# Closure of relations

## ① Reflexive closure

Let $\Delta = \{(a,a) \mid a \in A\}$ is the reflexive relation on the set A. The reflexive closure of relation R on set A is $R \cup \Delta$.

## ② Symmetric closure

Let R be a relation on set A, and let $R^{-1}$ be the inverse of R. The symmetric closure of relation R on set A is $R \cup R^{-1}$

## ③ Transitive closure.

Let R be a relation on set A. The Connectivity relation is defined as $R^* = \bigcup_{n=1}^{\infty} R^n$.
The transitive closure of R is $R^*$.

① Let the relation R be defined on the set $A = \{1,2,3,4\}$ as $R = \{(1,2)\ (2,3),\ (2,4)\}$. Compute the reflexive, symmetric and transitive closure of R.

**Solution**

For the given set $\Delta = \{(1,1)\ (2,2),\ (3,3)\ (4,4)\}$. The reflexive closure of R is

$R \cup \Delta = \{(1,1)\ (1,2)\ (2,2)\ (2,3),\ \cancel{(3,3)}\ (2,4)\ (3,3)\ (4,4)\}$

$R^{-1} = \{(2,1)(3,2)(4,2)\}$

The symmetric closure of R is $R \cup R^{-1}$

$= \{(1,2)(2,1)(2,3)(2,4)(3,2)(4,2)\}$

$R^1 = \{(1,2)(2,3)(2,4)\}$

$R^2 = \{(1,3)(1,4)\}$

$R^3 = \{(1,3)(1,4)\} = R^2$

Transitive Closure $R^* = R^1 \cup R^3$

$= \{(1,2)(1,3)(1,4)(2,3)(2,4)\}$

Q). Let R be a relation on a set $A = \{1,2,3,4\}$ with $R = \{(1,1),(1,4)(2,3)(3,1)(3,4)\}$ Find the reflexive, symmetric and transitive closure of R.

Solution

For the given set $\Delta = \{(1,1)(2,2)(3,3)(4,4)\}$

The reflexive closure of R is $R \cup \Delta$

$= \{(1,1)(1,4)(2,2)(2,3)(3,1)(3,3)(3,4)(4,4)\}$

The inverse of R is

$R^{-1} = \{(1,1)(1,3)(3,2)(4,1)(4,3)\}$

The symmetric closure of R is

$R \cup R^{-1} = \{(1,1)(1,3)(1,4)(2,3)(3,1)(3,2)(3,4)$
$(4,1)(4,3)\}$

$R^1 = \{ (1,1)(1,4)(2,3)(3,1)(3,4) \}$

$R^2 = \{ (1,1)(1,4)(2,1)(2,4)(3,1)(3,4) \}$

$R^3 = \{ (1,1)(1,4)(2,1)(2,4)(3,1)(3,4) \}$

$R^2 = R^3$.

The transitive closure is $R^* = R^1 \cup R^3$

$= \{ (1,1)(1,4)(2,1)(2,3)(2,4)(3,1)(3,4) \}$

(3) Find the transitive closure of the relation.

$\{ (1,3)(3,2)(2,4)(3,1)(4,1) \}$ on $\{1,2,3,4\}$

<u>Solution</u>

$R^1 = \{ (1,3)(3,2)(2,4)(3,1)(4,1) \}$

$R^2 = \{ (1,2)(1,1)(3,4)(2,1)(3,3)(4,3) \}$

$R^3 = \{ (1,1)(1,2)(2,1)(2,2)(3,3)(3,4)(4,3)$
$(4,4) \}$

$R^4 = \{ (1,1)(1,2)(2,1)(2,2)(3,3)(3,4)(4,3)$
$(4,4) \}$

$R^3 = R^4$.

Transitive closure $= R^1 \cup R^4$

$= \{ (1,1)(1,2)(1,3)(2,1)(2,2)(2,4)(3,1)$
$(3,2)(3,3),(3,4)(4,1)(4,3)(4,4) \}$

# Matrix representation of Relations

1) Let $A = \{1, 2, 3, 4\}$, $B = \{w, x, y, z\}$, $C = \{5, 6, 7\}$

Consider $R_1 = \{(1, x)\ (2, x)\ (3, y)\ (3, z)\}$ is a relation from A to B and $R_2 = \{(w, 5)(x, 6)\}$ is a relation from B to C. Find the relation matrices for $R_1$ and $R_2$.

**Solution**

$$M(R_1) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$M(R_2) = \begin{array}{c} \\ w \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccc} 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

The relation matrix for the relation R is defined as.

$$m_{ij} = \begin{cases} 1, & \text{if } (i, j) \in R. \\ 0, & \text{otherwise} \end{cases}$$

2) Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2)(1, 3)(2, 4)(3, 2)\}$

Find the relation matrix.

**Solution**

$$M(R) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$