1.5 Nested Quantifiers

Homework-4(Page-69)

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Question No.10

Let $F(x, y)$ be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody can fool Fred.
b) Evelyn can fool everybody.
c) Everybody can fool somebody.
d) There is no one who can fool everybody.
e) Everyone can be fooled by somebody.
f) No one can fool both Fred and Jerry.
g) Nancy can fool exactly two people.
h) There is exactly one person whom everybody can fool.
i) No one can fool himself or herself.
j) There is someone who can fool exactly one person besides himself or herself.

Answer No.10

(a)

"Everybody" means "All people in the world".

∀x F(x, Fred)

(b)

"Everybody" means "All people in the world".

∀y F (Evelyn, y)

(c)

"Everybody" means "All people in the world".

"Somebody"means "There exists a person in the world".

$\forall x \, \exists y \, F(x, y)$

(d)

"No one" means "There does not exists a person in the world.
"Everybody" means "All people in the world".

$\neg \exists x \, \forall y \, F(x, y)$

Note: This is logically equivalent with $\forall x \exists y \neg F(x, y)$ by De Morgan's law for qualfiers.

(e) "Everybody" means "All people in the world". "Somebody" means "There exists a person in the world". (Note: Everyone refers to y, while somebody refers to x and the statement of y occurs before the statement of x).

$\forall y \, \exists x \, F(x, y)$

(f) "No one" means "There does not exists a person in the world".

$\neg \exists x \, (F(x, \text{Fred}) \land F(x, \text{Jerry}))$

(g) We could rewrite the given sentence as "Nancy can fool two people y and z, and y and z cannot be the same person, and all people that Nancy can fool then have to be either y or z".

$\exists y \exists z \, (F(\text{Nancy}, y) \land F(\text{Nancy}, z) \land y \neq z \land \forall w(F(\text{Nancy}, w) \rightarrow (w = y \lor w = z)))$

(h) We could rewrite the given sentence as "There is a person y whom everybody can fool and all other people whom everybody can fool then have to be this person y".

"Everybody" means "All people in the world".

$$\exists y\,(\forall x\, F(x, y) \land \forall z\,(\forall w\, F(w, z) \to z = y))$$

(i) We could rewrite the given sentences as "Every person x can not fool x (himself / herself)

$$\forall x \neg F(x,x)$$

Note : This is logically equivalent with $\neg\exists x F(x,x)$ by De Morgan's Law for qualifiers.

(j) We could rewrite the given sentence as "There is a person x for whom there exists a person y that x can fool and for all other people z that can fool, z has to be the person y or z has to be x himself (herself)".

$$\exists x \exists y (L(x,y) \land \forall z\,(L(x, z) \to z = y \lor z=x))$$

Question No.28

Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a) $\forall x \exists y (x^2 = y)$
b) $\forall x \exists y (x = y^2)$
c) $\exists x \forall y (xy = 0)$
d) $\exists x \exists y (x + y \neq y + x)$
e) $\forall x (x \neq 0 \to \exists y (xy = 1))$
f) $\exists x \forall y (y \neq 0 \to xy = 1)$
g) $\forall x \exists y (x + y = 1)$

h) $\exists x \exists y(x + 2y = 2 \wedge 2x + 4y = 5)$
i) $\forall x \exists y(x + y = 2 \wedge 2x - y = 1)$

j) $\forall x \forall y \exists z(z = (x + y)/2)$

Answer No.28

a) True (let y = x 2)
b) False (no such y exists if x is negative)
c) True (let x = 0)
d) False (the commutative law for addition always holds)
e) True (let y = 1/x)
f) False (the reciprocal of y depends on y —there is not one x that works for all y)
g) True (let y = 1− x)
h) False (this system of equations is inconsistent)
i) False (this system has only one solution; if x = 0, for example, then no y satisfies y = 2∧−y = 1)
j) True (let z = (x + y)/2)

Question No.33

Rewrite each of these statements so that negations appearonly within predicates (that is, so that no negation
is outside a quantifier or an expression involving logical connectives).

a) $\neg \forall x \forall y P(x, y)$

b) $\neg \forall y \exists x P(x, y)$

c) $\neg \forall y \forall x(P(x, y) \vee Q(x, y))$

d) $\neg\, (\exists x \exists y . P(x, y) \wedge \forall x \forall y Q(x, y))$

e) $\neg \forall x(\exists y \forall z P(x, y, z) \wedge \exists z \forall y P(x, y, z))$

Answer No.33

Logical Equivalences:

$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$

Double negation law:

$$\neg(\neg p) \equiv p$$

De Morgan's laws:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

De Morgan's laws for Qualifiers:

$$\neg\exists x P(x) \equiv \forall x \neg P(x)$$

$$\neg\forall x P(x) \equiv \exists x \neg P(x)$$

Solution:

(a) Given,

$$\neg\forall x \forall y P(x, y)$$

Use De Morgan's Law for Qualifiers(twice):

$$\equiv \exists x \neg\forall y\, P(x,y)$$

$$\equiv \exists x \exists y \; \neg \, \mathrm{P(x,y)}$$

(b) Given,

$$\neg \forall y \exists x P(x, y)$$

Use De Morgan's Law for Qualifiers(twice):

$$\equiv \exists y \, \neg \exists x \mathrm{P(x,y)}$$

$$\equiv \exists y \forall x \, \neg \mathrm{P(x,y)}$$

(c) Given,

$$\neg \forall y \forall x (P(x, y) \lor Q(x, y))$$

Use De Morgan's Law for Qualifiers:

$$\equiv \exists y \, \neg \forall x [P(x, y) \lor Q(x, y)]$$

Use De Morgan's Law for Qualifiers:

$$\equiv \exists y \exists x \, \neg [P(x, y) \lor Q(x, y)]$$

Use De Morgan's Law :

$$\equiv \exists y \exists x \, [\, \neg P(x, y) \land \neg Q(x, y)]$$

(d) Given

$$\neg \, (\exists x \exists y . P(x, y) \land \forall x \forall y Q(x, y))$$

Use De Morgan's Law:

$$\equiv \neg\exists x\exists y \neg P(x,y) \wedge \forall x\forall y Q(x, y)$$

Use De Morgan's Law for Qualifiers:

$$\equiv\forall x \neg\exists y \neg P(x,y) \vee \exists x \neg\forall y Q(x, y)$$

Use De Morgan's Law for Qualifiers:

$$\equiv\forall x\forall y \neg[P(x, y)] \vee \exists x\exists y \neg Q(x,y)$$

Use the double negation law:

$$\equiv\forall x\forall y\ P(x, y) \vee \exists x\exists y \neg Q(x,y)$$

(e) Given,

$$\neg\forall x(\exists y\forall z P(x, y, z) \wedge \exists z\forall y P(x, y, z))$$

Use De Morgan's Law for Qualifiers:

$$\equiv\exists x \neg[\exists y\forall z P(x,y,z) \wedge \exists z\forall y P(x, y, z)]$$

Use De Morgan's Law

$$\equiv\exists x[\neg\exists y\forall z P(x,y,z) \vee \exists z\forall y P(x, y, z)]$$

Use De Morgan's Law for Qualifiers(twice):

$$\equiv\exists x[\forall y \neg\forall z P(x,y,z) \vee \forall z \neg\forall y P(x, y, z)]$$

Use De Morgan's Law for Qualifiers(twice):

$$\equiv\exists x[\forall y\exists z \neg P(x,y,z) \vee \forall z\exists y \neg P(x, y, z)]$$

1.6 Rules of Inference(Pg-82)

Question No:4

What rule of inference is used in each of these arguments?
a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

Answer No:4

(a) Let p be the proposition "Kangaroos live in Australia." and q be the proposition.
"Kangaroos are marsupials."

$p \wedge q$

---

q Simplification

(b) Let p be the proposition "Hotter than 100 degrees today." and q be the proposition "Pollution is dangerous."

$\neg p$

$p \vee q$

---

q Disjunctive syllogism

(c) Let p be the proposition "Linda is an excellent swimmer." and q be the proposition "Can work as a lifeguard."

p

$p \rightarrow q$

---

q Modus ponens

(d) Let p be the proposition "Steve will work at a computer company this summer." and q be the proposition "Steve will be a beach bum."

P

$p \vee q$ Addition

(e) Let p as the proposition "If I do not wake up". Let q as the proposition" I cannot go to work" and let r as a proposition " I will not get paid."

$p \rightarrow q$

$q \rightarrow r$

$p \rightarrow r$ Hypothetical Syllogism

Question No:5

Use rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job."

Answer No:5

INTERPRETATION SYMBOLS

Negation $\neg p$: not p

Disjunction $p \vee q$: p or q

Conjunction $p \wedge q$: p and q

Conditional statement $p \rightarrow q$: if p, then q

Biconditional statement $p \leftrightarrow q$: p if and only if q

RULES OF INFERENCE

Modus ponens

P

$p \rightarrow q$

$\therefore$ q

Let us assume:

P="Randy works hard"

q ="Randy is a dull boy"

r ="Randy will get the job"

We can then rewrite the given statements (1), (2) and (3) using the above interpretations.

Step		Reason
1.	p	Premise
2.	$p \rightarrow q$	Premise
3.	$q \rightarrow \neg r$	Premise
4.	q	Modens ponens from (1) and (2)
5.	$\neg r$	Modens ponens from (1) and (2)

¬r means “Randy will not get the job” and thus we obtained the implied conclusion.

Question No:6

Use rules of inference to show that the hypotheses “If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,” “If the sailing race is held, then the trophy will be awarded,” and “The trophy was not awarded” imply the conclusion “It rained.”

Answer No:6(**)

r. "It rains"

f: "It is foggy"

s: "The sailing race will be held"

d: "The lifesaving demonstration goes on"

t' "The trophy will be awarded."

Assign statements to propositional variables

Step	Reason
1. $(\neg r \lor \neg f) \rightarrow (s \land d)$	Premise 1
2. $s \rightarrow t$	Premise 2
3. $\neg t$	Premise 3
4. $\neg s$	Modus Tollens with Premise 2 and Premise 3
5. $\neg(s \land d) \rightarrow \neg(\neg r \lor \neg f)$	Use equivalency $p \rightarrow q \equiv \neg q \rightarrow \neg p$ with Premise 1
6. $(\neg s \land \neg d) \rightarrow (r \land f)$	De Morgan's Laws with 5
7. $(\neg s \land \neg d)$	Addition with 4
8. $(r \land f)$	Modus Ponens with 6 and 7
9. r	Simplification of 8

Thus,it rained

Question No:12

Show that the argument form with premises $(p \land t) \rightarrow (r \lor s)$, $q \rightarrow (u \land t)$, $u \rightarrow p$, and $\neg s$ and conclusion $q \rightarrow r$ is valid by first using Exercise 11 and then using rules of inference from Table 1.

Answer No:12

Rules of Inference:

Modus ponens:

$$\begin{array}{c} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$$

Simplification:

$$\frac{p \wedge q}{\therefore p}$$

Conjunction:

$$\frac{\begin{array}{l} p \\ q \end{array}}{\therefore (p \wedge q)}$$

Disjunctive syllogism

$$\frac{\begin{array}{l} p \vee q \\ \neg p \end{array}}{\therefore q}$$

<u>**Result previous exercise:**</u>

$$\frac{\begin{array}{l} P_1 \\ P_2 \\ P_3 \\ \cdots\cdots \\ P_n \end{array}}{\therefore q \to r}$$

is valid,if

$$\begin{array}{c} P_1 \\ P_2 \\ P_3 \\ \cdots\cdots \\ P_n \\ q \\ \hline \therefore r \end{array}$$

Solution:

Using the result of the previous exercise:

$$\begin{array}{l} (p \land t) \rightarrow (r \lor s) \\ q \rightarrow (u \land t) \\ u \rightarrow p \\ \neg s \\ \hline \therefore q \rightarrow r \end{array}$$

is valid ,if

$$\begin{array}{l} (p \land t) \rightarrow (r \lor s) \\ q \rightarrow (u \land t) \\ u \rightarrow p \\ \neg s \\ q \\ \hline \therefore r \end{array}$$

is valid.

We will thus first prove that the second inference is valid ,because then we know that the first inference is valid as well.

	Step	Reason
1.	$(p \wedge t) \rightarrow (r \vee s)$	Premise
2.	$q \rightarrow (u \wedge t)$	Premise
3.	$u \rightarrow p$	Premise
4.	$\neg s$	Premise
5.	q	Premise
6.	$u \wedge t$	Modus ponens from (2) and (5)
7.	u	Simplification from from (6)
8.	p	Modus ponens from (3) and (7)
9.	t	Simplification from from (6)
10.	$p \wedge t$	Conjunction from (8) and (9)
11.	$r \vee s$	Modus ponens from (1) and (10)
12.	r	Disjunctive syllogism from (11) and (4)

We have thus shown that,

$$
\begin{array}{l}
(p \wedge t) \rightarrow (r \vee s) \\
q \rightarrow (u \wedge t) \\
u \rightarrow p \\
\neg s \\
q \\
\hline
\therefore r
\end{array}
$$

is valid.

By the result of the previous exercise,we then know that,

$$
\begin{array}{c}
(p \wedge t) \rightarrow (r \vee s) \\
q \rightarrow (u \wedge t) \\
u \rightarrow p \\
\neg s \\
\hline
\therefore q \rightarrow r
\end{array}
$$

is also valid.