Section 2 Relationship and operations of events

- 一、Relationship and operations of events
- 二、Conclusions

一Relationship and operation of events

Notions:

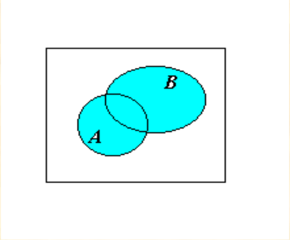
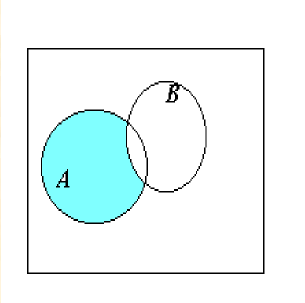
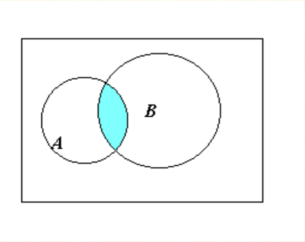
Sample point ω = **basic event**

Sample space Ω = **{sample point}**

= **certain event**

event = **subset of** Ω.

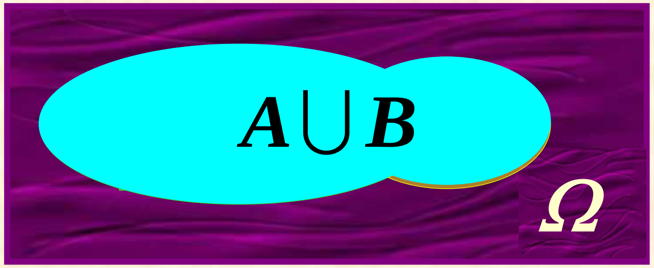
1.Operations(3 kinds）

operation	notation	Probability	Set theory	Venn graph
union	$A \cup B$	**At least A or B happens**	**A union B**	
minus	$A - B$	**A happens B not happen**	**A minus B**	
intersect	AB $or A \cap B$	**A happens and B happens**	**A intersects B**	

A ∪ B : at least one event happens

$$A \bigcup B = \{e \mid e \in A \quad or \quad e \in B\}.$$

e. g. **The door is suitable = both the length and height are suitable. Thus, {door not suitable} = {Length not suitable} U { height not suitable }**

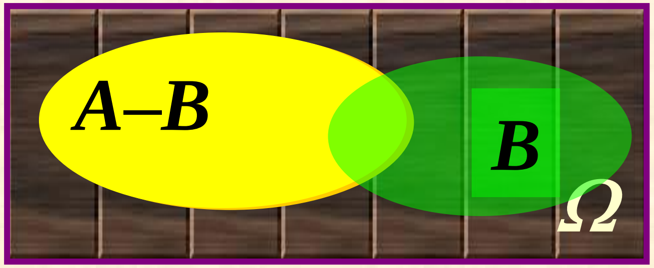


$A - B$: A happens, and B not happen.

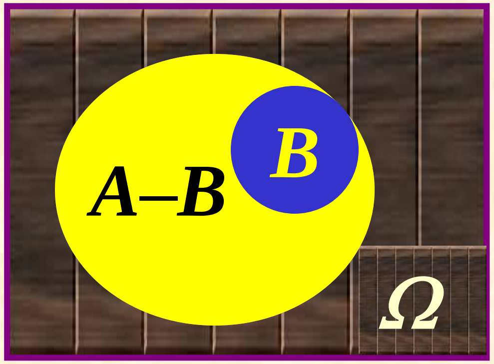
e.g., A: length is suitable. B: height is suitable , then A-B=length is suitable and the height is not suitable.

$A - B$

$B \not\subset A$



$B \subset A$



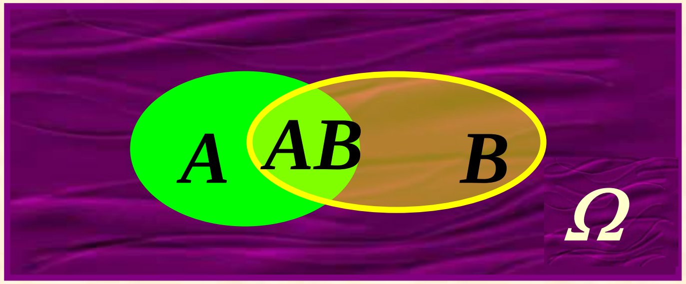
$$A \cap B, \quad A \cap B = \{e \mid e \in A \quad and \quad e \in B\}.$$

$$in \quad short \quad A \cdot B \; or \; AB.$$

E.g. A={ length suitable}, B={height suitable}

AB= "length suitable" and "height suitable" .

The intersection of A and B .



generalization ① $A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$:

At least one of $A_1, A_2, \cdots, A_n$ *happens*

$$A_1 \cup A_2 \cup \cdots \cup A_n \cdots = \bigcup_{i=1}^{\infty} A_i \ :$$

At least one of $A_1, A_2, \cdots, A_n, \cdots$ *happens.*

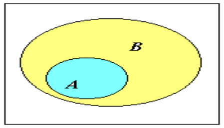
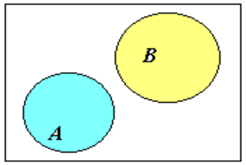
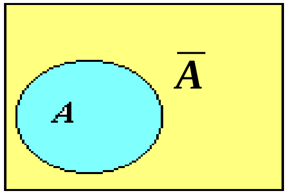
② $A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$:

All $A_1, A_2, \cdots, A_n$ happen.

$$A_1 \cap A_2 \cap \cdots \cap A_n \cdots = \bigcap_{i=1}^{\infty} A_i \ :$$

All $A_1, A_2, \cdots, A_n, \cdots$ *happen.*

2.Relationship(4 types）

relation-ship	notation	probability		Venn graph
include	$A \subset B$	A hapeens then B happens		
equal	$A = B$	$A \subset B$ $and \quad B \subset A$		
mutually exclude	$AB = \varnothing$	A and B do not happen at the same time		
Opposite	$\bar{A}$	A union $\bar{A}$ = sample space		

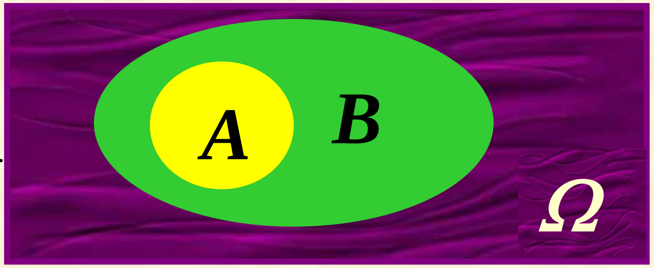
Include

B **includes** ***A*****,** **If** ***A*** **happens, then** ***B*** **happens .**

For short, $B \supset A$ *or* $A \subset B$.

e. g. **"B: length is suitable" includes**
"A: the door is suitable"

Graph ***B*** includes ***A***



Mutually exclusive

$$A \cap B = AB = \varnothing.$$

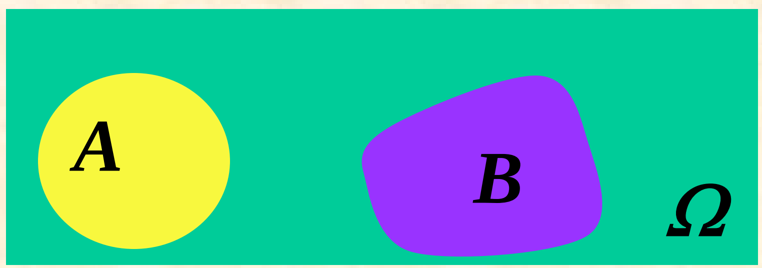
if A *happens, then B does not happen*, or if *B happens, then A does not happen*.

e.g. toss a coin, “A: head appears” and “B: tail appears”

e.g. Toss a die, observe the number.

"the die is 1" $\xleftrightarrow{\text{Mutually exclusive}}$ "the die is 2"

graph A and B are mutually exclusive



Note if $A \cap B = \varnothing$, $A \cup B$ can be denoted by $A+B$.

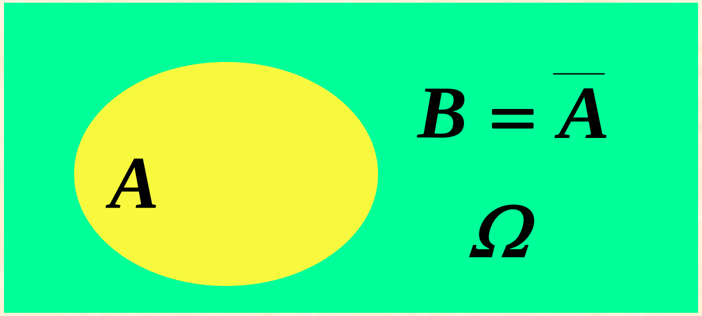
any event A and $\varnothing$ are mutually exclusive.

The opposite event A

if A : " A happens" , then "A does not happen" is *A's opposite event*, denoted by $\overline{A}$.

e.g. toss a die "no. is 1" $\xleftrightarrow{\text{opposite}}$ "no. is not 1."

Graph: A and B are opposite.



if A and B are opposite, then $A \cup B = \Omega$, $AB = \varnothing$.

Note. 1 °the relationship between mutually exclusive and opposite

$$\boxed{\textbf{ME} \quad \underset{\longleftarrow}{\not\longrightarrow} \quad \textbf{opposite}}$$

e.g. Toss a die, $A = \{2\}, B = \{5\}$

$\because AB = \varnothing \quad \therefore$ A and B mutually exclusive

but $A \cup B = \{2, 5\} \neq \Omega = \{1, 2, \cdots, 6\}$

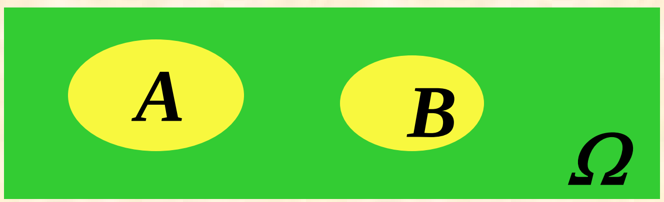
$\therefore$ A and B are not opposite.

$D = \{1, 3, 5\}$ and $G = \{2, 4, 6\}$ are opposite.

2 ° Ω and $\varnothing$ are opposite .

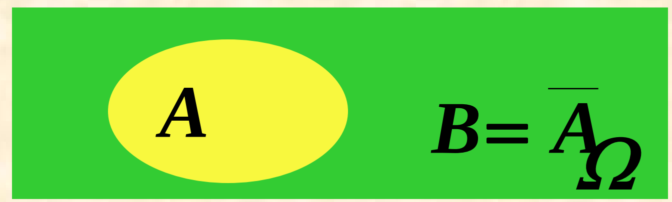
Difference between ME and opposition

A、B mutually exclusive



$AB=\varnothing,$

A、B opposite



$A\cup B=\Omega$ 且 $AB=\varnothing$.

Mutually exclusive $\underset{\longleftarrow}{\not\longrightarrow}$ opposite

3. Rules of operations of events

1.communication:

$$(1)\ A \cup B = B \cup A$$

$$(2)\ AB = BA$$

2. association:

$$(1)\ (A \cup B) \cup C = A \cup (B \cup C)$$

$$(2)\ (AB)C = A(BC)$$

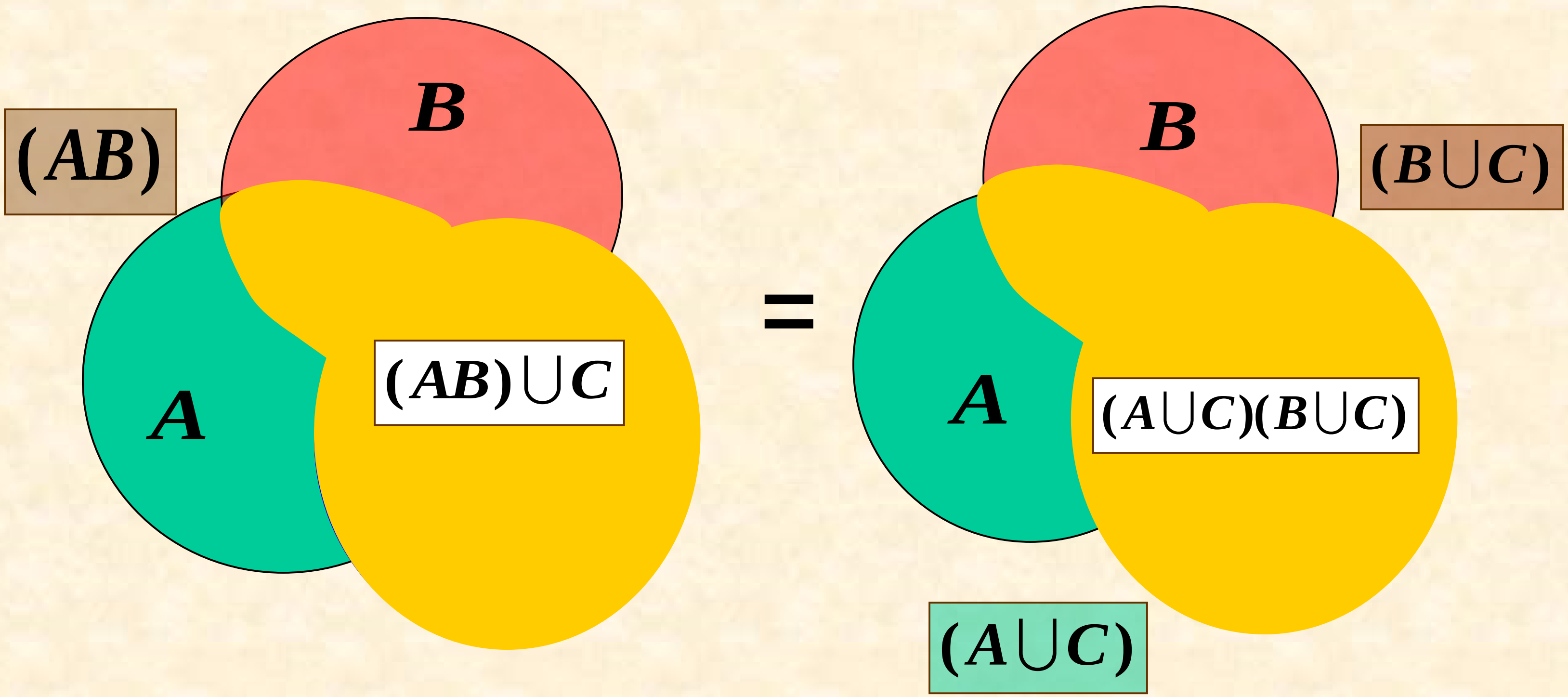
3.distribution

$$(1)\ (A \cup B)C = AC \cup BC$$

★ $$(2)\ (AB) \cup C = (A \cup C)(B \cup C)$$

$$A \cup (BC) = (A \cup B)(A \cup C)$$

Show the correction of formula ★ by veen graph



Note： just show the correction, the proof is not strict 。

4. (De Morgan theory)

$$(1)\ \overline{A \cup B} = \overline{A}\,\overline{B}$$

meaning: "A or B happen" 's opposite event is "both A and B do not happen".

$$(2)\ \overline{AB} = \overline{A} \cup \overline{B}$$

meaning: "both A and B happen" 's opposite event is "A does not happen or B does not happen.

generalization: $\overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} \overline{A_i}$

$$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$$

5. if $A \subset B$, then $A \bigcup B = B$, $AB = A$.

particularly, $A \bigcup \varnothing = A$, $A \bigcup \Omega = \Omega$

$$A\varnothing = \varnothing, \qquad A\Omega = A$$

e.g.1 **A，B are two events，try to prove：**

(1) $A\text{-}B=A\text{-}AB$

(2) $A\cup B=A+B\overline{A}=A\overline{B}+\overline{A}B+AB$

Proof : (1) $A-AB=A\overline{AB}$ $\quad (A-B=A\overline{B})$

$$=A(\overline{A}\cup\overline{B})$$

$$=A\overline{A}\cup A\overline{B}=\varnothing\cup A\overline{B}$$

$$=A\overline{B}=A-B$$

(2) $A+B\overline{A}=A\cup B\overline{A}$

$$=(A\cup B)(A\cup\overline{A})$$

$$=(A\cup B)\Omega=A\cup B$$

$$A\overline{B}+\overline{A}B+AB$$

$$=A(B+\overline{B})+\overline{A}B=A\Omega+B\overline{A}$$

$$=A+B\overline{A}=A\cup B$$

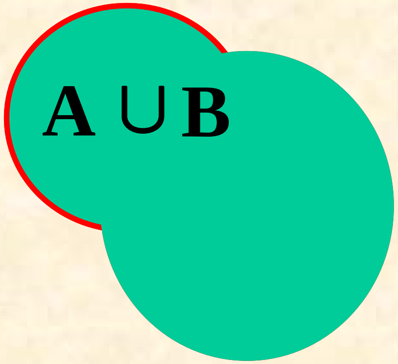
e.g.2 **true or false?**

$$(1)\ \overline{AB} = \overline{A}\,\overline{B}$$ ✗

A does not happen, or B does not happen

Both A, B do not happen

$$(2)\ A + (B - A) = B$$ ✗



Solution: **false.**

In fact, $A + (B - A) = A \cup B \neq B$

special case,

$$if \;\; A \subset B, \quad \text{then } A \bigcup B = B$$

Therefore, $\boldsymbol{A + (B - A) = A \bigcup B = B}$

$$\text{(3) } \boldsymbol{B(A - C) = BA - BC} \qquad \surd$$

Solution: true.

$$\boldsymbol{BA - BC = BA\overline{BC} = BA(\overline{B} \cup \overline{C})}$$

$$\boldsymbol{= BA(\overline{B} \cup \overline{C})}$$

$$\boldsymbol{= BA\overline{B} \cup BA\overline{C} = \varnothing \cup BA\overline{C}}$$

$$\boldsymbol{= BA\overline{C} = B(A - C)}$$

e.g. 3 Given A, B, C are events, describe the following events by operations of events A, B, C

(1) A happens and B, C do not happen.

solution: $A\overline{B}\,\overline{C}$ or $A\overline{B \cup C}$

(2) Only one event of A, B,C happens.

solution: $A\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C$

(3) Only two events of A,B,C happen

solution: $AB\overline{C} + A\overline{B}C + \overline{A}BC$

$$or\ AB \cup BC \cup AC - ABC$$

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(4) Not more than one event happens.

solution: $\overline{A}\,\overline{B}\,\overline{C}+A\overline{B}\,\overline{C}+\overline{A}B\overline{C}+\overline{A}\,\overline{B}C$

$$or \quad \overline{AB\cup BC\cup AC}.$$

e.g.4 **E = Choose one student from NPU，if**

A="the student selected is male"；

B="the student selected is in his/her first year"；

C="the student selected is athlete".

(1)Give the meaning of the event $A B \bar{C}$.

(2) Give the condition such that ABC=C?

(3) When $C \subset B$ always hold

Solution (1) $\boldsymbol{A B \bar{C}}$ means "the student selected is male, who is in his/her first year, but not athlete.

(2) $\because \boldsymbol{ABC \subset C}$

$\therefore ABC = C \;\; iff \;\; \boldsymbol{C \subset ABC}$

$moreover \because ABC \subset AB$

$\therefore ABC = C \;\; iff \;\; \boldsymbol{C \subset AB}$

$C \subset AB$ means "all the athletes are male and in his/her first year at NPU

(3) $when\ C \subset B$ always holds?

Solution: When all the athletes are in their first year, C is the subset of B, yielding

$$C \subset B.$$