Measurement of gravitational acceleration

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Acceleration of free-falling objects caused by the gravity is called gravitational acceleration (or acceleration of gravity). Gravitational acceleration can be expressed as

$$G=mg \tag{1}$$

where, m and g are the quality and gravitational acceleration of an object, respectively. The international unit of gravitational acceleration is m/s^2.

The direction of gravitational acceleration g is vertical downward. At the same place and height, the gravitational acceleration is a constant for any object. The value of gravitational acceleration decreases with the increase of altitude, and increases with the latitude.

The standard value of gravitational acceleration is g=9.80665 m/s^2, which is measured at sea level at latitude 45°. Generally, g is considered as an invariant constant of 9.80 m/s^2 in the normal calculations.

1. Measurement of gravitational acceleration using simple pendulum method

(1) Principle

One port of a non-extendable light rope is fixed, and another port hangs a tiny metal ball. When the quality of light rope is much less than that of metal ball, the ball diameter d is much less than the light rope length l, and the swinging angle θ of ball is very small, the equipment with a light rope and a metal ball is a simple pendulum. If the air resistance, rope quality and extension are neglectable, the motion of simple pendulum can be regarded as a harmonic oscillation when θ<5°. The vibration cycle (period) of harmonic oscillation is described as

$$\mathrm{T}=2\pi\sqrt{\frac{L}{g}} \tag{2}$$

where, L is the length of simple pendulum (i.e., $L= l+d/2$). Thus, the gravitational acceleration can be expressed as

$$g=\frac{4\pi^2 L}{\mathrm{T}^2} \tag{3}$$

Here, L and T can be measured using a meter ruler and a stopwatch (or phone), respectively. The error of T can be reduced through the continuously multiple record. For example, T=T'/N, where T' is the total time of simple pendulum and N is the cycle number.

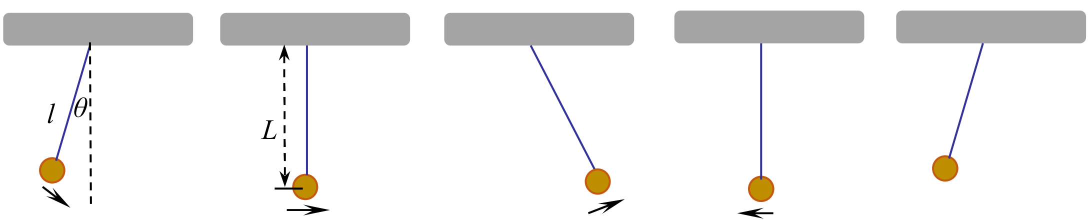


Figure 1. One vibration cycle

(2) Data processing

a. Calculation of uncertainty of measurement, write the complete expression of results.

b. Comparing the calculation results of gravitational acceleration with local recognized value (for example: g=9.79684m/s^2 at Xi'an), calculating relative error.

(3) Notes

a. Rope requirements: light, fine, non-extendable, and long enough. Ball requirements: high density, small size (for instance, a steel ball with 2 cm diameter)

b. Simple pendulum requirements: the upper end (port) should be fixed to avoid the movement of fixed point and the change of L.

c. Ball swinging requirements: keep swinging in a vertical plane to avoid conical pendulum and θ<5°.

2. Other methods for measurement of gravitational acceleration

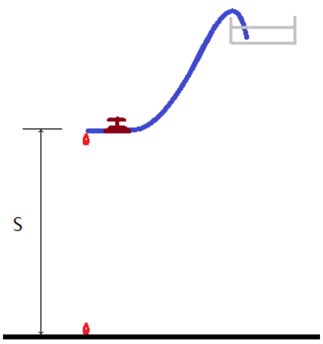
(1) Free-falling method

For the free-falling movement, the gravitation acceleration can be expressed as

$$g=\frac{2s}{t^2} \tag{4}$$

where, s and t are the free-falling height and time of an object, respectively. s and t can be measured using meter ruler and stopwatch (or dripping water), respectively. The falling time t is the difficulty in the experiment. Here, the dripping water method can be used for home experiments.

As shown in Fig. 2, the required instruments: a tap, water, and waterpipe. The dripping water method can be described as follows.



Let the water drops fall to the ground from a certain height s. The sound of each water drop can be heard, then the drop speed can be controlled by adjusting the tap until the second water drop starts to fall when the first water drop touches the ground. One can use the stopwatch to record the time T of 100 water drops. Thus, the falling time can be measured as t=T/100.

(2) Balancing method

One can measure the quality m and gravity G of an object using a balance and force meter, respectively. According to the equation g=G/m, we can calculate gravitational acceleration.

In summary, the methods to measure gravitational acceleration can be indirectly achieved from the physical equations with g. The instruments can be properly selected according to the equations.

Experiment requirements:

1. Tasks

a. Learn the principles and methods of measurement of gravitational acceleration.

b. Establish a simple pendulum system or others, measure multiple times.

c. Processing experiment data (results, calculation of uncertainty, comparison with recognized value), write report (3 minutes PPT and others).

2. Report requirements

a. Experiment principle: describe the principle and derive the equation.

b. Experiment instruments: list the instruments used, describe the instruments (give the pictures)

c. Experiment steps: list the experiment steps and offer a short video of doing experiment.

d. Experiment data and processing: list of experimental data, calculated results, uncertainty calculation, complete expression of results, comparison of measured *g* with recognized value (relative percentage error of *g*).

e. Result analysis: analyze the error reason, improved methods.

Experiment data:

	1	2	3	4
L				
T				
g				

Data processing: (n=4)

1. Best estimated value of L: $\overline{L}=\frac{1}{n}\sum_{i=1}^{n}L_i$

   Uncertainty of L

   Type A: $u_A(L)=S_{\overline{L}}=t_p\sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(L_i-\overline{L})^2}=$

   Type B: $u_B(L)=\frac{\Delta}{C}$ (Δ=0.5 mm for meter ruler, C=3 for normal distribution)

   Total uncertainty: $u_c(L)=\sqrt{u_A(L)^2+u_B(L)^2}$

2. Best estimated value of T: $\overline{\mathrm{T}}=\frac{1}{n}\sum_{i=1}^{n}\mathrm{T}_i$

   Uncertainty of T

   Type A: $u_A(\mathrm{T})=S_{\overline{\mathrm{T}}}=t_p\sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(\mathrm{T}_i-\overline{\mathrm{T}})^2}=$

Type B: $u_B(\mathrm{T}) = \dfrac{\Delta}{C}$ (Δ=0.2 s for stopwatch, C=$\sqrt{3}$ for uniform distribution)

Total uncertainty: $u_c(\mathrm{T}) = \sqrt{u_A(\mathrm{T})^2 + u_B(\mathrm{T})^2}$

3. Relative uncertainty of g: ($g = \dfrac{4\pi^2 L}{\mathrm{T}^2}$),

$$u_r(g) = \sqrt{\left(\frac{\partial \ln g}{\partial L}\right)^2 u_c^2(L) + \left(\frac{\partial \ln g}{\partial T}\right)^2 u_c^2(T)}$$

($\ln g = \ln(4\pi^2 L) - \ln \mathrm{T}^2 = \ln(4\pi^2) + \ln L - 2\ln \mathrm{T}$)

$$u_c(g) = u_r(g) \times \bar{g} =$$

4. The complete expression:

$$\begin{cases} g = \bar{g} \pm u_c(g) = \\ u_r = \dfrac{u_c(g)}{\bar{g}} \times 100\% = \end{cases} , p = 0.683$$

Table 2-2 t_p as a function of repetition time

n-1	1	2	3	4	5	6	7	8	9	10	15	20	30	40	∞
t_P	1.84	1.32	1.20	1.14	1.11	1.09	1.08	1.07	1.06	1.05	1.04	1.03	1.02	1.01	1

Table 2-3 Instrumental error distribution

Instruments	Micrometer	Vernier Caliper	Meter	Stopwatch	Ammeter
Error	Normal, 0.004mm	Rectangular 0.02/0.05mm	Normal, 0.5mm	Normal, 0.2s	Rectangular Range ×a %

C=$\sqrt{3}$ for uniform distribution, C=3 for normal distribution.

中文原版：

重力加速度的测量

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重力对自由下落的物体产生的加速度，称为重力加速度。如果以 m 表示物体的质量，以 g 表示重力加速度，重力 G 可表示为 G=mg, 在国际单位制中，重力加速度的单位是 m/s^2。

重力加速度 g 的方向总是竖直向下的。在同一地区的同一高度，任何物体的重力加速度都是相同的。重力加速度的数值随海拔高度增大而减小，也会随着纬度的升高而变大。

国际上将在纬度 45°的海平面精确测得物体的重力加速度 g=9.80665 m/s^2 作为重力加速度的标准值。在解决地球表面附近的问题中，通常将 g 作为常数，在一般计算中可以取 g=9.80 m/s^2。

一、单摆法测重力加速度

1、实验原理

将一根不可伸长的轻绳一端固定，在轻绳的另一端上悬挂一金属小球。若轻绳的质量远远小于金属小球的质量，并且金属小球的直径又比轻绳长度小得多，金属小球做幅角 θ 很小的摆动，这样的装置就是一个单摆。在忽略空气阻力、摆线质量和伸长等因素对实验结果的影响，同时单摆幅角很小($\theta<5°$)的情况下，单摆摆动可看作简谐振动，该简谐振动的振动周期为：

$$\mathrm{T}=2\pi\sqrt{\frac{l}{\mathrm{g}}}$$

式中 l 是单摆的摆长(摆线长加金属小球半径)，g 是重力加速度，T 是单摆摆动周期。则重力加速度 g 的计算公式为：

$$\mathrm{g}=\frac{4\pi^2 l}{\mathrm{T}^2}$$

用米尺多次测量摆长，用秒表多次测量 T（应采用累计放大法测量）。

2、数据处理

1、计算不确定度并完整表示结果

2、将测量结果与当地重力加速度的公认值进行比较计算百分百误差（西安地区 g=9.79684m/s^2）

3、注意事项

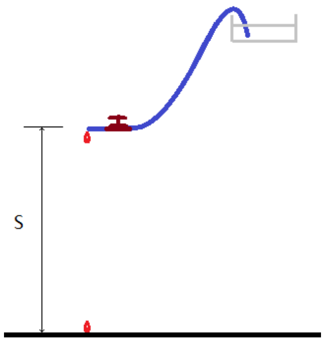
1、选择材料时应选择细、轻、不易伸长且长度较长的线；摆球应选用密度较大且尺寸较小的物品（例如是直径为 2 厘米左右的钢球）。

2、单摆悬线的上端应固定，以免摆动时发生摆线上固定点的移动、摆长改变等现象。

3、摆球摆动时，要使之保持在同一个竖直平面内，不要形成圆锥摆，且摆角小于 5°。

二、测量重力加速度的其它方法

1、落体法测量重力加速度



根据自由落体运动 $g=2s/t^2$，测量下落的高度和时间。高度可由米尺测出，时间的测量可用秒表、打点计时、频闪照片、滴水法、光电门等等测出。这种方法的难点是下落时间较难测量，学生在家方便实现的是滴水法：

让水滴从距地面一定高度的出水口中下落到地面上，可以听到水滴每次碰地面的声音，仔细地调整水阀来控制滴速，使第一滴水碰到地面的瞬间第二滴水正好从出水口处开始下落，用秒表记录 100 个水滴下落的时间，用米尺测量出水滴下落的高度，即可计算出 g。这种方法的关键是仔细调节水阀控制滴度满足要求。

2、平衡法

用天平测出物体的质量 m，用测力计测出重力 G，根据 $g=G/m$ 计算出 g。这种方法对于学生在家实验关键是测力计不好找，而且找到的测力计一般精度不高，导致 g 的测量精度不高。

总之，测重力加速度的基本思路是：只要是关系式中包含有 g 的，均可考虑能否根据该关系式测出其他的各物理量，从而间接地测出重力加速度。在设计时，根据公式的不同、仪器选择的不同、数据处理的不同也都可以引申出很多种测量的方法。

实验要求：

一、实验任务

1. 学习测定重力加速度的原理与方法
2. 搭建单摆实验系统，完成多次测量
3. 处理实验数据（实验结果，不确定计算，完整结果表示并与公认值进行比较），撰写实验报告

二、实验报告要求

1、实验原理：论述实验原理，推导实验公式。
2、实验仪器：列出实验仪器，实验装置描述（为减小误差，你在实验装置参数选择，装置措施等方面的考虑），附上装置照片。
3、实验步骤：列出实验步骤并附上实验过程短视频。
4、**实验数据与处理：**实验数据列表，计算结果、不确定计算并完整结果表示，将测量结果与公认值进行比较与公认值进行比较，计算百分百误差
5、实验结果分析：分析误差产生原因，改进措施，若可能最好附上改进后的实验及结果。
6、扩充内容(选作）:展示用其它方法完成重力加速度测量的情况。