Similarly

$$\boxed{H(Y/X) = -\sum_{j=1}^{m}\sum_{k=1}^{n} p(x_j, y_k) \log p(y_k/x_j)}$$

There are five entropies associated with two dimensional probability scheme. They are $H(X)$, $H(Y)$, $H(XY)$, $H(X/Y)$ and $H(Y/X)$.

Now for a two port communication system :

Let X represent the transmitter and Y is the receiver.

$H(X)$: Entropy of the transmitter.

$H(Y)$: Entropy of the receiver.

$H(XY)$: Entropy of the communication system as a whole.

$H(X/Y)$: A measure of information about transmitter knowing that Y is received.

$H(Y/X)$: A measure of information about receiver, knowing that X is transmitted.

The Relationship between different Entropies :

$$H(XY) = -\sum_{j=1}^{m}\sum_{k=1}^{n} p(x_j, y_k) \log p(x_j, y_k)$$

$$= -\sum_{j=1}^{m}\sum_{k=1}^{n} p(x_j, y_k) \log \left[p(x_j/y_k) \cdot p(y_k) \right]$$

$$= -\sum_{j=1}^{m}\sum_{k=1}^{n} p(x_j, y_k) \left[\log p(x_j/y_k) + \log p(y_k) \right]$$

$$= -\sum_{j=1}^{m}\sum_{k=1}^{n} p(x_j, y_k) \log p(x_j/y_k) + $$

$$- \sum_{j=1}^{m}\sum_{k=1}^{n} p(x_j, y_k) \log p(y_k).$$

$$= H(X/Y) - \sum_{k=1}^{n} \left[\sum_{j=1}^{m} p(x_j, y_k) \right] \log p(y_k).$$

$$= H(X/Y) - \sum_{k=1}^{n} p(y_k) \log p(y_k)$$ since $\sum_{j=1}^{m} p(x_j, y_k) = p(y_k)$

$$\boxed{H(XY) = H(X/Y) + H(Y)}$$

Similarly

$$\boxed{H(XY) = H(Y/X) + H(X)}$$

MUTUAL INFORMATION

Before the reception of a message, the state of knowledge at the receiver about the transmitted signal x_j, is known as a-priori probability, $p(x_j)$.

After the reception of the symbol y_k, knowing the transmitted signal x_j is the conditional probability $p(x_j/y_k)$, is known as a-posteriori probability

Then the amount of information, before the reception of y_k, ie the uncertainty is $-\log p(x_j)$.

After the reception of y_k, the uncertainty becomes $-\log p(x_j/y_k)$.

The information gained about x_j by the reception of y_k is the net reduction in its uncertainty known as Mutual Information.

$$I(x_j, y_k) = \text{initial uncertainty} - \text{final uncertainty}$$

$$= -\log p(x_j) - [-\log p(x_j/y_k)]$$

$$\boxed{I(x_j, y_k) = \log \frac{p(x_j/y_k)}{p(x_j)}}$$

<u>Mutual information</u> can also be interpreted as follows:

When an information is transmitted over the channel, an average amount of information is lost in the channel due to noise. The balance information received at the receiver with respect to an observed output symbol is the <u>mutual information</u>.

Properties:

① Mutual information of a channel is symmetric.

$$I(x_j, y_k) = \log \frac{p(x_j/y_k)}{p(x_j)}$$

$$P(A/B) = \frac{P(AB)}{P(B)}$$

$$= \log \frac{p(x_j, y_k)}{p(x_j)\cdot p(y_k)}$$

$$P(AB) = P(B/A)\cdot P(A)$$

$$= \log \frac{p(y_k/x_j)}{p(y_k)}$$

$$= I(y_k, x_j).$$

ie $$I(x_j, y_k) = I(y_k, x_j).$$

② The mutual information is always non-negative.

③ The mutual information of a channel may be expressed in terms of the entropy of the channel output.

④ The mutual information is also related to the joint entropy of the channel.

Amount of information or self information is treated as a special case of mutual information, ie when $x_j = y_k$.

$$I(x_j, x_j) = \log \frac{p(x_j/x_j)}{p(x_j)} = \log \frac{1}{p(x_j)} = I(x_j).$$