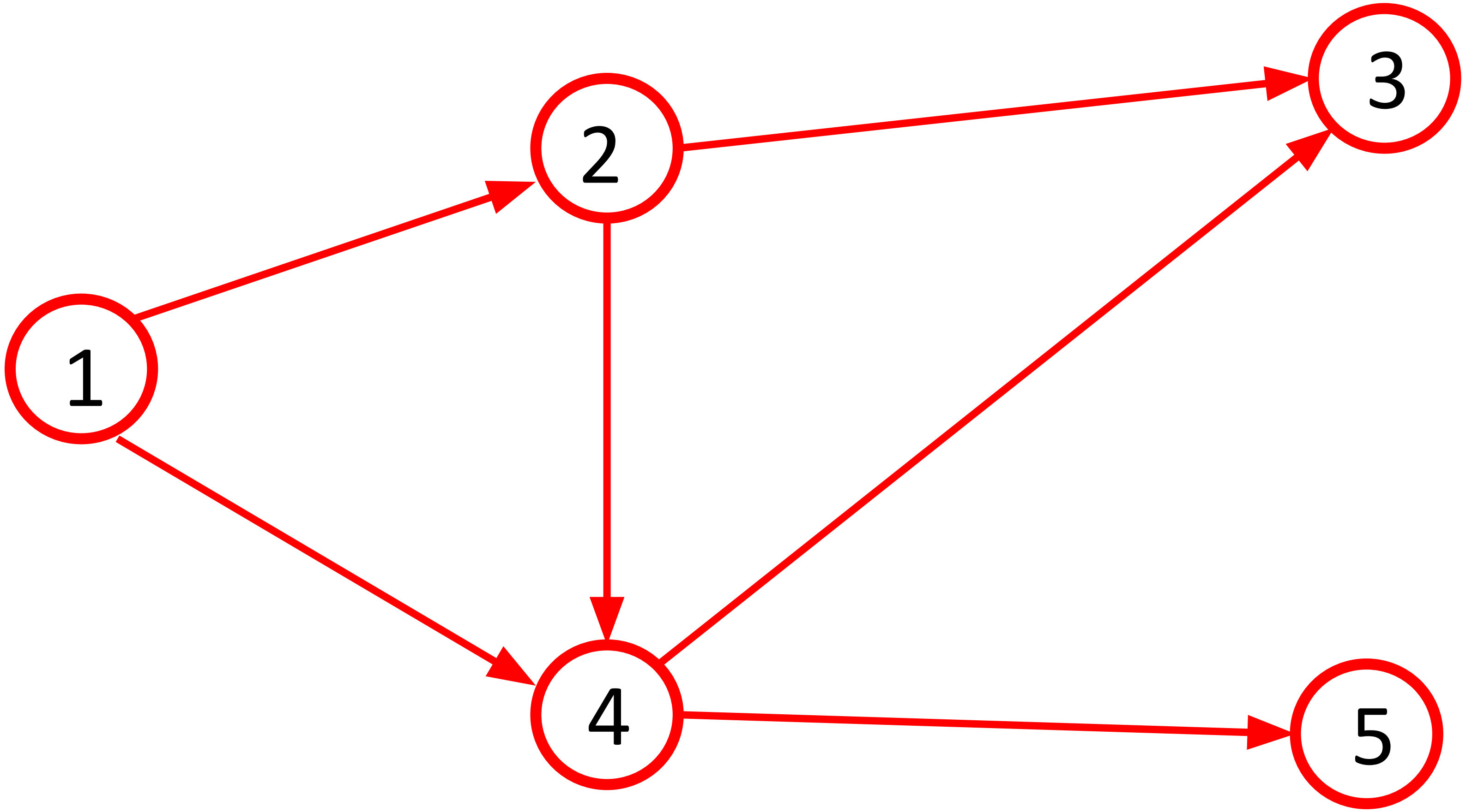
Topological Sort(or Topological Ordering)

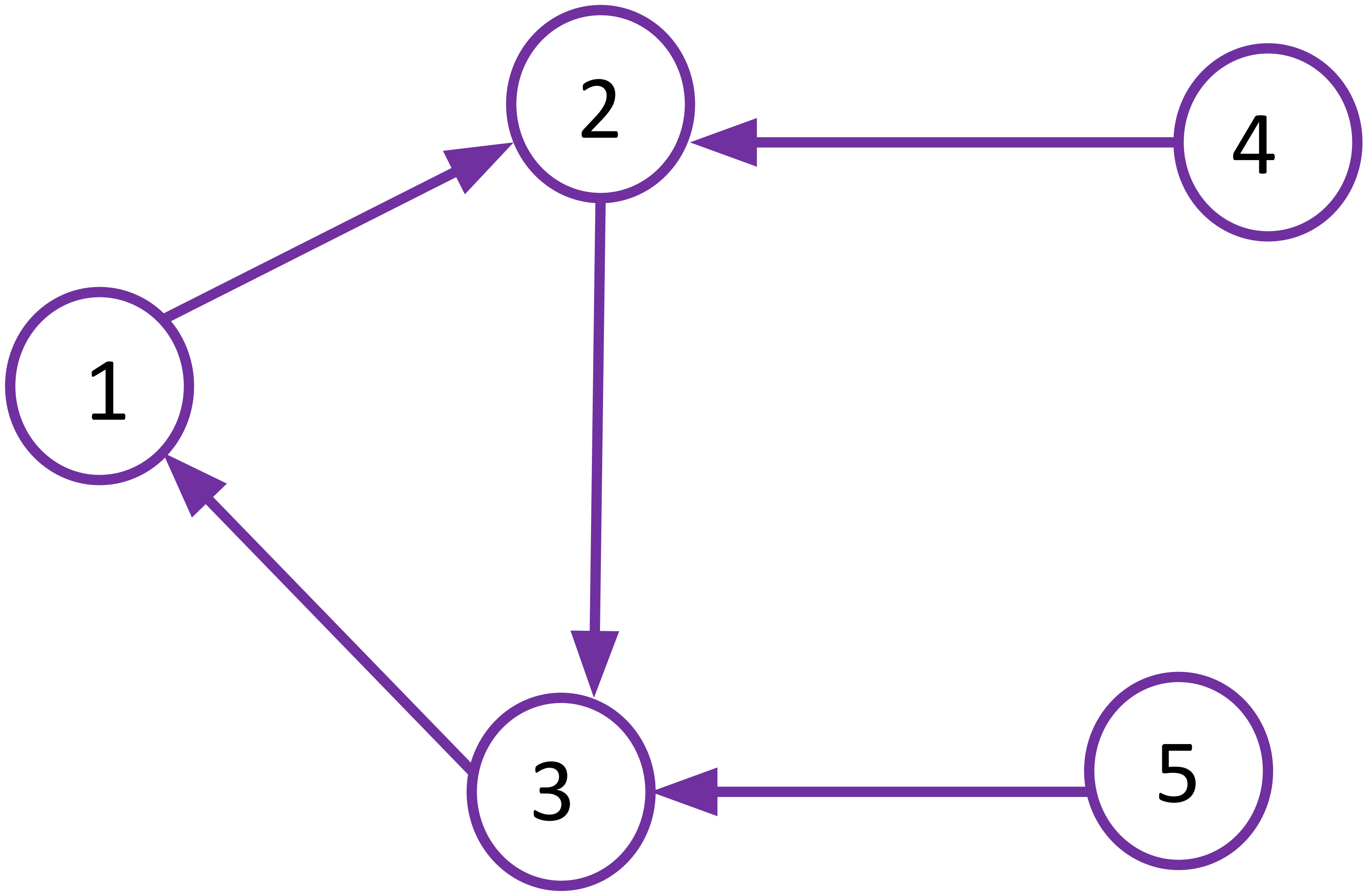
Topological Sort is a linear ordering of the vertices such that if there is an edge in the DAG going from vertex ‘u’ to vertex ‘v’, then ‘u’ comes before ‘v’ in the ordering.

Topological Sorting is possible if and only if the graph is a Directed Acyclic Graph.

DAG will have at least one topological ordering

There may exist multiple different topological orderings for a given directed acyclic graph.


1
2
3
4
5



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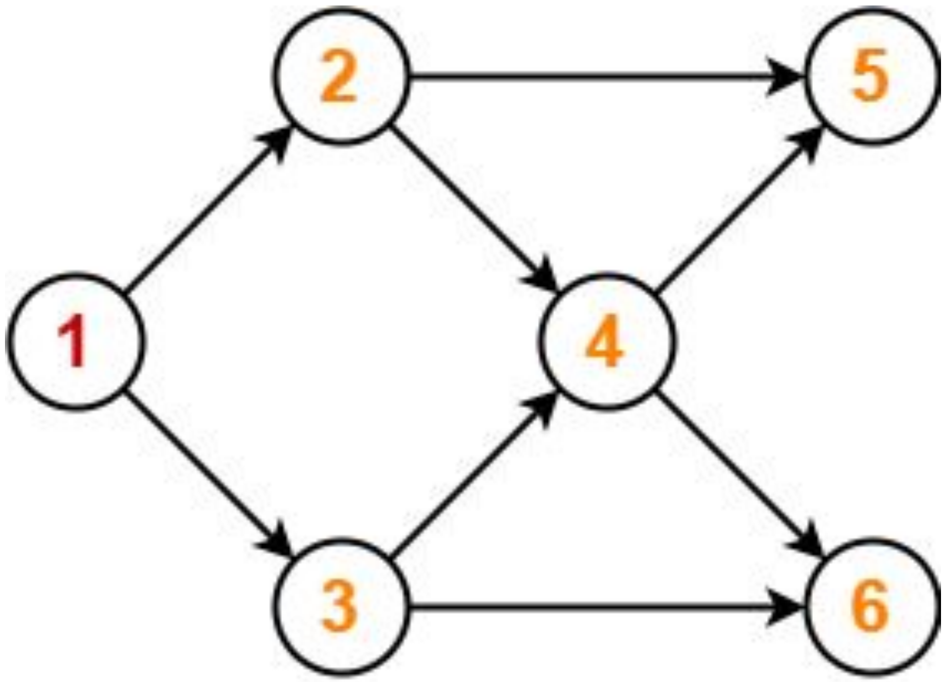
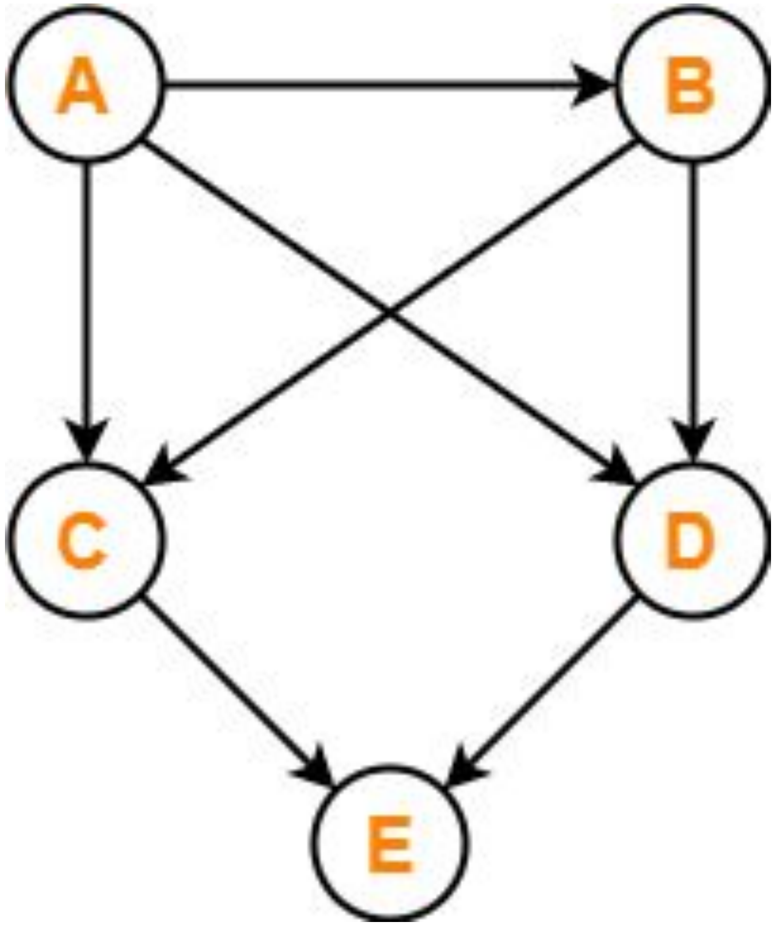


Topological Sort Example

For this graph, following 4 different topological orderings are possible-

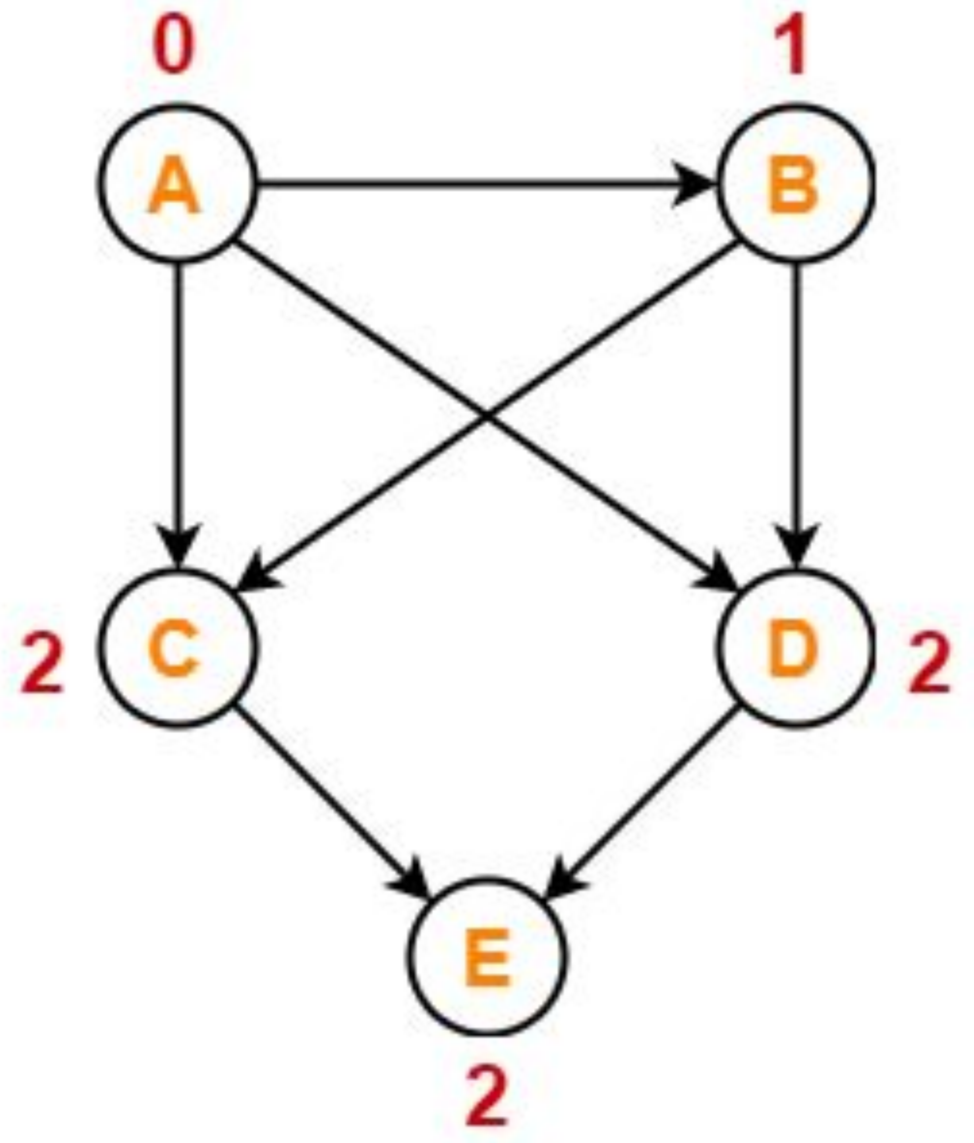
- **1 2 3 4 5 6**
- **1 2 3 4 6 5**
- **1 3 2 4 5 6**
- **1 3 2 4 6 5**

Find the number of different topological orderings possible for the given graph-



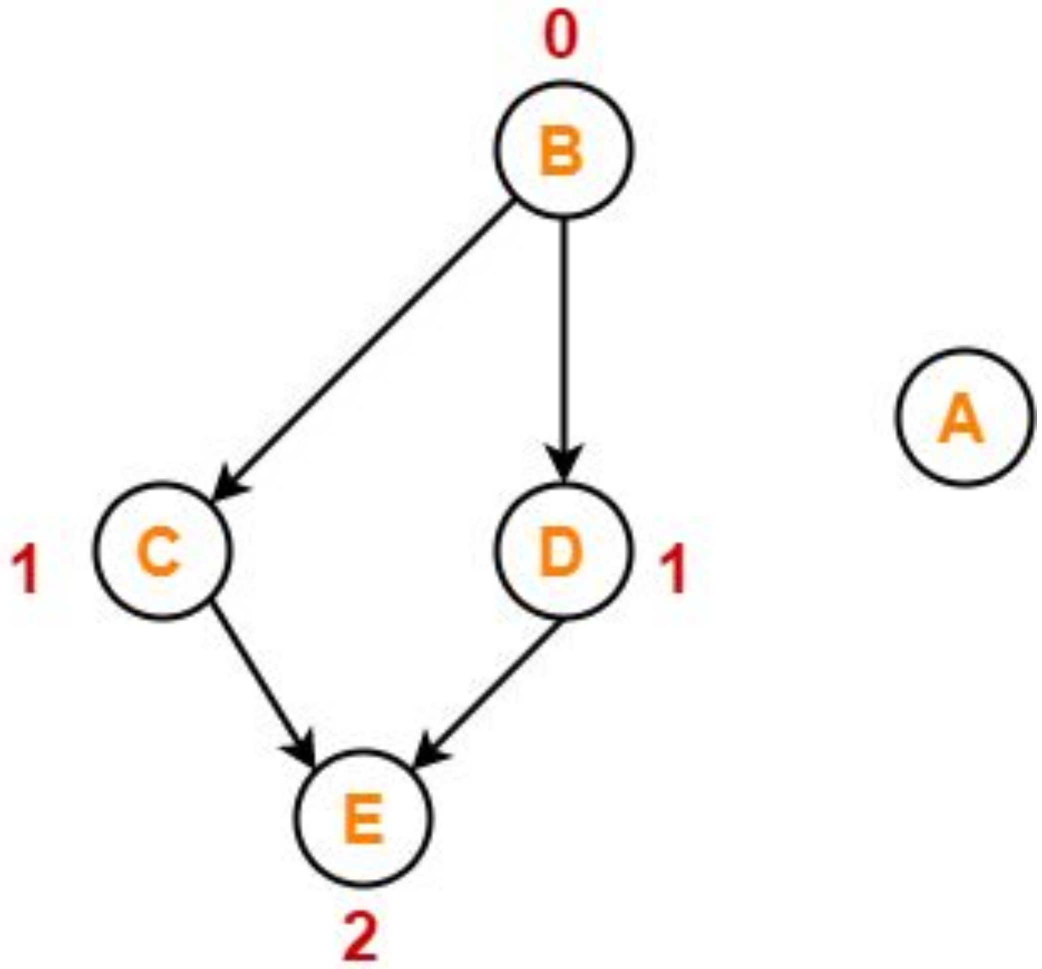
Step-01:

Write in-degree of each vertex-



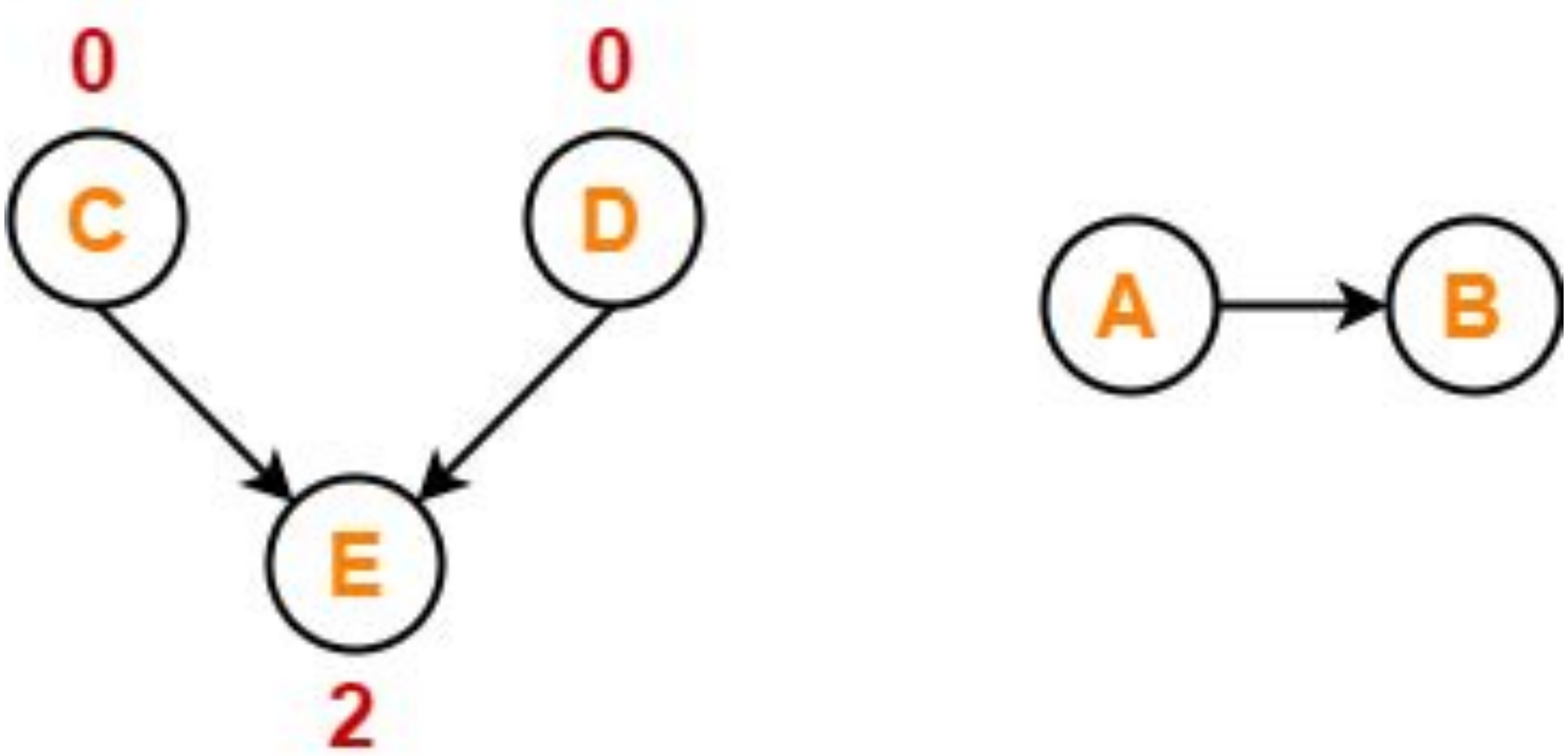
Step-02:

- Vertex-A has the least in-degree.
- So, remove vertex-A and its associated edges.
- Now, update the in-degree of other vertices.



Step-03:

- Vertex-B has the least in-degree.
- So, remove vertex-B and its associated edges.
- Now, update the in-degree of other vertices.



Step-04:

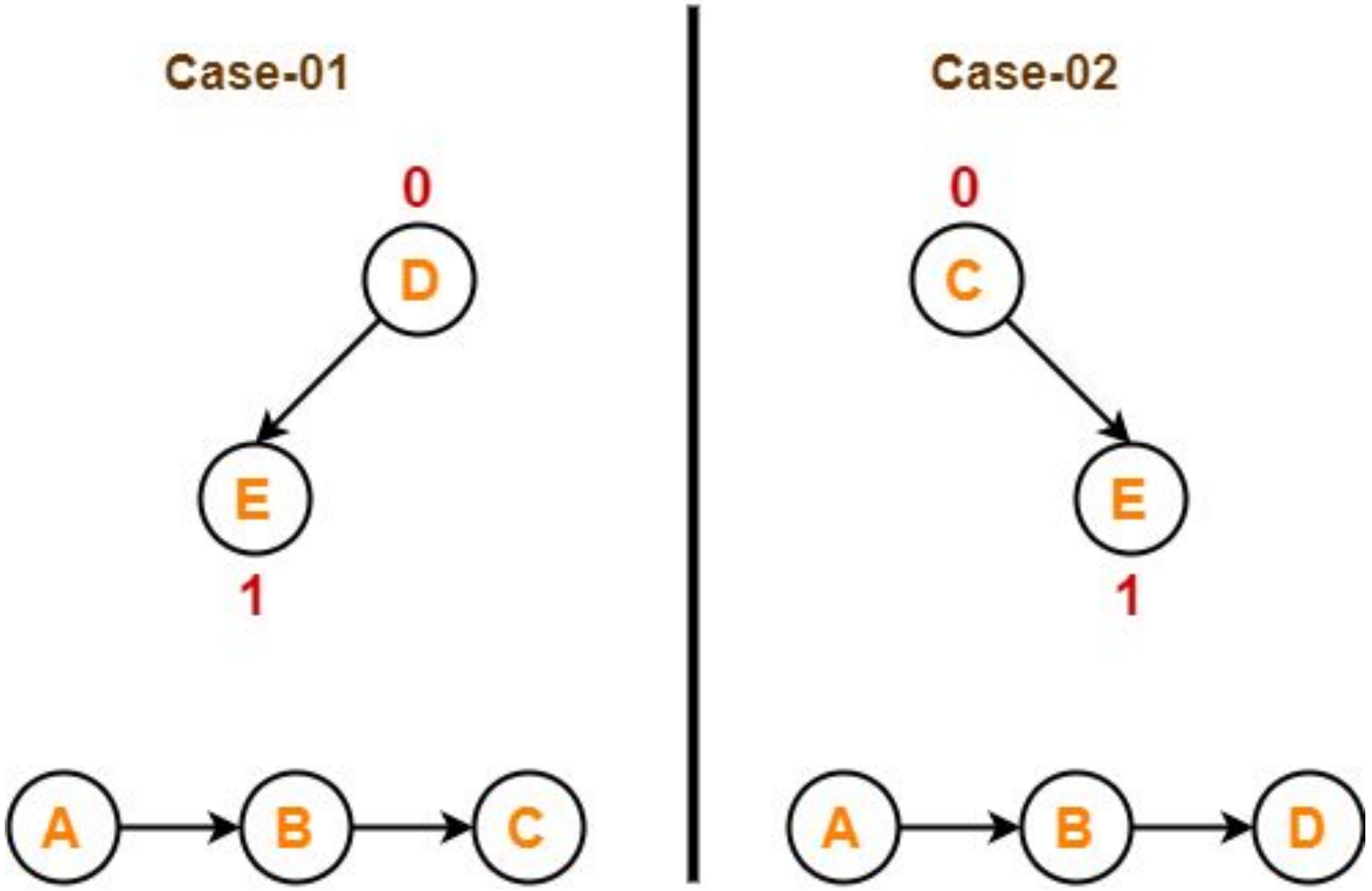
There are two vertices with the least in-degree. So, following 2 cases are possible-

In case-01,

- Remove vertex-C and its associated edges.
- Then, update the in-degree of other vertices.

In case-02,

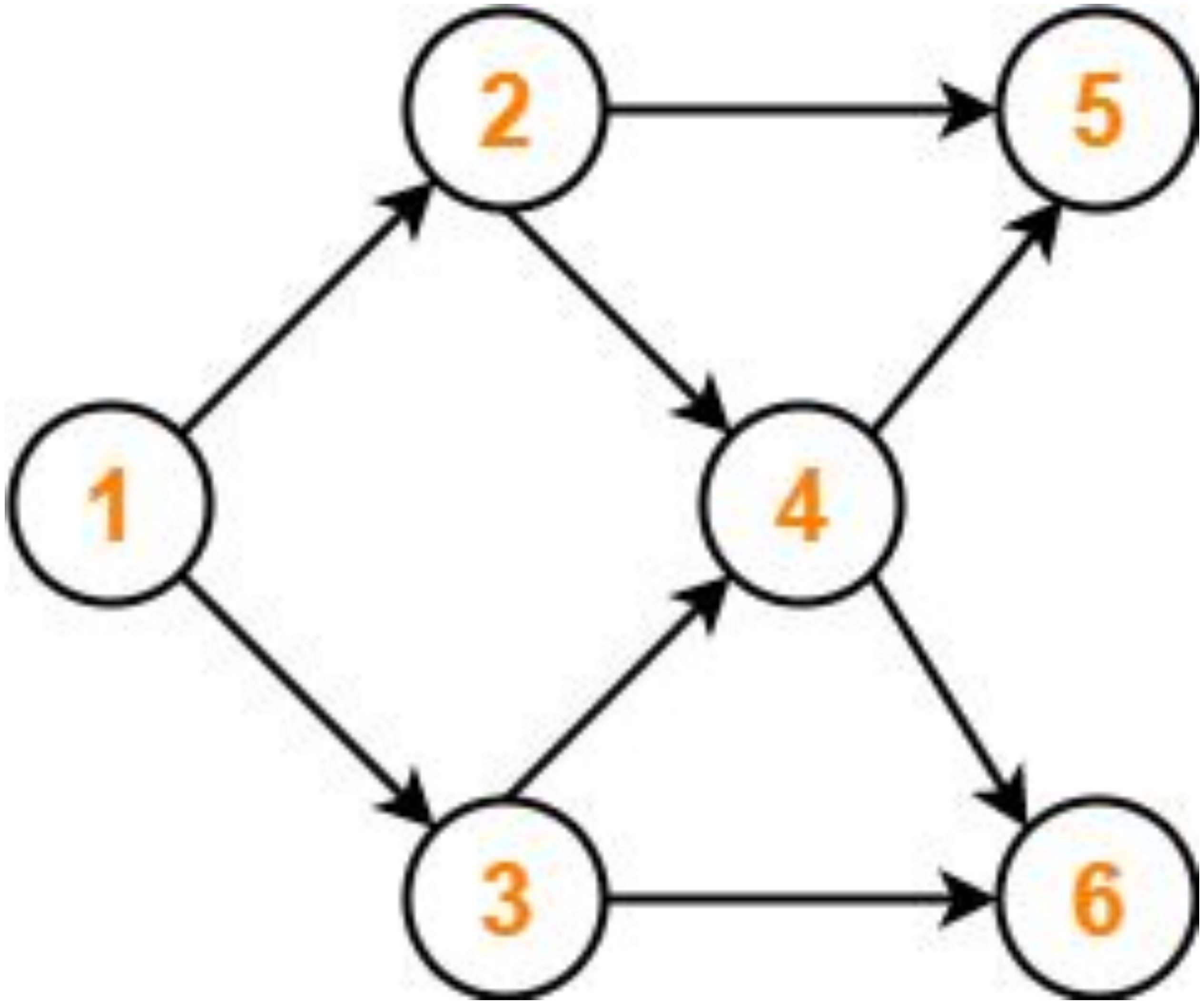
- Remove vertex-D and its associated edges.
- Then, update the in-degree of other vertices.



For the given graph, following **2** different topological orderings are possible-

- **A B C D E**
- **A B D C E**

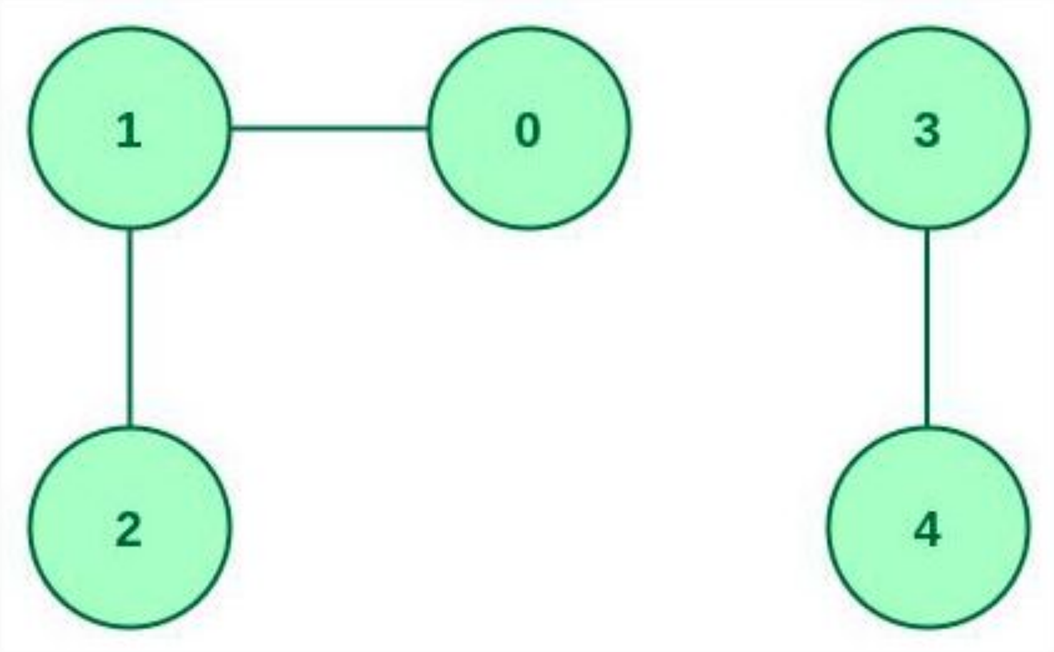
Find the number of different topological orderings possible for the given graph-



Connected Components

A connected component or simply component of an undirected graph is a subgraph in which each pair of nodes is connected with each other via a path.

Every vertex has a path to every other vertex in that component.



Example of an undirected graph

Output:

0 1 2

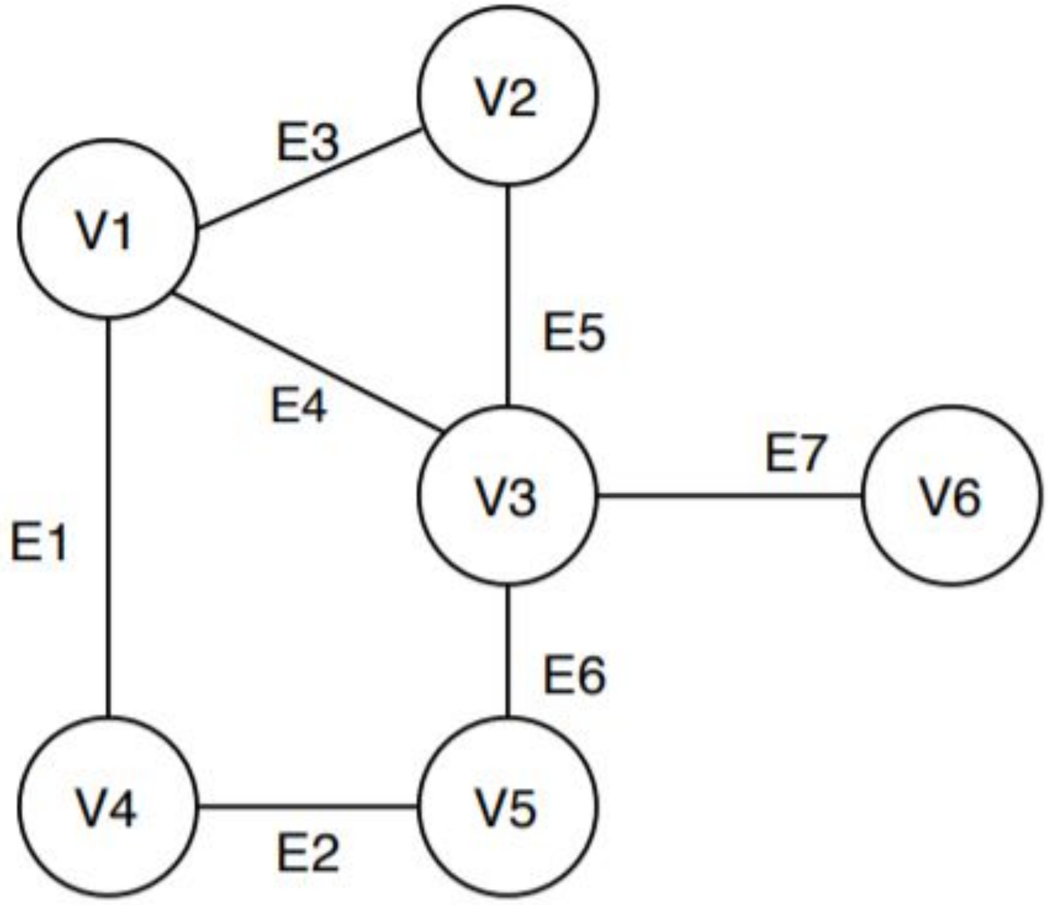
3 4

Explanation: *There are 2 different connected components.*

They are {0, 1, 2} and {3, 4}.

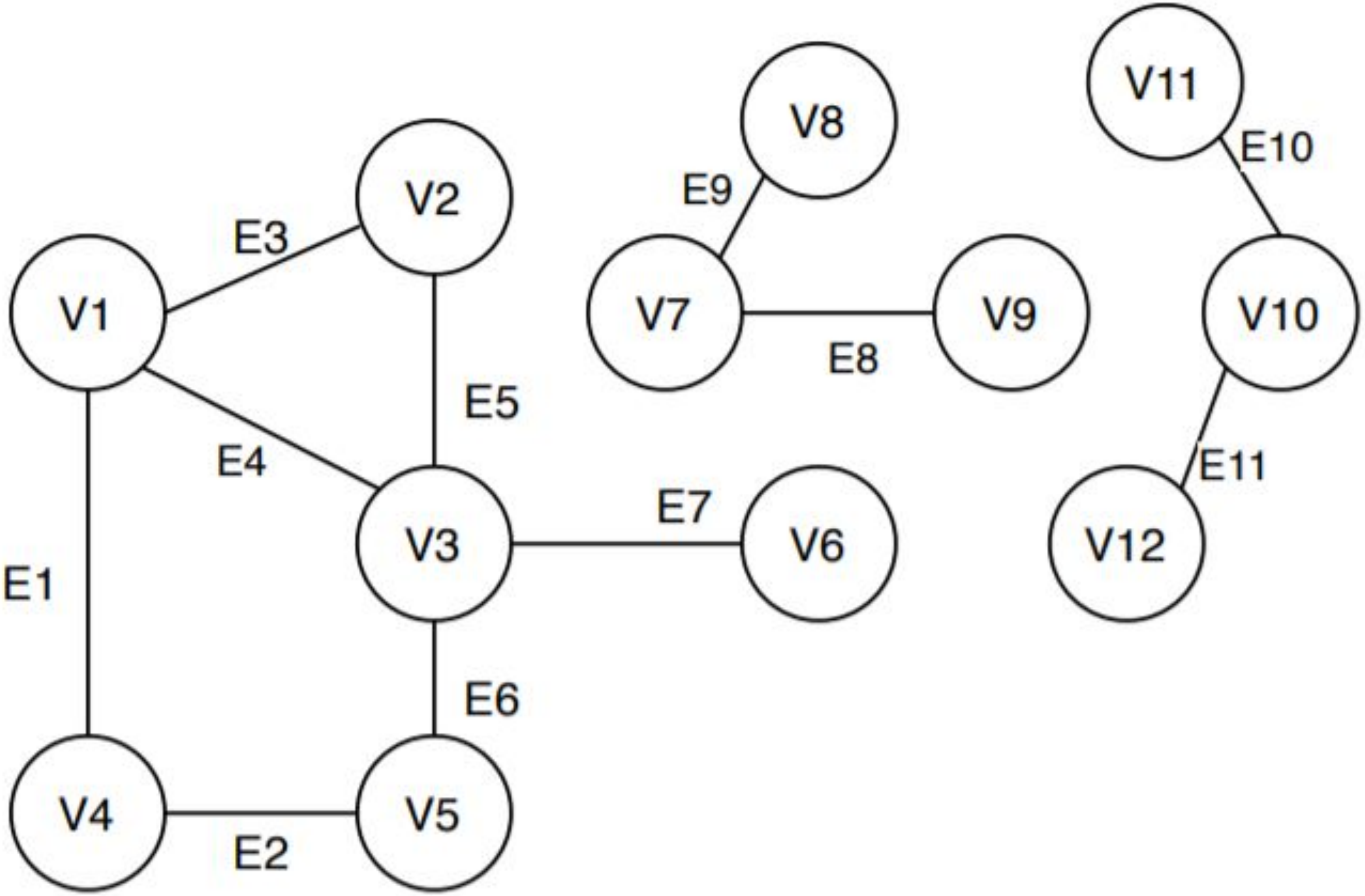
One Connected Component:-

In this example, the given undirected graph has one connected component:



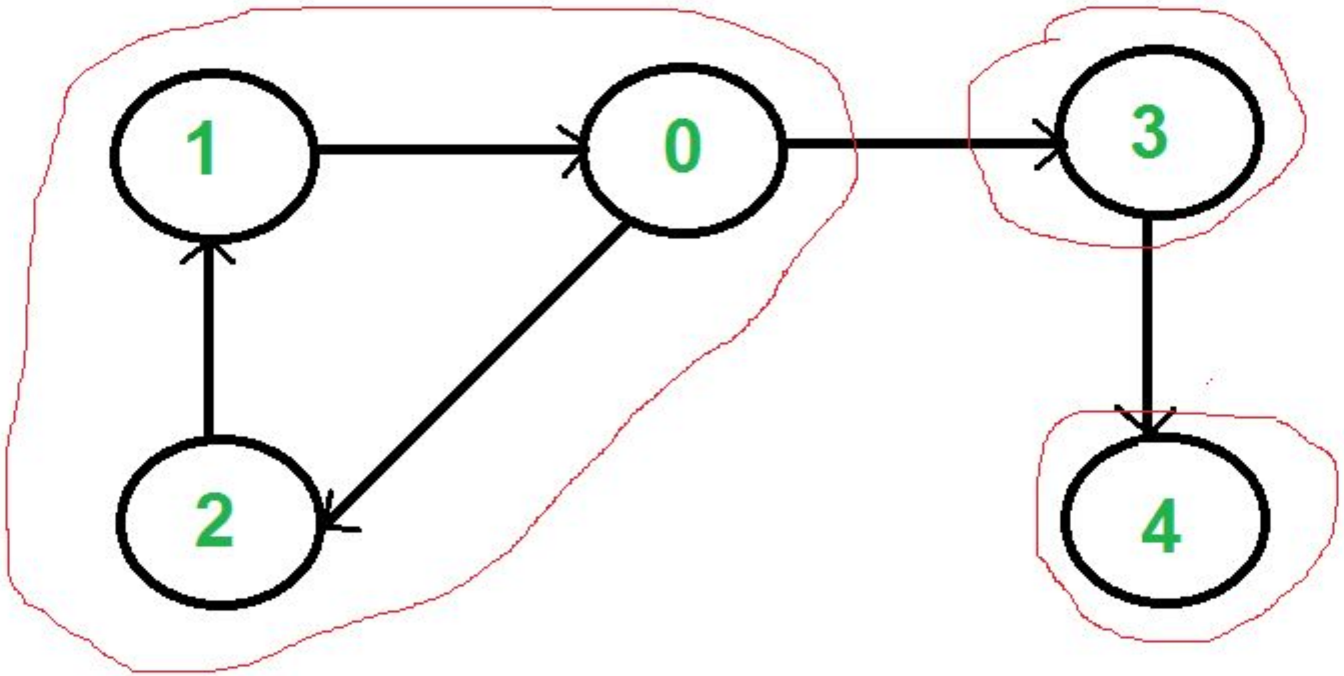
More Than One Connected Component

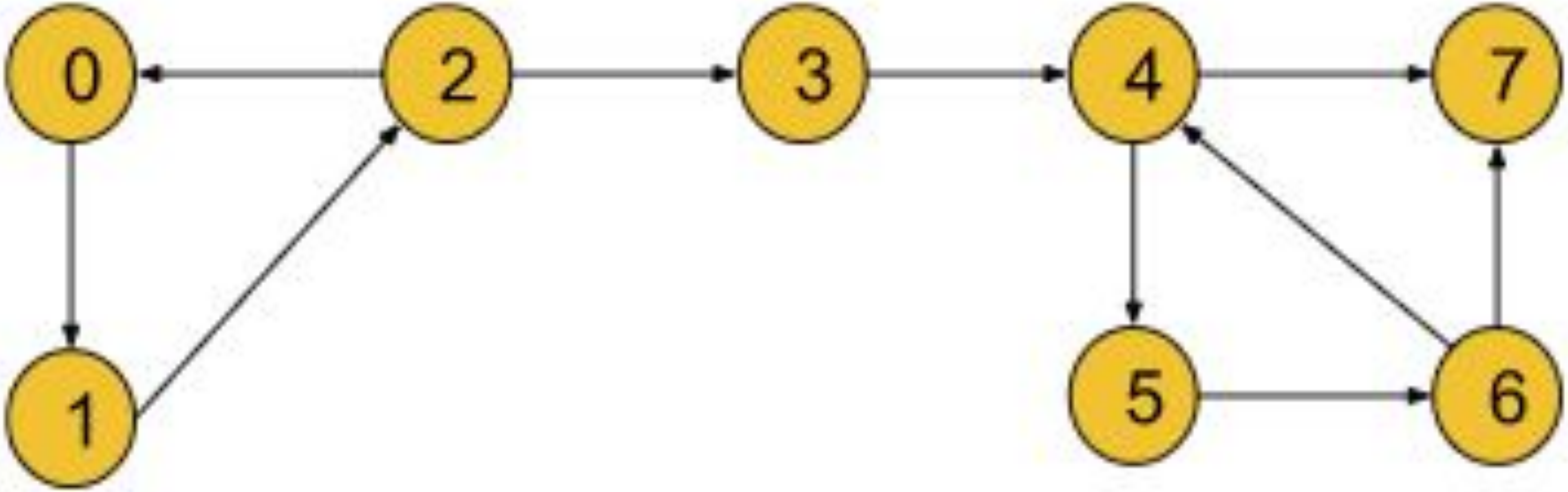
In this example, the undirected graph has three connected components:



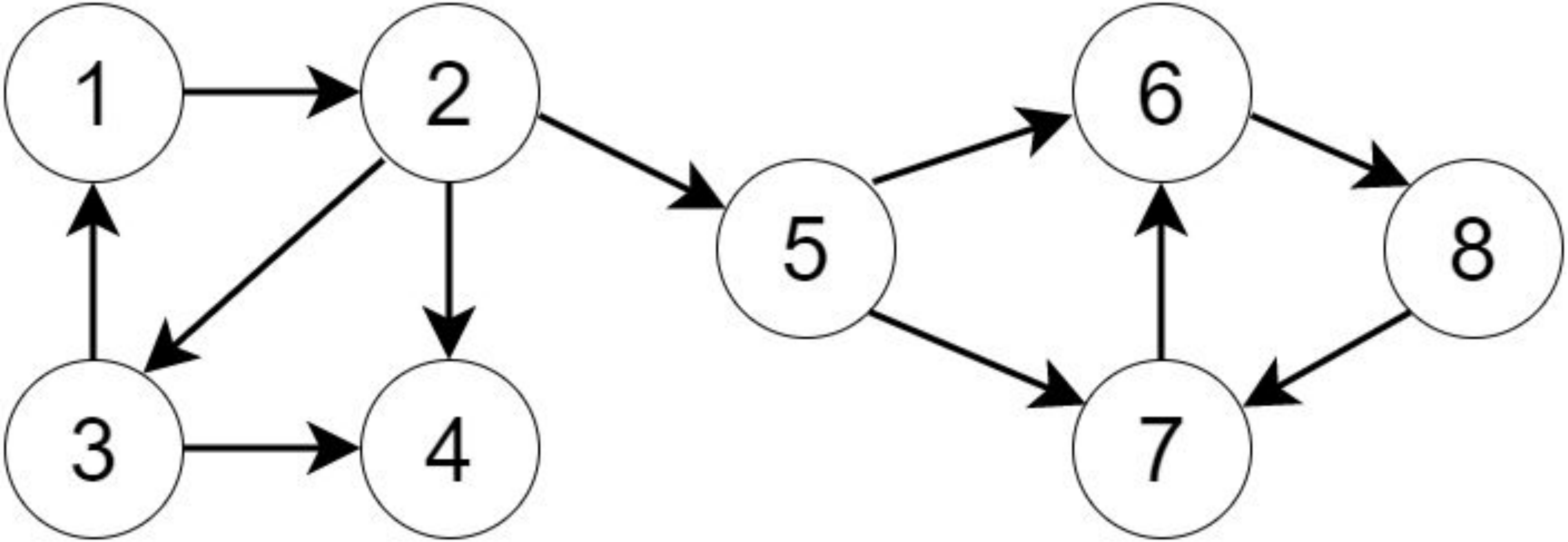
Strongly Connected Components

A directed graph is strongly connected if there is a path between all pairs of vertices.





0
2
3
4
7
1
5
6


1
2
3
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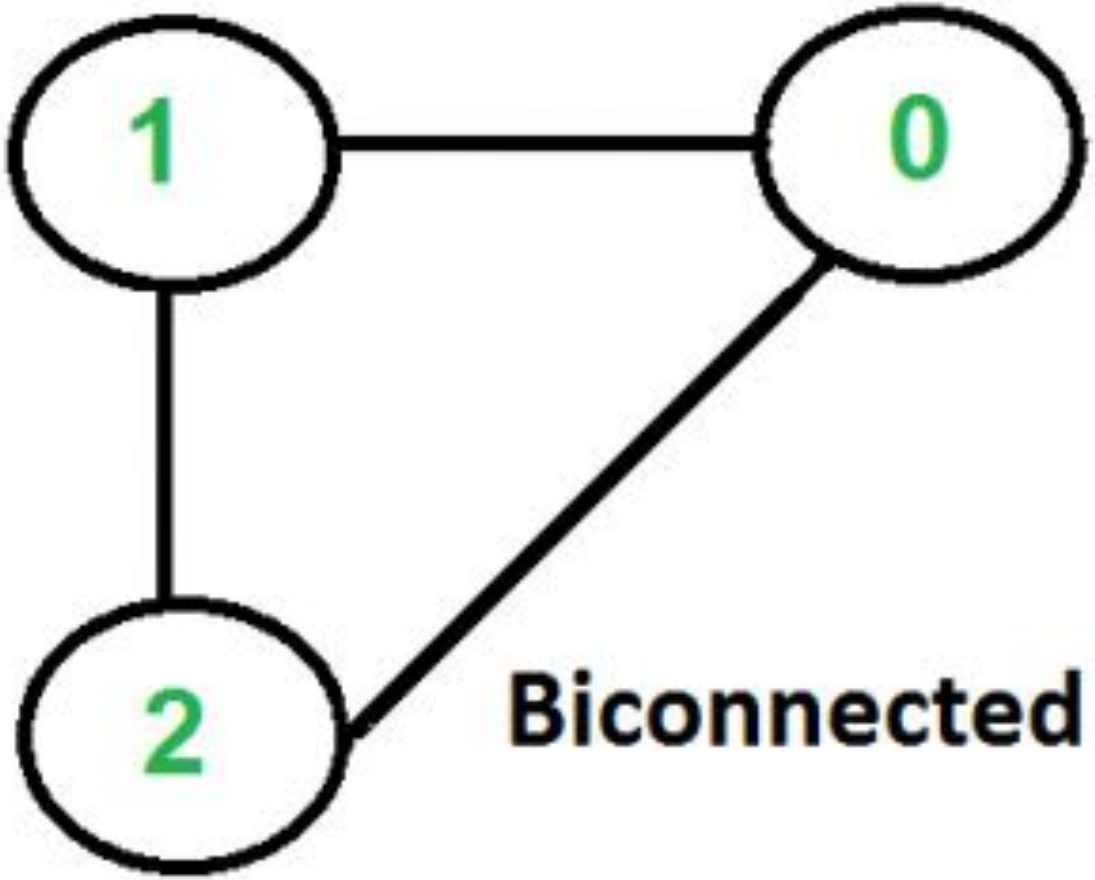
7
8

Biconnected Graph

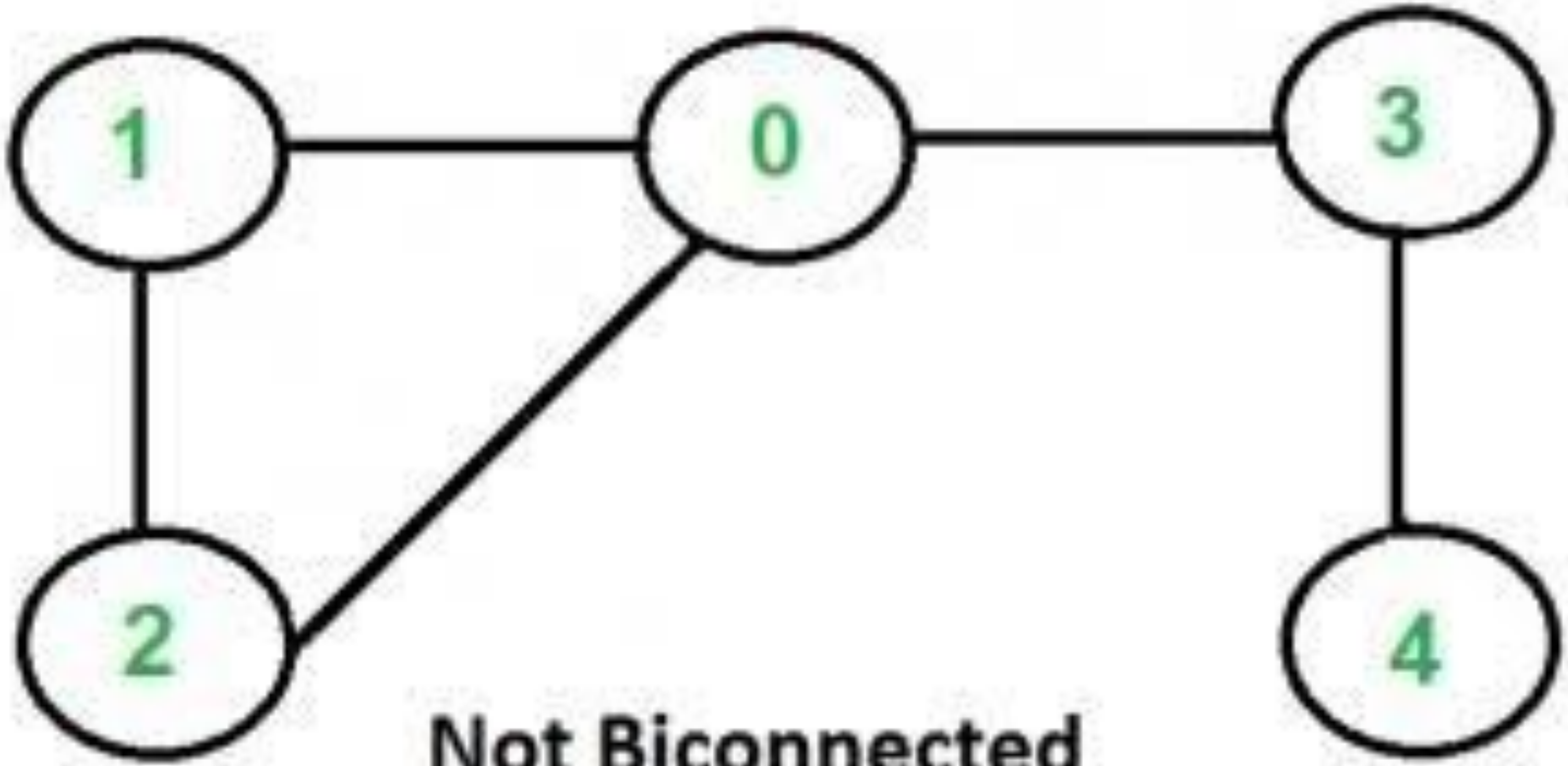
A graph is said to be Biconnected if:

1) It is connected, i.e. it is possible to reach every vertex from every other vertex, by a simple path.

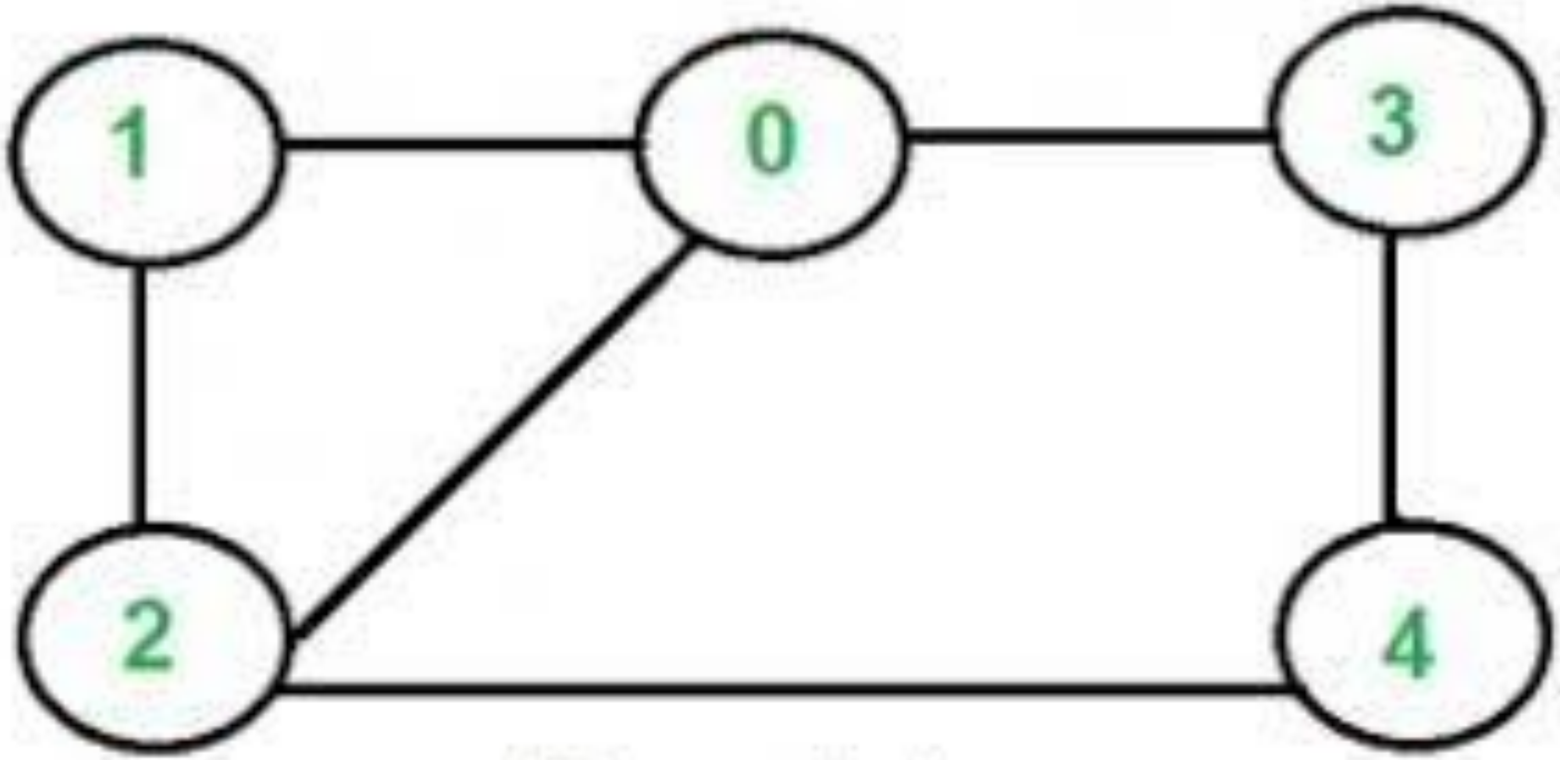
2) Even after removing any vertex the graph remains connected. That is there should not be any articulation point



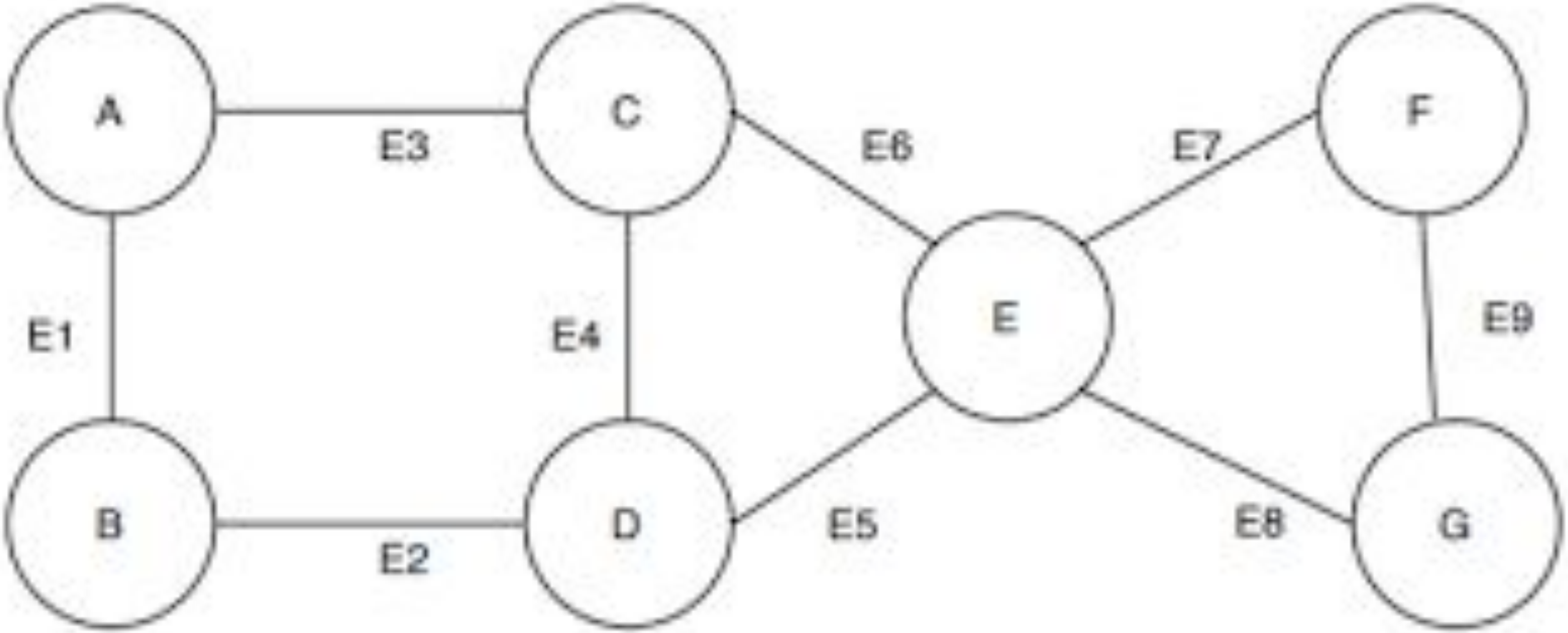
Biconnected

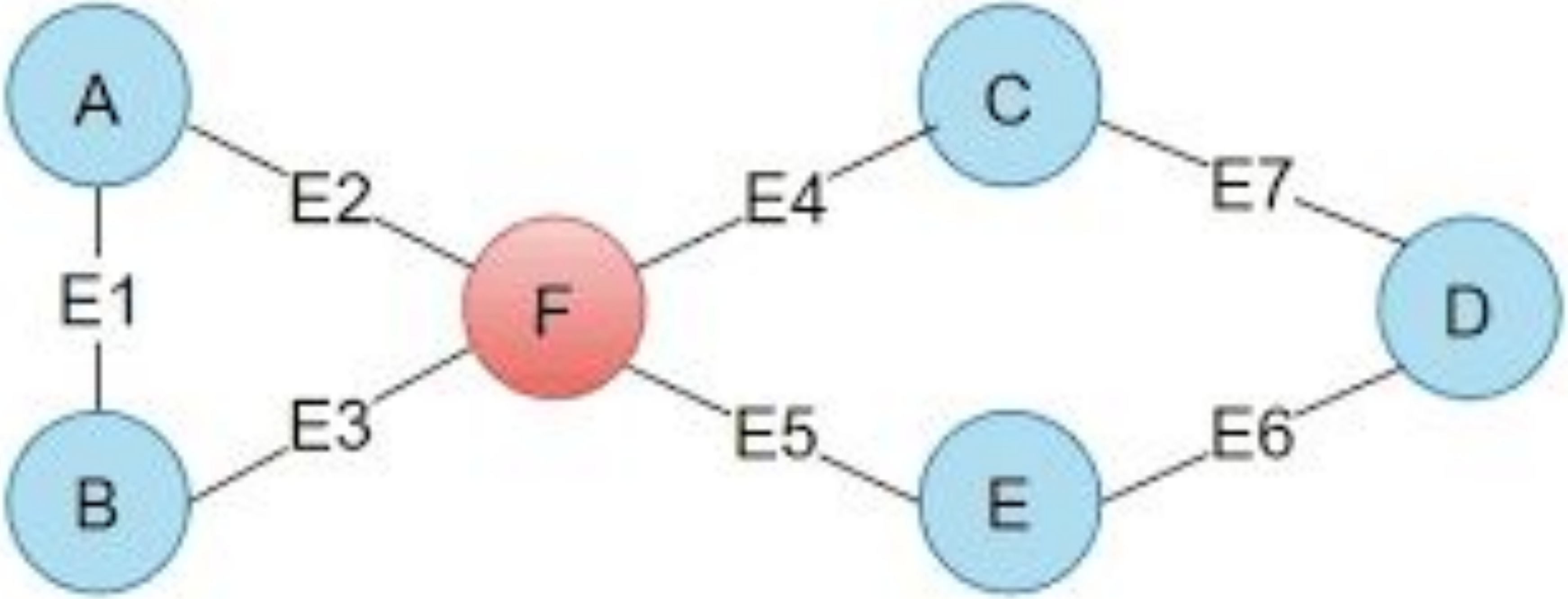


Not Biconnected

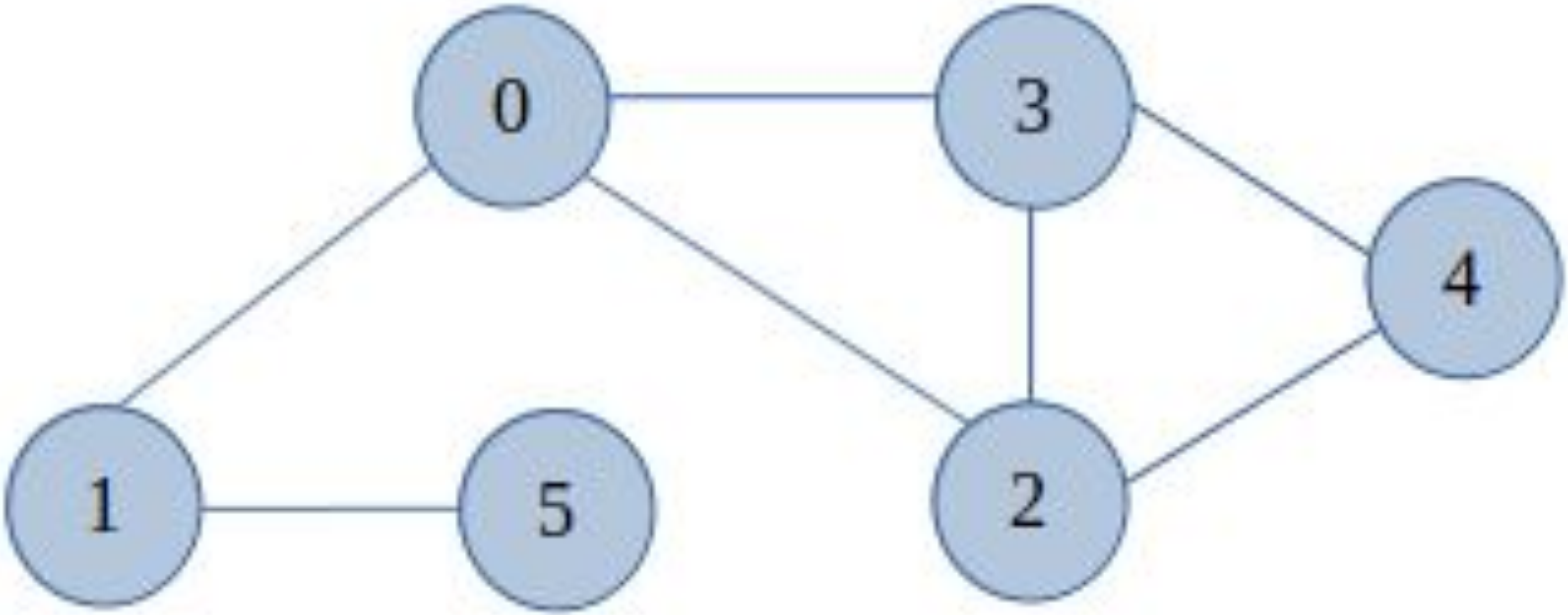


Biconnected

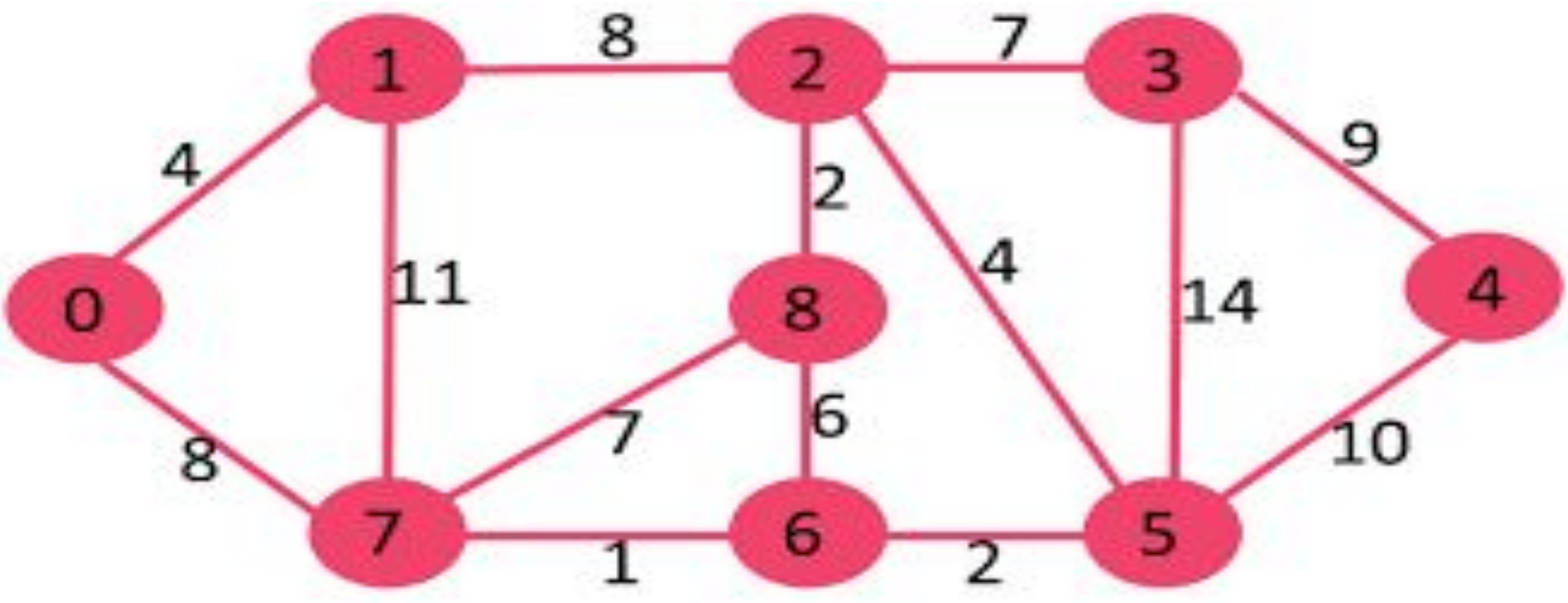

A
C
F
E3
E6
E7
E1
E4
E
E9
B
D
G
E2
E5
E8

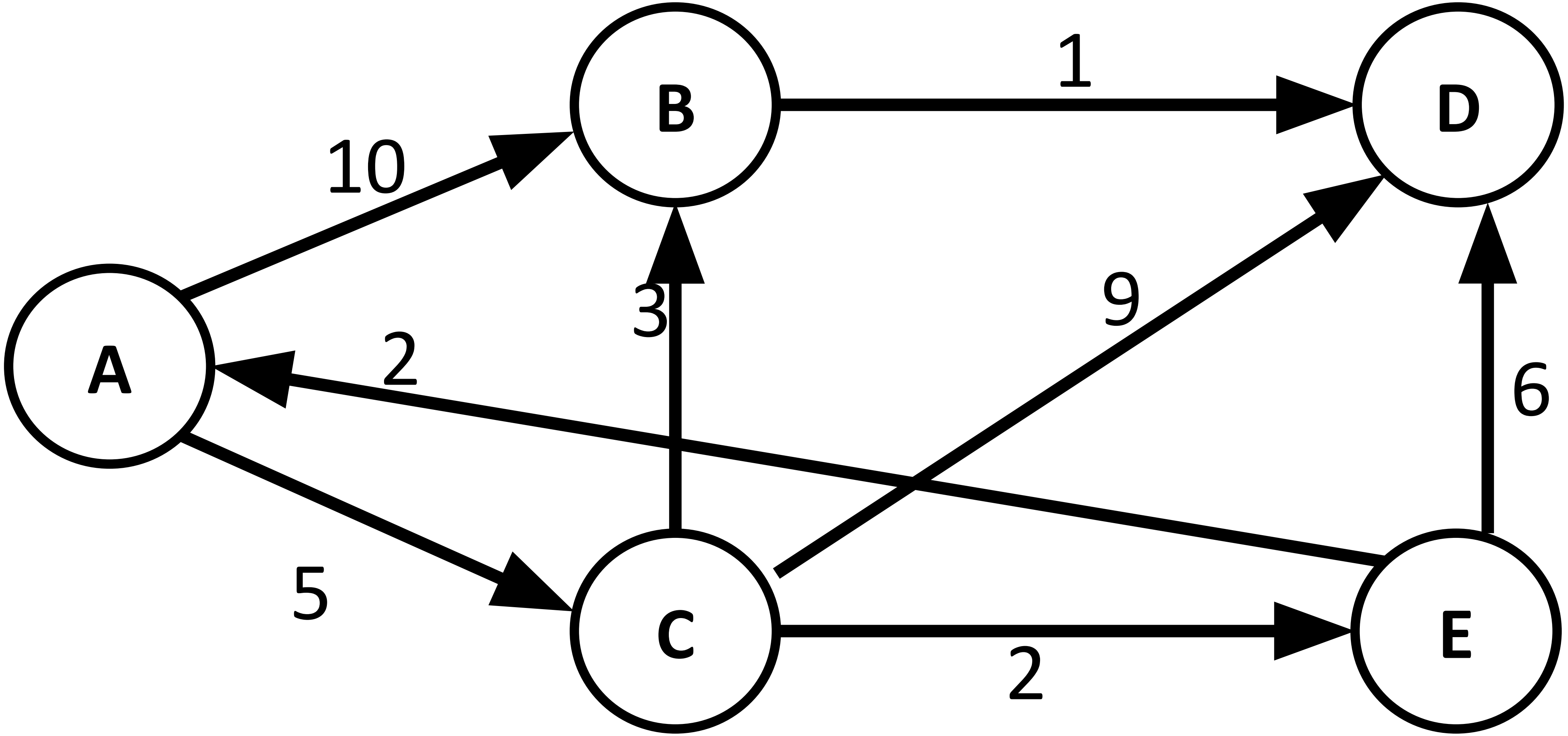


F is the Articulation Point.


0
3
4
1
5
2

Dijkstra's algorithm





A
B
C
D
E
10
5
1
3
2
9
2
6