Then $H_{max} = H(P=\frac{1}{2}) = \frac{1}{2}\log 2 + \frac{1}{2}\log 2 = 1$ bit/msg.

For binary case $(M=2)$, the entropy is maximum when $p=\frac{1}{2}$, ie when both the msgs are equally likely.

(6) → For an M-ary case, entropy is max when all the msgs are equally likely.

$$H_{max} = \sum_{k=1}^{M} p_k \log \frac{1}{p_k}$$

$$= \sum \frac{1}{M} \log M$$

$$H_{max} = \log M \text{ bits/msg} \quad \text{Since } p_k = \frac{1}{M}.$$

Properties of Entropy:

1. It is a continuous function of $p(x)$.
2. It is a symmetrical function of $p(x)$.
3. $0 \le H(x) \le \log M$.
4. $H(x) = 0$ for all $p(x) = 0$ except for one which must be unity.
5. $H(x) = \log M$ if all probabilities are equal; ie $p(x)_i = \frac{1}{M}$ for all i.

A binary source is emitting an independent sequence of '0's and '1's with probabilities p and 1 – p respectively. Plot the entropy of the source versus p.

Solution

From equation (1.4), the entropy of the binary source is given by

$$H(S) = \sum_{i=1}^{2} p_i \log \frac{1}{p_i}$$

$$= p_1 \log \frac{1}{p_1} + p_2 \log \frac{1}{p_2}$$

$$\therefore \qquad H(S) = p \log \frac{1}{p} + (1 - p) \log \frac{1}{(1 - p)}$$

Let $p = 0.1$, $\quad H(S) = 0.1 \log \frac{1}{0.1} + 0.9 \log \frac{1}{0.9} = 0.469$ bits/symbol.

Let $p = 0.2$, $\quad H(S) = 0.2 \log \frac{1}{0.2} + 0.8 \log \frac{1}{0.8} = 0.722$ bits/symbol.

Let $p = 0.3$, $\quad H(S) = 0.3 \log \frac{1}{0.3} + 0.7 \log \frac{1}{0.7} = 0.881$ bits/symbol.

Let $p = 0.4$, $\quad H(S) = 0.4 \log \frac{1}{0.4} + 0.6 \log \frac{1}{0.6} = 0.971$ bits/symbol.

Let $p = 0.5$, $\quad H(S) = 0.5 \log \frac{1}{0.5} + 0.5 \log \frac{1}{0.5} = 1$ bits/symbol.

Let $p = 0.6$, $\quad H(S) = 0.6 \log \frac{1}{0.6} + 0.4 \log \frac{1}{0.4} = 0.971$ bits/symbol.

Let $p = 0.7$, $\quad H(S) = 0.7 \log \frac{1}{0.7} + 0.3 \log \frac{1}{0.3} = 0.881$ bits/symbol.

Let $p = 0.8$, $\quad H(S) = 0.8 \log \frac{1}{0.8} + 0.2 \log \frac{1}{0.2} = 0.722$ bits/symbol.

Let $p = 0.9$, $\quad H(S) = 0.9 \log \frac{1}{0.9} + 0.1 \log \frac{1}{0.1} = 0.469$ bits/symbol.

For $p = 0$ and $p = 1$, $H(S) = 0$.

The above calculated values can be tabulated as shown in table 1.2.

p	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
H(S) bits/m-sym	0	0.469	0.722	0.881	0.971	1	0.971	0.88[illegible]	[illegible]	[illegible].469	0

Table 1.2 : Entropy values for various probabilities

The entropy H(S) can now be plotted as a function of p as shown in figure 1.2.