Complete HMM Forward/Backtracking Table

Given:

- **States**: Sunny, Cloudy, Rainy
- **Observations** (3 days):
  1. Day 1: Walk
  2. Day 2: Umbrella
  3. Day 3: Walk
- **Initial Distribution**: π(Sunny)=π(Cloudy)=π(Rainy)=1/3 ≈ 0.333
- **Transition Probabilities** P(To|From):

From\To	Sunny	Cloudy	Rainy
Sunny	0.25	0.50	0.25
Cloudy	0.33	0.33	0.33
Rainy	0.33	0.33	0.33

- **Emission Probabilities** P(Behavior|Weather):

Weather	Walk	Umbrella
Sunny	1.00	0.00
Cloudy	0.67	0.33
Rainy	0.33	0.67

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1. Forward Algorithm (α)

We compute $\alpha_t(s) = P(O_1,O_2,\ldots,O_t,\ q_t=s)$. Below is a table of **partial sums** and **final α values** for each day.

Day	Observation	Computation	α(Sunny)	α(Cloudy)	α(Rainy)
V0(?)	*Initial*	*π(S)=π(C)=π(R)=0.333*	0.333	0.333	0.333
1	Walk	$\alpha_1(S)=0.333\times1.00=0.333$ $\alpha_1(C)=0.333\times0.67=0.223$ $\alpha_1(R)=0.333\times0.33=0.110$	0.333	0.223	0.110
2	Umbrella	For **Sunny**: Sum = (0.333×0.25 + 0.223×0.33 + 0.110×0.33)=0.19314 → ×P(U	S)=0 → $\alpha_2(S)=0.000$ For **Cloudy**: Sum=…=0.27639 → ×0.33=0.09121 For **Rainy**: Sum=…=0.19314 → ×0.67=0.12940	0.000	0.0912
3	Walk	For **Sunny**: Sum=(0.000×0.25 + 0.0912×0.33 + 0.1294×0.33)=0.0728 → ×P(W	S)=1.00=0.0728 For **Cloudy**: Sum=…=0.0728 → ×0.67=0.0488 For **Rainy**: Sum=…=0.0728 → ×0.33=0.0240	0.0728	0.0488

Total Probability of observations (Walk, Umbrella, Walk):

[P(O) ;=; α_3(\text{Sunny}) + α_3(\text{Cloudy}) + α_3(\text{Rainy}) ;=; 0.0728 + 0.0488 + 0.0240 ;=; 0.1456.]

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2. Backward Algorithm (β)

We compute $\beta_t(s) = P(O_{t+1},\ldots,O_t \mid q_t=s)$. Initialize $\beta_t=1$ at the last time step, then move backward.

Day	Observation	Computation	β(Sunny)	β(Cloudy)	β(Rainy)

Day	Observation	Computation	β(Sunny)	β(Cloudy)	β(Rainy)
3	Walk	*No future obs, so $\beta_3(S)=\beta_3(C)=\beta_3(R)=1$*	1.0000	1.0000	1.0000
2	Umbrella	$\beta_2(S)=\sum[a(S\to j)\times P(Walk$	$j)\times\beta_3(j)]$ $=0.25\times1.0\times1 + 0.50\times0.67\times1 + 0.25\times0.33\times1=0.6675$ $\beta_2(C)=\ldots=0.66$ $\beta_2(R)=\ldots=0.66$	0.6675	0.6600
1	Walk	$\beta_1(S)=\sum[a(S\to j)\times P(Umbrella$	$j)\times\beta_2(j)]$ $=0.25\times0.00\times0.6675 + 0.50\times0.33\times0.66 +$ $0.25\times0.67\times0.66=0.21945$ $\beta_1(C)=\ldots\approx0.2178$ $\beta_1(R)=\ldots\approx0.2178$	0.2195	0.2178

$$ P(O) = \sum_{s \in \{\text{S,C,R}\}} \alpha_1(s)\,\beta_1(s) = 0.333\times0.2195 + 0.223\times0.2178 + 0.110\times0.2178 \approx 0.1456, $$

MDP Process Table

Iteration	State	V(s)	Q(State,C)	Q(State,A)	Policy(s)
0	Low	0.00	-	-	*N/A*
	Medium	0.00	-	-	*N/A*
	High	0.00	-	-	*N/A*
1	Low	-1.00	-1.18	-0.46	Aggressive (A)
	Medium	3.00	6.24	6.60	Aggressive (A)
	High	5.00	9.32	8.96	Conservative(C)
2	Low	-0.46	-0.1432	1.1276	Aggressive (A)
	Medium	6.60	9.6744	10.164	Aggressive (A)
	High	9.32	13.1432	12.6536	Conservative(C)
3	Low	1.12	1.62	3.26	Aggressive (A)
	Medium	10.16	12.94	13.48	Aggressive (A)
	High	13.14	16.54	16.007	Conservative(C)