Question 4

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a) $f(x) = \frac{1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\right)$ (probability density function of a Gaussian distribution)

$$f(x_{\frac{1}{2}}) = \frac{1}{2}\cdot\frac{1}{\sigma\sqrt{2\pi}}$$

$$\frac{1}{2\sigma\sqrt{2\pi}} = \frac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{1}{2}\left(\frac{x_{1/2}-\mu}{\sigma}\right)^2}$$

$$\ln\left(\frac{1}{2\sigma\sqrt{2\pi}}\right) = \ln\left(\frac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{1}{2}\left(\frac{x_{1/2}-\mu}{\sigma}\right)^2}\right)$$

$$\sqrt{\left(\frac{x_{1/2}-\mu}{\sigma}\right)^2} = \sqrt{-2\ln\left(\frac{1}{\sigma\sqrt{2\pi}}\right)}$$

$$\cancel{\sigma}\left(\frac{x_{1/2}-\mu}{\cancel{\sigma}}\right) = \sqrt{-2\ln\left(\frac{1}{\sigma\sqrt{2\pi}}\right)}\quad\sigma$$

$$x_{1/2}-\mu = \sigma\sqrt{-2\ln\left(1/\sigma\sqrt{2\pi}\right)}$$

$$+\mu \qquad\qquad +\mu$$

$$x_{1/2} = \sigma\sqrt{-2\ln\left(1/\sigma\sqrt{2\pi}\right)}+\mu$$

$$-2\ln\left(1/\sigma\sqrt{2\pi}\right)$$

$$\Rightarrow -2\ln\left(\frac{1}{\sigma}\cdot\frac{1}{\sqrt{2\pi}}\right)$$

$$\Rightarrow -2\ln\left(\frac{1}{\sigma}\right)+\ln(2\pi)$$

$$x_{1/2} = \sigma\sqrt{2\ln(\sigma)}$$

let $u = 2\ln(\sigma)$

$$x_{1/2} = \sigma\sqrt{u}$$

consider 2 values of $x_{1/2}$ positioned around μ:

(+) $x_{1/2} = \mu + \sigma\sqrt{u}$

(-) $x_{1/2} = \mu - \sigma\sqrt{u}$

$$\text{FWHM} = (+)x_{1/2}\ (-)x_{1/2}$$

$$= \mu+\sigma\sqrt{u} - (\mu - \sigma\sqrt{u})$$

$$= \cancel{\mu}+\sigma\sqrt{u} - \cancel{\mu}+\sigma\sqrt{u}$$

$$= 2\sigma\sqrt{u}$$

Substitute

$$\text{FWHM} = 2\sigma\sqrt{2\ln(2)}$$

$$\text{FWHM} = 2.3548\sigma$$

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b.) $y = kx^n$

$\log(y) = \log(kx^n)$

$\log(y) = \log(k) + \log(x^n)$

$\log(y) = \log(k) + n\log(x)$

→ this is similar to slope intercept form

$\log(y) = b + mx$

- Slope (m) corresponds to the exponent n
- intercept (b) corresponds to $\log(k)$
  - this is the value of $\log(y)$ when $\log(x)$ is 0