Bauhaus-University Weimar

Solving Thermoelasticity Problem For Rectangular Domain Using Finite Difference Method

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Project Task

- To solve Thermoelasticity problem for a rectangular domain using Matlab.

$$\begin{aligned} \mu\Delta\mathbf{u} + (\lambda+\mu)\operatorname{grad}\operatorname{div}\mathbf{u} &= (3\lambda+2\mu)\alpha_T \operatorname{grad}(T-T_0), \\ a^2\Delta T - \dot{T} &= 0. \end{aligned}$$

The code should allow:

- ✓ to choose arbitrary material parameters
- ✓ to choose arbitrary geometric parameters
- ✓ to choose arbitrary number of nodes in each directions
- ✓ to choose different types of boundary conditions

Procedure

- Solving the heat equation for the given rectangular domain .
- Determining the deformation of rectangular domain using Linear Elasticity Equation (Navier-Lamé equation) for the heat distribution calculated on the first procedure.

Solving The Heat Equation

For calculation of heat flow in the domain an implicit Finite Difference Method is chosen in this project.

Advantage :

- ✓ unconditionally stable
- ✓ accurate results for fewer time steps (large Δt)

Disadvantage:

- ✓ Implementation is more difficult
- ✓ More time consuming due to matrix manipulations
- ✓ Due to larger Δt, truncation error is also larger

Heat Equation

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

where: α – coefficient of thermal diffusivity

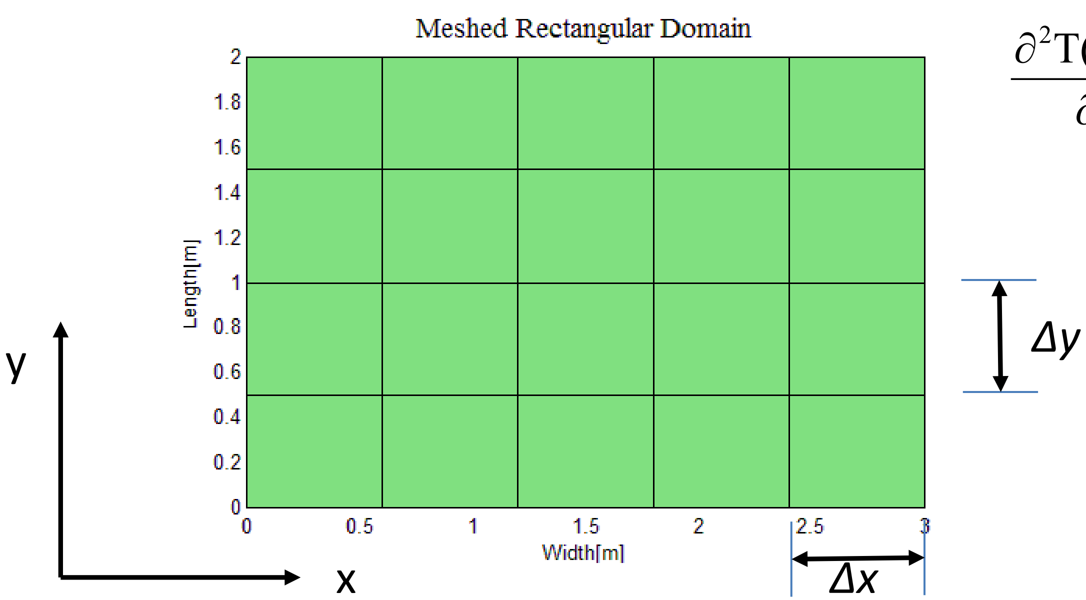
- ✓ First Order Approximation (for time)

$$\frac{\partial \mathrm{T}(x,y,t)}{\partial t} \approx \frac{\mathrm{T}_{i,j}^{k+1} - \mathrm{T}_{i,j}^{k}}{\Delta t} + O(\Delta t)$$

- ✓ Second Order Approximation (for x & y)

$$\frac{\partial^2 \mathrm{T}(x,y,t)}{\partial x^2} \approx \frac{\mathrm{T}_{i,j\text{-}1}^{k+1} - 2\mathrm{T}_{i,j}^{k+1} + \mathrm{T}_{i,j+1}^{k+1}}{(\Delta x)^2} + O\left(\Delta x^2\right)$$

$$\frac{\partial^2 \mathrm{T}(x,y,t)}{\partial y^2} \approx \frac{\mathrm{T}_{i\text{-}1,j}^{k+1} - 2\mathrm{T}_{i,j}^{k+1} + \mathrm{T}_{i+1,j}^{k+1}}{(\Delta y)^2} + O\left(\Delta y^2\right)$$



Boundary conditions

Boundary conditions:

Neumann boundary condition

$$\frac{\partial T}{\partial n} = f(t)$$

$$\mathrm{f(t) = c\_0 + c\_1 * t + c\_2 * t^2 + c\_3 * \sin(c\_4 * t) + c\_5 * \cos(c\_6 * t)}$$

Dirichlet boundary condition (fixed temperature)

Initial conditions:

Function on x and y direction:

$$\mathrm{a\_1 * y\^{}2 + a\_2 * y + a\_3 * x\^{}2 + a\_4 * x + a\_5 * x * y + a\_6 \ldots}$$
$$\mathrm{+ b\_3 * \sin(b\_1 * y + b\_2 * x) + b\_4 * \cos(b\_5 * y + b\_6 * x)}$$

Example 1:

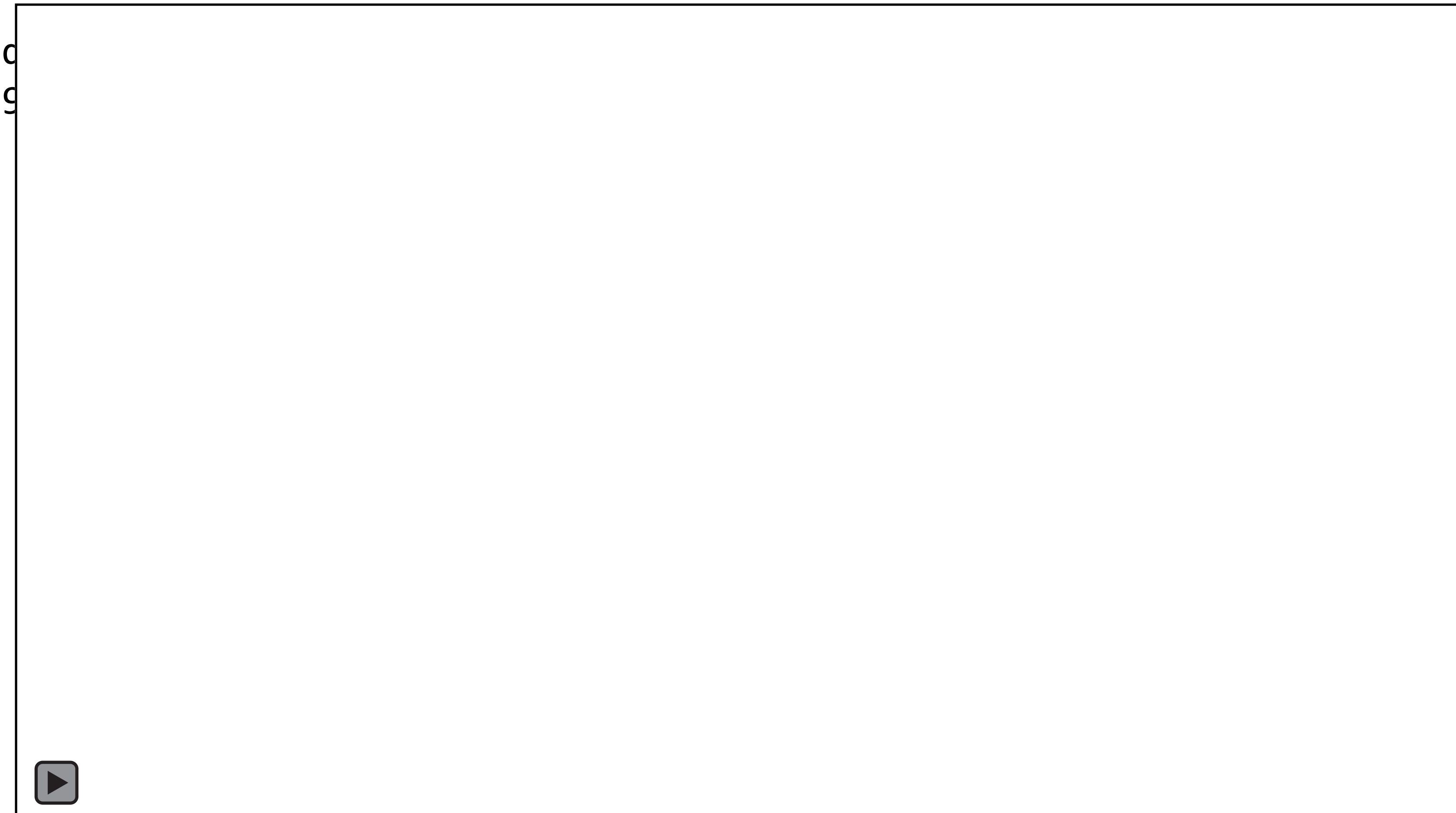
l= 3m w= 4m Mesh (30,40) $\alpha = 1.172 \times 10^{-5}$ (m^2/s) for steel

BC 4 Dirichlet boundaries (T =20°C for all boundaries)

IC=0

Elapsed

is 34.19

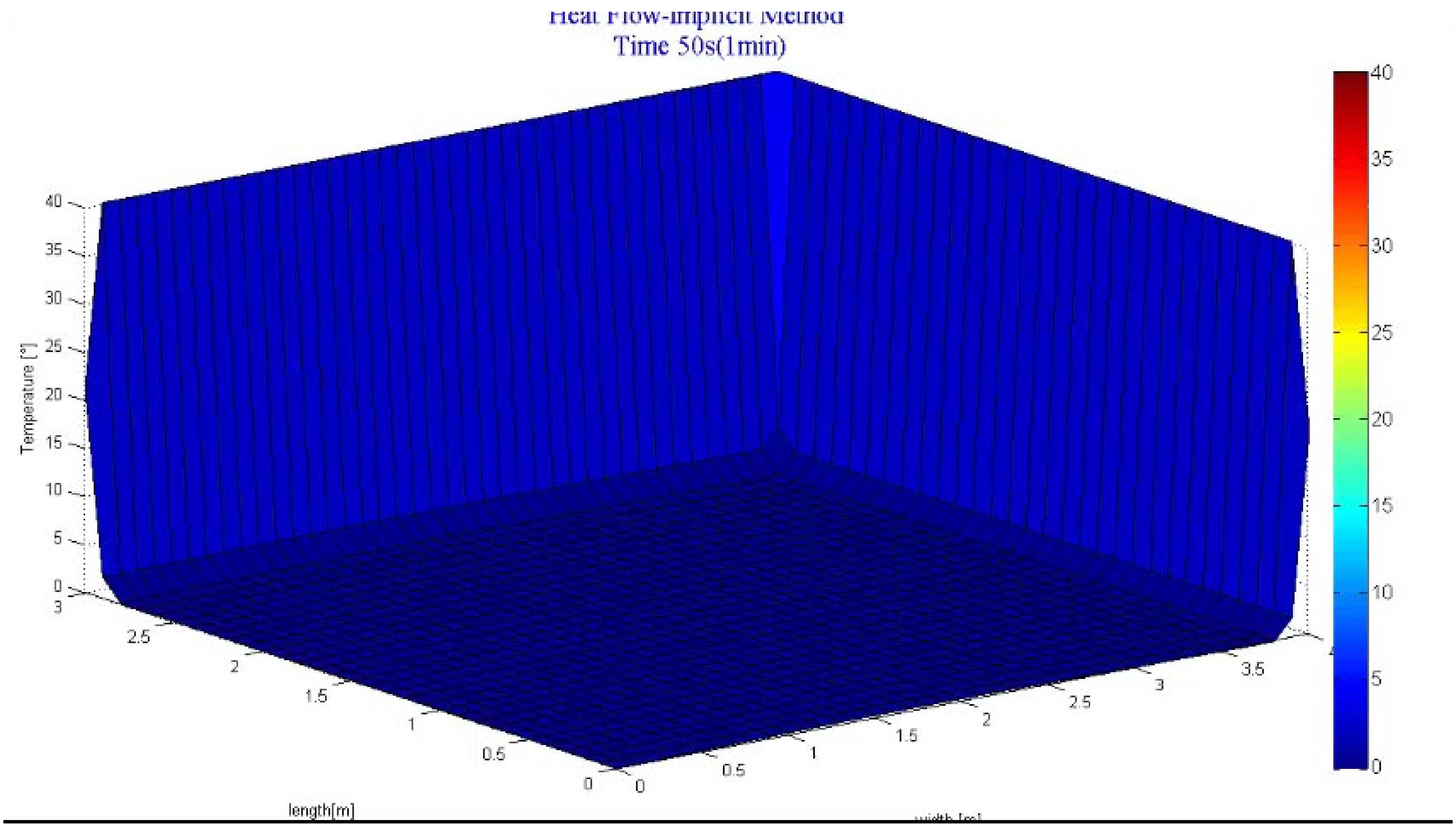


Example 2:

l= 3m w= 4m Mesh (30,40) $\alpha = 1.172 \times 10^{-5}$ (m^2/s) for steel

BC 2 Dirichlet boundaries (T =40°C for both boundaries) &2 Neumann boundaries(f(t)=0)

IC=0

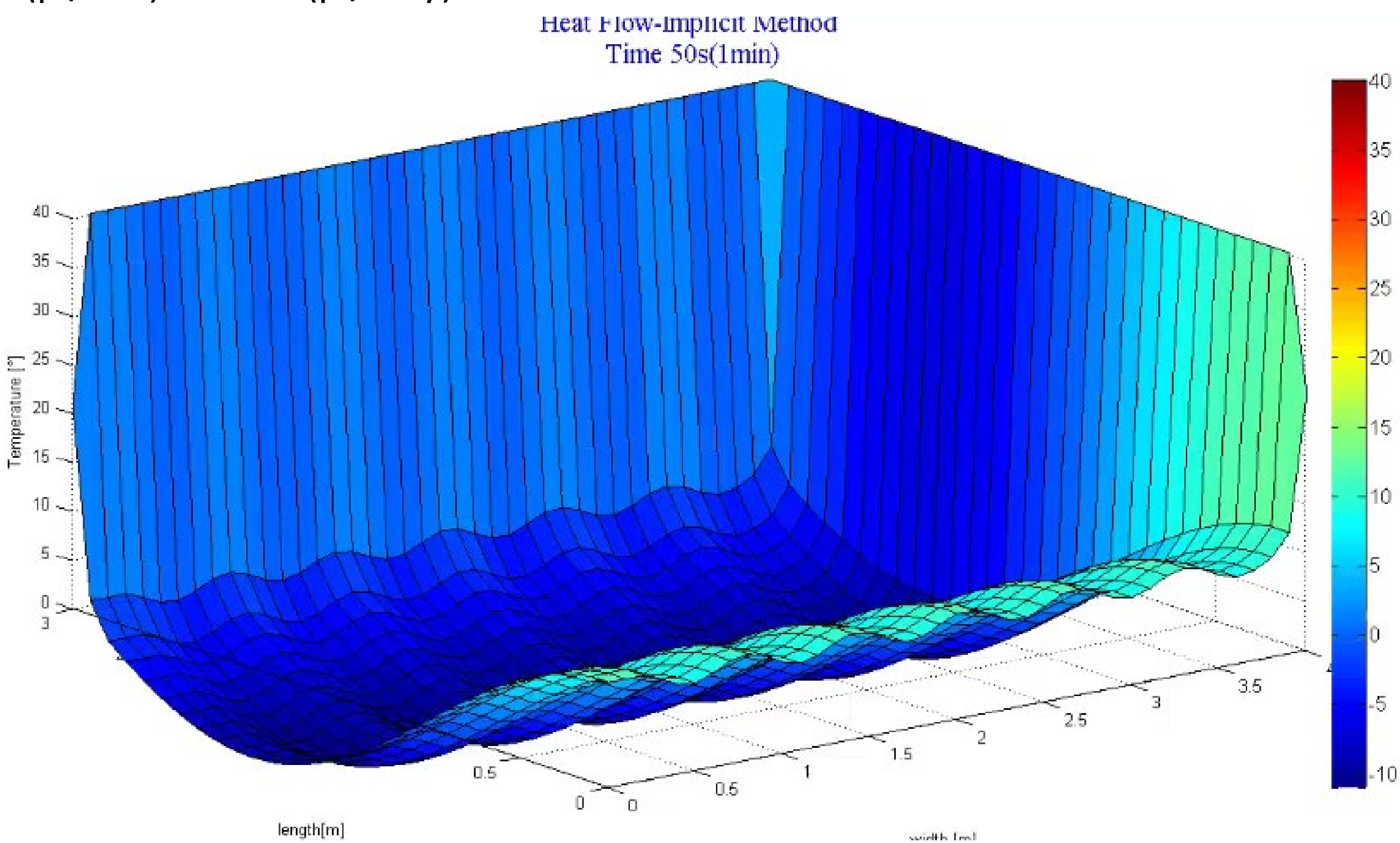


Example 3:

l= 3m w= 4m Mesh (30,40) α =1.172 × 10^{-5} (m^2/s) for steel

BC 2 Dirichlet boundaries (T =40°C for both boundaries) &2 Neumann boundaries(f(t)=0)

IC=10sin(pi/3*x) +10 cos(pi/2 *y)



Determining the deformation of rectangular domain using Linear Elasticity Equation

$$\mu\Delta\mathbf{u} + (\lambda+\mu)\operatorname{grad}\operatorname{div}\mathbf{u} = (3\lambda+2\mu)\alpha_T \operatorname{grad}(T-T_0),$$

$$\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+(\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\alpha_T(3\lambda+2\mu)[\frac{\partial T}{\partial x}-\frac{\partial T_o}{\partial x}]$$

&

$$\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+(\lambda+\mu)\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\alpha_T(3\lambda+2\mu)[\frac{\partial T}{\partial y}-\frac{\partial T_o}{\partial y}]$$

where: μ,λ– Lamé parameters

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

$$\mu = \frac{E}{2(1+\nu)}$$

where: μ shear modulus
ν Poisson ratio
E Young’s modulus

Boundary Conditions

Neumann boundary condition (free surface condition; vanishing of normal tractions)

$$T_j(\vec{u},\vec{n}) = \sigma_{ij} n_i = 0 \qquad \sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{pmatrix}$$

$$\sigma_{yy} = \lambda \frac{\partial u}{\partial x} + (\lambda + 2\mu)\frac{\partial v}{\partial y} = 0$$

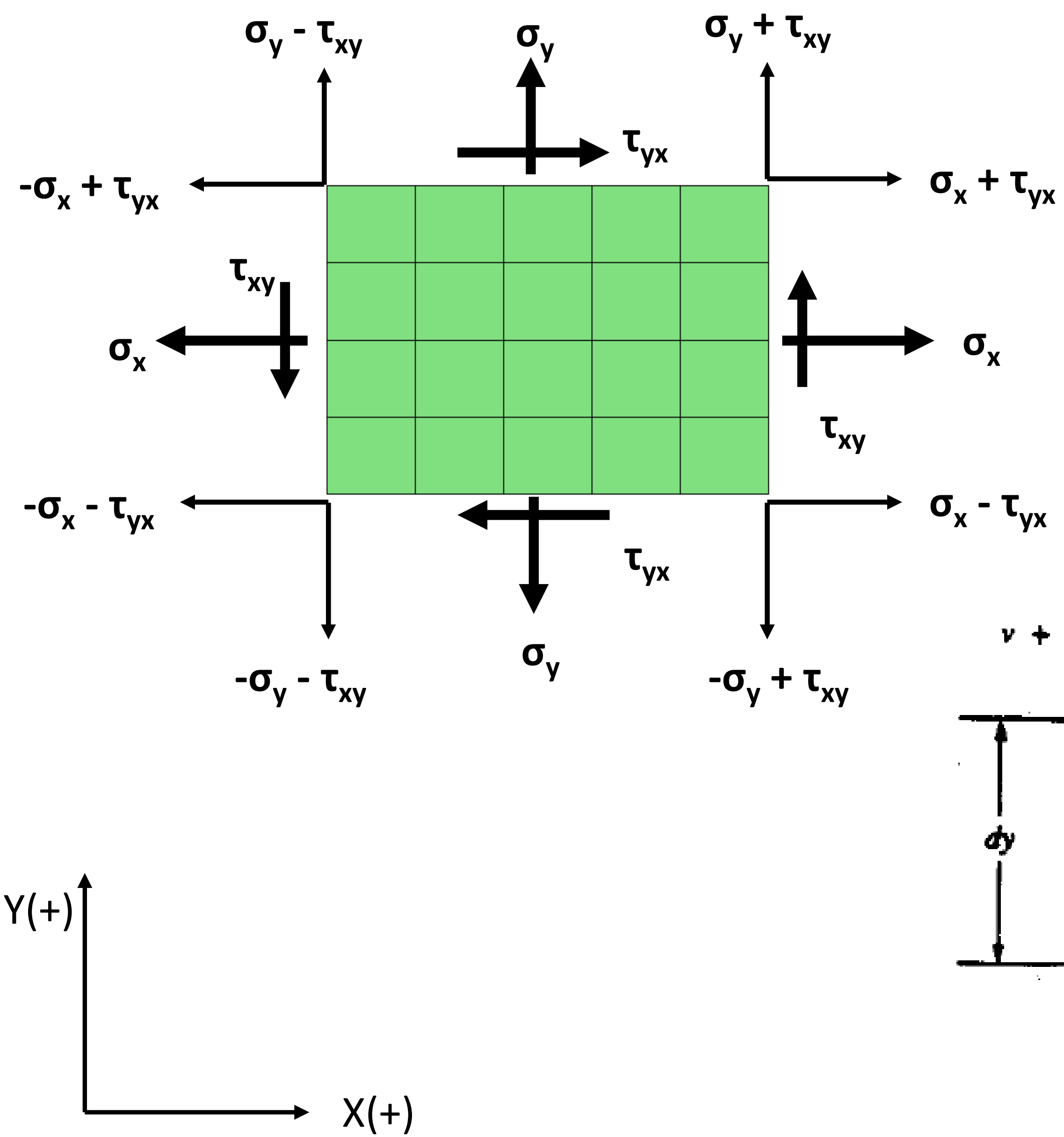
$$\tau_{yx} = \mu \frac{\partial v}{\partial x} + \mu \frac{\partial u}{\partial y} = 0$$

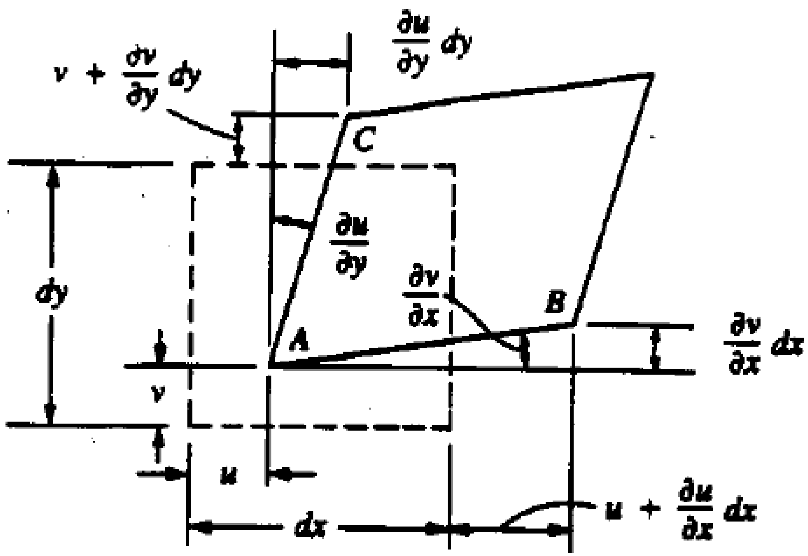
$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} = 0$$

$$\tau_{xy} = \mu \frac{\partial u}{\partial y} + \mu \frac{\partial v}{\partial x} = 0$$

Dirichlet boundary condition (fixed support x = 0)

2D Thermoelasticity





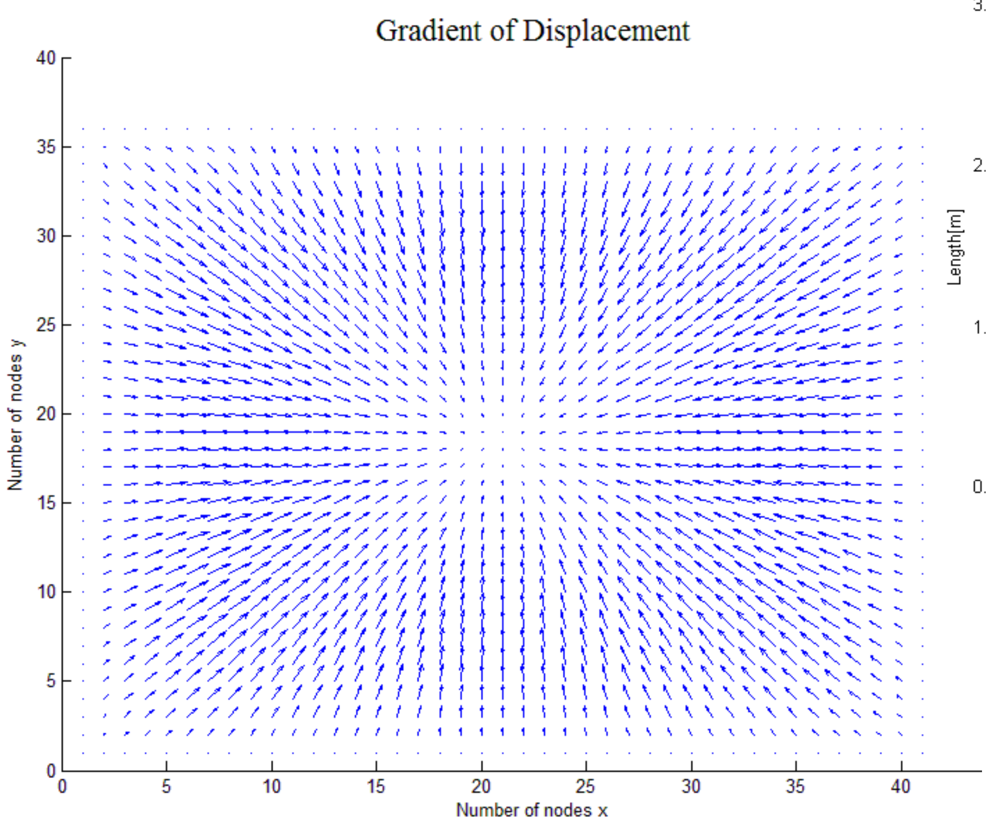
Example 4:

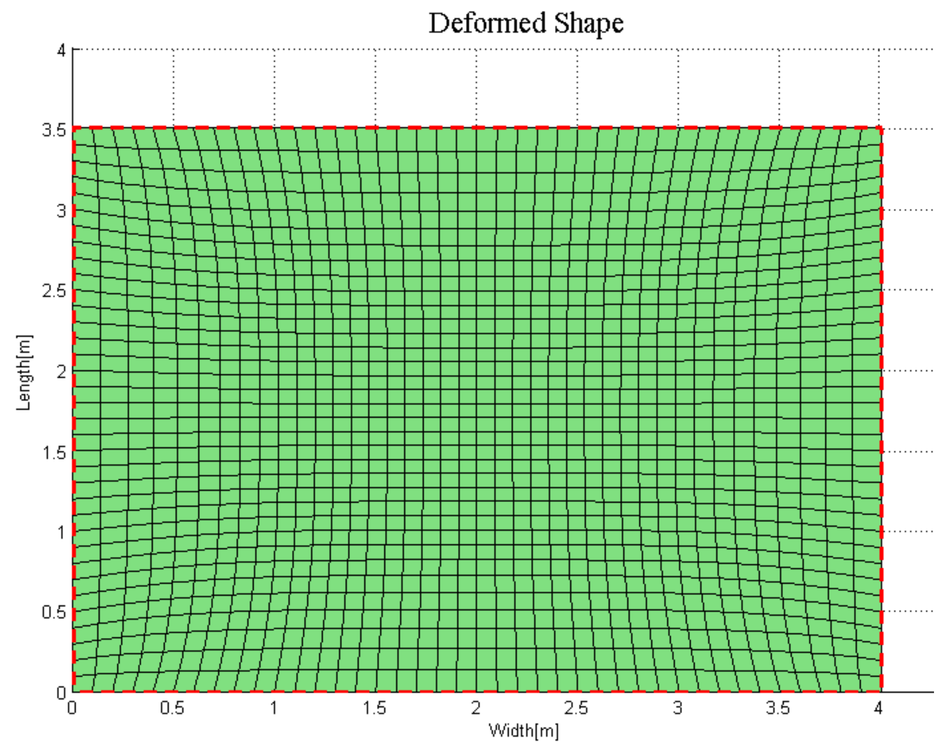
l= 3.5m w= 4m Mesh (35,40) α =1.172 × 10^{-5} (m^2/s) for steel

BC 4 Dirichlet boundaries (T =20°C for all boundaries)

IC=0

With four sides fixed





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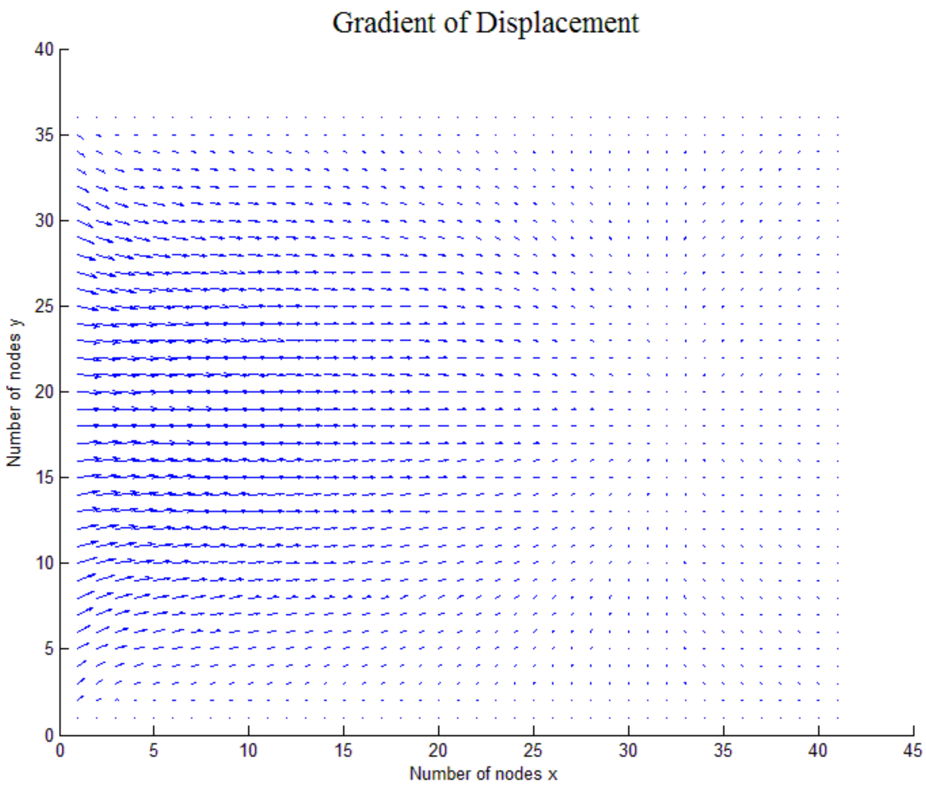
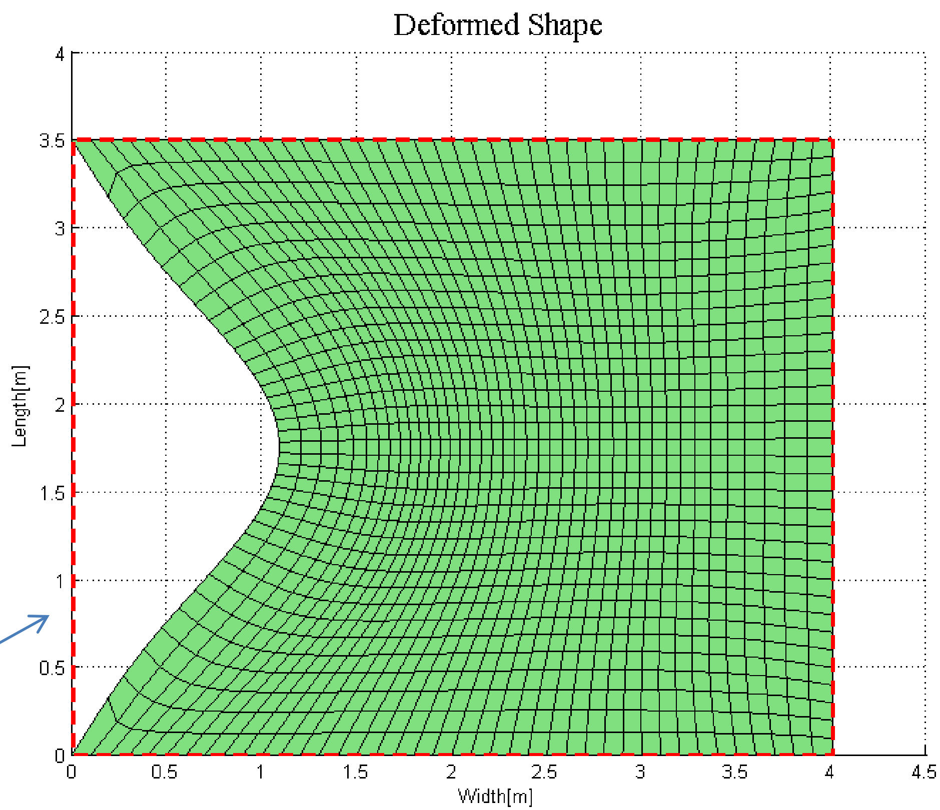
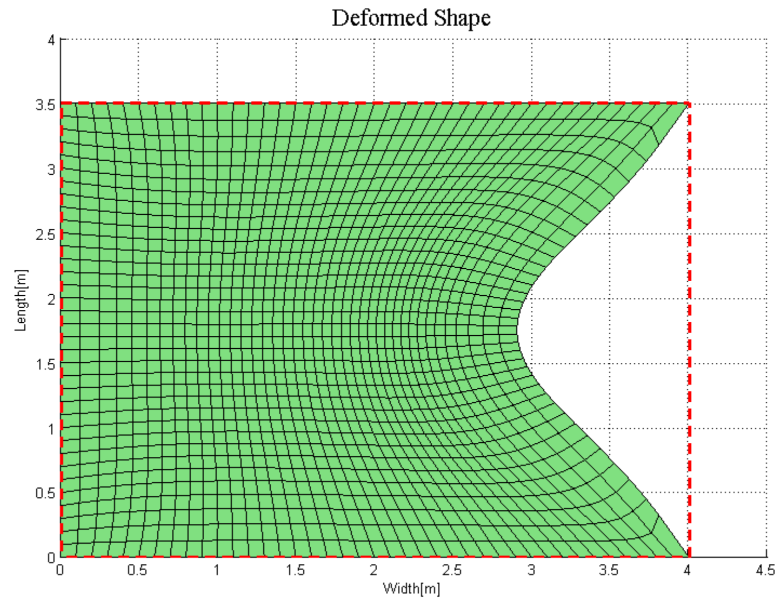




Free BC

Example 4: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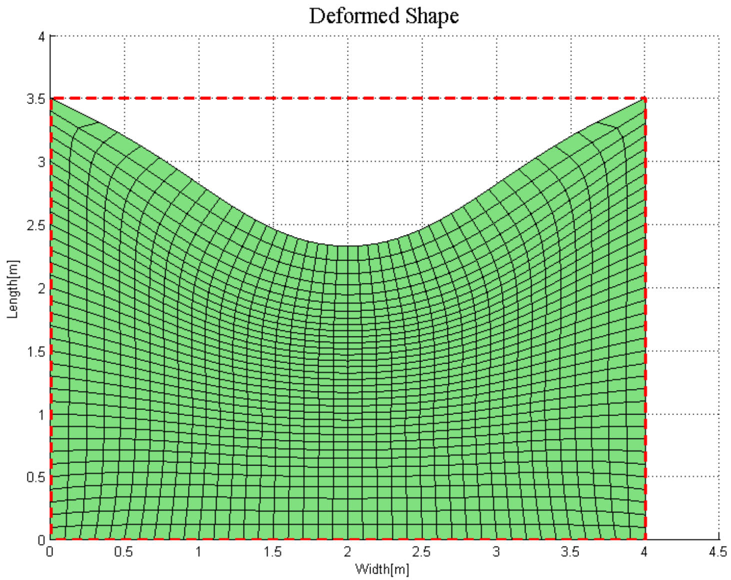
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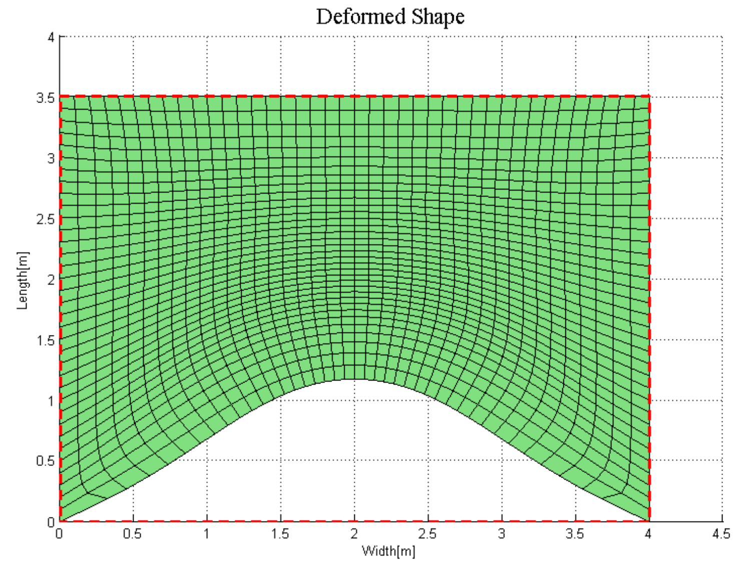
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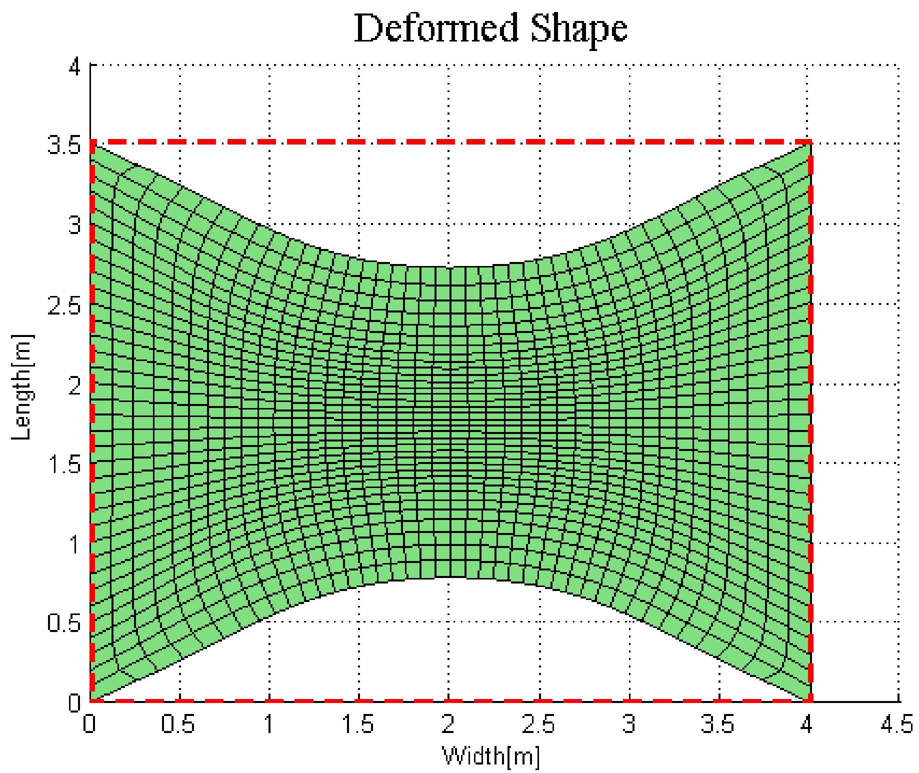
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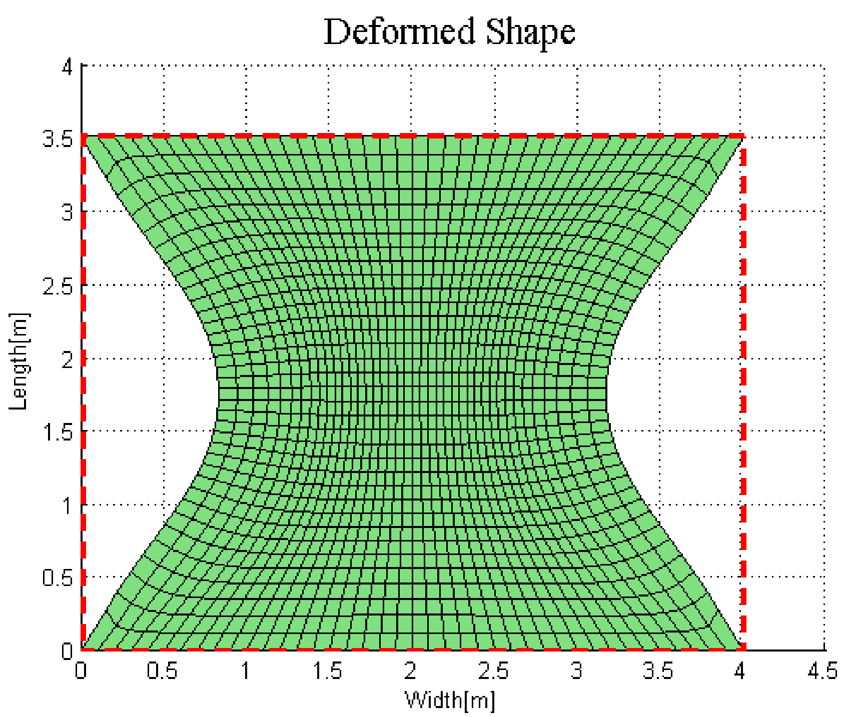
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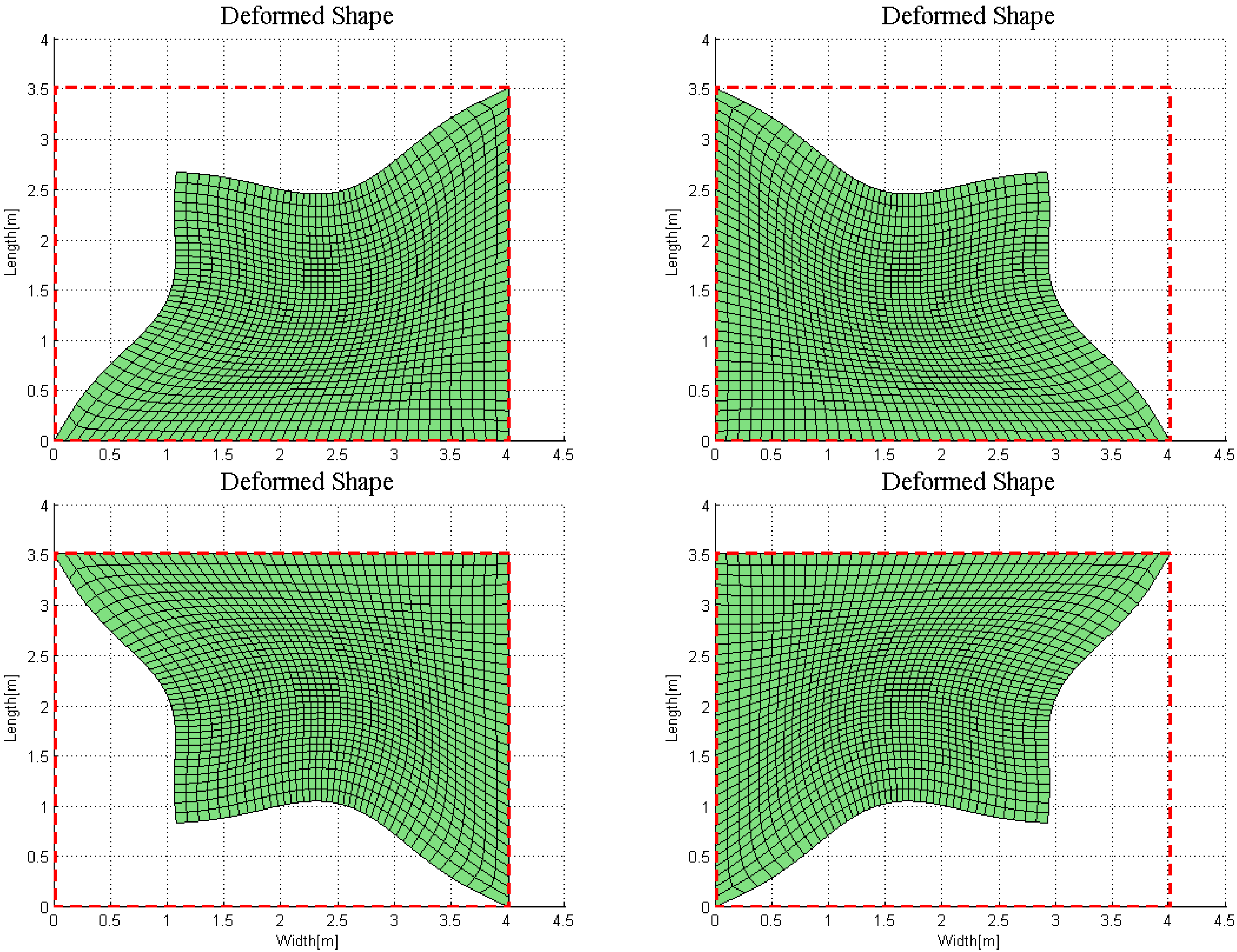
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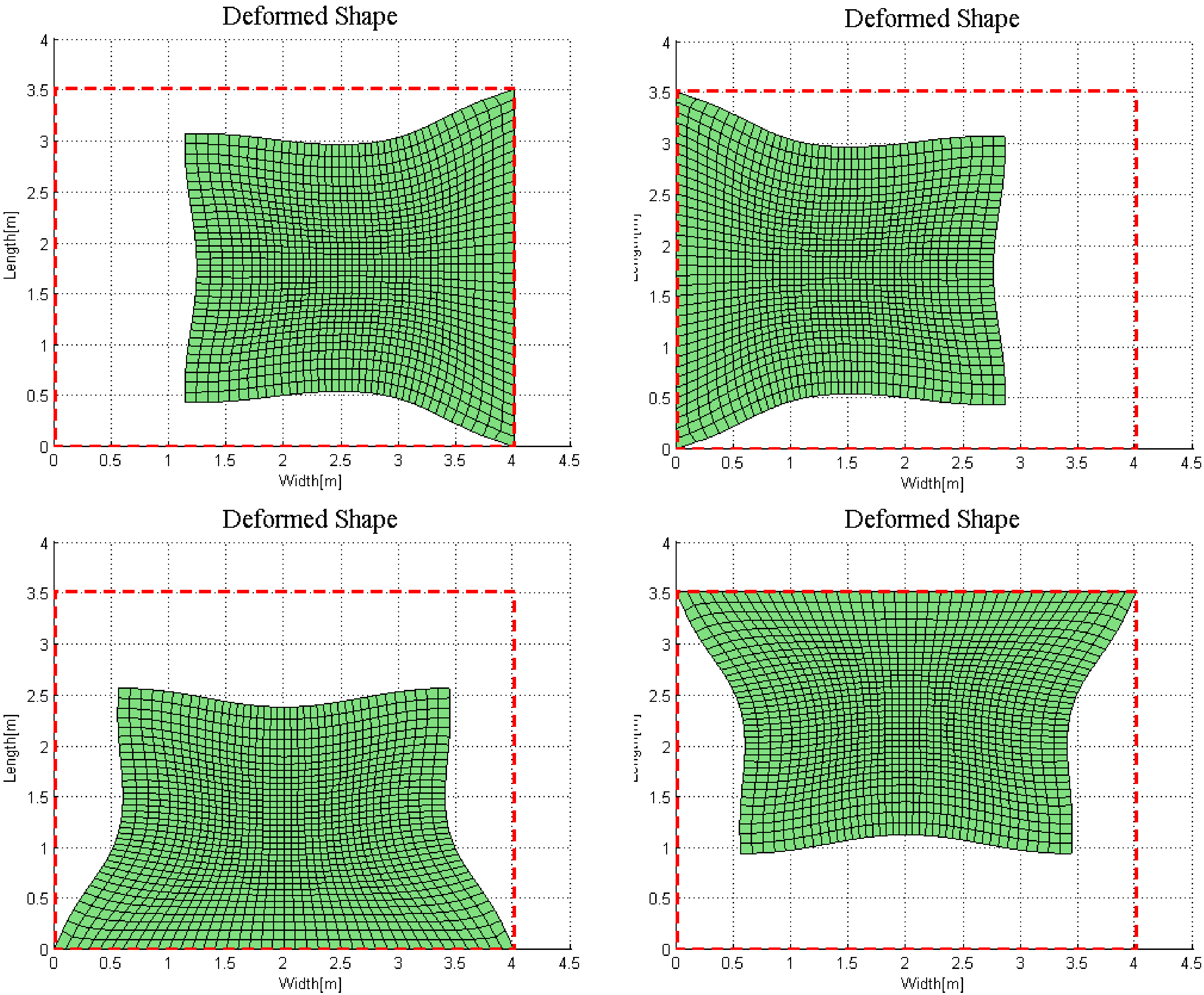
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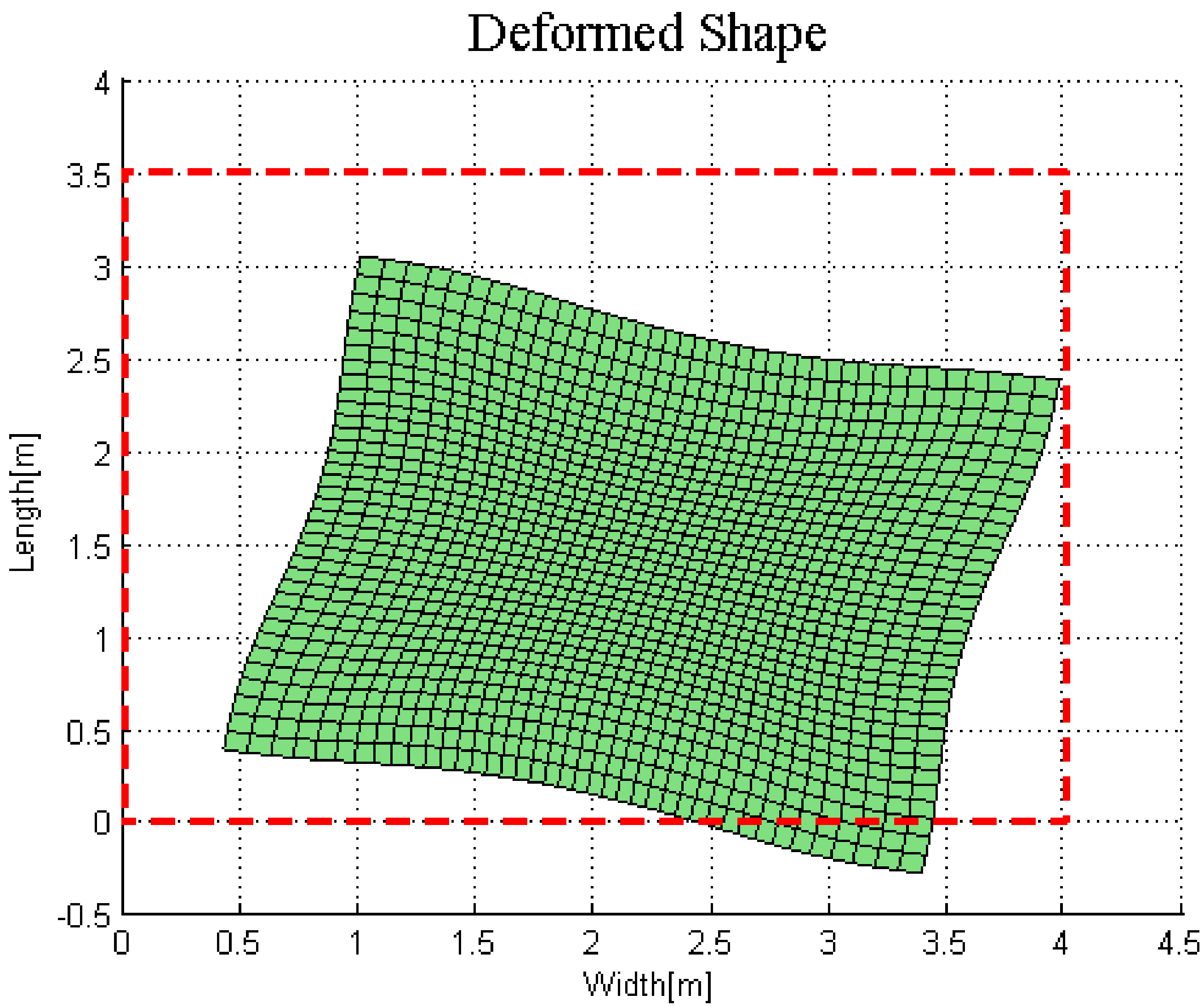

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THANK YOU
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