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Assignment-3

problem-2.10

Soln: Let $f = m(1,2,3,4,5,6,7)$

The canonical sum-of-product for f is given by.

$$f = \bar{x}_1\bar{x}_2x_3 + \bar{x}_1x_2\bar{x}_3 + \bar{x}_1x_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3 + x_1x_2x_3$$

It can be manipulated as follows:

$$f = (x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3 + x_1x_2x_3) + (\bar{x}_1x_2\bar{x}_3 + \bar{x}_1x_2x_3 + \bar{x}_1x_2x_3 + x_1x_2\bar{x}_3 + x_1x_2x_3) + (\bar{x}_1\bar{x}_2x_3 + \bar{x}_1x_2x_3 + x_1\bar{x}_2x_3 + x_1x_2x_3)$$

$$= x_1(\bar{x}_2\bar{x}_3 + \bar{x}_2x_3 + x_2\bar{x}_3 + x_2x_3) + x_2(\bar{x}_1\bar{x}_3 + \bar{x}_1x_3 + x_1\bar{x}_3 + x_1x_3) + x_3(\bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + x_1\bar{x}_2 + x_1x_3)$$

$$= x_1(\bar{x}_2(\bar{x}_3 + x_3) + x_2(\bar{x}_3 + x_3)) + x_2(\bar{x}_1(\bar{x}_3 + x_3) + x_1(\bar{x}_3 + x_3)) + x_3(\bar{x}_1(\bar{x}_2 + x_2) + x_1(\bar{x}_2 + x_2))$$

$$= x_1 + x_2 + x_3$$

Problem - 2.11

Soln: The canonical product-of-sums for f is,

$$f = (x_1+x_2+x_3)(x_1+x_2+\bar{x}_3)(x_1+\bar{x}_2+x_3)(x_1+\bar{x}_2+\bar{x}_3)$$
$$(\bar{x}_1+x_2+x_3)(\bar{x}_1+x_2+\bar{x}_3)(\bar{x}_1+\bar{x}_2+x_3)$$

It can be manipulated as follows;

$$f = (x_1(1+x_2+x_3)(1+x_2+\bar{x}_3)(1+\bar{x}_2+x_3)(1+\bar{x}_2+\bar{x}_3))$$
$$(x_2(x_1+1+x_3)(x_1+1+\bar{x}_3)(\bar{x}_1+1+x_3)(\bar{x}_1+1+\bar{x}_3))$$
$$(x_3(x_1+x_2+1)(x_1+\bar{x}_2+1)(\bar{x}_1+x_2+1)(\bar{x}_1+\bar{x}_2+1))$$

$$= (x_1.1.1.1.1)(x_2.1.1.1.1)(x_3.1.1.1.1)$$

$$= x_1x_2x_3$$

Problem - 2.12

Soln: Using algebraic manipulation for the minimum sum-of-products expression:

$$f = x_1x_3 + x_1\bar{x}_2 + \bar{x}_1x_2x_3 + \bar{x}_1\bar{x}_2\bar{x}_3$$

$$= x_1(\bar{x}_2+x_2)x_3 + x_1\bar{x}_2(\bar{x}_3+x_3) + \bar{x}_1x_2x_3 + \bar{x}_1\bar{x}_2\bar{x}_3$$

$$= x_1\bar{x}_2x_3 + x_1x_2x_3 + x_1\bar{x}_2\bar{x}_3 + \bar{x}_1x_2x_3 + \bar{x}_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3$$

$$= x_1x_3 + (x_1+\bar{x}_1)x_2x_3 + (x_1+\bar{x}_1)\bar{x}_2\bar{x}_3$$

$$= x_1x_3 + x_2x_3 + \bar{x}_2\bar{x}_3$$

problem - 2.20

Soln: The simplest sum-of-products implementation of the function is

$$f = \bar{x}_1x_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1x_2\bar{x}_3 + x_1x_2x_3$$

$$= (\bar{x}_1+x_1)x_2x_3 + x_1(\bar{x}_2+x_2)\bar{x}_3$$

$$= x_2x_3 + x_1\bar{x}_3$$

problem - 2.22

Soln: The simplest product-of-sums implementation of the function is,

$$f = (x_1+x_2+x_3)(x_1+\bar{x}_2+x_3)(\bar{x}_1+x_2+\bar{x}_3)$$

$$= ((x_1+x_3)+x_2)((x_1+x_3)+\bar{x}_2)(\bar{x}_1+x_2+\bar{x}_3)$$

$$= (x_1+x_3)(\bar{x}_1+x_2+\bar{x}_3)$$

Problem- 2.23

Soln: The simplest product-of-sum implementation of the function is,

$$f = (x_1+x_2+x_3)(x_1+x_2+\bar{x}_3)(\bar{x}_1+x_2+\bar{x}_3)(\bar{x}_1+\bar{x}_2+\bar{x}_3)$$

$$= ((x_1+x_2)+x_3)((x_1+x_2)+\bar{x}_3)((\bar{x}_1+\bar{x}_3)+x_2)((\bar{x}_1+\bar{x}_3)+\bar{x}_2).$$

$$= (x_1+x_2)(x_3+\bar{x}_3)(\bar{x}_1+\bar{x}_3)(x_2+\bar{x}_2)$$

$$= (x_1+x_2)(\bar{x}_1+\bar{x}_3).$$