Machine learning Homework- Dimensionality Reduction

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1 PCA and SVD

Problem 1:

The model $y = Ax$ is a noiseless transformation since we haven't included the bias.Given that
$p(z) = \mathcal{N}(z|0, I)$
$p(x|z) = \mathcal{N}(x|Wz + \mu, \phi)$
we come to a conclusion that x is a gaussian distribution which gives us results that mean of the distribution is μ and the variance is $WW^T\phi$.
from this we can get the distribution of y with mean and variances $A\mu$ and $AWW^TA^T + A\phi A^T$. From pattern matching we can say that the maximum likelihood can be written as $A\mu_x$ and AW_x and $A\phi_x A^T$.

Problem 2:

The representation of the new vector in the space is given by [0,3,0,0,4].So the new Matrix M looks like $\begin{Bmatrix} 1 & 1 & 1 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 \\ 4 & 4 & 4 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 3 & 0 & 0 & 4 \end{Bmatrix}$ The SVD of the matrix gives U,V,and S which gives the singular values and vector. Now to do dimensionality reduction we take only top K singular values of the output and we get a P = M*V. as P = [0,3,0,0,4]*V = [1.74,2.84] which mostly corresponds to the classic movie genre. A good recommendation of the genre would consists of Titanic and Casablanca. Since the previous data consists of Titanic we can recommend Casablanca.

Problem 5:

(a)The Autoencoder cannot have reconstruction error to be zero since it would correspond to PCA and hence has a linear mapping between latent space and the decoder output. This is not a desirable property.
(b) This is possible only in the case of a linear mapping conditions where the entire input is learnt as such.

Programming assignment 10: Dimensionality Reduction

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

%matplotlib inline
```

Exporting the results to PDF

Once you complete the assignments, export the entire notebook as PDF and attach it to your homework solutions. The best way of doing that is

1. Run all the cells of the notebook.
2. Download the notebook in HTML (click File > Download as > .html)
3. Convert the HTML to PDF using e.g. https://www.sejda.com/html-to-pdf or `wkhtmltopdf` for Linux (tutorial)
4. Concatenate your solutions for other tasks with the output of Step 3. On a Linux machine you can simply use `pdfunite`, there are similar tools for other platforms too. You can only upload a single PDF file to Moodle.

This way is preferred to using `nbconvert`, since `nbconvert` clips lines that exceed page width and makes your code harder to grade.

PCA Task

Given the data in the matrix X your tasks is to:

- Calculate the covariance matrix Σ.
- Calculate eigenvalues and eigenvectors of Σ.
- Plot the original data X and the eigenvectors to a single diagram. What do you observe? Which eigenvector corresponds to the smallest eigenvalue?
- Determine the smallest eigenvalue and remove its corresponding eigenvector. The remaining eigenvector is the basis of a new subspace.
- Transform all vectors in X in this new subspace by expressing all vectors in X in this new basis.

The given data X

In [2]:
```
X = np.array([(-3,-2),(-2,-1),(-1,0),(0,1),
              (1,2),(2,3),(-2,-2),(-1,-1),
              (0,0),(1,1),(2,2),  (-2,-3),
              (-1,-2),(0,-1),(1,0),  (2,1),(3,2)])
```

Task 1: Calculate the covariance matrix Σ

In [3]:
```
def get_covariance(X):
    """Calculates the covariance matrix of the input data.
```

```
    Parameters
    ----------
    X : array, shape [N, D]
        Data matrix.

    Returns
    -------
    Sigma : array, shape [D, D]
        Covariance matrix

    """
    # TODO
    Sigma = X.T.dot(X)/(X.shape[0]-1) #np.cov(X)

    return Sigma
```

Task 2: Calculate eigenvalues and eigenvectors of Σ.

In [4]:
```
def get_eigen(S):
    """Calculates the eigenvalues and eigenvectors of the input matrix.

    Parameters
    ----------
    S : array, shape [D, D]
        Square symmetric positive definite matrix.

    Returns
    -------
    L : array, shape [D]
        Eigenvalues of S
    U : array, shape [D, D]
        Eigenvectors of S

    """
    # TODO
    L,U = np.linalg.eigh(S)
    return L,U
```

Task 3: Plot the original data X and the eigenvectors to a single diagram.

Note that, in general if u_i is an eigenvector of the matrix M with eigenvalue λ_i then $\alpha \cdot u_i$ is also an eigenvector of M with the same eigenvalue λ_i, where α is an arbitrary scalar (including $\alpha = -1$).

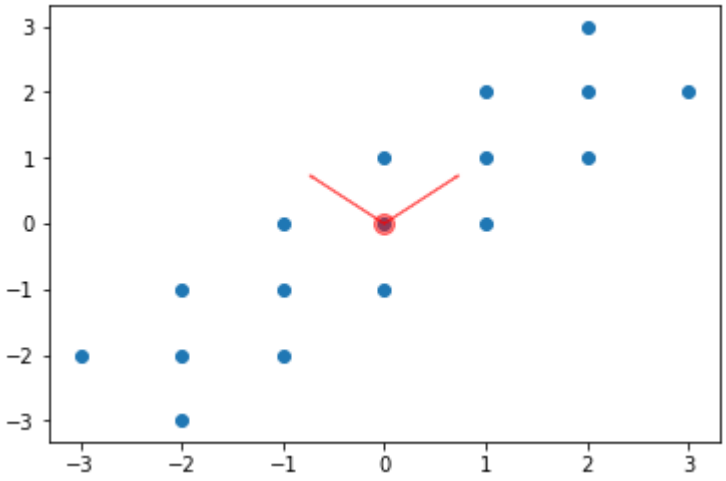
Thus, the signs of the eigenvectors are arbitrary, and you can flip them without changing the meaning of the result. Only their direction matters. The particular result depends on the algorithm used to find them.

In [5]:
```
# plot the original data
plt.scatter(X[:, 0], X[:, 1])

# plot the mean of the data
mean_d1, mean_d2 = X.mean(0)
plt.plot(mean_d1, mean_d2, 'o', markersize=10, color='red', alpha=0.5)

# calculate the covariance matrix
Sigma = get_covariance(X)
# calculate the eigenvector and eigenvalues of Sigma
L, U = get_eigen(Sigma)
```

```
plt.arrow(mean_d1, mean_d2, U[0, 0], U[1, 0], width=0.01, color='red', alpha=0.
5)
plt.arrow(mean_d1, mean_d2, U[0, 1], U[1, 1], width=0.01, color='red', alpha=0.
5);
```



What do you observe in the above plot? Which eigenvector corresponds to the smallest eigenvalue?

Write your answer here:

[YOUR ANSWER] The eigenvector corresponding to smallest eigenvalue is aligned whereas the eigenvector with the largest eigenvalue captures direction of high variance hence transformation can be done on it without loss of information.

Task 4: Transform the data

Determine the smallest eigenvalue and remove its corresponding eigenvector. The remaining eigenvector is the basis of a new subspace. Transform all vectors in X in this new subspace by expressing all vectors in X in this new basis.

In [6]:
```
def transform(X, U, L):
    """Transforms the data in the new subspace spanned by the eigenvector corre
sponding to the largest eigenvalue.

    Parameters
    ----------
    X : array, shape [N, D]
        Data matrix.
    L : array, shape [D]
        Eigenvalues of Sigma_X
    U : array, shape [D, D]
        Eigenvectors of Sigma_X

    Returns
    -------
    X_t : array, shape [N, 1]
        Transformed data

    """
    # TODO
    X_t = X.dot(U[:,0])
    return X_t
```

In [7]:
```
X_t = transform(X, U, L)
```

Task SVD

Task 5: Given the matrix M find its SVD decomposition $M = U \cdot \Sigma \cdot V$ and reduce it to one dimension using the approach described in the lecture.

In [8]:
```
M = np.array([[1, 2], [6, 3],[0, 2]])
```

In [9]:
```
def reduce_to_one_dimension(M):
    """Reduces the input matrix to one dimension using its SVD decomposition.

    Parameters
    ----------
    M : array, shape [N, D]
        Input matrix.

    Returns
    -------
    M_t: array, shape [N, 1]
        Reduce matrix.

    """
    # TODO
    U, sigma, V = np.linalg.svd(M, 0)
    Sigma = np.diag(sigma)

    # construct the truncated Sigma, here called Sigma_c
    Sigma_c = Sigma.copy()
    Sigma_c[1, 1] = 0

    M_t = U.dot(Sigma_c)

    # or alternatively and usually prefered
    M_t = M.dot(V[:, 0])
    return None
```

In [10]:
```
M_t = reduce_to_one_dimension(M)
```