logic

- Propositional logic
- Predicate logic
- Proof

Propositional logic

- The truth tables of five logic operators

Not and or conditional biconditional

- Logical equivalences

$P \to Q \Leftrightarrow \neg P \vee Q$ Implication

- Use truth table and propositional calculus(命题演算) to determine tautology or logical equivalence.

- Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

$$p \leftrightarrow q$$
$$= (p \to q) \wedge (q \to p)$$
$$= (\neg p \vee q) \wedge (\neg q \vee p)$$
$$= (\neg p \vee q) \wedge \neg q \vee (\neg p \vee q) \wedge p$$
$$= \neg p \wedge \neg q \vee \underbrace{q \wedge \neg q}_{F} \vee \underbrace{\neg p \wedge p}_{F} \vee q \wedge p$$
$$= \neg p \wedge \neg q \vee q \wedge p$$

Predicate logic

- Translating simple English sentences into predicates and quantifiers.

Eg： Express the statement“ Some students in this class do not like coffee.” using predicates and quantifiers where P(x) represents the statement that x is in this class, and Q(x) represents the statement that x likes coffee. (C)

A. $\forall x\ (P(x) \rightarrow \neg Q(x))$ B. $\forall x\ (P(x) \wedge \neg Q(x))$

C. $\exists x\ (P(x) \wedge \neg Q(x))$ D. $\exists x\ (P(x) \wedge Q(x))$

Predicate logic

- The order of quantifiers

Statement	When True?	When False
$\forall x \forall y P(x,y)$ $\forall y \forall x P(x,y)$	*P*(*x*,*y*) is true for every pair *x*,*y*.	There is a pair *x*, *y* for which *P*(*x*,*y*) is false.
$\forall x \exists y P(x,y)$	For every *x* there is a *y* for which *P*(*x*,*y*) is true.	There is an x such that *P*(*x*,*y*) is false for every *y*.
$\exists x \forall y P(x,y)$	There is an *x* for which *P*(*x*,*y*) is true for every *y*.	For every *x* there is a y for which *P*(x,y) is false.
$\exists x \exists y P(x,y)$ $\exists y \exists x P(x,y)$	There is a pair *x*, *y* for which *P*(*x*,*y*) is true.	*P*(x,y) is false for every pair *x*,*y*

- Assume that P(x, y) means "x + y = 0", where x and y are integers. Then the truth value of the statement $\exists y\ \forall x\ P(x, y)$ is ___false / 0_____.

Proof

Distinguish propositional proof and predicate proof

Use direct, contradiction, cp… methods to deduce

Using rules to build arguments

- Generally, if we verify the argument with quantifiers is valid, the first step, we use UI or EI rules to remove quantifiers, then, we use rules of inference or basic logic equivalences to proof, the last step, if there exists quantifiers in conclusion, here, we need to use UG or EG rules to add it.
- If there are existential and universal quantifiers in premises, EI rule is usually first used, then UI rule.

Construct an argument to show that the premises $\neg p \wedge q$, $r \to p$, $\neg r \to s$ and $s \to t$ lead to the conclusion t. (cp rule)

Proof :

1. $\neg p \wedge q$ P
2. $\neg p$ Simplification from 1.
3. $r \to p$ P
4. $\neg p \to \neg r$ Contrapositive identity from 3
5. $\neg r \to s$ P
6. $\neg p \to s$ Hypothetical syllogism from 4 and 5
7. $s \to t$ P
8. $\neg p \to t$ Hypothetical syllogism from 6 and 7
9. t Modus ponens from 2 and 8

Set

- power set, Cartesian product, Cardinality
- Can do set operations
- One-to-one, onto, bijective, inverse, composition

- Let A ={a,{a}} and its power set is P(A), which statement is false? (B)

A. $\{a\} \in P(A)$. B. $\{a\} \subseteq P(A)$. C. $\{\{a\}\} \in P(A)$. D. $\{\{a\}\} \subseteq P(A)$.

- Suppose that $f:[0,1] \rightarrow [a,b]$, where $a<b$, $f(x)=(b-a)x+a$. f is (B)

A. one-to-one, not onto B. one-to-one and onto

C. one-to-one D. onto

- Let A and B be sets. If $|A|=m$, $|B|=n$, then the number of the relations from A to B is ___ 2^{mn} ___.

Relations

- Definition, properties of relation

Reflexive, symmetric ,antisymmetric, transitive

- Composite of R and S
- Closures (reflexive symmetric transitive)

Reflexive closure of R is $R \cup I_A$

Symmetric closure of R is $R \cup R^{-1}$

If $|A|=n$, then the transitive closure of R is

$$\bigcup_{i=1}^{n} R^i = R \cup R^2 \cup \cdots \cup R^n$$

- Equivalence Relations
- Equivalence Classes
- Partitions
- Partial orderings
- Hasse diagram

- Let $R=\{(1,1), (2,1),(3,2),(4,3)\}$, then $R^2 = \underline{\{(1,1), (2,1), (3,1), (4,2)\}}$, and $R^3 = \underline{\{(1,1), (2,1),(3,1),(4,1)\}}$.

- Let $A=\{1,2,3,4\}$. $R_1=\{(1,1),(2,2),(2,3),(4,4)\}$ and $R_2 = \{(1,1),(2,2),(2,3),(3,2),(4,4)\}$ are the relations on the set A, then R_2 is (B) of R_1.

  A. reflexive closure B. symmetric closure

  C. transitive closure D. symmetric and transitive closure

- $A=\{a,b,c,d,e,f\}$, $R=\{a,a\rangle, \langle b,b\rangle, \langle c,c\rangle, \langle a,b\rangle, \langle b,a\rangle, \langle a,c\rangle, \langle c,a\rangle, \langle b,c\rangle, \langle c,b\rangle, \langle d,d\rangle, \langle e,e\rangle, \langle d,e\rangle, \langle e,d\rangle, \langle f,f\rangle\}$,

Equivalence classes are :

$$[a] = [b] = [c] = \{a,b,c\}$$
$$[d] = [e] = \{d,e\}$$
$$[f] = \{f\}.$$

Partition is: $\{[a],[\mathrm{d}],[\mathrm{f}]\}$

Suppose that A={1,2,3,4,6,12}, and R={(a,b) | a divides b}is a partial ordering on set A. Draw the Hasse diagram of R.

a) Find the minimal element, the maximal element, the greatest element, the least element, the least upper bound and the greatest lower bound of the subset {3,4,6,12}.

- The minimal element are 3, 4. The maximal element is 12.
- The greatest element is 12. This subset has no least element.
- Lub is 12 and glb is 1.

Graph

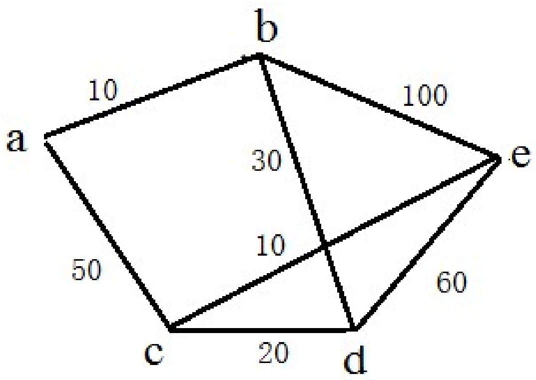
- the degree of vertices in the graph.
- handshaking theorem
- Adjacency matrix
- Determine isomorphism
- Shortest path method
- Euler theorem

- Suppose that A={1,2,3}. Then the matrix of R={(1,1),(1,2),(2,2),(2,3),(3,3)} on the set A is _____.

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Use Dijkstra's algorithm to find the shortest path from a to e.

The shortest path is 60. The path is a, c, e.



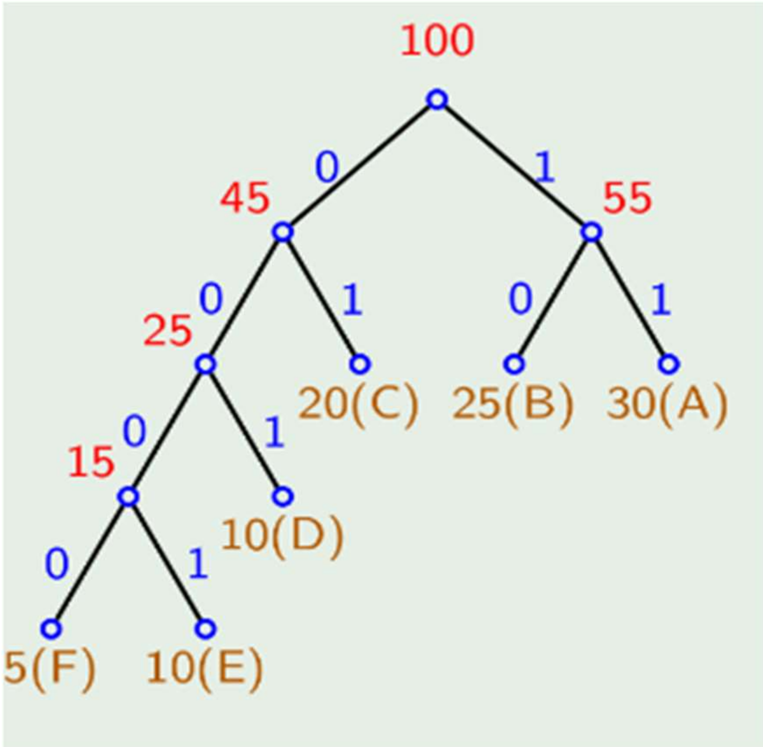
S	b	c	d	e
{a}	10	50	∞	∞
{a.b}	10	50	40	110
{a.b.d}	10	50	40	100
{a.b.d.c}	10	50	40	60

Tree

- Difinition
- Properties m=n-1
- Spanning tree
- Minimum spanning tree
- Prefix condes

Use Huffman coding to encode the following symbols with the frequencies listed:

A: 30% **B: 25%**
C: 20% **D : 10%**
E: 10% **F: 5%**



Counting

TABLE 1 Combinations and Permutations With and Without Repetition.

Type	*Repetition Allowed?*	*Formula*
r-permutations	No	$\dfrac{n!}{(n-r)!}$
r-combinations	No	$\dfrac{n!}{r!\,(n-r)!}$
r-permutations	Yes	n^r
r-combinations	Yes	$\dfrac{(n+r-1)!}{r!\,(n-1)!}$

Advanced counting

- The principle of inclusion-exclusion
- Solving linear recurrence relations
- Homogeneous, nonhomogeneous

Find characteristic equation

Get roots

Use initial conditions to get constants

Particular solution

Find the solutions of the recurrence relation $a_n = -5a_{n-1} - 6a_{n-2} + 42 \cdot 4^n$ with $a_1=56$ and $a_2=278$.

Hint: The recurrence relation has a solution of the form $a_n = C \cdot 4^n$, where C is constant.

a) How many solutions does the equation $x_1+x_2+x_3=17$ have, where x_1, x_2, x_3 are nonnegative integers?

b) Find the number of solutions with $2 \le x_1 \le 5$, $3 \le x_2 \le 6$ and $4 \le x_3 \le 7$.