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Neural Networks and Deep Learning – Summer Term 2020

Exercise sheet 2

Submission due: Wednesday, May 06, 13:15 sharp

Exercise 1 (Single-layer perceptron and Boolean functions with 2 inputs):

a) Show that the Boolean function XOR cannot be realized by a (single-layer) perceptron (with 2 inputs).

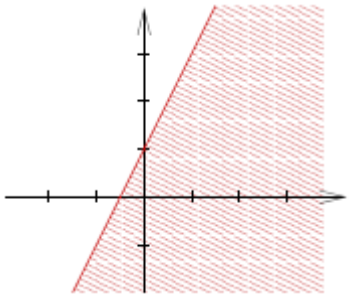
Note: The output y of a single-layer perceptron with 2 inputs x_1 and x_2, threshold θ and weights w_1 and w_2 is given by

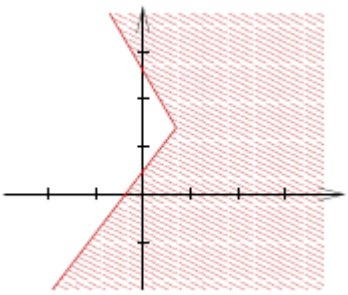
$$y = \Theta\left[x_1 w_1 + x_2 w_2 - \theta\right]$$ (Θ is the Heaviside function)

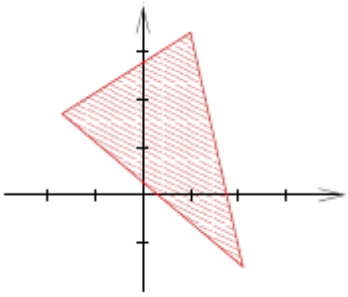
b) Give all Boolean functions with 2 inputs (i.e. for each Boolean function: the output for each input combination) and indicate whether they can be realized by a (single-layer) perceptron.

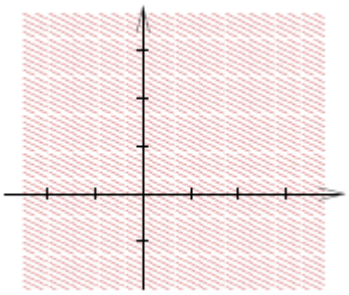
c) Select three Boolean functions with two inputs and give values for the synaptic weights w_1, w_2 and threshold θ so that the Boolean function is realized by a single-layer perceptron. Show for each of the three Boolean functions and each input pair that the Boolean function is indeed realized by the chosen combination of weights and threshold.

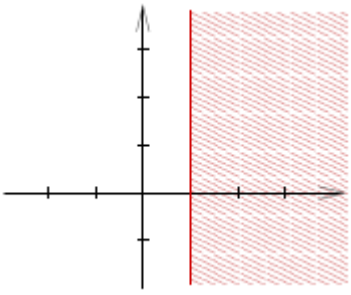
d) Which of the following partitioning of $\Re^2$ can be realized by a single-layer perceptron with two inputs? For those that can be realized, give weights and threshold of the perceptron. (Consider abscissa as x_1 and ordinate as x_2).

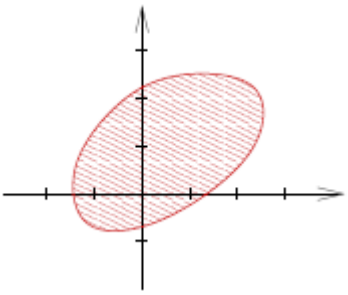
i) 

ii) 

iii) 

iv) 

v) 

vi) 

(From: Riedmiller)

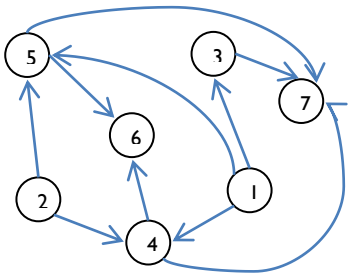
Exercise 2 (Types of neural networks, synaptic weight matrix):

a) Explain the following terms related to neural networks:

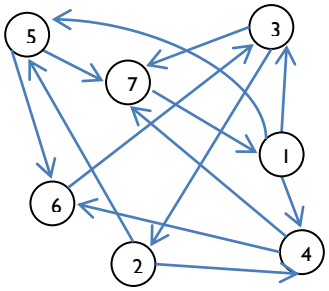
- Boolean function
- Feedforward neural network
- Recurrent neural network
- Multi-layer perceptron

b) Specify whether the following artificial neural networks are feedforward or recurrent neural networks and explain your selection.

i)



ii)

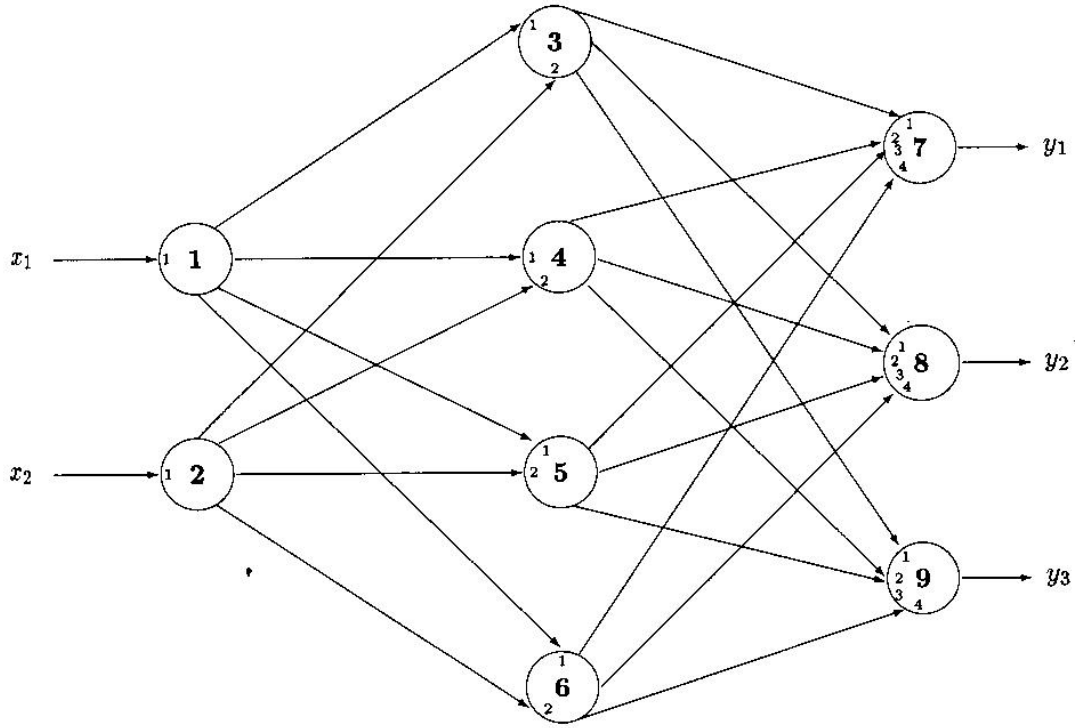


c) Using the neuron numbers from 1 to 7 given in the circles, fill out the following general weight matrix by marking the corresponding field entries. Example: Mark the field in row *i* and column *j* (weight w_{ij}) if there is a connection from neuron *j* to neuron *i*.

	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

Exercise 3 (Computing the output of a feedforward neural network):

a) Compute the output of the following feedforward neural network for the input x_1=3; x_2=1. Which neurons can be computed in parallel, which have to wait?



Note: The small numbers in each circle correspond to the components of the weight vector; see example below. In this part of the exercise, the threshold is set to θ =0 for all neurons.

Neuron	Activation function of neuron	Weight vector
1	Linear; c=1	(1)
2		(1)
3	Threshold element; θ =0	(1,-2)
4		(-1,0)
5		(3,2)
6		(0,2)
7	Linear; c=1	(0,2,-3,1)
8		(1,-2,3,8)
9		(0,2,3,-4)

c is the slope of the linear activation function: $f(h) = c \cdot h$

"Threshold element" means that the activation function is the Heaviside function

Example for weight vector of neuron 8:
1st component of weight vector (1) refers to connection neuron 3 → neuron 8
2nd component of weight vector (–2) refers to connection neuron 4 → neuron 8
3rd component of weight vector (3) refers to connection neuron 5 → neuron 8
4th component of weight vector (8) refers to connection neuron 6 → neuron 8

(Source: Stefan Hartmann, Cesar Research)

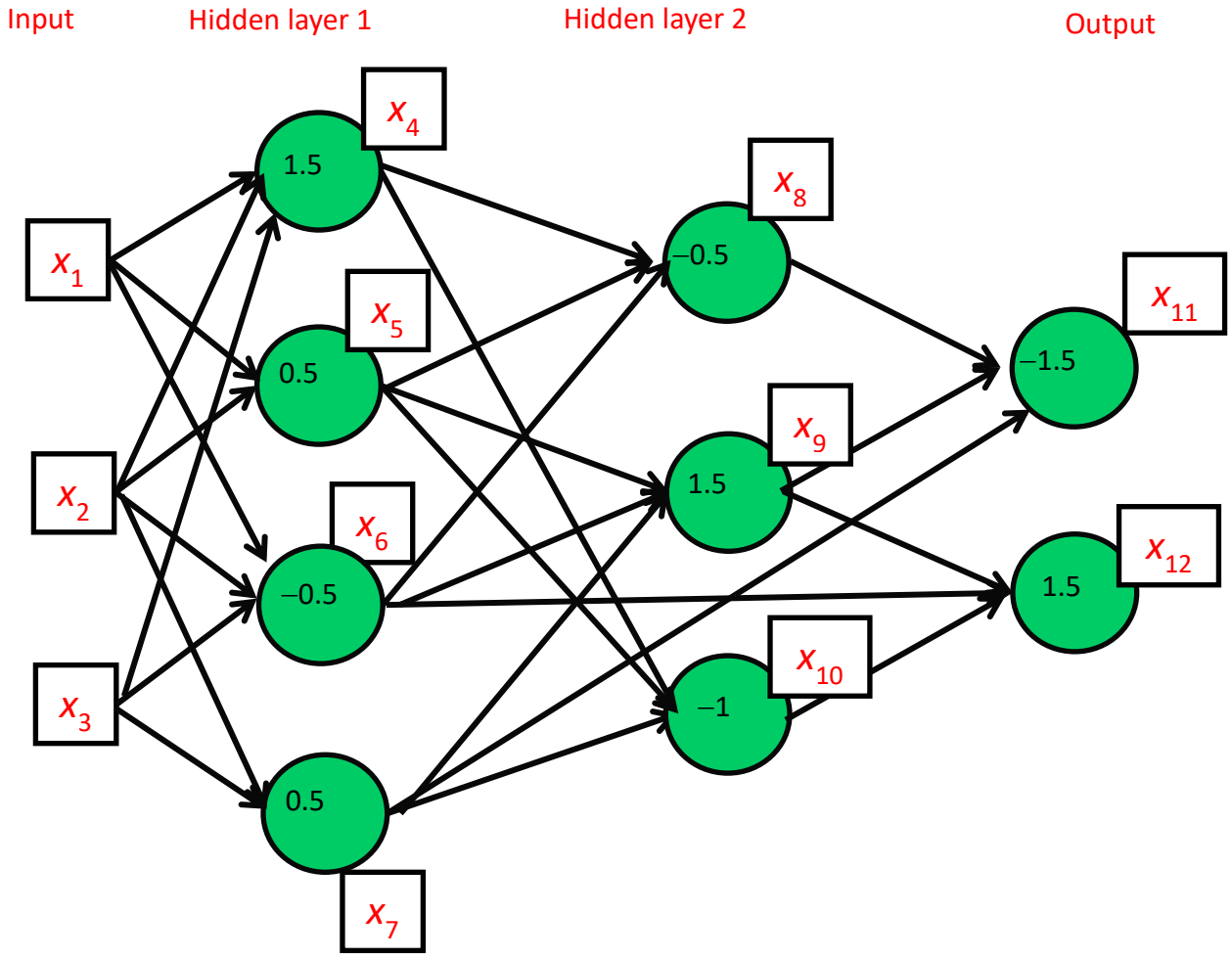
b) Assume the following weight matrix, where an entry w_{ij} (ith row, jth column) corresponds to the synaptic weight from neuron j to neuron i. (No entry means the synaptic weight is 0). Further assume that the activation function of the neurons of hidden layer 2 (neurons 8, 9 and 10) is linear (with slope c=1), whereas the activation function of all other neurons is a Heaviside step function. In this part of the exercise, the threshold θ of each node is indicated in the network graph as number in the corresponding neuron.

Compute the output of the following feedforward neural network for the inputs x_1=1, x_2=0, x_3=1 and x_1=0, x_2=1, x_3=1.

Weight matrix:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1												
2												
3												
4	–2	5	–4									
5	1	–2										
6	3	–1	6									
7		7	1									
8				–1	4	–2						
9					–3	5	1					
10				8	2		–3					
11							6	1	–2			
12						1			–4	3		

Network:



(Note: this is a feedforward neural network of second order)

Exercise 4 (Multi-layer perceptron and XOR):

a) Find a multi-layer perceptron which realizes the Boolean function XOR. Demonstrate that the found perceptron indeed performs XOR on all possible input pairs.

b) Find a perceptron with two (binary) inputs which realizes the function

$$F(x_1, x_2) = \begin{cases} 1 : x_1 + x_2 = 1 \\ 0 : else \end{cases}$$

Note: "+" denotes mathematical addition.