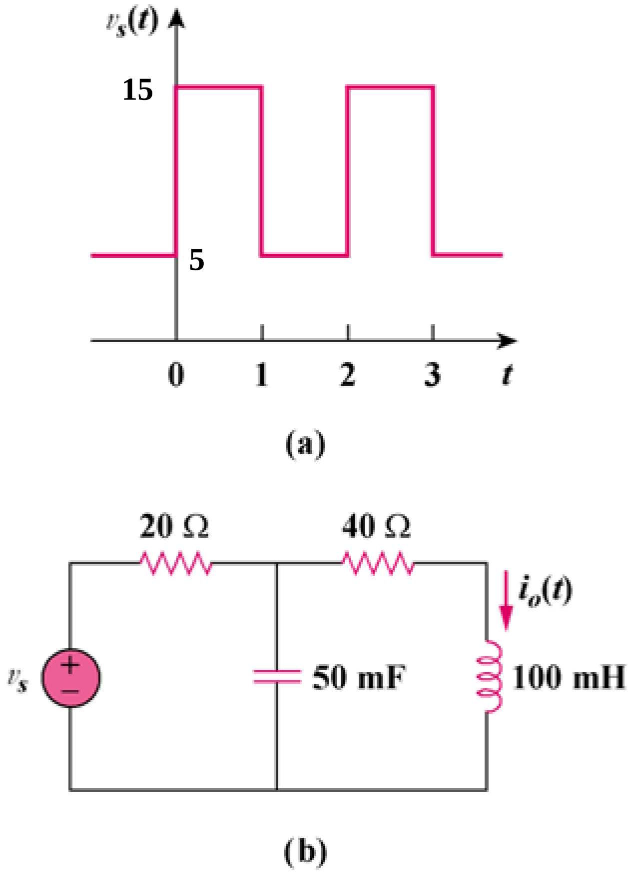


Figure 17.75 For Prob. 17.39.

Solution

Comparing $v_s(t)$ with f(t) in Figure 15.1, v_s is shifted by 2.5 and the magnitude is 5 times that of f(t).

Hence

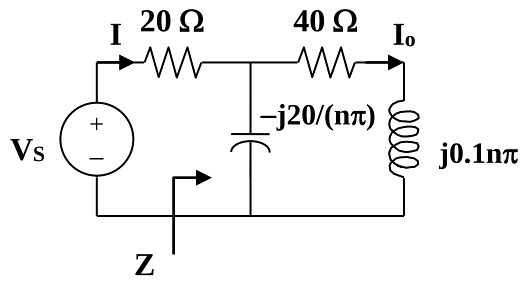
$$v_s(t) = 10 + \frac{20}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin(n\pi t), \qquad n = 2k - 1$$

$$T = 2, \ \omega_o = 2\pi//T = \pi, \ \omega_n = n\omega_o = n\pi$$

For the DC component, $\quad i_o = 10/(20 + 40) = 1/6$

For the kth harmonic, $\quad V_s = (20/(n\pi))\angle 0°$

100 mH becomes $j\omega_n L = jn\pi x0.1 = j0.1n\pi$
50 mF becomes $1/(j\omega_n C) = -j20/(n\pi)$



Let $Z = -j20/(n\pi)||(40 + j0.1n\pi) = \dfrac{-\dfrac{j20}{n\pi}(40 + j0.1n\pi)}{-\dfrac{j20}{n\pi} + 40 + j0.1n\pi}$

$$= \frac{-j20(40 + j0.1n\pi}{-j20 + 40n\pi + j0.1n^2\pi^2} = \frac{2n\pi - j800}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$Z_{in} = 20 + Z = \frac{802n\pi + j(2n^2\pi^2 - 1200)}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$I = \frac{V_s}{Z_{in}} = \frac{800n\pi + j2(n^2\pi^2 - 200)}{n\pi[802n\pi + j(2n^2\pi^2 - 1200)]}$$

$$I_o = \frac{-\dfrac{j20}{n\pi}I}{-\dfrac{j20}{n\pi} + (40 + j0.1n\pi)} = \frac{-j20I}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$= \frac{-j400}{n\pi[802n\pi + j(2n^2\pi^2 - 1200)]}$$

$$= \frac{400\angle -90° - \tan^{-1}\{(2n^2\pi^2 - 1200)/(802n\pi)\}}{n\pi\sqrt{(802)^2 + (2n^2\pi^2 - 1200)^2}}$$

Thus

$$i_o(t) = \left[\frac{1}{10} + \frac{400}{\pi}\sum_{k=1}^{\infty} I_n \sin(n\pi t - \theta_n)\right]A, \quad \mathbf{n = 2k - 1}$$

where $\boldsymbol{\theta_n = 90° + \tan^{-1}\dfrac{2n^2\pi^2 - 1200}{802n\pi}}$

$$I_n = \frac{2}{n\sqrt{(804n\pi)^2 + (2n^2\pi^2 - 1200)}}\,\text{A}$$

Solution 17.40

The signal in Fig. 17.76(a) is applied to the circuit in Fig. 17.76(b). Find $v_o(t)$.

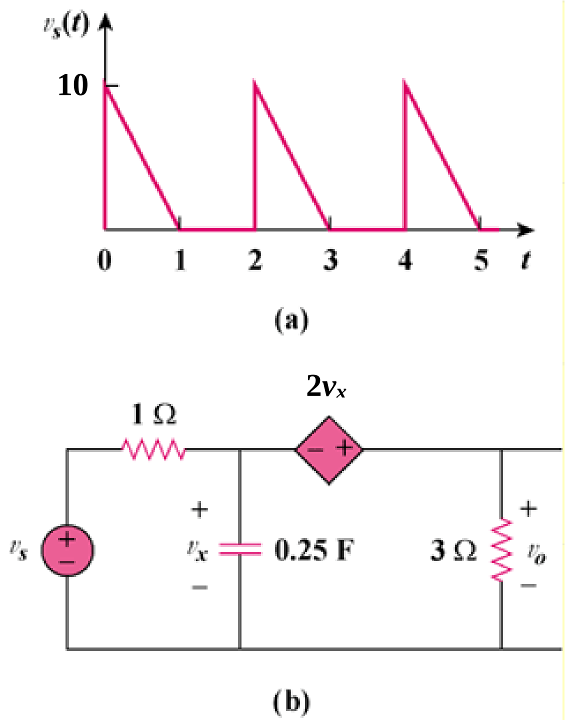


Figure 17.76
For Prob. 17.40.

Solution

T = 2, ω_o = $2\pi/T$ = π

$$a_o = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{2}\int_0^1 (10-10t)dt = 5\left[t-\frac{t^2}{2}\right]\Bigg|_0^1 = 2.5$$

$$a_n = \frac{2}{T}\int_0^T v(t)\cos(n\pi t)dt = \int_0^1 10(1-t)\cos(n\pi t)dt$$

$$= 10\left[\frac{1}{n\pi}\sin(n\pi t) - \frac{1}{n^2\pi^2}\cos(n\pi t) - \frac{t}{n\pi}\sin(n\pi t)\right]\Bigg|_0^1$$

$$= \frac{10}{n^2\pi^2}(1-\cos n\pi) = \begin{vmatrix} 0, & n = even \\ \dfrac{20}{n^2\pi^2}, & n = odd \end{vmatrix} = \frac{20}{\pi^2(2n-1)^2}$$

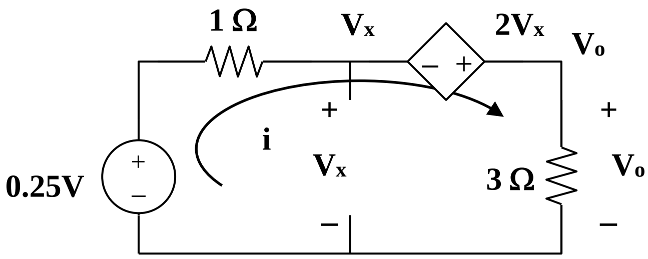
$$b_n = \frac{10}{T}\int_0^T v(t)\sin(n\pi t)dt = 10\int_0^1 (1-t)\sin(n\pi t)dt$$

$$= 10\left[-\frac{1}{n\pi}\cos(n\pi t) - \frac{1}{n^2\pi^2}\sin(n\pi t) + \frac{t}{n\pi}\cos(n\pi t)\right]\Bigg|_0^1 = \frac{10}{n\pi}$$

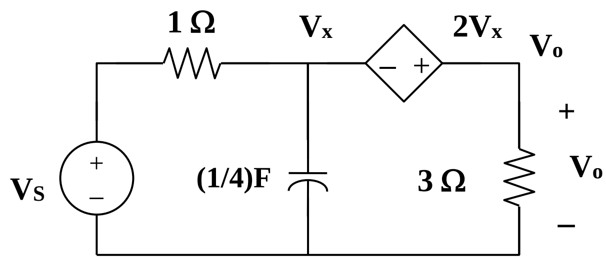
$$v_s(t) = 2.5 + \sum A_n \cos(n\pi t - \varphi_n)$$

where $\phi_n = \tan^{-1}\frac{\pi(2n-1)^2}{2n}$, $A_n = \sqrt{\frac{100}{n^2\pi^2} + \frac{400}{\pi^4(2n-1)^4}}$

For the DC component, $v_s = 2.5$. As shown in Figure (a), the capacitor acts like an open circuit.



(a)



(b)

Applying KVL to the circuit in Figure (a) gives

$$-2.5 - 2V_x + 4i = 0 \qquad (1)$$

But $\quad -2.5 + i + V_x = 0$ or $-5 + 2V_x + 2i = 0 \qquad (2)$

Adding (1) and (2), $\quad -7.5 + 6i = 0$ or $i = 1.25$

$$V_o = 3i = 3.75$$

For the nth harmonic, we consider the circuit in Figure (b).

$$\omega_n = n\pi,\ V_s = A_n\angle-\phi,\ 1/(j\omega_n C) = -j4/(n\pi)$$

At the supernode,

$$(V_s - V_x)/1 = -[n\pi/(j4)]V_x + V_o/3$$

$$V_s \;\; = [1 + jn\pi/4]V_x + V_o/3 \qquad (3)$$

But $\quad -V_x - 2V_x + V_o = 0$ or $\;\; V_o = 3V_x$

Substituting this into (3),

$$V_s = [1 + jn\pi/4]V_x + V_x \; = \; [2 + jn\pi/4]V_x$$

$$= \; (1/3)[2 + jn\pi/4]V_o \; = \; (1/12)[8 + jn\pi]V_o$$

$$V_o \; = \; 12V_s/(8 + jn\pi) \; = \; \frac{12A_n\angle -\phi}{\sqrt{64+n^2\pi^2}\angle \tan^{-1}(n\pi/8)}$$

$$V_o \; = \; \frac{12}{\sqrt{64+n^2\pi^2}}\sqrt{\frac{100}{n^2\pi^2}+\frac{400}{\pi^4(2n-1)^4}}\angle[\tan^{-1}(n\pi/8)-\tan^{-1}(\pi(2n-1)^2/(2n))]$$

Thus

$$v_o(t) \; = \; 3.75+\sum_{n=1}^{\infty}V_n\cos(n\pi t+\theta_n)\,\textbf{volts}$$

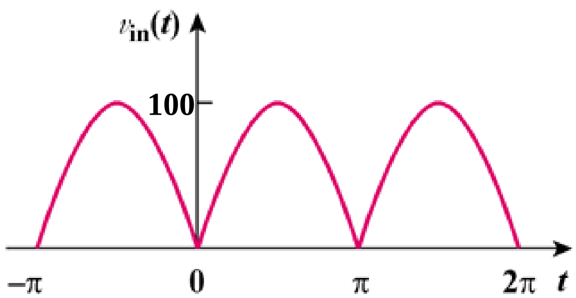
where

$$V_n \; = \; \frac{120}{\sqrt{64+n^2\pi^2}}\sqrt{\frac{1}{n^2\pi^2}+\frac{4}{\pi^4(2n-1)^4}} \;\text{ and}$$

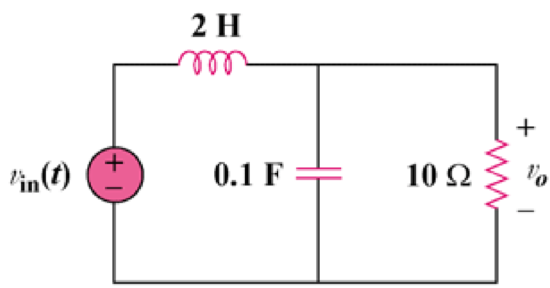
$$\theta_n \; = \; \mathbf{\tan^{-1}(n\pi/8) - \tan^{-1}(\pi(2n-1)^2/(2n))}$$

Solution 17.41

The full-wave rectified sinusoidal voltage in Fig. 17.77(a) is applied to the lowpass filter in Fig. 17.77(b). Obtain the output voltage $v_o(t)$ of the filter.



(a)



(b)

Figure 17.77
For Prob. 17.41.

Solution

For the full wave rectifier,

$$T = \pi, \ \omega_o = 2\pi/T = 2, \ \omega_n = n\omega_o = 2n$$

Hence

$$v_{in}(t) = \left[\frac{200}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{100}{4n^2-1}\cos(2nt)\right] volts$$

For the DC component,

$$V_{in} = 200/\pi$$

The inductor acts like a short-circuit, while the capacitor acts like an open circuit.

$$V_o = V_{in} = 200/\pi$$

For the nth harmonic,

$V_{in} = [-400/(\pi(4n^2 - 1))]\angle 0°$
2 H becomes $j\omega_n L = j4n$
0.1 F becomes $1/(j\omega_n C) = -j5/n$
$Z = 10||(-j5/n) = -j10/(2n - j)$
$V_o = [Z/(Z + j4n)]V_{in} = -j10V_{in}/(4 + j(8n - 10))$

$$= -\frac{j10}{4 + j(8n - 10)}\left(-\frac{400\angle 0°}{\pi(4n^2 - 1)}\right)$$

$$= \frac{4{,}000\angle\{90° - \tan^{-1}(2n - 2.5)\}}{\pi(4n^2 - 1)\sqrt{16 + (8n - 10)^2}}$$

Hence

$$v_o(t) = \left[\frac{200}{\pi} + \sum_{n=1}^{\infty} A_n \cos(2nt + \theta_n)\right] \textbf{volts}$$

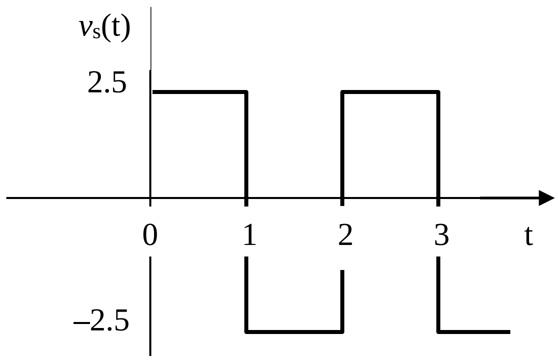
where

$$A_n = \frac{2{,}000}{\pi(4n^2 - 1)\sqrt{16n^2 - 40n + 29}}$$

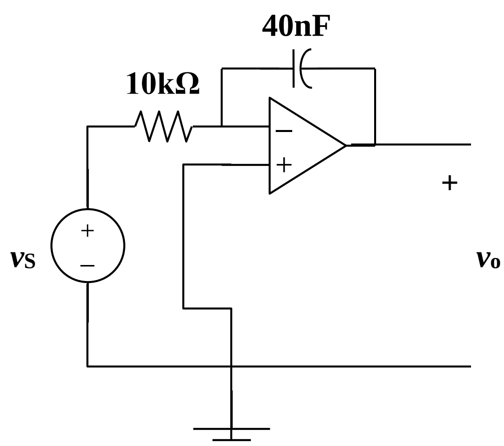
$$\theta_n = \mathbf{90° - \tan^{-1}(2n - 2.5)}$$

Solution 17.42

When the square wave in Fig. 17.78(a) is applied to the circuit in Fig. 17.78(b), find the Fourier series of $v_o(t)$



(a)



(b)

Figure 17.78
For Problem 17.42

Solution

$$v_s = \frac{10}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin n\pi t,\ \ \mathrm{n = 2k - 1}$$

$$\frac{V_s - 0}{R} = j\omega_n C(0 - V_o) \quad\longrightarrow\quad V_o = \frac{j}{\omega_n RC}V_s,\ \ \omega_n = n\omega_o = n\pi$$

For n = 0 (dc component), $V_o = 0$.

For the nth harmonic,

$$V_o = \frac{1\angle 90^o}{n\pi RC}\frac{10}{n\pi}\angle -90^o = \frac{10}{n^2\pi^2 x10^4 x40x10^{-9}} = \frac{2.5x10^4}{n^2\pi^2}$$

Hence,

$$v_o(t) = \underline{\frac{2.5x10^4}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{n^2}\cos(n\pi t)V, \ \text{n = 2k -1}}$$

Alternatively, we notice that this is an integrator so that

$$v_o(t) = -\frac{1}{RC}\int v_s dt = \underline{\frac{2.5x10^4}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{n^2}\cos n\pi t V, \ \text{n = 2k -1}}$$

Solution 17.43

(a) $$V_{rms} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)} = \sqrt{30^2 + \frac{1}{2}(20^2 + 10^2)} = \mathbf{33.91\ V}$$

(b) $$I_{rms} = \sqrt{6^2 + \frac{1}{2}(4^2 + 2^2)} = \mathbf{6.782\ A}$$

(c) $$P = V_{dc}I_{dc} + \frac{1}{2}\sum V_n I_n \cos(\Theta_n - \Phi_n)$$

$$= 30x6 + 0.5[20x4\cos(45^o\text{-}10^o) – 10x2\cos(\text{-}45^o\text{+}60^o)]$$

$$= 180 + 32.76 – 9.659 = \mathbf{203.1\ W}$$

Solution 17.44

Design a problem to help other students to better understand how to find the rms voltage across and the rms current through an electrical element given a Fourier series for both the current and the voltage. In addition, have them calculate the average power delivered to the element and the power spectrum.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The voltage and current through an element are respectively

$$v(t) = [30 \cos (t + 35°) + 10\cos(2t – 55°) + 4 \cos(3t - 10°)] \text{ V}$$

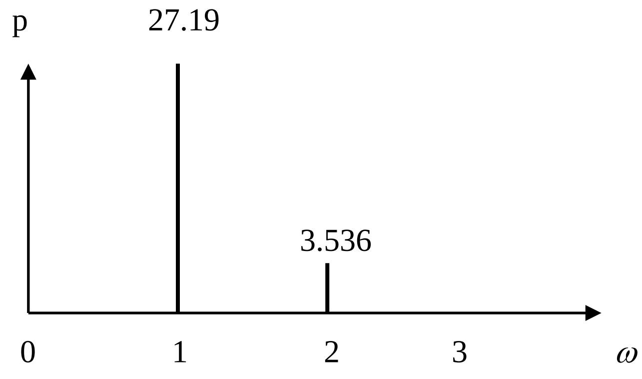
$$i(t) = [2 \cos (t–10°) + \cos(2t – 10°)] \text{ A}$$

(a) Find the average power delivered to the element
(b) Plot the power spectrum.

Solution

(a) $\quad p = vi = \frac{1}{2}\left[60\cos(-25^\circ) + 10\cos 45^\circ + 0\right] = 27.19 + 3.536 =$ **30.73 W**.

(b) The power spectrum is shown below.



Solution 17.45

$\omega_n = 1000n$

$$j\omega_n L = j1000n\text{x}2\text{x}10^{-3} = j2n$$

$$1/(j\omega_n C) = -j/(1000n\text{x}40\text{x}10^{-6}) = -j25/n$$

$$Z = R + j\omega_n L + 1/(j\omega_n C) = 10 + j2n - j25/n$$

$$I = V/Z$$

For $n = 1$, $V_1 = 100$, $Z = 10 + j2 - j25 = 10 - j23$

$$I_1 = 100/(10 - j23) = 3.987\angle 73.89°$$

For $n = 2$, $V_2 = 50$, $Z = 10 + j4 - j12.5 = 10 - j8.5$

$$I_2 = 50/(10 - j8.5) = 3.81\angle 40.36°$$

For $n = 3$, $V_3 = 25$, $Z = 10 + j6 - j25/3 = 10 - j2.333$

$$I_3 = 25/(10 - j2.333) = 2.435\angle 13.13°$$

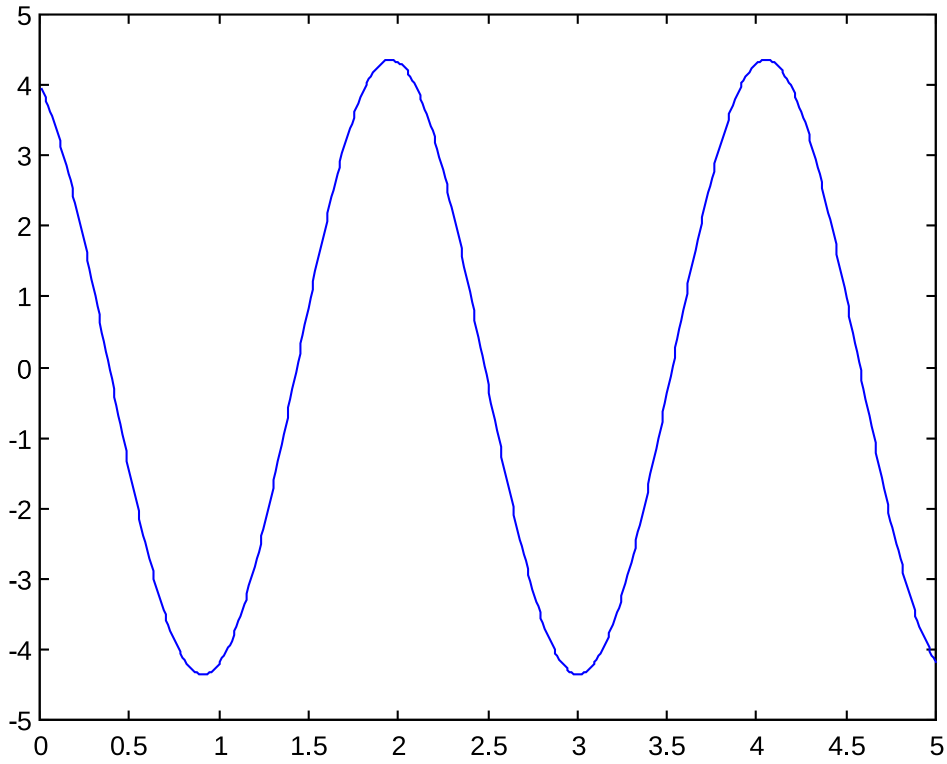
$$I_{rms} = \sqrt{0.5(3.987^2 + 3.81^2 + 2.435^2)} = \mathbf{4.263\ A}$$

$$p = R(I_{rms})^2 = \mathbf{181.7W}$$

Solution 17.46

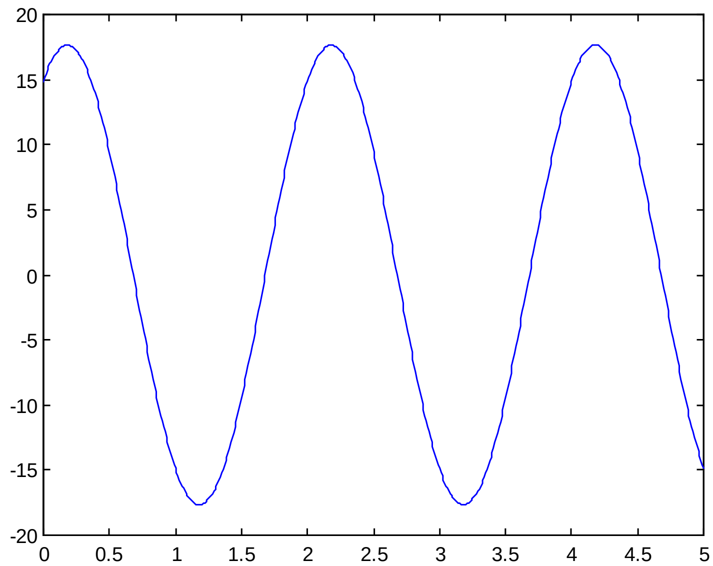
(a)The MATLAB commands are:

```
t=0:0.01:5;
y=5*cos(3*t) – 2*cos(3*t-pi/3);
plot(t,y)
```



(b) The MATLAB commands are:

```
t=0:0.01:5;
» x=8*sin(pi*t+pi/4)+10*cos(pi*t-pi/8);
» plot(t,x)
» plot(t,x)
```


20
15
10
5
0
-5
-10
-15
-20
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Solution 17.47

$$T = 2, \qquad \omega_o = 2\pi / T = \pi$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\left[\int_0^1 4dt + \int_1^2 (-2)dt\right] = \frac{1}{2}(4-2) = 1$$

$$P = Ri_{rms}^2 = \frac{R}{T}\int_0^T f^2(t)dt = \frac{R}{2}\left[\int_0^1 4^2\,dt + \int_1^2 (-2)^2\,dt\right] = 10R$$

The average power dissipation caused by the dc component is

$$P_0 = Ra_o^2 = R = \underline{10\%} \text{ of P}$$

Solution 17.48

(a) For the DC component, $i(t) = 20$ mA. The capacitor acts like an open circuit so that

$$v = Ri(t) = 2x10^3x20x10^{-3} = 40$$

For the AC component,

$$\omega_n = 10n, \ n = 1,2$$

$$1/(j\omega_nC) = -j/(10nx100x10^{-6}) = (-j/n)\ k\Omega$$

$$Z = 2||(-j/n) = 2(-j/n)/(2 - j/n) = -j2/(2n - j)$$

$$V = ZI = [-j2/(2n - j)]I$$

For $n = 1$, $$V_1 = [-j2/(2 - j)]16\angle 45^\circ = 14.311\angle -18.43^\circ \text{ mV}$$

For $n = 2$, $$V_2 = [-j2/(4 - j)]12\angle -60^\circ = 5.821\angle -135.96^\circ \text{ mV}$$

$$v(t) = \mathbf{40 + 0.014311cos(10t - 18.43^\circ) + 0.005821cos(20t - 135.96^\circ)\ V}$$

(b) $$p = V_{DC}I_{DC} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$$

$$= 20x40 + 0.5x10x0.014311cos(45^\circ + 18.43^\circ)$$
$$+0.5x12x0.005821cos(-60^\circ + 135.96^\circ)$$

$$= \mathbf{800.1\ mW}$$

Solution 17.49

(a) $$Z^2{}_{rms} = \frac{1}{T}\int_0^T z^2(t)dt = \frac{1}{2\pi}\left[\int_0^{\pi} 4dt + \int_{\pi}^{2\pi} 16dt\right] = \frac{1}{2\pi}(20\pi) = 10$$

Z_{rms} = **3.162**

(b)

$$Z^2{}_{rms} = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2) = 1 + \frac{1}{2}\sum_{\substack{n=1\\ n=odd}}^{\infty}\frac{144}{n^2\pi^2} = 1 + \frac{72}{\pi^2}\left(1 + 0 + \frac{1}{9} + 0 + \frac{1}{25} + ...\right) = 9.396$$

Z_{rms} = **3.065**

(c) $$\%\text{error} = \left(1 - \frac{3.065}{3.162}\right)x100 = \mathbf{3.068\%}$$

Solution 17.50

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt, \quad \omega_o = \frac{2n}{1} = \pi$$

$$= \frac{1}{2}\int_{-}^{1} te^{-jn\pi t}dt$$

Using integration by parts,

$$u = t \text{ and } du = dt$$

$$dv = e^{-jn\pi t}dt \text{ which leads to } v = -[1/(2jn\pi)]e^{-jn\pi t}$$

$$c_n = -\frac{t}{2jn\pi}e^{-jn\pi t}\Big|_{-1}^{1} + \frac{1}{2jn\pi}\int_{-1}^{1} e^{-jn\pi t}dt$$

$$= \frac{j}{n\pi}\left[e^{-jn\pi} + e^{jn\pi t}\right] + \frac{1}{2n^2\pi^2(-j)^2}e^{-jn\pi t}\Big|_{-1}^{1}$$

$$= [j/(n\pi)]\cos(n\pi) + [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})$$

$$c_n = \frac{j(-1)^n}{n\pi} + \frac{2j}{2n^2\pi^2}\sin(n\pi) = \frac{j(-1)^n}{n\pi}$$

Thus

$$f(t) = \sum_{n=-\infty}^{\infty} \mathbf{c_n e^{jn\omega_o t}} = \sum_{n=-\infty}^{\infty} \mathbf{(-1)^n \frac{j}{n\pi} e^{jn\pi t}}$$

Solution 17.51

Design a problem to help other students to better understand how to find the exponential Fourier series of a given periodic function.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Given the periodic function

$$f(t) = t^2, \qquad 0 < t < T$$

obtain the exponential Fourier series for the special case $T = 2$.

Solution

$$T = 2, \qquad \omega_o = 2\pi/T = \pi$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{1}{2}\int_0^2 t^2 e^{-jn\pi t}dt = \frac{1}{2}\frac{e^{-jn\pi t}}{(-jn\pi)^3}\left(-n^2\pi^2 t^2 + 2jn\pi t + 2\right)\Big|_0^2$$

$$c_n = \frac{1}{j2n^3\pi^3}(-4n^2\pi^2 + j4n\pi) = \frac{2}{n^2\pi^2}(1 + jn\pi)$$

$$\mathbf{f(t) = \underline{\sum_{n=-\infty}^{\infty} \frac{2}{n^2\pi^2}(1 + jn\pi)e^{jn\pi t}}}$$

Solution 17.52

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt, \quad \omega_o = \frac{2n}{1} = \pi$$

$$= \frac{1}{2}\int_{-}^{1} te^{-jn\pi t}dt$$

Using integration by parts,

$$u = t \text{ and } du = dt$$

$$dv = e^{-jn\pi t}dt \text{ which leads to } v = -[1/(2jn\pi)]e^{-jn\pi t}$$

$$c_n = -\frac{t}{2jn\pi}e^{-jn\pi t}\Big|_{-1}^{1} + \frac{1}{2jn\pi}\int_{-1}^{1} e^{-jn\pi t}dt$$

$$= \frac{j}{n\pi}\left[e^{-jn\pi} + e^{jn\pi}\right] + \frac{1}{2n^2\pi^2(-j)^2}e^{-jn\pi t}\Big|_{-1}^{1}$$

$$= [j/(n\pi)]\cos(n\pi) + [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})$$

$$c_n = \frac{j(-1)^n}{n\pi} + \frac{2j}{2n^2\pi^2}\sin(n\pi) = \frac{j(-1)^n}{n\pi}$$

Thus

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t} = \boldsymbol{\sum_{n=-\infty}^{\infty}(-1)^n \frac{j}{n\pi} e^{jn\pi t}}$$

Solution 17.53

$\omega_o = 2\pi/T = 2\pi$

$$c_n = \int_0^T e^{-t} e^{-jn\omega_o t} dt = \int_0^1 e^{-(1+jn\omega_o)t} dt$$

$$= \frac{-1}{1+j2\pi n} e^{-(1+j2n\pi)t}\Big|_0^1 = \frac{-1}{1+j2n\pi}\left[e^{-(1+j2n\pi)} - 1\right]$$

$= [1/(j2n\pi)][1 - e^{-1}(\cos(2\pi n) - j\sin(2n\pi))]$
$= (1 - e^{-1})/(1 + j2n\pi) = 0.6321/(1 + j2n\pi$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{\mathbf{0.6321e^{j2n\pi t}}}{\mathbf{1 + j2n\pi}}$$

Solution 17.54

$T = 4, \ \omega_o = 2\pi/T = \pi/2$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt$$

$$= \frac{1}{4}\left[\int_0^1 2e^{-jn\pi t/2}dt + \int_1^2 1e^{-jn\pi t/2}dt - \int_2^4 1e^{-jn\pi t/2}dt\right]$$

$$= \frac{j}{2n\pi}\left[2e^{-jn\pi/2} - 2 + e^{-jn\pi} - e^{-jn\pi/2} - e^{-j2n\pi} + e^{-jn\pi}\right]$$

$$= \mathbf{\frac{j}{2n\pi}\left[3e^{-jn\pi/2} - 3 + 2e^{-jn\pi}\right]}$$

$$f(t) = \sum_{n=-\infty}^{\infty} \mathbf{c_n e^{jn\omega_o t}}$$

Solution 17.55

$T = 2\pi, \ \omega_o = 2\pi/T = 1$

$$c_n = \frac{1}{T}\int_0^T i(t)e^{-jn\omega_o t}dt$$

But $$i(t) = \begin{cases} \sin(t), & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

$$c_n = \frac{1}{2\pi}\int_0^{\pi}\sin(t)e^{-jnt}dt = \frac{1}{2\pi}\int_0^{\pi}\frac{1}{2j}(e^{jt} - e^{-jt})e^{-jnt}dt$$

$$= \frac{1}{4\pi j}\left[\frac{e^{jt(1-n)}}{j(1-n)} + \frac{e^{-jt(1+n)}}{j(1+n)}\right]\Bigg|_0^{\pi}$$

$$= -\frac{1}{4\pi}\left[\frac{e^{j\pi(1-n)} - 1}{1-n} + \frac{e^{-j\pi(n+1)} - 1}{1+n}\right]$$

$$= \frac{1}{4\pi(n^2 - 1)}\left[e^{j\pi(1-n)} - 1 + ne^{j\pi(1-n)} - n + e^{-j\pi(1+n)} - 1 - ne^{-j\pi(1+n)} + n\right]$$

But $e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1 = e^{-j\pi}$

$$c_n = \frac{1}{4\pi(n^2 - 1)}\left[-e^{-jn\pi} - e^{-jn\pi} - ne^{-jn\pi} + ne^{-jn\pi} - 2\right] = \frac{1 + e^{-jn\pi}}{2\pi(1 - n^2)}$$

Thus

$$i(t) = \sum_{n=-\infty}^{\infty}\frac{\mathbf{1 + e^{-jn\pi}}}{\mathbf{2\pi(1 - n^2)}}\mathbf{e^{jnt}}$$

Solution 17.56

$c_o = a_o = 10, \ \omega_o = \pi$

$c_o = (a_n - jb_n)/2 = (1 - jn)/[2(n^2 + 1)]$

$$\mathbf{f(t) = 10 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{(1-jn)}{2(n^2+1)} e^{jn\pi t}}$$

Solution 17.57

$a_o = (6/–2) = –3 = c_o$

$c_n = 0.5(a_n – jb_n) = a_n/2 = 3/(n^3 – 2)$

$$f(t) = -3 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{3}{n^3 - 2} e^{j50nt}$$

Solution 17.58

$c_n = (a_n - jb_n)/2$, $(-1)^n = \cos(n\pi)$, $\omega_o = 2\pi/T = 1$

$c_n = [(\cos(n\pi) - 1)/(2\pi n^2)] - j\cos(n\pi)/(2n)$

Thus

$$f(t) = \frac{\pi}{4} + \sum \left(\frac{\cos(n\pi) - 1}{2\pi n^2} - j\frac{\cos(n\pi)}{2n} \right) e^{jnt}$$

Solution 17.59

For f(t), $T = 2\pi$, $\omega_o = 2\pi/T = 1$.

$$a_o = \text{DC component} = (1x\pi + 0)/2\pi = 0.5$$

For h(t), $T = 2$, $\omega_o = 2\pi/T = \pi$.

$$a_o = (2x1 - 2x1)/2 = 0$$

Thus by replacing $\omega_o = 1$ with $\omega_o = \pi$ and multiplying the magnitude by four, we obtain

$$h(t) = -\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{j4e^{-j(2n+1)\pi t}}{(2n+1)\pi}$$

Solution 17.60

From Problem 17.24,

$a_o = 0 = a_n,\ \ b_n = [2/(n\pi)][1 - 2\cos(n\pi)],\ \ c_o = \mathbf{0}$

$c_n = (a_n - jb_n)/2 = \mathbf{[j/(n\pi)][2\cos(n\pi) - 1],\ \ n \neq 0}$.

Solution 17.61

(a) $\omega_o = 1.$

$$f(t) = a_o + \sum A_n \cos(n\omega_o t - \phi_n)$$

$$= 6 + 4\cos(t + 50°) + 2\cos(2t + 35°) + \cos(3t + 25°) + 0.5\cos(4t + 20°)$$

$$= 6 + 4\cos(t)\cos(50°) - 4\sin(t)\sin(50°) + 2\cos(2t)\cos(35°) - 2\sin(2t)\sin(35°) + \cos(3t)\cos(25°) - \sin(3t)\sin(25°) + 0.5\cos(4t)\cos(20°) - 0.5\sin(4t)\sin(20°)$$

$$= \mathbf{6 + 2.571\cos(t) - 3.73\sin(t) + 1.635\cos(2t) - 1.147\sin(2t) + 0.906\cos(3t) - 0.423\sin(3t) + 0.47\cos(4t) - 0.171\sin(4t)}$$

(b) $$f_{rms} = \sqrt{a_o^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2}$$

$$f_{rms}^2 = 6^2 + 0.5[4^2 + 2^2 + 1^2 + (0.5)^2] = 46.625$$

$$f_{rms} = \mathbf{6.828}$$

Solution 17.62

(a)

$$f(t) = 12 + 10\cos(2\omega_o t + 90^o) + 8\cos(4\omega_o t - 90^o) + 5\cos(6\omega_o t + 90^o) + 3\cos(8\omega_o t - 90^o)$$

(b) f(t) is an **even** function of t.

Solution 17.63

This is an even function.

$T = 3,\ \omega_o = 2\pi/3,\ b_n = 0.$

$$f(t) = \begin{vmatrix} 1, & 0 < t < 1 \\ 2, & 1 < t < 1.5 \end{vmatrix}$$

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{3}\left[\int_0^1 1dt + \int_1^{1.5} 2\,dt\right] = (2/3)[1 + 1] = 4/3$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{3}\left[\int_0^1 1\cos(2n\pi t/3)dt + \int_1^{1.5} 2\cos(2n\pi t/3)dt\right]$$

$$= \frac{4}{3}\left[\frac{3}{2n\pi}\sin\left(\frac{2n\pi t}{3}\right)\bigg|_0^1 + \frac{6}{2n\pi}\sin\left(\frac{2n\pi t}{3}\right)\bigg|_1^{1.5}\right]$$

$$= [-2/(n\pi)]\sin(2n\pi/3)$$

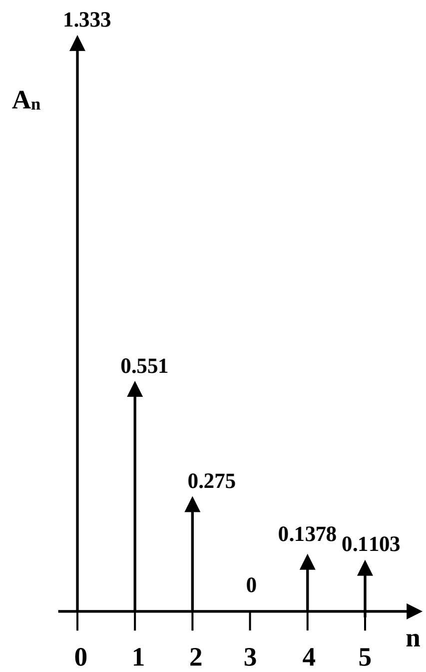
$$f_2(t) = \frac{4}{3} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{3n\pi}{3}\right)\cos\left(\frac{2n\pi t}{3}\right)$$

$a_o = 4/3 = 1.3333,\ \omega_o = 2\pi/3,\ a_n = -[2/(n\pi)]\sin(2n\pi t/3)$

$$A_n = \sqrt{a_n^2 + b_n^2} = \left|\frac{2}{n\pi}\sin\left(\frac{2n\pi}{3}\right)\right|$$

$A_1 = 0.5513,\ A_2 = 0.2757,\ A_3 = 0,\ A_4 = 0.1375,\ A_5 = 0.1103$

The amplitude spectra are shown below.



Solution 17.64

Design a problem to help other students to better understand the amplitude and phase spectra of a given Fourier series.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

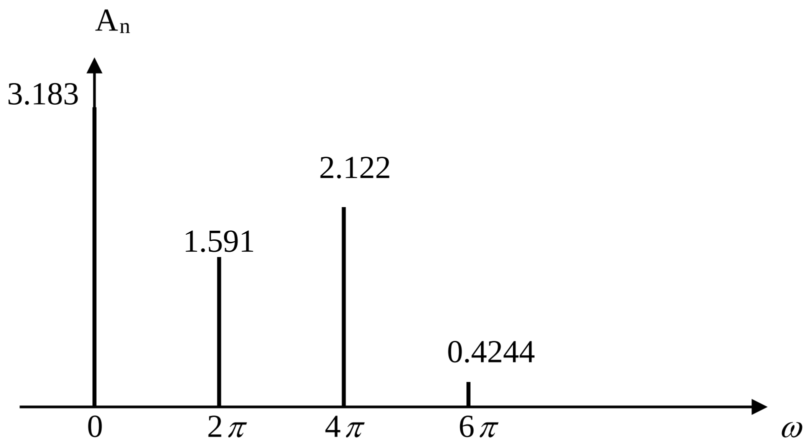
Given that

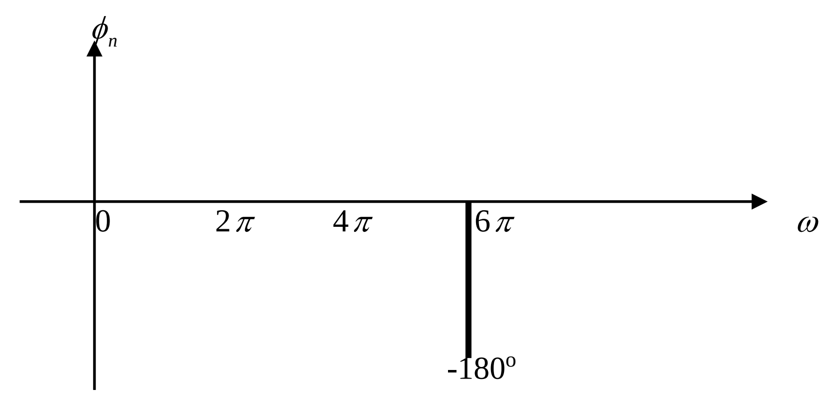
$$v(t) = (10/\pi)[1 + (1/2)\cos(2\pi t) + (2/3)\cos(4\pi t) – (2/15)\cos(6\pi t)] \text{ V}$$

draw the amplitude and phase spectra for $v(t)$.

Solution

The amplitude and phase spectra are shown below.





Solution 17.65

$a_n = 20/(n^2\pi^2)$, $b_n = -3/(n\pi)$, $\omega_n = 2n$

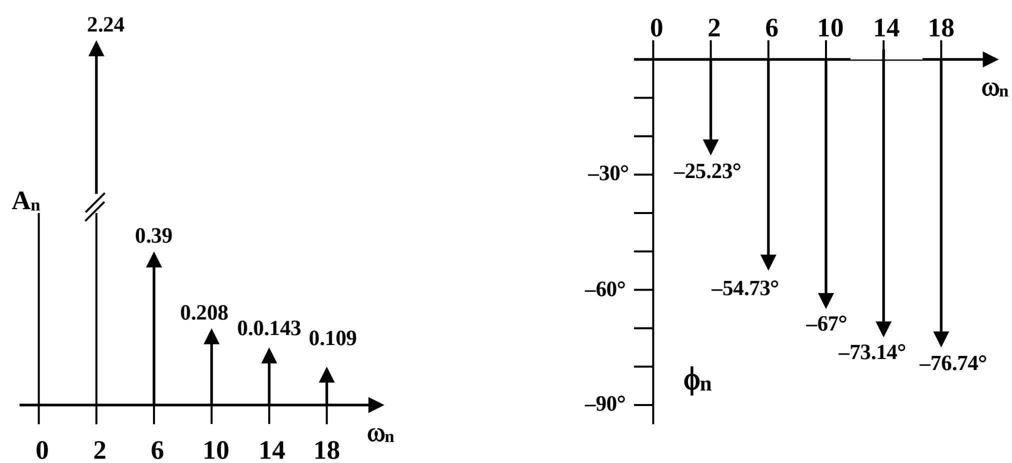
$$A_n = \sqrt{a_n^2 + b_n^2} = \sqrt{\frac{400}{n^4\pi^4} + \frac{9}{n^2\pi^2}}$$

$$= \frac{3}{n\pi}\sqrt{1 + \frac{44.44}{n^2\pi^2}}, \quad n = 1, 3, 5, 7, 9, \text{ etc.}$$

n	A_n
1	2.24
3	0.39
5	0.208
7	0.143
9	0.109

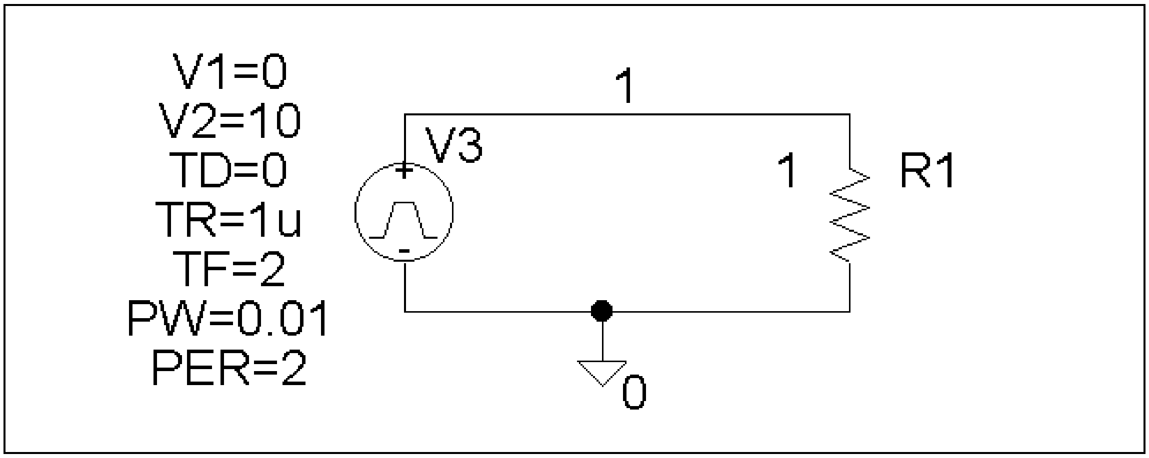
$\phi_n = \tan^{-1}(b_n/a_n) = \tan^{-1}\{[-3/(n\pi)][n^2\pi^2/20]\} = \tan^{-1}(-n\text{x}0.4712)$

n	ϕ_n
1	–25.23°
3	–54.73°
5	–67°
7	–73.14°
9	–76.74°
∞	–90°



Solution 17.66

The schematic is shown below. The waveform is inputted using the attributes of VPULSE. In the Transient dialog box, we enter Print Step = 0.05, Final Time = 12, Center Frequency = 0.5, Output Vars = V(1) and click enable Fourier. After simulation, the output plot is shown below. The output file includes the following Fourier components.



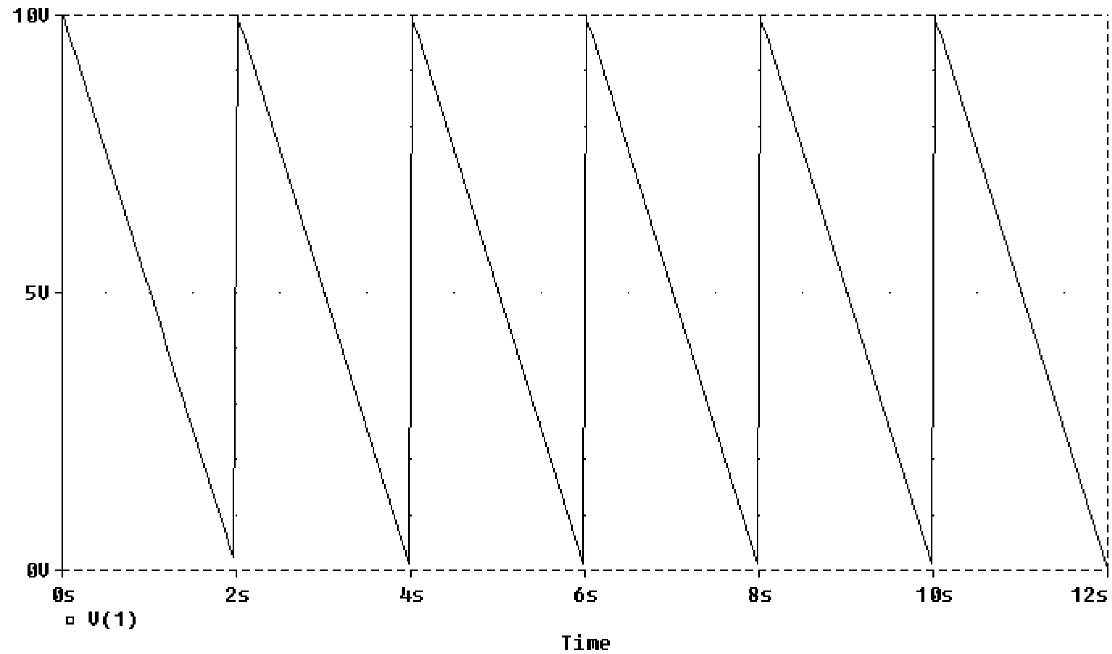
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 5.099510E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	3.184E+00	1.000E+00	1.782E+00	0.000E+00
2	1.000E+00	1.593E+00	5.002E-01	3.564E+00	1.782E+00
3	1.500E+00	1.063E+00	3.338E-01	5.347E+00	3.564E+00
4	2.000E+00	7.978E-01	2.506E-01	7.129E+00	5.347E+00
5	2.500E+00	6.392E-01	2.008E-01	8.911E+00	7.129E+00
6	3.000E+00	5.336E-01	1.676E-01	1.069E+01	8.911E+00
7	3.500E+00	4.583E-01	1.440E-01	1.248E+01	1.069E+01
8	4.000E+00	4.020E-01	1.263E-01	1.426E+01	1.248E+01
9	4.500E+00	3.583E-01	1.126E-01	1.604E+01	1.426E+01

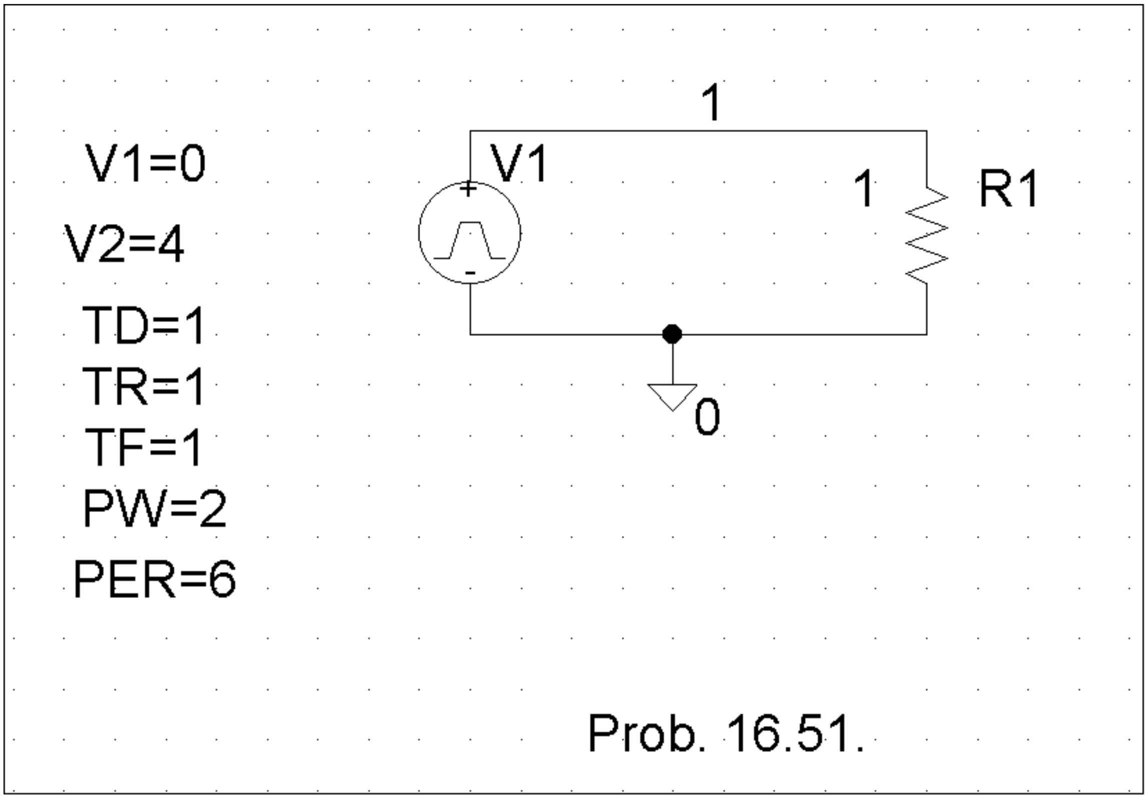
TOTAL HARMONIC DISTORTION = 7.363360E+01 PERCENT

From Prob. 17.4, we know the phase angle should be zero. Why do we have a phase angle equal to n(1.782)? The answer is actually quite straight forward. ***The angle comes from the approximation of the leading edge of the pulse. The graph shows an instantaneous rise whereas PSpice needs a finite rise time, thus artificially creating a phase shift.***


10V
5V
0V
0s
2s
4s
6s
8s
10s
12s
V(1)
Time

Solution 17.67

The Schematic is shown below. In the Transient dialog box, we type "Print step = 0.01s, Final time = 36s, Center frequency = 0.1667, Output vars = v(1)," and click Enable Fourier. After simulation, the output file includes the following Fourier components,



FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

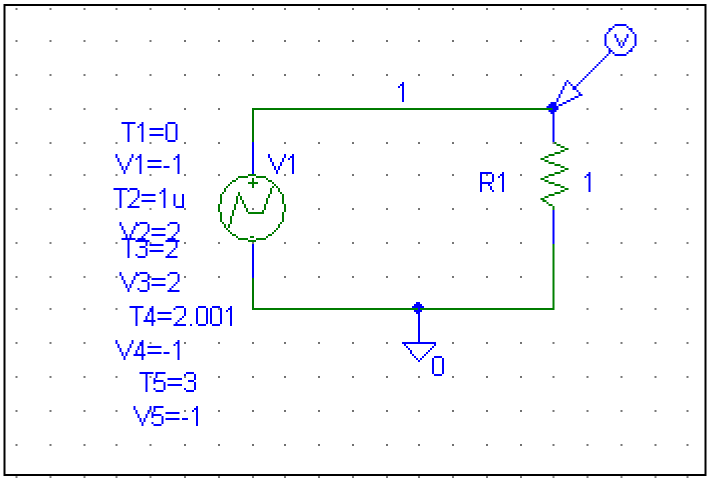
DC COMPONENT = 2.000396E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.667E-01	2.432E+00	1.000E+00	-8.996E+01	0.000E+00
2	3.334E-01	6.576E-04	2.705E-04	-8.932E+01	6.467E-01
3	5.001E-01	5.403E-01	2.222E-01	9.011E+01	1.801E+02
4	6.668E-01	3.343E-04	1.375E-04	9.134E+01	1.813E+02
5	8.335E-01	9.716E-02	3.996E-02	-8.982E+01	1.433E-01
6	1.000E+00	7.481E-06	3.076E-06	-9.000E+01	-3.581E-02
7	1.167E+00	4.968E-02	2.043E-02	-8.975E+01	2.173E-01
8	1.334E+00	1.613E-04	6.634E-05	-8.722E+01	2.748E+00
9	1.500E+00	6.002E-02	2.468E-02	9.032E+01	1.803E+02

TOTAL HARMONIC DISTORTION = 2.280065E+01 PERCENT

Solution 17.68

Since T=3, f =1/3 = 0.333 Hz. We use the schematic below.



We use VPWL to enter in the signal as shown. In the transient dialog box, we enable Fourier, select 15 for Final Time, 0.01s for Print Step, and 10ms for the Step Ceiling. When the file is saved and run, we obtain the Fourier coefficients as part of the output file as shown below.

Why is this problem wrong? Clearly the source is not periodic. The DC value must be +1!!!!!!!!!!

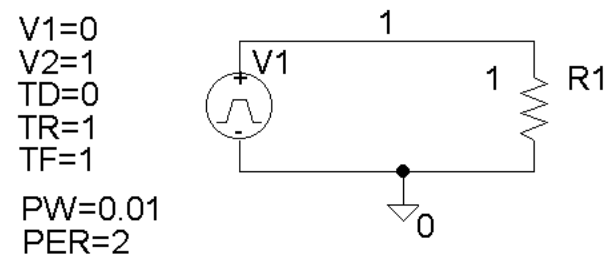
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = -1.000000E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	3.330E-01	1.615E-16	1.000E+00	1.762E+02	0.000E+00
2	6.660E-01	5.133E-17	3.179E-01	2.999E+01	-3.224E+02
3	9.990E-01	6.243E-16	3.867E+00	6.687E+01	-4.617E+02
4	1.332E+00	1.869E-16	1.158E+00	7.806E+01	-6.267E+02
5	1.665E+00	6.806E-17	4.215E-01	1.404E+02	-7.406E+02
6	1.998E+00	1.949E-16	1.207E+00	-1.222E+02	-1.179E+03
7	2.331E+00	1.465E-16	9.070E-01	-4.333E+01	-1.277E+03
8	2.664E+00	3.015E-16	1.867E+00	-1.749E+02	-1.584E+03
9	2.997E+00	1.329E-16	8.233E-01	-9.565E+01	-1.681E+03

Solution 17.69

The schematic is shown below. In the Transient dialog box, set Print Step = 0.05 s, Final Time = 120, Center Frequency = 0.5, Output Vars = V(1) and click enable Fourier. After simulation, we obtain V(1) as shown below. We also obtain an output file which includes the following Fourier components.

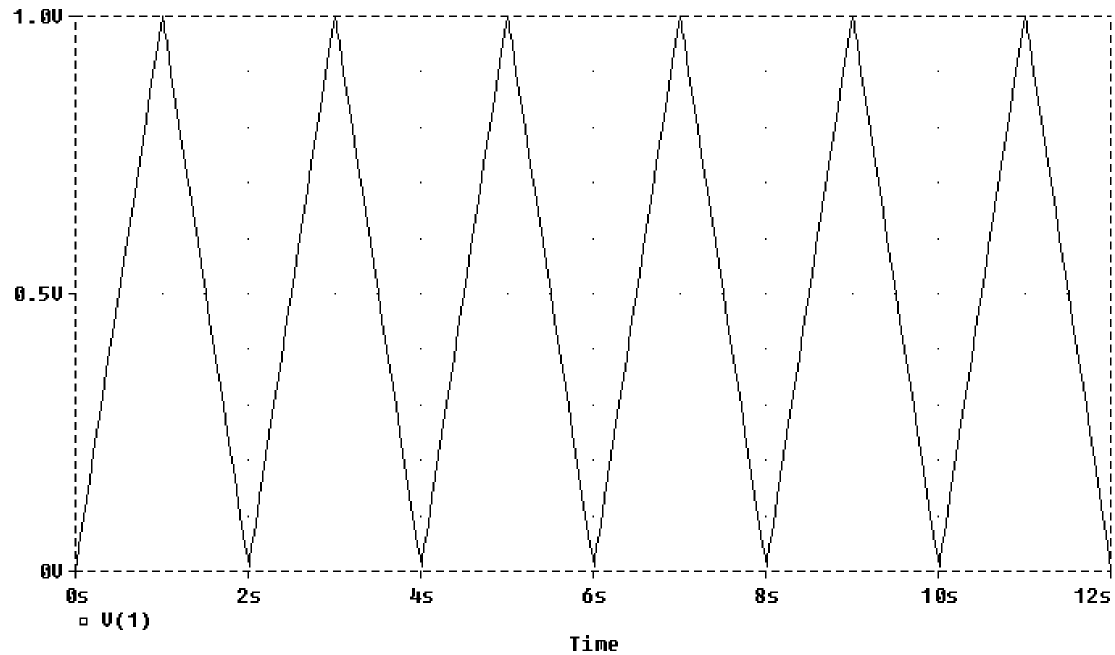


FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 5.048510E-01

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	4.056E-01	1.000E+00	-9.090E+01	0.000E+00
2	1.000E+00	2.977E-04	7.341E-04	-8.707E+01	3.833E+00
3	1.500E+00	4.531E-02	1.117E-01	-9.266E+01	-1.761E+00
4	2.000E+00	2.969E-04	7.320E-04	-8.414E+01	6.757E+00
5	2.500E+00	1.648E-02	4.064E-02	-9.432E+01	-3.417E+00
6	3.000E+00	2.955E-04	7.285E-04	-8.124E+01	9.659E+00
7	3.500E+00	8.535E-03	2.104E-02	-9.581E+01	-4.911E+00
8	4.000E+00	2.935E-04	7.238E-04	-7.836E+01	1.254E+01
9	4.500E+00	5.258E-03	1.296E-02	-9.710E+01	-6.197E+00

TOTAL HARMONIC DISTORTION = 1.214285E+01 PERCENT


1.0V
0.5V
0V
0s
2s
4s
6s
8s
10s
12s
V(1)
Time

Solution 17.70

Design a problem to help other students to better understand how to use *PSpice* to solve circuit problems with periodic inputs.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Rework Prob. 17.40 using *PSpice*.

Chapter 17, Problem 40.

The signal in Fig. 17.77(a) is applied to the circuit in Fig. 17.77(b). Find $v_o(t)$.

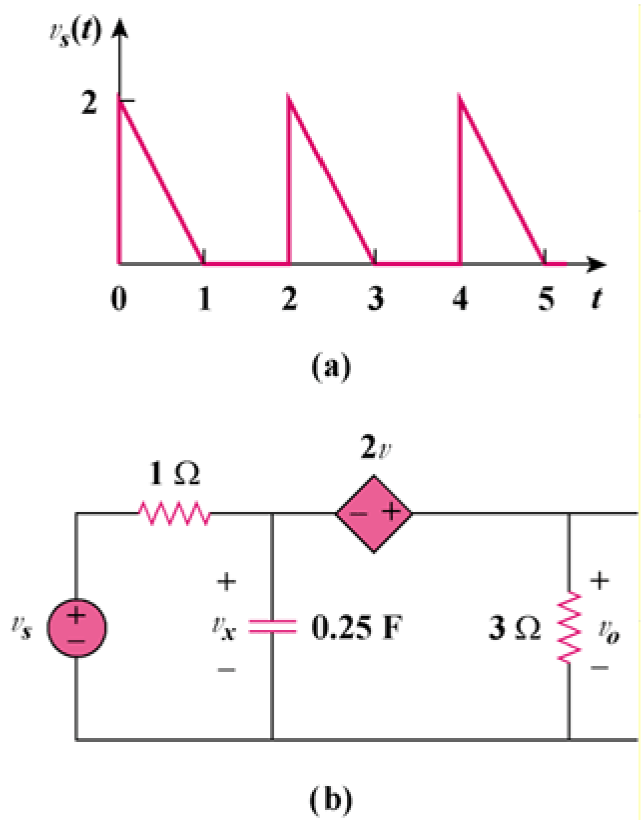
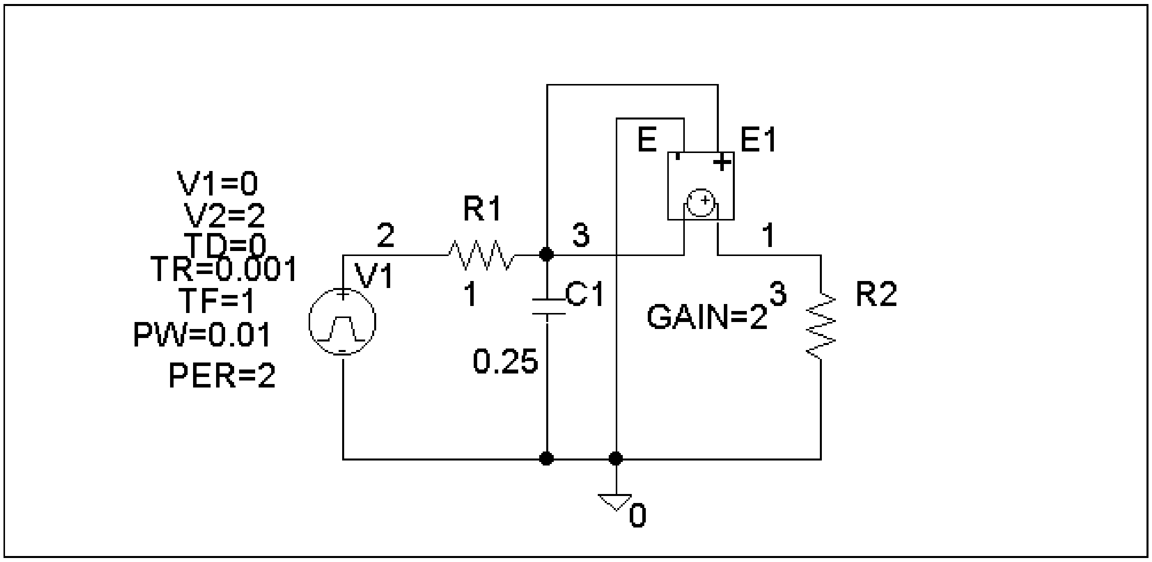
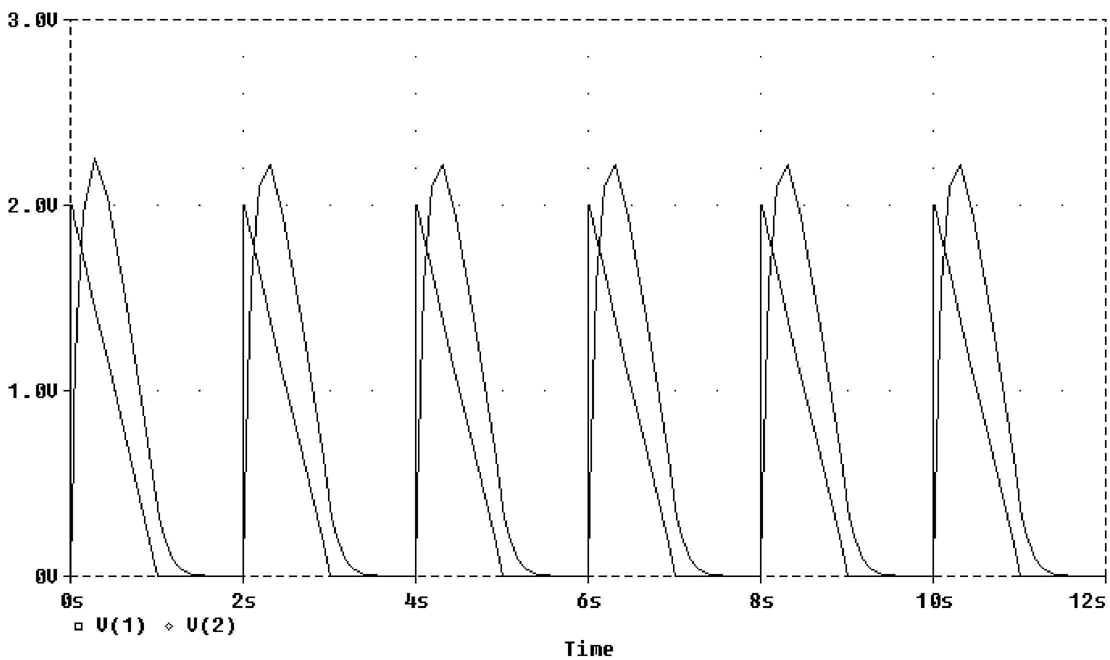


Figure 17.77

Solution

The schematic is shown below. In the Transient dialog box, we set Print Step = 0.02 s, Final Step = 12 s, Center Frequency = 0.5, Output Vars = V(1) and V(2), and click enable Fourier. After simulation, we compare the output and output waveforms as shown. The output includes the following Fourier components.





FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

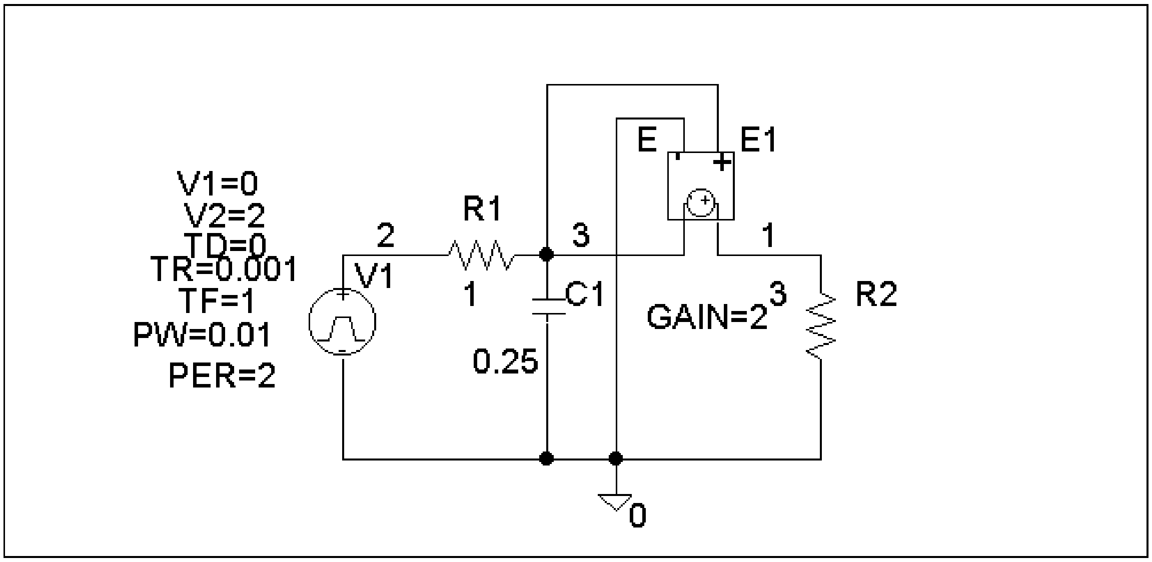
DC COMPONENT = 7.658051E-01

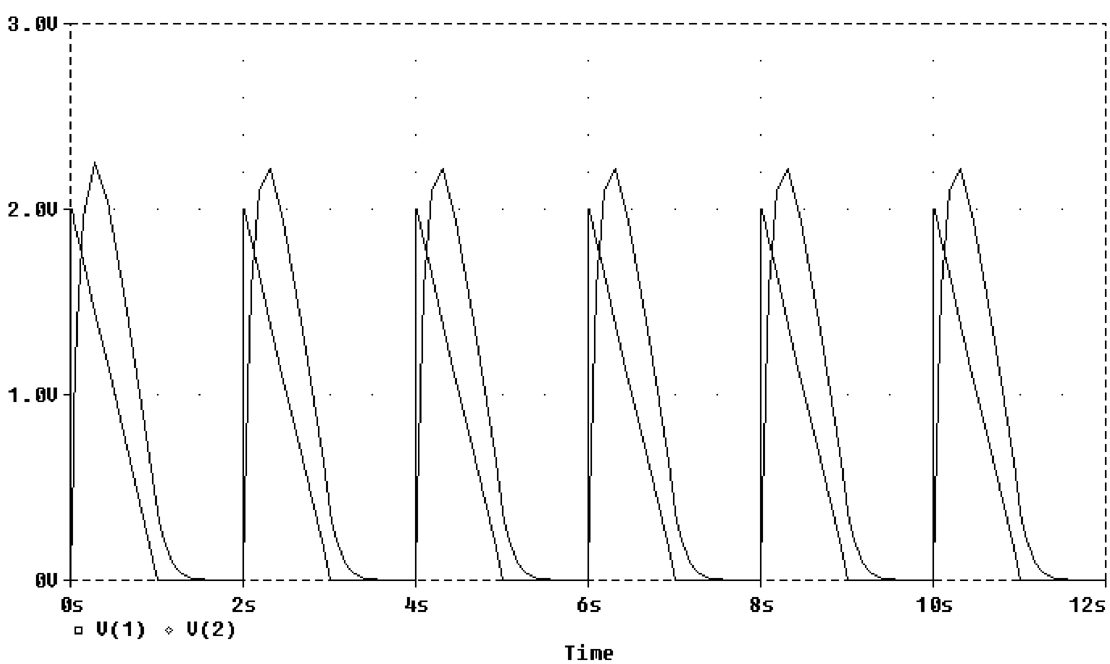
HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	1.070E+00	1.000E+00	1.004E+01	0.000E+00
2	1.000E+00	3.758E-01	3.512E-01	-3.924E+01	-4.928E+01
3	1.500E+00	2.111E-01	1.973E-01	-3.985E+01	-4.990E+01
4	2.000E+00	1.247E-01	1.166E-01	-5.870E+01	-6.874E+01
5	2.500E+00	8.538E-02	7.980E-02	-5.680E+01	-6.685E+01
6	3.000E+00	6.139E-02	5.738E-02	-6.563E+01	-7.567E+01
7	3.500E+00	4.743E-02	4.433E-02	-6.520E+01	-7.524E+01
8	4.000E+00	3.711E-02	3.469E-02	-7.222E+01	-8.226E+01
9	4.500E+00	2.997E-02	2.802E-02	-7.088E+01	-8.092E+01

TOTAL HARMONIC DISTORTION = 4.352895E+01 PERCENT

Solution 17.71

The schematic is shown below. In the Transient dialog box, we set Print Step = 0.02 s, Final Step = 12 s, Center Frequency = 0.5, Output Vars = V(1) and V(2), and click enable Fourier. After simulation, we compare the output and output waveforms as shown. The output includes the following Fourier components.





FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 7.658051E-01

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	1.070E+00	1.000E+00	1.004E+01	0.000E+00
2	1.000E+00	3.758E-01	3.512E-01	-3.924E+01	-4.928E+01
3	1.500E+00	2.111E-01	1.973E-01	-3.985E+01	-4.990E+01
4	2.000E+00	1.247E-01	1.166E-01	-5.870E+01	-6.874E+01
5	2.500E+00	8.538E-02	7.980E-02	-5.680E+01	-6.685E+01
6	3.000E+00	6.139E-02	5.738E-02	-6.563E+01	-7.567E+01
7	3.500E+00	4.743E-02	4.433E-02	-6.520E+01	-7.524E+01
8	4.000E+00	3.711E-02	3.469E-02	-7.222E+01	-8.226E+01
9	4.500E+00	2.997E-02	2.802E-02	-7.088E+01	-8.092E+01

TOTAL HARMONIC DISTORTION = 4.352895E+01 PERCENT

Solution 17.72

$T = 5, \ \omega_o = 2\pi/T = 2\pi/5$

f(t) is an odd function. $a_o = 0 = a_n$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_o t)dt = \frac{4}{5}\int_0^{10} 10\sin(0.4n\pi t)dt$$

$$= -\frac{8x5}{2n\pi}\cos(0.4\pi nt)\Big|_0^1 = \frac{20}{n\pi}[1-\cos(0.4n\pi)]$$

$$f(t) = \mathbf{\frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}[1-\cos(0.4n\pi)]\sin(0.4n\pi t)}$$

Solution 17.73

$$p = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum \frac{V_n^2}{R}$$

$= 0 + 0.5[(2^2 + 1^2 + 1^2)/10] =$ **300 mW**

Solution 17.74

(a)

$$A_n = \sqrt{a_n^2 + b_n^2}, \qquad \phi = \tan^{-1}(b_n/a_n)$$

$$A_1 = \sqrt{6^2 + 8^2} = 10, \qquad \phi_1 = \tan^{-1}(6/8) = 36.87°$$

$$A_2 = \sqrt{3^2 + 4^2} = 5, \qquad \phi_2 = \tan^{-1}(3/4) = 36.87°$$

$$i(t) = \mathbf{\{4 + 10\cos(100\pi t - 36.87°) - 5\cos(200\pi t - 36.87°)\}\ A}$$

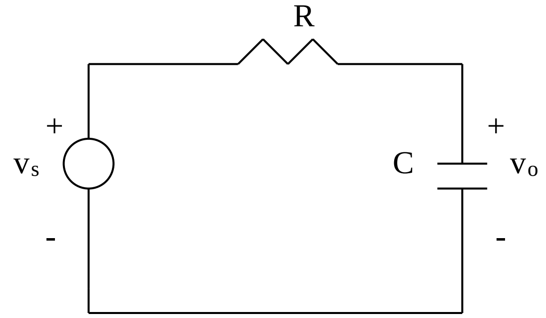
(b)

$$p = I_{DC}^2 R + 0.5\sum I_n^2 R$$

$$= 2[4^2 + 0.5(10^2 + 5^2)] = \mathbf{157\ W}$$

Solution 17.75

The lowpass filter is shown below.



$$v_s = \frac{A\tau}{T} + \frac{2A}{T}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi\tau}{T}\cos n\omega_o t$$

$$V_o = \frac{\frac{1}{j\omega_n C}}{R + \frac{1}{j\omega_n C}} V_s = \frac{1}{1 + j\omega_n RC} V_s, \qquad \omega_n = n\omega_o = 2n\pi / T$$

For n=0, (dc component), $V_o = V_s = \frac{A\tau}{T}$ (1)

For the nth harmonic,

$$V_o = \frac{1}{\sqrt{1 + \omega^2{}_n R^2 C^2} \angle \tan^{-1} \omega_n RC} \bullet \frac{2A}{nT}\sin\frac{n\pi\tau}{T}\angle - 90^o$$

When n=1, $|V_o| = \frac{2A}{T}\sin\frac{n\pi\tau}{T} \bullet \frac{1}{\sqrt{1 + \frac{4\pi^2}{T}R^2C^2}}$ (2)

From (1) and (2),

$$\frac{A\tau}{T} = 50x\frac{2A}{T}\sin\frac{\pi}{10}\frac{1}{\sqrt{1 + \frac{4\pi^2}{T}R^2C^2}} \longrightarrow \sqrt{1 + \frac{4\pi^2}{T}R^2C^2} = \frac{30.9}{\tau} = 3.09x10^4$$

$$1+\frac{4\pi^2}{T}R^2C^2=10^{10} \quad \longrightarrow \quad C=\frac{T}{2\pi R}10^5=\frac{10^{-2}x3.09x10^4}{4\pi x10^3}=\underline{24.59\,mF}$$

Solution 17.76

$v_s(t)$ is the same as f(t) in Figure 16.1 except that the magnitude is multiplied by 10. Hence

$$v_o(t) = 5 + \frac{20}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin(n\pi t), \quad n = 2k - 1$$

$$T = 2, \ \omega_o = 2\pi/T = 2\pi, \ \omega_n = n\omega_o = 2n\pi$$

$$j\omega_n L = j2n\pi; \ Z = R\|10 = 10R/(10 + R)$$

$$V_o = ZV_s/(Z + j2n\pi) = [10R/(10R + j2n\pi(10 + R))]V_s$$

$$V_o = \frac{10R\angle -\tan^{-1}\{(n\pi/5R)(10+R)\}}{\sqrt{100R^2 + 4n^2\pi^2(10+R)^2}}V_s$$

$$V_s = [20/(n\pi)]\angle 0°$$

The source current I_s is

$$I_s = \frac{V_s}{Z + j2n\pi} = \frac{V_s}{\frac{10R}{10+R} + j2n\pi} = \frac{(10+R)\frac{20}{n\pi}}{10R + j2n\pi(10+R)}$$

$$= \frac{(10+R)\frac{20}{n\pi}\angle -\tan^{-1}\{(n\pi/3)(10+R)\}}{\sqrt{100R^2 + 4n^2\pi^2(10+R)^2}}$$

$$p_s = V_{DC}I_{DC} + \frac{1}{2}\sum V_{sn}I_{sn}\cos(\theta_n - \phi_n)$$

For the DC case, L acts like a short-circuit.

$$I_s = \frac{5}{\frac{10R}{10+R}} = \frac{5(10+R)}{10R}, \ V_s = 5 = V_o$$

$$p_s = \frac{25(10+R)}{10R} + \frac{1}{2}\left[\left(\frac{20}{\pi}\right)^2 \frac{(10+R)\cos\left(\tan^{-1}\left(\frac{\pi}{5}(10+R)\right)\right)}{\sqrt{100R^2 + 4\pi^2(10+R)^2}}\right.$$

$$\left. + \left(\frac{10}{\pi}\right)^2 \frac{(10+R)^2\cos\left(\tan^{-1}\left(\frac{2\pi}{5}(10+R)\right)\right)}{\sqrt{100R^2 + 16\pi^2(10+R)^2}} + \cdots\right]$$

$$p_s = \frac{V_{DC}}{R} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{V_{on}}{R}$$

$$= \frac{25}{R} + \frac{1}{2}\left[\frac{100R}{100R^2 + 4\pi^2(10+R)^2} + \frac{100R}{100R^2 + 10\pi^2(10+R)^2} + \cdots\right]$$

We want $p_o = (70/100)p_s = 0.7p_s$. Due to the complexity of the terms, we consider only the DC component as an approximation. In fact the DC component has the largest share of the power for both input and output signals.

$$\frac{25}{R} = \frac{7}{10} \text{x} \frac{25(10+R)}{10R}$$

$100 = 70 + 7R$ which leads to $R = 30/7 =$ **<u>4.286 Ω</u>**

Solution 17.77

(a) For the first two AC terms, the frequency ratio is 6/4 = 1.5 so that the highest common factor is 2. Hence $\omega_o = 2$.

$$T = 2\pi/\omega_o = 2\pi/2 = \boldsymbol{\pi}$$

(b) The average value is the DC component = **–2**

(c) $$V_{rms} = \sqrt{a_o + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)}$$

$$V_{rms}^2 = (-2)^2 + \frac{1}{2}(10^2 + 8^2 + 6^2 + 3^2 + 1^2) = 121.5$$

V_{rms} = **11.02 V**

Solution 17.78

(a) $$p = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum\frac{V_n^2}{R} = \frac{V_{DC}^2}{R} + \sum\frac{V_{n,rms}^2}{R}$$

$= 0 + (40^2/5) + (20^2/5) + (10^2/5) =$ **420 W**

(b) 5% increase = (5/100)420 = 21

$p_{DC} = 21 \text{ W} = \dfrac{V_{DC}^2}{R}$ which leads to $V_{DC}^2 = 21R = 105$

$V_{DC} =$ **10.25 V**

Solution 17.79

From Table 17.3, it is evident that $a_n = 0$,

$$b_n = 4A/[\pi(2n - 1)], \quad A = 10.$$

A Fortran program to calculate b_n is shown below. The result is also shown.

```
C     FOR PROBLEM 17.79
         DIMENSION B(20)

         A = 10
         PIE = 3.142
         C = 4.*A/PIE
         DO 10 N = 1, 10
         B(N) = C/(2.*FLOAT(N) – 1.)
         PRINT *, N, B(N)
10       CONTINUE
         STOP
         END
```

n	b_n
1	12.731
2	4.243
3	2.546
4	1.8187
5	1.414
6	1.1573
7	0.9793
8	0.8487
9	0.7498
10	0.6700

Solution 17.80

From Problem 17.55,

$$c_n = [1 + e^{-jn\pi}]/[2\pi(1 - n^2)]$$

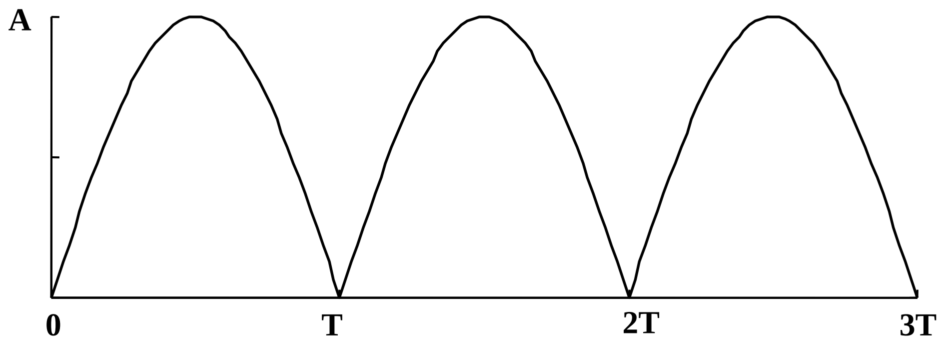
This is calculated using the Fortran program shown below. The results are also shown.

```
C       FOR PROBLEM 17.80
              COMPLEX X, C(0:20)

              PIE  =  3.1415927
              A  =  2.0*PIE
              DO  10  N = 0, 10
              IF(N.EQ.1) GO TO 10
              X  =  CMPLX(0, PIE*FLOAT(N))
              C(N)  =  (1.0 + CEXP(–X))/(A*(1 – FLOAT(N*N)))
              PRINT *, N, C(N)
10            CONTINUE
              STOP
              END
```

n	c_n
0	0.3188 + j0
1	0
2	$-0.1061 + j0$
3	0
4	$-0.2121\text{x}10^{-1} + j0$
5	0
6	$-0.9095\text{x}10^{-2} + j0$
7	0
8	$-0.5052\text{x}10^{-2} + j0$
9	0
10	$-0.3215\text{x}10^{-2} + j0$

Solution 17.81

(a)



$$f(t) = \frac{2A}{\pi} - \frac{4A}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cos(n\omega_o t)$$

The total average power is $p_{avg} = F_{rms}^2 R = F_{rms}^2$ since R = 1 ohm.

$$P_{avg} = F_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \mathbf{0.5A^2}$$

(b) From the Fourier series above

$$|c_o| = 2A/\pi, \ |c_n| = |a_n|/2 = 2A/[\pi(4n^2 - 1)]$$

n	ω_o	$\lvert c_n\rvert$	$\lvert c_o\rvert^2$ or $2\lvert c_n\rvert^2$	% power
0	0	$2A/\pi$	$4A^2/(\pi^2)$	81.1%
1	$2\omega_o$	$2A/(3\pi)$	$8A^2/(9\pi^2)$	18.01%
2	$4\omega_o$	$2A/(15\pi)$	$8A^2/(225\pi^2)$	0.72%
3	$6\omega_o$	$2A/(35\pi)$	$8A^2/(1225\pi^2)$	0.13%
4	$8\omega_o$	$2A/(63\pi)$	$8A^2/(3969\pi^2)$	0.04%

(c) **81.1%**

(d) **0.72%**

Solution 17.82

$$P = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{V_n^2}{R}$$

Assuming V is an amplitude-phase form of Fourier series. But

$$|A_n| = 2|C_n|, \quad c_o = a_o$$

$$|A_n|^2 = 4|C_n|^2$$

Hence,

$$P = \frac{c_o^2}{R} + 2\sum_{n=1}^{\infty}\frac{c_n^2}{R}$$

Alternatively,

$$P = \frac{V_{rms}^2}{R}$$

where

$$V_{rms}^2 = a_o^2 + \frac{1}{2}\sum_{n=1}^{\infty}A_n^2 = c_o^2 + 2\sum_{n=1}^{\infty}c_n^2 = \sum_{n=-\infty}^{\infty}c_n^2$$

$$= 10^2 + 2(8.5^2 + 4.2^2 + 2.1^2 + 0.5^2 + 0.2^2)$$

$$= 100 + 2x94.57 = 289.14$$

$P = 289.14/4 =$ **72.3 W**

Solution 18.1

Obtain the Fourier transform of the function in Fig. 18.26.

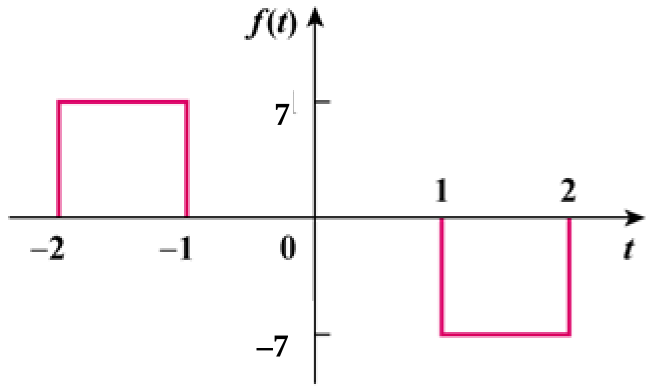


Figure 18.26
For Prob. 18.1.

Solution

$f(t) = 7u(t+2) – 7u(t+1) – 7u(t–1) + 7u(t–2)$
$f'(t) = 7\delta(t+2) – 7\delta(t+1) – 7\delta(t–1) + 7\delta(t–2)$
$j\omega F(\omega) = 7(e^{j2\omega} – e^{j\omega} – e^{–j\omega} + e^{–j2\omega}) = 14\cos(2\omega) – 14\cos(\omega)$ or

$$F(\omega) = \mathbf{14[\cos(2\omega) – \cos(\omega)]/(j\omega)}$$

Solution 18.2

Using Fig. 18.27, design a problem to help other students to better understand the Fourier transform given a wave shape.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

What is the Fourier transform of the triangular pulse in Fig. 18.27?

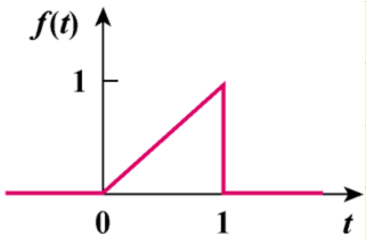
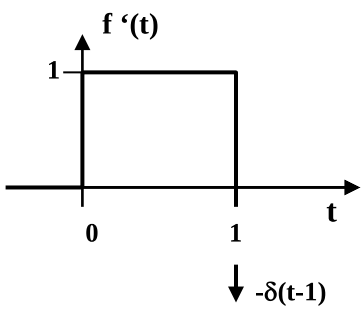
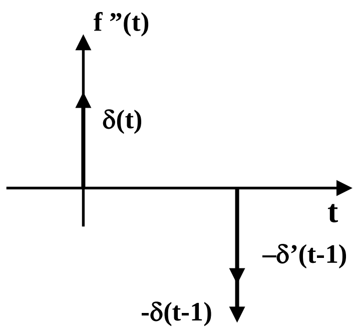


Figure 18.27

Solution

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$





$f''(t) = \delta(t) - \delta(t - 1) - \delta'(t - 1)$

Taking the Fourier transform gives

$$-\omega^2 F(\omega) = 1 - e^{-j\omega} - j\omega e^{-j\omega}$$

$$F(\omega) = \underline{\frac{(1 + j\omega)e^{-j\omega} - 1}{\omega^2}}$$

or $F(\omega) = \int_0^1 t\, e^{-j\omega t}\, dt$

But $\int x\, e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1) + c$

$$F(\omega) = \frac{e^{-j\omega}}{(-j\omega)^2}(-j\omega t - 1)\Big|_0^1 = \mathbf{\frac{1}{\omega^2}\left[(1 + j\omega)e^{-j\omega} - 1\right]}$$

Solution 18.3

Calculate the Fourier transform of the signal in Fig. 18.28.

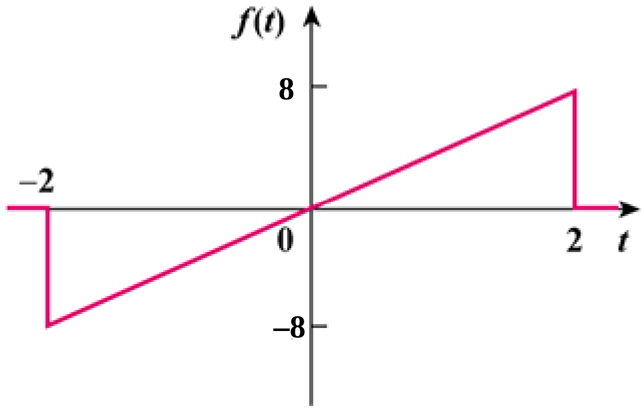


Figure 18.28
For Prob. 18.3.

Solution

$f(t) = 4t,\ -2 < t < 2, \qquad f'(t) = 4,\ -2 < t < 2$

$$F(\omega) = \int_{-2}^{2} 4t\, e^{-j\omega t} dt = \frac{4e^{-j\omega t}}{(-j\omega)^2}(-j\omega t - 1)\Big|_{-2}^{2}$$

$$= -(4/\omega^2)[e^{-j\omega 2}(-j\omega 2 - 1) - e^{j\omega 2}(j\omega 2 - 1)]$$

$$= -\frac{4}{\omega^2}\left[-j\omega 2\left(e^{-j\omega 2} + e^{j\omega 2}\right) + e^{j\omega 2} - e^{-j\omega 2}\right]$$

$$= -\frac{4}{\omega^2}\left(-j\omega 4\cos 2\omega + j2\sin 2\omega\right)$$

$F(\omega)$ = $\mathbf{(j8/\omega^2)[2\omega\cos(2\omega) - \sin(2\omega)]}$

Solution 18.4

Find the Fourier transform of the waveform shown in Fig. 18.29.

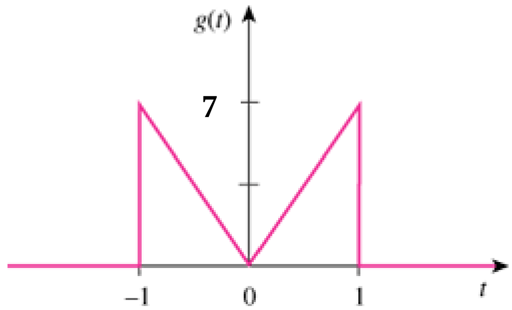
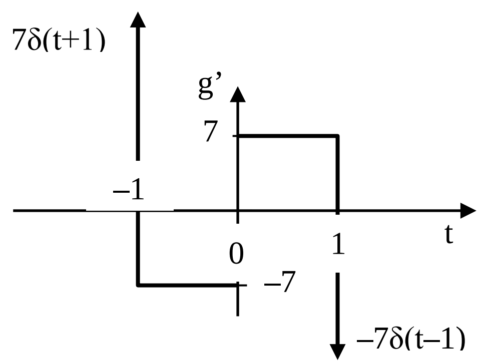
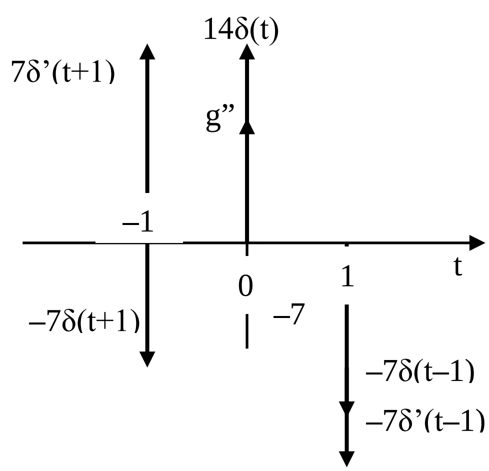


Figure 18.29
For Prob. 18.4.

Solution

We can solve the problem by following the approach demonstrated in Example 18.5.





$$g'' = -7\delta(t+1) + 7\delta'(t+1) + 14\delta(t) - 7\delta(t-1) - 7\delta'(t-1)$$

$$(j\omega)^2 G(\omega) = -7e^{j\omega} + 7j\omega e^{j\omega} + 14 - 7e^{-j\omega} - 7j\omega e^{-j\omega}$$
$$= -14\cos\omega - 14\omega\sin\omega + 14$$

G(ω) = **$(14/\omega^2)[\cos(\omega) + \omega\sin(\omega) - 1]$**

Solution 18.5

Obtain the Fourier transform of the signal shown in Fig. 18.30.

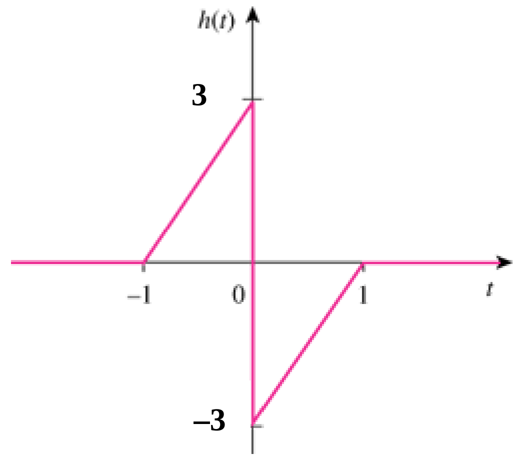
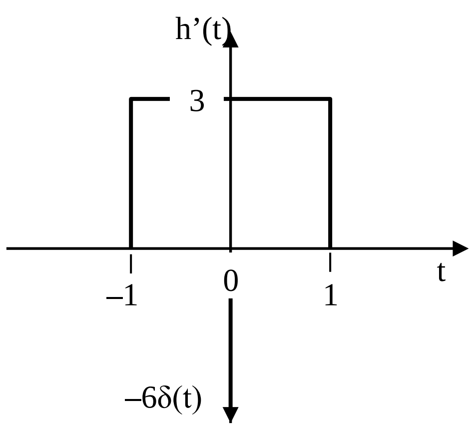
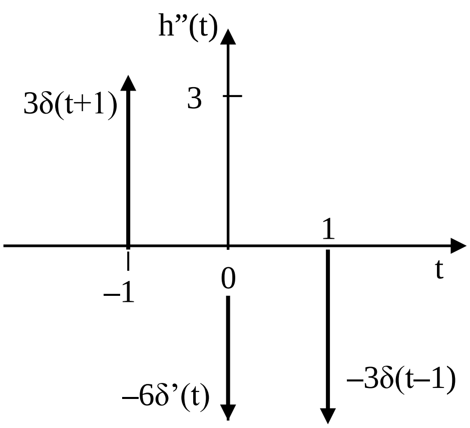


Figure 18.30
For Prob. 18.5.

Solution





$$h''(t) = 3\delta(t+1) - 3\delta(t-1) - 6\delta'(t)$$

$$(j\omega)^2 H(\omega) = 3[e^{j\omega} - e^{-j\omega} - 2j\omega] = 6j\sin\omega - 6j\omega$$

H(ω) = **(6j/ω)[1 – (1/ω)sin(ω)]**

Solution 18.6

Find the Fourier transform of each of the functions in Fig. 18.31.

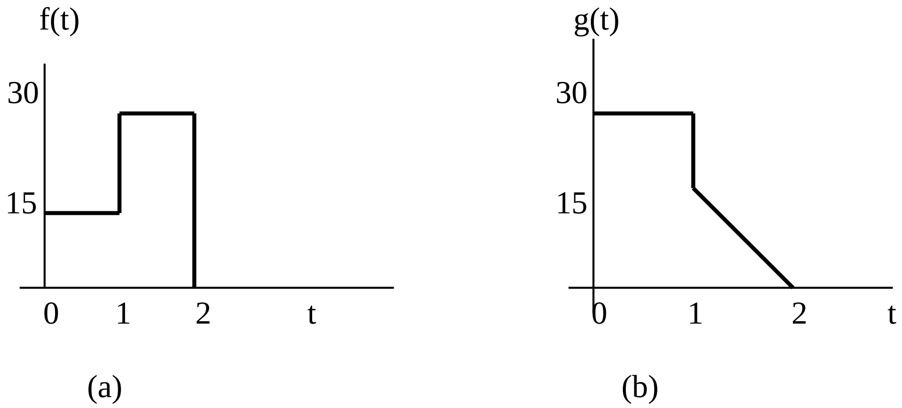
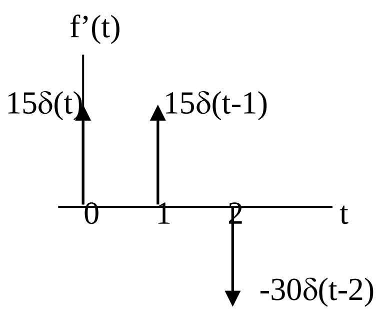


Figure 18.31
For Prob. 18.6.

Solutions

(a) The derivative of f(t) is shown below.



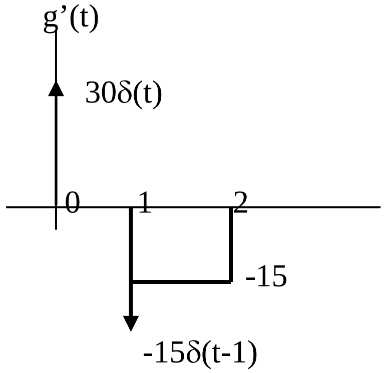
$f'(t) = 15[\delta(t) + \delta(t-1) - 2\delta(t-2)]$

Taking the Fourier transform of each term,

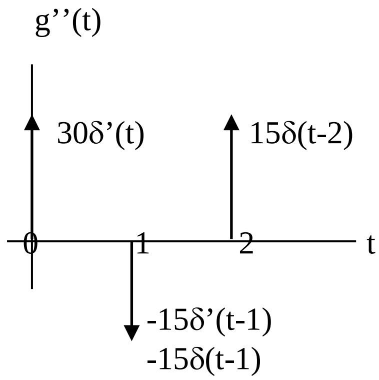
$j\omega F(\omega) = 15[1 + e^{-j\omega} - 2e^{-j\omega}]$ or

$$F(\omega) = \mathbf{(15/(j\omega))[\,1 + e^{-j\omega} - 2e^{-j\omega}]}$$

(b) The derivative of g(t) is shown below.



The second derivative of g(t) is shown below.



$g''(t) = 30\delta'(t) - 15\delta'(t-1) - 15\delta(t-1) + 15\delta(t-2)$

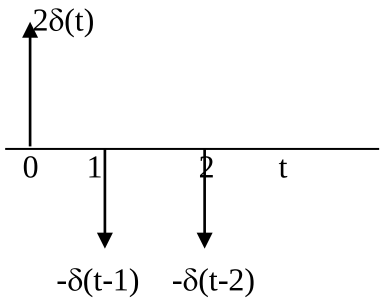
Take the Fourier transform of each term.

$(j\omega)^2G(j\omega) = 30j\omega - 15j\omega e^{-j\omega} - 15e^{-j\omega} + 15e^{-j2\omega}$ which leads to

$$G(j\omega) = \mathbf{[-30j\omega + 15j\omega e^{-j\omega} + 15e^{-j\omega} - 15e^{-j2\omega}]/\omega^2}$$

Solution 18.7

(a) Take the derivative of $f_1(t)$ and obtain $f_1'(t)$ as shown below.



$$f_1^{'}(t) = 2\delta(t) - \delta'(t-1) - \delta(t-2)$$

Take the Fourier transform of each term,

$$j\omega F_1(\omega) = 2 - e^{-j\omega} - e^{-j2\omega}$$

$$\underline{F_1(\omega) = \frac{2 - e^{-j\omega} - e^{-j2\omega}}{j\omega}}$$

(b) $f_2(t) = 5t$

$$F_2(\omega) = \int_{-\infty}^{\infty} f_2(t)e^{-j\omega}dt = \int_0^2 5te^{-j\omega}dt = \frac{5}{(-j\omega)^2}e^{-j\omega t}(-j\omega-1)\Big|_0^2$$

$$\underline{F_2(\omega) = \frac{5e^{-j2\omega}}{\omega^2}(1+j\omega 2) - \frac{5}{\omega^2}}$$

Solution 18.8

Obtain the Fourier transforms of the signals in Fig. 18.33.

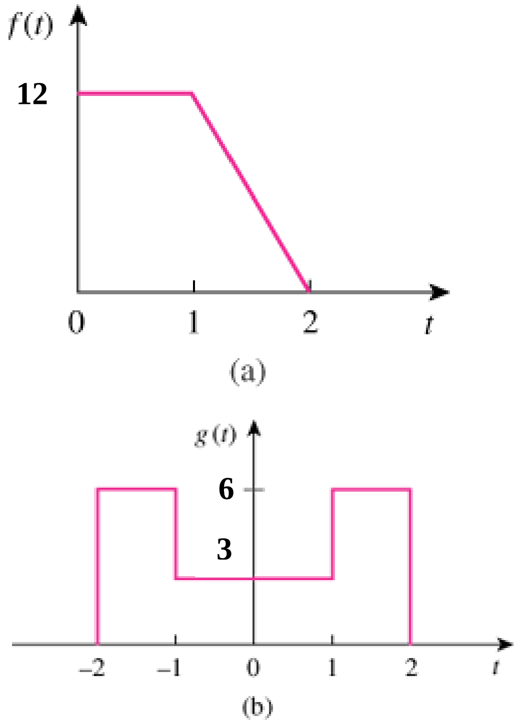


Figure 18.33
For Prob. 18.8.

Solution

(a)

$$F(\omega)=\int_0^1 12e^{-j\omega t}dt+\int_1^2(24-12t)e^{-j\omega t}dt$$

$$=\frac{12}{-j\omega}e^{-j\omega t}\Big|_0^1+\frac{24}{-j\omega}e^{-j\omega t}\Big|_1^2-\frac{12}{-\omega^2}e^{-j\omega t}(-j\omega t-1)\Big|_1^2$$

$$\underline{F(\omega)=\frac{12}{\omega^2}+\frac{12}{j\omega}e^{-j\omega}+\frac{12}{j\omega}-\frac{24}{j\omega}e^{-j2\omega}-\frac{12}{\omega^2}(1+j2\omega)e^{-j2\omega}}$$

(b) g(t) = 6[u(t+2) – u(t-2)] – 3[u(t+1) – u(t-1)]

$$G(\omega)=\underline{\frac{12\sin 2\omega}{\omega}-\frac{6\sin\omega}{\omega}}$$

Solution 18.9

Determine the Fourier transforms of the signals in Fig. 18.34.

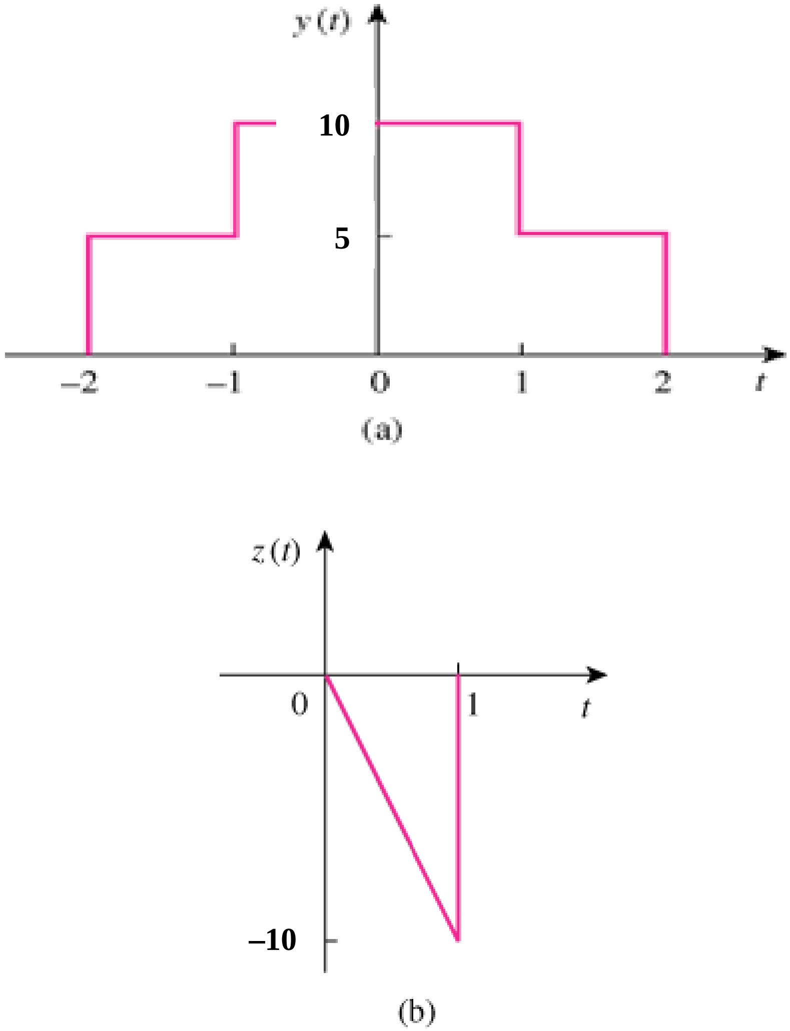


Figure 18.34
For Prob. 18.9.

Solution

(a) y(t) = 5u(t+2) – 5u(t-2) + 5[u(t+1) – u(t-1)]

Y(ω) = **(10/ω)[sin(2ω) + sin(ω)]**

(b) $$Z(\omega) = \int_0^1 (-10t)e^{-j\omega t}dt = \frac{-10e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_0^1 = \underline{\frac{10}{\omega^2} - \frac{10e^{-j\omega}}{\omega^2}(1 + j\omega)}$$

Solution 18.10

Obtain the Fourier transforms of the signals shown in Fig. 18.35.

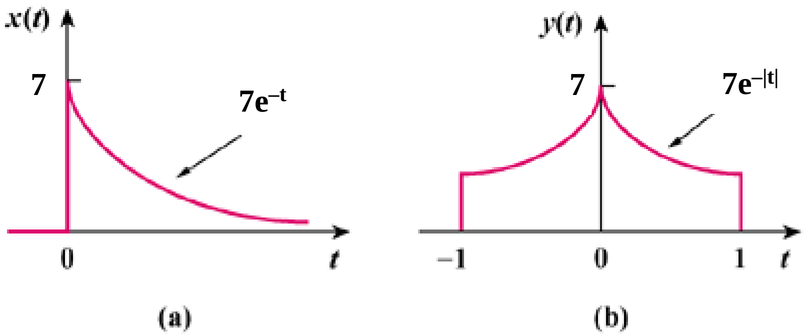


Figure 18.35
For Prob. 18.10.

Solution

(a) $x(t) = 7e^{-t}u(t)$

$X(\omega) =$ **$7/(1 + j\omega)$**

(b)
$$y(t) = \begin{bmatrix} e^{-t}, & t > 0 \\ e^{t}, & t < 0 \end{bmatrix}$$

$$Y(\omega) = \int_{-1}^{1} y(t)e^{-j\omega t}dt = \int_{-1}^{0} 7e^{t}e^{-j\omega}dt + \int_{0}^{1} 7e^{-t}e^{-j\omega t}dt$$

$$= \left.\frac{7e^{(1-j\omega)t}}{1-j\omega}\right|_{-1}^{0} + \left.\frac{7e^{-(1+j\omega)t}}{-(1+j\omega)}\right|_{0}^{1}$$

$$= \frac{14}{1+\omega^2} - 7e^{-1}\left[\frac{\cos\omega + j\sin\omega}{1-j\omega} + \frac{\cos\omega - j\sin\omega}{1+j\omega}\right]$$

$Y(\omega) =$ **$[14/(1+\omega^2)][1 - e^{-1}\{\cos(\omega) - \omega\sin(\omega)\}]$**

Solution 18.11

Find the Fourier transform of the “sine-wave pulse” shown in Fig. 18.36.

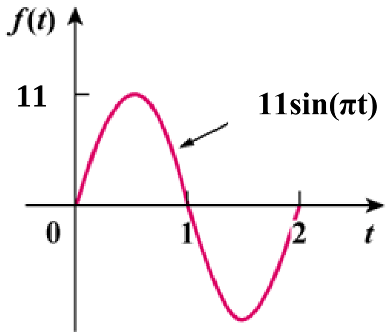


Figure 18.36
For Prob. 18.11.

Solution

f(t) = 11sin π t [u(t) - u(t - 2)]

$$F(\omega) = \int_0^2 11\sin\pi t\, e^{-j\omega t}\, dt = \frac{11}{2j}\int_0^2 \left(e^{j\pi} - e^{-j\pi}\right)e^{-j\omega t}\, dt$$

$$= \frac{11}{2j}\left[\int_0^2 (e^{+j(-\omega+\pi)t} + e^{-j(\omega+\pi)t})dt\right]$$

$$= \frac{11}{2j}\left[\frac{1}{-j(\omega-\pi)}e^{-j(\omega-\pi)t}\Big|_0^2 + \frac{e^{-j(\omega+\pi)t}}{-j(\omega+\pi)}\Big|_0^2\right]$$

$$= \frac{11}{2}\left(\frac{1-e^{-j2\omega}}{\pi-\omega} + \frac{1-e^{-j2\omega}}{\pi+\omega}\right)$$

$$= \frac{11}{2(\pi^2-\omega^2)}\left(2\pi + 2\pi e^{-j2\omega}\right)$$

$$\mathrm{F}(\omega) \;=\; \frac{11\pi}{\omega^2-\pi^2}\left(e^{-j\omega 2} - 1\right)$$

Solution 18.12

(a) $F_1(\omega) = \dfrac{10}{(3+j\omega)^2 + 100}$

(b) $F_2(\omega) = \dfrac{4+j\omega}{(4+j\omega)^2 + 100}$

Solution 18.13

(a) We know that $\mathcal{F}[\cos at] = \pi[\delta(\omega - a) + \delta(\omega + a)]$.

Using the time shifting property,

$$\mathcal{F}[\cos a(t - \pi/3a)] = \pi e^{-j\omega\pi/3a}[\delta(\omega - a) + \delta(\omega + a)] = \underline{\pi e^{-j\pi/3}\delta(\omega - a) + \pi e^{j\pi/3}\delta(\omega + a)}$$

(b) $\sin\pi(t+1) = \sin\pi t\cos\pi + \cos\pi t\sin\pi = -\sin\pi t$

g(t) = -u(t+1) sin (t+1)

Let x(t) = u(t)sin t, then $X(\omega) = \dfrac{1}{(j\omega)^2 + 1} = \dfrac{1}{1-\omega^2}$

Using the time shifting property,

$$G(\omega) = -\frac{1}{1-\omega^2}e^{j\omega} = \underline{\frac{e^{j\omega}}{\omega^2 - 1}}$$

(c) Let y(t) = 1 + Asin at, then $Y(\omega) = 2\pi\delta(\omega) + j\pi A[\delta(\omega + a) - \delta(\omega - a)]$

h(t) = y(t) cos bt

Using the modulation property,

$$H(\omega) = \frac{1}{2}[Y(\omega + b) + Y(\omega - b)]$$

$$H(\omega) = \underline{\pi[\delta(\omega + b) + \delta(\omega - b)] + \frac{j\pi A}{2}[\delta(\omega + a + b) - \delta(\omega - a + b) + \delta(\omega + a - b) - \delta(\omega - a - b)]}$$

(d) $$I(\omega) = \int_0^4 (1-t)e^{-j\omega t}dt = \frac{e^{-j\omega t}}{-j\omega} - \frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_0^4 = \underline{\frac{1}{\omega^2} - \frac{e^{-j4\omega}}{j\omega} - \frac{e^{-j4\omega}}{\omega^2}(j4\omega + 1)}$$

Solution 18.14

Design a problem to help other students to better understand finding the Fourier transform of a variety of time varying functions (do at least three).

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Fourier transforms of these functions:

(a) $f(t) = e^{-t} \cos(3t + \pi)\, u(t)$
(b) $g(t) = \sin \pi t\, [\, u(t + 1) - u(t-1)]$
(c) $h(t) = e^{-2t} \cos \pi t\, u(t-1)$
(d) $p(t) = e^{-2t} \sin 4t\, u(-t)$
(e) $q(t) = 4 \operatorname{sgn}(t - 2) + 3\delta(t) - 2\, u(t - 2)$

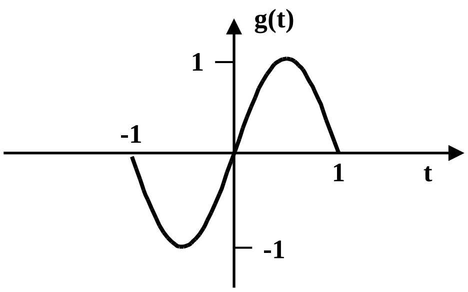
Solution

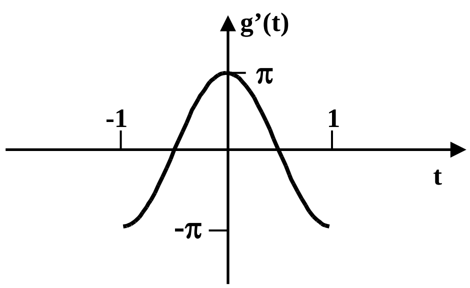
(a) $\cos(3t + \pi) = \cos 3t \cos \pi - \sin 3t \sin \pi = \cos 3t(-1) - \sin 3t(0) = -\cos(3t)$

$$f(t) = -e^{-t} \cos 3t\, u(t)$$

$$F(\omega) = \frac{-(1 + j\omega)}{(1 + j\omega)^2 + 9}$$

(b)





$$g'(t) = \pi \cos \pi t[u(t-1) - u(t-1)]$$
$$g''(t) = -\pi^2 g(t) - \pi\delta(t+1) + \pi\delta(t-1)$$
$$-\omega^2 G(\omega) = -\pi^2 G(\omega) - \pi e^{j\omega} + \pi e^{-j\omega}$$
$$\left(\pi^2 - \omega^2\right) G(\omega) = -\pi(e^{j\omega} - e^{-j\omega}) = -2j\pi \sin \omega$$
$$G(\omega) \;=\; \boldsymbol{\frac{2j\pi \sin \omega}{\omega^2 - \pi^2}}$$

Alternatively, we compare this with Prob. 17.7

f(t) = g(t - 1)

$F(\omega) = G(\omega)e^{-j\omega}$

$$G(\omega) = F(\omega)e^{j\omega} = \frac{\pi}{\omega^2 - \pi^2}\left(e^{-j\omega} - e^{j\omega}\right)$$
$$= \frac{-j2\pi \sin \omega}{\omega^2 - \pi^2}$$
$$G(\omega) \;=\; \boldsymbol{\frac{2j\pi \sin \omega}{\pi^2 - \omega^2}}$$

(c) $\cos \pi(t-1) = \cos \pi t \cos \pi + \sin \pi t \sin \pi = \cos \pi t(-1) + \sin \pi t(0) = -\cos \pi t$

Let $x(t) = e^{-2(t-1)} \cos \pi(t-1)u(t-1) = -e^2 h(t)$

and $y(t) = e^{-2t} \cos(\pi t)u(t)$

$$Y(\omega) = \frac{2 + j\omega}{(2 + j\omega)^2 + \pi^2}$$
$$y(t) = x(t-1)$$
$$Y(\omega) = X(\omega)e^{-j\omega}$$
$$X(\omega) = \frac{(2 + j\omega)e^{j\omega}}{(2 + j\omega)^2 + \pi^2}$$
$$X(\omega) = -e^2 H(\omega)$$
$$H(\omega) = -e^{-2} X(\omega)$$
$$= \boldsymbol{\frac{-(2 + j\omega)e^{j\omega-2}}{(2 + j\omega)^2 + \pi^2}}$$

(d) Let $x(t) = e^{-2t} \sin(-4t)u(-t) = y(-t)$

$p(t) = -x(t)$

where $y(t) = e^{2t} \sin 4t\, u(t)$

$$Y(\omega) = \frac{2 + j\omega}{(2 + j\omega)^2 + 4^2}$$
$$X(\omega) = Y(-\omega) = \frac{2 - j\omega}{(2 - j\omega)^2 + 16}$$

$$p(\omega) = -X(\omega) = \frac{\mathbf{j\omega - 2}}{\mathbf{(j\omega - 2)^2 + 16}}$$

(e) $$Q(\omega) = \frac{8}{j\omega}e^{-j\omega 2} + 3 - 2\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)e^{-j\omega 2}$$

$$Q(\omega) = \frac{\mathbf{6}}{\mathbf{j\omega}}\mathbf{e^{j\omega 2} + 3 - 2\pi\delta(\omega)e^{-j\omega 2}}$$

Solution 18.15

(a) $F(\omega) = e^{j3\omega} - e^{-j\omega 3} = \mathbf{2j\sin 3\omega}$

(b) Let $g(t) = 2\delta(t-1)$, $G(\omega) = 2e^{-j\omega}$

$$F(\omega) = \mathcal{F}\left(\int_{-\infty}^{t} g(t)\,dt\right)$$

$$= \frac{G(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

$$= \frac{2e^{-j\omega}}{j\omega} + 2\pi\delta(-1)\delta(\omega)$$

$$= \boldsymbol{\frac{2e^{-j\omega}}{j\omega}}$$

(c) $\mathcal{F}\left[\delta(2t)\right] = \frac{1}{2}\cdot 1$

$$F(\omega) = \frac{1}{3}\cdot 1 - \frac{1}{2}j\omega = \boldsymbol{\frac{1}{3} - \frac{j\omega}{2}}$$

Solution 18.16

Determine the Fourier transforms of these functions:

(a) $f(t) = 8/t^2$
(b) $g(t) = 4/(4 + t^2)$

Solution

(a) Using duality property

$$|t| \rightarrow \frac{-2}{\omega^2}$$

$$\frac{-4}{t^2} \rightarrow 4\pi|\omega|$$

or $$\frac{8}{t^2} \rightarrow -8\pi|\omega|$$

$$F(\omega) = F\left(\frac{8}{t^2}\right) = \mathbf{-8\pi|\omega|}$$

(b) $$e^{-a|t|} \longrightarrow \frac{2a}{a^2+\omega^2}$$

$$\frac{2a}{a^2+t^2} \longrightarrow 2\pi e^{-a|\omega|}$$

$$\frac{4}{a^2+t^2} \quad \text{leads to} \quad 2\pi e^{-2|\omega|}$$

$$G(\omega) = F\left(\frac{4}{4+t^2}\right) = \mathbf{2\pi e^{-2|\omega|}}$$

Solution 18.17

Find the Fourier transforms of:

(a) $2\cos 2t\, u(t)$
(b) $0.5\sin 10\, t\, u(t)$

Solution

(a) Since H(ω) $= F\left(2\cos\omega_0 t\ \ f(t)\right) = \left[F(\omega+\omega_0) + F(\omega-\omega_0)\right]$

where F(ω) $= F\left[\mathrm{u(t)}\right] = \pi\delta(\omega) + \dfrac{1}{\mathrm{j}\omega}$, $\omega_0 = 2$

$$H(\omega) = \left[\pi\delta(\omega+2) + \frac{1}{j(\omega+2)} + \pi\delta(\omega-2) + \frac{1}{j(\omega-2)}\right]$$

$$= \frac{\pi}{1}\left[\delta(\omega+2) + \delta(\omega-2)\right] - \frac{j}{1}\left[\frac{\omega+2+\omega-2}{(\omega+2)(\omega-2)}\right]$$

H(ω) = **π[δ(ω+2) + δ(ω–2)] – (j2ω)/(ω²–4)**

(b) G(ω) $= F\left[0.5\sin\omega_0 t\ \ f(t)\right] = 0.25j\left[F(\omega+\omega_0) - F(\omega-\omega_0)\right]$

where F(ω) $= F\left[\mathrm{u(t)}\right] = \pi\delta(\omega) + \dfrac{1}{\mathrm{j}\omega}$

$$G(\omega) = \frac{j}{4}\left[\pi\delta(\omega+10) + \frac{1}{j(\omega+10)} - \pi\delta(\omega-10) - \frac{1}{j(\omega-10)}\right]$$

$$= \frac{j\pi}{4}\left[\delta(\omega+10) - \delta(\omega-10)\right] + \frac{j}{4}\left[\frac{j}{\omega-10} - \frac{j}{\omega+10}\right]$$

= **(jπ/4)[δ(ω+10) – δ(ω–10)] –5/(ω²–100)**

Solution 18.18

(a) $F[f(t-t_o)] = \int\limits_{-\infty}^{\infty} f(t-t_o)e^{-j\omega t}dt$

Let $t - t_o = \lambda \quad \longrightarrow \quad t = \lambda + t_o, \quad dt = d\lambda$

$$F[f(t-t_o)] = \int\limits_{-\infty}^{\infty} f(\lambda)e^{-j\omega\lambda}e^{-j\omega t_o}d\lambda = e^{-j\omega t_o}F(\omega)$$

(b) Given that $\qquad f(t) = F^{-1}[F(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$

$$f'(t) = \frac{j\omega}{2\pi}\int\limits_{-\infty}^{\infty} F(\omega)e^{j\omega t}dt = j\omega F^{-1}[F(\omega)]$$

or

$$F[f'(t)] = j\omega F(\omega)$$

(c) This is a special case of the time scaling property when a = –1. Hence,

$$F[f(-t)] = \frac{1}{|-1|}F(-\omega) = F(-\omega)$$

(d) $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$

Differentiating both sides respect to ω and multiplying by t yields

$$j\frac{dF(\omega)}{d\omega} = j\int\limits_{-\infty}^{\infty}(-jt)f(t)e^{-j\omega t}dt = \int\limits_{-\infty}^{\infty} tf(t)e^{-j\omega t}dt$$

Hence,

$$j\frac{dF(\omega)}{d\omega} = F[tf(t)]$$

Chapter 18, Solution 19.

Find the Fourier transform of $f(t) = 2\cos 2\pi t[\,u(t) - u(t-1)]$.

Solution

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{j\omega t}\,dt = \int_0^1 \left(e^{j2\pi t} + e^{-j2\pi t}\right)e^{-j\omega t}\,dt$$

$$F(\omega) = \int_0^1 \left[e^{-j(\omega+2\pi)t} + e^{-j(\omega-2\pi)t}\right]dt$$

$$= \left[\frac{1}{-j(\omega+2\pi)}e^{-j(\omega+2\pi)t} + \frac{1}{-j(\omega-2\pi)}e^{-j(\omega-2\pi)t}\right]_0^1$$

$$= -\left[\frac{e^{-j(\omega+2\pi)}-1}{j(\omega+2\pi)} + \frac{e^{-j(\omega-2\pi)}-1}{j(\omega-2\pi)}\right]$$

But $\quad e^{j2\pi} = \cos 2\pi + j\sin 2\pi = 1 = e^{-j2\pi}$

$$F(\omega) = -\left(\frac{e^{-j\omega}-1}{j}\right)\left(\frac{1}{\omega+2\pi} + \frac{1}{\omega-2\pi}\right)$$

= **$[j2\omega/(\omega^2-4\pi^2)][e^{-j\omega}-1]$**

Solution 18.20

(a) $\quad F(c_n) = c_n\delta(\omega)$

$$F\left(c_n e^{jn\omega_o t}\right) = c_n\delta(\omega - n\omega_o)$$

$$F\left(\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}\right) = \boldsymbol{\sum_{n=-\infty}^{\infty} c_n\delta(\omega - n\omega_o)}$$

(b) $\quad T = 2\pi \qquad \omega_o = \frac{2\pi}{T} = 1$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}\,dt = \frac{1}{2\pi}\left(\int_0^\pi 1\cdot e^{-jnt}\,dt + 0\right)$$

$$= \frac{1}{2\pi}\left(-\frac{1}{jn}e^{jnt}\Big|_0^\pi\right) = \frac{j}{2\pi n}\left(e^{-jn\pi} - 1\right)$$

But $e^{-jn\pi} = \cos n\pi + j\sin n\pi = \cos n\pi = (-1)^n$

$$c_n = \frac{j}{2n\pi}\left[(-1)^n - 1\right] = \begin{cases} 0, & n = \text{even} \\ \frac{-j}{n\pi}, & n = \text{odd}, n \neq 0 \end{cases}$$

for n = 0

$$c_n = \frac{1}{2\pi}\int_0^\pi 1\,dt = \frac{1}{2}$$

Hence

$$f(t) = \frac{1}{2} - \sum_{\substack{n=-\infty \\ n\neq 0 \\ n=\text{odd}}}^{\infty} \frac{j}{n\pi}e^{jnt}$$

$$F(\omega) = \boldsymbol{\frac{1}{2}\delta\omega - \sum_{\substack{n=-\infty \\ n\neq 0 \\ n=\text{odd}}}^{\infty} \frac{j}{n\pi}\delta(\omega - n)}$$

Solution 18.21

Using Parseval's theorem,

$$\int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} | F(\omega) |^2 \, d\omega$$

If f(t) = u(t+a) – u(t+a), then

$$\int_{-\infty}^{\infty} f^2(t)dt = \int_{-a}^{a} (1)^2 dt = 2a = \frac{1}{2\pi}\int_{-\infty}^{\infty} 4a^2 \left(\frac{\sin a\omega}{a\omega}\right)^2 d\omega$$

or

$$\int_{-\infty}^{\infty} \left(\frac{\sin a\omega}{a\omega}\right)^2 \mathbf{d\omega = \frac{4\pi a}{4a^2} = \frac{\pi}{a}} \quad \text{as required.}$$

Solution 18.22

$$\mathcal{F}\left[f(t)\sin\omega_o t\right] = \int_{-\infty}^{\infty} f(t)\frac{\left(e^{j\omega_o t} - e^{-j\omega_o t}\right)}{2j} e^{-j\omega t}\, dt$$

$$= \frac{1}{2j}\left[\int_{-\infty}^{\infty} f(t) e^{-j(\omega-\omega_o)t}\, dt - \int_{-\infty}^{\infty} e^{-j(\omega+\omega_o)t}\, dt\right]$$

$$= \ \boldsymbol{\frac{1}{2j}\left[F(\omega-\omega_o) - F(\omega+\omega_o)\right]}$$

Solution 18.23

(a) f(3t) leads to $\frac{1}{3}\cdot\frac{10}{(2+j\omega/3)(5+j\omega/3)}=\frac{30}{(6+j\omega)(15+j\omega)}$

$$F\,[f(-3t)]=\mathbf{\frac{30}{(6-j\omega)(15-j\omega)}}$$

(b) f(2t) $\longrightarrow$ $\frac{1}{2}\cdot\frac{10}{(2+j\omega/2)(15+j\omega/2)}=\frac{20}{(4+j\omega)(10+j\omega)}$

f(2t-1) = f [2(t-1/2)] $\longrightarrow$ $\mathbf{\frac{20e^{-j\omega/2}}{(4+j\omega)(10+j\omega)}}$

(c) f(t) cos 2t $\longrightarrow$ $\frac{1}{2}F(\omega+2)+\frac{1}{2}F(\omega+2)$

$$=\mathbf{\frac{5}{[2+j(\omega+2)][5+j(\omega+2)]}+\frac{5}{[2+j(\omega-2)][5+j(\omega-2)]}}$$

(d) $F\,\mathbf{[f'(t)]=j\omega F(\omega)=\frac{j\omega 10}{(2+j\omega)(5+j\omega)}}$

(e) $\int_{-\infty}^{t} f(t)dt$ $\longrightarrow$ $\frac{F(\omega)}{j(\omega)}+\pi F(0)\delta(\omega)$

$$=\frac{10}{j\omega(2+j\omega)(5+j\omega)}+\pi\delta(\omega)\frac{x10}{2x5}$$

$$=\mathbf{\frac{10}{j\omega(2+j\omega)(5+j\omega)}+\pi\delta(\omega)}$$

Solution 18.24

(a) $$X(\omega) = F(\omega) + \mathcal{F}[3]$$
$$= \mathbf{6\pi\delta(\omega) + \frac{j}{\omega}\left(e^{-j\omega} - 1\right)}$$

(b) $$y(t) = f(t-2)$$
$$Y(\omega) = e^{-j\omega 2}F(\omega) = \mathbf{\frac{je^{-j2\omega}}{\omega}\left(e^{-j\omega} - 1\right)}$$

(c) If $h(t) = f\,'(t)$
$$H(\omega) = j\omega F(\omega) = j\omega\frac{j}{\omega}\left(e^{-j\omega} - 1\right) = \mathbf{1 - e^{-j\omega}}$$

(d) $g(t) = 4f\left(\frac{2}{3}t\right) + 10f\left(\frac{5}{3}t\right)$, $G(\omega) = 4x\frac{3}{2}F\left(\frac{3}{2}\omega\right) + 10x\frac{3}{5}F\left(\frac{3}{5}\omega\right)$

$$= 6\cdot\frac{j}{\frac{3}{2}\omega}\left(e^{-j3\omega/2} - 1\right) + \frac{6j}{\frac{3}{5}\omega}\left(e^{-j3\omega/5} - 1\right)$$

$$= \mathbf{\frac{j4}{\omega}\left(e^{-j3\omega/2} - 1\right) + \frac{j10}{\omega}\left(e^{-j3\omega/5} - 1\right)}$$

Solution 18.25

(a) $g(t) = \mathbf{5e^{2t}u(t)}$
(b) $h(t) = \mathbf{6e^{-2t}}$
(c) $X(\omega) = \dfrac{A}{s-1} + \dfrac{B}{s-2}, \qquad s = j\omega$

$$A = \frac{10}{1-2} = -10, \qquad B = \frac{10}{2-1} = 10$$

$$X(\omega) = \frac{-10}{j\omega - 1} + \frac{10}{j\omega - 2}$$

$x(t) = \mathbf{(-10e^{t}u(t)+10e^{2t})u(t)}$

(a) $\mathbf{5e^{2t}u(t)}$, **(b)** $\mathbf{6e^{-2t}}$ **, (c)** $\mathbf{(-10e^{t}u(t)+10e^{2t})u(t)}$

Solution 18.26

(a) $\underline{\mathbf{f(t) = e^{-(t-2)}u(t)}}$

(b) $\underline{\mathbf{h(t) = te^{-4t}u(t)}}$

(c) If $\mathbf{x(t) = u(t+1) - u(t-1)} \longrightarrow \mathbf{X(\omega) = 2\frac{\sin\omega}{\omega}}$

By using duality property,

$$\mathbf{G(\omega) = 2u(\omega+1) - 2u(\omega-1)} \longrightarrow \underline{\mathbf{g(t) = \frac{2\sin t}{\pi t}}}$$

Solution 18.27

(a) Let $F(s) = \dfrac{100}{s(s+10)} = \dfrac{A}{s} + \dfrac{B}{s+10}, \quad s = j\omega$

$$A = \frac{100}{10} = 10, \; B = \frac{100}{-10} = -10$$

$$F(\omega) = \frac{10}{j\omega} - \frac{10}{j\omega + 10}$$

$$f(t) \; = \; \mathbf{(5\,sgn(t) - 10e^{-10t})u(t)}$$

(b) $G(s) = \dfrac{10s}{(2-s)(3+s)} = \dfrac{A}{2-s} + \dfrac{B}{s+3}, \quad s = j\omega$

$$A = \frac{20}{5} = 4, \quad B = \frac{-30}{5} = -6$$

$$G(\omega) = \frac{4}{= -j\omega + 2} - \frac{6}{j\omega + 3}$$

$$g(t) \; = \; \mathbf{4e^{2t}u(-t) - 6e^{-3t}u(t)}$$

(c) $H(\omega) = \dfrac{60}{(j\omega)^2 + j40\omega + 1300} = \dfrac{60}{(j\omega + 20)^2 + 900}$

$$h(t) \; = \; \mathbf{2e^{-20t}\,\sin(30t)\,u(t)}$$

(d) $y(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} \dfrac{\delta(\omega)e^{j\omega t}d\omega}{(2 + j\omega)(j\omega + 1)} = \dfrac{1}{2}\pi \cdot \dfrac{1}{2} = \mathbf{\dfrac{1}{4}\pi}$

Solution 18.28

(a) $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\pi\delta(\omega)\,e^{j\omega t}}{(5+j\omega)(2+j\omega)}d\omega$$

$$= \frac{1}{2}\,\frac{1}{(5)(2)} = \frac{1}{20} = \mathbf{0.05}$$

(b) $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{10\delta(\omega+2)}{j\omega(j\omega+1)}e^{j\omega t}\,d\omega = \frac{10}{2\pi}\frac{e^{-j2t}}{(-j2)(-j2+1)}$$

$$= \frac{j5}{2\pi}\frac{e^{-j2t}}{1-j2} = \mathbf{\frac{(-2+j)e^{-j2t}}{2\pi}}$$

(c) $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{20\delta(\omega-1)e^{j\omega t}}{(2+j\omega)(3+5\omega)}d\omega = \frac{20}{2\pi}\frac{e^{jt}}{(2+j)(3+j)}$$

$$= \frac{20e^{jt}}{2\pi(5+5j)} = \mathbf{\frac{(1-j)e^{jt}}{\pi}}$$

(d) Let $$F(\omega) = \frac{5\pi\delta(\omega)}{(5+j\omega)} + \frac{5}{j\omega(5+j\omega)} = F_1(\omega) + F_2(\omega)$$

$$f_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{5\pi\delta(\omega)}{5+j\omega}e^{j\omega t}\,d\omega = \frac{5\pi}{2\pi}\cdot\frac{1}{5} = 0.5$$

$$F_2(s) = \frac{5}{s(5+s)} = \frac{A}{s} + \frac{B}{s+5}\ ,\ A = 1, B = -1$$

$$F_2(\omega) = \frac{1}{j\omega} - \frac{1}{j\omega+5}$$

$$f_2(t) = \frac{1}{2}\mathrm{sgn}(t) - e^{-5t} = -\frac{1}{2} + u(t) - e^{5t}$$

$$f(t) = f_1(t) + f_2(t) = \mathbf{u(t) - e^{-5t}}$$

Solution 18.29

(a) $f(t) = F^{-1}[\delta(\omega)] + F^{-1}\left[4\delta(\omega+3) + 4\delta(\omega-3)\right]$

$$= \frac{1}{2\pi} + \frac{4\cos 3t}{\pi} = \mathbf{\frac{1}{2\pi}\left(1 + 8\cos 3t\right)}$$

(b) If $h(t) = u(t+2) - u(t-2)$

$$H(\omega) = \frac{2\sin 2\omega}{\omega}$$

$$G(\omega) = 4H(\omega) \longrightarrow g(t) = \frac{1}{2\pi}\cdot\frac{8\sin 2t}{t}$$

$$g(t) = \mathbf{\frac{4\sin 2t}{\pi t}}$$

(c) Since the Fourier transform of cos(at) is $= \pi\delta(\omega+a) + \pi\delta(\omega-a)$

Using the reversal property,

$$2\pi\cos 2\omega \leftrightarrow \pi\delta(t+2) + \pi\delta(t-2)$$

or $F^{-1}\left[6\cos 2\omega\right] = \mathbf{3\delta(t+2) + 3\delta(t-2)}$

Solution 18.30

(a) $y(t) = \mathrm{sgn}(t) \longrightarrow Y(\omega) = \dfrac{2}{j\omega}, \qquad X(\omega) = \dfrac{1}{a + j\omega}$

$$\mathbf{H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2(a + j\omega)}{j\omega} = 2 + \frac{2a}{j\omega} \longrightarrow \underline{h(t) = 2\delta(t) + a[u(t) - u(-t)]}}$$

(b) $X(\omega) = \dfrac{1}{1 + j\omega}, \qquad Y(\omega) = \dfrac{1}{2 + j\omega}$

$$\mathbf{H(\omega) = \frac{1 + j\omega}{2 + j\omega} = 1 - \frac{1}{2 + j\omega} \longrightarrow \underline{h(t) = \delta(t) - e^{-2t}u(t)}}$$

(c) In this case, by definition, $\underline{\mathbf{h(t) = \quad y(t) = e^{-at}\sin bt\, u(t)}}$

Solution 18.31

(a) $$Y(\omega) = \frac{1}{(a + j\omega)^2}, \qquad H(\omega) = \frac{1}{a + j\omega}$$

$$\mathbf{X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{1}{a + j\omega} \quad \longrightarrow \quad \underline{x(t) = e^{-at}u(t)}}$$

(b) By definition, $\underline{\mathbf{x(t) = y(t) = u(t+1) - u(t-1)}}$

(c) $$Y(\omega) = \frac{1}{(a + j\omega)}, \qquad H(\omega) = \frac{2}{j\omega}$$

$$\mathbf{X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{j\omega}{2(a + j\omega)} = \frac{1}{2} - \frac{a}{2(a + j\omega)} \quad \longrightarrow \quad \underline{x(t) = \frac{1}{2}\delta(t) - \frac{a}{2}e^{-at}u(t)}}$$

Solution 18.32

(a) Since $\dfrac{e^{-j\omega}}{j\omega+1}$ $\qquad e^{-(t-1)}u(t-1)$

and $F(-\omega) \longrightarrow f(-t)$

$$F_1(\omega)=\frac{e^{j\omega}}{-j\omega+1} \longrightarrow f_1(t)=e^{-(-t-1)}u(-t-1)$$

$f_1(t)$ = $\mathbf{e^{(t+1)}u(-t-1)}$

(b) From Section 17.3,

$$\frac{2}{t^2+1} \longrightarrow 2\pi e^{-|\omega|}$$

If $F_2(\omega)=2e^{-|\omega|}$, then

$f_2(t)$ = $\mathbf{\dfrac{2}{\pi(t^2+1)}}$

(b) By partial fractions

$$F_3(\omega)=\frac{1}{(j\omega+1)^2(j\omega-1)^2}=\frac{\frac{1}{4}}{(j\omega+1)^2}+\frac{\frac{1}{4}}{(j\omega+1)}+\frac{\frac{1}{4}}{(j\omega-1)^2}-\frac{\frac{1}{4}}{j\omega-1}$$

Hence $f_3(t)=\dfrac{1}{4}\left(te^{-t}+e^{-t}+te^{t}-e^{t}\right)u(t)$

$$= \mathbf{\frac{1}{4}(t+1)e^{-t}u(t)+\frac{1}{4}(t-1)e^{t}u(t)}$$

(d) $f_4(t)=\dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}F_1(\omega)e^{j\omega t}\,d\omega=\dfrac{1}{2\pi}\int_{-\infty}^{\infty}\dfrac{\delta(\omega)e^{j\omega t}}{1+j2\omega}$ = $\mathbf{\dfrac{1}{2\pi}}$

Solution 18.33

(a) Let $x(t) = 2\sin \pi t[u(t+1) - u(t-1)]$

From Problem 17.9(b),

$$X(\omega) = \frac{4j\pi \sin \omega}{\pi^2 - \omega^2}$$

Applying duality property,

$$f(t) = \frac{1}{2\pi} X(-t) = \frac{2j\sin(-t)}{\pi^2 - t^2}$$

$$f(t) \;=\; \mathbf{\frac{2j\sin t}{t^2 - \pi^2}}$$

(b)

$$F(\omega) = \frac{j}{\omega}(\cos 2\omega - j\sin 2\omega) - \frac{j}{\omega}(\cos \omega - j\sin \omega)$$

$$= \frac{j}{\omega}\left(e^{j2\omega} - e^{-j\omega}\right) = \frac{e^{-j\omega}}{j\omega} - \frac{e^{j2\omega}}{j\omega}$$

$$f(t) = \frac{1}{2}\mathrm{sgn}(t-1) - \frac{1}{2}\mathrm{sgn}(t-2)$$

But $\mathrm{sgn}(t) = 2u(t) - 1$

$$f(t) = u(t-1) - \frac{1}{2} - u(t-2) + \frac{1}{2}$$

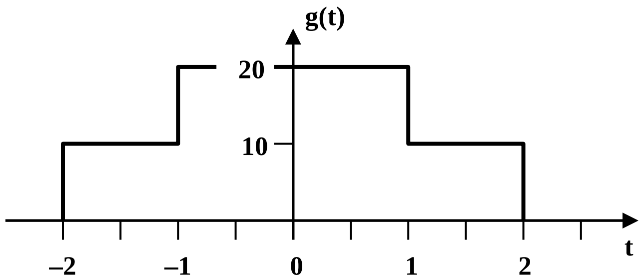
$$= \;\mathbf{u(t-1) - u(t-2)}$$

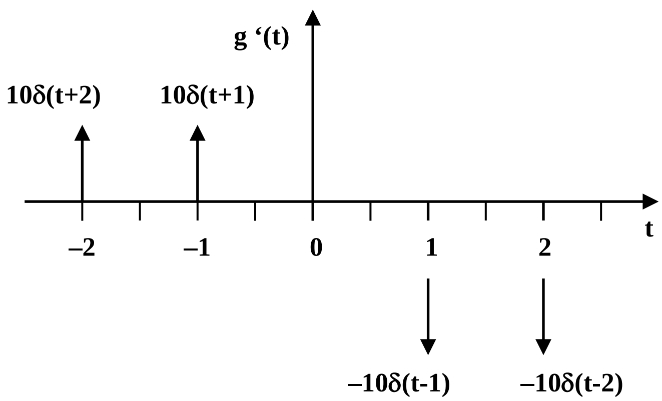
Solution 18.34

First, we find G(ω) for g(t) shown below.

$$g(t) = 10[u(t+2) - u(t-2)] + 10[u(t+1) - u(t-1)]$$
$$g'(t) = 10[\delta(t+2) - \delta(t-2)] + 10[\delta(t+1) - \delta(t-1)]$$

The Fourier transform of each term gives





$$j\omega G(\omega) = 10\left(e^{j\omega 2} - e^{-j\omega 2}\right) + 10\left(e^{j\omega} - e^{-j\omega}\right)$$
$$= 20j\sin 2\omega + 20j\sin\omega$$
$$G(\omega) = \frac{20\sin 2\omega}{\omega} + \frac{20\sin\omega}{\omega} = 40\ \text{sinc}(2\omega) + 20\ \text{sinc}(\omega)$$

Note that G(ω) = G(-ω).

$$F(\omega) = 2\pi G(-\omega)$$
$$f(t) = \frac{1}{2\pi}G(t)$$

= **(20/π)sinc(2t) + (10/π)sinc(t)**

Solution 18.35

(a) x(t) = f[3(t-1/3)]. Using the scaling and time shifting properties,

$$\mathbf{X}(\omega) = \frac{1}{3}\frac{1}{2 + j\omega/3}e^{-j\omega/3} = \underline{\frac{e^{-j\omega/3}}{(6 + j\omega)}}$$

(b) Using the modulation property,

$$\mathbf{Y}(\omega) = \frac{1}{2}[F(\omega+5) + F(\omega-5)] = \underline{\frac{1}{2}\left[\frac{1}{2 + j(\omega+5)} + \frac{1}{2 + j(\omega-5)}\right]}$$

(c) $$\mathbf{Z}(\omega) = j\omega F(\omega) = \underline{\frac{j\omega}{2 + j\omega}}$$

(d) $$\mathbf{H}(\omega) = F(\omega)F(\omega) = \underline{\frac{1}{(2 + j\omega)^2}}$$

(e) $$\mathbf{I}(\omega) = j\frac{d}{d\omega}F(\omega) = j\frac{(0 - j)}{(2 + j\omega)^2} = \underline{\frac{1}{(2 + j\omega)^2}}$$

Solution 18.36

The transfer function of a circuit is

$$H(\omega) = 10/(j\omega+2)$$

If the input signal to the circuit is $v_s(t) = e^{-4t}u(t)$ V, find the output signal. Assume all initial conditions are zero.

Solution

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \quad \longrightarrow \quad Y(\omega) = H(\omega)X(\omega)$$

$$x(t) = v_s(t) = e^{-4t}u(t) \quad \longrightarrow \quad X(\omega) = \frac{1}{4 + j\omega}$$

$$Y(\omega) = \frac{10}{(j\omega + 2)(4 + j\omega)} = \frac{10}{(s+2)(s+4)}, \; s = j\omega$$

$Y(\omega) = 10/[(j\omega+2)(4+j\omega)] = 10/[(s+2)(s+4)]$ where $s = j\omega$

$Y(s) = [A/(s+2)] + [B/(s+4)]$ where $A = 5$ and $B = -5$

$$Y(s) = \frac{A}{s+2} + \frac{B}{s+4}$$

$$A = \frac{2}{-2+4} = 1, \qquad B = \frac{2}{-4+2} = -1$$

$$Y(s) = \frac{1}{s+2} - \frac{1}{s+4}$$

$$y(t) = 5[e^{-2t} - e^{-4t}]u(t)$$

Please note, the units are not known since the transfer function does not give them. If the transfer function was a voltage gain then the units on y(t) would be volts.

Solution 18.37

$$2 \| j\omega = \frac{j2\omega}{2 + j\omega}$$

By current division,

$$H(\omega) = \frac{I_o(\omega)}{I_s(\omega)} = \frac{\dfrac{j2\omega}{2 + j\omega}}{4 + \dfrac{j2\omega}{2 + j\omega}} = \frac{j2\omega}{j2\omega + 8 + j4\omega}$$

$$H(\omega) \ = \ \boldsymbol{\frac{j\omega}{4 + j3\omega}}$$

Solution 18.38

Using Fig. 18.40, design a problem to help other students to better understand using Fourier transforms to do circuit analysis.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Suppose $v_s(t) = u(t)$ for $t>0$. Determine $i(t)$ in the circuit of Fig. 18.40 using Fourier transform.

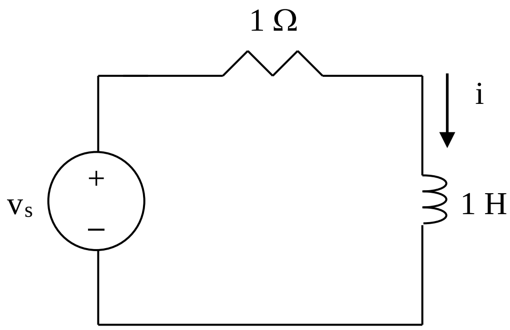


Figure 18.40 For Prob. 18.38.

Solution

$$V_s = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$I(\omega) = \frac{V_s}{1+j\omega} = \frac{1}{1+j\omega}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$

Let $I(\omega) = I_1(\omega) + I_2(\omega) = \dfrac{\pi\delta(\omega)}{1+j\omega} + \dfrac{1}{j\omega(1+j\omega)}$

$$I_2(\omega) = \frac{1}{j\omega(1+j\omega)} = \frac{A}{s} + \frac{B}{s+1}, \qquad s = j\omega$$

where $A = \dfrac{1}{1} = 1, \qquad B = \dfrac{1}{-1} = -1$

$$I_2(\omega) = \frac{1}{j\omega} + \frac{-1}{j\omega+1} \longrightarrow i_2(t) = \frac{1}{2}\operatorname{sgn}(t) - e^{-t}$$

$$I_1(\omega) = \frac{\pi\delta(\omega)}{1+j\omega}$$

$$i_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\pi\delta(\omega)}{1+j\omega} e^{j\omega t} d\omega = \frac{1}{2}\frac{e^{j\omega t}}{1+j\omega}\bigg|_{\omega=0} = \frac{1}{2}$$

Hence,

$$i(t) = i_1(t) + i_2(t) = \underline{\frac{1}{2} + \frac{1}{2}\text{sgn}(t) - e^{-t}}$$

Solution 18.39

Given the circuit in Fig. 18.41, with its excitation, determine the Fourier transform of *i*(*t*).

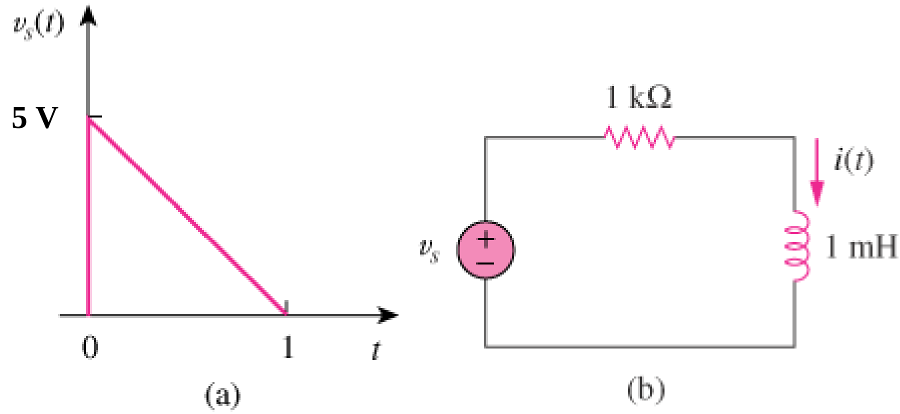


Figure 18.41
For Prob. 18.39.

Solution

$$V_s(\omega) = 5\int_{-\infty}^{\infty}(1-t)e^{-j\omega t}dt = \frac{5}{j\omega} + \frac{5}{\omega^2} - \frac{5}{\omega^2}e^{-j\omega}$$

$$I(\omega) = \frac{V_s(\omega)}{10^3 + j\omega x10^{-3}} = \underline{\frac{5x10^3}{10^6 + j\omega}\left(\frac{1}{j\omega} + \frac{1}{\omega^2} - \frac{1}{\omega^2}e^{-j\omega}\right)}$$

Solution 18.40

Determine the current $i(t)$ in the circuit of Fig. 18.42(b), given the voltage source shown in Fig. 18.42(a).

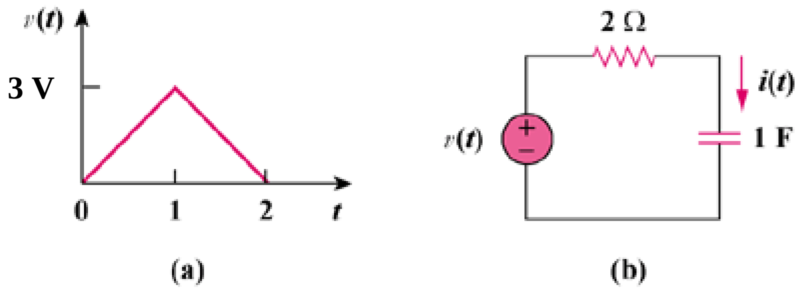


Figure 18.42
For Prob. 18.49.

Solution

$$\ddot{v}(t) = 3\delta(t) - 6\delta(t-1) + 3\delta(t-2)$$

$$-\omega^2 V(\omega) = 3 - 6e^{-j\omega} + 3e^{j\omega 2}$$

$$V(\omega) = \frac{3[1 - 2e^{-j\omega} + e^{-j\omega 2}]}{-\omega^2}$$

Now $$Z(\omega) = 2 + \frac{1}{j\omega} = \frac{1 + j2\omega}{j\omega}$$

$$I = \frac{V(\omega)}{Z(\omega)} = \frac{3[2e^{j\omega} - e^{j\omega 2} - 1]}{\omega^2} \cdot \frac{j\omega}{1 + j2\omega}$$

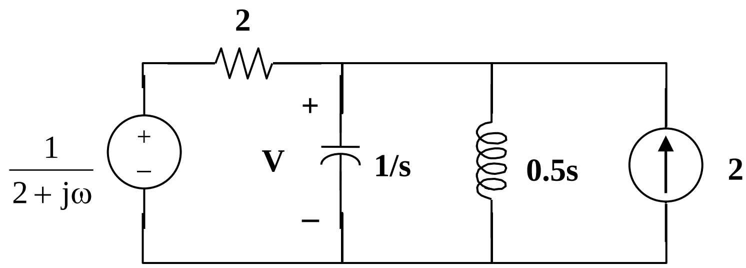
$$= \frac{3}{j\omega(0.5 + j\omega)}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

But $$\frac{3}{s(s+0.5)} = \frac{A}{s} + \frac{B}{s+0.5} \longrightarrow \text{A = 6, B = –6}$$

$$I(\omega) = \frac{6}{j\omega}\left(0.5 + 0.5e^{j\omega 2} - e^{-j\omega}\right) - \frac{6}{0.5 + j\omega}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

$$i(t) = \frac{3}{2}\mathrm{sgn}(t) + \frac{3}{2}\mathrm{sgn}(t-2) - 3\mathrm{sgn}(t-1) - 3e^{-0.5t}u(t) - 3e^{-0.5(t-2)}u(t-2) - 6e^{-0.5(t-1)}u(t-1)$$

Solution 18.41



$$\frac{V-\frac{1}{2+j\omega}}{2}+j\omega V+\frac{2V}{j\omega}-2=0$$

$$\left(j\omega-2\omega^2+4\right)V=j4\omega+\frac{j\omega}{2+j\omega}=\frac{-4\omega^2+j9\omega}{2+j\omega}$$

$$V(\omega)=\mathbf{\frac{2j\omega(4.5+j2\omega)}{(2+j\omega)(4-2\omega^2+j\omega)}}$$

Solution 18.42

By current division, $I_o = \frac{2}{2+j\omega} \cdot I(\omega)$

(a) For i(t) = 5 sgn (t),

$$I(\omega) = \frac{10}{j\omega}$$

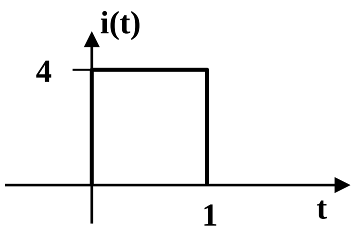
$$I_o = \frac{2}{2+j\omega} \cdot \frac{10}{j\omega} = \frac{20}{j\omega(2+j\omega)}$$

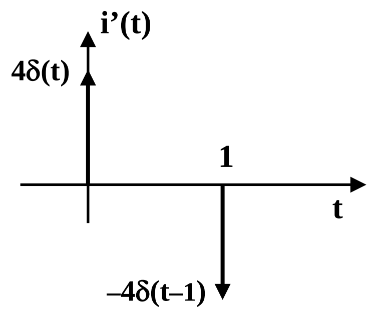
Let $I_o = \frac{20}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$, $A = 10$, $B = -10$

$$I_o(\omega) = \frac{10}{j\omega} - \frac{10}{2+j\omega}$$

$i_o(t)$ = $\mathbf{5\,sgn(t) - 10e^{-2t}u(t)A}$

(b)





$$i'(t) = 4\delta(t) - 4\delta(t-1)$$

$$j\omega I(\omega) = 4 - 4e^{-j\omega}$$

$$I(\omega) = \frac{4(1-e^{-j\omega})}{j\omega}$$

$$I_o = \frac{8(1-e^{-j\omega})}{j\omega(2+j\omega)} = 4\left(\frac{1}{j\omega} - \frac{1}{2+j\omega}\right)(1-e^{-j\omega})$$

$$= \frac{4}{j\omega} - \frac{4}{2+j\omega} - \frac{4e^{-j\omega}}{j\omega} + \frac{4e^{-j\omega}}{2+j\omega}$$

$i_o(t)$ = $\mathbf{2\,sgn(t) - 2\,sgn(t-1) - 4e^{-2t}u(t) + 4e^{-2(t-1)}u(t-1)A}$

Solution 18.43

$$20\,\text{mF} \quad\longrightarrow\quad \frac{1}{j\omega C} = \frac{1}{j20\text{x}10^{-3}\omega} = \frac{50}{j\omega}, \qquad i_s = 5e^{-t} \quad\longrightarrow\quad I_s = \frac{5}{1+j\omega}$$

$$V_o = \frac{40}{40+\frac{50}{j\omega}} I_s \bullet \frac{50}{j\omega} = \frac{250}{(s+1)(s+1.25)}, \qquad s = j\omega$$

$$V_o = \frac{A}{s+1} + \frac{B}{s+1.25} = 1000\left[\frac{1}{s+1} - \frac{1}{s+1.25}\right]$$

$$\mathbf{v_o(t) = \underline{1(e^{-1t} - e^{-1.25t})u(t)\ kV}}$$

Solution 18.44

If the rectangular pulse in Fig. 18.46(a) is applied to the circuit in Fig. 18.46(b), find v_o at t = 1 s.

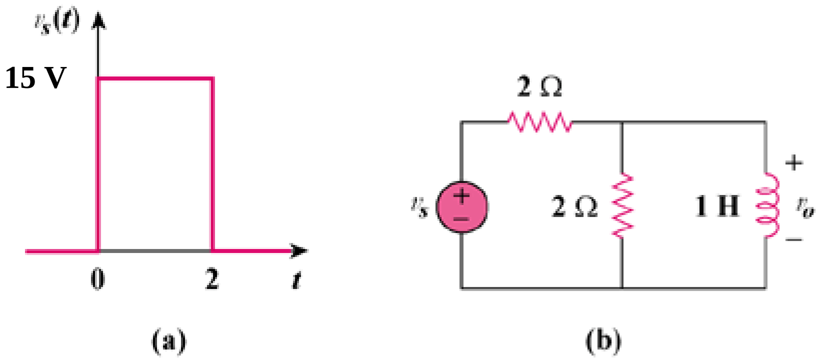
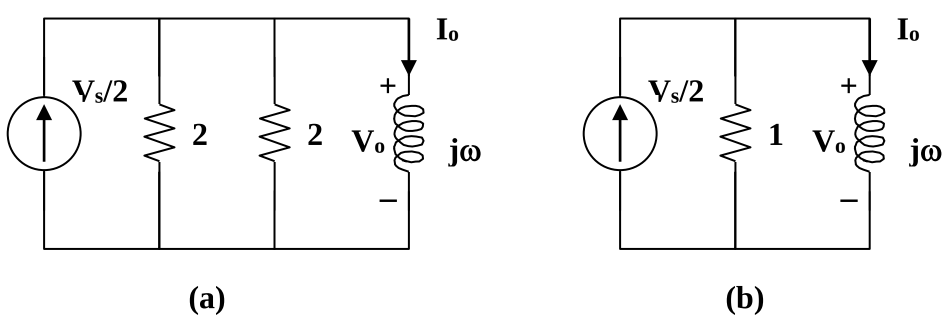


Figure 18.46
For Prob. 18.44.

Solution

$$1\text{H} \longrightarrow j\omega$$

We transform the voltage source to a current source as shown in Fig. (a) and then combine the two parallel 2Ω resistors, as shown in Fig. (b).



$$2\|2 = 1\Omega, \quad I_o = \frac{1}{1+j\omega}\cdot\frac{V_s}{2}$$

$$V_o = j\omega I_o = \frac{j\omega V_s}{2(1+j\omega)}$$

$$\dot{v}_s(t) = 15\delta(t) - 15\delta(t-2)$$

$$j\omega V_s(\omega) = 15 - 15e^{-j2\omega}$$

$$V_s(\omega) = \frac{15\left(1-e^{-j2\omega}\right)}{j\omega}$$

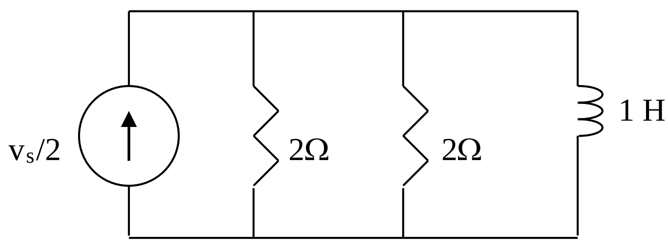
Hence $V_o = \frac{7.5\left(1-e^{-j2\omega}\right)}{1+j\omega} = \frac{7.5}{1+j\omega} - \frac{7.5}{1+j\omega}e^{-j2\omega}$

$$v_o(t) = 7.5e^{-t}u(t) - 7.5e^{-(t-2)}u(t-2)$$

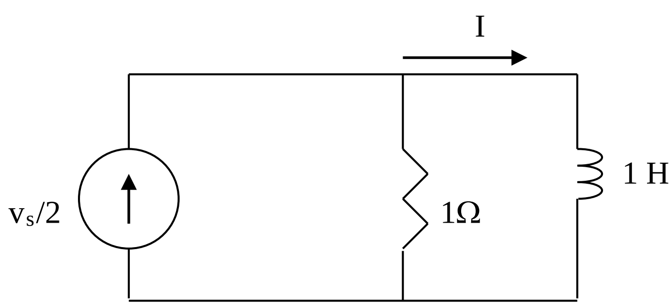
$$v_o(1) = 7.5e^{-1} - 0 = \mathbf{2.759V}$$

Solution 18.45

We may convert the voltage source to a current source as shown below.



Combining the two 2-Ω resistors gives 1 Ω. The circuit now becomes that shown below.



$$I = \frac{1}{1+j\omega} \cdot \frac{V_s}{2} = \frac{1}{1+j\omega} \cdot \frac{5}{2+j\omega} = \frac{5}{(s+1)(s+2)}, \qquad s = j\omega$$

$$= \frac{A}{s+1} + \frac{B}{s+2}$$

where $A = 5/1 = 5, \qquad B = 5/-1 = -5$

$$I = \frac{5}{s+1} - \frac{5}{s+2}$$

$$i(t) = \underline{5(e^{-t} - e^{-2t})u(t)}\,\text{A}$$

Solution 18.46

Determine the Fourier transform of $i_o(t)$ in the circuit of Fig. 18.48.

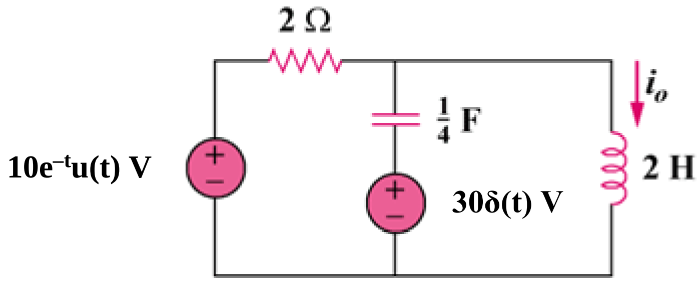


Figure 18.48
For Prob. 18.46.

Solution

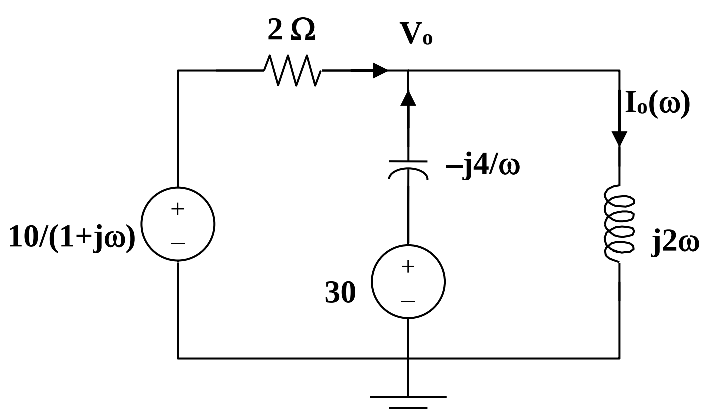
$\frac{1}{4}$F $\longrightarrow \dfrac{1}{j\omega\frac{1}{4}} = \dfrac{-j4}{\omega}$

$2H \longrightarrow j\omega 2$

$30\delta(t) \longrightarrow 30$

$10\,e^{-t}u(t) \longrightarrow \dfrac{10}{1+j\omega}$

The circuit in the frequency domain is shown below:



At node V_o, KCL gives

$$\frac{\frac{10}{1+j\omega}-V_o}{2}+\frac{30-V_o}{\frac{-j4}{\omega}}=\frac{V_o}{j2\omega}$$

$$\frac{20}{1+j\omega} - 2V_o + j\omega 30 - j\omega V_o = -\frac{j2V_o}{\omega}$$

$$V_o = \frac{\dfrac{20}{1+j\omega} + j\omega 30}{2 + j\omega - \dfrac{j2}{\omega}}$$

$$\mathrm{I_o}(\omega) = \frac{\mathrm{V_o}}{\mathrm{j}2\omega} = \frac{\dfrac{20+\mathrm{j}\omega 30 - 30\omega^2}{1+\mathrm{j}\omega}}{\mathrm{j}2\omega\left(2+\mathrm{j}\omega - \dfrac{\mathrm{j}2}{\omega}\right)} = \frac{10(2+3\mathrm{j}\omega - 3\omega^2)}{(1+\mathrm{j}\omega)(\mathrm{j}4\omega - 2\omega^2 + 4)} = \frac{10(2+3\mathrm{j}\omega - 3\omega^2)}{4 + \mathrm{j}4\omega - 2\omega^2 - 4\omega^2 - \mathrm{j}2\omega^3 + \mathrm{j}4\omega}$$

$$\mathrm{I_o}(\omega) = \frac{5[2+\mathrm{j}3\omega^2 - 3\omega^2]}{2 - 3\omega^2 + \mathrm{j}(4\omega - \omega^3)}$$

Solution 18.47

$$\frac{1}{2}F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{2}{j\omega}$$

$$I_o = \frac{1}{1+\dfrac{2}{j\omega}} I_s$$

$$V_o = \frac{2}{j\omega} I_o = \frac{\dfrac{2}{j\omega}}{1+\dfrac{2}{j\omega}} I_s = \frac{2}{2+j\omega}\frac{8}{1+j\omega}$$

$$= \frac{16}{(s+1)(s+2)}, \; s = j\omega$$

$$= \frac{A}{s+1} + \frac{B}{s+2}$$

where $A = 16/1 = 16$, $B = 16/(-1) = -16$

Thus,

$$v_o(t) = \mathbf{16(e^{-t} - e^{-2t})u(t)\ V.}$$

Solution 18.48

Find $i_o(t)$ in the op amp circuit of Fig. 18.50.

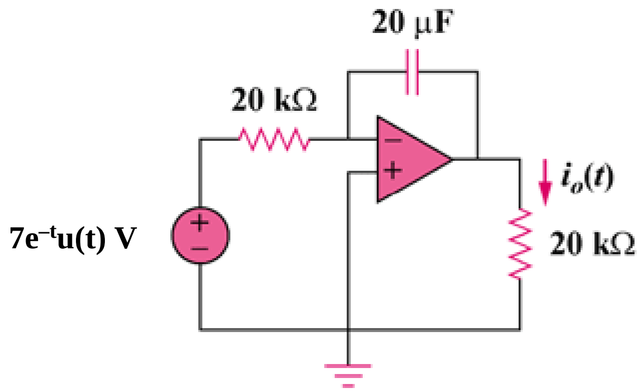


Figure 18.50
For Prob. 18.48.

Solution

As an integrator,

$$RC = 20\,x\,10^3\ x\ 20\,x\,10^{-6} = 0.4$$

$$v_o = -\frac{1}{RC}\int_o^t v_i\,d\tau$$

$$V_o = -\frac{1}{RC}\left[\frac{V_i}{j\omega} + \pi V_i(0)\delta(\omega)\right]$$

$$= -\frac{1}{0.4}\left[\frac{7}{j\omega(1+j\omega)} + 3.5\pi\delta(\omega)\right]$$

$$I_o = \frac{V_o}{20}mA = -0.125\left[\frac{7}{j\omega(2+j\omega)} + 3.5\pi\,\delta(\omega)\right]$$

$$= -\frac{0.4375}{j\omega} + \frac{0.4375}{2+j\omega} - 0.4375\pi\delta(\omega)$$

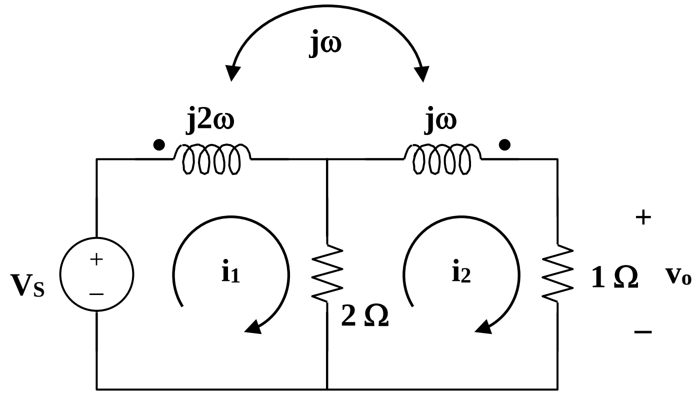
$$i_o(t) = -0.4375\,\mathrm{sgn}(t) + 0.4375e^{-2t}u(t) - \frac{0.4375}{2\pi}\int \pi\delta(\omega)e^{j\omega t}dt$$

$$= 0.4375 + 0.875u(t) + 0.4375e^{-2t}u(t) - \frac{0.4375}{2}$$

$i_o(t)$ = **[2.188 – 0.875u(t) + 0.4375e^{-2t}u(t)] mA**

Solution 18.49

Consider the circuit shown below:



$$V_s = \pi\left[\delta(\omega+1)+\delta(\omega-2)\right]$$

For mesh 1, $-V_s + (2+j2\omega)I_1 - 2I_2 - j\omega I_2 = 0$

$$V_s = 2(1+j\omega)I_1 - (2+j\omega)I_2 \qquad (1)$$

For mesh 2, $\quad 0 = (3+j\omega)I_2 - 2I_1 - j\omega I_1$

$$I_1 = \frac{(3+\omega)I_2}{(2+\omega)} \qquad (2)$$

Substituting (2) into (1) gives

$$V_s = 2\frac{2(1+j\omega)(3+j\omega)I_2}{2+j\omega} - (2+j\omega)I_2$$

$$V_s(2+\omega) = \left[2\left(3+j4\omega-\omega^2\right)-\left(4+j4\omega-\omega^2\right)\right]I_2$$

$$= I_2\left(2+j4\omega-\omega^2\right)$$

$$I_2 = \frac{(s+2)V_s}{s^2+4s+2},\ s = j\omega$$

$$V_o = I_2 = \frac{(j\omega+2)\pi\left[\delta(\omega+1)+\delta(\omega-1)\right]}{(j\omega)^2+j\omega 4+2}$$

$$v_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v_o(\omega)e^{j\omega t}d\omega$$

$$= \int_{-\infty}^{\infty}\frac{\frac{1}{2}(j\omega+2)e^{j\omega t}\delta(\omega+1)d\omega}{(j\omega)^2+j\omega 4+2} + \frac{\frac{1}{2}(j\omega+2)e^{j\omega t}\delta(\omega-1)d\omega}{(j\omega)^2+j\omega 4+2}$$

$$= \frac{\frac{1}{2}(-j+2)e^{jt}}{-1-j4+2} + \frac{\frac{1}{2}(j+2)e^{jt}}{-1+j4+2}$$

$$v_o(t) = \frac{\frac{1}{2}(2-j)(1+j4)}{17}e^{jt} + \frac{\frac{1}{2}(2-j)(1-j4)e^{jt}}{17}$$

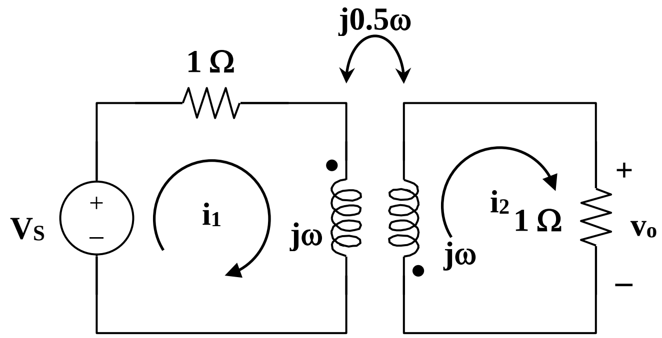
$$= \frac{1}{34}(6+j7)e^{jt} + \frac{1}{34}(6-j7)e^{jt}$$

$$= 0.271e^{-j(t-13.64°)} + 0.271e^{j(t-13.64°)}$$

$$v_o(t) = \mathbf{542\cos(t-13.64°)\,mV}$$

Solution 18.50

Consider the circuit shown below:



For loop 1,

$$-2+(1+j\omega)I_1 + j0.5\omega I_2 = 0 \qquad (1)$$

For loop 2,

$$(1+j\omega)I_2 + j0.5\omega I_1 = 0 \qquad (2)$$

From (2),

$$I_1 = \frac{(1+j\omega)I_2}{-j0.5\omega} = -2\frac{(1+j\omega)I_2}{j\omega}$$

Substituting this into (1),

$$2 = \frac{-2(1+j\omega)I_2}{j\omega} + \frac{j\omega}{2}I_2$$

$$2j\omega = -\left(4 + j4\omega - \frac{3}{2}\omega^2\right)I_2$$

$$I_2 = \frac{2j\omega}{4 + j4\omega - 1.5\omega^2}$$

$$V_o = I_2 = \frac{-2j\omega}{4 + j4\omega + 1.5(j\omega)^2}$$

$$V_o = \frac{\frac{4}{3}j\omega}{\frac{8}{3} + j\frac{8\omega}{3} + (j\omega)^2}$$

$$= \frac{-4\left(\frac{4}{3} + j\omega\right)}{\left(\frac{4}{3}+j\omega\right)^2 + \left(\frac{\sqrt{8}}{3}\right)^2} + \frac{\frac{16}{3}}{\left(\frac{4}{3}+j\omega\right)^2 + \left(\frac{\sqrt{8}}{3}\right)^2}$$

$$V_o(t) = \mathbf{-4e^{-4t/3}\cos\left(\frac{\sqrt{8}}{3}t\right)u(t) + 5.657e^{-4t/3}\sin\left(\frac{\sqrt{8}}{3}t\right)u(t)\,V}$$

Solution 18.51

In the frequency domain, the voltage across the 2-Ω resistor is

$$V(\omega) = \frac{2}{2+j\omega}V_s = \frac{2}{2+j\omega}\frac{10}{1+j\omega} = \frac{20}{(s+1)(s+2)}, \qquad s = j\omega$$

$$V(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = 20/1 = 20, \qquad B = 20/-1 = -20$$

$$V(\omega) = \frac{20}{j\omega+1} - \frac{20}{j\omega+2}$$

$$v(t) = \left(20e^{-t} - 20e^{-2t}\right)u(t)$$

$$W = \frac{1}{2}\int_0^\infty v^2(t)dt = 0.5\int 400\left(e^{-2t} + e^{-4t} - 3e^{-3t}\right)dt$$

$$= 200\left(\frac{e^{-2t}}{-2} + \frac{e^{-4t}}{-4} - \frac{2e^{-3t}}{-3}\right)\Bigg|_0^\infty = \mathbf{16.667\ J}.$$

Solution 18.52

For $F(\omega) = 3/(3+j\omega)$, find $J = \int_{-\infty}^{\infty} f^2(t)dt$.

Solution

$$J = 2\int_0^{\infty} f^2(t)\,dt = \frac{1}{\pi}\int_0^{\infty} \left|F(\omega)\right|^2 d\omega$$

$$= \frac{1}{\pi}\int_0^{\infty} \frac{9}{9^2+\omega^2}\,d\omega = \frac{9}{3\pi}\tan^{-1}(\omega/3)\Big|_0^{\infty} = \frac{9}{3\pi}\frac{\pi}{2} = \mathbf{(3/2)}$$

Solution 18.53

If $f(t) = e^{-2|t|}$, find $J = \int_{-\infty}^{\infty} \left|F(\omega)\right|^2 d\omega$.

$$J = \int_{-\infty}^{\infty} \left|F(\omega)\right|^2 d\omega = 2\pi\int_{-\infty}^{\infty} f^2(t)\,dt$$

$$f(t) = \begin{vmatrix} e^{2t}, & t < 0 \\ e^{-2t}, & t > 0 \end{vmatrix}$$

$$J = 2\pi\left[\int_{-\infty}^{0} e^{4t}dt + \int_{0}^{\infty} e^{-4t}dt\right] = 2\pi\left[\left.\frac{e^{4t}}{4}\right|_{-\infty}^{0} + \left.\frac{e^{-4t}}{-4}\right|_{0}^{\infty}\right] = 2\pi[(1/4) + (1/4)] = \boldsymbol{\pi}$$

Solution 18.54

Design a problem to help other students better understand finding the total energy in a given signal.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Given the signal $f(t) = 4\,e^{-t}\,u(t)$, what is the total energy in $f(t)$?

Solution

$$W_{1\Omega} = \int_{-\infty}^{\infty} f^2(t)\,dt = 16\int_0^{\infty} e^{-2t}\,dt = -8e^{-2t}\Big|_0^{\infty} = \mathbf{8\ J}$$

Solution 18.55

$f(t) = 5e^2e^{-t}u(t)$

$F(\omega) = 5e^2/(1 + j\omega)$, $|F(\omega)|^2 = 25e^4/(1 + \omega^2)$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^\infty |F(\omega)|^2 d\omega = \frac{25e^4}{\pi}\int_0^\infty \frac{1}{1+\omega^2} d\omega = \frac{25e^4}{\pi}\tan^{-1}(\omega)\Big|_0^\infty$$

$$= 12.5e^4 = \mathbf{682.5\ J}$$

or $$W_{1\Omega} = \int_{-\infty}^\infty f^2(t)\,dt = 25e^4\int_0^\infty e^{-2t}dt = 12.5e^4 = \mathbf{682.5\ J}$$

Solution 18.56

(a) $W = \int_{-\infty}^{\infty} V^2(t)dt = \int_{0}^{\infty} t^2 e^{-4t} dt = \frac{e^{-4t}}{(-4)^3}(16t^2 + 8t + 2)\Big|_0^{\infty} = \frac{2}{64} = \underline{0.0313 \text{ J}}$

(b) In the frequency domain,

$$V(\omega) = \frac{1}{(2 + j\omega)^2}$$

$$|V(\omega)|^2 = V(\omega)V^*(\omega) = \frac{1}{(4 + j\omega)^2}$$

$$W_o = \frac{1}{2\pi}\int_{-2}^{2} |V(\omega)|^2 d\omega = \frac{2}{2\pi}\int_{0}^{2} \frac{1}{(4 + \omega^2)^2} d\omega$$

$$= \frac{1}{\pi}\frac{1}{2\text{x}4}\left(\frac{\omega}{\omega^2 + 4} + 0.5\tan^{-1}(0.5\omega)\right)\Bigg|_0^2 = \frac{1}{32\pi} + \frac{1}{64} = 0.0256$$

$$\text{Fraction} = \frac{W_o}{W} = \frac{0.0256}{0.0313} = \underline{81.79\%}$$

Solution 18.57

$W_{1\Omega} = \int_{-\infty}^{\infty} i^2(t)\,dt = \int_{-\infty}^{0} 4e^{2t}dt = 2e^{2t}\Big|_{-\infty}^{0} =$ **2 J** or

$I(\omega) = 2/(1 - j\omega)$, $|I(\omega)|^2 = 4/(1 + \omega^2)$

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty}|I(\omega)|^2 d\omega = \frac{4}{2\pi}\int_{-\infty}^{\infty}\frac{1}{(1+\omega^2)}d\omega = \frac{4}{\pi}\tan^{-1}(\omega)\Big|_0^{\infty} = \frac{4}{\pi}\frac{\pi}{2} = \mathbf{2\ J}$$

In the frequency range, $-5 < \omega < 5$,

$$W = \frac{4}{\pi}\tan^{-1}\omega\Big|_0^5 = \frac{4}{\pi}\tan^{-1}(5) = \frac{4}{\pi}(1.373) = 1.7487$$

$W/W_{1\Omega} = 1.7487/2 = 0.8743$ or

87.43%

Solution 18.58

$\omega_m = 200\pi = 2\pi f_m$ which leads to $f_m = 100$ Hz

(a) $\omega_c = \pi x10^4 = 2\pi f_c$ which leads to $f_c = 10^4/2 =$ **5 kHz**

(b) lsb $= f_c - f_m = 5{,}000 - 100 =$ **4,900 Hz**

(c) usb $= f_c + f_m = 5{,}000 + 100 =$ **5,100 Hz**

Solution 18.59

$$H(\omega)=\frac{V_o(\omega)}{V_i(\omega)}=\frac{\dfrac{10}{2+j\omega}-\dfrac{6}{4+j\omega}}{2}=\frac{5}{2+j\omega}-\frac{3}{4+j\omega}$$

$$V_o(\omega)=H(\omega)V_i(\omega)=\left(\frac{5}{2+j\omega}-\frac{3}{4+j\omega}\right)\frac{4}{1+j\omega}$$
$$=\frac{20}{(s+1)(s+2)}-\frac{12}{(s+1)(s+4)},\qquad s=j\omega$$

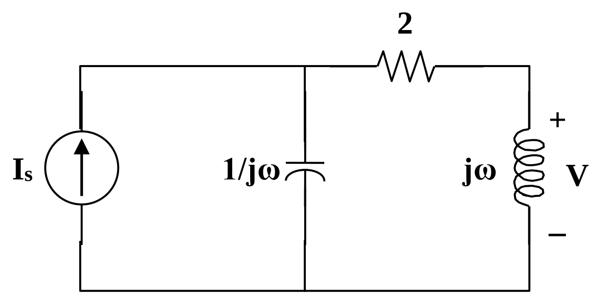
Using partial fraction,

$$V_o(\omega)=\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{s+1}+\frac{D}{s+4}=\frac{16}{1+j\omega}-\frac{20}{2+j\omega}+\frac{4}{4+j\omega}$$

Thus,

$$\mathbf{v_o(t)=\underline{\left(16e^{-t}-20e^{-2t}+4e^{-4t}\right)u(t)\,V}}$$

Solution 18.60



$$V = j\omega I_s \frac{\frac{1}{j\omega}}{\frac{1}{j\omega} + 2 + j\omega} = \frac{j\omega I_s}{1 - \omega^2 + j2\omega}$$

Since the voltage appears across the inductor, there is no DC component.

$$V_1 = \frac{2\pi\angle 90°8}{1 - 4\pi^2 + j4\pi} = \frac{50.27\angle 90°}{-38.48 + j12.566} = 1.2418\angle -71.92°$$

$$V_2 = \frac{4\pi\angle 90°5}{1 - 16\pi^2 + j8\pi} = \frac{62.83\angle 90°}{-156.91 + j25.13} = 0.3954\angle -80.9°$$

$$\mathbf{v(t) = \underline{1.2418\cos(2\pi t - 41.92°) + 0.3954\cos(4\pi t + 129.1°)\,mV}}$$

Solution 18.61

$y(t) = (2 + \cos \omega_o t)x(t)$

We apply the Fourier Transform

$$Y(\omega) = \mathbf{2}\boldsymbol{X}\mathbf{(\omega) + 0.5}\boldsymbol{X}\mathbf{(\omega+\omega_o) + 0.5}\boldsymbol{X}\mathbf{(\omega-\omega_o)}.$$

Solution 18.62

For the lower sideband, the frequencies range from

$$10{,}000{,}000 - 3{,}500 \text{ Hz} = \mathbf{9{,}996{,}500\ Hz} \text{ to}$$
$$10{,}000{,}000 - 400 \text{ Hz} = \mathbf{9{,}999{,}600\ Hz}$$

For the upper sideband, the frequencies range from

$$10{,}000{,}000 + 400 \text{ Hz} = \mathbf{10{,}000{,}400\ Hz} \text{ to}$$
$$10{,}000{,}000 + 3{,}500 \text{ Hz} = \mathbf{10{,}003{,}500\ Hz}$$

Solution 18.63

Since $f_n = 5$ kHz, $2f_n = 10$ kHz

i.e. the stations must be spaced 10 kHz apart to avoid interference.

$$\Delta f = 1600 - 540 = 1060 \text{ kHz}$$

The number of stations $= \Delta f / 10$ kHz $=$ **106 stations**

Solution 18.64

$\Delta f = 108 - 88 \text{ MHz} = 20 \text{ MHz}$

The number of stations = 20 MHz/0.2 MHz = **100 stations**

Solution 18.65

$\omega = 3.4$ kHz

$f_s = 2\omega =$ **6.8 kHz**

Solution 18.66

$\omega = 4.5$ MHz

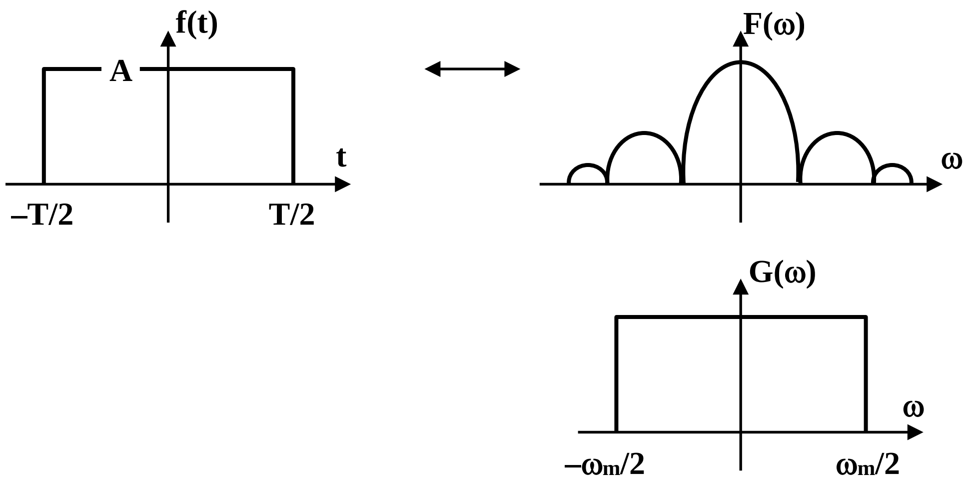
$f_c = 2\omega = 9$ MHz

$T_s = 1/f_c = 1/(9x10^6) = 1.11x10^{-7} =$ **111 ns**

Solution 18.67

We first find the Fourier transform of g(t). We use the results of Example 17.2 in conjunction with the duality property. Let Arect(t) be a rectangular pulse of height A and width T as shown below.

Arect(t) transforms to Atsinc($\omega^2/2$)



According to the duality property,

$A\tau \text{sinc}(\tau t/2)$ becomes $2\pi A\text{rect}(\tau)$

$g(t) = \text{sinc}(200\pi t)$ becomes $2\pi A\text{rect}(\tau)$

where $A\tau = 1$ and $\tau/2 = 200\pi$ or $T = 400\pi$

i.e. the upper frequency $\omega_u = 400\pi = 2\pi f_u$ or $f_u = 200$ Hz

The Nyquist rate $= f_s =$ **200 Hz**

The Nyquist interval $= 1/f_s = 1/200 =$ **5 ms**

Solution 18.68

The total energy is

$$W_T = \int_{-\infty}^{\infty} v^2(t)\,dt$$

Since v(t) is an even function,

$$W_T = \int_0^{\infty} 2500e^{-4t}dt = 5000\left.\frac{e^{-4t}}{-4}\right|_0^{\infty} = 1250 \text{ J}$$

$$V(\omega) = 50x4/(4 + \omega^2)$$

$$W = \frac{1}{2\pi}\int_1^5 |V(\omega)|^2 \; d\omega = \frac{1}{2\pi}\int_1^5 \frac{(200)^2}{(4+\omega^2)^2}d\omega$$

But $\int \frac{1}{(a^2+x^2)^2}dx = \frac{1}{2a^2}\left[\frac{x}{x^2+a^2}+\frac{1}{a}\tan^{-1}(x/a)\right]+C$

$$W = \frac{2x10^4}{\pi}\frac{1}{8}\left.\left[\frac{\omega}{4+\omega^2}+\frac{1}{2}\tan^{-1}(\omega/2)\right]\right|_1^5$$

$$= (2500/\pi)[(5/29) + 0.5\tan^{-1}(5/2) – (1/5) – 0.5\tan^{-1}(1/2) = 267.19$$

$W/W_T = 267.19/1250 = 0.2137$ or **21.37%**

Solution 18.69

The total energy is

$$W_T = \frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{400}{4^2+\omega^2}d\omega$$

$$= \frac{400}{\pi}\left[(1/4)\tan^{-1}(\omega/4)\right]_0^{\infty} = \frac{100}{\pi}\frac{\pi}{2} = 50$$

$$W = \frac{1}{2\pi}\int_{0}^{2}|F(\omega)|^2 d\omega = \frac{400}{2\pi}\left[(1/4)\tan^{-1}(\omega/4)\right]_0^{2}$$

$$= [100/(2\pi)]\tan^{-1}(2) = (50/\pi)(1.107) = 17.6187$$

$W/W_T = 17.6187/50 = 0.3524$ or **35.24%**

Solution 19.1

Obtain the z parameters for the network in Fig. 19.65.

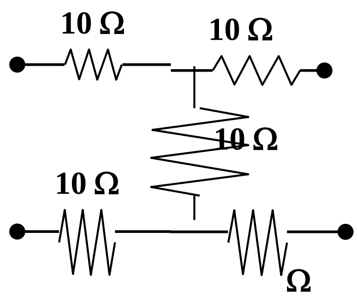
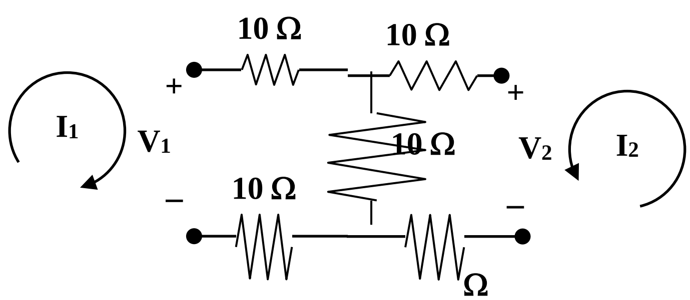


Figure 19.65
For Prob. 19.1.

Solution

Step 1. Label the circuit to allow us to determine the z-parameters.



The z-parameter equations are,

$V_1 = z_{11}I_1 + z_{12}I_2$ and $V_2 = z_{21}I_1 + z_{22}I_2$.

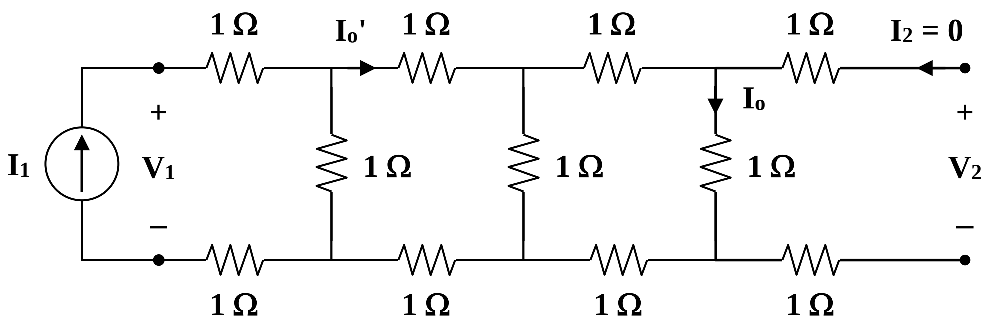
To determine z_{11} and z_{21}, let $I_1 = 1$ A and $I_2 = 0$. To determine z_{12} and z_{22}, let $I_1 = 0$ and $I_2 = 1$ A.

Step 2. For $I_1 = 1$ A and $I_2 = 0$, we get $V_1 = (10+10+10)1 = 30$ V and $V_2 = 10\text{x}1 = 10$ V. Therefore, $z_{11} = 30/1 =$ **30 Ω** and $z_{21} = 10/1 =$ **10 Ω**.

Next, for $I_1 = 0$ and $I_2 = 1$ A, we get $V_1 = 10\text{x}1 = 1$ V and $V_2 = (10+10+10)1$ 30 V. Therefore, $z_{12} = 10/1 =$ **10 Ω** and $z_{22} = 30/1 =$ **30 Ω**.

Solution 19.2

Consider the circuit in Fig. (a) to get $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$.



(a)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2 + 1 \,||\, [\,2 + 1 \,||\, (2+1)\,]$$

$$\mathbf{z}_{11} = 2 + 1 \,||\left(2 + \frac{3}{4}\right) = 2 + \frac{(1)(11/4)}{1 + 11/4} = 2 + \frac{11}{15} = 2.733$$

$$\mathbf{I}_o = \frac{1}{1+3}\mathbf{I}_o{}' = \frac{1}{4}\mathbf{I}_o{}'$$

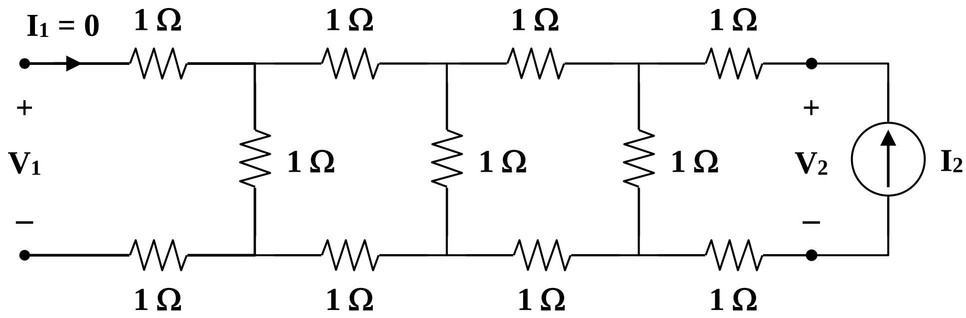
$$\mathbf{I}_o{}' = \frac{1}{1 + 11/4}\mathbf{I}_1 = \frac{4}{15}\mathbf{I}_1$$

$$\mathbf{I}_o = \frac{1}{4}\cdot\frac{4}{15}\mathbf{I}_1 = \frac{1}{15}\mathbf{I}_1$$

$$\mathbf{V}_2 = \mathbf{I}_o = \frac{1}{15}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{1}{15} = \mathbf{z}_{12} = 0.06667$$

To get $\mathbf{z}_{22}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 2 + 1 \| (2 + 1 \| 3) = \mathbf{z}_{11} = 2.733$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{2.733} & \mathbf{0.06667} \\ \mathbf{0.06667} & \mathbf{2.733} \end{bmatrix} \Omega$$

Solution 19.3

Find the z parameters of the circuit in Fig. 19.67.

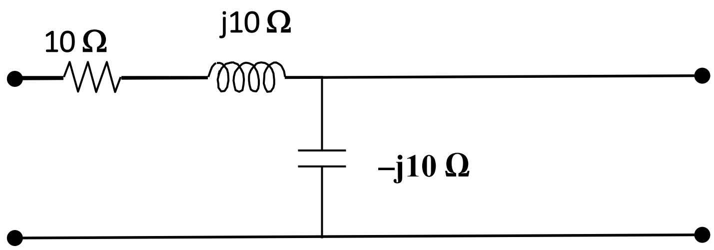
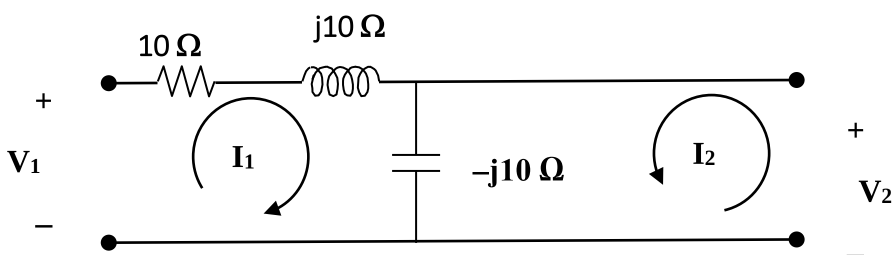


Figure 19.67
For Prob. 19.3.

Solution

Step 1. All we need to do is to set up the circuit description and solve for the z-parameters.



The z-parameter equations are $V_1 = z_{11}I_1 + z_{12}I_2$ and $V_2 = z_{21}I_1 + z_{22}I_2$.

All we need to do is to set $I_1 = 1$ A and $I_2 = 0$ and then $I_1 = 0$ and $I_2 = 1$ A to determine the z-parameters.

Step 2. For $I_1 = 1$ A and $I_2 = 0$ we get $V_1 = (10+j10–j10)1 = 10$ V and $V_2 = (–j10)1 = –j10$ V. Therefore, $z_{11} = 10/1 =$ **10 Ω** and $z_{21} =$ **–j10 Ω**. Next $V_1 = (–j10)1 = –j10$ V and $V_2 = (–j10)1 = –j10$ V. Therefore, $z_{12} = –j10/1 =$ **–j10 Ω** and $z_{22} = –j10/1 =$ **–j10 Ω**.

Solution 19.4

Using Fig. 19.68, design a problem to help other students to better understand how to determine z parameters from an electrical circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Calculate the *z* parameters for the circuit in Fig.19.68.

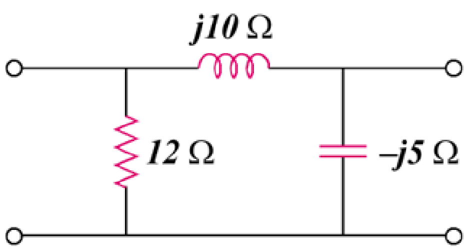
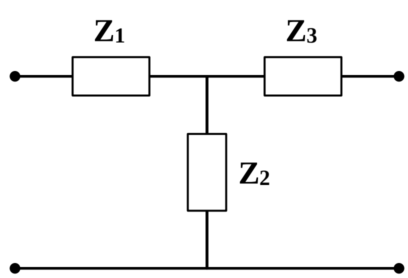


Figure 19.68

Solution

Transform the Π network to a T network.



$$\mathbf{Z}_1 = \frac{(12)(j10)}{12 + j10 - j5} = \frac{j120}{12 + j5}$$

$$\mathbf{Z}_2 = \frac{-j60}{12 + j5}$$

$$\mathbf{Z}_3 = \frac{50}{12 + j5}$$

The z parameters are

$$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{Z}_2 = \frac{(-j60)(12 - j5)}{144 + 25} = -1.775 - j4.26$$

$$\mathbf{z}_{11} = \mathbf{Z}_1 + \mathbf{z}_{12} = \frac{(j120)(12 - j5)}{169} + \mathbf{z}_{12} = 1.775 + j4.26$$

$$\mathbf{z}_{22} = \mathbf{Z}_3 + \mathbf{z}_{21} = \frac{(50)(12 - j5)}{169} + \mathbf{z}_{21} = 1.7758 - j5.739$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{1.775 + j4.26} & \mathbf{-1.775 - j4.26} \\ \mathbf{-1.775 - j4.26} & \mathbf{1.775 - j5.739} \end{bmatrix} \mathbf{\Omega}$$

Solution 19.5

Obtain the *z* parameters for the network in Fig.19.69 as functions of *s*.

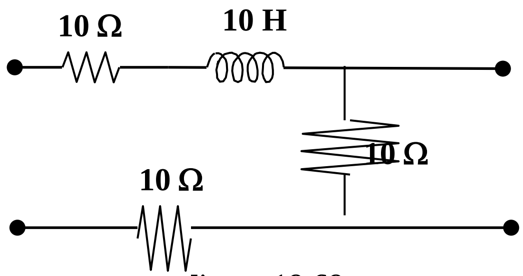
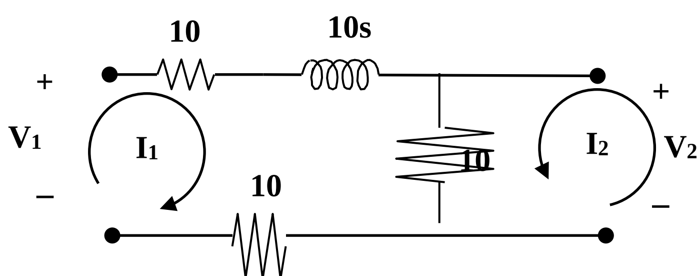


Figure 19.69
For Prob. 19.5

Solution

Step 1. We start by transforming the circuit into the s-domain and labeling the currents and voltages so that we can use the mesh equations to solve for the z-parameters.



Now we can write the z-parameter equations, $V_1 = z_{11}I_1 + z_{12}I_2$ and $V_2 = z_{21}I_1 + z_{22}I_2$.
To find the z-parameters we let $I_1 = 1/s$ and $I_2 = 0$ and then $I_1 = 0$ and $I_2 = 1/s$.

Step 2. $V_1 = (10+10s+10)(1/s) = 10(s+2)/s$ and $V_2 = 10/s$ which leads to, $z_{11} = (10(s+2)/s)/(1/s) =$ **$10(s+2)$** and $z_{21} = (10/s)/(1/s) =$ **10**.

Next, $V_1 = 10/s$ and $V_2 = 10/s$ which leads to, $z_{12} = (10/s)/(1/s) =$ **10** and $Z_{22} = (10/s)/(1/s) =$ **10**.

Solution 19.6

Compute the z parameters of the circuit in Fig. 19.70.

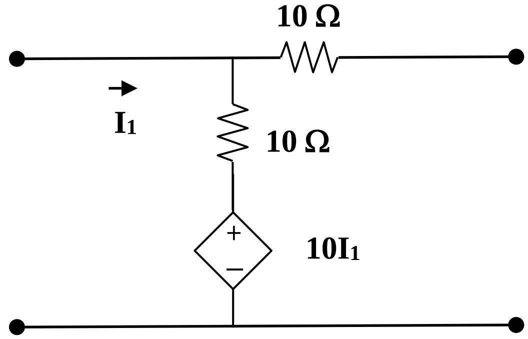
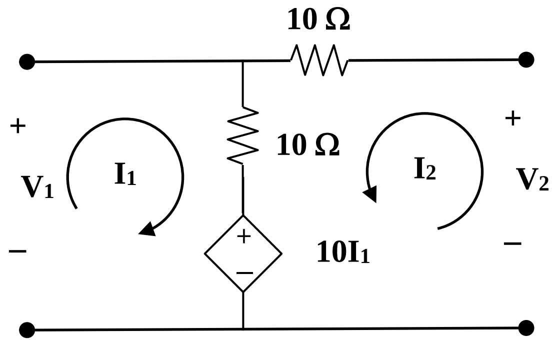


Figure 19.70
For Prob. 19.6 and 19.73.

Solution

Step 1. First we label the circuit so that we can find the z-parameters.



Next we write the mesh equations and then let I_1 = 1 A and I_2 = 0 and then I_1 = 0 and I_2 = 1A in order to find the z-parameters.

Step 2. V_1 = 10x1 + 10x1 = 20 V and V_2 = 10x1 + 10x1 = 20 V which leads to z_{11} = 20/1 = **20 Ω** and z_{21} = 20/1 = **20 Ω**. Finally, V_1 = 10x1 = 10 V and V_2 = (10+10)1 = 20 V which leads to z_{12} = 10/1 = **10 Ω** and z_{22} = 20/1 = **20 Ω**.

Solution 19.7

Calculate the impedance-parameter equivalent of the circuit in Fig. 19.71.

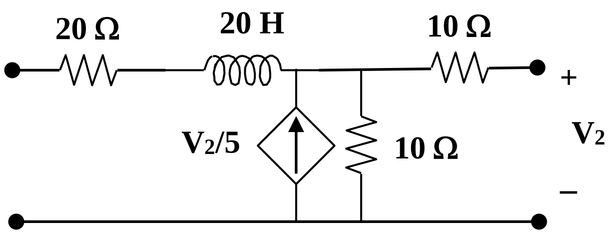
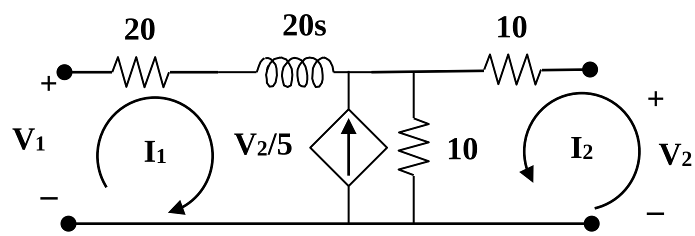


Figure 19.71
For Prob. 19.7.

Solution

Step 1. First we convert the circuit into the s-domain and then adding the identifying current and voltage notations so that we can solve for the z parameters using these equations, $V_1 = z_{11}I_1 + z_{12}I_2$ and $V_2 = z_{21}I_1 + I_2$.



Now we can find the z parameters by letting $I_1 = 1$ A and $I_2 = 0$ and then letting $I_1 = 0$ and $I_2 = 1$ A.

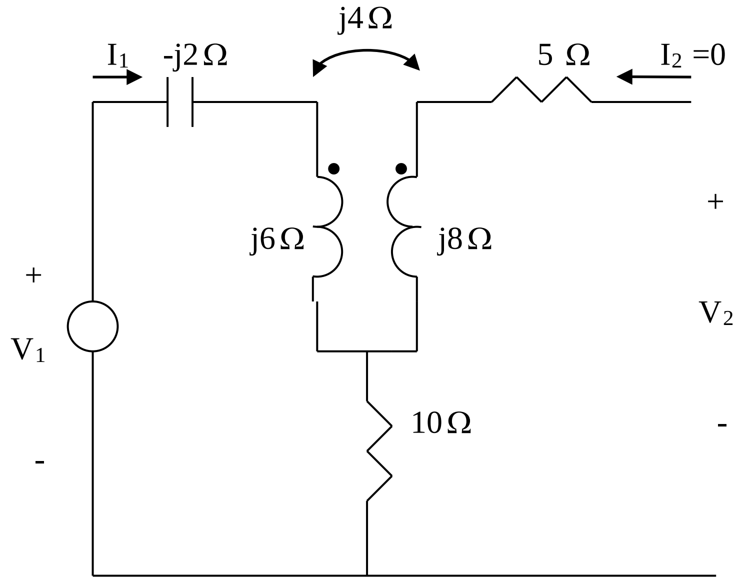
$V_1 = 20(s+1)1 + 10(1+V_2/5)$ and $V_2 = 10(1+V_2/5)$. Next we get, $V_1 = 10(1+V_2/5)$ and $V_2 = 10x1 + 10(1+V_2/5)$.

Step 2. $(1–2)V_2 = 10$ or $V_2 = –10$ V and $V_1 = 20s + 20 + 10(1–2) = 20s+20–10 = 20(s+0.5)$ which leads to $z_{11} =$ **$20(s+0.5)$** and $z_{21} =$ **$–10$**.

Finally, $V_2 = 10+10+2V_2$ or $V_2 = 20/(–1) = –20$ and $V_1 = 10(1–4) = –30$. $z_{12} =$ **$–30$** and $z_{22} =$ **$–20$**.

Solution 19.8

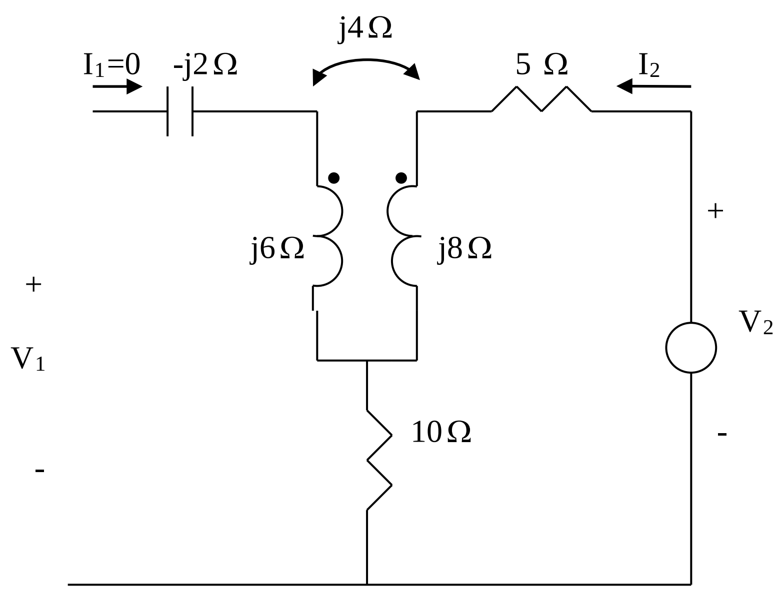
To get z_{11} and z_{21}, consider the circuit below.



$$V_1 = (10 - j2 + j6)I_1 \longrightarrow z_{11} = \frac{V_1}{I_1} = 10 + j4$$

$$V_2 = -10I_1 - j4I_1 \longrightarrow z_{21} = \frac{V_2}{I_1} = -(10 + j4)$$

To get z_{22} and z_{12}, consider the circuit below.



$$V_2 = (5+10+j8)I_2 \quad \longrightarrow \quad z_{22} = \frac{V_2}{I_2} = 15 + j8$$

$$V_1 = -(10+j4)I_2 \quad \longrightarrow \quad z_{12} = \frac{V_1}{I_2} = -(10+j4)$$

Thus,

$$\mathbf{[z]} = \underline{\begin{bmatrix} \mathbf{(10+j4)} & \mathbf{-(10+j4)} \\ \mathbf{-(10+j4)} & \mathbf{(15+j8)} \end{bmatrix} \boldsymbol{\Omega}}$$

Solution 19.9

$$\Delta_y = y_{11}y_{22} - y_{12}y_{21} = 0.5x0.4 - 0.2x0.2 = 0.16$$

$$z_{11} = \frac{y_{22}}{\Delta_y} = \frac{0.4}{0.16} = \mathbf{2.5\ \Omega}$$

$$z_{12} = \frac{-y_{12}}{\Delta_y} = \frac{0.2}{0.16} = \mathbf{1.25\ \Omega} = z_{21}$$

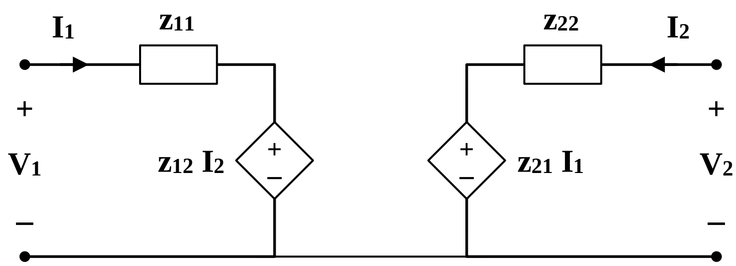
$$z_{22} = \frac{y_{11}}{\Delta_y} = \frac{0.5}{0.16} = \mathbf{3.125\ \Omega}$$

Thus,

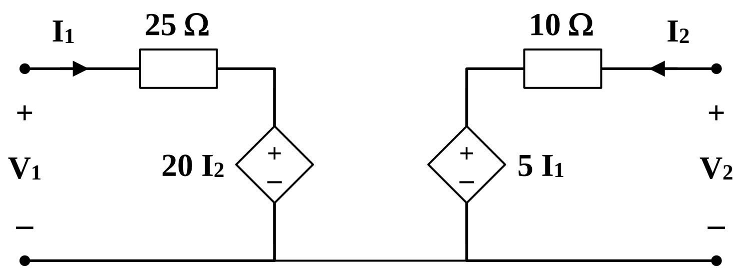
$$\mathbf{Z} = [\mathbf{z}] = \begin{bmatrix} \mathbf{2.5} & \mathbf{1.25} \\ \mathbf{1.25} & \mathbf{3.125} \end{bmatrix} \mathbf{\Omega}$$

Solution 19.10

(a) This is a non-reciprocal circuit so that **the two-port looks like the one shown in Figs. (a) and (b)**.

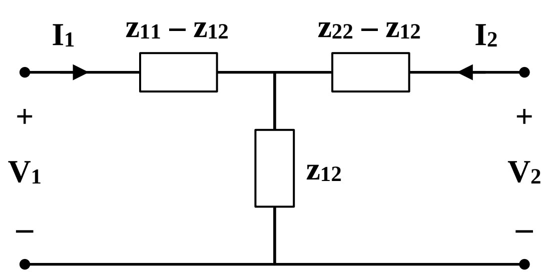


(a)



(b)

(b) This is a reciprocal network and **the two-port look like the one shown in Figs. (c) and (d)**.

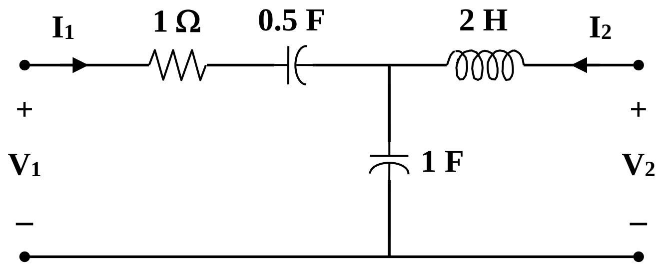


(c)

$$\mathbf{z}_{11} - \mathbf{z}_{12} = 1 + \frac{2}{s} = 1 + \frac{1}{0.5s}$$

$$\mathbf{z}_{22} - \mathbf{z}_{12} = 2s$$

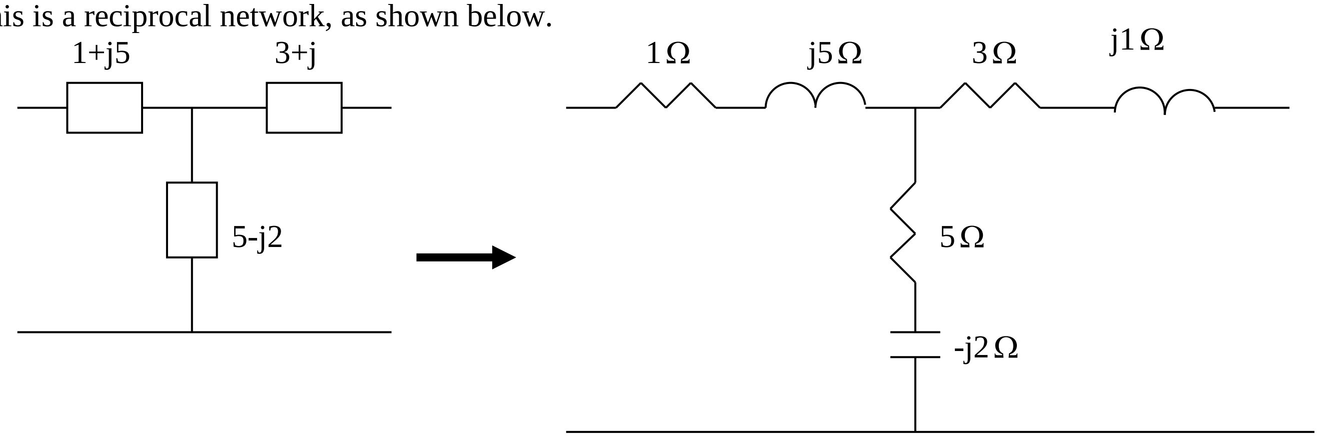
$$\mathbf{z}_{12} = \frac{1}{s}$$



(d)

Solution 19.11

This is a reciprocal network, as shown below.



Solution 19.12

For the circuit shown in Fig. 19.73, let

$$[z]=\begin{bmatrix}10 & -6\\ -4 & 12\end{bmatrix}\ \Omega$$

Find I_1, I_2, V_1, and V_2.

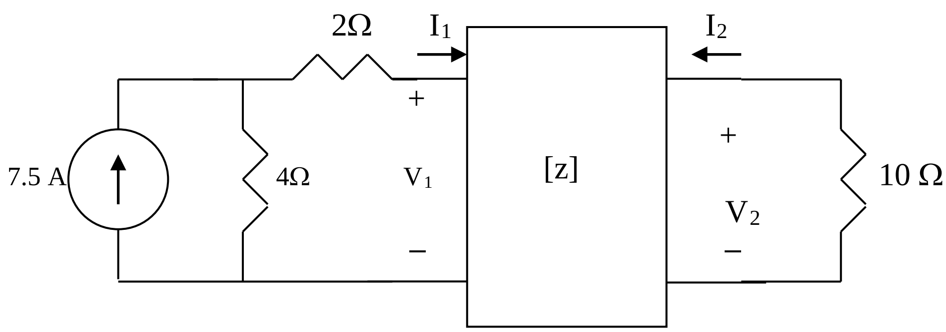
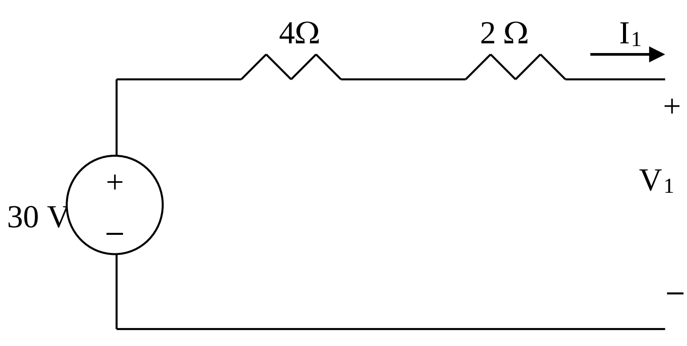


Figure 19.73
For Prob. 19.12.

Solution

$$V_1 = 10I_1 - 6I_2 \qquad (1)$$
$$V_2 = -4I_2 + 12I_2 \qquad (2)$$
$$V_2 = -10I_2 \qquad (3)$$

If we convert the current source to a voltage source, that portion of the circuit becomes what is shown below. Now we write an mesh equation.



$$-30 + (4+2)I_1 + V_1 = 0 \text{ or } V_1 = 30 - 6I_1 \qquad (4)$$

Substituting (3) and (4) into (1) and (2), we get

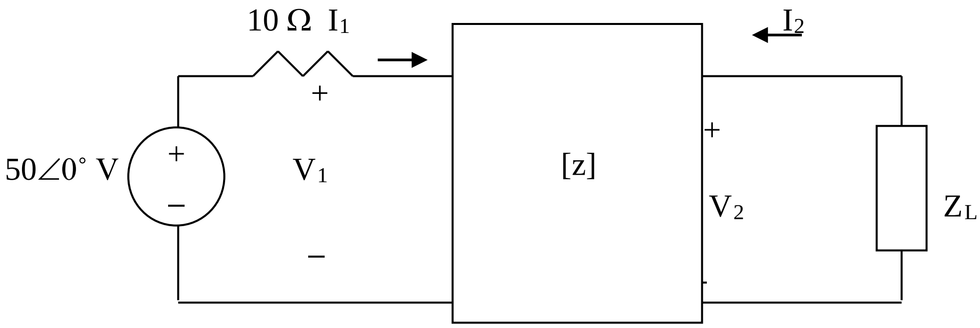
$30 - 6I_1 = 10I_1 - 6I_1$ or $16I_1 - 6I_2 = 30$ (5)
$-10I_2 = -4I_1 + 12I_2$ or $-4I_1 + 22I_2 = 0$ or $I_1 = 5.5I_2$ (6)

From (5) and (6),

$$88I_2 - 6I_2 = 82I_2 = 30 \text{ or } I_2 = \mathbf{365.9\ mA}$$
$$I_1 = 5.5I_2 = \mathbf{2.012\ A}$$
$$V_2 = -10I_2 = \mathbf{-3.659\ V}$$
$$V_1 = 30 - 6I_1 = \mathbf{17.928\ V}.$$

Solution 19.13

Consider the circuit as shown below.



$V_1 = 40I_1 + 60I_2$ (1)

$V_2 = 80I_1 + 100I_2$ (2)

$V_2 = -I_2 Z_L = -I_2(5 + j4)$ (3)

$50 = V_1 + 10I_1 \longrightarrow V_1 = 50 - 10I_1$ (4)

Substituting (4) in (1)

$50 - 10I_1 = 40I_1 + 60I_2 \longrightarrow 5 = 5I_1 + 6I_2$ (5)

Substituting (3) into (2),

$-I_2(5 + j4) = 80I_1 + 100I_2 \longrightarrow 0 = 80I_1 + (105 + j4)I_2$ (6)

Solving (5) and (6) gives

$I_2 = -7.423 + j3.299$ A

We can check the answer using MATLAB.

First we need to rewrite equations 1-4 as follows,

$$\begin{bmatrix} 1 & 0 & -40 & -60 \\ 0 & 1 & -80 & -100 \\ 0 & 1 & 0 & 5 + j4 \\ 1 & 0 & 10 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = A * X = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 50 \end{bmatrix} = U$$

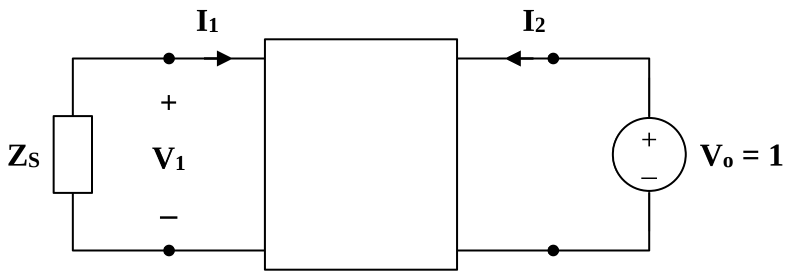
```
>> A=[1,0,-40,-60;0,1,-80,-100;0,1,0,(5+4i);1,0,10,0]
A =
  1.0e+002 *
   0.0100            0         -0.4000         -0.6000
        0       0.0100         -0.8000         -1.0000
        0       0.0100               0      0.0500 + 0.0400i
   0.0100            0          0.1000               0
```

```
>> U=[0;0;0;50]
U =
     0
     0
     0
    50
>> X=inv(A)*U
X =
 -49.0722 +39.5876i
  50.3093 +13.1959i
   9.9072 - 3.9588i
  -7.4227 + 3.2990i
```

$$P = |I_2|^2 5 = \mathbf{329.9\ W}.$$

Solution 19.14

To find $\mathbf{Z}_{\text{Th}}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad (1)$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \qquad (2)$$

But

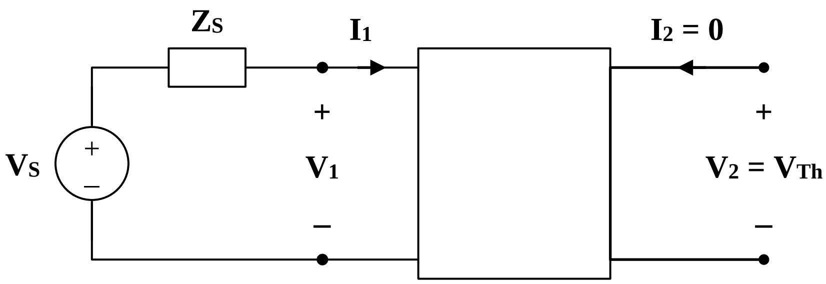
$$\mathbf{V}_2 = 1, \qquad \mathbf{V}_1 = -\mathbf{Z}_s\mathbf{I}_1$$

Hence, $$0 = (\mathbf{z}_{11} + \mathbf{Z}_s)\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \longrightarrow \mathbf{I}_1 = \frac{-\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s}\mathbf{I}_2$$

$$1 = \left(\frac{-\mathbf{z}_{21}\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s} + \mathbf{z}_{22}\right)\mathbf{I}_2$$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{I}_2} = \underline{\mathbf{z}_{22} - \frac{\mathbf{z}_{21}\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s}}$$

To find $\mathbf{V}_{\text{Th}}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = 0, \qquad \mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_s$$

Substituting these into (1) and (2),

$$\mathbf{V}_s - \mathbf{I}_1 \mathbf{Z}_s = \mathbf{z}_{11} \mathbf{I}_1 \quad \longrightarrow \quad \mathbf{I}_1 = \frac{\mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_2 = \mathbf{z}_{21} \mathbf{I}_1 = \frac{\mathbf{z}_{21} \mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_{Th} = \mathbf{V}_2 = \frac{\mathbf{z}_{21} \mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

Solution 19.15

For the two-port circuit in Fig. 19.76,

$$[z]=\begin{bmatrix}40 & 60\\ 80 & 120\end{bmatrix}\Omega$$

(a). Find Z_L for maximum power transfer to the load.
(b). Calculate the maximum power delivered to the load.

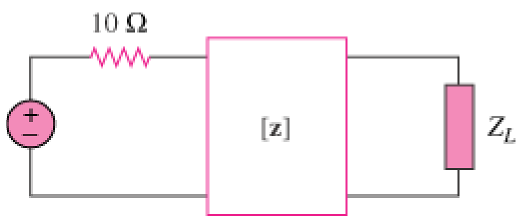
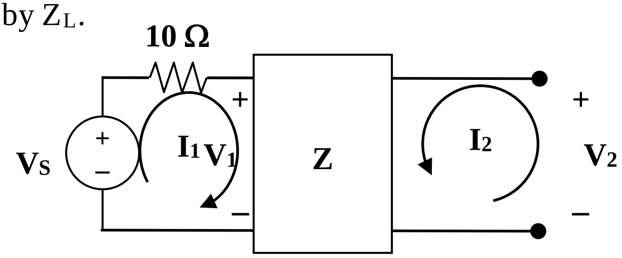


Figure 19.76
For Prob. 19.15.

Solution

(a) Since this is just a Thevenin equivalent circuit problem, we need to find V_{oc} and I_{sc} as seen by Z_L.



The defining equations for this circuit are $V_1 = z_{11}I_1+z_{12}I_2$ and $V_2 = z_{21}I_1 + z_{22}I_2$ or $V_1 = 40I_1 + 60I_2$ and $V_2 = 80I_1 + 120I_2$. Thus, for V_{oc} $V_1 = 240$ and $I_2 = 0$.

$240 – 10I_1 = 40I_1$ or $I_1 = 4.8$ A and $V_2 = V_{oc} = 80\text{x}4.8 = 384$ V.

For I_{sc} $V_2 = 0$, which gives us $240 –10I_1 = 40I_1 + 60I_2$ and $0 = 80I_1 + 120I_2$ or $I_1 = –1.5I_2$.

$240 = 50(–1.5I_2) + 60\ I_2 = –15I_2$ or $I_2 = –16\text{ A} = –I_{sc}$ or $I_{sc} = 16$ A and

$Z_{eq} = 384/16 = 24\ \Omega$. Thus,

$Z_L = \mathbf{24\ \Omega}$.

(b) $I = 384/(24+24) = 8$ A. $P_{max} = (8)^2\text{x}24 = \mathbf{1.536\ kW}$.

Problem 19.16

For the circuit in Fig.19.77, at w = 2 rad/s, z_{11} = 10 Ω, $z_{12} = z_{21}$ = j6 Ω, z_{22} = 4 Ω. Obtain the Thevenin equivalent circuit at terminals a-b and calculate v_o.

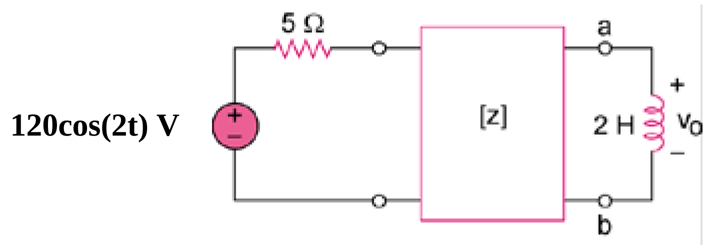
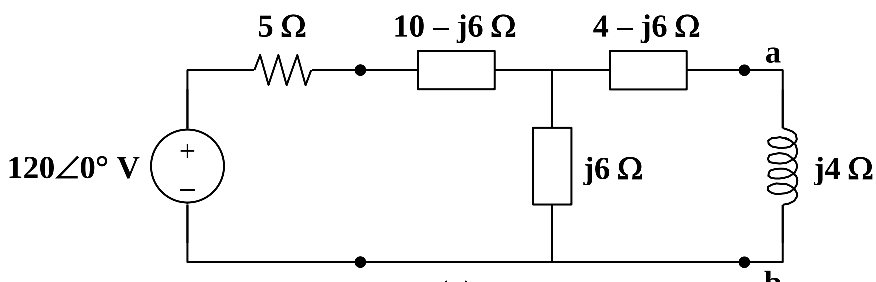


Figure 19.77
For Prob. 19.16.

Solution

As a reciprocal two-port, the given circuit can be represented as shown in Fig. (a).



(a)

At terminals a-b,

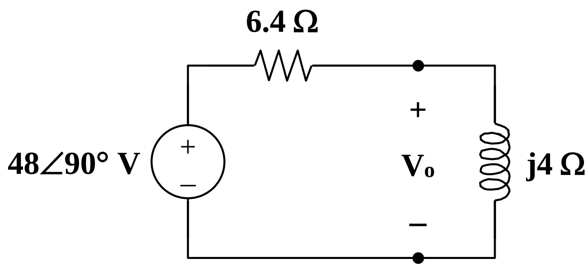
$$\mathbf{Z}_{Th} = (4 - j6) + j6 \,||\, (5 + 10 - j6)$$

$$\mathbf{Z}_{Th} = 4 - j6 + \frac{j6(15 - j6)}{15} = 4 - j6 + 2.4 + j6$$

$$\mathbf{Z}_{Th} = \mathbf{6.4\,\Omega}$$

$$\mathbf{V}_{Th} = \frac{j6}{j6 + 5 + 10 - j6}(120\angle 0°) = j48 = \mathbf{48\angle 90°\ V}.$$

The Thevenin equivalent circuit is shown in Fig. (b).



(b)

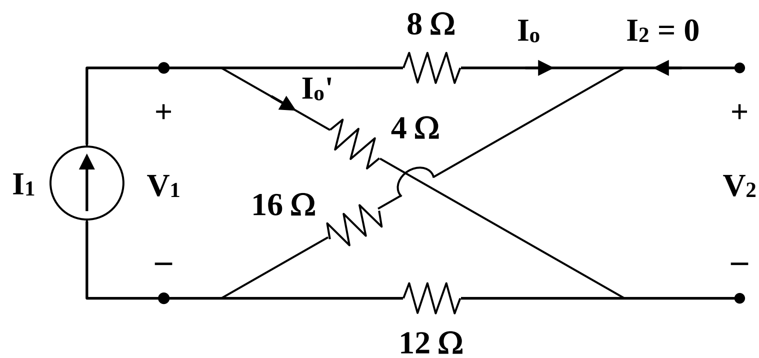
From this,

$$\mathbf{V}_o = \frac{j4}{6.4 + j4}(j48) = 25.4399\angle 148°$$

$v_o(t) =$ **25.44cos(2t + 148°) V**.

Solution 19.17

To obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



(a)

In this case, the 8-Ω and 16-Ω resistors are in series, since the same current, $\mathbf{I}_o$, passes through them. Similarly, the 4-Ω and 12-Ω resistors are in series, since the same current, $\mathbf{I}_o^{'}$, passes through them.

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = (8+16)\,||\,(4+12) = 24\,||\,16 = \frac{(24)(16)}{40} = \mathbf{9.6\ \Omega}$$

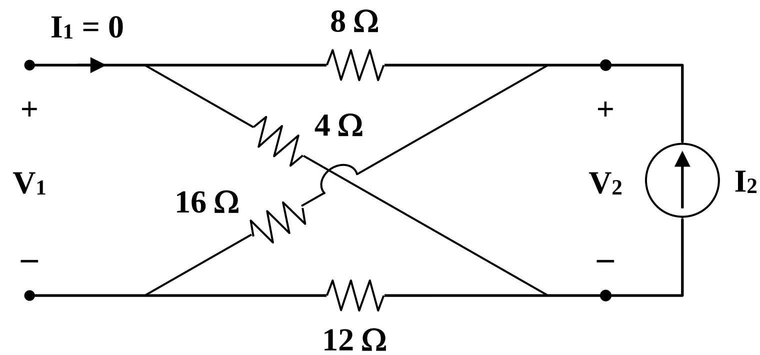
$$\mathbf{I}_o = \frac{16}{16+24}\mathbf{I}_1 = \frac{2}{5}\mathbf{I}_1 \qquad\qquad \mathbf{I}_o^{'} = \frac{3}{5}\mathbf{I}_1$$

But $$-\mathbf{V}_2 - 8\mathbf{I}_o + 4\mathbf{I}_o^{'} = 0$$

$$\mathbf{V}_2 = -8\mathbf{I}_o + 4\mathbf{I}_o^{'} = \frac{-16}{5}\mathbf{I}_1 + \frac{12}{5}\mathbf{I}_1 = \frac{-4}{5}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{-4}{5} = \mathbf{-0.8\ \Omega}$$

To get $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = (8+4)\,||\,(16+12) = 12\,||\,28 = \frac{(12)(28)}{40} = \mathbf{8.4\ \Omega}$$

$$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{-0.8\ \Omega}$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{9.6} & \mathbf{-0.8} \\ \mathbf{-0.8} & \mathbf{8.4} \end{bmatrix} \mathbf{\Omega}$$

We may take advantage of Table 18.1 to get [**y**] from [**z**].

$$\Delta_z = (9.6)(8.4) - (0.8)^2 = 80$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{8.4}{80} = \mathbf{0.105\ S} \qquad \mathbf{y}_{12} = \frac{-\mathbf{z}_{12}}{\Delta_z} = \frac{0.8}{80} = \mathbf{0.01\ S}$$

$$\mathbf{y}_{21} = \frac{-\mathbf{z}_{21}}{\Delta_z} = \frac{0.8}{80} = \mathbf{0.01\ S} \qquad \mathbf{y}_{22} = \frac{\mathbf{z}_{11}}{\Delta_z} = \frac{9.6}{80} = \mathbf{0.12\ S}$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{0.105} & \mathbf{0.01} \\ \mathbf{0.01} & \mathbf{0.12} \end{bmatrix} \mathbf{S}$$

Solution 19.18

Calculate the *y* parameters for the two-port in Fig.19.79.

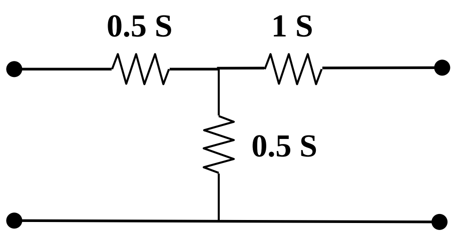
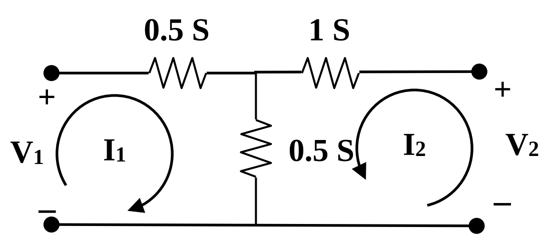


Figure 19.79
For Probs. 19.18 and 19.37.

Solution

Step 1. We first label the circuit so that we can determine the y parameters.



Now we can use the y parameter equations, $I_1 = y_{11}V_1 + y_{12}V_2$ and $I_2 = y_{21}V_1 + y_{22}V_2$, to determine the y parameters by letting $V_1 = 1$ V and $V_2 = 0$ and $V_1 = 0$ and $V_2 = 1$ V.

Step 2. Note that we can combine the 1 S resistor in parallel with the 0.5 S resistor and obtain 1.5 S. Now $I_1 = 1/[(1/0.5)+(1/1.5)] = 1/(2+0.66667) = 0.375$ A and $I_2 = –0.375(2/3)$ [current division] $= –0.25$ A. This leads to $y_{11} = 0.375/1 =$ **0.375 S** and $y_{21} =$ **–0.25 S**.

In the next case, $I_2 = 1/[(1/1)+(1/(0.5+0.5))] = 0.5$ A and again by current division, $I_1 = –0.5(1/2) =$ **–0.25 A**. This leads to $y_{12} = –0.25$ S and $y_{22} =$ **0.5 S**.

Solution 19.19

Using Fig. 19.80, design a problem to help other students to better understand how to find y parameters in the s-domain.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the *y* parameters of the two-port in Fig.19.80 in terms of *s*.

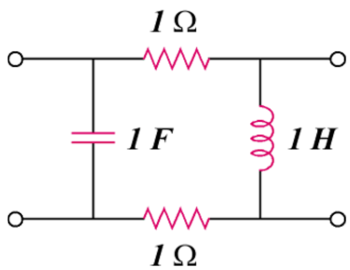
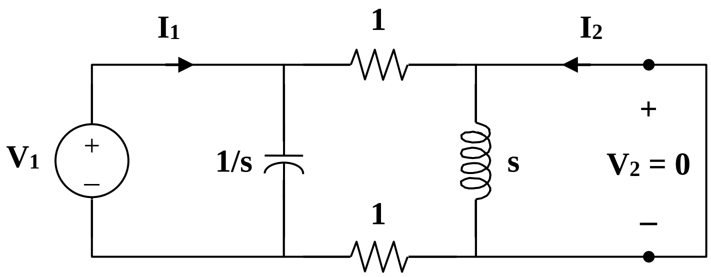


Figure 19.80

Solution

Consider the circuit in Fig.(a) for calculating $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$.



(a)

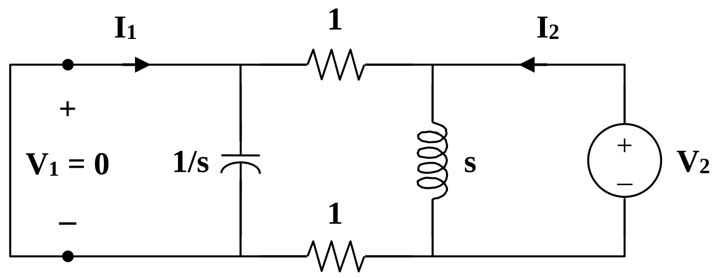
$$\mathbf{V}_1 = \left(\frac{1}{s} \parallel 2\right)\mathbf{I}_1 = \frac{2/s}{2+(1/s)}\mathbf{I}_1 = \frac{2}{2s+1}\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{2s+1}{2} = s + 0.5$$

$$\mathbf{I}_2 = \frac{(-1/s)}{(1/s)+2}\mathbf{I}_1 = \frac{-\mathbf{I}_1}{2s+1} = \frac{-\mathbf{V}_1}{2}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \text{-}0.5$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig.(b).



(b)

$$\mathbf{V}_2 = (\mathrm{s} \,\|\, 2)\mathbf{I}_2 = \frac{2\mathrm{s}}{\mathrm{s}+2}\mathbf{I}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{\mathrm{s}+2}{2\mathrm{s}} = 0.5 + \frac{1}{\mathrm{s}}$$

$$\mathbf{I}_1 = \frac{\text{-}\mathrm{s}}{\mathrm{s}+2}\mathbf{I}_2 = \frac{\text{-}\mathrm{s}}{\mathrm{s}+2}\cdot\frac{\mathrm{s}+2}{2\mathrm{s}}\mathbf{V}_2 = \frac{\text{-}\mathbf{V}_2}{2}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \text{-}0.5$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{s+0.5} & \mathbf{\text{-}\,0.5} \\ \mathbf{\text{-}\,0.5} & \mathbf{0.5+1/s} \end{bmatrix}\mathbf{S}$$

Solution 19.20

Find the *y* parameters for the circuit in Fig. 19.81.

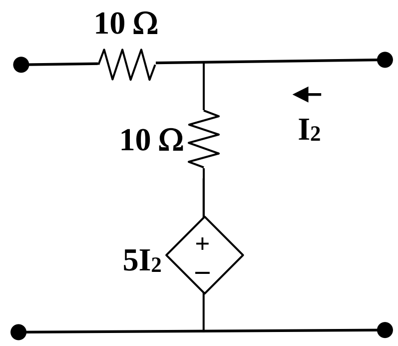
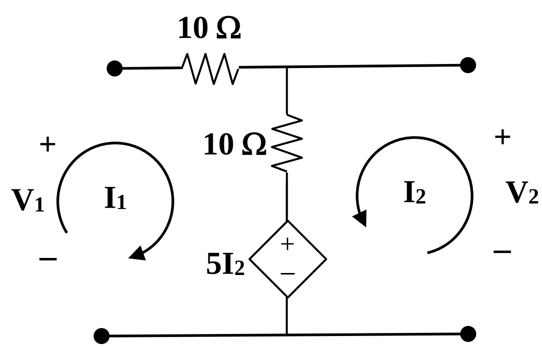


Figure 19.81
For Prob. 19.20.

Solution

Step 1. We label the circuit with the representative currents and voltages so we can use the y parameter equations to solve for the y parameters.



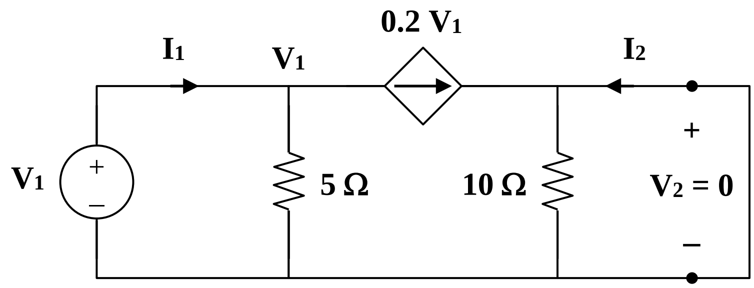
$I_1 = y_{11}V_1 + y_{12}V_2$ and $I_2 = y_{21}V_1 + y_{22}V_2$. Now we let $V_1 = 1$ V and $V_2 = 0$ and next we let $V_1 = 0$ and $V_2 = 1$ V which will allow us to determine the y parameters.

Step 2. For the first case $I_1 = 1/10 = 0.1$ A and $I_2 = 0.1 – 5I_2/10$ or $1.5I_2 = 0.1$ or $I_2 = 0.06667$ A which leads to $y_{11} = 0.1/1 =$ **0.1 S** and $y_{21} =$ **0.066667 S**.

In the second case, $I_1 = –1/10 = –0.1$ A and $I_2 =(1/10) + (1–5I_2)/10$ or $1.5I_2 = 0.2$ or $I_2 = 0.13333$ A which leads to $y_{12} =$ **–0.1 S** and $y_{22} =$ **0.13333 S**.

Solution 19.21

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, refer to Fig. (a).



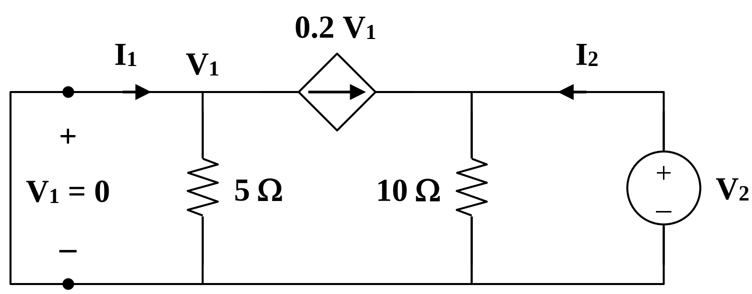
(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{5} + 0.2\mathbf{V}_1 = 0.4\mathbf{V}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 0.4$$

$$\mathbf{I}_2 = -0.2\mathbf{V}_1 \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.2$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig. (b).



(b)

Since $\mathbf{V}_1 = 0$, the dependent current source can be replaced with an open circuit.

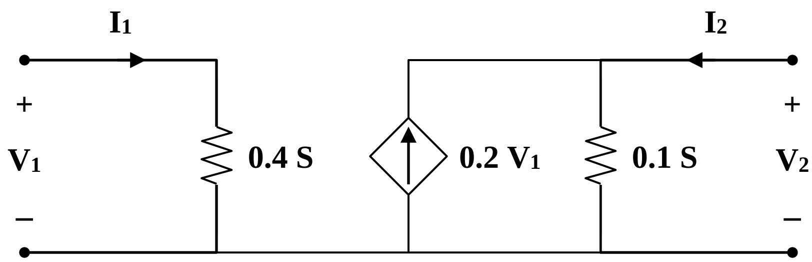
$$\mathbf{V}_2 = 10\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{10} = 0.1$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} 0.4 & 0 \\ -0.2 & 0.1 \end{bmatrix} \text{S}$$

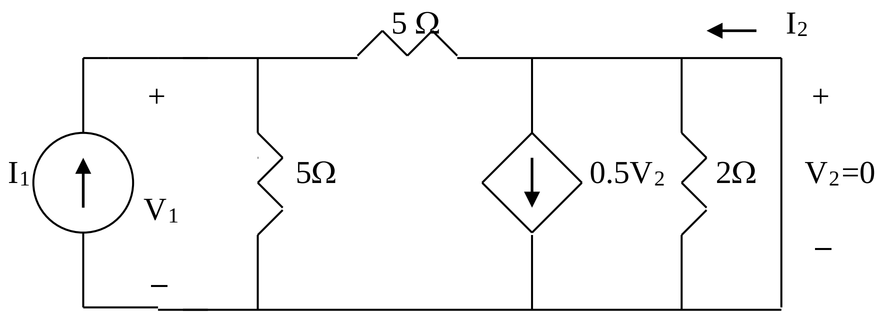
Consequently, **the y parameter equivalent circuit is shown in Fig. (c)**.



(c)

Solution 19.22

To obtain y_{11} and y_{21}, consider the circuit below.

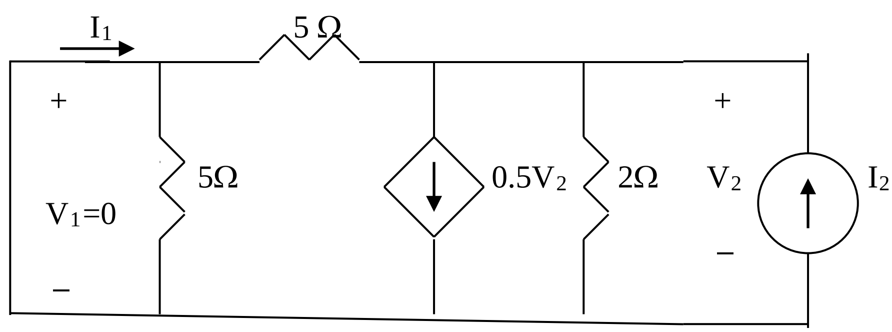


The 2-Ω resistor is short-circuited.

$$V_1 = 5\frac{I_1}{2} \quad \longrightarrow \quad y_{11} = \frac{I_1}{V_1} = \frac{2}{5} = 0.4$$

$$I_2 = \frac{1}{2}I_1 \quad \longrightarrow \quad y_{21} = \frac{I_2}{V_1} = \frac{\frac{1}{2}I_1}{2.5I_1} = 0.2$$

To obtain y_{12} and y_{22}, consider the circuit below.



At the top node, KCL gives

$$I_2 = 0.5V_2 + \frac{V_2}{2} + \frac{V_2}{5} = 1.2V_2 \quad \longrightarrow \quad y_{22} = \frac{I_2}{V_2} = 1.2$$

$$I_1 = -\frac{V_2}{5} = -0.2V_2 \quad \longrightarrow \quad y_{12} = \frac{I_1}{V_2} = -0.2$$

Hence,

$$[y] = \underline{\begin{bmatrix} 0.4 & -0.2 \\ 0.2 & 1.2 \end{bmatrix} \text{S}}$$

Solution 19.23

(a)

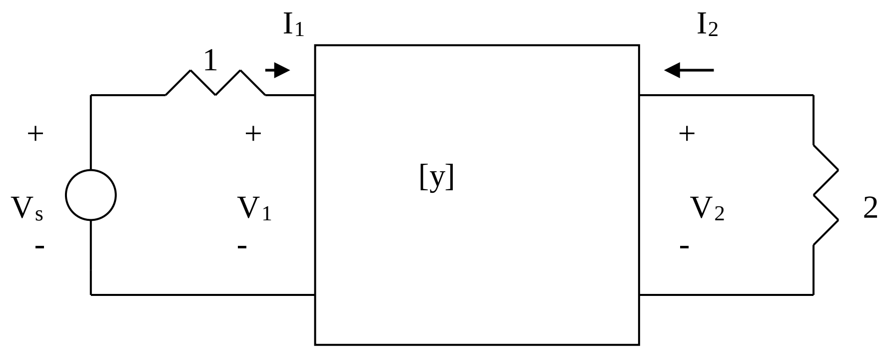
$$1/(-y_{12}) = 1 // \frac{1}{s} = \frac{1}{s+1} \qquad \longrightarrow \qquad y_{12} = -(s+1)$$

$$y_{11} + y_{12} = 1 \qquad \longrightarrow \qquad y_{11} = 1 - y_{12} = 1 + (s+1) = s+2$$

$$y_{22} + y_{12} = s \qquad \longrightarrow \qquad y_{22} = s - y_{12} = \frac{1}{s} + (s+1) = \frac{s^2+s+1}{s}$$

$$\underline{[y] = \begin{bmatrix} s+2 & -(s+1) \\ -(s+1) & \frac{s^2+s+1}{s} \end{bmatrix}}$$

(b) Consider the network below.



$$V_s = I_1 + V_1 \qquad (1)$$

$$V_2 = -2I_2 \qquad (2)$$

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad (3)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \qquad (4)$$

From (1) and (3)

$$V_s - V_1 = y_{11}V_1 + y_{12}V_2 \quad \longrightarrow \quad V_s = (1 + y_{11})V_1 + y_{12}V_2 \qquad (5)$$

From (2) and (4),

$$-0.5V_2 = y_{21}V_1 + y_{22}V_2 \quad \longrightarrow \quad V_1 = -\frac{1}{y_{21}}(0.5 + y_{22})V_2 \qquad (6)$$

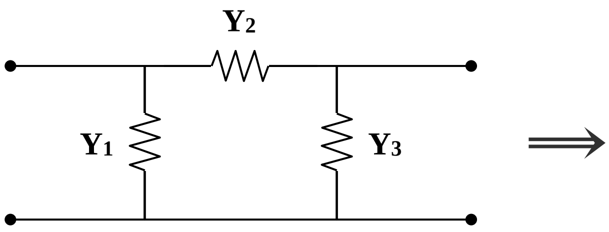
Substituting (6) into (5),

$$\mathbf{V_s = -\frac{(1+y_{11})(0.5+y_{22})}{y_{21}}V_2 + y_{12}V_2}$$
$$\mathbf{= \frac{2}{s} \quad \longrightarrow \quad V_2 = \frac{2/s}{\left[y_{12} - \frac{1}{y_{21}}(1+y_{11})(0.5+y_{22})\right]}}$$

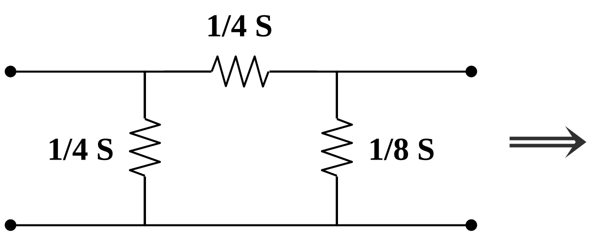
$$\mathbf{V_2 = \frac{2/s}{-(s+1) + \frac{1}{s+1}(1+s+2)\left(0.5 + \frac{s^2+s+1}{s}\right)} = \frac{2/s}{\frac{-s^3 - s^2 - s^2 - s + (s+3)(0.5s + s^2 + s + 1)}{s(s+1)}}}$$
$$\mathbf{= \frac{2(s+1)}{-s^3 - 2s^2 - s + s^3 + 1.5s^2 + s + 3s^2 + 4.5s + 3} = \frac{2(s+1)}{2.5s^2 + 4.5s + 3} = \frac{0.8(s+1)}{s^2 1.8s + 1.2}}$$

Solution 19.24

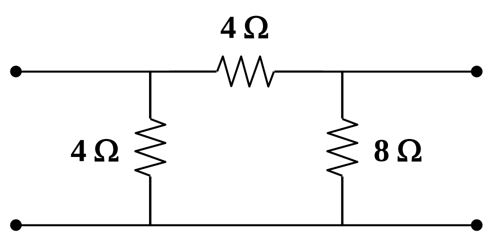
Since this is a reciprocal network, **a Π network is appropriate, as shown below**.



(a)



(b)



(c)

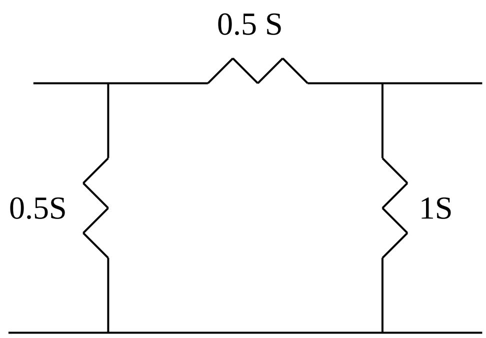
$$\mathbf{Y}_1 = \mathbf{y}_{11} + \mathbf{y}_{12} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}\,\text{S}, \qquad \mathbf{Z}_1 = \mathbf{4\,\Omega}$$

$$\mathbf{Y}_2 = \text{-}\mathbf{y}_{12} = \frac{1}{4}\,\text{S}, \qquad \mathbf{Z}_2 = \mathbf{4\,\Omega}$$

$$\mathbf{Y}_3 = \mathbf{y}_{22} + \mathbf{y}_{21} = \frac{3}{8} - \frac{1}{4} = \frac{1}{8}\,\text{S}, \qquad \mathbf{Z}_3 = \mathbf{8\,\Omega}$$

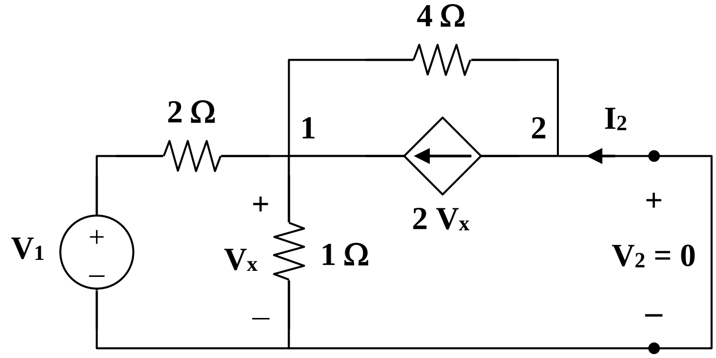
Solution 19.25

This is a reciprocal network and is shown below.



Solution 19.26

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig. (a).



(a)

At node 1,

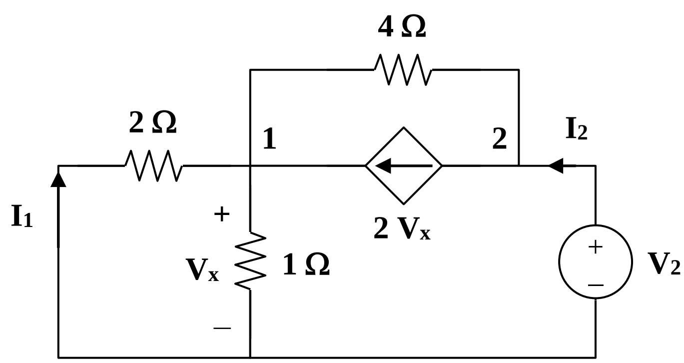
$$\frac{\mathbf{V}_1 - \mathbf{V}_x}{2} + 2\mathbf{V}_x = \frac{\mathbf{V}_x}{1} + \frac{\mathbf{V}_x}{4} \longrightarrow 2\mathbf{V}_1 = \text{-}\mathbf{V}_x \qquad (1)$$

But $$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_x}{2} = \frac{\mathbf{V}_1 + 2\mathbf{V}_1}{2} = 1.5\mathbf{V}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 1.5$$

Also, $$\mathbf{I}_2 + \frac{\mathbf{V}_x}{4} = 2\mathbf{V}_x \longrightarrow \mathbf{I}_2 = 1.75\mathbf{V}_x = \text{-}3.5\mathbf{V}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \text{-}3.5$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig.(b).



(b)

At node 2,

$$\mathbf{I}_2 = 2\mathbf{V}_x + \frac{\mathbf{V}_2 - \mathbf{V}_x}{4} \qquad (2)$$

At node 1,

$$2\mathbf{V}_x + \frac{\mathbf{V}_2 - \mathbf{V}_x}{4} = \frac{\mathbf{V}_x}{2} + \frac{\mathbf{V}_x}{1} = \frac{3}{2}\mathbf{V}_x \quad \longrightarrow \quad \mathbf{V}_2 = -\mathbf{V}_x \qquad (3)$$

Substituting (3) into (2) gives

$$\mathbf{I}_2 = 2\mathbf{V}_x - \frac{1}{2}\mathbf{V}_x = 1.5\mathbf{V}_x = -1.5\mathbf{V}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = -1.5$$

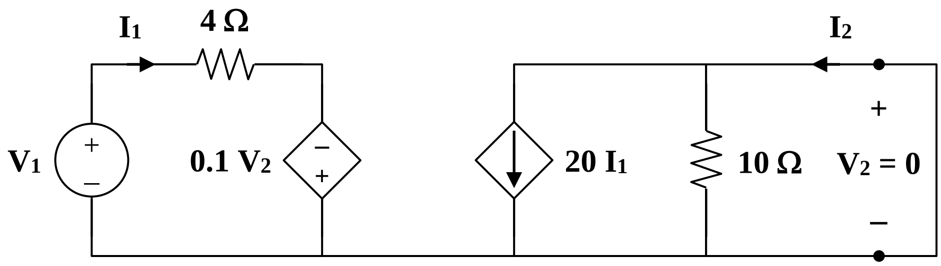
$$\mathbf{I}_1 = \frac{-\mathbf{V}_x}{2} = \frac{\mathbf{V}_2}{2} \quad \longrightarrow \quad \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0.5$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{1.5} & \mathbf{0.5} \\ \mathbf{-3.5} & \mathbf{-1.5} \end{bmatrix} \mathbf{S}$$

Solution 19.27

Consider the circuit in Fig. (a).

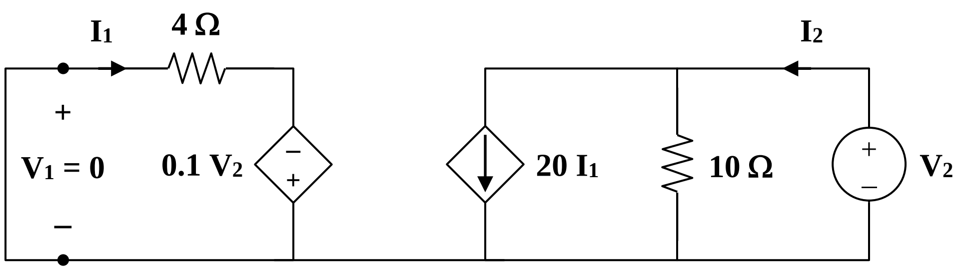


(a)

$$\mathbf{V}_1 = 4\mathbf{I}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{\mathbf{I}_1}{4\mathbf{I}_1} = 0.25$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 = 5\mathbf{V}_1 \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = 5$$

Consider the circuit in Fig. (b).



(b)

$$4\mathbf{I}_1 = 0.1\mathbf{V}_2 \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{0.1}{4} = 0.025$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 + \frac{\mathbf{V}_2}{10} = 0.5\mathbf{V}_2 + 0.1\mathbf{V}_2 = 0.6\mathbf{V}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = 0.6$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{0.25} & \mathbf{0.025} \\ \mathbf{5} & \mathbf{0.6} \end{bmatrix} \mathbf{S}$$

Alternatively, from the given circuit,

$$\mathbf{V}_1 = 4\mathbf{I}_1 - 0.1\mathbf{V}_2$$
$$\mathbf{I}_2 = 20\mathbf{I}_1 + 0.1\mathbf{V}_2$$

Comparing these with the equations for the h parameters show that

$$\mathbf{h}_{11} = 4, \qquad \mathbf{h}_{12} = \text{-}0.1, \qquad \mathbf{h}_{21} = 20, \qquad \mathbf{h}_{22} = 0.1$$

Using Table 18.1,

$$\mathbf{y}_{11} = \frac{\mathbf{1}}{\mathbf{h}_{11}} = \frac{\mathbf{1}}{\mathbf{4}} = \mathbf{0.25\,S}, \qquad \mathbf{y}_{12} = \frac{\mathbf{-\,h}_{12}}{\mathbf{h}_{11}} = \frac{\mathbf{0.1}}{\mathbf{4}} = \mathbf{0.025\,S}$$

$$\mathbf{y}_{21} = \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} = \frac{\mathbf{20}}{\mathbf{4}} = \mathbf{5\,S}, \qquad \mathbf{y}_{22} = \frac{\boldsymbol{\Delta}_{\mathbf{h}}}{\mathbf{h}_{11}} = \frac{\mathbf{0.4 + 2}}{\mathbf{4}} = \mathbf{0.6\,S}$$

as above.

Solution 19.28

In the circuit of Fig.19.65, the input port is connected to a 1-A dc current source and the right hand side of the circuit is left open ($I_2 = 0$). Calculate the power absorbed by the circuit by using the y parameters. Confirm your result by direct circuit analysis.

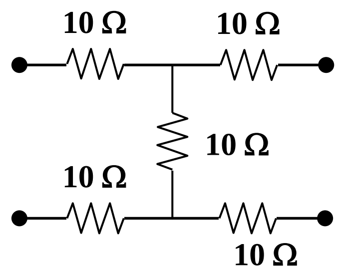
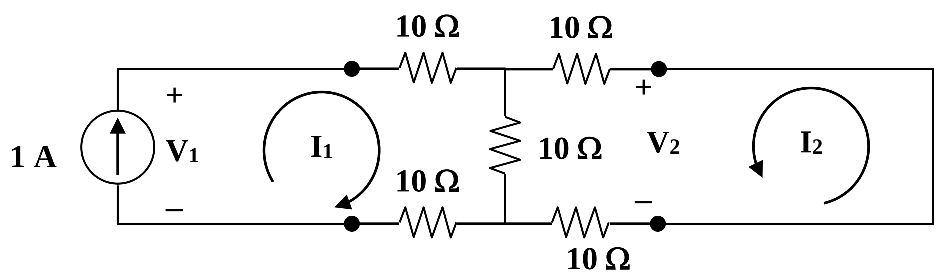


Figure 19.65 For Probs. 19.1 and 19.28.

Solution

Step 1. We obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by considering the circuit in Fig.(a) and using these equations. $y_{11}V_1 + y_{12}V_2 = I_1$ and $y_{21}V_1 + y_{22}V_2 = I_2$.



For $I_1 = 1$ A and $V_2 = 0$ gives us $I_1 = y_{11}V_1$ and $I_2 = y_{21}V_1$,
$V_1 = [10+[10x20/(10+20)]+10]1$, and $I_2 = –1x10/(10+20)$.

Next we let $I_2 = 1$ A and $V_1 = 0$ which gives us $I_1 = y_{12}V_2$ and $I_2 = y_{22}V_2$,
$V_2 = [10+[10x20/(10+20)]+10]1$, and $I_1 = –1x10/(10+20)$.

Step 2. $V_1 = 26.667$ V and $I_2 = –0.33333$ A which leads to $y_{11} = 1/26.667 = 37.5$ mS and $y_{21} = –0.33333/26.667 = –12.5$ mS.

$V_2 = 26.667$ V and $I_1 = –0.33333$ A leads to $y_{12} = –12.5$ mS and $y_{22} = 37.5$ mS. We now have $0.0375V_1 – 0.0125V_2 = 1$ and $–0.0125V_1 + 0.0375V_2 = 0$ or $V_2 = [0.0125/0.0375]V_1 = 0.33333V_1$ or $1 = 0.033333V_1$ or $V_1 = 30$ V and $P_{abs} = V_1I_1 = 30x1 =$ **30 W**.

We can check by recognizing that the 1 A current flows through 3 10 Ω resistors connected in series. This leads to $P_{abs} = (1)^2[10+10+10] =$ **30 W**. The answer checks!

Solution 19.29

In the bridge circuit of Fig.19.87, I_1 = 20 A and I_2 = –8 A.

a) Find V_1 and V_2 using y parameters.
b) Confirm the results in part (a) by direct circuit analysis.

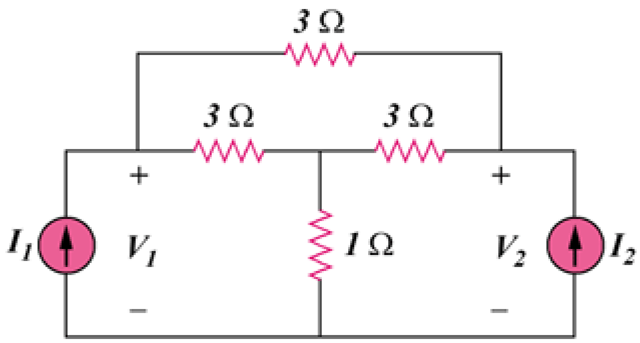
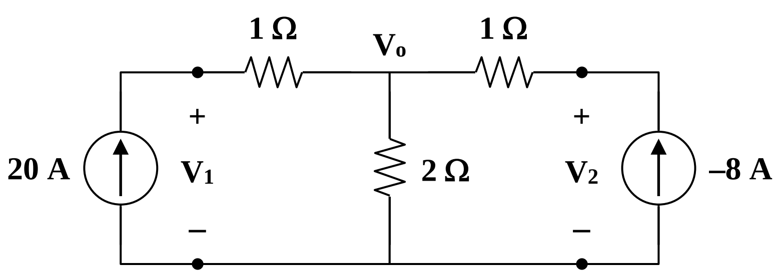


Figure 19.87
For Prob. 19.29.

Solution

(a) Transforming the Δ subnetwork to Y gives the circuit in Fig. (a).



(a)

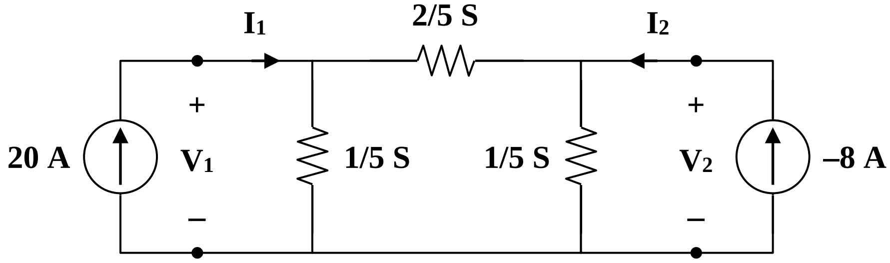
It is easy to get the z parameters

$$\mathbf{z}_{12} = \mathbf{z}_{21} = 2, \qquad \mathbf{z}_{11} = 1 + 2 = 3, \qquad \mathbf{z}_{22} = 3$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 9 - 4 = 5$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{3}{5} = \mathbf{y}_{22}, \qquad \mathbf{y}_{12} = \mathbf{y}_{21} = \frac{-\mathbf{z}_{12}}{\Delta_z} = \frac{-2}{5}$$

Thus, the equivalent circuit is as shown in Fig. (b).



(b)

$$\mathbf{I}_1 = 20 = \frac{3}{5}\mathbf{V}_1 - \frac{2}{5}\mathbf{V}_2 \quad \longrightarrow \quad 100 = 3\mathbf{V}_1 - 2\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = -8 = \frac{-2}{5}\mathbf{V}_1 + \frac{3}{5}\mathbf{V}_2 \quad \longrightarrow \quad -40 = -2\mathbf{V}_1 + 3\mathbf{V}_2$$

$$20 = \mathbf{V}_1 - 1.5\mathbf{V}_2 \quad \longrightarrow \quad \mathbf{V}_1 = 20 + 1.5\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$100 = 60 + 4.5\mathbf{V}_2 - 2\mathbf{V}_2 \quad \longrightarrow \quad \mathbf{V}_2 = \mathbf{16\ V}$$

$$\mathbf{V}_1 = 20 + 1.5\mathbf{V}_2 = \mathbf{44\ V}$$

(b) For direct circuit analysis, consider the circuit in Fig. (a).

For the main non-reference node,

$$20 - 8 = \frac{\mathbf{V}_o}{2} \quad \longrightarrow \quad \mathbf{V}_o = 24$$

$$20 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} \quad \longrightarrow \quad \mathbf{V}_1 = 20 + \mathbf{V}_o = \mathbf{44\ V}$$

$$-8 = \frac{\mathbf{V}_2 - \mathbf{V}_o}{1} \quad \longrightarrow \quad \mathbf{V}_2 = \mathbf{V}_o - 8 = \mathbf{16\ V}$$

Solution 19.30

(a) Convert to z parameters; then, convert to h parameters using Table 18.1.

$$\mathbf{z}_{11} = \mathbf{z}_{12} = \mathbf{z}_{21} = 60\,\Omega, \qquad \mathbf{z}_{22} = 100\,\Omega$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 6000 - 3600 = 2400$$

$$\mathbf{h}_{11} = \frac{\Delta_z}{\mathbf{z}_{22}} = \frac{2400}{100} = 24, \qquad \mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} = \frac{60}{100} = 0.6$$

$$\mathbf{h}_{21} = \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} = -0.6, \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}} = 0.01$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{24\,\Omega} & \mathbf{0.6} \\ \mathbf{-0.6} & \mathbf{0.01\ S} \end{bmatrix}$$

(b) Similarly,

$$\mathbf{z}_{11} = 30\,\Omega \qquad \mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{z}_{22} = 20\,\Omega$$

$$\Delta_z = 600 - 400 = 200$$

$$\mathbf{h}_{11} = \frac{200}{20} = 10 \qquad \mathbf{h}_{12} = \frac{20}{20} = 1$$

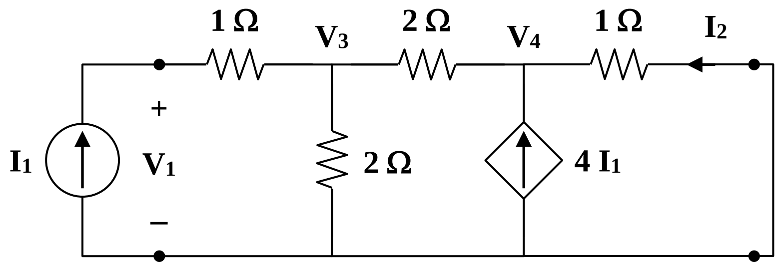
$$\mathbf{h}_{21} = -1 \qquad \mathbf{h}_{22} = \frac{1}{20} = 0.05$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{10\,\Omega} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0.05\ S} \end{bmatrix}$$

Solution 19.31

We get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by considering the circuit in Fig. (a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_3}{2} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{2} \longrightarrow 2\mathbf{I}_1 = 2\mathbf{V}_3 - \mathbf{V}_4 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_3 - \mathbf{V}_4}{2} + 4\mathbf{I}_1 = \frac{\mathbf{V}_4}{1}$$

$$8\mathbf{I}_1 = -\mathbf{V}_3 + 3\mathbf{V}_4 \longrightarrow 16\mathbf{I}_1 = -2\mathbf{V}_3 + 6\mathbf{V}_4 \qquad (2)$$

Adding (1) and (2),

$$18\mathbf{I}_1 = 5\mathbf{V}_4 \longrightarrow \mathbf{V}_4 = 3.6\mathbf{I}_1$$

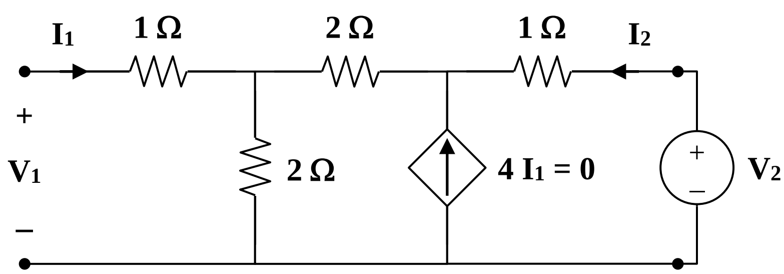
$$\mathbf{V}_3 = 3\mathbf{V}_4 - 8\mathbf{I}_1 = 2.8\mathbf{I}_1$$

$$\mathbf{V}_1 = \mathbf{V}_3 + \mathbf{I}_1 = 3.8\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 3.8\,\Omega$$

$$\mathbf{I}_2 = \frac{-\mathbf{V}_4}{1} = -3.6\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = -3.6$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in Fig. (b). The dependent current source can be replaced by an open circuit since $4\mathbf{I}_1 = 0$.



(b)

$$\mathbf{V}_1 = \frac{2}{2+2+1}\mathbf{V}_2 = \frac{2}{5}\mathbf{V}_2 \quad \longrightarrow \quad \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 0.4$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2+2+1} = \frac{\mathbf{V}_2}{5} \quad \longrightarrow \quad \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{5} = 0.2\text{ S}$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{38\,\Omega} & \mathbf{0.4} \\ \mathbf{-3.6} & \mathbf{0.2\ S} \end{bmatrix}$$

Solution 19.32

Using Fig. 19.90, design a problem to help other students to better understand how to find the *h* and *g* parameters for a circuit in the s-domain.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the *h* and *g* parameters of the two-port network in Fig.19.90 as functions of *s*.

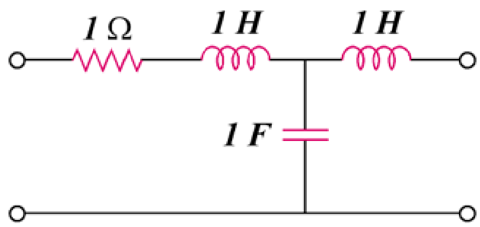
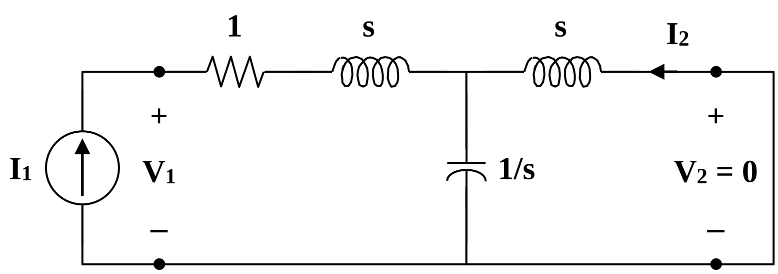


Figure 19.90

Solution

(a) We obtain $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by referring to the circuit in Fig. (a).



(a)

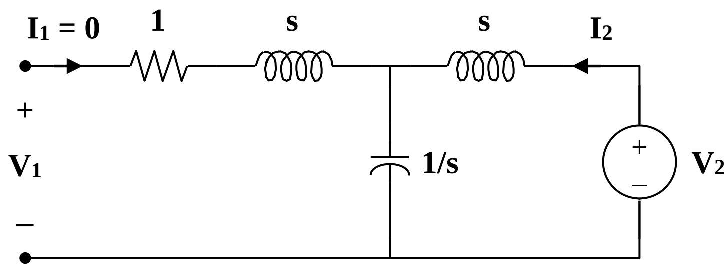
$$\mathbf{V}_1 = \left(1 + s + s \parallel \frac{1}{s}\right)\mathbf{I}_1 = \left(1 + s + \frac{s}{s^2+1}\right)\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = s + 1 + \frac{s}{s^2+1}$$

By current division,

$$\mathbf{I}_2 = \frac{-1/s}{s+1/s}\mathbf{I}_1 = \frac{-\mathbf{I}_1}{s+1} \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-1}{s^2+1}$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

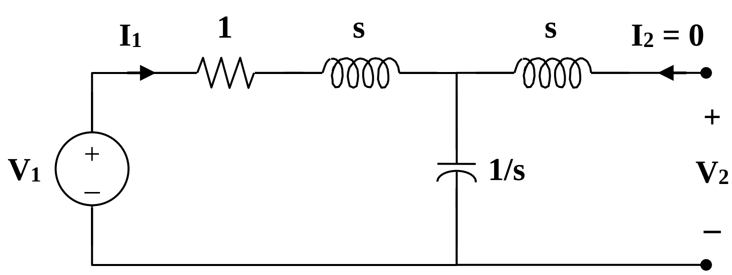
$$\mathbf{V}_1 = \frac{1/s}{s + 1/s}\mathbf{V}_2 = \frac{\mathbf{V}_2}{s^2 + 1} \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{s^2 + 1}$$

$$\mathbf{V}_2 = \left(s + \frac{1}{s}\right)\mathbf{I}_2 \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{s + 1/s} = \frac{s}{s^2 + 1}$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{s + 1 + \frac{s}{s^2 + 1}} & \mathbf{\frac{1}{s^2 + 1}} \\ \mathbf{\frac{-1}{s^2 + 1}} & \mathbf{\frac{s}{s^2 + 1}} \end{bmatrix}$$

(b) To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, refer to Fig. (c).

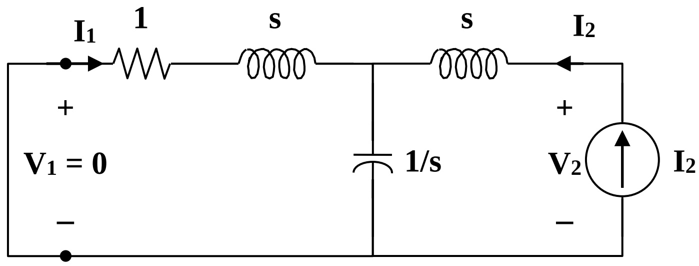


(c)

$$\mathbf{V}_1 = \left(1 + s + \frac{1}{s}\right)\mathbf{I}_1 \longrightarrow \mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{1 + s + 1/s} = \frac{s}{s^2 + s + 1}$$

$$\mathbf{V}_2 = \frac{1/s}{1 + s + 1/s}\mathbf{V}_1 = \frac{\mathbf{V}_1}{s^2 + s + 1} \longrightarrow \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{s^2 + s + 1}$$

To get $\mathbf{g}_{22}$ and $\mathbf{g}_{12}$, refer to Fig. (d).



(d)

$$\mathbf{V}_2 = \left(s + \frac{1}{s} \parallel (s+1) \right) \mathbf{I}_2 = \left(s + \frac{(s+1)/s}{1+s+1/s} \right) \mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = s + \frac{s+1}{s^2+s+1}$$

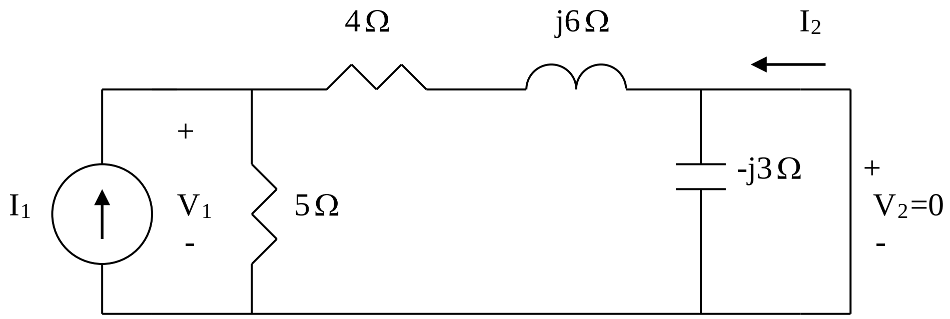
$$\mathbf{I}_1 = \frac{-1/s}{1+s+1/s} \mathbf{I}_2 = \frac{-\mathbf{I}_2}{s^2+s+1} \longrightarrow \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-1}{s^2+s+1}$$

Thus,

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{\mathbf{s}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} & \dfrac{-\mathbf{1}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} \\ \dfrac{\mathbf{1}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} & \mathbf{s}+\dfrac{\mathbf{s}+\mathbf{1}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} \end{bmatrix}$$

Solution 19.33

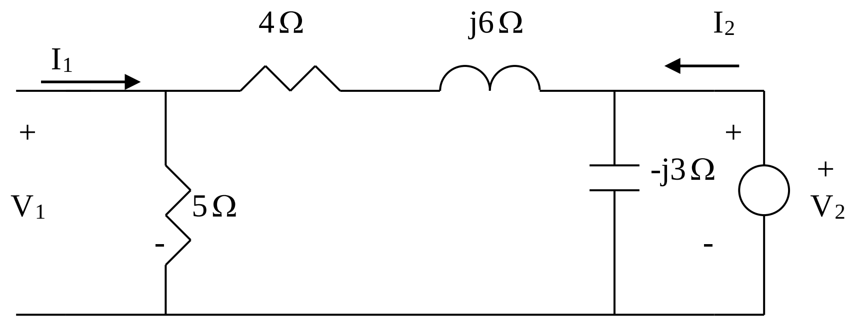
To get h_{11} and h_{21}, consider the circuit below.



$$V_1 = 5//(4+j6)I_1 = \frac{5(4+j6)I_1}{9+j6} \qquad h_{11} = \frac{V_1}{I_1} = 3.0769 + j1.2821$$

Also, $I_2 = -\dfrac{5}{9+j6}I_1 \quad \longrightarrow \quad h_{21} = \dfrac{I_2}{I_1} = -0.3846 + j0.2564$

To get h_{22} and h_{12}, consider the circuit below.



$$V_1 = \frac{5}{9+j6}V_2 \quad \longrightarrow \quad h_{12} = \frac{V_1}{V_2} = \frac{5}{9+j6} = 0.3846 - j0.2564$$

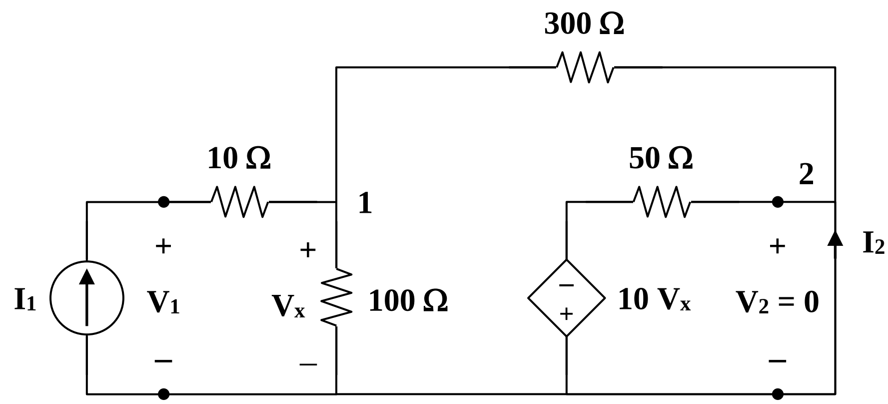
$$V_2 = -j3//(9+j6)I_2 \quad \longrightarrow \quad h_{22} = \frac{I_2}{V_2} = \frac{1}{-j3//(9+j6)} = \frac{9+j3}{-j3(9+j6)}$$
$$= 0.0769 + j0.2821$$

Thus,

$$\mathbf{[h]} = \begin{bmatrix} \mathbf{(3.077 + j1.2821)\ \Omega} & \mathbf{0.3846 - j0.2564} \\ \mathbf{-0.3846 + j0.2564} & \mathbf{(0.0769 + j0.2821)\ S} \end{bmatrix}$$

Solution 19.34

Refer to Fig. (a) to get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$.



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_x}{100} + \frac{\mathbf{V}_x - 0}{300} \longrightarrow 300\,\mathbf{I}_1 = 4\,\mathbf{V}_x \qquad (1)$$

$$\mathbf{V}_x = \frac{300}{4}\mathbf{I}_1 = 75\,\mathbf{I}_1$$

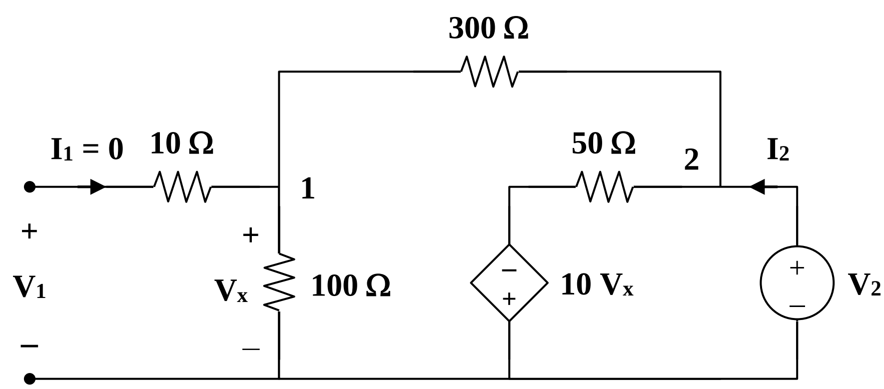
But $\mathbf{V}_1 = 10\,\mathbf{I}_1 + \mathbf{V}_x = 85\,\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \dfrac{\mathbf{V}_1}{\mathbf{I}_1} = 85\,\Omega$

At node 2,

$$\mathbf{I}_2 = \frac{0 + 10\,\mathbf{V}_x}{50} - \frac{\mathbf{V}_x}{300} = \frac{\mathbf{V}_x}{5} - \frac{\mathbf{V}_x}{300} = \frac{75}{5}\mathbf{I}_1 - \frac{75}{300}\mathbf{I}_1 = 14.75\,\mathbf{I}_1$$

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = 14.75$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

At node 2,

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{400} + \frac{\mathbf{V}_2 + 10\mathbf{V}_x}{50} \longrightarrow 400\mathbf{I}_2 = 9\mathbf{V}_2 + 80\mathbf{V}_x$$

But $\mathbf{V}_x = \dfrac{100}{400}\mathbf{V}_2 = \dfrac{\mathbf{V}_2}{4}$

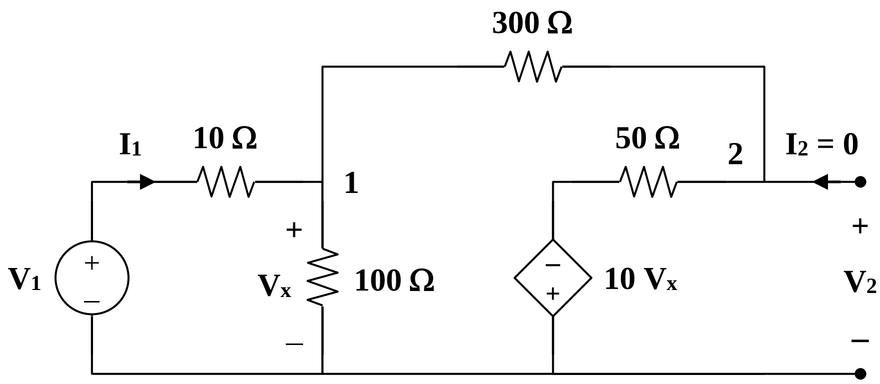
Hence, $400\mathbf{I}_2 = 9\mathbf{V}_2 + 20\mathbf{V}_2 = 29\mathbf{V}_2$

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{29}{400} = 0.0725\text{ S}$$

$$\mathbf{V}_1 = \mathbf{V}_x = \frac{\mathbf{V}_2}{4} \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{4} = 0.25$$

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{85\,\Omega} & \mathbf{0.25} \\ \mathbf{14.75} & \mathbf{0.0725\text{ S}} \end{bmatrix}$$

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, refer to Fig. (c).



(c)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_x}{100} + \frac{\mathbf{V}_x + 10\mathbf{V}_x}{350} \longrightarrow 350\mathbf{I}_1 = 14.5\mathbf{V}_x \qquad (2)$$

But $\mathbf{I}_1 = \dfrac{\mathbf{V}_1 - \mathbf{V}_x}{10} \longrightarrow 10\mathbf{I}_1 = \mathbf{V}_1 - \mathbf{V}_x$

or $\mathbf{V}_x = \mathbf{V}_1 - 10\mathbf{I}_1 \qquad (3)$

Substituting (3) into (2) gives

$$350\mathbf{I}_1 = 14.5\mathbf{V}_1 - 145\mathbf{I}_1 \longrightarrow 495\mathbf{I}_1 = 14.5\mathbf{V}_1$$

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{14.5}{495} = 0.02929 \text{ S}$$

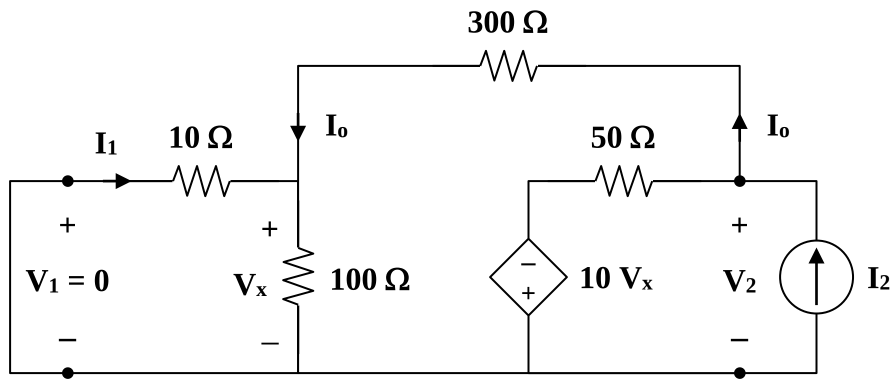
At node 2,

$$\mathbf{V}_2 = (50)\left(\frac{11}{350}\mathbf{V}_x\right) - 10\,\mathbf{V}_x = \text{-}8.4286\,\mathbf{V}_x$$

$$= \text{-}8.4286\,\mathbf{V}_1 + 84.286\,\mathbf{I}_1 = \text{-}8.4286\,\mathbf{V}_1 + (84.286)\left(\frac{14.5}{495}\right)\mathbf{V}_1$$

$$\mathbf{V}_2 = \text{-}5.96\,\mathbf{V}_1 \quad \longrightarrow \quad \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \text{-}5.96$$

To get $\mathbf{g}_{22}$ and $\mathbf{g}_{12}$, refer to Fig. (d).



(d)

$$10 \,||\, 100 = 9.091$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2 + 10\,\mathbf{V}_x}{50} + \frac{\mathbf{V}_2}{300 + 9.091}$$

$$309.091\,\mathbf{I}_2 = 7.1818\,\mathbf{V}_2 + 61.818\,\mathbf{V}_x \qquad (4)$$

But $$\mathbf{V}_x = \frac{9.091}{309.091}\mathbf{V}_2 = 0.02941\,\mathbf{V}_2 \qquad (5)$$

Substituting (5) into (4) gives

$$309.091\,\mathbf{I}_2 = 9\,\mathbf{V}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 34.34\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{309.091} = \frac{34.34\,\mathbf{I}_2}{309.091}$$

$$\mathbf{I}_1 = \frac{-100}{110}\mathbf{I}_o = \frac{-34.34\,\mathbf{I}_2}{(1.1)(309.091)}$$

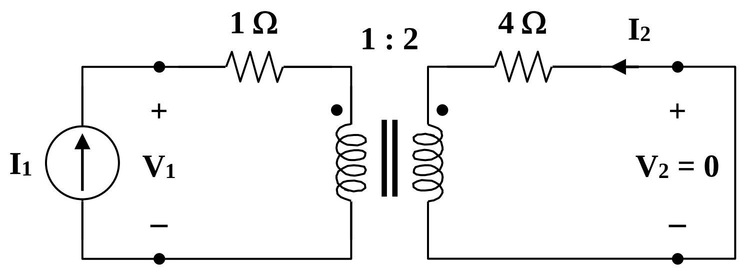
$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = -0.101$$

Thus,

$$[\mathbf{g}] = \begin{bmatrix} \mathbf{0.02929\ S} & \mathbf{-0.101} \\ \mathbf{-5.96} & \mathbf{34.34\ \Omega} \end{bmatrix}$$

Solution 19.35

To get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ consider the circuit in Fig. (a).



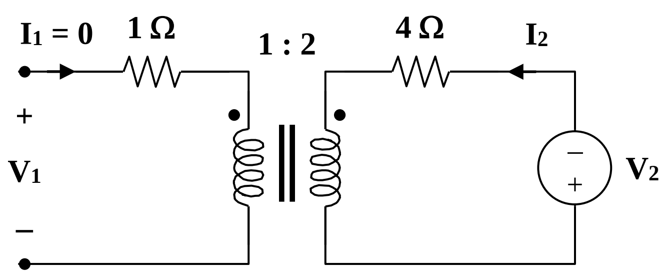
(a)

$$Z_R = \frac{4}{n^2} = \frac{4}{4} = 1$$

$$\mathbf{V}_1 = (1+1)\mathbf{I}_1 = 2\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2\,\Omega$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-N_2}{N_1} = -2 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-1}{2} = -0.5$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

Since $\mathbf{I}_1 = 0$, $\mathbf{I}_2 = 0$.

Hence, $\mathbf{h}_{22} = 0$.

At the terminals of the transformer, we have $\mathbf{V}_1$ and $\mathbf{V}_2$ which are related as

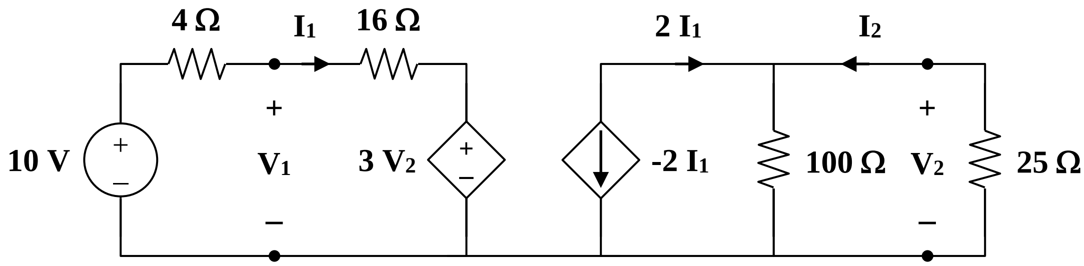
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n = 2 \quad \longrightarrow \quad \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{2} = 0.5$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{2\,\Omega} & \mathbf{0.5} \\ \mathbf{-\,0.5} & \mathbf{0} \end{bmatrix}$$

Solution 19.36

We replace the two-port by its equivalent circuit as shown below.



$$100 \,||\, 25 = 20\,\Omega$$

$$\mathbf{V}_2 = (20)(2\mathbf{I}_1) = 40\mathbf{I}_1 \qquad (1)$$

$$-10 + 20\mathbf{I}_1 + 3\mathbf{V}_2 = 0$$
$$10 = 20\mathbf{I}_1 + (3)(40\mathbf{I}_1) = 140\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{1}{14}, \qquad \mathbf{V}_2 = \frac{40}{14}$$

$$\mathbf{V}_1 = 16\mathbf{I}_1 + 3\mathbf{V}_2 = \frac{136}{14}$$

$$\mathbf{I}_2 = \left(\frac{100}{125}\right)(2\mathbf{I}_1) = \frac{-8}{70}$$

(a) $\dfrac{\mathbf{V}_2}{\mathbf{V}_1} = \dfrac{40}{136} = \mathbf{0.2941}$

(b) $\dfrac{\mathbf{I}_2}{\mathbf{I}_1} = \mathbf{-1.6}$

(c) $\dfrac{\mathbf{I}_1}{\mathbf{V}_1} = \dfrac{1}{136} = \mathbf{7.353 \times 10^{-3}\ S}$

(d) $\dfrac{\mathbf{V}_2}{\mathbf{I}_1} = \dfrac{40}{1} = \mathbf{40\,\Omega}$

Solution 19.37

The input port of the circuit in Fig.19.79 is connected to a 10-V dc voltage source while the output port is terminated by a 5-Ω resistor. Find the voltage across the 5-Ω resistor by using *h* parameters of the circuit. Confirm your result by using direct circuit analysis.

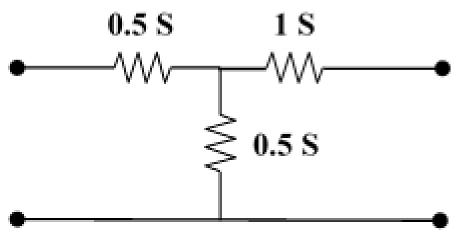
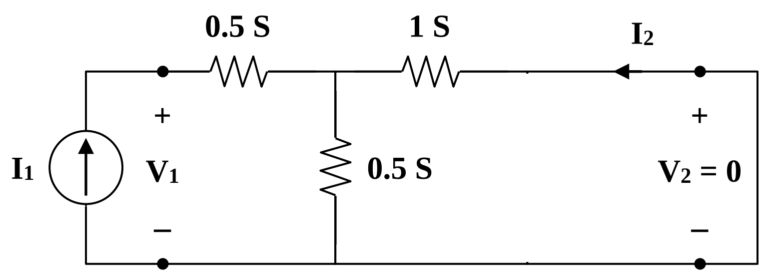


Figure 19.79
For Probs. 19.18 and 19.37.

Solution

We first obtain the h parameters. We need to start with the h parameter equations, $V_1 = h_{11}I_1 + h_{12}V_2$ and $I_2 = h_{21}I_1 + h_{22}V_2$. To get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ refer to Fig. (a).



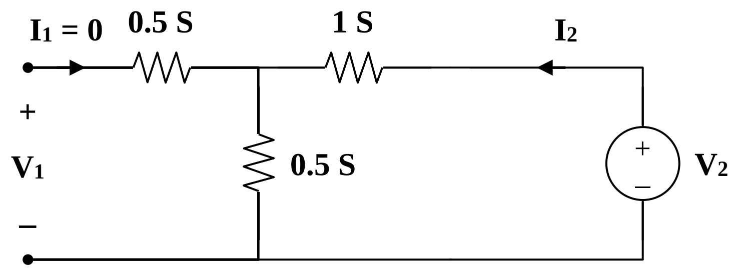
(a)

$$\mathbf{V}_1 = [(1/0.5) + (1/1.5)]\mathbf{I}_1 = [2 + 0.66667]\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2.667\ \Omega \text{ or}$$

$\mathbf{h_{11} = (8/3)\ \Omega}$

$$\mathbf{I}_2 = \frac{-2}{2+1}\mathbf{I}_1 = \frac{-2}{3}\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \mathbf{-(2/3)}$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in Fig. (b).

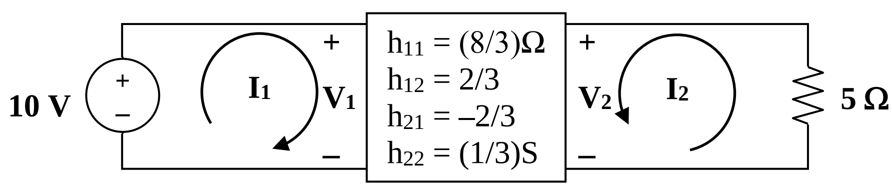


(b)

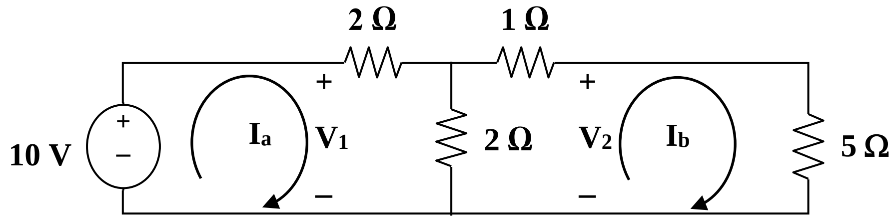
$$\mathbf{V}_2 = (2+1)\mathbf{I}_2 \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \textbf{(1/3) S}$$

$$\mathbf{V}_1 = \frac{2}{1+2}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \textbf{(2/3)}$$

$$[\mathbf{h}] = \begin{bmatrix} (8/3)\,\Omega & \frac{2}{3} \\ \frac{-2}{3} & \frac{1}{3}S \end{bmatrix}$$



We now have $V_1 = 10$ V and $V_2 = -5I_2$. Thus, we now have two equations with two unknowns. $10 = h_{11}I_1 + h_{12}(-5I_2)$ and $I_2 = h_{21}I_1 + h_{22}(-5I_2)$ or
$10 = (8/3)I_1 - (10/3)I_2$ and $I_2 = (-2/3)I_1 - (5/3)I_2$ or $(2/3)I_1 = -(8/3)I_2$ or $I_1 = -4I_2$ which leads to $10 = [-(32/3)-(10/3)]I_2 = -14I_2$ or $I_2 = -714.29$ mA. Therefore the power absorbed by the 5 Ω resistor $= (-0.71429)^2 5$
= **2.551 W**.



Checking using nodal analysis we get, $-10 + 2I_a + 2(I_a - I_b) = 0$ and $2(I_b - I_a) + 1I_b + 5I_b = 0$. From the second equation we get $I_a = 4I_b$ and then from the first equation we get $8I_b + 6I_b = 10$ or $I_b = 714.29$ mA. Finally we get 2.551 W and the answer agrees.

Solution 19.38

From eq. (19.75),

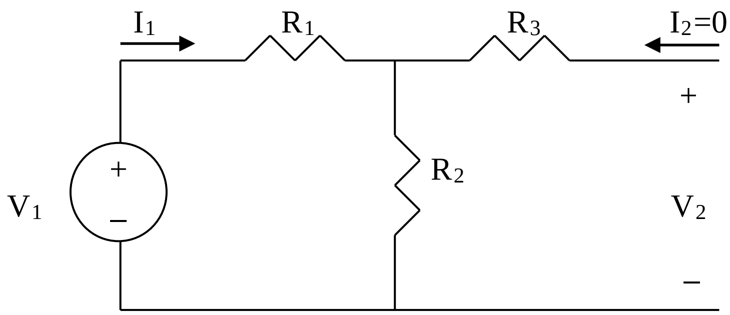
$$Z_{in} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1+h_{oe}R_L} = h_{11} - \frac{h_{12}h_{21}R_L}{1+h_{22}R_L} = 600 - \frac{0.04x30x400}{1+2x10^{-3}x400} = \underline{333.33\ \Omega}$$

From eq. (19.79),

$$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{0e} - h_{re}h_{fe}} = \frac{R_s + h_{11}}{(R_s + h_{11})h_{22} - h_{21}h_{12}} = \frac{2,000 + 600}{2600x2x10^{-3} - 30x0.04} = \underline{650\ \Omega}$$

Solution 19.39

We obtain g_{11} and g_{21} using the circuit below.

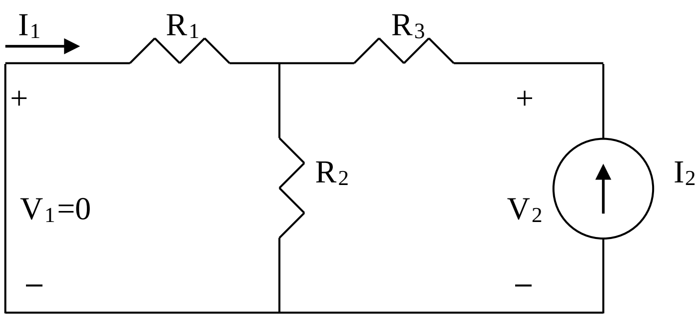


$$I_1 = \frac{V_1}{R_1 + R_2} \longrightarrow g_{11} = \frac{I_1}{V_1} = \underline{\frac{1}{R_1 + R_2}}$$

By voltage division,

$$V_2 = \frac{R_2}{R_1 + R_2} V_1 \longrightarrow g_{21} = \frac{V_2}{V_1} = \underline{\frac{R_2}{R_1 + R_2}}$$

We obtain g_{12} and g_{22} using the circuit below.



By current division,

$$I_1 = -\frac{R_2}{R_1 + R_2} I_2 \longrightarrow g_{12} = \frac{I_1}{I_2} = -\underline{\frac{R_2}{R_1 + R_2}}$$

Also,

$$V_2 = I_2(R_3 + R_1 // R_2) = I_2\left(R_3 + \frac{R_1 R_2}{R_1 + R_2}\right) \quad g_{22} = \frac{V_2}{I_2} = \underline{R_3 + \frac{R_1 R_2}{R_1 + R_2}}$$

$$\mathbf{g_{11} = \frac{1}{R_1 + R_2}, g_{12} = -\frac{R_2}{R_1 + R_2}}$$

$$\mathbf{g_{21} = \frac{R_2}{R_1 + R_2}, g_{22} = R_3 + \frac{R_1 R_2}{R_1 + R_2}}$$

Solution 19.40

Using Fig. 19.97, design a problem to help other students to better understand how to find *g* parameters in an ac circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the *g* parameters for the circuit in Fig.19.97.

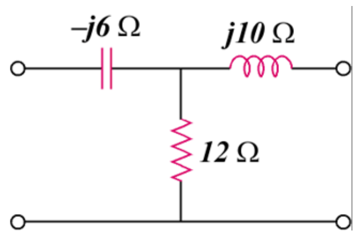
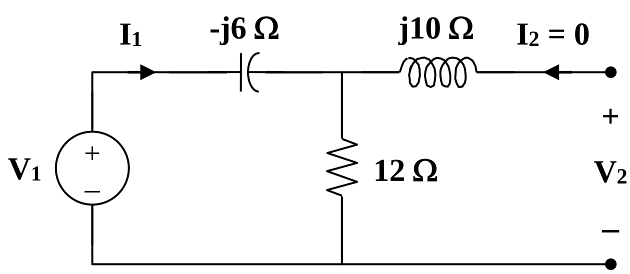


Figure 19.97

Solution

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, consider the circuit in Fig. (a).

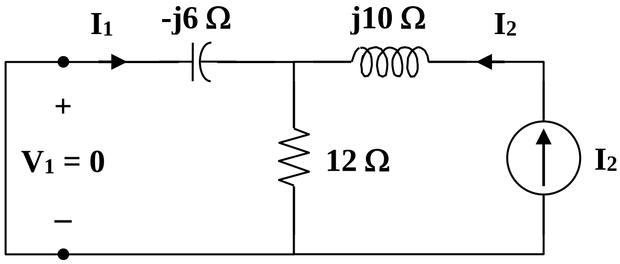


(a)

$$\mathbf{V}_1 = (12 - j6)\,\mathbf{I}_1 \quad \longrightarrow \quad \mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{12 - j6} = 0.0667 + j0.0333 \text{ S}$$

$$\mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{12\,\mathbf{I}_1}{(12 - j6)\,\mathbf{I}_1} = \frac{2}{2 - j} = 0.8 + j0.4$$

To get $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_1 = \frac{-12}{12 - j6}\mathbf{I}_2 \quad \longrightarrow \quad \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-12}{12 - j6} = -\mathbf{g}_{21} = -0.8 - j0.4$$

$$\mathbf{V}_2 = (j10 + 12 \,\|\, -j6)\mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = j10 + \frac{(12)(-j6)}{12 - j6} = 2.4 + j5.2\,\Omega$$

$$[\mathbf{g}] = \begin{bmatrix} \mathbf{0.0667 + j0.0333\ S} & \mathbf{-0.8 - j0.4} \\ \mathbf{0.8 + j0.4} & \mathbf{2.4 + j5.2\,\Omega} \end{bmatrix}$$

Solution 19.41

For the g parameters

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2 \qquad (2)$$

But $\mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_s$ and

$$\mathbf{V}_2 = -\mathbf{I}_2\mathbf{Z}_L = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

$$0 = \mathbf{g}_{21}\mathbf{V}_1 + (\mathbf{g}_{22} + \mathbf{Z}_L)\mathbf{I}_2$$

or

$$\mathbf{V}_1 = \frac{-(\mathbf{g}_{22} + \mathbf{Z}_L)}{\mathbf{g}_{21}}\mathbf{I}_2$$

Substituting this into (1),

$$\mathbf{I}_1 = \frac{(\mathbf{g}_{22}\mathbf{g}_{11} + \mathbf{Z}_L\mathbf{g}_{11} - \mathbf{g}_{21}\mathbf{g}_{12})}{-\mathbf{g}_{21}}\mathbf{I}_2$$

or

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-\mathbf{g}_{21}}{\mathbf{g}_{11}\mathbf{Z}_L + \Delta_g}$$

Also,

$$\begin{aligned}\mathbf{V}_2 &= \mathbf{g}_{21}(\mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_s) + \mathbf{g}_{22}\mathbf{I}_2 \\ &= \mathbf{g}_{21}\mathbf{V}_s - \mathbf{g}_{21}\mathbf{Z}_s\mathbf{I}_1 + \mathbf{g}_{22}\mathbf{I}_2 \\ &= \mathbf{g}_{21}\mathbf{V}_s + \mathbf{Z}_s(\mathbf{g}_{11}\mathbf{Z}_L + \Delta_g)\mathbf{I}_2 + \mathbf{g}_{22}\mathbf{I}_2\end{aligned}$$

But

$$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{\mathbf{Z}_L}$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_s - [\mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \Delta_g\mathbf{Z}_s + \mathbf{g}_{22}]\left[\frac{\mathbf{V}_2}{\mathbf{Z}_L}\right]$$

$$\frac{\mathbf{V}_2[\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \Delta_g\mathbf{Z}_s + \mathbf{g}_{22}]}{\mathbf{Z}_L} = \mathbf{g}_{21}\mathbf{V}_s$$

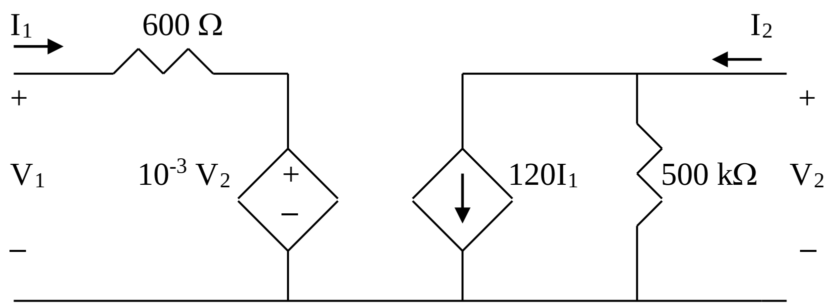
$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \Delta_g\mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{g}_{22}\mathbf{Z}_s - \mathbf{g}_{21}\mathbf{g}_{12}\mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{(1 + \mathbf{g}_{11}\mathbf{Z}_s)(\mathbf{g}_{22} + \mathbf{Z}_L) - \mathbf{g}_{12}\mathbf{g}_{21}\mathbf{Z}_s}$$

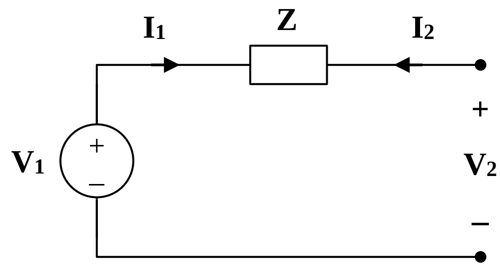
Solution 19.42

With the help of Fig. 19.20, we obtain the circuit model below.



Solution 19.43

(a) To find $\mathbf{A}$ and $\mathbf{C}$, consider the network in Fig. (a).



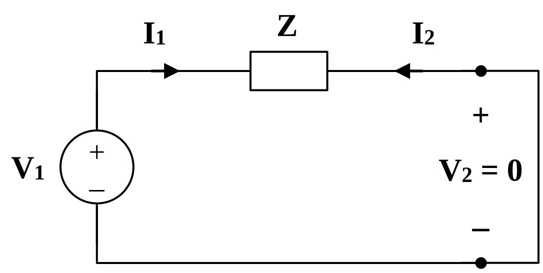
(a)

(a)

$$\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 1$$

$$\mathbf{I}_1 = 0 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0$$

To get $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



(b)

(a)

$$\mathbf{V}_1 = \mathbf{Z}\mathbf{I}_1, \qquad \mathbf{I}_2 = -\mathbf{I}_1$$

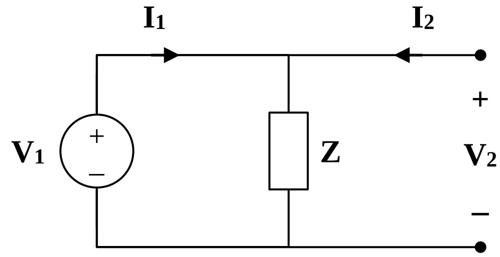
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{-\mathbf{Z}\mathbf{I}_1}{-\mathbf{I}_1} = \mathbf{Z}$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = 1$$

Hence,

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{1} & \mathbf{Z} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

(b) To find $\mathbf{A}$ and $\mathbf{C}$, consider the circuit in Fig. (c).



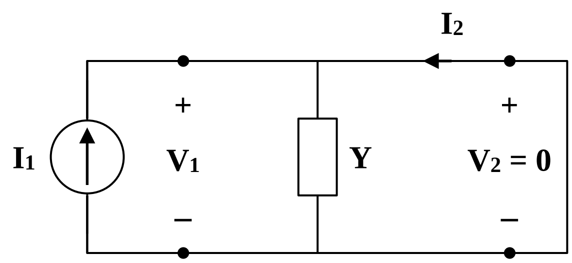
(c)

(a)

$$\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 1$$

$$\mathbf{V}_1 = \mathbf{Z}\,\mathbf{I}_1 = \mathbf{V}_2 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{\mathbf{Z}} = \mathbf{Y}$$

To get $\mathbf{B}$ and $\mathbf{D}$, refer to the circuit in Fig.(d).



(d)

(a)

$$\mathbf{V}_1 = \mathbf{V}_2 = 0 \qquad \mathbf{I}_2 = -\mathbf{I}_1$$

$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = 0, \qquad \mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = 1$$

Thus,

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{Y} & \mathbf{1} \end{bmatrix}$$

Solution 19.44

Using Fig. 19.99, design a problem to help other students to better understand how to find the transmission parameters of an ac circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the transmission parameters of the circuit in Fig.19.99.

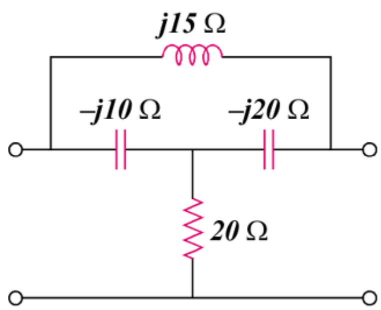
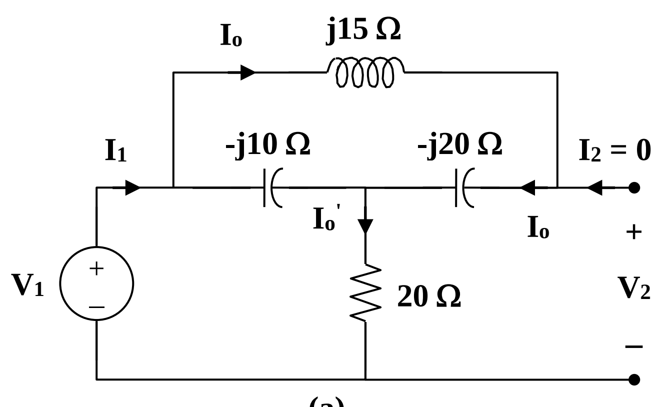


Figure 19.99

Solution

To determine $\mathbf{A}$ and $\mathbf{C}$, consider the circuit in Fig.(a).



(a)

$$\mathbf{V}_1 = [\,20 + (\text{-j10}) \,||\, (\text{j15} - \text{j20})\,]\mathbf{I}_1$$

$$\mathbf{V}_1 = \left[\,20 + \frac{(\text{-j10})(\text{-j5})}{\text{- j15}}\right]\mathbf{I}_1 = \left[\,20 - \text{j}\frac{10}{3}\right]\mathbf{I}_1$$

$$\mathbf{I}_o^{'} = \mathbf{I}_1$$

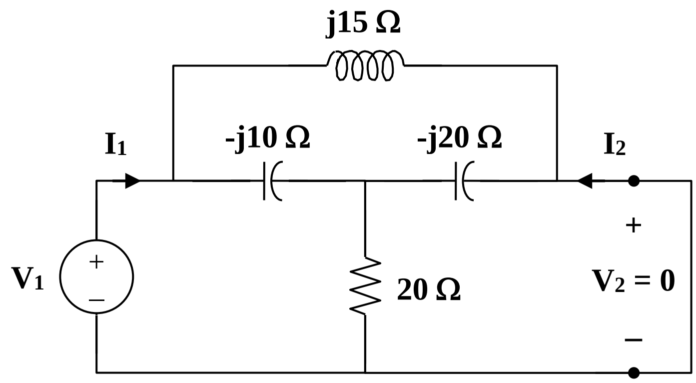
$$\mathbf{I}_o = \left(\frac{\text{- j10}}{\text{- j10} - \text{j5}}\right)\mathbf{I}_1 = \left(\frac{2}{3}\right)\mathbf{I}_1$$

$$\mathbf{V}_2 = (\text{-j20})\mathbf{I}_o + 20\mathbf{I}_o{}' \;=\; -\,j\frac{40}{3}\mathrm{I}_1 + 20\mathrm{I}_1 \;=\; \left(20 - j\frac{40}{3}\right)\mathrm{I}_1$$

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{(20 - \text{j}10/3)\,\mathbf{I}_1}{\left(20 - j\frac{40}{3}\right)\mathrm{I}_1} = 0.7692 + \text{j}0.3461$$

$$\mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{20 - j\frac{40}{3}} = \;0.03461 + \text{j}0.023$$

To find $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



(b)

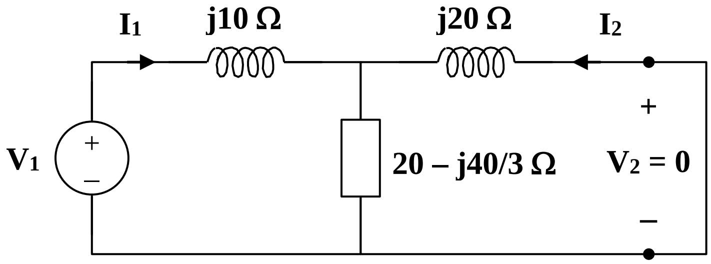
(a)

We may transform the Δ subnetwork to a T as shown in Fig. (c).

$$\mathbf{Z}_1 = \frac{(\text{j15})(\text{-j10})}{\text{j15} - \text{j10} - \text{j20}} = \text{j10}$$

$$\mathbf{Z}_2 = \frac{(\text{-j10})(\text{-j20})}{\text{- j15}} = \text{-j}\frac{40}{3}$$

$$\mathbf{Z}_3 = \frac{(j15)(-j20)}{-j15} = j20$$



(c)

$$-\mathbf{I}_2 = \frac{20 - j40/3}{20 - j40/3 + j20}\mathbf{I}_1 = \frac{3 - j2}{3 + j}\mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{3 + j}{3 - j2} = 0.5385 + j0.6923$$

$$\mathbf{V}_1 = \left[j10 + \frac{(j20)(20 - j40/3)}{20 - j40/3 + j20} \right]\mathbf{I}_1$$

$$\mathbf{V}_1 = [\, j10 + 2(9 + j7)\,]\mathbf{I}_1 = j\mathbf{I}_1(24 - j18)$$

$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{-j\mathbf{I}_1(24 - j18)}{\frac{-(3 - j2)}{3 + j}\mathbf{I}_1} = \frac{6}{13}(-15 + j55)$$

$$\mathbf{B} = -6.923 + j25.385\,\Omega$$

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{0.7692 + j0.3461} & \mathbf{(-6.923 + j25.38)\,\Omega} \\ \mathbf{(0.03461 + j0.023)\,S} & \mathbf{0.5385 + j0.6923} \end{bmatrix}$$

Solution 19.45

Find the ABCD parameters for the circuit in Fig. 19.100.

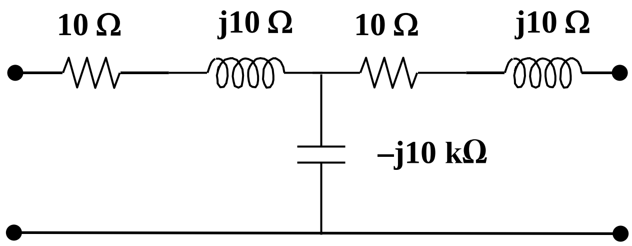
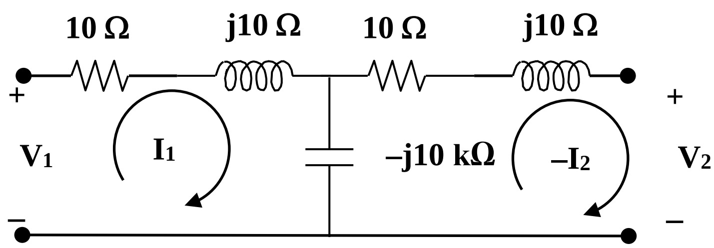


Figure 19.100
For Prob. 19.45.

Solution

Step 1. First we need to label all of the currents and voltages so we can apply the defining equations for the T parameters.



$V_1 = AV_2 - BI_2$ and $I_1 = CV_2 - DI_2$. To find the T parameters we let $I_1 = 1$ A and $I_2 = 0$ followed by letting $I_1 = 1$ A and $V_2 = 0$.

Step 2. $V_1 = (10+j10)1 - j10{,}000\text{x}1 \approx -j10$ kV and $V_2 = -j10$ kV which leads to A = **1** and C = 1/(–j10k) = **j100 μS**.

$V_1 \approx (10+j10+10+j10)1 = (20+j20)$ V and $I_2 \approx -1$ A which leads to B = **(20+j20) Ω** and D = **1**.

Solution 19.46

Find the transmission parameters for the circuit in Fig.19.101.

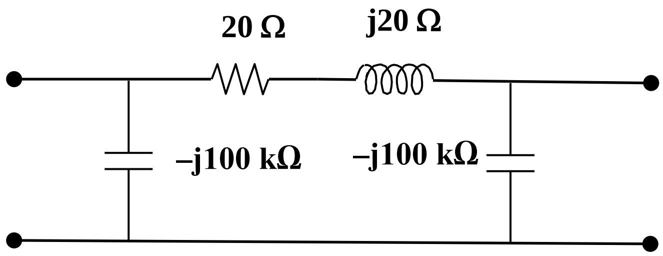
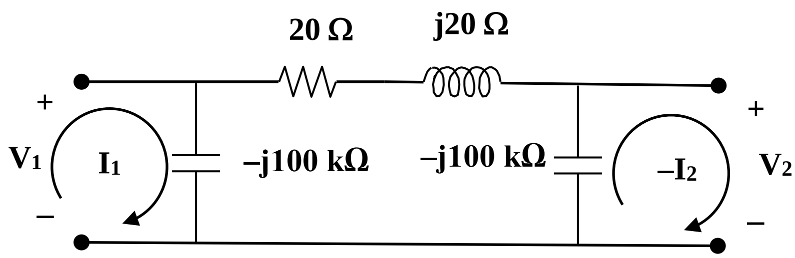


Figure 19.101
For Prob. 19.46.

Solution

Step 1. First we need to label all of the currents and voltages so we can apply the defining equations for the T parameters.



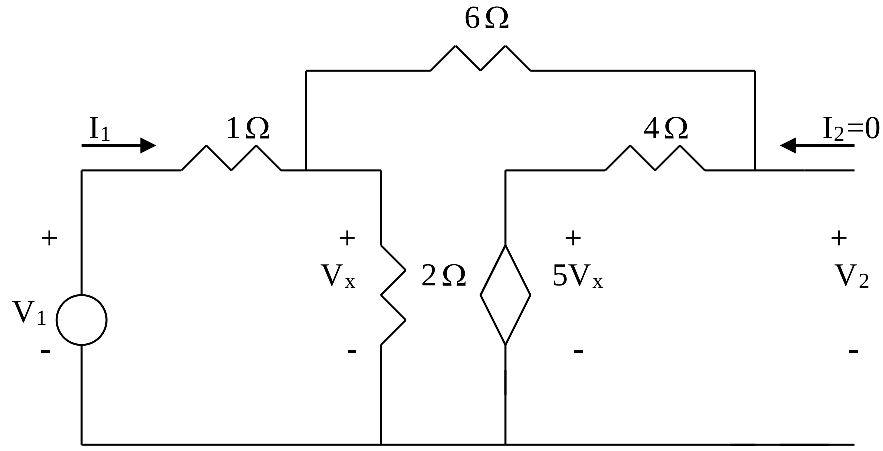
$V_1 = AV_2 - BI_2$ and $I_1 = CV_2 - DI_2$. To find the T parameters we let $I_1 = 1$ A and $I_2 = 0$ followed by letting $I_1 = 1$ A and $V_2 = 0$.

Step 2. Because the voltage drop across the (20+j20) Ω is relatively small it can be neglected which means that $V_1 \approx 1[(-j100k)(-j100k)]/(-j200k) = -j50$ kV and $V_2 \approx -j50$ kV which leads to A = 1 and C = 1/(–j50k) = **j20 µS**.

With $V_2 = 0$ we can neglect the current through the capacitor which gives us $V_1 \approx (20+j20)$ V and $I_2 = -1$ A which leads to B = **(20+j20) Ω** and D = **1**.

Solution 19.47

To get A and C, consider the circuit below.

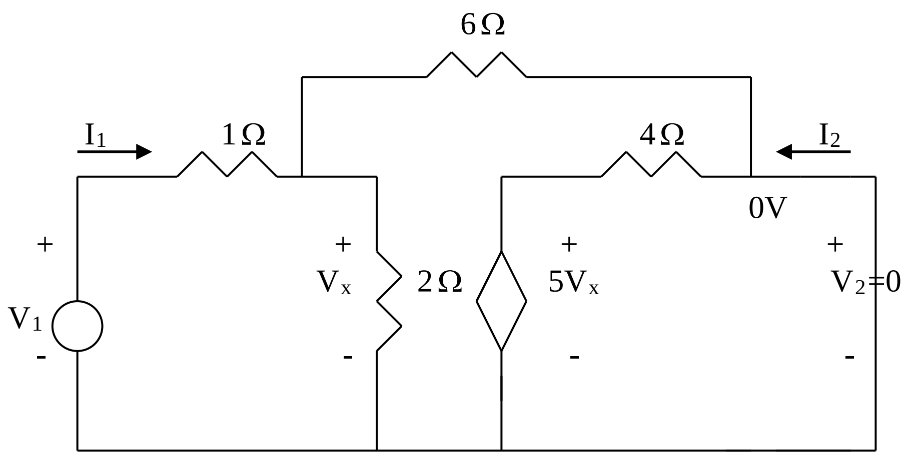


$$\frac{V_1 - V_x}{1} = \frac{V_x}{2} + \frac{V_x - 5V_x}{10} \longrightarrow V_1 = 1.1V_x$$

$$V_2 = 4(-0.4V_x) + 5V_x = 3.4V_x \longrightarrow A = \frac{V_1}{V_2} = 1.1/3.4 = 0.3235$$

$$I_1 = \frac{V_1 - V_x}{1} = 1.1V_x - V_x = 0.1V_x \longrightarrow C = \frac{I_1}{V_2} = 0.1/3.4 = 0.02941$$

To get B and D, consider the circuit below.



$$\frac{V_1 - V_x}{1} = \frac{V_x}{6} + \frac{V_x}{2} \longrightarrow V_1 = \frac{10}{6}V_x \qquad (1)$$

$$I_2 = -\frac{5V_x}{4} - \frac{V_x}{6} = -\frac{17}{12}V_x \qquad (2)$$

$$V_1 = I_1 + V_x \qquad (3)$$

From (1) and (3)

$$I_1 = V_1 - V_x = \frac{4}{6}V_x \longrightarrow D = -\frac{I_1}{I_2} = \frac{4}{6}(\frac{12}{17}) = 0.4706$$

$$B = -\frac{V_1}{I_2} = \frac{10}{6}(\frac{12}{17}) = 1.176$$

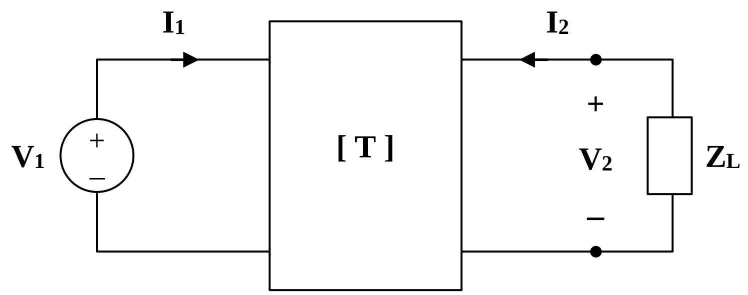
S

Ω

$$\underline{[T] = \begin{bmatrix} 0.3235 & 1.176\Omega \\ 0.02941S & 0.4706 \end{bmatrix}}$$

Solution 19.48

(a) Refer to the circuit below.



$$\mathbf{V}_1 = 4\mathbf{V}_2 - 30\mathbf{I}_2 \qquad (1)$$
$$\mathbf{I}_1 = 0.1\mathbf{V}_2 - \mathbf{I}_2 \qquad (2)$$

When the output terminals are shorted, $\mathbf{V}_2 = 0$.
So, (1) and (2) become

$\mathbf{V}_1 = \text{-}30\mathbf{I}_2$ and $\mathbf{I}_1 = \text{-}\mathbf{I}_2$

Hence,

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \mathbf{30\,\Omega}$$

(b) When the output terminals are open-circuited, $\mathbf{I}_2 = 0$.
So, (1) and (2) become

$\mathbf{V}_1 = 4\mathbf{V}_2$

$\mathbf{I}_1 = 0.1\mathbf{V}_2$ or $\mathbf{V}_2 = 10\mathbf{I}_1$

$\mathbf{V}_1 = 40\mathbf{I}_1$

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \mathbf{40\,\Omega}$$

(c) When the output port is terminated by a 10-Ω load, $\mathrm{V}_2 = \text{-}10\mathrm{I}_2$.
So, (1) and (2) become

$\mathbf{V}_1 = \text{-}40\mathbf{I}_2 - 30\mathbf{I}_2 = \text{-}70\mathbf{I}_2$

$\mathbf{I}_1 = \text{-}\mathbf{I}_2 - \mathbf{I}_2 = \text{-}2\mathbf{I}_2$

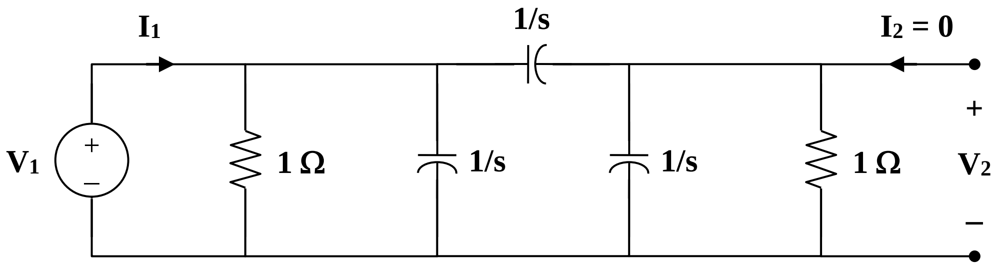
$\mathbf{V}_1 = 35\mathbf{I}_1$

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \mathbf{35\,\Omega}$$

Alternatively, we may use $\mathbf{Z}_{\text{in}} = \dfrac{\mathbf{A}\mathbf{Z}_{\text{L}} + \mathbf{B}}{\mathbf{C}\mathbf{Z}_{\text{L}} + \mathbf{D}}$

Solution 19.49

To get **A** and **C**, refer to the circuit in Fig.(a).



(a)

(a)

$$1 \| \frac{1}{s} = \frac{1/s}{1+1/s} = \frac{1}{s+1}$$

$$\mathbf{V}_2 = \frac{1 \| 1/s}{1/s + 1 \| 1/s} \mathbf{V}_1$$

$$\mathbf{A} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\frac{1}{s+1}}{\frac{1}{s} + \frac{1}{s+1}} = \frac{s}{2s+1}$$

$$\mathbf{V}_1 = \mathbf{I}_1 \left(\frac{1}{s+1}\right) \| \left(\frac{1}{s} + \frac{1}{s+1}\right) = \mathbf{I}_1 \left(\frac{1}{s+1}\right) \| \left(\frac{2s+1}{s(s+1)}\right)$$

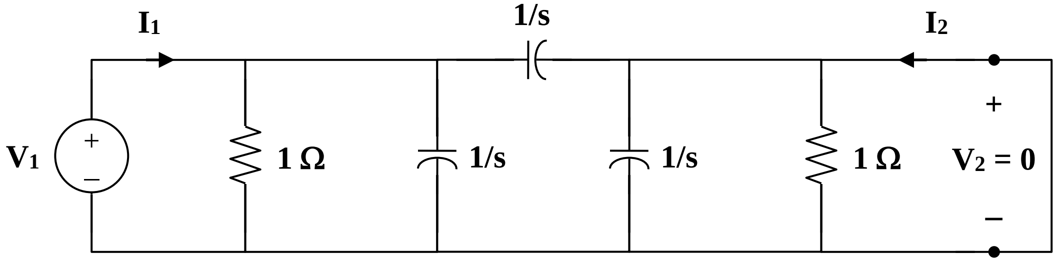
$$\frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\left(\frac{1}{s+1}\right) \cdot \left(\frac{2s+1}{s(s+1)}\right)}{\frac{1}{s+1} + \frac{2s+1}{s(s+1)}} = \frac{2s+1}{(s+1)(3s+1)}$$

But $\quad \mathbf{V}_1 = \mathbf{V}_2 \cdot \dfrac{2s+1}{s}$

Hence, $\quad \dfrac{\mathbf{V}_2}{\mathbf{I}_1} \cdot \dfrac{2s+1}{s} = \dfrac{2s+1}{(s+1)(3s+1)}$

$$\mathbf{C} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{(s+1)(3s+1)}{s}$$

To get $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



(b)

(a)

$$\mathbf{V}_1 = \mathbf{I}_1\left(1 \| \frac{1}{s} \| \frac{1}{s}\right) = \mathbf{I}_1\left(1 \| \frac{1}{2s}\right) = \frac{\mathbf{I}_1}{2s+1}$$

$$\mathbf{I}_2 = \frac{\frac{-1}{s+1}\mathbf{I}_1}{\frac{1}{s+1}+\frac{1}{s}} = \frac{-s}{2s+1}\mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{2s+1}{s} = 2+\frac{1}{s}$$

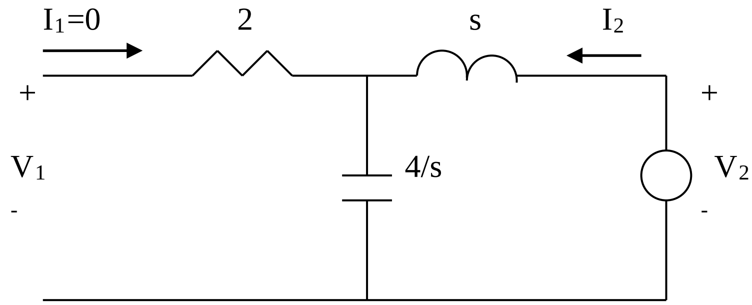
$$\mathbf{V}_1 = \left(\frac{1}{2s+1}\right)\left(\frac{2s+1}{-s}\right)\mathbf{I}_2 = \frac{\mathbf{I}_2}{-s} \longrightarrow \mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{1}{s}$$

Thus,

$$[\mathbf{T}] = \begin{bmatrix} \dfrac{\mathbf{2}}{\mathbf{2s+1}} & \dfrac{\mathbf{1}}{\mathbf{s}} \\ \dfrac{\mathbf{(s+1)(3s+1)}}{\mathbf{s}} & \mathbf{2+\dfrac{1}{s}} \end{bmatrix}$$

Solution 19.50

To get a and c, consider the circuit below.

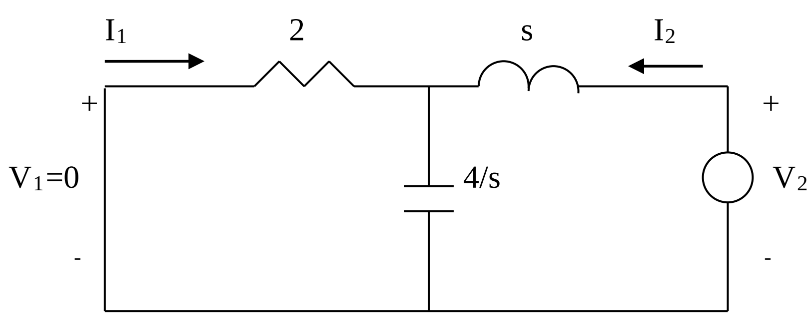


$$V_1 = \frac{4/s}{s+4/s}V_2 = \frac{4}{s^2+4}V_2 \quad \longrightarrow \quad a = V_2/V_1 = 1 + 0.25s^2$$

$$V_2 = (s + 4/s)I_2 \text{ or}$$

$$I_2 = \frac{V_2}{s+4/s} = \frac{(1+0.25s^2)V_1}{s+4/s} \quad \longrightarrow \quad c = \frac{I_2}{V_1} = \frac{s+0.25s^3}{s^2+4}$$

To get b and d, consider the circuit below.



$$I_1 = \frac{-4/s}{2+4/s}I_2 = -\frac{2I_2}{s+2} \quad \longrightarrow \quad d = -\frac{I_2}{I_1} = 1 + 0.5s$$

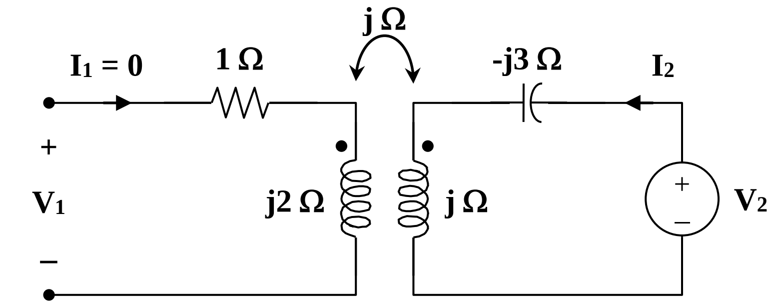
$$V_2 = (s + 2//\frac{4}{s})I_2 = \frac{(s^2+2s+4)}{s+2}I_2$$

$$= -\frac{(s^2+2s+4)}{s+2}\frac{(s+2)}{2}I_1 \quad \longrightarrow \quad b = -\frac{V_2}{I_1} = 0.5s^2 + s + 2$$

$$[\mathbf{t}] = \begin{bmatrix} 0.25s^2 + 1 & 0.5s^2 + s + 2 \\ \dfrac{0.25s^2 + s}{s^2 + 4} & 0.5s + 1 \end{bmatrix}$$

Solution 19.51

To get $\mathbf{a}$ and $\mathbf{c}$, consider the circuit in Fig. (a).



(a)

(a)

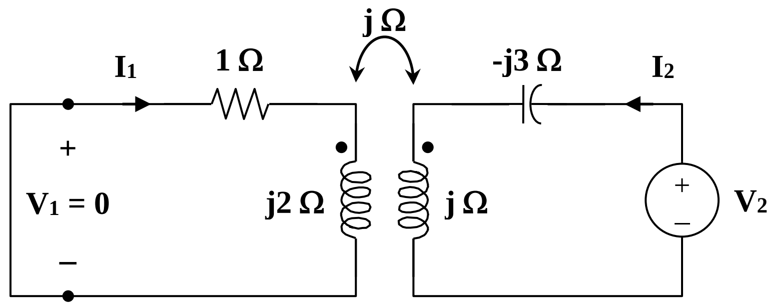
$$\mathbf{V}_2 = \mathbf{I}_2\,(\mathrm{j} - \mathrm{j}3) = \text{-j}2\,\mathbf{I}_2$$

$$\mathbf{V}_1 = \text{-j}\mathbf{I}_2$$

$$\mathbf{a} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{-\,\mathrm{j}2\,\mathbf{I}_2}{-\,\mathrm{j}\mathbf{I}_2} = 2$$

$$\mathbf{c} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{1}{-\,\mathrm{j}} = \mathrm{j}$$

To get $\mathbf{b}$ and $\mathbf{d}$, consider the circuit in Fig. (b).



(b)

(a)

For mesh 1,

$$0 = (1 + \mathrm{j}2)\,\mathbf{I}_1 - \mathrm{j}\,\mathbf{I}_2$$

or
$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{1 + \mathrm{j}2}{\mathrm{j}} = 2 - \mathrm{j}$$

$$\mathbf{d} = \frac{-\,\mathbf{I}_2}{\mathbf{I}_1} = \text{-}2 + \mathrm{j}$$

For mesh 2,

$$\mathbf{V}_2 = \mathbf{I}_2 (j - j3) - j\mathbf{I}_1$$
$$\mathbf{V}_2 = \mathbf{I}_1 (2 - j)(-j2) - j\mathbf{I}_1 = (-2 - j5)\mathbf{I}_1$$

$$\mathbf{b} = \frac{-\mathbf{V}_2}{\mathbf{I}_1} = 2 + j5$$

Thus,

$$[\mathbf{t}] = \begin{bmatrix} \mathbf{2} & \mathbf{2+j5} \\ \mathbf{j} & \mathbf{-2+j} \end{bmatrix}$$

Solution 19.52

It is easy to find the z parameters and then transform these to h parameters and T parameters.

$$[\mathbf{z}] = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix}$$

$$\begin{aligned} \Delta_z &= (R_1 + R_2)(R_2 + R_3) - R_2^2 \\ &= R_1R_2 + R_2R_3 + R_3R_1 \end{aligned}$$

(a) $$[\mathbf{h}] = \begin{bmatrix} \dfrac{\Delta_z}{\mathbf{z}_{22}} & \dfrac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \dfrac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \dfrac{1}{\mathbf{z}_{22}} \end{bmatrix} = \begin{bmatrix} \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2 + R_3} & \dfrac{R_2}{R_2 + R_3} \\ \dfrac{-R_2}{R_2 + R_3} & \dfrac{1}{R_2 + R_3} \end{bmatrix}$$

Thus,

$$\mathbf{h_{11} = R_1 + \frac{R_2R_3}{R_2 + R_3}}, \quad \mathbf{h_{12} = \frac{R_2}{R_2 + R_3} = -h_{21}}, \quad \mathbf{h_{22} = \frac{1}{R_2 + R_3}}$$

as required.

(b) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \dfrac{\Delta_z}{\mathbf{z}_{21}} \\ \dfrac{1}{\mathbf{z}_{21}} & \dfrac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \\ \dfrac{1}{R_2} & \dfrac{R_2 + R_3}{R_2} \end{bmatrix}$$

Hence,

$$\mathbf{A = 1 + \frac{R_1}{R_2}}, \quad \mathbf{B = R_3 + \frac{R_1}{R_2}(R_2 + R_3)}, \quad \mathbf{C = \frac{1}{R_2}}, \quad \mathbf{D = 1 + \frac{R_3}{R_2}}$$

as required.

Solution 19.53

For the z parameters,

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{12}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \qquad (2)$$

For **ABCD** parameters,

$$\mathbf{V}_1 = \mathbf{A}\,\mathbf{V}_2 - \mathbf{B}\,\mathbf{I}_2 \qquad (3)$$

$$\mathbf{I}_1 = \mathbf{C}\,\mathbf{V}_2 - \mathbf{D}\,\mathbf{I}_2 \qquad (4)$$

From (4),

$$\mathbf{V}_2 = \frac{\mathbf{I}_1}{\mathbf{C}} + \frac{\mathbf{D}}{\mathbf{C}}\mathbf{I}_2 \qquad (5)$$

Comparing (2) and (5),

$$\mathbf{z}_{21} = \frac{1}{\mathbf{C}}, \qquad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}}$$

Substituting (5) into (3),

$$\mathbf{V}_1 = \frac{\mathbf{A}}{\mathbf{C}}\mathbf{I}_1 + \left(\frac{\mathbf{AD}}{\mathbf{C}} - \mathbf{B}\right)\mathbf{I}_2$$

$$= \frac{\mathbf{A}}{\mathbf{C}}\mathbf{I}_1 + \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}}\mathbf{I}_2 \qquad (6)$$

Comparing (6) and (1),

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}} \qquad \mathbf{z}_{12} = \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}} = \frac{\Delta_T}{\mathbf{C}}$$

Thus,

$$[Z] = \begin{bmatrix} \dfrac{\mathbf{A}}{\mathbf{C}} & \dfrac{\mathbf{\Delta_T}}{\mathbf{C}} \\ \dfrac{\mathbf{1}}{\mathbf{C}} & \dfrac{\mathbf{D}}{\mathbf{C}} \end{bmatrix}$$

Solution 19.54

For the y parameters

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \qquad (2)$$

From (2),

$$\mathbf{V}_1 = \frac{\mathbf{I}_2}{\mathbf{y}_{21}} - \frac{\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2$$

or

$$\mathbf{V}_1 = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{12}}\mathbf{V}_2 + \frac{1}{\mathbf{y}_{21}}\mathbf{I}_2 \qquad (3)$$

Substituting (3) into (1) gives

$$\mathbf{I}_1 = \frac{-\mathbf{y}_{11}\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2 + \mathbf{y}_{12}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2$$

or

$$\mathbf{I}_1 = \frac{-\Delta_y}{\mathbf{y}_{21}}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2 \qquad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

clearly shows that

$$\mathbf{A} = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{21}}, \quad \mathbf{B} = \frac{-1}{\mathbf{y}_{21}}, \quad \mathbf{C} = \frac{-\Delta_y}{\mathbf{y}_{21}}, \quad \mathbf{D} = \frac{-\mathbf{y}_{11}}{\mathbf{y}_{21}}$$

as required.

Solution 19.55

For the z parameters

$$\mathbf{V}_1 = \mathbf{z}_{11}\,\mathbf{I}_1 + \mathbf{z}_{12}\,\mathbf{I}_2 \tag{1}$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\,\mathbf{I}_1 + \mathbf{z}_{22}\,\mathbf{I}_2 \tag{2}$$

From (1),

$$\mathbf{I}_1 = \frac{1}{\mathbf{z}_{11}}\mathbf{V}_1 - \frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}\mathbf{I}_2 \tag{3}$$

Substituting (3) into (2) gives

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}\mathbf{V}_1 + \left(\mathbf{z}_{22} - \frac{\mathbf{z}_{21}\,\mathbf{z}_{12}}{\mathbf{z}_{11}}\right)\mathbf{I}_2$$

or

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}\mathbf{V}_1 + \frac{\Delta_z}{\mathbf{z}_{11}}\mathbf{I}_2 \tag{4}$$

Comparing (3) and (4) with the following equations

$$\mathbf{I}_1 = \mathbf{g}_{11}\,\mathbf{V}_1 + \mathbf{g}_{12}\,\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\,\mathbf{V}_1 + \mathbf{g}_{22}\,\mathbf{I}_2$$

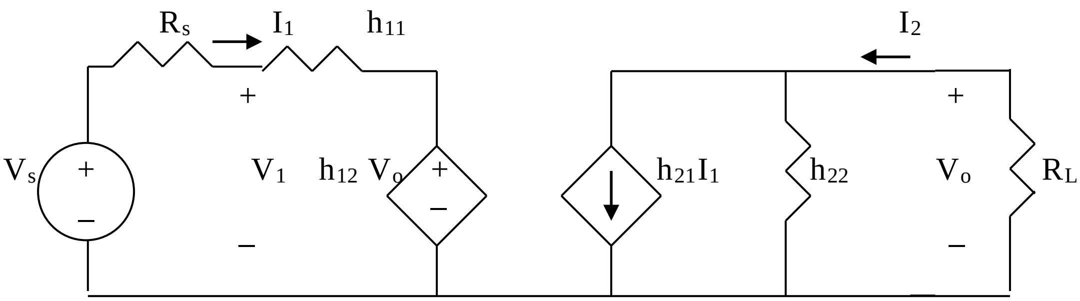
indicates that

$$\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}, \quad \mathbf{g}_{12} = \frac{-\,\mathbf{z}_{12}}{\mathbf{z}_{11}}, \quad \mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}, \quad \mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}$$

as required.

Solution 19.56

Using Fig. 19.20, we obtain the equivalent circuit as shown below.



We can solve this using MATLAB. First, we generate 4 equations from the given circuit. It may help to let $V_s = 10$ V.

$-10 + R_sI_1 + V_1 = 0$ or $V_1 + 1000I_1 = 10$
$-10 + R_sI_1 + h_{11}I_1 + h_{12}V_o = 0$ or $0.0001V_s + 1500 = 10$
$I_2 = -V_o/R_L$ or $V_o + 2000I_2 = 0$
$h_{21}I_1 + h_{22}V_o - I_2 = 0$ or $2\text{x}10^{-6}V_o + 100I_1 - I_2 = 0$

```
>> A=[1,0,1000,0;0,0.0001,1500,0;0,1,0,2000;0,(2*10^-6),100,-1]
A =
  1.0e+003 *
    0.0010         0    1.0000         0
         0    0.0000    1.5000         0
         0    0.0010         0    2.0000
         0    0.0000    0.1000   -0.0010
>> U=[10;10;0;0]
U =
    10
    10
     0
     0
>> X=inv(A)*U
X =
  1.0e+003 *
    0.0032
   -1.3459
    0.0000
    0.0007
```

Gain = V_o / V_s = –1,345.9/10 = **–134.59**.

There is a second approach we can take to check this problem. First, the resistive value of h_{22} is quite large, 500 kΩ versus R_L so can be ignored. Working on the right side of the circuit we obtain the following,

$$I_2 = 100I_1 \text{ which leads to } V_o = -I_2 x2k = -2x10^5 I_1.$$

Now the left hand loop equation becomes,

$$-V_s + (1000 + 500 + 10^{-4}(-2x10^5))I_1 = 1480I_1.$$

Solving for V_o/V_s we get,

$$V_o/V_s = -200{,}000/1480 = \mathbf{-134.14}.$$

Our answer checks!

Solution 19.57

$\Delta_T = (3)(7) - (20)(1) = 1$

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{\mathbf{A}}{\mathbf{C}} & \dfrac{\Delta_T}{\mathbf{C}} \\ \dfrac{1}{\mathbf{C}} & \dfrac{\mathbf{D}}{\mathbf{C}} \end{bmatrix} = \begin{bmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{1} & \mathbf{7} \end{bmatrix} \mathbf{\Omega}$$

$$[\mathbf{y}] = \begin{bmatrix} \dfrac{\mathbf{D}}{\mathbf{B}} & \dfrac{-\Delta_T}{\mathbf{B}} \\ \dfrac{-1}{\mathbf{B}} & \dfrac{\mathbf{A}}{\mathbf{B}} \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{7}}{\mathbf{20}} & \dfrac{\mathbf{-1}}{\mathbf{20}} \\ \dfrac{\mathbf{-1}}{\mathbf{20}} & \dfrac{\mathbf{3}}{\mathbf{20}} \end{bmatrix} \mathbf{S}$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{\mathbf{B}}{\mathbf{D}} & \dfrac{\Delta_T}{\mathbf{D}} \\ \dfrac{-1}{\mathbf{D}} & \dfrac{\mathbf{C}}{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{20}}{\mathbf{7}}\mathbf{\Omega} & \dfrac{\mathbf{1}}{\mathbf{7}} \\ \dfrac{\mathbf{-1}}{\mathbf{7}} & \dfrac{\mathbf{1}}{\mathbf{7}}\mathbf{S} \end{bmatrix}$$

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{\mathbf{C}}{\mathbf{A}} & \dfrac{-\Delta_T}{\mathbf{A}} \\ \dfrac{1}{\mathbf{A}} & \dfrac{\mathbf{B}}{\mathbf{A}} \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{1}}{\mathbf{3}}\mathbf{S} & \dfrac{\mathbf{-1}}{\mathbf{3}} \\ \dfrac{\mathbf{1}}{\mathbf{3}} & \dfrac{\mathbf{20}}{\mathbf{3}}\mathbf{\Omega} \end{bmatrix}$$

$$[\mathbf{t}] = \begin{bmatrix} \dfrac{\mathbf{D}}{\Delta_T} & \dfrac{\mathbf{B}}{\Delta_T} \\ \dfrac{\mathbf{C}}{\Delta_T} & \dfrac{\mathbf{A}}{\Delta_T} \end{bmatrix} = \begin{bmatrix} \mathbf{7} & \mathbf{20\,\Omega} \\ \mathbf{1\,S} & \mathbf{3} \end{bmatrix}$$

Solution 19.58

Design a problem to help other students to better understand how to develop the y parameters and transmission parameters, given equations in terms of the hybrid parameters.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A two-port is described by

$$\mathbf{V}_1 = \mathbf{I}_1 + 2\mathbf{V}_2, \quad \mathbf{I}_2 = -2\mathbf{I}_1 + 0.4\mathbf{V}_2$$

Find: (a) the *y* parameters, (b) the transmission parameters.

Solution

The given set of equations is for the h parameters.

$$[\mathbf{h}] = \begin{bmatrix} 1\,\Omega & 2 \\ -2 & 0.4\text{ S} \end{bmatrix} \qquad \Delta_h = (1)(0.4) - (2)(-2) = 4.4$$

(a) $$[\mathbf{y}] = \begin{bmatrix} \dfrac{1}{\mathbf{h}_{11}} & \dfrac{-\mathbf{h}_{12}}{\mathbf{h}_{11}} \\ \dfrac{\mathbf{h}_{21}}{\mathbf{h}_{11}} & \dfrac{\Delta_h}{\mathbf{h}_{11}} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{-2} & \mathbf{4.4} \end{bmatrix}\mathbf{S}$$

(b) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{-\Delta_h}{\mathbf{h}_{21}} & \dfrac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \dfrac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \dfrac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \begin{bmatrix} \mathbf{2.2} & \mathbf{0.5\,\Omega} \\ \mathbf{0.2\text{ S}} & \mathbf{0.5} \end{bmatrix}$$

Solution 19.59

$$\Delta_g = (0.06)(2) - (-0.4)(0.2) = 0.12 + 0.08 = 0.2$$

(a) $$[\mathbf{z}] = \begin{bmatrix} \dfrac{1}{\mathbf{g}_{11}} & \dfrac{-\mathbf{g}_{12}}{\mathbf{g}_{11}} \\ \dfrac{\mathbf{g}_{21}}{\mathbf{g}_{11}} & \dfrac{\Delta_g}{\mathbf{g}_{11}} \end{bmatrix} = \begin{bmatrix} \mathbf{16.667} & \mathbf{6.667} \\ \mathbf{3.333} & \mathbf{3.333} \end{bmatrix} \mathbf{\Omega}$$

(b) $$[\mathbf{y}] = \begin{bmatrix} \dfrac{\Delta_g}{\mathbf{g}_{22}} & \dfrac{\mathbf{g}_{12}}{\mathbf{g}_{22}} \\ \dfrac{-\mathbf{g}_{21}}{\mathbf{g}_{22}} & \dfrac{1}{\mathbf{g}_{22}} \end{bmatrix} = \begin{bmatrix} \mathbf{0.1} & \mathbf{-0.2} \\ \mathbf{-0.1} & \mathbf{0.5} \end{bmatrix} \mathbf{S}$$

(c) $$[\mathbf{h}] = \begin{bmatrix} \dfrac{\mathbf{g}_{22}}{\Delta_g} & \dfrac{-\mathbf{g}_{12}}{\Delta_g} \\ \dfrac{-\mathbf{g}_{21}}{\Delta_g} & \dfrac{\mathbf{g}_{11}}{\Delta_g} \end{bmatrix} = \begin{bmatrix} \mathbf{10\,\Omega} & \mathbf{2} \\ \mathbf{-1} & \mathbf{0.3\ S} \end{bmatrix}$$

(d) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{1}{\mathbf{g}_{21}} & \dfrac{\mathbf{g}_{22}}{\mathbf{g}_{21}} \\ \dfrac{\mathbf{g}_{11}}{\mathbf{g}_{21}} & \dfrac{\Delta_g}{\mathbf{g}_{21}} \end{bmatrix} = \begin{bmatrix} \mathbf{5} & \mathbf{10\,\Omega} \\ \mathbf{0.3\ S} & \mathbf{1} \end{bmatrix}$$

Problem 19.60

Comparing this with Fig. 19.5,

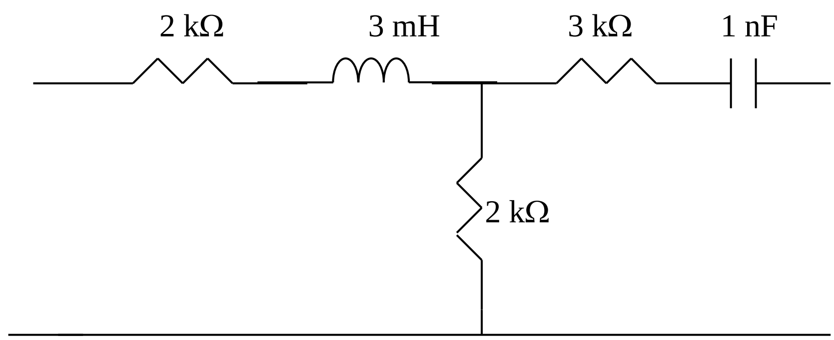
$$z_{11} - z_{12} = 4 + j3 - 2 = 2 + j3k\Omega$$

$z_{22} - z_{12} = 5 - j - 2 = 3 - j$ kΩ

$$X_L = 3x10^3 = \omega L \quad \longrightarrow \quad L = \frac{3x10^3}{10^6} = 3mH$$

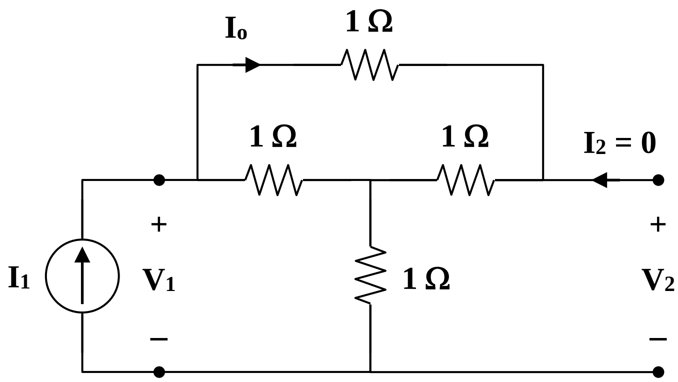
$X_C = 1x10^3 = 1/(\omega C)$ or $C = 1/(10^3 x 10^6) = 1$ nF

Hence, the resulting T network is shown below.



Solution 19.61

(a) To obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



(a)

(a)

$$\mathbf{V}_1 = \mathbf{I}_1[1+1\,||\,(1+1)] = \mathbf{I}_1\left(1+\frac{2}{3}\right) = \frac{5}{3}\mathbf{I}_1$$

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{5}{3}$$

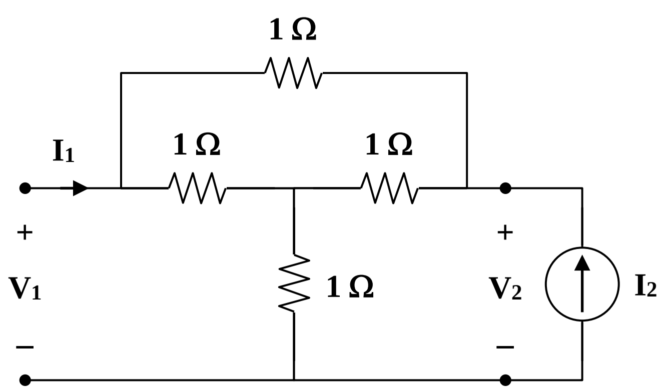
$$\mathbf{I}_o = \frac{1}{1+2}\mathbf{I}_1 = \frac{1}{3}\mathbf{I}_1$$

$$-\mathbf{V}_2 + \mathbf{I}_o + \mathbf{I}_1 = 0$$

$$\mathbf{V}_2 = \frac{1}{3}\mathbf{I}_1 + \mathbf{I}_1 = \frac{4}{3}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{4}{3}$$

To obtain $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

(a)

Due to symmetry, this is similar to the circuit in Fig. (a).

$$\mathbf{z}_{22} = \mathbf{z}_{11} = \frac{5}{3}, \qquad \mathbf{z}_{21} = \mathbf{z}_{12} = \frac{4}{3}$$

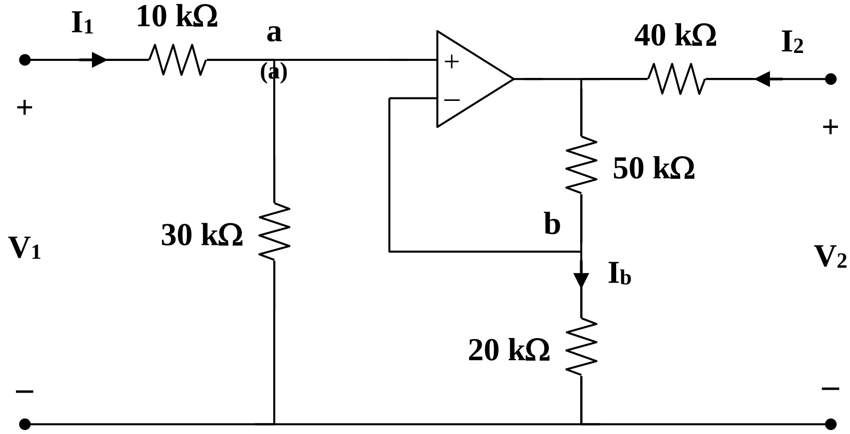
$$[\mathbf{z}] = \begin{bmatrix} \mathbf{\frac{5}{3}} & \mathbf{\frac{4}{3}} \\ \mathbf{\frac{4}{3}} & \mathbf{\frac{5}{3}} \end{bmatrix} \mathbf{\Omega}$$

(b) $$\mathbf{[h]} = \begin{bmatrix} \dfrac{\Delta_z}{\mathbf{z}_{22}} & \dfrac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \dfrac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \dfrac{\mathbf{1}}{\mathbf{z}_{22}} \end{bmatrix} = \begin{bmatrix} \mathbf{\frac{3}{5}\Omega} & \mathbf{\frac{4}{5}} \\ \mathbf{\frac{-4}{5}} & \mathbf{\frac{3}{5}S} \end{bmatrix}$$

(c) $$\mathbf{[T]} = \begin{bmatrix} \dfrac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \dfrac{\Delta_z}{\mathbf{z}_{21}} \\ \dfrac{\mathbf{1}}{\mathbf{z}_{21}} & \dfrac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \begin{bmatrix} \mathbf{\frac{5}{4}} & \mathbf{\frac{3}{4}\Omega} \\ \mathbf{\frac{3}{4}S} & \mathbf{\frac{5}{4}} \end{bmatrix}$$

Solution 19.62

Consider the circuit shown below.



Since no current enters the input terminals of the op amp,

$$\mathbf{V}_1 = (10+30)\times 10^3\,\mathbf{I}_1 \qquad (1)$$

But $\quad \mathbf{V}_a = \mathbf{V}_b = \dfrac{30}{40}\mathbf{V}_1 = \dfrac{3}{4}\mathbf{V}_1$

$$\mathbf{I}_b = \frac{\mathbf{V}_b}{20\times 10^3} = \frac{3}{80\times 10^3}\mathbf{V}_1$$

which is the same current that flows through the 50-kΩ resistor.

Thus, $\quad \mathbf{V}_2 = 40\times 10^3\,\mathbf{I}_2 + (50+20)\times 10^3\,\mathbf{I}_b$

$$\mathbf{V}_2 = 40\times 10^3\,\mathbf{I}_2 + 70\times 10^3 \cdot \frac{3}{80\times 10^3}\mathbf{V}_1$$

$$\mathbf{V}_2 = \frac{21}{8}\mathbf{V}_1 + 40\times 10^3\,\mathbf{I}_2$$

$$\mathbf{V}_2 = 105\times 10^3\,\mathbf{I}_1 + 40\times 10^3\,\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

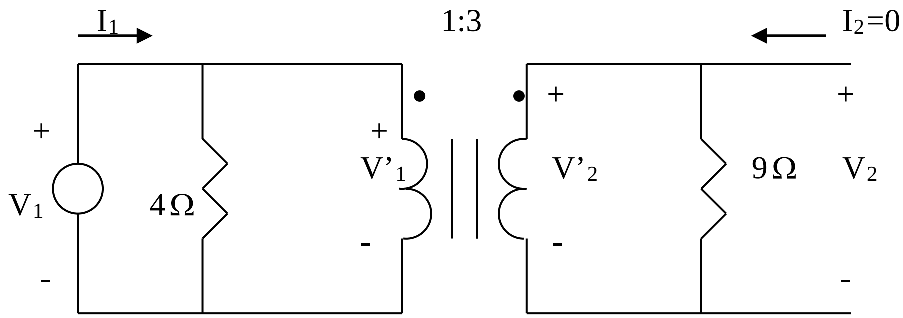
$$[\mathbf{z}] = \begin{bmatrix} \mathbf{40} & \mathbf{0} \\ \mathbf{105} & \mathbf{40} \end{bmatrix}\mathbf{k\Omega}$$

$$\boldsymbol{\Delta_z = z_{11}\,z_{22} - z_{12}\,z_{21} = 16\times 10^8}$$

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \dfrac{\Delta_z}{\mathbf{z}_{21}} \\ \dfrac{1}{\mathbf{z}_{21}} & \dfrac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \begin{bmatrix} \mathbf{0.381} & \mathbf{15.24\ k\Omega} \\ \mathbf{9.52\ \mu S} & \mathbf{0.381} \end{bmatrix}$$

Solution 19.63

To get z_{11} and z_{21}, consider the circuit below.

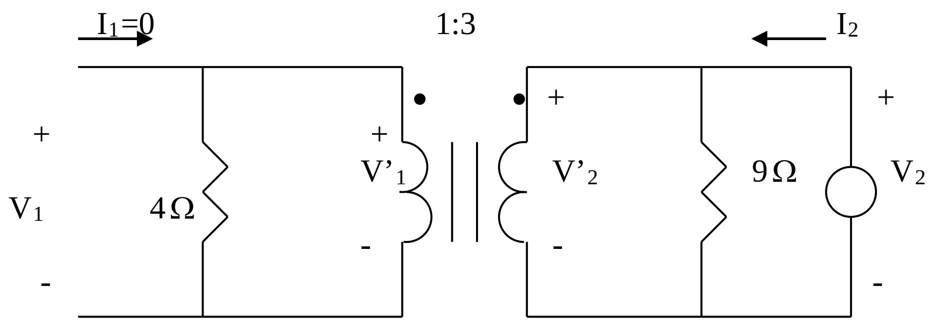


$$Z_R = \frac{9}{n^2} = 1, \qquad n = 3$$

$$V_1 = (4 // Z_R)I_1 = \frac{4}{5}I_1 \longrightarrow z_{11} = \frac{V_1}{I_1} = 0.8$$

$$V_2 = V_2' = nV_1' = nV_1 = 3(4/5)I_1 \longrightarrow z_{21} = \frac{V_2}{I_1} = 2.4$$

To get z_{21} and z_{22}, consider the circuit below.



$$Z_R' = n^2(4) = 36, \qquad n = 3$$

$$V_2 = (9 // Z_R')I_2 = \frac{9x36}{45}I_2 \longrightarrow z_{22} = \frac{V_2}{I_2} = 7.2$$

$$V_1 = \frac{V_2}{n} = \frac{V_2}{3} = 2.4I_2 \quad \longrightarrow \quad z_{21} = \frac{V_1}{I_2} = 2.4$$

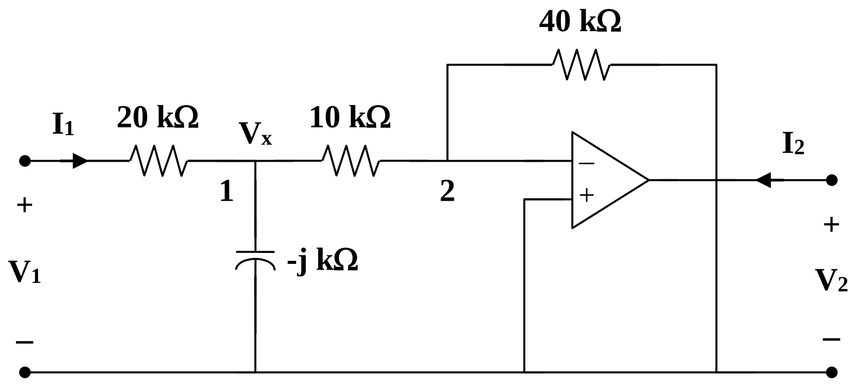
Thus,

$$\mathbf{[z]} = \underline{\begin{bmatrix} \mathbf{0.8} & \mathbf{2.4} \\ \mathbf{2.4} & \mathbf{7.2} \end{bmatrix} \boldsymbol{\Omega}}$$

Solution 19.64

$$1\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{-j}{(10^3)(10^{-6})} = -j\ k\Omega$$

Consider the op amp circuit below.



At node 1,

$$\frac{\mathbf{V}_1 - \mathbf{V}_x}{20} = \frac{\mathbf{V}_x}{-j} + \frac{\mathbf{V}_x - 0}{10}$$

$$\mathbf{V}_1 = (3 + j20)\,\mathbf{V}_x \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_x - 0}{10} = \frac{0 - \mathbf{V}_2}{40} \longrightarrow \mathbf{V}_x = \frac{-1}{4}\mathbf{V}_2 \qquad (2)$$

But
$$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_x}{20\times 10^3} \qquad (3)$$

Substituting (2) into (3) gives

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 + 0.25\,\mathbf{V}_2}{20\times 10^3} = 50\times 10^{-6}\,\mathbf{V}_1 + 12.5\times 10^{-6}\,\mathbf{V}_2 \qquad (4)$$

Substituting (2) into (1) yields

$$\mathbf{V}_1 = \frac{-1}{4}(3 + j20)\,\mathbf{V}_2$$

or
$$0 = \mathbf{V}_1 + (0.75 + j5)\,\mathbf{V}_2 \qquad (5)$$

Comparing (4) and (5) with the following equations

$$\mathbf{I}_1 = \mathbf{y}_{11}\,\mathbf{V}_1 + \mathbf{y}_{12}\,\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2$$

indicates that $\mathbf{I}_2 = 0$ and that

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{50 \times 10^{-6}} & \mathbf{12.5 \times 10^{-6}} \\ \mathbf{1} & \mathbf{0.75 + j5} \end{bmatrix} \mathbf{S}$$

$$\mathbf{\Delta_y = (77.5 + j25. - 12.5) \times 10^{-6} = (65 + j250) \times 10^{-6}}$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{1}{\mathbf{y}_{11}} & \dfrac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \dfrac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \dfrac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \begin{bmatrix} \mathbf{2 \times 10^4\ \Omega} & \mathbf{-0.25} \\ \mathbf{2 \times 10^4} & \mathbf{1.3 + j5\ S} \end{bmatrix}$$

Solution 19.65

The network consists of two two-ports in series. It is better to work with z parameters and then convert to y parameters. It is obvious that the upper 1 Ω resistor is shorted out by the top circuit so we are essentially left with 2 Ω connected to 3 Ω. This then produces the Z parameters

$$[\mathbf{z}] = \begin{bmatrix} 5\Omega & 3\Omega \\ 3\Omega & 3\Omega \end{bmatrix}$$

$$\Delta_z = 15 - 9 = 6$$

$$[\mathbf{y}] = \begin{bmatrix} \dfrac{\mathbf{z}_{22}}{\Delta_z} & \dfrac{-\mathbf{z}_{12}}{\Delta_z} \\ \dfrac{-\mathbf{z}_{21}}{\Delta_z} & \dfrac{\mathbf{z}_{11}}{\Delta_z} \end{bmatrix} = \begin{bmatrix} \mathbf{0.5} & \mathbf{-0.5} \\ \mathbf{-0.5} & \dfrac{\mathbf{5}}{\mathbf{6}} \end{bmatrix} \mathbf{S}$$

Solution 19.66

Since we have two two-ports in series, it is better to convert the given y parameters to z parameters.

$$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21} = (2\times 10^{-3})(10\times 10^{-3}) - 0 = 20\times 10^{-6}$$

$$[\mathbf{z}_a] = \begin{bmatrix} \dfrac{\mathbf{y}_{22}}{\Delta_y} & \dfrac{-\mathbf{y}_{12}}{\Delta_y} \\ \dfrac{-\mathbf{y}_{21}}{\Delta_y} & \dfrac{\mathbf{y}_{11}}{\Delta_y} \end{bmatrix} = \begin{bmatrix} 500\,\Omega & 0 \\ 0 & 100\,\Omega \end{bmatrix}$$

$$[\mathbf{z}] = \begin{bmatrix} 500 & 0 \\ 0 & 100 \end{bmatrix} + \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix} = \begin{bmatrix} 600 & 100 \\ 100 & 200 \end{bmatrix}$$

i.e.
$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

or
$$\mathbf{V}_1 = 600\mathbf{I}_1 + 100\mathbf{I}_2 \qquad (1)$$
$$\mathbf{V}_2 = 100\mathbf{I}_1 + 200\mathbf{I}_2 \qquad (2)$$

But, at the input port,
$$\mathbf{V}_s = \mathbf{V}_1 + 60\mathbf{I}_1 \qquad (3)$$

and at the output port,
$$\mathbf{V}_2 = \mathbf{V}_o = -300\mathbf{I}_2 \qquad (4)$$

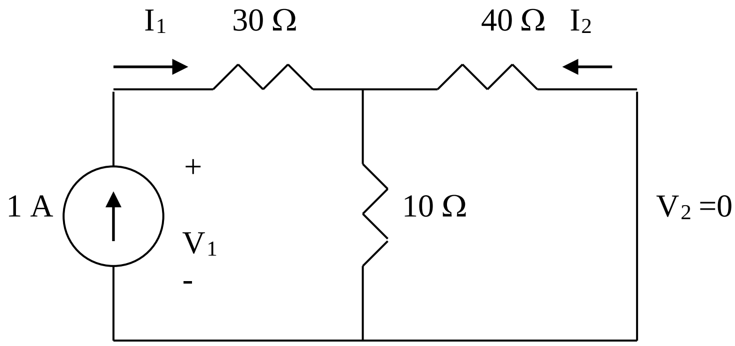
From (2) and (4),
$$100\mathbf{I}_1 + 200\mathbf{I}_2 = -300\mathbf{I}_2$$
$$\mathbf{I}_1 = -5\mathbf{I}_2 \qquad (5)$$

Substituting (1) and (5) into (3),
$$\mathbf{V}_s = 600\mathbf{I}_1 + 100\mathbf{I}_2 + 60\mathbf{I}_1$$
$$= (660)(-5)\mathbf{I}_2 + 100\mathbf{I}_2$$
$$= -3200\mathbf{I}_2 \qquad (6)$$

From (4) and (6),
$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{-300\mathbf{I}_2}{-3200\mathbf{I}_2} = \mathbf{0.09375}$$

Solution 19.67

We first the y parameters, To find y_{11} and y_{21} consider the circuit below.

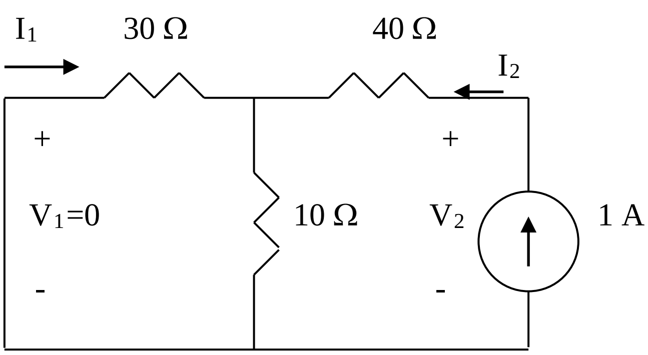


$$V_1 = I_1(30 + 10 // 40) = 38I_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{1}{38}$$

By current division,

$$I_2 = \frac{-10}{50}I_1 = -0.2I_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = \frac{-0.2I_1}{38I_1} = \frac{-1}{190}$$

To find y_{22} and y_{12} consider the circuit below.



$$V_2 = (40 + 10 // 30)I_2 = 47.5I_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = \frac{2}{93}$$ y_{22} = 2/95

By current division,

$$I_1 = -\frac{10}{30+10}I_2 = -\frac{I_2}{4} \longrightarrow y_{12} = \frac{I_1}{V_2} = \frac{-\frac{1}{4}I_2}{47.5I_2} = -\frac{1}{190}$$

$$[y] = \begin{bmatrix} 1/38 & -1/190 \\ -1/190 & 2/95 \end{bmatrix}$$

For three copies cascaded in parallel, we can use MATLAB.

```
>> Y=[1/38,-1/190;-1/190,2/95]
Y =
    0.0263   -0.0053
   -0.0053    0.0211
>> Y3=3*Y
Y3 =
    0.0789   -0.0158
   -0.0158    0.0632
>> DY=0.0789*0.0632-0.0158*0.158
DY =
    0.0025
>> T=[0.0632/0.0158,1/0.0158;DY/0.0158,0.0789/0.0158]
T =
    4.0000   63.2911
    0.1576    4.9937
```

$$T = \underline{\underline{\begin{bmatrix} \mathbf{4} & \mathbf{63.29\,\Omega} \\ \mathbf{0.1576\,S} & \mathbf{4.994} \end{bmatrix}}}$$

Solution 19.68

For the upper network N_a, $[\mathbf{y}_a] = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$

and for the lower network N_b, $[\mathbf{y}_b] = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

For the overall network,

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}$$

$$\Delta_y = 36 - 9 = 27$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{1}{\mathbf{y}_{11}} & \dfrac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \dfrac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \dfrac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \begin{bmatrix} \mathbf{\dfrac{1}{6}\,\Omega} & \mathbf{\dfrac{1}{2}} \\ \mathbf{\dfrac{1}{2}} & \mathbf{\dfrac{9}{2}\,S} \end{bmatrix}$$

Solution 19.69

We first determine the y parameters for the upper network N_a.
To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig. (a).

$$n = \frac{1}{2}, \qquad \mathbf{Z}_R = \frac{1/s}{n^2} = \frac{4}{s}$$

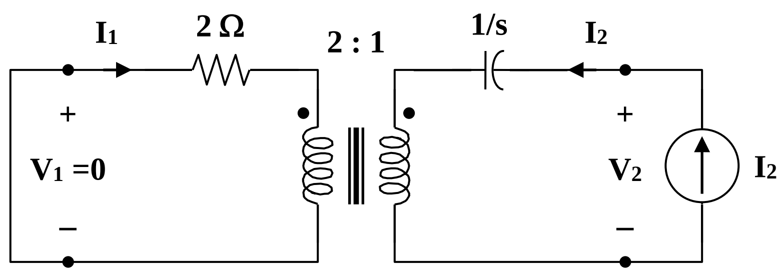
$$\mathbf{V}_1 = (2 + \mathbf{Z}_R)\mathbf{I}_1 = \left(2 + \frac{4}{s}\right)\mathbf{I}_1 = \left(\frac{2s+4}{s}\right)\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{s}{2(s+2)}$$

$$\mathbf{I}_2 = \frac{-\mathbf{I}_1}{n} = -2\mathbf{I}_1 = \frac{-s\mathbf{V}_1}{s+2}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-s}{s+2}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig. (b).



(b)

(a)

$$\mathbf{Z}_R{}' = (n^2)(2) = \left(\frac{1}{4}\right)(2) = \frac{1}{2}$$

$$\mathbf{V}_2 = \left(\frac{1}{s} + \mathbf{Z}_R{}'\right)\mathbf{I}_2 = \left(\frac{1}{s} + \frac{1}{2}\right)\mathbf{I}_2 = \left(\frac{s+2}{2s}\right)\mathbf{I}_2$$

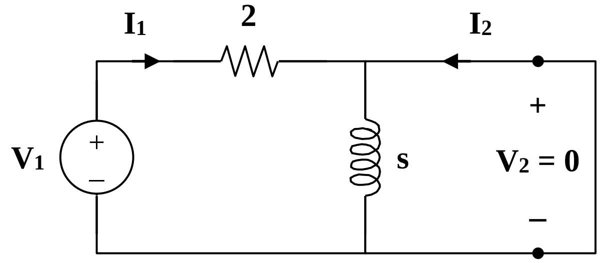
$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{2s}{s+2}$$

$$\mathbf{I}_1 = -n\mathbf{I}_2 = \left(\frac{-1}{2}\right)\left(\frac{2s}{s+2}\right)\mathbf{V}_2 = \left(\frac{-s}{s+2}\right)\mathbf{V}_2$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-s}{s+2}$$

$$[\mathbf{y}_a] = \begin{bmatrix} \frac{s}{2(s+2)} & \frac{-s}{s+2} \\ \frac{-s}{s+2} & \frac{2s}{s+2} \end{bmatrix}$$

For the lower network N_b, we obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by referring to the network in Fig. (c).



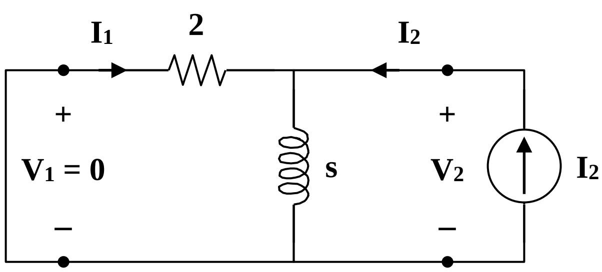
(c)

(a)

$$\mathbf{V}_1 = 2\mathbf{I}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{2}$$

$$\mathbf{I}_2 = -\mathbf{I}_1 = \frac{-\mathbf{V}_1}{2} \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{2}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig. (d).



(d)

(a)

$$\mathbf{V}_2 = (s \,\|\, 2)\mathbf{I}_2 = \frac{2s}{s+2}\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{s+2}{2s}$$

$$\mathbf{I}_1 = -\mathbf{I}_2 \cdot \frac{-s}{s+2} = \left(\frac{-s}{s+2}\right)\left(\frac{s+2}{2s}\right)\mathbf{V}_2 = \frac{-\mathbf{V}_2}{2}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{2}$$

$$[\mathbf{y}_b] = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & (s+2)/2s \end{bmatrix}$$

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} \dfrac{\mathbf{s+1}}{\mathbf{s+2}} & \dfrac{\mathbf{-(3s+2)}}{\mathbf{2(s+2)}} \\ \dfrac{\mathbf{-(3s+2)}}{\mathbf{2(s+2)}} & \dfrac{\mathbf{5s^2+4s+4}}{\mathbf{2s(s+2)}} \end{bmatrix}$$

Solution 19.70

We may obtain the g parameters from the given z parameters.

$$[\mathbf{z}_a] = \begin{bmatrix} 25 & 20 \\ 5 & 10 \end{bmatrix}, \qquad \Delta_{z_a} = 250 - 100 = 150$$

$$[\mathbf{z}_b] = \begin{bmatrix} 50 & 25 \\ 25 & 30 \end{bmatrix}, \qquad \Delta_{z_b} = 1500 - 625 = 875$$

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{1}{z_{11}} & \dfrac{-z_{12}}{z_{11}} \\ \dfrac{z_{21}}{z_{11}} & \dfrac{\Delta_z}{z_{11}} \end{bmatrix}$$

$$[\mathbf{g}_a] = \begin{bmatrix} 0.04 & -0.8 \\ 0.2 & 6 \end{bmatrix}, \qquad [\mathbf{g}_b] = \begin{bmatrix} 0.02 & -0.5 \\ 0.5 & 17.5 \end{bmatrix}$$

$$[\mathbf{g}] = [\mathbf{g}_a] + [\mathbf{g}_b] = \begin{bmatrix} \mathbf{0.06\ S} & \mathbf{-1.3} \\ \mathbf{0.7} & \mathbf{23.5\ \Omega} \end{bmatrix}$$

Solution 19.71

This is a parallel-series connection of two two-ports. We need to add their g parameters together and obtain z parameters from there.

For the transformer,

$$V_1 = \frac{1}{2}V_2, \quad I_1 = -2I_2$$

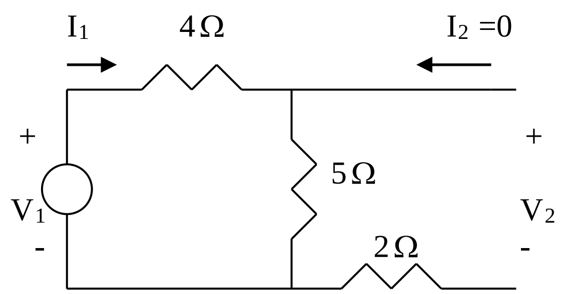
Comparing this with

$$V_1 = AV_2 - BI_2, \qquad I_1 = CV_2 - DI_2$$

shows that

$$[T_{b1}] = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$$

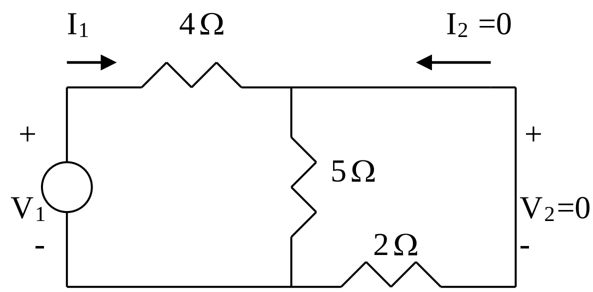
To get A and C for T_{b2}, consider the circuit below.



$$V_1 = 9I_1, \qquad V_2 = 5I_1$$

$$A = \frac{V_1}{V_2} = 9/5 = 1.8, \qquad C = \frac{I_1}{V_2} = 1/5 = 0.2$$

We obtain B and D by looking at the circuit below.



$$I_2 = -\frac{5}{7}I_1 \qquad \longrightarrow \qquad D = -\frac{I_1}{I_2} = 7/5 = 1.4$$

$$V_1 = 4I_1 - 2I_2 = 4(-\frac{7}{5}I_2) - 2I_2 = -\frac{38}{5}I_2 \qquad \longrightarrow \qquad B = -\frac{V_1}{I_2} = 7.6$$

$$[T_{b2}] = \begin{bmatrix} 1.8 & 7.6 \\ 0.2 & 1.4 \end{bmatrix}$$

$$[T] = [T_{b1}][T_{b2}] = \begin{bmatrix} 0.9 & 3.8 \\ 0.4 & 2.8 \end{bmatrix}, \qquad \Delta_T = 1$$

$$[g_b] = \begin{bmatrix} C/A & -\Delta_T/A \\ 1/A & B/A \end{bmatrix} = \begin{bmatrix} 0.4444 & -1.1111 \\ 1.1111 & 4.2222 \end{bmatrix}$$

From Prob. 19.52,

$$[T_a] = \begin{bmatrix} 1.8 & 18.8 \\ 0.1 & 1.6 \end{bmatrix}$$

$$[g_a] = \begin{bmatrix} C/A & -\Delta_T/A \\ 1/A & B/A \end{bmatrix} = \begin{bmatrix} 0.05555 & -0.5555 \\ 0.5555 & 10.4444 \end{bmatrix}$$

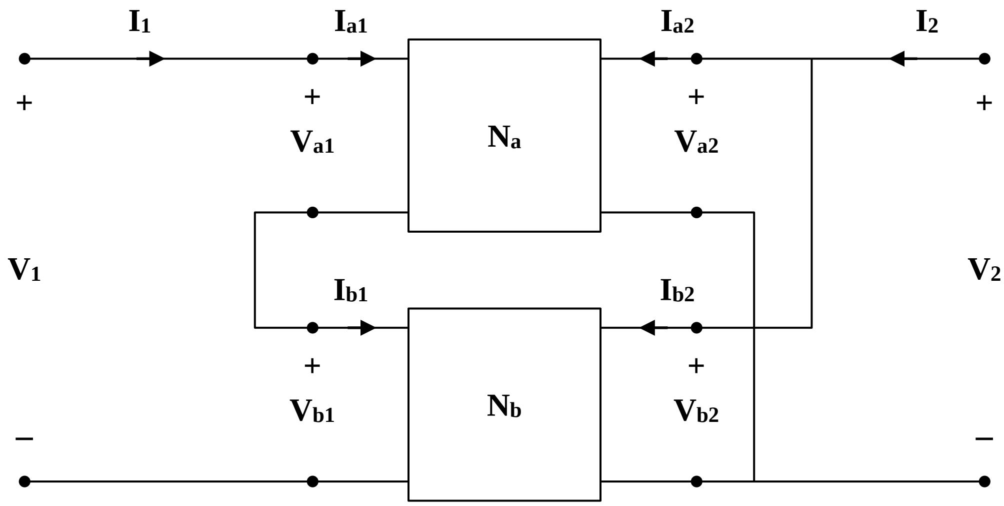
$$[g] = [g_a] + [g_b] = \begin{bmatrix} 0.4999 & -1.6667 \\ 1.6667 & 14.667 \end{bmatrix}$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} 1/g_{11} & -g_{21}/g_{11} \\ g_{21}/g_{11} & \Delta_g/g_{11} \end{bmatrix} = \underline{\begin{bmatrix} 2 & -3.334 \\ 3.334 & 20.22 \end{bmatrix}\Omega}$$

Solution 19.72

Consider the network shown below.



$$\mathbf{V}_{a1} = 25\mathbf{I}_{a1} + 4\mathbf{V}_{a2} \quad (1)$$
$$\mathbf{I}_{a2} = -4\mathbf{I}_{a1} + \mathbf{V}_{a2} \quad (2)$$
$$\mathbf{V}_{b1} = 16\mathbf{I}_{b1} + \mathbf{V}_{b2} \quad (3)$$
$$\mathbf{I}_{b2} = -\mathbf{I}_{b1} + 0.5\mathbf{V}_{b2} \quad (4)$$

$$\mathbf{V}_1 = \mathbf{V}_{a1} + \mathbf{V}_{b1}$$
$$\mathbf{V}_2 = \mathbf{V}_{a2} = \mathbf{V}_{b2}$$
$$\mathbf{I}_2 = \mathbf{I}_{a2} + \mathbf{I}_{b2}$$
$$\mathbf{I}_1 = \mathbf{I}_{a1}$$

Now, rewrite (1) to (4) in terms of $\mathbf{I}_1$ and $\mathbf{V}_2$

$$\mathbf{V}_{a1} = 25\mathbf{I}_1 + 4\mathbf{V}_2 \quad (5)$$
$$\mathbf{I}_{a2} = -4\mathbf{I}_1 + \mathbf{V}_2 \quad (6)$$
$$\mathbf{V}_{b1} = 16\mathbf{I}_{b1} + \mathbf{V}_2 \quad (7)$$
$$\mathbf{I}_{b2} = -\mathbf{I}_{b1} + 0.5\mathbf{V}_2 \quad (8)$$

Adding (5) and (7),

$$\mathbf{V}_1 = 25\mathbf{I}_1 + 16\mathbf{I}_{b1} + 5\mathbf{V}_2 \quad (9)$$

Adding (6) and (8),

$$\mathbf{I}_2 = -4\mathbf{I}_1 - \mathbf{I}_{b1} + 1.5\mathbf{V}_2 \quad (10)$$

$$\mathbf{I}_{b1} = \mathbf{I}_{a1} = \mathbf{I}_1 \quad (11)$$

Because the two networks N_a and N_b are independent,

$$\mathbf{I}_2 = -5\mathbf{I}_1 + 1.5\mathbf{V}_2$$

or $$\mathbf{V}_2 = 3.333\mathbf{I}_1 + 0.6667\mathbf{I}_2 \qquad (12)$$

Substituting (11) and (12) into (9),

$$\mathbf{V}_1 = 41\mathbf{I}_1 + \frac{25}{1.5}\mathbf{I}_1 + \frac{5}{1.5}\mathbf{I}_2$$

$$\mathbf{V}_1 = 57.67\mathbf{I}_1 + 3.333\mathbf{I}_2 \qquad (13)$$

Comparing (12) and (13) with the following equations

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

indicates that

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{57.67} & \mathbf{3.333} \\ \mathbf{3.333} & \mathbf{0.6667} \end{bmatrix}\boldsymbol{\Omega}$$

Alternatively,

$$[\mathbf{h}_a] = \begin{bmatrix} 25 & 4 \\ -4 & 1 \end{bmatrix}, \qquad [\mathbf{h}_b] = \begin{bmatrix} 16 & 1 \\ -1 & 0.5 \end{bmatrix}$$

$$[\mathbf{h}] = [\mathbf{h}_a] + [\mathbf{h}_b] = \begin{bmatrix} 41 & 5 \\ -5 & 1.5 \end{bmatrix} \qquad \Delta_h = 61.5 + 25 = 86.5$$

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{\Delta_h}{\mathbf{h}_{22}} & \dfrac{\mathbf{h}_{12}}{\mathbf{h}_{22}} \\ \dfrac{-\mathbf{h}_{21}}{\mathbf{h}_{22}} & \dfrac{1}{\mathbf{h}_{22}} \end{bmatrix} = \begin{bmatrix} \mathbf{57.67} & \mathbf{3.333} \\ \mathbf{3.333} & \mathbf{0.6667} \end{bmatrix}\boldsymbol{\Omega}$$

as obtained previously.

Solution 19.73

Three copies of the circuit shown in Fig. 19.70, are connected in cascade. Determine the z parameters.

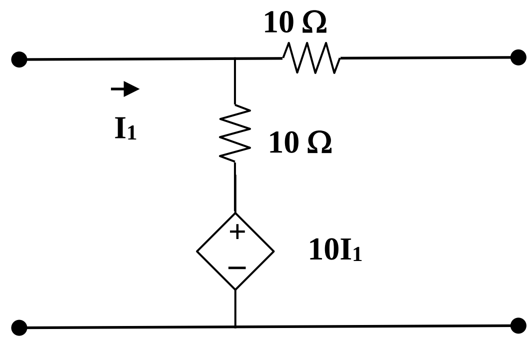
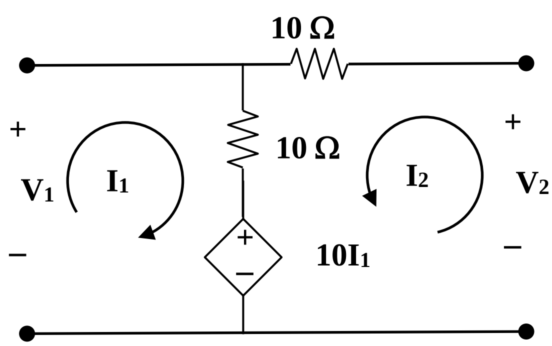


Figure 19.70
For Prob. 19.6 and 19.73.

Solution

First we need to find the z parameters of this circuit.

Step 1. First we label the circuit so that we can find the z-parameters.



Next we write the mesh equations and then let $I_1 = 1$ A and $I_2 = 0$ and then $I_1 = 0$ and $I_2 = 1$A in order to find the z-parameters.

Step 2. $V_1 = 10\text{x}1 + 10\text{x}1 = 20$ V and $V_2 = 10\text{x}1 + 10\text{x}1 = 20$ V which leads to $z_{11} = 20/1 = \mathbf{20\ \Omega}$ and $z_{21} = 20/1 = \mathbf{20\ \Omega}$. Finally, $V_1 = 10\text{x}1 = 10$ V and $V_2 = (10+10)1 = 20$ V which leads to $z_{12} = 10/1 = \mathbf{10\ \Omega}$ and $z_{22} = 20/1 = \mathbf{20\ \Omega}$.

In matrix form we have, $\mathbf{Z} = \begin{bmatrix} 20 & 10 \\ 20 & 20 \end{bmatrix} \mathbf{\Omega}$.

Next we know that for cascaded circuits the result of multiplying the T parameters together. So, we convert the z parameters to T parameters and then multiply them together three times and converting the T parameters to z parameters and we are finished. $\Delta Z = 400–200 = 200$. This leads to $A = z_{11}/z_{21} = 20/20 = 1$, $B = \Delta Z/20 = 10\ \Omega$, $C = 1/20 = 0.05$ S, and $D = 20/20 = 1$. In matrix form,

$$T = \begin{bmatrix} 1 & 10\Omega \\ 0.05\text{S} & 1 \end{bmatrix}$$

Using MATLAB we get,

```
>> T=[1,10;0.05,1]

T =

    1.0000   10.0000
    0.0500    1.0000

>> F=T*T*T

F =

    2.5000   35.0000
    0.1750    2.5000
```

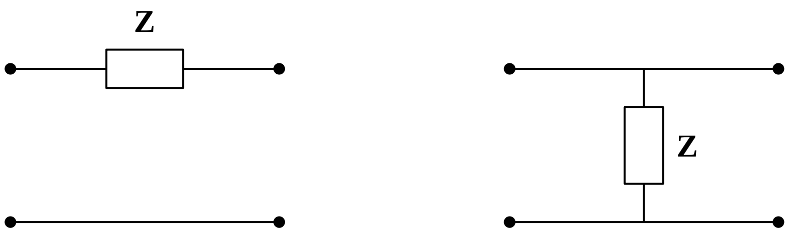
$z_{11} = A/C = 2.5/0.175 = 14.286\ \Omega$, $z_{12} = \Delta T/C = (6.25–6.125)/0.175 = 714.29\ \text{m}\Omega$,
$z_{21} = 1/C = 5.714\ \Omega$, and $z_{22} = D/C = 2.5/0.175 = 14.286\ \Omega$.
In matrix form we have,

$$Z = \begin{bmatrix} 14.286\mathbf{\Omega} & 714.29\mathbf{m\Omega} \\ 5.714\mathbf{\Omega} & 14.286\mathbf{\Omega} \end{bmatrix}.$$

Solution 19.74

From Prob. 18.35, the transmission parameters for the circuit in Figs. (a) and (b) are

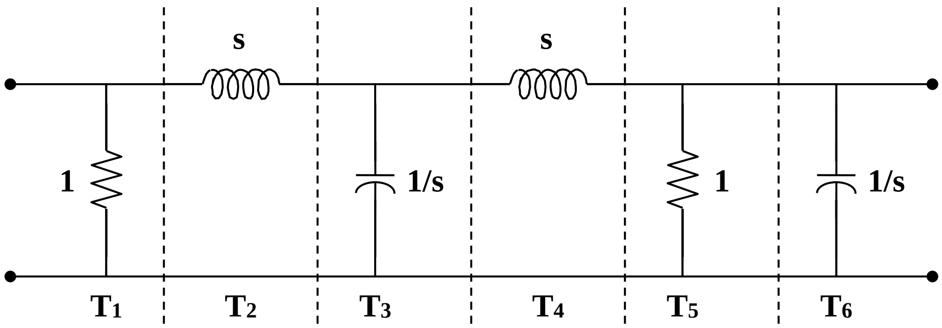
$$[\mathbf{T}_a] = \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix}, \qquad [\mathbf{T}_b] = \begin{bmatrix} 1 & 0 \\ 1/\mathbf{Z} & 1 \end{bmatrix}$$



(a) **(b)**

(a)

We partition the given circuit into six subcircuits similar to those in Figs. (a) and (b) as shown in Fig. (c) and obtain $[\mathbf{T}]$ for each.



$$[\mathbf{T}_1] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \qquad [\mathbf{T}_2] = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}, \qquad [\mathbf{T}_3] = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

$$[\mathbf{T}_4] = [\mathbf{T}_2], \qquad [\mathbf{T}_5] = [\mathbf{T}_1], \qquad [\mathbf{T}_6] = [\mathbf{T}_3]$$

$$[\mathbf{T}] = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4][\mathbf{T}_5][\mathbf{T}_6] = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4]\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

$$= [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4]\begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix} = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3]\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix}$$

$$= [\mathbf{T}_1][\mathbf{T}_2]\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} s^2+s+1 & s \\ s+1 & 1 \end{bmatrix}$$

$$= [\mathbf{T}_1]\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s^2+s+1 & s \\ s^3+s^2+2s+1 & s^2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} s^4+s^3+3s^2+2s+1 & s^3+2s \\ s^3+s^2+2s+1 & s^2+1 \end{bmatrix}$$

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{s^4+s^3+3s^2+2s+1} & \mathbf{s^3+2s} \\ \mathbf{s^4+2s^3+4s^2+4s+2} & \mathbf{s^3+s^2+2s+1} \end{bmatrix}$$

Note that $\mathbf{AB} - \mathbf{CD} = 1$ as expected.

Solution 19.75

(a) We convert $[z_a]$ and $[z_b]$ to T-parameters. For N_a, $\Delta_z = 40 - 24 = 16$.

$$[T_a] = \begin{bmatrix} z_{11}/z_{21} & \Delta_z/z_{21} \\ 1/z_{21} & z_{22}/z_{21} \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0.25 & 1.25 \end{bmatrix}$$

For N_b, $\Delta_y = 80 + 8 = 88$.

$$[T_b] = \begin{bmatrix} -y_{22}/y_{21} & -1/y_{21} \\ -\Delta_y/y_{21} & -y_{11}/y_{21} \end{bmatrix} = \begin{bmatrix} -5 & -0.5 \\ -44 & -4 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} -186 & -17 \\ -56.25 & -5.125 \end{bmatrix}$$

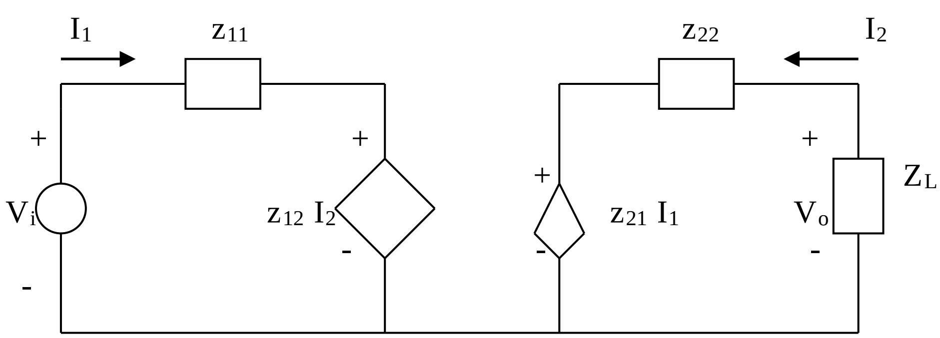
We convert this to y-parameters. $\Delta_T = AD - BC = -3.$

$$\mathbf{[y]} = \begin{bmatrix} \mathbf{D/B} & \mathbf{-\Delta_T/B} \\ \mathbf{-1/B} & \mathbf{A/B} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0.3015} & \mathbf{-0.1765} \\ \mathbf{0.0588} & \mathbf{10.94} \end{bmatrix}\mathbf{S}}$$

(b) The equivalent z-parameters are

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 3.3067 & 0.0533 \\ -0.0178 & 0.0911 \end{bmatrix}$$

Consider the equivalent circuit below.



$$V_i = z_{11}I_1 + z_{12}I_2 \qquad (1)$$

$$V_o = z_{21}I_1 + z_{22}I_2 \qquad (2)$$

But $V_o = -I_2 Z_L \quad \longrightarrow \quad I_2 = -V_o / Z_L \qquad (3)$

From (2) and (3) ,

$$V_o = z_{21}I_1 - z_{22}\frac{V_o}{Z_L} \quad \longrightarrow \quad I_1 = V_o\left(\frac{1}{z_{21}} + \frac{z_{22}}{Z_L z_{21}}\right) \qquad (4)$$

Substituting (3) and (4) into (1) gives

$$\mathbf{\frac{V_i}{V_o} = \left(\frac{z_{11}}{z_{21}} + \frac{z_{11}z_{22}}{z_{21}Z_L}\right) - \frac{z_{12}}{Z_L} = -194.3} \quad \longrightarrow \quad \underline{\mathbf{\frac{V_{o.}}{V_i} = -0.0051}}$$

Solution 19.76

To get z_{11} and z_{21}, we open circuit the output port and let $I_1 = 1A$ so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \qquad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 3.849, \qquad z_{21} = V_2 = 1.122$$

Similarly, to get z_{22} and z_{12}, we open circuit the input port and let $I_2 = 1A$ so that

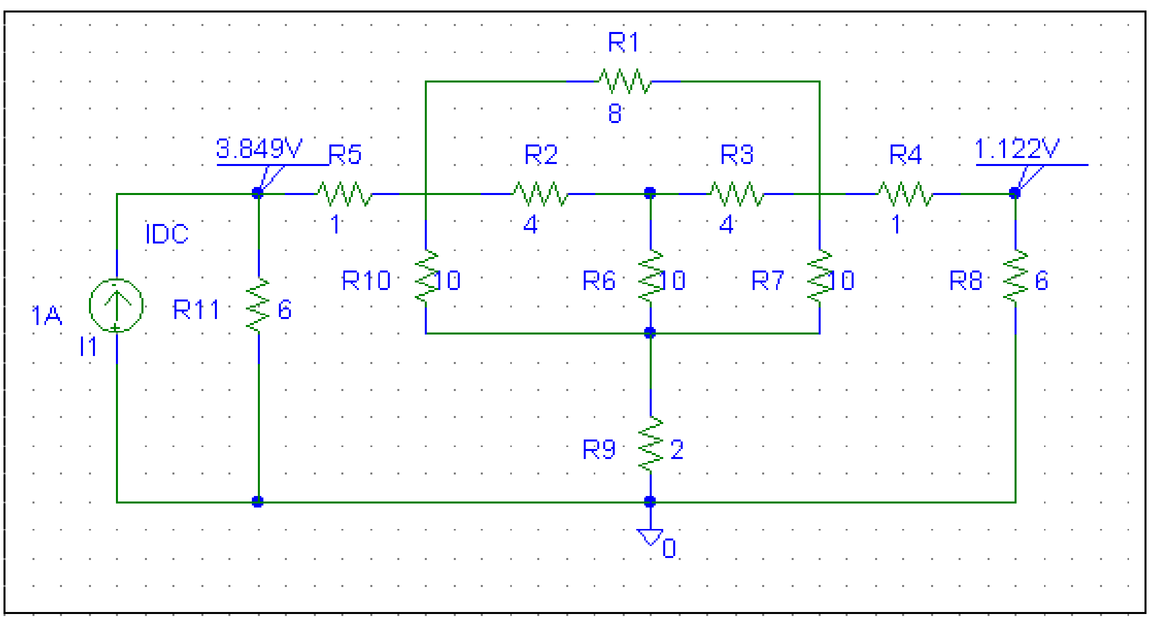
$$z_{12} = \frac{V_1}{I_2} = V_1, \qquad z_{22} = \frac{V_2}{I_2} = V_2$$

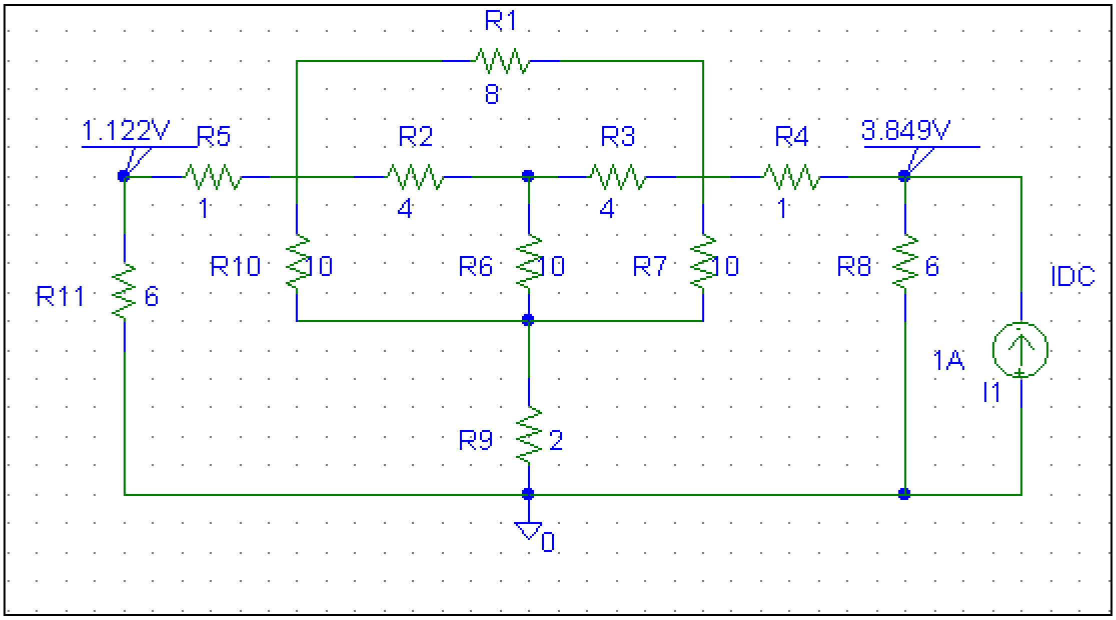
The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 1.122, \qquad z_{22} = V_2 = 3.849$$

Thus,

$$\underline{[z] = \begin{bmatrix} \mathbf{3.949} & \mathbf{1.122} \\ \mathbf{1.122} & \mathbf{3.849} \end{bmatrix}\Omega}$$




R1
8
1.122V
R5
1
R2
4
R3
4
R4
1
3.849V
R10
10
R6
10
R7
10
R8
6
R11
6
IDC
1A
I1
R9
2
0

Solution 19.77

We follow Example 19.15 except that this is an AC circuit.

(a) We set $V_2 = 0$ and $I_1 = 1$ A. The schematic is shown below. In the AC Sweep Box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

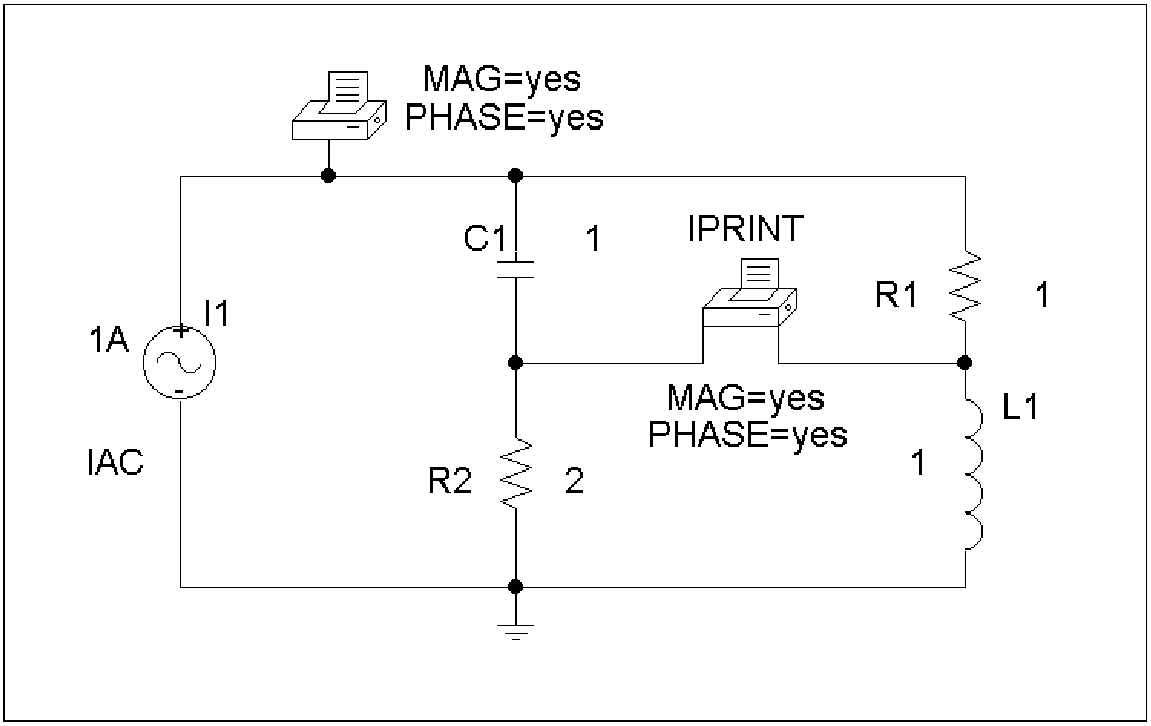
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	3.163 E–.01	–1.616 E+02

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	9.488 E–01	–1.616 E+02

From this we obtain

$$h_{11} = V_1/1 = 0.9488\angle{-161.6°}$$

$$h_{21} = I_2/1 = 0.3163\angle{-161.6°}.$$



(b) In this case, we set $I_1 = 0$ and $V_2 = 1$V. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	3.163 E–.01	1.842 E+01

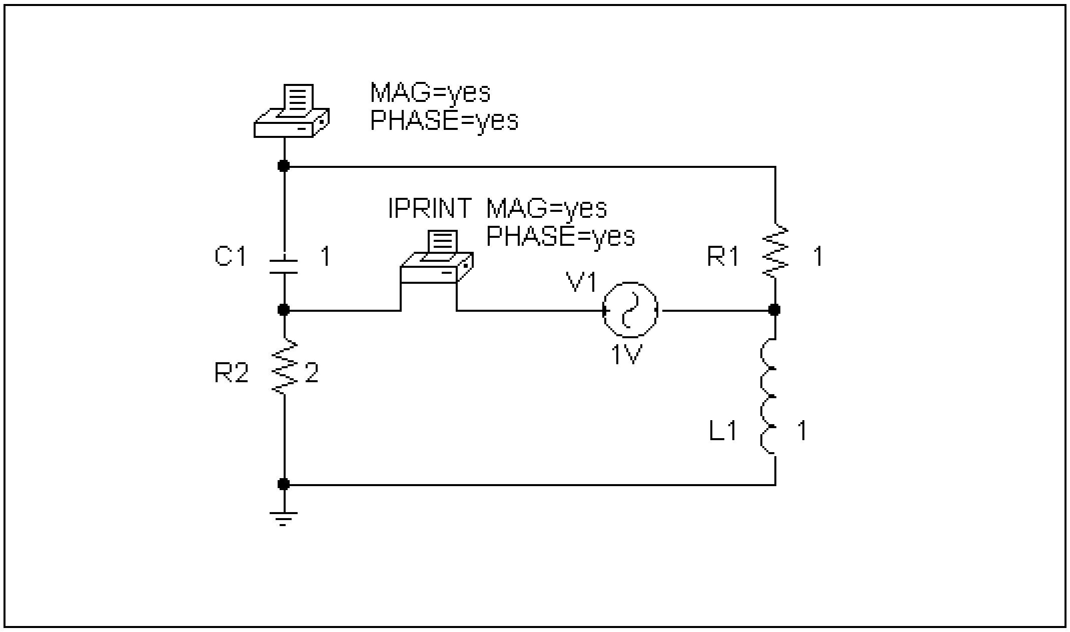
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	9.488 E–01	–1.616 E+02

From this,

$$h_{12} = V_1/1 = 0.3163\angle 18.42°$$

$$h_{21} = I_2/1 = 0.9488\angle -161.6°.$$

Thus, $$[h] = \begin{bmatrix} \mathbf{0.9488\angle -161.6°\,\Omega} & \mathbf{0.3163\angle 18.42°} \\ \mathbf{0.3163\angle -161.6°} & \mathbf{0.9488\angle -161.6°\,S} \end{bmatrix}$$



Solution 19.78

For h_{11} and h_{21}, short-circuit the output port and let I_1 = 1A. $f = \omega / 2\pi = 0.6366$. The schematic is shown below. When it is saved and run, the output file contains the following:

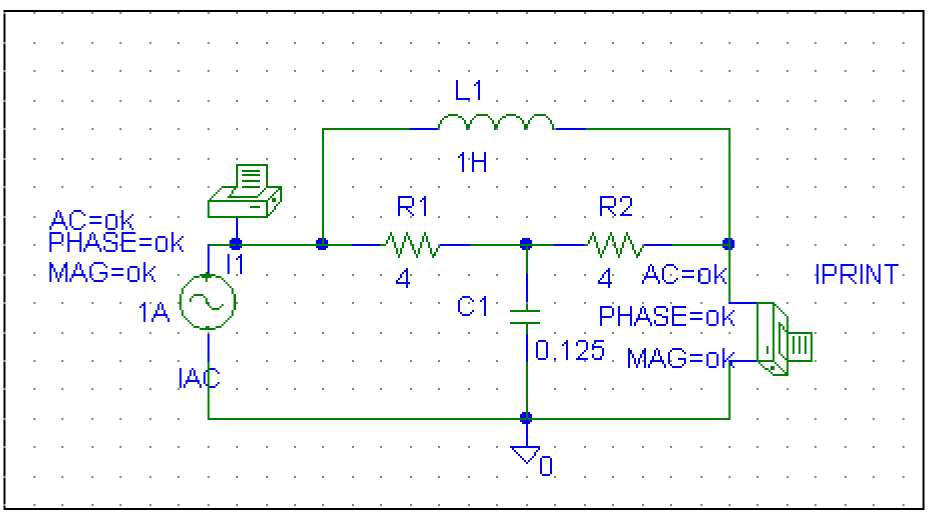
```
FREQ        IM(V_PRINT1)IP(V_PRINT1)


   6.366E-01   1.202E+00   1.463E+02


  FREQ        VM($N_0003) VP($N_0003)


   6.366E-01   3.771E+00  -1.350E+02
```

From the output file, we obtain

$$I_2 = 1.202\angle 146.3^o, \qquad V_1 = 3.771\angle -135^o$$

so that

$$h_{11} = \frac{V_1}{1} = 3.771\angle -135^o, \qquad h_{21} = \frac{I_2}{1} = 1.202\angle 146.3^o$$



For h_{12} and h_{22}, open-circuit the input port and let V_2 = 1V. The schematic is shown below. When it is saved and run, the output file includes:

```
  FREQ        VM($N_0003) VP($N_0003)
```

```
  6.366E-01   1.202E+00  -3.369E+01

 FREQ        IM(V_PRINT1)IP(V_PRINT1)


  6.366E-01   3.727E-01  -1.534E+02
```

From the output file, we obtain

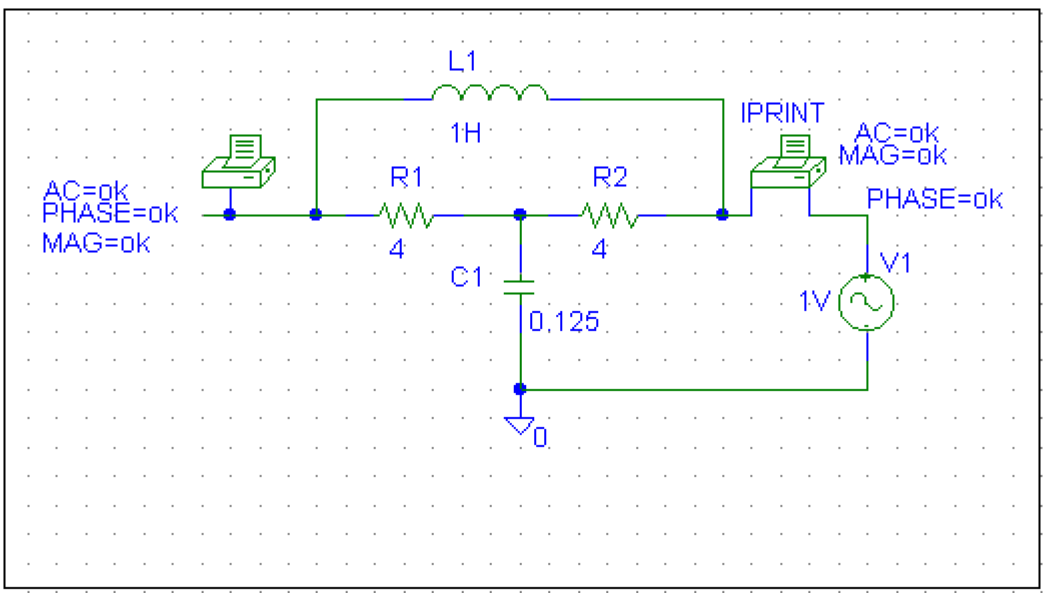
$$I_2 = 0.3727\angle -153.4^o, \qquad V_1 = 1.202\angle -33.69^o$$

so that

$$h_{12} = \frac{V_1}{1} = 1.202\angle -33.69^o, \qquad h_{22} = \frac{I_2}{1} = 0.3727\angle -153.4^o$$

Thus,

$$\underline{\mathbf{[h]} = \begin{bmatrix} \mathbf{3.771\angle -135^o\ \Omega} & \mathbf{1.202\angle -33.69^o} \\ \mathbf{1.202\angle 146.3} & \mathbf{0.3727\angle -153.4^o\ S} \end{bmatrix}}$$



Solution 19.79

We follow Example 19.16.

(a) We set $I_1 = 1$ A and open-circuit the output-port so that $I_2 = 0$. The schematic is shown below with two VPRINT1s to measure V_1 and V_2. In the AC Sweep box, we enter Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

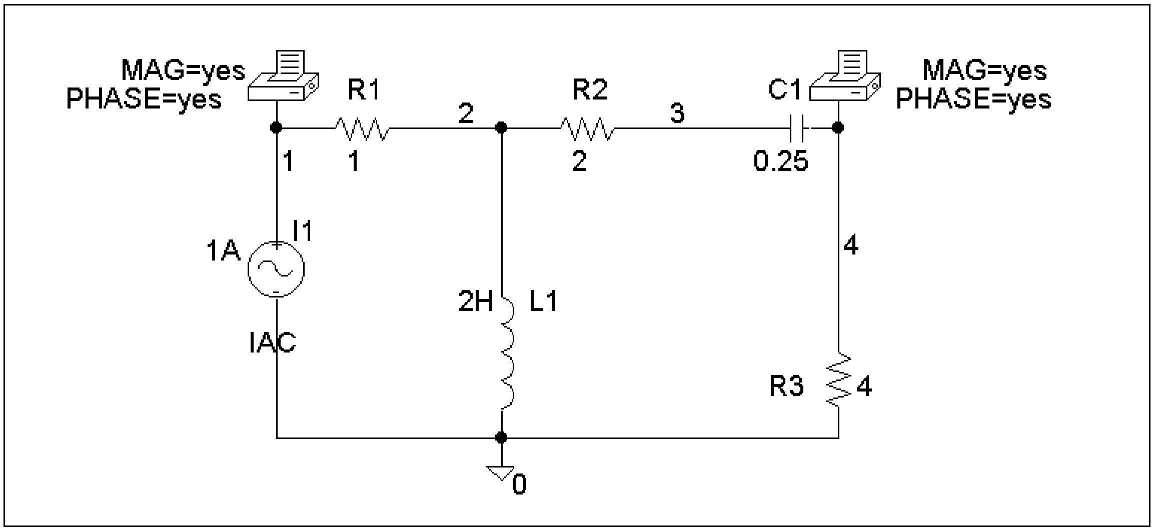
FREQ	VM(1)	VP(1)
3.183 E–01	4.669 E+00	–1.367 E+02

FREQ	VM(4)	VP(4)
3.183 E–01	2.530 E+00	–1.084 E+02

From this,

$$z_{11} = V_1/I_1 = 4.669\angle{-136.7°}/1 = 4.669\angle{-136.7°}$$

$$z_{21} = V_2/I_1 = 2.53\angle{-108.4°}/1 = 2.53\angle{-108.4°}.$$



(b) In this case, we let $I_2 = 1$ A and open-circuit the input port. The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

FREQ	VM(1)	VP(1)
3.183 E–01	2.530 E+00	–1.084 E+02

FREQ	VM(2)	VP(2)
3.183 E–01	1.789 E+00	–1.534 E+02

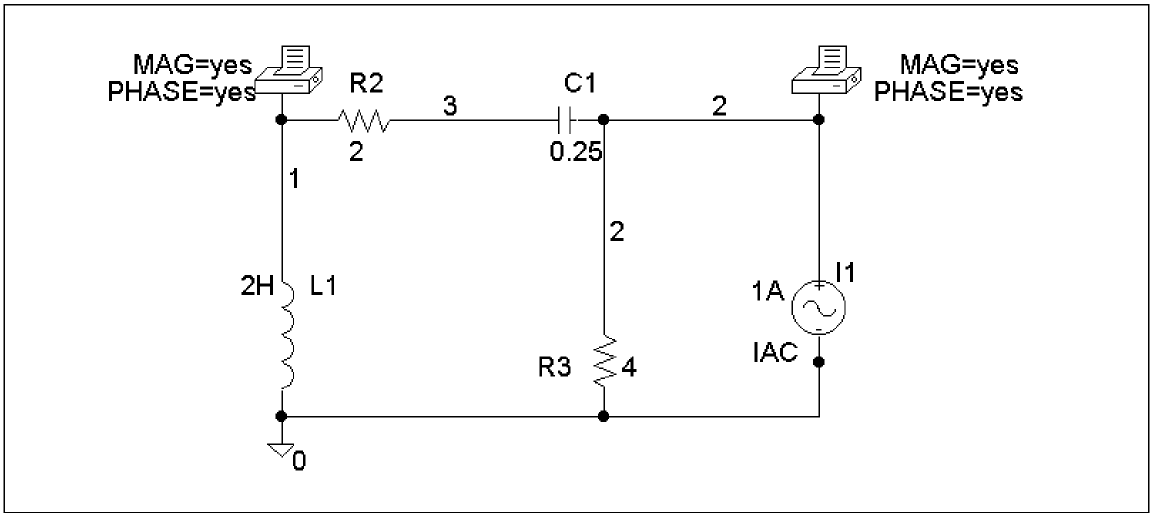
From this,

$$z_{12} = V_1/I_2 = 2.53\angle{-108.4°}/1 = 2.53\angle{-108..4°}$$

$$z_{22} = V_2/I_2 = 1.789\angle{-153.4°}/1 = 1.789\angle{-153.4°}.$$

Thus,

$$[z] = \begin{bmatrix} \mathbf{4.669\angle -136.7°} & \mathbf{2.53\angle -108.4°} \\ \mathbf{2.53\angle -108.4°} & \mathbf{1.789\angle -153.4°} \end{bmatrix}\boldsymbol{\Omega}$$



Solution 19.80

Use *PSpice* or *MultiSim* to find the *z* parameters of the circuit in Fig. 19.71. Assume ω = 1 rad/s.

Problem 19.7

Calculate the impedance-parameter equivalent of the circuit in Fig. 19.71.

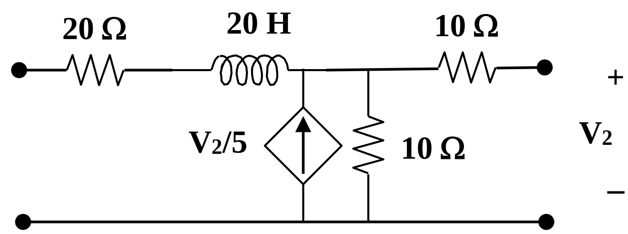


Figure 19.71
For Prob. 19.7 and 19.80.

Solution

Please note that the Multisim simulation runs better with a higher input frequency, so the frequency of the input current was scaled by 100 (15.91 Hz), and the inductance was scaled by 1/100 (0.200 H). So the impedance of the inductor is still j20 ohm.

To get z_{11} and z_{21}, we open circuit the output port and let I_1 = 1A so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \qquad z_{21} = \frac{V_2}{I_1} = V_2$$

Multisim gives us $V_1/1 = z_{11}$ = **(10+j20) Ω** = **(22.36∠63.43°) Ω** and $V_2/1 = z_{21}$ = **–10 Ω**.

Similarly, to get z_{22} and z_{12}, we open circuit the input port and let I_2 = 1A so that

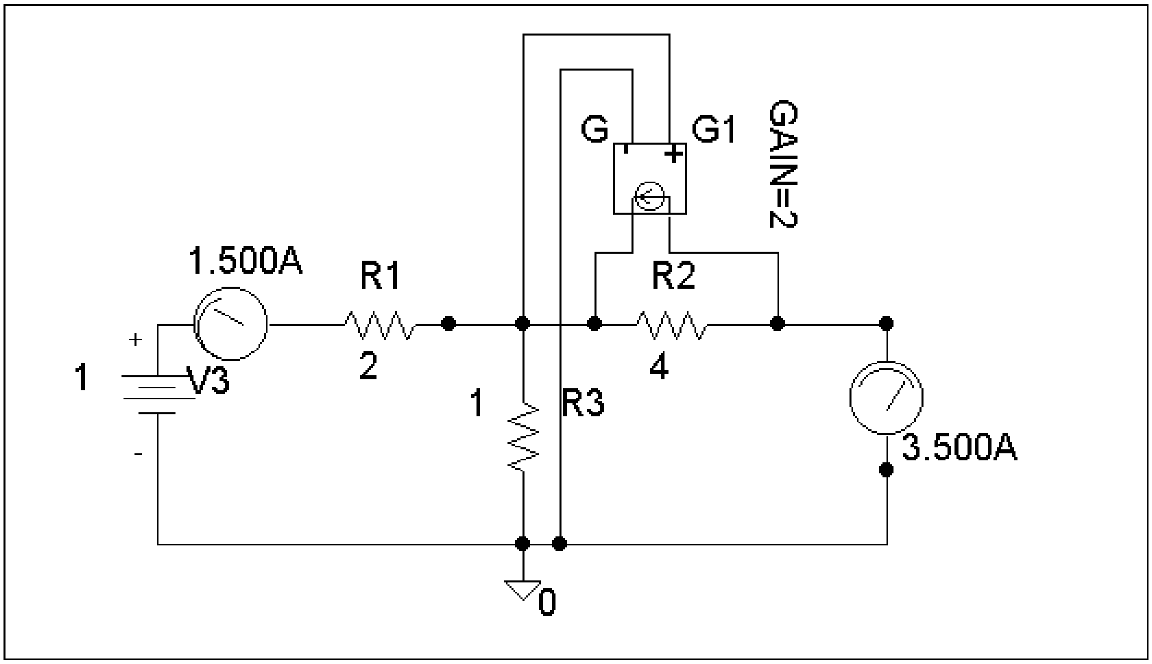
$$z_{12} = \frac{V_1}{I_2} = V_1, \qquad z_{22} = \frac{V_2}{I_2} = V_2$$

Multisim gives us $V_1/1 = z_{12}$ = **–30 Ω** and $V_2/1$ = **–20 Ω**.

Solution 19.81

(a) We set $V_1 = 1$ and short circuit the output port. The schematic is shown below. After simulation we obtain

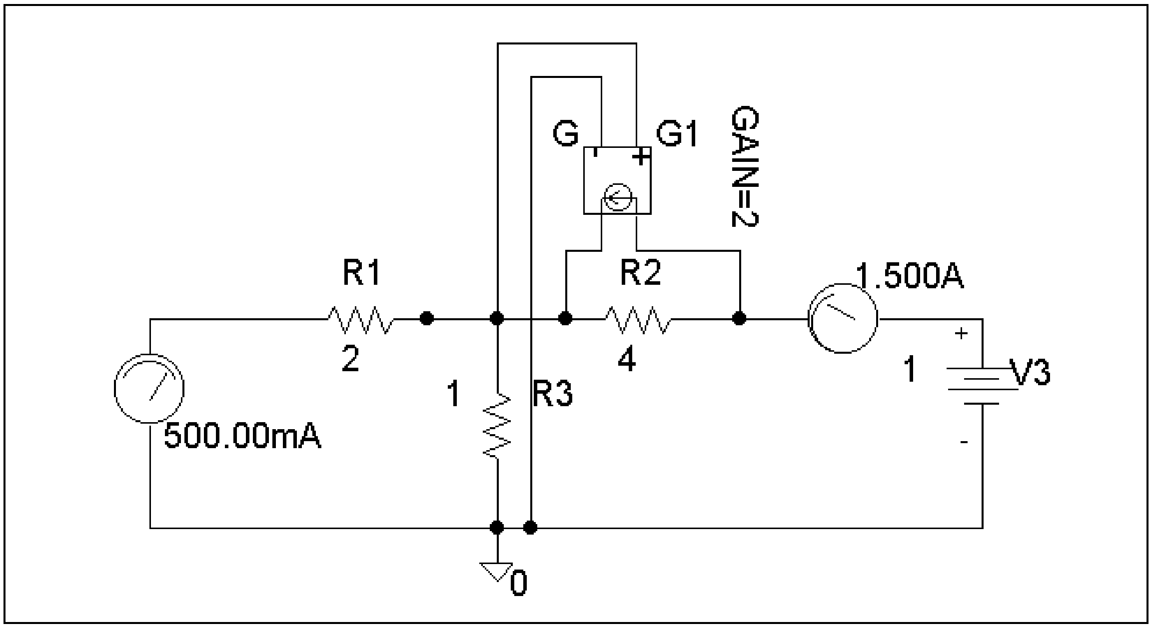
$$y_{11} = I_1 = 1.5, \quad y_{21} = I_2 = 3.5$$



(b) We set $V_2 = 1$ and short-circuit the input port. The schematic is shown below. Upon simulating the circuit, we obtain

$$y_{12} = I_1 = -0.5, \quad y_{22} = I_2 = 1.5$$

$$[Y] = \begin{bmatrix} \mathbf{1.5} & \mathbf{-0.5} \\ \mathbf{3.5} & \mathbf{1.5} \end{bmatrix} \mathbf{S}$$



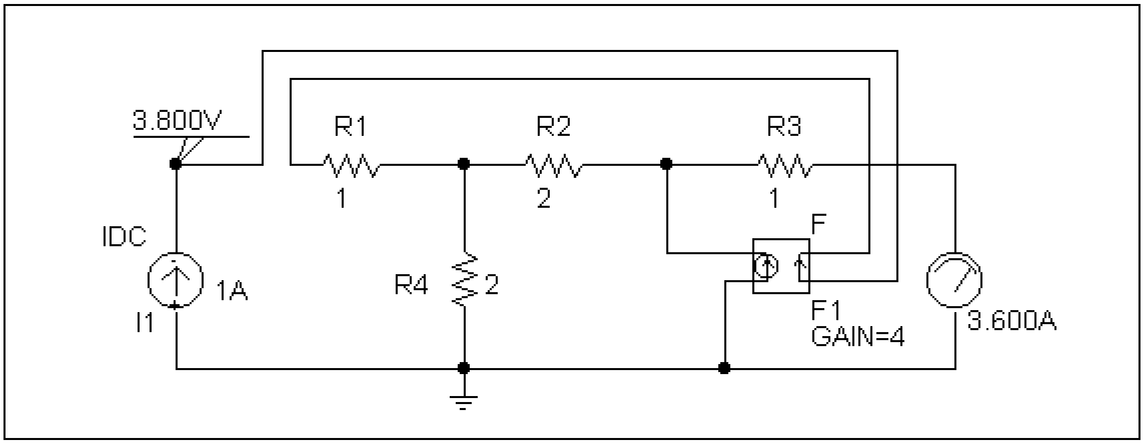
G
G1
GAIN=2
R1
R2
1.500A
2
4
1
R3
1
V3
500.00mA
0

Solution 19.82

We follow Example 19.15.

(a) Set $V_2 = 0$ and $I_1 = 1A$. The schematic is shown below. After simulation, we obtain

$$h_{11} = V_1/1 = 3.8, \; h_{21} = I_2/1 = 3.6$$

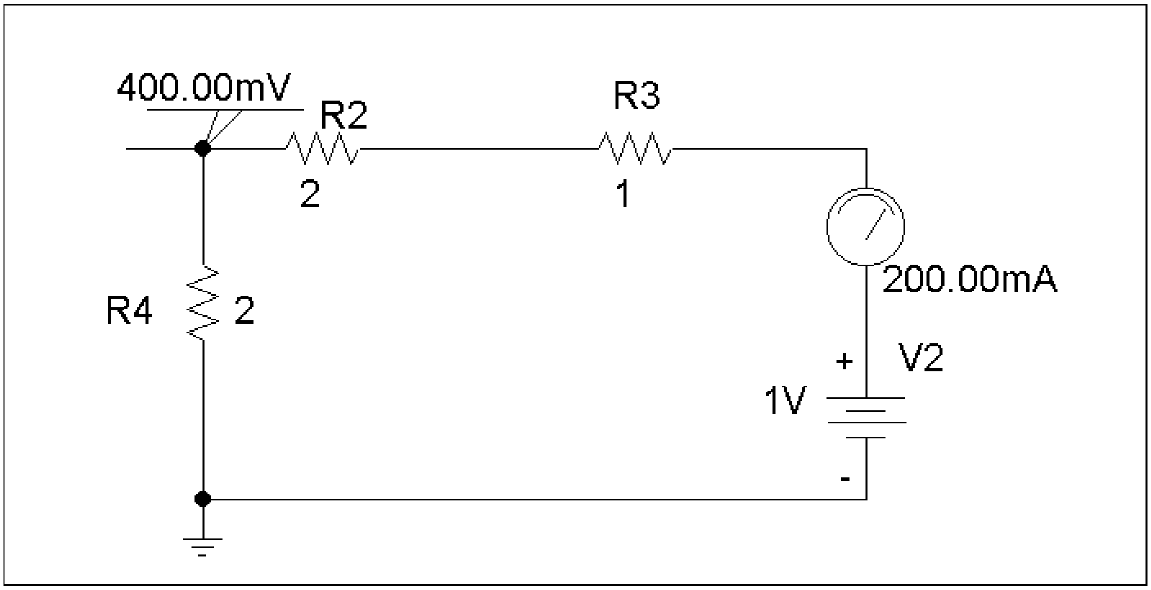


(b) Set $V_1 = 1$ V and $I_1 = 0$. The schematic is shown below. After simulation, we obtain

$$h_{12} = V_1/1 = 0.4, \; h_{22} = I_2/1 = 0.25$$

Hence,

$$[h] = \begin{bmatrix} \mathbf{3.8\Omega} & \mathbf{0.4} \\ \mathbf{3.6} & \mathbf{0.25\,S} \end{bmatrix}$$


400.00mV
R2
2
R3
1
R4
2
200.00mA
+
V2
1V
-

Solution 19.83

To get A and C, we open-circuit the output and let $I_1 = 1A$. The schematic is shown below. When the circuit is saved and simulated, we obtain $V_1 = 11$ and $V_2 = 34$.

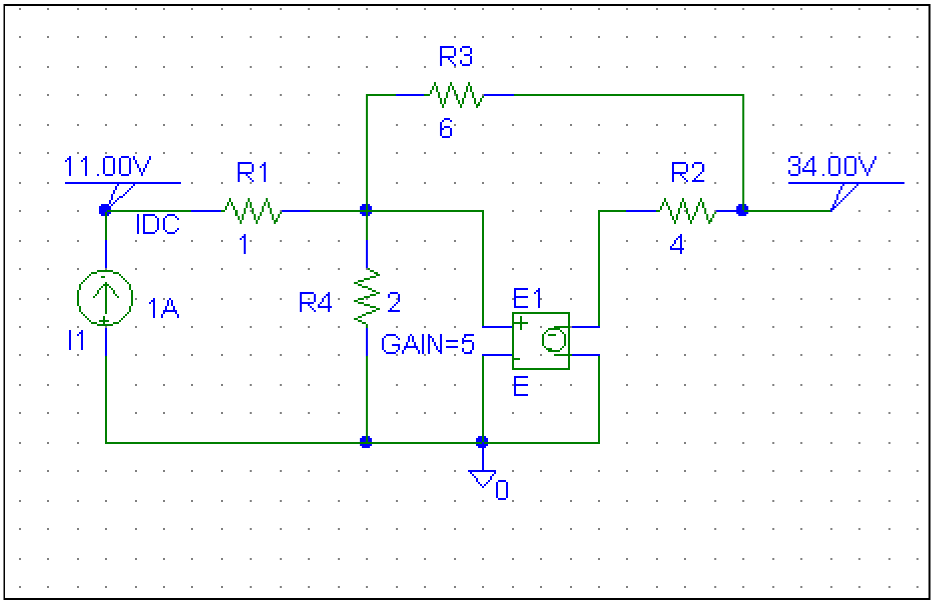
$$A = \frac{V_1}{V_2} = 0.3235, \qquad C = \frac{I_1}{V_2} = \frac{1}{34} = 0.02941$$

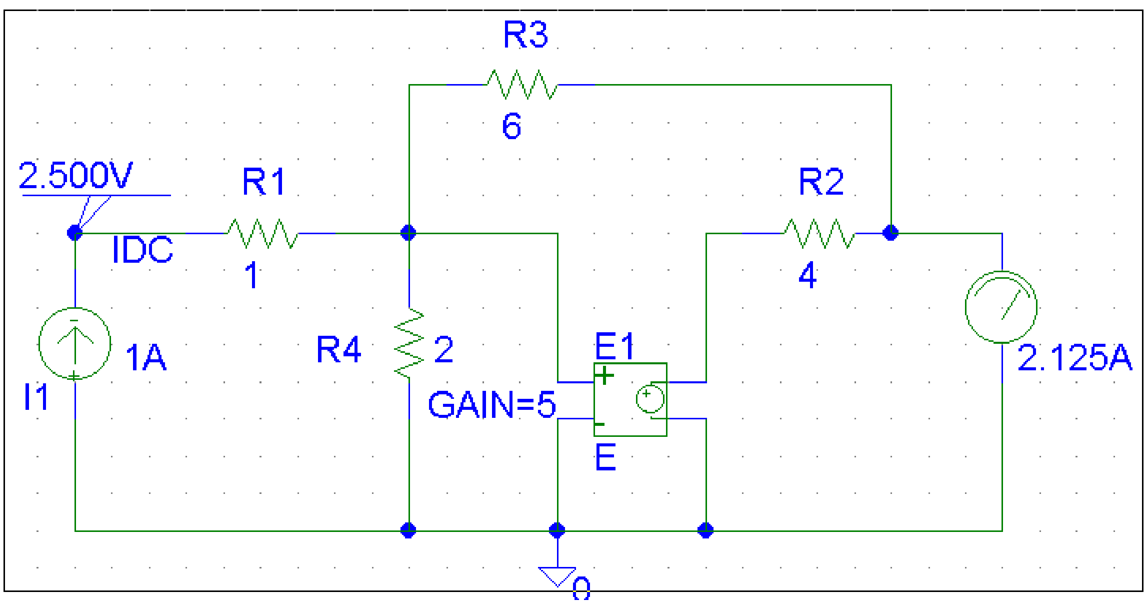
Similarly, to get B and D, we open-circuit the output and let $I_1 = 1A$. The schematic is shown below. When the circuit is saved and simulated, we obtain $V_1 = 2.5$ and $I_2 = -2.125$.

$$B = -\frac{V_1}{I_2} = \frac{2.5}{2.125} = 1.1765, \qquad D = -\frac{I_1}{I_2} = \frac{1}{2.125} = 0.4706$$

Thus,

$$\underline{\mathbf{[T]} = \begin{bmatrix} \mathbf{0.3235} & \mathbf{1.1765\,\Omega} \\ \mathbf{0.02941\,S} & \mathbf{0.4706} \end{bmatrix}}$$


R3
6
11.00V
R1
IDC
1
R2
34.00V
4
1A
I1
R4
2
E1
GAIN=5
E
0

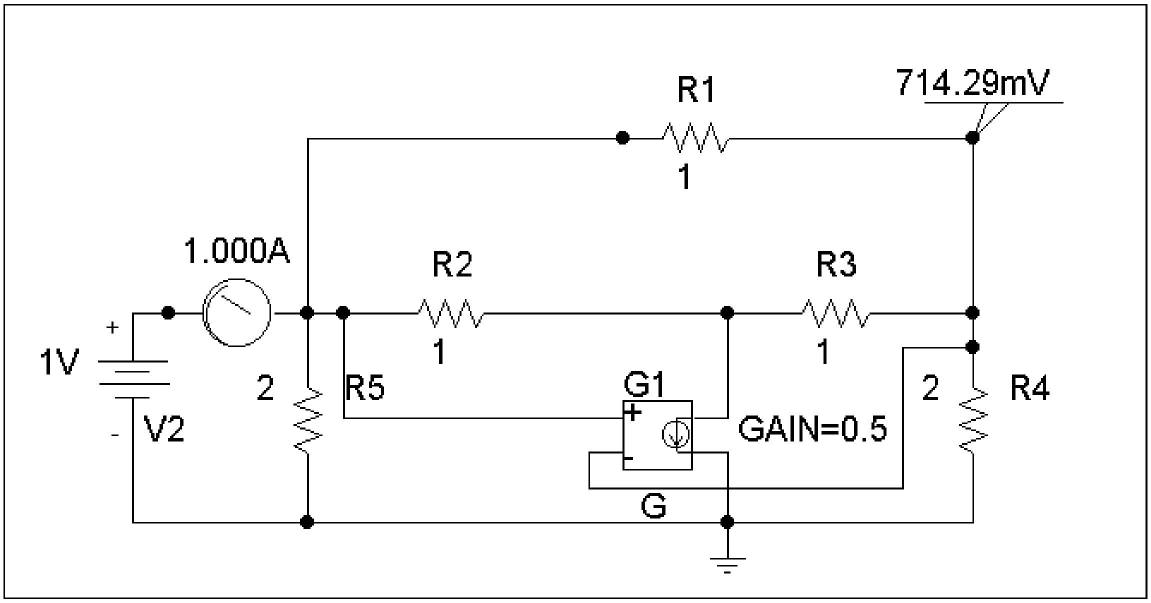

R3
6
2.500V
R1
IDC
1
R2
4
1A
I1
R4
2
E1
GAIN=5
E
2.125A
0

Solution 19.84

(a) Since $A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ and $C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$, we open-circuit the output port and let $V_1 = 1$ V. The schematic is as shown below. After simulation, we obtain

$$A = 1/V_2 = 1/0.7143 = 1.4$$

$$C = I_2/V_2 = 1.0/0.7143 = 1.4$$

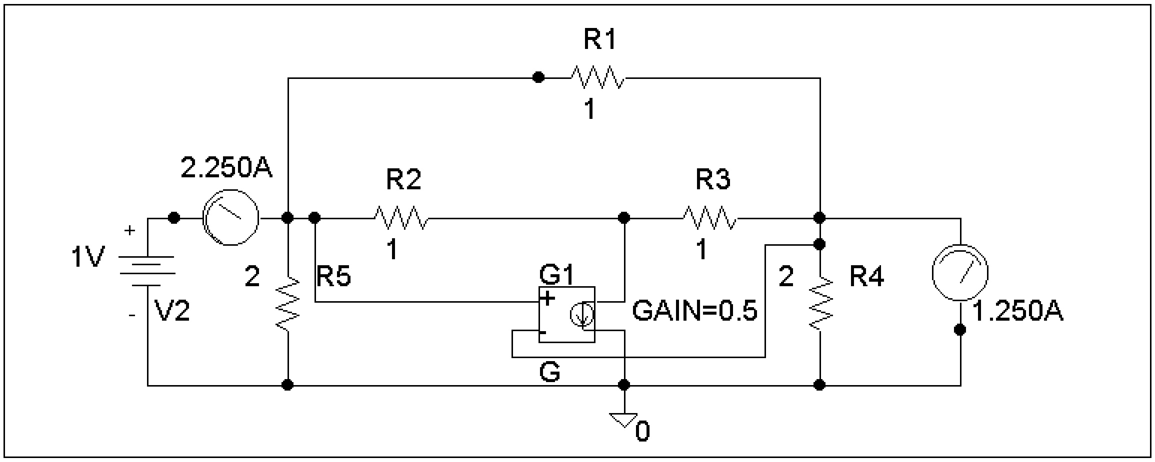


(b) To get B and D, we short-circuit the output port and let $V_1 = 1$. The schematic is shown below. After simulating the circuit, we obtain

$$B = -V_1/I_2 = -1/1.25 = -0.8$$

$$D = -I_1/I_2 = -2.25/1.25 = -1.8$$

Thus $$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \mathbf{1.4} & \mathbf{-0.8\Omega} \\ \mathbf{1.4S} & \mathbf{-1.8} \end{bmatrix}$$



R1
1
2.250A
R2
R3
1V
1
1
2
R5
G1
2
R4
V2
GAIN=0.5
1.250A
G
0

Solution 19.85

(a) Since $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$ and $C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$, we let V_1 = 1 V and open-circuit the output port. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

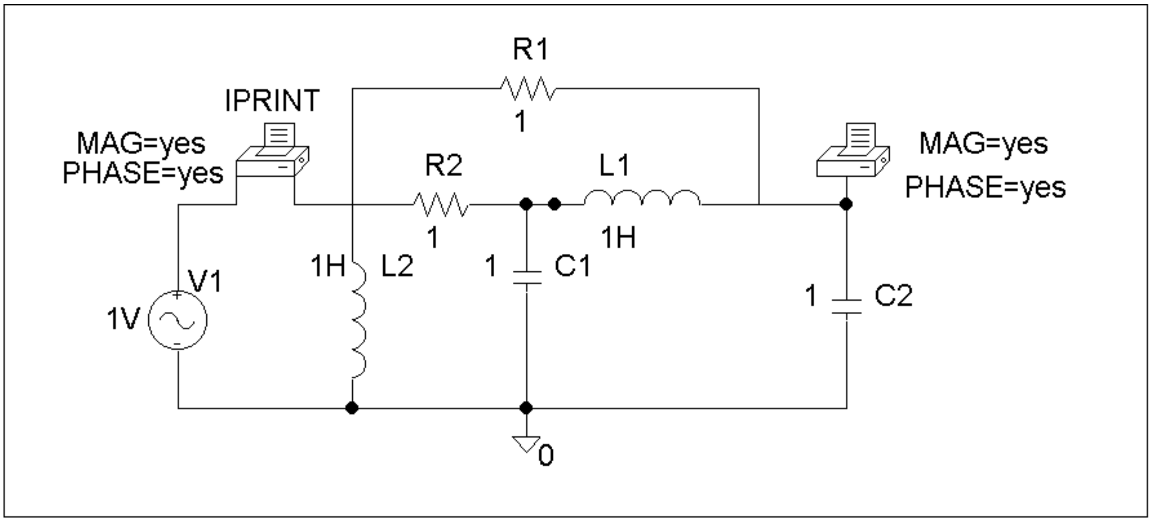
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	6.325 E–01	1.843 E+01

FREQ	VM($N_0002)	VP($N_0002)
1.592 E–01	6.325 E–01	–7.159 E+01

From this, we obtain

$$A = \frac{1}{V_2} = \frac{1}{0.6325\angle -71.59^\circ} = 1.581\angle 71.59^\circ$$

$$C = \frac{I_1}{V_2} = \frac{0.6325\angle 18.43^\circ}{0.6325\angle -71.59^\circ} = 1\angle 90^\circ = j$$



(b) Similarly, since $B = \left.\dfrac{V_1}{I_2}\right|_{V_2=0}$ and $D = -\left.\dfrac{I_1}{I_2}\right|_{V_2=0}$, we let V_1 = 1 V and short-circuit the output port. The schematic is shown below. Again, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. After simulation, we get an output file which includes the following results:

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	5.661 E–04	8.997 E+01

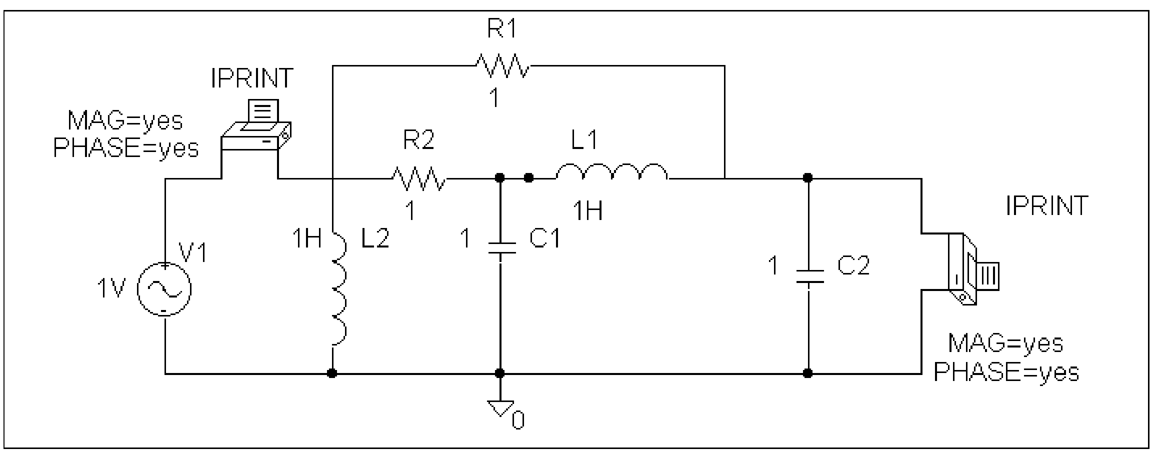
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	9.997 E–01	–9.003 E+01

From this,

$$B = -\frac{1}{I_2} = -\frac{1}{0.9997\angle -90^\circ} = -1\angle 90^\circ = -j$$

$$D = -\frac{I_1}{I_2} = -\frac{5.661\text{x}10^{-4}\angle 89.97^\circ}{0.9997\angle -90^\circ} = 5.561\text{x}10^{-4}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \mathbf{1.581\angle 71.59^\circ} & \mathbf{-\,j\Omega} \\ \mathbf{jS} & \mathbf{5.661x10^{-4}} \end{bmatrix}$$



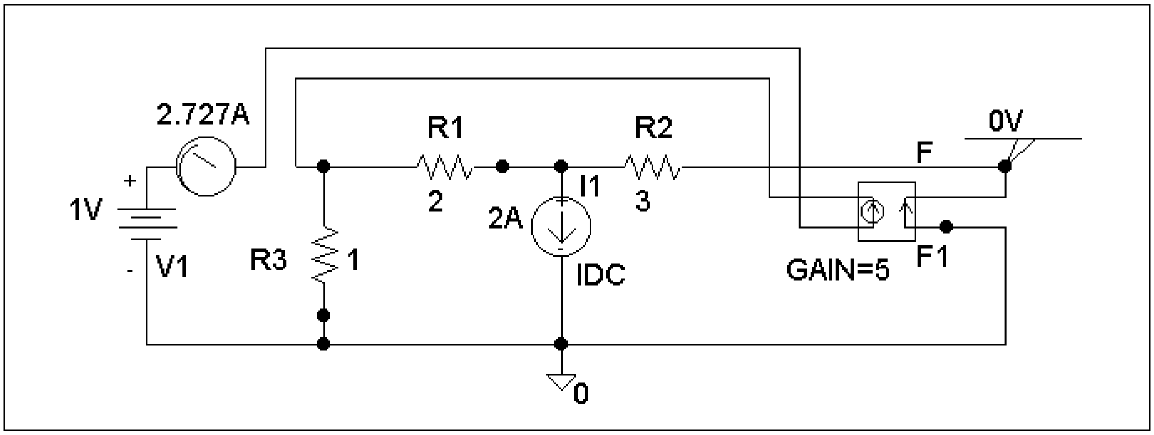
Solution 19.86

(a) By definition, $g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$, $g_{21} = \left.\frac{V_1}{V_2}\right|_{I_2=0}$.

We let $V_1 = 1$ V and open-circuit the output port. The schematic is shown below. After simulation, we obtain

$$g_{11} = I_1 = 2.7$$

$$g_{21} = V_2 = 0.0$$



(b) Similarly,

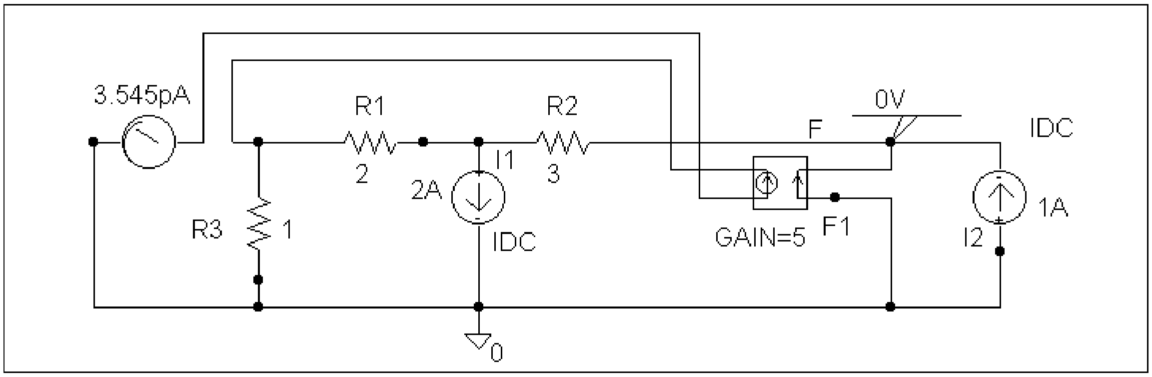
$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}, \quad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

We let $I_2 = 1$ A and short-circuit the input port. The schematic is shown below. After simulation,

$$g_{12} = I_1 = 0$$

$$g_{22} = V_2 = 0$$

Thus $$[g] = \begin{bmatrix} \mathbf{2.727S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$


3.545pA
R1
R2
0V
F
IDC
I1
2
3
2A
1A
R3
1
IDC
GAIN=5
F1
I2
0

Solution 19.87

(a) Since $a = \left.\dfrac{V_2}{V_1}\right|_{I_1=0}$ and $c = \left.\dfrac{I_2}{V_1}\right|_{I_1=0}$,

we open-circuit the input port and let $V_2 = 1$ V. The schematic is shown below. In the AC Sweep box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

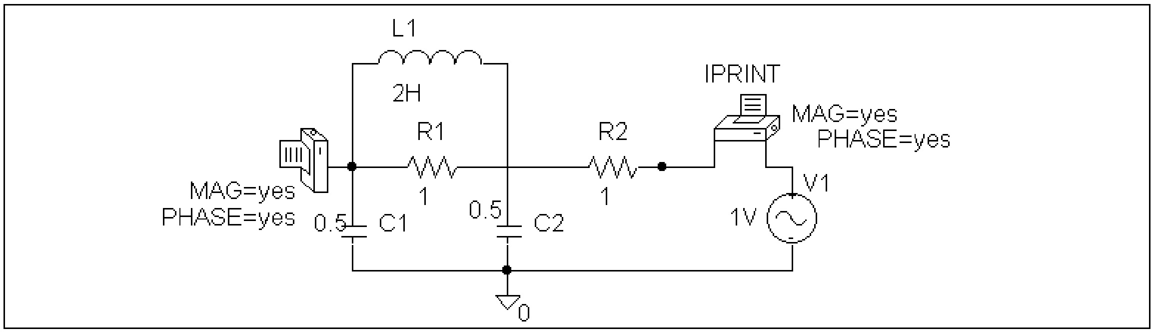
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	5.000 E–01	1.800 E+02

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	5.664 E–04	8.997 E+01

From this,

$$a = \frac{1}{5.664\text{x}10^{-4}\angle 89.97^\circ} = 1765\angle -89.97^\circ$$

$$c = \frac{0.5\angle 180^\circ}{5.664\text{x}10^{-4}\angle 89.97^\circ} = -882.28\angle -89.97^\circ$$



(b) Similarly,

$$b = -\frac{V_2}{I_1}\bigg|_{V_1=0} \quad \text{and} \quad d = -\frac{I_2}{I_1}\bigg|_{V_1=0}$$

We short-circuit the input port and let $V_2 = 1$ V. The schematic is shown below. After simulation, we obtain an output file which includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	5.000 E–01	1.800 E+02

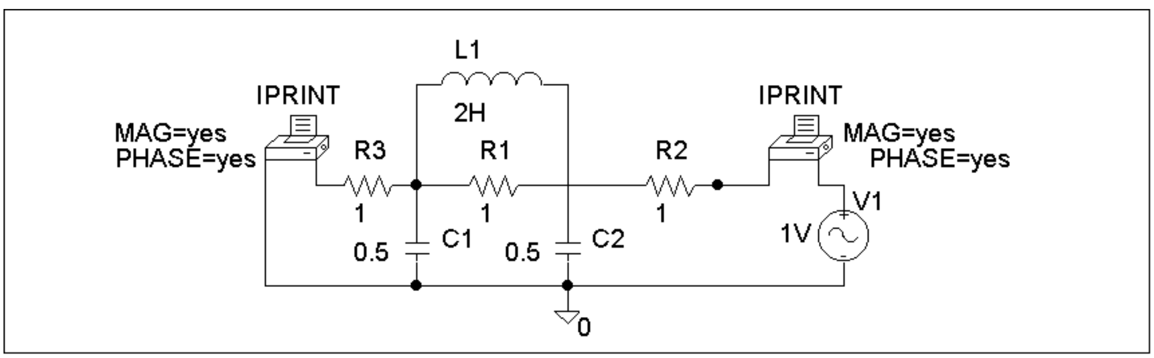
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	5.664 E–04	–9.010 E+01

From this, we get

$$b = -\frac{1}{5.664\text{x}10^{-4}\angle -90.1^\circ} = -j1765$$

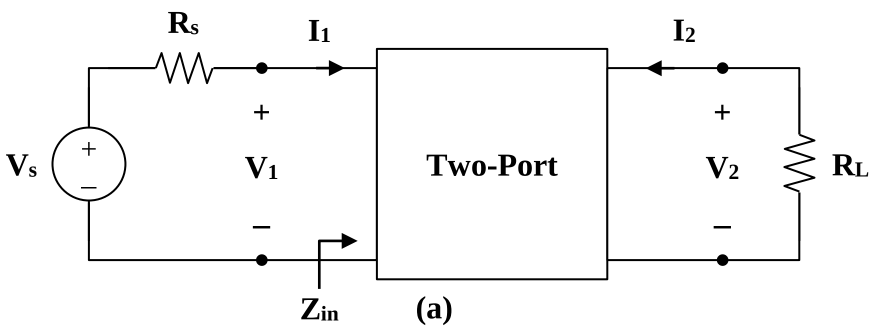
$$d = -\frac{0.5\angle 180^\circ}{5.664\text{x}10^{-4}\angle -90.1^\circ} = j888.28$$

Thus $$[\mathbf{t}] = \begin{bmatrix} \mathbf{-j1765} & \mathbf{-j1765\Omega} \\ \mathbf{j888.2S} & \mathbf{j888.2} \end{bmatrix}$$



Solution 19.88

To get Z_{in}, consider the network in Fig. (a).



(a)

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \qquad (2)$$

But $$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{R_L} = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + 1/R_L} \qquad (3)$$

Substituting (3) into (1) yields

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\cdot\left(\frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + 1/R_L}\right), \qquad \mathbf{Y}_L = \frac{1}{R_L}$$

$$\mathbf{I}_1 = \left(\frac{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)\mathbf{V}_1, \qquad \Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}$$

or $$Z_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{y_{22}} + \mathbf{Y_L}}{\mathbf{\Delta_y} + \mathbf{y_{11}}\mathbf{Y_L}}$$

$$A_i = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{\mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2}{\mathbf{I}_1} = \mathbf{y}_{21}Z_{in} + \left(\frac{\mathbf{y}_{22}}{\mathbf{I}_1}\right)\left(\frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)$$

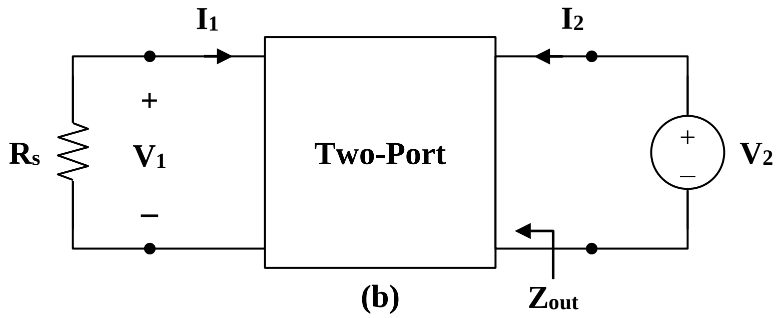
$$= \mathbf{y}_{21}Z_{in} - \frac{\mathbf{y}_{22}\mathbf{y}_{21}Z_{in}}{\mathbf{y}_{22} + \mathbf{Y}_L} = \left(\frac{\mathbf{y}_{22} + \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}\right)\left(\mathbf{y}_{21} - \frac{\mathbf{y}_{22}\mathbf{y}_{21}}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)$$

$$A_i = \frac{\mathbf{y_{21}}\mathbf{Y_L}}{\mathbf{\Delta_y} + \mathbf{y_{11}}\mathbf{Y_L}}$$

From (3),

$$A_v = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{-\mathbf{y}_{21}}{\mathbf{y}_{22} + \mathbf{Y}_L}$$

To get Z_{out}, consider the circuit in Fig. (b).



(b)

(a)

$$Z_{out} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2} \qquad (4)$$

But $\quad \mathbf{V}_1 = -R_s\,\mathbf{I}_1$

Substituting this into (1) yields

$$\mathbf{I}_1 = -\mathbf{y}_{11}\,R_s\,\mathbf{I}_1 + \mathbf{y}_{12}\,\mathbf{V}_2$$

$$(1 + \mathbf{y}_{11}\,R_s)\,\mathbf{I}_1 = \mathbf{y}_{12}\,\mathbf{V}_2$$

$$\mathbf{I}_1 = \frac{\mathbf{y}_{12}\,\mathbf{V}_2}{1 + \mathbf{y}_{11}\,R_s} = \frac{-\mathbf{V}_1}{R_s}$$

or $\quad \dfrac{\mathbf{V}_1}{\mathbf{V}_2} = \dfrac{-\mathbf{y}_{12}\,R_s}{1 + \mathbf{y}_{11}\,R_s}$

Substituting this into (4) gives

$$Z_{out} = \frac{1}{\mathbf{y}_{22} - \dfrac{\mathbf{y}_{12}\,\mathbf{y}_{21}\,R_s}{1 + \mathbf{y}_{11}\,R_s}}$$

$$= \frac{1 + \mathbf{y}_{11}\,R_s}{\mathbf{y}_{22} + \mathbf{y}_{11}\,\mathbf{y}_{22}\,R_s - \mathbf{y}_{21}\,\mathbf{y}_{22}\,R_s}$$

$$Z_{out} = \frac{\mathbf{y}_{11} + \mathbf{Y}_s}{\Delta_y + \mathbf{y}_{22}\,\mathbf{Y}_s}$$

Solution 19.89

$$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + (2640 \times 16 \times 10^{-6} - 2.6 \times 10^{-4} \times 72) \cdot 10^5}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + 1824} = \mathbf{-1613}$$

$$\text{dc gain} = 20 \log \left| A_v \right| = 20 \log (1613) = \mathbf{64.15\ dB}$$

Solution 19.90

(a) $$Z_{in} = h_{ie} - \frac{h_{re}\, h_{fe}\, R_L}{1 + h_{oe}\, R_L}$$

$$1500 = 2000 - \frac{10^{-4} \times 120\, R_L}{1 + 20 \times 10^{-6}\, R_L}$$

$$500 = \frac{12 \times 10^{-3}}{1 + 2 \times 10^{-5}\, R_L}$$

$$500 + 10^{-2}\, R_L = 12 \times 10^{-3}\, R_L$$

$$500 \times 10^2 = 0.2\, R_L$$

$$R_L = \mathbf{250\ k\Omega}$$

(b) $$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie}\, h_{oe} - h_{re}\, h_{fe}) R_L}$$

$$A_v = \frac{-120 \times 250 \times 10^3}{2000 + (2000 \times 20 \times 10^{-6} - 120 \times 10^{-4}) \times 250 \times 10^3}$$

$$A_v = \frac{-30 \times 10^6}{2 \times 10^3 + 7 \times 10^3} = \mathbf{-\ 3333}$$

$$A_i = \frac{h_{fe}}{1 + h_{oe}\, R_L} = \frac{120}{1 + 20 \times 10^{-6} \times 250 \times 10^3} = \mathbf{20}$$

$$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie}) h_{oe} - h_{re}\, h_{fe}} = \frac{600 + 2000}{(600 + 2000) \times 20 \times 10^{-6} - 10^{-4} \times 120}$$

$$Z_{out} = \frac{2600}{40}\ k\Omega = \mathbf{65\ k\Omega}$$

(c) $$A_v = \frac{\mathbf{V}_c}{\mathbf{V}_b} = \frac{\mathbf{V}_c}{\mathbf{V}_s} \longrightarrow \mathbf{V}_c = A_v \mathbf{V}_s = -3333 \times 4 \times 10^{-3} = \mathbf{-\ 13.33\ V}$$

Solution 19.91

$R_s = 1.2\ \text{k}\Omega, \qquad R_L = 4\ \text{k}\Omega$

(a) $$\mathbf{A_t = \frac{-h_{fe}\, R_L}{h_{ie} + (h_{ie}\, h_{oe} - h_{re}\, h_{fe})\, R_L}}$$

$$\mathbf{A_t = \frac{-80 \times 4 \times 10^3}{1200 + (1200 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80) \times 4 \times 10^3}}$$

$\mathbf{A_t = \dfrac{-32000}{1248} = -25.64}$ This is just the gain for the transistor. If we calculate the gain for the circuit we get $A_t = V_o/V_{be}$ and $V_{be} = V_s[1.2k/(1.2k+2k)] = 0.375$, thus,
$\mathbf{V_A}$ = (0.375)(–25.64) = **–9.615.**

(b) $$A_i = \frac{h_{fe}}{1 + h_{oe}\, R_L} = \frac{80}{1 + 20 \times 10^{-6} \times 4 \times 10^3} = \mathbf{74.07}$$

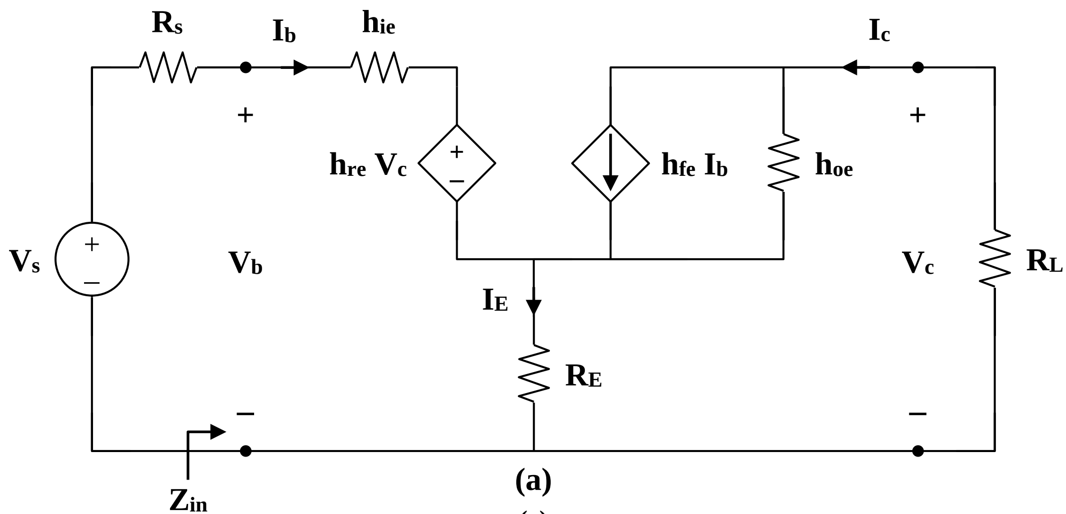
(c) $$Z_{in} = h_{ie} - h_{re}\, A_i$$
$$Z_{in} = 1200 - 1.5 \times 10^{-4} \times 74.074 \cong \mathbf{1.2\ k\Omega}$$

(d) $$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie})\, h_{oe} - h_{re}\, h_{fe}}$$
$$Z_{out} = \frac{1200 + 1200}{2400 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80} = \frac{2400}{0.0468} = \mathbf{51.28\ k\Omega}$$

(a) –25.64 for the transistor and –9.615 for the circuit, (b) 74.07, (c) 1.2 kΩ, (d) 51.28 kΩ

Solution 19.92

Due to the resistor $R_E = 240\,\Omega$, we cannot use the formulas in section 18.9.1. We will need to derive our own. Consider the circuit in Fig. (a).



(a)

(a)

$$\mathbf{I}_E = \mathbf{I}_b + \mathbf{I}_c \qquad (1)$$

$$\mathbf{V}_b = h_{ie}\,\mathbf{I}_b + h_{re}\,\mathbf{V}_c + (\mathbf{I}_b + \mathbf{I}_c)R_E \qquad (2)$$

$$\mathbf{I}_c = h_{fe}\,\mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + 1/h_{oe}} \qquad (3)$$

But $$\mathbf{V}_c = -\mathbf{I}_c R_L \qquad (4)$$

Substituting (4) into (3),

$$\mathbf{I}_c = h_{fe}\,\mathbf{I}_b - \frac{R_L}{R_E + 1/h_{oe}}\mathbf{I}_c$$

or $$A_i = \frac{\mathbf{I}_c}{\mathbf{I}_b} = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L} \qquad (5)$$

$$A_i = \frac{100(1 + 240\text{x}30\text{x}10^{-6})}{1 + 30\times 10^{-6}(4{,}000 + 240)}$$

$$A_i = \underline{\mathbf{79.18}}$$

From (3) and (5),

$$\mathbf{I}_c = \frac{h_{fe}(1+R_E)h_{oe}}{1+h_{oe}(R_L+R_E)}\mathbf{I}_b = h_{fe}\,\mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + \frac{1}{h_{oe}}} \qquad (6)$$

Substituting (4) and (6) into (2),

$$\mathbf{V}_b = (h_{ie}+R_E)\mathbf{I}_b + h_{re}\,\mathbf{V}_c + \mathbf{I}_c\,R_E$$

$$\mathbf{V}_b = \frac{\mathbf{V}_c\,(h_{ie}+R_E)}{\left(R_E+\dfrac{1}{h_{oe}}\right)\left[\dfrac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)} - h_{fe}\right]} + h_{re}\,\mathbf{V}_c - \frac{\mathbf{V}_c}{R_L}R_E$$

$$\frac{1}{A_v} = \frac{\mathbf{V}_b}{\mathbf{V}_c} = \frac{(h_{ie}+R_E)}{\left(R_E+\dfrac{1}{h_{oe}}\right)\left[\dfrac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)} - h_{fe}\right]} + h_{re} - \frac{R_E}{R_L} \qquad (7)$$

$$\frac{1}{A_v} = \frac{(4000+240)}{\left(240+\dfrac{1}{30\text{x}10^{-6}}\right)\left[\dfrac{100(1+240\text{x}30\text{x}10^{-6})}{1+30\times10^{-6}\times4240} - 100\right]} + 10^{-4} - \frac{240}{4000}$$

$$\frac{1}{A_v} = -6.06\text{x}10^{-3} + 10^{-4} - 0.06 = -0.066$$

$$A_v = \mathbf{-15.15}$$

From (5),

$$\mathbf{I}_c = \frac{h_{fe}}{1+h_{oe}R_L}\mathbf{I}_b$$

We substitute this with (4) into (2) to get

$$\mathbf{V}_b = (h_{ie}+R_E)\mathbf{I}_b + (R_E - h_{re}R_L)\mathbf{I}_c$$

$$\mathbf{V}_b = (h_{ie}+R_E)\mathbf{I}_b + (R_E - h_{re}R_L)\left(\frac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)}\mathbf{I}_b\right)$$

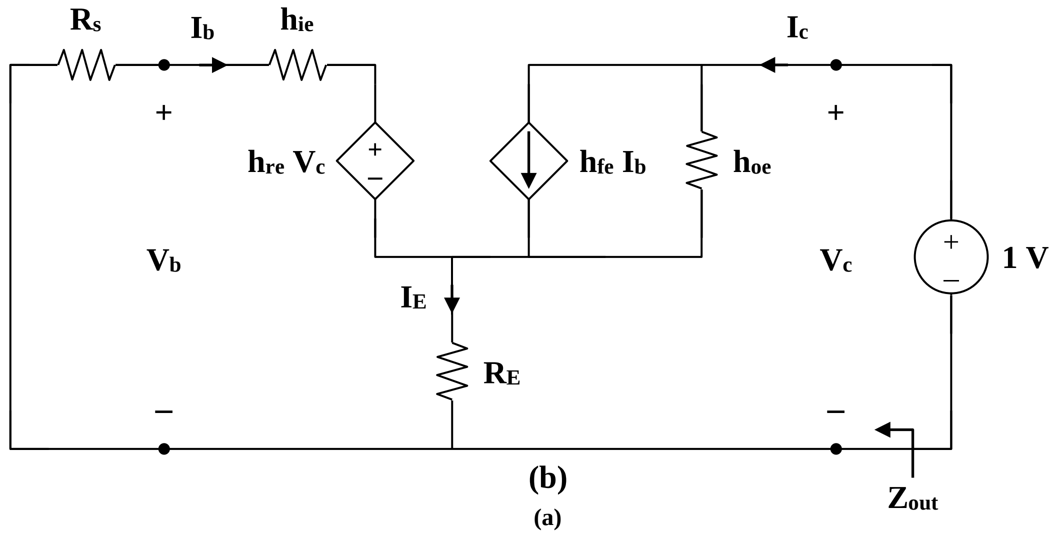
$$Z_{in} = \frac{\mathbf{V}_b}{\mathbf{I}_b} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re}R_L)(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)}$$

(8)

$$Z_{in} = 4000 + 240 + \frac{(100)(240\times10^{-4}\times4\times10^{3})(1+240\text{x}30\text{x}10^{-6})}{1+30\times10^{-6}\times4240}$$

$$Z_{in} = \mathbf{12.818\ k\Omega}$$

To obtain Z_{out}, which is the same as the Thevenin impedance at the output, we introduce a 1-V source as shown in Fig. (b).



(b)

(a)

From the input loop,

$$\mathbf{I}_b(R_s + h_{ie}) + h_{re}\mathbf{V}_c + R_E(\mathbf{I}_b + \mathbf{I}_c) = 0$$

But $\qquad \mathbf{V}_c = 1$

So,

$$\mathbf{I}_b(R_s + h_{ie} + R_E) + h_{re} + R_E\mathbf{I}_c = 0 \qquad (9)$$

From the output loop,

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{R_E + \dfrac{1}{h_{oe}}} + h_{fe}\mathbf{I}_b = \frac{h_{oe}}{R_E h_{oe} + 1} + h_{fe}\mathbf{I}_b$$

or

$$\mathbf{I}_b = \frac{\mathbf{I}_c}{h_{fe}} - \frac{h_{oe}/h_{fe}}{1 + R_E h_{oe}} \qquad (10)$$

Substituting (10) into (9) gives

$$(R_s + R_E + h_{ie})\left(\frac{\mathbf{I}_c}{h_{fe}}\right) + h_{re} + R_E\mathbf{I}_c - \frac{(R_s + R_E + h_{ie})\left(h_{oe}/h_{fe}\right)}{1 + R_E h_{oe}} = 0$$

$$\frac{R_s + R_E + h_{ie}}{h_{fe}}\mathbf{I}_c + R_E\mathbf{I}_c = \frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\left(\frac{h_{oe}}{h_{fe}}\right) - h_{re}$$

$$\mathbf{I}_c = \frac{(h_{oe}/h_{fe})\left[\dfrac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] - h_{re}}{R_E + (R_s + R_E + h_{ie})/h_{fe}}$$

$$Z_{out} = \frac{1}{\mathbf{I}_c} = \frac{R_E\, h_{fe} + R_s + R_E + h_{ie}}{\left[\dfrac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] h_{oe} - h_{re}\, h_{fe}}$$

$$Z_{out} = \frac{240 \times 100 + (1200 + 240 + 4000)}{\left[\dfrac{1200 + 240 + 4000}{1 + 240 \text{x} 30 \text{x} 10^{-6}}\right] \times 30 \times 10^{-6} - 10^{-4} \times 100}$$

$$Z_{out} = \frac{24000 + 5440}{0.152} = \mathbf{193.7\ k\Omega}$$

Solution 19.93

We apply the same formulas derived in the previous problem.

$$\frac{1}{A_v} = \frac{(h_{ie} + R_E)}{\left(R_E + \frac{1}{h_{oe}}\right)\left[\frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe}\right]} + h_{re} - \frac{R_E}{R_L}$$

$$\frac{1}{A_v} = \frac{(2000 + 200)}{(200 + 10^5)\left[\frac{150(1 + 0.002)}{1 + 0.04} - 150\right]} + 2.5\times10^{-4} - \frac{200}{3800}$$

$$\frac{1}{A_v} = -0.004 + 2.5\times10^{-4} - 0.05263 = -0.05638$$

$$A_v = \mathbf{-17.74}$$

$$A_i = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} = \frac{150(1 + 200\text{x}10^{-5})}{1 + 10^{-5}\times(200 + 3800)} = \mathbf{144.5}$$

$$Z_{in} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re}R_L)(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)}$$

$$Z_{in} = 2000 + 200 + \frac{(150)(200 - 2.5\times10^{-4}\times3.8\times10^3)(1.002)}{1.04}$$

$$Z_{in} = 2200 + 28966$$

$$Z_{in} = \mathbf{31.17\ k\Omega}$$

$$Z_{out} = \frac{R_E h_{fe} + R_s + R_E + h_{ie}}{\left[\frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right]h_{oe} - h_{re}h_{fe}}$$

$$Z_{out} = \frac{200\times150 + 1000 + 200 + 2000}{\left[\frac{3200\times10^{-5}}{1.002}\right] - 2.5\times10^{-4}\times150} = \frac{33200}{-0.0055}$$

$$Z_{out} = \mathbf{-6.148\ M\Omega}$$

Solution 19.94

We first obtain the **ABCD** parameters.

Given $\quad [\mathbf{h}] = \begin{bmatrix} 200 & 0 \\ 100 & 10^{-6} \end{bmatrix}, \qquad \Delta_h = \mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{12}\mathbf{h}_{21} = 2\times10^{-4}$

$$[\mathbf{T}] = \begin{bmatrix} \dfrac{\Delta_h}{\mathbf{h}_{21}} & \dfrac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \dfrac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \dfrac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}$$

The overall **ABCD** parameters for the amplifier are

$$[\mathbf{T}] = \begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}\begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix} \cong \begin{bmatrix} 2\times10^{-8} & 2\times10^{-2} \\ 10^{-10} & 10^{-4} \end{bmatrix}$$

$$\Delta_T = 2\times10^{-12} - 2\times10^{-12} = 0$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{\mathbf{B}}{\mathbf{D}} & \dfrac{\Delta_T}{\mathbf{D}} \\ \dfrac{-1}{\mathbf{D}} & \dfrac{\mathbf{C}}{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} 200 & 0 \\ -10^4 & 10^{-6} \end{bmatrix}$$

Thus, $\quad h_{ie} = 200, \quad h_{re} = 0, \quad h_{fe} = -10^4, \quad h_{oe} = 10^{-6}$

$$A_v = \frac{(10^4)(4\times10^3)}{200 + (2\times10^{-4} - 0)\times4\times10^3} = \mathbf{2\times10^5}$$

$$Z_{in} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L} = 200 - 0 = \mathbf{200\,\Omega}$$

Solution 19.95

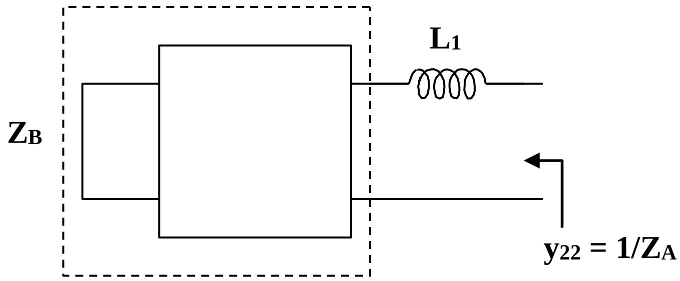
Let $\mathbf{Z}_A = \dfrac{1}{\mathbf{y}_{22}} = \dfrac{s^4 + 10s^2 + 8}{s^3 + 5s}$

Using long division,

$$\mathbf{Z}_A = s + \frac{5s^2 + 8}{s^3 + 5s} = sL_1 + \mathbf{Z}_B$$

i.e. $\quad L_1 = 1\text{ H} \quad$ and $\quad \mathbf{Z}_B = \dfrac{5s^2 + 8}{s^3 + 5s}$

as shown in Fig (a).



(a)

(a)

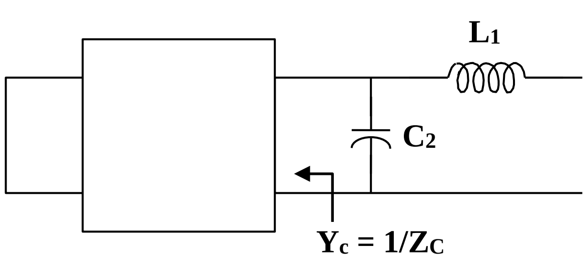
$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = \frac{s^3 + 5s}{5s^2 + 8}$$

Using long division,

$$\mathbf{Y}_B = 0.2s + \frac{3.4s}{5s^2 + 8} = sC_2 + \mathbf{Y}_C$$

where $\quad C_2 = 0.2\text{ F} \quad$ and $\quad \mathbf{Y}_C = \dfrac{3.4s}{5s^2 + 8}$

as shown in Fig. (b).



(b)

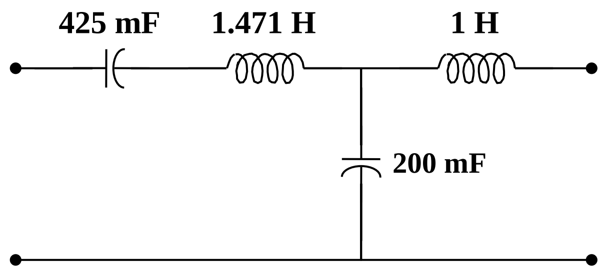
(a)

$$\mathbf{Z}_C = \frac{1}{\mathbf{Y}_C} = \frac{5s^2 + 8}{3.4s} = \frac{5s}{3.4} + \frac{8}{3.4s} = sL_3 + \frac{1}{sC_4}$$

i.e. an inductor in series with a capacitor

$$L_3 = \frac{5}{3.4} = 1.471\text{ H} \quad \text{and} \quad C_4 = \frac{3.4}{8} = 0.425\text{ F}$$

Thus, **the LC network is shown in Fig. (c)**.

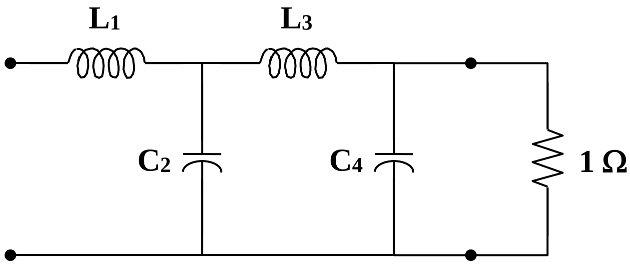


(c)

(a)

Solution 19.96

This is a fourth order network which can be realized with the network shown in Fig. (a).



(a)

(a)

$$\Delta(s) = (s^4 + 3.414s^2 + 1) + (2.613s^3 + 2.613s)$$

$$H(s) = \frac{\dfrac{1}{2.613s^3 + 2.613s}}{1 + \dfrac{s^4 + 3.414s^2 + 1}{2.613s^3 + 2.613s}}$$

which indicates that

$$\mathbf{y}_{21} = \frac{-1}{2.613s^3 + 2.613s}$$

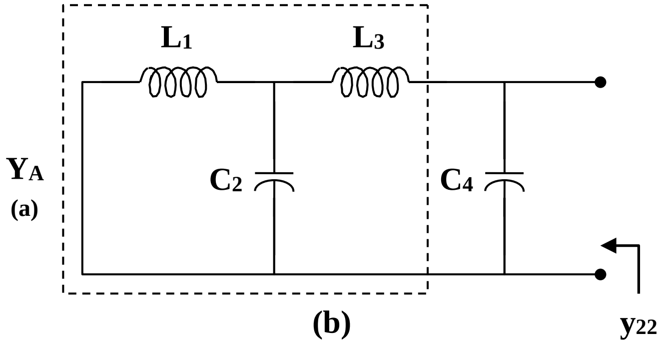
$$\mathbf{y}_{22} = \frac{s^4 + 3.414s + 1}{2.613s^3 + 2.613s}$$

We seek to realize $\mathbf{y}_{22}$. By long division,

$$\mathbf{y}_{22} = 0.383s + \frac{2.414s^2 + 1}{2.613s^3 + 2.613s} = sC_4 + \mathbf{Y}_A$$

i.e. $C_4 = 0.383\text{ F}$ and $\mathbf{Y}_A = \dfrac{2.414s^2 + 1}{2.613s^3 + 2.613s}$

as shown in Fig. (b).



(b)

(a)

(a)

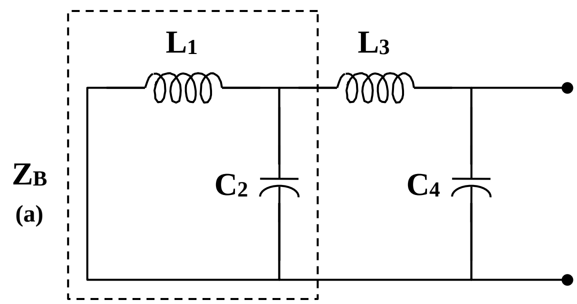
$$\mathbf{Z}_A = \frac{1}{\mathbf{Y}_A} = \frac{2.613s^3 + 2.613s}{2.414s^2 + 1}$$

By long division,

$$\mathbf{Z}_A = 1.082s + \frac{1.531s}{2.414s^2 + 1} = sL_3 + \mathbf{Z}_B$$

i.e. $L_3 = 1.082$ H and $\mathbf{Z}_B = \dfrac{1.531s}{2.414s^2 + 1}$

as shown in Fig.(c).



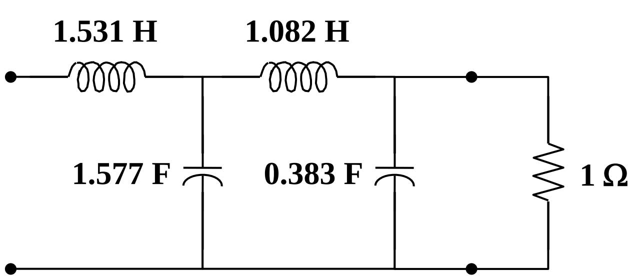
(c)

(a)

$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = 1.577s + \frac{1}{1.531s} = sC_2 + \frac{1}{sL_1}$$

i.e. $C_2 = 1.577$ F and $L_1 = 1.531$ H

Thus, **the network is shown in Fig. (d)**.



(d)

(a)

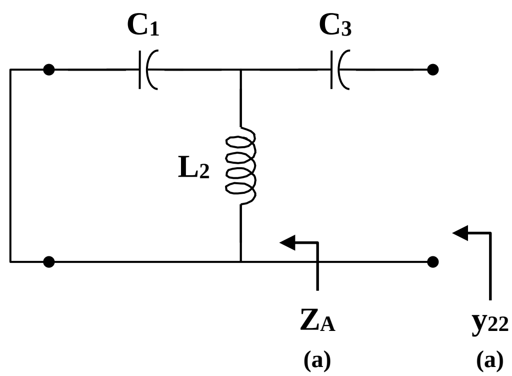
Solution 19.97

$$H(s) = \frac{s^3}{(s^3+12s)+(6s^2+24)} = \frac{\dfrac{s^3}{s^3+12s}}{1+\dfrac{6s^2+24}{s^3+12s}}$$

Hence,

$$\mathbf{y}_{22} = \frac{6s^2+24}{s^3+12s} = \frac{1}{sC_3} + \mathbf{Z}_A \qquad (1)$$

where $\mathbf{Z}_A$ is shown in the figure below.



We now obtain C_3 and $\mathbf{Z}_A$ using partial fraction expansion.

Let $$\frac{6s^2+24}{s(s^2+12)} = \frac{A}{s} + \frac{Bs+C}{s^2+12}$$

$$6s^2+24 = A(s^2+12)+Bs^2+Cs$$

Equating coefficients :

s^0 : $24 = 12A \longrightarrow A = 2$

s^1: $0 = C$

s^2 : $6 = A + B \longrightarrow B = 4$

Thus,

$$\frac{6s^2+24}{s(s^2+12)} = \frac{2}{s} + \frac{4s}{s^2+12} \qquad (2)$$

Comparing (1) and (2),

$$C_3 = \frac{1}{A} = \frac{1}{2}\ \text{F}$$

$$\frac{1}{\mathbf{Z}_A} = \frac{s^2 + 12}{4s} = \frac{1}{4}s + \frac{3}{s} \qquad (3)$$

But
$$\frac{1}{\mathbf{Z}_A} = sC_1 + \frac{1}{sL_2} \qquad (4)$$

Comparing (3) and (4),

$$C_1 = \frac{1}{4}\ \text{F} \qquad \text{and} \qquad L_2 = \frac{1}{3}\ \text{H}$$

Therefore,

$$C_1 = \mathbf{250\ mF}, \qquad L_2 = \mathbf{333.3\ mH}, \qquad C_3 = \mathbf{500\ mF}$$

Solution 19.98

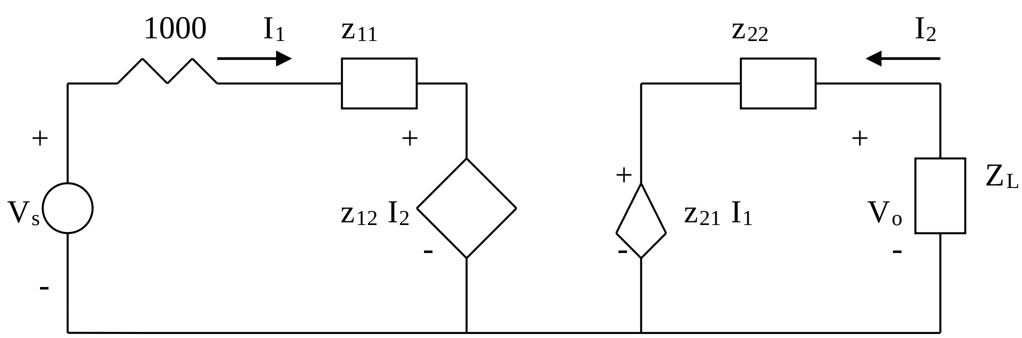
$$\Delta_h = 1 - 0.8 = 0.2$$

$$[T_a] = [T_b] = \begin{bmatrix} -\Delta_h / h_{21} & -h_{11}/h_{21} \\ -h_{22}/h_{21} & -1/h_{21} \end{bmatrix} = \begin{bmatrix} -0.001 & -10 \\ -2.5\text{x}10^{-6} & -0.005 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} 2.6\text{x}10^{-5} & 0.06 \\ 1.5\text{x}10^{-8} & 5\text{x}10^{-5} \end{bmatrix}$$

We now convert this to z-parameters

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 1.733\text{x}10^{3} & 0.0267 \\ 6.667\text{x}10^{7} & 3.33\text{x}10^{3} \end{bmatrix}$$



$$V_s = (1000 + z_{11})I_1 + z_{12}I_2 \quad (1)$$

$$V_o = z_{22}I_2 + z_{21}I_1 \quad (2)$$

But $V_o = -I_2Z_L \longrightarrow I_2 = -V_o / Z_L$ (3)

Substituting (3) into (2) gives

$$I_1 = V_o\left(\frac{1}{z_{21}} + \frac{z_{22}}{z_{21}Z_L}\right) \quad (4)$$

We substitute (3) and (4) into (1)

$$\mathbf{V_s = (1000 + z_{11})\left(\frac{1}{z_{11}} + \frac{z_{22}}{z_{21}Z_L}\right)V_o - \frac{z_{12}}{Z_L}V_o}$$

$$\mathbf{= 7.653x10^{-4} - 2.136x10^{-5} = \underline{744\mu V}}$$

Solution 19.99

$$\mathbf{Z}_{ab} = \mathbf{Z}_1 + \mathbf{Z}_3 = \mathbf{Z}_c \parallel (\mathbf{Z}_b + \mathbf{Z}_a)$$

$$\mathbf{Z}_1 + \mathbf{Z}_3 = \frac{\mathbf{Z}_c(\mathbf{Z}_a + \mathbf{Z}_b)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{1}$$

$$\mathbf{Z}_{cd} = \mathbf{Z}_2 + \mathbf{Z}_3 = \mathbf{Z}_a \parallel (\mathbf{Z}_b + \mathbf{Z}_c)$$

$$\mathbf{Z}_2 + \mathbf{Z}_3 = \frac{\mathbf{Z}_a(\mathbf{Z}_b + \mathbf{Z}_c)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{2}$$

$$\mathbf{Z}_{ac} = \mathbf{Z}_1 + \mathbf{Z}_2 = \mathbf{Z}_b \parallel (\mathbf{Z}_a + \mathbf{Z}_c)$$

$$\mathbf{Z}_1 + \mathbf{Z}_2 = \frac{\mathbf{Z}_b(\mathbf{Z}_a + \mathbf{Z}_c)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{3}$$

Subtracting (2) from (1),

$$\mathbf{Z}_1 - \mathbf{Z}_2 = \frac{\mathbf{Z}_b(\mathbf{Z}_c - \mathbf{Z}_a)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{4}$$

Adding (3) and (4),

$$\mathbf{Z}_1 = \underline{\frac{\mathbf{Z_b Z_c}}{\mathbf{Z_a + Z_b + Z_c}}} \tag{5}$$

Subtracting (5) from (3),

$$\mathbf{Z}_2 = \frac{\mathbf{Z_a Z_b}}{\mathbf{Z_a + Z_b + Z_c}} \tag{6}$$

Subtracting (5) from (1),

$$\mathbf{Z}_3 = \frac{\mathbf{Z_c Z_a}}{\mathbf{Z_a + Z_b + Z_c}} \tag{7}$$

Using (5) to (7)

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a\mathbf{Z}_b\mathbf{Z}_c(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)}{(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)^2}$$

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{8}$$

Dividing (8) by each of (5), (6), and (7),

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

as required. Note that the formulas above are not exactly the same as those in Chapter 9 because the locations of $\mathbf{Z}_b$ and $\mathbf{Z}_c$ are interchanged in Fig. 18.122.