Multiply throughout by r^{-i};

$$n_i r^{-i} + n_1 r^{-1} + n_2 r^{-2} + \dots + n_{(i-1)} r^{-(i-1)} \leq 1$$

Rearrange ;

$$n_i r^{-i} + n_{(i-1)} r^{-(i-1)} + n_{(i-2)} r^{-(i-2)} + \dots + n_2 r^{-2} + n_1 r^{-1} \leq 1$$

Write as summation :

$$\sum_{m=1}^{i} n_m r^{-m} \leq 1$$

Rewrite the equation, since actual number of messages n_i has to be integer.

$$\sum_{m=1}^{i} n_m r^{-m} = n_1 r^{-1} + n_2 r^{-2} + \dots + n_i r^{-i}.$$

$$= \underbrace{r^{-1} + r^{-1} + \dots + r^{-1}}_{n_1 \text{ terms}} + \underbrace{r^{-2} + r^{-2} + \dots + r^{-2}}_{n_2 \text{ terms}} + \dots + \underbrace{r^{-i} + r^{-i} + \dots + r^{-i}}_{n_i \text{ terms}} \leq 1$$

$$\sum_{m=1}^{i} n_m r^{-m} = \underbrace{\sum_{j=1}^{n_1} r^{-1}}_{\text{length } l_1} + \underbrace{\sum_{j=1}^{n_2} r^{-2}}_{\text{length } l_2} + \dots + \underbrace{\sum_{j=1}^{n_i} r^{-i}}_{\text{length } l_i} \leq 1$$

Combining all groups; $n_1 + n_2 + \dots + n_i = q$

$$\boxed{\sum_{i=1}^{q} r^{-l_i} \leq 1} \longrightarrow \text{Hence Proved.}$$

Note : unit of information : bits (BInary unITS).

unit of wordlength : binits (BINary digITS).

eg:

Source symbol	Code I	Code J	Code K	Code L	Code M
S_1	00	0	0	0	0
S_2	01	100	10	100	10
S_3	10	110	110	110	110
S_4	11	111	111	11	11

use kraft inequality :

For code I : $s_1 \longrightarrow l_1 = 2$ binits

$s_2 \longrightarrow l_2 = 2$ binits

$s_3 \longrightarrow l_3 = 2$ binits

$s_4 \longrightarrow l_4 = 2$ binits

Then $\sum_{i=1}^{q} 2^{-l_i} = \sum_{i=1}^{4} 2^{-l_i} = 2^{-l_1} + 2^{-l_2} + 2^{-l_3} + 2^{-l_4}$

$= 2^{-2} + 2^{-2} + 2^{-2} + 2^{-2} = \underline{\underline{1}}$

Prefix values for s_1 and s_2 are '0' and s_3 and s_4 are '1', which are not other code words. ie it satisfying prefix property. So it is possible to construct an instantaneous code.

For code J : $s_1 \longrightarrow l_1 = 1$ binits

$s_2, s_3, s_4 \longrightarrow l_2 = l_3 = l_4 = 3$ binits

Then $\sum_{i=1}^{4} 2^{-l_i} = 2^{-1} + 2^{-3} + 2^{-3} + 2^{-3} = 7/8 < 1$

Also satisfying prefix property ; So possible to construct an instantaneous code.

For code K : $s_1 \longrightarrow l_1 = 1$ binits

$s_2 \longrightarrow l_2 = 2$ binits

$s_3, s_4 \rightarrow l_3 = l_4 = 3$ binits

Then $\sum_{i=1}^{4} 2^{-l_i} = 2^{-1} + 2^{-2} + 2^{-3} + 2^{-3} = 1$: Instantaneous.

For code L :
$s_1 \longrightarrow l_1 = 1$ binits
$s_2, s_3 \longrightarrow l_2 = l_3 = 3$ binits
$s_4 \longrightarrow l_4 = 2$ binits

$$\sum_{i=1}^{4} r^{-l_i} = 2^{-1} + 2^{-3} + 2^{-3} + 2^{-2} = \underline{\underline{1}} \; ;$$

prefix property not satisfying ; Not instantaneous .

For code M :
$s_1 \longrightarrow l_1 = 1$ binits
$s_2, s_4 \rightarrow l_2 = l_4 = 2$ binits
$s_3 \longrightarrow l_3 = 3$ binits

$$\sum_{i=1}^{4} r^{-l_i} = 2^{-1} + 2^{-2} + 2^{-2} + 2^{-3} = 9/8 > 1.$$

Not satisfying Kraft inequality, so it is impossible to construct a code with prefix property.

HW 1. Identify the instantaneous codes and construct their individual decision trees.

Symbol	Code N	Code O	Code P	Code Q.
s_1	0	0	0	00
s_2	10	01	01	01
s_3	110	001	011	10
s_4	1110	0010	110	110
s_5	1111	0011	111	111

Ans : instantaneous, not, not, instantaneous

Decision Tree (Code Tree) :

For code N : It has one initial state and five terminal states corresponding to s_1, s_2, s_3, s_4 and s_5.