Step 8 : As we coming backward, the recomposition of one code word is done in order to form new codewords.

Step 9 : This procedure continues till we get codewords corresponding to each of the source symbols. If any dummy symbols are used, they are discarded.

Q8. Given the msgs $s_1, s_2, s_3$ and $s_4$ with respective probabilities 0.4, 0.3, 0.2 and 0.1. Construct a binary code by applying Huffman encoding procedure. Determine the efficiency and Redundancy of the code?

Sol: Step 1 : Arrange the symbols in non-increasing order of probabilities.

| $s_1$ | $s_2$ | $s_3$ | $s_4$ |
|---|---|---|---|
| 0.4 | 0.3 | 0.2 | 0.1 |
| ↑ | ↑ | ↑ | ↑ |
| $P_1$ | $P_2$ | $P_3$ | $P_4$ |



Step 2 : Calculate '$\alpha$' using $q = r + (r-1)\alpha$

Given $q = 4$ and $r = 2$ (binary code).

$$\alpha = ?$$

$$4 = 2 + (2-1)\alpha$$

$$\alpha = 2 \text{ ; An integer.}$$

Here $\alpha$ shows the number of reduced sources. or the number of stages we have to proceed to reduce the given sources to '$r$' symbols.

Step 3 : Form reduced source "$s_a$" by combining last "$r = 2$" symbols.

**Table 1:**

| Source symbol | prob | code | Source "Sa" prob | Source "Sa" code |
|---|---|---|---|---|
| $s_1$ | 0.4 | | 0.4 | |
| $s_2$ | 0.3 | | 0.3 | |
| $s_3$ | 0.2 | | 0.3 | |
| $s_4$ | 0.1 | | | |

Two choises for placing the "composite symbol". One way is placing the "composite symbol" "as low as possible." as shown in Table 1. The second way is placing the composite symbol "as high as possible" as shown in Table 2.

**Table 2:**

| Source symbol | prob | code | Source "Sa" prob | Source "Sa" code |
|---|---|---|---|---|
| $s_1$ | 0.4 | | 0.4 | |
| $s_2$ | 0.3 | | 0.3 | |
| $s_3$ | 0.2 | | 0.3 | |
| $s_4$ | 0.1 | | | |



Step 4 and step 5: Follow the first method to form reduced source "$S_b$" by combining last "$r = 2$" symbols of "$S_a$".

**Table 3:**

| Source symbol | prob | code | Source "Sa" prob | Source "Sa" code | Source "Sb" prob | Source "Sb" code |
|---|---|---|---|---|---|---|
| $s_1$ | 0.4 | | 0.4 | | 0.6 | |
| $s_2$ | 0.3 | | 0.3 | | 0.4 | |
| $s_3$ | 0.2 | | 0.3 | | | |
| $s_4$ | 0.1 | | | | | |

Follow the method by placing the composite symbol "as low as possible".