Discrete Mathematics

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Importance

- Discrete mathematics is the part of mathematics devoted to the study of discrete objects.
- According to this course, You can develop mathematical maturity(ability to understand and create mathematical arguments). This is important for further studies.
- It is gateway to more advanced courses including data structures, algorithms, database, and operating systems.

Application

- Logic has many applications to computer science, such as the design of computer circuits, the construction of computer programs, the verification of the correctness programs
- Sets and relations can be applied to Database Management System(add delete combine……)
- Determine whether two computers are connected by a communications link
- Design the shortest path from one server to another server

How to learn it?

- The best way to learn a mathematics is understand it and do **a lot of exercises.**
  - Knowledge—learn the concepts, rules and how to proof them.
  - Skill—learn how to abstract the real world problem into mathematic model, how to apply the knowledge solve problems.

Credit

- Daily practice (homework and attendance)
- Final examination
- The homework will mainly focus on routine exercises.

Key to the Exercises

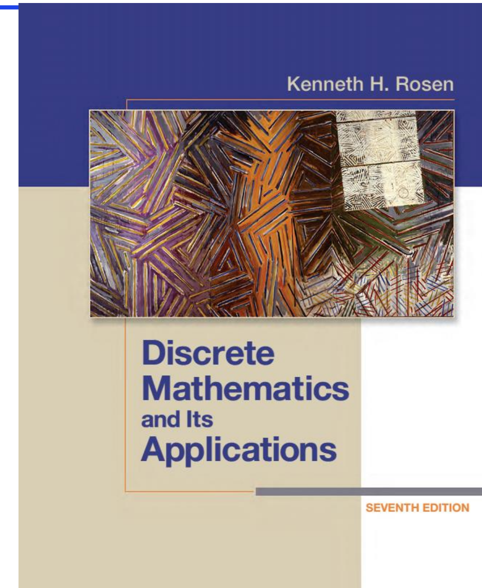
no marking	A routine exercise
*	A difficult exercise
**	An extremely challenging exercise
☞	An exercise containing a result used in the book (Table 1 on the following page shows where these exercises are used.)
(*Requires calculus*)	An exercise whose solution requires the use of limits or concepts from differential or integral calculus

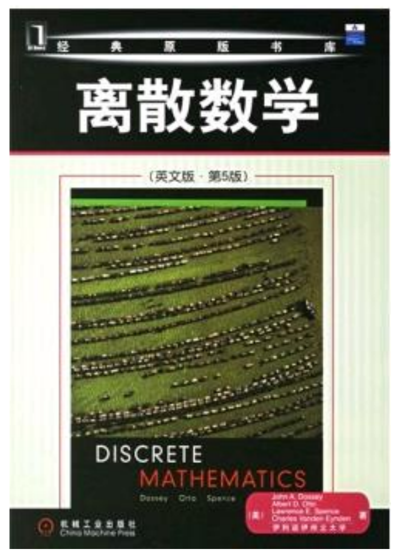
Homework

- Assistant instructor help me check your work;
- Upload homework to qq group on every Friday ;
- If you don’t load qq, send your work to assistant instructor by email.

Reference

- Text：
  - Discrete Mathematics and its Application, Kenneth H.Rosen（8th version)， 2021
- Other reference Text
  - Discrete Mathematics for Computer Sciences, Clifford stein,2010
  - Discrete mathematics, John, A.Dassey , 2007





机械工业出版社
CHINA MACHINE PRESS

Schedule

1. Chapter 1 Logic and Proofs
2. Chapter 2 Sets, Functions, Sequences, Sums and Matrices
3. Chapter 9 Relations
4. Chapter 6 Counting
5. Chapter 8 Advance counting
6. Chapter 10 Graphs
7. Chapter 11 Trees

Discrete Mathematics

The Foundations: Logic and Proofs

Contents

Logic

Proof

Foundations of Logic

- Propositional logic (§ 1.1-1.3): 命题逻辑
  - Basic definitions. (§ 1.1)
  - Applications (§ 1.2)
  - Equivalence rules (§ 1.3)
- Predicate logic (§ 1.4-1.5) 谓词逻辑
  - Predicates.
  - Quantified predicate expressions.
  - Equivalences

Propositions（命题）

- **Proposition** is a declarative sentence（陈述句）that is either *true* or *false*,but not both.
  - true = T (or 1)
  - false = F (or 0)

Example

- ‘The moon is made of green cheese’
- ‘go to town!’
- X – *imperative (祈使句）*
- ‘What time is it?’
- X – *interrogative （疑问句）*

Example

- “Beijing is the capital of China.
- $1 + 2 = 2$.

But, the following are **NOT** propositions:

- “Who’s there?” (interrogative)
- “y= x+1” （uncertain truth value）

Example

- Which of the following are statements?
  - (a) The earth is round.
  - (b) 2+3=5
  - (c) Do you speak English?
  - (d) $3-x=5$
  - (e) Take two aspirins.
  - (f) The sun will come out tomorrow.

Symbol

- Letters are used to denote propositions p, q, r, s, . . .
- p: The earth is round
- Can we get more propositions from those that we have? How can get?
- q: The earth is not round
- p has no relation with q?

Logical Operators(逻辑运算）

- **Negation（否定）**
- **Conjunction（合取）**
- **Disjunction （析取）**
- **Exclusive OR （异或）**
- **Implication （蕴涵）**
- **Bi implication （等价）**

Negation

Definition1:

Let p be a proposition. The statement"It is not the case that p" is another proposition, called the negation of p. The negation of p is denoted by¬p. It is read "not p".

E.g. p : I have brown hair.

$\neg p$: I do **not** have brown hair.

真值表TRUTH TABLES

- Write all possible truth value of proposition in the first column
- The second column shows the truth value of ¬ p corresponding to the truth value of p in the same row

p	$\neg p$
T	F
F	T

Operand column

Result column

Conjunction合取

Definition2:

Let p and q be propositions. The conjunction of p and q, denoted by p $\wedge$q, is the proposition “p and q”.

E.g. p:I will have salad for lunch.

q:I will have steak for dinner.

$p \wedge q$: I will have salad for lunch **and** I will have steak for dinner.

Truth Table for Conjunction

- The conjunction is true when both p and q are true.
- Note that a conjunction $p_1 \wedge p_2 \wedge \ldots \wedge p_n$ of n propositions will have 2^n rows in its truth table.

Operand columns

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Disjunction析取

Definition3:

Let p and q be propositions. The disjunction of p and q, denoted by p $\vee$ q, is the proposition "p or q".

E.g. *p*:My car has a bad engine.

q: My car has a bad door.

$p \vee q$: Either my car has a bad engine, **or** my car has a bad door."

Truth Table for Disjunction

- *p∨q is true* means that *p* is true, or *q* is true, **or both** are true!
- Note that it is also called *inclusive or*（兼或）, because it **includes** the possibility that both *p* and *q* are true.

p	q	$p \lor q$
F	F	F
F	T	**T**
T	F	**T**
T	T	T

Note difference from AND

Exclusive Or

Definition4:

Let p and q be propositions. The exclusive or of p and q, denoted by p ⊕ q, is the proposition.

p : I will earn an A in this course.

q : I will drop this course.

$p \oplus q$: I will either earn an A in this course, or I will drop it (but not both!)”

Truth Table for Exclusive-Or

- Note that $p \oplus q$ *is true* means *p* is true, or *q* is true, but **not both**!
- This operation is called *exclusive or,* because it **excludes** the possibility that both *p* and *q* are true.

p	q	$p \oplus q$
F	F	F
F	T	T
T	F	T
T	T	**F**

Note difference from OR.

Conditional statement (implication)

Definition 5: Let p and q be propositions. The conditional statement $p \to q$ is the propositions "if p, then q."

--The statement *p* is called the antecedent (前件）, hypothesis（假设） condition.

--The statement *q* is called the *consequence* (后件)or *conclusion*（结论）.

E.g., let *p* = "You study hard."

q = "You will get a good grade."

$p \to q$ = "If you study hard, then you will get a good grade."

Truth Table for Implication

- It can be think of a promise.
- $p \rightarrow q$ is **false** <u>only</u> when p is true but q is **not** true.
- $p \rightarrow q$ does **not** require that p or q **<u>are true</u>**!
- p may be has no cause-and-effect relation with q.
- E.g.1 If Tom get 100 scores in final exam, he will invite his friends to dinner.
- E.g.2 "(1=0) $\rightarrow$ pigs can fly" is TRUE!

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	**F**
T	T	T

The <u>only</u> False case!

Different Ways of Expressing $p \rightarrow q$

- "*p* implies *q*"
- "if *p*, then *q*"
- "if *p*, *q*"
- "when *p*, *q*"
- "whenever *p*, *q*"
- "*q* if *p*"
- "*q* when *p*"
- "*q* whenever *p*"
- "*p* only if *q*"
- "*p* is sufficient for *q*"
- "*q* is necessary for *p*"
- "*q* follows from *p*"
- "*q* is implied by *p*"
- "*q* unless $\neg p$"

Biconditional Statement

Definition 6:

Let p and q be propositions. The biconditional statement p $\leftrightarrow$ q is the propositions “p if and only if q.” (Bi-implication)

p = “Barack Obama won the *2012* presidential *election*.”

q = “Barack Obama was president for all of 2013.”

p $\leftrightarrow$ *q* = “If, and only if, Barack Obama won the *2012* presidential *election*, Barack Obama was president for all of 2013.”

Truth Table for Bi-implications

- P is necessary and sufficient for q.
- $p \leftrightarrow q$ means that p and q have the **same** truth value.

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

Converse, Contrapositive, and Inverse

- From $p \to q$ we can form new conditional statements .

  $q \to p$ is the **converse** of $p \to q$ （逆命题)

  $\neg q \to \neg p$ is the **contrapositive** of $p \to q$（逆反命题)

  $\neg p \to \neg q$ is the **inverse** of $p \to q$(反命题)

Example: Find the converse, inverse, and contrapositive of "If it is raining, I will not go to town."

Solution:

converse: If I do not go to town, then it is raining.

inverse: If it is not raining, then I will go to town.

contrapositive: If I go to town, then it is not raining.

Contrapositive

- One of these has the *same meaning* (same truth table) as $p \rightarrow q$. Can you figure out which?

原命题为：p－－＞q

converse 逆命题为：q－－＞p

inverse 否命题为：非p－－＞非q

逆否命题为：非q－－＞非p

Contrapositive

How do we know for sure?

Proving the equivalence of $p \to q$ and its contrapositive using truth tables:

p		q	$\neg q$		$\neg p$	$p \to q$	$\neg q \to \neg p$
F	$\to$	F	T	$\to$	T	T	T
F	$\to$	T	F	$\to$	T	T	T
T	$\not\to$	F	T	$\not\to$	F	F	F
T	$\to$	T	F	$\to$	F	T	T

Propositions in Propositional Logic

- Atomic propositions: *p*, *q*, *r*, …
  (I had salad for lunch)
- Complex propositions : built up from atoms using operators: $p \wedge q$
  (I had salad for lunch **and** I had steak for dinner)

propositions

Atoms propositions： can not be divided into smaller propositions

Complex propositions： built up from atoms or complex propositions using operators

Compound Propositions

- Use atomic propositions and logic operators to form compound propositions ;
- Use truth table to get the truth values of compound propositions;

$$(P \vee q) \rightarrow \neg R$$

Precedence of logical operators

- Priority level

  $\neg \quad \wedge \quad \vee \quad \rightarrow \quad \leftrightarrow$

- $\neg p \wedge q$ means the conjunction of $\neg p$ and q
- Precedence of logical operators can reduce the number of paretheses of compound propositions.
- $(p \wedge q) \vee r$ means $p \wedge q \vee r$
- $(P \vee q) \rightarrow \neg r$ means $p \vee q \rightarrow \neg r$

Truth Tables For Compound Propositions

- Construction of a truth table:
- Rows
  - Need a row for every possible combination of values for the atomic propositions.
- Columns
  - Need a column for the compound proposition (usually at far right)
  - Need a column for the truth value of each expression that occurs in the compound proposition as it is built up.
    - This includes the atomic propositions

Example

- Construct a truth table for

$(p \vee q) \rightarrow \neg r$

p	**q**	**r**	**¬r**	**p ∨ q**	**p ∨ q → ¬r**
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T

Exercise

- 1. If value of p,q,r,s is 1,1,0,0, calculate the value of formula:

$$(P \vee (Q \rightarrow (R \wedge \neg P))) \leftrightarrow (Q \vee \neg S)$$

- 2. Give the truth table of following fomula:

$$(P \wedge Q \rightarrow R) \rightarrow P$$

answer

- 1. $(P \vee (Q \rightarrow (R \wedge \neg P))) \leftrightarrow (Q \vee \neg S)$
  $\Leftrightarrow (1 \vee (1 \rightarrow (0 \wedge \neg 1))) \leftrightarrow (1 \vee \neg 1)$
  $\Leftrightarrow 1$
- 2.
-

P	Q	R	P∧Q	P∧Q->R	(P∧Q->R) ->P
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	0	1	0
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

Logic and bit operations

- Computers represent information using bits
- A bit has two possible values, it can be used to represent a truth value
- 1 represents T, 0 represents F
- Bit operations correspond to the logical connectives
- $\wedge$ $\vee$ $\oplus$
- OR AND XOR

Bitwise Operations

- Boolean operations can be extended to operate on bit strings as well as single bits.
- E.g.:

  01 1011 0110

  <u>11 0001 1101</u>

  Bit-wise OR

  Bit-wise AND

  Bit-wise XOR

Homework

- § 1.1 2, 13, 19, 29