CSE 318 Assignment-03

Solving the Max-cut problem by GRASP

1905066 - Abir Muhtasim

GRASP (greedy randomized adaptive search procedure) is a randomized multistart iterative method for computing good quality solutions of combinatorial optimization problems. Each GRASP iteration is usually made up of a construction phase, where a feasible solution is constructed, and a local search phase which starts at the constructed solution and applies iterative improvement until a locally optimal solution is found. The construction phase of GRASP is a semi-greedy or randomized greedy algorithm.

Semi-Greedy

For each candidate vertex v, we apply a score function to v. In this way, we obtain a ranking of all elements according to their score values. Next, well-ranked elements are placed in a restricted candidate list (RCL), and we may select one element randomly selected from the RCL to add to (partially constructed) solution.

the candidate elements are vertices which are not yet assigned to set X and set Y. Let $V = V - \{X \cup Y\}$ be the set of all candidate elements. The cut-off value is denoted by $\mu = score_{min} + \alpha * (score_{max} - score_{min})$, where α is a parameter such that 0 ≤ α ≤ 1, and the restricted candidate list consists of all vertices whose value of the greedy function is greater than or equal to μ. A vertex is randomly selected from the restricted candidate list.

For each vertex v, we define -

$$\sigma_x(v) = \sum_{u \epsilon Y} w_{uv} \quad \text{and} \quad \sigma_y(v) = \sum_{u \epsilon X} w_{uv}$$

After choosing a vertex v from the RCL, if $\sigma_x(v) > \sigma_y(v)$, then v is place in X. Otherwise v is placed in Y.

Greedy

Greedy is calculated by setting α = 1 in the semi greedy method.

Heuristics

For the scoring of the vertices, we used two heuristics from this paper.

SG1

$$score(i) = max(\sigma_x(i), \sigma_y(i))$$

SG3

$$score(i) = |\sigma_x(i) - \sigma_y(i)|$$

Randomized

For comparing the initial solution in the construction phase of the semi-greedy and greedy method a randomized method was also implemented. Initially, both partitions X and Y are empty. For each vertex $v \in V$, place v in partition X or partition Y with uniform randomness, i.e., with probability . The procedure terminates when all vertices are placed either in X or Y.

For comparing the construction algorithms, we used 20 iterations in the Greedy, Semi-greedy and Randomized methods and took the avarage. We used 500 iterations in the GRASP.

Performance Analysis:

- We can see that the Simple Randomized method gives a fairly decent result when there are no edge with negative weight. But performs very poorly when there are negative edges.

- We can see SG1 gives a better result than SG3 when approximating the max cut with greedy approach. But SG3 gives a better result than SG1 when approximating the max cut with Semi-Grady approach. Also SG3 performs better than SG1 in GRASP.

- We can see the GRASP gives about 85% of the best known value when there are negative edges. But when there are no negative edges GRASP gives about 95% - 100% of the best known value.

- For SG3 the number of avarage iterations is about half compared to SG1. This is because SG3 gives a better initial semi-greedy solution than SG1. So it converges to local optima faster.

Result Link : GRASP

		Problem				Constructive algorithm			Local search		GRASP		Known Best Solution
Name	|V| or n	|E| or m	min edge	max edge	Simple Randomized	Simple Greedy	Semi Greedy	Avg iterations	Avg improvements	Avg value	Iterations	Best Value	
G1	800	19176	1	1	9608	11265	11112	122	252	11371	500	11482	12078
G2	800	19176	1	1	9568	11307	11113	119	247	11374	500	11482	12084
G3	800	19176	1	1	9613	11225	11108	120	250	11368	500	11488	12077
G4	800	19176	1	1	9583	11286	11144	122	254	11388	500	11531	
G5	800	19176	1	1	9568	11233	11142	120	253	11376	500	11498	
G6	800	19176	-1	1	71	1701	1608	138	291	1917	500	2033	
G7	800	19176	-1	1	-104	1654	1445	133	283	1743	500	1844	
G8	800	19176	-1	1	-82	1588	1475	137	292	1751	500	1851	
G9	800	19176	-1	1	-9	1713	1488	134	284	1790	500	1900	
G10	800	19176	-1	1	-97	1668	1452	132	283	1744	500	1873	
G11	800	1600	-1	1	19	488	430	13	25	454	500	508	627
G12	800	1600	-1	1	-3	480	413	13	25	442	500	490	621
G13	800	1600	-1	1	18	488	444	14	28	468	500	526	645
G14	800	4694	1	1	2361	2951	2927	30	38	2960	500	2993	3187
G15	800	4661	1	1	2335	2925	2900	30	38	2942	500	2973	3169
G16	800	4672	1	1	2346	2950	2906	31	39	2946	500	2982	3172
G17	800	4667	1	1	2324	2928	2908	30	38	2943	500	2980	
G18	800	4694	-1	1	30	835	728	63	102	843	500	913	
G19	800	4661	-1	1	-47	744	657	63	103	759	500	832	
G20	800	4672	-1	1	-11	776	702	60	98	789	500	862	
G21	800	4667	-1	1	-41	753	676	65	105	782	500	846	
G22	2000	19990	1	1	9995	12794	12603	170	300	12877	500	13015	14123
G23	2000	19990	1	1	10018	12763	12609	174	308	12872	500	13025	14129
G24	2000	19990	1	1	9997	12784	12534	170	301	12874	500	12997	14131
G25	2000	19990	1	1	9978	12793	12610	168	297	12878	500	13019	
G26	2000	19990	1	1	9961	12810	12670	173	304	12866	500	13009	
G27	2000	19990	-1	1	-17	2613	2411	215	387	2840	500	2991	
G28	2000	19990	-1	1	-58	2699	2470	216	392	2801	500	2950	
G29	2000	19990	-1	1	-1	2744	2445	216	389	2896	500	3039	
G30	2000	19990	-1	1	49	2669	2473	216	392	2900	500	3063	
G31	2000	19990	-1	1	-33	2692	2395	217	391	2816	500	2985	
G32	2000	4000	-1	1	10	1186	1078	33	66	1132	500	1234	1560
G33	2000	4000	-1	1	-7	1176	1012	33	67	1103	500	1218	1537
G34	2000	4000	-1	1	-22	1196	1036	34	68	1099	500	1220	1541
G35	2000	11778	1	1	5879	7404	7348	68	88	7429	500	7499	8000
G36	2000	11766	1	1	5882	7384	7345	70	90	7420	500	7478	7996
G37	2000	11785	1	1	5894	7371	7341	70	90	7432	500	7487	8009
G38	2000	11779	1	1	5894	7396	7347	70	91	7427	500	7485	
G39	2000	11778	-1	1	-1	2011	1775	157	262	2040	500	2194	
G40	2000	11766	-1	1	-37	2014	1794	156	260	2021	500	2158	
G41	2000	11785	-1	1	-8	2050	1794	153	254	2027	500	2174	
G42	2000	11779	-1	1	62	2022	1864	159	266	2110	500	2248	
G43	1000	9990	1	1	4987	6411	6265	89	158	6425	500	6533	7027
G44	1000	9990	1	1	4988	6362	6245	87	152	6423	500	6517	7022
G45	1000	9990	1	1	4994	6394	6259	83	146	6425	500	6503	7020
G46	1000	9990	1	1	5000	6396	6288	86	153	6424	500	6505	
G47	1000	9990	1	1	5016	6340	6291	90	158	6431	500	6534	
G48	3000	6000	1	1	2987	6000	5255	29	65	5464	500	6000	6000
G49	3000	6000	1	1	3001	6000	5453	29	64	5484	500	6000	6000
G50	3000	6000	1	1	2986	5880	5251	28	63	5472	500	5880	5988
G51	1000	5909	1	1	2959	3714	3676	37	46	3721	500	3757	
G52	1000	5916	1	1	2949	3715	3680	37	47	3724	500	3767	
G53	1000	5914	1	1	2956	3714	3669	34	44	3722	500	3763	
G54	1000	5916	1	1	2963	3698	3671	36	46	3719	500	3754	

		Problem				Constructive algorithm			Local search		GRASP		Known Best Solution
Name	|V| or n	|E| or m	min edge	max edge	Simple Randomized	Simple Greedy	Semi Greedy	Avg iterations	Avg improvements	Avg value	Iterations	Best Value	
G1	800	19176	1	1	9580	11070	11288	62	124	11407	500	11531	12078
G2	800	19176	1	1	9589	11273	11265	63	129	11412	500	11552	12084
G3	800	19176	1	1	9598	11249	11297	59	118	11406	500	11543	12077
G4	800	19176	1	1	9580	11010	11286	60	121	11427	500	11540	
G5	800	19176	1	1	9594	11010	11289	61	124	11414	500	11550	
G6	800	19176	-1	1	77	1551	1850	66	135	1956	500	2097	
G7	800	19176	-1	1	-73	1535	1683	64	131	1793	500	1895	
G8	800	19176	-1	1	-106	1521	1689	63	129	1803	500	1911	
G9	800	19176	-1	1	-66	1521	1687	65	132	1837	500	1948	
G10	800	19176	-1	1	-86	1422	1631	62	129	1789	500	1882	
G11	800	1600	-1	1	9	414	469	6	11	488	500	536	627
G12	800	1600	-1	1	0	452	484	6	11	480	500	528	621
G13	800	1600	-1	1	16	408	502	7	13	501	500	554	645
G14	800	4694	1	1	2347	2938	2937	19	23	2976	500	3017	3187
G15	800	4661	1	1	2345	2862	2939	17	21	2960	500	2998	3169
G16	800	4672	1	1	2342	2912	2954	19	24	2961	500	3003	3172
G17	800	4667	1	1	2331	2893	2929	20	25	2958	500	3010	
G18	800	4694	-1	1	25	695	879	25	38	890	500	960	
G19	800	4661	-1	1	-66	605	791	24	37	809	500	875	
G20	800	4672	-1	1	-22	730	805	24	36	837	500	905	
G21	800	4667	-1	1	-47	653	791	25	38	830	500	892	
G22	2000	19990	1	1	10014	12667	12794	95	165	12954	500	13152	14123
G23	2000	19990	1	1	9981	12817	12820	93	162	12959	500	13107	14129
G24	2000	19990	1	1	9993	12402	12802	90	155	12962	500	13108	14131
G25	2000	19990	1	1	10010	12392	12787	91	156	12965	500	13149	
G26	2000	19990	1	1	9991	12326	12812	98	169	12945	500	13129	
G27	2000	19990	-1	1	-13	2741	2790	102	178	2946	500	3135	
G28	2000	19990	-1	1	-55	2478	2762	96	168	2913	500	3088	
G29	2000	19990	-1	1	48	2621	2889	99	173	3008	500	3188	
G30	2000	19990	-1	1	77	2562	2772	99	172	3017	500	3199	
G31	2000	19990	-1	1	-19	2686	2635	94	163	2932	500	3109	
G32	2000	4000	-1	1	10	1098	1136	17	34	1210	500	1314	1560
G33	2000	4000	-1	1	-11	968	1211	17	33	1187	500	1300	1537
G34	2000	4000	-1	1	-40	950	1158	17	33	1189	500	1296	1541
G35	2000	11778	1	1	5883	7354	7455	43	55	7470	500	7566	8000
G36	2000	11766	1	1	5875	7376	7379	39	49	7468	500	7546	7996
G37	2000	11785	1	1	5898	7377	7438	40	51	7477	500	7571	8009
G38	2000	11779	1	1	5888	7326	7376	40	50	7474	500	7554	
G39	2000	11778	-1	1	12	1752	2145	52	80	2172	500	2303	
G40	2000	11766	-1	1	-53	1891	2048	51	79	2154	500	2287	
G41	2000	11785	-1	1	-18	1682	2096	49	76	2160	500	2285	
G42	2000	11779	-1	1	45	1787	2167	50	78	2239	500	2365	
G43	1000	9990	1	1	4984	6332	6335	47	80	6466	500	6560	7027
G44	1000	9990	1	1	4989	6225	6319	48	82	6464	500	6567	7022
G45	1000	9990	1	1	5011	6243	6412	52	89	6459	500	6563	7020
G46	1000	9990	1	1	5003	6121	6360	48	84	6463	500	6554	
G47	1000	9990	1	1	4977	6183	6412	51	87	6465	500	6551	
G48	3000	6000	1	1	3012	5208	5453	23	52	5492	500	6000	6000
G49	3000	6000	1	1	3005	4910	5489	25	57	5478	500	6000	6000
G50	3000	6000	1	1	3007	4864	5300	26	59	5450	500	5880	5988
G51	1000	5909	1	1	2963	3712	3729	23	29	3738	500	3790	
G52	1000	5916	1	1	2953	3602	3708	23	29	3742	500	3790	
G53	1000	5914	1	1	2959	3699	3707	23	28	3741	500	3799	
G54	1000	5916	1	1	2960	3679	3716	23	28	3739	500	3788	