MATHCOUNTS®

2025-2026 School Handbook

RESOURCES + INFO EVERY COACH NEEDS

Look through this handbook and your School Competition Kit.

Log in to your ***Coach Dashboard*** (instructions emailed to you).

Hold weekly, biweekly or monthly practices with your Mathletes.
Use *Coach Dashboard* resources to:

Recruit Students

- How to promote competition math in your school
- How to support competitors of different backgrounds and skill levels

Run Meetings

- What to do during a math team practice
- Suggested calendars
- Tips for making practices fun and easy to plan

Get Math Resources

- Problem of the Week
- Pre-written Practice Plans
- Mini video lessons
- How to use this handbook during team practices

Hold the 2026 School Competition (PDF available Nov. 6, 2025 on the *Coach Dashboard*)

Get ready for the Chapter Competition! On your *Coach Dashboard*, find:

- Past School + Chapter competitions
- Chapter coordinator contact info
- How to add competitor info for the chapter event
- What to expect on Competition Day

Take your Mathletes to the 2026 Chapter Competition!

LOG IN TO THE COACH DASHBOARD

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2025-2026 COMPETITION SERIES COACHES!
FIND PROBLEMS #201-250 (+ THEIR SOLUTIONS) AT
mathcounts.org/coaches

THIS YEAR'S HANDBOOK PROBLEMS

11 WARM-UPS
10 questions per Warm-Up
no calculators used

Warm-Ups prepare students particularly for the Sprint Round.

6 WORKOUTS
10 questions per Workout
calculators used

Workouts prepare students particularly for the Target Round.

3 STRETCHES
Number of questions and use of calculators vary by Stretch

Each Stretch covers a particular math topic that could be covered in any round. These help prepare students for all rounds: Sprint, Target, Team or Countdown.

vs

Throughout this handbook, look for special Countdown Round problems top Mathletes tackled in last year's Chapter C and State S competitions.

HIGHLIGHTED RESOURCES

Coaching made easy for as little as

$20.42 a month

Get a full year of access to OPLET for just $295.

Competition Series schools get a $5 discount for each registered chapter competitor (up to $50 off)!

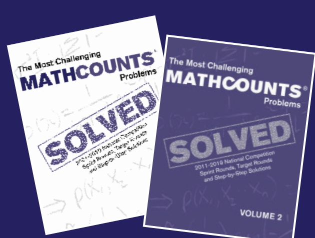
OPLET is...

an amazing **problem library** of over 15,000 past MATHCOUNTS competition and handbook problems.

Coaches, Mathletes + teachers use OPLET to create customized quizzes, flashcards, worksheets and Problems of the Day in a snap!

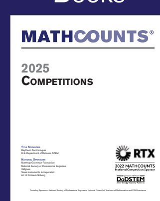
mathcounts.org/myoplet

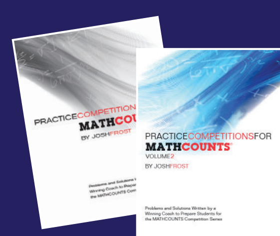
Level up your MATHCOUNTS practice!



Most Challenging MATHCOUNTS Problems Solved Vol. 1 & Vol. 2

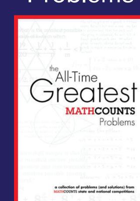
MATHCOUNTS Competitions Books





Practice Competitions for MATHCOUNTS Vol. 1 & Vol. 2

All-Time Greatest MATHCOUNTS Problems



mathcounts.org/store

WONDERING WHERE TO START?

FIND MORE RESOURCES AT **MATHCOUNTS.ORG/COACHES**

GUIDE FOR NEW COACHES AT **MATHCOUNTS.ORG/NEWCOACH**

2025–2026 CRITICAL DATES

Aug. 15	Registration and kit fulfillment begin for both MATHCOUNTS programs. Register at mathcounts.org/reg.
Oct. 31	**Silver and Gold Level Applications open** on the Club Leader Dashboard (mathcounts.org/dashboard).
Nov. 5	**Early Bird Registration Deadline**: $40/student for schools
Nov. 5	**2026 School Competition** released to coaches on the Coach Dashboard (mathcounts.org/dashboard).
Nov. 5	**Non-school competitor (NSC) registration opens** for students whose official school of record will not register for the Competition Series and will not support their participation through the school.
Dec. 15	**Regular Registration Deadline:** $45/student for schools and $70/NSC. After this date, registration will cost $50/student for schools and $80/NSC. While many local coordinators will accept new registrations after this date, please register on time to ensure your students' participation and avoid late fees.
early Jan.	If you haven't received details about your upcoming chapter competition from your chapter coordinator or on the Coach Dashboard, **contact your local or state coordinator.** Find coordinator contact info at mathcounts.org/findmycoordinator.
Feb. 1-28	**2026 Chapter Competitions:** Coaches must complete their Competitor Roster on the Coach Dashboard before their chapter competition (specific date determined by the local coordinator).
Mar. 1-31	**2026 State Competitions**
Apr. 15	**Deadline to qualify for Silver and Gold Level prizes and drawings.** Find applications and info on the Club Leader Dashboard.
May 10-11	**2026 RTX MATHCOUNTS National Competition** in Orlando, FL
May 15	**Silver and Gold Level drawing winners and Gold Level Honor Roll announced.**

Chance to Win Stretch

1. __________ % Kima flips a fair nickel twice and will win \$2 only if both flips come up tails. What is the probability that Kima will win \$2? Express your answer as a percent.

2. __________ Raja has a fair standard six-sided die with four sides labeled “4” and two sides labeled “2.” He gives the die to Ravi and tells him to roll it twice. Raja tells him if he rolls a 4 followed by a 2, he will win \$42. What is Ravi’s chance of winning the \$42? Express your answer as a common fraction.

3. __________ Rosalie, Sara and Trent each flip three fair coins. A player wins a free ticket to the zoo if all three of their coin flips match (all heads or all tails). What is the probability that exactly one of them wins a ticket? Express your answer as a decimal to the nearest hundredth.

4. __________ A bowl contains two red, two green and two purple candies. Christopher randomly selects two candies without replacement. If he does not pick both a red and a green candy, he wins two extra candies. If the odds of winning to not winning is given as ratio a:b in simplest form, what is the value of $a - b$?

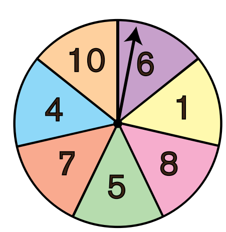
5. __________ Blake has three cards—one labeled Y, one labeled T and one labeled O. He shuffles them and places them face down. He then flips them over one at a time, from left to right. If the cards appear in the order T-O-Y, he wins a toy from The Toy Temple. What is the probability that Blake wins? Express your answer as a common fraction.

6. __________ Mark and Mary each roll a fair standard six-sided die simultaneously. The player with the higher number wins the round. If they roll the same number, they roll again until there is a winner. What is the probability that Mary wins given that Mark rolls a 2? Express your answer as a common fraction.

7. __________ A randomly selected card from a shuffled standard 52-card deck lands face down on the floor. If Granger correctly guesses that it is a face card (Jack, Queen and King), he wins two new decks of cards. What is the probability that Granger wins? Express your answer as a common fraction.

8. __________ Karina randomly selects two cards, without replacement, from a standard 52-card deck. She wins free tickets to her two favorite rides at the county fair if the cards are either the same color or the same rank (2–10, J, Q, K, A). What is the probability that Karina wins? Express your answer as a decimal to the nearest hundredth.

9. __________ % The spinner shown is divided into seven equal sections labeled with numbers. A player calls out “even” or “odd” before spinning, and if the spinner lands on a number matching the call, the player wins Jenna calls out “even” and spins. What is the probability that she wins? Express your answer to the nearest percent.

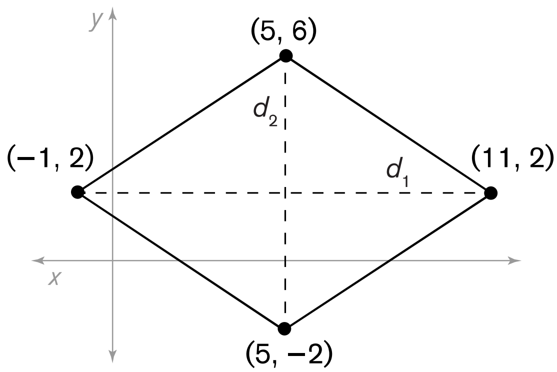


10. __________ A fair standard six-sided die has each side painted a different color. Sara rolls the die three times, and she wins two movie tickets if all three rolls show different colors. What is the probability that Sara wins? Express your answer as a common fraction.

Area on the Coordinate Plane Stretch

Consider the equation $2|x-5| + 3|y-2| = 12$. Since both $2|x-5|$ and $3|y-2|$ are always non-negative, their maximum values must occur when the other term is zero, as their sum must always equal 12.

We can start by analyzing the x-term. If $3|y-2| = 0$, then $y = 2$. That means $2|x-5| = 12$, so $|x-5| = 6$, and $x = -1$ or $x = 11$. This gives us the two points **(−1, 2)** and **(11, 2)**, the left and right vertices. Now we can analyze the y-term. If $2|x-5| = 0$, then $x = 5$. That means $3|y-2| = 12$, so $|y-2| = 4$, and $y = -2$ or $y = 6$. This gives us the two points **(5, −2)** and **(5, 6)**, the top and bottom vertices.



These four points form a diamond-shaped figure. We can use its diagonals to find the area. The horizontal diagonal connects the left and right vertices and has length $d_1 = \sqrt{(-1-11)^2 + (2-2)^2} =$ **12 units**. The vertical diagonal connects the bottom and top vertices and has length $d_2 = \sqrt{(5-5)^2 + (-2-6)^2} =$ **8 units**.

Now we can use the formula for the area of a diamond (or any quadrilateral with perpendicular diagonals):

$$\text{Area} = (d_1 \times d_2)/2 = (12 \times 8)/2 = \mathbf{48\ units^2}$$

11. ________ units² What is the area, in square units, of the region enclosed by $y = -|x-5| + 4$ and $y = |x-5| - 6$?

12. ________ units² What is the area, in square units, of the region enclosed by $y = |x-20|$ and $y = 10$?

13. ________ units² What is the area, in square units, of the region enclosed by $|x-3| + |y+5| = 8$?

14. ________ units² What is the area, in square units, of the region enclosed by $2|x-3| + 4|y+5| = 8$?

15. ________ units What is the absolute difference of the lengths of the diagonals, in units, of the region enclosed by $3|x+2| + 2|y-1| = 12$?

16. ________ units² What is the area, in square units, of the region enclosed by $(x-3)^2 + (y+5)^2 = 8$? Express your answer in terms of π.

17. ________ The area of the region enclosed by $|x+2| + |y-a| = b$ is 20 units². The product ab is 30. What is the value of a? Express your answer in simplest radical form.

18. ________ If the region enclosed by $2|x-3| + 5|y+8| = a$ has an area of 80 units², what is the value of a?

19. ________ units² A circle is inscribed in the figure described by the equation $5|x-3| + 12|y+5| = 60$. What is the area of this circle, in square units? Express your answer as a common fraction in terms of π.

20. ________ units² A square is inscribed in the figure described by the equation $(x+6)^2 + (y+7)^2 = 5$. What is the area of this square, in square units?

Simplifying Radicals Stretch

Square roots, cube roots and other higher-order roots are called **radicals**. Expressions containing radicals, such as $\sqrt{2}$, $4\sqrt{3}$, $\frac{\sqrt{6}}{2}$, $9+3\sqrt{7}$ and $\sqrt[3]{10}$, are typically expressed in **simplest radical form**.

Simplify radicals by factoring out perfect powers: move **perfect squares** out from under square root symbols, **perfect cubes** out from under cube root symbols and so on.

$$\sqrt{18} = \sqrt{9\times 2} = \sqrt{9}\times\sqrt{2} = 3\sqrt{2} \qquad \sqrt[3]{54} = \sqrt[3]{27\times 2} = \sqrt[3]{27}\times\sqrt[3]{2} = 3\sqrt[3]{2}$$

Eliminate radicals from denominators (**rationalize**), and simplify both the numerator and denominator if possible.

$$\frac{6}{\sqrt{3}} \rightarrow \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

21.__________ What is $\sqrt{8}$? Express your answer in simplest radical form.

22.__________ What is $\sqrt[3]{81}$? Express your answer in simplest radical form.

23.__________ What is $\sqrt{150}-\sqrt{24}$? Express your answer in simplest radical form.

24.__________ What is $\frac{\sqrt{3}}{2}\times\frac{\sqrt{6}}{3}$? Express your answer as a common fraction in simplest radical form.

25.__________ What is $\frac{1}{\sqrt{7}}$? Express your answer as a common fraction in simplest radical form.

If the denominator is a **binomial** containing a square root, use its **conjugate** to eliminate the radical. The conjugate of $(a+\sqrt{b})$ is $(a-\sqrt{b})$, and multiplying these expressions gives a **difference of squares**: $(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$.

$$\frac{5}{2-\sqrt{2}} \rightarrow \frac{5}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{5(2+\sqrt{2})}{4-2} = \frac{10+5\sqrt{2}}{2}$$

26.__________ What is $\frac{1}{3-2\sqrt{2}}$? Express your answer in the form $a+b\sqrt{c}$.

27.__________ What is $\left(\frac{2-\sqrt{2}}{2}\right)^2$? Express your answer in the form $\frac{a-b\sqrt{c}}{d}$.

28.__________ What is $\frac{3+\sqrt{2}}{\sqrt{2}}$? Express your answer in the form $\frac{a+b\sqrt{c}}{d}$.

29.__________ What is $(\sqrt{7}+\sqrt{5})\times(\sqrt{7}-\sqrt{5})$?

30.__________ What is $\dfrac{1}{1+\dfrac{1}{2+\sqrt{3}}}$? Express your answer in the form $\frac{a+b\sqrt{c}}{d}$.

Warm-Up 1

31.__________ Elmina wants to estimate the sum 5394 + 32,147 + 792 to check if her final answer is reasonable. She rounds each number to the nearest thousand before adding them. What is her estimated sum?

32.__________ An arithmetic sequence has a common difference between consecutive terms. For example, the sequence 5, 12, 19, 26, 33 is an arithmetic sequence with a common difference of 7. The sum of the first six terms in a particular arithmetic sequence is 42. If the first term is 2, what is the fourth term?

33.__________ Mr. Huang's class creates a banner to hang around the walls of his classroom. The banner repeats the 26 letters of the alphabet. What is the 137th letter written on the banner?

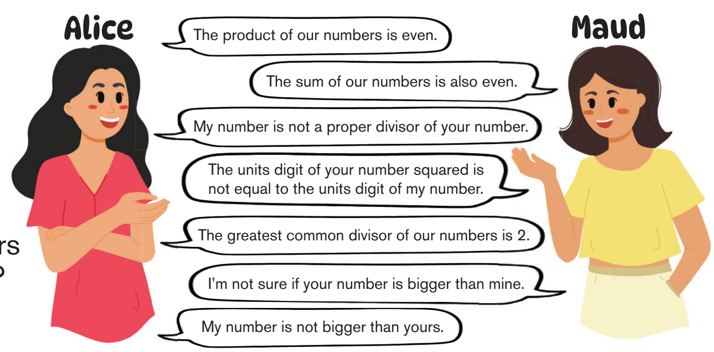
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z A B C

34.__________ If $x + 3 = 10$, what is $x^2 + 3^2$?

35.__________ fluid ounces

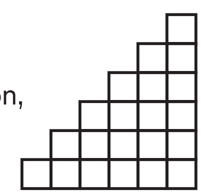
Nadine started with 1 gallon of water. She removed 2 quarts, then added 3 pints and finally removed 4 cups. How many fluid ounces of water are left?

36.__________ Friends Alice and Maud each think of a different integer from 1 to 10, inclusive. They have the following conversation in front of Nina, who confirms to them that the two integers are different. What is Alice's integer?



37.__________ cups Alex is making soup. The recipe's final step, after blending ingredients into a puree, says: "For every $\frac{3}{4}$ cup of puree, add $\frac{1}{3}$ cup of milk and $\frac{1}{2}$ cup of water." If Alex has 3 cups of puree, how many more cups of water than milk does he need to add? Express your answer as a common fraction.

38.__________ units Stephen has 21 unit square tiles, which he aligns in a triangular formation, shown right. What is the perimeter of his figure, in units?



39.__________ If Dante cut his blueberry pie into 6 equal slices, and Joe ate $1\frac{1}{2}$ pieces, what fraction of the pie did Joe eat? Express your answer as a common fraction.

40.__________ What is the units digit of the sum $1^2 + 2^2 + 3^2 + 4^2 + 5^2$?

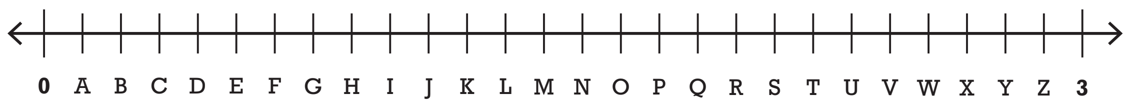
Warm-Up 2

41. ________ froops On the island of Barteria, 5 glorps can be traded for 3 plaps, and 2 plaps can be traded for 1 froop. Brian gives Tina 35 glorps in exchange for 15 plaps. How many froops would Tina have to give Brian in addition to the plaps to make the trade fair?

42. ________ What is the value of $\dfrac{11^2 - 1}{5^2 - 1}$?

43. ________ Elliott Middle School has 3 grades, 6th through 8th, with 6 teachers per grade. There are 148, 152 and 168 students in each grade, respectively. What is the ratio of teachers to students in the entire school? Express your answer as a common fraction.

44. ________ The letters of the alphabet are equally spaced on a number line between 0 and 3, as shown. Which letter occurs at 2 on the number line?



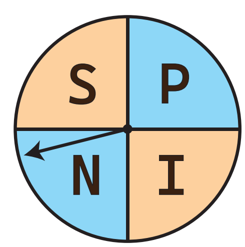
45. ________ If 57 percent of 200 equals 300 percent of x, what is the value of x?

46. ________ grams Todd's trainer recommends that he consume 2400 calories a day and that 30% of his calories should come from protein. If 1 gram of protein contains 4 calories, how many grams of protein should Todd consume daily?

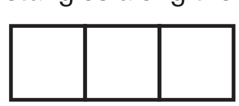
47. ________ integers How many positive four-digit integers exist such that the product of the digits is 7!?

48. ________ If $5x + 9 = 37$, what is the value of $15x + 16$?

49. ________ spins Leonard has a spinner with four equal parts labeled S, P, I and N, which he spins 72 times. What is the expected number of spins that land on the letter P?



50. ________ rectangles How many rectangles with integer side lengths can be traced out of a 1×3 grid of unit squares, with the sides of the rectangles along the grid lines?

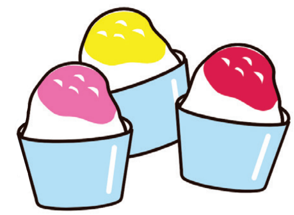


Warm-Up 3

51. ________ % What percent of the positive divisors of 2000 are perfect squares?

52. ________ questions If six MATHCOUNTS question writers can write 822 questions in one year, how many questions can one writer write in four years, assuming all writers work at the same rate?

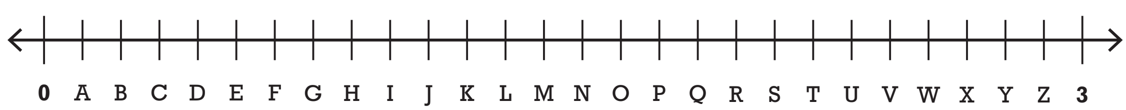
53. ________ combin-ations A shaved ice cart has 5 syrup flavors and 3 toppings. A medium shaved ice includes 2 pumps of syrup—either the same flavor twice or two different flavors—and at most 1 topping, which could also be none. How many different shaved ice combinations are possible?



54. ________ If $5x + 6 = 3x + 10$ and $ax - 4 = 0$, what is the value of a?

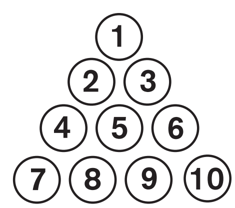
55. ________ miles Jimmy drove at 65 mi/h for 1 hour and 35 mi/h for some number of hours. If Jimmy's average speed during the trip was 45 mi/h, how many total miles did Jimmy drive?

56. ________ The letters of the alphabet are equally spaced on a number line between 0 and 3, as shown. What math word is represented by $\frac{1}{9}$, $2\frac{2}{3}$, 1 and $2\frac{1}{9}$, with letters in that order?



57. ________ What is the units digit of $2 + 2^0 + 2^{0+2} + 2^{0+2+5}$?

58. ________ Geo arranges the counting numbers in a triangular pattern: 1 at the top, 2 and 3 in the second row, 4, 5 and 6 in the third row, and so on. What is the sum of the first and last numbers in the seventh row?



59. ________ What is the value of $\sqrt{20\sqrt{25}}$?

60. ________ degrees The angles of a triangle form an arithmetic progression, and the smallest angle is 35 degrees. What is the degree measure of the largest angle of the triangle?

Warm-Up 4

61. __________ Amy is picking her outfit for the day. She has a red, blue and black version each of her headband, sweater and skirt. If she selects each item at random, what is the probability that at least 2 pieces are the same color? Express your answer as a common fraction.

62. __________ What is the units digit of $2026^{2025} + 2025^{2026}$?

63. __________ ft³ A cube has surface area, in square feet, equal to the total length of its edges, in feet. What is the volume of the cube, in cubic feet?

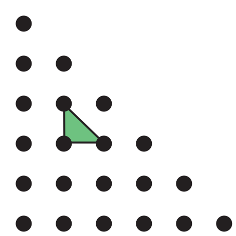
64. __________ Professor Plum teaches a class of 30 students. He accidentally left out Scarlet's score of 100 when first calculating the class average on the final exam. After including her score, the average increased by 1 point. What is the correct class average with Scarlet's score included?

65. __________ A game called MATHS is played on a diagram with 5 squares. A counter starts on square T, and a coin is tossed. If the coin lands heads up, the counter moves 1 square to the right; if it lands tails up, the counter moves 1 square to the left. What is the probability that after 2 tosses, the counter returns to square T? Express your answer as a common fraction.

66. __________ What is the least integer $n > 1$ for which $n!$ is divisible by n^2?

67. __________ minutes Micah's flight was originally scheduled to depart Portland at 2:40 p.m. and arrive in San Francisco at 4:10 p.m. However, due to weather delays, the plane didn't depart Portland until 3:15 p.m. His connecting flight in San Francisco is scheduled to depart at 5:05 p.m. Assume the flight will take the expected amount of time and that the connecting flight departs on time. How many minutes after his plane lands in San Francisco will his connecting flight depart?

68. __________ triangles The points in the triangular array shown right are 1 unit apart both horizontally and vertically. How many isosceles right triangles, with legs of length 1 unit, can be drawn by connecting 3 points in the array?



69. __________ If $2a + b = 13$, what is the value of $6a + 3b$?

70. __________ Beginning with the number 100, Arlo begins counting down by 7s until he reaches a non-positive number. He then begins counting up by 6s until he reaches a number that is at least 100. What is the last number that Arlo counts?

Warm-Up 5

71. ________ cm^2 Right triangle ABC has integer side lengths, with AB = 20 cm and AC = 25 cm. What is the least possible area of the triangle, in square centimeters?

72. ________ If x and y are integers, $x + y = 20$ and $\frac{x}{y} = 3$, what is the value of $x - y$?

73. ________ A numeric palindrome is a positive integer, not ending in 0, that reads the same forwards and backwards. For example, 3, 55 and 17071 are all palindromes. What is the least palindrome that is divisible by 12?

74. ________ feet In the capital of Mathlandia stands a building known as the Gauss Monument, which prominently features a regular heptadecagon (a 17-sided polygon). A scale model of the Gauss Monument is 3 feet tall, with one side of the model's heptadecagon measuring 2 inches. If the real heptadecagon has a perimeter of 510 feet, how tall is the actual Gauss Monument, in feet?

75. ________ What is the sum of $123_{\text{four}} + 1234_{\text{five}}$ written in base ten?

76. ________ The sum of the first 5 positive perfect squares is equal to the sum of the first n positive integers. What is the value of n?

77. ________ integers How many positive three-digit integers have a hundreds digit and a units digit that are each either one more or one less than the tens digit?

78. ________ cupcakes It takes Cory 14 minutes to put frosting on 2 dozen cupcakes. It takes Dory 15 minutes to put frosting on 3 dozen cupcakes. If they work together to put frosting on 1 dozen cupcakes, how many of the cupcakes will Cory put frosting on?

79. ________ What is the absolute difference between the mean and median of the five numbers 2, 5, 11, 14 and 23?

80. ________ Garrett's passcode has four digits. He knows the digits are 2, 6, 8 and 7, but he cannot recall their proper sequence. What is the probability that he will enter the correct passcode on his first try? Express your answer as a common fraction.

Warm-Up 6

81.__________ A list of four positive integers has a median of 5. What is the least possible mean of the list?

82.__________ If $x + x^2 = 6$, what is the least possible value of integer x?

83.__________ minutes A bathtub has a faucet that fills it completely in 10 minutes and a drain that empties it completely in 15 minutes. The bathtub is currently half full. If the drain is open while the faucet is on, how long will it take to fill the tub completely?

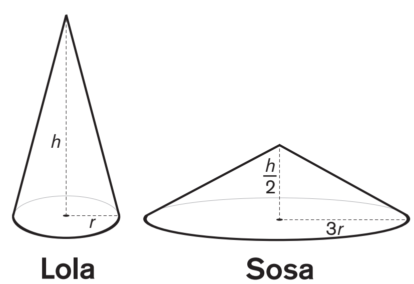
84.__________ Anu randomly cuts a ribbon of length 14 inches into two pieces. What is the probability that the difference in length between the two pieces is 3 inches or less? Express your answer as a common fraction.

85.__________ Matt and his friend Matics order a pizza that is $\frac{2}{3}$ vegetarian and $\frac{1}{3}$ pepperoni. Matt eats $\frac{1}{4}$ of the vegetarian part of the pizza and $\frac{2}{3}$ of the pepperoni part. Matics eats $\frac{2}{3}$ of the vegetarian part of the pizza and $\frac{1}{4}$ of the pepperoni part. How much of the pizza is left? Express your answer as a common fraction.

86.__________ A cyclist has completed two-thirds of his route when he gets a flat tire. He walks the rest of the way and spends twice as much time walking as he did riding. How many times as fast does he ride his bicycle as he walks?

87.__________ Let $f(x) = \sqrt{x+1}$ and $g(x) = x^2 + 1$. What is $g(f(8))$?

88.__________ Lola and Sosa are making cones out of clay in their art class. Sosa's cone has a radius that is three times that of Lola's cone and a height that is half that of Lola's cone. What is the ratio of the volume of Sosa's cone to the volume of Lola's cone? Express your answer as a common fraction.



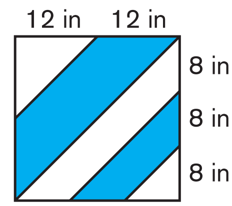
89.__________ clanks If 6 clanks are in 1 clink and 12 clinks are in 9 clonks, how many clanks are in 1 clonk?

90.__________ Two fair standard six-sided dice are rolled. What is the probability that the number showing on one of them is twice the number showing on the other? Express your answer as a common fraction.

Warm-Up 7

91. __________ Katie rolls two fair standard six-sided dice and records the positive difference between the numbers only if the two dice show different numbers. If both dice show the same number, she does not record anything. James then rolls a standard four-sided die. What is the probability that James and Katie obtain the same number? Express your answer as a common fraction.

92. __________ The flag of the local rowing club is a square, shown right, and all diagonal lines are parallel. What portion of the flag is shaded? Express your answer as a common fraction.



93. __________ combinations A florist is creating an arrangement using roses, tulips, peonies and daffodils. The arrangement must include exactly 9 flowers, with at least 1 of each type, and the order of the flowers doesn't matter. How many different combinations of flowers are possible?

94. __________ students Mr. Campo asks the 30 students in his Statistics class what foreign language they are taking and records their responses in a pie chart. However, it turns out 2 of the students study both Spanish and Latin, and therefore got counted in both of those sectors. If the Spanish sector of the pie chart is 180 degrees, how many of Mr. Campo's students are taking just Spanish?

95. __________ What ratio of the divisors of $6!$ are also divisors of 2^6? Express your answer as a common fraction.

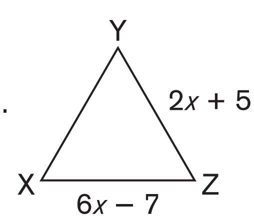
96. __________ What is the units digits of $202^5 + 2^{0 + 25}$?

97. (,) Point A has coordinates $(14, -4)$, and point B has coordinates $(-10, 8)$. Point B is $\frac{3}{5}$ of the way from point A to point C, along segment AC. What are the coordinates of point C? Express your answer as an ordered pair.

98. __________ groups Five families, each with three teenagers, are on a camping trip. Seven teenagers are needed to go into town to buy supplies. If the group must include at least one teenager from each family, how many different groups of seven can be chosen?

C 99. __________ What is the sum of all distinct prime number divisors of $\frac{22!}{20!}$?

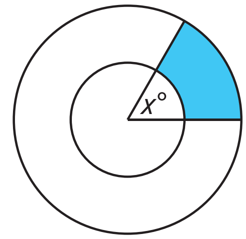
C 100. __________ feet In equilateral triangle XYZ, shown right, $YZ = 2x + 5$ and $XZ = 6x - 7$. If all lengths given are in feet, what is the length of XY?



Warm-Up 8

101. ______ coins Brennan has some pennies, nickels and dimes. If he can pay any amount of money from 1 cent to 25 cents in exact change, what is the least total number of coins he could have?

102. ______ degrees The dartboard shown right consists of two circles. The radius of the outer circle is twice the radius of the inner circle. The shaded region represents $\frac{1}{8}$ of the area of the dartboard. What is the measure of the unknown angle x? Express your answer in degrees.

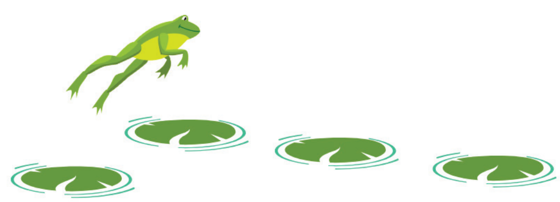


103. ______ arc-seconds There are 60 arcminutes in a degree and 60 arcseconds in 1 arcminute. If a circle rotates completely once every 24 hours, how many arcseconds does the circle rotate every second?

104. ______ triangles How many non-congruent triangles with integer side lengths have perimeter of at most 7?

105. ______ A square has a perimeter that exceeds its area by a positive integer n. What is the sum of the possible distinct values of n?

106. ______ What integer x satisfies $\frac{x-1}{x-4} = \frac{x-3}{x-5}$?



107. ______ ways A frog is trying to hop from one shore to another, using 18 lily pads arranged in a line across the pond. The frog can hop either 2 lily pads (double hop, which must skip over 1 lily pad) or 3 lily pads (triple hop, which must skip over 2 lily pads); in other words, the frog cannot jump from shore to the first lily pad, nor from the last lily pad to the shore. If the frog makes it across in the fewest hops possible, how many different ways can the frog hop from one shore to the other?

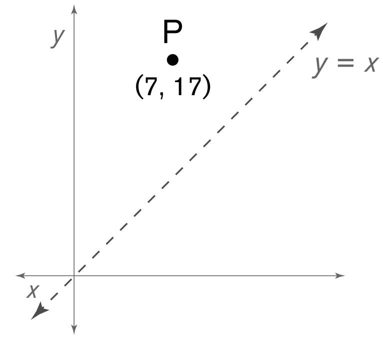
108. ______ cups Rachel uses a pumpkin bread recipe that calls for 4 eggs and $3\frac{1}{2}$ cups of flour. If she uses 7 eggs, how many cups of flour should she use? Express your answer as a mixed number.

109. ______ What is the probability that a randomly chosen positive integer divisor of 32 is a perfect square? Express your answer as a common fraction.

110. ______ dimes Leona has twelve coins totaling 85 cents. If she only has dimes and nickels, how many dimes does she have?

Warm-Up 9

111.__________ Andrew rolls a fair standard six-sided die five times. What is the probability that the sum of the five die rolls is divisible by 3? Express your answer as a common fraction.



112.__________ units The point P has coordinates (7, 17). If point Q is the reflection of point P across the line $y = x$, and point R is the reflection of point P across the y-axis, what is the length of segment QR?

113.__________ What is $111^2 - 99^2$?

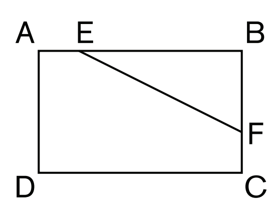
114.__________ The number 2401 is a perfect square whose nonzero digits are all powers of 2. What is the largest three-digit perfect square with this property?

115.__________ A fair standard six-sided die is rolled three times and the sum of the three numbers thrown is 15. What is the probability that the number on the first roll is 4? Express your answer as a common fraction.

116.__________ % A small hose fills a swimming pool in 20 hours, and a large hose fills the same pool in 16 hours. Starting with an empty pool, the small hose is turned on at 8:00 a.m., and the large hose is turned on at 12:00 p.m. Both hoses run until 4:00 p.m., when they are both turned off. What percent of the pool is filled by 4:00 p.m.?

117.__________ Person A tells the truth 75% of the time, while Person B tells the truth 80% of the time. If both describe the same incident, what is the probability that their statements will contradict each other, meaning one tells the truth and the other lies? Express your answer as a common fraction.

118.__________ An arithmetic sequence consists entirely of positive integers. The third term is equal to the sum of the first two terms, and the fourth term is equal to the product of the first two terms. What is the fifth term of this sequence?



C 119.__________ cm^2 Rectangle ABCD has AB = 5 cm and BC = 3 cm. Points E and F lie on sides AB and BC, respectively, so that AE = CF = 1 cm. What is the area of pentagon AEFCD, in square centimeters?

C 120.__________ What is the value of $13^2 - 12^2 - 5^2$?

Warm-Up 10

121. ________ ft^2 A dog is tied to a vertical pole with a 13-foot-long leash. The leash is tied to the pole 5 feet above the ground, which is level and without obstacles. What is the total area the dog can explore, in square feet?

122. ________ What is the integer closest to $\sqrt{2026} + \sqrt[4]{2026}$?

123. ________ A bag contains 4 red marbles and 2 blue marbles, and 2 marbles are drawn at random with replacement. The ratio of the probabilities that 0, 1 or 2 red marbles are drawn is a:b:c in simplest form, where a, b and c are positive integers with no common factor larger than 1. What is the value of $100a + 10b + c$?

124. ________ What is the 2026th digit after the decimal in the expansion of the fraction $\frac{3}{14}$?

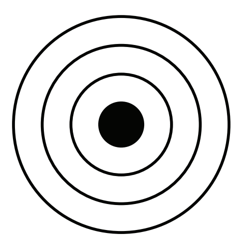
125. ________ A triangle has two sides with lengths 7 and 12. If the third side has an integer length, what is the absolute difference between the greatest possible perimeter of this triangle and the least possible perimeter of this triangle?

126. ________ If $a = \frac{2}{3}$, what is the exact value of $(a+a^{-1})^3 - a^3 - a^{-3}$? Express your answer as a common fraction .

127. ________ A line passing through the point P(−5, 4) intersects the x-axis at A and the y-axis at B. The ratio of the lengths PA and PB is 1:2. What is the slope of this line? Express your answer as a common fraction.

128. ________ arrange-ments Two parents and their six children go to a restaurant and are seated at a circular table. If the parents insist on sitting next to one another, how many seating arrangements are possible? Note that rotations and reflections are considered distinct.

C 129. ________ % Christopher is practicing archery with a target consisting of a center circle (bullseye) with a 6 cm diameter and three outer rings, each with a width of 4 cm. The bullseye is what percent of the target area?



S 130. ________ rows Lyle has 120 toy soldiers, which he wants to arrange in a rectangular formation with no more than 20 soldiers in any row or column. What is the sum of all possible numbers of rows that Lyle's formation can have?

Warm-Up 11

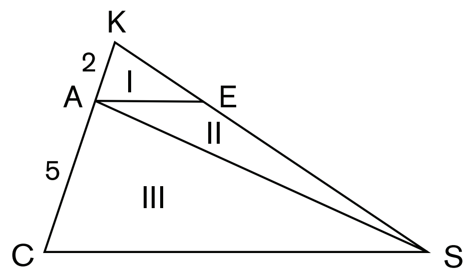
131.________ What is the least positive integer that does not divide the number $9! + 10! = 3{,}991{,}680$?

132.________ A class of 12 students has 3 tables, each seating 4 students. If 2 students are randomly selected to present homework problems at the blackboard, what is the probability that they are sitting at the same table? Express your answer as a common fraction.

133.________ Suppose that a circle of radius r units has area A square units and A is at most 20 greater than r. What is the greatest possible integer value of A?

134.________ If the graphs of $x + 2y + a = 0$ and $3x + by - 9 = 0$ intersect at $(-2, 3)$, what is the value of $a + b$?

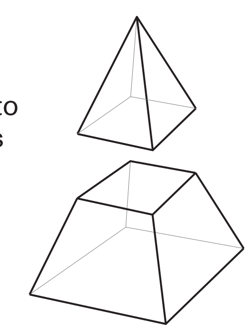
135. $________ Benjamin bakes cakes in a triangular pan shaped like triangle CKS. He cuts along segment AE, which is parallel to CS, dividing segment CK into segments of 2 inches and 5 inches, and then cuts along segment AS, creating a total of three cake pieces. If he sells all three pieces at the same price per square inch and prices piece II at $2, how much will he charge for piece III?



136.________ The number 27 is the cube of a prime positive integer. What is the sum of all such integers that are at most 2025?

137.________ digits How many digits in the 2025-digit number 202520252025…20252 are prime?

138.________ cm³ A pyramid with volume 40 cm^3 has its tip cut off by a plane parallel to the base of the pyramid. If the two pieces have equal height, what is the absolute difference, in cubic centimeters, between the volumes of the pieces?



139.________ If $2^5 \times 8^3 \times 16^2 = 4^m$, what is the value of m?

140.________ What is the value of $\left(2-\frac{1}{3}\right)\times\left(2-\frac{1}{5}\right)\times\left(2-\frac{1}{7}\right)$? Express your answer as a common fraction.

Workout 1

141. ________ calories A certain brand of almond milk has 30 calories per cup. If there are 8 fluid ounces in a cup, and the carton holds 100 fluid ounces, how many calories are in a full carton of almond milk?

142. ________ digits What is the total number of digits used when the first 500 positive even integers are written?

143. ________ in^2 Two photographs have equal areas of 16 square inches—one is a square measuring 4 inches by 4 inches, and the other is a rectangle measuring 2 inches by 8 inches. Each photograph is surrounded by a wooden frame that is 1 inch wide on all sides. What is the absolute difference between the areas of the two frames?

144. ________ miles A car can travel from Sprintville to Countdown City in 3 hours, traveling at a constant rate. If it travels at another constant rate that is 10 mi/h faster on the return trip, it can travel back from Countdown City to Sprintville in 20 fewer minutes. How many miles are between Sprintville and Countdown City?

145. ________ Three distinct prime numbers form an arithmetic sequence. If their sum is 111, what is the least possible value of their product?

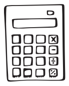
146. ________ A number is chosen at random from the set of consecutive integers {1, 2, 3, …, 25}. What is the probability that the number chosen is a divisor of 5!? Express your answer as a common fraction.

147. ________ What is the sum of all positive palindromes less than 2025 that are divisible by 99?

148. ________ inches The average height of a professional basketball player is 78 inches, and there are typically 15 players per team. The average height of a professional baseball player is 74 inches, and there are typically 26 players per team. What is the expected absolute difference, in inches, between the combined heights of the players on a professional basketball team and the combined heights of the players on a professional baseball team?

C 149. ________ integers How many positive integers are divisors of 18, but not of 63?

C 150. ________ The first three terms of a sequence are 4, 7 and 13. Each term after the first is one less than twice the previous term. What is the sixth term in the sequence?



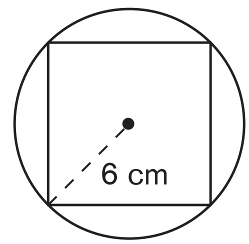
Workout 2

151.__________ The letters in the word MATHLETE are arranged in a random order. What is the probability that no two consecutive letters are the same? Express your answer as a common fraction.

152.__________ Consider a five-term arithmetic sequence that starts at 6 and increases by 5 with each term. Another sequence is added to the first, term by term, resulting in the sequence 25, 25, 25, 25, 25. What is the absolute difference between the first and last term of the second sequence?

153.__________ What is the sum of the first 10 integers that have 3 distinct digits?

154.__________ cm A square is inscribed in a circle with radius 6 cm, shown right. What is the length of a side of the square? Express your answer as a decimal to the nearest tenth.



155.__________ An even number N has 16 positive integer divisors. If one of the divisors is chosen at random, the probability that it is even is $\frac{3}{4}$. What is the least possible value of N?

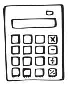
156.__________ Let $f(x) = \frac{237}{2x - 37}$. What the sum of all integer values of x such that $f(x)$ is also an integer?

157.__________ Each positive integer 1 to 100, inclusive, is divided by 17. What is the sum of all 100 remainders?

158.__________ A right triangle has a 30-degree interior angle and an integer length for its shortest side. The side lengths of this triangle have a product whose square is S. What is the sum of the possible values of S that are at most 2025?

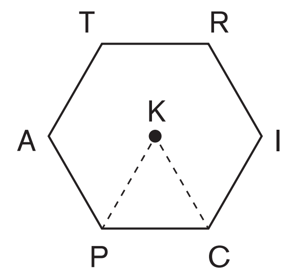
159.__________ combinations Justin has an unlimited supply of dimes and quarters. In how many different combinations of these coins can he pay exactly $5.00 for a board game?

160.__________ What is the least positive perfect square that is a multiple of 24?



Workout 3

161.________ cm^2 A rectangle has area 20 cm^2 and perimeter 26 cm. If each of its side lengths is increased by 1 cm, how many square centimeters are now in its area?



162.________ in^2 Regular hexagon PATRIC has center K. If heptagon PATRICK has perimeter 28 inches, how many square inches are in its area? Express your answer in simplest radical form.

163.________ ways How many ways are there to rearrange the letters in the word TOPOLOGY?

164.________ If d distinct positive integers sum to 27 for some positive integer d, what is the sum of the possible values of d?

165.________ $units^2$ The graphs of $x + 2y = 4$ and $3x + 5y = 9$ intersect at $(-2, 3)$. What is the area of the triangle formed by these two lines and the x-axis? Express your answer as a common fraction.

166.________ cm^2 George has a right rectangular prism with three distinct integer edge lengths, and its volume is 2026 cm^3. In square centimeters, what is the surface area of George's prism?

167.$________ During their fundraiser, the math club kept track of how many bags of each type of popcorn they sold. The results are shown in the table below. In dollars, how much did the math club earn from selling popcorn?

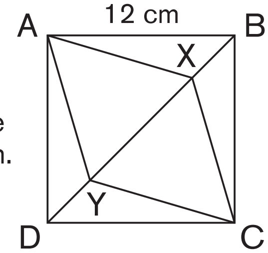
Popcorn Type	**Kettle**	**Sea Salt**	**Cheddar**	**Butter**	**Spicy Herb**
Number Sold	12	13	21	11	16
Price per Bag	\$3.50	\$2.75	\$4.00	\$2.75	\$3.50

168.________ digits The sum of the digits of an integer N is 2026. What is the minimum number of digits that N has?

169.________ If $x \clubsuit y = \frac{x^2}{y}$, what is the value of $6 \clubsuit 4$?

170.________ hours Sam reads an average of 8 pages of a book in 10 minutes. At this rate, how many hours will it take Sam to read the entire 192-page book?

Workout 4



171. ________ cm Square ABCD has side length 12 cm, and rhombus AXCY inside square ABCD has area half the area of square ABCD. What is the side length of rhombus AXCY? Express your answer in simplest radical form.

172. $ ________ At Al's Grocer, 5 bananas and 3 oranges cost \$10.92, while 4 bananas and 5 oranges cost \$13.91. What is the cost of a dozen bananas?

173. ________ arrange-ments How many visually distinguishable arrangements of 4 lights in a single-file line, each either red or green, are there such that no green light is directly between two red lights?

174. ________ The digits of the number 2027 sum to 11, and 2027 is two more than a perfect square. What is the largest positive integer less than 2027 with this property?

175. ________ in^3 Rick rolls a rectangular 6 inch by 8 inch sheet of paper up, edge to edge, to form a cylinder. Depending on how he rolls the paper, there are two different cylinders he can make. What is the absolute difference between their volumes? Express your answer as a common fraction in terms of π.

176. ________ % Barter Bob's sells its Peanut & Cocoa Cluster Bites cereal in a 10-ounce box for \$3.49. In January 2026, they plan to introduce a new 14-ounce box priced at \$4.49. From a price-per-ounce perspective, by what percentage has the cost decreased compared to the old box? Express your answer to the nearest tenth of a percent.

177. ________ numbers A five-digit number is created by using each of the digits 1, 3, 5, 7 and 9 exactly once. Each of the digits in the tens and thousands positions is larger than its neighboring digits. How many such five-digit numbers can be formed?

178. ________ What is the sum of all positive integers less than or equal to 100 that have the property that their value divided by the sum of their digits is 3?

C 179. ________ in^3 The sum of the lengths of all edges of a cube is 84 in. What is the volume of the cube, in cubic inches?

S 180. ________ If x, y and z are positive integers for which $x + y + z = 20$ and $x^2 + y^2 + z^2 = 222$, what is the value of $xy + xz + yz$?

Workout 5

181. ________ cm^2 A rectangular piece of paper has perimeter 30 cm. Carl cuts the paper into two smaller rectangular pieces, one with perimeter 26 cm and one with perimeter 14 cm. What is the area of the original piece of paper, in square centimeters?

182. ________ % Jarnail's average score for the first 4 tests in her math class was 86%. If she scored 96% on the 5th test, what was her average score for all 5 tests?

183. ________ prisms How many 2 × 4 × 4 right rectangular prisms can fit inside a cube with side length 10?

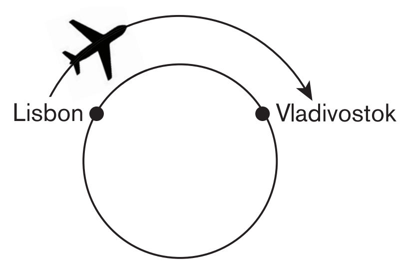
184. ________ Three letters are chosen at random, without replacement, from the word MATHCOUNTS and each written on their own slip of paper. The slips of paper are then tossed into a bag. From the bag, two slips of paper are drawn at random without replacement. What is the probability that the two drawn slips have different letters on them? Express your answer as a common fraction.

185. ________ What is the sum of all positive integer perfect squares up to and including 2025 that have a units digit of 4 and a square root that is a multiple of 4?

186. ________ inches The ratio of a cube's surface area, in square inches, to its volume, in cubic inches, is 3:10. What is the side length of the cube, in inches?

187. ________ Angelica places the letters that spell the word COTTONELLE in a hat. She then picks the letters out one at a time to create a new string of letters. What is the probability that the new string of letters has, for each duplicated letter, the two occurrences of the letter adjacent to one another? One such string of letters to include would be LLTTEECNOO. Express your answer as a common fraction.

188. ________ miles The circumference of the Earth is 24,901 miles, and the distance between Lisbon, Portugal and Vladivostok, Russia is one-fourth the circumference of the Earth. If a plane flies between those two cities at an altitude of 34,000 feet, approximately how many miles longer is the length of the plane's flight in the air than the distance between these two cities on the ground? There are 5,280 feet in a mile. Express your answer to the nearest integer.



189. ________ What is the greatest of six consecutive odd integers whose sum is 216?

190. ________ minutes Larry can make 7 pizzas in 9 minutes. Harry can make 7 pizzas in 12 minutes. Mary can make 4 pizzas in 6 minutes. Working together, how many minutes will it take the three friends to make 730 pizzas?

Workout 6

191. ________ If x and y are positive integers that satisfy $2^x \times 4^y = 1024$ and $4^x \times 2^y = 256$, what is the value of $x + y$?

192. ________ Dorina rolls a fair six-sided die. If the number she rolls equals n, what is the expected value of $n!$? Express your answer as a decimal to the nearest tenth.

193. ________ Let $a_1, a_2, a_3, \ldots$ be a sequence with $a_1 = 1$. For each integer $n \geq 2$, the term a_n is defined as the product of all previous terms plus 2: $a_n = a_1 \times a_2 \times a_3 \times \ldots \times a_{n-1} + 2$. What is the units digit of a_{2025}?

194. ________ choices If four distinct positive integers sum to 20, how many such choices of positive integers, without regard to order, include at least one integer that is divisible by 5?

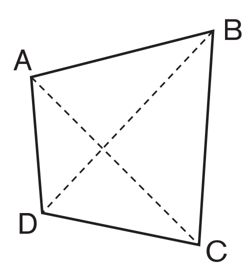
195. ________ A standard deck of cards has 4 suits: clubs, hearts, spades and diamonds, with 13 cards in each suit. Jaris deals 3 cards from a standard deck to David. What is the probability that David receives exactly 1 heart? Express your answer as a common fraction.

196. ________ cents Water costs \$6.70 per 100 cubic feet. To the nearest hundredth of a cent, what is the cost of a glass of water, assuming that the inner diameter of the glass is 2 inches and the height of the water in the glass is 5 inches?

197. ________ What is the maximum value of n such that 4^n is a factor of $120!$?

198. ________ Let S be the sum of all of the positive integers less than or equal to 2026 that are of the form $n^2 + 1$ for some positive integer n. What is the remainder upon dividing S by 100?

199. ________ cm^2 In convex quadrilateral ABCD, the area of triangle ABC is 20 cm^2, the area of triangle ADC is 18 cm^2, and the area of triangle BCD is 25 cm^2. What is the area of triangle BAD, in square centimeters?



200. ________ Jon and Tim each flip a fair coin 4 times. What is the probability that Jon obtained the same number of heads as Tim? Express your answer as a common fraction.

COMPETITION COACH TOOLKIT

This is a collection of lists, formulas and terms that Mathletes frequently use to solve problems like those found in this handbook. There are many others we could have included, but we hope you find this collection to be a useful reference.

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50
$\frac{1}{3}$	$0.\overline{3}$	$33.\overline{3}$
$\frac{1}{4}$	0.25	25
$\frac{1}{5}$	0.2	20
$\frac{1}{6}$	$0.1\overline{6}$	$16.\overline{6}$
$\frac{1}{8}$	0.125	12.5
$\frac{1}{9}$	$0.\overline{1}$	$11.\overline{1}$
$\frac{1}{10}$	0.1	10
$\frac{1}{11}$	$0.\overline{09}$	$9.\overline{09}$
$\frac{1}{12}$	$0.08\overline{3}$	$8.\overline{3}$

n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375

Common Arithmetic Series

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 3 + 5 + 7 + \cdots + (2n-1) = n^2$$

$$2 + 4 + 6 + 8 + \cdots + 2n = n^2 + n$$

Combinations & Permutations

$${}_nC_r = \frac{n!}{r!(n-r)!} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

Sequences & Series

For an **arithmetic sequence** with common difference d:

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

For a **geometric sequence** with common ratio r:

$$a_n = a_1 r^{(n-1)}$$

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right) \quad \text{for } r \neq 1$$

Prime Numbers

2	43
3	47
5	53
7	59
11	61
13	67
17	71
19	73
23	79
29	83
31	89
37	97
41	

Divisibility Rules

- 2: units digit is 0, 2, 4, 6 or 8
- 3: sum of digits is divisible by 3
- 4: two-digit number formed by tens and units digits is divisible by 4
- 5: units digit is 0 or 5
- 6: number is divisible by both 2 and 3
- 8: three-digit number formed by hundreds, tens and units digits is divisible by 8
- 9: sum of digits is divisible by 9
- 10: units digit is 0

Difference of Squares

$$a^2 - b^2 = (a+b)(a-b)$$

Distance Traveled

Distance = Rate × Time

Geometric Mean

$$\frac{a}{x} = \frac{x}{b} \quad \text{and} \quad x = \sqrt{ab}$$

Sum & Difference of Cubes

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Binomial Theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Quadratic Formula

For $ax^2 + bx + c = 0$, where $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Parity of Sums, Differences & Products

even ± even = even	even × even = even
odd ± odd = even	odd × odd = odd
even ± odd = odd	even × odd = even

Triangular Numbers

$$T_n = \sum_{k=1}^{n} k$$

Circles

Circumference	Area
$2\pi r = \pi d$	πr^2

For radius r

Arc Length	Sector Area
$\frac{x}{360}(2\pi r)$	$\frac{x}{360}(\pi r^2)$

For central angle of x degrees

Pythagorean Triples

(3, 4, 5)

(5, 12, 13)

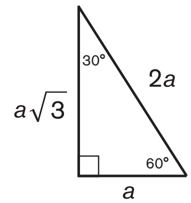
(7, 24, 25)

(8, 15, 17)

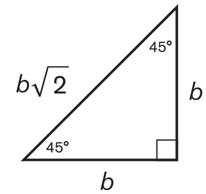
(9, 40, 41)

(12, 35, 37)

Special Right Triangles

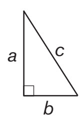


30-60-90 Right Triangle



45-45-90 Right Triangle

Pythagorean Theorem



$a^2 + b^2 = c^2$

Triangle Inequality

For a triangle with side lengths a, b and c:

$a + b > c$
$a + c > b$
$b + c > a$

Given A(x_1, y_1) and B(x_2, y_2)

Distance from A to B $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope of $\overline{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

Area of Polygons

Square	side length s	s^2
Rectangle	length l, width w	lw
Parallelogram	base b, height h	bh
Trapezoid	bases b_1, b_2, height h	$\frac{1}{2}(b_1 + b_2)h$
Rhombus	diagonals d_1, d_2	$\frac{1}{2}d_1 d_2$
Triangle	base b, height h	$\frac{1}{2}bh$
Triangle *Heron's formula*	semiperimeter s, side lengths a, b, c	$\sqrt{s(s-a)(s-b)(s-c)}$
Equilateral Triangle	side length s	$\frac{s^2\sqrt{3}}{4}$

Polygon Angles (*n* sides)

Sum of the interior angle measures:

$180(n - 2)$

Central angle measure of a regular polygon:

$\frac{360}{n}$

Interior angle measure of a regular polygon:

$\frac{180(n-2)}{n}$ or $180 - \frac{360}{n}$

Solid	Dimensions	Surface Area	Volume
Cube	side length s	$6s^2$	s^3
Rectangular Prism	length l, width w, height h	$2(lw + wh + lh)$	lwh
Cylinder	circular base radius r, height h	$2\pi rh + 2\pi r^2$	$\pi r^2 h$
Cone	circular base radius r, height h	$\pi r^2 + \pi r \times \sqrt{r^2 + h^2}$	$\frac{1}{3}\pi r^2 h$
Sphere	radius r	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Pyramid	base area B, height h		$\frac{1}{3}Bh$

Equation of a Line

Standard Form

$\boxed{Ax + By = C}$

Slope-Intercept Form

$\boxed{y = mx + b}$

m = slope b = y-intercept

Point-Slope Form

$\boxed{y - y_1 = m(x - x_1)}$

m = slope (x_1, y_1) = point on the line

Vocabulary & Terms

The following list is representative of terminology used in the problems but **should not** be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference
absolute value
acute angle
additive inverse *(opposite)*
adjacent angles
apex
arithmetic mean
arithmetic sequence
base ten
binary
binomial theorem
bisect
box-and-whisker plot
center
chord
circumscribe
coefficient
collinear
common divisor
common factor
common fraction
complementary angles
congruent
convex
coordinate plane/system
coplanar
counting numbers
counting principle
diagonal of a polygon
diagonal of a polyhedron
digit sum
dilation
direct variation
divisor
domain of a function
edge
equiangular
equidistant
expected value
exponent
exterior angle of a polygon
factor
finite
frequency distribution
frustum
function
GCF (GCD)
geometric sequence
hemisphere
image(s) of a point(s)
(under a transformation)
improper fraction
infinite series
inscribe
integer
interior angle of a polygon
intersection
inverse variation
irrational number
isosceles
lateral edge
lateral surface area
lattice point(s)
LCM
median of a data set
median of a triangle
mixed number
mode(s) of a data set
multiplicative inverse *(reciprocal)*
natural number
obtuse angle
ordered pair
origin
palindrome
parallel
Pascal's Triangle
percent increase/decrease
perpendicular
planar
polyhedron
polynomial
prime factorization
principal square root
proper divisor
proper factor
proper fraction
quadrant
quadrilateral
random
range of a data set
range of a function
rate
ratio
rational number
ray
real number
reciprocal *(multiplicative inverse)*
reflection
regular polygon
relatively prime
revolution
right angle
right polyhedron
rotation
scalene triangle
scientific notation
sector
segment of a circle
segment of a line
semicircle
semiperimeter
sequence
set
significant digits
similar figures
slope
space diagonal
square root
stem-and-leaf plot
supplementary angles
system of equations/inequalities
tangent figures
tangent line
term
transformation
translation
triangle inequality
triangular numbers
trisect
twin primes
union
unit fraction
variable
whole number
y-intercept

Forms of Answers

The following rules explain acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Competition answers will be scored in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lb 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

Geometric figures may not be drawn to scale and lengths of geometric figures should be assumed to be measured in "units" unless otherwise stated.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not have a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle of diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc* or *a.bc* (dollars), where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* are both zero, in which case they both may be omitted. Answers in the form ($) *a.bc* or *a.bc* (dollars) should be rounded to the nearest cent, unless otherwise specified. Examples:

Acceptable Forms: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:

Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3

Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

SOLUTIONS

The solutions provided here are only *possible* solutions. It is very likely that you or your students will come up with additional—and perhaps more elegant—solutions. Happy solving!

Chance to Win Stretch

1. Each flip independently has a probability of 1/2 chance of landing tails, so the chance of getting two tails is $1/2 \times 1/2 = 1/4$ or **25**%.

2. The probability of rolling a 4 is 4/6 and the probability of rolling a 2 is 2/6. The probability of rolling a 4 followed by a 2 is $4/6 \times 2/6 = 8/36 =$ **2/9**.

3. Each person has a $2/8 = 1/4$ chance of getting all heads or all tails, since there are 2 matching outcomes (HHH and TTT) out of 8 total outcomes for 3 coin flips. The probability that exactly 1 person gets all heads or all tails is $3 \times (1/4) \times (3/4) \times (3/4) = 27/64 \approx$ **0.42**.

4. There are 6 candies, and we are choosing 2. That's "6 choose 2" $= 6!/(2! \times 4!) = (6 \times 5)/(2 \times 1) = 15$ total pairs. The pairs that include both a red and a green are (R1, G1), (R1, G2), (R2, G1) and (R2, G2), for a total of 4. So the number of pairs *not* containing both a red and a green is $15 - 4 = 11$. The odds of winning to not winning are 11:4, so $a - b = 11 - 4 =$ **7**.

5. There are $3! = 3 \times 2 \times 1 = 6$ ways to order the letters Y, T and O. Only 1 of these is T-O-Y, so the probability is **1/6**.

6. If Mark rolls a 2, Mary can roll a 3, 4, 5 or 6 to win. That's 4 out of 6 outcomes. The probability is $4/6 =$ **2/3**.

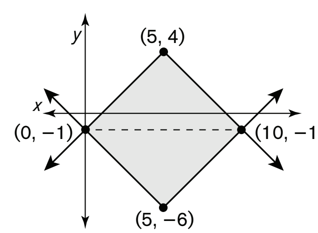
7. There are 12 face cards in a deck (4 Jacks, 4 Queens, 4 Kings) out of 52 total cards. So the probability is $12/52 =$ **3/13**.

8. There are "52 choose 2" $= 52!/(2! \times 50!) = (52 \times 51)/(2 \times 1) = 1326$ total 2-card combinations. There are 26 red cards and 26 black cards, so "26 choose 2" $= 26!/(2! \times 24!) = (26 \times 25)/(2 \times 1) = 325$ each, for a total of 650 combinations that are the same color. There are 13 ranks, and for each rank, "4 choose 2" $= 4!/(2! \times 2!) = (4 \times 3)/(2 \times 1) = 6$ ways to pick 2 cards. So there are $13 \times 6 = 78$ combinations that are the same rank. Some pairs are double-counted (same color and same rank), like 2 red 8s. There are 13 such overlaps for red cards, and likewise 13 for black cards, so the total number of winning outcomes is $650 + 78 - 26 = 702$. That means that the probability that Karina wins is $702/1326 \approx$ **0.53**.

9. Since the spinner has 7 equal sections and Jenna calls "even," we need to know how many even numbers appear. The even numbers on the spinner are 4, 6, 8 and 10. So the probability that Jenna wins is $4/7 \approx 0.5714$, which is **57**% when rounded to the nearest percent.

10. Sara wants 3 different colors. The first roll can be any of the 6 colors. For the second roll, 5 colors remain, and for the third, 4 colors remain. The probability is $(6/6) \times (5/6) \times (4/6) = 1 \times 5/6 \times 2/3 = 10/18 =$ **5/9**.

Area on the Coordinate Plane Stretch



11. Both given equations are absolute value functions and together form a symmetric, enclosed region. The first graph is an upside-down V with vertex at (5, 4). The second is a right-side-up V with vertex at (5, −6). To find where the graphs intersect, we set the expressions equal and solve for x: $-|x - 5| + 4 = |x - 5| - 6 \rightarrow 4 = 2|x - 5| - 6 \rightarrow 10 = 2|x - 5| \rightarrow |x - 5| = 5$. This gives us $x = 0$ and $x = 10$. Substituting either value back into either equation gives us the points of intersection (0, −1) and (10, −1). These two graphs enclose a region that is split horizontally at $y = -1$, forming two congruent isosceles triangles. Each triangle has a base of 10 units and height of 5 units. The area of one triangle is $1/2 \times 10 \times 5 = 25$ units^2, so the total area enclosed by the graphs is $2 \times 25 =$ **50** units^2.

12. This absolute value function has a vertex at (20, 0). To find where the graph intersects the horizontal line $y = 10$, we substitute 10 for y, giving us $10 = |x - 20|$. Solving this equation gives $x = 10$ and $x = 30$, so the region enclosed between the graph is an isosceles triangle with a base of 20 units and a height of 10 units. The area of this triangle is $1/2 \times 20 \times 10 =$ **100** units^2.

13. The area of a rhombus is given by $(d_1 \times d_2)/2$, so we need the lengths of its diagonals. Since $|x - 3|$ and $|y + 5|$ are always non-negative, their maximum values are 8. For the horizontal diagonal, solving $|x - 3| \le 8$ gives $-5 \le x \le 11$, so its length is $11 - (-5) = 16$ units. For the vertical diagonal, solving $|y + 5| \le 8$ gives $-13 \le y \le 3$, so its length is $3 - (-13) = 16$ units. Thus, the area of the rhombus is $(16 \times 16)/2 =$ **128** units^2.

14. We find the length of the horizontal diagonal by solving $2|x - 3| \le 8$, which simplifies to $|x - 3| \le 4$. Since $-1 \le x \le 7$, the length of the horizontal diagonal is 8 units. Similarly, $4|y + 5| \le 8$ simplifies to $|y + 5| \le 2$, so $-7 \le y \le -3$ and the length of the vertical diagonal is 4 units. Thus, the area of the rhombus is $(8 \times 4)/2 =$ **16** units^2.

15. We find the length of the horizontal diagonal by solving $3|x + 2| \le 12$, which simplifies to $|x + 2| \le 4$. Since $-6 \le x \le 2$, the length of the horizontal diagonal is 8 units. Similarly, $2|y - 1| \le 12$ simplifies to $|y - 1| \le 6$, so $-5 \le y \le 7$ and the length of the vertical diagonal is 12 units. Thus, the absolute difference of the diagonals is $|12 - 8| =$ **4** units.

16. The area of a circle is given by $A = \pi r^2$. Since the equation represents a circle with radius $\sqrt{8}$ units, the area is $\pi \times (\sqrt{8})^2 =$ **8π** units².

17. The graph of $|x + 2| + |y - a| = b$ is a rhombus centered at $(-2, a)$. Because the absolute value expressions have no coefficients, the diagonals of the rhombus are both $2b$ units long. The area of a rhombus is given by $A = (d_1 \times d_2)/2$, so we have $20 = (2b \times 2b)/2 \rightarrow 40 = 4b^2 \rightarrow 10 = b^2 \rightarrow b = \sqrt{10}$. We are given that $ab = 30$, so $a = 30/b = 30/\sqrt{10} =$ **3√10**.

18. The equation $2|x - 3| + 5|y + 8| = a$ can be rewritten as $|x - 3|/(a/2) + |y + 8|/(a/5) = 1$, which is the equation of a rhombus. The diagonals are $2a/2 = a$ and $2a/5$. So, the area is $(a \times 2a/5)/2 = (a^2 \times 2)/(5 \times 2) = a^2/5$. Setting this equal to 80, we have $a^2/5 = 80 \rightarrow a^2 = 400 \rightarrow a =$ **20**.

19. The inradius (radius of the inscribed circle) of a rhombus is given by r = area/semiperimeter. We'll start by finding the lengths of the diagonals of the given rhombus. We find the length of the horizontal diagonal by solving $5|x - 3| \le 60$, which simplifies to $|x - 3| \le 12$. Since $-9 \le x \le 15$, the length of the horizontal diagonal is 24 units. Similarly, $12|y + 5| \le 60$ simplifies to $|y + 5| \le 5$, so $-10 \le y \le 0$ and the length of the vertical diagonal is 10 units. The area of the rhombus is $(24 \times 10)/2 = 120$ units². Each side of the rhombus is $\sqrt{(12^2 + 5^2)} = 13$ units, so the perimeter is 52 units and the semiperimeter is 26 units. That means the inradius is $120/26 = 60/13$ units, and the area of circle is $\pi \times (60/13)^2 =$ **3600π/169** units².

20. A square inscribed in a circle has its diagonal equal to the circle's diameter, which is twice the radius, or $2\sqrt{5}$. To find the area of a square given its diagonal d, we use the formula $A = (1/2) \times d^2$, so $(1/2) \times (2\sqrt{5})^2 = (1/2) \times 4 \times 5 =$ **10** units².

Simplifying Radicals Stretch

21. To simplify, we can write $8 = 4 \times 2$. Then $\sqrt{8} = \sqrt{(4 \times 2)} = \sqrt{4} \times \sqrt{2} =$ **2√2**.

22. To simplify $\sqrt[3]{81}$, we can write $81 = 27 \times 3$. Then $\sqrt[3]{81} = \sqrt[3]{(27 \times 3)} = \sqrt[3]{27} \times \sqrt[3]{3} =$ **$3\sqrt[3]{3}$**.

23. First, simplify each square root: $\sqrt{150} = \sqrt{(25 \times 6)} = 5\sqrt{6}$ and $\sqrt{24} = \sqrt{(4 \times 6)} = 2\sqrt{6}$. Now subtract: $5\sqrt{6} - 2\sqrt{6} =$ **3√6**.

24. To multiply $(\sqrt{3}/2) \times (\sqrt{6}/3)$, we can multiply the numerators and multiply the denominators: $\sqrt{3} \times \sqrt{6} = \sqrt{18} = \sqrt{(9 \times 2)} = 3\sqrt{2}$ and $2 \times 3 = 6$, so the expression becomes $(3\sqrt{2})/6$. This simplifies to **√2/2**.

25. To eliminate the radical from the denominator of $1/\sqrt{7}$, we can multiply by $\sqrt{7}/\sqrt{7}$: $1/\sqrt{7} \times \sqrt{7}/\sqrt{7} =$ **√7/7**.

26. To simplify $1/(3 - 2\sqrt{2})$, we can multiply by the numerator and denominator by the conjugate $(3 + 2\sqrt{2})$: $(1 \times (3 + 2\sqrt{2}))/((3 - 2\sqrt{2})(3 + 2\sqrt{2})) = (3 + 2\sqrt{2})/(9 - 8) = (3 + 2\sqrt{2})/1 =$ **3 + 2√2**.

27. We square $((2 - \sqrt{2})/2)^2$: $((2 - \sqrt{2})/2)^2 = (2 - \sqrt{2})^2/4 = (4 - 4\sqrt{2} + 2)/4 = (6 - 4\sqrt{2})/4 =$ **(3 − 2√2)/2**.

28. To simplify $(3 + \sqrt{2})/\sqrt{2}$, we can multiply by the numerator and denominator by $\sqrt{2}$: $((3 + \sqrt{2}) \times \sqrt{2})/\sqrt{(\sqrt{2} \times \sqrt{2})} = (3\sqrt{2} + \sqrt{2})/2 =$ **(2 + 3√2)/2**.

29. Multiplying conjugates gives us a difference of squares, so $(\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5}) = (\sqrt{7})^2 - (\sqrt{5})^2 = 7 - 5 =$ **2**.

30. We begin by rationalizing the denominator of the fraction in the denominator by multiplying its numerator and denominator by the conjugate: $(1 \times (2 - \sqrt{3}))/((2 + \sqrt{3})(2 - \sqrt{3})) = (2 - \sqrt{3})/(4 - 3) = 2 - \sqrt{3}$. The expression becomes $1/(1 + 2 - \sqrt{3}) = 1/(3 - \sqrt{3})$. We rationalize again by multiplying by the conjugate: $(1 \times (3 + \sqrt{3}))/((3 - \sqrt{3})(3 + \sqrt{3})) = (3 + \sqrt{3})/(9 - 3) =$ **(3 + √3)/6**.

Warm-Up 1

31. If Elmina rounds each number to the nearest thousand before adding, she should get a sum of 5000 + 32,000 + 1000 = **38,000**.

32. Since we know that the sum of the first six terms of the arithmetic sequence is 42, we can calculate that the average of the terms is $42 \div 6 = 7$. Given that the first term is 2, this is enough to work out that the six terms must be 2, 4, 6, 8, 10, 12, where the average of 7 is half-way between the 6 and the **8**. Alternatively, some mathletes may know that the sum of an arithmetic sequence is the average of the first and last terms times the number of terms. Since we know that the average is 7, we can calculate that the sum of the first and last terms must be $2 \times 7 = 14$ and the last term must be $14 - 2 = 12$. The common difference must be added five times to get from the first to the sixth term, so that difference must be $(12 - 2) \div 5 = 10 \div 5 = 2$. Now we write out the six terms 2, 4, 6, 8, 10, 12, and we see that the fourth term is **8**.

33. We can divide 137 by 26 to find that the quotient is 5 and the remainder is 7. For this problem, we are more interested in the remainder, because the 137th letter on the banner is simply the 7th letter of the alphabet, which is **G**.

34. If $x + 3 = 10$, then $x = 7$. The value of $x^2 + 3^2$ is $7^2 + 3^2 = 49 + 9 =$ **58**.

35. There are 4 quarts in 1 gallon, so there are 2 quarts remaining after Nadine removed 2 quarts. There are 2 pints in 1 quart, so those 2 quarts are 4 pints. Nadine added 3 pints to this, so that makes 7 pints. There are 2 cups in 1 pint, so those 7 pints are 14 cups. Nadine removed 4 cups, so she ends up with 10 cups, or **80** fluid ounces, since there are 128/16 = 8 fluid ounces in a cup. We could also convert all quantities to ounces at the start, in which case we could compute 128 – 64 + 48 – 32 = **80** fluid ounces.

36. Alice chooses a number A and Maud chooses a number M, both from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, with $M \neq A$, as was confirmed up front by Nina. Alice can know for sure that AM is even if and only if her number A is even. Then, with A advertised to be even, Maud can know for sure that $A + M$ is even if and only if her number M is even. At this stage, A is an element of {2, 4, 6, 8, 10} and M is an element of {2, 4, 6, 8, 10}. If $A = 2$, then M being even and greater than 2 means that A is a proper divisor of M; if $A = 4$, then M could be 8 as far as Alice knows, making A a proper divisor of M. Such an issue does not occur for $A \geq 6$, so Alice can validly make such an assertion about A not being a proper divisor of M if and only if A is 6, 8 or 10. The remaining candidate (A, M) pairs are (6, 2), (6, 4), (6, 8), (6, 10), (8, 2), (8, 4), (8, 6), (8, 10), (10, 2), (10, 4), (10, 6) and (10, 8). Now we deal with the units digit of A^2 not equaling the units digit of M. Of our remaining candidate ordered pairs, only (8, 4) violates this congruence relation. Therefore, Maud can validly make her assertion if and only if $M \neq 4$, eliminating 3 ordered pairs from being candidates, leaving (6, 2), (6, 8), (6, 10), (8, 2), (8, 6), (8, 10), (10, 2), (10, 6) and (10, 8). Alice's assertion about the greatest common divisor provides no new information for eliminating any of the currently remaining 9 candidate ordered pairs, because gcd(A, M) = 2 for all 9 ordered pairs. This step is useless. Every remaining candidate pair with $M \leq 6$ has $A \geq 8$, so Alice's number is definitely greater; every remaining ordered pair with $M = 10$ has $A \leq 8$, so Alice's number is definitely not greater. The only value that would cause Maud to have uncertainty is $M = 8$, for which A could be 6 (less than M) or 10 (greater than M). Alice's response that $A \ngeq M = 8$ requires $A = 6$, so Alice's integer must be **6**.

37. Since 4 times 3/4 is 3, Alex will need to quadruple his recipe. He will need 4/2 cups of water and 4/3 cups of milk. Converting both of these quantities to sixths, we see that Alex will need 12/6 cups of water and 8/6 cups of milk, which is 4/6 or **2/3** more cups of water than milk.

38. If we look at this formation from each of the four sides, we can see a total of 6 units per side, so the perimeter of the figure is 4 × 6 = **24** units.

39. Joe ate 3/2 × 1/6 = 3/12 = **1/4** of the pie.

40. To find the units digit of the sum $1^2 + 2^2 + 3^2 + 4^2 + 5^2$, we take the units digits of the squares and add them: 1 + 4 + 9 + 6 + 5 = 25, which has a units digit **5**.

Warm-Up 2

41. Given that 5 glorps can be traded for 3 plaps, we can multiply both of these by 7 to find that Brian's 35 glorps should be worth 21 plaps. Tina only gave Brian 15 plaps, so she still owes him 21 – 15 = 6 plaps. Since 2 plaps can be traded for 1 froop, we can triple both of these to find that 6 plaps should be worth 3 froops. That means Tina would have to give Brian **3** froops to make the trade fair.

42. The value of $(11^2 - 1) \div (5^2 - 1)$ is $(121 - 1) \div (25 - 1) = 120 \div 24 =$ **5**.

43. There are 3 × 6 = 18 teachers and 148 + 152 + 168 = 468 students at Elliott Middle School. The ratio of teachers to students is 18 to 468, which simplifies to **1/26** as a common fraction

44. There are 26 letters in the alphabet, but there are 27 intervals from 0 to 3 on the number line. We want the letter at 2, which will be 2/3 × 27 = 18 intervals from 0. The 18th letter of the alphabet is **R**.

45. Fifty-seven percent of 200 is 114. If 114 is 300 percent of x, then x must be 114 ÷ 3 = **38**.

46. Todd should consume 30/100 × 2400 = 720 calories of protein each day. Since 1 gram of protein contains 4 calories, Todd should consume 720 ÷ 4 = **180** grams of protein daily.

47. The value of seven factorial is 7! = 1 × 2 × 3 × 4 × 5 × 6 × 7 = 5040, and its prime factorization is $2^4 \times 3^2 \times 5 \times 7$. The positive four-digit integer will have to include the digits 5 and 7, but there is no way we can contain four factors of 2 and two factors of 3 in the other two digits. If we use the digit 9, we would still need $2^4 = 16$, which requires two more digits. If we use two digits of 6 along with the digits 5 and 7, then we have left out two more factors of 2. The answer is that there are **0** (zero) positive four-digit integers such that the product is 7!.

48. If $5x + 9 = 37$, then $5x = 28$. That means $15x = 28 \times 3 = 84$, so $15x + 16 = 84 + 16 =$ **100**.

49. We would expect Leonard's spinner to land on the letter P one quarter of the time, which is $72 \div 4 =$ **18** spins.

50. There are 3 one-by-one rectangles, 2 one-by-two rectangles and 1 one-by-three rectangles, for a total of $3 + 2 + 1 =$ **6** rectangles.

Warm-Up 3

51. The prime factorization of 2000 is $2^4 \times 5^3$. There are $(4 + 1)(3 + 1) = 5 \times 4 = 20$ total positive divisors of 2000. To specify a perfect square divisor, we must choose even exponents for each prime factor. For the factor of 2, we can choose an exponent of 0, 2 or 4 (3 options). For the factor of 5, we can choose 0 or 2 (2 options). This gives us $3 \times 2 = 6$ perfect square divisors. Therefore, the percent of divisors that are perfect squares is $6/20 = 3/10 = 30/100 =$ **30**%. Alternatively, we can list the divisors of 2000, as shown in the array to the right. The 6 perfect square divisors of 2000 are 1, 4, 16, 25, 100 and 400. These 6 perfect squares represent **30**% of the 20 divisors.

1	5	**25**	125
2	10	50	250
4	20	**100**	500
8	40	200	1000
16	80	**400**	2000

52. If all writers work at the same rate, then the 822 questions written by 6 writers in one year means that each writer wrote $822 \div 6 = 137$ questions in that one year. In four years, one writer can write $137 \times 4 =$ **548** questions.

53. There are 5 ways to give 2 pumps of the same flavor and then "five choose two" $= (5 \times 4) \div 2 = 10$ ways to pick two different flavors. Then there are $3 + 1 = 4$ possible toppings, including no topping. That makes $(5 + 10) \times 4 = 15 \times 4 =$ **60** combinations of shaved ice.

54. If we subtract $3x + 6$ from both sides of the first equation, we get $2x = 4$, to which the solution is $x = 2$. Substituting this value into the second equation, we get $a(2) - 4 = 0$, which simplifies to $2a = 4$ and then $a =$ **2**.

55. Suppose Jimmy drove at 35 mi/h for h hours. Then he covered $65 + 35h$ miles in $(h + 1)$ hours, so $65 + 35h = 45(h + 1)$. Expanding and subtracting $35h + 45$ from both sides gives $20 = 10h$, so $h = 2$. We can calculate the total distance Jimmy drove in two ways: $45 \times (2 + 1) = 45 \times 3 =$ **135** miles *or* $35 \times 2 + 65 \times 1 = 70 + 65 =$ **135** miles.

56. There are 27 intervals from 0 to 3 on this number line, which is 9 intervals per unit. Since $1/9 \times 9 = 1$, the letter at 1/9 is the first letter, which is A. Since $2\ 2/3 \times 9 = 8/3 \times 9 = 24$, the next letter is the 24th letter, which is X. Likewise, the letter at 1 is the $1 \times 9 =$ 9th letter, which is I, and the letter at 2 1/9, is the $2\ 1/9 \times 9 = 19/9 \times 9 =$ 19th letter, which is S. The math word is **AXIS**.

57. To find the units digit of the expression, we can add just the units digit of each term. The first term 2 has a units digit of 2. The second term is $2^0 = 1$, with a units digit 1. The third term $2^{0+2} = 2^2 = 4$ has a units digit 4. The fourth term $2^{0+2+5} = 2^7 = 128$ has a units digit 8. Adding the units digits of each term gives $2 + 1 + 4 + 8 = 15$, so the units digit of the sum is **5**.

58. The numbers at the far right of each row of Geo's arrangement are known as the triangular numbers. The nth triangular number can be calculated as $n(n + 1) \div 2$, so the last number of the sixth row is $6(6 + 1) \div 2 = 21$ and the last number of the seventh row is $7(7 + 1) \div 2 = 28$. Knowing this, we can say that the sum of the first and last numbers of the seventh row is $22 + 28 =$ **50**.

59. To evaluate $\sqrt{(20\sqrt{25})}$, we start with the innermost radical sign: $\sqrt{25} = 5$. Now we have $\sqrt{(20 \times 5)} = \sqrt{100} =$ **10**.

60. The terms of an arithmetic progression must have a common difference, which we will call d. The three angles are 35, $35 + d$ and $35 + 2d$, and their sum must be 180 degrees. We need to solve the equation $35 + 35 + d + 35 + 2d = 180$. This simplifies to $105 + 3d = 180$, which simplifies further to $3d = 75$ and finally $d = 25$. The three angles are 35, 60 and 85, with the measure of the largest angle being **85** degrees.

Warm-Up 4

61. Amy can pick any of $3 \times 3 \times 3 = 27$ outfits, so 27 will be our working denominator to put pieces together, though we may need to reduce it in the final answer. Rather than count the number of ways that at least 2 pieces of her outfit are the same color, we can more easily count the complement of this condition, which is the number of ways that all 3 pieces are different colors. Amy can pick one of 3 colors for the headband, then either of the 2 remaining colors for the sweater, and then she has to pick the 1 remaining color for the skirt, so there are $3 \times 2 \times 1 = 6$ outfits with no matching colors. This means there are $27 - 6 = 21$ outfits with at least 2 pieces that are the same color, so the desired probability is 21/27 or **7/9**.

62. Since $6 \times 6 = 36$, all powers of numbers ending in 6 end in 6. Similarly, all powers of numbers ending in 5 end in 5, since $5 \times 5 = 25$. The units digit of $2026^{2025} + 2025^{2026}$ will be equal to the units digit of $6 + 5 = 11$, which is **1**.

63. Let's say that the cube has an edge length of x. There are 12 congruent edges on a cube, so the total length of the edges is $12x$. There are 6 congruent faces on the cube, so the total surface area of the cube is $6x^2$. Since these two quantities are equal on this particular cube, we can solve the equation $12x = 6x^2$ for x. In general, we should be careful about dividing both sides of an equation by a variable since we might inadvertently divide by zero. In this case, however, we can be sure that the cube has an edge length greater than zero. We can divide both sides of our equation by x and we get $12 = 6x$, so $x = 2$. We now know the cube has an edge length of 2 feet and we can calculate that the volume is $2 \times 2 \times 2 =$ **8** ft^3.

64. Let's imagine that 29 of Professor Plum's students have actually brought plums to school and have redistributed then so that everyone has the same number of plums. This is the average number of plums. Imagine further that Scarlet arrives late with 100 plums. When Scarlet gives 1 plum to each of the other 29 students, the 100 − 29 = 71 plums that she keeps for herself just happens to be the same number that everyone else has. This is the new average. Although the scores on the final exam are not actually redistributed like we imagined with the plums, the concept of an average is the number you would get if you did redistribute. Therefore, the correct class average on the final exam must be **71**. Alternatively, we can use algebra and let x equal the first average. Then $29x$ is the total score without Scarlet and $29x + 100$ is the total score with Scarlet. If we divide the total with Scarlet by 30, we get one more than the old average, which we write as $x + 1$. Our equation can be solved as follows: $(29x + 100)/30 = x + 1 \rightarrow 29x + 100 = 30(x + 1) \rightarrow 29x + 100 = 30x + 30 \rightarrow 100 = x + 30 \rightarrow x = 70$. So the first average without Scarlet was 70 and the correct class average with Scarlet is 70 + 1 = **71**.

65. There are 2 × 2 = 4 equally likely possible outcomes for the coin that is tossed twice. We will list the outcomes as HH, HT, TH and TT. Outcome HH means that the coin lands on heads twice and the counter would end up at S. Outcomes HT and TH result in the counter returning to square T. And outcome TT leaves the counter on square M. In 2 out of 4 outcomes, the counter returns to square T, so the probability is 2/4 or **1/2**.

66. Let's try some small values of n and work our way up. If $n = 2$, we get $2! = 1 \times 2 = 2$, which is not divisible by $2^2 = 4$. If $n = 3$, we get $3! = 1 \times 2 \times 3 = 6$, which is not divisible by $3^2 = 9$. If $n = 4$, we get $4! = 1 \times 2 \times 3 \times 4 = 24$, which is not divisible by $4^2 = 16$. If $n = 5$, we get $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$, which is not divisible by $5^2 = 25$. Finally, if $n = 6$, we get $6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$, which is in fact divisible by $6^2 = 36$. So our answer is $n =$ **6**.

67. The original flight was expected to be 4:10 − 2:40 = 1:30 hours long, which is 1 hour and 30 minutes. If we add this amount of time to the 3:15 departure time, we get an arrival time of 3:15 + 1:30 = 4:45 p.m. Micah's connecting flight will leave 5:05 – 4:45 = 0:20 hour or **20** minutes later.

68. To count the number of isosceles right triangles with legs of 1 unit, we consider all four orientations. There are 15 triangles with the right angle at the bottom-left (◣), 10 triangles with the right angle at the bottom-right (◢), 10 triangles with the right angle at the top-left (◤), and 10 triangles with the right angle at the top-right (◥). Thus, there are 15 + 10 + 10 + 10 = **45** isosceles right triangles, with legs of 1 unit, in the array.

69. If we triple both sides of the first equation, we get $6a + 3b =$ **39**.

70. Arlo counts 100, 93, 86, etc. until he gets to 100 – 15 × 7 = 100 – 105 = –5. He then starts adding 6, giving him 1, 7, 13, etc. until he gets to 1 + 17 × 6 = 1 + 102 = **103**.

Warm-Up 5

71. A Pythagorean triple is a set of 3 integers that satisfy the Pythagorean theorem, $a^2 + b^2 = c^2$. If these numbers are the side lengths of a triangle, then we know that it is a right triangle. The most common example is the 3-4-5 triple, since $3^2 + 4^2 = 5^2$. To find a Pythagorean triple with the numbers 20 and 25, we multiply the 3-4-5 triple by 5, which gives us the lengths 15, 20 and 25. Since AB = 20 cm and AC = 25 cm, side BC must equal 15 cm. These 3 side lengths form a right triangle because $15^2 + 20^2 = 25^2$. Since the legs of a right triangle are the 2 shorter sides, the legs must be 15 cm and 20 cm, not 20 and 25, since $20^2 + 25^2 = 1025$, which is not a perfect square. So AB and BC are the legs and are perpendicular to each other, and we can calculate the area as 1/2 × 15 × 20 = **150** cm^2.

72. It is helpful to think of the ratio x/y as 3/1, so we think of it as 3 + 1 = 4 parts. The sum of x and y is 20, so each part is 20 ÷ 4 = 5. The numbers must be $x = 15$ and $y = 5$. The value of $x - y$ is 15 – 5 = **10**.

73. To find the least numeric palindrome divisible by 12, we list multiples of 12 in order until we reach one that reads the same forwards and backwards. The first such number is **252**.

74. Given that the real heptadecagon has a perimeter of 510 feet, we can calculate that each side of the real heptadecagon is 510 ÷ 17 = 30 feet. We are told that one side of the model heptadecagon is 2 inches, which is 1/6 of a foot. The scale factor of the model must be 1/6 ÷ 30 = 1/180, which is to say that all linear measurements on the actual Gauss Monument must be 180 times as long as the corresponding measurements on the scale model. Since the scale model is 3 feet tall, the real monument must be 180 × 3 feet = **540** feet tall.

75. The base ten value of 123_{four} is $1 \times 4^2 + 2 \times 4 + 3 = 16 + 8 + 3 = 27$, and the base ten value of 1234_{five} is $1 \times 5^3 + 2 \times 5^2 + 3 \times 5 + 4 = 125 + 50 + 15 + 4 = 194$. The desired sum is 27 + 194 = **221**.

76. The sum of the first 5 positive perfect squares is 1 + 4 + 9 + 16 + 25 = 55. This is also the sum of the first 10 positive integers, since 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55, so the value of n is **10**.

77. We will consider each of the possible tens digits and decide how many three-digit integers meet the conditions. For a tens digit of 0, there is only 1 three-digit integer and that's 101. For a tens digit of 1, there are 2 three-digit integers: 210 and 212. There are 4 three-digit integers with a tens digit of 2: 121, 123, 321 and 323. For tens digits of 3 through 8, there are likewise 4 three-digit integers. Finally, there is just 1 three-digit integers with a tens digit of 9 and that's 898. In all, there are 1 + 2 + 7 × 4 + 1 = **32** integers.

78. Cory can frost 2 dozen cupcakes in 14 minutes, which is a rate of 1 dozen every 7 minutes or 12/7 cupcakes per minute. Dory can frost 3 dozen cupcakes in 15 minutes, which is a rate of 1 dozen every 5 minutes or 12/5 cupcakes per minute. Together, they can frost 12/7 + 12/5 = 60/35 + 84/35 = 144/35 cupcakes per minute or 144 cupcakes in 35 minutes. That's 12 dozen cupcakes, so 1 dozen cupcakes would take 35/12 minutes, which is just under 3 minutes. At Cory's rate of 12/7 cupcakes per minute, he will put frosting on 12/7 × 35/12 = **5** cupcakes.

79. The mean of the numbers is (2 + 5 + 11 + 14 + 23) ÷ 5 = 55 ÷ 5 = 11, and the median of the numbers is 11. The absolute difference between the mean and the median is |11 − 11| = **0**.

80. There are 4! =24 possible four-digit passcodes using those numbers, so the probability that Garrett guesses the correct one on his first try is **1/24**.

Warm-Up 6

81. In order for the median to be 5, the average of the middle two numbers must be 5, so their sum must be 10. The greatest element can't be any less than 5, and the least element can't be any less than 1. Therefore the best we can do is 1, 5, 5, 5; the mean is 16/4 = **4**.

82. We can rewrite the equation $x + x^2 = 6$ as $x^2 + x - 6 = 0$ and solve by factoring. This quadratic factors easily as $(x + 3)(x - 2) = 0$, so the solutions are $x = -3$ and $x = 2$. Therefore, the least possible value of integer x is **−3**.

83. The size of the bathtub is not stated, so we can make up a convenient capacity to help us think through this problem. Let's use the LCM of 10 and 15 and suppose that the bathtub holds 30 gallons of water when filled completely. In this case, the faucet would fill the bathtub at a rate of 30 ÷ 10 = 3 gallons per minute, and the drain would empty it at a rate of 30 ÷ 15 = 2 gallons per minute. With the faucet on and the drain open, the bathtub would fill at a rate of 3 − 2 = 1 gallon per minute. Since the tub is already half full, it must already have 15 gallons in it and need 15 more gallons to fill completely. At the rate of 1 gallon per minute, that will take **15** minutes.

84. Let's suppose for a moment that Anu cuts the ribbon so that the difference is exactly 3 inches. Then the shorter piece is (14 − 3) ÷ 2 = 11 ÷ 2 = 5.5 inches long, and the longer piece is 5.5 + 3 = 8.5 inches long. If Anu cuts the ribbon anywhere between the lengths of 5.5 and 8.5 inches, the difference will be less than 3 inches, so the probability is **3/14**.

85. Matt eats 1/4 × 2/3 + 2/3 × 1/3 = 2/12 + 2/9 = 6/36 + 8/36 = 14/36 of the pizza and Matics eats 2/3 × 2/3 + 1/4 × 1/3 = 4/9 + 1/12 = 16/36 + 3/36 = 19/36 of the pizza. That's a total of 14/36 + 19/36 = 33/36 = 11/12, so **1/12** of the pizza must be left.

86. The cyclist took twice as long to walk half the distance, so he must ride his bicycle 2 ÷ 1/2 = **4** times as fast as he walks.

87. We can evaluate $g(f(8))$ by evaluating $f(8)$ first and then using the output of the f function as the input of the g function. Evaluating $f(8)$, we have $f(8) = \sqrt{(8 + 1)} = \sqrt{9} = 3$. Then evaluating $g(3)$, we have $g(3) = 3^2 + 1 = 9 + 1 =$ **10**.

88. The formula for the volume of a cone is $V_{Cone} = (1/3)\pi r^2 h$, where r is the radius and h is the height. Since Sosa's cone has a radius 3 times that of Lola's, its base area is $3^2 = 9$ times as large. Since the height of Sosa's cone is half that of Lola's cone, the ratio of the volumes is **9/2**.

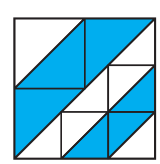
89. We can rewrite the ratio of 6 clanks to 1 clink as 72 clanks to 12 clinks. Now, since we are told that 12 clinks are in 9 clonks, we can see that 72 clanks are in 9 clonks, which means that **8** clanks are in 1 clonk.

90. There are 6 × 6 = 36 possible outcomes when rolling two dice. If we treat the dice as different colors, there are 6 outcomes where one number is twice the other: (1, 2), (2, 4), (3, 6) and, with reversed colors, (2, 1), (4, 2), (6, 3). Thus, the probability of rolling such a pair is 6/36 = **1/6**.

Warm-Up 7

−	**1**	**2**	**3**	**4**	**5**	**6**
1		1	2	3	4	5
2	1		1	2	3	4
3	2	1		1	2	3
4	3	2	1		1	2
5	4	3	2	1		1
6	5	4	3	2	1	

91. The grid at right shows the positive differences between Katie's two dice that are rolled. When James rolls his four-sided (a.k.a. tetrahedral) die, the probability is 1/4 that he rolls each of the four numbers 1, 2, 3 and 4. There are 10 ways for Katie's dice to have a difference of 1, so the probability that she gets a 1 is 10/36 or 5/18. The probability that Katie gets a difference of 1 and James gets a 1 is 5/18 × 1/4 = 5/72. The probability that they both obtain a 2 is 4/18 × 1/4 = 4/72, that they both obtain a 3 is 3/18 × 1/4 = 3/72, and that they both obtain a 4 is 2/18 × 1/4 = 2/72. The total probability that they match is 5/72 + 4/72 + 3/72 + 2/72 = 14/72 = **7/36**. Note that James cannot roll a 0 or a 5, so the probability of a match is zero if Katie rolls either of those differences.



92. We can subdivide the figure into congruent triangles, shown right. The shaded portion of the flag is 3/4 × 1/2 + 3/9 × 1/2 = 3/8 + 3/18 = 3/8 + 1/6 = 9/24 + 4/24 = **13/24**. Alternatively, to compute the area of the larger shaded region, notice that it is the area of an isosceles right triangle with base 24 in minus the area of an isosceles right triangle with base 12 in, so this area is 1/2 × 24 × 24 − 1/2 × 12 × 12 = 12 × 24 − 12 × 6 = 12 × (24 − 6) = 12 × 18 = 216 in^2. Similarly, the area of the smaller shaded region is the area of an isosceles right triangle with base 16 in minus the area of an isosceles right triangle with base

8 in, so this area is $1/2 \times 16 \times 16 - 1/2 \times 8 \times 8 = 8 \times 16 - 8 \times 4 = 8 \times (16 - 4) = 8 \times 12 = 96$ in^2. Therefore the total shaded area is $216 + 96 = 312$ in^2. The area of the square is (24×24) in^2, so the portion of the square that is shaded is $312/(24 \times 24) = (3 \times 8 \times 13)/(24 \times 24) =$ **13/24**.

93. Since the flower arrangement must have at least 1 of each type of flower, we can begin with 1 rose, 1 tulip, 1 peony and 1 daffodil. We have to count how many ways we can choose the other $9 - 4 = 5$ flowers. For this we can use the "stars and bars" method, where each star represents a flower in a certain bin and each bar represents the partitions separating those bins. The order of our bins will be roses, tulips, peonies, daffodils, so the string of 8 symbols *****||| represents 5 roses, no tulips, no peonies and no daffodils. Another string of 8 symbols might be *|*|*|**, which represents 1 rose, 1 tulip, 1 peony and 2 daffodils, or |**|***|, which represents no roses, 2 tulips, 3 peonies and no daffodils. In this way, all possible combinations can be represented. Since there are always 8 symbols, we can think of this a "8 choose 3", where we are choosing 3 of the symbols to be bars instead of stars. The value of "8 choose 3" is $(8 \times 7 \times 6) \div (3 \times 2 \times 1) = 56$, so there are **56** possible combinations of flowers.

94. The Spanish sector of the pie chart is 180 degrees, which is half of the pie chart. Since 2 students were counted for 2 languages, $30 + 2 = 32$ students are represented by the pie chart. Half of 32 is 16, but 2 of the students taking Spanish are also taking Latin, so there must be $16 - 2 =$ **14** students taking just Spanish in Mr. Campo's Statistics class.

95. The value of 6! is $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$, and the prime factorization of 720 is $2^4 \times 3^2 \times 5^1$. To find how many positive divisors 720 has, we count all the different ways we can form a product using these prime factors. Each exponent can range from 0 up to its maximum in the factorization: the exponent on 2 can be 0 through 4 (5 choices), the exponent on 3 can be 0 through 2 (3 choices), and the exponent on 5 can be 0 or 1 (2 choices). Multiplying these together, we can calculate that 720 has $5 \times 3 \times 2 = 30$ positive divisors. Now we consider the number 2^6, whose divisors are the powers of 2 from 2^0 to 2^6, for a total of 7 divisors. However, since 720 only contains 2^4, the only powers of 2 that divide both 720 and 2^6 are $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$ and $2^4 = 16$. So exactly 5 of the 30 divisors of 720 are also divisors of 2^6, and the desired fraction is $5/30 =$ **1/6**.

96. The units digit of $202^5 + 2^{0+25}$ is the same as the units digit of $2^5 + 2^{25}$. The powers of 2 are 2, 4, 8, 16, 32, 64, 128, 256, etc., with the units digits repeating every fourth term. Since 5 and 25 are both 1 more than a multiple of 4, both the fifth and the twenty-fifth powers of 2 will have a units digit of 2. The units digit of $2^5 + 2^{25}$ is just the units digit of $2 + 2$, which is **4**.

97. We can find the coordinates of point C by looking at the *x*- and *y*-coordinates separately. From point A to point B, the *x*-coordinate changes from 14 to −10, a difference of −24. Since point B is 3/5 of the way from point A to point C along segment AC, the *x*-coordinate point C must be $-24 \times 5/3 = -40$ away from point A, making it $14 - 40 = -26$. For the *y*-coordinate, point A to point B changes from −4 to 8, a difference of 12. Thus, the *y*-coordinate of point C must be $12 \times (5/3) = 20$ away from point A, making it $-4 + 20 = 16$. Therefore, the coordinates of point C are (**−26**, **16**).

98. To meet the requirements, the group must either include 3 teenagers from 1 family and 1 teenager from each of the other 4 families, or include 2 teenagers from each of 2 families and 1 teenager from each of the other 3 families. In the first case, there are 5 ways to choose the family contributing 3 teenagers, and $3^4 = 81$ ways to choose 1 teenager from each of the other 4 families, for a total of $5 \times 81 = 405$ groups. In the second case, there are "5 choose 2" = 10 ways to choose the 2 families contributing 2 teenagers each and $3^5 = 243$ ways to choose the teenagers for the 5 families, for a total of $10 \times 243 = 2430$ groups. Adding both cases gives $405 + 2430 =$ **2835** possible groups.

99. We can start by simplifying the expression: $22!/20! = (22 \times 21 \times 20!)/20! = 22 \times 21$. The prime factorization of 22 is 2×11, and the prime factorization of 21 is $2 \times 3 \times 7$. That means the sum of the distinct prime number divisors of 22×21 is $2 + 11 + 3 + 7 =$ **23**.

100. Since all sides of an equilateral triangle are congruent, we can equate the stated lengths of YZ and XZ and solve for *x*. We have $2x + 5 = 6x - 7$. If we subtract $2x$ from both sides, we get $5 = 4x - 7$. Then we add 7 to both sides, which gives $12 = 4x$. Next, dividing both sides by 4, we get $x = 3$. Substituting this value into the expression for side YZ gives a length of $2 \times 3 + 5 = 6 + 5 = 11$ feet. The length of XY must also be **11** feet.

Warm-Up 8

101. Brennan will need 4 pennies to make 1, 2, 3 and 4 cents. If he also has 1 nickel, he can make 5 through 9 cents. Adding a dime allows him to make amounts from 10 through 19 cents. With his last coin being another dime, Brennan can make any amount from 20 to 25 cents. So, with just **7** coins—4 pennies, 1 nickel and 2 dimes—Brennan can pay any amount from 1 to 25 cents in exact change.

102. The area of the inner circle is πr^2 and the area of the outer circle is $\pi(2r)^2 = 4\pi r^2$. The area of the annulus (or ring) is $4\pi r^2 - \pi r^2 = 3\pi r^2$. We need some fraction of $3\pi r^2$ to be equal to 1/8 of $4\pi r^2$. That fraction should have a denominator of 360 so that the numerator is the number of degrees in the unknown angle *x*. We need to solve the following equation for *x*: $x/360 \times 3\pi r^2 = 1/8 \times 4\pi r^2$. This simplifies to $x/120 = 1/2$, so $x =$ **60** degrees.

103. There are $360 \times 60 \times 60$ arcseconds in a full circle and $24 \times 60 \times 60$ seconds in 24 hours. The circle will rotate $360 \div 24 =$ **15** arcseconds every second.

104. The triangle inequality theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. The list to the right shows the only **5** non-congruent triangles with integer side lengths and a perimeter of at most 7.

(1, 1, 1)
(1, 2, 2)
(1, 3, 3)
(2, 2, 2)
(2, 2, 3)

105. A square with a side length of *s* has a perimeter of $4s$ and an area of s^2. We are told that the perimeter exceeds the area by a positive integer *n*, so we have the equation $4s = s^2 + n$. Rearranging gives $s^2 - 4s = -n$, and completing the square yields $(s - 2)^2 = 4 - n$. Since the square of a real number is nonnegative, we must have $4 - n \geq 0$, so $n \leq 4$. Since n is a positive integer, the possible values of n are 1, 2, 3 and 4. Each of these values gives a real solution for s, so all are valid. Therefore, the sum of all possible distinct values of n is 1 + 2 + 3 + 4 = **10**.

106. We can take the cross product of the given equation to get $(x - 1)(x - 5) = (x - 3)(x - 4)$. We can now expand both sides of the equation to get $x^2 - 6x + 5 = x^2 - 7x + 12$. We can subtract x^2 from both sides of the equation to get $-6x + 5 = -7x + 12$, and finally we can add $7x - 5$ to both sides to get x = **7**. Note that substituting $x = 7$ into the original equation does not make any denominator equal to zero, so it is not an extraneous solution.

107. If the frog is to get across the river in the fewest hops, he should take as many triple hops as possible. With 18 lily pads in the pond, it will take 19 hops to get across. The frog can do 5 triple hops and 2 double hops, since 5 × 3 + 2 × 2 = 19. The sequence TTTTTDD describes one way the frog could get across the pond and TTTTDTD describes another. We want to count all the ways that 2 of these 7 characters can be D instead of T. This is just “7 choose 2,” which is 7 × 6 ÷ 2 = 21. Therefore, there are **21** different ways the frog can hop from one shore to the other.

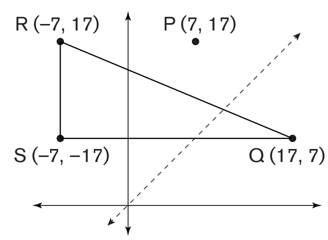
108. The ratio of flour to eggs in Rachel’s pumpkin bread recipe is 3.5/4 or 7/8, which means there are 7/8 as many cups of flour as there are eggs in the recipe. If Rachel uses 7 eggs, she should use 7/8 × 7 = 49/8 = **6 1/8** cups of flour.

109. The positive integer divisors of 32 are 1, 2, 4, 8, 16 and 32, of which three are perfect squares: 1, 4 and 16. If one of the divisors is chosen at random, the probability is 3/6 or **1/2** that the chosen divisor is a perfect square.

110. If Leona had 85 cents entirely in nickels, then she would have 85 ÷ 5 = 17 coins. Each time she trades 2 nickels for 1 dime, she reduces the total number of coins by 1 because 2 coins become 1 coin. Since she ends up with 12 coins, the total number of trades must be 17 − 12 = 5. That means she traded 10 nickels (5 trades × 2 nickels each) for 5 dimes. Therefore, she must have **5** dimes.

Warm-Up 9

111. There are $6^5 = 7776$ possible outcomes when rolling the die five times, and directly counting those with a sum divisible by 3 would be tedious. Instead, consider the first four rolls. There are $6^4 = 1296$ possible sequences, and their sums range from 4 to 24. If it’s a multiple of 3 already, then we want the last roll of the die to be a 3 or a 6, or 1/3 of the possible rolls of the die. If the sum is 1 more than a multiple of 3 (1 mod 3), then we want the last roll of the die to be a 2 or a 5, or 1/3 of the possible rolls of the die. If the sum is 2 more than a multiple of 3 (2 mod 3), then we want the last roll of the die to be a 1 or a 4, or 1/3 of the possible rolls of the die. In each case, exactly two of the six outcomes for the last roll make the total sum divisible by 3. Thus, regardless of the first four rolls, the probability is 2/6 = **1/3** that the sum of all five rolls is divisible by 3.



112. Reflecting across the line $y = x$ causes the x and y to switch places, so the coordinates of point Q are (17, 7). Reflecting across the y-axis changes the sign of the x-coordinate, so the coordinates of point R are (−7, 17). The horizontal distance from point Q to point R is 17 − (−7) = 17 + 7 = 24 units, and the vertical distance is 17 − 7 = 10 units. By the Pythagorean theorem, QR = $\sqrt{(24^2 + 10^2)} = \sqrt{(576 + 100)} = \sqrt{676}$ = **26** units.

113. The expression $111^2 - 99^2$ is a difference of squares, so it factors as (111 + 99)(111 − 99), which then simplifies to 210 × 12 = **2520**.

114. The single-digit powers of 2 are 1, 2, 4 and 8. The only perfect square number in the 8 hundreds is 29^2 = **841**, and all its digits are powers of 2.

115. There are 10 possible ways to get a sum of 15 on three rolls of a standard die. The distinct permutations are 366, 636, 663, 456, 465, 546, 564, 645, 654 and 555. The probability is 2/10 = **1/5** that the first roll is a 4.

116. If the small hose takes 20 hours to fill the pool, it would fill 1/20 of the pool each hour. Likewise, if the large hose takes 16 hours to fill the pool, it would fill 1/16 of the pool each hour. Since the small hose is on for 4 hours by itself, the pool was already 4/20 full at 12:00 p.m. when the large hose was turned on. Together, the two hoses will fill 1/20 + 1/16 = 4/80 + 5/80 = 9/80 of the pool each hour, or 4 × 9/80 = 9/20 of the pool in 4 hours. By 4:00 p.m., the pool must be 4/20 + 9/20 = 13/20 or **65**% full.

117. The probability that person A tells the truth and person B lies is 3/4 × 1/5 = 3/20, and the probability that person A lies and person B tells the truth is 1/4 × 4/5 = 4/20. Therefore, the probability that one tells the truth and the other lies is 3/20 + 4/20 = **7/20**.

118. Let’s call the first term of this sequence a and the second term b. The third term is then $a + b$, and the fourth term is ab. Since this is an arithmetic sequence, there must be a common difference between terms. Subtracting the second term from the third term, we get $(a + b) - b = a$, which shows that this common difference is equal to the first term, a. The difference between the second term and the first term must also be a, so we get $b - a = a$, which means that $b = 2a$. Now we can look at the difference between the fourth and the third terms, which is $ab - (a + b) = a$, and substitute $2a$ for b. This gives us the equation $2a^2 - (a + 2a) = a$, which we can simplify to $2a^2 - 4a = 0$, then $a^2 - 2a = 0$. We can factor this as $a(a - 2) = 0$, so $a = 0$ and $a = 2$. Since we know that the sequence consists “entirely of positive integers,” we can discard the $a = 0$ solution and keep the $a = 2$ solution. The first four terms of the sequence are 2, 4, 6, 8, and the fifth term is **10**.

119. The area of rectangle ABCD is $5 \times 3 = 15$ cm^2. The area of triangle EBF is $1/2 \times 4 \times 2 = 4$ cm^2. The area of pentagon AEFCD is the difference $15 - 4 =$ **11** cm^2.

120. The value of $13^2 - 12^2 - 5^2$ is $169 - 144 - 25 =$ **0**.

Warm-Up 10

121. Imagine that the dog is standing as far from the pole as possible. In this position, the 5-foot pole is one leg of a right triangle, the 13-foot leash is the hypotenuse, and the radius of the circular area he can explore is the other leg. Some mathletes will recognize this as a 5-12-13 Pythagorean triple, but if not, we can determine the length of the radius as follows: $r = \sqrt{(13^2 - 5^2)} = \sqrt{(169 - 25)} = \sqrt{144} = 12$. Thus, the dog can explore a circle with radius 12 feet, giving an area of $12^2 \times \pi =$ **144π** ft^2.

122. Since $45 \times 45 = 2025$, the square root of 2026 must be only slightly more than 45, and the fourth root of 2026 is equal to the square root of this number slightly more than 45. The nearest perfect square to 45 is 49, so $\sqrt[4]{2026} \approx 7$. The integer closest to $\sqrt{2026} + \sqrt[4]{2026}$ is $45 + 7 =$ **52**.

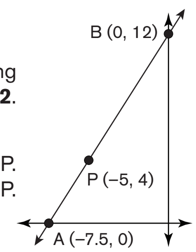
123. The probability that 0 reds are drawn at random from the bag is $2/6 \times 2/6 = 4/36 = 1/9$. The probability that 1 red is drawn at random from the bag is $4/6 \times 2/6 + 2/6 \times 4/6 = 8/36 + 8/36 = 16/36 = 4/9$. The probability that 2 reds are drawn at random is $4/6 \times 4/6 = 16/36 = 4/9$. The ratio a:b:c is 1:4:4, so the value of $100a + 10b + c$ is **144**.

124. If we divide 3 by 14, we get $0.2\overline{142857}$, which has a single non-repeating digit in the tenths place and then a six-digit repeating pattern. This means that the 2026th digit after the decimal point is the 2025th digit in the repeating pattern. Since 2025 is 3 more than a multiple of 6, we want the third digit in the repeating pattern, which is **2**.

125. The triangle inequality theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. This means that the third side must be less than $12 + 7 = 19$ and greater than $12 - 7 = 5$. The greatest possible perimeter is $12 + 7 + 18 = 37$ and the least possible perimeter is $12 + 7 + 6 = 25$. So, the absolute difference is $|37 - 25| =$ **12**.

126. Since $(x + y)^3 = x^3 + 3x^3y + 3xy^2 + y^3$, we know that $(a + a^{-1})^3 = a^3 + 3a^2a^{-1} + 3aa^{-2} + a^{-3} = a^3 + 3a + 3a^{-1} + a^{-3}$. Subtracting $a^3 + a^{-3}$ gives $3a + 3a^{-1} = 3(a + a^{-1})$. Letting $a = 2/3$, so $a^{-1} = 3/2$, yields $3(2/3 + 3/2) = 3(2 \times 2 + 3 \times 3)/(3 \times 2) = (4 + 9)/2 =$ **13/2**.

127. Since the point P splits segment AB in a 1:2 ratio, the x-coordinate of point A needs to be 3/2 times the x-coordinate of point P. Thus, $x = 3/2 \times (-5) = -7.5$, so point A is $(-7.5, 0)$. Similarly, the y-coordinate of point B must be 3 times the y-coordinate of point P. Thus, $y = 3 \times 4 = 12$, so point B is $(0, 12)$. The slope of line AB is $(12 - 0)/(0 - (-7.5)) = 12/7.5 = 24/15 =$ **8/5**.



128. Since rotations and reflections are considered different, we treat each of the 8 seats around the circular table as distinct. The 2 parents must sit next to each other, so we begin by choosing 1 of the 8 adjacent seat pairs around the circle—there are 8 such pairs because the table wraps around. For each pair, there are 2 ways to arrange the parents (either a sits first and b second, or vice versa). Once the parents are seated, we must arrange the 6 children (c, d, e, f, g and h) in the remaining 6 seats, which can be done in $6! = 720$ ways. Multiplying these together, we find the total number of seating arrangements is $8 \times 2 \times 720 =$ **11,520** seating arrangements.

129. The diameter of the center circle (bullseye) is 6 cm, so its radius is 3 cm and its area is $3^2 \times \pi = 9\pi$ cm^2. The radius of the whole target is $3 + 4 + 4 + 4 = 15$ cm, so its area is $15^2 \times \pi = 225\pi$ cm^2. The bullseye is $9\pi/(225\pi) = 1/25$ of the whole target, and 1/25 is **4**%.

130. Lyle can arrange his toy soldiers into 6 rows of 20, 8 rows of 15 or 10 rows of 12. For each of these arrangements, he can rotate the arrangements of the soldiers by 90 degrees, essentially making the rows into columns and the columns into rows. The sum of all possible numbers of rows, with no more than 20 soldiers in any row or column, is $6 + 8 + 10 + 12 + 15 + 20 =$ **71** rows.

Warm-Up 11

131. We can start by factoring the expression: $9! + 10! = 1 \times 9! + 10 \times 9! = 9! \times (1 + 10) = 9! \times 11$. This tells us that any divisor of 9! or 11 will divide the entire expression. All positive integers from 1 through 9 are divisors of 9!, so they divide the expression. The number 10 is 2×5, and both 2 and 5 are divisors of 9!, so 10 divides 9!, and therefore also divides $9! \times 11$. Since 11 appears explicitly as a divisor, it also divides the expression. Now consider 12: it is 2×6, and both 2 and 6 are divisors of 9!, so 12 divides 9!. Next comes 13, which is a prime number greater than 11. It does not divide 9! or 11. Therefore, 13 does not divide $9! \times 11$. So, the least positive integer that does not divide $9! + 10! = 3{,}991{,}680$ is **13**.

132. There are a total of "12 choose 2" $= 12!/(2! \times 10!) = (12 \times 11)/(2 \times 1) = 66$ ways to choose 2 students from the class. To find the number of favorable outcomes—pairs of students who are sitting at the same table—we consider each table separately. Each of the 3 tables has 4 students, and from each table there are "4 choose 2" $= 4!/(2! \times 2!) = (4 \times 3)/(2 \times 1) = 6$ ways to choose 2 students from that table. Since there are 3 tables, the total number of favorable outcomes is $3 \times 6 = 18$. So, the probability that the 2 randomly selected students are sitting at the same table is $18/66 =$ **3/11**.

133. Let's try some different radii and see what happens. If $r = 2$, then we have $4\pi - 2 \approx 12.57 - 2 = 10.57$, which is less than 20. If $r = 3$, we have $9\pi - 3 \approx 28.27 - 3 = 25.27$, which is too much. The area A can be as much as 20 greater than r, but not more. Because $r = 2$ works, then $20 + 2 = 22$ works for A; $r = 3$ does not work, so $20 + 3 = 23$ does not work for A. Therefore, **22** is the greatest possible integer value of A.

134. Since we are given $(-2, 3)$ as the intersection of these two lines, we can simply substitute the values of x and y, and solve for the unknown values of a and b, as shown below. So, the value of $a + b$ is $-4 + 5 =$ **1**.

$$\begin{array}{cc} x + 2y + a = 0 & 3x + by - 9 = 0 \\ (-2) + 2(3) + a = 0 & 3(-2) + b(3) - 9 = 0 \\ 4 + a = 0 & 3b - 15 = 0 \\ a = -4 & b = 5 \end{array}$$

135. Triangles AKE and CKS are similar with a scale factor of 2 to 7. Therefore the ratio of lengths CS and AE is 7/2. If we say that segment AE is $2x$ units, then segment CS must be $7x$ units. Piece II and piece III are triangles with the same height, so the ratio of their areas is equal to the ratio of their bases. Since Benjamin prices piece II at \$2, he will charge $2 \times 7/2 =$ **\$7** for piece III.

136. The sum of all cubes of positive prime integers less than or equal to 2025 is $2^3 + 3^3 + 5^3 + 7^3 + 11^3 = 8 + 27 + 125 + 343 + 1331 =$ **1834**.

137. Let's remove the 2 at the end of this 2025-digit number, making it a 2024-digit number with exactly $2024 \div 4 = 506$ copies of the number 2025. Each copy of the number 2025 has 3 prime digits, so there would be $506 \times 3 = 1518$ prime digits. Adding back the final 2 increases the count by 1, giving a total of **1519** prime digits.

138. The small pyramid that is cut off at the top is similar to the original pyramid because the cutting plane is parallel to the base. Since the two resulting pieces have equal height, the smaller pyramid has half the height of the original. This means the scale factor between the smaller pyramid and the original is 1:2. Because volume scales with the cube of the scale factor, the volumes are in the ratio $1^3:2^3$, or 1:8. So, the smaller pyramid has volume $1/8 \times 40 = 5$ cm^3, and the remaining piece has volume $40 - 5 = 35$ cm^3. The absolute difference between these volumes is $|35 - 5| =$ **30** cm^3.

139. Since 8 is 2^3 and 16 is 2^4, we can rewrite the expression $2^5 \times 8^3 \times 16^2$ as $2^5 \times 2^9 \times 2^8 = 2^{22}$. This is the same as 4^{11}, so m is **11**.

140. The value of the expression is $(6/3 - 1/3) \times (10/5 - 1/5) \times (14/7 - 1/7) = 5/3 \times 9/5 \times 13/7 = 3 \times 13/7 =$ **39/7**.

Workout 1

141. There are $100 \div 8 = 12.5$ cups in 100 fluid ounces, so there are $12.5 \times 30 =$ **375** calories in a full carton of almond milk.

142. There are 4 single-digit positive even integers, namely 2, 4, 6 and 8. From 10 to 99, there are $99/2 - (10 - 1)/2 = 99/2 - 9/2 = 90/2 = 45$ two-digit positive even integers. From 100 to 999, there are $999/2 - (100 - 1)/2 = 999/2 - 99/2 = 900/2 = 450$ three-digit positive even integers. That's 499 positive even integers so far, so we just need the 1 four-digit number 1000. The number of digits used to write these 500 positive even integers is $4 \times 1 + 45 \times 2 + 450 \times 3 + 1 \times 4 = 4 + 90 + 1350 + 4 =$ **1448** digits.

143. For the square photo, including the 1-inch frame on all sides, the outer dimensions are $4 + 2 = 6$ in by $4 + 2 = 6$ in. The total area of the photo with the frame is $6 \times 6 = 36$ in^2, so the area of the frame is $36 - 16 = 20$ in^2. For the rectangular photo, the outer dimensions are $2 + 2 = 4$ in by $8 + 2 = 10$ in. The total area of the photo with the frame is $4 \times 10 = 40$ in^2, so the area of the frame is $40 - 16 = 24$ in^2. That means the absolute difference between the areas of the two frames is $|24 - 20| =$ **4** in^2.

144. We can solve this problem algebraically as shown below, but we can also solve it with some proportional reasoning. Let's say the original rate of speed is r mi/h and the distance is d miles. At rate r, it takes 3 hours (9/3 hours) to go the distance d. At rate $(r + 10)$, it takes 2 hours 40 minutes (8/3 hours) to go the same distance. If it takes 8/9 of the time to go the same distance, then the car must be traveling 9/8 as fast. This means that the extra 10 mi/h is the 1/8 of r, so r must be $8 \times 10 = 80$ mi/h. Traveling for 3 hours at 80 mi/h is **240** miles, so this is the distance between Sprintville and Countdown City.

$$3r = (3 - 1/3)(r + 10)$$
$$9r = 8r + 80$$
$$r = 80$$

145. Since the sequence of primes is arithmetic, the middle number must be $111 \div 3 = 37$. Now we need to find two primes equidistant from 37, one less and one more. The numbers 31 and 43 are both prime and both 6 away, so the sequence could be 31, 37, 43. Other candidates are 13, 37, 61 or 7, 37, 67 or 3, 37, 71. The last of these has the least possible product, which is **7881**.

1	**3**	**5**	**15**
2	**6**	**10**	30
4	**12**	**20**	60
8	**24**	40	120

146. The value of 5! is 120 and its divisors are shown in the array to the right. Twelve of these divisors are also elements of the set {1, 2, 3, …, 25}, so the probability is **12/25**.

147. To find the sum of all positive palindromes less than 2025 that are divisible by 99, we first note that the positive multiples of 99 less than 2025 are 99×1 through 99×20, which gives the numbers 99, 198, 297, 396, 495, 594, 693, 792, 891, 990, 1089, 1188, 1287, 1386, 1485, 1584, 1683, 1782, 1881 and 1980. Scanning the list of 20 values, we see that only 99 and 1881 are palindromes. Their sum is $99 + 1881 =$ **1980**.

148. Since the average height of a professional basketball player is 78 inches and there are typically 15 players per team, the combined height of the players on a basketball team is $15 \times 78 = 1170$ inches. Since the average height of a professional baseball player is 74 inches and there are typically 26 players per team, the combined height of the players on a baseball team is $26 \times 74 = 1924$ inches. The absolute difference between these two totals is $|1924 - 1170| =$ **754** inches.

149. The divisors of 18 are the odd numbers 1, 3, 9 and the even numbers 2, 6, 18. Only the evens are not divisors of 63, so the answer is **3** integers.

150. The first six terms of the sequence are 4, 7, 13, 25, 49, 97. The answer is **97**.

Workout 2

151. There would be $8! = 40{,}320$ different arrangements if all the letters of MATHLETE were different, but the word has two T's and two E's, so there are only $40{,}320 \div 4 = 10{,}080$ unique arrangements of the eight letters. To find out how many of these arrangements have the two T's next to each other, we can pretend that we have TT glued together as a single item. There are then $7!/2 = 2520$ arrangements with TT. Likewise, there are 2520 arrangements with EE. We should note that we have double counted the arrangements that have both TT and EE. There are $6! = 720$ of those. To find the number of arrangements without TT or EE, we calculate $10{,}080 - 2520 - 2520 + 720 = 5760$, adding back in the arrangements that we subtracted twice. The probability that a random arrangement of the letters has no two consecutive letters is thus $5760/10{,}080 =$ **4/7**.

152. The five terms of the first sequence are 6, 11, 16, 21, 26. The five terms of the second sequence must be 19, 14, 9, 4, –1. The absolute difference between the first and last terms of the second sequence is $|19 - (-1)| =$ **20**.

153. The first 10 positive three-digit integers with distinct digits are 102, 103, 104, 105, 106, 107, 108, 109, 120 and 123. Their sum is **1087**.

154. Since the radius is 6 cm, the diameter—and thus the diagonal of the square—is 12 cm. The side length of a square is its diagonal divided by $\sqrt{2}$, so the side length is $12 \div \sqrt{2} \approx$ **8.5** cm.

155. A number with 16 factors can have a prime factorization of the form p^{15}, $p^7 \times q$, $p^3 \times q^3$, $p^3 \times q \times r$, or $p \times q \times r \times s$. The least number of the first form is $2^{15} = 32768$, which has 15/16 of its factors even. The least number of the second form is $2^7 \times 3 = 128 \times 3 = 384$, with $14/16 = 7/8$ of its factors even. The least number of the third form is $2^3 \times 3^3 = 8 \times 27 = 216$, with $12/16 = 3/4$ of its factors even, making it a candidate for the least such number. The least number of the fourth form is $2^3 \times 3 \times 5 = 120$, also with $12/16 = 3/4$ of its factors even, so this is our least candidate so far. The least number of the last form is $2 \times 3 \times 5 \times 7 = 210$, with $8/16 = 1/2$ of its factors even. Thus, the number N must be **120**.

156. The only way $f(x)$ can be an integer is if the value of the denominator, $2x - 37$, is a divisor of the numerator, 237. The divisors of 237 are 1, 3, 79, 237 and their opposites, –1, –3, –79, –237. Solving an equation for each of these possible denominators, we get the corresponding x-values 19, 20, 58, 137 and 18, 17, –21, –100. The sum of these values is $269 - 121 =$ **148**.

157. The first 17 positive integers have remainders of 1, 2, 3, … 16 and 0 when divided by 17. The sum of these first 17 remainders is $16 \times 17 \div 2 = 136$. If we were continuing until 102, this cycle would repeat 6 times and the total would be $6 \times 136 = 816$. Since we are summing the remainders only to 100, we need to subtract from 816 the last two remainders of 16 and 0. The answer is $816 - 16 - 0 =$ **800**.

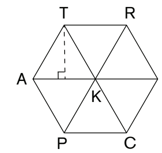
158. The square root of 2025 is 45, so the product of the side lengths of our triangle should not exceed 45. At the low end, we can assign the least positive integer 1 to the shortest side of this 30-60-90 triangle. The side length, in this case, would be 1, $\sqrt{3}$ and 2 and their product would be $2\sqrt{3}$. The square of this product would be $S_1 = (2\sqrt{3})^2 = 12$. The next set of possible side lengths is 2, $2\sqrt{3}$ and 4, with a product of $16\sqrt{3}$ and a square $S_2 = (16\sqrt{3})^2 = 768$. Then we have 3, $3\sqrt{3}$ and 6, with a product of $54\sqrt{3}$ and a square $S_3 = (54\sqrt{3})^2 = 8748$, which is too big. The sum of the possible values of S is $12 + 768 =$ **780**.

159. Justin can use an even number of quarters from 0 to 20 and complete the rest of the $5.00 with dimes. Including zero, that's 11 even numbers, so there are **11** combinations of quarters and dimes he can use to pay $5.00 in the board game.

160. The prime factorization of 24 is $2^3 \times 3$. To make a perfect square, we need an even number of each prime factor, so we just need to multiply 24 by 2 and 3. The answer is $24 \times 6 =$ **144**.

Workout 3

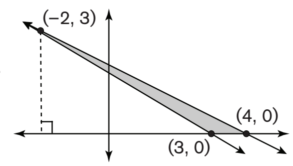
161. Suppose the lengths of the sides of the rectangle are a and b. Then $ab = 20$ and $a + b = 26/2 = 13$. The new rectangle will have area $(a + 1)(b + 1) = ab + a + b + 1$. So, the area of the rectangle is $20 + 13 + 1 =$ **34** cm^2.



162. The perimeter of heptagon PATRICK is 28 inches, so each segment must be $28 \div 7 = 4$ inches. The area of the heptagon consists of 5 equilateral triangles. Each equilateral triangle has a base of 4 inches, a height of $2\sqrt{3}$ inches, and an area of $1/2 \times 4 \times 2\sqrt{3} = 4\sqrt{3}$ square inches. Thus, the area of heptagon PATRICK is $5 \times 4\sqrt{3} =$ **$20\sqrt{3}$** in^2.

163. There are 8 letters in the word TOPOLOGY, but 3 letters are the same, so there are $8! \div 3! = 40{,}320 \div 6 =$ **6720** ways to rearrange the letters.

164. At one extreme, the set could contain the number 27 by itself, so $d = 1$ is a possibility. At the other extreme, the set could contain 2, 3, 4, 5, 6 and 7, whose sum is 27, so $d = 6$ is also a possibility. With some checking, we can confirm that d can also be any natural number between 1 and 6. The sum of the possible values of d is thus $1 + 2 + 3 + 4 + 5 + 6 =$ **21**.



165. The line $x + 2y = 4$ intersects the x-axis at (4, 0), the line $3x + 5y = 9$ intersects the x-axis at (3, 0), and their point of intersection is (–2, 3). The triangle formed by these three points has a base on the x-axis of length 1 unit and a height of 3 units. Thus, the area of this triangle is $(1 \times 3) \div 2 =$ **3/2** $units^2$.

166. The prime factorization of 2026 is 2×1013, so the only possible dimensions for George's prism are 1 cm by 2 cm by 1013 cm. Therefore, the surface area is $2(1 \times 2 + 1 \times 1013 + 2 \times 1013) = 2(2 + 1013 + 2026) = 2 \times 3041 =$ **6082** cm^2.

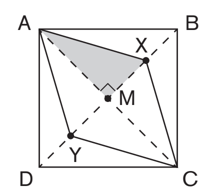
167. The math club earned $12 \times 3.50 + 13 \times 2.75 + 21 \times 4.00 + 11 \times 2.75 + 16 \times 3.50 = 42 + 35.75 + 84 + 30.25 + 56 =$ $**248**.

168. Since $2026 \div 9 = 225\ 1/9$, the integer N could have 225 digits of 9 and 1 digit of 1. That's **226** digits, which is the minimum.

169. The value of $6 \clubsuit 4$ is $6^2 \div 4 = 36 \div 4 =$ **9**.

170. The 192-page book could be divided into $192 \div 8 = 24$ sets of 8 pages, so it will take Sam $24 \times 10 = 240$ minutes or $240 \div 60 =$ **4** hours to read the book.

Workout 4



171. Points X and Y lie on diagonal BD, with the diagonals of the rhombus intersecting at point M. That means point X is halfway between points B and M, and point Y is halfway between points D and M. Square ABCD has side length 12 cm, so diagonal AC has length $12\sqrt{2}$ cm. Segment AM is half of diagonal AC, so its length is $6\sqrt{2}$ cm. Segment XM is one quarter of diagonal AC, so its length is $3\sqrt{2}$ cm. Since triangle AMX is a right triangle, we can use the Pythagorean theorem to solve for the length of hypotenuse AX, which is also the side length of rhombus AXCY: $AX = \sqrt{(3\sqrt{2})^2 + (6\sqrt{2})^2} = \sqrt{18 + 72} = \sqrt{90} =$ **$3\sqrt{10}$** cm.

172. Let's suppose we go to Al's Grocer and buy 5 bananas and 3 oranges five times. We would have 25 bananas and 15 oranges and the cost would be 5 × \$10.92 = \$54.60. Let's also suppose that we buy 4 bananas and 5 oranges three times. We would have 12 bananas and 15 oranges and the cost would be 3 × \$13.91 = \$41.73. Since we purchase 15 oranges in both scenarios, the extra \$54.60 – \$41.73 = \$12.87 must be the cost of the extra 25 – 12 = 13 bananas. Each banana must cost \$12.87 ÷ 13 = \$0.99, so a dozen bananas would cost 12 × \$0.99 = $**11.88**.

173. Each of 4 lights can be red or green, so there are $2^4 = 16$ possible arrangements of the lights. Only 4 of these arrangements have a single green in between two red lights, namely GRGR, RGRG, RGRR and RRGR. Therefore, there are $16 - 4 =$ **12** arrangements that qualify.

174. We want the largest number with these conditions, so we should go backwards in time, looking at the sum of the digits for the years that are two more than a perfect square year. The number 2027 is two more than 45^2, so let's look at $44^2 + 2$, which is $1936 + 2 = 1938$ with a sum of digits of 21. Let's try $43^2 + 2 = 1849 + 2 = 1851$ with a sum of digits of 15. We continue in this way until we find $39^2 + 2 = 1521 + 2 = 1523$, with a sum of digits of 11. Our answer is **1523**.

175. If Rick rolls his sheet of paper so that the circumference of the circular base is 6 in, then the diameter will be $6/\pi$ in and the radius will be $3/\pi$ in. The cylinder will have a height of 8 in and a volume of $\pi \times (3/\pi)^2 \times 8 = 72/\pi$ in^3. If Rick rolls his sheet of paper the other way, the diameter of the base will be $8/\pi$ in and the radius $4/\pi$ in. The cylinder will have a height of 6 in and a volume of $\pi \times (4/\pi)^2 \times 6 = 96/\pi$ in^3. The absolute difference in these volumes is $|96/\pi - 72/\pi| =$ **$24/\pi$** in^3.

176. The 10-ounce box of cereal costs 3.49 ÷ 10 ≈ \$0.349 per ounce. The new 14-ounce box costs 4.49 ÷ 14 ≈ \$0.3207 per ounce. The cost decrease is about 0.349 – 0.3207 = 0.0283. Compared to \$0.349 per ounce, this is a percent decrease of about 0.0283 ÷ 0.349 × 100 ≈ **8.1**%. Note that if we round the unit prices to the nearest cent before calculating the percent decrease, our answer will disagree in the tenths of a percent.

177. If we place the 7 in the tens place and 9 in the thousands place, the remaining 3 digits can be arranged in $3! = 6$ ways. Switching their positions, putting the 7 in the thousands place and the 9 in the tens place, gives another 6 numbers, for a total of 12 so far. We can also form 4 more numbers by placing the 7 at either end of the number with the 9 next to it: 15,397, 35,197, 79,351 and 79,153. In total, this gives $12 + 4 =$ **16** five-digit numbers.

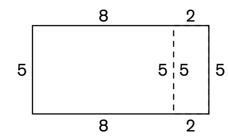
178. Suppose the tens digit of the number is a and the units digit is b. Then the number is $10a + b$, and the given condition is equivalent to $10a + b = 3(a + b)$. Expanding and subtracting $3a + b$ from both sides yields $7a = 2b$. Since b must be a single digit divisible by 7, it equals 7. That means a must equal 2. The only such number is 27, so the requested sum is **27**.

179. There are 12 edges on a cube, so this cube must have an edge length of $84 \div 12 = 7$ in. The volume of the cube is $7^3 =$ **343** in^3.

180. We are given that $x + y + z = 20$ and $x^2 + y^2 + z^2 = 222$, and we are asked to find the value of $xy + xz + yz$. We can use the identity $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + xz + yz)$. Substituting the known values, we get $20^2 = 222 + 2(xy + xz + yz)$, or $400 = 222 + 2(xy + xz + yz)$. Subtracting 222 from both sides gives $178 = 2(xy + xz + yz)$, and dividing both sides by 2 yields $xy + xz + yz = 89$. So, the value of $xy + xz + yz$ is **89**.

Workout 5

181. When Carl cuts the paper into two smaller rectangular pieces, 10 cm are added to the total perimeter. We know this because $26 + 14 - 30$ is 10. The cut must have been 5 cm long, since both sides of the cut become part of the new perimeter. The other sides of the smaller piece of paper must be $(14 - 2 \times 5) \div 2 = 4 \div 2 = 2$ cm each, and the other sides of the bigger piece must be $(26 - 2 \times 5) \div 2 = 16 \div 2 = 8$ cm each. The original piece of paper is a 5 cm by 10 cm rectangle with an area of **50** cm^2.



182. To find Jarnail's overall average, we assume that each test is equally weighted (for example, each out of 100 points). Then her total score for the first 4 tests was $4 \times 86 = 344$. Adding the fifth test score of 96 gives a total of $344 + 96 = 440$. Dividing by 5 tests, her new average is $440 \div 5 =$ **88**%.

183. Assume the prisms are right rectangular prisms aligned with the cube's sides. The cube has volume $10^3 = 1000$, and each $2 \times 4 \times 4$ prism has volume 32, so the theoretical maximum number of prisms is $1000 \div 32 = 31.25$. However, this can't be achieved because 4 does not divide evenly into 10. One possible packing is as follows: place a 2-by-2 arrangement of prisms lying flat in a corner of the cube. This covers an 8×8 area with a height of 2, allowing for 5 such layers, or $4 \times 5 = 20$ prisms. Then stand up 4 more prisms along each of two adjacent walls, adding 8 more for a total of 28. Some space remains, but no additional prisms can fit. While this may not be the only packing method, it appears to be the most efficient under these constraints. So **28** prisms can fit inside the cube.

184. Although the problem describes a multi-step process, the outcome is equivalent to randomly choosing 2 letters without replacement from the 10 letters in the word MATHCOUNTS. This is because we ultimately want the probability that 2 drawn slips (from 3 randomly chosen letters) are different, and the order of selection does not affect this probability. There are 10 letters in MATHCOUNTS, so the number of ways to choose 2 letters is "10 choose 2" $= (10 \times 9)/2 = 45$. Only 1 of these pairs is the two T's, so the probability that the two letters are different is $1 - 1/45 =$ **44/45**.

185. If the square root of a perfect square number is a multiple of 4, then the perfect square must be a multiple of 4^2, which is 16. We need to multiply 16 by other perfect squares to find those that have a last digit of 4. We know that the only single-digit multiples of 6, the units digit of 16, that end in 4 are 6×4 and 6×9, so we need to find the perfect squares that end in 4 or 9. Here's what we find: $16 \times 4 = 64$, $16 \times 9 = 144$, $16 \times 49 = 784$, $16 \times 64 = 1024$, $16 \times 144 = 2304$. The last of these is greater than 2025, so the sum we want is $64 + 144 + 784 + 1024 =$ **2016**.

186. If s is the side length of a cube, its surface area is $6s^2$ and its volume is s^3. The ratio of the surface area to volume is therefore $6s^2/s^3 = 6/s$. Setting this equal to 3/10 gives $6/s = 3/10$, which simplifies to $60 = 3s$, so $s = 20$. Thus, the side length of the cube is **20** inches.

187. There are 10 letters in the word COTTONELLE, so there are $10! = 3{,}628{,}800$ ways to choose the letters from the hat. Since there are 4 pairs of letters that are the same, there are only $10! \div 2^4 = 3{,}628{,}800 \div 16 = 226{,}800$ ways that are "distinguishable." We will accept any possible order of the 4 repeated letters, so we will treat them as pairs that are locked together. We calculate that the 4 pairs and 2 other letters can be arranged in $6! = 720$ ways. The desired probability is $720/226{,}800 =$ **1/315**.

188. The plane's altitude of 34,000 feet is about $34{,}000 \div 5280 \approx 6.44$ miles above the earth. Since the circumference of a circle is $2\pi r$, this extra distance from the center of the earth would add about $2 \times 3.14 \times 6.44 \approx 40.44$ miles to a full circle or about $40.44 \div 4 \approx$ **10** miles to the trip from Lisbon, Portugal to Vladivostok, Russia.

189. Since the six consecutive odd integers have a sum of 216, the median of the six integers must be $216 \div 6 = 36$. The integers must be 31, 33, 35, 37, 39 and 41, with **41** being the greatest of them.

190. Let's use 36 minutes as our unit of time for this problem, which is the least common multiple of the number of minutes given for each person. Larry can make 7 pizzas in 9 minutes, which is 28 pizzas in 36 minutes. Harry can make 7 pizzas in 12 minutes, which is 21 pizzas in 36 minutes. And Mary can make 4 pizzas in 6 minutes, which is 24 pizzas in 36 minutes. Together they make $28 + 21 + 24 = 73$ pizzas in 36 minutes, so they can make ten times as many pizzas in ten times as much time. That's 730 pizzas in **360** minutes.

Workout 6

191. We can express the equation $2^x \times 4^y = 1024$ as $2^x \times 2^{2y} = 2^{10}$ and then $2^{x + 2y} = 2^{10}$. If this is true, then $x + 2y = 10$. In a similar manner, we can rewrite the equation $4^x \times 2^y = 256$ as $2^{2x} \times 2^y = 2^8$ and then $2^{2x + y} = 2^8$. This means that $2x + y = 8$. Now we have a system of two equations with two unknowns. If we add these two equations together, we get $3x + 3y = 18$. Dividing both sides by 3, we get $x + y =$ **6**.

192. Each of the numbers 1 through 6 on the die are equally likely to be the number n that Dorina rolls. The expected value of $n!$ is the average of these six factorial values as follows: $(1! + 2! + 3! + 4! + 5! + 6!) \div 6 = (1 + 2 + 6 + 24 + 120 + 720) \div 6 = 873 \div 6 =$ **145.5**.

193. The first few terms of the sequence are: 1, 3, 5, 17, 257, …, and since we are only interested in the units digit of the 2025th term, we only need to keep track of the units digit. These "modulo ten" values are: 1, 3, 5, 7, 7, …. At this point, we can see that each new term will be a product of a bunch of numbers and 5, which will always end in 5. Adding 2 then gives a units digit of 7. Thus, the units digit of the 2025th term is **7**.

194. We can create an organized list, as shown right. There are 23 possible combinations of four distinct positive integers that sum to 20. The **10** choices in bold include at least one integer divisible by 5.

1, 2, 3, 14	1, 3, 4, 12	1, 4, 7, 8	**2, 4, 5, 9**
1, 2, 4, 13	**1, 3, 5, 11**	**1, 5, 6, 8**	2, 4, 6, 8
1, 2, 5, 12	**1, 3, 6, 10**	2, 3, 4, 11	**2, 5, 6, 7**
1, 2, 6, 11	1, 3, 7, 9	**2, 3, 5, 10**	**3, 4, 5, 8**
1, 2, 7, 10	**1, 4, 5, 10**	2, 3, 6, 9	3, 4, 6, 7
1, 2, 8, 9	1, 4, 6, 9	2, 3, 7, 8	

195. The total number of ways for Jaris to deal 3 cards to David is "52 choose 3" $= 52!/(3! \times 49!) = (52 \times 51 \times 50)/(3 \times 2 \times 1) = 22{,}100$. This will be the denominator of our probability. To count the favorable outcomes, we choose 1 heart and 2 non-hearts: "13 choose 1" × "39 choose 2" $= (13!/(1! \times 12!)) \times (39!/(2! \times 37!)) = 13 \times ((39 \times 38)/(2 \times 1)) = 9633$. This is the numerator. Thus, the probability is 9,633/22,100 = **741/1700**.

196. The water forms a cylinder with radius 1/12 of a foot and height 5/12 of a foot. Therefore the volume of the glass of water in cubic feet is $\pi \times (1/12)^2 \times (5/12)$. The water costs 6.7 cents per cubic foot, so the cost of the glass of water is $\pi \times (1/12)^2 \times (5/12) \times 6.7 \approx$ **0.06** cents.

197. To answer this question, we will count the factors of 2 in 120!. There are 60 multiples of 2, 30 multiples of 4, 15 multiples of 8, 7 multiples of 16, 3 multiples of 32, and 1 multiple of 64, so there are $60 + 30 + 15 + 7 + 3 + 1 = 116$ multiples of 2. Since 4 is just 2×2, we now know that 120! has $116 \div 2 = 58$ factors of 4. The maximum value of n is therefore **58**.

198. First, we determine the largest value of n such that $n^2 + 1 \leq 2026$. Solving $n^2 \leq 2025$, we find that the greatest possible value of n is 45. We are adding the values of $n^2 + 1$ for all integers from 1 to 45. This means we are adding together all the perfect squares from 1^2 to 45^2 and then adding 1 for each of the 45 terms. The sum of the first n perfect squares is given by $(n(n + 1)(2n + 1))/6$, so the sum of the first 45 perfect squares is $(45 \times 46 \times 91)/6 = 31{,}395$. Adding 1 for each of the 45 terms gives a sum of $31{,}395 + 45 = 31{,}440$. The remainder when S is divided by 100 is **40**.

199. The combined areas of ABC and ADC cover the entire quadrilateral, so we know that the total area is $20 + 18 = 38$ cm^2. Likewise, the combined areas of BCD and BAD cover the entire quadrilateral, so the total area of BAD must be $38 - 25 =$ **13** cm^2.

200. We will consider 5 cases. There is only 1 way for all 4 flips of a coin to be heads (HHHH) and there are $2^4 = 16$ possible outcomes for the 4 flips, so the probability that they both get 4 heads is $1/16 \times 1/16 = 1/256$. There are 4 ways to get 3 heads (HHHT, HHTH, HTHH and THHH), so the probability that they both get 3 heads is $4/16 \times 4/16 = 16/256$. There are 6 ways to get 2 heads (HHTT, HTHT, HTTH, THHT, THTH and TTHH), so the probability that they both get 2 heads is $6/16 \times 6/16 = 36/256$. There are 4 ways to get only 1 head (HTTT, THTT, TTHT and TTTH), so the probability that they both get 1 head is $4/16 \times 4/16 = 16/256$. Finally, there is only 1 way to get 0 (zero) heads (TTTT), so the probability that they both get 0 heads is $1/16 \times 1/16 = 1/256$. The total probability for all 5 cases is $(1 + 16 + 36 + 16 + 1)/256 = 70/256 =$ **35/128**.

PROBLEM INDEX

It is very difficult to categorize many of the problems in this handbook. MATHCOUNTS problems often straddle multiple categories and cover several concepts, but in this index, we have placed each problem in exactly one category and mapped it to exactly one Common Core State Standard (CCSS). **In this index, code 9 (3) 7.SP.3 would refer to problem #9 with difficulty rating 3 mapped to CCSS 7.SP.3.** The difficulty rating and CCSS mapping are explained below.

DIFFICULTY RATING: Our scale is 1-7, with 7 being most difficult. These general ratings are only approximations:
- **1, 2 or 3**: Appropriate for students just starting the middle school curriculum; 1 concept; 1- or 2-step solution.
- **4 or 5**: Knowledge of some middle school topics necessary; 1-2 concepts; multi-step solution.
- **6 or 7**: Knowledge of advanced middle school topics and/or problem-solving strategies necessary; multiple and/or advanced concepts; multi-step solution.

COMMON CORE: We align our problems to the NCTM Standards for Grades 6-8, however we also have mapped these problems to CCSS because 41 states, D.C., 4 territories and the Dept. of Defense Education Activity (DoDEA) have voluntarily adopted it. Our CCSS codes contain (in this order):

1. **Grade level** in the K-8 Standards for Mathematical Content (SMC). Courses that are in the high school SMC instead have the first letter of the course name.
2. **Domain** within the grade level or course and then the **individual standard**.

Here are 2 examples:
- *6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6*
- *G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry*

Some math concepts are not specifically mentioned in CCSS. For problems using these concepts, we use the code of a related standard, when possible. Some of our problems are based on concepts outside the scope of CCSS or are based on concepts in the K-5 SMC but are more difficult than a grade K-5 problem. When appropriate, we coded these problems SMP for the CCSS Standards for Mathematical Practice.

MEASUREMENT

35	(2)	5.MD.1
67	(3)	6.RP.3
103	(4)	6.RP.3
181	(3)	6.G.1
188	(4)	7.G.4
196	(5)	7.G.6

PERCENTS & FRACTIONS

39	(2)	5.NF.1
45	(3)	7.RP.3
46	(3)	6.RP.3
85	(3)	5.NF.6
176	(3)	6.RP.3

LOGIC

36	(4)	SMP
101	(3)	SMP
173	(4)	SMP
177	(4)	SMP

STATISTICS & DATA

64	(5)	6.SP.5
79	(3)	6.SP.5
81	(4)	6.SP.5
94	(3)	6.SP.5
148	(2)	6.SP.5
167	(2)	6.RP.3
182	(2)	6.SP.5

GENERAL MATH

31	(1)	4.NBT.3

ALGEBRAIC EXPRESSIONS & EQUATIONS

21	(2)	8.EE.2
22	(2)	8.EE.2
23	(2)	8.EE.2
24	(3)	8.EE.2
25	(3)	8.EE.2
26	(4)	A-REI.1
27	(4)	A-SSE.1
28	(4)	A-REI.1
29	(3)	A-SSE.2
30	(5)	A-REI.1
34	(3)	6.EE.2
48	(4)	8.EE.7
54	(4)	8.EE.7
59	(3)	8.EE.2
69	(3)	7.EE.3
72	(4)	7.EE.3
82	(4)	8.EE.7
87	(4)	F-IF.2
106	(5)	A-REI.1
110	(3)	7.EE.4
113	(3)	A-SSE.2
120	(3)	6.EE.2
122	(5)	8.EE.2
126	(5)	A-SSE.2
139	(5)	8.EE.1
140	(4)	6.EE.2
156	(5)	8.EE.7
159	(3)	7.EE.4
169	(2)	6.EE.2
172	(4)	8.EE.8
180	(4)	A-APR.3
191	(4)	8.EE.1

Nt

NUMBER THEORY

40	(3)	5.NBT.6
42	(3)	6.EE.2
51	(5)	6.NS.4
57	(4)	6.EE.2
62	(4)	6.EE.2
66	(5)	6.NS.4
73	(3)	6.NS.4
75	(4)	7.NS.2
77	(5)	6.NS.4
95	(5)	6.NS.4
96	(5)	6.EE.2
99	(4)	6.NS.4
114	(3)	6.NS.4
124	(5)	6.NS.4
130	(4)	6.NS.4
131	(5)	6.NS.4
136	(5)	6.EE.2
137	(4)	4.NB.1
142	(3)	5.NBT.1
145	(4)	6.NS.4
147	(4)	6.NS.4
149	(2)	6.NS.4
153	(3)	6.NS.4
155	(5)	6.NS.4
157	(4)	6.NS.4
160	(4)	6.NS.4
164	(4)	6.NS.4
168	(4)	6.NS.3
174	(5)	7.NS.3
178	(3)	6.NS.3
185	(4)	6.NS.4
187	(6)	7.SP.8
189	(3)	6.EE.6
194	(5)	7.NS.3
197	(6)	8.EE.1
198	(4)	7.NS.3

Sp

SEQUENCES, SERIES & PATTERNS

32	(4)	A-SSE.4
33	(2)	4.OA.5
44	(3)	4.OA.5
56	(2)	4.OA.5
58	(2)	A-SSE.1
70	(3)	4.OA.5
76	(4)	A-SSE.4
107	(5)	S-IF.3
118	(5)	A-SSE.1
150	(2)	A-SSE.1
152	(2)	A-SSE.1
193	(5)	8.F.4

Pc

PROBABILITY, COUNTING & COMBINATORICS

1	(1)	7.SP.8
2	(2)	7.SP.8
3	(5)	7.SP.8
4	(5)	7.SP.8
5	(2)	S-CP.1
6	(2)	7.SP.8
7	(3)	7.SP.7
8	(4)	S-CP.1
9	(1)	7.SP.7
10	(4)	S-CP.9
47	(6)	S-CP.9
49	(2)	7.SP.7
50	(2)	SMP
53	(5)	7.SP.8
61	(4)	7.SP.8
65	(3)	7.SP.8
80	(3)	7.SP.8
84	(4)	S-CP.1
90	(5)	7.SP.8
91	(6)	7.SP.8
93	(5)	S-CP.9
98	(6)	S-CP.9
109	(3)	7.SP.7
111	(6)	S-CP.9
115	(5)	7.SP.8
117	(5)	S-CP.1
123	(5)	7.SP.8
128	(5)	S-CP.9
132	(4)	7.SP.8
146	(4)	7.SP.7
151	(6)	S-CP.9
163	(4)	S-CP.9
184	(4)	7.SP.7
192	(4)	7.SP.7
195	(6)	7.SP.7
200	(5)	7.SP.8

Pg

PLANE GEOMETRY

38	(1)	4.MD.3
60	(4)	7.G.5
68	(3)	5.G.1
71	(4)	8.G.7
92	(5)	6.G.1
100	(3)	7.G.5
102	(6)	7.G.5
104	(4)	7.G.6
105	(5)	7.G.4
119	(4)	6.G.1
121	(4)	G.GMD.1
125	(4)	7.G.6
129	(4)	7.G.4
133	(5)	7.G.4
135	(7)	7.G.1
143	(4)	6.G.1
154	(3)	8.G.7
158	(5)	8.G.7
161	(5)	7.G.6
162	(5)	8.G.9
171	(5)	G-CO.11
199	(4)	6.G.1

Cg

COORDINATE GEOMETRY

11	(5)	G-GPE.7
12	(3)	G-GPE.7
13	(5)	G-GPE.7
14	(6)	G-GPE.7
15	(5)	G-GPE.7
16	(3)	7.G.4
17	(7)	G-GPE.7
18	(5)	G-GPE.7
19	(7)	7.G.4
20	(7)	G-GPE.1
97	(6)	G-GPE.6
112	(5)	G.CO.4
127	(6)	G.GPE.6
134	(4)	8.EE.8
165	(5)	8.G.1

Pr

PROPORTIONAL REASONING

37	(3)	6.RP.3
41	(4)	7.RP.3
43	(3)	6.RP.1
52	(2)	6.RP.3
55	(4)	6.RP.3
74	(4)	7.RP.2
78	(4)	6.RP.3
83	(4)	7.RP.3
86	(4)	6.RP.3
89	(3)	6.RP.3
108	(2)	7.RP.3
116	(5)	7.RP.3
141	(2)	6.SP.3
144	(4)	7.RP.3
170	(2)	6.RP.3

Sg

SOLID GEOMETRY

63	(4)	G-GMD.3
88	(4)	G-GMD.3
138	(6)	G-GMD.3
166	(5)	7.G.6
175	(4)	G-GMD.3
179	(2)	5.MD.5
183	(4)	6.G.2
186	(4)	6.G.2

Ps

PROBLEM SOLVING (MISCELLANEOUS)

190	(3)	6.RP.3

COACHES:

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ANSWERS

CHANCE TO WIN STRETCH

#	Answer	Level
1.	25	(1)
2.	2/9	(2)
3.	0.42	(5)
4.	7	(5)
5.	1/6	(2)
6.	2/3	(2)
7.	3/13	(3)
8.	0.53	(4)
9.	57	(1)
10.	5/9	(4)

AREA ON THE COORDINATE PLANE STRETCH

#	Answer	Level
11.	50	(5)
12.	100	(3)
13.	128	(5)
14.	16	(6)
15.	4	(5)
16.	8π	(3)
17.	$3\sqrt{10}$	(7)
18.	20	(5)
19.	$3600\pi/169$	(7)
20.	10	(7)

SIMPLIFYING RADICALS STRETCH

#	Answer	Level
21.	$2\sqrt{2}$	(2)
22.	$3\sqrt[3]{3}$	(2)
23.	$3\sqrt{6}$	(2)
24.	$\sqrt{2}/2$	(3)
25.	$\sqrt{7}/7$	(3)
26.	$3 + 2\sqrt{2}$	(4)
27.	$(3 - 2\sqrt{2})/2$	(4)
28.	$(2 + 3\sqrt{2})/2$	(4)
29.	2	(3)
30.	$(3 + \sqrt{3})/6$	(5)

WARM-UP 1

#	Answer	Level
31.	38,000	(1)
32.	8	(4)
33.	G	(2)
34.	58	(3)
35.	80	(2)
36.	6	(4)
37.	2/3	(3)
38.	24	(1)
39.	1/4	(2)
40.	5	(3)

WARM-UP 2

#	Answer	Level
41.	3	(4)
42.	5	(3)
43.	1/26	(3)
44.	R	(3)
45.	38	(3)
46.	180	(3)
47.	0	(6)
48.	100	(4)
49.	18	(2)
50.	6	(2)

WARM-UP 3

#	Answer	Level
51.	30	(5)
52.	548	(2)
53.	60	(5)
54.	2	(4)
55.	135	(4)
56.	AXIS	(2)
57.	5	(4)
58.	50	(2)
59.	10	(3)
60.	85	(4)

WARM-UP 4

#	Answer	Level
61.	7/9	(4)
62.	1	(4)
63.	8	(4)
64.	71	(5)
65.	1/2	(3)
66.	6	(5)
67.	20	(3)
68.	45	(3)
69.	39	(3)
70.	103	(3)

WARM-UP 5

#	Answer	Level
71.	150	(4)
72.	10	(4)
73.	252	(3)
74.	540	(4)
75.	221	(4)
76.	10	(4)
77.	32	(5)
78.	5	(4)
79.	0	(3)
80.	1/24	(3)

WARM-UP 6

#	Answer	Level
81.	4	(4)
82.	−3	(4)
83.	15	(4)
84.	3/14	(4)
85.	1/12	(3)
86.	4	(4)
87.	10	(4)
88.	9/2	(4)
89.	8	(3)
90.	1/6	(5)

WARM-UP 7

#	Answer	Level
91.	7/36	(6)
92.	13/24	(5)
93.	56	(5)
94.	14	(3)
95.	1/6	(5)
96.	4	(5)
97.	(−26, 16)	(6)
98.	2835	(6)
99.	23	(4)
100.	11	(3)

WARM-UP 8

#	Answer	Level
101.	7	(3)
102.	60	(6)
103.	15	(4)
104.	5	(4)
105.	10	(5)
106.	7	(5)
107.	21	(5)
108.	6 1/8	(2)
109.	1/2	(3)
110.	5	(3)

WARM-UP 9

#	Answer	Level
111.	1/3	(6)
112.	26	(5)
113.	2520	(3)
114.	841	(3)
115.	1/5	(5)
116.	65	(5)
117.	7/20	(5)
118.	10	(5)
119.	11	(4)
120.	0	(3)

WARM-UP 10

#	Answer	Level
121.	144π	(4)
122.	52	(5)
123.	144	(5)
124.	2	(5)
125.	12	(4)
126.	13/2	(5)
127.	8/5	(6)
128.	11,520	(5)
129.	4	(4)
130.	71	(4)

WARM-UP 11

#	Answer	Level
131.	13	(5)
132.	3/11	(4)
133.	22	(5)
134.	1	(4)
135.	7	(7)
136.	1834	(5)
137.	1519	(4)
138.	30	(6)
139.	11	(5)
140.	39/7	(4)

WORKOUT 1

#	Answer	Level
141.	375	(2)
142.	1448	(3)
143.	4	(4)
144.	240	(4)
145.	7881	(4)
146.	12/25	(4)
147.	1980	(4)
148.	754	(2)
149.	3	(2)
150.	97	(2)

WORKOUT 2

#	Answer	Level
151.	4/7	(6)
152.	20	(2)
153.	1087	(3)
154.	8.5	(3)
155.	120	(5)
156.	148	(5)
157.	800	(4)
158.	780	(5)
159.	11	(3)
160.	144	(4)

WORKOUT 3

#	Answer	Level
161.	34	(5)
162.	$20\sqrt{3}$	(5)
163.	6720	(4)
164.	21	(4)
165.	3/2	(5)
166.	6082	(5)
167.	248	(2)
168.	226	(4)
169.	9	(2)
170.	4	(2)

WORKOUT 4

#	Answer	Level
171.	$3\sqrt{10}$	(5)
172.	11.88	(4)
173.	12	(4)
174.	1523	(5)
175.	$24/\pi$	(4)
176.	8.1	(3)
177.	16	(4)
178.	27	(3)
179.	343	(2)
180.	89	(4)

WORKOUT 5

#	Answer	Level
181.	50	(3)
182.	88	(2)
183.	28	(4)
184.	44/45	(4)
185.	2016	(4)
186.	20	(4)
187.	1/315	(6)
188.	10	(4)
189.	41	(3)
190.	360	(3)

WORKOUT 6

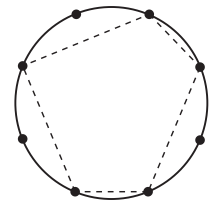
#	Answer	Level
191.	6	(6)
192.	145.5	(4)
193.	7	(5)
194.	10	(5)
195.	741/1700	(6)
196.	0.06	(5)
197.	58	(6)
198.	40	(4)
199.	13	(4)
200.	35/128	(5)

* *The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

Warm-Up 12

201.__________ Andrea rolls a fair standard six-sided die five times. What is the probability that the product of the five die rolls is divisible by 3? Express your answer as a common fraction.

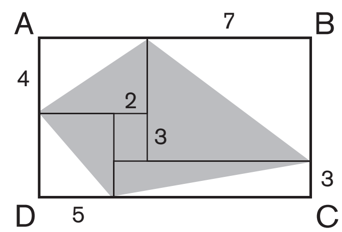
202.__________ penta-gons If 8 points are drawn on a circle, how many distinct convex pentagons can be drawn using some of these points as vertices? One such pentagon is shown right.



203.__________ If $\dfrac{1}{x-1} - \dfrac{1}{2x-2} = 2027$, what is the value of x? Express your answer as a common fraction.

204.__________ palin-dromes A numeric palindrome is a number that reads the same forward and backward. How many three-digit numeric palindromes are multiples of 4?

205.__________ units^2 Rectangle ABCD is shown, containing a shaded interior quadrilateral. From each vertex of this quadrilateral, segments are drawn perpendicular to the sides of the rectangle until they intersect another perpendicular segment. The small rectangle formed inside the quadrilateral measures 2 units by 3 units. What is the area of the shaded region, in square units?



206.__________ moves There are 7 cubes colored red, orange, yellow, green, blue, indigo and violet, arranged in this order from left to right, with red on the far left and violet on the far right. Salvatore is rearranging the cubes, but the only move allowed is swapping the position of adjacent cubes. What is the minimum number of moves needed to reverse the order, so that the violet cube is on the left and the red cube is on the right?

207.__________ Charles bakes 32 blueberry muffins, and he arranges them in an 8 by 4 rectangular grid. While Charles isn't looking, Ray randomly selects 2 muffins and eats them. What is the probability that the 2 muffins Ray eats are adjacent, meaning they occupy adjacent squares in the grid that share a side? Express your answer as a common fraction.

208.__________ units^2 A right rectangular prism with integer side lengths has a face diagonal with length $\sqrt{50}$ units and volume 100 cubic units. What is the surface area of this prism, in square units?

C 209.__________ The sum of three numbers is 50 and the sum of five other numbers is 70. What is the average (or mean) of all eight numbers?

S 210.__________ If $(x + 1)^2 = 6x$, what is the value of $(x - 2)^2$?

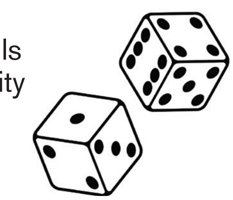
Warm-Up 13

211. ________ What is the closest integer to $10 + \frac{10}{3} + \frac{10}{9} + \frac{10}{27} + \frac{10}{81} + \frac{10}{243} + \frac{10}{729}$?

212. ________ If $4^{3x-1} = 7$, what is the value of 4^{2x}? Express your answer in simplest radical form.

213. ________ The ratio of the number of square inches in the surface area of a cube to the number of cubic inches in the volume of a cube is r. The ratio of the number of square feet in the surface area of a cube to the number of cubic feet in the volume of a cube is kr. What is the value of k?

214. ________ Omar rolls two fair standard six-sided dice and sums the results. Jamie rolls a fair twelve-sided die, with sides numbered 1 to 12. What is the probability that Jamie's roll is greater than Omar's sum? Express your answer as a common fraction.



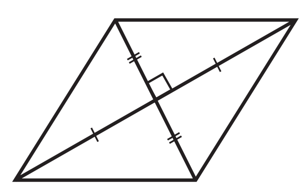
215. ________ What is the units digit of the number 7^{2025}?

216. ________ If a, b and c are positive integers, $a < b < c < 12$ and $\frac{a}{12} + \frac{b}{12} + \frac{c}{12} = 2\frac{1}{4}$, what is the maximum possible value of $a \times b \times c$?

217. ________ What positive integer is equal to 32 times the sum of its digits?

218. ________ It is known that $2^a = 3$. What is the value of 4^{3a-1}? Express your answer as a common fraction.

219. ________ cm How many centimeters are in the perimeter of a rhombus with diagonals of lengths 24 cm and 32 cm?



220. ________ If x and y are integers for which $2x - y = 3$ and $x + 2y = 4$, what is the value of $x + y$?

Warm-Up 14

221. ________ A cube has side length 1. Zach randomly picks two different vertices of the cube. What is the expected value of the square of the distance between them? Express your answer as a common fraction.

222. ________ What is the remainder when 7^{59} is divided by 8?

223. ________ digits If the number $8^{21} \times 9^{15}$ is written in base 12, how many digits does it have?

224. ________ A regular octagon has a perimeter of 48. Regular hexagon ABCDEF has the same area as the regular octagon. The square of the length of side AB takes the form $a\sqrt{3}+b\sqrt{6}$. What is the value of $\frac{b^2}{a}$?

225. ________ The product of $\frac{12^3}{\gcd(12^3,4^4)}$ and a positive integer x is a multiple of $\operatorname{lcm}(12^3,4^4)$. What is the least possible value of x?

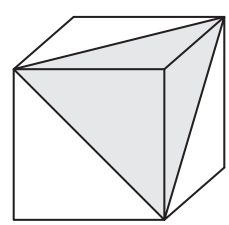
226. ________ people For a game, a group of friends write their names on slips of paper, one name per slip. The names are put in a bowl, and then each person removes a slip of paper from the bowl. On average, how many people will draw their own name?

227. ________ seconds Zaila can run $\frac{2}{3}$ of a lap in $\frac{3}{2}$ minutes. Yael can run $\frac{3}{4}$ of a lap in $\frac{4}{3}$ minutes. Xavion can run $\frac{4}{5}$ of a lap in $\frac{5}{4}$ minutes. In a four-lap race, how many seconds will the third-place finisher finish behind the winner of the race?

228. ________ zeros How many zeros are at the end of the base 10 number 200! when it is expanded and written in base 14?

229. ________ Valeria makes a list of all the positive integers n that produce only integer solutions to the quadratic equation $x^2 - 20x + n = 0$. What is the absolute difference between the greatest and least values on Valeria's list?

230. ________ cm^2 Three of the vertices of a particular cube can be joined to form an equilateral triangle with area $4\sqrt{3}$ cm^2. What is the surface area of this cube, in square centimeters?



Workout 7

231. ______ integers How many positive integers are divisors of 20^{26} but not divisors of 20^{25}?

232. ______ What is the value of $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \frac{31}{32} + \frac{63}{64} + \frac{127}{128} + \frac{255}{256} + \frac{511}{512}$? Express your answer as a common fraction.

233. ______ in^2 What is the area, in square inches, of a right triangle with altitudes of lengths 420 inches, 580 inches and 609 inches?

234. ______ What is the value of $11^2 - 22^2 + 33^2 - 44^2 + 55^2 - 66^2 + 77^2 - 88^2 + 99^2$?

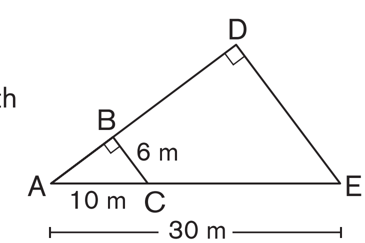
235. ______ Let a and b be two different positive integers with $a < b$. If the sum of the positive integers from a to b, inclusive, is exactly 100, what is the sum of the possible values of $b - a$?

236. ______ The elements of a set of distinct positive integers sum to 20. If k is a positive integer for which 10^k, but not 10^{k+1}, divides the product of the integers in the set, what is the largest possible value of k?

237. ______ million cases In 1912, it was said that the canneries along the Columbia River packed enough salmon to supply four pounds to every person on the planet. The population at the time was estimated to be 1,800,000,000 people. The salmon was packed in cases that contained 48 one-pound cans each. Using those estimates, how many million cases of salmon were packed by the Columbia River canneries in 1912?

238. ______ permutations How many permutations of the digits in 2026 are divisible by 4?

239. ______ meters What is the perimeter, in meters, of triangle ADE shown right, with BC = 6 m, AC = 10 m and AE = 30 m?



240. ______ integers How many positive integers less than or equal to 70 have at least three distinct prime factors?

Workout 8

241. ________ Natalie flips a fair coin four times. She then says to Thinula, "At least two of the four coin flips were heads." What is the probability that at least three of the four coin flips were heads? Express your answer as a common fraction.

242. ________ sides The interior angles of a convex polygon all measure an integer number of degrees, and their angle measures form a nonconstant arithmetic sequence. What is the largest number of sides the polygon could have?

243. ________ integers How many positive integers from 1 through 999, inclusive, do not include two or more prime digits that sum to at least 14?

244. ________ A hotel has 54 stories, where story s, for $1 \le s \le 54$, contains s rooms numbered 1 to s. Each room has a capacity equal to its room number. If no room is more than half full, how much of the hotel's total capacity can be occupied? Express your answer as a common fraction.

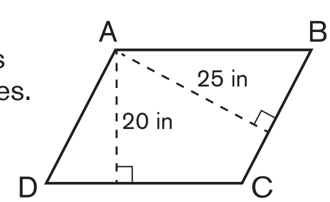
245. ________ What is the sum of all the roots in the equation $2\left(x-\frac{1}{x}\right)^2-7x+\frac{7}{x}+6=0$? Express your answer as a common fraction.

246. (,) The graph of the function $y = x^2$ is shifted, but not rotated or stretched. If the new graph passes through the points (12, 6) and (16, 6), what is the new vertex of the graph? Express your answer as an ordered pair.

247. ________ What is the sum of all positive integers k for which there is no positive integer strictly between $\sqrt[3]{k}$ and $\sqrt{k}$?

248. ________ Consider all nonnegative integer values of n and k together satisfying ${}_{11}C_2 + {}_{11}C_3 = {}_nC_k$. What is the least possible value of $n + k$?

249. ________ inches In parallelogram ABCD, the distance between sides AB and CD is 20 inches, and the distance between sides AD and BC is 25 inches. If parallelogram ABCD has area 700 square inches, what is the length of its longer side, in inches?



250. ________ Riley divides the number x by 24 and gets 15 as her answer. Gavin multiplies the same number x by 20. What answer should he get?

Warm-Up 12

201. If any of the five rolls of Andrea's fair die come up with a 3 or a 6, then the product of all five rolls will be divisible by 3. We can more easily calculate the complementary probability that the product is NOT divisible by 3. For this to be the case, all five rolls would have to land on 1, 2, 4 or 5, and the probability of this is $(4/6)^5 = (2/3)^5 = 32/243$. Thus the probability that the product is divisible by 3 is $1 - 32/243 =$ **211/243**.

202. For any selection of 5 of the 8 points on the circle, there is only one way to connect these points so that the pentagon is convex. The number of distinct convex pentagons is just "8 choose 5," but this value is more easily calculated by choosing 3 of the 8 points to be excluded. That value is "8 choose 3" = (8 × 7 × 6) ÷ (3 × 2 × 1) = **56** pentagons.

203. This problem can be solved algebraically as follows:

$$1/(x - 1) - 1/(2x - 2) = 2027$$
$$2/(2x - 2) - 1/(2x - 2) = 2027$$
$$1/(2x - 2) = 2027$$
$$1 = 2027(2x - 2)$$
$$1 = 4054x - 4054$$
$$4055 = 4054x$$
$$x = 4055/4054$$

We want to make sure this is not an extraneous solution—that is, a solution that would make any denominator zero. The only value of x that would cause a denominator to be zero is $x = 1$, so our answer is indeed **4055/4054**.

204. A three-digit number is divisible by 4 if the last two digits are divisible by 4. We can build three-digit palindromes from the multiples of 4 that are less than 100, attaching a hundreds digit to match the units digit. The resulting list will not be in order from least to greatest, but we only need to know the count. Also, we will strike out those multiples of 4 that would need a hundreds digit of zero: ~~000~~, 404, 808, 212, 616, ~~020~~, 424, 828, 232, 636, ~~040~~, 444, 848, 252, 656, ~~060~~, 464, 868, 272, 676, ~~080~~, 484, 888, 292, 696. These **20** palindromes that are multiples of 4 can be thought of as the 25 multiples of 4 less than 100 minus the 5 multiples of 20 because they end in zero.

205. The overall dimensions of rectangle ABCD are 3 + 3 + 4 = 10 units vertically and 5 + 2 + 7 = 14 units horizontally. The shaded region is half of the four larger rectangles plus all of the smallest rectangle. Half the area of ABCD is 10 × 14 ÷ 2 = 70 units^2. If we add the other half of the 2-by-3 rectangle, we find that the shaded region must be 70 + 3 = **73** units^2.

206. We will use the strategy known as "start with a simpler problem" and we will use the letters ROYGBIV to represent the 7 cubes. To reverse the 2 letters RO, we make just one move, ending with OR. To reverse the 3 letters ROY, we swap the Y with the O and then the Y with the R to get YRO. Then we just swap the R and the O to get YOR. That's 2 + 1 = 3 moves. To reverse the 4 letters ROYG, we make 3 moves to get the G all the way to the left to get GROY. Then we already know that it takes 3 moves to turn ROY into YOR, so it must take 3 + 3 = 6 moves to turn ROYG into GYOR. The pattern is clear: We need to make $n - 1$ moves to get the nth letter from one end to the other, then we add the number of moves it took to move $n - 1$ letters. The table shown right continues the pattern to 7 letters (or cubes), and the answer is **21** moves.

Number of Cubes	Number of Moves
2	1
3	3
4	6
5	10
6	15
7	21

207. We will consider 3 cases based on the position of the first muffin Ray selects on the grid. First, if Ray selects a corner muffin, there are 4 corner muffins in total. After choosing 1 corner muffin, there are 2 muffins adjacent to it that could be chosen second. So the probability in this case is (4/32) × (2/31). Second, if Ray selects a muffin on an edge but not a corner, there are 16 such edge muffins. Each of these has 3 adjacent muffins. The probability for this case is (16/32) × (3/31). Third, if Ray selects a muffin in the middle of the grid, there are 12 such muffins. Each middle muffin has 4 adjacent muffins. The probability for this case is (12/32) × (4/31). Adding the probabilities of these 3 mutually exclusive cases gives the total probability that the 2 muffins Ray eats are adjacent: (4/32 × 2/31) + (16/32 × 3/31) + (12/32 × 4/31) = 8/(32 × 31) + 48/(32 × 31) + 48/(32 × 31) = (8 + 48 + 48)/(32 × 31) = 104/(32 × 31). We can simplify this fraction by dividing numerator and denominator by 4: 104 ÷ 4 = 26, 32 ÷ 4 = 8, so the simplified probability is 26/(8 × 31) = 26/248. Further simplification divides numerator and denominator by 2: 26 ÷ 2 = 13, 248 ÷ 2 = 124. Thus, the probability that the 2 muffins Ray eats are adjacent is **13/124**.

208. We can start our search for the dimensions of this prism with the factor triples of 100. None of these with a side length of 1 unit will work for this problem, so let's just list the following triples: 2 × 2 × 25, 2 × 5 × 10 and 4 × 5 × 5. In this last triple, we can spot a pair of side lengths with a sum of squares equal to 50. The diagonal of a 5 × 5 face is $\sqrt{(5^2 + 5^2)} = \sqrt{(25 + 25)} = \sqrt{50}$ units. The prism must have side lengths of 4, 5 and 5 units. The two square bases each have an area of 5 × 5 = 25 units^2, and the four rectangular faces each have an area of 4 × 5 = 20 units^2. The surface area of the prism is 2 × 25 + 4 × 20 = 50 + 80 = **130** units^2.

209. The average of the eight numbers is (50 + 70) ÷ 8 = 120 ÷ 8 = **15**.

210. First, we can expand the left side of the equation to get $x^2 + 2x + 1 = 6x$, and then we can subtract $6x$ from both sides to get $x^2 - 4x + 1 = 0$. This is not factorable, so we can complete the square to solve the equation, as follows:

$$x^2 - 4x + 1 = 0$$
$$x^2 - 4x = -1$$
$$x^2 - 4x + 4 = -1 + 4$$
$$(x - 2)^2 = 3$$

At this point, we don't even need to continue solving. Since the problem asks for the value of $(x - 2)^2$, the answer is **3**.

Warm-Up 13

211. We can factor out 10 to rewrite the expression as 10(1 + 1/3 + 1/9 + 1/27 + 1/81 + 1/243 + 1/729). The expression in parentheses is the sum of the first 7 terms of a geometric series with first term 1 and common ratio 1/3. Instead of adding each term individually, we can use the formula for the sum of the first n terms of a geometric series: $S_n = a(1 - r^n)/(1 - r)$. Here, $a = 1$ (our first term), $r = 1/3$ (the common ratio) and $n = 7$ (the number of terms in the series). Substituting into the summation formula, we have $S_7 = (1 - (1/3)^7)/(1 - 1/3) = (1 - 1/2187)/(2/3) = (2186/2187) \times (3/2)$. Because 2186/2187 is very close to 1, the sum is approximately (3/2). Multiplying by 10, the value of the original expression is about 10 × (3/2) = 15, so the closest integer is **15**.

212. We can rewrite the equation $4^{3x-1} = 7$ as $4^{3x} \times 4^{-1} = 4^{3x} \div 4 = 7$, then as $4^{3x} = 28$, and finally as $4^x = \sqrt[3]{28}$. Since we want to know the value of 4^{2x}, we can just square both sides of this last equation. That gives us $(4^x)^2 = (\sqrt[3]{28})^2$, which we can rewrite as $4^{2x} = \sqrt[3]{(28^2)}$. Since 28 is $2^2 \times 7$, 28^2 must be $2^4 \times 7^2$ or $2^3 \times (2 \times 7^2)$. This means we can rewrite $\sqrt[3]{(28^2)}$ as $\sqrt[3]{(2^3 \times (2 \times 7^2))}$ = $\mathbf{2\sqrt[3]{98}}$, which is the value of 4^{2x}.

213. Let's say that the side length of a cube is s inches. Then the ratio of the number of square inches in the surface area of the cube to the number of cubic inches in the volume of the cube is $6s^2/s^3$, which simplifies to $6/s$. This ratio, we are told, is r. Now, changing this to a ratio of square feet to cubic feet will be a little different. Our side length of s inches is $s/12$ feet, so the number of square feet in the surface area of the cube to the number of cubic feet in the volume of the cube is $6(s/12)^2/(s/12)^3$. This simplifies to $6/(s/12)$ or $12 \times (6/s)$, so the value of k is **12**.

214. The table at right shows the sums of all 36 possible rolls that Omar can make with the two fair dice. With his twelve-sided die, Jamie has an equal chance of getting each number from 1 to 12. For each possible sum that Omar can get, from 2 to 12, we will multiply that probability by the probability that Jamie will roll a greater number. The final probability is the sum of all these products: (1/36 × 10/12) + (2/36 × 9/12) + (3/36 × 8/12) + (4/36 × 7/12) + (5/36 × 6/12) + (6/36 × 5/12) + (5/36 × 4/12) + (4/36 × 3/12) + (3/36 × 2/12) + (2/36 × 1/12) + (1/36 × 0/12) = 10/432 + 18/432 + 24/432 + 28/432 + 30/432 + 30/432 + 20/432 + 12/432 + 6/432 + 2/432 + 0/432 = 180/432 = **5/12**.

+	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

215. The units digit of powers of 7 follows a repeating cycle of 4: 7, 9, 3, 1. Dividing 2025 by 4 gives a remainder of 1, which means 7^{2025} falls in the first position of the cycle. Therefore, the units digit of 7^{2025} is **7**.

216. Let's rewrite 2 1/4 as 9/4 and then as 27/12. That means we want $a + b + c = 27$, where a, b and c are positive integers and $a < b < c < 12$. We want them as close as possible to each other to maximize the product $a \times b \times c$. If we make $a = 8$, $b = 9$ and $c = 10$, then our product is 8 × 9 × 10 = **720**.

217. Let the number be n, and let the sum of its digits be s. We are told that $n = 32 \times s$. This means that n must be a multiple of 32, and the number must also be relatively small, since multiplying even a moderate digit sum by 32 gets large quickly. The maximum possible digit sum for a 2-digit number is 9 + 9 = 18, and anything larger would make n quite big. So we try $s = 18$ and compute 32 × 18 = 576. The sum of the digits of 576 is 5 + 7 + 6 = 18, which matches! So, the integer is **576**.

218. The expression 4^{3a-1} can be rewritten as $4^{3a} \times 4^{-1} = 4^{3a} \div 4 = 2^{6a} \div 4 = (2^a)^6 \div 4$. Since we are told that $2^a = 3$, we can substitute 3 for 2^a in the last expression. The result is $3^6 \div 4$ = **729/4**.

219. Inside the rhombus, there are four congruent right triangles whose legs are half the length of the diagonals, so they are 24 ÷ 2 = 12 cm and 32 ÷ 2 = 16 cm. The length of a side of the rhombus, which is also the hypotenuse of one of these right triangles, must be 20 cm, since 12-16-20 is a multiple of a 3-4-5 triangle. We can verify that the side length is $\sqrt{(12^2 + 16^2)} = \sqrt{(144 + 256)} = \sqrt{400} = 20$ cm. The perimeter of the rhombus is 4 × 20 = **80** cm.

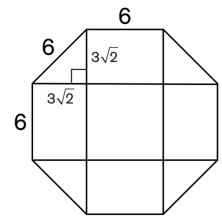
220. This is a system of two equations with two unknowns: $2x - y = 3$ and $x + 2y = 4$. If we double both sides of the first equation, we get $4x - 2y = 6$. Now we can eliminate the y term by adding the two equations. We get $5x = 10$, so $x = 2$. Substituting this value back into the first equation, we get $2(2) - y = 3$, for which $y = 1$ is the solution. The value of $x + y$ is 2 + 1 = **3**.

Warm-Up 14

221. The first pick can be any of the 8 vertices of the cube. After choosing the first vertex, there are 7 remaining vertices equally likely to be picked as the second vertex. From that first vertex, there are 3 vertices that are 1 unit away, moving along an edge of the cube; there are 3 vertices that are $\sqrt{2}$ units away, moving diagonally across a face of the cube; and there is 1 vertex that is $\sqrt{3}$ units away, moving through the interior of the cube along the space diagonal. The squares of these distances are 1, 2 and 3, respectively, and the expected value of these squares is $(3 \times 1 + 3 \times 2 + 1 \times 3) \div 7 =$ **12/7**.

222. Here are the first few powers of 7: 7, 49, 343, 2401. Dividing each of these by 8, we get the following remainders: 7, 1, 7, 1. This pattern continues, with every odd power of 7 being 7 more than a multiple of 8 and every even power being 1 more than a multiple of 8. Since 59 is odd, the remainder is **7** when 7^{59} is divided by 8.

223. The number $8^{21} \times 9^{15}$ can be rewritten as follows: $2^{63} \times 3^{30} = 2^3 \times (2^{60} \times 3^{30}) = 8 \times (4^{30} \times 3^{30}) = 8 \times 12^{30}$. If this number were to be written in base 12, it would be an 8 followed by 30 zeros, so it would have **31** digits.



224. The edge length of a regular octagon with perimeter of 48 is $48 \div 8 = 6$. The octagon is composed of a single square in the middle, four rectangles and four triangles, as shown in the diagram. That means the total area is $6 \times 6 + 4 \times (6 \times 3\sqrt{2}) + 4 \times (3\sqrt{2} \times 3\sqrt{2} \div 2) = 36 + 72\sqrt{2} + 36 = 72 + 72\sqrt{2}$ square units. Regular hexagon ABCDEF is composed of 6 equilateral triangles, so the area of one of these would be $(72 + 72\sqrt{2}) \div 6 = 12 + 12\sqrt{2}$. The formula for the area of an equilateral triangle is $A = \sqrt{3}/4 \times s^2$, where s is the side length. We know the area and we are looking for the square of the side length, so we need to solve for s^2 as follows: $s^2 = (4/\sqrt{3})(12 + 12\sqrt{2}) = ((4\sqrt{3})/3)(12 + 12\sqrt{2}) = 16\sqrt{3} + 16\sqrt{6}$. This expression is in the form $a\sqrt{3} + b\sqrt{6}$, with $a = 16$ and $b = 16$. Computing for b^2/a, we get $16^2/16 =$ **16**.

225. We can rewrite 12^3 as $2^6 \times 3^3$ and 4^4 as 2^8, so $\gcd(12^3, 4^4) = \gcd(2^6 \times 3^3, 2^8) = 2^6$. The quotient $12^3 \div 2^6$ is 3^3. Similarly, we can evaluate $\operatorname{lcm}(12^3, 4^4)$ as $\operatorname{lcm}(2^6 \times 3^3, 2^8) = 2^8 \times 3^3$. We are looking for the least positive integer x such that $3^3x = 2^8 \times 3^3$. That would be 2^8, which is **256**.

ABCD 4	BACD 2	CABD 1	DABC 0
ABDC 2	BADC 0	CADB 0	DACB 1
ACBD 2	BCAD 1	CBAD 2	DBAC 1
ACDB 1	BCDA 0	CBDA 1	DBCA 2
ADBC 1	BDAC 0	CDAB 0	DCAB 0
ADCB 2	BDCA 1	CDBA 0	DCBA 0

226. If there were just one person playing this game, the person would be sure to pick the slip of paper with their own name from the bowl. If there were two people playing, then half the time they would both pick their own name and half the time neither would pick their own name. On average, $(2 + 0) \div 2 = 1$ person would pick their own name. With three people, there are $3! = 6$ ways the slips of paper could be picked. Let's say these are ABC, where all three get their own name, ACB, BAC and CBA where one person gets their own name, and then BCA and CAB where nobody gets their own name. On average, $(3 + 1 + 1 + 1 + 0 + 0) \div 6 = 1$ person gets their own name. With four people, there are $4! = 24$ ways the slips of paper could be picked. The table shows the 24 possible orders with each entry showing the number of people getting their own slip. There is 1 way that all four people get their own name, 0 ways that three people get their own name, 6 ways that two people get their own name, 8 ways that one person gets their own name, and 9 ways than nobody gets their own name. On average, $(1 \times 4 + 0 \times 3 + 6 \times 2 + 8 \times 1 + 9 \times 0) \div 24 = 24 \div 24 = 1$ person gets their own name. It would seem that on average, regardless of the number of people, **1** person would get their own name playing this game. Alternatively, suppose there are n people in the group. The average number of times that any particular person gets their own name is $1/n$, because they have a $1/n$ chance of drawing their own name and a $(n - 1)/n$ chance of not drawing their own name. It is a theorem of probability theory that the expectation of a sum is the sum of the expectations (even when the events involved are not independent, which is the case here). Therefore the expected number of people who get their own name is n times $1/n$, which equals **1**.

227. For Zaila to run one full lap, she would have to run 3/2 as far as her 2/3 laps, so her time would be $3/2 \times 3/2 = 9/4$ minutes. Similarly, Yael's time for one lap would be $4/3 \times 4/3 = 16/9$ minutes and Xavion's time for one lap would be $5/4 \times 5/4 = 25/16$ minutes. In a four-lap race, the times would be Zaila at $4 \times 9/4 = 9$ minutes, Yael at $4 \times 16/9 = 64/9 = 7\ 1/9$ minutes, and Xavion at $4 \times 25/16 = 6\ 1/4$ minutes. The third-place finisher, Zaila, will be $9 - 6\ 1/4 = 2\ 3/4$ minutes behind the winner, Xavion. That's a difference of $2\ 3/4 \times 60 =$ **165** seconds.

228. The number 200! will have many more factors of 2 than factors of 7, so we only need to count the factors of 7 to figure out how many zeros are at the end of the base-14 version of this number. Since $7 \times 28 = 196$ and $49 \times 4 = 196$, there are $28 + 4 = 32$ factors of 7 in 200!. Each factor of 7 will combine with a factor of 2 to make a higher power of 14, resulting in **32** zeros at the end of the number.

229. Let's use the quadratic formula to solve for x in terms of n: $x = (-(-20) \pm \sqrt{((-20)^2 - 4 \times 1 \times n)})/(2 \times 1) = 20/2 \pm \sqrt{(400 - 4n)}/2$. For this formula to produce only integer solutions, we need $400 - 4n$ to be an even square number. Setting this equal to the largest even square less than 400, we get $400 - 4n = 324$, for which $4n = 76$ and $n = 19$. This is the least value of n. At the other extreme, we get $400 - 4n = 0$, for which $4n = 400$ and $n = 100$. This is the greatest value of n. The absolute difference is $|100 - 19| =$ **81**.

230. The sides of the equilateral triangle must be the diagonals of three faces that come together at one vertex of the cube. The formula for the area of an equilateral triangle is $A = \sqrt{3}/4 \times a^2$, where a is the side length. Our triangle has an area of $4\sqrt{3}$ cm^2, so we have $\sqrt{3}/4 \times a^2 = 4\sqrt{3}$. We can multiply both sides of the equation by $4/\sqrt{3}$, which gives us $a^2 = 16$. Since a side length must be positive, we know that $a = 4$ cm. This is the diagonal of a face of the cube, so the edge length of the cube must be $4/\sqrt{2} = 2\sqrt{2}$ cm. The cube has six faces, so the surface area of the cube is $6(2\sqrt{2})^2 = 6 \times 8 =$ **48** cm^2.

Workout 7

231. The prime factorization of 20^{26} is $4^{26} \times 5^{26} = 2^{52} \times 5^{26}$. Since 2^0 and 5^0 are also divisors of this number, it has 53 × 27 = 1431 divisors. Since 20^{25} has one less divisor of 20, it has two fewer divisors of 2 and one less divisor of 5. Its prime factorization is $2^{50} \times 5^{25}$ and it has 51 × 26 = 1326 divisors. All of these divisors are also divisors of 20^{26}, so there must be 1431 – 1326 = **105** integers that are divisors of 20^{26} but not divisors of 20^{25}.

232. The expression has 9 fractions, each of which is 1 minus a unit fraction, and the pattern of those unit fractions is the inverse powers of 2. We can evaluate this expression as follows: 9 – (1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64 + 1/128 + 1/256 + 1/512) = 9 – 511/512 = 8 1/512 = **4097/512**.

233. The altitude to the hypotenuse must be the shortest of the lengths given, so the legs of the right triangle are 580 inches and 609 inches. The area of the triangle is 580 × 609 ÷ 2 = 176,610 in^2. To verify that this configuration actually forms a right triangle, we use the area and the given shortest altitude (420 inches) to solve for the hypotenuse: 176,610 = (1/2) × c × 420, which gives c = 841 inches. We then check the Pythagorean Theorem and find that $580^2 + 609^2 = 841^2$, confirming that the triangle is indeed right. Therefore, the area is **176,610** in^2.

234. We can factor out 11^2 and evaluate the expression as follows: $11^2(1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + 9^2) = 11^2(1 - 4 + 9 - 16 + 25 - 36 + 49 - 64 + 81) = 11^2(165 - 120) = 121 \times 45 =$ **5445**.

235. No two consecutive positive integers could have a sum of 100, since an even plus an odd is odd. The sum of three consecutive positive integers is always a multiple of 3 and the sum of four consecutive positive integers is always 2 more than a multiple of 4, so neither of those work. Our first possible set of integers with a sum of 100 is 18 + 19 + 20 + 21 + 22 = 100. Since a is the first of these numbers and b is the last, the difference $b - a$ is 22 – 18 = 4. Now consider six consecutive integers. The average would be 100 ÷ 6 ≈ 16.67, which is not an integer or half-integer, so a set of six integers is not possible. For seven integers, the average would be 100 ÷ 7 ≈ 14.29—also not an integer or half-integer. With eight integers, the average would be 100 ÷ 8 = 12.5, a half-integer, which means we can place eight consecutive integers symmetrically around that value. This gives the integers 9, 10, 11, 12, 13, 14, 15 and 16, which sum to 100. In this case, $b - a$ = 16 – 9 = 7. Any set of more than eight consecutive integers would require the average to be less than 12.5, which would push the smallest number below 9. Trying nine integers, for example, gives an average of 100 ÷ 9 ≈ 11.11, making the middle number too small for the surrounding integers to stay positive and still sum to 100. So larger sets do not work. These are the only two solutions, so the sum of the possible values of $b - a$ is 4 + 7 = **11**.

236. For the product of the integers in the set to be divisible by a power of 10, we will need factors of 5 and 2. Since the integers must be distinct, we will want to use 5 and 10 so that we get two factors of 5. We cannot include a third multiple of 5 because 5 + 10 + 15 = 30, which exceeds the total allowed sum of 20. Then we will need a second factor of 2 to pair with the second factor of 5. The rest of the sum does not matter. If we use 2, 3, 5 and 10, the sum is 20 and the product is 300. The largest possible power of 10 that will divide 300 is 10^2, so the value of k is **2**.

237. If every person on the planet in 1912 could have gotten four pounds of salmon, then 1,800,000,000 × 4 = 7,200,000,000 pounds were packed. Since each case contained 48 one-pound cans, the Columbia River canneries packed 7,200,000,000 ÷ 48 = 150,000,000, or **150** million cases.

238. A number is divisible by 4 if and only if the two-digit number formed by the last two digits of that number is divisible by 4. Considering multiples of 4, we realize that a number ending in 2 or 6 is divisible by 4 only when the tens digit is odd, and 2026 has no odd digits. The only permutations of 2026 that end in zero are divisible by 4. These **3** permutations are 2260, 2620 and 6220.

239. Triangles ABC and ADE are similar. Triangle ABC is a right triangle, so using the Pythagorean Theorem, we find $AB = \sqrt{(AC^2 - BC^2)} = \sqrt{(10^2 - 6^2)} = \sqrt{(100 - 36)} = 8$ m. Since AE is three times as long as AC, AD must be 3 × 8 = 24 m and DE must be 3 × 6 = 18 m. So, the perimeter of triangle ADE is 24 + 18 + 30 = **72** meters.

240. The least positive integer with three distinct prime factors is 2 × 3 × 5 = 30. The next few are 2 × 3 × 7 = 42, $2^2 \times 3 \times 5 = 60$, 2 × 3 × 11 = 66, and 2 × 5 × 7 = 70. That makes **5** integers less than or equal to 70 that have at least three distinct prime factors.

Workout 8

241. There are $2^4 = 16$ outcomes for the four flips of a fair coin. When Natalie says that at least two of the four coin flips were heads, she rules out the possibility of four tails or three tails. There is only 1 way to get four tails, TTTT, and there are 4 ways to get three tails, HTTT, THTT, TTHT and TTTH. The denominator of our probability is the other 16 – (1 + 4) = 11 possible outcomes. For the numerator, we want to count the ways that at least three of the flips are heads. We can see that there is 1 way to get all heads, HHHH, and 4 ways to get three heads, HHHT, HHTH, HTHH and THHH. The probability is thus **5/11**.

242. If the interior angles of this convex polygon form a nonconstant arithmetic sequence, then the exterior angles also form a nonconstant arithmetic sequence. The advantage of considering the exterior angles is that we know that their sum must be 360 degrees. Suppose, for example, that the polygon has 15 exterior angles with an average angle of 360/15 = 24 degrees. The constant difference for the arithmetic sequence could be 1, so the exterior angles would have measures 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30 and 31 degrees and the interior angles would have the corresponding supplementary angle measures. The example works, but we can do better. There could be 18 exterior angles with an average of 360/18 = 20 degrees. The constant difference would have to be 2 this time, and the exterior angles would have measures 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35 and 37 degrees. The interior angles would be 177, 175, 173, etc., … until 143 degrees. These are all obtuse angles, so this polygon would be convex as required. If there were more than 18 sides, there would be angles with non-integer or negative measures, so the largest number of sides is **18** sides.

243. We will count the positive integers that do include two or more prime digits that sum to at least 14 and then subtract these from the 999 integers there are from 1 to 999 inclusive. There are 10 positive integers that look like 77_, where the blank can be any of 10 positive integers. Likewise, there are 10 integers that look like 7_7 and another 10 integers that look like _77, allowing for a leading zero to signify two-digit numbers. That seems like 30 integers, but the same number 777 has been counted in each of the three patterns, thus three times, so the number of distinct patterns at at least two 7s is 30 − 2 = 28. There are 6 ways to rearrange the digits 257 and 6 ways to rearrange the digits 357, so that's 12 more integers. There are 3 ways to rearrange the digits 557 and then just one way to write 555, which is 4 more. In all, there are 28 + 12 + 4 = 44 positive integers that *do* include two or more prime digits that sum to at least 14, so there must be 999 − 44 = **955** integers that do not.

244. We need to find the total capacity of the hotel for our denominator, and the maximum occupancy if no room is more than half full for our numerator. Story 1 has 1 room with a capacity of 1 person. Story 2 has 2 rooms with capacities of 1 and 2, so the capacity of story 2 is 1 + 2 = 3 people. Story 3 has 3 rooms with capacities of 1, 2 and 3, which is 6 people. Most mathletes will recognize this pattern of triangular numbers. Story 54 has 54 rooms with capacities of 1 through 54 people. The sum of the first 54 counting numbers can be calculated as "55 choose 2", which is (55 × 54) ÷ (2 × 1) = 1485, so that's 1485 people on the 54th story. The total capacity of the hotel is the sum of the first 54 triangular numbers. This is known as a tetrahedral number and can be calculated as "56 choose 3", which is (56 × 55 × 54) ÷ (3 × 2 × 1) = 27,720 people. (Search the internet for the "hockey stick pattern in Pascal's triangle"). For the numerator of this fraction, we have to find the maximum occupancy if no room is more than half full. If we just divide 27,720 by 2, which is 13,860 people, we get an overcount since the rooms with an odd number capacity must be rounded down. Let's count the number of odd rooms. All 54 stories have a room 1 with a capacity of 1 person. There are 52 stories with a room 3 with a capacity of 3 people, 50 stories with a room 5 with a capacity of 5 people, etc., until we get to 2 stories with a room 53. In all, there are 54 + 52 + 50 + … + 2 = (54 + 2) × 27 ÷ 2 = 756 odd numbered rooms with an odd number capacity. The whole number of people not more than half the capacity of an odd number is 0.5 people less than the actual half, so we need to subtract 756 × 0.5 = 378 from the earlier calculation of 13,860 as half the capacity. The result is 13,482 people, so our final fraction of the total capacity of the building is 13,482/27,720. To simplify this fraction, let's find the prime factorization of both numerator and denominator. That would be (2 × 3 × 3 × 7 × 107)/(2 × 2 × 2 × 3 × 3 × 5 × 7 × 11). Canceling the common factors, we get a simplified answer of **107/220**.

245. The given equation $2(x - 1/x)^2 - 7x + 7/x + 6 = 0$ can be rewritten as $2(x - 1/x)^2 - 7(x - 1/x) + 6 = 0$. The expression "$x - 1/x$" appears more than once in the equation. To simplify things, we let $u = x - 1/x$, which allows us to rewrite the equation in terms of a single variable. Substituting into the equation, we get $2u^2 - 7u + 6 = 0$. This is a quadratic equation in u, which factors as $(2u - 3)(u - 2) = 0$. So the possible values of u are $u = 2$ and $u = 3/2$. We now return to the original substitution and solve for x in each case. First, suppose $x - 1/x = 2$. Multiplying both sides by x eliminates the fraction and gives $x^2 - 1 = 2x$, which rearranges to $x^2 - 2x - 1 = 0$. Solving this quadratic using the quadratic formula gives two roots: $x = 1 + \sqrt{2}$ and $x = 1 - \sqrt{2}$. Next, suppose $x - 1/x = 3/2$. Multiplying both sides by x gives $x^2 - 1 = 3x/2$, or $2x^2 - 3x - 2 = 0$. Solving this quadratic yields the roots $x = 2$ and $x = -1/2$. Altogether, the four roots of the original equation are: $1 + \sqrt{2}$, $1 - \sqrt{2}$, 2 and $-1/2$. To find the sum of all the roots, we add them together: $1 + \sqrt{2} + 1 - \sqrt{2} + 2 + (-1/2) = 1 + 1 + 2 - 1/2 = 4 - 1/2 =$ **7/2**.

246. If the function $y = x^2$ is shifted, but not rotated or stretched, then the function can be rewritten as $y = (x - h)^2 + k$. We can plug in the coordinates of the two points given to get a system of two equations with the two unknowns h and k, and then solve that system. Alternatively, we might notice that the two points, (12, 6) and (16, 6), have the same y-coordinate. The symmetry of a parabola requires that the axis of symmetry be halfway between these two points, which would be at $x = 14$. The value of h must also be 14, since 14 − 14 = 0, zeroing out the squared term. Now we can plug in the x and y values from either of the points given, along with $h = 14$, and solve for k as follows: $6 = (12 - 14)^2 + k \rightarrow 6 = (-2)^2 + k \rightarrow 6 = 4 + k \rightarrow k = 2$. The equation of the new graph is $y = (x - 14)^2 + 2$, and the vertex is (**14**, **2**).

247. There are no positive integers strictly between $\sqrt[3]{k}$ and $\sqrt{k}$ for k = 1, 2, 3, 4, 8 and 9. For k = 5, 6 and 7, the positive integer 2 is between the cube root and the square root of each number. For any positive integer k greater than 9, there is always a positive integer between the cube root of k and the square root of k. The desired sum is thus 1 + 2 + 3 + 4 + 8 + 9 = **27**.

248. By Pascal's rule, for all integers n and k with $0 \le k \le n - 1$, ${}_nC_k + {}_nC_{k+1} = {}_{n+1}C_{k+1}$. The left side of the equation in the problem is a special case of the left side of Pascal's rule with $n = 11$ and $k = 2$, so the sum is ${}_{12}C_3 = 12!/(3! \times 9!) = (12 \times 11 \times 10)/(3 \times 2 \times 1) = 220$. Thus, we already have one solution to the equation and a consequent value $n + k = 12 + 3 = 15$. To find the least possible value of $n + k$, we can check whether there are other values of n and k such that ${}_nC_k = 220$ and $n + k < 15$. Note that ${}_nC_k = 220$ implies that $n!/(k!(n - k!)) = 220$. Since 220 is divisible by 11, n must be at least 11 for $n!$ to be divisible by 11. We only need to check values of n such that $11 \le n \le 14$, and for each of these, all values of k between 0 and n. For $n = 11$, we have ${}_{11}C_0 = 1$, ${}_{11}C_1 = 11$, ${}_{11}C_2 = 55$, ${}_{11}C_3 = 165$ and ${}_{11}C_4 = 330$, none of which equal 220. For $n = 12$, we have ${}_{12}C_2 = 66$ and ${}_{12}C_3 = 220$; the latter works and is the solution we already found. For $n = 13$, we have ${}_{13}C_3 = 286$ and ${}_{13}C_4 = 715$, which are both too large. Finally, for $n = 14$, we have ${}_{14}C_3 = 364$ and ${}_{14}C_4 = 1001$, both of which are too large. There are no other values of n and k with ${}_nC_k = 220$, so the least possible value of $n + k$ is **15**.

249. Let's say that side CD has a length of x and side BC has a length of y. Then $x = 700 \div 20 = 35$ and $y = 700 \div 25 = 28$. Therefore, longer side is **35** inches.

250. Working Riley's division problem backwards, we can find that x is 15 × 24 = 360. So, Gavin should get 360 × 20 = **7200**.

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