AMC 8

Math Club

2008-2009

THE MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

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Please give us Feedback!

This year marks the fifth year we have produced this Math Club Guide. Please take a few minutes and give us some feedback on its content. We'd like to know which sections have been helpful, and which sections could use some "beefing up." For each of the major sections (in black) please give us two sets of information: whether you read the material or not, and scale of usefulnes with 0 being not useful, and 5 being very useful . If you would like to give feedback on the sub-headings (smaller, in gray), that would also be appreciated.

We have this questionaire available on line at:

http://www.unl.edu/amc/mathclub/2008AMC8questionaire.shtml

with electronic delivery. Or, copy or tear out this page, and fill in and mail to us at:

Math Club 8 Questionaire
American Mathematics Competitions
University of Nebraska – Lincoln
1740 Vine Street
Lincoln, NE 68588-0658

I am a (position);	math teacher	math supervisor	mentor	parent/guardian	student
School size:	0-100	101-250	251-500	501-1000	1000+
School setting:	city	urban	suburban	sm community	rural
Type:	public	private	home school	club/circle	individual

Grades included in school: K 1 2 3 4 5 6 7 8 9 10 11 12 13

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We know some schools purchase the Math Club book every year, and others purchase it only once. If you purchase it every year, we'd like to know why you continue to purchase it:

General comments:

If you would like to give us your name and contact information (address, email, etc.), you may do so here, but it is not required.

Club Organization

Guidelines

Academic Guidelines

1. Especially at first, keep it casual and fun - you want them to come back for the next meeting.
2. The math club should have a balanced focus between mathematical enrichment topics and mathematical problem-solving, with the balance determined by both the interests of the participants and the club sponsor.
3. In both cases, the students should be actively involved in the problem-solving and topic presentations. This guideline means that students should be able to explain their problem and to demonstrate the solution or topic to the others in the club.
4. Let the students teach each other - they will learn communications skills and will retain the structure of the problem better if they have to show each other how the problem is solved, and even what alternatives there are to solving it. "Teaching time" will present itself as student discussions evolve, so lecture only when the situation presents itself.

Selecting problems and solutions may depend on the goals of the math club sponsor and the problem-solving ability of the club members. Some sponsors will specifically select topics and problems of an appropriate difficulty and assign them. Other may elect to use a "cookie-jar" approach where students select at random from a larger collection.

All students should be encouraged to participate and benefit. After several meetings you should be able to tell most students strengths: Deep thinker? Quick? Creative? Consensus builder? Good communicator? Set up groups to balance the qualities, and they will gel into a good problem solving unit.

Even if a math club is officially noncompetitive, informal competition among students will occur. The participants will quickly recognize who among them are good problem solvers, who are quick problem solvers, who are deep thinkers about mathematical problems, and who can explain things well. But this sort of competition is healthy, friendly and constructive, and even leads to cooperative efforts among the participants.

Administrative Guidelines

1. All club members should be participants.
2. It is ideal for students to work together in small groups.
3. Providing snacks is an excellent incentive for students to attend.
4. Vary the location of your sessions to allow for a less regimented, more relaxing atmosphere.
5. Vary the content of the meetings, so they don't become predictable, routine or boring. Students often think of math class as containing lots of repetition, so we need to have the club be something they want to participate in, because it is can be unpredictable, and interesting.
6. Schedule the meetings weekly, even if you don't have a special event planned. The first few meetings should be informal. Hold icebreakers that allow students to get to know each other.

One way that Sliffe Award winning teachers have encouraged a high level of participation is to have students work in groups of (for instance) 5 on a group of 5 problems. (Use an array of AMC problems of varying difficulties, for example.) Each of the 5 students in the group should be able to present and explain any one of the 5 problems assigned to the group. This way the students learn that part of doing mathematics is sharing insights, ideas, and experiences solving similar problems. Teamwork develops as the students work on topics and problems. Students who have difficulty in solving a problem may have other skills that result in a clearer explanation or effective presentation of a topic.

Informal surveys of Sliffe Award winning teachers show that they are about evenly divided among having Math Club

or AMC contest practice sessions before and after the school day. Much depends on the specific school schedule and other extra-curricular activities. These teachers suggested having doughnuts at before-school gatherings and pizza as an incentive for after-school gatherings.

Club Advisor

⌘ Keep a short journal/notebook. Make notations each week for:

- What worked well, and what didn't in the practice sessions.
- The format and rules of any meets and contests your students participate in.
- If you can, keep a copy of the questions posed at each event, to use to prepare your students the next year.
- Talk to other coaches/teachers at the events, and jot down any new ideas for practice sessions, or procedures which have worked well for others

Next year you can look this over, and give guidance to your students before an event, going over the rules, and any tricks you thought up which might help this year. If you do this for several years, you will have a treasure trove of useful ideas and questions.

⌘ Find alternate means of transportation for students who normally need to take the bus, or ride in car pools - this may mean helping parents organize a math carpool for the mornings or evenings they have meetings.

⌘ Get help by having other teachers assist when you need an extra hand at the practice sessions, and get a parent volunteer list which you can call on to get help with arrangements or other details (like a scrapbook, awards program, handle press/newsletter articles, etc.).

⌘ If there is a Math Department at a local College or University, you can see if they sponsor a student MAA chapter, which could take on your club as a community service project. A listing of these is at <http://www.maa.org/students/chapter_index.html>.

⌘ You can also call the local community service clubs (Rotary, Kiwanis, Masons, etc.), to see if there is a way they could mentor/sponsor your group. They might be able to help with practice sessions or provide snacks once a month. They could also be used as a sponsor, helping with details, and having a dinner with the students at the end of the school year.

Publicity and the Math Club

To encourage new students and continue participation once they have become regulars, strong local support is needed. The rewards of participation should be visible and enumerated upon to invigorate the program.

⌘ Set up a display when school opens in the fall with T-shirts, photos of last year's activities, posters or banners. List names of events you will be participating in this year, and show any trophies you have.

⌘ Design posters for your first meeting with interesting math questions and post them all over school. Refer students to the first meeting for the answers.

⌘ Select a specific group of students to serve as PR representatives:

- Have your returning students design a skit for a beginning of the year school assembly or pep rally.
- Let a committee of 2-4 students be in charge of posters for weekly meetings, and writing regular updates for the school newspaper, as well as sending the information to your local newspaper when you do well -- pictures with a trophy always look nice

⌘ Have an announcement made in all the math classes about the start-up of the club, and the first meetings

⌘ Ask other teachers to help recruit students for the club, and have them recommend a list of possible students so you can extend a personal invitation.

⌘ Extend a personal invitation to students you feel might benefit - sometimes we all need a little push, and sometimes knowing a teacher wants you there is the little push that's needed.

- Be sure parents know about your club - and that all students are welcome. Put a blurb in the school's parent newsletter or email. Give a presentation to the school parents organization, outlining how participation in a math club will help their children. You can also request funds for event registrations from PTO's/PTA's.
- Keep parents up-to-date on the schedule for all the club's travel plans: meets, field trips, social gatherings, etc. and arrange for a sign-up if you will need help with transportation.
- Send parents emails or update fliers, have reports at PTO/PTA, open houses and the school newsletter.
- Use the local and school newspapers to high light club events and contests they attend. Use individual students to communicate their success stories.
- Organize a pep rally for your math team, or have them included in another pep rally.
- Be sure to recognize the students with a school award programs.
- Encourage teachers of lower grades to look for students who seem to like, or are good in math, to encourage them to participate in the club's activities.
- Take plenty of pictures during practice sessions and at meets/contests. You can assign this task to a student or students who like to take photos - even if it's with their camera phones. Keep these photos and any articles and make a scrapbook. Then when local press would like a picture, you will have a variety to choose from.
- Once you have an established group, you can find local corporate sponsors who enjoy helping a positive, beneficial cause. They can help with anything from t-shirts to pencils, food and drinks to travel funds (or gas).

Problem-solving

Mathematical problems from the (book and web) resources suggested elsewhere in this manual are not always immediately solvable by all clubs, or all club members. Even the sample problems included in this package are not always immediately solvable. Here are some problem solving coaching tips:

- Replace large numbers with smaller ones. Use numbers that have fewer factors, or are easily divisible.
- Replace continuous variables with discrete variables. For example: "If a problem involves time or distance which are continuous variables, can they be substituted by variables that vary in discrete steps?"
- Reduce the number of pieces in play. Does the problem change significantly, for example: "If ten individuals are replaced with five?"
- Make a smaller playing-field. For instance, if the problem is on an 8x8 checkerboard, is the problem easier on a 4x4 checkerboard?
- Make a manipulative to illustrate the problem, for instance use beans or chips for counting problems, make cardboard constructions or wire sculptures for space geometry problems.
- Use an interactive computer program (such as Geometer's Sketchpad) to illustrate the problem dynamically.
- Use a calculator or computer to simulate, especially with probability problems.
- Remove time or step restrictions, for instance if a problem asks for solutions with a specific number of steps.

Ideas to consider when preparing

- How will you choose your participants for the AMC (8, 10, 12) or other competitions and math meets? Some teachers involve whole grade levels, some teachers involve specific classes, some teachers pick their schools' most talented students, etc.
- What resources do you use to train your student participants for the AMC (8, 10, 12)?
  - Prior years' tests in booklet form or on CD

- Contest Problem Books I-IX from MAA Publications
- Other problem and contest books
- Text books and curricular resources
- Materials available on the Web.

⌘ Will you prepare your own solutions and training materials for training your student participants? Will you provide the AMC Solution Pamphlets, or will you have the students create their own solutions, and compile them into a booklet themselves, as part of their training?

⌘ How will you integrate your current textbooks and curricular materials with your contest training, if at all?

⌘ When will you and your participants prepare?

- Before school?
- After school?
- During school – (e.g. Lunchtime, study period, special class)
- Math Club or similar extracurricular group?

⌘ Will you use the AMC (8, 10, 12) to prepare your students for other contests and activities or vice versa?

⌘ How will you archive and classify the problems they will work on? Will you use NCTM standards, Mathworld standards or some other classification method? How will you store them, to have them easily available? Is there a way to create a database of questions with Excel, or some other program?

⌘ Will you train students in cooperative groups or other collaborative strategies for an individually based competition such as the AMC (8, 10, 12)?

⌘ Do your students cooperatively help each other with training, or do students compete independently, or a mix of the two strategies?

⌘ Will you have timed "practice contests" to prepare your students for the actual competitions? Will you use previous copies of competitions to provide these practice contests?

⌘ What was it that first turned you on to math? What turned your fellow teachers on to math? Is there any way of duplicating those experiences for your students?

Club Ideas

Club activities

- Find a "real-world" application of mathematics in your school. For example, on MathForum.org, one teacher suggested "I run a math club for elementary students (gr. 4 and 5) in my school, and one of our first activities this year will be to measure the cafeteria, and the tables, and find ways to rearrange them to be able to fit more tables without too much crowding. Our cafeteria doesn't have enough tables to accommodate the number of students who need to eat, so this activity will provide a real life connection to mathematics." (Teacher2Teacher forum, Q&A 568). Other ideas might involve sports measurements and timings, planting gardens and trees, and measuring the ratio of classroom space to public space such as hallways.
- Games and tournaments of games, especially mathematical games such as 3-dimensional tic-tac-toe, nim and all the variants of nim, checkers and chess. Another teacher on MathForum.org says "strategy games are a big favorite for my middle school students. There are many different games that are quite applicable to mathematics. We have mini tournaments with our group. The game does not make that much difference as long as it is one that involves strategy or probability so that a discussion can evolve from the game. It can be as simple as a version of tic-tac-toe or nim to something more complex such as chess."
- On March 14, i.e. 3/14, organize a Pi Day at your school. Some clubs even celebrate at 3 pm., (i.e. 3/14 , 1500 hrs.) and serve pies. Of course, circle measurement, circumference and area computation are elementary activities, but discussion of pi can range to irrationality and how to calculate and approximate the value of pi.
- Talk to a local college or university Department of Mathematics and invite a mathematician to give a demonstration of current mathematics.
- Alternatively, connect with medical, pharmaceutical, or agricultural researchers and learn how they use mathematics. Marketing organizations sometimes use polling and statistical analysis. Engineering and software firms are good sources of mathematically trained employees.
- Ask friends, colleagues, and neighbors for the names of local businesses with employees who use mathematics in interesting or different ways who can come speak to your club.
- Arrange field trips (students always enjoy getting away from school) to visit sites that use math and have someone there give a tour and talk about math use in this field. You can develop a problem worksheet that relates to the business, and have the students discuss the problems with their guide.
- Have your students participate in a "World Wide Web mathematical scavenger hunt". Have the students look for additional problem resources, formula pages, pages about pi, e, the golden ratio, mathematical bloopers and fallacies including "circle squarers" and "angle trisectors".
- Use <mathdl.maa.org/mathDL/46> and <turnbull.mcs.st-and.ac.uk> for math history and events that occurred on a specific day.
- Have a birthday party for mathematicians born in the month (See calendar section). With more mature students, a funeral oration or eulogy may be appropriately used on the anniversary of the death of a famous mathematician.
- Have students think about interesting problems to choose or write for each other, then have them go through and solve them either as a group activity, or as a homework assignment to be gone over as a group during the next class.
- Assign individual problems for oral group presentation, for practice.
- Have the principal or another teacher the students respect come and give them a short pep talk.

- ⌘ Organize a competition between the students and the teachers at your school, and invite the student body to attend.
- ⌘ Hold reunions to inspire your students, by exposing them to successful former members.
- ⌘ One way to help high school students to master a subject is to have them teach it to someone else. Taking your high school club to a feeder middle school, two or three times a year, and having them plan the curriculum for those meeting, can reinforce basic concepts in them.

Suggestions from High School Sliffe nominations

Each year in the spring the top 60 teams on the AMC 12 are asked to nominate a teacher they think deserves to win the Edyth May Sliffe Award for Distinguished MathematicsTeaching in the High School. From these nominations, over the past 5 years, we have gleaned a variety of suggestions for the Math Club Coach. We understand that no one teacher could do all of these things, but we hope you can find a suggestion or two which you can incorporate into your program, *(We have paraphrased most comments, additional information from us on a student comment are in italics.)*

Contests

- ⌘ Registering for competitions and math leagues ahead of time to provide plenty of on-task practice.
- ⌘ Applying for maximum funding from the high school promptly to minimize student and coach expenses.
- ⌘ Support the club against the hardships of cut budgets and uncooperative students.
- ⌘ Send out public announcements and written notifications to students, to ensure they will be present and prepared for the test.
- ⌘ Organize your own mathematics tournament:
  - Registration
  - Event coordination
  - Organization
- ⌘ Our teacher coaches the Math and Science Club, for which he ... exhaustively labors in search for a multitude of practice tests.
- ⌘ She manages registration for our numerous contest through out the year, and deals effectively with the organizational hassles of over 600 AMC competitors and over 100 AIME competitiors every year. Every week in which we do not have contests she prepares a practice. Her practices through out the year give us a great deal of additional preparation for the AMC contests.
- ⌘ In preparation for the AMC tests or any other contest arrange to reserve a section of the school large enough for the math students on campus so that they could take the contest in a monitored, organized manner. *(Sometimes this means putting in your request in the spring, when building use plans are being drawn up for the next year)*
- ⌘ Whenever the competitions are over, our teacher always explains the answers so that everyone in the class can understand them.
- ⌘ At the end of the year, organize an award ceremony for whom ever won any sort of award in the various contests the club has participated in.

Clubs

- ⌘ She is respected by the team for doing things such as including in the formal description of our math team, “Fun! Fun! Fun!” and taking the team out to after-math-meet dinners.
- ⌘ As a High School Teacher, approach your “Feeder” middle schools and help them start or improve a math club, as a Middle School Teacher, approach your High School and ask for “assistants” from the students there, who

can periodically attend the club meetings and mentor the students.

- Post notices on the "Daily Bulletin" and broadcast math events on the intercom virtually every day. Be very generous with praise and frequently announce the math team's achievements school-wide.
- Our teacher understands that many students participate solely for enjoyment and maintains a relaxed, friendly atmosphere at club events.
- Rather than only encouraging the top students, he encourages everyone to to participate. It does not make much of a difference to encourage those who would already participate, even if the school did not offer the contest but instead he encourages every common student to attempt new opportunities. This provides the initial push everyone needs to spark interest in mathematics. As the most difficult part of improving in the area of mathematics for students who are not mathematically inclined is discouragement caused by a strong mathematical environment, he provides the encouragement necessary.
- The most important element of the encouragement he provides is not reward for high scores, but rather rewards for sincere effort. For instance, a student who is enrolled in a regular math class (as opposed to an honors class) would tend to have been discouraged in mathematics into holding low self esteem. Rather than awarding the top scores in the entire school, he awards the top scores in each section. This matches performance to environment and prevents students who are not mathematically inclined from being discouraged. We find this method of encouraging mathematical participation effective in nurturing every minuscule of mathematical interest and preventing it from being suppressed in the process!
- Create a fun, enjoyable environment (with snacks!) which inspires many students and instills in them a great love for math for many years.
- She brings character to our team, instilling a love of learning rather than a thirst for victory.
- Our Math Team's ideals: learning, solving and competing for life rather than for score.
- Our Teacher has given us all a love and enthusiasm for mathematics, connecting its beauty to specific problems and encouraging us to practice not for the contest, but for a command of careful problem-solving technique and for our own enjoyment.
- Since her arrival at the position, our focus has shifted from score pursuit to developing an appreciation for unusual mathematics and divergent thinking. Her gift - not our score - will last a lifetime.
- She started a tutor program in our school in which volunteer student tutors help other students with classes.
- Although our sponsor takes pride in winning, he never pressures the team. Rather, he sticks to his motto that math competitions are about learning cool math and having fun in the process.
- Spend time working with the student leadership of your group to plan meetings and seek out new competitions.
- Work with frustrated students after school all week long to help them through the more difficult problems.
- Every morning, our teacher arrives at school an hour early to practice with the team.
- He motivates dozens of students to attend the math team's Early-bird class at 6 a.m., and he is often at school working with team members until our night practices finish at 7 p.m.
- He encourages us at every turn to be leaders, to take charge and master new math concepts in a cooperative environment.
- She has often encouraged students who could not make it to the competition to come and see her afterwards to look at the problems on the test and see which ones they can or cannot solve. This way they do not fall behind students from other schools who attend the meet.

- ⌘ At a school with nearly seven hundred AMC participants and over 150 AIME qualifiers, the AMC and AIME practices were the most popular practices of the year, with over one hundred students in attendance. Because so many people attended these practices, there was a wide range of experience and ability. To satisfy the needs of all students, the coaches organized and ran a system in which three different levels of practice were offered simultaneously. As a result, math team veterans could be challenged by harder problems, while new members could also improve their math skills through problems appropriate to their level of experience.
- ⌘ Provided resources and materials with which we could practice, always taking time to help us work through a complicated problem. As more students got involved, he opted to offer an independent study course, meeting weekly after school, during which he would give us problems at the AIME level and higher to prepare us for competitions and help us gain a deeper understanding of mathematics.
- ⌘ To encourage students who do not participate in the Math Club to participate he offered extra credit points to students in his normal classes who attended the Mu Alpha Theta competitions.
- ⌘ Organizes math practices before and after school, as well as on Saturdays and six days a week during the summer.
- ⌘ Making packets with practice problems, formulas, and strategies to help us improve as math students.
- ⌘ Distribute challenging sets of problems in practices each week, that force students to think about math in ways that would prove invaluable on the contests.
- ⌘ Instruct students on the finer points of competition strategy, including time management, strategies for double checking work and ways of dividing up problems on team rounds.
- ⌘ Providing our team members with past contests and time-limit advice.
- ⌘ Ask area businesses for small items to be used as prizes in practice competitions.
- ⌘ Some of the events she's made possible for us are: a math competition for elementary school students; two meets that we host every year for other high schools; and huge turnouts for the competitions we attend (including the State Competition).
- ⌘ He often co-teaches a free "Math Days" camp during the summer with a local college professor. This not only keeps our ties with the University math department strong, it also provides an excellent math camp for any high school student in the area who wishes to apply and come.
- ⌘ She keeps in touch with all the math team members, even in the summer. Last summer, she organized a barbecue for the math team, including old math team alumni.
- ⌘ If there is a University or College nearby, talk to them about someone who might be willing to mentor several students through a series of weekly problem solving seminars.
- ⌘ He is always up to date with the latest news, and is truly understanding about the time commitment that his athletes have towards their respective sports.
- ⌘ At the end of the year plan a party, with a musical theme.

  Example: Mathematical Morsels and Mayhem. A teacher composed an hour and a half long musical about some of the greatest figures in mathematical history. Most of the songs are easy to recognize, but with a mathematical twist. He is constantly adding and revising this show as he gets new ideas. Most recently, he has added a song in which Fermat is in constant sorrow (based on music from Oh Brother, Where Art Thou?) because the margin in his book is too small to fit his lovely proof of what is now known as Fermant's Last Theorem.

Classroom

- ⌘ Tell an appropriate number of math-oriented jokes in class to keep students interested during every moment of class.

- Each Monday after school, we meet in her clsassroom and she provides us study materials from past exams. Often she tells us stories relating to the topics we are doing, such as her calculus cats. There's the integral cat, who's tail is bent like an integration sign. There's also the cat of continuity that glides, not walks across the floor.

  Example: 3-D boxing important equations and demonstrating the concept of the absolute value by absolute valuing his depressed students to transform them into extra happy pupils.

  Example: In AP Statistics for example, he runs ongoing statistics projects which the class does as a whole, and he uses statistical analysis to evaluate test grades. He does an excellent job in pacing difficult material and shows incredible enthusiasm for the subject.

- Teach math beyond the curricula by assigning various projects in which students research unique mathematical topics, including tensegrity and stellated icosahedrons made from origami paper.

- *He has developed* an unique curriculum characterized by ingenuity and novelty, with a harmonious combination of theoretical and applied methods. For example, one of the first problems we approached was to find the volume of an irregular tetrahedron with given side-lengths. Students were given no explicit instruction on how to do the problem. In the end, our class found several different solutions, each with its own merits. While our mathematical backgrounds were more developed than those of many of our classmates, we were astounded by some of the solutions, which included application of technologies ranging from number crunching with CAS to physical modeling with CAD. We ourselves solved the problem two ways using Cartesian analytic geometry and Euclidean geometry.

- Work with the school to allow students who are bored with their math classes, to take higher level classes, so they can be challenged properly at their level.

- Along with some other teachers, he convinced the school's administration to start offering math electives in logic and number theory.

- Subscribe to many notable mathematics periodicals to allow students to study independently.

- Among the many memorable moments he has spent with us, our personal favorites were undoubtedly the lessons on the "why" instead of the "how." In this regard, the most important thing he taught us was that there exists no dichotomy between creativity and precision, herein lies the elegance of the mathematical proof.

- He never presents a formula or result without first making sure we understand the theory behind it. He derives why these methods work, focusing on making sure that people understand the concepts going into the proof rather than being able to regurgitate a formula on an examination. His proofs are always elegant, well thought out, and beautifully presented, and he has instilled in many of his students a great appreciation for mathematical aesthetics.

- Once a week, our teacher has each of her students hand in a short journal entry about their understanding of the material taught in class. She always takes the trouble to write personalized responses, which include words of encouragement, tips for solving tricky problems, explanations of difficult concepts, and much more.

- For the dedicated students and those most interested in math and science, our teacher helps them find challenges beyond the high school level. She is in constant contact with a nearby National Laboratory, finding mentors and thesis projects for her students. In the past, she has introduced her students to advanced fields such as robotics, particle physics, and neuro-circuitry, always with extra emphasis in mathematical modeling and programming.

- Provides various resources and advice to students, allowing them to learn eclectic areas of mathematics.

  Example: Discrete Math and Linear Algebra classes, allow students to explore number theory and graph theory, subjects not discussed in regular high school math classes.

- Alternatively, provide independent study courses on specific topics for those students who finished all the

regular classes.

Example: Several of us had become more advanced than the Geometry class that was the top class offered at our school. Our teacher created a special class for us, allowing us to meet during her prep period to study Algebra 2. She encouraged us to take an active role in designing the class, assigning our own homework and scheduling our own tests. She trusted us and expected us to take the math as seriously as she did, and we worked hard to meet her expectations.

⌘ Over the summer, offer one or two levels of a math problem solving course that meets for four weeks.

Example: one section geared toward underclassmen in the morning and the advanced problem solving course in the afternoon. Each night, the teacher assigns problems from a work book and for the first hour and a half, we solve and discuss these problems as a class. After a quick break where we can buy soda and make popcorn, we are led back into the room for the contests. Each day, a new set of teams are picked and awarded points for problems from numerous problem sets that they get right. At the end of the course, there is an extensive award ceremony where many students are awarded prizes ranging from bookmarks to t-shirts to coffee mugs (items he collects through the year, at NCTM and such).

⌘ Our teacher refuses to assign homework and instead calls it homefun. Additionally, tests are known as parties. He does, however, administer tests, which are days when students bring in snacks and soft drinks. On tests days, we relax and enjoy an hour of chess, card games, or circular tic-tac-toe, a modification of the classic game with added intricate strategy.

⌘ Our teacher goes to great lengths to ensure that every student has a thorough understanding of the subject matter before advancing to the next topic. Because he turns every class into an open discussion, nobody leaves the classroom feeling left out. He also advances our mathematical thought process with each successive problem by teaching us new tricks to attack harder problems. By linking multiple concepts learned in the past, he ensures that we always have a complete understanding of all the materials learned throughout the year.

⌘ His teaching methodology is very similar to his chalk board erasing methods. In order to erase a chalk board and have the end result look orderly, start by making one long swipe across the entire top of the chalk board. Follow this swipe by making vertical erasures going across the entire board, never leaving a sliver uncovered and taking as many strokes as necessary. Clean up the job with one swipe across the top, then the left edge, and finally one across the bottom edge. Not only does this look aesthetically pleasing, but it also sums up his teaching philosophy. By making that first initial swipe, he sets down the foundation for the problem, listing everything that needs to be known to reach the answer. All the vertical swipes going from left to right represent the multiple steps to solving the problems. He covers every aspect of the problem just like his eraser eventually goes over every square inch of the board. By the time the final clean up swipes are reached, the problem has been virtually solved by the class with his guidance. Those swipes are like him making sure that the actual question was answered and that all loose ends are cleaned up.

⌘ Our teacher's senior students are to understand enough to teach others or to write a thorough paper on math concepts. This is something she requires of all her students in the school's highest-level course. What she teaches is not equations, or certainly not a list of rules learned by rote, but something far greater; in her class the knowledge she imparts to us transcends the normal conception of math and becomes something far greater: Understanding.

⌘ Our teacher is perhaps most notable for her in-class curriculum, the work that affects all highly motivated mathematicians at our school, not just the competitors. "Elements of Mathematics" which, in addition to standard algebra and geometry, teaches probability, number theory, field theory, set theory, and formal logic. In this class she teaches how to write proofs and how to solve problems, skills that, in high school, are generally reserved for the most elite competitors. Instead, through her, any skilled and motivated student can learn advanced techniques for tackling problems, techniques which are powerful and versatile, and which extend beyond the problem at hand, for they are heuristic, not algorithmic. She opens the techniques of the "great"

to students who would otherwise be merely "good".

- Our teacher is a math teacher who is not satisfied with merely presenting to his students what the textbook says; he seeks to instill a thorough understanding of every topic in us, and to do so, he devotes a large amount of time to writing solutions to hard problems, tutorials or difficult topics, and to integrating the use of technology in his pre-calculus and calculus course. We remember the class's amazement at seeing endless rows of math files on his laptop that he wrote during his teaching career of more than thirty years. We still remember how the beautiful graphs of what were boring equations fascinated us and deepened our understanding of conics.
- When teaching a course in mathematics, our teacher draws from sources that he has gathered from all corners of the world...He picks only the sources that teach the material in the best way, and finds them through exchange students and the Internet.
- He works harder than anyone else to make sure that math competitions are open to any one and he is also very helpful to those who are dedicated to the competitions.
- Always ready with an open door, a friendly smile and free snacks.

General comments:

- From Eugene, Oregon to Vancouver, British Columbia is about an 8-hour drive, making it both slightly shorter and less fun than an USAMO exam. As the coach of both our state and middle school MathCounts teams, she drove the eight of us up there for a regional competition in 2001. After getting a special bus driver's license and renting a small bus, she was rewarded with eight hours of listening to us play Mafia.
- Our teacher contributes to the team atmosphere that any good math team possesses. She spearheaded the effort to purchase math team T-shirts, the first time in many years that our team has made that decision. She takes pictures of the team at competitions and victories. And she organizes frequent pizza parties to, let's just say, fuel our mathematical efforts. A math team can be successful but boring. Thanks to her, ours is successful and awesome.
- She organized meetings with a nearby University professor, contacted contest organizers to register our team and provided the all important communications center for our team, sending out announcements, collecting permission forms and making sure that our achievements were recognized by the school. Since then she has spent countless weekends driving us to math competitions around the state and she has spent the weeks inbetween hunting us down to secure registration information and permission slips.
- He volunteered his time and energy to support students' extra-curricular activities, even working hands-on with the robotics team despite having to walk with a cane. His selflessness and determination in spite of the obstacles he faced continue to be a motivation for our success no matter how difficult the task.

Calendar

The following pages contain a 2008-2009 school year calendar, for your use. Take the calendar, mark your own school holidays, etc. When you know the days the Math Club will meet, the dates of math competitions, Math Counts, etc., you can add those to the calendar and post a copy in your room, and elsewhere, as reminders for the club.

Below we have taken each week, and made suggestions for Math Club activities for each week. Take your school calendar and mix-and-match up activities based on the amount of time you will have, and what is being covered in the students classrooms

For the week beginning:

August, 2008

31 -- Organizational Meeting

September, 2008

7 -- Favorite brain teasers

14 -- "Easy" Practice Questions & Group Discussion

21 -- September Mini Quiz

28 -- "Algebra" Practice Questions & Group Discussion

October, 2008

5 -- "Algebra" Mini quiz

12 -- Discussion of answers to "Algebra" Mini quiz

19 -- October Mini Quiz

26 -- "Geometry" Practice Questions & Group Discussion

November, 2008

2 -- Discussion of elections & statistics, related problems

9 -- AMC 8 Contest on Tuesday, Nov. 13 &/or discuss last years AMC contest questions.

16 -- November Mini Quiz

23 -- *Break*

30 -- "Problem Solving" Practice Questions & Group Discussion

December, 2008

7 -- December Mini Quiz

14 -- Holiday Party

21, 38 -- *Break,* might try a Reunion party with former students who are home for the holidays.

January, 2009

4 -- Practice Questions & Group Discussion

11 -- Winter Snowballs - Problems with multiple steps- have each student solve 1 step of a different problem and then pass it on to the next student to solve the next step, then to the next student, etc.

18 -- January Mini Quiz

25 -- www scavenger hunt

February, 2009

1 -- February Mini Quiz

8 -- AMC 10A or AMC 12A on Tuesday, Feb. 12 or Practice Questions & Group Discussion

15 -- Discuss & rework AMC 10A/12A Questions

22 -- AMC 10B or AMC 12B on Wednesday, Feb 27 or Practice Questions & Group Discussion

March, 2009

1 -- Discuss & rework AMC 10B/12B Questions

8 -- Pi Party

15 -- AIME I on Tuesday, March 18 or Practice Questions & Group Discussion

22 -- March Mini Quiz

29 -- AIME II on Wednesday, Apr. 2 or Practice Questions & Group Discussion

April, 2009

5 -- Practice Questions & Group Discussion

12 -- Assign former USAMO Questions to groups, have them present the solutions to the group

19 -- April Mini Quiz

26 -- USAMO on Tuesday and Wednesday, April 29 & 30

May, 2009

3 -- May Basket - basket of candy with math problems wrapped around each one - you solve that problem and you can get another candy/problem

10 -- Practice Questions & Group Discussion

17 -- May Mini Quiz

24 -- Math Aawards Ceremony/Banquet

September

M	1	*Marin Mersenne*	*Died*	*1648*	**Labor Day**
T	2	*William Rowan Hamilton*	*Died*	*1865*	**Ramadan begins** *(Islamic)*
W	3	*Solomon Lefschetz*	*Born*	*1884*	
T	4	*Heinrich Bruns*	*Born*	*1848*	
F	5	*Jean Montucla*	*Born*	*1725*	
S	6	*John E Littlewood*	*Died*	*1977*	
S	7	*George Pólya*	*Died*	*1985*	**GrandParents Day**
M	8	*Joseph Liouville*	*Died*	*1882*	
T	9	*Frank Morley*	*Born*	*1860*	
W	10	*Gabrielle du Châtelet*	*Died*	*1749*	
T	11	*Felice Casorati*	*Died*	*1890*	
F	12	*Haskell Curry*	*Born*	*1900*	
S	13	*Constantin Carathéodory*	*Born*	*1873*	
S	14	*Giovanni Cassini*	*Died*	*1712*	
M	15	*Abu Rayhan al-Biruni*	*Born*	*973*	
T	16	*James Jeans*	*Died*	*1946*	
W	17	*Bernhard Riemann*	*Born*	*1826*	
T	18	*Adrien-Marie Legendre*	*Born*	*1752*	
F	19	*Gaspard-Gustave de Coriolis*	*Died*	*1843*	**Talk like a Pirate Day**
S	20	*Moritz Pasch*	*Died*	*1930*	
S	21	*Girolamo Cardano*	*Died*	*1576*	
M	22	*Michael Faraday*	*Born*	*1791*	**Autumnal Equinox**
T	23	*William Wallace*	*Born*	*1768*	
W	24	*Max Noether*	*Born*	*1844*	
T	25	*Johann Lambert*	*Died*	*1777*	
F	26	*Hermann Grassmann*	*Died*	*1877*	**Native American Day**
S	27	*Hans Hahn*	*Born*	*1879*	
S	28	*Pierre de Maupertuis*	*Born*	*1698*	
M	29	*Friedrich Engel*	*Died*	*1941*	**Rosh Hashanah** *(Jewish)*
T	30	*Ernst Hellinger*	*Born*	*1883*	

2008

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AMC 10 / AMC 12 Math Club 2008-2009

October

W	1	John Campbell	Died	1924		
T	2	Arthur Erdélyi	Born	1908		Eid ul-Fitr *(Islamic)*
F	3	Edouard Lucas	Died	1891		
S	4	Max Planck	Died	1947		
S	5	Bernard Bolzano	Born	1781		
M	6	Richard Dedekind	Born	1831		
T	7	Niels Bohr	Born	1885		
W	8	Brian Hartley	Died	1994		
T	9	Johann Andrea von Segner	Born	1704		Yom Kippur *(Jewish)*
F	10	Georgii Pfeiffer	Died	1946		
S	11	Alfréd Haar	Born	1885		
S	12	Piero della Francesca	Died	1492		
M	13	Kurt Reidemeister	Born	1893	Columbus Day Thanksgiving (Canada)	
T	14	Jules Richard	Died	1956		Sukkot *(Jewish)*
W	15	Evangelista Torricelli	Died	1608		
T	16	William Gosset	Died	1937	National Boss's Day	
F	17	Jacques Hadamard	Died	1963		
S	18	Charles Babbage	Died	1871	Sweetest Day	
S	19	Subrahmanyan Chandrasekhar	Born	1910		
M	20	Christopher Wren	Born	1632	AMC 8 - Last Day Stage 1 Registration	
T	21	Eduard Heine	Died	1881		
W	22	Reinhold Baer	Died	1979		
T	23	John Greenlees Semple	Died	1985		
F	24	Wilhelm Weber	Born	1804	United Nations Day	
S	25	Evariste Galois	Born	1811		
S	26	Georg Frobenius	Born	1849		
M	27	Ulugh Beg	Died	1449		
T	28	John Wallis	Died	1703		
W	29	Jean d'Alembert	Died	1783		
T	30	Harold Davenport	Born	1907		
F	31	Karl Weierstrass	Born	1815	Halloween	

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AMC 8 Math Club 2008-2009

November

S	1	Giambattista della Porta	Born	1535	
S	2	George Boole	Born	1815	Daylight Savings Time ends
M	3	George Chrystal	Died	1911	AMC 8 - Last Day Stage 2 Registration (with expedited Shipping)
T	4	Kurt Hirsch	Died	1986	Election Day
W	5	James Clerk Maxwell	Died	1879	
T	6	Alexander Weinstein	Died	1979	
F	7	Thomas De Lagny	Born	1660	
S	8	Gottlob Frege	Born	1848	
S	9	Hermann Weyl	Born	1885	National School Lunch Week
M	10	Elwin Christoffel	Born	1829	
T	11	Henry Whitehead	Born	1904	Veteran's Day / Armistice Day
W	12	John William Strutt(Lord Rayleigh)	Born	1842	AMC 8 - Last Day Final Stage Registration (with overnight shipping)
T	13	Max Dehn	Born	1878	
F	14	Gottfried Leibniz	Died	1716	
S	15	Michel Chasles	Born	1793	
S	16	Eugenio Beltrami	Born	1835	
M	17	August Möbius	Born	1790	
T	18	Adolf Hurwitz	Died	1919	AMC 8 MIDDLE SCHOOL CONTEST
W	19	Heinz Hopf	Born	1894	*window days*
T	20	Benoit Mandelbrot	Born	1924	
F	21	Francesco Tricomi	Died	1978	
S	22	Arthur Eddington	Died	1944	
S	23	Isaac Todhunter	Born	1820	
M	24	Gerhard Gentzen	Born	1909	
T	25	Edouard Goursat	Died	1936	
W	26	Ruth Moufang	Died	1977	
T	27	Abraham de Moivre	Died	1754	Thanksgiving Day - (U.S.)
F	28	Eduard Helly	Died	1943	
S	29	Mary Somerville	Died	1872	
S	30	Bonaventura Cavalieri	Died	1647	

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December

M	1	Nikolai Ivanovich Lobachevskii	Born	1792		
T	2	L E J Brouwer	Died	1966		
W	3	Felix Bernstein	Died	1956		
T	4	Thomas Hobbes	Died	1679		
F	5	Paul Painlevé	Born	1863		
S	6	Walther von Dyck	Born	1856		
S	7	Luigi Cremona	Born	1830	Pearl Harbor Day	
M	8	Jacques Hadamard	Born	1865		
T	9	Grace Hopper	Born	1906		Eid al-Adha *(Islamic)*
W	10	Carl Jacobi	Born	1804		
T	11	Max Born	Born	1882		
F	12	Ludwig Sylow	Born	1832		
S	13	Niccolo Fontana Tartaglia	Died	1557		
S	14	Tycho Brahe	Born	1546		
M	15	János Bolyai	Born	1802	AMC 10/12 A - Last Day Stage 1 Registration	
T	16	Viktor Yakovlevich Bunyakovskii	Born	1804		
W	17	Sophus Lie	Born	1842		
T	18	Bernard Bolzano	Died	1848		
F	19	Helmut Wielandt	Born	1910		
S	20	Emil Artin	Died	1962		
S	21	Paul Gordan	Died	1912	Winter Solstice	Hanukkah *(Jewish, 21-28)*
M	22	Otto Hölder	Born	1859		
T	23	Edward Sang	Died	1890		
W	24	Charles Hermite	Born	1822		
T	25	Antoni Zygmund	Born	1900		Christmas Day *(Christian)*
F	26	Charles Babbage	Born	1791	Boxing Day	Kwanzaa *(26-Jan 1)*
S	27	Jacob Bernoulli	Born	1654		
S	28	John von Neumann	Born	1903		
M	29	Brook Taylor	Died	1731		Muharram - New Year *(Islamic)*
T	30	Philip Hall	Died	1982		
W	31	Ludolph Van Ceulen	Died	1610		

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January

T	1	Johann Bernoulli	Died	1748	New Years Day
F	2	George Airy	Died	1892	
S	3	Carle Runge	Died	1927	
S	4	Isaac Newton	Born	1643	
M	5	Camille Jordan	Born	1838	
T	6	Georg Cantor	Died	1918	
W	7	Emile Borel	Born	1871	Ashura *(Islamic)*
T	8	Galileo Galilei	Died	1642	
F	9	Maria Gaetana Agnesi	Died	1799	
S	10	Adrien-Marie Legendre	Died	1833	
S	11	Emanuel Lasker	Died	1941	
M	12	Pierre Fermat	Died	1665	
T	13	Erhard Schmidt	Born	1876	
W	14	Charles Dodgson	Died	1898	
T	15	Ludwig Schläfli	Born	1814	
F	16	Bill Boone	Born	1920	
S	17	Leonard Dickson	Died	1954	
S	18	Charles Dupin	Died	1873	
M	19	Aleksandr Gennadievich Kurosh	Born	1908	Martin L. King Day
T	20	André-Marie Ampère	Born	1775	Inauguration Day
W	21	John Couch Adams	Died	1892	
T	22	Harald Bohr	Died	1951	
F	23	David Hilbert	Born	1862	
S	24	Percy Heawood	Died	1955	
S	25	Joseph-Louis Lagrange	Born	1736	
M	26	Arthur Cayley	Died	1895	AMC 10/12 - Last Day Stage 2 Registration Australia Day / Chinese New Year
T	27	Janos Bolyai	Died	1860	
W	28	Louis Mordell	Born	1888	
T	29	Eduard Kummer	Born	1810	
F	30	Edward Sang	Born	1805	
S	31	Joost Bürgi	Died	1632	

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February

S	1	George Stokes	Died	1903	Super Bowl Sunday
M	2	Jacques Binet	Born	1786	Groundhog Day
T	3	Gaston Julia	Born	1893	AMC 10/12 A - Last Day Final Stage Registration (with overnight shipping)
W	4	Hendrik Lorentz	Died	1928	
T	5	Jean-Marie Duhamel	Born	1797	
F	6	Nicolaus(II) Bernoulli	Born	1695	
S	7	G H Hardy	Born	1877	
S	8	Daniel Bernoulli	Born	1700	Tu B'Shevat (Jewish)
M	9	Lipót Fejér	Born	1880	
T	10	Sofia Kovalevskaya	Died	1891	AMC 10 A / AMC 12 A Contests
W	11	Claude Chevalley	Born	1909	
T	12	Richard Dedekind	Died	1916	
F	13	Lejeune Dirichlet	Born	1805	
S	14	Edmund Landau	Born	1877	Valentines Day
S	15	Galileo Galilei	Born	1564	
M	16	Francis Galton	Born	1822	Presidents Day
T	17	Adolphe Quetelet	Died	1874	
W	18	Leone Alberti	Born	1404	AMC 10/12 B - Last Day Final Stage Registration (with overnight shipping)
T	19	Karl Weierstrass	Died	1897	
F	20	Ludwig Boltzmann	Born	1844	
S	21	Girard Desargues	Born	1591	
S	22	Adolphe Quetelet	Born	1796	
M	23	Gaston Darboux	Died	1917	
T	24	Felix Bernstein	Born	1878	Mardi Gras
W	25	Christopher Wren	Died	1723	AMC 10 B / AMC 12 B Contests
					Ash Wednesday (Christian)
T	26	François Arago	Born	1786	
F	27	John Arbuthnot	Died	1735	
S	28	Joost Bürgi	Born	1552	

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March

S	1	Isaac Todhunter	Died	1884		
M	2	Charles De la Vallée Poussin	Died	1962	Read Across America Day Dr. Suess Birthday	
T	3	Emil Artin	Born	1898		
W	4	Eugène Cosserat	Born	1866		
T	5	Pierre-Simon Laplace	Died	1827		
F	6	Ferdinand von Lindemann	Died	1939		
S	7	Axel Thue	Died	1922		
S	8	George Chrystal	Born	1851	Daylight Saving Time Begins	
M	9	Max Zorn	Died	1993		Purim *(Jewish)* Mawlid al-Nabi *(Islamic)*
T	10	John Playfair	Born	1748		
W	11	Helge von Koch	Died	1924		
T	12	George Berkeley	Born	1685		
F	13	Siegfried Aronhold	Died	1884		
S	14	Albert Einstein	Born	1879		
S	15	James Joseph Sylvester	Died	1897		
M	16	Eduard Heine	Born	1821		
T	17	Daniel Bernoulli	Died	1782	AIME I St. Patrick's Day	
W	18	Augustus De Morgan	Died	1871		
T	19	Jacob Wolfowitz	Born	1910		
F	20	Ludwig Schläfli	Died	1895	Vernal Equinox	
S	21	Joseph Fourier	Born	1768		
S	22	Irving Kaplansky	Born	1917		
M	23	Jurij Vega	Born	1754		
T	24	Marston Morse	Born	1892		
W	25	Christopher Clavius	Born	1538		
T	26	Paul Erdös	Born	1913		
F	27	Karl Pearson	Born	1857		
S	28	Ernst Hellinger	Died	1950		
S	29	Tullio Levi-Civita	Born	1873		
M	30	Stefan Banach	Born	1892		
T	31	René Descartes	Born	1596		

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April

W	1	Marie-Sophie Germain	Born	1776	AIME II All Fools Day
T	2	Paul Cohen	Born	1934	
F	3	Stan Ulam	Died	1909	
S	4	John Napier	Died	1617	
S	5	Joseph Bertrand	Died	1900	Palm Sunday *(Christian)*
M	6	Niels Abel	Died	1829	
T	7	Paul du Bois-Reymond	Died	1889	
W	8	Marshall Stone	Born	1903	
T	9	Elie Cartan	Born	1869	Passover *(Jewish)*
F	10	Ehrenfried Tschirnhaus	Born	1651	Good Friday *(Christian)*
S	11	Andrew Wiles	Born	1953	
S	12	Wolfgang Krull	Died	1971	Easter *(Christian)*
M	13	Francesco Severi	Born	1879	
T	14	Christiaan Huygens	Born	1629	
W	15	Leonardo da Vinci	Born	1452	Income Taxes Due
T	16	Gotthold Eisenstein	Born	1823	
F	17	Arthur Schönflies	Born	1853	
S	18	Lars Ahlfors	Born	1907	
S	19	Evgeny Slutsky	Born	1880	
M	20	Giuseppe Peano	Died	1932	
T	21	Teiji Takagi	Born	1875	
W	22	Otto Hesse	Born	1811	Administrative Professionals Day Earth Day
T	23	Johann Hudde	Born	1628	
F	24	Henry Dudeney	Died	1930	
S	25	Felix Klein	Born	1849	
S	26	Srinivasa Ramanujan	Died	1920	
M	27	Paul Gordan	Born	1837	
T	28	J Willard Gibbs	Died	1903	USAMO Day 1
W	29	Henri Poincaré	Born	1854	USAMO Day 2
T	30	Carl Friedrich Gauss	Born	1777	

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AMC 8 Math Club 2008-2009

May

F	1	Gabriel Lamé	Died	1870	**May Day**
S	2	D'Arcy Thompson	Born	1860	
S	3	Vito Volterra	Born	1860	
M	4	Isaac Barrow	Died	1677	
T	5	Lejeune Dirichlet	Died	1859	**Cinco de Mayo** **National Teachers Day**
W	6	Elie Cartan	Died	1951	
T	7	Alexis Clairaut	Born	1713	
F	8	Henry Whitehead	Died	1960	
S	9	Gaspard Monge	Born	1746	
S	10	Wilhelm Killing	Born	1847	**Mother's Day**
M	11	Richard Feynman	Born	1918	
T	12	Jacques Binet	Died	1856	
W	13	Lazare Carnot	Born	1753	
T	14	Rudolf Lipschitz	Born	1832	
F	15	Brian Hartley	Born	1939	
S	16	Pafnuty Chebyshev	Born	1821	
S	17	Alexis Clairaut	Died	1765	
M	18	Bertrand Russell	Born	1872	
T	19	Joseph Larmor	Died	1942	
W	20	Henry White	Born	1861	
T	21	Edouard Goursat	Born	1858	
F	22	Irmgard Flügge-Lotz	Died	1974	
S	23	Augustin-Louis Cauchy	Died	1857	
S	24	Nicolaus Copernicus	Died	1543	
M	25	Karl Peterson	Born	1828	**Memorial Day**
T	26	Abraham de Moivre	Born	1667	
W	27	Arthur Schönflies	Died	1928	
T	28	Jacopo Riccati	Born	1676	
F	29	Finlay Freundlich	Born	1885	**Shavou'ot** *(Jewish)* **Pentecost** *(Christian)*
S	30	Eugène Catalan	Born	1814	
S	31	Eugène Cosserat	Died	1931	

2009

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AMC 10 / AMC 12 Math Club 2008-2009

June

M	1	Kurt Hensel	Died	1941	
T	2	Otto Schreier	Died	1929	
W	3	Heinz Hopf	Died	1971	
T	4	Eugenio Beltrami	Died	1899	
F	5	John Couch Adams	Born	1819	
S	6	Max Zorn	Born	1906	
S	7	Alan Turing	Died	1954	
M	8	Giovanni Cassini	Born	1625	
T	9	John E Littlewood	Born	1885	
W	10	Luigi Cremona	Died	1903	
T	11	Wilhelm Meyer	Died	1934	
F	12	Zygmunt Janiszewski	Born	1888	
S	13	William Gosset	Born	1876	
S	14	Colin Maclaurin	Died	1746	Flag Day
M	15	Nikolai Chebotaryov	Born	1894	
T	16	Julius Petersen	Born	1839	
W	17	Maurits Escher	Born	1898	
T	18	Charles Weatherburn	Born	1884	
F	19	Blaise Pascal	Born	1623	Juneteenth
S	20	Helena Rasiowa	Born	1917	
S	21	Siméon Poisson	Born	1781	Father's Day Summer Solstice
M	22	Hermann Minkowski	Born	1864	
T	23	Wilhelm Weber	Died	1891	
W	24	Oswald Veblen	Born	1880	
T	25	Cornelius Lanczos	Died	1974	
F	26	William Thomson	Born	1824	
S	27	Max Dehn	Died	1952	
S	28	Henri Lebesgue	Born	1875	
M	29	Witold Hurewicz	Born	1904	
T	30	William Oughtred	Died	1660	

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July

W	1	Jean-Victor Poncelet	Born	1788	Canada Day
T	2	William Burnside	Born	1852	
F	3	Henry Baker	Born	1866	
S	4	Oscar Zariski	Died	1986	Independence Day
S	5	Henry Scheffé	Died	1977	
M	6	Alfred Kempe	Born	1849	
T	7	Gösta Mittag-Leffler	Died	1927	
W	8	Johann Regiomontanus	Died	1476	
T	9	George Darwin	Born	1845	
F	10	Roger Cotes	Born	1682	International Mathematical Olympiad - IMO Bremen, Germany
S	11	Nicole Oresme	Died	1382	July 10-22, 2009
S	12	Ernst Fischer	Born	1875	
M	13	John Dee	Born	1527	
T	14	Augustin Fresnel	Died	1827	Bastille Day
W	15	Stephen Smale	Born	1930	
T	16	Jacob Wolfowitz	Died	1981	
F	17	Wilhelm Lexis	Born	1837	
S	18	Hendrik Lorentz	Born	1853	
S	19	Egor Ivanovich Zolotarev	Died	1878	
M	20	Bernhard Riemann	Died	1866	
T	21	John Leech	Born	1926	
W	22	Wilhelm Bessel	Born	1784	
T	23	Ernest Brown	Died	1938	
F	24	Hans Hahn	Died	1934	
S	25	Johann Benedict Listing	Born	1808	
S	26	Gottlob Frege	Died	1925	Parent's Day
M	27	Johann Bernoulli	Born	1667	
T	28	Gaspard Monge	Died	1818	
W	29	Ronald Fisher	Died	1962	
T	30	Julia B Robinson	Died	1985	
F	31	Ernst Meissel	Born	1826	

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AMC 10 / AMC 12 Math Club 2008-2009

August

S	1	Ivar Bendixson	Born	1861	
S	2	László Kalmár	Died	1976	**Friendship Day**
M	3	Georg Frobenius	Died	1917	
T	4	Otto Hesse	Died	1874	
W	5	Niels Abel	Born	1802	
T	6	Nicolas Malebranche	Born	1638	
F	7	Gabor Szego	Died	1985	
S	8	Paul Dirac	Born	1902	
S	9	John Fields	Died	1932	
M	10	Oswald Veblen	Died	1960	
T	11	Enrico Betti	Died	1892	
W	12	Erwin Schrodinger	Born	1887	
T	13	George Stokes	Born	1819	
F	14	Edmond Laguerre	Died	1886	
S	15	Edward Waring	Died	1798	
S	16	Jacob Bernoulli	Died	1705	
M	17	Ivar Fredholm	Died	1927	
T	18	Brook Taylor	Born	1685	
W	19	Jean Baptiste Delambre	Died	1822	
T	20	Thomas Simpson	Born	1710	
F	21	Edward Copson	Born	1901	
S	22	Edward Cocker	Died	1676	**Ramadan begins** *(Islamic)*
S	23	Charles Augustin Coulomb	Died	1806	
M	24	Karen Uhlenbeck	Born	1942	
T	25	Helmut Hasse	Born	1898	
W	26	Johann Lambert	Born	1728	
T	27	Giuseppe Peano	Born	1858	
F	28	Maxime Bocher	Born	1867	
S	29	Otto Holder	Died	1937	
S	30	Carle Runge	Born	1856	
M	31	Herbert Turnbull	Born	1885	

2009

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AMC 8 Math Club 2008-2009

Practice questions for the AMC 8

AMC 8 Contest questions categories

These tables illustrate the distribution of the various types of questions on the AMC 8 in the last nine years.

NCTM Standards:

	1999	2000	2001	2002	2003	2004	2005	2006	2007	Avg
Algebra	4	3	4	2	1	0	2	3	3	2.44
Analyze change in various contexts			1							
Model and solve contextualized problems using various				1						
Represent and analyze mathematical situations and			1		1				1	
Relate and compare different forms of representation for a				1						
Understand patterns, relations, and functions							2		1	
Use mathematical models to represent and understand				1				2		2
Use proportionality and a basic understanding of			1							
Use symbolic algebra to represent situations and to solve	4		1		1					
Write equivalent forms of equations, inequalities, and										
Data Analysis & Probability	4	1	3	8	3	6	2	1	3	3.44
Compute probabilities for simple compound events, using		1								
Develop and evaluate inferences and predictions that are			2		4		2		1	
Discuss and understand the correspondence between data	1									
Find, use, and interpret measures of center and spread,	1		1		2		1			
Select and use appropriate statistical methods to analyze						2				
Solve problems that arise in mathematics and in other										
Understand and apply basic concepts of probability.				1		2		1		3
Understand and use appropriate terminology to describe				1						
Use proportionality and a basic understanding of probability	2									
Geometry	11	10	7	7	9	4	9	7	7	7.88
Analyze characteristics and properties of 2- and 3- dimen.	1		3		1		2		2	
Apply transformations and use symmetry to analyze math.		1				1		1		2
Describe sizes, positions, and orientations of shapes under			1							
Draw geometric objects with specified properties, such as										
Examine the congruence, similarity, and line or rotational	1		1							1
Precisely describe, classify, and understand relationships	1	1								2
Specify locations and describe spatial relationships using						1				1
Understand relationships among the angles, side lengths,	4	8		1						2
Understand the meaning and effects of arithmetic operations	1									
Use coordinate geometry to represent and examine the			1							
Use geometric models to represent and explain numerical	2									
Use two-dimensional representations of three-dimensional	1				3					
Use visual tools such as networks to represent and solve					1					
Use visualization, spatial reasoning, and geometric model.			1	5	2	1	2		1	
Measurement	0	0	4	0	1	3	1	0	0	1.00
Apply appropriate techniques, tools, and formulas to deter.			1			1				
Develop and use formulas to determine the circumference					1					
Solve simple problems involving rates and derived			1							
Understand measurable attributes of objects and the units,						1	1			
Understand relationships among units and convert from			2							
Use mathematical models to represent and understand						1				
Number & Operations	4	8	4	3	7	7	5	7	11	6.22
Compare and order fractions, decimals, and percents		1								
Compute fluently and make reasonable estimates								1		
Develop, analyze, and explain methods for solving problems		1								
Develop meaning for percents greater than 100 and less than 1			1							
Model and solve contextualized problems using various		1								
Select appropriate methods and tools for computing with	1									
Understand and use ratios and proportions to represent	1		1	1	1	1			1	
Understand meanings of operations and how they relate	1	2		1		2	2	1	3	
Understand numbers, ways of representing numbers,	1	2	1		1	3	3	3	4	
Use factors, multiples, prime factorization, and relatively		1	1		3	1		1	1	
Work flexibly with fractions, decimals, and percents to				1	2			1	2	
Problem Solving	2	3	3	5	4	5	6	7	1	4.00
Analyze change in various contexts										
Apply and adapt a variety of appropriate strategies to	1	1	3	4	4	1	1	4		
Build new mathematical knowledge through problem solving.								1		
Instructional programs from pre-kindergarten through				1						
Monitor and reflect on the process of mathematical problem						1				
Solve problems that arise in mathematics and in other	1	2				3	5	2	1	
TOTALS	25	25	25	25	25	25	25	25	25	25.00

MathWorld.com Classifications:

	1999	2000	2001	2002	2003	2004	2005	2006	2007	Avg
Algebra	1	0	0	0	1	0	1	0	0	0.33
Linear Algebra					1					
Sums	1									
Vector Algebra							1			
Applied Mathematics	0	0	0	0	1	2	1	0	0	0.44
Business					1					
Data Visualization						1				
Game Theory						1				
Optimization							1			
Calculus & Analysis	2	6	1	2	1	1	2	0	3	2.00
Differential Geometry		3					2			
Functions		1							1	
Inequalities									1	
Special Functions	2	2	1	2	1	1			1	
Discrete Mathematics	1	0	6	6	4	4	2	1	1	2.77
Combinatorics	1		1	2	1	3	2		1	
Computer Science				3						
Graph Theory				1	3			1		
Point Latices			5			1				
Foundations of Math	0	0	3	0	1	1	0	0	1	0.66
Logic			1		1	1			1	
Set Theory			2							
Geometry	8	7	2	6	10	3	8	8	8	6.66
Distance			1							
General Geometry		1								
Gweometric Construction	1									
Geometric Similarity			1							
Line Geometry				1			1			
Plane Geometry	4	4		3	6	3	6	5	8	
Projective Geometry					1			2		
Solid Geometry	1			2	2					
Symmetry							1			
Transformations								1		
Trigonometry	2	2			1					
History & Terminology	2	0	0	0	0	0	0	0	0	0.22
Terminology	2									
Number Theory	10	8	10	9	6	11	10	15	9	9.77
Arithmetic	10	7	7	4	3	11	5	9	8	
Congruences				1			2			
Constants										
Diophantine Equations			1	2						
Divisors					1		1			
Factoring			1		1					
Integers			1					1		
Numbers								1		
Number Theoretic Functions					1			1		
Parity							1			
Prime numbers								1		
Rational numbers		1								
Rounding								1		
Sequences							1			
Special Numbers				2				1	1	
Probability & Statistics	1	1	1	2	0	3	1	0	2	1.22
Probability	1	1	1	2		2	1		2	
Rank Statistics						1				
Recreational Mathematics	0	3	2	0	1	0	0	1	1	0.88
Cryptograms		1			1			1		
Folding			2							
Games		1								
Mathematical Records									1	
Puzzles		1								
Totals	25	25	25	25	25	25	25	25	25	25.00

Topical Practice Quizes

There are 9 quizes (mini-contests). Each cover a specific topic:

There are 50 new problem worksheets, available in this 2008 Math Club book.We have listed the problems by subject matter just before the worksheets begin. The questions have been sorted into the following two listings:

NCTM Standards, with divisions:

- Algebra
- Data Analysis & Probability
- Geometry
- Measurement
- Number & Operations
- Problem Solving

and MathWorld.com Classifications, with divisions:

- Algebra
- Applied Math
- Calculus & Analysis
- Discrete Math
- Foundations of Math
- Geometry
- Number Theory
- Probability & Statistics
- Recreational Mathematics

In Appendix IV we have all the problem worksheets from previous Math Club publications listed in these same two categories. These worksheets are available in the problems section on the Math Club web site.

Averages

1. Which of the following sets of whole numbers has the largest average?

   (A) Multiples of 2 between 1 and 101 **(B)** Multiples of 3 between 1 and 101

   (C) Multiples of 4 between 1 and 101 **(D)** Multiples of 5 between 1 and 101

   (E) Multiples of 6 between 1 and 101

2. A fifth number, n, is added to the set of numbers $\{3, 6, 9, 10\}$ to make the mean of the set of five numbers equal to its median. The number of possible values for n is:

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** More than 4

3. The number N is between 9 and 17. The average of 6, 10, and N could be

   (A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

4. The average (arithmetic mean) of 10 different positive whole numbers is 10. The largest possible value of any of these numbers is:

   (A) 10 **(B)** 50 **(C)** 55 **(D)** 90 **(E)** 91

5. Five test scores have a mean (average score) of 90, a median (middle score) of 91, and a mode (most frequent score) of 94. The sum of the two lowest test scores is:

   (A) 170 **(B)** 171 **(C)** 176 **(D)** 177

   (E) Not determined by the information given

6. The arithmetic mean (average) of four numbers is 85. If the largest of these numbers is 97, then the mean of the remaining three numbers is:

   (A) 81.0 **(B)** 82.7 **(C)** 83.0 **(D)** 84.0 **(E)** 84.3

7. The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element in this collection:

   (A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

Counting, II

1. Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

   (A) 22 **(B)** 25 **(C)** 27 **(D)** 28 **(E)** 729

2. Nebraska, the home of the AMC, changed its license plate scheme. Each old license plate consisted of a letter followed by four digits. Each new license plate consists of three letters followed by three digits. By how many times is the number of possible license plates increased?

   (A) $\frac{26}{10}$ **(B)** $\frac{26^2}{10^2}$ **(C)** $\frac{26^2}{10}$ **(D)** $\frac{26^3}{10^3}$ **(E)** $\frac{26^3}{10^2}$

3. A restaurant offers three desserts, and exactly twice as many appetizers as main courses. A dinner consists of an appetizer, a main course, and a dessert. What is the least number of main courses that the restaurant should offer so that a customer could have a different dinner each night in the year 2003, which is not a leap year?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

4. How many distinct four-digit numbers can be formed by rearranging the four digits in 2004?

   (A) 4 **(B)** 6 **(C)** 16 **(D)** 24 **(E)** 81

5. Ms. Hamilton's eighth-grade class wants to participate in the annual three-person-team basketball tournament. Lance, Sally, Joy and Fred are chosen for the team. In how many ways can three starters be chosen?

   (A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

6. Henry's Hamburger Heaven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties, and any collection of condiments. How many different kinds of hamburgers can be ordered?

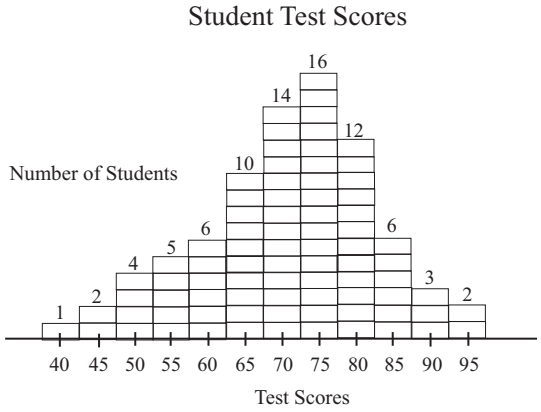
   (A) 24 **(B)** 256 **(C)** 768 **(D)** $40,320$ **(E)** $120,960$

7. The Little Twelve Basketball Conference has two divisions, with six teams in each division. Each team plays each of the other teams in its own division twice and every team in the other division once. How many conference games are scheduled?

   (A) 80 **(B)** 96 **(C)** 100 **(D)** 108 **(E)** 192

Distribution, II

1. Consider this histogram of the scores for 81 students taking a test.



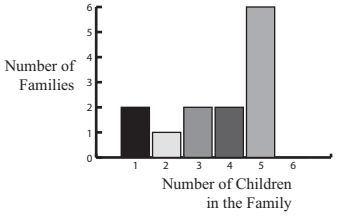
The median is in the interval labeled.

(A) 60 **(B)** 65 **(C)** 70 **(D)** 75 **(E)** 80

2. The arithmetic mean (average) of four numbers is 85. If the largest of these numbers is 97, then the mean of the remaining three numbers is:

(A) 81.0 **(B)** 82.7 **(C)** 83.0 **(D)** 84.0 **(E)** 84.3

3. The graph shows the distribution of the number of children in the families of the students in Ms. Jordan's English class. The median number of children in the family for this distribution is



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

4. There is a set of five positive integers whose average (mean) is 5, whose median is 5, and whose only mode is 8. What is the difference between the largest and smallest integers in the set?

(A) 3 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

5. The average age of the 40 members of a computer science camp is 17 years. There are 20 girls, 15 boys, and 5 adults. If the average age of the girls is 15 and the average age of the boys is 16, what is the average age of the adults?

(A) 26 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 30

Probability, II

1. Diana and Apollo each roll a standard die obtaining a number at random from 1 to 6. What is the probability that Diana's number is larger than Apollo's number?

   (A) $\frac{1}{3}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{17}{36}$ **(E)** $\frac{1}{2}$

2. A point is chosen at random from within a circular region. What is the probability that the point is closer to the center of the region than it is to the boundary of the region?

   (A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

3. A pair of 8-sided dice have sides numbered 1 through 8. Each side has the same probability of landing face up. The probability that the product of the two numbers on the sides that land face up exceeds 36 is:

   (A) $\frac{5}{32}$ **(B)** $\frac{11}{64}$ **(C)** $\frac{3}{16}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{2}$

4. Tamika selects two different numbers at random from the set 8,9,10 and adds them. Carlos takes two different numbers at random from the set 3,5,6 and multiplies them. What is the probability that Tamika's result is greater than Carlos' result?

   (A) $\frac{4}{9}$ **(B)** $\frac{5}{9}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{2}{3}$

5. A complete cycle of a traffic light takes 60 seconds. During each cycle the light is green for 25 seconds, yellow for 5 seconds, and red for 30 seconds. At a randomly chosen time, what is the probability that the light is NOT green?

   (A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{5}{12}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{7}{12}$

6. Keiko tosses one penny and Ephraim tosses two pennies. The probability that Ephraim gets the same number of heads that Keiko gets is:

   (A) $\frac{1}{4}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

Probability/Statistics, II

1. Five test scores have a mean (average score) of 90, a median (middle score) of 91, and a mode (most frequent score) of 94. The sum of the two lowest test scores is:

   (A) 170 **(B)** 171 **(C)** 176 **(D)** 177

   (E) Not determined by the information given

2. The arithmetic mean (average) of four numbers is 85. If the largest of these numbers is 97, then the mean of the remaining three numbers is:

   (A) 81.0 **(B)** 82.7 **(C)** 83.0 **(D)** 84.0 **(E)** 84.3

3. A gumball machine contains 9 red, 7 white, and 8 blue gumballs. the least number of gumballs a person must buy to be SURE of getting four gumballs of the same color is:

   (A) 8 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 18

4. Diana and Apollo each roll a standard die obtaining a number at random from 1 to 6. What is the probability that Diana's number is larger than Apollo's number?

   (A) $\frac{1}{3}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{17}{36}$ **(E)** $\frac{1}{2}$

5. A point is chosen at random from within a circular region. What is the probability that the point is closer to the center of the region than it is to the boundary of the region?

   (A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

6. Tamika selects two different numbers at random from the set 8,9,10 and adds them. Carlos takes two different numbers at random from the set 3,5,6 and multiplies them. What is the probability that Tamika's result is greater than Carlos' result?

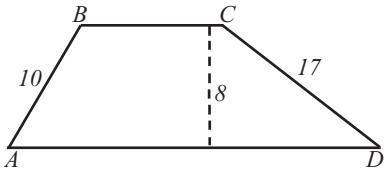
   (A) $\frac{4}{9}$ **(B)** $\frac{5}{9}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{2}{3}$

7. The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element in this collection:

   (A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

Pythagorean, II

1. The area of trapezoid $ABCD$ is 164 cm^2. The altitude is 8 cm, AB is 10 cm, and CD is 17 cm. What is BC, in centimeters?

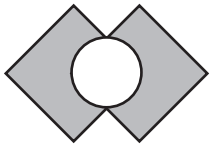


(A) 9 **(B)** 10 **(C)** 12 **(D)** 15 **(E)** 20

2. Many television screens are rectangles that are measured by the length of their diagonals. The ratio of the horizontal length to the height in a standard television screen is 4:3. The horizontal length of a "27-inch" television screen is closest to which of the following?

(A) 20 **(B)** 20.5 **(C)** 21 **(D)** 21.5 **(E)** 22

3. Two 4×4 squares intersect at right angles, bisecting their intersecting sides, as shown. The circle's diameter is the segment between the two points of intersection. What is the area of the shaded region created by removing the circle from the squares?



(A) $16 - 4\pi$ **(B)** $16 - 2\pi$ **(C)** $28 - 4\pi$ **(D)** $28 - 2\pi$ **(E)** $32 - 2\pi$

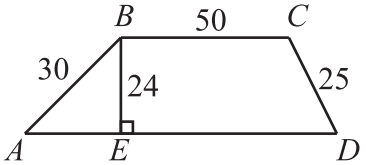
4. Minneapolis-St. Paul International Airport is 8 miles southwest of downtown St. Paul and 10 miles southeast of downtown Minneapolis. Which of the following is closest to the number of miles between downtown St. Paul and Minneapolis?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

5. Bill walks $\frac{1}{2}$ mile south, then $\frac{3}{4}$ mile east, and finally $\frac{1}{2}$ mile south. How many miles is he, in a direct line, from his starting point?

(A) 1 **(B)** $1\frac{1}{4}$ **(C)** $1\frac{1}{2}$ **(D)** $1\frac{3}{4}$ **(E)** 2

6. What is the perimeter of trapezoid $ABCD$?



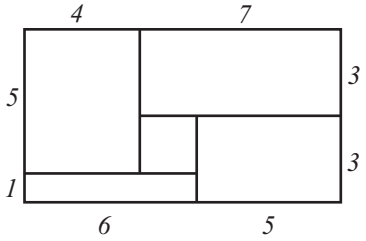
(A) 180 **(B)** 188 **(C)** 196 **(D)** 200 **(E)** 204

Rectangles, II

1. In trapezoid $ABCD$, $\overline{AB}$ and $\overline{CD}$ are perpendicular to $\overline{AD}$, with $AB + CD = BC$, $AB < CD$, and $AD = 7$. What is $AB \cdot CD$?

   (A) 12 **(B)** 12.25 **(C)** 12.5 **(D)** 12.75 **(E)** 13

2. Rose fills each of the rectangular regions of her rectangular flower bed with a different type of flower. The lengths, in feet, of the rectangular regions in her flower bed are as shown in the figure. She plants one flower per square foot in each region. Asters cost \$1 each, begonias \$1.50 each, cannas \$2 each, dahlias \$2.50 each, and Easter lilies \$3 each. What is the least possible cost, in dollars, for her garden?

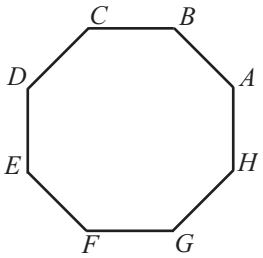


   (A) 108 **(B)** 115 **(C)** 132 **(D)** 144 **(E)** 156

3. Many television screens are rectangles that are measured by the length of their diagonals. The ratio of the horizontal length to the height in a standard television screen is 4:3. The horizontal length of a "27-inch" television screen is closest to which of the following?

   (A) 20 **(B)** 20.5 **(C)** 21 **(D)** 21.5 **(E)** 22

4. A regular octagon $ABCDEFGH$ has an area of one square unit. What is the area of rectangle $ABEF$?



   (A) $1 - \frac{\sqrt{2}}{2}$ **(B)** $\frac{\sqrt{2}}{4}$ **(C)** $\sqrt{2} - 1$ **(D)** $\frac{1}{2}$ **(E)** $\frac{1+\sqrt{2}}{4}$

Sequences, II

1. Terri produces a sequence of positive integers by following three rules. She starts with a positive integer, then applies the appropriate rule to the result, and continues in this fashion.
   Rule 1: If the integer is less than 10, multiply it by 9.
   Rule 2: If the integer is even and greater than 9, divide it by 2.
   Rule 3: If the integer is odd and greater than 9, subtract 5 from it.
   A sample sequence: 23, 18, 9, 81, 76, . . . Find the 98th term of the sequence that begins 98, 49, . . .

   (A) 6 **(B)** 11 **(C)** 22 **(D)** 27 **(E)** 54

2. The Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, . . . start with two 1s and each term afterwards is the sum of its two predecessors. Which of the ten digits is the last to appear in the units position of a number in the Fibonacci sequence?

   (A) 0 **(B)** 4 **(C)** 6 **(D)** 7 **(E)** 9

3. Let $\{a_k\}$ be a sequence of integers such that $a_1 = 1$ and $a_{m+n} = a_m + a_n + mn$ for all positive integers m and n. Then a_{12} is

   (A) 45 **(B)** 56 **(C)** 67 **(D)** 78 **(E)** 89

4. A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

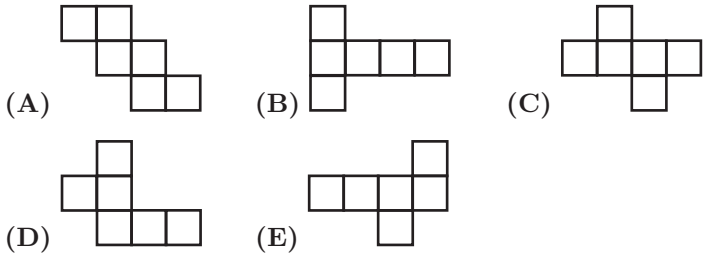
   (A) 1 **(B)** 4 **(C)** 36 **(D)** 48 **(E)** 81

5. In the sequence 2001,2002,2003, . . ., each term after the third is found by subtracting the previous term from the sum of the two terms that precede that term. For example, the fourth term is 2001+2002-2003=2000. What is the 2004th term in this sequence?

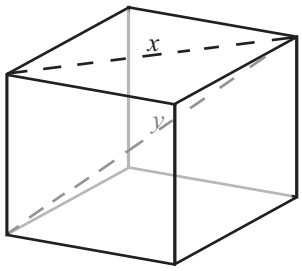
   (A) -2004 **(B)** -2 **(C)** 0 **(D)** 4003 **(E)** 6007

Solid Geometry, II

1. Which pattern of identical squares could NOT be folded along the lines shown to form a cube?

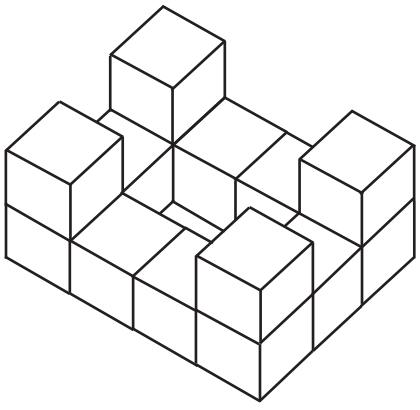


2. A cube has eight vertices and 12 edges. A segment, such as x, which joins two vertices not joined by an edge is called a diagonal. Segment y is also a diagonal. How many diagonals does a cube have?



(A) 6 **(B)** 8 **(C)** 12 **(D)** 14 **(E)** 16

3. Fourteen white cubes are put together to form the figure illustrated. The complete surface of the figure, including the bottom, is painted red. The figure is then separated into individual cubes. How many of the cubes have exactly four red faces?



(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

Answers

Averages, p. 32

1.	(D)	1986 AMC 8	Problem #25	H	12.46
2.	(C)	1988 AMC 8	Problem #21	H	17.65
3.	(B)	1989 AMC 8	Problem #17	MH	38.37
4.	(C)	1991 AMC 8	Problem #19	H	12.53
5.	(B)	1992 AMC 8	Problem #13	H	15.06
6.	(A)	1993 AMC 8	Problem #15	M	47.21
7.	(D)	2002 AMC 10 A	Problem #21	MH	34.20

Counting, II p. 33

1.	(D)	2003 AMC 10 A	Problem #21	H	18.76
2.	(C)	2003 AMC 10 B	Problem #10	MH	33.34
3.	(E)	2003 AMC 10 B	Problem #16	M	48.24
4.	(B)	2004 AMC 8	Problem #2	M	44.49
5.	(B)	2004 AMC 8	Problem #4	M	51.14
6.	(C)	2004 AMC 10 A	Problem #12	E	83.13
7.	(B)	2005 AMC 8	Problem #14	MH	35.54

Distributions, II p.34

1.	(C)	1993 AMC 8	Problem #11	MH	34.63
2.	(A)	1993 AMC 8	Problem #15	M	47.21
3.	(D)	1995 AMC 8	Problem #19	MH	21.80
4.	(D)	1997 AMC 8	Problem #14	MH	21.91
5.	(C)	1999 AMC 8	Problem #13	MH	28.13
6.	(B)	2000 AMC 8	Problem #23	MH	29.51

Probability, II p.35

1.	(B)	1995 AMC 8	Problem #20	H	16.12
2.	(A)	1996 AMC 8	Problem #25	H	16.54
3.	(A)	1997 AMC 8	Problem #20	H	15.76
4.	(A)	1998 AMC 8	Problem #19	H	15.57
5.	(E)	1999 AMC 8	Problem #10	M	46.73
6.	(B)	2000 AMC 8	Problem #22	H	17.78

Probability/Statistics, II p. 36

1.	(B)	1992 AMC 8	Problem #13	H	15.06
2.	(A)	1993 AMC 8	Problem #15	M	47.21
3.	(C)	1994 AMC 8	Problem #21	H	18.81
4.	(B)	1995 AMC 8	Problem #20	H	16.12
5.	(A)	1996 AMC 8	Problem #25	H	16.54
6.	(A)	1998 AMC 8	Problem #19	H	15.57
7.	(D)	2002 AMC 10 A	Problem #21	H	6.26

Pythagorean, II p. 37

1.	(B)	2003 AMC 8	Problem #21	MH	32.62
2.	(D)	2003 AMC 10 B	Problem #6	MH	38.48
3.	(D)	2004 AMC 8	Problem #25	H	13.12
4.	(A)	2004 AMC 10 B	Problem #8	E	98.16
5.	(B)	2005 AMC 8	Problem #7	MH	22.26
6.	(A)	2005 AMC 8	Problem #19	MH	38.79

Rectangles, II p. 38

1.	(B)	2001 AMC 10	Problem #24	[H	7.21
2.	(A)	2003 AMC 10 B	Problem #4	E	82.92
3.	(D)	2003 AMC 10 B	Problem #6	MH	38.48
4.	(D)	2003 AMC 10 B	Problem #23	H	10.86
5.	(C)	2005 AMC 8	Problem #13	MH	24.92

Sequences, II p.39

1.	(D)	1998 AMC 8	Problem #22	MH	23.01
2.	(C)	2000 AMC 10	Problem #6	MH	26.36
3.	(D)	2002 AMC 10 B	Problem #23	H	6.58
4.	(A)	2004 AMC 10 A	Problem #18	MH	32.65
5.	(C)	2004 AMC 10 B	Problem #19	E	84.84

Solid Geometry, II p. 40

1.	(D)	1992 AMC 8	Problem #20	M	44.13
2.	(E)	1997 AMC 8	Problem #17	MH	30.36
3.	(B)	2003 AMC 8	Problem #13	M	54.30
4.	(B)	2003 AMC 8	Problem #15	MH	33.38

Practice Worksheets

NCTM Standards Listing

The folllowing list shows the range of of Practice Questions difficulty, with E = Easy, M-E = Medium-easy, M = Medium, M-H = Medium-hard, and H = Hard, and the corresponding NCTM Standards they fill.

Q#	Page	Diff	NCTM Standard	Definitions
m07-20	63	MH-26.60	Algebra	Represent and analyze mathematical situations and structures using algebraic symbols.
m07-04	47	M-46.66	Algebra	Use mathematical models to represent and understand quantitative relationships.
m07-01	44	E-84.11	Algebra	Use mathematical models to represent and understand quantitative relationships.
m99-18	86	MH-27.12	Algebra	Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
m99-19	87	MH-35.91	Algebra	Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
m99-17	85	M-42.22	Algebra	Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
m99-16	84	M-44.75	Algebra	Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
m99-13	81	MH-28.13	Data Analysis & Probability	Find, use, and interpret measures of center and spread, including mean and interquartile range.
m07-25	68	H-12.89	Data Analysis & Probability	Understand and apply basic concepts of probability.
m07-24	67	H-17.24	Data Analysis & Probability	Understand and apply basic concepts of probability.
m99-12	80	M-44.16	Data Analysis & Probability	Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.
m99-04	72	E-92.30	Data Analysis & Probability	Discuss and understand the correspondence between data sets and their graphical representations.
m99-10	78	M-46.73	Data Analysis & Probability	Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.
m99-20	88	M-40.59	Geometry	Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
m99-05	73	MH-34.76	Geometry	Draw geometric objects with speci¯ed properties, such as side lengths or angle measures.
m07-11	54	MH-35.82	Geometry	Examine the congruence, similarity, and line or rotational symmetry of objects using transformations.
m07-23	66	MH-24.48	Geometry	Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
m99-02	70	M-45.79	Geometry	Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
m07-08	51	M-55.65	Geometry	Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
m07-16	59	MH-25.94	Geometry	Specify locations and describe spatial relationships using coordinate geometry and other representational systems.
m07-14	57	H-17.20	Geometry	Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
m99-23	91	H-17.98	Geometry	Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
m99-25	93	H-18.36	Geometry	Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
m99-21	89	MH-24.66	Geometry	Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
m99-14	82	MH-25.65	Geometry	Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
m07-12	55	M-44.14	Geometry	Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
m99-24	92	MH-21.10	Geometry	Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers.
m99-07	75	MH-20.30	Geometry	Use geometric models to represent and explain numerical and algebraic relationships.
m99-09	77	M-54.17	Geometry	Use geometric models to represent and explain numerical and algebraic relationships.
m99-08	76	ME-65.38	Geometry	Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.
m07-22	65	MH-22.95	Geometry	Use visualization, spatial reasoning, and geometric modeling to solve problems.
m99-22	90	MH-32.34	Number & Operations	Understand and use ratios and proportions to represent quantitative relationships.
m07-02	45	ME-65.97	Number & Operations	Understand and use ratios and proportions to represent quantitative relationships.
m07-10	53	MH-22.12	Number & Operations	Understand meanings of operations and how they relate to one another.
m07-15	58	M-42.74	Number & Operations	Understand meanings of operations and how they relate to one another.
m07-18	61	M-45.39	Number & Operations	Understand meanings of operations and how they relate to one another.
m99-01	69	E-80.22	Number & Operations	Understand meanings of operations and how they relate to one another.
m99-15	83	MH-23.36	Number & Operations	Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
m07-13	56	H-16.25	Number & Operations	Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
m07-19	62	MH-22.97	Number & Operations	Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
m07-07	50	M-57.88	Number & Operations	Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
m07-09	52	ME-76.94	Number & Operations	Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
m07-03	46	ME-63.43	Number & Operations	Use factors, multiples, prime factorization, and relatively prime numbers to solve problems.
m07-06	49	MH-28.83	Number & Operations	Work flexibly with fractions, decimals, and percents to solve problems.
m07-17	60	MH-33.75	Number & Operations	Work flexibly with fractions, decimals, and percents to solve problems.
m99-03	71	ME-64.72	Number & Operations	Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods.
m07-21	64	H-12.47	Probability	Understand and apply basic concepts of probability.
m99-06	74	E-86.55	Problem Solving	Apply and adapt a variety of appropriate strategies to solve problems.
m99-11	79	MH-35.64	Problem Solving	Solve problems that arise in mathematics and in other contexts.
m07-05	48	E-84.94	Problem Solving	Solve problems that arise in mathematics and in other contexts.

MathWorld.com Classifications

The following list shows the Mathworld.com Classifications for the various questions, with the corresponding NCTM Standard category and the difficulty level.

Q#	Page	Diff-%	Mathworld.com Classification
m99-11	79	MH-35.64	Algebra > Sums
m07-10	53	MH-22.12	Calculus & Analysis > Functions > Unary Operation
m07-15	58	M-42.74	Calculus & Analysis > Inequalities > Inequality
m99-13	81	MH-28.13	Calculus & Analysis > Special Functions > Means > Arithmetic Mean
m07-07	50	M-57.88	Calculus & Analysis > Special Functions > Means > Arithmetic Mean
m99-24	92	MH-21.10	Calculus & Analysis > Special Functions > Powers
m07-04	47	M-46.66	Discrete Mathematics > Combinatorics > General Combinatorics > Counting Generalized Principle
m99-15	83	MH-23.36	Discrete Mathematics > Combinatorics > Permutations > Combination
m07-13	56	H-16.25	Foundations of Mathematics > Logic > General Logic > Venn Diagram
m99-20	88	M-40.59	Geometry > Geometric Construction
m07-25	68	H-12.89	Geometry > Plane Geometry > Circles > Circle
m07-16	59	MH-25.94	Geometry > Plane Geometry > Circles > Circle
m99-23	91	H-17.98	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area
m99-25	93	H-18.36	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area
m99-05	73	MH-34.76	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area
m07-12	55	M-44.14	Geometry > Plane Geometry > Polygons > Hexagram
m99-14	82	MH-25.65	Geometry > Plane Geometry > Quadrilaterals > Isosceles Trapezoid
m07-08	51	M-55.65	Geometry > Plane Geometry > Quadrilaterals > Trapezoid
m99-05	73	MH-34.76	Geometry > Plane Geometry > Squares
m07-22	65	MH-22.95	Geometry > Plane Geometry > Squares > Square
m07-23	66	MH-24.48	Geometry > Plane Geometry > Squares > Square
m07-11	54	MH-35.82	Geometry > Plane Geometry > Tiling > Tessellation
m07-14	57	H-17.20	Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Isosceles Triangle
m07-23	66	MH-24.48	Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Triangle
m99-21	89	MH-24.66	Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Triangle
m99-20	88	M-40.59	Geometry > Solid Geometry > Polyhedra > Cubes
m99-08	76	ME-65.38	Geometry > Solid Geometry > Polyhedra > Cubes
m99-21	89	MH-24.66	Geometry > Trigonometry > Angles
m99-02	70	M-45.79	Geometry > Trigonometry > Angles
m99-04	72	E-92.30	History & Terminology > Terminology > Diagram
m99-06	74	E-86.55	History & Terminology > Terminology > Order
m99-09	77	M-54.17	Number Theory > Arithmetic > Addition & Subtraction
m07-03	46	ME-63.43	Number Theory > Arithmetic > Addition & Subtraction > Addition
m99-03	71	ME-64.72	Number Theory > Arithmetic > Addition & Subtraction > Addition
m99-01	69	E-80.22	Number Theory > Arithmetic > Addition & Subtraction > Subtraction
m99-07	75	MH-20.30	Number Theory > Arithmetic > Fractions
m99-22	90	MH-32.34	Number Theory > Arithmetic > Fractions
m07-03	46	ME-63.43	Number Theory > Arithmetic > Fractions
m99-03	71	ME-64.72	Number Theory > Arithmetic > Fractions
m07-20	63	MH-26.60	Number Theory > Arithmetic > Fractions > Percent
m99-18	86	MH-27.12	Number Theory > Arithmetic > Fractions > Percent
m07-06	49	MH-28.83	Number Theory > Arithmetic > Fractions > Percent
m07-17	60	MH-33.75	Number Theory > Arithmetic > Fractions > Percent
m99-12	80	M-44.16	Number Theory > Arithmetic > Fractions > Percent
m99-16	84	M-44.75	Number Theory > Arithmetic > Fractions > Percent
m07-02	45	ME-65.97	Number Theory > Arithmetic > Fractions > Ratio
m07-01	44	E-84.11	Number Theory > Arithmetic > General Arithmetic > Arithmetic
m07-05	48	E-84.94	Number Theory > Arithmetic > General Arithmetic > Arithmetic
m99-18	86	MH-27.12	Number Theory > Arithmetic > Multiplication & Division
m99-19	87	MH-35.91	Number Theory > Arithmetic > Multiplication & Division
m99-17	85	M-42.22	Number Theory > Arithmetic > Multiplication & Division
m07-18	61	M-45.39	Number Theory > Arithmetic > Multiplication & Division > Multiplication
m99-24	92	MH-21.10	Number Theory > Arithmetic > Multiplication & Division > Remainder
m07-19	62	MH-22.97	Number Theory > Special Numbers > Figurate Numbers > Square Numbers > Square
m99-10	78	M-46.73	Probability & Statistics > Probability
m07-25	68	H-12.89	Probability & Statistics > Probability > Probability
m07-21	64	H-12.47	Probability & Statistics > Probability > Probability
m07-24	67	H-17.24	Probability & Statistics > Probability > Probability
m07-09	52	ME-76.94	Recreational Mathematics > Mathematical Records > Latin Square

m07-01

Theresa's parents have agreed to buy her tickets to see her favorite band if she spends an average of 10 hours per week helping around the house for 6 weeks. For the first 5 weeks she helps around the house for 8, 11, 7, 12 and 10 hours. How many hours must she work during the final week to earn the tickets?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

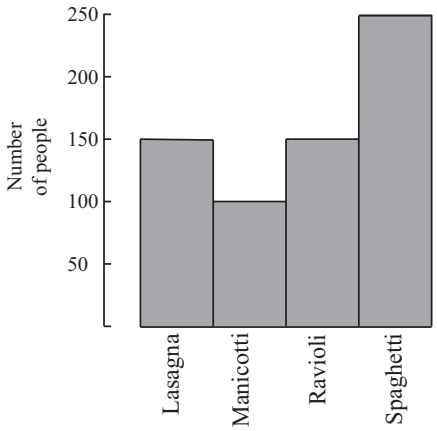
2007 AMC 8, Problem #1—
"Theresa needs to help around the house for a total of $10 \times 6 = 60$ **hours."**

Solution (D) The first 5 weeks Theresa works a total of $8 + 11 + 7 + 12 + 10 = 48$ hours. She has promised to work $6 \times 10 = 60$ hours. She must work $60 - 48 = 12$ hours during the final week.

Difficulty: Easy
NCTM Standard: Algebra Standard for Grades 6–8: use mathematical models to represent and understand quantitative relationships.
Mathworld.com Classification: Number Theory > Arithmetic > General Arithmetic > Arithmetic

m07-02

Six-hundred fifty students were surveyed about their pasta preferences. The choices were lasagna, manicotti, ravioli and spaghetti. The results of the survey are displayed in the bar graph. What is the ratio of the number of students who preferred spaghetti to the number of students who preferred manicotti?



(A) $\frac{2}{5}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{4}$ **(D)** $\frac{5}{3}$ **(E)** $\frac{5}{2}$

2007 AMC 8, Problem #2—
"Represent quantitative relationships with ratios."

Solution (E) The ratio of the number of students who preferred spaghetti to the number of students who preferred manicotti is $\frac{250}{100} = \frac{5}{2}$.

Difficulty: Medium-easy
NCTM Standard: Number and Operations Standard for Grades 6–8: understand and use ratios and proportions to represent quantitative relationships.
Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Ratio

m07-03

What is the sum of the two smallest prime factors of 250?

(A) 2 **(B)** 5 **(C)** 7 **(D)** 10 **(E)** 12

2007 AMC 8, Problem #3—
"Write out the prime factorization of 250."

Solution (C) The prime factorization of 250 is $2 \cdot 5 \cdot 5 \cdot 5$. The sum of 2 and 5 is 7.

Difficulty: Medium-easy
NCTM Standard: Number and Operations Standard for Grades 6–8: use factors, multiples, prime factorization, and relatively prime numbers to solve problems.
Mathworld.com Classification: Number Theory > Prime Numbers > Prime Factorization

m07-04

A haunted house has six windows. In how many ways can Georgie the Ghost enter the house by one window and leave by a different window?

(A) 12 **(B)** 15 **(C)** 18 **(D)** 30 **(E)** 36

2007 AMC 8, Problem #4—
"After Georgie picks the first window, how many choices does he have for picking the second window?"

Solution (D) Georgie has 6 choices for the window in which to enter. After entering, Georgie has 5 choices for the window from which to exit. So altogether there are $6 \times 5 = 30$ different ways for Georgie to enter one window and exit another.

Difficulty: Medium
NCTM Standard: Algebra Standard for Grades 6–8: use mathematical models to represent and understand quantitative relationships.
Mathworld.com Classification: Discrete Mathematics > Combinatorics > General Combinatorics > Counting Generalized Principle

m07-05

Chandler wants to buy a $\$500$ mountain bike. For his birthday, his grandparents send him $\$50$, his aunt sends him $\$35$ and his cousin gives him $\$15$. He earns $\$16$ per week for his paper route. He will use all of his birthday money and all of the money he earns from his paper route. In how many weeks will he be able to buy the mountain bike?

(A) 24 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

2007 AMC 8, Problem #5—
"How many dollars does Chandler have to earn from his paper route?"

Solution (B) For his birthday, Chandler gets $50 + 35 + 15 = 100$ dollars. Therefore, he needs $500 - 100 = 400$ dollars more. It will take Chandler $400 \div 16 = 25$ weeks to earn 400 dollars, so he can buy his bike after 25 weeks.

Difficulty: Easy
NCTM Standard: Problem Solving for Grades 6–8: solve problems that arise in mathematics and in other contexts.
Mathworld.com Classification: Number Theory > Arithmetic > General Arithmetic > Arithmetic

m07-06

The average cost of a long-distance call in the USA in 1985 was 41 cents per minute, and the average cost of a long-distance call in the USA in 2005 was 7 cents per minute. Find the approximate percent decrease in the cost per minute of a long-distance call.

(A) 7 **(B)** 17 **(C)** 34 **(D)** 41 **(E)** 80

2007 AMC 8, Problem #6—
"Percentage decreased $= \frac{\text{price difference}}{\text{old price}}$**."**

Solution (E) The difference in the cost of a long-distance call per minute from 1985 to 2005 was $41 - 7 = 34$ cents. The percent decrease is $100 \times \frac{34}{41} \approx 100 \times \frac{32}{40} = 100 \times \frac{8}{10} = 80\%$.

Difficulty: Medium-hard
NCTM Standard: Number and Operations for Grades 6–8: work flexibly with fractions, decimals, and percents to solve problems.
Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

m07-07

The average age of 5 people in a room is 30 years. An 18-year-old person leaves the room. What is the average age of the four remaining people?

(A) 25 **(B)** 26 **(C)** 29 **(D)** 33 **(E)** 36

2007 AMC 8, Problem #7—
"What is the sum of the ages of the people in the room originally?"

Solution (D) Originally the sum of the ages of the people in the room is $5 \times 30 = 150$. After the 18-year-old leaves, the sum of the ages of the remaining people is $150 - 18 = 132$. So the average age of the four remaining people is $\frac{132}{4} = 33$ years.

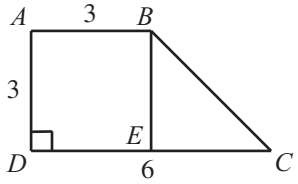
OR

The 18-year-old is 12 years younger than 30, so the four remaining people are an average of $\frac{12}{4} = 3$ years older than 30.

Difficulty: Medium
NCTM Standard: Number and Operations for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.
Mathworld.com Classification: Calculus and Analysis > Special Functions > Means > Arithmetic Mean

m07-08

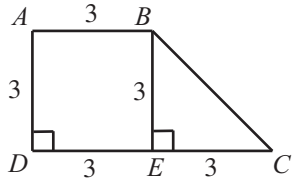
In trapezoid $ABCD$, $\overline{AD}$ is perpendicular to $\overline{DC}$, $AD = AB = 3$, and $DC = 6$. In addition, E is on $\overline{DC}$, and $\overline{BE}$ is parallel to $\overline{AD}$. Find the area of $\triangle BEC$.



(A) 3 **(B)** 4.5 **(C)** 6 **(D)** 9 **(E)** 18

2007 AMC 8, Problem #8—
"Triangle BEC is a right triangle."

Solution (B) Note that $ABED$ is a square with side 3. Subtract DE from DC, to find that $\overline{EC}$, the base of $\triangle BEC$, has length 3. The area of $\triangle BEC$ is $\frac{1}{2} \cdot 3 \cdot 3 = \frac{9}{2} = 4.5$.



OR

The area of the $\triangle BEC$ is the area of the trapezoid $ABCD$ minus the area of the square $ABED$. The area of $\triangle BEC$ is $\frac{1}{2}(3+6)3 - 3^2 = 13.5 - 9 = 4.5$.

Difficulty: Medium
NCTM Standard: Geometry for Grades 6–8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
Mathworld.com Classification: Geometry > Plane Geometry > Quadrilaterals > Trapezoid

m07-09

To complete the grid below, each of the digits 1 through 4 must occur once in each row and once in each column. What number will occupy the lower right-hand square?

1		2	
2	3		
			4

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** cannot be determined

2007 AMC 8, Problem #9—
"The number in the last column of the second row must be 1."

Solution (B) The number in the last column of the second row must be 1 because there are already a 2 and a 3 in the second row and a 4 in the last column. By similar reasoning, the number above the 1 must be 3. So the number in the lower right-hand square must be 2. This is not the only way to find the solution.

1		2	3
2	3		1
			4
			2

The completed square is

1	4	2	3
2	3	4	1
3	2	1	4
4	1	3	2

Difficulty: Medium-easy
NCTM Standard: Number and Operations for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.
Mathworld.com Classification: Recreational Mathematics > Mathematical Records > Latin Square

m07-10

For any positive integer n, define $\boxed{n}$ to be the sum of the positive factors of n. For example, $\boxed{6} = 1 + 2 + 3 + 6 = 12$. Find $\boxed{\boxed{11}}$.

(A) 13 **(B)** 20 **(C)** 24 **(D)** 28 **(E)** 30

2007 AMC 8, Problem #10—
"$\boxed{\mathbf{11}} = 1 + 11 = 12$."

Solution (D) First calculate $\boxed{11} = 1 + 11 = 12$. So

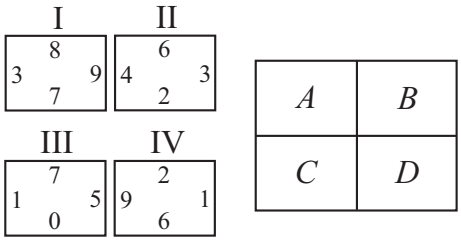
$$\boxed{\boxed{11}} = \boxed{12} = 1 + 2 + 3 + 4 + 6 + 12 = 28$$

.

Difficulty: Medium-hard
NCTM Standard: Number and Operations for Grades 6–8: understand meanings of operations and how they relate to one another.
Mathworld.com Classification: Calculus and Analysis > Functions > Unary Operation

m07-11

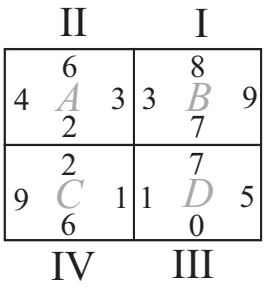
Tiles I, II, III and IV are translated so one tile coincides with each of the rectangles A, B, C and D. In the final arrangement, the two numbers on any side common to two adjacent tiles must be the same. Which of the tiles is translated to Rectangle C?



(A) I **(B)** II **(C)** III **(D)** IV **(E)** cannot be determined

2007 AMC 8, Problem #11—
"Because Tile III has a 0 on the bottom edge and there is no 0 on any other tile, Tile III must be placed on C or D."

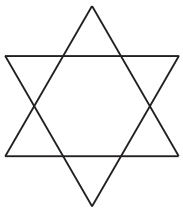
Solution (D) Because Tile III has a 0 on the bottom edge and there is no 0 on any other tile, Tile III must be placed on C or D. Because Tile III has a 5 on the right edge and there is no 5 on any other tile, Tile III must be placed on the right, on D. Because Tile III has a 1 on the left edge and only Tile IV has a 1 on the right edge, Tile IV must be placed to the left of Tile III, that is, on C.



Difficulty: Medium-hard
NCTM Standard: Geometry for Grades 6–8: examine the congruence, similarity, and line or rotational symmetry of objects using transformations.
Mathworld.com Classification: Geometry > Plane Geometry > Tiling > Tessellation

m07-12

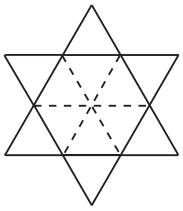
A unit hexagram is composed of a regular hexagon of side length 1 and its 6 equilateral triangular extensions, as shown in the diagram. What is the ratio of the area of the extensions to the area of the original hexagon?



(A) $1:1$ **(B)** $6:5$ **(C)** $3:2$ **(D)** $2:1$ **(E)** $3:1$

2007 AMC 8, Problem #12—
"Use diagonals to cut the hexagon into 6 congruent triangles."

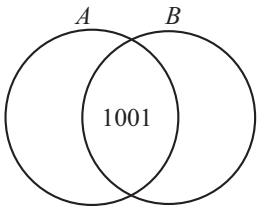
Solution (A) Use diagonals to cut the hexagon into 6 congruent triangles. Because each exterior triangle is also equilateral and shares an edge with an internal triangle, each exterior triangle is congruent to each interior triangle. Therefore, the ratio of the area of the extensions to the area of the hexagon is 1:1.



Difficulty: Medium
NCTM Standard: Geometry for Grades 6–8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
Mathworld.com Classification: Geometry > Plane Geometry > Polygons > Hexagram

m07-13

Sets A and B, shown in the Venn diagram, have the same number of elements. Their union has 2007 elements and their intersection has 1001 elements. Find the number of elements in A.



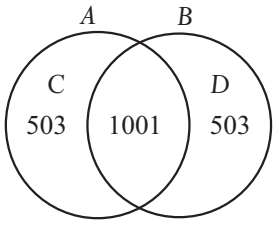
(A) 503 **(B)** 1006 **(C)** 1504 **(D)** 1507 **(E)** 1510

2007 AMC 8, Problem #13—
"The sum of elements in A and B is $2007 + 1001 = 3008$."

Solution (C) Let C denote the set of elements that are in A but not in B. Let D denote the set of elements that are in B but not in A. Because sets A and B have the same number of elements, the number of elements in C is the same as the number of elements in D. This number is half the number of elements in the union of A and B minus the intersection of A and B. That is, the number of elements in each of C and D is

$$\frac{1}{2}(2007 - 1001) = \frac{1}{2} \cdot 1006 = 503.$$

Adding the number of elements in A and B to the number in A but not in B gives $1001 + 503 = 1504$ elements in A.



OR

Let x be the number of elements each in A and B. Then $2x - 1001 = 2007$, $2x = 3008$ and $x = 1504$.

Difficulty: Hard
NCTM Standard: Number and Operations for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.
Mathworld.com Classification: Foundations of Mathematics > Logic > General Logic > Venn Diagram

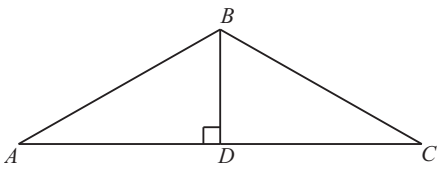
m07-14

The base of isosceles $\triangle ABC$ is 24 and its area is 60. What is the length of one of the congruent sides?

(A) 5 **(B)** 8 **(C)** 13 **(D)** 14 **(E)** 18

2007 AMC 8, Problem #14—
"Draw $\overline{BD}$ to be the altitude from B to $\overline{AC}$."

Solution (C) Let $\overline{BD}$ be the altitude from B to $\overline{AC}$ in $\triangle ABC$.



Then $60 =$ the area of $\triangle ABC = \frac{1}{2} \cdot 24 \cdot BD$, so $BD = 5$. Because $\triangle ABC$ is isosceles, $\triangle ABD$ and $\triangle CBD$ are congruent right triangles. This means that $AD = DC = \dfrac{24}{2} = 12$. Applying the Pythagorean Theorem to $\triangle ABD$ gives

$$AB^2 = 5^2 + 12^2 = 169 = 13^2, \text{ so } AB = 13.$$

Difficulty: Hard

NCTM Standard: Geometry for Grades 6–8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Isosceles Triangle

m07-15

Let a, b and c be numbers with $0 < a < b < c$. Which of the following is impossible?

(A) $a + c < b$ **(B)** $a \cdot b < c$ **(C)** $a + b < c$ **(D)** $a \cdot c < b$ **(E)** $\frac{b}{c} = a$

2007 AMC 8, Problem #15—
"Note that a, b, c are all positive numbers."

Solution (A) Because $b < c$ and $0 < a$, adding corresponding sides of the inequalities gives $b < a+c$, so (A) is impossible. To see that the other choices are possible, consider the following choices for a, b, and c:

(B) and (C): $a = 1$, $b = 2$, and $c = 4$;

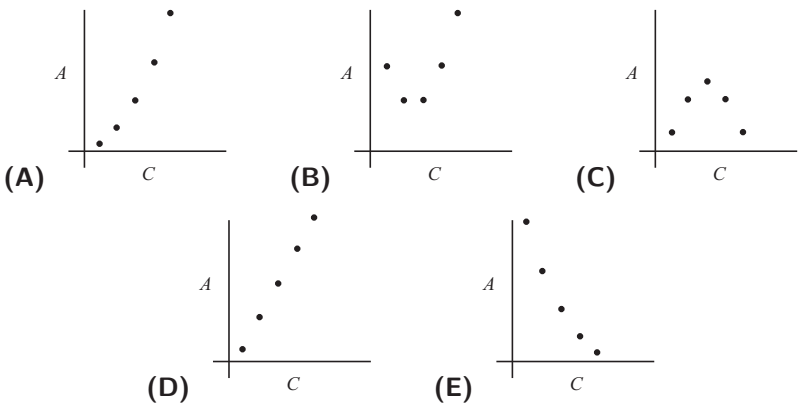
(D): $a = \frac{1}{3}$, $b = 1$, and $c = 2$;

(E): $a = \frac{1}{2}$, $b = 1$, and $c = 2$.

Difficulty: Medium
NCTM Standard: Number and Operations for Grades 6–8: understand meanings of operations and how they relate to one another.
Mathworld.com Classification: Calculus and Analysis > Inequalities > Inequality

m07-16

Amanda Reckonwith draws five circles with radii 1, 2, 3, 4 and 5. Then for each circle she plots the point (C, A), where C is its circumference and A is its area. Which of the following could be her graph?



2007 AMC 8, Problem #16—
"Find the circumferences and areas for the five circles."

Solution (A) The circumferences of circles with radii 1 through 5 are 2π, 4π, 6π, 8π and 10π, respectively. Their areas are, respectively, $\pi, 4\pi$, 9π, 16π and 25π. The points $(2\pi, \pi)$, $(4\pi, 4\pi)$, $(6\pi, 9\pi)$, $(8\pi, 16\pi)$ and $(10\pi, 25\pi)$ are graphed in **(A)**. It is the only graph of an increasing quadratic function, called a parabola .

Difficulty: Medium-hard
NCTM Standard: Geometry for Grades 6–8: specify locations and describe spatial relationships using coordinate geometry and other representational systems.
Mathworld.com Classification: Geometry > Plane Geometry > Circles > Circle

m07-17

A mixture of 30 liters of paint is 25% red tint, 30% yellow tint and 45% water. Five liters of yellow tint are added to the original mixture. What is the percent of yellow tint in the new mixture?

(A) 25 **(B)** 35 **(C)** 40 **(D)** 45 **(E)** 50

2007 AMC 8, Problem #17—
"There are $0.30(30) = 9$ liters of yellow tint in the original 30-liter mixture."

Solution (C) There are $0.30(30) = 9$ liters of yellow tint in the original 30-liter mixture. After adding 5 liters of yellow tint, 14 of the 35 liters of the new mixture are yellow tint. The percent of yellow tint in the new mixture is $100 \times \frac{14}{35} = 100 \times \frac{2}{5}$ or 40%.

Difficulty: Medium-hard
NCTM Standard: Number and Operations for Grades 6–8: work flexibly with fractions, decimals, and percents to solve problems.
Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

m07-18

The product of the two 99-digit numbers

$$303{,}030{,}303{,}\ldots{,}030{,}303 \quad \text{and} \quad 505{,}050{,}505{,}\ldots{,}050{,}505$$

has thousands digit A and units digit B. What is the sum of A and B?

(A) 3 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 10

2007 AMC 8, Problem #18—
"To find A and B, it is sufficient to consider only $303 \cdot 505$, because 0 is in the thousands place in both factors."

Solution (D) To find A and B, it is sufficient to consider only $303 \cdot 505$, because 0 is in the thousands place in both factors.

$$\begin{array}{r} \cdots 303 \\ \times \quad \cdots 505 \\ \hline \cdots 1515 \\ \cdots 1500 \\ \hline \cdots 3015 \end{array}$$

So $A = 3$ and $B = 5$, and the sum is $A + B = 3 + 5 = 8$.

Difficulty: Medium

NCTM Standard: Number and Operations for Grades 6–8: understand meanings of operations and how they relate to one another.

Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division > Multiplication

m07-19

Pick two consecutive positive integers whose sum is less than 100. Square both of those integers and then find the difference of the squares. Which of the following could be the difference?

(A) 2 **(B)** 64 **(C)** 79 **(D)** 96 **(E)** 131

2007 AMC 8, Problem #19—

"One of the squares of two consecutive integers is odd and the other is even, so their difference must be odd."

Solution (C) One of the squares of two consecutive integers is odd and the other is even, so their difference must be odd. This eliminates A, B and D. The largest consecutive integers that have a sum less than 100 are 49 and 50, whose squares are 2401 and 2500, with a difference of 99. Because the difference of the squares of consecutive positive integers increases as the integers increase, the difference cannot be 131. The difference between the squares of 40 and 39 is 79.

OR

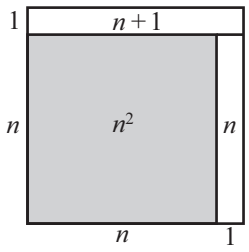
Let the consecutive integers be n and $n+1$, with $n \leq 49$. Then

$$(n+1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1 = n + (n+1).$$

That means the difference of the squares is an odd number. Therefore, the difference is an odd number less than or equal to $49 + (49+1) = 99$, and choice C is the only possible answer. The sum of $n = 39$ and $n + 1 = 40$ is 79.

Note: The difference of the squares of any two consecutive integers is not only odd but also the sum of the two consecutive integers. Every positive odd integer greater than 1 and less than 100 could be the answer.

Seen in geometric terms, $(n+1)^2 - n^2$ looks like



Difficulty: Medium-hard

NCTM Standard: Number and Operations for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Mathworld.com Classification: Number Theory > Special Numbers > Figurate Numbers > Square Numbers > Square

m07-20

Before district play, the Unicorns had won 45% of their basketball games. During district play, they won six more games and lost two, to finish the season having won half their games. How many games did the Unicorns play in all?

(A) 48 **(B)** 50 **(C)** 52 **(D)** 54 **(E)** 60

2007 AMC 8, Problem #20—

"Won half games $= 50\% = \frac{45\% \cdot \textbf{non-district games} + 6}{\textbf{non-district games} + 6 + 2}$ **."**

Solution (A) Because 45% is the same as the simplified fraction $\frac{9}{20}$, the Unicorns won 9 games for each 20 games they played. This means that the Unicorns must have played some multiple of 20 games before district play. The table shows the possibilities that satisfy the conditions in the problem.

Before District Play			**After District Play**		
Games Played	Games Won	Games Lost	Games Played	Games Won	Games Lost
20	9	11	28	15	13
40	**18**	**22**	**48**	**24**	**24**
60	27	33	68	33	35
80	36	44	88	42	46
...	...	...	...	...	...

Only when the Unicorns played 40 games before district play do they finish winning half of their games. So the Unicorns played $24 + 24 = 48$ games.

OR

Let n be the number of Unicorn games before district play. Then $0.45n+6 = 0.5(n+8)$. Solving for n yields

$$\begin{aligned} 0.45n + 6 &= 0.5n + 4, \\ 2 &= 0.05n, \\ 40 &= n. \end{aligned}$$

So the total number of games is $40 + 8 = 48$.

Difficulty: Medium-hard

NCTM Standard: Algebra for Grades 6–8: represent and analyze mathematical situations and structures using algebraic symbols.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

m07-21

Two cards are dealt from a deck of four red cards labeled A, B, C, D and four green cards labeled A, B, C, D. A winning pair is two of the same color or two of the same letter. What is the probability of drawing a winning pair?

(A) $\frac{2}{7}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{5}{8}$

2007 AMC 8, Problem #21—
"After the first card is dealt, there are seven left. How many of the remaining cards are winners?"

Solution (D) After the first card is dealt, there are seven left. The three cards with the same color as the initial card are winners and so is the card with the same letter but a different color. That means four of the remaining seven cards form winning pairs with the first card, so the probability of winning is $\frac{4}{7}$.

Difficulty: Hard
NCTM Standard: Probability for Grades 6–8: understand and apply basic concepts of probability.
Mathworld.com Classification: Probability and Statistics > Probability > Probability

m07-22

A lemming sits at a corner of a square with side length 10 meters. The lemming runs 6.2 meters along a diagonal toward the opposite corner. It stops, makes a 90° right turn and runs 2 more meters.

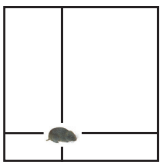
A scientist measures the shortest distance between the lemming and each side of the square. What is the average of these four distances in meters?

(A) 2 **(B)** 4.5 **(C)** 5 **(D)** 6.2 **(E)** 7

2007 AMC 8, Problem #22—

"Wherever the lemming is inside the square, the sum of the distances to the two horizontal sides is 10 meters and the sum of the distances to the two vertical sides is 10 meters."

Solution (C) Wherever the lemming is inside the square, the sum of the distances to the two horizontal sides is 10 meters and the sum of the distances to the two vertical sides is 10 meters. Therefore the sum of all four distances is 20 meters, and the average of the four distances is $\frac{20}{4} = 5$ meters.



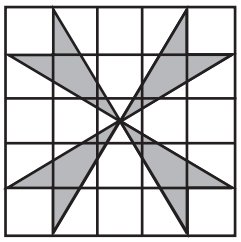
Difficulty: Medium-hard

NCTM Standard: Geometry for Grades 6–8: use visualization, spatial reasoning, and geometric modeling to solve problems.

Mathworld.com Classification: Geometry > Plane Geometry > Squares > Square

m07-23

What is the area of the shaded pinwheel shown in the 5×5 grid?



(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

2007 AMC 8, Problem #23—
"Find the area of the unshaded portion."

Solution (B) Find the area of the unshaded portion of the 5×5 grid, then subtract the unshaded area from the total area of the grid. The unshaded triangle in the middle of the top of the 5×5 grid has a base of 3 and an altitude of $\frac{5}{2}$. The four unshaded triangles have a total area of $4 \times \frac{1}{2} \times 3 \times \frac{5}{2} = 15$ square units. The four corner squares are also unshaded, so the shaded pinwheel has an area of $25 - 15 - 4 = 6$ square units.

Difficulty: Medium-hard

NCTM Standard: Geometry for Grades 6–8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.

Mathworld.com Classification: Geometry > Plane Geometry > Squares > Square
Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Triangle

m07-24

A bag contains four pieces of paper, each labeled with one of the digits 1, 2, 3 or 4, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

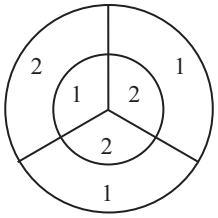
2007 AMC 8, Problem #24—
"A number is a multiple of three when the sum of its digits is a multiple of 3."

Solution (C) A number is a multiple of three when the sum of its digits is a multiple of 3. If the number has three distinct digits drawn from the set $\{1, 2, 3, 4\}$, then the sum of the digits will be a multiple of three when the digits are $\{1, 2, 3\}$ or $\{2, 3, 4\}$. That means the number formed is a multiple of three when, after the three draws, the number remaining in the bag is 1 or 4. The probability of this occurring is $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

Difficulty: Hard
NCTM Standard: Data Analysis and Probability for Grades 6–8: understand and apply basic concepts of probability.
Mathworld.com Classification: Probability and Statistics > Probability > Probability

m07-25

On the dart board shown in the figure, the outer circle has radius 6 and the inner circle has radius 3. Three radii divide each circle into three congruent regions, with point values shown. The probability that a dart will hit a given region is proportional to the area of the region. When two darts hit this board, the score is the sum of the point values in the regions. What is the probability that the score is odd?



(A) $\frac{17}{36}$ **(B)** $\frac{35}{72}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{37}{72}$ **(E)** $\frac{19}{36}$

2007 AMC 8, Problem #25—
"Find the area of each area."

Solution (B) The outer circle has area 36π and the inner circle has area 9π, making the area of the outer ring $36\pi - 9\pi = 27\pi$. So each region in the outer ring has area $\frac{27\pi}{3} = 9\pi$, and each region in the inner circle has area $\frac{9\pi}{3} = 3\pi$. The probability of hitting a given region in the inner circle is $\frac{3\pi}{36\pi} = \frac{1}{12}$, and the probability of hitting a given region in the outer ring is $\frac{9\pi}{36\pi} = \frac{1}{4}$. For the score to be odd, one of the numbers must be 1 and the other number must be 2. The probability of hitting a 1 is

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{12} = \frac{7}{12},$$

and the probability of hitting a 2 is

$$1 - \frac{7}{12} = \frac{5}{12}.$$

Therefore, the probability of hitting a 1 and a 2 in either order is

$$\frac{7}{12} \cdot \frac{5}{12} + \frac{5}{12} \cdot \frac{7}{12} = \frac{70}{144} = \frac{35}{72}.$$

Difficulty: Hard

NCTM Standard: Data Analysis and Probability for Grades 6–8: understand and apply basic concepts of probability.

Mathworld.com Classification: Geometry > Plane Geometry > Circles > Circle
Probability and Statistics > Probability > Probability

m99-01

$(6 \ ? \ 3) + 4 - (2 - 1) = 5$. To make this statement true, the question mark between the 6 and the 3 should be replaced by

(A) $\div$ **(B)** $\times$ **(C)** $+$ **(D)** $-$ **(E)** None of these

1999 AMC 8, Problem #1—
"Simplify first, to find what $(6 \ ? \ 3)$ **should equal."**

Solution

Answer (A):

$$\begin{aligned}
&(6 \ ? \ 3) + 4 - (2 - 1) = 5 && \\
&(6 \ ? \ 3) + 4 - 1 = 5 && (\text{subtract:} 2 - 1 = 1) \\
&(6 \ ? \ 3) + 3 = 5 && (\text{subtract:} 4 - 1 = 3) \\
&(6 \ ? \ 3) = 2 && (\text{subtract 3 from both sides}) \\
&(6 \div 3) = 2 &&
\end{aligned}$$

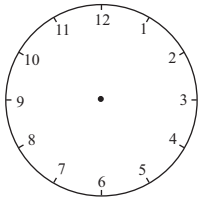
The other operations produce the following result:

$$\begin{aligned}
(6 + 3) + 4 - (2 - 1) &= 9 + 4 - 1 = 12 \\
(6 - 3) + 4 - (2 - 1) &= 3 + 4 - 1 = 6 \\
(6 \times 3) + 4 - (2 - 1) &= 18 + 4 - 1 = 21
\end{aligned}$$

Difficulty: Easy
NCTM Standard: Number and Operations Standard for Grades 6-8: Understand meanings of operations and how they relate to one another.
Mathworld.com Classification: Number Theory > Arithmetic > Addition and Subtraction > Subtraction

m99-02

What is the degree measure of the smaller angle formed by the hands of a clock at 10 o'clock?



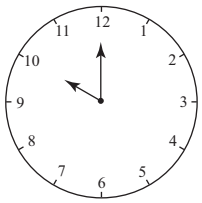
(A) 30 **(B)** 45 **(C)** 60 **(D)** 75 **(E)** 90

1999 AMC 8, Problem #2—

"Find out how many degrees each of the twelve spaces on a clock measures."

Solution

Answer (C): There are $360°$(degrees) in a circle and twelve spaces on a clock. This means that each space measures $30°$. At 10 o'clock the hands point to 10 and 12. They are two spaces or $60°$ apart.



Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 6-8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.

Mathworld.com Classification: Geometry > Trigonometry > Angles

m99-03

Which triplet of numbers has a sum NOT equal to 1 ?

(A) $(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$ **(B)** $(2, -2, 1)$ **(C)** $(0.1, 0.3, 0.6)$ **(D)** $(1.1, -2.1, 1.0)$ **(E)** $(-\frac{3}{2}, -\frac{5}{2}, 5)$

1999 AMC 8, Problem #3—
"Find the sum of all triplets."

Solution

Answer (D): $1.1 + (-2.1) + 1.0 = 0$. The other triplets add to 1 .

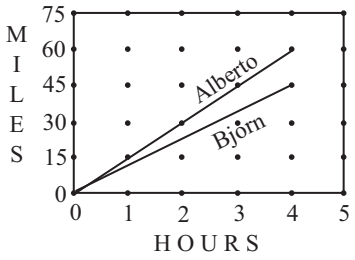
Difficulty: Medium-easy

NCTM Standard: Number and Operations Standard for Grades 6-8: select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions
Number Theory > Arithmetic > Addition and Subtraction > Addition

m99-04

The diagram shows the miles traveled by bikers Alberto and Bjorn. After four hours about how many more miles has Alberto biked then Bjorn?



(A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

1999 AMC 8, Problem #4—
"After four hours, how many miles do Alberto and Bjorn each bike?"

Solution

Answer (A): Four hours after starting, Alberto has gone about 60 miles and Bjorn has gone about 45 miles. Therefore, Alberto has biked about 15 more miles.

Difficulty: Easy
NCTM Standard: Data Analysis and Probability Standard for Grades 6-8: discuss and understand the correspondence between data sets and their graphical representations.
Mathworld.com Classification: History and Terminology > Terminology > Diagram

m99-05

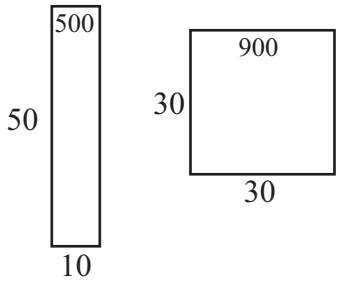
A rectangular garden 50 feet long and 10 feet wide is enclosed by a fence. To make the garden larger, while using the same fence, its shape is changed to a square. By how many square feet does this enlarge the garden?

(A) 100 **(B)** 200 **(C)** 300 **(D)** 400 **(E)** 500

1999 AMC 8, Problem #5—
"The square will have the same perimeter as the rectangular garden."

Solution

Answer (D): The area of the garden was 500 square feet(50×10) and its perimeter was 120 feet, $2 \times (50 + 10)$. The square garden is also enclosed by 120 feet of fence so its sides are each 30 feet long. The square garden's area is 900 square feet (30×30). and this has increased the garden area by 400 square feet.



Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6-8: draw geometric objects with specified properties, such as side lengths or angle measures.

Mathworld.com Classification: Geometry > Plane Geometry > Squares
Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area

m99-06

Bo, Coe, Flo, Jo, and Moe have different amounts of money. Neither Jo nor Bo has as much money as Flo. Both Bo and Coe have more than Moe. Jo has more than Moe, but less than Bo. Who has the least amount of money?

(A) Bo **(B)** Coe **(C)** Flo **(D)** Jo **(E)** Moe

1999 AMC 8, Problem #6—
"Cross out the name, that have a greater amount of money."

Solution

Answer (E): From the second sentence, Flo has more than someone so she can't have the least. From the third sentence both Bo and Coe have more than someone so that eliminates them. And, from the fourth sentence, Jo has more than someone, so that leaves only poor Moe!

Difficulty: Easy
NCTM Standard: Problem Solving Standard for Grades 6-8: apply and adapt a variety of appropriate strategies to solve problems.
Mathworld.com Classification: History and Terminology > Terminology > Order

m99-07

The third exit on a highway is located at milepost 40 and the tenth exit is at milepost 160. There is a service center on the highway located three-fourths of the way from the third exit to the tenth exit. At what milepost would you expect to find this service center?

(A) 90 **(B)** 100 **(C)** 110 **(D)** 120 **(E)** 130

1999 AMC 8, Problem #7—
"The service center is located at a milepost equal to $40 +$ $(\frac{3}{4}$**of the difference in mileage between the 3rd and the 10th exit**$)$**."**

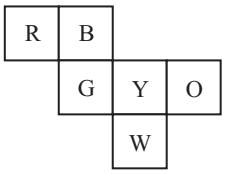
Solution

Answer (E): There are $160 - 40 = 120$ miles between the third and tenth exits, so the service center is at milepost $40 + (\frac{3}{4})120 = 40 + 90 = 130$.

Difficulty: Medium-hard
NCTM Standard: Geometry Standard for Grades 6-8: use geometric models to represent and explain numerical and algebraic relationships.
Mathworld.com Classification: Geometry > Line Geometry > Lines > Real Line

m99-08

Six squares are colored, front and back, (R=red, B=blue, O=orange, Y=yellow, G=green, and W=white). They are hinged together as shown, then folded to form a cube. The face opposite the white face is

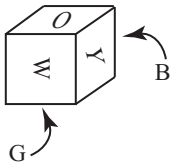


(A) B **(B)** G **(C)** O **(D)** R **(E)** Y

1999 AMC 8, Problem #8—
"Set G as the base, form the cube."

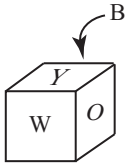
Solution

Answer (A):



When G is arranged to be the base, B is the back face and W is the front face. Thus, B is opposite W.

OR



Let Y be the top and fold G, O, and W down. Then B will fold to become the back face and be opposite W.

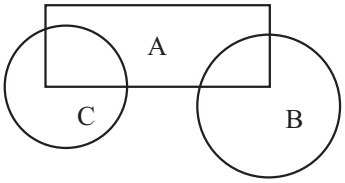
Difficulty: Medium-easy

NCTM Standard: Geometry Standard for Grades 6-8: use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.

Mathworld.com Classification: Geometry > Solid Geometry > Polyhedra > Cubes

m99-09

Three flower beds overlap as shown. Bed A has 500 plants, bed B has 450 plants, and bed C has 350 plants. Beds A and B share 50 plants, while beds A and C share 100. The total number of plants is

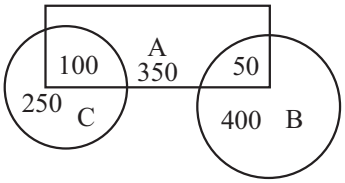


(A) 850 **(B)** 1000 **(C)** 1150 **(D)** 1300 **(E)** 1450

1999 AMC 8, Problem #9—
"Plants shared by two beds have been counted twice."

Solution

Answer (C): Bed A has 350 plants it doesn't share with B or C. Bed B has 400 plants it doesn't share with A or C. And C has 250 it doesn't share with A or B. The total is $350 + 400 + 250 + 50 + 100 = 1150$ plants.



OR

Plants shared by two beds have been counted twice, so the total is $500 + 450 + 350 - 50 - 100 = 1150$.

Difficulty: Medium
NCTM Standard: Geometry Standard for Grades 6-8: use geometric models to represent and explain numerical and algebraic relationships.
Mathworld.com Classification: Number Theory > Arithmetic > Addition and Subtraction

m99-10

A complete cycle of a traffic light takes 60 seconds. During each cycle the light is green for 25 seconds, yellow for 5 seconds, and red for 30 seconds. At a randomly chosen time, what is the probability that the light will NOT be green?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{5}{12}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{7}{12}$

1999 AMC 8, Problem #10—
"The probability of not green $= 1 -$ **the probability of green."**

Solution

Answer (E):

$$\frac{\text{time not green}}{\text{total time}} = \frac{\text{R} + \text{Y}}{\text{R} + \text{Y} + \text{G}} = \frac{35}{60} = \frac{7}{12}.$$

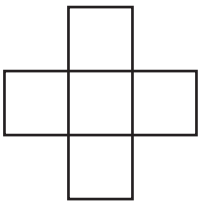
OR

The probability of green is $\frac{25}{60} = \frac{5}{12}$. so the probability of not green is $1 - \frac{5}{12} = \frac{7}{12}$.

Difficulty: Medium
NCTM Standard: Data Analysis and Probability Standard for Grades 6-8: use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.
Mathworld.com Classification: Probability and Statistics > Probability

m99-11

Each of the five numbers 1,4,7,10, and 13 is placed in one of the five squares so that the sum of the three numbers in the horizontal row equals the sum of the three numbers in the vertical column. The largest possible value for the horizontal or vertical sum is

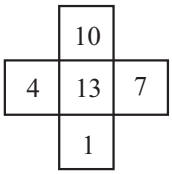


(A) 20 **(B)** 21 **(C)** 22 **(D)** 24 **(E)** 30

1999 AMC 8, Problem #11—
"The sum of horizontal numbers + The sum of vertical numbers = The sum of five numbers + The center number."

Solution

Answer (D): The largest sum occurs when 13 is placed in the center. This sum is $13 + 10 + 1 = 13 + 7 + 4 = 24$. Note: Two other common sums, 18 and 21, are possible.



OR

Since the horizontal sum equals the vertical sum, twice this sum will be the sum of the five numbers plus the number in the center. When the center number is 13, the sum is the largest, $\frac{10+4+1+7+2(13)}{2} = \frac{48}{2} = 24$. The other four numbers are divided into two pairs with equal sums.

Difficulty: Medium-hard
NCTM Standard: Problem Solving Standard for Grades 6-8: solve problems that arise in mathematics and in other contexts.
Mathworld.com Classification: Algebra > Sums

m99-12

The ratio of the number of games won to the number of games lost(no ties) by the Middle School Middies is $\frac{11}{4}$. To the nearest whole percent, what percent of its games did the team lose?

(A) 24 **(B)** 27 **(C)** 36 **(D)** 45 **(E)** 73

1999 AMC 8, Problem #12—
"The ratio of the number of games lost to the number of games played is $\frac{4}{11+4}$."

Solution

Answer (B): The Won/Lost ratio is 11/4 so, for some number N, the team won $11N$ games and lost $4N$ games. Thus, the team played $15N$ games and the fraction of games lost is $\frac{4N}{15N} = \frac{4}{15} \approx 0.27 = 27\%$.

Difficulty: Medium
NCTM Standard: Data Analysis and Probability Standard for Grades 6-8: use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.
Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

m99-13

The average age of the 40 members of a computer science camp is 17 years. There are 20 girls, 15 boys, and 5 adults. If the average age of the girls is 15 and the average age of the boys is 16, what is the average age of the adults?

(A) 26 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 30

1999 AMC 8, Problem #13—
"The sum of the adult's ages = The sum of all ages − (The sum of the girls' ages + The sum of the boys' ages)."

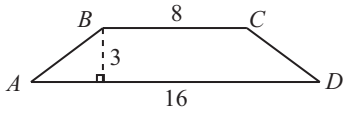
Solution

Answer (C): The sum of all ages is $40 \times 17 = 680$. The sum of the girls' ages is $20 \times 15 = 300$ and the sum of the boys' ages is $15 \times 16 = 240$. The sum of the five adults' ages is $680 - 300 - 240 = 140$. Therefore, their average is $\frac{140}{5} = 28$.

Difficulty: Medium-hard
NCTM Standard: Data Analysis and Probability Standard for Grades 6-8: find, use, and interpret measures of center and spread, including mean and interquartile range.
Mathworld.com Classification: Calculus and Analysis > Special Functions > Means > Arithmetic Mean

m99-14

In trapezoid $ABCD$, the side AB and CD are equal. The perimeter of $ABCD$ is

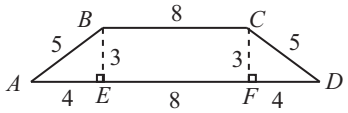


(A) 27 **(B)** 30 **(C)** 32 **(D)** 34 **(E)** 48

1999 AMC 8, Problem #14—
"Using the Pythagorean Theorem, $AB^2 = AE^2 + EB^2$."

Solution

Answer (D): When the figure is divided, as shown the unknown sides are the hypotenuses of right triangles with legs of 3 and 4. Using the Pythagorean Theorem yields $AB = CD = 5$. The total perimeter is $16 + 5 + 8 + 5 = 34$.



Difficulty: Medium-hard
NCTM Standard: Geometry Standard for Grades 6-8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
Mathworld.com Classification: Geometry > Plane Geometry > Quadrilaterals > Isosceles Trapezoid

m99-15

Bicycle license plates in Flatville each contain three letters. The first is chosen from the set C, H, L, P, R, the second from A, I, O, and the third from D, M, N, T.

When Flatville needed more license plates, they added two new letters. The new letters may both be added to one set or one letter may be added to one set and one to another set. What is the largest possible number of ADDITIONAL license plates than can be made by adding two letters?

(A) 27 **(B)** 30 **(C)** 32 **(D)** 34 **(E)** 48

1999 AMC 8, Problem #15—
"How many license plates could originally be made? Where can the two letters be placed so the most new license plates will be created?"

Solution

Answer (D): Before new letters were added, five different letters could have been chosen for the first position, three for the second, and four for the third. This means that $5 \cdot 3 \cdot 4 = 60$ plates could have been made.
If two letters are added to the second set, then $5 \cdot 5 \cdot 4 = 100$ plates can be made. If one letter is added to each of the second and third sets, then $5 \cdot 4 \cdot 5 = 100$ plates can be made. None of the other four ways to place the two letters will create as many plates. So, $100 - 60 = 40$ ADDITIONAL plates can be made.
Note: Optimum results can usually be obtained in such problems by making the factors as nearly equal as possible.

Difficulty: Medium-hard
NCTM Standard: Number and Operations Standard: Understand numbers, ways of representing numbers, relationships among numbers, and number systems
Mathworld.com Classification:
Discrete Mathematics > Combinatorics > Permutations > Combination

m99-16

Tori's mathematic test had 75 problems: 10 arithmetic, 30 algebra, and 35 geometry problems. Although she answered 70% of the arithmetic, 40% of the algebra, and 60% of the geometry problems correctly, she did not pass the test because she got less than 60% of the problems right. How many more questions would she have needed to answer correctly to earn a 60% passing grade?

(A) 1 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

1999 AMC 8, Problem #16—
"Calculate the total number of questions Tory has answered correctly, and subtract it from $60\%(75)$**."**

Solution

Answer (B): Since $70\%(10)+40\%(30)+60\%(35)=7+12+21=40$, she answered 40 questions correctly. She needed $60\%(75)=45$ to pass, so she needed 5 mo43 correct answers.

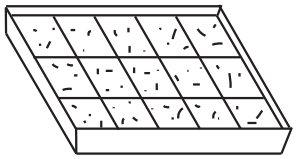
Difficulty: Medium
NCTM Standard: Algebra Standard for Grades 6-8: use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

m99-17

Problems 17, 18, and 19 refer to the following:

Cookies For a Crowd

At Central Middle School the 108 students who take the AMC $\rightarrow$ 8 meet in the evening to talk about problems and eat an average of two cookies apiece. Walter and Gretel are baking Bonnie's Best Bar Cookies this year. Their recipe, which makes a pan of 15 cookies, list these items:
$1\frac{1}{2}$ cups of flour, 2 eggs, 3 tablespoons butter, $\frac{3}{4}$ cups sugar, and 1 package of chocolate drops. They will make only full recipes, not partial recipe.



Walter can buy eggs by the half-dozen. How many half-dozens should be buy to make enough cookies? (Some eggs and some cookies may be left over.)

(A) 1 **(B)** 2 **(C)** 5 **(D)** 7 **(E)** 15

1999 AMC 8, Problem #17—
"There are a total of 216 cookies that will be consumed, each recipe makes 15 cookies. So $216 \div 15 \approx 15$ recipes are needed."

Solution

Answer (C): One recipe makes 15 cookies, so $216 \div 15 = 14.4$ recipes are needed, but this must be rounded up to 15 recipes to make enough cookies. Each recipe requires 2 eggs. So 30 eggs are needed. This is 5 half-dozens.

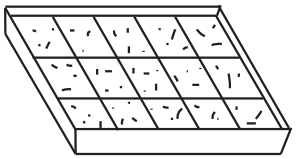
Difficulty: Medium
NCTM Standard: Algebra Standard for Grades 6-8: use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division

m99-18

Problems 17, 18, and 19 refer to the following:

Cookies For a Crowd

At Central Middle School the 108 students who take the AMC $\rightarrow$ 8 meet in the evening to talk about problems and eat an average of two cookies apiece. Walter and Gretel are baking Bonnie's Best Bar Cookies this year. Their recipe, which makes a pan of 15 cookies, list these items:
$1\frac{1}{2}$ cups of flour, 2 eggs, 3 tablespoons butter, $\frac{3}{4}$ cups sugar, and 1 package of chocolate drops. They will make only full recipes, not partial recipe.



They learn that a big concert is scheduled for the same night and attendance will be down 25%. How many recipes of cookies should they make for their smaller party?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

1999 AMC 8, Problem #18—
"The number of Cookies that need to be prepared is $108(75\%)\times 2$**."**

Solution

Answer (E): The $108(0.75) = 81$ students need 2 cookies each so 162 cookies are to be baked. Since $162 \div 15 = 10.8$, Walter and Gretel must bake 11 recipes. A few leftovers are a good thing!

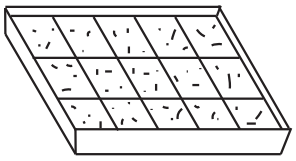
Difficulty: Medium-hard
NCTM Standard: Algebra Standard for Grades 6-8: use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division
Number Theory > Arithmetic > Fractions > Percent

m99-19

Problems 17, 18, and 19 refer to the following:

Cookies For a Crowd

At Central Middle School the 108 students who take the AMC $\rightarrow$ 8 meet in the evening to talk about problems and eat an average of two cookies apiece. Walter and Gretel are baking Bonnie's Best Bar Cookies this year. Their recipe, which makes a pan of 15 cookies, list these items:
$1\frac{1}{2}$ cups of flour, 2 eggs, 3 tablespoons butter, $\frac{3}{4}$ cups sugar, and 1 package of chocolate drops. They will make only full recipes, not partial recipe.



The drummer gets sick. The concert is cancelled. Walter and Gretel must make enough pans of cookies to supply 216 cookies. There are 8 tablespoons in a stick of butter. How many sticks of butter will be needed? (Some butter may be left over, of course.)

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

1999 AMC 8, Problem #19—
"They will have to bake 15 recipes of cookies."

Solution

Answer (B): Since $216 \div 15 = 14.4$, they will have to bake 15 recipes. This requires $15 \times 3 = 45$ tablespoons of butter. So, $45 \div 8 = 5.625$, and 6 sticks are needed.

Difficulty: Medium-hard
NCTM Standard: Algebra Standard for Grades 6-8: use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division

m99-20

Figure 1 is called a "stack map." The numbers tell how many cubes are stacked in each position. Fig. 2 shows these cubes, and Fig. 3 shows the view of the stacked cubes as seen from the front.

Which of the following is the front view for the stack map in Fig. 4?

3	4
2	1

Figure 1

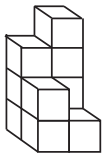


Figure 2

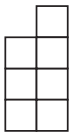
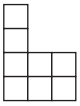
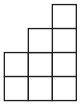
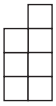
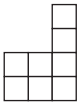
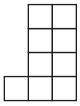


Figure 3

2	2	4
1	3	1

Figure 4

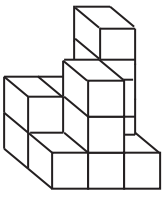
(A)  **(B)**  **(C)**  **(D)**  **(E)** 

1999 AMC 8, Problem #20—

"The front view shows the larger of the numbers of cubes in the front or back stack in each column."

Solution

Answer (B): The front view shows the larger of the numbers of cubes in the front or back stack in each column. Therefore the desired front view will have, from left to right, 2, 3, and 4 cubes. This is choice B.



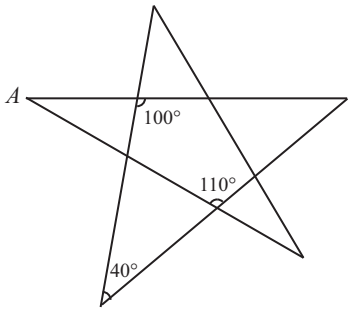
Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6-8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Mathworld.com Classification: Geometry > Solid Geometry > Polyhedra > Cubes
Geometry > Geometric Construction

m99-21

The degree measure of angle A is



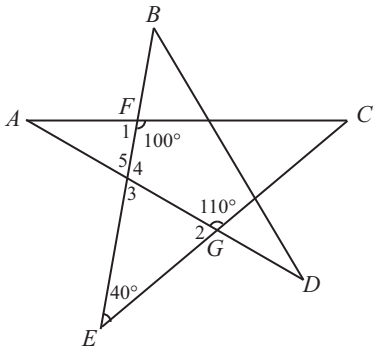
(A) 20 **(B)** 30 **(C)** 35 **(D)** 40 **(E)** 45

1999 AMC 8, Problem #21—

"$\angle C = 180^\circ - \angle E - \angle F$**, and** $\angle G = 110^\circ$**."**

Solution

Answer (B): Since $\angle 1$ forms a straight line with angle 100°, $\angle 1 = 80^\circ$. Since $\angle 2$ forms a straight line with angle 110°, $\angle 2 = 70^\circ$. Angle 3 is the third angle in a triangle with $\angle E = 40^\circ$ and $\angle 2 = 70^\circ$, so $\angle 3 = 180^\circ - 40^\circ - 70^\circ = 70^\circ$.Angle $4 = 110^\circ$ since it forms a straight angle with $\angle 3$. Then $\angle 5$ forms a straight angle with $\angle 4$, so $\angle 5 = 70^\circ$. (Or $\angle 3 = \angle 5$ because they are vertical angles.) Therefore, $\angle A = 180^\circ - \angle 1 - \angle 5 = 180^\circ - 80^\circ - 70^\circ = 30^\circ$.



OR

The angle sum in $\triangle CEF$ is 180°, so $\angle C = 180^\circ - 40^\circ - 100^\circ = 40^\circ$. In $\triangle ACG$, $\angle G = 110^\circ$ and $\angle C = 40^\circ$, so $\angle A = 180^\circ - 110^\circ - 40^\circ = 30^\circ$.

Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 6-8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Trigonometry > Angles
Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Triangle

m99-22

In a far-off land three fish can be traded for two loaves of bread and a loaf of bread can be traded for four bags of rice. How many bags of rice is one fish worth?

(A) $\frac{3}{8}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3}{4}$ **(D)** $2\frac{2}{3}$ **(E)** $3\frac{1}{3}$

1999 AMC 8, Problem #22—

"One fish is worth $\frac{2}{3}$ of a loaf of bread, and each loaf of bread is worth four bags of rice."

Solution

Answer (D): One fish is worth $\frac{2}{3}$ of a loaf of bread and $\frac{2}{3}$ of a loaf of bread is worth $\frac{2}{3} \cdot 4 = \frac{8}{3} = 2\frac{2}{3}$ bags of rice.

OR

$$\begin{aligned} 3F &= 2B \\ \frac{3}{2}F &= B = 4R \\ (\frac{2}{3})(\frac{3}{2}) &= \frac{2}{3}(4R) \\ F &= \frac{8}{3}R = 2\frac{2}{3}R. \end{aligned}$$

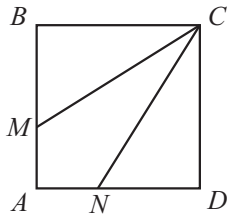
Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 6-8: understand and use ratios and proportions to represent quantitative relationships.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions

m99-23

Square $ABCD$ has sides of length 3. Segments CM and CN divide the square's area into three equal part. How long is segment CM ?

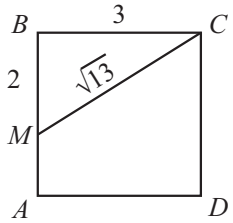


(A) $\sqrt{10}$ **(B)** $\sqrt{12}$ **(C)** $\sqrt{13}$ **(D)** $\sqrt{14}$ **(E)** $\sqrt{15}$

1999 AMC 8, Problem #23—
"Area of $\triangle MBC = (3 \times 3)\frac{1}{3} = \frac{1}{2}(MB)(BC)$**."**

Solution

Answer (C): One-third of the square's area is 3, so triangle MBC has area $3 = \frac{1}{2}(MB)(BC)$. Since side BC is 3, side MB must be 2. The hypotenuse CM of this right triangle is $\sqrt{2^2 + 3^2} = \sqrt{13}$.



Difficulty: Hard
NCTM Standard: Geometry Standard for Grades 6-8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
Mathworld.com Classification: Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area

m99-24

When 1999^{2000} is divided by 5, the remainder is

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0

1999 AMC 8, Problem #24—
"Since any positive integer(expressed in base ten) is some multiple of 5 plus its last digit, its remainder when divided by 5 can be obtained by knowing its last digit."

Solution

Answer (D): Since any positive integer(expressed in base ten) is some multiple of 5 plus its last digit, its remainder when divided by 5 can be obtained by knowing its last digit.

Note that 1999^1 ends in 9, 1999^2 ends in 1, 1999^3 ends in 9, 1999^4 ends in 1, and this alternation of 9 and 1 endings continues with all even powers ending in 1. Therefore, the remainder when 1999^{2000} is divided by 5 is 1 .

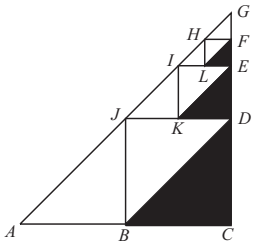
Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6-8: understand the meaning and effects of arithmetic operations with fractions, decimals, and integers.

Mathworld.com Classification: Calculus and Analysis > Special Functions > Powers
Number Theory > Arithmetic > Multiplication and Division > Remainder

m99-25

Points B, D, and J are midpoints of the sides of right triangle ACG. Points K, E, I are midpoints of the sides of triangle JDG, etc. If the dividing and shading process is done 100 times(the first three are shown) and $AC = CG = 6$, then the total area of the shaded triangles is nearest



(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

1999 AMC 8, Problem #25—

"At each stage the area of the shaded triangle is one-third of the trapezoidal region not containing the smaller triangle being divided in the next step."

Solution

Answer (A): At each stage the area of the shaded triangle is one-third of the trapezoidal region not containing the smaller triangle being divided in the next step. Thus, the total area of the shaded triangles comes closer and closer to one-third of the area of the triangular region ACG and this is $\frac{1}{3} \cdot \frac{1}{2} \cdot 6 \cdot 6 = 6$. The shaded areas for the first six stages are: 4.5, 5.625, 5.906, 5.976, 5.994, and 5,668.

These are the calculations for the first three steps.

$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} = 4.5$

$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} + \frac{1}{2} \cdot \frac{6}{4} \cdot \frac{6}{4} = 4.5 + 1.125 = 5.625$

$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} + \frac{1}{2} \cdot \frac{6}{4} \cdot \frac{6}{4} + \frac{1}{2} \cdot \frac{6}{8} \cdot \frac{6}{8} = 5.625 + 0.281 = 5.906$

Difficulty: Hard

NCTM Standard: Geometry Standard for Grades 6-8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area

Resources

Web sites with useful information

Competitions

http://archives.math.utk.edu/contests/ - list of competitions at varying levels
http://bigcheese.math.sc.edu/contest/ - University of South Carolina HS Math Contest
http://cemc.uwaterloo.ca/ - Canadian Mathematics Competitions
http://courses.ncssm.edu/math/Mathcon.htm - North Carolina State HS Mathematics Contest
http://www.maml.org/ - Massachusetts Association of Mathematics Leagues
http://math.furman.edu/tournament/tournament.html - Furman University Wylie Mathematics Tournament, Greenville, SC
http://math.uww.edu/mathmeet/ - Purple Comet M/HS Math Meet
http://mathcircle.berkeley.edu/ - problems, high school
http://nciml.org/NCIMLCoachesGuide.pdf - Nassau County Interscholastic Mathematics League Long Island, NY
http://regentsprep.org/Regents/math/math-a.cfm#1 - – competitions prep
http://web.mit.edu/hmmt/ - Harvard-MIT Mathematics Tournament, MA
http://www.amatyc.org/SML/ - American Mathematical Association of Two Year Colleges Student Mathematics League
http://www.arml.com/ - American Regions Math League, past questions, high school
http://www.cms.math.ca/CMS/Competitions/IMTS/ - questions, high school
http://www.cms.math.ca/Competitions/CMO/ - Canadian Mathematical Olympiad
http://www.cms.math.ca/Competitions/COMC/ - Canadian Open Mathematics Challenge
http://www.comap.com/undergraduate/contests/mcm/ - Mathematical Contest in Modeling
http://www.imo.org.yu - mathematical olympiads, for those who prepare for math competitions or love problem mathematics.
http://www.lehigh.edu/%7edmd1/hs.html - Lehigh University High School Math Contest, PA
http://www.maml.org/ - Massachusetts Association of Mathematics Leagues
http://www.mandelbrot.org/ - Mandelbrot
http://www.math.fau.edu/MathematicsCompetition/Home.htm - Internet High School Math Competition, Florida Atlantic University
http://www.math.okstate.edu/%7ehsc/ - Oklahoma State University HS Math Contest
http://www.math.umd.edu/highschool/mathcomp/ - University of Maryland HS Mathematics Competition
http://www.math.wisc.edu/%7etalent/ - University of Wisconsin Mathematics, Engineering and Science Talent Search
http://www.mathcounts.org/ - MathCounts grouop competition for Middle Schools
http://mathforum.org/library/view/41814.html- Maritime Mathematics Competition, University of Prince Edward Island
http://www.mathleague.com/ - Math League (New England)
http://www.mathleague.com/reglist/REGIL.HTM - Illinois Math League
http://www.mathpropress.com/competitions.html - - links to multiple contests' problems
http://www.ruf.rice.edu/~eulers/RMT.html - Rice University Math Tournament, Houston, TX
http://www.testprepreview.com/thea_practice.htm - – THEA practice tests (Texas Higher Education Assessment)
http://www.uccs.edu/~olympiad/ - Colorado Math Olympiad
http://www.unl.edu/amc/a-activities/a7-problems/problemdir.shtml - problems, high school
http://www.unl.edu/amc/a-activities/a7-problems/putnamindex.shtml - questions, college
http://www.usamts.org/ - USA Mathematical Talent Search

General

http://education.jlab.org/indexpages/ - games
http://mathcounts.org/ - middle school competition website
http://mathforum.org/ - for students and teachers, math resource
http://mathworld.wolfram.com/ - Wolfram MathWorld - math resource for anyone
http://milan.milanovic.org/math/english/contents.html - Rasko Jovanovic`s World of Mathematics
http://www.cms.math.ca/Competitions/MOCP/ - Mathematical Olympiads Correspondence Program
http://www.curiousmath.com/ - Curious Math
http://www.maa.org/ - The Mathematical Association of America
http://www.mathaware.org/ - Mathematics Awareness
http://www.mathfrog.ca - free site of Fun Resources and Online Games for teachers, parents & students, grades 4-6
http://www.mathsisfun.com - mainly K-12 site, it sometimes helps to have basic concepts explained, + plenty of math puzzles.
http://www.mathispower.com/ - Math is Power
http://www.mathlinks.ro/Forum/portal.php - MathLinks EveryOne
http://www.mathpath.org/ - Math Path
http://www.nctm.org/ - lessons, resources, etc for teachers
http://www.visualmathlearning.com/ - interactive tutorial -pre-algebra students, games, puzzles, + animated manipulatives

Reference

http://amser.org/ - Applied Math and Science Education Repository
http://eqworld.ipmnet.ru - World of Mathematical Equations, post high school
http://historical.library.cornell.edu/math/ - online math books
http://math.usask.ca/emr/menu.html - math readiness, high school

http://mathforum.org/dr.math/ - ask math questions, 7-12
http://www.artofproblemsolving.com/ - student resource, high school
http://www.circusofpatterns.com - research in mathematics number patterns, a series of mathematical charts math
http://www.curiousmath.com/ - tips, tricks, facts, etc; middle school
http://www.cut-the-knot.org/ - extra math reference, 7-12
http://www.sosmath.com/ - classroom math reference, 7-12
http://www.webmath.com/ - classroom math questions, 7-10

Math History

http://aleph0.clarku.edu/~djoyce/mathhist/mathhist.html - math history, regional, chronological
http://library.thinkquest.org/22584/ - math history, biographies
http://members.aol.com/jeff570/mathword.html - history of mathematical terms
http://www-gap.dcs.st-and.ac.uk/~history/BiogIndex.html - math biographies

Teaching

http://euler.slu.edu/Dept/SuccessinMath.html - How to study (mainly college)
http://gametheory.net/ - Game Theory - resource for educators
http://jwilson.coe.uga.edu/emt725/PSsyn/PSsyn.html - Teaching problem solving
http://mathforum.org/ - Math Forum @ Drexel University
http://www.claymath.org/index.php - Clay Mathematics Institute
http://www.lessonplanz.com/ - Lesson plans for teachers
http://www.mathaware.org/ - List of professional websites
http://www.mathgoodies.com/ - Worksheets, lessons, articles; middle school
http://www.mathsolutions.com/ - Math Solutions Professional Development
http://www.mathswap.ca - web site for teachers to share materials with each other, 9-12
http://www.pbs.org/teachersource/recommended/math/lk_problemsolving.shtm - List of teaching resources for k-12
http://www.totallyfreemath.com/ - GetMath - Terry Wesner's page – algebra reference materials
http://www.wiredmath.ca - free student/teacher mathematics resources, on-line games + drills, for 7-9; English + French.

Mentoring

http://jhuniverse.jhu.edu/%7egifted/set/ - Study of Execptional Talent, middle school - Johns Hopkins University
http://www.awesomemath.org/ - Awesome Math
http://www.tip.duke.edu/ - Talent Identification Program, middle school - Duke University

Books

http://store.doverpublications.com/ - Dover Books
http://www.amazon.com/exec/obidos/tg/browse/-/13884/ - Amazon.com Mathematics Books
http://www.ams.org/bookstore/ - American Mathematical Society Bookstore
http://www.artofproblemsolving.com/ - Art of Problem Solving
http://www.cms.math.ca/Publications/ - Canadian Mathematical Society
http://www.geocities.com/asoifer/orderform - Center for Excellence in Mathematical Education
http://www.mathpropress.com/ - MathPro Press
http://www.nctm.org/publications/ - NCTM Book Store
http://www.springeronline.com/ - Springer Publishing
http://www.tarquin-books.demon.co.uk/ - Tarquin Books
http://www.whfreeman.com/generalreaders/sal.asp - W. H. Freeman Publishers
https://enterprise.maa.org/ecomtpro/timssnet/common/tnt_frontpage.cfm - Mathematical Association of America Bookstore

Journals & Magazines

http://www.informs.org/index.php?c=31&kat=-+INFORMS+Journals - Interfaces
http://journals.cms.math.ca/CRUX/welcome.html - CRUX Mathematicorum with Mathematical Mayhem
http://komal.elte.hu/info/bemutatkozas.e.shtml - KöMaL
http://olympiads.win.tue.nl/ioi/misc/miq.html - Mathematics and Informatics Quarterly
http://plus.maths.org.uk/issue18/index.html - Plus Magazine ..living Mathematics
http://www.artofproblemsolving.com/ - Art of Problem Solving
http://interfaces.journal.informs.org/ - Interfaces
http://www.joma.org/jsp/index.jsp - Journal of Online Mathematics and its Applications
http://www.maa.org/mathhorizons/ - mMath Horizons
http://www.whitehouse.gov/kids/math/ - White House Math

Circles

http://comet.lehman.cuny.edu/mathcircle/ - Lehman College Math Circle, City University of New York
http://mathcircle.berkeley.edu/ - Berkley Math Circle, Berkley, CA area
http://mathcircles.fiu.edu/ - Florida International University, University Park, Miami, FL
http://www.sdmathcircle.org/Welcome.php - Math Circle in San Diego
http://www.math.uci.edu/%7emathcirc/ - Math Circle at University of California, Irvine
http://www.ma.utexas.edu/users/smmg/ - University of Texas at Austin Saturday Morning Math Group

http://www.math.ucla.edu/~radko/circles - UCLA Math Circle
http://www.math.utah.edu/mathcircle/ - University of Utah Math Circle, Salt Lake City, UT
http://www.mathlessons.com/ - offers free Math Lessons Classifieds, Goal - make it easier to find what's available and fits them best.
http://www.mualphatheta.org/ - Mu Alpha Theta, a national organization which has local chapters, similar to a Math Circle
http://www.southalabama.edu/mathstat/non-css-mathcircle.shtml - University of South Alabama, Mobile, AL
http://www.themathcircle.org/ - The Math Circle, Boston, MA area

Fairs/Scholarships

http://www.ams.org/prizes/epsilon-award.html - AMS Epsilon Awards for Young Scholars Programs
http://www.ditdservices.org/Articles.aspx?ArticleID=36&NavID=1_0 - Davidson Institute Programs & Scholarships
http://www.questbridge.org/ - Quest Bridge
http://www.sciserv.org/isef/ - Intel International Science and Engineering Fair
http://www.sciserv.org/sts/ - Intel Science Talent Search
http://www.siemens-foundation.org/ - Siemens Competition in Math, Science & Technology

Summer Camps

http://dimacs.rutgers.edu/ysp/ - Young Scholars Program in Discrete Mathematics, Rutgers University, NJ
http://math.bu.edu/people/promys/ - PROMYS-Program in Mathematics for Young Scientists
http://math.stanford.edu/sumac/ - Stanford University Mathematics Camp
http://mrsec.uchicago.edu/outreach/ysp.html - 4 week Summer camp at University of Chicago
http://www.awesomemath.org/ - Awesome Math
http://www.cee.org/rsi/index.shtml - Research Science Institute, MIT
http://www.hcssim.org/ - Hampshire College Summer Studies in Mathematics, Amherst, MA
http://www.math.lsa.umich.edu/mmss/ - University of Michigan Math and Science Scholars
http://www.math.ohio-state.edu/ross/ - Ross Program
http://www.mathcamp.org/ - Canada/USA Mathcamp

Miscellaneous

These sites provide additional mathematics information, including problems. There are problems listed on many of the competition sites, listed in the events section, but problem lists without an attached event are listed here.

http://aleph0.clarku.edu/~djoyce/java/elements/elements.html - Euclid's Elements
http://gams.nist.gov/ - Guide to Available Mathematical Software
http://knotplot.com/-The Knot Plot Site
http://math.cofc.edu/faculty/kasman/MATHFICT/ - Mathematical Fiction
http://math.furman.edu/~mwoodard/ - Mark Woodard's Mathematical Quotation Server (Mark.Woodard@furman.edu)
http://olympiads.win.tue.nl/imo/soviet/RusMath.html - Soviet Union Olympiad problems
http://primes.utm.edu/ - The Prime Pages
http://puzzleshq.com/ - Puzzles HQ
http://random.mat.sbg.ac.at/ - Random Number Generation
http://wims.unice.fr/ - Interactive mathematics on the internet
http://www.aimsedu.org/ - AIMS Foundation (Activities Integrating Mathematics & Science)
http://www.ams.org/ - American Mathematical Society
http://www.ams.org/mathweb/ - Math on the Web, by the American Mathematical Society
http://www.c3.lanl.gov/mega-math/ - Mega Mathematics
http://www.c3.lanl.gov/mega-math/workbk/knot/knot.html - Untangling the Mathematics of Knots
http://www.cms.math.ca/Competitions/IMTS/ - International Mathematical Talent Search - discontinued in 2001, but all problems listed
http://knotplot.com/- The Knot Plot Site
http://www.emis.de/ - European Mathematical Information Service
http://www.e-tutor.com/ - Online mathematics Dictionary
http://www.geocities.com/CapeCanaveral/Lab/4661/ - Olympiad Math Madness
http://www.imo.org.yu/ - The IMO Collection
http://www.lib.uwaterloo.ca/discipline/math/ - Mathematics Research Guide, University of Waterloo, Waterloo, Ontario, Canada
http://www.math.fsu.edu/Virtual/index.php - WWW virtual Mathematical Library, located at Florida State University
http://www.math.kth.se/~shapiro/problem.html - Mathematical Problem Solving
http://www.math.psu.edu/MathLists/Contents.html - Mathematics Information Servers at Penn State U.
http://www.math-atlas.org/welcome.html - Mathematical Atlas
http://www.mathlinks.ro/Forum/ - MathLinks Math
http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html - Fibonacci Numbers and the Golden Section
http://www.oakland.edu/enp/ - The Erdös Number Project
http://www.qbyte.org/puzzles/ - Nicks Mathematical Puzzles
http://www.research.att.com/~njas/sequences/ - Sloane's On-Line Encyclopedia of Integer Sequences
http://www.siam.org/ - SIAM [Society for Industrial and Applied Mathematics
http://www.superarco.com - Grapphing Calculator +
http://www.testprepreview.com/thea_practice.htm - THEA Online Courses, Practice Tests
http://www.w3.org/Math/ - W3C Math Home, What is MathML?
http://www-groups.dcs.st-and.ac.uk/~history/ - The MacTutor History of Mathematics archive

Books & publications for broadening student skills

Algebra

Posamentier, Alfred, and Charles Salkind. *Challenging Problems in Algebra.* (New York: Dover, 1988), 272 pgs.

Calculus

Calculus topics are not covered on the AMC 10, AMC 12, AIME, and USAMO contests, but the following book is a resource for teachers and enrichment topics.

Jackson, Michael B., and John R. Ramsay. Review of Resources for *Calculus: Problems for Student Snvestigation.* (Mathematics Teacher, 1993), 86(4) p.

Fractals

Peitgen, H. O., H. Jürgens, D. Saupe, E. Maletsky, T. Perciante, and L. Yunker. *Fractals for the Classroom: Strategic Activities Volume One.* (Springer-Verlag, 0), 450 pgs.

Peitgen, H. O., and D. Saupe. *Fractals for the Classroom: Strategic Activities Volume Three.* (New York: Springer-Verlag, 2005), 128 pgs.

Geometry

The following books provide more advanced content on Euclidean geometry. The topics covered in these books typcially appears on the AMC 12, AIME and USAMO contests.

Coxeter, H. S. M. ., and S. L. Greitzer. *Geometry Revisited.* (Washington D.C.: Mathematical Association of America, 1967), 207 pgs.

Hahn, Liang-shin. *Complex Numbers and Geometry.* (Washington D.C.: Mathematical Association of America, 1994), 203 pgs.

Honsberger, Ross. *Episodes in Nineteenth and Twentieth Century Euclidean Geometry.* (Washington D.C.: Mathematical Association of America, 1995), 188 pgs.

Pedoe, Dan. *Circles: A Mathematical View Revised Edition.* (Washington D.C.: Mathematical Association of America, 1995), 137 pgs.

The following books provide explanations of problems in geometry from antiquity to modern times. These books are excellent sources of supplementary and enrichment topics in geometry.

Banchoff, Thomas F. *Beyond the Third Dimension: Geometry, Computer Graphics, and Higher Dimensions.* (New York: Freeman Scientific American Library Series Paperback., 1990), 210 pgs.

Bold, B. *Famous Problems of Geometry and How to Solve Them.* (New York: Dover, 1964), 128 pgs.

Klee, Victor, and Stan Wagon. *Old and New Unsolved Problems in Plane Geometry and Number Theory, rev. ed..* (Washington D.C.: Mathematical Association of America, 1991), 352 pgs.

Klein, Felix. *Famous Problems of Elementary Geometry: The Duplication of the Cube, the Trisection of the Angle, and the Quadrature of the Circle.* In Famous Problems and Other Monographs. (New York: Dover, 1980), 112 pgs.

Leapfrogs Group. *Images of Infinity.* (England: Tarquin, 1992), 96 pgs.

Posamentier, Alfred, and Charles Salkind. *Challenging Problems in Geometry.* (New York: Dover, 1997), 256 pgs.

AMC10/12/AIME

All of the following books are excellent sources of study for AMC 10 and AMC 12 level problems and solutions. Some books may also contain mathematics and problems of the level contained in the AIME contest.

Artino, Ralph, Anthony Gaglione, and Neil Shell. *Contest Problem Book IV, 1973-1982, The.* (Washington D.C.: Mathematical Association of America, 1983), 198 pgs.

Barry, Donald, and Thomas Kilkelly. *ARML contests and Power Contests from 1995-2003.*

Berzsenyi, George, and Stephen B. Maurer. *Contest Problem Book V: American High School Mathematics Examination and American Invitational Mathematics Examinations 1983-1988, The.* (Washington D.C.: Mathematical Association of America, 1997), 308 pgs.

Faires, J. Douglas. *First Steps for Math Olympians.* (Washington D.C.: Mathematical Association of America, 2006), 320 pgs.

Gardiner, A. *Mathematical Olympiad Handbook: An Introduction to Problem Solving Based on the First 32 British Mathematical Olympiads 1965-1996, The.* (Oxford, England: Oxford University Press, 1997), 248 pgs.

Gardiner, A. *More mathematical challenges.* (Cambridge U.K: New York Cambridge University Press, 1997), 144 pgs.

Kessler, Gilbert, and Lawrence Zimmerman. *ARML-NYSML contests 1989-1994.* (MathPro Press, 1995), 208 pgs.

Kessler, Gilbert, and Lawrence Zimmerman. *NYSML-ARML Contests 1983-1988.* (Washington DC: National Council of Teachers of Mathematics, 1989), 148 pgs.

Kürschák, József, and Gyorgy Hajos. *Hungarian Problem Book, Based on the Eötvös Competitions, Vol. 1: 1894-1905.* (New York: Random House, 1963), 111 pgs.

Kürschák, József, and Gyorgy Hajos. *Hungarian Problem Book, Based on the Eötvös Competitions, Vol. 2: 1906-1928.* (New York: Random House, 1963), 120 pgs.

Kürshák, Hájos, and Surányi Neukomm. *Hungarian Problem Book I, (Eötvös Competition 1894-1905).* (Washington D.C.: Mathematical Association of America, 1967), 111 pgs.

Kürshák, Hájos, and Surányi Neukomm. *Hungarian Problem Book II, (Eötvös Competition 1906-1928).* (Washington D.C.: Mathematical Association of America, 1967), 120 pgs.

Liu, Andy. *Hungarian Problem Book III, (Eötvös Competition 1929-1943).* (Washington D.C.: Mathematical Association of America, 2001), 163 pgs.

Patrick, David. *Introduction to Counting & Probability.* (Art of Problem Solving, 2005),

Patrick, David. *Introduction to Counting & Probability, Solutions Manual.* (Art of Problem Solving, 2005),

Reiter, Harold. *Contest Problem Book VII: American Mathematics Competitions 1995-2000 Contests, The.* (Washington D.C.: Mathematical Association of America, 2006),

Salkind, Charles T. *Contest Problem Book I: Problems from the Annual High School Contests 1950-1960., The.* (New York: Random House, 1961), 154 pgs.

Salkind, Charles T. *Contest Problem Book II: Problems from the Annual High School Contests 1961-1965., The.* (Washington D.C.: Mathematical Association of America, 1966), 112 pgs.

Salkind, Charles T., and James M. Earl. *Contest Problem Book III: Annual High School Contests 1966-1972, The.* (Washington D.C.: Mathematical Association of America, Book IV compiled by Artino et al., 1973), 186 pgs.

Schneider, Leo J., comp.and Ed.. *Contest Problem Book VI: American High School Mathematics Examinations 1989-1994, The.* (Washington D.C.: Mathematical Association of America, 2000), 212 pgs.

IMO/USAMO

These books contain advanced level problems, solutions and topics at the level of the USAMO and the IMO. These are essential for understanding the format and content of the problems and solutions of these proof-essay style contests. Books containing Putnam Contest problems may include topics typically covered in University-level mathematics courses, as well as problems which can be solved with more elementary mathematics.

Alexanderson, Gerald L., Leonard Klosinski, and Loren Larson. *William Lowell Putnam Mathematical Competition, Problems and Solutions: 1965-1984, The.* (Washington D.C.: Mathematical Association of America, 1986), 151 pgs.

Andreescu, Titu, and Zuming Feng. *102 Combinatorial Problems from Training of the USA IMO Team.* (Boston: Birkhäuser, 2003), 116 pgs.

Andreescu, Titu, and Razvan Gelca. *Mathematical Olympiad Challenges.* (Boston, MA: Birkhäuser, 2000), 260 pgs.

Andreescu, Titu, and Zuming Feng. *Mathematical Olympiads: Problems and Solutions from Around the World 1998-1999.* (Washington D.C.: Mathematical Association of America, 2000),

Andreescu, Titu, and Zuming Feng. *Mathematical Olympiads: Problems and Solutions from Around the World 1999-2000.* (Washington D.C.: Mathematical Association of America, 2002), 280 pgs.

Andreescu, Titu, and Zuming Feng. *USA and International Mathematical Olympiads 2000.* (Washington D.C.: Mathematical Association of America, 2001), 120 pgs.

Andreescu, Titu, Zuming Feng, and George Lee. *USA and International Mathematical Olympiads 2001.* (Washington D.C.: Mathematical Association of America, 2002), 130 pgs.

Andreescu, Titu, Zuming Feng, and George Lee. *Mathematical Olympiads 2000-2001: Problems and Solutions from Around the World.* (Washington D.C.: Mathematical Association of America, 2003), 292 pgs.

Andreescu, Titu, and Zuming Feng. *USA and International Mathematical Olympiads 2002.* (Washington D.C.: Mathematical Association of America, 2003),

Andreescu, Titu, and Zuming Feng. *USA and International Mathematical Olympiads 2003.* (Washington D.C.: Mathematical Association of America, 2004), 104 pgs.

Andreescu, Titu, Zuming Feng, and Po Shen Loh. *USA and International Mathematical Olympiads 2004.* (Washington D.C.: Mathematical Association of America, 2005), 100 pgs.

Djukic, D., V. Z. Jankovic, I. Matic, and N. Petrovic. *IMO Compendium, The.* (New York: Springer Verlag, 2006), 760 pgs.

Feng, Zuming, Cecil Rousseau, and Melanie Wood. USA and International Mathematical Olympiads 2005. (Washington D.C.: Mathematical Association of America, 2006), 100 pgs.

Fomin, Dmitry, and Alexey Kirichenko. *Leningrad Mathematical Olympiads 1987-1991.* (MathPro Press, 1994), 197 pgs.

Gleason, A. M., R. E.Greenwood, and L. M. Kelly. *William Lowell Putnam Mathematical Competition, Problems and Solutions: 1938-1964, The.* (, 1980), 652 pgs.

Greitzer, Samuel L. *International Mathematical Olympiads, 1959-1977.* (Washington D.C.: Mathematical Association of America, 1978), 204 pgs.

Ivanov, O. A. *Easy as "Pi?": An Introduction to Higher Math.* (Springer-Verlag, 1998), 187 pgs.

Klamkin, Murray S. *International Mathematical Olympiads, 1978-1985 and Forty Supplementary Problems.* (Washington D.C.: Mathematical Association of America, 1986), 154 pgs.

Klamkin, Murray S. *USA Mathematical Olympiads, 1972-1986.* (Washington D.C.: Mathematical Association of America, 1988), 180 pgs.

Savchev, Svetoslav, and Titu Andreescu. *Mathematical Miniatures.* (Washington D.C.: Mathematical Association of America, 2003), 230 pgs.

Shkliarskii, David Oskarovich, N. N. Chentzov, and I. M. Yaglom. *USSR Olympiad Problem Book: Selected Problems and Theorems of Elementary Mathematics, The.* (New York: Dover, 1993), 452 pgs.

Slinko, A.M. *USSR Mathematical Olympiads 1989-1992.* (Australian Mathematics Trust, 1997),

Steele, J. Michael. *Cauchy-Schwarz Master Class, The.* (Washington D.C.: Mathematical Association of America, 2004), 316 pgs.

Tao, Terence. *Solving Mathematical Problems: A Personal Perspective.* (Oxford, England: Oxford University Press, 2006), 150 pgs.

Higher Mathematics

These books cover topics in number theory, combinatorics, calculus, and advanced algebra. Some of the problems were unsolved at the time of writing. Topics may be useful for the AMC 12 and AIME contests, but mostly serve for mathematical enrichment and advancement.

Alexanderson, Gerald L., Leonard F. Klosinski, and Loren C. Larson. *William Lowell Putnam Mathematical Competition, Problems and Solutions 1965-1984, The.* (Washington D.C.: Mathematical Association of America, 2003), 168 pgs.

Barbeau, Edward J., Murray S. Klamkin, and William O.J. Moser. *Five Hundred Mathematical Challenges.* (Washington D.C.: Mathematical Association of America, 1995), 236 pgs.

Barbeau, Edward. *Power Play.* (Washington D.C.: Mathematical Association of America, 1997), 212 pgs.

Biggs, William. *Ants, Bikes, and Clocks: Problem Solving for Undergraduates.* (SIAM, 2004), 174 pgs.

Chang, Gengzhe, and Thomas W. Sederberg. *Over and Over Again.* (Washington D.C.: Mathematical Association of America, 1998), 323 pgs.

Fraga, Robert, Ed.. *Calculus Problems for a New Century.* (Washington D.C.: Mathematical Association of America, 0), 488 pgs.

Gelca, Razvan, and Titu Andreescu. *Putnam and Beyond.* (New York: Springer Verlag, 2006), 550 pgs.

Gleason, A. M., R. E. Greenwood, and L. M. Kelly. *William Lowell Putnam Mathematical Competition, Problems and Solutions 1938-1964, The.* (Washington D.C.: Mathematical Association of America, 2003), 673 pgs.

Graham, L.A.. *Ingenious Mathematical Problems and Methods.* (New York: Dover, 0), 254 pgs.

Halmos, Paul Robert. *Problems for Mathematicians Young and Old.* (Washington D.C.: Mathematical Association of America, 1991), 328 pgs.

Herman, Jiri, Radan Kucera, and Jaromir Simsa. *Equations and Inequalities: Elementary Problems and Theorems in Algebra and Number Theory.* (New York: Springer-Verlag, 2000), 344 pgs.

Honsberger, Ross. *From Erdös to Kiev.* (Washington D.C.: Mathematical Association of America, 1995), 267 pgs.

Honsberger, Ross. *Mathematical Chestnuts from Around the World.* (Washington D.C.: Mathematical Association of America, 2001), 220 pgs.

Honsberger, Ross. *Mathematical Gems I.* (Washington D.C.: Mathematical Association of America, 1973),

Honsberger, Ross. *Mathematical Gems II.* (Washington D.C.: Mathematical Association of America, 1976), 191 pgs.

Honsberger, Ross. *Mathematical Gems III.* (Washington D.C.: Mathematical Association of America, 1985), 132 pgs.

Honsberger, Ross. *Mathematical Morsels.* (Washington D.C.: Mathematical Association of America, 1979), 262 pgs.

Honsberger, Ross, Ed. *Mathematical Plums.* (Washington D.C.: Mathematical Association of America, 1979), 191 pgs.

Honsberger, Ross. *More Mathematical Morsels.* (Washington D.C.: Mathematical Association of America, 1991), 344 pgs.

Kedlaya, Kiran, Bjorn Poonen, and Ravi Vakil. *William Lowell Putnam Mathematical Competition 1985-2000: Problems, Solutions, and Commentary, The.* (Washington D.C.: Mathematical Association of America, 2002), 354 pgs.

Konhauser, Joseph D. E., Dan Velleman, and Stan Wagon. *Which Way Did The Bicycle Go? ... And Other Intriguing Mathematical Mysteries.* (Washington D.C.: Mathematical Association of America, 1996), 256 pgs.

Lavász, Lázlo. *Combinatorial Problems and Exercises.* (Budapest: Akadéminal Kiadó, 1979), 636 pgs.

Ogilvy, C.S. *Tomorrow's Math, Unsolved Problems for the Amateur.* (New York: Oxford University Press, 1962),

Posamentier, Alfred, and Charles Salkind. *Challenging Problems in Algebra.* (New York: Dover, 1997), 272 pgs.

Shanks, Daniel. *Solved and Unsolved Problems in Number Theory, 4th ed.* (New York: Chelsea, 1993), 305 pgs.

Steinhaus, Hugo. *One Hundred Problems in Elementary Mathematics.* (New York: Dover, 1979), 174 pgs.

Székely, Gábor J., Ed. *Contests in Higher Mathematics: Miklós Schweitzer Competitions 1962-1991.* (New York: Springer-Verlag, 1996), 584 pgs.

Yaglom, A.M., and I. M. Yaglom. *Challenging Mathematical Puzzles with Elementary Solutions, Vol. 1: Combinatory Analysis and Probability Theory.* (New York: Dover, 1987), 231 pgs.

Yaglom, A.M., and I. M. Yaglom. *Challenging Mathematical Puzzles with Elementary Solutions, Vol. 2: Problems from Various Branches of Mathematics.* (New York: Dover, 1987), 223 pgs.

Problem Solving and Proving

These are books which cover the general area of creative thinking, problem solving, solution-writing and discovery. This collection of books is generally at the level of the problem solving that occurs in the AMC 10, AMC 12, and AIME, but without directly addressing the style and content of the AMC contests.

D'Angelo, John P., Douglas B. West. *Mathematical Thinking: Problem-Solving and Proofs, 2nd ed.* (Upper Saddle River NJ: Prentice-Hall, 2000), 412 pgs.

Devlin, Keith. *Life by the Numbers.* (New York: John Wiley & Sons, 1998), 214 pgs.

Devlin, Keith. *Mathematics: the Science of Patterns. Part 1: Intro to Fractals and Chaos, part 2: Complex Systems and Mandelbrot Set.* (New York: Freeman Science American Library, 1994), 224 pgs.

Dunham, William. *Journey through Genius.* (New York: John Wiley & Sons, 1990), 320 pgs.

Engel, Arthur. *Problem-Solving Strategies.* (New York: Springer-Verlag, 1998), 416 pgs.

Gillman, Rick. *A Friendly Mathematics Competition: Thirty-Five Years of Teamwork in Indiana.* (Washington D.C.: Mathematical Association of America, 2003), 196 pgs.

Graham, L.A.. *Surprise Attack in Mathematical Problems, The.* (New York: Dover, 1968), 126 pgs.

Hardy, Kenneth, Kenneth S. Williams. *Green Book of Mathematical Problems, The.* (New York: Dover, 1997), 184 pgs.

Hardy, Kenneth, and Kenneth S. Williams. *Red Book of Mathematical Problems, The.* (New York: Dover, 1996), 192 pgs.

Hayes, David E., and Tatiana Shubin. *Mathematical Adventures for Students and Amateurs.* (Washington D.C.: Mathematical Association of America, 2004), 304 pgs.

Hildebrandt, Stefan, and Anthony Tromba. Parsimonious Universe, The. (New York: Springer-Verlag, 1996), 330 pgs.

Honsberger, Ross. *In Pólya's Footsteps: Miscellaneous Problems and Essays.* (Washington D.C.: Mathematical Association of America, 1997), 212 pgs.

Honsberger, Ross. *Mathematical Diamonds.* (Washington D.C.: Mathematical Association of America, 2003), 256 pgs.

Larson, Loren C.. *Problem-Solving Through Problems.* (New York: Springer-Verlag, 1983), 352 pgs.

Lehoczky, Sandor, and Richard Rusczyk. *Art of Problem Solving, Volumes I and II, The.* (Alpine, California 91903-2185: AoPS Incorporated,),

Rabinowitz, Stanley. *Index to Mathematical Problems 1980-1984.* (Mathpro Press, 1992), 532 pgs.

Shkliarskii, David Oskarovich, N. N. Chentzov, and I. M. Yaglom. *USSR Olympiad Problem Book: Selected Problems and Theorems of Elementary Mathematics, The.* (New York: Dover, 1962), 452 pgs.

Soifer, Alexander. *Mathematics as Problem Solving.* (Colorado Springs CO: Center for Excellence in Mathematical Education, 1987),

Tanton, James Stuart. *Solve this: Math Activities for Students and Clubs.* (Washington, DC: Mathematical Association of America,, 2001), 240 pgs.

Tietze, Heinrich. *Famous Problems of Mathematics: Solved and Unsolved Mathematics Problems from Antiquity to Modern Times.* (New York: Graylock Press, 1965), 367 pgs.

Trigg, Charles W. *Mathematical Quickies: 270 Stimulating Problems with Solutions.* (New York: Dover, 1985), 210 pgs.

Ulam, Stanislaw M. A. *Collection of Mathematical Problems.* (New York: Interscience Publishers, 1960), 150 pgs.

Vaderlind, Paul, Richard Guy, and Loren C. Larson. *Inquisitive Problem Solver, The.* (Washington D.C.: Mathematical Association of America, 2002), 344 pgs.

Zeitz, Paul. *Art and Craft of Problem Solving, The.* (New York: John Wiley & Sons, 1999), 384 pgs.

Puzzles

Puzzles are a venerable topic in recreational mathematics, but typically puzzles involve a "trick", so while these texts are valuable for encouraging creative thinking and mathematical enrichment, the puzzles covered here do not directly contribute to improvement on the AMC contests. These books are especially rich sources for mathematics enrichment topics.

Clessa, J.J. *Math and Logic Puzzles for PC Enthusiasts.* (New York: Dover, 1996), 144 pgs.

Costello, Matthew J. *Greatest Puzzles of All Time, The.* (New York: Dover, 1988), 192 pgs.

Dudeney, Henry Ernest. *536 Puzzles and Curious Problems.* (New York: Scribner, 1967), 428 pgs.

Dudeney, Henry Ernest. *Amusements in Mathematics.* (New York: Dover, 1917), 258 pgs.

Dudeney, Henry Ernest. *Canterbury Puzzles and Other Curious Problems, 7th ed., The.* (London: Thomas Nelson and Sons, 1949), 256 pgs.

Dudeney, Henry Ernest. *Modern Puzzles.* (Thomas Nelson & Sons Ltd., 1938),

Dudeney, Henry Ernest. *Puzzle Mine, A.* (Thomas Nelson & Sons Ltd., 1951),

Emmet, E.R. *101 Brain Puzzles: A Treasury of Unique Mind-Stretching Puzzles.* (New York: Harper and Row, 1970), 254 pgs.

Fujii, J.N. *Puzzles and Graphs.* (Washington DC: National Council of Teachers of Mathematics, 1966),

Gamow, George, and M.Stern. *Puzzle-math.* (New York: Viking, 1958),

Gardner, Martin. *Colossal Book of Mathematics: Classic Puzzles, Paradoxes, and Problems, The.* (New York: W.W. Norton, 2001), 704 pgs.

Gardner, Martin. *My Best Mathematical and Logic Puzzles.* (New York: Dover, 1994), 96 pgs.

Kordemsky, Boris A. *Moscow Puzzles: 359 Mathematical Recreations, The.* (New York: Dover, 1992), 320 pgs.

Krusemeyer, Mark, and Loren Larson. *Wohascum County Problem Book, The.* (Washington D.C.: Mathematical Association of America, 1993), 243 pgs.

Loyd, Sam Jr. *Mathematical Puzzles of Sam Loyd, Vol. 1.* (New York: Dover, 1959), 165 pgs.

Loyd, Sam Jr. *More Mathematical Puzzles of Sam Loyd, Vol. 2.* (New York: Dover, 1960), 177 pgs.

Loyd, Sam Jr. Sam Loyd's Cyclopedia of 5,000 Puzzles, Tricks, and Conundrums. (Pinacle, 1976), 384 pgs.

Mott-Smith, Geoffrey. *Mathematical Puzzles for Beginners and Enthusiasts, 2nd rev. ed.* (New York: Dover, 1954), 248 pgs.

O'Beirne, T. H. *Puzzles and Paradoxes: Fascinating Excursions in Recreational Mathematics.* (New York: Dover, 1984), 238 pgs.

Olivastro, Dominic. *Ancient Puzzles: Classic Brainteasers and Other Timeless Mathematical Games of the Last Ten Centuries.* (New York: Bantam, 1993), 288 pgs.

Wells, David Graham. *Penguin Book of Curious and Interesting Puzzles, The.* (London: Penguin Books, 1992), 400 pgs.

Winkler, Peter. *Mathematical Puzzles: A Connoisseur's Collection.* (A K Peters, 2004), 163 pgs.

Wylie, C.R. Jr. *101 Puzzles in Thought and Logic.* (New York: Dover, 1957), 107 pgs .

Appendix I

Formulas and Definitions

Formulas and Definitions
Algebra, geometry, trig, calc, other

Exponents
$a^0 = 1$, ($a \neq 0$)
$(a^m)(a^n) = a^{m+n}$
$(ab)^m = a^m b^m$
$(a^m)^n = a^{mn}$
$a^{m/n} = \sqrt[n]{a^m}$
$(a^m)/(a^n) = a^{m-n}$
$a^{-m} = 1/(a^m)$

Quadratic Formula
In an equation like: $ax^2 + bx + c = 0$
$x = (-b \pm \sqrt{(b^2-4ac)})/ (2a)$

Binomial Theorem (Reference Pascal's Triangle)
$(a + b)^1 = a + b$
$(a + b)^2 = a^2 + 2ab + b^2$
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Difference of Squares
$a^2 - b^2 = (a - b) (a + b)$

Rules of Zero
$0/x = 0$ where x is not equal to 0.
$a^0 = 1$
$0^a = 0$
$a \times 0 = 0$
$a/0$ is undefined

Probability
Probabilities are never greater than 1 or less than 0.
$$\mathbf{0 \leq P(E) \leq 1}$$
Complement:
$$\boldsymbol{P(\sim E) = 1 - P(E)}$$
P (A & B) = P(A) P(B) for A, B independent events.
For AND, multiply individual probabilities. Check for replacement.
P (A Or B) = P(A) + P(B) – P(A & B)
For OR, add the individual probabilities and subtract the overlap.

Factorial:
$n! = n(n-1)(n-2)(n-3)...1$

Appendix II

The "Elusive Formulas" - Part 1

Geometry

a, b, c, s = sides
b_1, b_2 = bases
h = height

l = length
A = area
R, r = radii

C = circumference
V = volume
P = perimeter

Triangle
Sum of angles = 180°
P = a + b + c
$A = (1/2)b_1 h$

Right Triangle
$P = b + h + \sqrt{(b^2 + h^2)}$
$A = (b_1 h/2)$
45° - 45° - 90°
$H = L\sqrt{2}$
30° – 60° – 90°
L opposite 30° = (1/2)(H)
L opposite 60° = $(\sqrt{(3)}/2)(H)$

Equilateral Triangle
P = 3s
$A = s^2 \sqrt{(3)}/4$
$A = h^2 \sqrt{(3)}/3$

Pythagorean Theorem
$a^2 + b^2 = c^2$

Pythagorean Triples: 3,4, 5; 5,12, 13; 8, 15, 17

Heron's Formula
$A = \sqrt{(s(s - a)(s - b)(s - c))}$
S = semi perimeter = (a + b + c)/2

Square
P = 4s
$A = s^2$
Rectangle
P = 2(l + w)
A = lw

Parallelogram
P = 2(l + w)
$A = b_1 h$

Trapezoid
P=a + b + c + d
$A = (h/2)(b_1 + b_2)$

Circle
Number of degrees = 360°
Number of radians = 2π
$A = \pi r^2$
$C = 2\pi r$

Theta, θ, is in radians
Arc of a circle = r θ
Segment of a circle = $r^2 [\theta - \sin(\theta)]/2$
Sector of a circle = $r^2\theta/2$

Radians to Degrees
Multiply radians by 180/π

Degrees to Radians
Multiply degrees by π/180

Ellipse
$A = Rr\pi$

Solids

Angles
(Answers will be in degrees unless otherwise noted)

Sum of Interior Angles: 180(n-2)
Sum of Exterior Angles: 360
Each Interior Angle (regular poly): 180(n-2)/n
Each Exterior Angle (regular poly): 360/n
Sum of angles of triangle: 180
Measure of exterior angle of triangle: the sum of the two non-adjacent interior angles.
The sum of any two sides of a triangle is greater than the third side
To convert a degree measure to radians multiply by π/180
To convert a radian measure to degrees multiply by 180/π
Complementary angles are two angles whose sum is 90
Supplementary angles are two angles whose sum is 180.

Appendix II - The "Elusive Formulas" continued

Slope Formula:
m, m_1, m_2 = slopes
$m = (y_1 - y_2)/(x_1 - x_2)$ = rise/run

Slope-Intercept Method:
Y = mx + b
b = y-intercept

Point-Slope Method:
$y - y_1 = m(x - x_1)$
(x_1, y_1) is a point on the line

Standard Form:
Ax + By = C
where A and B are not both zero

Distance Formula:
$d = ((x_1 - x_2)^2 + (y_1 - y_2)^2)^{(1/2)}$

Midpoint Formula:
$(x, y) = ((x_1 + x_2)/2, (y_1 + y_2)/2)$

Parallel lines: $m_1 = m_2$
Perpendicular lines: $m_1 m_2 = -1$

d = rt; distance = rate x time

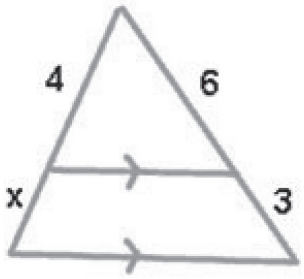
i = prt; interest = principal x interest rate x time

Equations of Circles and Parabolas
Circle, center at origin:
$x^2 + y^2 = r^2$

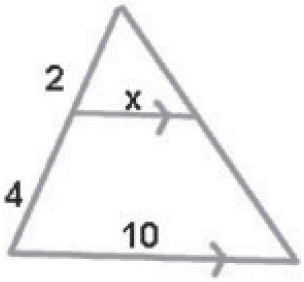
Circle, center at (h,k):
$(x-h)^2 + (y-k)^2 = r^2$

Parabola:
$y = ax^2 + bx + c$

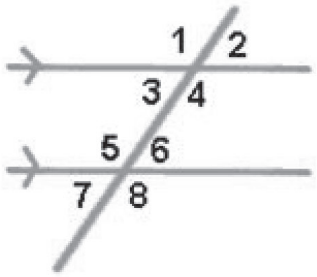
Two triangles are similar if the corresponding angles are congruent and the corresponding sides are in proportion.



$$\frac{4}{(4+x)} = \frac{6}{9}$$
$$36 = 24 + 6x$$
$$12 = 6x$$
$$2 = x$$



$$\frac{x}{10} = \frac{2}{6}$$
$$6x = 20$$
$$x = \frac{20}{6}$$
$$x = \frac{10}{3}$$



Corresponding angles are equal. \1=\5, \2=\6, \3=\7, \4=\8
Alternate Interior angles are equal. \3=\6, \4=\5
Alternate Exterior angles are equal. \1=\8, \2=\7
Same side interior angles are supplementary. \3+\5=180, \4+\6=180

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent} = \frac{\sin\theta}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$= 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$
$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

$$\sin A \cdot \sin B = \frac{1}{2}\left[-\cos(A+B) + \cos(A-B)\right]$$

$$\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \cdot \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

Appendix II - The "Elusive Formulas" continued

$$y = \sin^{-1} x = \arcsin x$$

$$D: -1 \le x \le 1$$

$$R: -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

$$y = \cos^{-1} x = \arccos x$$

$$D: -1 \le x \le 1$$

$$R: 0 \le y \le \pi$$

unit circle
amplitude
period
phase shift
vertical shift
graphs of 6 basics

Complex Numbers

$$r = \sqrt{a^2 + b^2} \text{ and } \tan\theta = \frac{b}{a}$$

Area of Triangle

$$K = \frac{1}{2}ab\sin C$$

Conics

General Form

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

where A, C, D, E, F $\in$ I

Standard Form

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = \pm 1$$

$$y - k = a(x-h)^2$$

$$x - h = a(y-k)^2$$

Appendix II - The "Elusive Formulas" continued

Distance
1 foot = 12 inches
1 yard = 3 feet
1 mile = 5,280 feet
1 mile ≈ 1.61 kilometers
1 inch = 2.54 centimeters
1 foot = 0.3048 meters
1 meter = 1,000 millimeters
1 meter = 100 centimeters
1 kilometer = 1,000 meters
1 kilometer ≈ 0.62 miles

Area
1 square foot = 144 square inches
1 square yard = 9 square feet
1 acre = 43,560 square feet

Volume
1 cup = 8 fluid ounces
1 quart = 4 cups
1 gallon = 4 quarts
1 gallon = 231 cubic inches
1 liter ≈ 0.264 gallons
1 cubic foot = 1,728 cubic inches
1 cubic yard = 27 cubic feet
1 board foot = 1 inch by 12 inches by 12 inches

Weight
1 ounce ≈ 28.350 grams
1 pound = 16 ounces
1 pound ≈ 453.592 grams
1 milligram = 0.001 grams
1 kilogram = 1,000 grams
1 kilogram ≈ 2.2 pounds
1 ton = 2,000 pounds

Electricity
1 kilowatt-hour = 1,000 watt-hours
amps = watts ÷ volts

Temperature
°C = (5/9)(°F – 32)
°F = (°C)(9/5) + 32

Appendix II - The "Elusive Formulas" continued

Definitions

Multiplication Principle.
If one of K objects must be chosen and 1 of M other objects must be chosen and 1 of N other objects must be chosen then there are KMN ways to do this.

Permutations
A permutation is an arrangement of a number of objects in all possible ways. Order counts and without replacement. The formula for the number of permutations of n things taken r at a time:

$$_nP_r = \frac{n!}{(n-r)!}$$

Permutations Of Objects Not All Different
Given n objects of which r are identical and s are identical and t are identical, the number of permutations is

$$\frac{n!}{r!\,s!\,t!}$$

Combinations
A combination is similar to a permutation, except that order does not count (and still no replacement). The formula for the number of combinations of n things taken r at a time is:

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

Arrangements With Replacement
The number of arrangements with replacement of n things taken r at a time is n^r.

Fundamental Rule of Probability.
If there are n equally likely outcomes in a sample space, and event E consists of k outcomes, then the probability of E is: P(E)=k/n.

Independent Events.
"Independent" means the outcome of one event does not affect the outcome of the other. If A and B are independent: $P(A \text{ and } B) = P(A) * P(B)$

Dependent Events; Conditional Probability
If two events A and B are not independent, then the probability of A and then B involves the conditional probability of B given that A has happened:

$$P(A \text{ and B}) = P(A) * P(B|A)$$

Mutually Exclusive Events.
"Mutually exclusive" means the two events cannot both happen. If A and B are mutually exclusive:

$$P(A \text{ or } B) = P(A) + P(B)$$

Complementary Events
A and (not A) are complementary events. The sum of their probabilities is 1.

$P(A) + P(not\ A) = 1$. Equivalently: $P(A) = 1 - P(not\ A)$.

Expected Value
Assume an experiment has n mutually exclusive events (E_1, E_2, …, E_n) with probabilities ($P(E_1)$, $P(E_2)$, …, $P(E_n)$), and assume the probabilities add up to 1 (that is, there are no other possible events). If each event has a numeric value associated with it, called the payoff, ($Pay(E_1)$, $Pay(E_2)$, …, $Pay(E_n)$), then the expected value of the experiment is:

$$EV = \sum_{i=1}^{n} P(E_i) * Pay(E_i) = P$$

Expected value is a weighted average of the values associated with each event, where the weights are the probabilities.

Binomial Probability
If an experiment can have only two outcomes (like flipping a coin), and one outcome (called "success") has probability p and the other outcome (called "failure") has probability q = 1-p, then the probability of k successes in n trials is:

$$P(k, n, p) = {}_nC_k\, p^k q^{n-k}$$

Note that if p = q =1/2, then

$$P(n, k, p) = {}_nC_k\, p^k q^{n-k} = {}_nC_k \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{n-k} = {}_nC_k \left(\frac{1}{2}\right)^n = \frac{{}_nC_k}{2^n}$$

Also note that

$$\sum_{k=0}^{n} {}_nC_k = 2^n$$

http://www.math.com/tables/

Appendix

III: The "Elusive Formulas" - Part 2

The "Elusive Formulas"[2]

2nd Edition: finalized August 1, 2001
Original Edition: finalized May 23, 2001

Section A – Symbol Table

$\forall$	for all
$\exists$	there exists
$\varnothing$	the empty set
$\in$	is an element of
$\notin$	is not an element of
$\mathbb{N}$, $\mathbb{N}^+$	the set of natural numbers
$\mathbb{Z}$	the set of integers
$\mathbb{Q}$	the set of rational numbers
$\mathbb{R}$	the set of real numbers
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\vee$	or
$\wedge$	and
$\cap$	union
$\cup$	intersection
$\Rightarrow$	implies
$\Leftrightarrow$, iff	is equivalent to
$\sum_{i=1}^{n} a_i$	$a_1+a_2+a_3+a_4+a_5+...+a_n$
$\prod_{i=1}^{n} a_i$	$a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5 \cdot \ldots \cdot a_n$
(a,b) = d	d is the gcd of a and b
[a,b] = d	d is the lcm of a and b
$\tau(a)$	number of factors of a
$\sigma(a)$	sum of the factors of a
$\varphi(a)$	Euler Phi Function
$\mu(a)$	Mobius Function
$\lvert a \rvert$	absolute value of a
$\lfloor a \rfloor$	greatest integer function
$\lceil a \rceil$	least integer function
a:b:c	ratio of a to b to c
a:b:c::d:e:f	ratio of a to b to c=ratio of d to e to f
π	pi ≈ 3.141592653589793…
e	euler number ≈ 2.718281828459…
$\log_b(a) = c$	$b^c = a$
$\log(a) = c$	$10^c = a$
n!	n(n–1)(n–2)(n–3)(n–4)…3×2×1
${}_nP_r$	$\frac{n!}{r!}$ = n(n–1)(n–2)…(n–r+1)
${}_nC_r$ or $\binom{n}{r}$	$\frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)...(n-r+1)}{n(n-1)(n-2)...(2)(1)}$

$a \equiv b \pmod{c}$ a and b leave the same remainder when divided by c.

Appendix III - The "Elusive Formulas" continued

Section B – Algebra

- $(a \pm b)^3 = a^3 \pm b^3$ iff $a = 0$ or $b = 0$ or $(a \pm b) = 0$
- $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$
- $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
- $a^4 + b^4 + c^4 - 2a^2b^2 - 2b^2c^2 - 2c^2a^2 = -16s(s - a)(s - b)(s - c)$ when $2s = a+b+c$
- $a^n + b^n = (a + b)(a^{n-1} + b^{n-1}) - ab(a^{n-2} + b^{n-2})$
- $a^n \pm b^n = (a \pm b)(a^{n-1} \mp a^{n-2}b + a^{n-3}b^2 \mp a^{n-4}b^3 + \ldots + a^2b^{n-3} \mp ab^{n-2} + b^{n-1})$ [$a^n + b^n$ is only true for odd n.]
- $(a \pm b)^n = {}_nC_0a^n \pm {}_nC_1a^{n-1}b + {}_nC_2a^{n-2}b^2 \pm {}_nC_3a^{n-3}b^3 + {}_nC_4a^{n-4}b^4 \pm \ldots \pm {}_nC_{n-2}a^2b^{n-2} + {}_nC_{n-1}ab^{n-1} + {}_nC_nb^n$
- $a(a+1)(a+2)(a+3) = (a^2+3a+1)^2 - 1$

Arithmetic Series: If $a_1, a_2, a_3, \ldots, a_n$ are in arithmetic series with common difference d:	
n^{th} term in terms of m^{th} term	$a_n = a_m + (n - m)d$
Sum of an arithmetic series up to term n	$\sum_{i=1}^{n} a_i = \frac{n(a_1 + a_n)}{2} = \frac{n(2a_1 + (n-1)d)}{2}$
Geometric Series: If $a_1, a_2, a_3, \ldots, a_n$ are in geometric series with common ratio r:	
n^{th} term of a geometric series	$a_n = a_1r^{n-1}$
Sum of a non-constant ($r \neq 1$) geometric series up to term n	$\sum_{i=1}^{n} a_i = \frac{a_1(1 - r^n)}{1 - r}$
Sum of an infinite geometric series	$\sum_{i=1}^{\infty} a_i = \frac{a_1}{1 - r}$ iff $\lvert r \rvert < 1$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$	$\sum_{i=1}^{n} i^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$

If $P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + a_{n-3}x^{n-3} + \ldots + a_1x + a_0 = 0$, a_i is a constant, then	
Sum of roots taken one at a time (the sum of the roots)	$\sum r_i = \frac{-a_{n-1}}{a_n}$
Sum of roots taken two at a time	$\sum_{i \neq j} r_ir_j = \frac{a_{n-2}}{a_n}$
Sum of roots taken p at a time	$\sum_{i \neq j \neq \ldots \neq k} r_ir_j\ldots r_k = (-1)^p \frac{a_{n-p}}{a_n}$
Rational Root Theorem	
If $P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + a_{n-3}x^{n-3} + \ldots + a_1x + a_0$ is a polynomial with integer coefficients and $\frac{b}{c}$ is a rational root of the equation $P(x) = 0$ (where $(b, c) = 1$), then $b \mid a_0$ and $c \mid a_n$.	

- If P(x) is a polynomial with real coefficients and $P(a + bi) = 0$, then $P(a - bi) = 0$.
- If P(x) is a polynomial with rational coefficients and $P(a + b\sqrt{c}) = 0$, then $P(a - b\sqrt{c}) = 0$.

Appendix III - The "Elusive Formulas" continued

Section C – Number Theory

- Number Theory mainly concerns $\mathbb{Z}$ and $\mathbb{N}$, all variables exist in $\mathbb{Z}$ unless stated otherwise

Divisibility: $\forall a,b\in\mathbb{Z}$, $a\neq 0$: $a \mid b \Leftrightarrow \exists k\in\mathbb{Z}$ such that $ak = b$		
$1\mid a,\ a\mid 0,\ a\mid(\pm a)$	$a\mid b \Rightarrow a\mid bc$	$a\mid b \wedge b\mid c \Rightarrow a\mid c$
$a\mid 1 \Leftrightarrow a=\pm 1$	$a\mid b \wedge a\mid c \Rightarrow a\mid(b\pm c)$	$a\mid bc \wedge (a,b)=1 \Rightarrow a\mid c$
$a\mid b \wedge b\mid a \Leftrightarrow a=\pm b$	$a\mid b \wedge c\mid d \Rightarrow ab\mid cd$	$a\mid c \wedge b\mid c \wedge (a,b)=1 \Rightarrow ab\mid c$
Modulo Congruence: $\forall a,b,m\in\mathbb{Z}$, $m\neq 0$: $a \equiv b \pmod m \Leftrightarrow m \mid (a-b)$		
Suppose that $a\equiv b \pmod m$, $c\equiv d \pmod m$, and p is prime; then:		
$a\pm g \equiv c\pm g \pmod m$	$a\pm b \equiv c\pm d \pmod m$	$(g,p)=1 \Rightarrow g^{p-1} \equiv 1 \pmod p$
$ag \equiv cg \pmod m$	$ab \equiv cd \pmod m$	$(p-1)! \equiv -1 \pmod p$
$(g,m)=1 \Rightarrow g^{\varphi(m)} \equiv 1 \pmod m$	$hf\equiv hg \pmod m \wedge (m,h)=1 \Rightarrow f\equiv g \pmod m$	

Fibonacci Sequence

- Sequence of integers beginning with two 1's and each subsequent term is the sum of the previous 2 terms.
- 1,1,2,3,5,8,13,21,34,55,89,144, ...
- $F(1)=F(2)=1$, for $n\geq 3$, $F(n)=F(n-1)+F(n-2)$
- Let ψ = Golden Ratio = $\dfrac{\left(\sqrt{5}+1\right)}{2}$, then $F(n) = \dfrac{\psi^n - (-\psi)^{-n}}{\sqrt{5}}$
- $F(n)\bullet F(n+3) - F(n+1)\bullet F(n+2) = (-1)^n$

Farey Series [F_n]

- Ascending sequence of irreducible fractions between 0 and 1 inclusive whose denominator is $\leq n$
- $F_3 = \frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}$; $F_7 = \frac{0}{1}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{2}{7}, \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{1}{2}, \frac{4}{7}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{1}{1}$
- if $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ are successive terms in F_n, then $bc–ad = de–cf = 1$ and $\frac{c}{d} = \frac{a+e}{b+f}$

Number Theory Functions

The following number theory functions have the property that if $(a,b)=1$, then $f(a\times b)=f(a)\times f(b)$

Tau Function: Number of factors of n: $\tau(n) = \prod_{i=1}^{m}\left(1+\alpha_i\right)$

Sigma Function: Sum of factors of n: $\sigma(n) = \prod_{i=1}^{m}\left(\sum_{j=0}^{\alpha_i}\left(p_i^{\,j}\right)\right) = \prod_{i=1}^{m}\left(\frac{p_i^{\,1+\alpha_i}-1}{p_i-1}\right)$

Euler Phi Function: Number of integers between 0 and n that are relatively prime to n

$$\varphi(n) = \prod_{i=1}^{m}\left(p_i^{\,\alpha_i} - p_i^{\,\alpha_i-1}\right) = n\prod_{i=1}^{m}\left(1-\frac{1}{p_i}\right)$$

Mobius Function: $\mu(n) = \begin{cases} 0 & \text{if n is divisible by any square} \geq 1 \\ \text{otherwise:} & \\ 1 & \text{if n is has an even number of prime factors} \\ -1 & \text{if n is has an odd number of prime factors} \end{cases}$

Appendix III - The "Elusive Formulas" continued

Divisibility Rules		
Given integer k expressed in base $n \geq 2$, $k = a_0 + a_1 n + a_2 n^2 + a_3 n^3 + \ldots = \sum_{i=0}^{\infty}\left(a_i n^i\right)$, $0 \leq a_i < n$		
Note: $\left(\overline{a_m a_{m-1} \ldots a_0}\right)_n = \sum_{i=0}^{\infty}\left(a_i n^i\right)$, secondary subscript omission implies base 10: $\overline{a_m a_{m-1} \ldots a_0} = \sum_{i=0}^{\infty}\left(10^i a_i\right)$		
Divisor (d)		**Criterion**
Basic/Specific	3, 9	If $a_0 + a_1 + a_2 + a_3 + a_4 + \ldots$ is divisible by 3 or 9
	11	If $a_0 - a_1 + a_2 - a_3 + a_4 - \ldots$ is divisible by 11
	7, 13	If $\overline{a_2 a_1 a_0} - \overline{a_5 a_4 a_3} + \overline{a_8 a_7 a_6} - \overline{a_{11} a_{10} a_9} + \ldots$ is divisible by 7 or 13
	2^m, 5^m	If $\overline{a_{m-1} a_{m-2} a_{m-3} \ldots a_0}$ is divisible by 2^m or 5^m
	7	Truncate rightmost digit and subtract twice the value of said digit from the remaining integer. Repeat this process until divisibility test becomes trivial.
General	$d \mid n^m$	If $\left(\overline{a_{m-1} a_{m-2} a_{m-3} a_{m-4} \ldots a_0}\right)_n$ is divisible by d
	factor of $n^m - 1$	If $\left(\overline{a_{m-1} a_{m-2} \ldots a_1 a_0}\right)_n + \left(\overline{a_{2m-1} a_{2m-2} \ldots a_{m+1} a_m}\right)_n + \left(\overline{a_{3m-1} a_{3m-2} \ldots a_{2m+1} a_{2m}}\right)_n + \ldots$ is divisible
	factor of $n^m + 1$	If $\left(\overline{a_{m-1} a_{m-2} \ldots a_1 a_0}\right)_n - \left(\overline{a_{2m-1} a_{2m-2} \ldots a_{m+1} a_m}\right)_n + \left(\overline{a_{3m-1} a_{3m-2} \ldots a_{2m+1} a_{2m}}\right)_n - \ldots$ is divisible
	$d = xy$, $(x,y)=1$	(x | k and y | k) $\Leftrightarrow$ d | k
	$d \mid kn \pm 1$	Truncate rightmost digit and add $\mp k$ times the value of said digit from the remaining integer. Repeat this process until divisibility test becomes trivial.

Section D –Logarithms

For b an integer >1, $\log_b(a) = c \Leftrightarrow b^c = a$	$\log_b(b) = 1$	$\log_b(1) = 0$
$\log(a^c) = c \log(a)$	$a^{\log_a(b)} = b$	$\log\left(\frac{ab}{c}\right) = \log(a) + \log(b) - \log(c)$
$\log_a(b) \cdot \log_b(c) = \log_a(c)$	$\log_a(b) \cdot \log_b(a) = 1$	$a^{\log(b)} = b^{\log(a)}$

Section E – Analytic Geometry

Distance between line ax + by + c = 0 and point (x_0, y_0) in 2D plane:	Distance between the plane ax + by + cz + d = 0 and point (x_0, y_0, z_0) in 3D space:
$\frac{\lvert x_0 a + y_0 b + c \rvert}{\sqrt{a^2 + b^2}}$	$\frac{\lvert x_0 a + y_0 b + z_0 c + d \rvert}{\sqrt{a^2 + b^2 + c^2}}$

Section F – Inequalities

- $\mathbb{R}^+$: the set of all positive real numbers; $\mathbb{R}^-$: the set of all negative real numbers
- $a^2 + b^2 \geq 2ab$; $a^2 + b^2 + c^2 \geq ab + bc + ca$; $3(a^2 + b^2 + c^2 + d^2) \geq 2(ab + bc + cd + da + ac + bd)$
- The "quadratic-arithmetic-geometric-harmonic mean inequality:" for $a_i > 0$

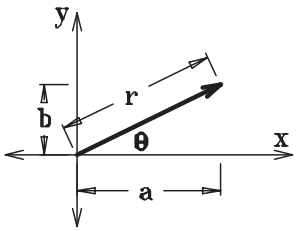
  $$\frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_n}} \leq \sqrt[n]{a_1 a_2 a_3 \ldots a_n} \leq \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n} \leq \sqrt{\frac{a_1^2 + a_2^2 + a_3^2 + \ldots + a_n^2}{n}}$$, with equalities holding iff $a_1 = a_2 = a_3 = a_4 = \ldots = a_n$.
- If constant $k>1$ and large x: $1 < k^{1/x} < x^{1/x} < \log(x) < x^{1/k} < x < x\log(x) < x^k < x^{\log(x)} < k^x < x! < x^x$

Appendix III - The "Elusive Formulas" continued

- Cauchy-Schwarz Inequality- For 2nd degree: $(a_1b_1+a_2b_2)^2 \le (a_1^2+a_2^2)(b_1^2+b_2^2)$ with equality holding iff $a_1:a_2::b_1:b_2$. In general, for any 2 sequences of real numbers, a_i and b_i, each of length n:
  $(a_1b_1+a_2b_2+a_3b_3+\ldots+a_nb_n)^2 \le (a_1^2+a_2^2+a_3^2+\ldots+a_n^2)(b_1^2+b_2^2+b_3^2+\ldots+b_n^2)$ with equality holding iff $a_1:a_2:a_3:\ \ldots\ :a_n::b_1:b_2:b_3:\ \ldots\ :b_n$.
- Chebyshev's Inequality- If $0\le a_1\le a_2\le a_3\le \ldots \le a_n$, $0\le b_1\le b_2\le b_3\le \ldots \le b_n$, then:
  $(a_1+a_2+a_3+\ldots+a_n)\,(b_1+b_2+b_3+\ldots+b_n) \le n\bullet(a_1b_1+a_2b_2+a_3b_3+\ldots+a_nb_n)$
- Jensen's Inequality- For a convex function f(x): $f(a_1)+f(a_2)+f(a_3)+\ldots+f(a_n) \ge n\bullet f\left(\frac{a_1+a_2+\ldots+a_n}{n}\right)$. More generally, if $b_1+b_2+\ldots+b_n=1$ and $b_i>0$, then: $b_1f(a_1)+b_2f(a_2)+b_3f(a_3)+\ldots+b_nf(a_n) \ge f(b_1a_1+b_2a_2+b_3a_3+\ldots+a_n)$

Section G – Number Systems

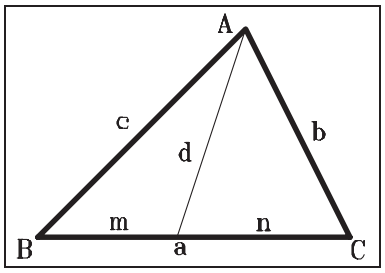
- $\mathbb{N}$ = natural numbers: 1, 2, 3, 4, 5, ...
- Algebraic numbers: numbers that can be solutions to polynomial equations with integer coefficients: $\sqrt{2}$, $\sqrt[5]{23}$, $\sqrt{\sqrt[5]{23}+\sqrt[7]{5}}$, ...
- Transcendental numbers: numbers that cannot be solutions to polynomials: *e*, π, ...
  - π is the ratio of the length of the circumference to the length of the diameter of a circle
  - $e = \lim_{x\to\infty}\left(\left(1+\frac{1}{x}\right)^x\right)$
- if we define the square root of –1 to be *i*, then:
  - $\mathbb{C}$ = complex numbers = a+b*i*, where $a,b\in\mathbb{R}$

Complex Numbers in Rectangular & Polar		$a^2 + b^2 = r^2$; $\tan\theta = \frac{b}{a}$; $a = r\bullet\cos\theta$; $b = r\bullet\sin\theta$ $Z = a + bi = r\bullet\text{cis}\,\theta$ (polar form of a complex number) The magnitude of Z, represented by $\lvert a+bi\rvert = \sqrt{a^2+b^2}$
	$e^{i\theta} = \cos\theta + i\sin\theta = \text{cis}\,\theta$	$\text{cis}\,(\theta+\gamma) = \text{cis}\,\theta\bullet\text{cis}\,\gamma$
	$(a+bi)^n = (r\,\text{cis}\,\theta)^n = r^n\bullet\text{cis}(n\theta)$	$\text{cis}\,(\theta-\gamma) = \frac{\text{cis}\,\theta}{\text{cis}\,\gamma}$

Appendix III - The "Elusive Formulas" continued

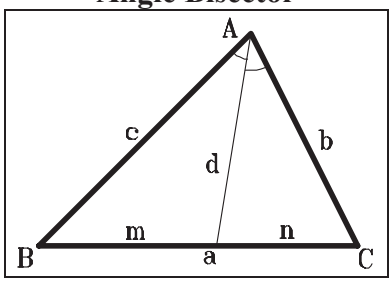
Section H – Euclidean Geometry I (The Triangle)

Stewart's Theorem



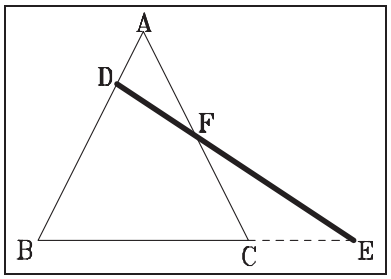
$man + dad = bmb + cnc$

Angle Bisector



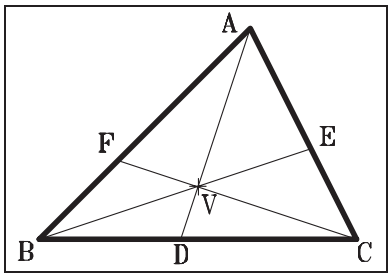
$bm = cn; \qquad d^2 = bc - mn$

Menelaus' Theorem



$\overline{AD}\cdot\overline{BE}\cdot\overline{CF} = \overline{DB}\cdot\overline{EC}\cdot\overline{FA}$

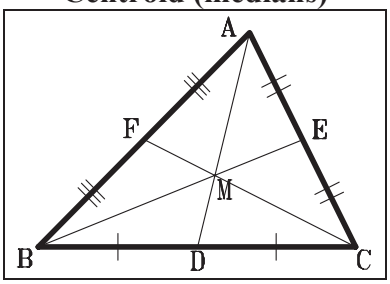
Ceva's Theorem



$\overline{AF}\cdot\overline{BD}\cdot\overline{CE} = \overline{AE}\cdot\overline{BF}\cdot\overline{CD}$

$$\frac{\overline{VD}}{\overline{AD}} + \frac{\overline{VE}}{\overline{BE}} + \frac{\overline{VF}}{\overline{CF}} = 1$$

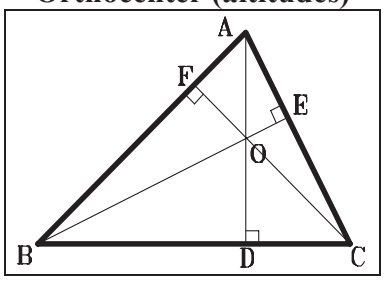
Centroid (medians)



$$\frac{\overline{AM}}{\overline{MD}} = \frac{\overline{BM}}{\overline{ME}} = \frac{\overline{CM}}{\overline{MF}} = 2$$

$K_{AFM} = K_{FBM} = K_{BDM} = K_{DCM} = K_{CEM} = K_{EAM} = \frac{1}{6} K_{ABC}$

Orthocenter (altitudes)

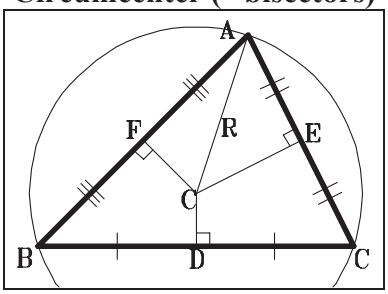


$\Delta_{AFC} \sim \Delta_{AEB} \sim \Delta_{OEC} \sim \Delta_{OFB}$

$\Delta_{BDA} \sim \Delta_{BFC} \sim \Delta_{OFA} \sim \Delta_{ODC}$

$\Delta_{CEB} \sim \Delta_{CDA} \sim \Delta_{ODB} \sim \Delta_{OEA}$

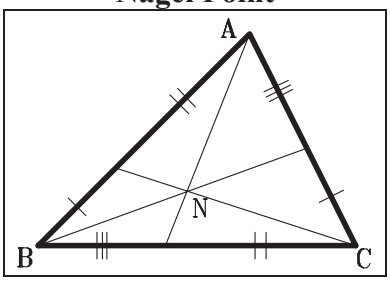
Circumcenter (⊥bisectors)



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

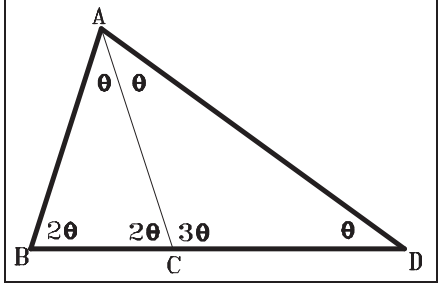
Extended Law of Sines

Nagel Point



Joins semi-perimeter points to vertices

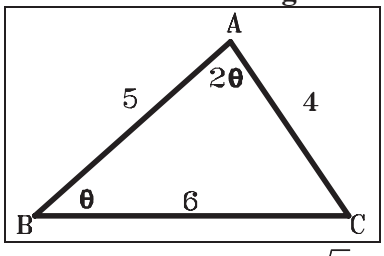
Golden Triangle



$\Delta_{ABC} \sim \Delta_{DAB}$; $\theta = 36° = \pi/5$

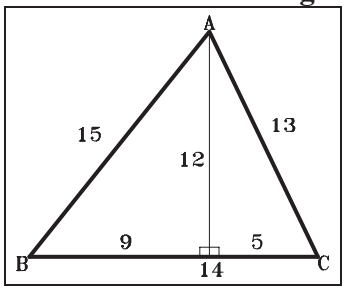
$$\frac{\overline{CD}}{\overline{BC}} = \frac{\overline{BC} + \overline{CD}}{\overline{CD}} = \frac{\sqrt{5}+1}{2}$$

The 4-5-6 Triangle



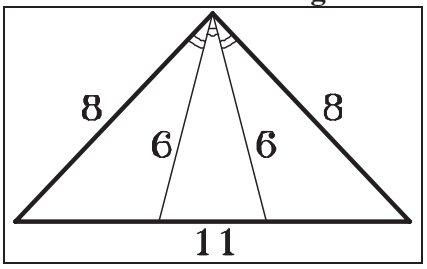
$A = 2B; \qquad K = \frac{15\sqrt{7}}{4}$

The 13-14-15 Triangle



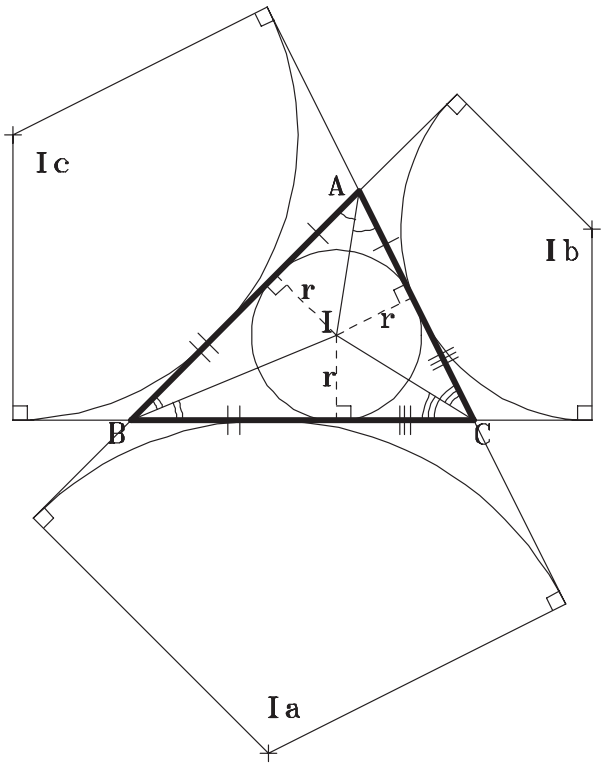
$K = 84; \; R = \frac{65}{8}; \; r = 4$

The 8-8-11 Triangle



Trisectors of the largest angle has length 6

Appendix III - The "Elusive Formulas" continued



A Triangle and Its Circles

Δ_{ABC} has sides a, b and c and angles A, B, and C.
The radius of the inscribed circle is r.
The radius of the circumscribed circle is R.
The area of the triangle is K.
The semi-perimeter of the triangle is s.
The altitude to sides a, b, c are h_a, h_b, h_c respectively.
The angle bisectors to angles A, B, C are t_a, t_b, t_c respectively.
The medians to side a, b, c are m_a, m_b, m_c respectively.
The circles tangent to each line $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CA}$ and directly next to sides a, b, c are called ex-circles I_a, I_b, I_c respectively.
The radii to ex-circles I_a, I_b, I_c are r_a, r_b, r_c respectively.
The distance from I to circumcenter is d.

Area Formulas of the Triangle					
$K=\frac{c \cdot h_c}{2}$	$K=\frac{ab\sin C}{2}$	$K=\frac{c^2 \sin A \sin B}{2\sin C}$	$K=\frac{abc}{4R}$	$K=rs$	$K=\sqrt{s(s-a)(s-b)(s-c)}$
For planar triangle with vertices $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, $P_3(x_3\ y_3)$					
$K=\frac{1}{2}\left\|\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}\right\|$			Coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$		

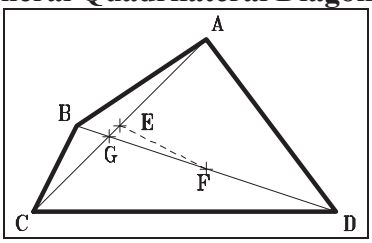
Basic Edge Inequalities	a+b>c, b+c>a, c+a>b
Basic Angle Identities	A+B+C = 180°, {a,b,c}⊂ (0,π)
Law of Cosines	$a^2 + b^2 = c^2 + 2ab\cos C$
Law of Tangents	tan(A)tan(B)tan(C) = tan(A)+tan(B)+tan(C)

Assorted Identities			
$r_a r_b + r_b r_c + r_c r_a = s^2$	$D^2 = R^2 - 2Rr$	$4m_c{}^2 = 2a^2 + 2b^2 + c^2$	$r_a + r_b + r_c - r = 4R$
$r = \frac{c\sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{C}{2}}$	$r_c = \frac{K}{s-c}$	$r^2 = \frac{(s-a)(s-b)(s-c)}{s}$	$\frac{1}{r} = \frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c}$
$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$	$\tan\frac{C}{2} = \frac{r}{s-c}$	$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$	$\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$
$t_c = \frac{2\sqrt{a \cdot b \cdot s(s-c)}}{a+b}$	$t_c = \frac{2ab\cos\frac{C}{2}}{a+b}$	$\frac{3}{4} \le \frac{m_a + m_b + m_c}{a+b+c} \le 1$	$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$

Appendix III - The "Elusive Formulas" continued

Section I – Euclidean Geometry II (The Quadrilateral)

General Quadrilateral Diagonals



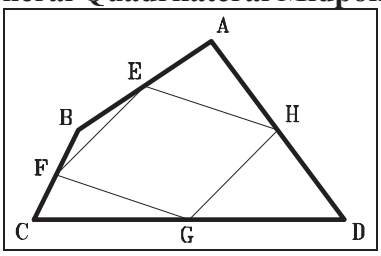
E and F are midpoints of $\overline{AC}$ and $\overline{BD}$

$K_{GAB} \bullet K_{GCD} = K_{GBC} \bullet K_{GDA}$

$K = \frac{1}{2}\overline{AC}\,\overline{BD}\sin\angle AGB$

$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{DA}^2 = \overline{AC}^2 + \overline{BD}^2 + 4\overline{EF}^2$

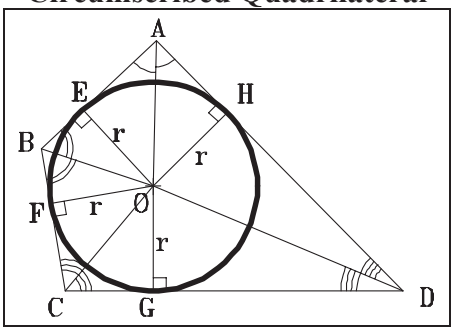
General Quadrilateral Midpoints



If $\frac{\overline{AH}}{\overline{HD}} = \frac{\overline{DG}}{\overline{GC}} = \frac{\overline{CF}}{\overline{FB}} = \frac{\overline{BE}}{\overline{EA}} = n$

Then: $\frac{K_{EFGH}}{K_{ABCD}} = \frac{n^2+1}{(n+1)^2}$

Circumscribed Quadrilateral

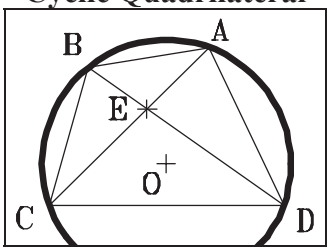


$\overline{AB} + \overline{CD} = \overline{BC} + \overline{AD} = s$; $K_{ABCD} = rs$

If $Quad_{ABCD}$ is also cyclic, then

$K = \sqrt{\overline{AB}\,\overline{CD}\,\overline{BC}\,\overline{AD}}$

Cyclic Quadrilateral



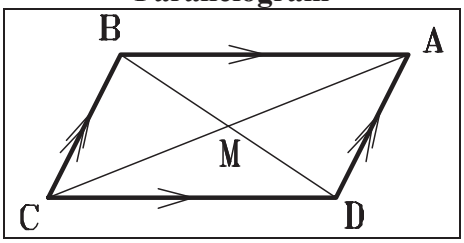
$A + C = B + D = 180°$

$K_{ABCD} = \sqrt{\left(s-\overline{AB}\right)\left(s-\overline{BC}\right)\left(s-\overline{CD}\right)\left(s-\overline{DA}\right)}$

$\overline{BC}\,\overline{AD} + \overline{AB}\,\overline{CD} = \overline{BD}\,\overline{AC}$

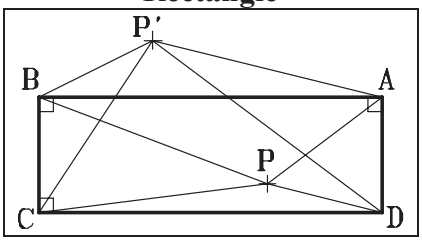
$\overline{AC}\left(\overline{BC}\,\overline{CD} + \overline{DA}\,\overline{AB}\right) = \overline{BD}\left(\overline{AB}\,\overline{BC} + \overline{CD}\,\overline{DA}\right)$

Parallelogram



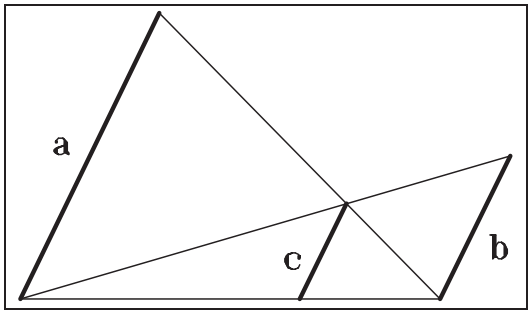
$2(\overline{BC}^2 + \overline{BA}^2) = \overline{BD}^2 + \overline{AC}^2$

Rectangle



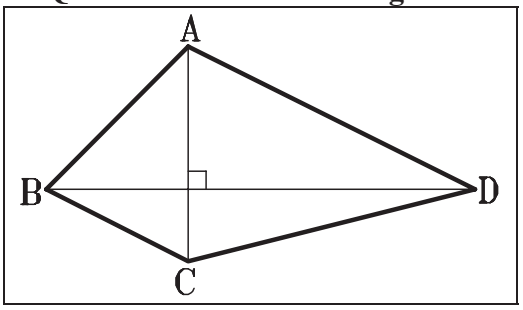
For all point P: $\overline{PA}^2 + \overline{PC}^2 = \overline{PB}^2 + \overline{PD}^2$

Three Pole Problem



if a || b || c, then $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$

Quadrilateral with ⊥ Diagonals



$\overline{AC} \perp \overline{BD} \Rightarrow K = \frac{1}{2}\overline{AC}\,\overline{BD}$

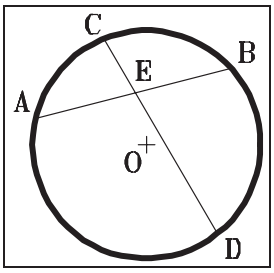
$\overline{AB}^2 + \overline{CD}^2 = \overline{BC}^2 + \overline{DA}^2$

Ptolemy's Theorem:

In any $Quad_{ABCD}$, $\overline{BD}\,\overline{AC} \le \overline{BC}\,\overline{AD} + \overline{AB}\,\overline{CD}$, with equality holding iff $Quad_{ABCD}$ is cyclic.

Section J – Euclidean Geometry III (The Circle)

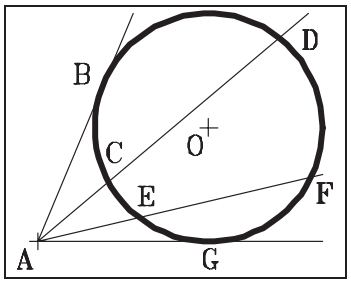
Circles



$\angle AEC = \angle BED = \frac{1}{2}\left(\overset{\frown}{AC} + \overset{\frown}{BD}\right)$

Power of the point: $\overline{AE} \cdot \overline{BE} = \overline{CE} \cdot \overline{DE}$

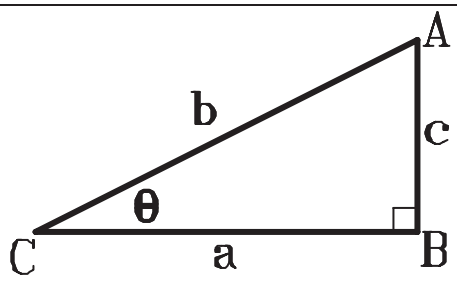
Circles 2



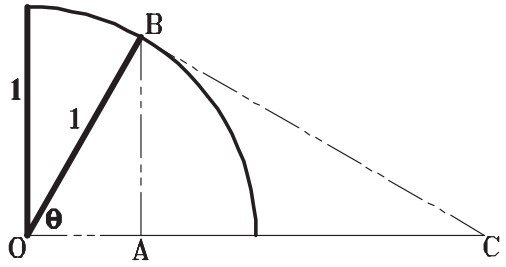
$\overline{AB} = \overline{AG}$; $\angle DAF = \frac{1}{2}\left(\overset{\frown}{DF} - \overset{\frown}{CE}\right)$

$\overline{AB}^2 = \overline{AD} \cdot \overline{AC} = \overline{AF} \cdot \overline{AE}$

Section K – Trigonometry



$\sin\theta = {}^{c}/_{b}$; $\cos\theta = {}^{a}/_{b}$; $\tan\theta = {}^{c}/_{a}$



$\sin\theta = \overline{AB}$; $\cos\theta = \overline{OA}$; $\tan\theta = \overline{BC}$

θ	15°	18°	30°	36°	45°	54°	60°	75°
sin θ	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{5}-1}{4}$	${}^{1}/_{2}$	$\frac{\sqrt{2(5-\sqrt{5})}}{4}$	${}^{\sqrt{2}}/_{2}$	$\frac{\sqrt{5}+1}{4}$	${}^{\sqrt{3}}/_{2}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$
cos θ	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{2(5+\sqrt{5})}}{4}$	${}^{\sqrt{3}}/_{2}$	$\frac{\sqrt{5}+1}{4}$	${}^{\sqrt{2}}/_{2}$	$\frac{\sqrt{2(5-\sqrt{5})}}{4}$	${}^{1}/_{2}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$
tan θ	$2-\sqrt{3}$	$\frac{(\sqrt{5}-1)\sqrt{2}}{2\sqrt{5+\sqrt{5}}}$	${}^{\sqrt{3}}/_{3}$	$\frac{\sqrt{2(5-\sqrt{5})}}{\sqrt{5}+1}$	1	$\frac{(\sqrt{5}+1)\sqrt{2}}{2\sqrt{5-\sqrt{5}}}$	$\sqrt{3}$	$2+\sqrt{3}$

Pythagorean	Odd-Even Functions	Summation of Angles
$\sin^2\theta + \cos^2\theta = 1$	$\sin(-\theta) = -\sin(\theta)$	$\sin(\theta \pm \gamma) = \sin(\theta)\cos(\gamma) \pm \cos(\theta)\sin(\gamma)$
$1 + \tan^2\theta = \sec^2\theta$	$\cos(-\theta) = \cos(\theta)$	$\cos(\theta \pm \gamma) = \cos(\theta)\cos(\gamma) \mp \sin(\theta)\sin(\gamma)$
$1 + \cot^2\theta = \csc^2\theta$	$\tan(-\theta) = -\tan(\theta)$	$\tan(\theta \pm \gamma) = \frac{\tan(\theta) \pm \tan(\gamma)}{1 \mp \tan(\theta)\tan(\gamma)}$

Multiple Angles			
	$\sin 2\theta = 2\sin\theta\cos\theta$	$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$	$\sin 4\theta = 4 \bullet \sin\theta \bullet \cos\theta \bullet (\cos^3\theta - \sin\theta)$
	$\cos 2\theta = \cos^2\theta - \sin^2\theta$	$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$	$\cos 4\theta = \sin^4\theta + \cos^4\theta - 6\cos^2\theta \bullet \sin^2\theta$
	$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$	$\tan 3\theta = \frac{\tan^3\theta - 3\tan\theta}{3\tan^2\theta - 1}$	$\tan 4\theta = \frac{4\tan\theta(1-\tan^2\theta)}{\tan^4\theta - 6\tan^2\theta + 1}$

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www.nysml.org/Files/Formulas.pdf

Appendix III - The "Elusive Formulas" continued

Sum to Product	Product to Sum
$\sin\theta \pm \sin\gamma = 2\sin\left(\frac{\theta \pm \gamma}{2}\right)\cos\left(\frac{\theta \mp \gamma}{2}\right)$	$\sin\theta \bullet \sin\gamma = \frac{1}{2}[\cos(\theta-\gamma) - \cos(\theta+\gamma)]$
$\cos\theta + \cos\gamma = 2\cos\left(\frac{\theta+\gamma}{2}\right)\cos\left(\frac{\theta-\gamma}{2}\right)$	$\cos\theta \bullet \cos\gamma = \frac{1}{2}[\cos(\theta-\gamma) + \cos(\theta+\gamma)]$
$\cos\theta - \cos\gamma = -2\sin\left(\frac{\theta+\gamma}{2}\right)\sin\left(\frac{\theta-\gamma}{2}\right)$	$\sin\theta \bullet \cos\gamma = \frac{1}{2}[\sin(\theta-\gamma) + \sin(\theta+\gamma)]$
$\tan\theta \pm \tan\gamma = \frac{\sin(\theta \pm \gamma)}{\cos\theta \square \cos\gamma}$	$\tan\theta \bullet \tan\gamma = \frac{\cos(\theta-\gamma) - \cos(\theta+\gamma)}{\cos(\theta-\gamma) + \cos(\theta+\gamma)}$

Square Identities	Cube Identities	½ Angle Identities	tan (θ/2) Identities
$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$	$\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$	$\sin\theta/2 = \pm\sqrt{\frac{1-\cos\theta}{2}}$	$\sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$
$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$	$\cos^3\theta = \frac{3\cos\theta + \cos 3\theta}{4}$	$\cos\theta/2 = \pm\sqrt{\frac{1+\cos\theta}{2}}$	$\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$
$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$	$\tan^3\theta = \frac{3\sin\theta - \sin 3\theta}{3\cos\theta + \cos 3\theta}$	$\tan\theta/2 = \pm\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$	

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Programs Used:

Math Type 4, 5
CadKey 5
Geometer's Sketchpad 3, 4
Microsoft Word XP
Mathematica 4.1

References:

IMSA – Noah Sheets
Bronx Science High School – Formula Sheets, Math Bulletin

Appendix IV

NCTM Standards - AMC 8 Worksheets

This is a listing of all the Worksheets produced thus far (2004-05, 2005-06, 2006-07 and 2007-08) in an NCTM categorized list for easy reference.

Q #	Difficulty	Standard	Definition	MC yr	page
m01-17	MH - 30.26	Algebra	Analyze change in various contexts.	05-06	60
m01-17	MH - 30.26	Algebra	Analyze change in various contexts.	05-06	60
m00-05	H - 14.24	Algebra	Model and solve contextualized problems using various . . .	07-08	67
m00-20	MH - 25.69	Algebra	Relate and compare different forms of representation for a . . .	07-08	82
m07-20	MH - 26.60	Algebra	Represent and analyze mathematical situations and . . .	08-09	63
m01-12	MH - 29.85	Algebra	Represent and analyze mathematical situations and . . .	05-06	58
sb06-03	ME - 78.81	Algebra	Represent and analyze mathematical situations and . . .	06-07	92
m02-11	ME - 67.55	Algebra	Represent, analyze, and generalize a variety of patterns . . .	04-05	61
sa06-02	MH - 32.49	Algebra	Understand patterns, relations, and functions.	06-07	77
m06-09	MH - 36.92	Algebra	Understand patterns, relations, and functions.	07-08	46
m05-08	M - 52.69	Algebra	Understand patterns, relations, and functions.	06-07	58
m05-12	M - 53.86	Algebra	Understand patterns, relations, and functions.	06-07	62
ta06-06	ME - 79.96	Algebra	Understand patterns, relations, and functions.	06-07	81
m06-22	H - 17.16	Algebra	Use mathematical models to represent and understand . . .	07-08	59
ta06-08	MH - 27.81	Algebra	Use mathematical models to represent and understand . . .	06-07	83
m00-23	MH - 29.51	Algebra	Use mathematical models to represent and understand . . .	07-08	85
sa06-22	MH - 35.17	Algebra	Use mathematical models to represent and understand . . .	06-07	89
sb06-12	M - 45.86	Algebra	Use mathematical models to represent and understand . . .	06-07	99
m06-03	M - 46.39	Algebra	Use mathematical models to represent and understand . . .	07-08	40
m07-04	M - 46.66	Algebra	Use mathematical models to represent and understand . . .	08-09	47
m07-01	E - 84.11	Algebra	Use mathematical models to represent and understand . . .	08-09	44
m99-18	MH - 27.12	Algebra	Use symbolic algebra to represent situations and to solve . . .	08-09	86
m99-19	MH - 35.91	Algebra	Use symbolic algebra to represent situations and to solve . . .	08-09	87
m99-17	M - 42.22	Algebra	Use symbolic algebra to represent situations and to solve . . .	08-09	85
m99-16	M - 44.75	Algebra	Use symbolic algebra to represent situations and to solve . . .	08-09	84
m02-17	M - 58.00	Algebra	Use symbolic algebra to represent situations and to solve . . .	04-05	65
m03-04	ME - 74.39	Algebra	Use symbolic algebra to represent situations and to solve . . .	04-05	72
m01-03	E - 89.49	Algebra	Use symbolic algebra to represent situations and to solve . . .	04-05	44
m00-21	H - 7.90	Data Analysis & Probability	Compute probabilities for simple compound events, using . . .	07-08	83
m05-22	H - 17.63	Data Analysis & Probability	Develop and evaluate inferences and predictions that are . . .	06-07	72
m01-19	H - 19.06	Data Analysis & Probability	Develop and evaluate inferences and predictions that are . . .	05-06	61
m03-24	MH - 26.05	Data Analysis & Probability	Develop and evaluate inferences and predictions that are . . .	05-06	78
m02-06	MH - 26.63	Data Analysis & Probability	Develop and evaluate inferences and predictions that are . . .	05-06	65
m01-20	MH - 26.87	Data Analysis & Probability	Develop and evaluate inferences and predictions that are . . .	05-06	62
m02-10	M - 43.90	Data Analysis & Probability	Develop and evaluate inferences and predictions that are . . .	05-06	69
m03-17	M - 49.96	Data Analysis & Probability	Develop and evaluate inferences and predictions that are . . .	05-06	75
m04-05	M - 59.14	Data Analysis & Probability	Develop and evaluate inferences and predictions that are . . .	05-06	84
m04-10	M - 60.21	Data Analysis & Probability	Develop and evaluate inferences and predictions that are . . .	05-06	89
m02-09	ME - 78.30	Data Analysis & Probability	Develop and evaluate inferences and predictions that are . . .	05-06	68
m02-08	E - 89.23	Data Analysis & Probability	Develop and evaluate inferences and predictions that are . . .	05-06	67
m99-04	E - 92.30	Data Analysis & Probability	Discuss and understand the correspondence between . . .	08-09	72
m01-21	MH - 23.28	Data Analysis & Probability	Find, use, and interpret measures of center and spread, . . .	04-05	54
m99-13	MH - 28.13	Data Analysis & Probability	Find, use, and interpret measures of center and spread, . . .	08-09	81
m02-18	MH - 32.19	Data Analysis & Probability	Find, use, and interpret measures of center and spread, . . .	04-05	66
m02-03	M - 40.80	Data Analysis & Probability	Find, use, and interpret measures of center and spread, . . .	04-05	58
m03-07	M - 49.72	Data Analysis & Probability	Find, use, and interpret measures of center and spread, . . .	04-05	74
m04-09	MH - 38.66	Data Analysis & Probability	Select and use appropriate statistical methods to analyze data.	05-06	88
m04-11	M - 60.74	Data Analysis & Probability	Select and use appropriate statistical methods to analyze data.	05-06	90
m02-21	H - 3.24	Data Analysis & Probability	Understand and apply basic concepts of probability, compute . . .	04-05	89
m07-21	H - 12.47	Data Analysis & Probability	Understand and apply basic concepts of probability.	08-09	64
m07-25	H - 12.89	Data Analysis & Probability	Understand and apply basic concepts of probability.	08-09	68
m04-22	H - 16.20	Data Analysis & Probability	Understand and apply basic concepts of probability.	05-06	101
m07-24	H - 17.24	Data Analysis & Probability	Understand and apply basic concepts of probability.	08-09	67
m04-21	MH - 25.22	Data Analysis & Probability	Understand and apply basic concepts of probability.	05-06	100
ta06-18	MH - 29.08	Data Analysis & Probability	Understand and apply basic concepts of probability.	06-07	88
ta06-13	M - 50.78	Data Analysis & Probability	Understand and apply basic concepts of probability.	06-07	86
m06-17	MH - ‘26.89	Data Analysis & Probability	Understand and apply basic concepts of probability.	07-08	54
m02-12	ME - 78.39	Data Analysis & Probability	Understand and use appropriate terminology to describe . . .	04-05	62
m01-18	H - 14.17	Data Analysis & Probability	Use proportionality and a basic understanding of . . .	04-05	53
m99-12	M - 44.16	Data Analysis & Probability	Use proportionality and a basic understanding of . . .	08-09	80
m99-10	M - 46.73	Data Analysis & Probability	Use proportionality and a basic understanding of . . .	08-09	78
m05-16	H - 19.42	Data Analysis & Probability	Understand and apply basic concepts of probability.	06-07	66
m01-09	H - 12.08	Geometry	Analyze characteristics and properties of two- and three-dim . . .	05-06	57
m04-25	H - 13.12	Geometry	Analyze characteristics and properties of two- and three-dim . . .	05-06	104
m06-21	H - 19.56	Geometry	Analyze characteristics and properties of two- and . . .	07-08	58
m06-07	MH - 20.87	Geometry	Analyze characteristics and properties of two- and . . .	07-08	44
m05-23	MH - 22.18	Geometry	Analyze characteristics and properties of two- and three-dim . . .	06-07	73
m05-07	MH - 22.26	Geometry	Analyze characteristics and properties of two- and three-dim . . .	06-07	57
m04-24	MH - 24.99	Geometry	Analyze characteristics and properties of two- and three-dim . . .	05-06	103
m05-15	MH - 26.87	Geometry	Analyze characteristics and properties of two- and three-dim . . .	06-07	65

Appendix IV - Worksheet NCTM Standards continued

sb06-10	MH	27.33	Geometry	Analyze characteristics and properties of two- and three-dim . . .	06-07	97
m03-21	MH	32.62	Geometry	Analyze characteristics and properties of two- and three-dim . . .	05-06	76
tb06-04	MH	33.11	Geometry	Analyze characteristics and properties of two- and three-dim . . .	06-07	93
m05-19	MH	38.79	Geometry	Analyze characteristics and properties of two- and three-dim . . .	06-07	69
m01-08	MH	38.97	Geometry	Analyze characteristics and properties of two- and three-dim . . .	05-06	56
m05-09	M	40.37	Geometry	Analyze characteristics and properties of two- and three-dim . . .	06-07	59
m99-20	M	40.59	Geometry	Analyze characteristics and properties of two- and three-dim . . .	08-09	88
m01-07	M	43.14	Geometry	Analyze characteristics and properties of two- and three-dim . . .	05-06	55
m03-06	M	47.69	Geometry	Analyze characteristics and properties of two- and three-dim . . .	05-06	71
m06-19	M	52.37	Geometry	Analyze characteristics and properties of two- and . . .	07-08	56
m06-06	M	55.10	Geometry	Analyze characteristics and properties of two- and . . .	07-08	43
m05-04	M	56.1	Geometry	Analyze characteristics and properties of two- and three-dim . . .	06-07	54
m06-04	ME	66.91	Geometry	Analyze characteristics and properties of two- and . . .	07-08	41
m02-15	E	78.10	Geometry	Analyze characteristics and properties of two- and three-dim . . .	04-05	86
m00-19	H	15.04	Geometry	Apply transformations and use symmetry to analyze mathe . . .	07-08	81
m06-18	MH	22.81	Geometry	Apply transformations and use symmetry to analyze . . .	07-08	55
m03-25	MH	28.34	Geometry	Apply transformations and use symmetry to analyze mathe . . .	05-06	79
tb06-06	MH	35.67	Geometry	Apply transformations and use symmetry to analyze mathe . . .	06-07	95
m05-03	M	43.6	Geometry	Apply transformations and use symmetry to analyze mathe . . .	06-07	53
m06-05	M	63.37	Geometry	Apply transformations and use symmetry to analyze . . .	07-08	42
sa06-07	ME	72.98	Geometry	Apply transformations and use symmetry to analyze mathe . . .	06-07	82
m01-16	H	13.60	Geometry	Describe sizes, positions, and orientations of shapes under . . .	04-05	52
m99-05	MH	34.76	Geometry	Draw geometric objects with specified properties, such as . . .	08-09	73
m07-11	MH	35.82	Geometry	Examine the congruence, similarity, and line or rotational . . .	08-09	54
m01-23	M	42.29	Geometry	Examine the congruence, similarity, and line or rotational . . .	04-05	56
m07-23	MH	24.48	Geometry	Precisely describe, classify, and understand relationships . . .	08-09	66
m99-02	M	45.79	Geometry	Precisely describe, classify, and understand relationships . . .	08-09	70
m07-08	M	55.65	Geometry	Precisely describe, classify, and understand relationships . . .	08-09	51
m00-04	E	91.47	Geometry	Precisely describe, classify, and understand relationships . . .	07-08	66
m07-16	MH	25.94	Geometry	Specify locations and describe spatial relationships . . .	08-09	59
m04-14	MH	37.25	Geometry	Specify locations and describe spatial relationships . . .	05-06	93
m00-25	H	16.65	Geometry	Understand relationships among the angles, side lengths, . . .	07-08	87
m07-14	H	17.20	Geometry	Understand relationships among the angles, side lengths, . . .	08-09	57
m00-24	H	17.48	Geometry	Understand relationships among the angles, side lengths, . . .	07-08	86
m00-22	H	17.78	Geometry	Understand relationships among the angles, side lengths, . . .	07-08	84
m99-23	H	17.98	Geometry	Understand relationships among the angles, side lengths, . . .	08-09	91
m99-25	H	18.36	Geometry	Understand relationships among the angles, side lengths, . . .	08-09	93
m99-21	MH	24.66	Geometry	Understand relationships among the angles, side lengths, . . .	08-09	89
m99-14	MH	25.65	Geometry	Understand relationships among the angles, side lengths, . . .	08-09	82
m02-13	MH	26.83	Geometry	Understand relationships among the angles, side lengths, . . .	04-05	63
m00-13	MH	37.04	Geometry	Understand relationships among the angles, side lengths, . . .	07-08	75
m00-15	MH	37.89	Geometry	Understand relationships among the angles, side lengths, . . .	07-08	77
m00-16	MH	38.33	Geometry	Understand relationships among the angles, side lengths, . . .	07-08	78
m07-12	M	44.14	Geometry	Understand relationships among the angles, side lengths, . . .	08-09	55
m00-18	M	45.27	Geometry	Understand relationships among the angles, side lengths, . . .	07-08	80
m00-06	M	53.32	Geometry	Understand relationships among the angles, side lengths, . . .	07-08	68
m99-24	MH	21.10	Geometry	Understand the meaning and effects of arithmetic . . .	08-09	92
m01-11	M	47.16	Geometry	Use coordinate geometry to represent and examine the . . .	04-05	49
m99-07	MH	20.30	Geometry	Use geometric models to represent and explain numerical . . .	08-09	75
m99-09	M	54.17	Geometry	Use geometric models to represent and explain numerical . . .	08-09	77
m03-15	MH	33.38	Geometry	Use two-dimensional representations of three-dimensional . . .	04-05	79
m03-13	M	54.30	Geometry	Use two-dimensional representations of three-dimensional . . .	04-05	77
m99-08	ME	65.38	Geometry	Use two-dimensional representations of three-dimensional . . .	08-09	76
m03-01	ME	77.36	Geometry	Use two-dimensional representations of three-dimensional . . .	04-05	69
m03-18	H	18.49	Geometry	Use visual tools such as networks to represent and solve . . .	04-05	0
m02-16	H	19.22	Geometry	Use visualization, spatial reasoning, and geometric . . .	04-05	87
m01-24	H	20.00	Geometry	Use visualization, spatial reasoning, and geometric . . .	05-06	63
m05-25	MH	22.05	Geometry	Use visualization, spatial reasoning, and geometric . . .	06-07	75
m07-22	MH	22.95	Geometry	Use visualization, spatial reasoning, and geometric . . .	08-09	65
m05-13	MH	24.92	Geometry	Use visualization, spatial reasoning, and geometric . . .	06-07	63
m02-23	MH	26.10	Geometry	Use visualization, spatial reasoning, and geometric . . .	05-06	70
m02-22	H	27.95	Geometry	Use visualization, spatial reasoning, and geometric . . .	04-05	90
m03-10	MH	31.86	Geometry	Use visualization, spatial reasoning, and geometric . . .	05-06	74
ta06-12	M	48.04	Geometry	Use visualization, spatial reasoning, and geometric . . .	06-07	85
tb06-05	M	49.54	Geometry	Use visualization, spatial reasoning, and geometric . . .	06-07	94
m02-01	E	52.17	Geometry	Use visualization, spatial reasoning, and geometric . . .	04-05	69
m03-08	M	58.65	Geometry	Use visualization, spatial reasoning, and geometric . . .	05-06	72
m02-20	MH	30.73	Geometry	Use visualization, spatial reasoning, and geometric . . .	04-05	88
m04-15	MH	36.74	Geometry	Use visualization, spatial reasoning, and geometric . . .	05-06	94
m01-13	MH	25.41	Measurement	Apply appropriate techniques, tools, and formulas to . . .	05-06	59
m04-01	E	90.53	Measurement	Apply appropriate techniques, tools, and formulas to . . .	05-06	80
m03-22	H	14.15	Measurement	Develop and use formulas to determine the circumference . . .	04-05	84
m01-15	M	56.47	Measurement	Solve simple problems involving rates and derived . . .	04-05	51
sb06-13	MH	28.06	Measurement	Understand measurable attributes of objects and the units, . . .	06-07	100
sa06-14	MH	39.42	Measurement	Understand measurable attributes of objects and the units, . . .	06-07	87
sb06-09	ME	77.07	Measurement	Understand measurable attributes of objects and the units, . . .	06-07	96
m01-05	M	58.46	Measurement	Understand relationships among units and convert from . . .	04-05	46
m04-12	MH	21.74	Measurement	Use mathematical models to represent and understand . . .	05-06	91
m05-17	M	45.49	Measurement	Understand measurable attributes of objects and the units, . . .	06-07	67

Appendix IV - Worksheet NCTM Standards continued

m04-16	H	20.25	Measurement	Understand measurable attributes of objects and the units,	05-06	95
m01-01	E	82.37	Measurement	Understand relationships among units and convert from	04-05	42
m00-03	MH	34.12	Number & Operations.	Compare and order fractions, decimals, and percents	07-08	65
m06-01	E	93.65	Number & Operations.	Compute fluently and make reasonable estimates.	07-08	38
m01-10	MH	32.20	Number & Operations.	Develop meaning for percents greater than 100 and less than 1.	04-05	48
m00-10	MH	26.83	Number & Operations.	Develop, analyze, and explain methods for solving problems	07-08	72
m00-01	E	93.09	Number & Operations.	Model and solve contextualized problems using various	07-08	63
m99-03	ME	64.72	Number & Operations.	Select appropriate methods and tools for computing with	08-09	71
m02-24	MH	30.08	Number & Operations.	Understand and use ratios and proportions to represent	04-05	68
m99-22	MH	32.34	Number & Operations.	Understand and use ratios and proportions to represent	08-09	90
m01-06	MH	36.05	Number & Operations.	Understand and use ratios and proportions to represent	04-05	47
m03-03	ME	65.56	Number & Operations.	Understand and use ratios and proportions to represent	04-05	71
m07-02	ME	65.97	Number & Operations.	Understand and use ratios and proportions to represent	08-09	45
m04-03	ME	72.69	Number & Operations.	Understand and use ratios and proportions to represent	05-06	82
m06-24	H	15.39	Number & Operations.	Understand meanings of operations and how they relate	07-08	61
m00-17	MH	21.73	Number & Operations.	Understand meanings of operations and how they relate	07-08	79
m07-10	MH	22.12	Number & Operations.	Understand and use ratios and proportions to represent	08-09	53
m00-14	MH	22.47	Number & Operations.	Understand meanings of operations and how they relate	07-08	76
m07-15	M	42.74	Number & Operations.	Understand and use ratios and proportions to represent	08-09	58
m07-18	M	45.39	Number & Operations.	Understand and use ratios and proportions to represent	08-09	61
m04-07	M	46.05	Number & Operations.	Understand meanings of operations and how they relate	05-06	86
sa06-09	M	54.85	Number & Operations.	Understand meanings of operations and how they relate	06-07	84
sb06-02	M	56.22	Number & Operations.	Understand meanings of operations and how they relate	06-07	91
m02-07	ME	61.62	Number & Operations.	Understand meanings of operations and how they relate	05-06	66
m99-01	E	80.22	Number & Operations.	Understand meanings of operations and how they relate	08-09	69
sa06-03	E	84.08	Number & Operations.	Understand meanings of operations and how they relate	06-07	78
sa06-01	E	99.06	Number & Operations.	Understand meanings of operations and how they relate	06-07	76
m99-15	MH	23.36	Number & Operations.	Understand numbers, ways of representing numbers,	08-09	83
m06-25	H	15.48	Number & Operations.	Understand numbers, ways of representing numbers,	07-08	62
m07-13	H	16.25	Number & Operations.	Understand numbers, ways of representing numbers,	08-09	56
m06-11	H	17.51	Number & Operations.	Understand numbers, ways of representing numbers,	07-08	48
m06-23	H	20.76	Number & Operations.	Understand numbers, ways of representing numbers,	07-08	60
m01-25	MH	20.99	Number & Operations.	Understand numbers, ways of representing numbers,	05-06	64
m00-11	MH	22.49	Number & Operations.	Understand numbers, ways of representing numbers,	07-08	73
m07-19	MH	22.97	Number & Operations.	Understand numbers, ways of representing numbers,	08-09	62
m04-02	M	44.49	Number & Operations.	Understand numbers, ways of representing numbers,	05-06	81
m00-02	M	46.78	Number & Operations.	Understand numbers, ways of representing numbers,	07-08	64
m03-09	M	50.74	Number & Operations.	Understand numbers, ways of representing numbers,	05-06	73
m04-08	M	51	Number & Operations.	Understand numbers, ways of representing numbers,	05-06	87
m04-04	M	51.14	Number & Operations.	Understand numbers, ways of representing numbers,	05-06	83
sb06-01	M	57.67	Number & Operations.	Understand numbers, ways of representing numbers,	06-07	90
m07-07	M	57.88	Number & Operations.	Understand numbers, ways of representing numbers,	08-09	50
m07-09	ME	76.94	Number & Operations.	Understand numbers, ways of representing numbers,	08-09	52
m01-18	H	14.17	Number & Operations.	Use factors, multiples, prime factorization, and relatively	04-05	53
m06-10	MH	23.19	Number & Operations.	Use factors, multiples, prime factorization, and relatively	07-08	47
m03-12	MH	25.37	Number & Operations.	Use factors, multiples, prime factorization, and relatively	04-05	76
m00-07	MH	28.77	Number & Operations.	Use factors, multiples, prime factorization, and relatively	07-08	69
m03-19	MH	32.84	Number & Operations.	Use factors, multiples, prime factorization, and relatively	04-05	82
m04-19	MH	36.62	Number & Operations.	Use factors, multiples, prime factorization, and relatively	05-06	98
m03-02	M	41.09	Number & Operations.	Use factors, multiples, prime factorization, and relatively	04-05	70
m07-03	ME	63.43	Number & Operations.	Use factors, multiples, prime factorization, and relatively	08-09	46
m01-02	E	86.57	Number & Operations.	Use factors, multiples, prime factorization, and relatively	04-05	43
m06-12	M	43.32	Number & Operations.	Work flexibly with fractions, decimals, and percents	07-08	49
m02-14	H	16.68	Number & Operations.	Work flexibly with fractions, decimals, and percents to	04-05	64
m03-11	H	19.91	Number & Operations.	Work flexibly with fractions, decimals, and percents to	04-05	75
m07-06	MH	28.83	Number & Operations.	Work flexibly with fractions, decimals, and percents to	08-09	49
m07-17	MH	33.75	Number & Operations.	Work flexibly with fractions, decimals, and percents to	08-09	60
m03-05	ME	71.27	Number & Operations.	Work flexibly with fractions, decimals, and percents to	04-05	73
m04-06	M	51.62	Number & Operations.	Understand meanings of operations and how they relate	05-06	85
m05-11	ME	61.33	Number & Operations.	Understand meanings of operations and how they relate	06-07	61
m05-01	ME	65.26	Number & Operations.	Understand meanings of operations and how they relate	06-07	51
m05-18	MH	30.44	Number & Operations.	Understand numbers, ways of representing numbers,	06-07	68
m05-06	MH	31.19	Number & Operations.	Understand numbers, ways of representing numbers,	06-07	56
m05-02	ME	64.02	Number & Operations.	Understand numbers, ways of representing numbers,	06-07	52
m03-20	H	13.81	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	04-05	83
m03-23	H	16.89	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	05-06	77
m00-12	H	19.31	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	07-08	74
m05-21	MH	23.24	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	06-07	71
m06-16	MH	23.46	Problem Solving.	Apply and adapt a variety of appropriate strategies to	07-08	53
tb06-11	MH	26.28	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	06-07	98
m02-19	MH	27.67	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	04-05	67
m03-14	MH	36.89	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	04-05	78
m01-14	MH	37.40	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	04-05	50
m02-05	MH	39.28	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	04-05	60
m04-18	M	41.93	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	05-06	97
m06-15	M	43.67	Problem Solving.	Apply and adapt a variety of appropriate strategies to	07-08	52
m01-22	M	48.08	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	04-05	55
m01-04	M	50.08	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	04-05	45
m02-04	M	53.33	Problem Solving.	Apply and adapt a variety of appropriate strategies to solve	04-05	59

m03-16	M - 55.70	Problem Solving	Apply and adapt a variety of appropriate strategies to solve . . .	04-05	80
m06-08	M - 59.60	Problem Solving	Apply and adapt a variety of appropriate strategies to . . .	07-08	45
sa06-04	ME - 66.42	Problem Solving	Apply and adapt a variety of appropriate strategies to solve . . .	06-07	79
m02-02	ME - 69.27	Problem Solving	Apply and adapt a variety of appropriate strategies to solve . . .	04-05	57
m06-14	E - 79.27	Problem Solving	Apply and adapt a variety of appropriate strategies to . . .	07-08	51
m99-06	E - 86.55	Problem Solving	Apply and adapt a variety of appropriate strategies to . . .	08-09	74
m06-13	MH - 26.59	Problem Solving	Build new mathematical knowledge through problem . . .	07-08	50
m02-25	H - 31.61	Problem Solving	Instructional programs from pre-kindergarten through . . .	04-05	91
m04-13	MH - 24.43	Problem Solving	Monitor and reflect on the process of mathematical problem . . .	05-06	92
m05-24	H - 16	Problem Solving	Solve problems that arise in mathematics and in other contexts.	06-07	74
m04-23	MH - 21.48	Problem Solving	Solve problems that arise in mathematics and in other contexts.	05-06	102
m00-09	MH - 23.64	Problem Solving	Solve problems that arise in mathematics and in other contexts.	07-08	71
m04-17	MH - 23.72	Problem Solving	Solve problems that arise in mathematics and in other contexts.	05-06	96
m04-20	MH - 24.29	Problem Solving	Solve problems that arise in mathematics and in other contexts.	05-06	99
m05-20	MH - 25.35	Problem Solving	Solve problems that arise in mathematics and in other contexts.	06-07	70
m06-20	MH - 31.90	Problem Solving	Solve problems that arise in mathematics and in other contexts.	07-08	57
m05-14	MH - 35.54	Problem Solving	Solve problems that arise in mathematics and in other contexts.	06-07	64
m99-11	MH - 35.64	Problem Solving	Solve problems that arise in mathematics and in other contexts.	08-09	79
m05-05	ME - 61.09	Problem Solving	Solve problems that arise in mathematics and in other contexts.	06-07	55
sa06-05	ME - 69.79	Problem Solving	Solve problems that arise in mathematics and in other contexts.	06-07	80
m00-08	ME - 79.26	Problem Solving	Solve problems that arise in mathematics and in other contexts.	07-08	70
m07-05	E - 84.94	Problem Solving	Solve problems that arise in mathematics and in other contexts.	08-09	48
m06-02	E - 85.73	Problem Solving	Solve problems that arise in mathematics and in other contexts.	07-08	39
m05-10	E - 84.82	Problem Solving	Solve problems that arise in mathematics and in other contexts.	06-07	60

Appendix V

MathWorld.com - AMC 8 Worksheets

This is a listing of all the Worksheets produced thus far (2004-05, 2005-06, 2006-07 and 2007-08) in a MathWorld.com categorized list for easy reference.

Q #	Difficulty	Classification	MC yr	page
sb06-12	M - 45.86	Algebra > Linear Algebra > General Linear Algebra > Linear Algebra	06-07	99
m03-04	ME - 74.39	Algebra > Linear Algebra > Linear Systems of Equations	04-05	72
ta06-08	MH - 27.81	Algebra > Linear Algebra > Linear Systems of Equations > Linear System of Equations	06-07	83
sb06-03	ME - 78.81	Algebra > Linear Algebra > Linear Systems of Equations > Linear System of Equations	06-07	92
ta06-06	ME - 79.96	Algebra > Polynomials > Polynomial	06-07	81
sa06-01	E - 99.06	Algebra > Products > Product	06-07	76
m99-11	MH - 35.64	Algebra > Sums	08-09	79
sb06-01	M - 57.67	Algebra > Sums > Sum	06-07	90
m05-17	M - 45.49	Algebra > Vector Algebra > Speed	06-07	67
m03-09	M - 50.74	Applied Mathematics > Business > Economics > Marginal Analysis	05-06	73
m04-23	MH - 21.48	Applied Mathematics > Data Visualization > Function Graph	05-06	102
m04-05	M - 59.14	Applied Mathematics > Game Theory > Game	05-06	84
m05-24	H - 16	Applied Mathematics > Optimization > Global Optimization	06-07	74
m00-05	E - 14.24	Calculus & Analysis > Calculus > Maxima and Minima > Maximum	07-08	67
sa06-04	ME - 66.42	Calculus & Analysis > Calculus > Maxima and Minima > Maximum	06-07	79
m00-15	MH - 37.89	Calculus & Analysis > Differential Geometry > Differential Geometry of Curves > Perimeter	07-08	77
m00-16	MH - 38.33	Calculus & Analysis > Differential Geometry > Differential Geometry of Curves > Perimeter	07-08	78
m00-18	M - 45.27	Calculus & Analysis > Differential Geometry > Differential Geometry of Curves > Perimeter	07-08	80
m05-04	M - 56.1	Calculus & Analysis > Differential Geometry > Differential Geometry of Curves > Perimeter	06-07	54
m05-10	E - 84.82	Calculus & Analysis > Differential Geometry > Differential Geometry of Curves > Velocity	06-07	60
m00-22	H - 17.78	Calculus & Analysis > Differential Geometry > Differential Geometry of Surfaces > Surface Area	07-08	84
m00-05	H - 14.24	Calculus & Analysis > Functions > Period	07-08	67
m07-10	MH - 22.12	Calculus & Analysis > Functions > Unary Operation	08-09	53
m07-15	M - 42.74	Calculus & Analysis > Inequalities > Inequality	08-09	58
sb06-10	MH - 27.33	Calculus & Analysis > Inequalities > Triangle Inequality	06-07	97
m00-23	MH - 29.51	Calculus & Analysis > Special Functions > Means	07-08	85
m01-21	MH - 23.28	Calculus & Analysis > Special Functions > Means > Arithmetic Mean	04-05	54
m99-13	MH - 28.13	Calculus & Analysis > Special Functions > Means > Arithmetic Mean	08-09	81
m02-18	MH - 32.19	Calculus & Analysis > Special Functions > Means > Arithmetic Mean	04-05	66
m04-09	MH - 38.66	Calculus & Analysis > Special Functions > Means > Arithmetic Mean	05-06	88
m02-03	M - 40.80	Calculus & Analysis > Special Functions > Means > Arithmetic Mean	04-05	58
m03-07	M - 49.72	Calculus & Analysis > Special Functions > Means > Arithmetic Mean	04-05	74
m07-07	M - 57.88	Calculus & Analysis > Special Functions > Means > Arithmetic Mean	08-09	50
m99-24	MH - 21.10	Calculus & Analysis > Special Functions > Powers	08-09	92
m00-14	MH - 22.47	Calculus & Analysis > Special Functions > Powers	07-08	76
m02-19	MH - 27.67	Combinatorics > Enumeration	04-05	67
m04-14	MH - 37.25	Discrete Mathematics > Point Lattices > Geoboard	05-06	93
m01-14	MH - 37.40	Discrete Mathematics > Combinatorics > Enumeration	04-05	50
m07-04	M - 46.66	Discrete Mathematics > Combinatorics > General Combinatorics > Counting Generalized Principle	08-09	47
m02-15	E - 78.10	Discrete Mathematics > Combinatorics > Lattice Paths & Polygons > Lattice Polygons > Pick's Theorem	04-05	86
m04-17	MH - 23.72	Discrete Mathematics > Combinatorics > Partitions > Partition	05-06	96
m99-15	MH - 23.36	Discrete Mathematics > Combinatorics > Permutations > Combination	08-09	83
ta06-18	MH - 29.08	Discrete Mathematics > Combinatorics > Permutations > Combination	06-07	88
m05-14	MH - 35.54	Discrete Mathematics > Combinatorics > Permutations > Combination	06-07	64

m04-04	M	- 51.14	Discrete Mathematics > Combinatorics > Permutations > Combination	05-06	83
tb06-11	MH	- 26.28	Discrete Mathematics > Combinatorics > Permutations > Factorial	06-07	98
m05-21	MH	- 23.24	Discrete Mathematics > Combinatorics > Permutations > Permutation	06-07	71
m03-16	M	- 55.70	Discrete Mathematics > Combinatorics > Permutations > Permutation	04-05	80
m02-10	M	- 43.90	Discrete Mathematics > Computer Science > Data Structures > Database	05-06	69
m02-09	ME	- 78.30	Discrete Mathematics > Computer Science > Data Structures > Database	05-06	68
m02-08	E	- 89.23	Discrete Mathematics > Computer Science > Data Structures > Database	05-06	67
m03-23	H	- 16.89	Discrete Mathematics > Graph Theory > Circuits > Graph Cycle	05-06	77
m06-20	MH	- 31.90	Discrete Mathematics > Graph Theory > Directed Graphs > Tournament	07-08	57
m03-24	MH	- 26.05	Discrete Mathematics > Graph Theory > General Graph Theory > Graph	05-06	78
m02-06	MH	- 26.63	Discrete Mathematics > Graph Theory > General Graph Theory > Graph	05-06	65
m03-18	H	- 18.49	Discrete Mathematics > Graph Theory > Graph Properties > Graph Distance	04-05	0
m01-23	M	- 42.29	Discrete Mathematics > Point Lattices	04-05	56
m01-11	M	- 47.16	Discrete Mathematics > Point Lattices > Pick's Theorem	04-05	49
m01-09	H	- 12.08	Discrete Mathematics > Point Lattices > Square Grid	05-06	57
m01-08	MH	- 38.97	Discrete Mathematics > Point Lattices > Square Grid	05-06	56
m01-07	M	- 43.14	Discrete Mathematics > Point Lattices > Square Grid	05-06	55
m04-18	M	- 41.93	Discrete Mathematics> Combinatorics > Partitions > Partition	05-06	97
m01-20	MH	- 26.87	Foundations of Mathematics > Logic > General Logic > Logic	05-06	62
m03-17	M	- 49.96	Foundations of Mathematics > Logic > General Logic > Logic	05-06	75
m04-13	MH	- 24.43	Foundations of Mathematics > Logic > General Logic > True	05-06	92
m07-13	H	- 16.25	Foundations of Mathematics > Logic > General Logic > Venn Diagram	08-09	56
m01-25	MH	- 20.99	Foundations of Mathematics > Set Theory > General Set Theory > Set Theory	05-06	64
m01-12	MH	- 29.85	Foundations of Mathematics > Set Theory > Set Properties > Operation	05-06	58
sa06-02	MH	- 32.49	Foundations of Mathematics > Set Theory > Set Properties > Operation	06-07	77
sb06-02	M	- 56.22	Foundations of Mathematics > Set Theory > Set Properties > Operation	06-07	91
tb06-05	M	- 49.54	Geometry > Computational Geometry > Packing Problems > Packing	06-07	94
m02-15	E	- 78.10	Geometry > Computational Geometry > Triangulation > Triangulation	04-05	86
m01-19	H	- 19.06	Geometry > Distance > Distance	05-06	61
sa06-14	MH	- 39.42	Geometry > Distance > Distance	06-07	87
ta06-12	M	- 48.04	Geometry > Distance > Locus	06-07	85
m00-12	H	- 19.31	Geometry > General Geometry	07-08	74
m99-20	M	- 40.59	Geometry > Geometric Construction	08-09	88
m01-06	MH	- 36.05	Geometry > Geometric Similarity > Similarity	04-05	47
m05-23	MH	- 22.18	Geometry > Line Geometry > Concurrence > Inscribed	06-07	73
m02-01	E	- 52.17	Geometry > Line Geometry > Lines > Circle-Line Intersection;	04-05	69
m01-23	M	- 42.29	Geometry > Line Geometry > Lines > Midpoint	04-05	56
tb06-06	MH	- 35.67	Geometry > Plane Geometry > Arcs > Semicircle	06-07	95
m06-07	MH	- 20.87	Geometry > Plane Geometry > Circles	07-08	44
m07-25	H	- 12.89	Geometry > Plane Geometry > Circles > Circle	08-09	68
m05-25	MH	- 22.05	Geometry > Plane Geometry > Circles > Circle	06-07	75
m07-16	MH	- 25.94	Geometry > Plane Geometry > Circles > Circle	08-09	59
m02-01	E	- 52.17	Geometry > Plane Geometry > Circles > Circle-Line Intersection	04-05	69
tb06-04	MH	- 33.11	Geometry > Plane Geometry > Circles > Concentric Circles	06-07	93
m04-25	H	- 13.12	Geometry > Plane Geometry > Circles > Diameter	05-06	104
m02-20	MH	- 30.73	Geometry > Plane Geometry > Geometric Similarity > Congruent	04-05	88
sa06-07	ME	- 72.98	Geometry > Plane Geometry > Geometric Similarity >Congruent	06-07	82
m03-22	H	- 14.15	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area	04-05	84
m00-19	H	- 15.04	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area	07-08	81
m00-25	H	- 16.65	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area	07-08	87
m99-23	H	- 17.98	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area	08-09	91
m99-25	H	- 18.36	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area	08-09	93
m03-25	MH	- 28.34	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area	05-06	79
m99-05	MH	- 34.76	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area	08-09	73
m00-16	MH	- 38.33	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area	07-08	78
m00-18	M	- 45.27	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area	07-08	80
m00-06	M	- 53.32	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area	07-08	68
m03-08	M	- 58.65	Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area	05-06	72
m07-12	M	- 44.14	Geometry > Plane Geometry > Polygons > Hexagram	08-09	55
m05-13	MH	- 24.92	Geometry > Plane Geometry > Polygons > Polygon	06-07	63
m99-14	MH	- 25.65	Geometry > Plane Geometry > Quadrilaterals > Isosceles Trapezoid	08-09	82
m04-24	MH	- 24.99	Geometry > Plane Geometry > Quadrilaterals > Parallelogram	05-06	103
m05-09	M	- 40.37	Geometry > Plane Geometry > Quadrilaterals > Quadrilateral	06-07	59
m03-21	MH	- 32.62	Geometry > Plane Geometry > Quadrilaterals > Trapezoid	05-06	76
m05-19	MH	- 38.79	Geometry > Plane Geometry > Quadrilaterals > Trapezoid	06-07	69
m07-08	M	- 55.65	Geometry > Plane Geometry > Quadrilaterals > Trapezoid	08-09	51
m06-10	MH	- 23.19	Geometry > Plane Geometry > Rectangles	07-08	47
m06-06	M	- 55.10	Geometry > Plane Geometry > Rectangles	07-08	43
m99-05	MH	- 34.76	Geometry > Plane Geometry > Squares	08-09	73
m06-05	M	- 63.37	Geometry > Plane Geometry > Squares	07-08	42
m07-22	MH	- 22.95	Geometry > Plane Geometry > Squares > Square	08-09	65
m07-23	MH	- 24.48	Geometry > Plane Geometry > Squares > Square	08-09	66
m03-06	M	- 47.69	Geometry > Plane Geometry > Squares > Square	05-06	71
m02-23	MH	- 26.10	Geometry > Plane Geometry > Tiling > Recreational Mathematics > Tiling	05-06	70
m07-11	MH	- 35.82	Geometry > Plane Geometry > Tiling > Tessellation	08-09	54
m04-15	MH	- 36.74	Geometry > Plane Geometry > Tiling > Tiling	05-06	94
m06-19	M	- 52.37	Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles	07-08	56
m07-14	H	- 17.20	Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Isosceles Triangle	08-09	57
m05-15	MH	- 26.87	Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Isosceles Triangle	06-07	65

Appendix V - Worksheet MathWorld Classifications continued

m05-07	MH - 22.26	Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Right Tri	06-07	57
m03-10	MH - 31.86	Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Right Triangle	05-06	74
m07-23	MH - 24.48	Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Triangle	08-09	66
m99-21	MH - 24.66	Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Triangle	08-09	89
m01-23	M - 42.29	Geometry > Plane Geometry > Tris > Special Tris > Other Tris	04-05	56
m02-16	H - 19.22	Geometry > Plane Geometry > Tris > Special Tris > Other Tris > Right Tris	04-05	87
m03-15	MH - 33.38	Geometry > Projective Geometry > Map Projections > Orthogonal Projection	04-05	79
m06-18	MH - 22.81	Geometry > Solid Geometry > Polyhedra > Cubes	07-08	55
m99-20	M - 40.59	Geometry > Solid Geometry > Polyhedra > Cubes	08-09	88
m99-08	ME - 65.38	Geometry > Solid Geometry > Polyhedra > Cubes	08-09	76
m03-01	ME - 77.36	Geometry > Solid Geometry > Polyhedra > Cubes > Cube	04-05	69
m02-22	H - 27.95	Geometry > Solid Geometry > Polyhedra > Cubes > Polycube	04-05	90
m03-15	MH - 33.38	Geometry > Solid Geometry > Polyhedra > Cubes > Polycube	04-05	79
m03-13	M - 54.30	Geometry > Solid Geometry > Polyhedra > Cubes > Polycube	04-05	77
m06-21	H - 19.56	Geometry > Solid Geometry > Volume	07-08	58
m02-13	MH - 26.83	Geometry > Solid Geometry > Volume > Volume	04-05	63
m05-03	M - 43.6	Geometry > Symmetry > Symmetry	06-07	53
m06-04	ME - 66.91	Geometry > Transformations > Rotations	07-08	41
m00-24	H - 17.48	Geometry > Trigonometry > Angles	07-08	86
m99-21	MH - 24.66	Geometry > Trigonometry > Angles	08-09	89
m00-13	MH - 37.04	Geometry > Trigonometry > Angles	07-08	75
m99-02	M - 45.79	Geometry > Trigonometry > Angles	08-09	70
m03-20	H - 13.81	Geometry > Trigonometry > Angles > Angle	04-05	83
m99-04	E - 92.30	History & Terminology > Terminology > Diagram	08-09	72
m99-06	E - 86.55	History & Terminology > Terminology > Order	08-09	74
m06-22	H - 17.16	Number Theory > Arithmetic > Addition & Subtraction	07-08	59
m99-09	M - 54.17	Number Theory > Arithmetic > Addition & Subtraction	08-09	77
m07-03	ME - 63.43	Number Theory > Arithmetic > Addition & Subtraction > Addition	08-09	46
m99-03	ME - 64.72	Number Theory > Arithmetic > Addition & Subtraction > Addition	08-09	71
m99-01	E - 80.22	Number Theory > Arithmetic > Addition & Subtraction > Subtraction	08-09	69
m00-01	E - 93.09	Number Theory > Arithmetic > Addition & Subtraction > Subtraction	07-08	63
m00-20	MH - 25.69	Number Theory > Arithmetic > Addition and Subtraction	07-08	82
m06-02	E - 85.73	Number Theory > Arithmetic > Addition and Subtraction	07-08	39
m04-16	H - 20.25	Number Theory > Arithmetic > Fractions	05-06	95
m99-07	MH - 20.30	Number Theory > Arithmetic > Fractions	08-09	75
m99-22	MH - 32.34	Number Theory > Arithmetic > Fractions	08-09	90
m06-09	MH - 36.92	Number Theory > Arithmetic > Fractions	07-08	46
m06-03	M - 46.39	Number Theory > Arithmetic > Fractions	07-08	40
m07-03	ME - 63.43	Number Theory > Arithmetic > Fractions	08-09	46
m99-03	ME - 64.72	Number Theory > Arithmetic > Fractions	08-09	71
m04-01	E - 90.53	Number Theory > Arithmetic > Fractions > Directly Proportional	05-06	80
m04-20	MH - 24.29	Number Theory > Arithmetic > Fractions > Fraction	05-06	99
m02-25	H - 31.61	Number Theory > Arithmetic > Fractions > Fraction	04-05	91
m00-03	MH - 34.12	Number Theory > Arithmetic > Fractions > Fraction	07-08	65
sa06-05	ME - 69.79	Number Theory > Arithmetic > Fractions > Fraction	06-07	80
sb06-09	ME - 77.07	Number Theory > Arithmetic > Fractions > Fraction	06-07	96
m02-14	H - 16.68	Number Theory > Arithmetic > Fractions > Percent	04-05	64
m05-22	H - 17.63	Number Theory > Arithmetic > Fractions > Percent	06-07	72
m03-11	H - 19.91	Number Theory > Arithmetic > Fractions > Percent	04-05	75
m01-13	MH - 25.41	Number Theory > Arithmetic > Fractions > Percent	05-06	59
m07-20	MH - 26.60	Number Theory > Arithmetic > Fractions > Percent	08-09	63
m99-18	MH - 27.12	Number Theory > Arithmetic > Fractions > Percent	08-09	86
m07-06	MH - 28.83	Number Theory > Arithmetic > Fractions > Percent	08-09	49
m02-24	MH - 30.08	Number Theory > Arithmetic > Fractions > Percent	04-05	68
m01-17	MH - 30.26	Number Theory > Arithmetic > Fractions > Percent	05-06	60
m01-10	MH - 32.20	Number Theory > Arithmetic > Fractions > Percent	04-05	48
m07-17	MH - 33.75	Number Theory > Arithmetic > Fractions > Percent	08-09	60
m06-12	M - 43.32	Number Theory > Arithmetic > Fractions > Percent	07-08	49
m99-12	M - 44.16	Number Theory > Arithmetic > Fractions > Percent	08-09	80
m99-16	M - 44.75	Number Theory > Arithmetic > Fractions > Percent	08-09	84
m04-07	M - 46.05	Number Theory > Arithmetic > Fractions > Percent	05-06	86
m04-06	M - 51.62	Number Theory > Arithmetic > Fractions > Percent	05-06	85
m06-08	M - 59.60	Number Theory > Arithmetic > Fractions > Percent	07-08	45
m05-11	ME - 61.33	Number Theory > Arithmetic > Fractions > Percent	06-07	61
m02-07	ME - 61.62	Number Theory > Arithmetic > Fractions > Percent	05-06	66
m05-02	ME - 64.02	Number Theory > Arithmetic > Fractions > Percent	06-07	52
m03-03	ME - 65.56	Number Theory > Arithmetic > Fractions > Percent	04-05	71
m03-05	ME - 71.27	Number Theory > Arithmetic > Fractions > Percent	04-05	73
m00-04	E - 91.47	Number Theory > Arithmetic > Fractions > Percent	07-08	66
m06-13	MH - 26.59	Number Theory > Arithmetic > Fractions > Proportional	07-08	50
m00-10	MH - 26.83	Number Theory > Arithmetic > Fractions > Proportional	07-08	72
m06-15	M - 43.67	Number Theory > Arithmetic > Fractions > Proportional	07-08	52
m04-12	MH - 21.74	Number Theory > Arithmetic > Fractions > Ratio	05-06	91
sb06-13	MH - 28.06	Number Theory > Arithmetic > Fractions > Ratio	06-07	100
m07-02	ME - 65.97	Number Theory > Arithmetic > Fractions > Ratio	08-09	45
m04-03	ME - 72.69	Number Theory > Arithmetic > Fractions > Ratio	05-06	82
sa06-03	E - 84.08	Number Theory > Arithmetic > Fractions > Ratio	06-07	78
m01-15	M - 56.47	Number Theory > Arithmetic > General Arithmetic > Arithmetic	04-05	51
m01-05	M - 58.46	Number Theory > Arithmetic > General Arithmetic > Arithmetic	04-05	46

m01-01	E	- 82.37	Number Theory > Arithmetic > General Arithmetic > Arithmetic	04-05 42
m07-01	E	- 84.11	Number Theory > Arithmetic > General Arithmetic > Arithmetic	08-09 44
m07-05	E	- 84.94	Number Theory > Arithmetic > General Arithmetic > Arithmetic	08-09 48
m01-03	E	- 89.49	Number Theory > Arithmetic > General Arithmetic > Arithmetic	04-05 44
m99-18	MH	- 27.12	Number Theory > Arithmetic > Multiplication & Division	08-09 86
m00-07	MH	- 28.77	Number Theory > Arithmetic > Multiplication & Division	07-08 69
m99-19	MH	- 35.91	Number Theory > Arithmetic > Multiplication & Division	08-09 87
m99-17	M	- 42.22	Number Theory > Arithmetic > Multiplication & Division	08-09 85
m06-14	E	- 79.27	Number Theory > Arithmetic > Multiplication & Division	07-08 51
m05-01	ME	- 65.26	Number Theory > Arithmetic > Multiplication & Division > Division	06-07 51
m00-01	E	- 93.09	Number Theory > Arithmetic > Multiplication & Division > Division	07-08 63
m07-18	M	- 45.39	Number Theory > Arithmetic > Multiplication & Division > Multiplication	08-09 61
m00-02	M	- 46.78	Number Theory > Arithmetic > Multiplication & Division > Reciprocal	07-08 64
m06-23	H	- 20.76	Number Theory > Arithmetic > Multiplication & Division > Remainder	07-08 60
m99-24	MH	- 21.10	Number Theory > Arithmetic > Multiplication & Division > Remainder	08-09 92
m04-19	MH	- 36.62	Number Theory > Arithmetic > Multiplication & Division > Remainder	05-06 98
m05-06	MH	- 31.19	Number Theory > Arithmetic > Number Bases > Digit	06-07 56
m04-02	M	- 44.49	Number Theory > Arithmetic > Number Bases > Digit	05-06 81
m04-08	M	- 51	Number Theory > Arithmetic > Number Bases > Digit	05-06 87
m04-10	M	- 60.21	Number Theory > Arithmetic > Number Bases > Sexagesimal	05-06 89
m02-05	MH	- 39.28	Number Theory > Congruences > Congruence	04-05 60
m05-05	ME	- 61.09	Number Theory > Congruences > Mod	06-07 55
m05-20	MH	- 25.35	Number Theory > Congruences > Residue	06-07 70
m00-03	MH	- 34.12	Number Theory > Constants > Pi	07-08 65
sa06-22	MH	- 35.17	Number Theory > Diophantine Equations > Diophantine Equation	06-07 89
m01-22	M	- 48.08	Number Theory > Diophantine Equations > Diophantine Equation	04-05 55
m02-17	M	- 58.00	Number Theory > Diophantine Equations > Diophantine Equation	04-05 65
m02-02	ME	- 69.27	Number Theory > Diophantine Equations > Diophantine Equation	04-05 57
m05-18	MH	- 30.44	Number Theory > Divisors > Divisible	06-07 68
m03-12	MH	- 25.37	Number Theory > Divisors > Divisor	04-05 76
m01-02	E	- 86.57	Number Theory > Factoring > Factorization	04-05 43
m03-02	M	- 41.09	Number Theory > Factoring > Least Prime Factor	04-05 70
m01-04	M	- 50.08	Number Theory > Integers > Even Numbers	04-05 45
m06-17	MH	-‘26.89	Number Theory > Integers > Odd Number	07-08 54
m06-16	MH	- 23.46	Number Theory > Number Theoretic Functions > Least Common Multiple	07-08 53
m03-19	MH	- 32.84	Number Theory > Number Theoretic Functions > Least Common Multiple	04-05 82
m06-22	H	- 17.16	Number Theory > Numbers > Small Numbers	07-08 59
m05-08	M	- 52.69	Number Theory > Parity > Odd Number	06-07 58
m06-10	MH	- 23.19	Number Theory > Prime Numbers > Prime Factorization > Factoring	07-08 47
m06-25	H	- 15.48	Number Theory > Prime Numbers > Prime Number Properties	07-08 62
m00-11	MH	- 22.49	Number Theory > Rational Numbers > Digit	07-08 73
m06-01	E	- 93.65	Number Theory > Rounding	07-08 38
m05-12	M	- 53.86	Number Theory > Sequences > Sequence	06-07 62
sa06-09	M	- 54.85	Number Theory > Special Numbers > Digit-Related Numbers > Consecutive Number Sequences	06-07 84
m06-11	H	- 17.51	Number Theory > Special Numbers > Figurate Numbers > Square Numbers	07-08 48
m02-11	ME	- 67.55	Number Theory > Special Numbers > Figurate Numbers > Square Numbers	04-05 61
m07-19	MH	- 22.97	Number Theory > Special Numbers > Figurate Numbers > Square Numbers > Square	08-09 62
m02-04	M	- 53.33	Number Theory > Special Numbers > Palindromic Numbers > Palindromic Number	04-05 59
m01-21	MH	- 23.28	Probability & Statistics > Descriptive Statistics > Statistical Median	04-05 54
m00-21	H	- 7.90	Probability & Statistics > Probability	07-08 83
m99-10	M	- 46.73	Probability & Statistics > Probability	08-09 78
m06-17	MH	-‘26.89	Probability & Statistics > Probability	07-08 54
m02-21	H	- 3.24	Probability & Statistics > Probability > Coin Tossing	04-05 89
m07-21	H	- 12.47	Probability & Statistics > Probability > Probability	08-09 64
m07-25	H	- 12.89	Probability & Statistics > Probability > Probability	08-09 68
m01-18	H	- 14.17	Probability & Statistics > Probability > Probability	04-05 53
m04-22	H	- 16.20	Probability & Statistics > Probability > Probability	05-06 101
m07-24	H	- 17.24	Probability & Statistics > Probability > Probability	08-09 67
m05-16	H	- 19.42	Probability & Statistics > Probability > Probability	06-07 66
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