CHAPTER 1·7:

Ex 8:

Let n be a perfect square, so that means:

$$\exists c \in \mathbb{N}^* / c = \sqrt{n} \Rightarrow c^2 = n$$

$$\Rightarrow c^2+2 = n+2$$

$$\Rightarrow \sqrt{c^2+2} = \sqrt{n+2}$$

Now, let's check if $\sqrt{c^2+2}$ is a natural number ($\sqrt{c^2+2} \in \mathbb{N}$?):

- We know that, $c \geq 1 \Rightarrow c > \frac{1}{2}$

$$\Rightarrow 2c > 1 \Rightarrow 1+2c > 2$$

And we know that, $2 > 0$

So, $0 < 2 < 1+2c$

We add c^2: $c^2 < c^2+2 < c^2+1+2c$

Square Root all : $\sqrt{c^2} < \sqrt{c^2+2} < \sqrt{c^2+1+2c}$

We know that, $\sqrt{c^2} = c$

And, $\sqrt{c^2+1+2c} = \sqrt{(c+1)^2} = c+1$

So, : $c < \sqrt{c^2+2} < c+1$

* $\sqrt{c^2+2}$ is between 2 successive natural numbers and different from both of them.

So, $\sqrt{c^2+2} \notin \mathbb{N}$ and then $n+2$ is not a perfect square.

Ex 20

a – Proof By Contraposition

Let's Prove that if n is odd, then $3n+2$ is odd –

n odd $\iff$ $n = 2k+1$, For some integer k

$\Rightarrow 3n = 3(2k+1) = 6k+3$

$\Rightarrow 3n+2 = 6k+5 = 6k+4+1 = 2(3k+2)+1$

Let, $z = 3k+2$

So, $3n+2 = 2z+1$, then $3n+2$ is odd.

b – Proof By Contradiction

Let's suppose that n is odd, then, $n = 2k+1$ for some integer, k $\Rightarrow 3n = 6k+3$

$\Rightarrow 3n+2 = 6k+5 = 2(3k+2)+1$

then, $3n+2$ is odd.

— Contradiction —

Because, $3n+2$ is even

so, it means n is even

a – We proved $\neg q \rightarrow \neg p$ is true

So, $p \rightarrow q$

b – We proved $\neg p \rightarrow F$ is true

So, the contrapositive $T \rightarrow P$ is true.

In both proofs, we proved: if ~~any~~ $3n+2$ is even then 'n' is even.

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Ex: 28

Proof By Contraposition

Let's Prove: n is odd $\Rightarrow 7n+4$ is odd

We have n odd, so $n = 2k+1$, For some integer k

$\Rightarrow 7n = 14k+7 \Rightarrow 7n+4 = 14k+11$

$= 2 \times 7k + 10 + 1 = 2(7k+5)+1$

Let's Pose, $z = (7k+5)$

So, $7n+4 = 2z+1$

then, $7n+4$ is odd

* So, We proved $\neg q \Rightarrow p$ is true,

that means $p \Rightarrow q$ is true,

So: $7n+4$ is even implies n is even.

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Chapter 1.8:

Ex; 6

- Universal quantification: $\forall x\ P(x)$
  - negation: $\neg \forall x\ P(x) \Leftrightarrow \exists x\ \neg P(x)$
- Existential quantification: $\exists x\ P(x)$
  - negation: $\neg \exists x\ P(x) \Leftrightarrow \forall x \neg P(x)$

Exercise: Prove that:

$$(a \longrightarrow b) \wedge (a \longrightarrow c), \neg(b \wedge c), d \vee a \Rightarrow d$$

Proof By Contradiction:

Suppose $\neg d$, then d is False, and then $d \vee a$ is true implies a is true. We know that $(a \longrightarrow b) \wedge (a \rightarrow c)$ is true. So, $a \rightarrow b$ is true and $a \longrightarrow c$ is true. And, we know that a is true. So, b and c must be true both. b and c true means $b \wedge c$ is true implies $\neg(b \wedge c)$ is False.

– Contradiction –

Because, we have $\neg(b \wedge c)$ as a premise, it's true.

So, our supposition is False. that means d is true.

So, We proved: $(a \longrightarrow b) \wedge (a \longrightarrow c), \neg(b \wedge c), d \vee a \Rightarrow d$

CHAPTER 2.1

Ex 7:

a) YES
b) NO
c) NO

Ex 9:

a) YES
b) NO
c) YES
d) NO
e) NO
f) NO

Ex 12:

a) TRUE
b) TRUE
c) FALSE
d) TRUE
e) TRUE
f) TRUE
g) TRUE

Ex 13:

a) TRUE

b) TRUE

c) FALSE

d) TRUE

e) TRUE

f) FALSA

Ex 22:

a) 0

b) 1

c) 2

d) 3

Ex 23:

a) $\{\emptyset, \{a\}\}$

b) $\{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

c) $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

Ex 29:

a) $A \times B = \{(a,y), (a,z), (b,y), (b,z), (c,y), (c,z), (d,y), (d,z)\}$

b) $B \times A = \{(y,a), (y,b), (y,c), (y,d), (z,a), (z,b), (z,c), (z,d)\}$

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Ex 34:

a) $A \times B \times C = \{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0),$
$(b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1),$
$(c, y, 0), (c, y, 1)\}$

b) $C \times B \times A = \{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b),$
$(0, y, c), (1, x, a), (1, x, b), (1, x, c), (1, y, a),$
$(1, y, b), (1, y, c)\}$

c) $C \times A \times B = \{(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x),$
$(0, c, y), (1, 0, x), (1, 0, y), (1, b, x), (1, b, y),$
$(1, c, x), (1, c, y)\}$

d) $B \times B \times B = \{(x, x, x), (x, x, y), (x, y, x), (x, y, y),$
$(y, x, x), (y, x, y), (y, y, x), (y, y, y)\}$