Year / Sem : **III / V**

Sub. Code & Subject : MA2265- **Discrete Mathematics** (DM)

TWO MARK QUESTION & ANSWERS

Unit-I&II

1.Define negation?

If p is a statement, then negation of p written as ~p (or ~p } or 7p and read as "not p". the truth table is as follows

p	~p
T F	F T

Example: P: madras is a city

~p:madras is not a city or it is not the case that madras is a city.

2.define conjunction?

The conjunction if 2 statements P and Q is the statement P^Q which is read as "P and Q". the statement P^Q has a truth value T whenever both P and Q have the truth table T; otherwise it has truth value F. the conjunction is defined by the truth table as follows.

P	Q	P^Q
T T F F	T F T F	T F F F

Example : P : jack went up the hill

Q: jill went up the hill

P^Q: jack and jill went up the hill

3.define disjunction?

The disjunction of 2 statements P and Q is the statement P v Q which is read as "P or Q". the statement P v Q has a truth value F only when both P and Q have truth value F otherwise it is true. The disjunction is defined by following table.

P	Q	P v Q

T	T	T
T	F	T
F	T	T
F	F	F

Example :

1.I shall go to market or a cinema.

2. there is something wrong with the bulb or wiring.

4.state molecular statements?

Those statements which contain one or more atomic statements and some connectives are called molecular statements.

Examples;

~P,P^~Q,P v Q.

5. Define conditional and biconditional?

If P and Q are any two statements then the statement P-> Q which is read as "if P then Q " is called a conditional statement. Here P is called antecedent and Q is called consequent.

Truth table:

P	Q	P-.>
T	T	T
T	F	F
F	T	T
F	F	T

Example:

P: it is hot.

Q: 5+3=8.

p-> is false only when P is true and Q is false. Otherwise p->Q is true.

Biconditional

If P and Q are any two statements , then the statement P<-> which is read as " P if only if Q" is called Biconditional statement , the statement p<->Q has the truth value T whenever both P and Q have identical truth values . the truth table for biconditional is as follows;

P	Q	P
T	T	T
T	F	F
F	T	F
f	F	T

6.define Tautology and Contradiction?

A statement formula which is true regardless of the truth values of the statements which replace the variables in it is called a universally valid formula or a tautology or a logical truth.

Example:
P v~P

P	~P	PV~P
T F	F T	T T

Contradiction:

A statement formula which is false regardless of the truth table values of the statements which replace variables in it is called a contradiction.

Example:

P^~P

Truth table for PV~P

P	~P	P^~P
T F	F T	F F

7.define Duality law?

Two formula A and A* are duals of each other if other if either one can be obtained from the other by replacing^ and v are also called duals of each other.

8.prove the following implications :

(i) (P^Q)=>(P->Q); (ii)P=>(Q->p)

Assume the consequent to be false (i.e)P->Q is false . by definition of conditional P is True and Q is false

(ii) Assume Q->P is false Q is True but P is false. Therefore P is false.

9.define DNF and CNF

DNF:

A formula which is equivalent to a given formula and which consists of sum of elementary products is called a disjunctive normal form of the given formula.

CNF:

A formula which is equivalent to a given formula and which consists of product of elementary sums is called conjuctive normal form of the given formula.

10.state inference theory?

Rule P: a given premises may be introduced at any stage in the derivative.

Rule T: a formula S may be introduced in a derivation if S is tautologically implied by one or more of the preceding formula in to derivation.

Rule CP: if the conclusion is the form R->S then we include R is an additional premises and derive S from the given act of premises and R.this rule is called rule CP.

11.define PDNF and PCNF

PDNF: a formula which is equivalent to a given formula which is consists of sum its minterms is called PDNF.

PCNF: a formula which is equivalent to a given formula which consists of product of maxterms is called PCNF.

12.construct the truth table for (q ^ (P->Q))->P)

Solution;

P	Q	P->Q	Q^(P->Q)	(Q^(P->Q))->P
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

13 .construct the truth table for (P^Q)v()7p^Qv(P^7Q)v(7p^7Q)

Solution;

p	Q	7P	7Q	P^Q	7P^Q	p^7Q	7P^7Q	T	R	S
T	T	F	F	T	F	F	F	T	F	T
T	F	F	T	F	F	T	F	F	T	T
F	T	T	F	F	T	F	F	T	F	T
T	F	T	T	F	F	F	T	F	T	T

14.construct the truth table for (PvQ)v7P

Solution:

P	Q	PVQ	7P	(PVQ)V7P
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

15.construct the truth table for (P->Q)^(q->P)

Solution:

P	Q	P->Q	Q->P	(P->Q)^(Q->P)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	F	T

16.construct the truth table for(P->Q)<->(7PVQ)

Solution:

P	Q	7P	P->Q	7PVQ	(P->Q)<->(7PVQ)
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

17.find the PDNF for 7PvQ

Sol:
7pvQ⇔(7P^T)v(T^Q)
⇔(7P^(Qv7Q))v((Pv7P)^Q)
⇔(7P^Q)v(7P^7Q)v(P^Q)v(7P^Q)
⇔(P^Q)v(7P^Q)v(7p^7Q)

18.obtain the PDNF for P->((P->Q)^7(7Qv7P))

Solution;
P->((P->Q^7(QV7P))
⇔P->((7PVQ)^(Q^P))
⇔7PV(7PVQ)^(Q^P))
⇔7P^(QV7Q)V(F^Q)V(P^Q)
⇔(7P^Q)V(7P^7Q)V(P^Q)

UNIT-III&IV

1.Define a simple statement function.

A simple statement function of one variable is defined to be anexpression consisting of a predicate symbol and an individual variable. Such a statement function becomes a statement when the variable is replased by the name of any object.

Ex:

If “X is a teacher” is denoted by T(x),it is a statement function.if X is replased by John,then “Johan is a teacher” is statement .

2.Define a compound statement function.

A compound stament function is obtained by combining one or more simple statement functions by logical connectives.

Ex:

M(x)^H(x)
M(x)->H(x)
M(x)v 7H(x)

An extension of this idea to the statement functions of two or more variables is straight forward.

3.Define universal Quantifiers and existential Quantifiers.

Universal Quantifiers:

The universal Quantification of P(x) is the proposition.”P(x) is true for all values of x in the universe of discourse”.

The notation ¥x P(x) denotes the universal quantification of P(x).here ¥ is called the universal quantifier.

Existential Quantifier:

The existential Quantification of P(x) is the proposition.” There exists an element x in the universe of discourse such that P(x) is true”.

We use the notation ∃x P(x) for the existential quantification of p(x).here ∃ is called the existential quantifier.

4. what are the rules of Quantifier?

1.Rule US (Universal specification)
2.Rule UG (Universal generalization)
3.Rule ES (Existential Specification)
4.Rule EG (Existential Specification)

5.what are the rules of inference?

1. Rule P

A given premises may be introduced at any stage in the derivation.

2. Rule T

A formula S may be introduced in a derivation if S is tautologically implied by one or more of the proceeding formulae in the derivation .

3. Rule CP

If we can derive S from R and a set of given premises,then we can derive R->S from the set of premises alone.

6. symbolize the expression "x is the father of the mother of y"

P(x) : x is a person
F(x,y) : x is a father of y
M(x,y) : x is a mother of y

We symbolize this as$(\exists z)(p(z)\wedge F(x,z)\wedge M(z,y))$

7. Express the statement, "Some people who trust others are rewarded" in symbolic form.

$(\exists x)[P(x)\wedge T(x)\wedge R(x)]$
P(x) : x is aperson
T(x) : x trusts others
R(x) : x is rewarded

8. Give an example of free and bound variable in predicate logic.

(¥x) P(x,y) : x is a bound variable
Y is a free variable

9. Define statement function of one variable. When it will become a statement?

Statement function of one variable is defined to be an expression consisting of a predicate symbol and an individual variable. The statement function becomes a statement, when the variable is replaced by the name of an object.

10. Use quantifiers to express the associate law for multiplication of real numbers.

Universe of discourse: Set of real numbers.
P(x,y,z): (x*y)*z
Q(x,y,z): x*(y*z)
$(x)(y)(z)(P(x,y,z)\Leftrightarrow Q(x,y,z))$

11. Define simple statement function.

A simple statement function contains a predicate symbol followed by one (or) more variables. It gives the statement when the variables are replaced by objects from a designated set.

EX: R(x) : x is Red
Q(x,y) : x+y=10

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12. Express the statement "For every 'x' there exist a 'y' such that $x^2 + y^2 \geq 100$" in symbolic form.

Universe of discourse = Set of all integers.

(x)(∃y)($x^2 + y^2 \geq 100$)

13.Give an example to show that (∃x) (A(x)∧ B(x)) need not be a conclusion from(∃x) a(x) B(x).

Let the universe of discourse be the set of all integers.

Let A(x) : 2x + 1 = 5 and B(x) :$x^2 = 9$.

The statements (∃x) A(x) and(∃x)B(x) are true. The statement (∃x)(A(x)∧B(x)) is false, because there is no integer 'a' such that 2a+1=5and $a^2 = 9$. But the state

14.show that ~P(a,b)follows logically from (x),(y)(P(x.y)->W(x,y)and ~W(a,b)

(i) (x)(y)(P(x,y)->W(x,y) rule P
(ii) (y)(P(a,y)->W(a,y)) US,(i)
(iii) P(a,b)->W(a,b) US ,(ii)
(iv) ~W(a,b) rule P(v) ~P(a,b) rule T(iii,iv)

15. show that (x) (p(x))->Q(x))^(x)(Q(x))->R(x))=>(x)(p(x))->R(x))

Solution;

Sno	**Premises**	**Rule**	**reason**
1	(x)(p(x))->Q(x))	P	Given premises
2	P(a)->Q(a)	T	From(1),US rule
3	(x)(Q(x)->R(x))	P	GP
4	Q(a)->R(a)	T	From(3),US rule
5	P(a)->R(a)	T	From(2),(4)(p->Q),(Q->R)=>P->R
6	(x)(p(x)->R(x))	T	From(5),UG rule

16.using CP or otherwise obtain the following implication

(¥x)(P(x))->Q(x)),(¥x)(R(x)-> ~Q(x))=>(¥x) r(x)->~P(x)

Solution:

Sno	Premises	rules	Reason
1	(¥x)(p(x)->Q(x)	P	Given premises
2	P(a)->Q(a)	T	US rule
3	R(x)	P	Additional premises
4	(¥x)(R(x)->~Q(x))	P	GP
5	R(a)->~Q(a)	T	US rule
6	~(~Q(a)->~R(a)	P	From (5) P->Q⇔~Q->~P
7	Q(a)->~R(a)	T	From(6)
8	P(a)->~R(a)	T	From(2)&(7)P->Q
9	~(~R(a))->~P(a)	T	From(8)
10	R(a)->~P(a)	T	From(9)
11	(¥x)(R(x)->~P(x)	T	UG rule

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17.which of the following are statements?
i.(x)(P(x)vQ(x))^R.
ii.(x)(P(x)^Q(x))^S(x)
solution:
i.is not a statement
ii.is a statement

18.using CP ot otherwise obtain the following implication:
(x)(P(x)->Q(x)),
(x)(R(x)->7Q(x))=>(x)(R(x)->7P(x))
Sol:
(x)(P(x)->Q(x)) rule P
2.(x) R(x)->7Q(x) rule T
3.Q(x)->7R(x) rule T,,2,3
4.R(x)->7P(x) rule T,4

Unit-V

1.define function.
Let X and Y be any two sets A relation f from X to Y is called a function if for every x €X there is a unique y€ Y such that (x, y)€f.

2. define graph of a function.
With each function we can associate a graph ,which is a diagrammatic representation of a function.if the domain x and codomain Y of a function f are finite,we can represent such a function as follows
We draw a circle for each element x of X and each element y of Y and join x with y by a directed line, directed from x to y ,if (x ,y)€f
.
3. define identity map
A mapping I_x:X->X is called an identify map if
I_x={(x,x)|{x€X}

4. define commutative property
A binary operation f:X x X ->X is said to be commutative if for every x, y €X ,f(x, y)=f(y, x).

5 define distributive
A binary operation f: X*X->X denote by * is said to be , distributive over the operation g:X*X->X, denoted by ^{o} if for every x, y, z €X.
X*(y^{o}z))=(x*Y)^o(x*z)

6. define idempotent
Let * be a binary operation on X an element a €X is called idempotent with respect to * if a*a=a.

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7. define primitive recursion

A function is called primitive recursive if and if it can be obtained from the initial functions by a finite number of operations of composition and recursion.

8. define onto (or) surjective (or)surjection

A mapping of f:X->Y is called onto if the range r_f =Y; otherwise it is called into.

9.Define injective

A mapping f:X->Y is called one-to -one if it is both one-to-one .such a mapping is also called a one- to- one correspondence between X and Y.

10. define graph of functions

With each function ,we can associate a graph , which is a diagrammatic representation of a function if the domain X and codomain Y of a function f are finite , we can represent such a function as follows.

We draw a circle for each element x of X and for each element of Y and join x with y by a directed line directed from x to y, if(x, y)€f.

11. define identity map

A mapping I_x=X->X is called an identity map if

I_x={(x, x)|{x€ X}

12. define inverse function

If it is a function from X to Y if f , the converse of f , given by (y, x)€f whenever (x, y) €f, need not be a function from Y to X.

13.define binary and n- ary operation

Let X be a set and f be a mapping f:X x X->X then f is called a binary operation on X. in general, a mapping f: X^n ->Xis called an n-ary operation and n is called the order of the operation . for n=1,f:X->X is called a unary operation.

14 define primitive recursion

A function is called primitive recursive if and only if it can be obtained from the initial functions by a finite number of operations of composition and recursion.

15.Define permutation

A bijection from a set A to itself is called permutation of A.

16.define function.

Let x and Y be any two set. A relation F From X to Y is called a function if every X € there is a unique Y such that (X,Y) € F.

For a from F : X→ Y , if (X, Y)€ F, then Xs c argument and corresponding Y a X . The pair (X, Y) € F, is also written as Y=F (X), X is then said to be mapped into XY.

17.define into

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Led F: X → Y such that there is at least one element b € Y which has no pre image under F, then F is said to be a pinto be a into function from X into Y .
Clear F : X → Y is an into function if (X) = Y.

18.definr one to one
A mapping F : X → Y is called one (injection, or 1-1) if distinct element of X are mapped into element of Y .
In other ,F is one to one if
X1# F(X2) X1 = X2 .

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