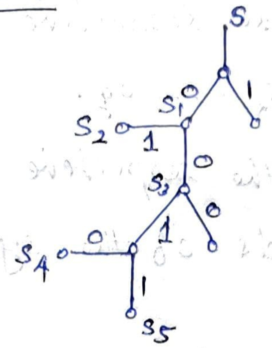
For code 'O' :



HW: Draw for code P and code Q?

eg: If the received sequence $R = 0,110,1111,1110,10,\ldots$

Decode using decision tree (code N): After decoding each symbol, the decoder resets to initial state.

Code words : $S_1\ S_3\ S_5\ S_4\ S_2$:

HW2: Which sets of word length given below are acceptable for the existence of an instantaneous code given $X = \{0,1,2\}$

Word Length (l_i)	Number of words with lgt l_i		
	Code R	Code S	Code T.
l_1	2	2	1
l_2	1	2	4
l_3	2	2	6
l_4	4	3	0
l_5	1	1	0

Construction of Instantaneous Codes :

By applying Kraft inequality, we can know if an instantaneous code can be constructed or not, keeping in mind the prefix property which says that "no complete word of a code be a prefix of any other code word".

Code Efficiency And Redundancy :

Let the average length of code be L, then

$$\boxed{L = \sum_{i=1}^{q} p_i l_i} \text{ bits/msg.}$$

where $p_i \rightarrow p_1, p_2 \cdots p_q$, the respective probabilities of the source symbols $s_1, s_2 \cdots s_q$.

$l_i \rightarrow l_1, l_2 \cdots l_q$, the respective word lengths in binits of the codewords of the symbols $s_1, s_2 \cdots s_q$

The entropy is given by :

$$\boxed{H(S) = -\sum_{j=1}^{q} p_j \log p_j} \text{ bits/msg.}$$

$H(S) \rightarrow$ for binary codes.

$H_r(S) \rightarrow$ for r-ary codes.

Then $\boxed{H_r(S) = \dfrac{H(S)}{\log_2 r}}$ where $H(S)$ in bits/msg and r is the number of different symbols used in code alphabet.

Code Efficiency (η_c) :

$$\eta_c = \frac{H(S)}{L} \text{ ; for binary codes.}$$

$$\eta_c = \frac{H_r(S)}{L} \text{ ; for r-ary codes.}$$

Here, $L \geq H(S)$; So $\eta_c \leq 100\%$.

Code Redundancy (R_c) :

$$\boxed{R_c = 1 - \eta_c}$$

Here η_c and R_c are expressed as percentage.

Q1. A source has an alphabet $S = \{s_1, s_2, s_3, s_4, s_5\}$ with probabilities $P = \{1/2, 1/6, 1/6, 1/9, 1/18\}$. find code efficiency and code redundancy, when coded with ① Code u and ② Code v.

Source symbol	Code u	Code v.
S_1	0	00
S_2	10	01
S_3	110	10
S_4	1110	110
S_5	1111	111

Sol. ① Average length ; $L = \sum_{i=1}^{5} p_i l_i$

$$= p_1 l_1 + p_2 l_2 + p_3 l_3 + p_4 l_4 + p_5 l_5$$

Here $l_1 = 1$; $l_2 = 2$; $l_3 = 3$; $l_4 = l_5 = 4$.

$$L = \tfrac{1}{2} \times 1 + \tfrac{1}{6} \times 2 + \tfrac{1}{6} \times 3 + \tfrac{1}{9} \times 4 + \tfrac{1}{18} \times 4$$

$$= 2 \text{ binits/msg}$$

$$H(S) = -\sum_{j=1}^{5} p_j \log p_j$$

$$= -\left[\tfrac{1}{2}\log\tfrac{1}{2} + 2 \times \tfrac{1}{6}\log\tfrac{1}{6} + \tfrac{1}{9}\log\tfrac{1}{9} + \tfrac{1}{18}\log\tfrac{1}{18}\right]$$

$$= 1.94553 \text{ bits/msg.}$$

Code Efficiency ; $\eta_c = \dfrac{H(S)}{L} = \dfrac{1.94553}{2} \times 100 = 97.28\%$

Code Redundancy, $R_c = (1 - 0.9728) \times 100 = 2.72\%$

② $l_1 = l_2 = l_3 = 2$; $l_4 = l_5 = 4$

$$L = \sum_{i=1}^{5} p_i l_i$$

$$= \tfrac{1}{2} \times 2 + \tfrac{1}{6} \times 2 + \tfrac{1}{6} \times 2 + \tfrac{1}{9} \times 4 + \tfrac{1}{18} \times 4 = 2.16667 \text{ binits/msg}$$