Machine learning Homework- Constrained Optimisation & SVM

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December 2, 2018

Problem 1:

We construct the Lagrangian function as $\mathcal{L}(\theta, \alpha) = f_0(\theta) + \alpha f_1(\theta)$
forcing $\nabla_\theta \mathcal{L} = 0$. We get $\theta_1 = -\frac{1}{2\alpha}$ and $\theta_2 = \frac{\sqrt{3}}{2\alpha}$
substituting back the θ in the Lagrangian function and minimising with respect to α, we get $\alpha = \pm\frac{1}{2}$ we take only the positive values of $\alpha = 1/2$.
Therefore $\theta_1 = -1$
$\theta_2 = \sqrt{3}$

Problem 2:

Solved in a Jupyter Notebook attached at the end of problem of Problem 5.

Problem 3:

- Both use hyperplane for classification
- SVM have margins and perceptrons do not have them

Problem 4:

As we use KKT conditions for finding the minima of the contraint optimisation problem. Therefore, We staisfy the Slater theorem, which states if the constraint functions are affine, the duality gap is zero.

Problem 5:

1. from $g(\alpha)$ we know that $\alpha Q \alpha^T$ is equivalent to $-\sum_{i=1}^{N}\sum_{j=1}^{N} y_i y_j \alpha_i \alpha_j \mathbf{x}_i \mathbf{x}_j$. By rearranging the scalars we get,$-\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i y_i y_j \mathbf{x}_i \mathbf{x}_j \alpha_j$.
   Therefore $Q = (-yy^T(hadamard)XX^T)$
2. We know that $Q = -p^T p$ and also we know that $p^T p$ is positive semi definite due to its symmetric nature. So $a^t(p^T p)a \geq 0$ but we negative sign also. Therefore, Q is negative semi definite.
3. The negative semi definiteness allows the concave optimisation to be a maximisation problem.