

Note : Unit of information : bits (Binary units).
Unit of wordlength : binits (Binary digits).

eg:

Source symbol	Code I	Code J	Code K	Code L	Code M
s_1	00	0	0	0	0
s_2	01	100	10	100	10
s_3	10	110	110	110	110
s_4	11	111	111	11	11

Use Kraft inequality :

For code I : $s_1 \rightarrow l_1 = 2 \text{ binits}$

$s_2 \rightarrow l_2 = 2 \text{ binits}$

$s_3 \rightarrow l_3 = 2 \text{ binits}$

$s_4 \rightarrow l_4 = 2 \text{ binits}$

$$\text{Then } \sum_{i=1}^4 2^{-l_i} = \sum_{i=1}^4 2^{-2} = 2^{-2} + 2^{-2} + 2^{-2} + 2^{-2} = 2^{-2} + 2^{-2} + 2^{-2} + 2^{-2} = 1$$

Prefix values for s_1 and s_2 are '0' and s_3 and s_4 are '1', which are not other code words. i.e. it satisfying prefix property. So it is possible to construct an instantaneous code.

For code J : $s_1 \rightarrow l_1 = 1 \text{ binits}$

$s_2, s_3, s_4 \rightarrow l_2 = l_3 = l_4 = 3 \text{ binits}$

$$\text{Then } \sum_{i=1}^4 2^{-l_i} = 2^{-1} + 2^{-3} + 2^{-3} + 2^{-3} = 7/8 < 1$$

Also satisfying prefix property ; So possible to construct an instantaneous code.

For code K :

$s_1 \rightarrow l_1 = 1 \text{ binits}$

$s_2 \rightarrow l_2 = 2 \text{ binits}$

$s_3, s_4 \rightarrow l_3 = l_4 = 3 \text{ binits}$

$$\text{Then } \sum_{i=1}^4 2^{-l_i} = 2^{-1} + 2^{-2} + 2^{-3} + 2^{-3} = 1 \quad : \text{Instantaneous.}$$

For code L: $s_1 \rightarrow l_1 = 1 \text{ bits}$

$s_2, s_3 \rightarrow l_2 = l_3 = 3 \text{ bits}$

$s_4 \rightarrow l_4 = 2 \text{ bits}$

$$\sum_{i=1}^4 r^{-l_i} = 2^{-1} + 2^{-3} + 2^{-3} + 2^{-2} = 1;$$

prefix property not satisfying; Not instantaneous.

For code M: $s_1 \rightarrow l_1 = 1 \text{ bits}$

$s_2, s_4 \rightarrow l_2 = l_4 = 2 \text{ bits}$

$s_3 \rightarrow l_3 = 3 \text{ bits}$

$$\sum_{i=1}^4 r^{-l_i} = 2^{-1} + 2^{-2} + 2^{-2} + 2^{-3} = 9/8 > 1.$$

Not satisfying Kraft inequality, so it is impossible to construct a code with prefix property.

Ques 1. Identify the instantaneous codes and construct their individual decision trees.

Symbol	Code N	Code O	Code P	Code Q
s_1	0	0	0	00
s_2	10	01	01	01
s_3	110	001	011	10
s_4	1110	0010	110	110
s_5	1111	0011	111	111

Ans: ↑ ↑ ↑ ↑
 instantaneous not not instantaneous

Decision Tree (code tree):

For code N: It has one initial state and five terminal states corresponding to s_1, s_2, s_3, s_4 and s_5 .

