Chapter 1 Random event

- 1、 **Origin and application of probability**
- 2、 Random pheomenon

- 3、 Random experiment
- 4、 Sample space, Sample point
- 5、 Random event
- 6、 **Conclusion**

一、 Origin and application of Probability

1. Origin of Probality

In 1654, a French gambler called Mere ask Pascal one question "A game is played between two person. The one who firstly scores three points wins the game. Given one player win *1 point, while the other 0 point*, and the game stops. How to distribute the total bet 64 pistoles". Pascal and Fermat communicated and solved this problem, which builds the foundation for the theory of probability.

2. Application of Probability

 Probability is one **branch** of Mathematics, which studies the **laws of random phenomenon**. It has been applied in almost all areas in today's society: forecasting the weather, predicting earth quakes, describing the effect of medicines, and so on.

二、 **Random Phenomenon**

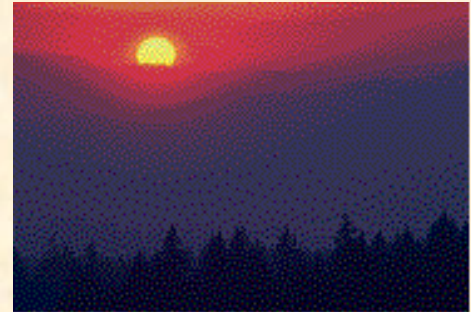Phenomenon in Real life: Certain; (random)uncertain

1.Certain Phenomenon

Definition:Given some conditions, the phenomenon must happen.

Example:

"**The Sun rises in the east**",

"**Water flows downwards from above**"

"**The same charges repel each other**"

Characteristic of the certain phenomenon ☞ **Conditions determine the outcome**

2. Random Phenomenon

Given the same conditions，the outcome will depend on **chance** and **can not** be predicted with certainty

Example1 "toss a coin, and observe that if the head appears".

The outcomes can be **Positive** or **negative**.

Example2 "fire a gun to a target many times, observe the location of the bullet".

Outcomes: "the location can be different".

Example3 "toss a die, observe the number showing on the top face"

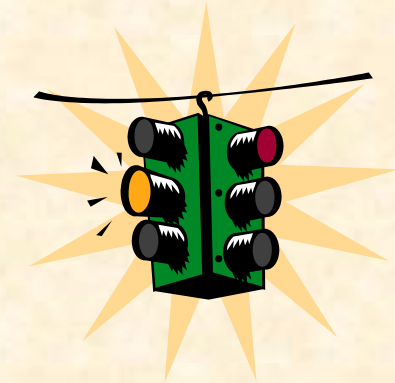Outcome:

"1", "2", "3", "4", "5" or "6".

Example4 "Select one product and observe if it is good".

Outcome:

good, bad

Example5 "Cross the road, and observe the traffic lights".
Outcome: red, green or yellow.

Example6 "Observe the service life of one fixed bulb" Outcome could be 0 or many years.

3. Classification of random phenomenon

Unrepeatable ： **phenomenon cannot appear in same condition（example 6）**

Repeatable： phenomenon can appear in same condition (example 1-5)

Characteristic of random phenomenon   **Condition can not decide the outcomes**

Note

1. **Random phenomenon** shows the **uncertain** relationship between **conditions** and the **outcomes**

2. **Observe once, the outcome is** uncertain, but if you experiment many times, the outcomes have laws. Probability is a course to study this kind laws of random phenomenon.

How to study random phenomenon?

By performing random experiments.

question   what is random experiments?

三、 **Random experiment**

Definition: the experiment with the following **three properties** is called **random experiment**.

1. the E can be done **repeatedly** under the same condition;

2. The outcome is more than one, and the outcomes are known;

3. Before the experiment, the outcome is uncertain

Note

1. Random experiment is denoted by E .

Example "**Toss a coin, and observe if the head side appears or not**".

Analysis

(1) The experiment can be conducted repeatedly in the same conditions.

(2) Outcomes are two:

head，tail;

(3) Before the experiment, we can

not predict the outcome.   **Conclusion: YES.**

Similarly, the following experiment are also E.

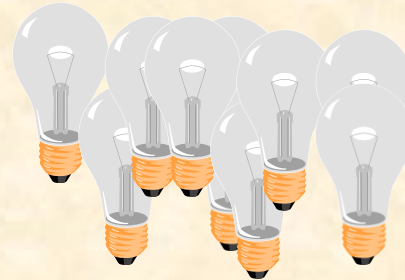**1."toss a balance die,observe the number".**

2."select 3 products, count the

number of good product".

3. Count the no. of people waiting for bus.

4. Test the service life of one bulb from a pile of bulbs.

四、 **Sample space, sample point**

Question: **How to describe the outcome of the E?**

Set theory provides us with useful approaches.

Definition. **The set of all possible outcomes of a E is called the sample space, denoted by** Ω.

The element of the Ω **is called sample point, i.e.** *E's outcome*(**which cannot be decomposed**), **denoted by** ω.

Example1. Toss a coin, observe if the head happens ,

$$\Omega_1 = \{H, T\}.$$

Example2. Toss a die, observe the no. on the top face.

$$\Omega_2 = \{1, 2, 3, 4, 5, 6\}.$$

Example3. Select 3 products, and observe the situation of the products(good or bad)

Denote good product by Z and bad one by C .

$$\Omega_3 = \{\, ZZZ,\ ZZC,\ ZCZ,\ CZZ,$$
$$ZCC,\ CCZ,\ CZC,\ CCC\}.$$

Example4 Count the no. waiting for the bus at 10 o'clock .

$$\Omega_4 = \{0, 1, 2, \cdots\}.$$

Example5 Observe the average temperature in April.

$$\Omega_5 = \{t \mid T_1 < t < T_2\}.$$

Example6 Select one bulb from a pile of bulbs, and test the service life.

$$\Omega_6 = \{t \mid t \geq 0\}.$$

Example7 Record the no. of the 120 emergency call in Xi'an last night.

$$\Omega_7 = \{0, \quad 1, \quad 2, \cdots\}.$$

class exercise

What are the sample spaces?

1.Toss three dies, and observe the sum of the number of the dies .

2. An experiment consists of tossing a coin and then flipping it a second time if a head occurs. If a tail occurs on the first flip then a die is tossing once .

Note   1. With different Es, sample spaces are different .

2. For the same E, if we are interested in different aims, then the sample spaces are different.

Example : "**toss one coin there times**" ,

if we observe if positive side occurs, then

$$\Omega = \{HHH , HHT , HTH , THH ,$$
$$HTT , TTH , THT , TTT \}.$$

If we observe the number of the head happing, then

$$\Omega = \{0, \quad 1, \quad 2, \quad 3\}.$$

The first step to describe the random experiment is to built the sample space.

五、The concept of random event

For the E，normally we pay attention if some event will happen or not, Which is random event.

1. Basic conception

(Random) event   the subset of the Ω.

Example toss a die and observe the number.

the number on the face of the die is "1",
"2", ... ,"6", "less than 4", "even" are all events.

Certain event: the event that will always happen is called certain event.

Example: tossing a die, the number "less or equal to 6"is certain event.

The sample space is also certain event.

impossible event: the event which can not happen.

Example   Toss a die "**the number is greater than 6** " is impossible event.

> Certain event is the opposite of impossible, vise versa, they are complementary events.

2. note   ↔

(1) **normally, we use capital letter $A,B, C, …$to denote events.**

For example, tossing a die, and observe the number.

we can suppose A = "number less than 4",

B = "number more than 5" and so on .

(2) Relation among E, sample space and events

Every E has one sample space, and the subset of sample space is called event.

Experiment $\longrightarrow$ Sample space $\xrightarrow{\text{subset}}$ event

Event $\left\{\begin{array}{l} \text{Certain event} \\ \\ \\ \text{Impossible event} \end{array}\right.$

六、Conclusion

1.Characteristic of random phenomenon:

Condition can not decide outcomes.

2. Phenomenon can be studied by experiment.

Experiment
$$\begin{cases} \textbf{(1) repeated ;} \\ \textbf{(2) The outcomes is more than 2,} \\ \quad \text{and the sample space is konwn;} \\ \\ \textbf{(3)Before the exercise, the outcome is} \\ \textbf{uncertain.} \end{cases}$$