Abigail López Saavedra
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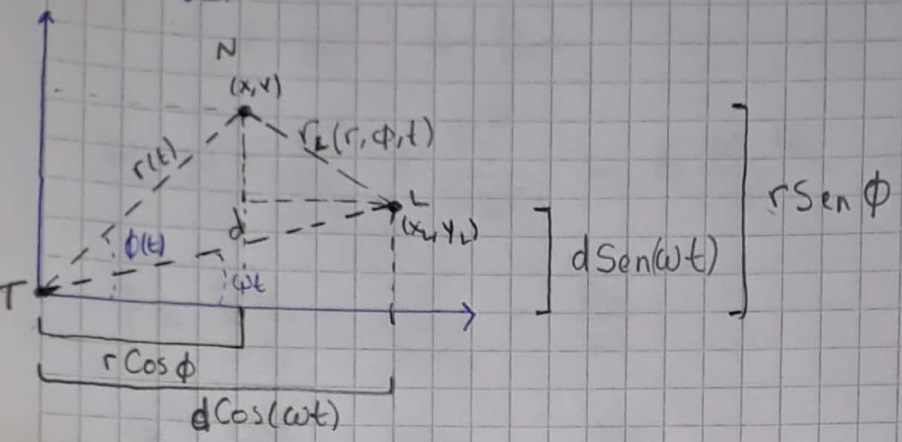
Métodos Computacionales II
Rocket. Sem. 4.



c) $r_L^2(r,\phi,t) = (r\,Sen\,\phi - d\,Sen(\omega t))^2 + (d\cos(\omega t) - r\cos\phi)^2$

$\Rightarrow r_L^2(r,\phi,t) = \underline{r^2 Sen^2\phi} - 2rd\,Sen\phi\,Sen\omega t + d^2 Sen^2(\omega t) + d^2\cos^2(\omega t)$
$\qquad - 2rd\cos\phi\cos\omega t + \underline{r^2\cos^2\phi}$

$\Rightarrow r_L^2(r,\phi,t) = r^2(t) + d^2 - 2r(t)d\,[Sen\phi\,Sen\omega t + \cos\phi\cos\omega t]$

$\Rightarrow r_L^2(r,\phi,t) = r^2(t) + d^2 - 2r(t)d\cos(\phi - \omega t)$

$\Rightarrow r_L(r,\phi,t) = \sqrt{r^2(t) + d^2 - 2r(t)d\cos(\phi - \omega t)}$ //

d) $H := p_i\dot{q}_i - L$ $\qquad q_1 = r, \quad q_2 = \phi$

$L = T + V$

$\rightsquigarrow T = \frac{1}{2}m_N v_N^2 + \frac{1}{2}m_N\omega^2 = \frac{1}{2}m_N v_r^2 + \frac{1}{2}m_N\frac{v_\phi^2}{r^2}$

$p = mv \rightsquigarrow v^2 = \frac{p^2}{m^2}$

$\Rightarrow T = \frac{p_r^2}{2m_N} + \frac{p_\phi^2}{2m_N r^2}$

$V = G\frac{m_N m_T}{r} + G\frac{m_N m_L}{r_L}$

$\rightsquigarrow H = p_r\dot{r} + p_\phi\dot{\phi} - \left[\frac{p_r^2}{2m_N} + \frac{p_\phi^2}{2m_N r^2} + G\frac{m_N m_T}{r} + G\frac{m_N m_L}{r_L}\right]$

$$\Rightarrow H = \underbrace{\frac{p_r^2}{m_N}}_{\dot r = \frac{p_r}{m}} + \underbrace{\frac{p_\phi^2}{m r^2}}_{\dot\phi = \frac{p_r}{m r^2}} - \left[\frac{p_r^2}{2m_N} + \frac{p_\phi^2}{2m_N r^2} + G\frac{m_N M_T}{r} + G\frac{m_N M_L}{r_L}\right]$$

$$\Rightarrow H = \frac{p_r^2}{2m_N} + \frac{p_\phi^2}{2m_N r^2} - G\frac{M_N M_T}{r} - G\frac{m_N M_L}{r_L} \checkmark$$

$$r_L = \sqrt{r^2 + d^2 - 2r(t)d\cos(\phi-\omega t)}$$

e) $$\dot r = \frac{\partial H}{\partial p_r} = \frac{\cancel{2}p_r}{\cancel{2}m_N} = \frac{p_r}{m}$$

$$\dot\phi = \frac{\partial H}{\partial p_\phi} = \frac{\cancel{2}p_\phi}{\cancel{2}m_N r^2} = \frac{p_\phi}{m_N r^2}$$

$$\dot p_r = -\frac{\partial H}{\partial r} = -\left[-\frac{\cancel{2}p_\phi^2}{\cancel{2}m_N r^3} + \frac{G M_N M_T}{r^2} + \frac{1}{\cancel{2}}\frac{G m_N M_L}{r_L^2 \underbrace{(\sqrt{(\,)})}_{r_L}}(2r - 2d\cos(\phi-\omega t))\right]$$

$$= \frac{p_\phi^2}{m_N r^3} - \frac{G m_N M_T}{r^2} - \frac{G m_N M_L}{r_L^3}(r - d\cos(\phi-\omega t))$$

$$\dot p_\phi = -\frac{\partial H}{\partial \phi} = -\left[\frac{1}{\cancel{2}}\frac{G m_N M_L}{r_L^2 \underbrace{(\sqrt{(\,)})}_{r_L}}(\cancel{2} r d \,\mathrm{sen}(\phi-\omega t))\right]$$

$$= -\frac{G m_N M_L}{r_L^3} r d \sin(\phi-\omega t) \checkmark$$

f) Si: $\tilde r = \frac{r}{d}$, $\phi = \phi$, $\tilde p_r = \frac{p_r}{md}$, $\tilde p_\phi = \frac{p_\phi}{md^2}$

$$\dot r = d\dot{\tilde r} = \frac{p_r}{m} = \frac{\tilde p_r \cancel{m} d}{\cancel{m}} = \tilde p_r d \Rightarrow \dot{\tilde r} = \tilde p_r$$

$$\left(\tilde r_L = \sqrt{\tilde r^2 d^2 + d^2 - 2\tilde r d^2 \cos(\phi-\omega t)}\right.$$
$$= d\sqrt{\tilde r^2 + 1 - 2\tilde r\cos(\phi-\omega t)}$$

$$\dot\phi = \frac{p_\phi}{m_n r^2} = \frac{\tilde p_\phi \cancel{m} d^2}{\cancel{m_n} r^2} = \frac{\tilde p_\phi d^2}{r^2} = \frac{\tilde p_\phi}{\tilde r^2}$$

$$\dot p_r = \cancel{md}\,\dot{\tilde p}_r = \frac{(m d^2 \tilde p_\phi)^2}{m_n(\tilde r d)^3} - G M_T\left[\frac{m_n}{(d\tilde r)^2} + \frac{M_L m_n}{M_T d^3 \tilde r_L^3}(\tilde r d - d\cos(\phi-\omega t))\right]$$

$$= \frac{\tilde p_\phi^2}{\tilde r^3}(\cancel{md}) - \frac{G M_T \cancel{m_n d}}{d^2 \cdot d}\left[\frac{1}{\tilde r^2} + \frac{\mu}{\tilde r'^3}(\tilde r - \cos(\phi-\omega t))\right]$$

$$\Rightarrow \dot{\tilde p}_r = \frac{\tilde p_\phi^2}{\tilde r^3} - \frac{G M_T}{d^3}\left[\frac{1}{\tilde r^2} + \frac{\mu}{\tilde r'^3}(\tilde r - \cos(\phi-\omega t))\right] \checkmark$$

$$\Rightarrow md^2 \dot{\tilde{P}}_\phi = -6 \frac{m M_L}{r_L^3} r d \sin(\phi - \omega t)$$

$$= -6 \frac{m M_L}{d^3 \tilde{r}_L^{\prime 3}} \tilde{r} d^2 \sin(\phi - \omega t)$$

$$\Rightarrow \cancel{md^2} \dot{\tilde{P}}_\phi = -6 \frac{\cancel{m} M_L \tilde{r}}{d \tilde{r}_L^{\prime 3}} \sin(\phi - \omega t) \left(\frac{d^2}{d^2}\right) = -\frac{6 M_L}{d^3 \tilde{r}_L^{\prime 3}} \tilde{r} \sin(\phi - \omega t)$$

g) $$\tilde{P}_r^0 = \frac{P_r}{md} = \frac{1}{d} \cdot \frac{dr}{dt} = \frac{1}{d} \frac{d\sqrt{x^2+y^2}}{dt} = \frac{1}{d} \frac{1}{2} \frac{1}{\sqrt{x^2+y^2}} \left(2x \frac{\partial x}{\partial t} + 2y \frac{\partial y}{\partial t}\right)$$

$$= \frac{x\dot{x} + y\dot{y}}{rd}$$

$$= \frac{\cancel{r}\cos(\phi) \cdot \dot{r}\cos(\theta) + \cancel{r}\,\mathrm{Sen}(\phi) \cdot \dot{r}\,\mathrm{Sen}(\theta)}{\cancel{r} d}$$

$$= \frac{\dot{r}}{d} (\cos\phi \cos\theta + \mathrm{Sen}\phi\, \mathrm{Sen}\theta)$$

$$= \dot{\tilde{r}} \cos(\theta - \phi)$$

$$= \tilde{v}_0 \cos(\theta - \phi)$$

$$\tilde{P}_\phi^0 = \frac{P_\phi}{md^2} = \frac{r^2}{md^2} \frac{d\phi}{dt} = \tilde{r}^2 \frac{d}{dt} (\arctan(y/x))$$

$$= \tilde{r}^2 \frac{1}{1 + (y^2/x^2)} \cdot \frac{d}{dt} \left(\frac{y}{x}\right)$$

$$= \tilde{r}^2 \cdot \frac{1}{\frac{x^2+y^2}{x^2}} \left(\frac{\dot{y}}{x} - \frac{y}{x^2} \dot{x}\right)$$

$$= \frac{\tilde{r}^2}{r^2} \cdot (x\dot{y} - y\dot{x}) =$$

$$= \frac{\tilde{r}^2}{r^2} (r\cos\phi\, \dot{r}\,\mathrm{Sen}\theta + r\,\mathrm{Sen}\phi\, \dot{r} \cos\theta)$$

$$= \frac{\tilde{r}^2}{r} \cdot \dot{r} (\sin(\theta - \phi)) = \frac{\tilde{r}^2}{\tilde{r} \cdot d} \cdot \dot{\tilde{r}} d \sin(\theta - \phi)$$

$$\Rightarrow \tilde{P}_\phi^0 = \tilde{r}_0 \tilde{v}_0 \sin(\theta - \phi)$$