FREQUENCY FILTRATION OF MEDICAL IMAGES

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Digital Image Processing

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- The guidelines lay out the fundamentals of utilizing the mathematical software package **MATLAB** to process biomedical images in the frequency domain. The use of common frequency filtering methods on initial biomedical images is considered. The work's sequence and the prerequisites for the report is handed out.

CONTENTS

OBJECTIVE: Using the computer calculating program MATLAB, we will study frequency filtering algorithms for biological images.

1. THEORETICAL BASES OF WORK

1.1 INTRODUCTION TO FOURIER ANALYSIS

The Fourier transform is used to execute the Fourier transform on a function of two variables — a discrete image function in Frequency-based picture Enhancement approaches. The equality gives the direct discrete Fourier transform of the function f (x, y) of an image of size M × N:

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)},$$

Where: u = 0, 1, 2, …, M – 1; v = 0, 1, 2, …, N – 1.

The inverse Fourier transform is determined by the expression:

$$f(x,y) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(u,v) e^{i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}.$$

The variables u and v are known as transform variables or frequency variables, while the variables x and y are known as spatial variables or picture variables. To ease computer implementation, the values M and N are usually even, and the center of the Fourier transform is at the point with coordinates: u = (M/2) + 1, v = (N/2) + 1. At a position (u, v) = (0, 0), the Fourier transform value is equal to:

$$F(0,0) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y).$$

Thus, if f (x, y) is the image, then the Fourier value is the transformation at the origin equal to the average brightness in the image. The values of F (0, 0) are often called the constant component of the spectrum.

Due to the fact that the function f (x, y) is real, the spectrum of the Fourier transform of the image has the property of symmetry. The following relationships between the samples in the spatial and frequency domains are valid:

$$\Delta u = \frac{1}{M\Delta x}, \quad \Delta v = \frac{1}{N\Delta y}.$$

The Fourier spectrum of the image consists of pixels having a large dynamic range of brightness. The image reproduction system, as a rule, is not able to correctly display such a large range of intensity values, which leads to the fact that in the usual display of the Fourier spectrum, a significant number of details are lost. In this regard, in order to improve the visual perception of halftones, the image of the spectrum is subjected to a logarithmic transformation.

Image 1 shows a white rectangle with dimensions of 20 × 40 pixels overlaid on a black backdrop with size of 512 × 512 pixels, as well as the image's centered Fourier spectrum. Before performing the Fourier transform, multiply the original picture by (–1) x + y in order to center the spectrum.

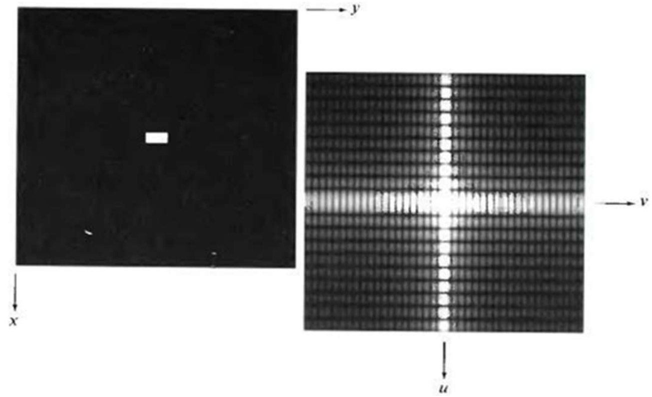

Fig.1 On the left is a white rectangle with dimensions of 20 × 40 pixels, on a black backdrop with dimensions of 512 × 512 pixels, and on the right is a centered Fourier spectrum after performing the logarithmic adjustment.

1.2 BASICS OF FREQUENCY FILTERING

The frequency domain of a digital picture is simply the space in which the Fourier transform variables (u, v) take values. Because the frequency of a signal is directly connected to its rate of change, it is intuitively obvious that the frequencies in the Fourier transform correspond to fluctuations in brightness in the image. The average picture brightness corresponds to the most slowly changing (constant) frequency component (u = 0, v = 0).

The slowly changing picture components are determined by the low frequencies corresponding to the Fourier transform points around the origin. Higher frequencies begin to correlate to more and more fast variations in brightness as you travel away from the origin, which are the borders of objects and other visual characteristics defined by abrupt changes in brightness, such as noise in the image. The frequency domain picture filtering technique consists of the following steps:

1. The original picture is multiplied by x + y to center its Fourier transform.
2. The direct discrete Fourier transform (DFT) of the original picture is determined as F (u, v).
3. The filter function H is multiplied by the function F (u, v) (u, v).
4. The inverse DFT is computed using the result of step 3.
5. The important element of the step 4 result is underlined.
6. Step 5's output is multiplied by x + y.

The filter function H (u, v), also known as the filter transfer characteristic, suppresses some conversion frequencies while leaving others unaltered. A structural schematic of the key phases of image filtering in the frequency domain is shown in Fig. 2. In addition to multiplying the picture by (-1) x + y, brightness scaling procedures, normalizing the size of the original image, changing the input data format to a floating-point format, and a variety of additional operations can be performed during the preprocessing step.

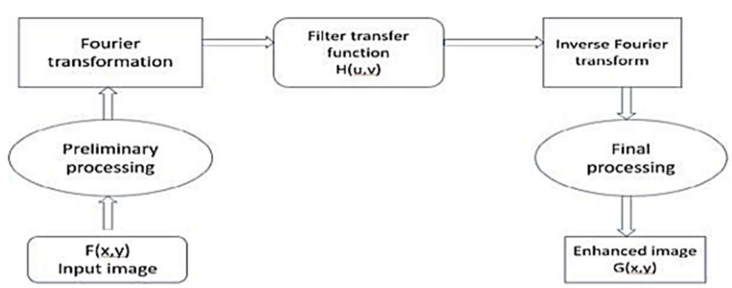

Fig. 2. The main stages of image filtering in the frequency domain

1.3 BASIC FREQUENCY FILTERS AND THEIR PROPERTIES

The most common types of filters are low-pass filters, which attenuate high frequencies while bypassing low frequencies, and high-pass filters, which have the opposite features. The appearance of dominating brightness values on smooth regions of the image is caused by low Fourier transform frequencies, whereas high frequencies are mostly responsible for contours and noise. The image has fewer clear features than the original after applying low-pass filtering. High-pass filtering reduces brightness variations within vast smooth sections of the picture while highlighting transition zones with fast brightness fluctuations, i.e., image contours. Such a picture is usually sharper than the original.

Because high-frequency filters nearly totally suppress the constant component F (0, 0), which controls the average brightness of the output picture after processing with such a filter, the image after processing with such a filter seems quite dark. To overcome this limitation, a constant equal to half the filter's height is added to the filter's transfer function. A filter plug, often known as a notch filter, is another form of frequency filter. It reads a brightness value from an image.

This brightness value at the origin is typically the image's average brightness. The image's average value Brightness cannot be precisely equal to zero since this would require some parts of the image to have negative values, and the methods for displaying the Information cannot function with negative brightness values. To eliminate the conflict, the lowest negative number is set to zero (black level), and the remaining values are increased accordingly.

1.4 SMOOTHING FREQUENCY FILTERS

As previously stated, contours and other sudden changes in brightness in the picture, including noise, contribute significantly to the high-frequency components of its Fourier transform. Image smoothing in the frequency domain is accomplished by attenuating the high-frequency components of an image's Fourier range. The generalized model of picture frequency domain filtering may be expressed by the following equality:

$$G\,(u, v) = H\,(u, v) \cdot F\,(u, v)$$

Where F (u, v) is the Fourier transform of the picture to be filtered, and H (u, v) is the filter's transfer function, which weakens the high-frequency components F (u, v) and creates the function G (u, v) Consider three types of low-pass filters: ideal, Butterworth, and Gaussian. According on the kind of transfer function, these filters range from highly sharp (ideal) filters to very smooth filters (Gaussian filter). The Butterworth filter is the only one studied, and it is distinguished by the order of the filter - a parameter that defines the slope of the filter's transfer function. For tiny filter order values, the transfer function has a smooth, close-shaped form.

1.4.1 PERFECT LOW PASS FILTERS

An ideal low-pass filter is one that cuts off any high-frequency components of the Fourier transform that are positioned further away from the origin of the centered picture than some set distance D_0. This filter is also known as a two-dimensional ideal low-pass filter, and it has the following transfer function:

$$H(u,v) = \begin{cases} 1, at\ D(u,v)\ > D_0 \\ 0, at\ D(u,v)\ > D_0 \end{cases}$$

Where D_0 – is the specified non-negative amount, the point of the filter profile at which the transition from H (u, v) = 1 to H (u, v) = 0 is made, known as the cutoff frequency; D (u, v) is the distance from the point (u, v) to the origin - the center of the frequency rectangle. Because the Fourier transform is centered, the distance D (u, v) is given by the formula:

$$D(u,v) = [(u - m/2)^2 + (v - n/2)^2]^{0,5}$$

where m and n are the dimensions of the source picture Figure 3 depicts an ideal low-pass filter in the shape of is fermentations, as well as the filter's radial profile:

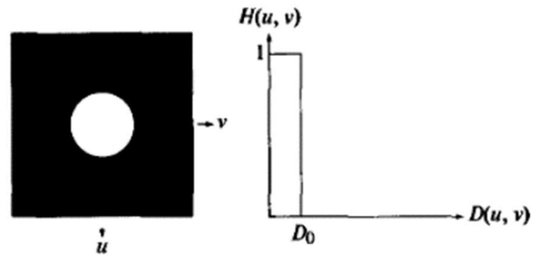

Fig. 3. On the left is a representation of an ideal low-pass filter as an image; on the right is a radial filter profile

The filter's ideality is highlighted by the fact that all frequencies inside a circle of radius D_0 pass unmodified, but frequencies beyond the circle are entirely muted. Because an ideal low-pass filter has symmetry with respect to the origin, it is sufficient to create one radial profile - a function of the distance from the origin - to uniquely define a filter. The filter's transfer function in the coordinates H (u, v) is derived by rotating the profile by 3600 around the origin. One method for determining the circles in which a particular amount of the total picture energy PT is encapsulated is to pick a reference set of cutoff frequency oppositionists. The total energy will be defined as (u, v); u = 0, 1, 1…M – 1; v = 0, 1, 1…N – 1.

$$P_T = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} P(u, v)$$

Where P (u, v) is defined as: P (u, v) = | F (u, v) |2, and F (u, v) is the Fourier transform of the original f (x, y).

The frequency r (α) is defined as the radius of the circle centered in the center of the frequency rectangle containing α percent of the energy of the image spectrum, i.e.:

$$\alpha = 100 \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \frac{P(u,v)}{P_T}.$$

Selecting a sub-optimal filter cut-off frequency might result in severe distortion of the processed picture: choosing a cut-off frequency that is too low will erase most of the visual features. While choosing a somewhat high cutoff frequency will result in a significantly different output picture from the original, the noise level will remain same.

When utilizing an ideal low-pass filter, undesired effects such as blurring and the appearance of false contours occur; however, when the breadth of the utilized filter in the frequency domain is reduced, the blurring effects are amplified.

The unwanted consequences of an ideal low-pass filter are known as "ringing" (or the Gibbs effect), and they manifest as the emergence of illusory contours around true ones. The structure of the false loops becomes thinner. When the cut-off high-frequency component's energy diminishes

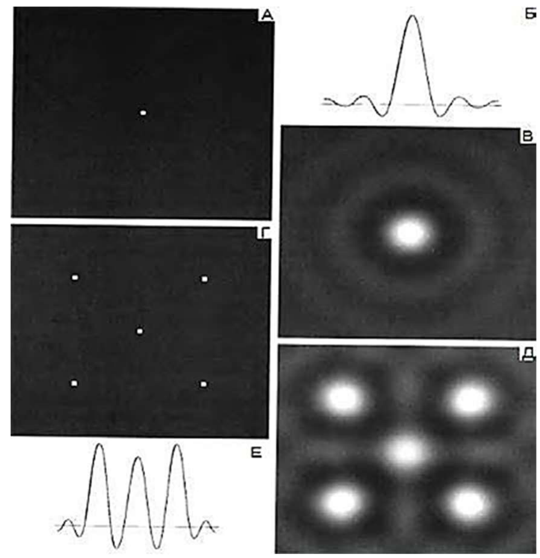

Fig. 4. A is the image of an ideal low-pass filter, Б is the brightness profile in the spatial domain, B is the image of the ideal low-pass filter in the spatial domain, Γ is a conditional image consisting of 5 bright points in the spatial domain, Д is the convolution of images B and Д, E -brightness profile filtered image Д

The convolution theorem, which states that there is a spatial counterpart of an ideal low-pass filter, can be used to explain the appearance of misleading contours. The spatial function of such a filter may be computed using the inverse Fourier transform of the low-pass filter's transfer function and is a sequence of circular concentric rings of varying brightness, resulting mostly in the appearance of false contours: Figure 4 depicts the Gibbs effect in action with an ideal low-pass filter.

The idea of brightness profile refers to the reliance of pixel brightness change on the number of pixels positioned on a horizontal line, often passing through the center of the picture; pixel counting along a straight line begins at the leftmost pixel and ends at the rightmost pixel. Thus, the filtering findings provided using an ideal low-pass filter reveal that perfect low-pass filters have little practical utility.

1.4.2 BUTTERWORTH LOW PASS FILTERING

The formula for the Butterworth low pass filter transfer function n with a cutoff frequency at a distance D_0 from the origin is:

$$H(u,v) = \frac{1}{1 + [D(u,v) / D_0]^{2n}}$$

The radial profiles of the Butterworth low-pass filter's transfer function, depending on the order of the filter, are illustrated in Fig. 5.

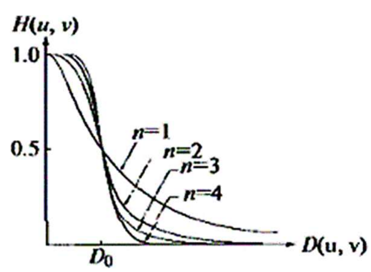

Fig. 5. Radial profiles of the transfer function of the low-frequency Butterworth filter depending on the order of the filter

The Butterworth low-pass filter's transfer function features a gap that specifies the exact border between the transmitted and cut-off frequencies, unlike a perfect low-pass filter. Butterworth low-pass filters have a lesser occurrence of unwanted blurring effects and the appearance of spurious loops as compared to perfect low-pass filters. As the order of the low-frequency Butterworth filter grows, the development of blur effects increases. The second-order Butterworth low-pass filter is commonly regarded as the best choice for attaining a balance of low-pass filtering efficacy and acceptable false contour and image blurring.

Figure 6 depicts the brightness profiles of pictures of low-frequency Butterworth filters with the same cutoff frequency of 1, 2, 5, and 20.

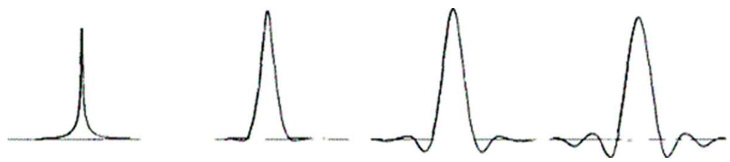

Fig.6.Fromleftto right: brightness profiles of low-frequency Butterworth filters with the same cutoff frequency of the order of 1, 2, 5, and 20 respectively

1.4.3 GAUSSIAN LOW-PASS FILTERS

Gaussian low-pass filters are defined as follows in the two-dimensional case:

$$H(u,v) = e^{-\frac{D^2(u,v)}{2\sigma^2}}$$

When D (u, v) = D_0 (D_0 is the filter cutoff frequency), the filter transfer function is 0.667 of its highest value. Figure 7 depicts the radial profiles of a Gaussian filter for a variety of D_0 values.

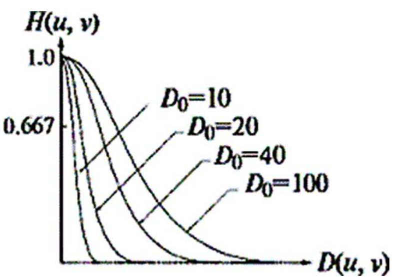

Fig. 7. Radial profiles of the transfer function of the low-frequency Gaussian filter at various values of the cutoff frequency *D*0

The inverse Fourier transform of a Gaussian function is also a Gaussian function, as is well known. The spatial Gaussian low-pass filter obtained by inverse Fourier transform of the frequency Gaussian filter will be positive and will not have concentric rings, resulting in the complete absence of Gibbs effect manifestations in the processed image, which is the main advantage of Gaussian low-pass filters over Butterworth filters.

However, a low-frequency Gaussian filter frequently gives less smoothing at the same cutoff frequency as Butterworth filters, therefore Butterworth filters appear to be a better choice in those circumstances when tight control of the transition zone from low to high frequencies is necessary.

1.5 FREQUENCY SHARPENING FILTERS

High-frequency components of the Fourier transform of the picture are related with contours and other rapid changes in brightness in the image. The picture can be sharpened in the frequency domain by applying a high-pass filtering process that suppresses the low-frequency components while leaving the high-frequency section of the Fourier transform alone.

The following relationship may be used to calculate the transfer function of high-pass filters:

$$H_{\text{hp}}\ (\text{u, v}) = 1 - H_{\text{hp}}\ (\text{u, v})$$

Where H_{hp} (u, v) – The equivalent low-pass filter's transfer function. Ideal high-pass filters, Butterworth high-pass filters, and Gaussian high-pass filters are examples of high-pass filters.

1.5.1 IDEAL HIGH PASS FILTERS

The formula for determining two-dimensional ideal high-pass filters is:

$$H(u,v) = \begin{cases} 0, \text{при } D(u,v) \leq D_0 \\ 1, \text{при } D(u,v) > D_0 \end{cases}$$

Where D (u, v) is the distance from the point with the coordinates (u, v) to the center of the frequency rectangle, and D_0 is the cutoff frequency (origin). Figure 8 depicts the radial profile of the ideal high-pass filter's transfer function, the spatial domain function of the ideal high-pass filter, and the accompanying brightness profile.

Fig. 8. From left to right: radial profile of the transfer function of an ideal high-pass filter; perfect high pass filter function in the spatial domain; corresponding brightness profile

The Gibbs effect, which causes "ringing" in the processed picture, is characteristic of perfect high-pass and low-pass filters. Ideal high-pass filters, like ideal low-pass filters, have essentially little practical applicability.

1.5.2 BUTTERWORTH HIGH PASS FILTERS

The Butterworth high-pass filter of order n with a cutoff frequency at a distance D_0 from the origin has the following transfer function:

$$H(u,v) = \frac{1}{1 + [\,D_0 + \text{D}(\text{u,v})\,]^{2n}}$$

Figure 9 depicts the radial profile of the second-order high-frequency Butterworth filter's transfer function, the spatial domain function of the high-frequency Butterworth filter, and the brightness profile of the spatial filter function.

Compared to ideal high-frequency Butterworth filters, high-frequency Butterworth filters cause substantially less distortion of object boundaries. The distortions of object boundaries clearly rise as the order of the Butterworth high-pass filter is increased.

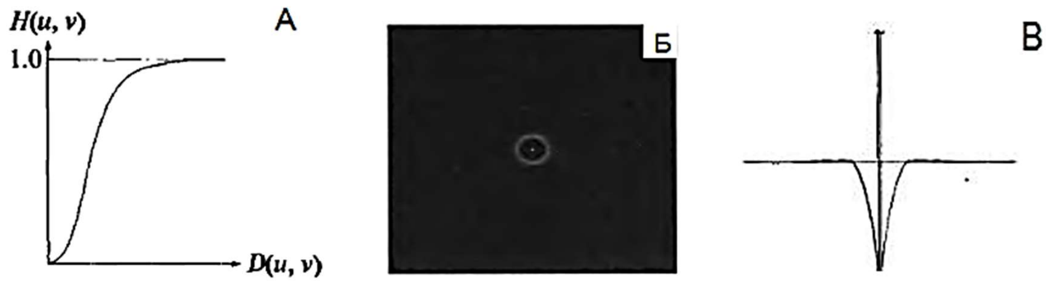

Fig. 9. A is the radial profile of the transfer function of the high-frequency Butterworth filter of the second order, Б is the function of the high-frequency Butterworth filter in the spatial domain, B is the brightness profile of the spatial filter function

1.5.3 GAUSSIAN HIGH PASS FILTERS

The expression specifies the transfer function of a Gaussian high-pass filter with a cutoff frequency at a distance D_0 from the origin:

$$H(u,v) = 1 - e^{-\frac{D^2(u,v)}{2{D_0}^2}}$$

Figure 10 depicts the radial profile of the Gaussian high-pass filter's transfer function, the function of the high-frequency Gaussian filter in the spatial domain, and the brightness profile of the Gaussian filter's spatial function. Even for small objects and narrow bands, the high-frequency Gaussian filter delivers high-quality filtering with no distortion.

Fig. 10. A is the radial profile of the transfer function of the high-frequency Gaussian filter, B is the function of the high-frequency Gaussian filter in the spatial domain, B is the brightness profile of the spatial function of the filter

1.6 UNSHARP MASKING. HIGH PASS FILTERING WITH INCREASING FREQUENCY RESPONSE

Since the high-frequency filters used to analyze picture data eliminate the constant component (zero component) of its Fourier transform, images acquired using high-pass filtering have an average background brightness value near to zero. To overcome this flaw, you can use "image underlay" to add a

little portion of the original picture to the filtering output. In certain circumstances, it appears that increasing the original image's contribution to the final filtering output is required.
This method is a generalization of the unsharp masking method and is known as high-pass filtering with increased frequency responsiveness. Unsharp masking is the process of creating a sharp picture by eliminating the original's smoothed duplicate from the original. This signifies that there is high-pass filtering in frequency transformations, which is performed by subtracting from the original picture f (x, y) the result of its low-pass filtering f_{lp} *(x, y)*: $f_{hp}(x, y) = f(x, y) - f_{lp}(x, y)$.

The following statement, which describes high-pass filtering with an increased frequency response, is a generalization of the previous expression: $f_{hb}(x, y) = A \cdot f(x, y) - f_{lp}(x, y)$.

Filtering with a higher frequency response allows you to alter the original image's contribution to the final processing outcome.

1.7 HIGH PASS FILTERING

It is beneficial to increase the high-frequency component of image processing while tackling specific image processing difficulties. The filter with high-frequency amplification has the following transfer function:

$$H_{hfe}(u, v) = a + b \cdot H_{hp}(u, v)$$

Where $H_{hp}(u, v)$ is the low-pass filter's transfer function. A typical value of an is in the range of 0.25 to 0.5, and a typical value of b is in the range of 1.5 to 2.5 (it's vital to remember that b > a). The method's name comes from the fact that high frequencies are amplified when b > 1.

Sharpening radiographic pictures is the most common use of filtering with high-frequency amplification in biomedical image processing jobs. Because x-rays cannot be focused like light beams, most x-rays seem fuzzy. Because the brightness of X-ray pictures is frequently moved to the dark area, image processing approaches to boost the brightness and contrast of the image are required.

1.8 IMPLEMENTATION OF FREQUENCY FILTERING METHODS IN THE SYSTEM OF COMPUTER COMPUTING MATLAB

The **fft2** function in the **MATLAB** package is used to do a direct Fourier transform, and it has the following syntax: **F = fft2(f)**, where f is the source picture and **F** denotes the Fourier transform of a two-dimensional **function (x, y)**.

The following command must be run to retrieve the image's spectrum: **S** stands for **S = abs(F)** The abs command determines the absolute value (module) of a complicated function F. Use the **fftshift** function with the following syntax to shift the Fourier transform's origin to the frequency domain's center: **Fc = fftshift(F)**, with **Fc** being the centered Fourier transform.

Because the dynamic range of the Fourier transform picture is so large, it is important to execute brightness range conversion procedures in order to display the Fourier transform accurately on the display. The following is a logarithmic transformation that may be used to do this operation: **S2 = log (1 + abs (Fc)).**

The function **ifft2** is used to perform the inverse Fourier transform, which has the following syntax: **F = ifft2(F)**, where **F** is the Fourier transform and **f** is the associated picture. Following the inverse Fourier transform, the real component of the result must be selected using the command of the following form: **F = real(ifft2(F))**.

For creating matrices and vectors, **MATLAB** has special functions. The easiest approach to specify a one-dimensional array (vector) is to use the syntax: **[name] = X1:dX:Xk**, where name is the name of the variable that will be assigned to the produced array, **X1** is the value of the first element, **Xk** is the value of the last element, and **dX** is the step that each subsequent element is formed using (by default it is 1). This is a row vector. To create a column vector, enter **[name] = [X1:dX:Xk]'**, where the symbol' denotes the matrix's transpose.

Forming two-dimensional arrays of the coordinate grid is required while tackling digital image processing challenges. The **meshgrid** function in the **MATLAB** system is useful for this, as it creates output arrays that determine the coordinates of the nodes of the rectangle indicated by the input vectors. The domain of a two-variable function is defined by the produced rectangle. The syntax for the **meshgrid** function is as follows: **[X, Y] = meshgrid (x, y).** The matrix X's rows are copies of the vector x, while the matrix Y's columns are copies of the vector y. The use of such arrays simplifies the computation of two-variable functions by allowing you to perform operations on them.

Let's say x = (0, 1, 2) and y = (0, 1). Then,

$$X = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}, \qquad Y = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

The creation of such arrays simplifies the calculation of two-variable functions by permitting the use of array operations.
As a result, to apply frequency filtering on the initial picture f (x, y), the following procedures must be performed:

1) Using the size command, determine the size of the picture f (x, y); produce vectors of frequency coordinates u and v with sizes equal to the vectors of spatial coordinates x and y;

2) Using the fft2 command, do a straightforward Fourier transform on the original picture to acquire the frequency representation F (u, v);

3) Set the cutoff frequency in pixels (as a percentage of the maximum horizontal or vertical picture size); construct the transfer characteristic of the needed filter H (u, v), which is a two-dimensional matrix whose dimensions are the same as the original image's matrix;

4) By elementwise multiplying the transfer characteristic of the filter H (u, v) and the Fourier transform of the original picture F (u, v), you may get the frequency representation of the filtered image G (u, v).

5) Using the ifft2 command, obtain a spatial representation of the filtered picture g (x, y) by performing an inverse Fourier transform on the frequency representation G (u, v).

6) Using the real command, choose the genuine component of the preceding stage's result.

2. PROCEDURE FOR PERFORMANCE OF WORK

1. To acquire the spectrum image in the frequency domain, upload the xraychest.jpg image of your chest and perform a Fourier transform on the original image. Visual perception can be improved by using the graduation logarithmic transformation.

2. Apply a high-pass filtering approach with a greater frequency response to the original chest x-ray picture xraychest.jpg.

2.1. Filter the original chest X-ray image using a low-pass filter: xraychest.jpg

2.2. By selecting parameter, A, you acquire the optimum sharpening outcome by increasing the frequency responsiveness.

2.3. An image acquired in step 2.2 was submitted to the histogram equalization method to seek MATLAB system assistance in performing this procedure by running the command doc haystack.

3. Using high-pass filtering techniques, sharpen the original chest x-ray picture xraychest.jpg.

3.1 Filter the original chest x-ray picture, xraychest.jpg, via a high-pass filter.

3.2. We accomplish high frequency amplification by selecting parameters a and b.

3.3. The histogram equalization process is used to the picture acquired in step 3.2.

3. CONTENTS OF THE REPORT

1. The goal of the work.

2. Listings of MATLAB programs (M-files) for each challenge.

3. Analyze the findings by comparing them to the theory.

4. CHECK-UP QUESTIONS

1. Fourier transform of an image in two dimensions

2. The fundamentals of picture frequency filtering.

3. Smoothing and frequency filtering

4. Sharpening filters for frequencies.

5. Filtering with increasing frequency characteristics for high-pass unsharp masking.

6. High pass filtering.

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I appreciated doing things flawlessly and autonomously. I am delighted to be doing it. It's a terrific attempt that will help you learn more about a tough topic.

REMARKS AND GRADE (BY THE INSTRUCTOR)

Instructor Signature:
Grading Date: