Definition and Notation

Contents

String

- A **String** is a finite sequence of symbols, and these symbols come from a finite alphabet.
- An **Alphabet** is a finite set, often denoted by $\sum$
  - Each element of an **alphabet** is a **symbol**.
  - A **string** over $\sum$ is any finite-length sequence of **symbols** from $\sum$
  - String sometimes referred to as words
  - The length of a string is W is the number of symbols in W, denoted by $|W|$.

> **GIST:** set of Symbols makes alphabet. A combination of those symbols of Alphabet gives string or word. Count of symbols in a string is the length.

- For any alphabet $\sum$ there is a special string called the empty string denoted by ϵ, its length is 0.
- Exponent Notation: Shortcut for repeated symbols
  - z^k to denote strings of Z's of length k, Example $2^3 = 222$
    - $2^0 = \epsilon$ Empty string.

Kleene Star

- For any **Alphabet** $\sum$ the set of all string over $\sum$ is denoted by $\sum^*$
  - One way to think about it, for each k>=0, list all string of lenmgth k you can make using symbols from $\sum$

String Contcatenation

- string are concatenated by putting them side to side.
- it uses dot notation.
- THe length of the concatenation string $W_1 \cdot W_2$: $|W_1 \cdot W_2|$ = $|W_1| + |W_2|$
- Concatenation the empty string does nothing,
- For a string W and integer $k \geq 0$,
  the value of W^K is W concatenated to itself k times.

Prefix

- String P is a **prefix** of string Y if $Y = P \cdot Z$ where Z is a string

Some facts

1. Empty string is a Prefix of every string
2. Every string is a prefix of itself

- A Prefix P of Y is called propper prefix if $P \neq \epsilon$ and $P \neq Y$

Reverse

- For any string W, the reverse W^R is reverse of W.
- If $W = W^R$ then it is a palindrome.

Sets

- A set is an **unordered** collection of **distinct** elements.
- A set of string is a language
  - A **language** can be **finite** or **infinite**
    But **alphabets** and **string** are **finite**.
- For any two sets A and B, write $A \subseteq B$ to say A is a subset of B.
  - Formal definition: $x \in A \implies x \in B$
  - Two sets A and B are equal, $A \subseteq B$ and $B \subseteq A$

- The size of a finite ser S is the number of elements in S, written as $|S|$

Union, Intersection, Substraction, Complement, Product, Concentation of language

Name	Def
Union	$A \cup B = \{x \| (x \in A) \bigvee (x \in B)\}$
Intersection	$A \cap B = \{x \| (x \in A) \bigwedge (x \in B)\}$
Substraction	$A - B = \{x \| (x \in A) \bigwedge (x \notin B)\}$
Complement	If U is the set of Universal element $\bar{A} = U - A$
Product	$A \times B = \{(a, b) \| (a \in A) \bigwedge (b \in B)\}$
Concatenation of Language	$AB = A \cdot B = \{W_1 \cdot W_2 \| (W_1 \in A) \bigwedge (W_2 \in A)\}$

Powers of Language

- A^K denotes the set of string obtained by concatenating A with itself K times.

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- $A* = A^0 \cup A^1 \cup A^2 \cup A^3 \cup A^4 \cup A^5 \cup A^6 \cdots$
  - $A*$ contains an infinite number of strings, but each string in $A*$ is finite

DeMorgan's laws

For any sets A and B

- $\overline{A \cup B} = \bar{A} \cap \bar{B}$
- $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Strings vs. Sets

Strings	Sets
Order matters $the \neq teh$	no ordering $\{t, h, e\} = \{t, e, h\}$

Strings	Sets
Repetations allowed `0100` is a valid string	no repetitions {0, 1, 0, 0} is not a set

Function Notation

- function takes one input and produces one output. It must be well defined.