$$I(x_j) = f\left\{\frac{1}{p(x_j)}\right\} \quad \text{and} \quad I(y_k) = f\left\{\frac{1}{p(y_k)}\right\}.$$

So we use a function which converts multiplication into addition in the RHS. Logarithm is one such function.

$$I(x_j, y_k) = \log\left[\frac{1}{p(x_j)\cdot p(y_k)}\right]$$

$$= \log\left[\frac{1}{p(x_j)}\right] + \log\left[\frac{1}{p(y_k)}\right]$$

$$= I(x_j) + I(y_k)$$

Then the basic equation defining "the amount of information" or "self-information" is given by:

$$\boxed{I(x_j) = \log\left[\frac{1}{p(x_j)}\right] = -\log p(x_j)}$$

Different units of Information:

If the base of the logarithm is '2', then the units are called "BITS", which is the short form of "BInary unITS". If the base is '10', the units are "HARTLEYS" or "DECITS". If the base is 'e', then the units are "NATS" and in general if the base is 'r', the unit are called "r-ary units".

Conversion of information units:

$\log_2 2 = 1$; $\log_2 e = 1.4426$; $\log_2 10 = 3.3219$

$\ln 2 = 0.6932$; $\ln e = 1$; $\ln 10 = 2.3026$

$\log_{10} 2 = 0.3010$; $\log_{10} e = 0.4342$; $\log_{10} 10 = 1.$

$\Downarrow$ $\Downarrow$ $\Downarrow$

1 bit = 0.6932 nat = 0.3010 decit

1 nat = 1.4426 bits = 0.4342 decit

1 decit = 3.3219 bits = 2.3026 nats.

Q.1: The binary symbols '0' and '1' are transmitted with probabilities 1/4 and 3/4 respectively. find the corresponding self-information ?

Sol: Self information in a '0' =>

$$I_0 = -\log P_0 = -\log(\tfrac{1}{4}) = \underline{\underline{2 \text{ bits}}}$$

Self information in a '1' =>

$$I_1 = -\log P_1 = -\log(\tfrac{3}{4}) = \underline{\underline{0.415 \text{ bits}}}$$

$$\text{ie } -\log_2(\tfrac{3}{4}) = -\frac{\log_{10}(\frac{3}{4})}{\log_{10} 2} = \underline{\underline{0.415 \text{ bit}}}.$$

Properties :

1. Self information of any message cannot be negative.
2. Lowest possible self-information is 'zero'.
3. More information is carried by a less likely message.
4. When independent symbols are transmitted, the total self information must be equal to the sum of individual self-informations.

ENTROPY

In information theory, Entropy is a measure of the uncertainity associated with a random variable. The entropy of a discrete random variable X is defined by,

$$\boxed{H(X) = -\sum_{x \in X} p(x) \log p(x).}$$

In a communication system, "the average information per individual message" is known as entropy of source.

$$\text{Entropy} = \frac{\text{Total Amount of Information}}{\text{Number of messages.}}$$

Let there be M different messages;

$$M = \{ m_1, m_2, m_3, \ldots \ldots m_M \} \leftarrow \text{diff. msgs.}$$

with respective probabilities P;

$$P = \{ P_1, P_2, P_3, \ldots \ldots P_M \} \leftarrow \text{resp. prob. of occurance.}$$

Then consider a ~~time~~ interval T, "L" messages are generated where $L \gg M$.

~~first~~ consider the msg m_1 only;

The number of times the msg m_1 has occured

$$n_1 = P_1 \cdot L$$

The amount of information in msg m_1

$$I_1 = \log \frac{1}{P_1}$$

The total amount of information in all m_1 msgs;

$$I_t(m_1) = n_1 \cdot I_1$$

$$I_t(m_1) = P_1 \cdot L \cdot \log \frac{1}{P_1}$$

m_2 No of occurance $\Rightarrow n_2 = P_2 \cdot L$; $I_2 = \log \frac{1}{P_2} \Rightarrow I_t(m_2) = P_2 \cdot L \log \frac{1}{P_2}$.

m_3 " $n_3 = P_3 \cdot L$; $I_3 = \log \frac{1}{P_3} \Rightarrow I_t(m_3) = P_3 \cdot L \cdot \log \frac{1}{P_3}$.

⋮

m_M " $n_M = P_M \cdot L$; $I_M = \log \frac{1}{P_M} \Rightarrow I_t(m_M) = P_M \cdot L \cdot \log \frac{1}{P_M}$.

Then the total amount of information in all L msgs;

$$I_t = I_t(m_1) + I_t(m_2) + \ldots \ldots + I_t(m_M).$$

$$= P_1 \cdot L \cdot \log \frac{1}{P_1} + P_2 \cdot L \cdot \log \frac{1}{P_2} + \ldots \ldots + P_M \cdot L \cdot \log \frac{1}{P_M}$$