A1

Contents

Q1

W_1 and W_3 not found for L^4
W_3 not found for L^*

Q2

❓ When calculating L^* do we need to consider $L^0 = \epsilon$ in the shortlex order
ϵ, 01, 010, 100, 0101, 01001, 01010, 01100, 10001, 010010, 010100, 010101, 100010, 100100, 0100101, 0101001, 0101010, 0101100, 0110001

Q3

A^*B^* = $\{0\}^*\{1\}^*$ = Strings 0^n1^m where $n, m \geq 0$
$(AB)^* = (01)^*$ = Even length strings, with equal number of 0 and 1, starting with 0, ending in 1, and no consecutive same symbols.
$(A \cap B)^*$ = set of empty string.

$(A \cup B)^* AB$ = Binary string ending in 01.
$(A^* \cup AB)^* - (BAB)^*$ = Binary String that don't end with 1 and no consecutive 1 in the string.

Q4

a. $G \times (H \cup K) = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3)\}$
b. $G \times (H \cap K) = \{a,b\} \times \{2\} = (a,2),(b,2)$
c. $\mathscr{P}((G \times H) \cap (G \times K)) = \{\phi, \{(a,2),(b,2)\}, \{(a,2)\}, \{(b,2)\}\}$

Q5

💢 Try proof by cases

- Let's L is an arbitrary language. L = $\{x_1, x_2, ...\}$ with length greater than 1.
- Now for strings x_1 and x_2 in L, by the given definition we $x_1 \cdot x_2 \in L$. ....(1)
  - If that's not true then L is not repeat avoiding. [case 1]
  - If that's true then **also** $x_1 \cdot x_2 \cdot x_1 \in L$. [case 2]
    - If that's not true then L is not repeat avoiding. [case 2.1]
    - If that's true then $x_1 \cdot x_2 \cdot x_1 \cdot x_2 \in L$. [case 2.2]
      - If case 2.2 not true false then L is not repead [case 2.2.1]
      - If case 2.2 is true then proposition 1 is false, as $x_1 \cdot x_2 \in L$ and thus viewing all cases it

Refined

Let's L is an arbitrary language. L = $\{x_1, x_2, ...\}$ where $|L| > 1$.
Let's consider 2 strins x_1 and x_2, where $x_1 \in L\ and\ x_2 \in L$
Now We consider 2 cases,
Case 1: $x_1 \cdot x_2 \in L$
If case 1 is true then,
By proposition 2, $x_1 \cdot x_2 \cdot x_1$ should be in L
case a $x_1 \cdot x_2 \cdot x_1 \in L$
If case a is true then, by propostion 2, $x_1 \cdot x_2 \cdot x_1 \cdot x_2 \in L$
then it goes against propostion 1 therefore L is not repead avoiding.
case b $x_1 \cdot x_2 \cdot x_1 \notin l$
If Case B then L is not repeat avoiding
Case 2: $x_1 \cdot x_2 \notin L$
If case 2 is true then L is not repeat avoiding.

Q6

$S = \{00, 10, 010, 01001\}$

a

Picking the longest:
010010 $\in S^*$
$010010 \in S^*$

$\because$ the string is concatenation of 010 to itself and $010 \in S$
First picking the longest prefix 01001 from S,
We have ~~01001~~0, After this we are left 0, and $0 \notin S$, therefore for W = 010010 the algorithm returns No, though the string $010010 \in S^*$

b

01001 = this is the last string in S.
$01001 \in S^*$

$01001 \in S, \therefore 01001 \in S^*$
First picking the shortest prefix $W_1 \in S$ such that $W_1 \cdot y = 01001$

We picked 010, therefore the string becomes ~~010~~01, but $01 \notin S$ therefore we couldn't cross off all symbols of string W. Hence our algorithm output is No. But as we have showed before string $W \in S^*$

Q7

a

W_1, W_2 two strings $|W_1| \geq 1$ and $|W_2| = n$

Base case: For n = 1,
W_2^R = W_2 Since $x^R = x$

> ❓ If $|W_2| = 1$, Can we apply $(W_1 \cdot W_2)^R = W_2 \cdot W_1^R$ **DONE**

$(W_1 \cdot W_2)^R = W_2 \cdot W_1^R$

Inductive Hypothesis: For n = m,
We assume, for W_1, W_m where W_1 is an arbitrary string and $|W_m| = m$,

$(W_1 \cdot W_m)^R = W_m^R \cdot W_1^R$

Inductive Step

Now let's consider for W_{m+1} where, $|W_{m+1}| = m + 1$,
We need to proof, $(W_1 \cdot W_{m+1})^R = W_{m+1}^R \cdot W_1^R$
Lets consider $W_{m+1} = W_q \cdot x$ and $|W_q| = m$

$(W_1 \cdot W_{m+1})^R = (W_1 \cdot W_q \cdot x)^R$
let, $W_1 \cdot W_q = W$
Then, $(W_1 \cdot W_q \cdot x)^R = (W \cdot x)^R$
$\implies (W_1 \cdot W_q \cdot x)^R = xW^R$ [$\because$ Statement (2)]
$\implies (W_1 \cdot W_q \cdot x)^R = x(W_1 \cdot W_q)^R$ By replacing value of W with $W_1 \cdot W_q$

Using our assumption,
$\implies (W_1 \cdot W_q \cdot x)^R = x(W_1 \cdot W_q)^R = x \cdot W_q^R \cdot W_1^R$ **[Both W_q, W_m are arbitrary string with same length]**

$(W_q \cdot x)^R = x \cdot W_q^R$
$\implies (W_{m+1})^R = x \cdot W_q^R$

Hence, $(W_1 \cdot W_q \cdot x)^R = x(W_1 \cdot W_q)^R = x \cdot W_q^R \cdot W_1^R$
$\implies (W_1 \cdot W_q \cdot x)^R = x(W_1 \cdot W_q)^R = (W_{m+1})^R \cdot W_1^R$
Start is here iam going to be here.

b

Base Case
$(W_1 \cdot W_2)^R = W_2^R \cdot W_1^R$

Induction Hypothesis
$(W_1 \cdot W_2 \cdots W_k)^R = W_k^R \cdots W_2^R \cdot W_1^R$

We need to prove, $(W_1 \cdot W_2 \cdots W_k \cdot W_{k+1})^R = W_{k+1}^R \cdot W_k^R \cdots W_2^R \cdot W_1^R$

L.H.S = $(W_1 \cdot W_2 \cdots W_k \cdot W_{k+1})^R$
= $(W \cdot W_{k+1})^R$, where $W = W_1 \cdot W_2 \cdots W_k$
= $W_{k+1}^R \cdot W^R$ [$\because proposition\ 1$]
= $W_{k+1}^R \cdot (W_1 \cdot W_2 \cdots W_k)^R$ [replacing W with it's value]
= $W_{k+1}^R \cdot W_k^R \cdots W_2^R \cdot W_1^R$ [Applying induction hypothesis]
= R.H.S

c

❔ Since k has no upper limit, kinda like limit can we use Induction?
❔ Is **x non-empty**?
❔ We can't use the definition of reverse, but can we use the definition of palindrome? that is, **if X is a palindrome then $X^R = X$**
❔ For a power of string, can we say $x^m \cdot x = x \cdot x^m$
❔ Can we use $|X^R| == |X|$
Proof If X is a palindrome, then X^n is a palindrome.

If X is a palindrome,
Base Case: for n = 1,

x^n = x, x,
there fore x^n is a palindrome.

Induction Hypothesis for n = m
Assume, the statement is true. i.e. If x is a palindrome, then x^m is also a palindrome.

induction step for n = m+1
We need to prove, the statement is true.

$x^{m+1} = x^m \cdot x$
$\therefore (x^{m+1})^R = (x^m \cdot x)^R = x^R \cdot (x^m)^R$ [**from 4a**]
$\therefore (x^{m+1})^R = x \cdot x^m$ [**Since x is a plaindome and using Induction hypothesis**]
$\therefore (x^{m+1})^R = x^{m+1}$
Hence x^{m+1} is a palindrome.

d

Contrapositive: If x is not a palindrome then x^n is not a palindrome.
Base case
for n = 1,
$x^1 = x$, hence if x is not a palindrome then $x \neq x^R$

Induction Hypothesis for n = m
Assune if x is not a palindrome then, x^m is not a palindrome

induction step for n = m+1,
$x^{m+1} = x^m \cdot x$
$\therefore (x^{m+1})^R = (x^m \cdot x)^R = x^R \cdot (x^m)^R$ [**from 4a**]

Now $x^{m+1} = x^m \cdot x = x \cdot x^m$
Between $x^R \cdot (x^m)^R$ and $x \cdot x^m$ the suffix (x$^{m)}$R and x^m is not same and it has same length,

therefire we can conclude that $x^R \cdot (x^m)^R \neq x \cdot x^m$ or $x^R \cdot (x^m)^R \neq x^{m+1}$
hence $x^{m+1} \neq (x^{m+1})^R$, therfore x^{m+1} is not a palindrome.