

**CSE221**  
**Lab Assignment 03**  
**Fall 2023**

**Submission Guidelines:**

1. You can code all of them either in Python, CPP, or Java. But you should choose a specific language for all tasks.
2. For each task write separate python files like task1.py, task2.py, and so on.
3. For each problem, take input from files called "**inputX.txt**" and output at "**outputX.txt**", where X is the task number.
4. Add a hand written explanation of 3-4 lines for each of your solutions in a separate document. You may compile all of your explanations in a single file.
5. Finally zip all the files and rename this zip file as per this format: **LabSectionNo\_ID\_CSE221LabAssignmentNo\_Fall2023.zip**  
[Example: **LabSection01\_21101XXX\_CSE221LabAssignment03\_Fall2023.zip**]  
]
6. Don't copy from your friends.
7. You MUST follow all the guidelines, naming/file/zippping convention stated above.

*Failure to follow instructions will result in a straight 50% mark deduction.*

## **Task 01: [Points 10]**

In this problem, you will be given a list of numbers. You have to sort the list using the Merge Sort algorithm.

### **Pseudocode of Merge Sort Algorithm:**

```
def merge(a, b):
    # write your code here
    # Here a and b are two sorted list
    # merge function will return a sorted list after merging a and b

def mergeSort(arr):
    if len(arr) <= 1:
        return arr
    else:
        mid = len(arr)//2
        a1 = mergeSort(.....) # write the parameter
        a2 = mergeSort(.....) # write the parameter
        return merge(a1, a2)   # complete the merge function above
```

**Note:** You already have coded the merge function. Do you know in which task?

### **Input**

The first line contains an integer  $N$  ( $1 \leq N \leq 10^5$ ), denoting the length of Alice's sorted list. In the next line, there will be  $N$  integers separated by space.

### **Output:**

You have to sort the number using the Merge Sort algorithm and show the sorted list.

### **Sample Input/Output:**

Sample Input 1	Sample Output 1
8 9 5 4 6 1 3 2 9	1 2 3 4 5 6 9 9

Sample Input 2	Sample Output 2
1 10	10
Sample Input 3	Sample Output 3
6 8 1 4 2 1 3	1 1 2 3 4 8
Sample Input 4	Sample Output 4
7 7 6 5 4 3 2 1	1 2 3 4 5 6 7

### **Task 02 [10 Points]:**

Alice and you are playing with a list of **N** non negative integers. Today you will try to find out the maximum number of the list. Alice writes the following code to find the maximum number.

```
maxValue = arr[0]
for i in range(1,N):
    if maxValue < arr[i]:
        maxValue = arr[i]
```

Recently you have learned merge sort. Now, you are thinking if you can use the divide and conquer approach to find out the maximum from the given list.

**Please note, you are not allowed to sort the given list.** The motive for this task is not sorting but to use the concepts of divide and conquer approach.

### **Input**

The first line contains an integer  $N$  ( $1 \leq N \leq 10^5$ ), denoting the length of Alice's list. In the next line, there will be  $N$  integers separated by space.

**Output:**

You have to find out the maximum value from the list using the divide and conquer approach.

**Sample Input/Output:**

Sample Input 1	Sample Output 1
8 1 7 13 4 5 7 13 12	13
Sample Input 2	Sample Output 2
7 5 15 2 3 10 1 9	15
Sample Input 3	Sample Output 3
1 9	9
Sample Input 4	Sample Output 4
6 5 2 3 10 1 9	10

What is the time complexity of your code?

**Task 03 [10 Points]:**

Somewhere in the universe, the Biannual Regional Alien Competition is taking place.

There are **N** aliens standing in a line. You will be given a permutation of N, which denotes the height of each alien. A sequence of N numbers is called a permutation if it contains all integers from 1 to N exactly once. For example, the sequences [3,1,4,2], [1] and [2,1] are permutations, but [1,2,1], [0,1] and [1,3,4] – are not.

In the competition, for each alien, the judge wants to count how many aliens are standing on its right side with a strictly smaller height. Then the judge wants to add up all the counts. To do this, the judge writes the following piece of code.

```
count = 0
for i in range(n):
    for j in range(i+1,n):
        if H[i] > H[j]:
            count+=1
```

However, their algorithm wasn't efficient at all. Hence, the alien calls you to write a better solution for the program.

More formally, you have to count how many pairs of aliens are standing in the line such that  $H[i] > H[j]$  and  $i < j$ . Here,  $A$  is a permutation of the aliens' heights. And  $i, j$  denote the Aliens' positions.

### Input

The first line contains a single integer  $1 \leq N \leq 10^6$  - the number of total aliens.

The next line contains  $N$  integers  $H_1, H_2, \dots, H_n (1 \leq H_i \leq N)$  - the height of the  $i$ -th alien. It is guaranteed that the given heights will be a permutation of  $N$ .

### Output

Print a single integer, which denotes the total number of pairs  $(i, j)$  such that  $i < j$  and  $H_i > H_j$ .

### Sample Input/Output:

Sample Input 1	Sample Output 1
5	0

1 2 3 4 5	
Sample Input 2	Sample Output 2
5 5 4 3 2 1	10
Sample Input 3	Sample Output 3
8 2 7 4 1 5 6 8 3	11

### Sample Input 3 Explanation:

In the sample input 3, the following pairs on alien's heights satisfy the condition: (2,1), (7,4), (7,1), (7,5), (7,6), (7,3), (4,1), (4,3), (5,3), (6,3), (8,3)

### Task 04 [10 points]

You are given a list A of N integers. You have to choose two indices i and j such that  $1 \leq i < j \leq N$  and  $A[i] + A[j]^2$  is maximum possible. Here, we are considering 1-based indexing.

Write a code which will find the maximum value of  $A[i] + A[j]^2$  in  $O(N \log N)$ .

### Input

The first line contains a single integer  $1 \leq N \leq 10^6$  - the length of the list.

The next line contains N integers  $A_1, A_2, \dots, A_n$  ( $-10^8 \leq A_i \leq 10^8$ ) separated by a space.

### Output

Print a single integer - which denotes the maximum possible value of  $A[i] + A[j]^2$ .

**Sample Input/Output:**

Sample Input 1	Sample Output 1
5 9 6 5 8 2	73
Sample Input 2	Sample Output 2
8 5 10 4 -3 1 6 -10 2	110
Sample Input 3	Sample Output 3
7 -5 -2 -6 -7 -1 8 2	63

**Task 05 [10 Points]**

In this problem, you will be given a list of numbers. You have to sort the list using the Quick Sort algorithm in ascending order.

**Pseudocode of Quick Sort Algorithm:**

QUICKSORT( $A, p, r$ )

```
1  if  $p < r$ 
2       $q = \text{PARTITION}(A, p, r)$ 
3      QUICKSORT( $A, p, q - 1$ )
4      QUICKSORT( $A, q + 1, r$ )
```

PARTITION( $A, p, r$ )

```
1   $x = A[r]$ 
2   $i = p - 1$ 
3  for  $j = p$  to  $r - 1$ 
4      if  $A[j] \leq x$ 
5           $i = i + 1$ 
6          exchange  $A[i]$  with  $A[j]$ 
7  exchange  $A[i + 1]$  with  $A[r]$ 
8  return  $i + 1$ 
```

[The code snippet has been taken from the book: Introduction to Algorithms]

### Input

The first line contains an integer  $N$  ( $1 \leq N \leq 10^5$ ), denoting the length of Alice's list. In the next line, there will be  $N$  integers separated by space.

### Output:

You have to sort the number using the Quick Sort algorithm in ascending order and show the sorted list.

### Sample Input/Output:

Sample Input 1	Sample Output 1
8 9 5 4 6 1 3 2 9	1 2 3 4 5 6 9 9
Sample Input 2	Sample Output 2
1 10	10
Sample Input 3	Sample Output 3
6 8 1 4 2 1 3	1 1 2 3 4 8



Sample Input 4	Sample Output 4
7 7 6 5 4 3 2 1	1 2 3 4 5 6 7

### Task 06 [10 Points]

In this problem, you will be given a list of numbers. You have to find the **k**-th smallest value from the list without sorting using the Partition function of Quick sort.

We will consider the 1-based indexing of the list.

#### Input

The first line contains an integer  $N$  ( $1 \leq N \leq 10^6$ ), denoting the length of the list.

The next line contains  $N$  integers  $A_1, A_2, \dots, A_n$  ( $1 \leq A_i \leq 10^6$ ) separated by a space.

The third line contains a single integer  $Q$  ( $1 \leq Q \leq 100$ ) - which denotes the number of queries you have to answer.

Each of the next  $Q$  lines will contain a single integer  $K$  ( $1 \leq K \leq N$ ).

#### Output:

For each query, you have to find the  $K$ -th smallest number from the given list.

#### Sample Input/Output:

Sample Input 1	Sample Output 1
9 // Total Elements 10 11 10 6 7 9 8 15 2 4 // Total queries	9 7 6

5 3 2 7	10
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