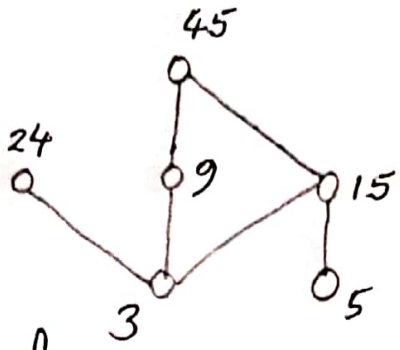
ABID ALI
ABIDALI_ 2019380141

e) Find all upper bounds of {3, 5}
15, 45



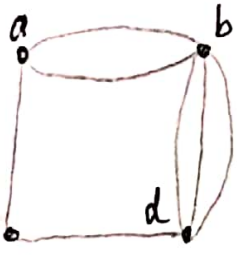
f) Find the least upper bounds of {3, 5}, if in exists. 15

g) Find all lower bounds of {15, 45} = 3, 5, 15

h) Find the greatest lower bound of {15, 45} if exists
15

4, 7

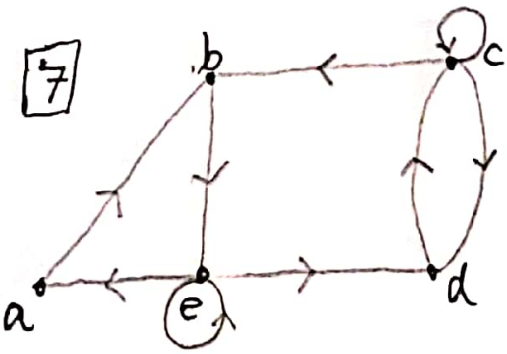
4



- undirected graph
- no loops
- has multiple edges

} Multigraph

7



- Directed graph
- 2 loops
- has multiple edges

} ⇒ Directed Multigraph

Pendent is a vertex with degree one

10.2 2, 8

2 No. of vertices : 5
No. of edges : 13

$\deg(a) = 6$ $\deg(d) = 5$
$\deg(b) = 6$ $\deg(e) = 3$
$\deg(c) = 6$

No isolated and pendent vertices

8 No. of vertices : 4
No. of edges : 8

The in-degree in this Graph are $\deg^-(a) = 2$
$\deg^-(b) = 3$
$\deg^-(c) = 2$
$\deg^-(d) = 1$

The out-degree are: $\deg^+(a) = 2$
$\deg^+(b) = 4$
$\deg^+(c) = 1$
$\deg^+(d) = 1$

10.3 5, 7, 11, 15, 17, 20, 39, 40, 41

5

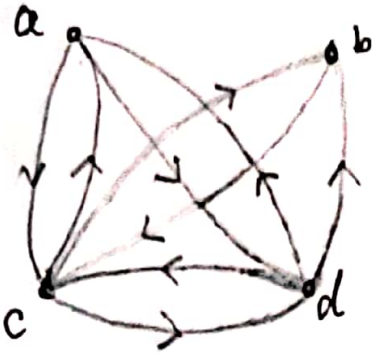
	a	b	c	d
a	0	1	1	1
b	1	0	0	1
c	1	0	0	1
d	1	1	1	0

7

	a	b	c	d
a	1	1	1	1
b	0	0	0	1
c	1	1	0	0
d	0	1	1	1

11 draw a graph with the given adjacently matrix

	a	b	c	d
a	0	0	1	1
b	0	0	1	0
c	1	1	0	1
d	1	1	1	0

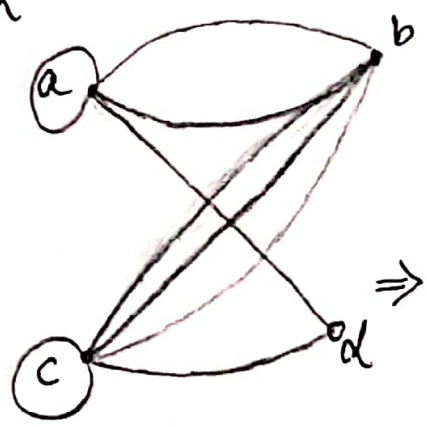


15

	a	b	c	d
a	1	0	2	1
b	0	1	1	2
c	2	1	1	0
d	1	2	0	1

17 Draw an undirected graph

	a	b	c	d
a	1	2	0	1
b	2	0	3	0
c	0	3	1	1
d	1	0	1	0



20

	a	b	c	d
a	1	1	1	1
b	0	1	0	1
c	1	0	1	0
d	1	1	1	1

39 G_1, 5 vertices, 5 edges $\simeq$ G_2 5 vertices, 5 edges

By looking at it we can tell it is isomorphic.

40 $G_1 \not\cong G_2$

vertices : $|V_1| = 5$ $|V_2| = 5$

edges : $|E_1| = 7$ $|E_2| = 7$

Degrees : $\deg(u_2) = 2$ $\deg(v_2) = 4$

$\deg(u_n) = 3\ (n, 3, 4, 5, 1)$ Not isomorphic

because, G_2 has a vertex with deg = 4 and G_1 does not.

41 $G_1 \cong G_2$

G_1

vertices : $|V_1| = 7$

edges : $|E_1| = 7$

deg (all) = 2

G_2

vertices : $|V_2| = 7$

edges : $|E_2| = 7$

deg (all) = 2

	u_1	u_2	u_3	u_4	u_5	u_6	u_7
u_1	0	1	0	0	0	0	1
u_2	1	0	1	0	0	0	0
u_3	0	1	0	1	0	0	0
u_4	0	0	1	0	1	0	0
u_5	0	0	0	1	0	1	0
u_6	0	0	0	0	1	0	1
u_7	1	0	0	0	0	1	0

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
1	0	0	1	0	0	1	0
2	0	0	0	1	0	0	1
3	1	0	0	0	1	0	0
4	0	1	0	0	0	1	0
5	0	0	1	0	0	0	1
6	1	0	0	1	0	0	0
7	0	1	0	0	1	0	0

$u_1 = v_1$ / $u_2 = v_3$ / $u_3 = v_5$ / $u_4 = v_7$ / .....

10.4

P724/3, 4, 5, 11, 14(a)(b), 19(b)(c), 20

3 Not connected (there is a vertex not connected to any other

4 It is connected.

5 Not connected. It has 2 groups with 3 vertices but nothing connects the 2 groups.

11 Determine whether each of these graphs is strongly connected and if not whether it is weekly connected.

a) $V = \{a, b, c, d, e\}$ $E = \{(b,a), (b,c), (b,e), (c,d), (d,b), (e,a)\}$

★ The graph is not strongly connected, $deg^+(a) = 0$. So, no path from a to b.

★ It is weekly connected ⇒ The underlying undifected graph is connected (has only one conected component)

b) ★ Same as "a". This connected graph is not strongly connected, no path from c to a (no edges with c as their initia vertex).

★ But it is weekly connected, because it's underlying undifected graph has 1 connected component.

c) ☆ Not strongly connected (for example, no path from c to a) ($\deg^+(c)=0$)

☆ It is also not weekly connected, because the underlying undirected graph has more than 2 components (not connects)

14 Find the strongly connected components of each of these graphs

a) $\{a, b, e\}$, $\{c\}$, $\{d\}$

b) $\{f\}$, $\{a\}$, $\{b\}$, $\{c, d, e\}$

19 Find the number of paths of length n between two different vertices in K_4 if n is 4 vertices $\Rightarrow 4 \times 4$

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

a) $n=2$ $\quad A^2 = A \cdot A_2$

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 3 \\ 2 & 2 & 2 & 3 \end{bmatrix}$$

2 paths of length 2 between every pair of different vertices.

b) $n=3$ $\quad A^3 = A^2 \cdot A$

$$\begin{bmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 7 & 7 & 7 \\ 7 & 6 & 7 & 7 \\ 7 & 7 & 6 & 7 \\ 7 & 7 & 7 & 6 \end{bmatrix} \Rightarrow 7$$

c) $n=4 \quad A^4 = A^3 \cdot A$

$$\begin{bmatrix} 6 & 7 & 7 & 7 \\ 7 & 6 & 7 & 7 \\ 7 & 7 & 6 & 7 \\ 7 & 7 & 7 & 6 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 21 & 20 & 20 & 20 \\ 20 & 21 & 20 & 20 \\ 20 & 20 & 21 & 20 \\ 20 & 20 & 20 & 21 \end{bmatrix} \Rightarrow \boxed{20}$$

P. 751 2, 17

2. Find the length of a shortest path between a and z in the given weighted graph

$z \to a \xrightarrow{2} b \xrightarrow{2} c \xrightarrow{1} d \xrightarrow{2} z \Rightarrow$ length : 7

17. The weighted graphs show some major roads in New Jersey. Part (a) shows the distances between cities Part (b) shows the tolls.

a) Find a shortest route in distance between Newark and Camden, b) between Newmark and Cape May.

a) Newark ⟶ Woodbridge ⟶ Camden

b) Newark ⟶ Woodbridge ⟶ Camden ⟶ Cape May

b) Find a least expensive route in terms of total tools

a) Newark ⟶ woodbridge ⟶ Camden

b) Newark ⟶ woodbridge ⟶ Campden ⟶ Cape May