ABID ALI_201938014

# Ex-2.4

3) What are the terms $a_0, a_1, a_2$ and $a_3$ of the sequence $\{a_n\}$, where $a_n$ equals:

a) $2^n+1$ : $a_0 = 2^0+1 = 2 / a_1 = 2^1+1 = 3 / a_2 = 2^2+1 = 5 / a_3 = 2^3+1 = 9$

b) $(n+1)^{n+1}$ : $a_0 = (0+1)^{0+1} = 1 / a_1 = (1+1)^{1+1} = 4 / a_2 = (2+1)^{2+1} = 27$

$a_3 = (3+1)^{3+1} = 256$

c) $\lfloor n/2 \rfloor$ : $a_0 = \lfloor \frac{0}{2} \rfloor = 0 / a_1 = \lfloor \frac{1}{2} \rfloor = \lfloor 0.5 \rfloor = 0 / a_2 = \lfloor \frac{2}{2} \rfloor = 1$

$a_3 = \lfloor 3/2 \rfloor = \lfloor 1.5 \rfloor = 1$

d) $\lfloor n/2 \rfloor + \lceil n/2 \rceil$ : $a_0 = \lfloor \frac{0}{2} \rfloor + \lceil \frac{0}{2} \rceil = 0 / a_1 = \lfloor 0.5 \rfloor + \lceil 0.5 \rceil = 1$

$a_2 = \lfloor \frac{2}{2} \rfloor + \lceil \frac{2}{2} \rceil = 2 / a_3 = \lfloor 3/2 \rfloor + \lceil 3/2 \rceil = 3$

12) Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if,

a) $a_n = 0$, So this is a constant value. Therefore,

$-3a_{n-1} + 4a_{n-2} = 3 \times 0 + 4 \times 0 = 0$

b) $a_n = 1$, one is constant. Therefore,

$-3 \times 1 + 4 \times 1 = -3 + 4 = 1$

~~c) $a_n = 6(-4)^n + 3 \Rightarrow -3a_{n-1} + 4a_{n-2}$~~

c) $a_n = (-4)^n \Rightarrow -3a_{n-1} + 4a_{n-2} = -3(-4)^{n-1} + 4(-4)^{n-2}$

$= 12(-4)^{n-2} + 4(-4)^{n-2} = 16(-4)^{n-2} = 4^2(-4)^{n-2} = (-4^n)$

d) $a_n = 2(-4)^n + 3 \Rightarrow -3a_{n-1} + 4a_{n-2} =$

$= -3\left(2(-4)^{n-1} + 3\right) + 4\left(2(-4)^{n-2} + 3\right) = -3\left(-8(-4)^{n-2} + 3\right) + 4\left(2(-4)^{n-2} + 3\right)$

$= 24(-4)^{n-2} - 9 + 8(-4)^{n-2} + 12 = \underbrace{32}_{\underbrace{2\times4\times4}_{4^2}}(-4)^{n-2} + 3 = 2(-4)^n + 3$

16 Find the solution to each of these recurrence relations with the given initial conditions.

a) $a_n = a_{n-1}$, $a_0 = 5$

$a_1 = -a_0 = 5$
$a_2 = -a_1 = 5$
$a_3 = -a_2 = 5$
$a_4 = -a_3 = 5$
$a_n = -a_{n-1} = (-1)^n \times 5$, $n \geq 1$

b) $a_n = a_{n-1} + 3$, $a_0 = 1$

$a_1 = 1 + 3$
$a_2 = (1+3) + 3 = 1 + 2\cdot3$
$a_3 = (1+2\cdot3) + 3 = 1 + 3\cdot3$
$a_n = 1 + 3n = \boxed{3n+1}$

c) $a_n = a_{n-1} - n$, $a_0 = 4$

$a_1 = 4 - 1 = 3$ / $a_2 = a_1 - 2 = 3 - 2 = 1$ / $a_3 = a_2 - 3 = 1 - 3 = -2$

$a_4 = a_3 - 4 = -2 - 4 = -6$ / $a_5 = a_4 - 5 = 10$

$\Rightarrow a_n = a_{n-1} - n$

$= a_{n-2} - (n-1) - n = a_{n-3} - (n-2) - (n-1) - n$

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$$\Rightarrow a_0 - 1 - 2 - \cdots - (n-2) - (n-1) - n$$

$$= 4 - [1+2+3+\cdots+n] = \boxed{4 - \frac{n(n+1)}{2} = a_n}$$

d) $a_n = 2a_{n-1} - 3$, $a_0 = -1$ $\quad \underbrace{-1, -5}_{-4}, \underbrace{-5, -13}_{-8} \cdots 0 \cdots$

① ②

$a_1 = 2(-1) - 3$

$a_2 = 2(2(-1) - 3) - 3 = 2 \cdot 2(-1) - 2 \cdot (+3) - 3 = -2^2 - (-3)(2^1 + 2^0)$

$a_3 = 2(2[2 \cdot (-1) - 3] - 3) - 3 = 2 \cdot 2 \cdot 2(-1) - 2^2 \cdot (+3) - 2(+3) - 3$

$= 2^3(-1) - 3(2^2 + 2^1 + 2^0)$

$a_n = -2^n - 3[1 + 2 + 2^2 + \cdots + 2^{n-1}]$

$= -2^n - 3\left(\frac{2^n - 1}{1}\right) \Rightarrow -2^n - 3 \cdot 2^n + 3 = \boxed{-4 \times 2^n + 3}$

e) $a_n = (n+1)a_{n-1}$, $a_0 = 2$ $\{4, 12, 48, 240 \cdots\}$

$a_1 = (1+1) \times 2 = 2 \times 2$

$a_2 = (2+1) \times (1+1) \times 2 = 3 \times 2 \times 2$

$a_3 = (3+1) \times \{(2+1)(1+1)2\} = 4 \times (3 \times 2 \times 2)$

$a_4 = (4+1) \times (\underbrace{4 \times 3 \times 2 \times 1}_{4!} \times 2)$

$$\boxed{a_n = (n+1)\, n! \times 2}$$

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f) $a_n = 2n a_{n-1}$ , $a_0 = 3$ $\{6, 24, 144, 1152 \ldots\ldots\}$

$a_1 = 2 \times 1 \times 3$

$a_2 = 2(2\times3)\times 2 = 2^2 \times 3 \times 2 = 4 \times 3 \times 2$

$a_3 = 2\times3(2^3\times3) = 2^4 \times 3 \times 3$

$a_4 = 2\times2^2(2^4\times3^2) = 2^5 \times 3^2 \times 4$

$a_5 = 2\times5(2^7\times3^2) = 2^6 \times 3 \times 5 \times 4 \times 3$

$a_n = 2n a_{n-1} = 2^2 n(n-1) a_{n-2} = 2^3 n(n-1)(n-2) a_{n-3}$

$= \ldots\ldots = 2^n n(n-1)(n-2) \ldots\ldots 1 . a_0 = 2^n n! \, 3$

$\Rightarrow \boxed{a_n = 2^n \times 3 n!}$

g) $a_n = -a_{n-1} + n - 1$ , $a_0 = 7$ $\{-7, 8, -6, 9, -5 \ldots$

$a_1 = -7 \cancel{+1-1}$

$a_2 = 7 \cancel{+2-1}$

$a_3 = -8 \cancel{+3^2-1}$

$a_4 = +6 + 3$

$a_5 = -9 + 5 - 1 = -5$

$\boxed{(-1)^n \times 7 + (n-1)}$

32) Find the value of each of these sums

a) $\sum_{j=0}^{8}(1+(-1)^j) = \sum_{j=0}^{8} 1 + \sum_{j=0}^{8}(-1)^j = 1(8) + \frac{1(-1)^{-2}-1}{\frac{-1-1}{-2}}$

$= 8+1 = 9$

b) $\sum_{j=0}^{8}(3^j - 2^j) = \sum_{j=0}^{8} 3^j - \sum^{8} 2^j = \frac{1\times 3^9-1}{3-1} - \frac{1\times 2^9-1}{2-1} = 9330$

c) $\sum_{j=0}^{8}(2\cdot 3^j + 3\cdot 2^j) = \sum_{j=0}^{8} 2\cdot 3^j + \sum_{j=0}^{8} 3\cdot 2^j = \frac{2\times 3^9-2}{2} + \frac{3\times 2^9-3}{1}$

$= 19682 + 1533 = 21215$

d) $\sum_{j=0}^{8}(2^{j+1} - 2^j) = \sum_{j=0}^{8} 2\times 2^j + \sum_{j=0}^{8}(-1)(-2^j)$

$= \frac{2\times 2^9-2}{2-1} + \frac{(-1)(2)^9+1}{2-1} = 512$

$2\times 512 \quad -512$

# Ex-25

2

a) the integers greater than 10, $= S \longrightarrow Z^+$
This is countably infinite, because it can be arranged in a sequence.

$|S| = \aleph_0 - 10 \quad S = \{11, 12, 13, \cdots\}$
$= \aleph_0$

it is one to one because $f(n) = f(m) \Rightarrow n = m$

$S = \{11, 12, 13, \cdots n\}$

let's imagine $f(n) = 2n = X = \{22 \quad 24 \quad 26 \cdots 2n\}$
So, this is also subjective.
So, it is bijective.

b) The odd negative integers. $A: \{-1, -3, -7, \cdots\}$
Countable infinite $f: Z^+ \longrightarrow A, f(N) - (2N-1)$
f is an tone, because if $f(a) = f(b)$
then: $f: -(2a-1) = -(2b-1) \Rightarrow a = b$

c) the integers with absolute value less than 1000000
.. This set is finite

d) the real numbers between 0 & 2, uncountable

e) the set $A \times Z^+$, where $A = \{2, 3\}$: countably infinte
let's say: $f: S \rightarrow Z^+$, defined by $f(2,k) = 2k, f(3, j) = 2j+1$

① $f(2,k) = f(2,j) \Rightarrow k = j$
② $f(3,k) = f(3,j) \Rightarrow k = j$
③ $f(2,k) = f(3,j) \Rightarrow 2k = 2j+1$, this is not possible.

So, we can see that this is one to one.

f) the integers that are multiplied of 10.
countable infinite

$$f: \mathbb{Z}^+ \to S \quad f(n) = \begin{cases} 5^0 n & n = 1 \\ & n \text{ is even} \\ -5(n+1) & n \text{ is odd } \& \ n \neq 1 \end{cases}$$

it is one to one, because $f(a) = f(b)$

$$S = \{ \ldots\ldots -10, -5, 0, 5, 10 \ldots \}$$
$$N = \{ 1, 2, 3, 4, 5, \ldots\ldots \}$$

$\boxed{10}$ Give an example of two uncountable sets A & B. Such that $A - B$ is:

a) finite: let $A = B = \mathbb{R}$ : uncountable
then $A - B = \phi$ : finite

b) Countably infinite: i.e + $A = \mathbb{R}$ & $B = \mathbb{R} - \mathbb{Z}$ is uncountable.
then $A - B = \mathbb{R} - \mathbb{R} + \mathbb{Z} = \mathbb{Z} \Rightarrow$ countable infinite

c) Uncountable: let $A = \mathbb{C}$ & $B = \mathbb{R}$, both uncountable
Then $A - B$ is the set of a complex non real number
$A - B$ contains a subset of a purely imaginary numbers, which is uncountable. Which makes $A - B$ uncountable

# EXERCISE – CH – 2.6

**Ex 4:**

a) $A \times B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}$

b) $A \times B = \begin{bmatrix} 4 & -1 & -7 & 6 \\ -7 & -5 & 8 & 5 \\ 4 & 0 & 7 & 3 \end{bmatrix}$

c) $A \times B = \begin{bmatrix} 2 & 0 & -3 & -4 & -1 \\ 24 & -7 & 20 & 23 & 2 \\ -10 & 4 & -17 & -24 & -3 \end{bmatrix}$

**Ex. 9:**

$$A+(B+C) = a_{ij} + (b_{ij} + c_{ij})$$
$$= (a_{ij}+b_{ij}) + c_{ij} = (A+B)+C$$

**Ex. 10:**

a) AB is $3 \times 5$ matrix

b) Undefined because number of columns of B not equal to number of row of A $(5 \neq 3)$

c) AC is $3 \times 4$ matrix.

d) Undefined because number of columns of C not equal to number of row of A $(4 \neq 3)$

e) Undefined because number of columns of B is not equal to number of rows of C $(5 \neq 4)$

f) it's a $4 \times 5$ matrix.

Ex. 27:

a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Ex. 29:

a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = A^{[2]}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = A^{[3]}$

c) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A \vee A^{[2]} \vee A^{[3]}$

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