Guarantee of Integrity

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Examination Paper of Northwestern Polytechnical University

Score:	

1st Semester of the Academic Year 2020-2021

Course School School of Computer Science

Course Name Discrete Mathematics

Date of Exam 2021.1.4 Duration and time 2 hours

NOTICE：Write all answers on answer sheet.

I. Choose the right answer（2 points for each, total 40 points）

1. How many propositions in the following statements?
   1. Do not pass go. 2. Everyone can pass the math exam.
   3. Who is Santa Claus? 5. Xi'an is clod in winter.
   6. 2+3=4 7. Discrete mathematics is difficult.
   A.2 B. 3 C. 4 D. 5
2. Let p and q be the propositions
   p: The earth is round: q: There are more than one way from Xi'an to Peking.
   Which one represents if the earth is round, there are more than one way from Xi'an to Peking?
   A. $(p \wedge q)$ B.$q \rightarrow p$ C.$p \rightarrow q$ D. $p \leftrightarrow q$
3. Let P(x) denote the statement "x passed the exam", which represent nobody in the class passed the exam?
   A.$\forall x P(x)$ B.$\forall x \neg P(x)$ C. $\exists x\, P(x)$ D. $\exists x \neg P(x)$
4. N is the set of natural numbers, P is the set of positive integers. What is the cardinality of N∩P?
   A. 0 B. 1 C. 2 D. infinite
5. Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 7$. What is the inverse function of f?
   A. x-7 B. (7-x)/2 C. (x-7)/2 D. does not exist
6. How many rows appear in a truth table for the compound propositions?
   $(p \vee \neg r) \wedge (q \vee r)$
   A. 4 B.8 C.16 D.32
7. What is the negation of 2+2>3?
   A. 2+2=4 B. 2+2<3 C. 2+2=4 D. 2+2<=3
8. Which integer is the remainder of -1011 mod 11?
   A. -10 B. 10 C. -1 D. 1
9. Convert the binary expansion of $(10000100001)_2$ into hexadecimal expansion.
   A. $(421)_{16}$ B. $(321)_{16}$ C. $(221)_{16}$ D. $(121)_{16}$

10. How many functions are $O(x^2)$

a) $f(x) = 10$ b) $f(x) = 3x^2 + 7$ c) $f(x) = x^3 + x^2 + 1$ d) $f(x) = 5 \log x$ e) $f(x) = |x|$

A. 1 B. 3 C.4 D. 5

11. How many bit strings of length eight start with a 1 bit or end with two 0s?

A.64 B. 128 C. 160 D. 192

12. How many subsets of {a,b,c,d}?

A. 3 B. 8 C. 9 D. 16

13. How many ways are there to select 5 players and 2 bench members from a 10-member team to make a team for basketball match?

A. C(10,7) B.C(10,5)*C(10,2) C.C(10,5)*C(5,2) D.P(20,7)

14. Which relation is a partial ordering on the set of integers?

A. R = {(a,b) | a + b ⩽ 3}. B. $R = \{(a,b) \mid a \leq b\}$

C. $R = \{(a,b) \mid a = b^2\}$ D. $R = \{(a,b) \mid a = b + 1\}$

15. How many permutations of the letters ABCDEFGH start with BC?

A. 5! B. 6! C. 7! D. 8!

16. Find $5^{2021} \bmod 19$.(Tip:Fermat's little theorem)

A. 5 B. 9 C. 11 D. 16

17. There are 500 students in CS major, 220 have taken course A, 170 have taken course B, 180 have taken course C, 50 have taken course A and B, 60 have taken course B and C, 40 have taken course A and C. How many students have taken all the three courses?

A. 20 B. 40 C. 80 D. 160

18. How many different license plates can be made if each plate is made of two uppercase English letters and three digits, and the letters and digits can be at different position?

A. $26*10^4$ B. $C(5,2)*26*26*10^3$ C. $26*26*10^3$ D. 10^5

19. Which is the inverse of 7 modulo 26?

A. 15 B. 14 C. 11 D. 16

20. How many edges in a complete graph with 6 vertices?

A. 12 B. 15 C.18 D. 24

II. Answer the question(5 points for each, total 30 points)

1. Use the Euclidean algorithm to find gcd(1527, 14163).
2. All books are identified by an International Standard Book Number (ISBN-10), a 10-digit code x1x2 ...x10, assigned by the publisher. a check digit that is either a digit or the letter X (used to represent 10). This check digit is selected so that:

$$x_{10} = \sum_{i=1}^{9} x_i \pmod{11},$$

   1) The first nine digits of the ISBN-10 of a book are 007288018. What is the check digit?
   2) (b) Is 084930249X a valid ISBN-10?
3. How many solutions does the equation x1+x2+x3 =17 have,
   1) Where x1, x2 and x3 are natural numbers?
   2) Where x1,x2 and x3 are positive numbers?

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4. If the original message is "DISCRETE" , the encrypted message is "TSQOLYVY", and the encrypt algorithm is affine ciphers($f(p) = (ap + b)$ **mod** 26). Give the value of a and b.
5. Find solution of the recurrence relation: f(n)=2*f(n-1)-*f(n-2),with f(1)=2,f(2)=6.
6. Find all solutions, if any, to the system of congruence $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, and $x \equiv 1 \pmod 7$. x is an integer.

III. Proof(5 points for each, total 30 points)

1. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.
2. Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.
3. Devise an algorithm that finds the index of x(x is a number) in an integers list.
4. Prove that 3 divides $n^3 - n$ when n is a positive integer.
5. Prove that there is no solution in integers x and y to the equation $2x^2 + 5y^2 = 16$.
6. Show that log n! is O(n log n).