

**HOME WORK**

**8.1,8.2,8.3,8.4**



**November 20, 2020**

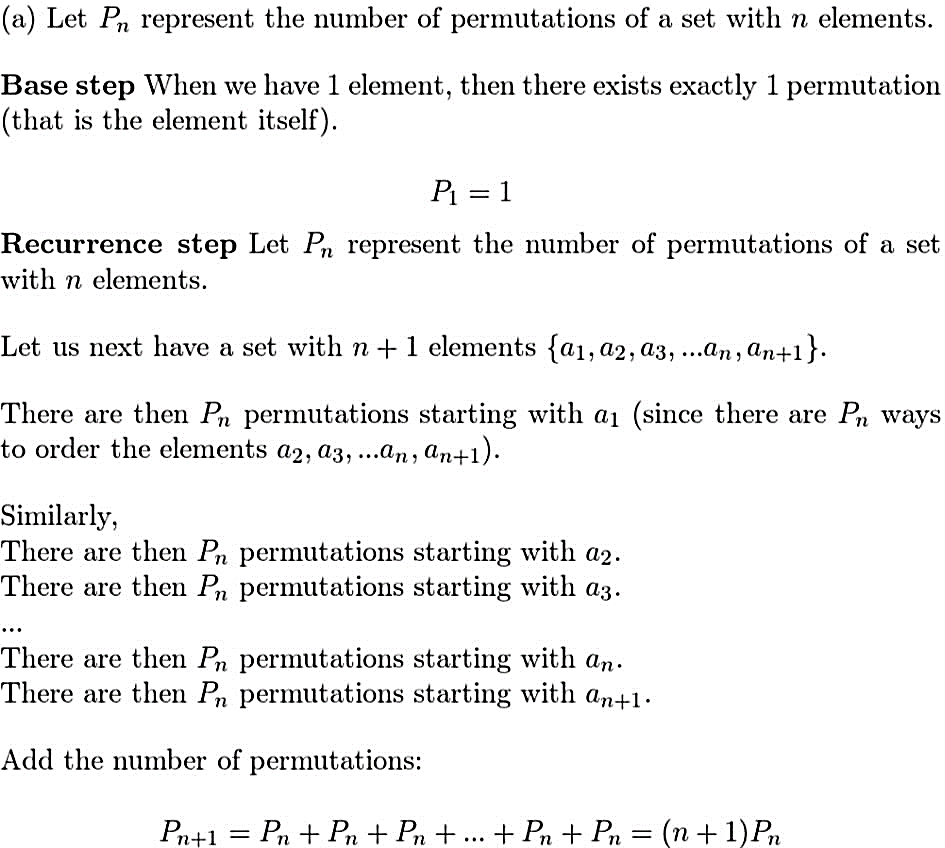
**RAIHAN MD RAKIBUL ISLAM**

**2020380029**

**8.1**

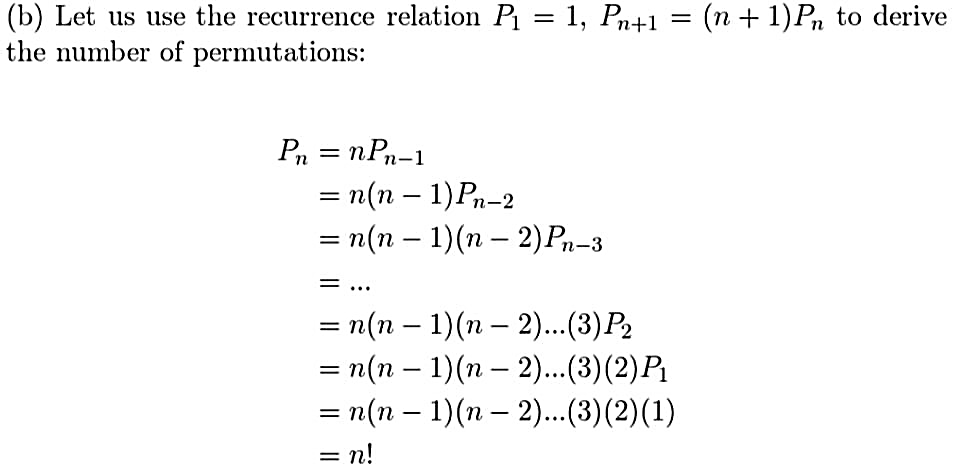
**2. a)** Find a recurrence relation for the number of permutations of a set with n elements.

**Sol:**



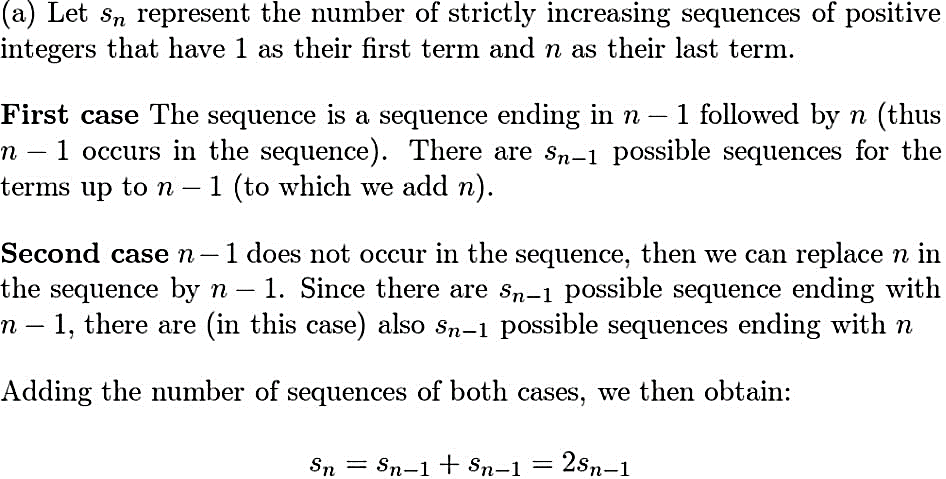
**b)** Use this recurrence relation to find the number of permutations of a set with n elements using iteration.

**Sol:**



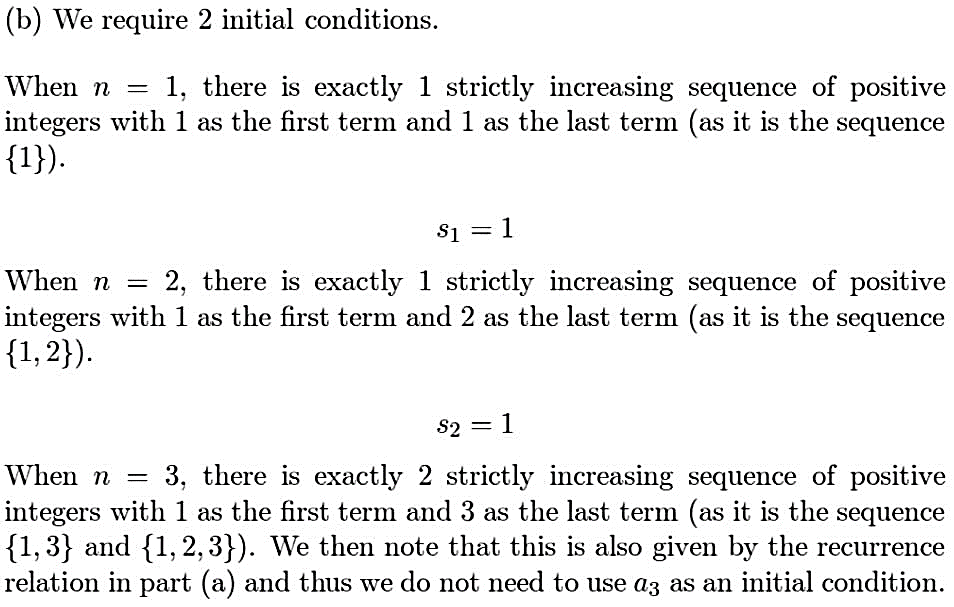
**6. a)** Find a recurrence relation for the number of strictly increasing sequences of positive integers that have 1 as their first term and n as their last term, where n is a positive integer. That is, sequences a1, a2, . . . , ak, where a1 = 1, ak = n, and aj < aj+1 for j = 1, 2, . . . , k − 1.

**Sol:**



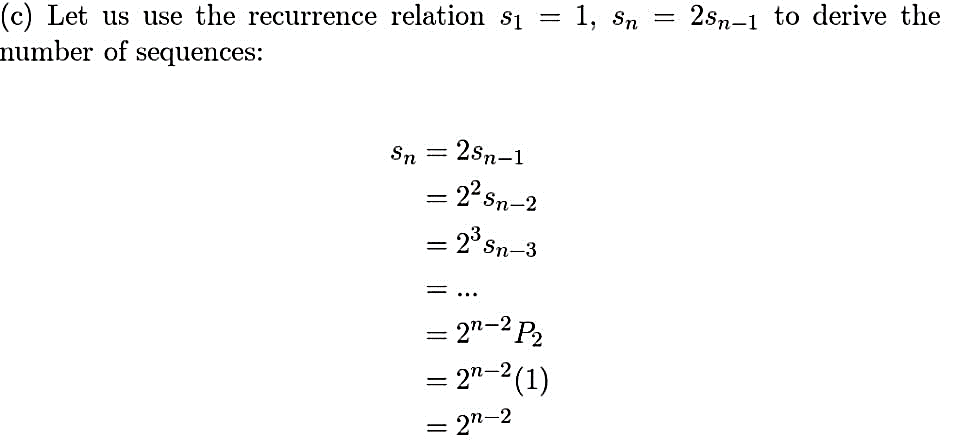
**b)** What are the initial conditions?

**Sol:**



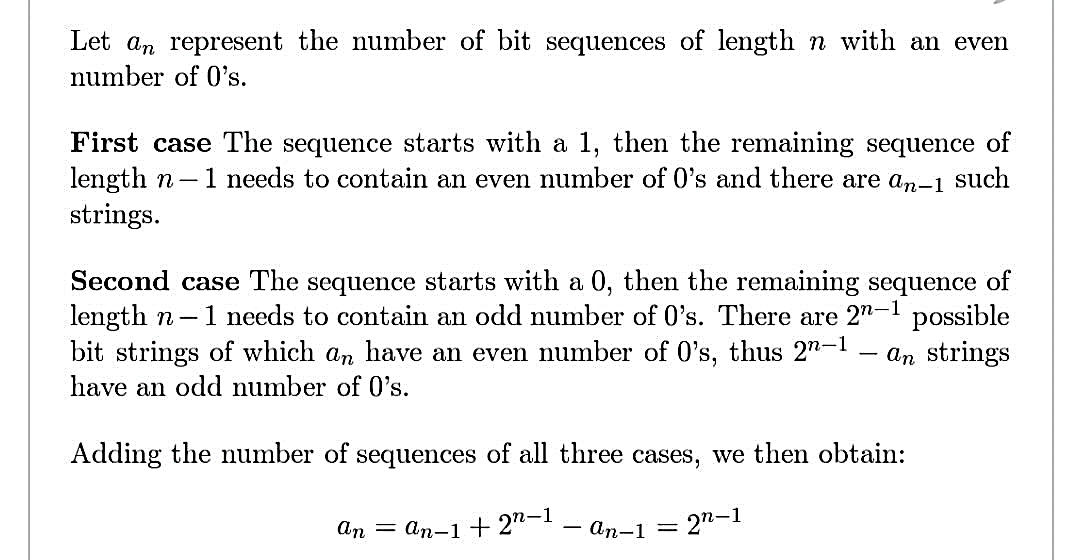
**c)** How many sequences of the type described in (a) are there when n is an integer with n ≥ 2?

**Sol:**



**24.** Find a recurrence relation for the number of bit sequences of length n with an even number of 0s.

**Sol:**

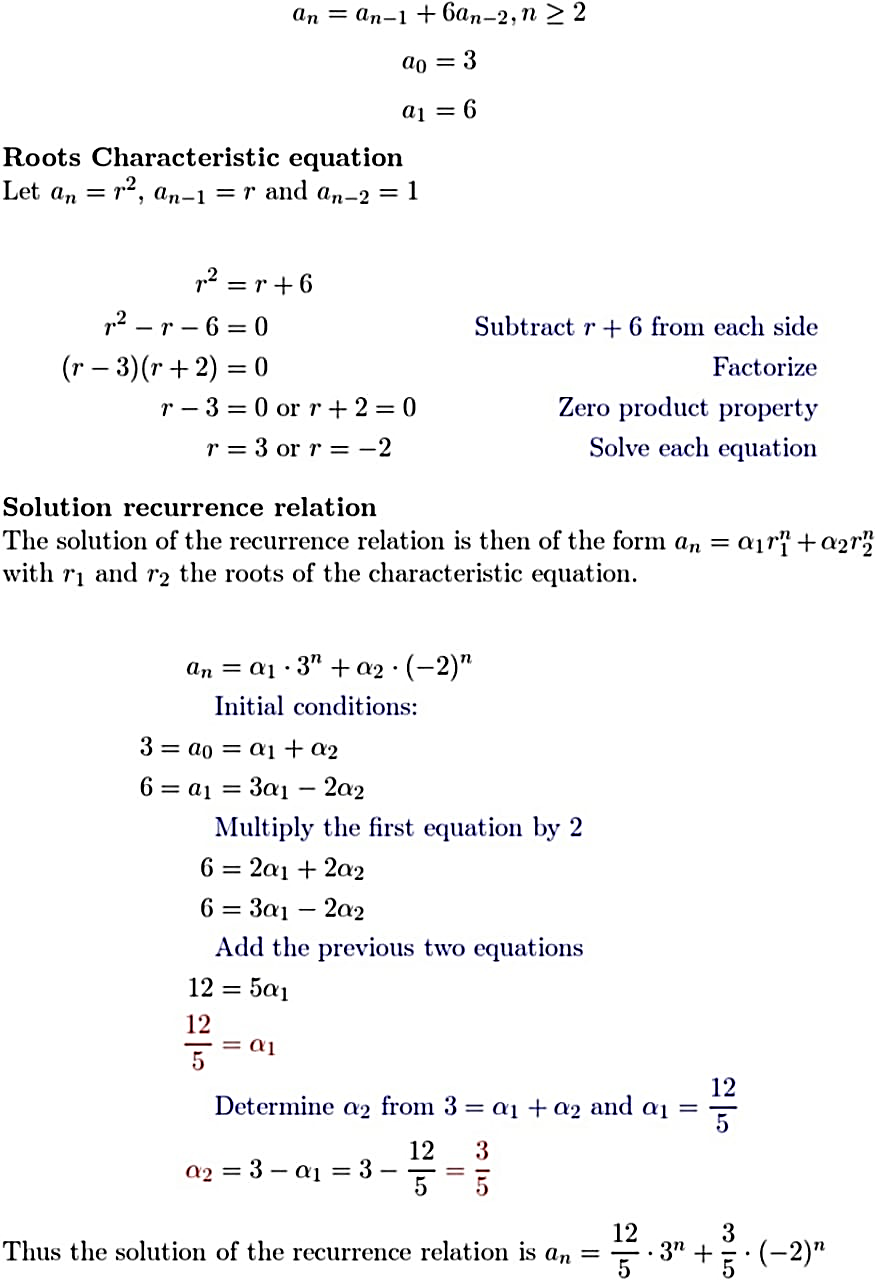


**8.2**

4. Solve these recurrence relations together with the initial conditions given.

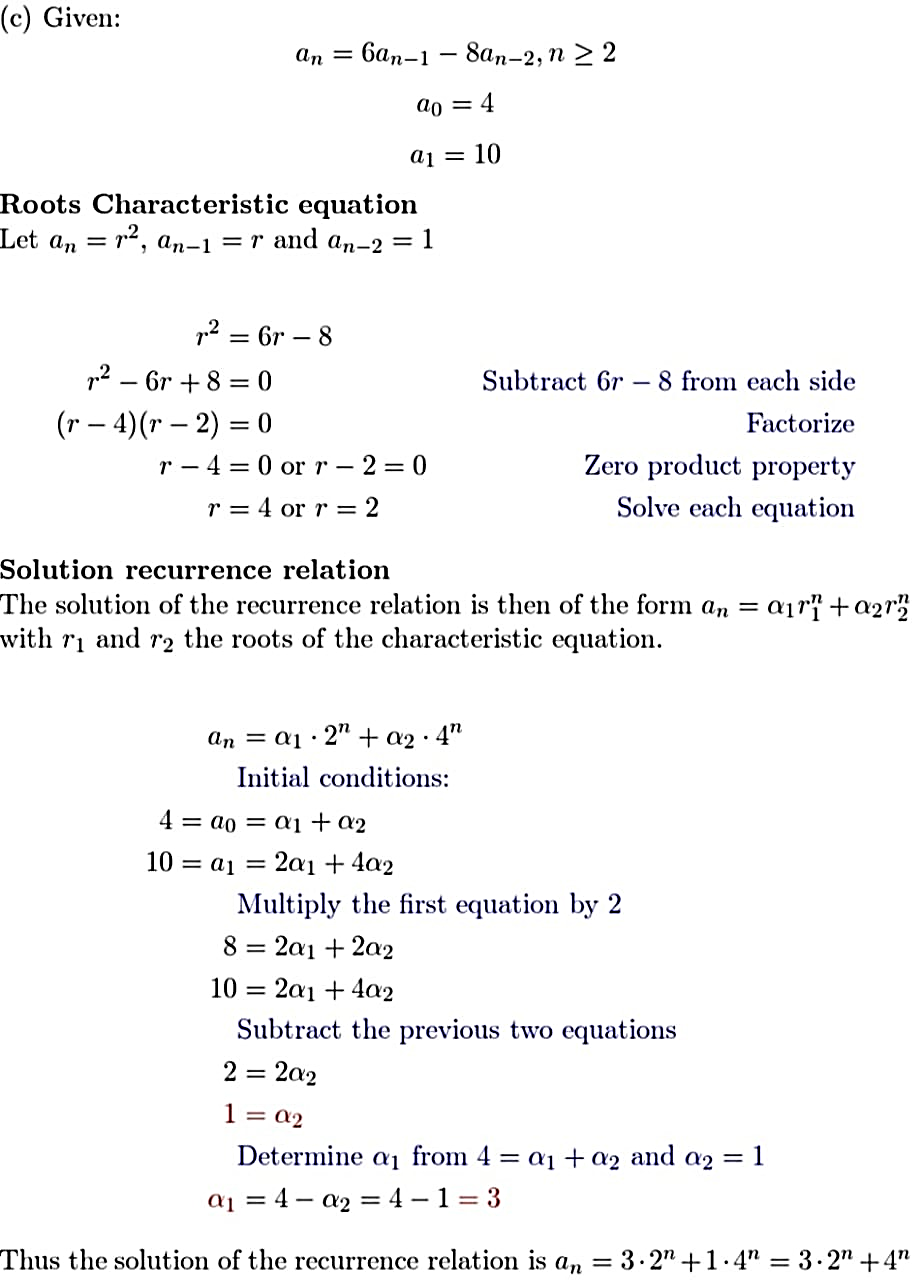
a) = + 6 for n ≥ 2, = 3, = 6

**Sol:**



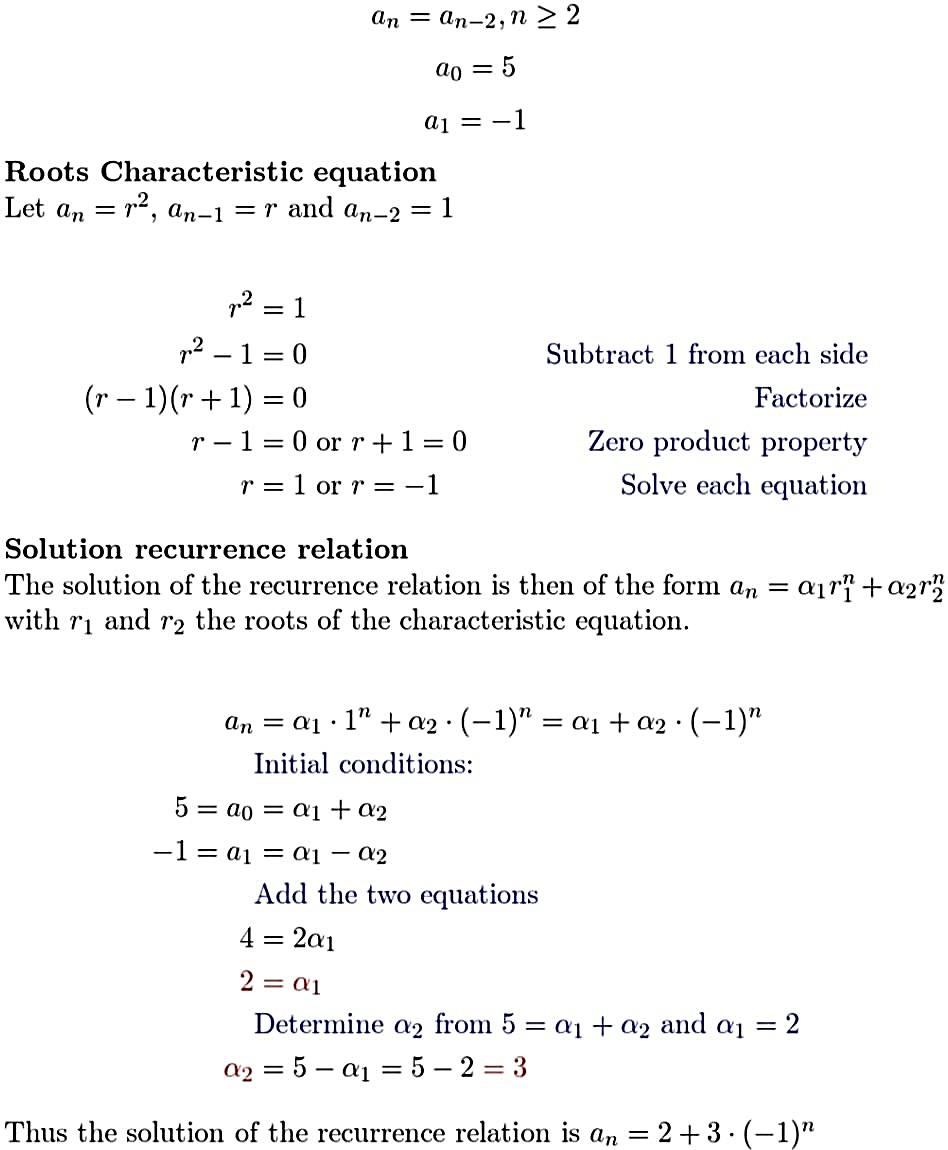
c) = 6 − 8 for n ≥ 2, = 4, = 10

**Sol:**



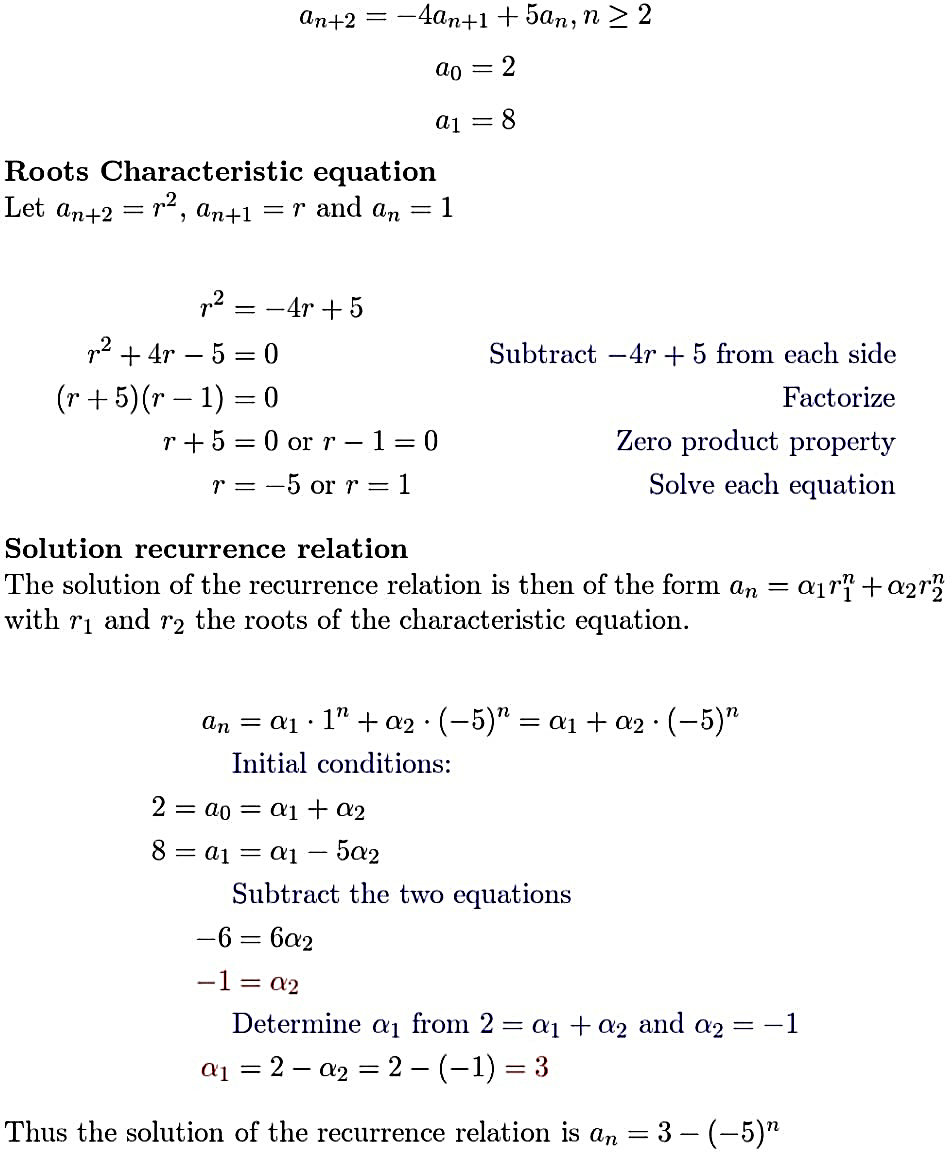
e) = for n ≥ 2, = 5, = −1

**Sol:**



g) = −4 + 5 for n ≥ 0, = 2, = 8

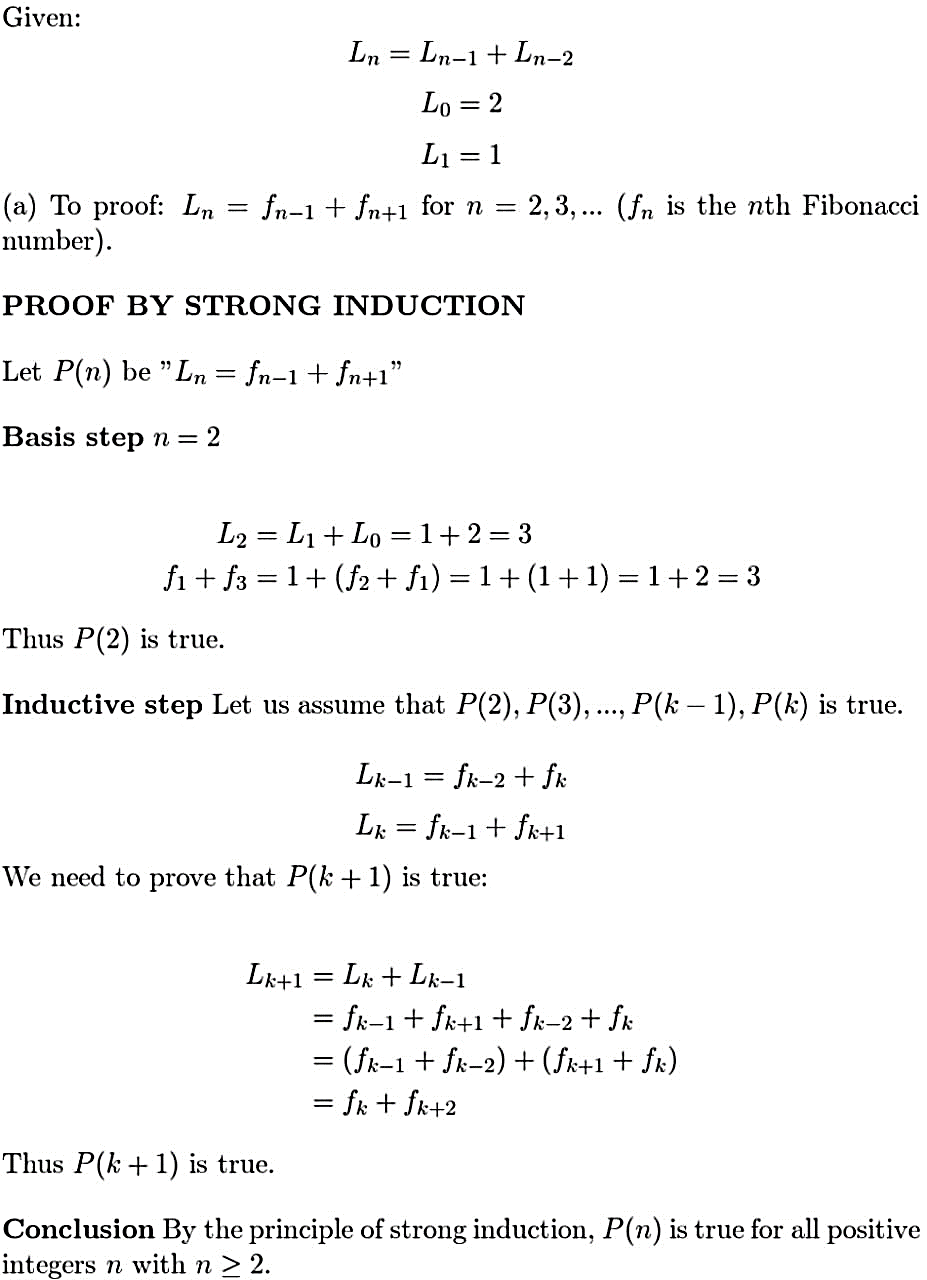
**Sol:**



11. The Lucas numbers satisfy the recurrence relation = + , and the initial conditions = 2 and = 1.

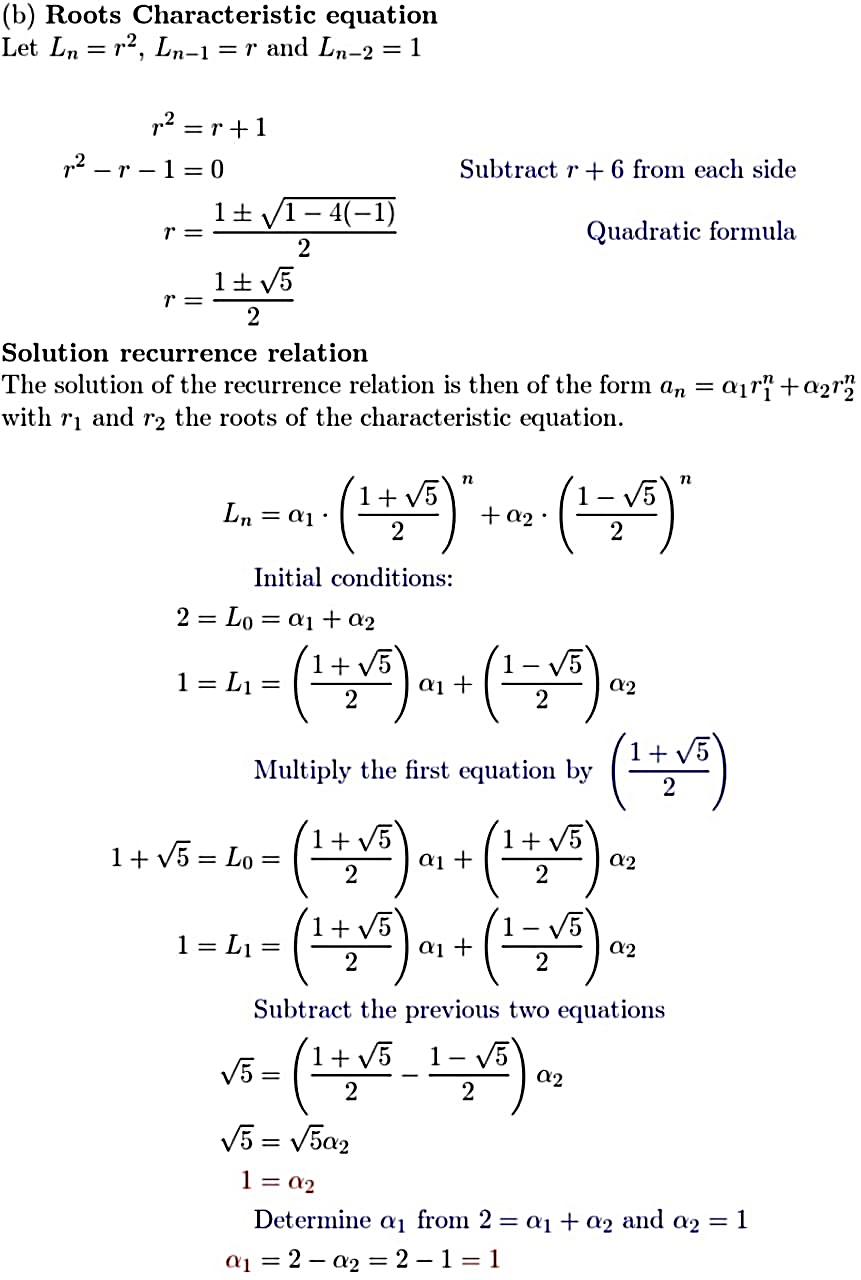
a) Show that = fn−1 + fn+1 for n = 2, 3, . . . , where is the nth Fibonacci number.

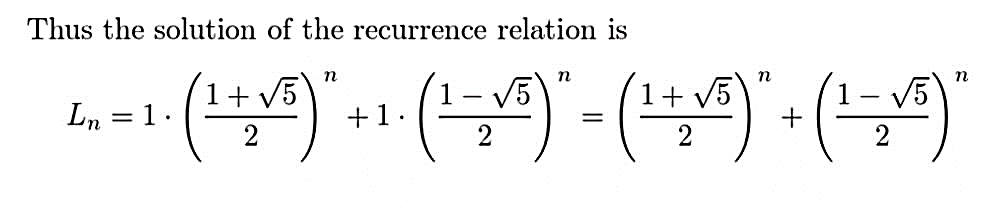
**Sol:**



b) Find an explicit formula for the Lucas numbers.

**Sol:**

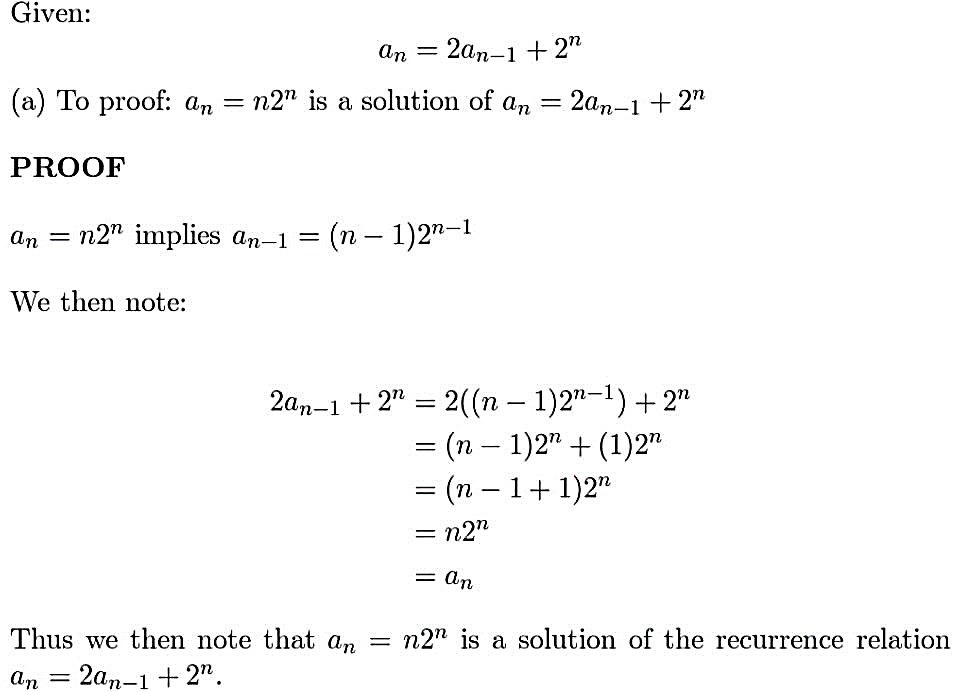




24. Consider the nonhomogeneous linear recurrence relation = 2 + .

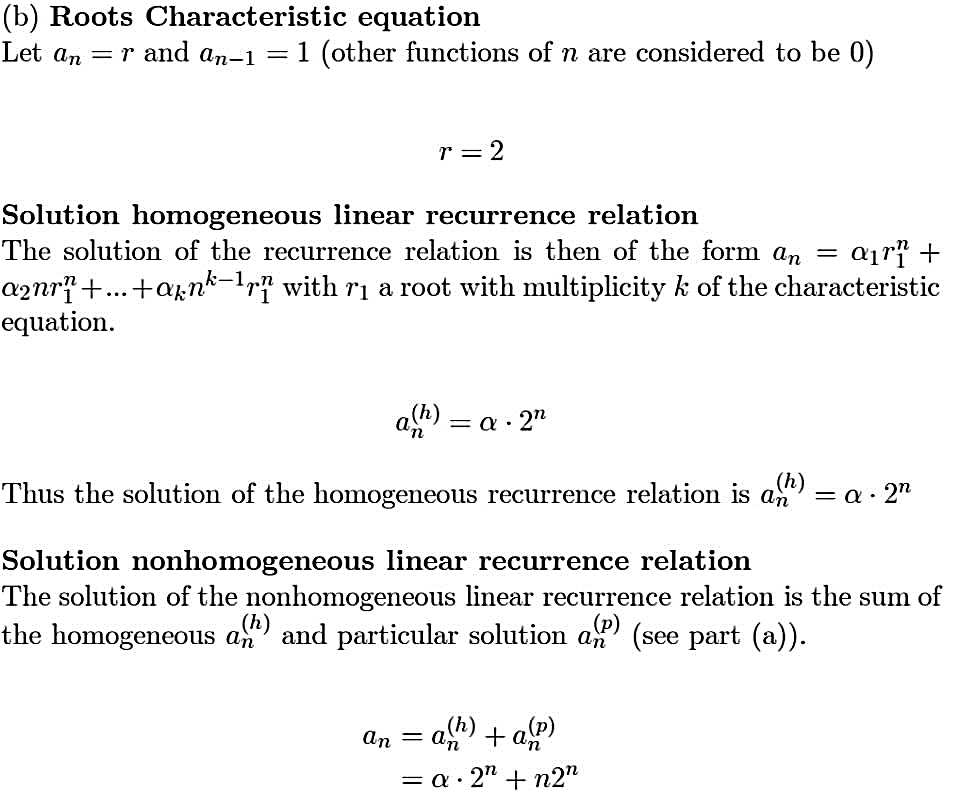
a) Show that = is a solution of this recurrence relation.

**Sol:**



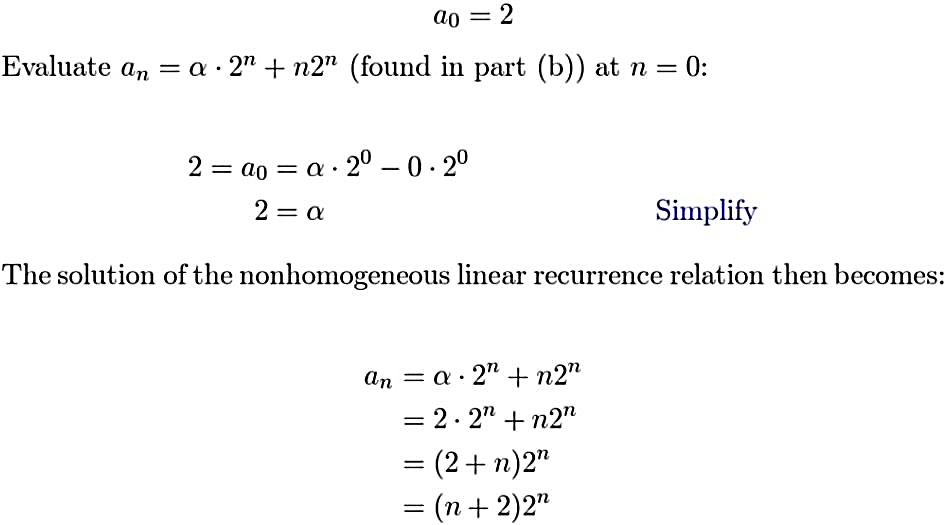
b) Use Theorem 5 to find all solutions of this recurrence relation.

**Sol:**



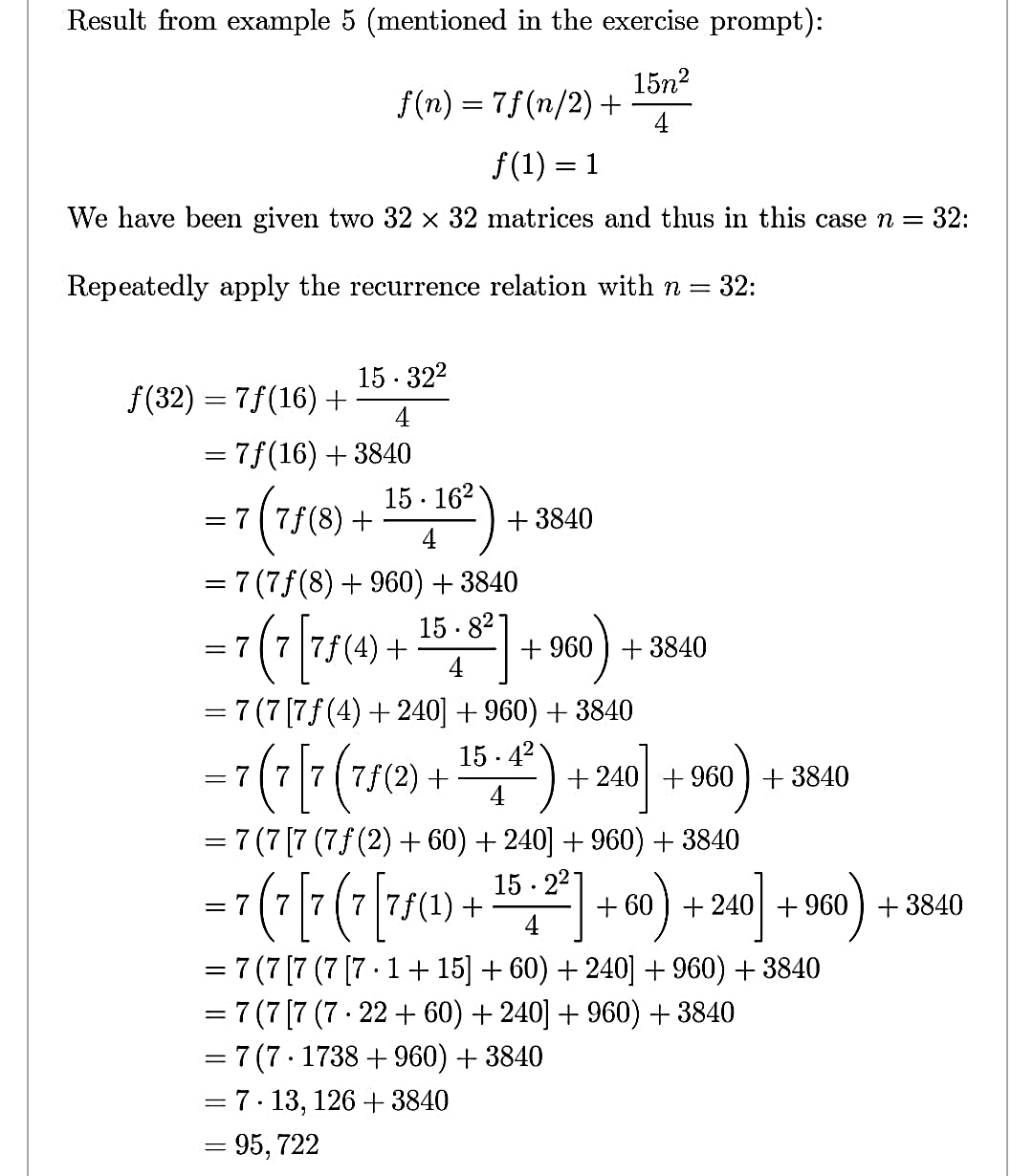
c) Find the solution with = 2.

**Sol:**



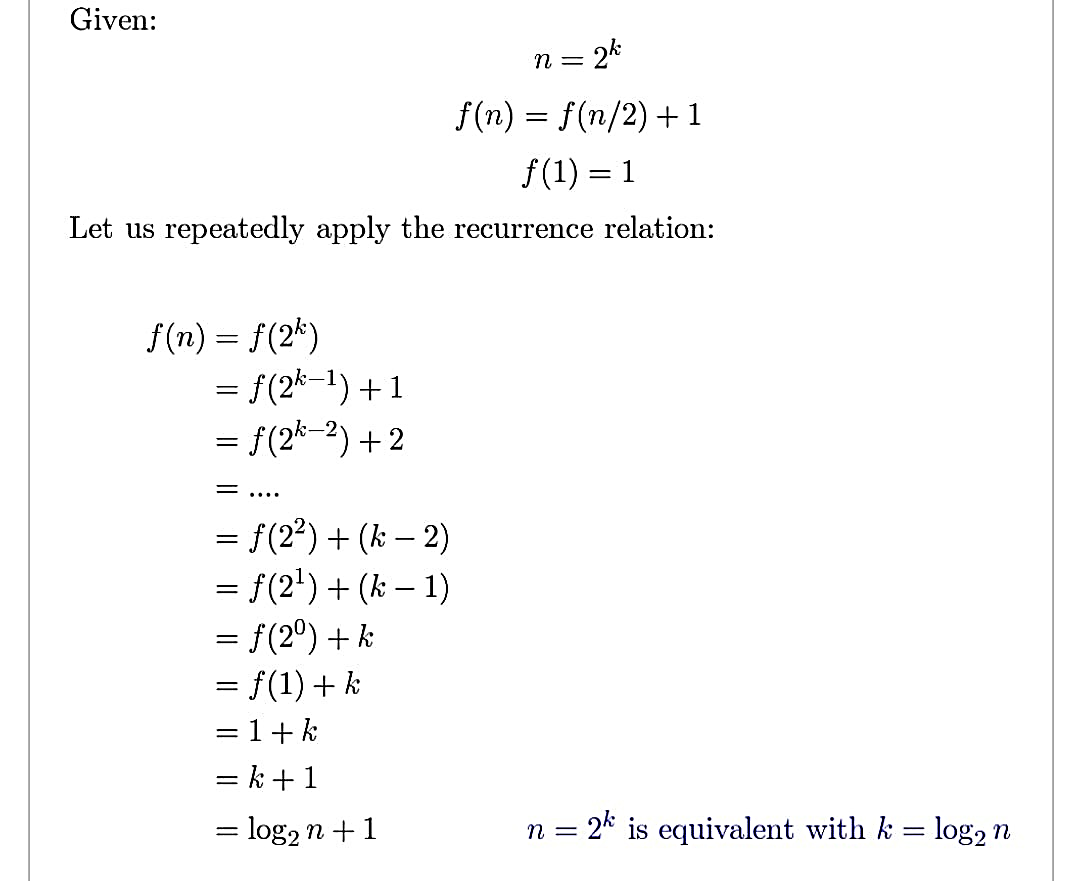
**8.3**

6. How many operations are needed to multiply two 32 × 32 matrices using the algorithm referred to in Example 5?

**Sol:**

10. Find f (n) when n = , where f satisfies the recurrence relation f (n) = f (n/2) + 1 with f (1) =1.

**Sol:**

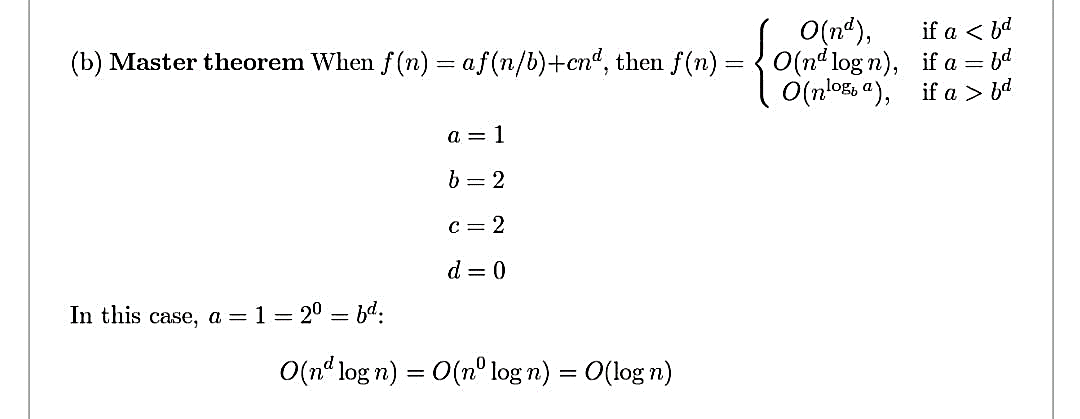


20. a) Set up a divide-and-conquer recurrence relation for the number of modular multiplications required to compute mod m, where a, m, and n are positive integers, using the recursive algorithms from Example 4 in Section 5.4.

**Sol:**

**b)** Use the recurrence relation you found in part (a) to construct a big-*O* estimate for the number of modular multiplications used to compute **mod** *m* using the recursive algorithm.

**Sol:**

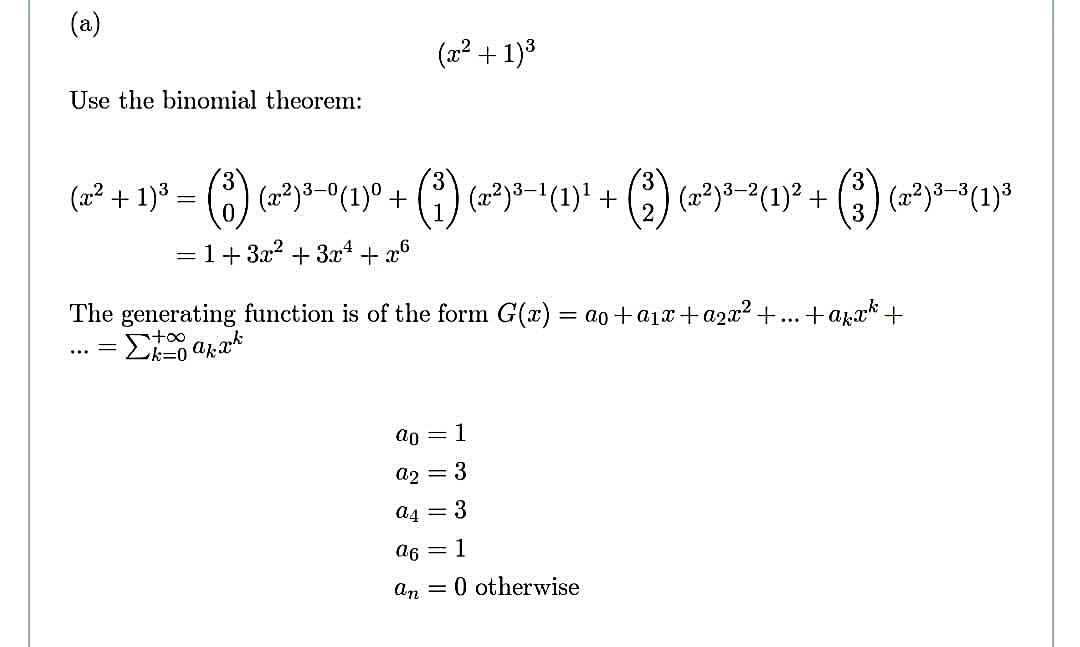


**8.4**

8. For each of these generating functions, provide a closed formula for the sequence it determines.

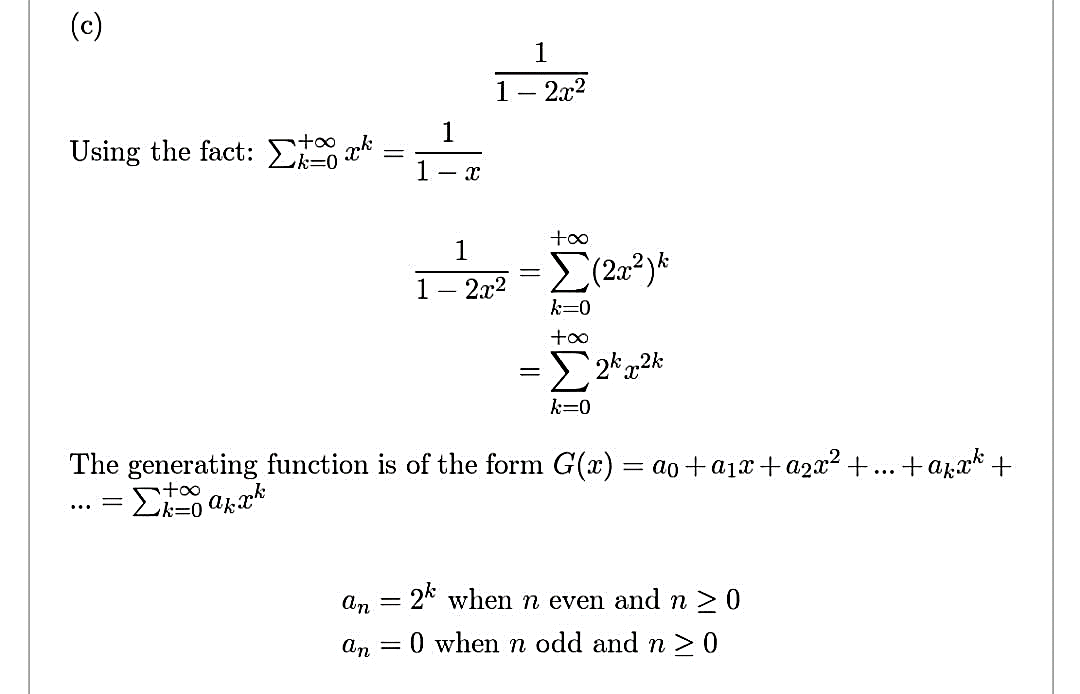
a)

**Sol:**



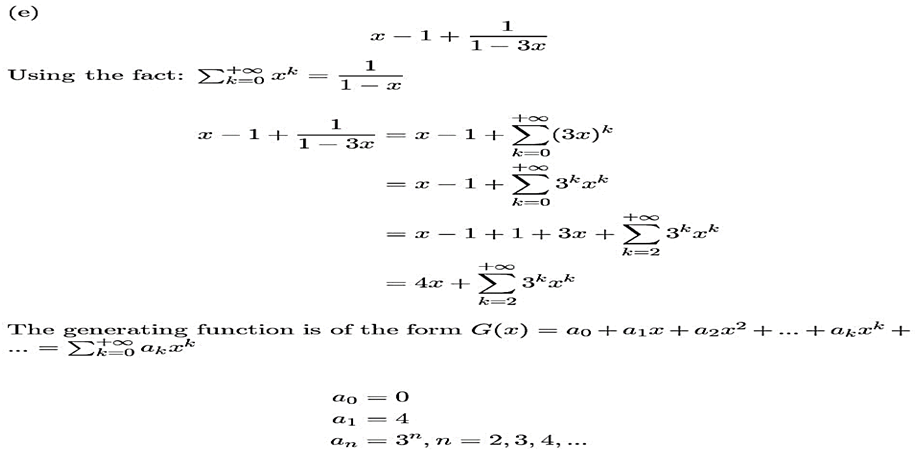
c) 1/(1 − 2)

**Sol:**



e) x − 1 + (1/(1 − 3x))

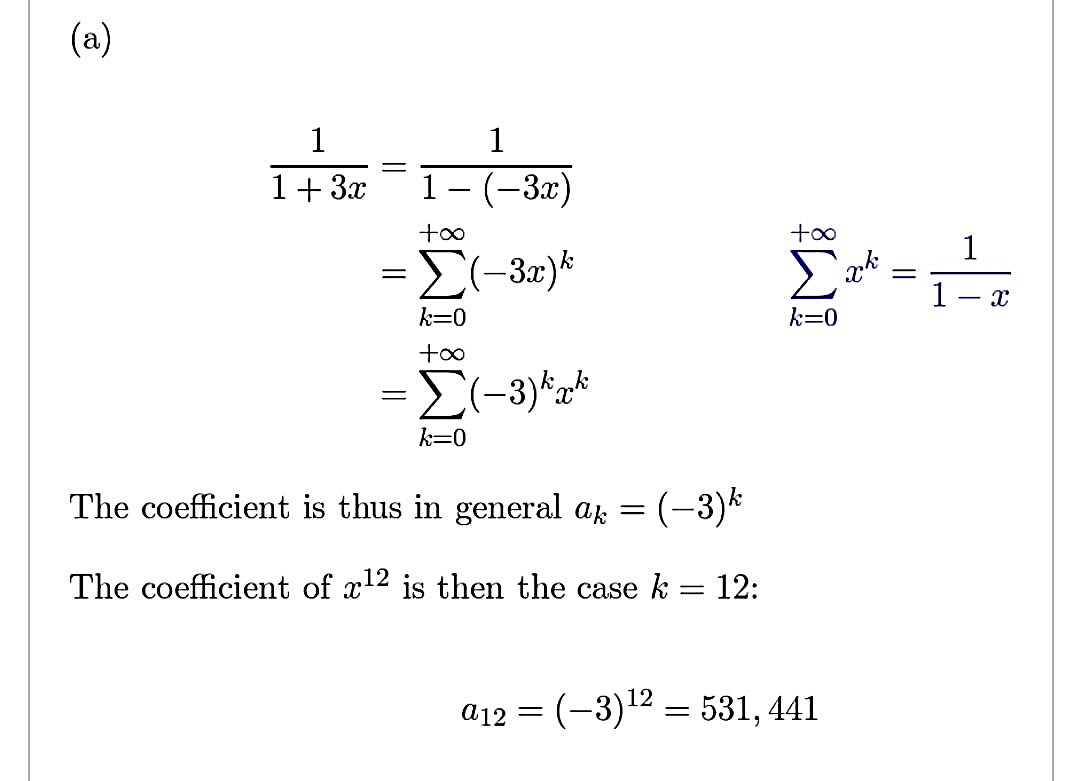
**Sol:**



12.Find the coefficient of x12 in the power series of each of these functions.

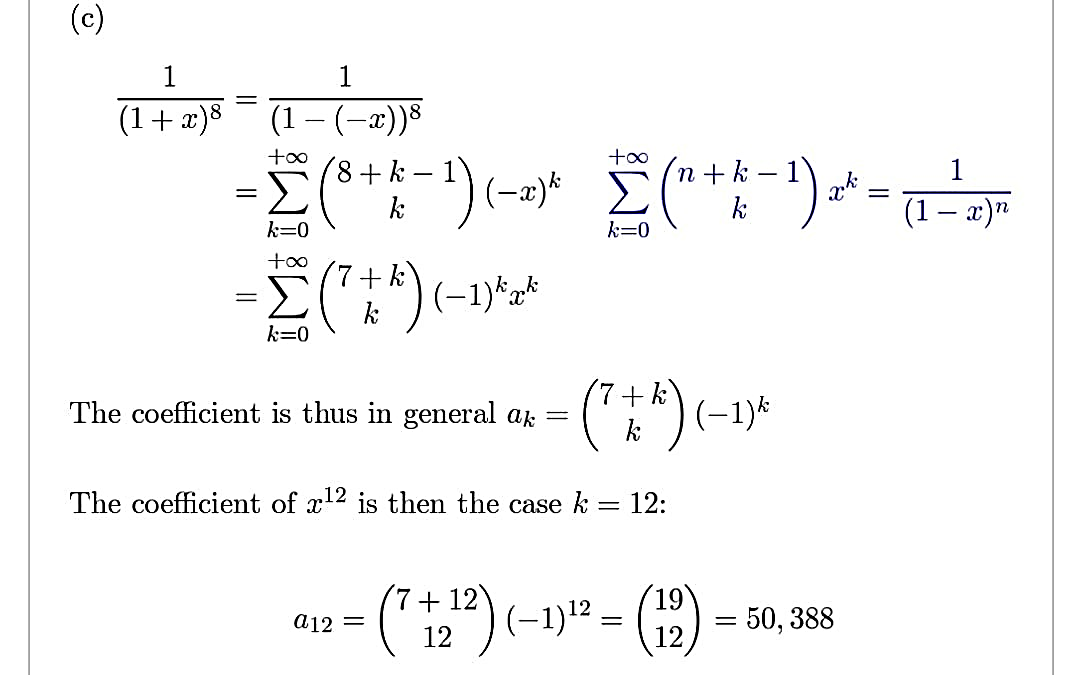
a) 1/(1 + 3x)

**Sol:**



c) 1/

**Sol:**



e)

**Sol:**

