Machine learning Homework- Linear Classification

Abinav Ravi Venkatakrishnan - 03694216 and Abhijeet Parida - 03679676

November 18, 2018

Problem 1:

1. **Given:** Prior $p(y=1) = \frac{1}{2}$
   Likelihood function $= p(x|y=1) = Expo(x|\lambda_1)$
   By Bayes theorem we know that

   $$p(y=1|x) = \frac{p(x|y=1)p(y=1)}{p(x)} \tag{1}$$

   this corresponds to

   $$posterior = \frac{likelihood * prior}{evidence} \tag{2}$$

   Since the Prior is a constant function so the Posterior is similar to the likelihood.
   So the Posterior is also of the exponential distribution.

2. The first derivative of the exponential distribution is at $x=1$; which gives the maxima value as $\frac{\lambda_1}{2} exp(-\lambda_1)$

Problem 2:

We would like to map the given domain to [-1,1] for logistic regression. Now once the mapping is done the Negative log likelihood will look like the heaviside function.

The problem of heaviside function is that at $\mathbf{x} = 0$ The derivative is not defined. Therefore a smoothening of this boundary is required and can be achieved via regularization.

Problem 3:

The softmax loss for two class is given by Eqn.3.

$$E_w = -\sum_{i=1}^{N} \sum_{c=1}^{2} y_{ic} ln \frac{exp(w_c^T x)}{\sum_c w_c^T x} \tag{3}$$

The sigmoid for a class is

$$p(y=1|x) = \sigma(w^T x) \tag{4}$$

Since it is given that the problem is of binary classification. Softmax loss function for the same can be written as

$$p(y=1|x) = \frac{exp(w_1^T x)}{exp(w_1^T x + w_2^T x)} \tag{5}$$

$$= \frac{1}{1 + exp(w_2 - w_1)^T x} \tag{6}$$

$$= \sigma((w_2 - w_1)^T x) \tag{7}$$

Problem 4:

We can take the $\phi(x_1, x_2) = tan(angle(x_1, x_2))$. This is because Tan function is positive valued in the First and Third quadrant and negative values in the Second and Fourth quadrant. therefore the separating line can be $\phi(x_1, x_2) = 0$