Matrix for the composite of relations

Suppose that $X=\{x_1,x_2,\ldots,x_m\}$, $Y=\{y_1,y_2,\ldots,y_n\}$, $Z=\{z_1,z_2,\ldots,z_p\}$, R is a relation from X to Y, and S is a relation from Y to Z. $M_R=[a_{ij}]$ is an $m\times n$ matrix and $M_S=[b_{ij}]$ is a $n\times p$ matrix.

The matrix for $S\circ R$ is $M_{S\circ R}=[c_{ij}]=M_R\cdot M_S$,

where $c_{ij}=\bigvee_{k=1}^{n} a_{ik}\wedge b_{kj}$

$$i=1,2,\cdots m;\ j=1,2,\cdots p$$

Example

- $X=\{1,2\}, Y=\{a,b,c\}, Z=\{\alpha,\beta\}$
- $R=\{<1,a>,<1,b>,<2,c>\}$， $S=\{<a,\beta>,<b,\beta>\}$

$$M_R=\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \qquad M_S=\begin{bmatrix}0 & 1\\ 0 & 1\\ 0 & 0\end{bmatrix}$$

$$M_{R\cdot S}=M_R\cdot M_S=\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}0 & 1\\ 0 & 1\\ 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$$

homework

P626

8. a)

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$R^n \subseteq R \iff R$ is transitive.

$n = 1, 2, 3 \cdots$

$$M_{R^2} = M_{R \circ R} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

①. The number of ones in M_{R^2} is less than or equal to the number of ones in M_R.

② If there is a 1 in M_{R^2}, there is a 1 in the orginal matrix M_R at corresponding position.

M_{R^2} doesn't satisfy these two conditions. So $R^2 \nsubseteq R$.

So R is not transtive.

homework

P145.

32. a) : $A=\{1\}$, $B=\{2\}$. $C=\{1, 2\}$

$A \cup C = B \cup C$. ~~B~~ but $A \neq B$

b) $A=\{1\}$ $B=\{2\}$, $C=\phi$

$A \cap C = B \cap C = \phi$ but $A \neq B$

c) $(A \cup C = B \cup C) \wedge (A \cap C = B \cap C) \Rightarrow A = B$

proof: 1) $A \subseteq B$.

Suppose that $x \in A$. If $x \in C$, then $x \in A \cap C = B \cap C$, so $x \in B$. On the other hand. If $x \notin C$. because $x \in A \cup C = B \cup C$. We also get $x \in B$. So $A \subseteq B$.

Using the same way. we get $B \subseteq A$.

So $A = B$.

homework

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b). $A_1 = \{0, 1\}$, $A_2 = \{0. 2\}$, $A_3 = \{0, 3\}$, ...

$\cup A_i = \{0. 1. 2, 3. \cdots\} = N$ (Natural numbers)

$\cap A_i = \{0\}$

c) $A_1 = (0, 1)$, $A_2 = (0, 2)$, $A_3 = (0, 3)$, ...

where $A_1 \subset A_2 \subset A_3 \subset \cdots$

$\cup A_i = (0. +\infty) = R^+$

$\cap A_i = (0. 1)$

d) $A_1 = (1. \infty)$, $A_2 = (2. \infty)$, $A_3 = (3, \infty)$ ....

where $A_1 \supset A_2 \supset A_3 \supset \cdots$

$\cup A_i = (1. \infty)$

$\cap A_i = \phi$

homework

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$\neg(\forall x\, P(x)) \Leftrightarrow \exists x \neg P(x)$

$\neg(\exists x\, P(x)) \Leftrightarrow \forall x \neg P(x)$

a)

$$\begin{aligned}\neg\exists z\forall y\forall x\, T(x,y,z) &\equiv \forall z\neg\forall y\forall x\, T(x,y,z)\\ &\equiv \forall z\exists y\neg\forall x\, T(x,y,z)\\ &\equiv \forall z\exists y\exists x\, \neg T(x,y,z)\end{aligned}$$

b)

$$\begin{aligned}\neg(\exists x\exists y\, P(x,y)\wedge\forall x\forall y\, Q(x,y)) &\equiv \neg\exists x\exists y\, P(x,y)\vee\neg\forall x\forall y\, Q(x,y)\\ &\equiv \forall x\neg\exists y\, P(x,y)\vee\exists x\neg\forall y\, Q(x,y)\\ &\equiv \forall x\forall y\,\neg P(x,y)\vee\exists x\exists y\,\neg Q(x,y)\end{aligned}$$

c)

$$\begin{aligned}\neg\exists x\exists y(Q(x,y)\leftrightarrow Q(y,x)) &\equiv \forall x\neg\exists y(Q(x,y)\leftrightarrow Q(y,x))\\ &\equiv \forall x\forall y\neg(Q(x,y)\leftrightarrow Q(y,x))\\ &\equiv \forall x\forall y(\neg Q(x,y)\leftrightarrow Q(y,x))\end{aligned}$$

d)

$$\begin{aligned}\neg\forall y\exists x\exists z\,(T(x,y,z)\vee Q(x,y)) &\equiv \exists y\neg\exists x\exists z\,(T(x,y,z)\vee Q(x,y))\\ &\equiv \exists y\forall x\neg\exists z\,(T(x,y,z)\vee Q(x,y))\\ &\equiv \exists y\forall x\forall z\,\neg(T(x,y,z)\vee Q(x,y))\\ &\equiv \exists y\forall x\forall z\,(\neg T(x,y,z)\wedge\neg Q(x,y))\end{aligned}$$