Chandra's®

# BLUE BOOK

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1) let p be the statement "Maria learns disvate mathemathic" and q the statement "Maria will find a good job" Express the statement $p \rightarrow q$ as a statement in English

from the defination of conditional statement we see that when p is the statement "Maria learns discreate Mathematics" and q is the statement "Maria will find a good job" $p \to q$ represents the statement "if maria learns discreate mathematics then she will find a good job." There are many way to express this conditional statement in English. Among the most natural of there are "maria will find a good job when she learns the discreate mathematics"

"For maria to get a good job it is sufficient for her to learn discreate mathematics"

and

"Maria will find a good job unless she does not learn discreate mathematics".

2) construct the truth table of the compond Proportion
$(p \vee \sim q) \rightarrow (p \wedge q)$

Decause this truth table involves two proportional variable p and q there are four rows in this truth table one for each of the pairs of truth values TT, TF, FF, FF. the first two colomns we find the truth values of ~q needed to find the truth values of $p \vee \sim q$ found in the fourth column the fifth column gives the truth value of $p \wedge q$ finally the truth value $(p \vee \sim q) \rightarrow (p \wedge q)$ is found in the last column the reesulting truth table is shown table.

$(p \vee \sim q) \rightarrow (p \wedge q)$

| p | q | ~q | $p \vee \sim q$ | $p \wedge q$ | $(p \vee \sim q) \rightarrow (p \wedge q)$ |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | T | T | F | F |
| F | T | F | F | F | T |
| F | F | T | F | F | F |

3) Show that $\sim(p \vee q)$ and $\sim p \wedge \sim q$ are logically equivalent

The truth table for there componend proportions are displayed in table 2. Because the truth value of the compound proporition $\sim(p \vee q)$ and $\sim p \wedge \sim q$ agree for all possible combination of the truth value of p and q it follows all possible combination of the truth values of p and q it follow that $\sim(p \vee q) \rightarrow (\sim p \wedge \sim q)$ it is a tatology and that there compound proposition are logically equivalent.

$\sim(p \vee q)$ and $\sim p \wedge \sim q$

| p | q | $p \vee q$ | $\sim(p \vee q)$ | $\sim p$ | $\sim q$ | $\sim p \wedge \sim q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | T | F | F | F | F |
| F | T | T | F | T | T | F |
| F | F | F | T | T | T | T |

4) Prove by PMI that $1+2+3+\cdots+n = \frac{n(n+1)}{2}$

let $P(n)$ be the statement that $1+2+3+\cdots+n = \frac{n(n+1)}{2}$

step (1) : Put $n=1$

$LHS = 1$

$RHS = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

$LHS = RHS$ $\therefore P(1)$ is true

step (2) : Assume that $P(n)$ is true for $n=k$

let $P(k)$ is true

$1+2+3+\cdots+k = \frac{k(k+1)}{2}$

step (3) : now we have to prove that $P(n)$ is true for $n=(k+1)$ LHS of $P(k+1)$

$1+2+3+\cdots+k+k-1 = \frac{(k+1)(3k+2)}{2}$

$1+2+3+\cdots+k+k+1 = \frac{k(k+1)}{2} + k+1$

$= \frac{(k+1)(k+1)}{2} = RHS$

$P(n)$ is true for $n \in N$

By PMI $P(A)$ is true $\forall n \in N$ = prove

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5) find the contrapositive the converse and the inverse of the conditional statement.

" The home team wins whenever it is raining".

Because "q whenever p" is one of the way to express the conditional statement $p \to q$ the orginal statement can be rewritten as

" It is raining then the home team wins"

Cosequently the contrapositive of this conditional statement is

" it is ~~home~~ home team does not win then it is not raining"

The inverse is

" if it is not raining then the home team does not win"

Only the contrapositive is equivalent to orginal statement.

6) what is the truth value of $\forall x P(x)$ where $P(x)$ is the statement $x^2 < 10$ and the domain consist of the +ve integers not exceeding 4?

The statement $\forall x P(x)$ is the same as the conjuction $P(1) \cap P(2) \cap P(3) \cap P(4)$

Because the domain consist of the integer 1, 2, 3 and 4

Because $P(4)$ which is the statement "$4^2 < 10$" is false it follow that $\forall x P(x)$ is false

Similarly when the element of domain are $x_1, x_2, \ldots, x_n$ where $n$ is a +ve integer the existential qualification $\exists x P(x)$ is the same as the disjuction

$$P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$$

Because this disjunction is true if and only if at least one of $P(x_1)$ $P(x_2)$ ... $P(x_n)$ is true

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7) what do the statement $\forall x < 0\ (x^2 > 0)$, $\forall y \neq 0\ (y^2 \neq 0)$, and $\exists z > 0\ (z^2 = 2)$ mean where the domain in each case consists of the real numbers?

The statement $\forall x < 0\ (x^2 > 0)$ statement that for every real number $x$ with $x < 0$ $x^2 > 0$ that is it starts "the square of a negative real number is +ve" this statement is the same as $\forall x\ (x < 0 \rightarrow x^2 > 0)$

The statement $\forall y \neq 0\ (y^2 \neq 0)$ state that for every real number $y$ wit $y \neq 0$ we have $y^3 \neq 0$ that is it states "the cube of every non zero real number is nonzero" This statement is equivalent to $\forall y\ (y \neq 0 \rightarrow y^3 \neq 0)$

finally the statement $\exists_z > 0\ (z^2 = 2)$ state that there exist a real number $z$ with $z > 0$ such that $z^2 = 2$ that is it state "there is a +ve square root of 2" this statement is equivalent to $\exists_z\ (z > 0 \wedge z^2 = 2)$

8) Express the statement "Every student in this class has studied calculus using predicates and quantifiers?

$1^{st}$, we rewrite the statement so that we can clearly identify the appropriate quantifiers to use.
Doing so, we obtain
"for every student in this class that student has studied calculus"
next, we introduce a variable $x$ so that our statement becomes "for every student $x$ in this class $x$ has studied calculas".
continuing we introduce $(x)$ which is the statement "$x$ has studied calculus" consequently if the domain for $x$ consists of the students in the class we can translate our statement as $\forall x\, c(x)$

finally, when we are interested in the background of people in subjects besides calculus we may prefer to use the two variable $(x, y)$ for the statement "student $x$ has studied subject $y$" Then we should replace $c(x)$ by $a\,(x, \text{calculate})$ in both approaches to obtain $\forall x\, Q\,(x, \text{calculus})$.

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9) combination A forma buys 3 cows 2 pig 4 hens from a man who has 6 cow 5 pig and 8 hens How many choice does forma has

$^6C_3$ = cows
$^3C_2$ = pigs
$^4C_8$ = hens

$$^6C_3 \times {}^3C_2 \times {}^4C_8 = \frac{6!}{3!\,3!} \times \frac{5!}{2!\,3!} \times \frac{8!}{4! \times 4!}$$

$$= \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{5 \times 4}{1 \times 2} \times \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2}$$

$$= 20 \times 10 \times 70$$

$$= \underline{\underline{14000}}$$

10) 6 men, 5 women to form the comitte of 5 membas should have 2 women

total = 11

memba = 5

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$${}^{3}C_{2} \times {}^{6}C_{3} = \frac{5!}{2!\,3!} \times \frac{6!}{3!\,3!}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1} \times \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 3 \times 2 \times 1}$$

$$= \frac{5 \times 4}{2 \times} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1}$$

$$= \frac{20}{2} \times \frac{120}{6}$$

$$= 10 \times 20$$

$$= \underline{\underline{200}}$$