

No: \_\_\_\_\_

# Examination Paper of Northwestern Polytechnical University

1<sup>st</sup> Semester of the Academic Year 2019-2020

Score:

Course School 2019 School of Computer Science

Course Name Discrete Mathematics

Date of Exam 2019.11.30 Duration and time 2 hours

NOTICE: Write all answers on answer sheet.

## I. Choose the right answer (2 points for each, total 20 points)

- How many propositions in the following statements?
  - Do not pass go.
  - The moon is made of green cheese.
  - What time is it?
  - $4 + x = 5$
  - Today is rain.
  - Tommorow is cloudy.

A.1 B. 2 C. 3 D. 4
- Let  $p$  and  $q$  be the propositions  
 $p$ : Charry is good at Chinese:  $q$ : Charry is good at Mathematics  
 Which one represent if Charry is good at Chinese then she is good at Mathematics?  

A.  $p \wedge q$  B.  $p \vee q$  C.  $p \rightarrow q$  D.  $p \leftrightarrow q$
- Let  $P(x)$  denote the statement “ $x$  passed the exam”, which represent not everyone in the class passed the exam?  

A.  $P(\text{David})$  B.  $\exists x P(x)$  C.  $\forall x \neg P(x)$  D.  $\exists x \neg P(x)$
- What is the cardinality of  $\{\emptyset, \{\emptyset\}, \{\emptyset, \emptyset\}\}$ ?  

A. 0 B. 1 C. 2 D. 3
- Let  $f$  and  $g$  be the functions from the set of integers to the set of integers defined by  $f(x) = 2x + 7$ . What is the inverse function of  $f$ ?  

A.  $2x+7$  B.  $7-2x$  C.  $1/2*(7-x)$  D. does not exist
- How many rows appear in a truth table for the compound propositions?  
 $(p \vee \neg r) \wedge (q \vee r)$   

A. 8 B. 16 C. 32 D. 24
- What is the negation of  $2+2=3$ ?  

A.  $2+2=3$  B.  $2+1=3$  C.  $2+2=4$  D.  $2+2 < 3$
- Decide which integer is remainder of  $-101 \bmod 11$ ?  

A. -2 B. 2 C. -9 D. 9
- Convert the binary expansion of  $(1000000001)_2$  into hexadecimal expansion.  

A.  $(101)_{16}$  B.  $(201)_{16}$  C.  $(301)_{16}$  D.  $(401)_{16}$
- How many functions are  $O(x)$   
 $a) f(x) = 10$   $b) f(x) = 3x + 7$   $c) f(x) = x^2 + x + 1$   $d) f(x) = 5 \log x$   $e) f(x) = |x|$   

A. 2 B. 3 C. 4 D. 5

## II. Answer the question(6 points for each, total 36 points)

- Suppose that the universal set is  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ . Express each

of these sets with bit strings where the  $i$ th bit in the string is 1 if  $i$  is in the set and 0 otherwise.

a)  $\{3, 4, 5\}$  b)  $\{1, 3, 6, 10\}$  c)  $\{2, 3, 4, 7, 8, 9\}$

2. Let  $S = \{-1, 0, 1, 2, 4, 7\}$ . Find  $f(S)$  if

a)  $f(x) = 1$ . b)  $f(x) = 2x + 1$ . c)  $f(x) = x^2 + 2x$

3. Find the inverse of 7 modulo 26.

4. Periodicals are identified using an International Standard Serial Number (ISSN). An ISSN consists of two blocks of four digits. The last digit in the second block is a check digit. This check digit is determined by the congruence  $d_8 \equiv 3d_1 + 4d_2 + 5d_3 + 6d_4 + 7d_5 + 8d_6 + 9d_7 \pmod{11}$ . When  $d_8 \equiv 10 \pmod{11}$ , we use the letter X to represent  $d_8$  in the code.

1) Determine the last number of 1570-868.

2) Check if 1059-1027 is a valid ISSN.

5. What is the best order to form the product ABCD if A, B, C, and D are matrices with dimensions  $30 \times 10$ ,  $10 \times 60$ ,  $60 \times 50$ , and  $50 \times 30$ , respectively? How many multiply operation to get the final result?

6. Use the Euclidean algorithm to find  $\gcd(1529, 14039)$ .

### III. Proof (4 points for No.1, 8 points for each of others, total 44 points)

1. Show that  $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$  is a tautology.

2. Devise an algorithm that finds the max number of all the integers in a list.

3. Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

4. Use mathematical induction to prove following statement.

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n},$$

5. Prove that 5 divides  $n^5 - n$  whenever  $n$  is a nonnegative integer.

6. Determine whether  $\log n!$  is  $\Theta(n \log n)$ . Justify your answer.