Relationship between Entropy & Mutual Information :

Average of mutual information ie entropy corresponding to mutual information.

$$I(XY) = I(x_j, y_k)$$

$$= \sum_{j=1}^{m} \sum_{k=1}^{n} p(x_j y_k)\, I(x_j y_k)$$

$$= \sum_{j=1}^{m} \sum_{k=1}^{n} p(x_j y_k) \log \frac{p(x_j y_k)}{p(x_j)}$$

$$= \sum_{j=1}^{m} \sum_{k=1}^{n} p(x_j y_k) \left[\log p(x_j/y_k) - \log p(x_j)\right]$$

$$= -\sum_{j=1}^{m} \sum_{k=1}^{n} p(x_j y_k) \log p(x_j) - \left[-\sum_{j=1}^{m} \sum_{k=1}^{n} p(x_j y_k) \log p(x_j/y_k)\right]$$

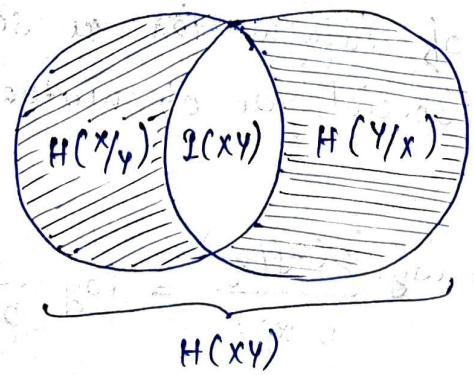
$$= -\sum_{j=1}^{m} \left[\sum_{k=1}^{n} p(x_j y_k)\right] \log p(x_j) - H(X/Y).$$

$$= -\sum_{j=1}^{m} p(x_j) \log p(x_j) - H(X/Y).$$

$$I(XY) = H(X) - H(X/Y)$$

$$= H(X) + H(Y) - H(XY)$$

$$= H(Y) - H(Y/X)$$



Illustrating entropy relations.

$I(XY)$ indicates the measure of information transferred through the channel, also known as transferred information or transinformation of the channel. $H(X/Y)$ gives the information lose in the channel. $H(Y/X)$ is the noise added in the channel ie measure of noise or error due to the channel.

Q7. find the given matrix is which type. ie $p(XY)$ or $p(X/Y)$ or $p(Y/X)$.

$$\begin{bmatrix} 0.1 & 0.3 & 0.2 & 0.4 \\ 0.3 & 0.1 & 0.1 & 0.5 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix}$$

(*) **Note** : $P(XY)$: Sum of all the entries is 1.

$P(X/Y)$: Sum of all the columns are 1.

$P(Y/X)$: Sum of all the rows are 1.

Case 1 : Sum of all the entries is $4 > 1$, So it cannot be $P(XY)$.

Case 2 : Check sum of all columns :

$$\left.\begin{aligned} C_1 &= 0.8 < 1 \\ C_2 &= 1 \\ C_3 &= 0.5 < 1 \\ C_4 &= 1.7 > 1 \end{aligned}\right\}$$

So it will not be $p(X/Y)$.

Case 3 : Check sum of all rows :

$$\left.\begin{aligned} R_1 &= 1 \\ R_2 &= 1 \\ R_3 &= 1 \\ R_4 &= 1 \end{aligned}\right\} = 1$$

; It will be $p(Y/X)$.

Q.8. A channel has the following channel matrix. Find p_1, p_2, p_3 and p_4 and complete $P(B/A)$.

$$P(B/A) = \begin{bmatrix} 0.2 & p_1 & 0.3 & p_2 \\ 2p_1 & 0.1 & p_2 & 0.3 \\ p_3 & 0.4 & 0.2 & p_4 \\ 0.3 & 3p_3 & 0.1 & p_4 \end{bmatrix}$$

Sol: * Sum of all the rows of $P(B/A)$ equals 1.

So. $0.2 + p_1 + 0.3 + p_2 = 1$ ——— (1)

$2p_1 + 0.1 + p_2 + 0.3 = 1$ ——— (2)

$p_3 + 0.4 + 0.2 + p_4 = 1$ ——— (3)

$0.3 + 3p_3 + 0.1 + p_4 = 1$ ——— (4)

from (1) $\longrightarrow$ $p_1 + p_2 = 0.5$ $\longrightarrow$ (5)

(2) $\longrightarrow$ $2p_1 + p_2 = 0.6$ $\longrightarrow$ (6)

(3) $\longrightarrow$ $p_3 + p_4 = 0.4$ $\longrightarrow$ (7)

(4) $\longrightarrow$ $3p_3 + p_4 = 0.6$ $\longrightarrow$ (8).

from (5) & (6) $\Longrightarrow$ $p_1 = 0.1$

$p_2 = 0.5 - 0.1 = 0.4$

from (7) & (8) $\Longrightarrow$ $2p_3 = 0.2$

$p_3 = 0.1$

$p_4 = 0.4 - 0.1 = 0.3.$

$$P(B/A) = \begin{bmatrix} 0.2 & 0.1 & 0.3 & 0.4 \\ 0.2 & 0.1 & 0.4 & 0.3 \\ 0.1 & 0.4 & 0.2 & 0.3 \\ 0.3 & 0.3 & 0.1 & 0.3 \end{bmatrix} //.$$