Experiment at home: Young's Modulus of elastic filaments measured by optical lever amplification method

Experiment tasks and requirement

1. Prepare requirements needed in this experiment.
2. Understand the meaning of Young's Modulus.
3. Use common household supplies to set up testing device for young's modulus measurement (Require shooting the entire device layout).
4. Deduce measurement formula of young's modulus according to the existing conditions and devices (Require shooting the readings when taking the measurement).
5. Deal with experimental data (including measurement results, uncertainty transmission, complete results presentation), and write experimental reports.
6. Improvement requirements: what other devices can be built to measure young's modulus of elastic filaments? What are the advantages and disadvantages of these devices? Make a comparison list. Other experimental measurement methods that can be implemented in the home can also be tried and compared with the method of this experiment.

Experiment objectives

1. Measure micro elongation by optical lever amplification method.
2. Process data by successive subtracting method.
3. Learn the transfer of uncertainty.
4. Determine the measurement error limit according to the actual situation.
5. Analysis the source of errors.

Introduction of theory and method

Young's modulus principle and derivation of measurement formula

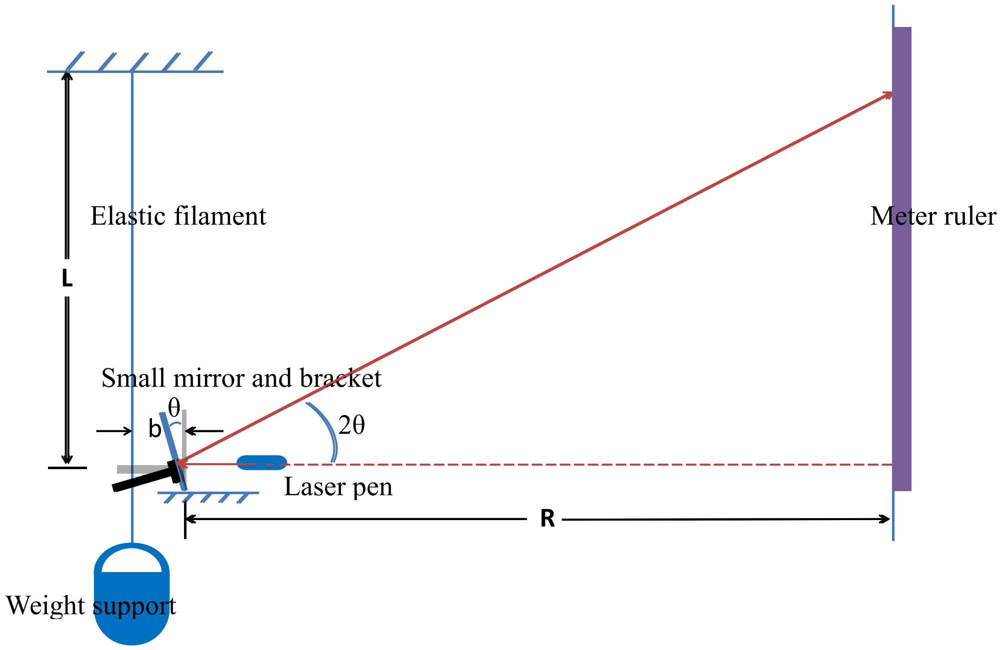
The young's modulus is defined as:

$$\frac{F}{S} = E\frac{\Delta L}{L}$$

In this formula, *E* is the young's modulus of the object to be measured, the external force *F* during installation is derived from the mass of weight m (coins, eggs, rice, bottled water, *etc.* can be used as weights according to the tensile coefficient of the filaments to be tested), *S* is the cross-sectional area of the object to be measured, which can be obtained from the its diameter *D*, so the definition of young's modulus can be expressed as:

$$\mathrm{E} = \frac{4mgL}{\pi D^2 \Delta L}$$

[Training 1] the schematic diagram of young's modulus measuring device is built with the existing objects in the home, as shown in the following figure.



It is necessary to ensure that the theta in a small angle in the measurement, thus

$$\tan\theta = \frac{\Delta L}{b} \approx \theta$$

$$\tan 2\theta = \frac{l}{R} \approx 2\theta$$

$$\Delta L = \frac{bl}{2R}$$

Substitute into the definition formula and get the measurement formula:

$$E = \frac{8mgLR}{\pi D^2 bl}$$

The above formula is the measurement formula of young's modulus of the object to be measured. Since telescope is not used, it is different from the formula in the textbook. Among them, R is the vertical distance between the small mirror and the wall and measured with a steel ruler.

Experimental steps (to be modified according to the actual situation) :

A) Set up the experimental device. Immobilize the bracket on the back of the little mirror and hang the elasticity filaments to be measured in high place. Find a suitable location to tie the bracket leg behind the small mirror, where the mirror is placed on a table at the same height with this location. The lower part of the elastic filaments to be measured is tied to a weight holder (such as a small bucket) to make the weight holder suspended. It is best to keep the bracket leg behind the small mirror basically level in the state of the empty plate of the weight. The key point is to ensure the stability of the small mirror and bracket device. Suitable items can be found as limit device.

B) Place the laser pointer at an appropriate position on the desktop, aim at the small mirror, and let the reflection spot reflect to the lower position of the wall. Fix the meter ruler vertically on the wall and pass through the spot position.

C) Since the experimental device is built at home, it needs to be predicted once (only observe the reading pattern, not the accurate reading) to check the existing problems of the device and make improvement and correction with household supplies.

D) Weigh the standby weights by electronic scales and select the 5 weights with the closest mass (note that the mass of the weights should match the tensile coefficient of the elastic filaments to be measured).

E) Measure l by placing weights in the bucket one by one and read the cursor position on the meter ruler on the wall, then take out the weights and read the cursor position on the meter ruler one by one.

F) Measure the diameter D of the object to be measured by the method of multi-coil winding.

G) Read the distance b from the small mirror to the bracket wrapped around the object to be measured many times with a meter ruler, read the distance L from the hanging place to the winding bracket with a meter ruler once, and read the vertical distance R from the small mirror to the reading meter ruler with a meter ruler once.

H) Process data by successive subtracting method.

Notes for experiment

A) If the laser pointer is used as the light source, please pay attention to safety, and the laser (including the reflected light of the laser) should not illuminate the human eye.

B) The weights should be placed with great care to ensure the stability of the entire experimental system.

Data recording and processing

A) The elastic filaments shall be tightly twined for n turns (up to the cm magnitude after being tightly twined), and measured once with a meter ruler, Unit: cm

n turns	1 turn

then D= cm

Set the error limit [training 4], make $nD=D_n$, then

$$u_B(D_n) = \frac{\Delta}{C} = \frac{0.05cm}{3} = 0.01667\ cm$$

$$u_B(D) = \frac{1}{n} u_B(D_n) = \qquad cm$$

$$u_C(D) = u_B(D) = \frac{\Delta}{C} = \qquad cm$$

B) The position of the elastic filaments twined on the bracket and its vertical distance from the small mirror is labeled as b and measured for three times, Unit: cm

1	2	3

$$\overline{b} = \quad \text{cm}$$

$$u_A(b) = t_p \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{3} (b_i - \overline{b})^2} = \quad cm$$

$$u_B(b) = \frac{\Delta}{C} = \quad cm$$

$$u_C(b) = \sqrt{u_A(b)^2 + u_B(b)^2} = \quad cm$$

C) The vertical distance between the mirror and the steel ruler is measured once, R= cm

Due to the long distance, it is difficult to measure horizontally, so the measurement cannot reach the precision of the steel ruler, so the error limit is 3 cm [training 4].

$$u_C(R) = u_B(R) = \frac{\Delta}{C} = \quad cm$$

D) The original length of the object to be measured is measured once, L= cm

Because the measurement cannot reach the accuracy of the meter ruler, the error limit is set to 0.3 cm [training 4]

$$u_C(L) = u_B(L) = \frac{\Delta}{C} = \quad \text{cm}$$

E) Measure the mass of the weights [training 4] unit: g

1	2	3	4	5

$$\overline{m} = \quad g$$

Weights are regarded as same weight, due to mass difference of lightest and the heaviest weight differ Δm = g, so the error is limited to

$$\Delta = \frac{\Delta m}{2} = \quad g$$

$$u_C(m) = u_B(m) = \frac{\Delta}{C} = \quad \text{g}$$

F) Measure l [training 2]

serial number	up	down	average	$l_{i+3} - l_i$	average
l_0					
l_1					
l_2					
l_3					
l_4					
l_5					

Set $l_{i+3} - l_i = x$, thus

$$u_A(x) = t_p \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{3}(x_i - \bar{x})^2} = \qquad cm$$

Because the laser cursor used for reading is as large as about 0.5 cm, the error limit [training 4] is set

$$\Delta = 0.5 \text{ cm}$$

The error limit can be determined according to the actual situation, thus

$$u_B(x) = \frac{\Delta}{C} = \qquad cm$$

$$u_C(x) = \sqrt{u_A(x)^2 + u_B(x)^2} = \qquad cm$$

Due to

$$l = \frac{1}{3}x$$

Thus

$$\bar{l} = \frac{1}{3}\bar{x} = \qquad cm$$

$$u_C(l) = \frac{1}{3}u_C(x) = \qquad cm$$

Relative uncertainty of young's modulus of the object to be measured

$$u_r = \sqrt{\left(\frac{\partial lnE}{\partial m}\right)^2 u_C^2(m) + \left(\frac{\partial lnE}{\partial L}\right)^2 u_C^2(L) + \left(\frac{\partial lnE}{\partial R}\right)^2 u_C^2(R) + \left(\frac{\partial lnE}{\partial D}\right)^2 u_C^2(D) + \left(\frac{\partial lnE}{\partial b}\right)^2 u_C^2(b) + \left(\frac{\partial lnE}{\partial l}\right)^2 u_C^2(l)}$$

$$= \sqrt{\left(\frac{1}{\bar{m}}\right)^2 u_C^2(m) + \left(\frac{1}{L}\right)^2 u_C^2(L) + \left(\frac{1}{R}\right)^2 u_C^2(R) + \left(-\frac{2}{\bar{D}}\right)^2 u_C^2(D) + \left(-\frac{1}{\bar{b}}\right)^2 u_C^2(b) + \left(-\frac{1}{\bar{l}}\right)^2 u_C^2(l)}$$

$$=$$

Since the gravitational acceleration in xi 'an is g = 9.7944 m/s^2, unify the number of significant figures of the input data so that the data in the input formula remains three to four significant figures. In order to ensure unit unification, all data are converted into international standard units Kg and m.

$$\bar{E} = \frac{8\bar{m}gLR}{\pi D^2 \bar{b}\bar{l}} = \frac{8 \times \qquad Kg \times 9.794\, m/s^2 \times \qquad m \times \qquad m}{3.142 \times (\qquad m)^2 \times \qquad m \times \qquad m} = \qquad N/m^2$$

Absolute uncertainty of young's modulus of the object to be measured

$$u_C(E) = u_r \cdot \bar{E} = \qquad N/m^2$$

The complete expression of the result is:

$$\begin{cases} E = \qquad N/m^2 \\ u_r(E) = \left|\frac{\quad}{\quad}\right| \times 100\% = \qquad \%,\ p = 0.683 \end{cases}$$

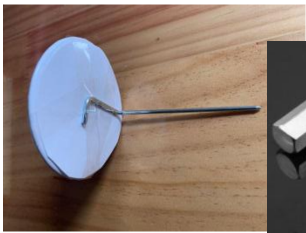
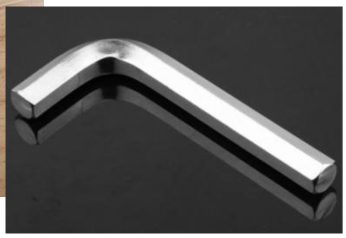
Data processing requirements:

1. The significant figures of the intermediate step can be reserved for four figures in advance, note that this step follows the format of **physical quantity name** = **symbol** = **data input** = **result (unit)**.

2. Calculate the directly measured class A, class B and synthetic uncertainty.
3. The relative uncertainty transfer formula is used to calculate the uncertainty of young's modulus, see data processing requirements part for details.
4. For the complete result, please refer to the data processing requirements in the introduction for the specific format and requirements.

Selection or manufacture of experimental instruments and equipment

1. The small mirror bracket can be made with an internal six-sided screwdriver, or it can be made into the desired shape by itself with steel wire.

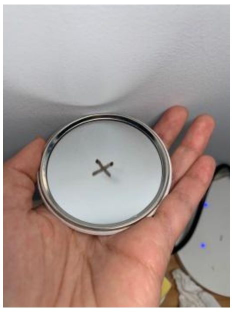
2. Meter ruler can be replaced by steel ruler, tape rule, *etc*.


3. The weight holder may be a keg or a homemade weight holder.

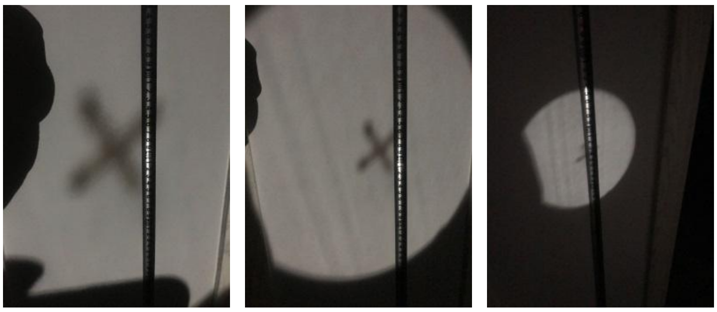
4. Weights can be used in eggs, mineral water, *etc*.


5. Laser pointer *etc.* can be used as parallel light source.

If there is no laser pointer, try using a flashlight or a cellphone light source, using a crosshairs drawn on a small mirror as the cursor.



But the light spot obtained by using a flashlight or mobile phone light source to reflect light on the wall through a small mirror may be large, the closer the light source is to the mirror, the bigger the spot, but the stronger the light, the farther the light source is from the mirror, the smaller the spot, but the weaker the light. Following images from left to right are the light source getting further and further away from the small mirror, the light spots are getting smaller and less intense, respectively. The distance between the light source and the small mirror can be adjusted according to the actual situation to make the clearest observation filaments.



Experimental report requirements

1. **Experimental principle:** [training 1] discuss the experimental principle.
2. **Experimental equipments:** list the ready-made or self-made experimental equipments and attach the photos of the equipments.
3. **Experimental steps:** list the experimental steps and attach photos or videos of the experimental process.
4. **Experimental data and processing:** [training 4] calculate the uncertainty of direct measurement and the uncertainty transfer formula to deduce the uncertainty of indirect measurement.
5. **Analysis of experimental results:** [training 5] analyze the causes of errors, take improvement measures, and then re-experiment and record the results.

Note: the video or photo taken must have your own identification (such as head, student ID card, signature, *etc.*)