1

a

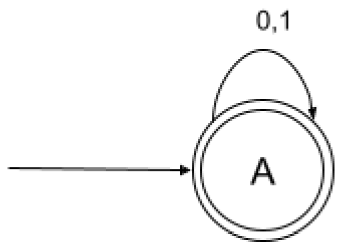
True
Let L is a finite language, and Let |L| = n, for some $n \geq 0$
For n = 0, $L = \phi$ and it is regular.
For $n > 0$,
lets , $L = \{a_1, a_2, a_3, ...a_n\}$ where $a_k\ for\ k \geq 1$ is the individual string in L.
Now, for each string $a_k \in L$ we generate a new Language, $L_k = a_k$ for $k \geq 1$.
We can draw a DFA for a language that has a single string. therefore all L_k for $k \geq 1$ is regular.
Since, $L = L_1 \cup L_2 \cup ..... \cup L_k$ and set of regular language is closed under Union operation.
therefore L is a regular language.

b

False
We will disprove the satement with an example,
Let, $L = \{W \in \{0, 1\}^*$| W contains an even number of 0's$\}$
This is a regular language as we have constructed a DFA for this language, in Lecture 7, but it is a infinite language.
Therefore there exists a regular language that is finite.

c

False
We will disprove the satement with an example,
Language of all binary string is regular, we can draw a DFA. with one state.
$L = \{W \in \{0, 1\}^*\}$, L is regular. As shown by the DFA below.
DFA:



But $L' = \{W \in \{0,1\}*|W = 0^k 1^k, \text{for } k \geq 0\}$ is not regular though $L' \subset L$
Hence the statement is false.

d

False
We will disprove the satement with an example,

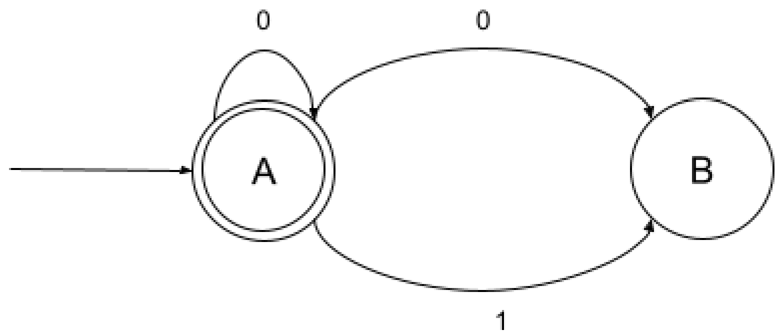
Lets consider infinitely many language with only one string, Such that, $L_k = 0^k 1^k$ where, $k \geq 0$,
For All $k \geq 0$ such L_k is regular as a language with one string is regular.
But if we take union of such infinite number of language where, $L = L_0 \cup L_1 \cup L_2 \cup L_3 \cup .... = \{W \in \{0,1\}*|W = 0^k 1^k, \text{for } k \geq 0\}$ Which we have proved that a non regular language in lecture 10.
Hence the statement is false.

e

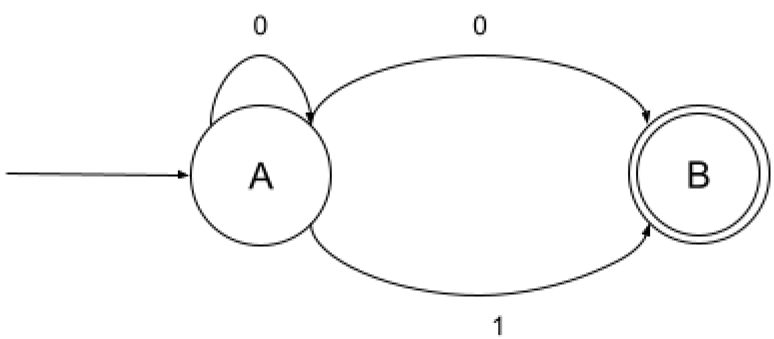
True
Let, We have n regular languages, $L_1, L_2, L_3, ..., L_n$
We consider , $L = L_1 \cup L_2 \cup L_3 \cup ...... \cup L_n$
Since, Regular language are closed under union, therefore L is regular

f

False
We will disprove the satement with an example,



NFA 1



NFA 2

NFA 2 is generated from NFA 1 using the algorithm described in the statement.

Here If we pick a string $x = 0$ then x is accepted by both the NFA's.
Hence NFA 2 is not valid NFA for complement of L where L is accepted by NFA 1.

g

False
We will disprove the satement with an example,
L = {00, 1, 10, 11}
A = 00
B = 1
X = 0

Accordign to definition string Equivalence, $\forall$,X if $AX \in L$ if and only if $BX \in L$, but in our example, $AX \notin L$ but $BX \in L$, Hence the statement is false.

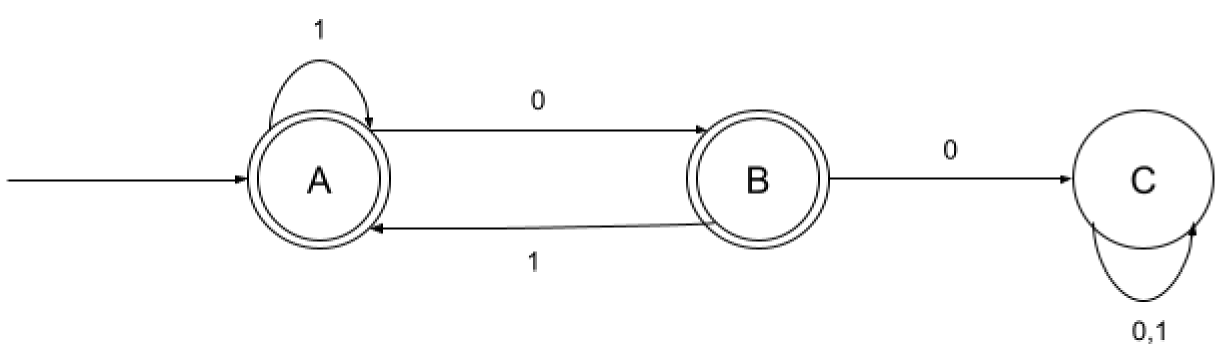
2

a

$\mathscr{L}(M) = \{W \in 0,1^* | W\ has\ no\ consecutive\ 0's\}$

b

> We will reference this diagram for our proof. This DFA is constructed from the given DFA description.



After running the DFA on input W, the following three Invariants are true.
Invariant 1: If the current state is A, then W has no consecutive 0's and if $|W| \geq 1$ then, the last symbol of Y is 1
Invariant 2: If the current state is B, then W has no consecutive 0's and last symbol of W is 0.
Invariant 2: If the current state is C, then W has at least one instance of consecutive 0's.

We will prove that the, the DFA accepts if $W \in \mathscr{L}(M)$

Proof:

We proceed by induction on the length of W.

Base case

For the base case, $W = \epsilon$, notice that we are in $A$and W has no 2 consecutive 0's. hence out invariant 1 is true.

When $W = \epsilon$ we are not in state B and C, so invariant 2 and 3 are vacuously true.

Induction hypothesis

Assume that, after running the DFA on any string Y of length $k \geq 0$, the following statements are true:

1. If the current state A, then Y has no consecutive 0's and if $|Y| \geq 1$ then, the last symbol of Y is 1.
2. If the current state B, then Y has no consecutive 0's. Last symbol of Y is 0.
3. If the current state C, then Y has at least one consecutive 2 0's.

Inductive step

Let's consider a string W of length $k+1$, such that, $W = Y \cdot z$, where z is the final symbol of W.

Invariant 1

Suppose that after reading $W = Y \cdot z$, the current state is A,
From the diagram of machine, we conclude that the machine was in state A or B after reading Y,
For z = 1, we consider,
Case 1: consider the machine was in State A,
By induction hypothesis, Y has no consecutive 0's, hence $W = Y \cdot z$ also has no consecutive 0's.
Case 1: consider the machine was in State B,
By induction hypothesis, Y has no consecutive 0's, hence $W = Y \cdot z$ also has no consecutive 0's.

Invariant 2

Suppose that after reading $W = Y \cdot z$, the current state is B,
From the diagram of machine, we conclude that the machine was in state A after reading Y,
For z = 0,
By induction hypothesis,
Y has no consecutive 0's and the last symbol of Y is 1. therefore, $W = Y \cdot z$ has no consecutive 0's.

Invariant 3

Direct Proof: Suppose that after reading $W = Y \cdot z$, the current state is C,
There 2 cases of z for whcih we can be in state c

Case 1

z = 0
From the diagram of machine, we conclude that the machine was in state B or C after reading Y,

- **Case 1.1** (machine was in state B)
  - By induction hypothesis, Y has no 2 consecutive 0 and the last symbol of the string is 0.
  - Since z = 0, $W = Y \cdot z$ has at least one consecutive 0's.
- **Case 1.2** (machine was in state C)
  - By induction hypothesis, Y has at least one consecutive 2 0's.
  - therefore $W = Y \cdot z$ also has at least one consecutive 2 0's.

Case 2

z = 1
From the diagram of machine, we conclude that the machine was in state C after reading Y,

By induction hypothesis Y has at least one consecutive 0's.
therfore $W = Y \cdot z$ has at least one consecutive 0's.

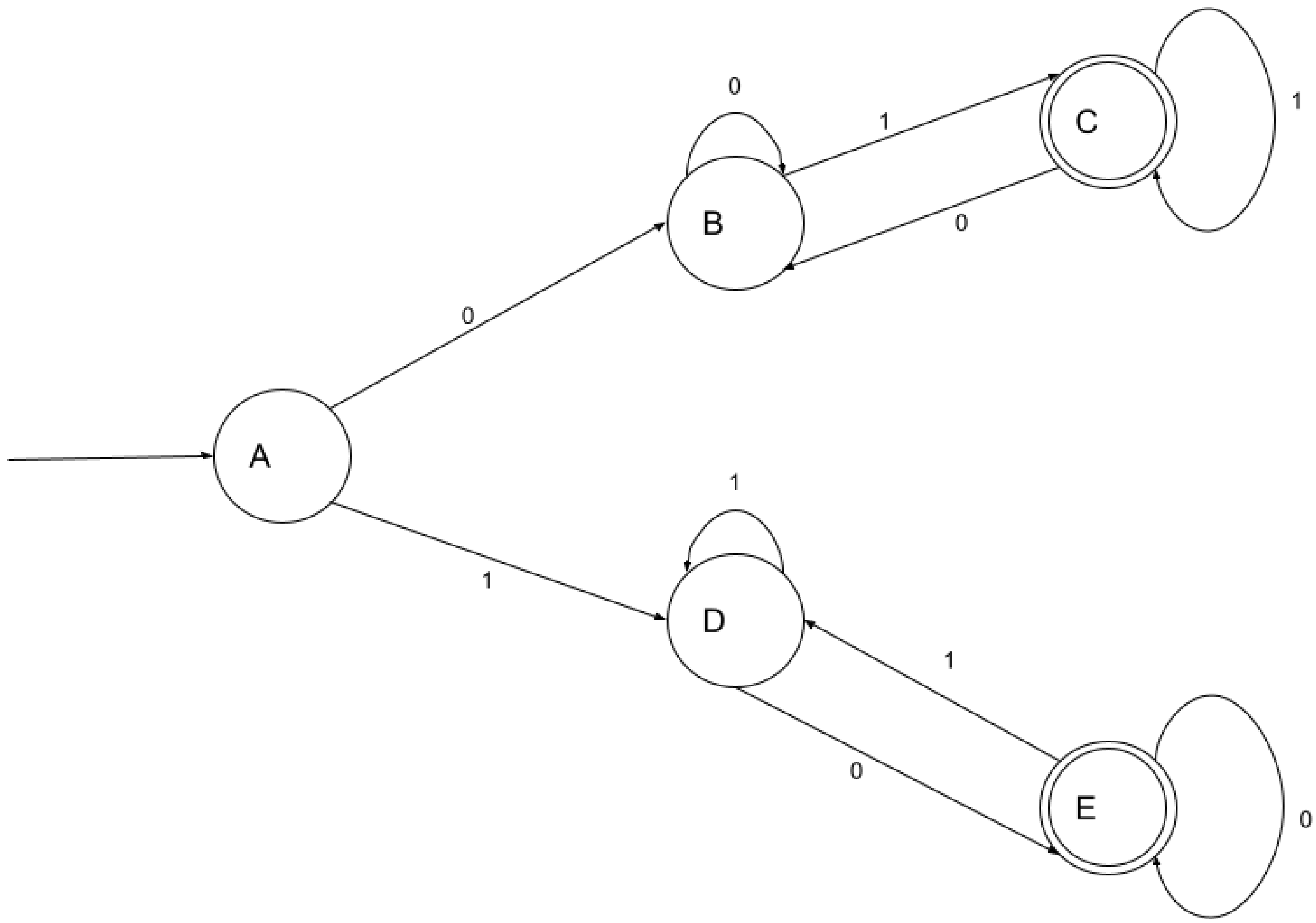
Hence by induction principle all the invariants are proved.

- From the machine description we see that the accepting states are $A,\ B$
- We proved invariant 1: if the machine is in state **A**, then the string has no consecutive 2 0's.
- We proved invariant 2: if the machine is in state **B**, then the string has no consecutive 2 0's.
- We proved invariant 3: if the machine is in state **C**, then the string has at least 1 consecutive 2 0's.

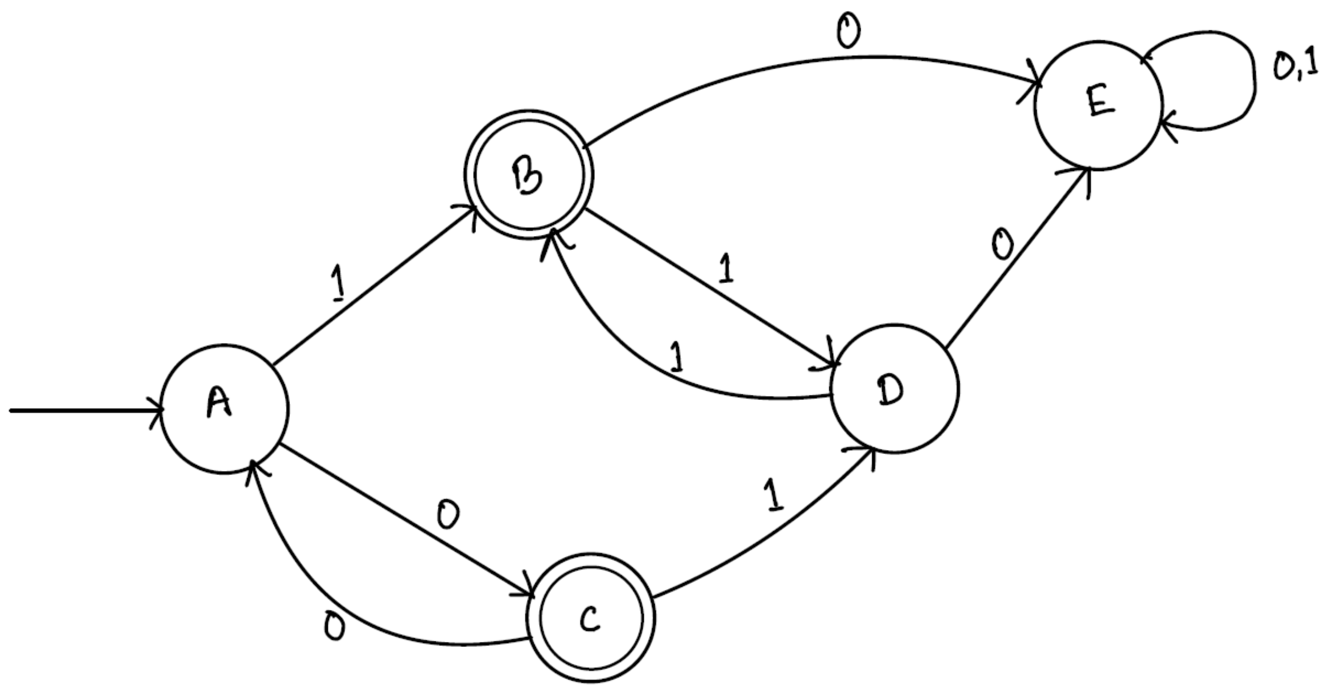
In other words M accepts $W \implies W$ has no consecutive 0's, and, M rejects $W \implies W$ has consecutive 0's. Thus proving our statement in part a.

3

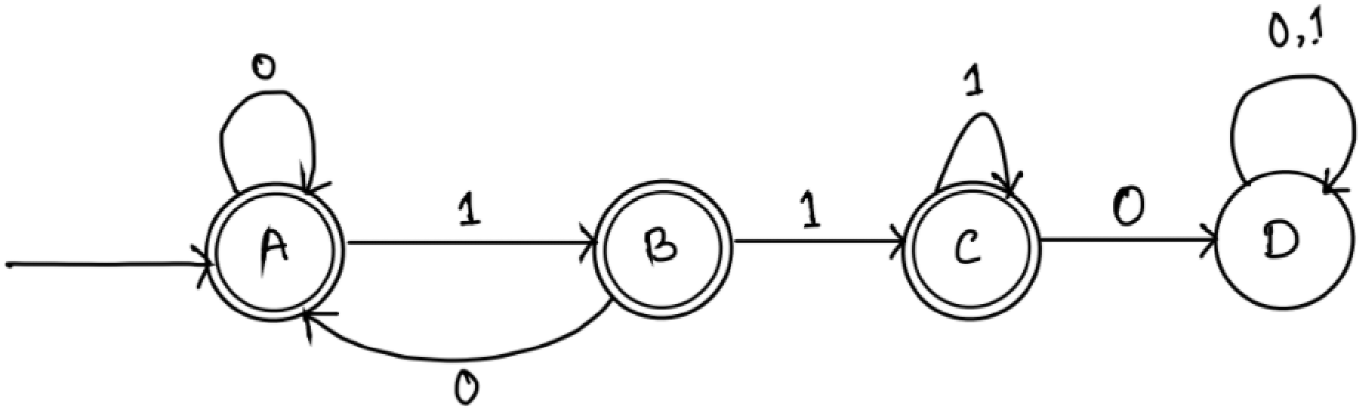
a



b



c



4

a

All the Myhill-Nerode equivalence classes induced by L are as follows,

$$C_1 = \{W \in \{0,1\}^* \| W = 1^k \; where \; k \geq 1\}$$
$$C_2 = \{W \in \{0,1\}^* \| W = \epsilon\}$$
$$C_3 = \{W \in \{0,1\}^* \| W = 0^k \; where \; k \geq 1\}$$
$$C_4 = \{W \in \{0,1\}^* \| W = 1^k 0 \; where \; k \geq 1\}$$
$$C_5 = \{W \in \{0,1\}^* \| W = 1^k 0 \; where \; k \geq 1\}$$

Two prove these are the Myhill-Nerode equivalence classes of L,
We need to prove that,

1. All strings in C_1 are equivalent to each other with respect to L.
2. All strings in C_2 are equivalent to each other with respect to L.
3. All strings in C_3 are equivalent to each other with respect to L.
4. All strings in C_4 are equivalent to each other with respect to L.
5. All strings in C_5 are equivalent to each other with respect to L.
6. There exists, a string in C_1, that is not equivalent to C_2 with respect to L.
7. There exists, a string in C_1, that is not equivalent to C_3 with respect to L.
8. There exists, a string in C_1, that is not equivalent to C_4 with respect to L.
9. There exists, a string in C_1, that is not equivalent to C_5 with respect to L.
10. There exists, a string in C_2, that is not equivalent to C_3 with respect to L.
11. There exists, a string in C_2, that is not equivalent to C_4 with respect to L.
12. There exists, a string in C_2, that is not equivalent to C_5 with respect to L.
13. There exists, a string in C_3, that is not equivalent to C_4 with respect to L.
14. There exists, a string in C_3, that is not equivalent to C_5 with respect to L.
15. There exists, a string in C_4, that is not equivalent to C_5 with respect to L.

We will use the following statement to check if a String belongs in L or not.
If a string is of the form 0^n where $n \geq 0$ or of the form 1^m0 where $m \geq 1$ then the string is in L.

Proof: All strings in C_1 are equivalent to each other with respect to L.
Let A and B be 2 arbritary string in C_1,
Consider string X, where
case 1 X is of the form 1^k0 where $k \geq 0$
then both AX and BX are of the form 1^m0 where $m \geq 1$
therefore, AX and $BX \in L$
case 2 X is not of the form 1^k0 where $k \geq 0$
then none of the AX or BX are of the form 1^m0 where $m \geq 1$ or 0^n where $n \geq 0$
therefore, AX, $BX \notin L$
This concludes the proof that an arbitrary A and B from C_1 are equivalent with respect to L.

Proof: All strings in C_2 are equivalent to each other with respect to L.
Since C_2 has only one element therefore the statement is vacuously true

Proof: All strings in C_3 are equivalent to each other with respect to L.
Let A and B be 2 arbritary string in C_3,
Consider string X, where
case 1 $X \in \{W \in \{0,1\}^* | W = 0^n \ for \ n \geq 0\}$

then both AX and BX are of the form 0^m where $m \geq 0$ and thus $AX, BX \in L$
case 2 X contains at aleast one 1 as symbol.
then none of the AX or BX are of the form 1^m0 where $m \geq 1$ or 0^n where $n \geq 0$
therefore, both AX and BX are not in L, as for a string starting with 0 can only contain 0, but X has at least 1 and thus both AX and BX at least one or more 1 and L has no such string.

This concludes the proof that an arbitrary A and B from C_3 are equivalent with respect to L.

Proof: All strings in C_4 are equivalent to each other with respect to L.

Let A and B be 2 arbritary string in C_4,
Consider string X, where
case 1 X = ϵ
then both AX and BX are of the form 1^m0 where $m \geq 1$

case 2 X a non empty string,
then none of the AX or BX are of the form 1^m0 where $m \geq 1$ or 0^n where $n \geq 0$

therefore, both AX and $BX \notin L$

This concludes the proof that an arbitrary A and B from C_4 are equivalent with respect to L.

Proof: All strings in C_5 are equivalent to each other with respect to L.
Let A and B be 2 arbritary string in C_5,
Consider an arbritary string X,
then none of the AX or BX are of the form 1^m0 where $m \geq 1$ or 0^n where $n \geq 0$

i.e. for all X AX and $BX \notin L$
This concludes the proof that an arbitrary A and B from C_5 are equivalent with respect to L.

Proof: There exists, a string in C_1, that is not equivalent to C_2 with respect to L.
We chose, A = 11 from C_1 and B = ϵ from C_2,
for X = ϵ
$A \cdot X \notin L$
$B \cdot X \in L$
therfore, C_1, C_2 forms seperate equivalence class

Proof: There exists, a string in C_1, that is not equivalent to C_3 with respect to L.
We chose A = 11 from C_1 and B = 00 from C_3,
for X = 00
$A \cdot X \notin L$
$B \cdot X \in L$

therfore, C_1, C_3 forms seperate equivalence class

Proof: There exists, a string in C_1, that is not equivalent to C_4 with respect to L.
We chose A = 11 from C_1 and B = 110 from C_4,
for X = 0
$A \cdot X \in L$
$B \cdot X \notin L$
therfore, C_1, C_2 forms seperate equivalence class

Proof: There exists, a string in C_1, that is not equivalent to C_5 with respect to L.
We chose A = 11 from C_1 and B = 101 from C_5,
for X = 0
$A \cdot X \in L$
$B \cdot X \notin L$

therfore, C_1, C_5 forms seperate equivalence class

Proof: There exists, a string in C_2, that is not equivalent to C_3 with respect to L.
We chose A = ϵ from C_2 and B=0 from C_3,
for X = 10
$A \cdot X \in L$
$B \cdot X \in L$
therfore, C_2, C_3 forms seperate equivalence class

Proof: There exists, a string in C_2, that is not equivalent to C_4 with respect to L.
We chose A = ϵ from C_2 and B = 10 from C_4,
for X = 0
$A \cdot X \in L$
$B \cdot X \notin L$
therfore, C_2, C_4 forms seperate equivalence class

Proof: There exists, a string in C_2, that is not equivalent to C_5 with respect to L.
We chose A = ϵ from C_2 and B = 101 from C_4,
for X = 0
$A \cdot X \in L$
$B \cdot X \notin L$
therfore, C_2, C_5 forms seperate equivalence class

Proof: There exists, a string in C_3, that is not equivalent to C_4 with respect to L.
We chose A = 00 from C_3 and B = 10 from C_4,
for X = 0

$A \cdot X \in L$
$B \cdot X \notin L$
therfore, C_3, C_4 forms seperate equivalence class

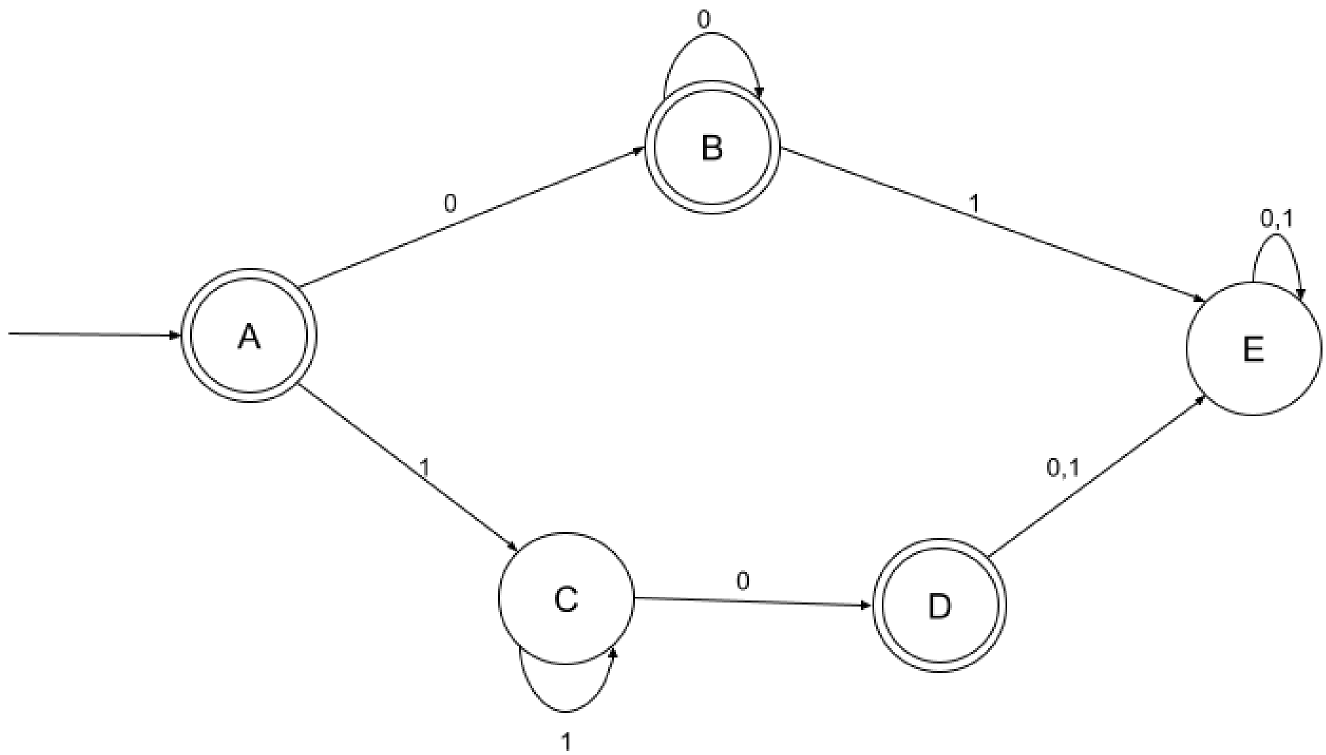
Proof: There exists, a string in C_3, that is not equivalent to C_5 with respect to L.
We chose A = 11 from C_3 and B = 101 from C_5,
for X = 0
$A \cdot X \in L$
$B \cdot X \notin L$
therfore, C_3, C_5 forms seperate equivalence class

Proof: There exists, a string in C_4, that is not equivalent to C_5 with respect to L.
We chose 10 from C_4 and 101 from C_5,
for X = ϵ
$A \cdot X \in L$
$B \cdot X \notin L$
therfore, C_4, C_5 forms seperate equivalence class

Thus we have proved that, C_1, C_2, C_3, C_4, C_5 are the 5 Myhill-nerode equivalence classes induced by the given L.

b

The smalled DFA of L has 5 states.
We are drawing the corresponding DFA with 5 states.


0
B
0
1
0,1
A
E
1
0,1
C
0
D
1