

1. Sean tres puntos $A = [2, 1, 3]$, $B = [4, 3, 1]$ y $C = [3, 2, 4]$:

Q = escala el punto A en X, Y y Z por un factor de 1.43 y trasládalo usando las coordenadas de B .

Escala del punto A

$$A' = A \cdot S \quad S = 1.43$$

$$A' = \begin{bmatrix} 1.43 & 0 & 0 & 0 \\ 0 & 1.43 & 0 & 0 \\ 0 & 0 & 1.43 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2(1.43) \\ 1(1.43) \\ 3(1.43) \\ 1(1) \end{bmatrix} = \begin{bmatrix} 2.86 \\ 1.43 \\ 4.29 \\ 1 \end{bmatrix} = \text{Escalar} \\ A' = (2.86, 1.43, 4.29)$$

Trasladar A'

$$T = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2.86 \\ 1.43 \\ 4.29 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.86 + 0 + 0 + \\ 0 + 1.43 + 0 + \\ 0 + 0 + 4.29 + 1 \\ 0 + 0 + 0 + 1 \end{bmatrix} = \begin{bmatrix} 6.86 \\ 4.43 \\ 5.29 \\ 1 \end{bmatrix} = \text{Traslación} \\ Q = (6.86, 4.43, 5.29)$$

Rota el punto C , 45° en el eje X . Toma como punto pivote al punto Q

Traslación al punto Q

$$\begin{bmatrix} 1 & 0 & 0 & -6.8 \\ 0 & 1 & 0 & -4.43 \\ 0 & 0 & 1 & -5.29 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 + 0 + 0 - 6.86 \\ 0 + 2 + 0 - 4.43 \\ 0 + 0 + 4 - 5.29 \\ 0 + 0 + 0 + 1 \end{bmatrix} = \begin{bmatrix} -3.86 \\ -2.43 \\ -1.29 \\ 1 \end{bmatrix} = \text{Traslación} \\ C = (-3.86, -2.43, -1.29)$$

Rotación

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \frac{\pi}{4} & -\sin \frac{\pi}{4} & 0 \\ 0 & \sin \frac{\pi}{4} & \cos \frac{\pi}{4} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3.86 \\ -2.43 \\ -1.29 \\ 1 \end{bmatrix} = \begin{bmatrix} -3.86(1) \\ -2.43(\frac{\sqrt{2}}{2}) + 1.29(\frac{\sqrt{2}}{2}) \\ -2.43(\frac{\sqrt{2}}{2}) - 1.29(\frac{\sqrt{2}}{2}) \\ 1 \end{bmatrix} = \begin{bmatrix} -3.86 \\ -0.81 \\ -2.63 \\ 1 \end{bmatrix} = \text{Rotación} \\ C = (-3.86, -0.81, -2.6)$$

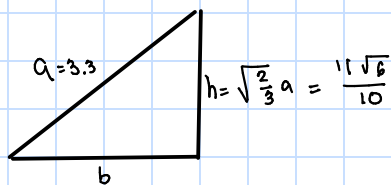
Traslación al punto original

$$\begin{bmatrix} 1 & 0 & 0 & 6.89 \\ 0 & 1 & 0 & 4.43 \\ 0 & 0 & 1 & 5.29 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3.86 \\ -0.81 \\ -2.63 \\ 1 \end{bmatrix} = \begin{bmatrix} -3.86 + 6.89 \\ -0.81 + 4.43 \\ -2.63 + 5.29 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.62 \\ 2.66 \\ 1 \end{bmatrix} =$$

Rotación de C con pivote
 $C = (3, 3.62, 2.66)$

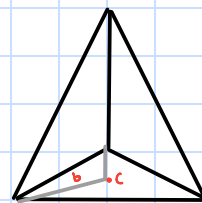
2. Considera una pirámide triangular regular. Su lado $a = 3.3$ y su altura $h = 23a$. La base de la pirámide está centrada en el punto $C = (-1.812, -6.824, 5.247)$

Encuentra los vértices de la pirámide.



$$b = \sqrt{(3.3)^2 - \left(\frac{11\sqrt{6}}{10}\right)^2} = \frac{11\sqrt{3}}{10} \approx 1.90$$

$$b = 1.90$$



Especificaciones:

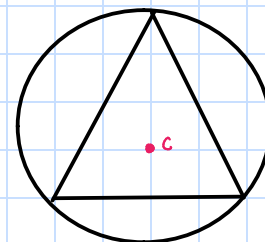
- Ecuación esfera: $x^2 + y^2 + z^2 = r^2$
- $x \rightarrow -3.71 \leq x \leq 0.09$
- $x = -1$
- $z = 0$

$$C = (-1.812, -6.824, 5.247)$$

$$(-1 + 1.812)^2 + (y + 6.824)^2 = (1.90)^2$$

$$0.659 + (y + 6.824)^2 = 3.61$$

$$y_1 = -8.54 \quad y_2 = -5.11$$



▷ Sistema de ecuaciones

$$(x + 1)^2 + (y + 5.11)^2 = 3.3^2$$

$$x = -3.704$$

$$y = -6.997$$

$$(x + 1.812)^2 + (y + 6.824)^2 = 1.9^2$$

$$x = -0.745$$

$$y = -8.396$$

$$P_2 = (-3.704, -6.997, 5.247)$$

$$P_3 = (-0.745, -8.396, 5.247)$$

▷ Cúspide

$$P_4 = C + h = (-1.812, -6.824, 7.941)$$

Encuentra la nueva posición de cada vértice, cuando la pirámide es rotada respecto al eje Y, por -15° . Usa como pivote el centroide de la pirámide (el centroide es el promedio de cada componente de cada vértice).

$$\text{centroide} = \left(\frac{-1-3.704-0.745-1.812}{4}, \frac{-5.106-6.997-8.396-6.824}{4}, \frac{5.247+5.247+5.247+7.941}{4} \right) = (-1.81, -6.8, 5.92)$$

▷ Tetraedro $\rightarrow \frac{1}{4}$ de la altura

$$\text{centroide} = (-1.812, -6.824, 5.247 + \frac{1}{4}(\frac{11\sqrt{6}}{10})) \rightarrow \text{centroide} = (-1.812, -6.824, 5.92)$$

► Rotar P_1

Traduzir al origen

$$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -5.106 \\ 5.247 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.812 \\ 1.718 \\ -0.68 \\ 1 \end{bmatrix}$$

Rotar $-15^\circ = -\frac{\pi}{12}$

$$P_1 = \begin{bmatrix} \cos(-\frac{\pi}{12}) & 0 & \sin(-\frac{\pi}{12}) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-\frac{\pi}{12}) & 0 & \cos(-\frac{\pi}{12}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.812 \\ 1.718 \\ -0.68 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.960 \\ 1.718 \\ -0.447 \\ 1 \end{bmatrix}$$

Traduzir al ponto original

$$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1.812 \\ 0 & 1 & 0 & 0 & -6.824 \\ 0 & 0 & 1 & 0 & 5.92 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.960 \\ 1.718 \\ -0.447 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.852 \\ -5.106 \\ 5.473 \\ 1 \end{bmatrix}$$

$$P_1 = (-0.852, -5.106, 5.473)$$

► Rotar P_2

Traduzir al origen

$$P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3.704 \\ -6.997 \\ 5.247 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.892 \\ -0.173 \\ -0.673 \\ 1 \end{bmatrix}$$

Rotar $-15^\circ = -\frac{\pi}{12}$

$$P_2 = \begin{bmatrix} \cos(-\frac{\pi}{12}) & 0 & \sin(-\frac{\pi}{12}) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-\frac{\pi}{12}) & 0 & \cos(-\frac{\pi}{12}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1.892 \\ -0.173 \\ -0.673 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.653 \\ -0.173 \\ -1.139 \\ 1 \end{bmatrix}$$

Traduzir al ponto original

$$P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1.812 \\ 0 & 1 & 0 & 0 & -6.824 \\ 0 & 0 & 1 & 0 & 5.92 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1.653 \\ -0.173 \\ -1.139 \\ 1 \end{bmatrix} = \begin{bmatrix} -3.465 \\ -6.997 \\ 4.781 \\ 1 \end{bmatrix}$$

$$P_2 = (-3.465, -6.997, 4.781)$$

► Rotar P_3

Traduzir al origen

$$P_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.745 \\ -8.396 \\ 5.247 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.067 \\ -1.572 \\ -0.673 \\ 1 \end{bmatrix}$$

Rotar $-15^\circ = -\frac{\pi}{12}$

$$P_3 = \begin{bmatrix} \cos(-\frac{\pi}{12}) & 0 & \sin(-\frac{\pi}{12}) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-\frac{\pi}{12}) & 0 & \cos(-\frac{\pi}{12}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.067 \\ -1.572 \\ -0.673 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.205 \\ -1.572 \\ -0.374 \\ 1 \end{bmatrix}$$

Traduzir al ponto original

$$P_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1.812 \\ 0 & 1 & 0 & 0 & -6.824 \\ 0 & 0 & 1 & 0 & 5.92 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1.205 \\ -1.572 \\ -0.374 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.607 \\ -8.396 \\ 5.546 \\ 1 \end{bmatrix}$$

$$P_3 = (-0.607, -8.396, 5.546)$$

► Rotar P_4

Traduzir al origen

$$P_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1.812 \\ -6.824 \\ 7.941 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2.021 \\ 1 \end{bmatrix}$$

Rotar $-15^\circ = -\frac{\pi}{12}$

$$P_4 = \begin{bmatrix} \cos(-\frac{\pi}{12}) & 0 & \sin(-\frac{\pi}{12}) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-\frac{\pi}{12}) & 0 & \cos(-\frac{\pi}{12}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2.021 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.523 \\ 0 \\ 1.952 \\ 1 \end{bmatrix}$$

Traduzir al ponto original

$$P_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1.812 \\ 0 & 1 & 0 & 0 & -6.824 \\ 0 & 0 & 1 & 0 & 5.92 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -0.523 \\ 0 \\ 1.952 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.335 \\ -6.824 \\ 7.872 \\ 1 \end{bmatrix}$$

$$P_4 = (-2.335, -6.824, 7.872)$$