# <u>Homework -3</u>

## 1.4 Predicates and Quantifiers

### Question No.10

Let $C(x)$ be the statement "$x$ has a cat," let $D(x)$ be the statement "$x$ has a dog," and let $F(x)$ be the statement "$x$ has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

**a)** A student in your class has a cat, a dog, and a ferret.
**b)** All students in your class have a cat, a dog, or a ferret.
**c)** Some student in your class has a cat and a ferret, but not a dog.
**d)** No student in your class has a cat, a dog, and a ferret.
**e)** For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

### Answer No.10

Given:

C(x) ="x has a cat".
D(x) ="x has a dog"
F(x) ="T has a ferret"

INTERPRETATION SYMBOLS

Negation $\neg p$: not p

Disjunction $p \vee q$: p or q

Conjunction p ^ q: p and q

Existential quantification $\exists xP(x)$: There exists an element x in the domain such that P (x)
Universal quantification $\forall xP(x)$: P (x) for all values of x in the domain.

Solution:
Rewriting the given English sentences using the above interpretations of the symbols.

**a) A student in your class has a cat, a dog, and a ferret.**

**$\exists x\ (C(x) \wedge D(x) \wedge F(x))$**

**b) All students in your class have a cat, a dog, or a ferret.**

**$\forall x\ (C(x) \vee D(x) \vee F(x))$**

**c) Some student in your class has a cat and a ferret, but not a dog.**

**$\exists x\ (C(x) \wedge F(x) \wedge \neg D(x))$**

**d) No student in your class has a cat, a dog, and a ferret.**

**$\neg \exists x\ (C(x) \wedge D(x) \wedge F(x))$**

**e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.**

**$(\exists x\ C(x)) \wedge (\exists x\ D(x)) \wedge (\exists x\ F(x))$**

(a) $\exists x(C(x) \wedge D(x) \wedge F(x))$

(b) $\forall x(C(x) \vee D(x) \vee F(x))$

(c) $\exists x(C(x) \wedge F(x) \wedge \neg D(x))$

(d) $\neg\exists x(C(x) \wedge D(x) \wedge F(x))$

(e)$(\exists x C(x)) \wedge (\exists x D(x)) \wedge (\exists x F(x))$

## Question No.12

Let $Q(x)$ be the statement “$x + 1 > 2x$.” If the domain consists of all integers, what are these truth values?

**a)** $Q(0)$
**b)** $Q(-1)$
**c)** $Q(1)$
**d)** $\exists x Q(x)$
**e)** $\forall x Q(x)$
**f )** $\exists x.Q(x)$
**g)** $\forall x.Q(x)$

## Answer No.12

**a)** $Q(0)$
0+1>2*0 that means 1>0 **Therefore,Q(0) True**

**b)** $Q(-1)$
-1+1>-2 that means 0>-2 **Therefore,Q(-1) True**

**c)** $Q(1)$
1+1=2 and 2 is not greater than 2 **Therefore,Q(1) True**

**d)** $\exists x Q(x)$
Because Q(0) exist and it's true **Therefore,The statement is True**

**e)** $\forall x Q(x)$
Because the statement is false **Therefore,The statement is False**

**f)** $\exists x.Q(x)$
if x = 3 so 3+1=< is true **Therefore,The statement is True**
so the statement is true.

**g)** $\forall x.Q(x)$
Consider negation of Q(0) gives **Therefore,The statement is False**
x+1=<2x
That means 0+1=<0 that is false.

## Question No.18

Suppose that the domain of the propositional function $P(x)$ consists of the integers −2, −1, 0, 1, and 2. Write out each of these propositions using disjunctions, conjunctions, and negations.

**a)** $\exists x P(x)$
**b)** $\forall x P(x)$
**c)** $\exists x.P(x)$
**d)** $\forall x.P(x)$
**e)** $.\exists x P(x)$
**f )** $.\forall x P(x)$

## Answer No.18

**a) ∃xP(x)**

∃*xP*(*x*) means that there exists a value of x for which P (x) is true, thus P (—2) is true or P (—1) is true or P (O) is true or P (1) is true or P (2) is true. Using the above interpretation of symbols, we can rewrite the proposition then as:

**P (-2) v P (-1) v P (0) v P (1) v P (2)**

**b) ∀xP(x)**

∀xP(x)means that for all possible values of ax: P (T) is true, thus P (—2) is true and P (—1) is true and P (O) is true and P (1) is true and P (2) is true. Using the above interpretation of symbols, we can rewrite the proposition then as:

**P (-2) ∧ P (-1) ∧ P (0) ∧ P (1) ∧ P (2)**

**c) ∃x¬P(x)**

∃x ¬P(x)means that there exists a value of T for which ¬P (T) is true, thus ¬P(—2) is true or ¬ P(—1) is true or ¬P (O) is true or ¬ (1) is true or ¬ (2) is true. Using the above interpretation of symbols, we can rewrite the proposition then as:

**¬P (-2) v ¬P (-1) v ¬P (0) v ¬P (1) v ¬P (2)**

**d) ∀x¬P(x)**

∀x ¬P(x) means that for all possible values of at: ¬P (x) is true, thus ¬P (—2) is true and ¬P (—1) is true and ¬ P(O) is true and ¬P (1) is true and ¬P (2) is true. Using the above interpretation of symbols, we can rewrite the proposition then as:

**¬P (-2) ∧ ¬P (-1) ∧ ¬P (0) ∧ ¬P (1) ∧ ¬P (2)**

**e) ¬∃xP(x)**

¬∃xP(x)) is the negation of EXP (T) in part (a), thus we can just take the negation of the result in part (a):

**$\neg$ (P (-2) v P (-1) v P (0) v P (1) v P (2))**

**f) $\neg\forall xP(x)$**

$\neg\forall xP(x))$ is the negation of $\forall xP(x))$ in part (b), thus we can just take the negation of the result in part (b):

**$\neg$ (P (-2) $\wedge$ P (-1) $\wedge$ P (0) $\wedge$ P (1) $\wedge$ P (2))**

## Question No.21

For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

**a)** Everyone is studying discrete mathematics.
**b)** Everyone is older than 21 years.
**c)** Every two people have the same mother.
**d)** No two different people have the same grandmother.

## Answer No.21

For each of these statements and a domain for which the statement is true and a domain for which the statement is false

Domain 1 = Domain that will make it True
Domain 2 = Domain that will make it false

a) Everyone is studying discrete mathematics."

Domain 1 "Everyone in the Discrete Math class"

Domain 2 "Everyone in the school"

Domain 1 will make the statement true

Domain 2 will make the statement false because some students will not have to take discrete math.

b)"Everyone is older than 21 years. "

Domain 1 "People who are eligible to be president "

Domain 2 "Students in a kindergarten class"

Domain 1 will make the statement true because the age requirement for president is 35.

Domain 2 will make the statement false because kindergartners are not 21 years of age

c)
"Every two people have the same mother."

Domain 1 "Twins in the world"

Domain 2 "Everyone in the world"

Domain 1 will make the statement true because twins have the same mother

Domain 2 will make the statement false because not everyone has the same mother

d) V.V.I
" No two different people have the same grandmother. "

Domain 1 "Hilary Clinton and Bernie Sanders"

Domain 2 "Everyone in the world"

Domain 1 will make the statement true because Hilary and Bernie do not have the same grandmother

Domain 2 will make the statement false because there are siblings who can have the same grandmother.

## Question No.23

Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

**a)** Someone in your class can speak Hindi.

**b)** Everyone in your class is friendly.

**c)** There is a person in your class who was not born in California.

**d)** A student in your class has been in a movie.

**e)** No student in your class has taken a course in logic programming.

## Answer No.23

a)

$\exists x P(x)$

$\exists x(P(x) \wedge A(x))$

There exists a person in you class that can speak Hindi.
Let P (x) be "x speaks Hindi" and A(x) be "x is in your class".

b)

$\forall x(Q(x))$

$\forall x(A(x) \longrightarrow Q(x))$

Everyone means **∀x** and if x is in your class then x has to be friendly.
Let Q(x) be "T is friendly" and A(x) be "x is in your class"

c)

$\exists x(\neg R(x))$

$\exists x(A(x) \wedge \neg R(x))$

d)

$\exists x(S(x))$

$\exists x(A(x) \wedge S(x))$

There is a student means $\exists x$ and x has to be in your class and has to have been in a movie.
Let S(x) be "x has been in a movie" and A(x) be "x is in your class"

e)

$\neg(\exists x T(x))$

$\neg(\exists x T(x))\wedge A(x)))$

No student means $\neg(\exists x)$ and T has to be in your class and has to have taken a course in logic programming.
Let T (x) be "x has taken a course in logic programming" and A(x) be "x is in your class"

## Question No.29

Express each of these statements using logical operators,predicates, and quantifiers.
**a)** Some propositions are tautologies.
**b)** The negation of a contradiction is a tautology.
**c)** The disjunction of two contingencies can be a tautology.

## Answer No.29

a)
$\exists x(P(x))$
Let the domain be the collection of all propositions.
Let P (x) mean "x is a tautology".

b)

$\forall x(Q(x) \longrightarrow P(\neg x))$

Let the domain be the collection of all propositions.
Let P (x) mean "x is a tautology" and let Q(x) mean “x is a contradictor”.

$\neg x$ is the negation of the proposition x.

c) **$\exists x \exists y((R(x) \wedge R(y)) \rightarrow (P(x \vee y)))$**

Let the domain be the collection of all propositions.
Let P (x) mean "x is a tautology", R(x) mean "x is a contingency"

d) **$\forall x \forall y((P(x) \wedge P(y)) \rightarrow (P(x \wedge y)))$**

Let the domain be the collection of all propositions.
Let P (x) mean "x is a tautology".

## Question No.45

Determine whether $\forall x(P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

## Answer No.45

Assume that the domain contains y and z for which P (y) is false and Q(y) is false and P (z) is true and Q(z) is false.

Then $\forall x(P(x) \rightarrow Q(x))$ is false (because it isn't true if x = z) and and $\forall x P(x) \rightarrow \forall x Q(x)$ is true (because $\forall x P(x)$ is false).

Thus we have found a situation where the two propositions do not contain the same truth value and thus the two propositions are not logically equivalent.

Not logically equivalent

## Question No.46

Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x\, P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

## Answer No.46

Assume that the domain contains y and z for which
P (y) is false
Q(y) is true
P (z) is true
Q(z) is false.

Then $\forall x(P(x) \leftrightarrow Q(x))$ is false (because it isn't true if a: z) and $\forall x\, P(x) \leftrightarrow \forall x Q(x)$ is true (because $\forall x\, P(x)$ is false and $\forall x Q(x)$ is false, since they both have the same truth value, the biconditional statement is true).

Thus we have found a situation where the two propositions have a different truth value and thus the two propositions are not logically equivalent.