回归分析（非线性回归）

多项式回归，分类型变量回归，分类型和数值型混合回归

虽然是非线性回归，但X Y是线性关系

- 简单线性回归：

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

- 多项式回归：

$$y_i = \beta_0 + \beta_1 {x_i}^1 + \beta_2 {x_i}^2 + \cdots + \varepsilon_i$$

- 非线性关系：

$$y_i = \beta_0 + \beta_1 x_1 + {\beta_1}^2 x_2 + \varepsilon_i$$

虽然是非线性回归，但X Y是线性关系

- 简单线性回归：

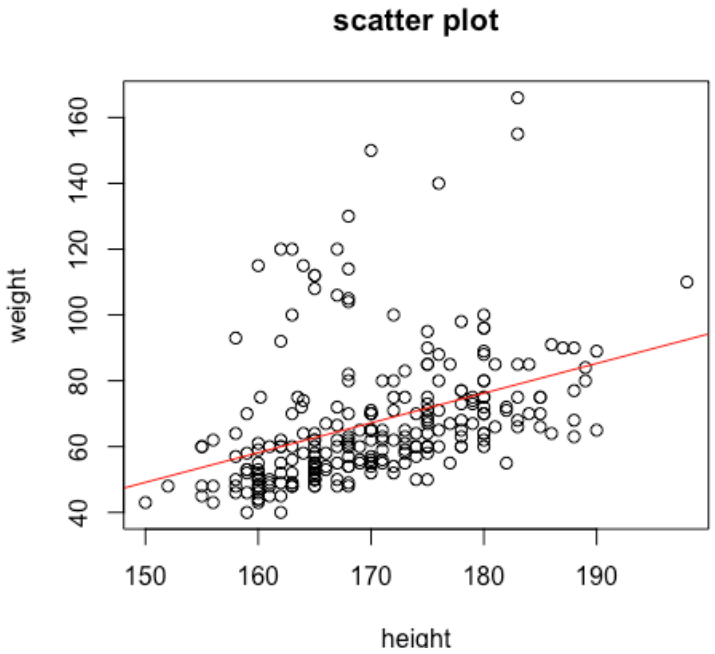
$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

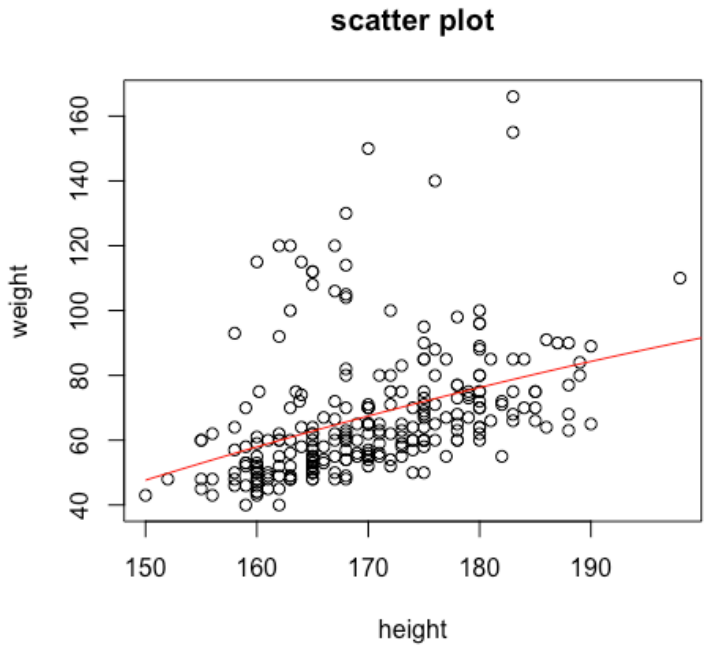
- 二次多项式回归：

$$y_i = \beta_0 + \beta_1 {x_i}^1 + \beta_2 {x_i}^2 + \varepsilon_i$$

- 三次多项式回归：

$$y_i = \beta_0 + \beta_1 {x_i}^1 + \beta_2 {x_i}^2 + \beta_3 {x_i}^3 + \varepsilon_i$$


scatter plot
weight
height
40
60
80
100
120
140
160
150
160
170
180
190


scatter plot
weight
height
40
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80
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120
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160
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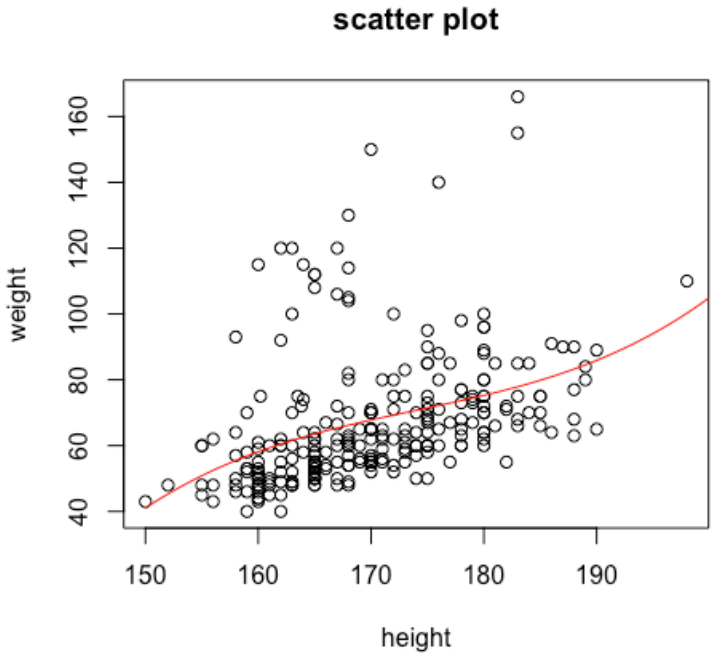
180
190


scatter plot
weight
height
40
60
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100
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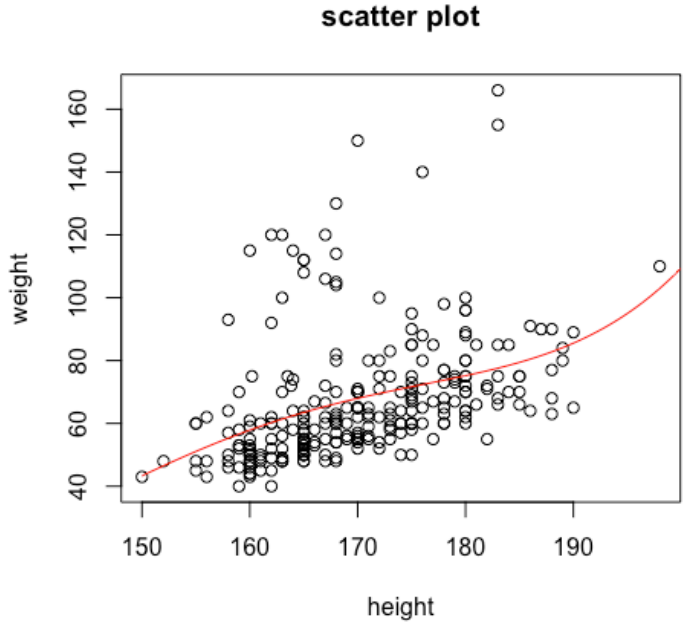
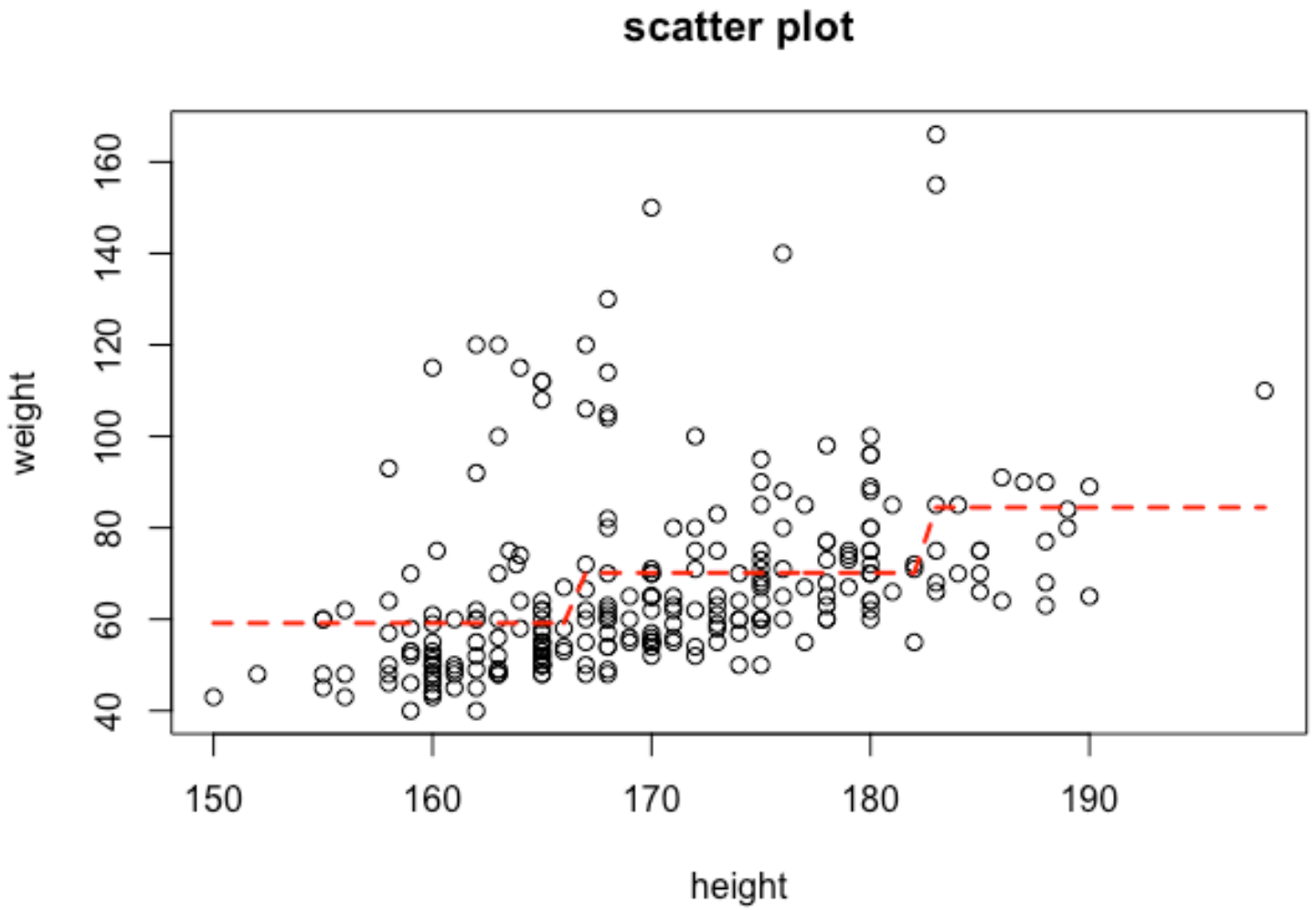
140
160
150
160
170
180
190

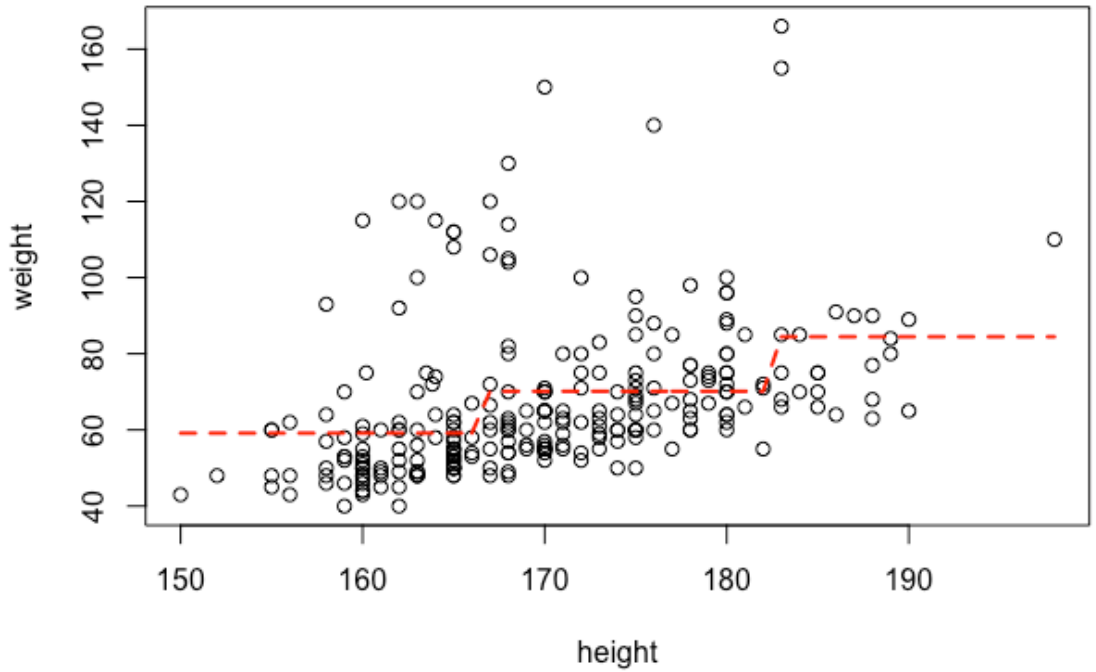

scatter plot
weight
height
40
60
80
100
120
140
160
150
160
170
180
190

分类型变量回归（跃阶函数）



```
Call:
lm(formula = weight ~ cut(height, 3))

Coefficients:
            (Intercept)  cut(height, 3)(166,182]  cut(height, 3)(182,198]
                  59.17                    10.98                    25.29
```



$$y = 59.17 + 10.98 \cdot I(\text{身高属于}(166,182]) + 25.29 \cdot I(\text{身高属于}(182,198]) = \begin{cases} 59.17\ \text{kg} & (\text{身高属于}150,166) \\ 70.15\text{kg} & (\text{身高属于}166,182) \\ 84.46\ kg & (\text{身高属于}182,198) \end{cases}$$

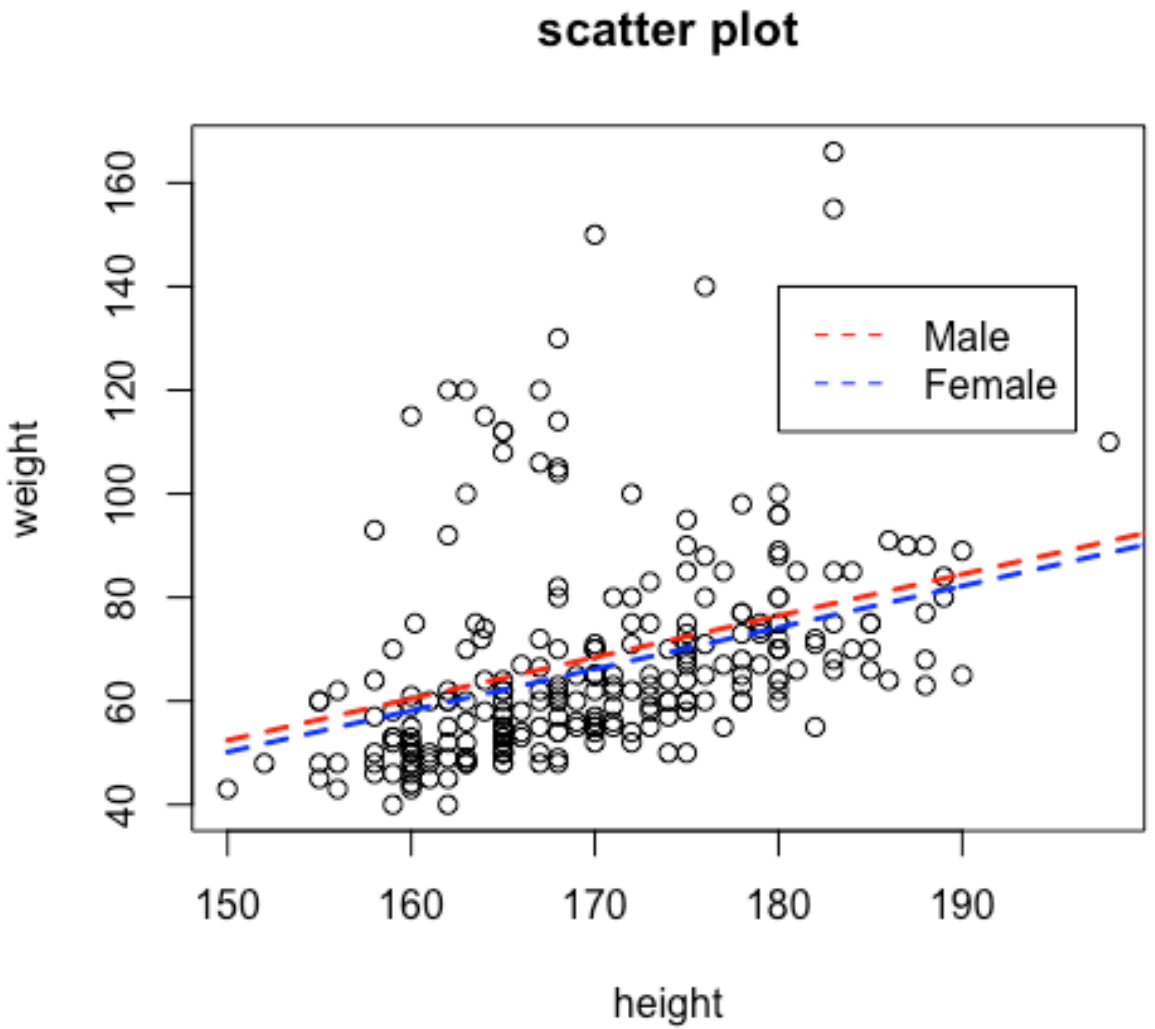
多分类型变量回归

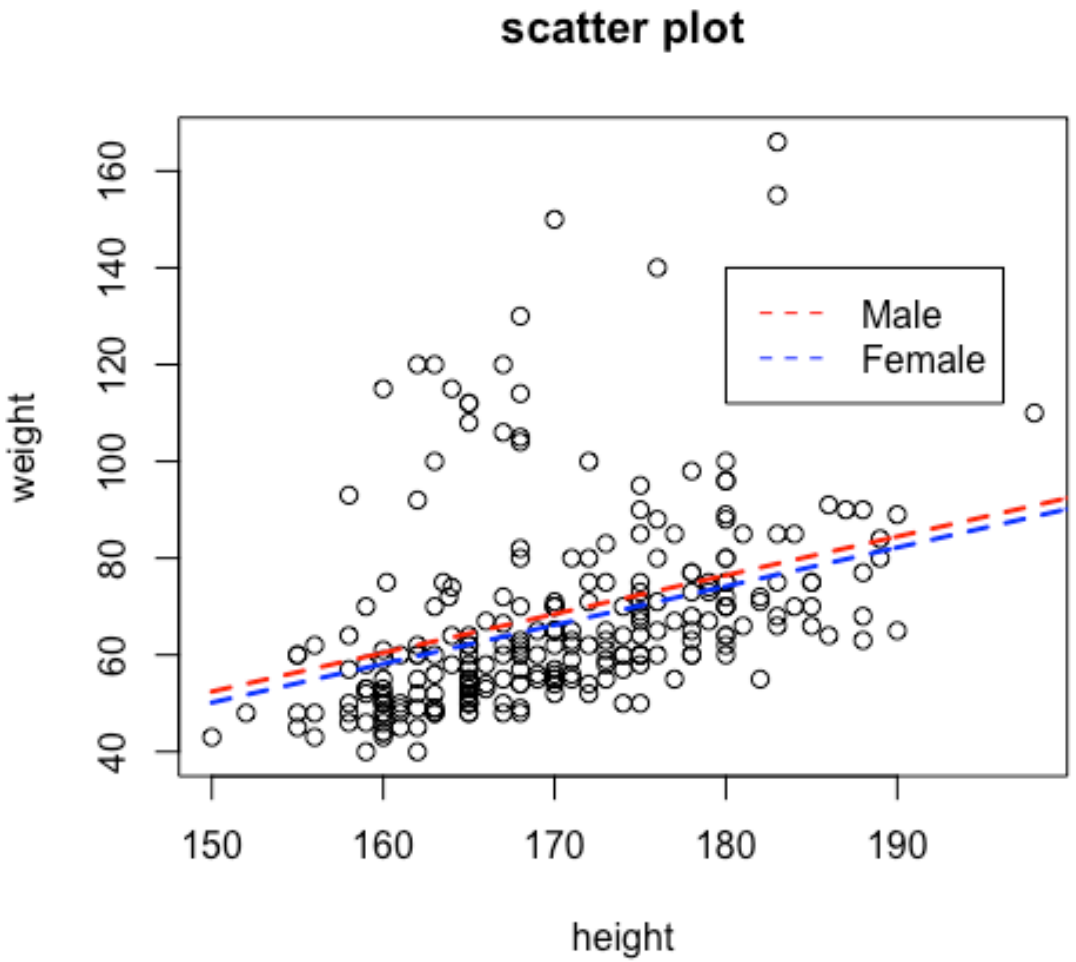
```
Call:
lm(formula = weight ~ cut(height, 3) + gender)

Coefficients:
            (Intercept)  cut(height, 3)(166,182]  cut(height, 3)(182,198]              genderMale
                 59.069                    7.923                   20.395                   4.994
```

$$weight$$
$$= 59.069 + 7.923 \cdot I(\text{身高属于}(166,182]) + 20.395 \cdot I(\text{身高属于}(182,198]) + 4.994 \cdot I(\text{男性})$$

分类型数值型混合回归


scatter plot
weight
height
40
60
80
100
120
140
160
150
160
170
180
190
Male
Female



$$weight = -69.9777 + 0.8009 * height + 2.2707 \cdot I(\text{男性})$$

$$= \begin{cases} \text{如果为男性：}(-69.9777 + 2.2707) + 0.8009 * height \\ \text{如果为女性：} -69.9777 + 0.8009 * height \end{cases}$$

```
Call:
lm(formula = weight ~ height + gender)

Coefficients:
(Intercept)       height   genderMale
   -69.9777       0.8009       2.2707
```

分类型数值型混合回归（含交互作用）

```
Call:
lm(formula = weight ~ height * gender)

Coefficients:
      (Intercept)             height         genderMale  height:genderMale
         -57.3487             0.7241            -24.1839             0.1547
```

$$weight = -57.3487 + 0.7241 * height - 24.1839 \cdot I(\text{男性}) + 0.1547 * I(\text{男性}) * height$$

$$= \begin{cases} \text{如果为男性：} (-57.3487 - 24.1839) + (0.7241 + 0.1547) * height \\ \text{如果为女性：} -57.3487 + 0.7241 * height \end{cases}$$