Q1. A source has an alphabet $S = \{s_1, s_2, s_3, s_4, s_5\}$ with probabilities $P = \{1/2, 1/6, 1/6, 1/9, 1/18\}$. find code efficiency and code redundancy, when coded with ① Code u and ② Code v.

Source symbol	Code u	Code v.
S_1	0	00
S_2	10	01
S_3	110	10
S_4	1110	110
S_5	1111	111

Sol. ① Average length ; $L = \sum_{i=1}^{5} p_i l_i$

$= p_1 l_1 + p_2 l_2 + p_3 l_3 + p_4 l_4 + p_5 l_5$

Here $l_1 = 1$; $l_2 = 2$; $l_3 = 3$; $l_4 = l_5 = 4$.

$L = \frac{1}{2} \times 1 + \frac{1}{6} \times 2 + \frac{1}{6} \times 3 + \frac{1}{9} \times 4 + \frac{1}{18} \times 4$

$= 2$ binits/msg

$H(S) = -\sum_{j=1}^{5} p_j \log p_j$

$= -\left[\frac{1}{2}\log\frac{1}{2} + 2 \times \frac{1}{6}\log\frac{1}{6} + \frac{1}{9}\log\frac{1}{9} + \frac{1}{18}\log\frac{1}{18}\right]$

$= 1.94553$ bits/msg.

Code Efficiency ; $\eta_c = \frac{H(S)}{L} = \frac{1.94553}{2} \times 100 = 97.28\%$

Code Redundancy, $R_c = (1 - 0.9728) \times 100 = 2.72\%$

② $l_1 = l_2 = l_3 = 2$; $l_4 = l_5 = 4$

$L = \sum_{i=1}^{5} p_i l_i$

$= \frac{1}{2} \times 2 + \frac{1}{6} \times 2 + \frac{1}{6} \times 2 + \frac{1}{9} \times 4 + \frac{1}{18} \times 4 = 2.16667$ binits/msg

$$H(s) = 1.94553 \text{ bits/msg}.$$

$$\eta_c = \frac{H(s)}{L} = \frac{1.94553}{2.16667} \times 100 = 89.79\%$$

$$R_c = (1 - 0.8979)\,100 = 10.21\%$$

SHANNON'S FIRST THEOREM (Noiseless coding Theorem):

Shannon suggested that the length 'l_i' using the formula,

$$l_i = \log_r \frac{1}{P_i}$$

ie $$l_i = \frac{\log_2 \frac{1}{P_i}}{\log_2 r}$$

If 'l_i' be a fraction then it is rounded off to next integer.

$$\log_r \frac{1}{P_i} \le l_i \le 1 + \log_r \frac{1}{P_i}.$$

$$\frac{\log \frac{1}{P_i}}{\log r} \le l_i \le 1 + \frac{\log \frac{1}{P_i}}{\log r}$$

Multiply through out by P_i.

$$\frac{P_i \log \frac{1}{P_i}}{\log r} \le P_i l_i \le P_i + \frac{P_i \log \frac{1}{P_i}}{\log r}.$$

Take summation for all i varying from 1 to q: