AMC / AIME Formula Sheet Adapted from NYC Contest Problem Book (Annotated 9/1/2015)

Example Question from NYC Contest Problem Book:

ALGEBRA

1.1 If $a:b = c:d$, then:

Important Ratio Equivalents

1.11 $(a + b):b = (c + d):d$

1.12 $(a + b):(a - b) = (c + d):(c - d)$

1.13 $a:b = c:d = (a + c):(b + d)$

1.21 Binomial Theorem: If $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$, (for $k \leq n$),

then: $(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

Rarely tested but good to know Pascal's triangle picture of coefficients:

1
1 1
1 2 1
1 3 3 1

Row sum = power of 2: 1 4 6 4 1

1.22 $(a + b)^3 = a^3 + b^3$ if and only if $a = 0$ or $b = 0$ or $a + b = 0$.

1.311 Factor Theorem: For $P(x)$ a polynomial in x and r a real or complex number, $x - r$ divides $P(x)$ if and only if $P(r) = 0$.

1.312 (Corollary) The polynomial $P(x)$ divides the polynomial $Q(x)$ if and only if every (complex) root of $P(x) = 0$ is also a root of $Q(x) = 0$ (including multiplicities of distinct roots).

1.321 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

1.322 $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

1.33 $x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \ldots + xy^{n-2} + y^{n-1})$

1.11
S81-#1, p. 34
F82-#16, p. 42

1.12
S75-#22, p. 3
F77-#8, p. 16

1.21
F75-#30, p. 3
S76-#4, p. 7
S77-#30, p. 15

1.22
S75-#2, p. 1
S76-#30, p. 9
F77-#10, p. 16

1.311
S75-#18, p. 2
F75-#8, p. 4
F75-#10, p. 4
F76-#2, p. 10
F77-#20, p. 17
F78-#18, p. 22
S80-#10, p. 29

1.312
F77-#20, p. 17

1.321
S77-#6, p. 13
S77-#8, p. 13
F77-#26, p. 17
S79-#21, p. 25
F81-#4, p. 37
F81-#9, p. 37
F81-#18, p. 38

1.322
S80-#26, p. 30
F81-#4, p. 37
F81-#18, p. 38

1.33
S77-#25, p. 14
F77-#7, p. 16
F78-#8, p. 21
S80-#19, p. 30
F80-#14, p. 32

1.34 For odd n, $x^n + y^n = (x + y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \ldots - xy^{n-2} + y^{n-1})$.

1.34
S79-#10, p. 24

1.4 If $P(x) = x^n - a_{n-1}x^{n-1} + a_{n-2}x^{n-2} - \ldots + (-1)^{n-1}a_1 x + (-1)^n a_o = 0$ has roots $r_1, r_2, r_3, \ldots, r_n$, then:

$$a_{n-1} = \sum_{i=1}^{n} r_i \qquad \text{(the sum of the roots)}$$

$$a_{n-2} = \sum_{1 \leq i < j \leq n} r_i r_j \qquad \text{(the sum of the roots "taken two at a time")}$$

$$a_{n-3} = \sum_{1 \leq i < j < k \leq n} r_i r_j r_k \qquad \text{(the sum of the roots "taken three at a time")}$$

$$\vdots$$

$$a_o = r_1 r_2 r_3 \ldots r_n$$

These Vieta formuli are
are frequently tested,
especially for quadratics

1.4
S75-#30, p. 3
F75-#6, p. 4
F75-#8, p. 4
F75-#9, p. 4
F77-#13, p. 16
F77-#21, p. 17
F78-#24, p. 22
S79-#17, p. 24
S79-#20, p. 25
S84-#27, p. 51

1.51 Arithmetic progression: If $a_1, a_2, \ldots, a_n$ are in arithmetic progression with common difference d, then:

1.511 $a_n = a_1 + (n - 1)d$

Front-to-Back grouping is often
a good method to sum a series

1.511
S76-#7, p. 7
F80-#10, p. 32

1.512 $\displaystyle\sum_{i=1}^{n} a_i = \frac{n}{2}[2a_1 + (n - 1)d] = \boxed{\frac{n}{2}[a_1 + a_n]}$

1.512
F75-#11, p. 4
S79-#29, p. 26
F79-#7, p. 26

1.52 Geometric progression: If $a_1, a_2, \ldots, a_n$ are in geometric progression with common ratio r, then:

1.521 $a_n = a_1 r^{n-1}$

1.522 $\displaystyle\sum_{i=1}^{n} a_i = \frac{a_1(1 - r^n)}{1 - r}$

1.522
S82-#8, p. 40
S82-#18, p. 40

1.523 $\displaystyle\sum_{i=1}^{\infty} a_i = \frac{a_1}{1 - r}$ if $|r| < 1$

1.523
S76-#1, p. 6
S76-#16, p. 8
S76-#15, p. 11
F76-#19, p. 11
S77-#12, p. 13
S78-#24, p. 20
S79-#25, p. 28
F81-#10, p. 37
F81-#14, p. 37
F82-#26, p. 43

1.53 $\sum_{i=1}^{n} i = n(n+1)/2$

For AIME, sum of squares = n(n+1)(2n+1)/6

1.53
S78-#6, p.
S78-#8, p.
S78-#20, p.
S78-#28, p.
S78-#23, p.

1.6 De Moivre's Theorem: For any complex number r cis θ (or $r\cos\theta + ir\sin\theta$),

$$(r \text{ cis } \theta)^n = r^n \text{ cis } (n\theta).$$

1.6
F78-#18, p. 22
F78-#30, p. 23

1.7 Rational Root Theorem: If $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_o$ is a polynomial with integer coefficients and p/q is a rational root of the equation $P(x) = 0$ (where p and q are relatively prime), then p must divide a_o, while q must divide a_n.

1.7
F75-#6, p. 4
F75-#10, p. 4
S76-#6, p. 7
S80-#30, p. 31

1.81 If $P(x)$ is a polynomial with real coefficients and $P(a + bi) = 0$ (where i is the imaginary unit and a,b are real), then $P(a - bi) = 0$ as well.

1.82 If $P(x)$ is a polynomial with rational coefficients and $P(a + b\sqrt{c}) = 0$ (where a, b, and c are rational and c is positive but not a perfect square), then $P(a - b\sqrt{c}) = 0$.

1.81
S80-#10, p. 29
S80-#20, p. 30

1.82
F77-#28, p. 17
S80-#10, p. 29

NUMBER THEORY

Skip this section for AMC-10/12. Sometimes useful for AIME

2.1 Pythagorean Triples: All positive integer solutions to the equation $a^2 + b^2 = c^2$ are given by:

$$a = k(m^2 - n^2)$$
$$b = k(2mn)$$
$$c = k(m^2 + n^2)$$

where m, n, and k are arbitrary positive integers, with $m > n$. For relatively prime triples (a,b,c), we need $k = 1$, and m and n relatively prime and of opposite parity.

2.1
F75-#25, p. 6
S76-#9, p. 7
F76-#30, p. 12
S78-#10, p. 18

2.2 Fermat's "Last Theorem" for $n = 3$: The equation $a^3 + b^3 = c^3$ has no solutions in integers unless $abc = 0$.

2.2
S78-#9, p. 18

2.3 Fermat's "Little" Theorem: If p is prime and a is relatively prime to p, then $a^{p-1} - 1$ is a multiple of p.

Skip this section for AMC-10/12

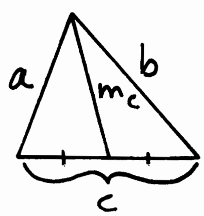
2.3
F82-#22, p. 43
F82-#29, p. 43

GEOMETRY

3.1 The medians of a triangle are concurrent at a point (the centroid) that divides each median in the ratio 2:1.

3.1
F77-#23, p. 17

3.2 For median m_c to side c, $m_c^{\,2} = \frac{1}{2}(a^2 + b^2) - \frac{1}{4}c^2$.



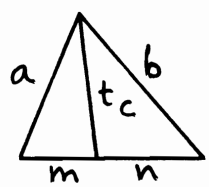
Not seen on AMC in last 10 years

3.2
S76-#20, p. 8

3.3 If angle bisector t_c divides side c into segments of lengths m and n, then:

3.31 $a:b = m:n$

Very Important



Less Important

3.32 $ab - mn = t_c^{\,2}$

3.3
F75-#5, p. 4
S76-#2, p. 6
F78-#12, p. 21
F81-#26, p. 39
S83-#12, p. 45
F84-#29, p. 54

3.32
S76-#2, p. 6

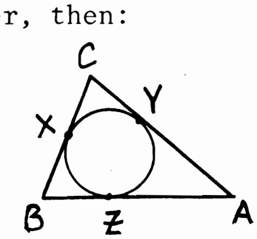
3.4 The "Extended Law of Sines": If R is the circumradius of triangle ABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$

Should also Include Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos(C)$

3.4
F78-#16, p. 21
F79-#24, p. 28
F79-#30, p. 28
F81-#16, p. 38
F81-#30, p. 39
F84-#15, p. 53

3.5 If X, Y, and Z are the points of contact of the sides of triangle ABC with its inscribed circle and s is the triangle's semiperimeter, then:

$CX = CY = s - c$
$BX = BZ = s - b$
$AY = AZ = s - a$



3.5
S77-#22, p. 14
S78-#30, p. 20
F82-#18, p. 43
F84-#3, p. 52

3.6 If a, b, c are the sides of triangle ABC, k its area, s its semiperimeter, R its circumradius, and r its inradius, then:

3.61 $k = \sqrt{s(s - a)(s - b)(s - c)}$ (Hero's Formula)

3.62 $k = rs$

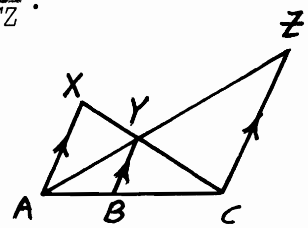
3.63 $k = abc/4R$

3.64 $k = (1/2)ab \sin C$

All four area formuli come up in AMC/AIME

3.7 In an equilateral triangle, the sum of the distances from any interior point to the three sides is always equal to the altitude of the triangle.

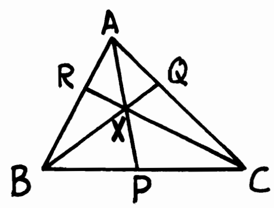
3.81 If $AX \| BY \| CZ$, then $\frac{1}{BY} = \frac{1}{AX} + \frac{1}{CZ}$.



3.82 Ceva's Theorem: If AP, BQ, CR are concurrent at X (as shown), then:

3.821 $\frac{AR}{RB} \cdot \frac{BP}{PC} \cdot \frac{CQ}{QA} = 1$

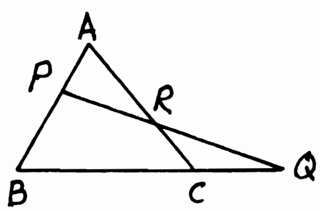
3.822 $\frac{XP}{AP} + \frac{XQ}{BQ} + \frac{XR}{CR} = 1$



3.83 Menelaus' Theorem: If "transversal" PR intersects the sides of triangle ABC in P, Q, and R (as shown), then:

$\frac{PP}{PB} \cdot \frac{BQ}{QC} \cdot \frac{CR}{RA} = 1$

"PP" should read "AP"



Both Rarely Tested in Last 10 yrs

3.84 In parallelogram $ABCD$, $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$.

Skip for AMC

3.6
S75-#3, p. 1
F76-#7, p. 10
F77-#25, p. 17
S78-#3, p. 18
S80-#18, p. 30
S81-#21, p. 35
S83-#26, p. 46

3.62
S75-#3, p. 1
S78-#2, p. 18
S80-#18, p. 30
F82-#18, p. 43

3.63, 3.64
S78-#3, p. 18
S79-#2, p. 23
S79-#15, p. 24
S79-#18, p. 24
F79-#4, p. 26
F79-#15, p. 27
S80-#12, p. 29
S82-#4, p. 39
S83-#22, p. 45
S83-#30, p. 46

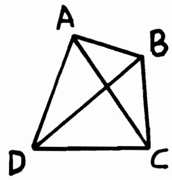
3.7
S76-#14, p. 7

3.81
S77-#14, p. 14

3.83
S77-#1, p. 13

3.84
F80-#4, p. 31
F80-#12, p. 32

3.91 Ptolemy's Theorem: In any quadrilateral $ABCD$, $AC \cdot BD \leq AB \cdot CD + AD \cdot BC$, with equality holding if and only if quadrilateral $ABCD$ can be inscribed in a circle.



Skip for AMC

3.92 For a quadrilateral to be cyclic (to possess a circumscribing circle), it is necessary and sufficient that its opposite angles be supplementary.

3.93 In a circle of radius r, the length of a chord which subtends an inscribed angle θ is $2r \sin \theta$.

TRIGONOMETRY

The "Pythagorean" Identities:

Very Useful

4.11 $\sin^2 x + \cos^2 x = 1$

4.12 $\tan^2 x + 1 = \sec^2 x$

4.13 $\cot^2 x + 1 = \csc^2 x$

Double and Half-Angle Formulas:

4.21 $\sin 2x = 2 \sin x \cos x$

4.22 $\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$

TRIGONOMETRY FORMULI TO REMEMBER

I would remember only the Pythagorean, double angle and sum angle formula, Derive the half angle and difference angle formula from these as needed.

Don't bother memorizing the (sin(x) + cos(y)) and sin(x)cos(y) type formula. These last two are very rarely useful. Spend your time elsewhere

3.91
F75-#9, p. 4
F75-#15, p. 5
F77-#3, p. 15
F77-#12, p. 16
S81-#20, p. 35
S81-#26, p. 36

3.92
F77-#12, p. 16
F81-#30, p. 39

3.93
F83-#30, p. 49
F84-#15, p. 53

4.11
F76-#12, p. 11
F76-#14, p. 11
S79-#8, p. 23
S79-#21, p. 25
F79-#5, p. 26
S80-#17, p. 30
F80-#19, p. 32
F81-#13, p. 37
S83-#10, p. 44
S83-#28, p. 46
F83-#13, p. 47
S84-#11, p. 50
S84-#18, p. 50
F84-#7, p. 52

4.12
F82-#19, p. 43

4.21
F76-#14, p. 11
S78-#13, p. 19
S78-#22, p. 19
F78-#30, p. 23
S79-#8, p. 23
S79-#18, p. 24
F79-#22, p. 28
F81-#29, p. 39
S82-#11, p. 40
F82-#30, p. 43
S84-#18, p. 50
F84-#7, p. 52

4.22
F78-#30, p. 23
S81-#8, p. 34
F81-#29, p. 39
S82-#11, p. 40
S84-#18, p. 50

4.23 $\tan x = (\sin 2x)/(1 + \cos 2x) = (1 - \cos 2x)/\sin 2x$

4.24 $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$; $\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$

4.25 $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$; $\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$

Very Useful

Addition Formulas:

4.311 $\sin (x + y) = \sin x \cos y + \cos x \sin y$

4.312 $\sin (x - y) = \sin x \cos y - \cos x \sin y$

4.313 $\cos (x + y) = \cos x \cos y - \sin x \sin y$

4.314 $\cos (x - y) = \cos x \cos y + \sin x \sin y$

Useful

4.321 $\sin x + \sin y = 2 \sin[(x + y)/2]\cos[(x - y)/2]$

4.322 $\sin x - \sin y = 2 \cos[(x + y)/2]\sin[(x - y)/2]$

4.323 $\cos x + \cos y = 2 \cos[(x + y)/2]\cos[(x - y)/2]$

4.324 $\cos x - \cos y = -2 \sin[(x + y)/2]\sin[(x - y)/2]$

4.331 $\sin x \sin y = (1/2)[\cos (x - y) - \cos (x + y)]$

4.332 $\cos x \sin y = (1/2)[\sin (x + y) - \sin (x - y)]$

4.333 $\cos x \cos y = (1/2)[\cos (x + y) + \cos (x - y)]$

4.334 $\sin x \cos y = (1/2)[\sin (x + y) + \sin (x - y)]$

Not Useful For AMC

4.341 $\tan (x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

Very Useful

4.23
F76-#14, p. 11

4.24
F76-#14, p. 11
S77-#4, p. 13
F79-#5, p. 26
F79-#10, p. 27

4.311
S77-#23, p. 14
S77-#30, p. 15
F84-#6, p. 52

4.312
S77-#30, p. 15
F82-#30, p. 43

4.313
F76-#25, p. 9
F84-#6, p. 52
F84-#21, p. 53

4.314
F80-#27, p. 33
F82-#24, p. 43
S83-#28, p. 46

4.321
S77-#30, p. 15
F77-#17, p. 16
S79-#4, p. 23
F82-#30, p. 43
F83-#4, p. 46

4.322
S82-#27, p. 41
F82-#30, p. 43

4.323
F77-#9, p. 16
F77-#17, p. 16
F78-#10, p. 21
F83-#4, p. 46

4.331
F78-#10, p. 21

4.341
F76-#3, p. 10
S78-#12, p. 19
F79-#10, p. 27
F79-#14, p. 27
F81-#22, p. 38
S82-#24, p. 41

4.342 $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

4.4 $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\dfrac{A + B}{1 - AB}\right)$

4.51 $\sin 3x = 3 \sin x - 4 \sin^3 x$

4.52 $\cos 3x = 4 \cos^3 x - 3 \cos x$

Not tested
Don't Bother

MISCELLANY

For $a, b, c > 0$, $a, b, c \neq 1$:

5.11 $(\log_a b)(\log_b c) = \log_a c$ (the "chain rule" for logarithms)

5.12 $\log_a b = \dfrac{1}{\log_b a}$

Very important for changing bases in a logarithm problem.

If $[x]$ denotes the greatest integer no larger than x, then:

5.21 $[(x + m)/n] = [([x] + m)/n]$, for m, n integers, and $n > 0$.

5.22 For nonintegral $x > 0$, if $[x] = n$, then $[-x] = -n - 1$.

INEQUALITIES

6.1 If $x > 0$, $f(x) = x + 1/x$ attains its minimum value when $x = 1$.

Very Useful

6.2 The "arithmetic-geometric mean inequality:" for $a, b > 0$, $(a + b)/2 \geq \sqrt{ab}$, with equality holding if and only if $a = b$. In general, for $a_1, a_2 \ldots a_n \geq 0$, $\frac{1}{n}(a_1 + a_2 + \ldots + a_n) \geq \sqrt[n]{a_1 a_2 \ldots a_n}$, with equality holding if and only if $a_1 = a_2 = \ldots = a_n$.

6.3 If $a + b$ remains constant, the product ab is largest when $a = b$.

These four inequalities often creep into AMC-12 problems

4.342
F79-#10, p. 27
F79-#11, p. 27

4.4
S77-#13, p. 13
F81-#17, p. 38

4.51
S79-#28, p. 25

5.11
S78-#7, p. 18
S78-#17, p. 19
F78-#11, p. 21

5.12
F79-#28, p. 28
S80-#13, p. 29
F82-#14, p. 42
F83-#17, p. 48

5.22
F75-#1, p. 3
S77-#29, p. 15

6.1
S76-#6, p. 7
F81-#2, p. 36
F82-#27, p. 43
S83-#30, p. 46

6.3
F76-#7, p. 10
S78-#29, p. 20
F80-#21, p. 33

6.4 If ab remains constant, the sum $a + b$ is smallest when $a = b$.

Very Useful

6.4
F82-#9, p. 42
F82-#15, p. 42
F82-#21, p. 43
F84-#16, p. 53

7. The Inclusion-Exclusion Principle: If we use $|A|$ to denote the number of elements in the set A, then:

7.1 $$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |A_i| - \sum_{1 \le i < j \le n} |A_i \cap A_j| + \sum_{1 \le i < j < k \le n} |A_i \cap A_j \cap A_k| - \dots + (-1)^{n+1}\left|\bigcap_{i=1}^{n} A_k\right|$$

In particular,

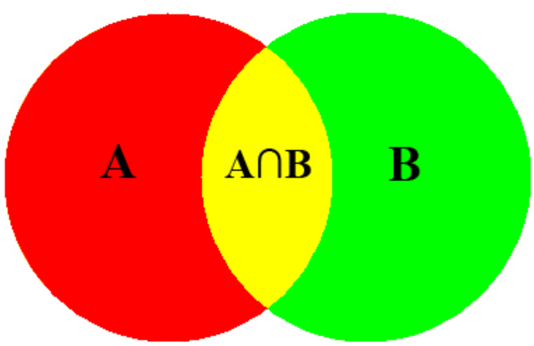
7.2 $|A \cup B| = |A| + |B| - |A \cap B|$

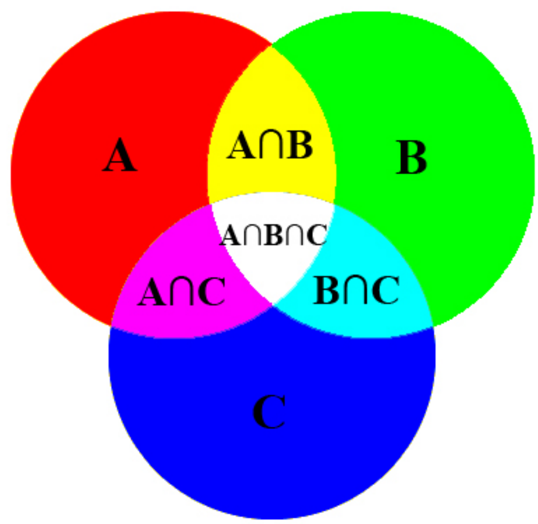
7.3 $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

7.1
S82-#25, p. 41

7.2
S77-#2, p. 13

7.3
F82-#6, p. 42





Chords, Secants & Tangents

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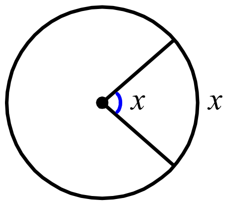

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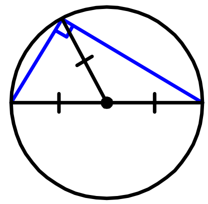

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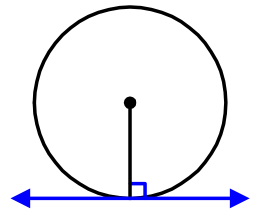

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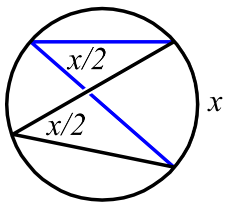

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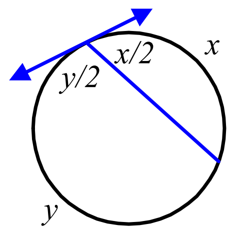

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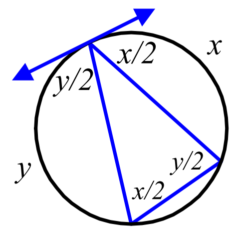

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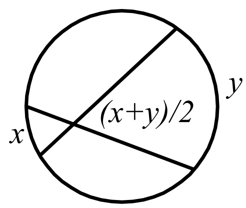

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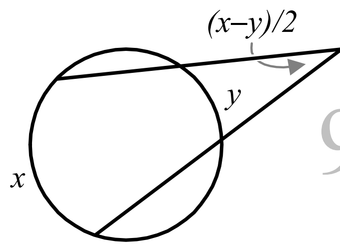

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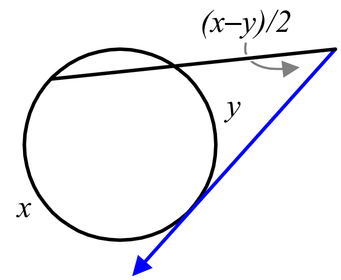

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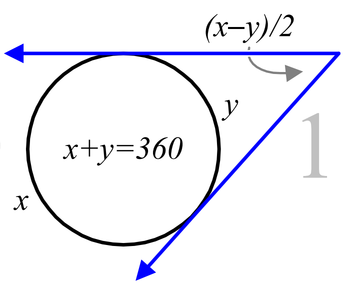

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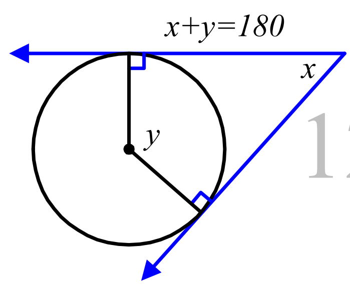

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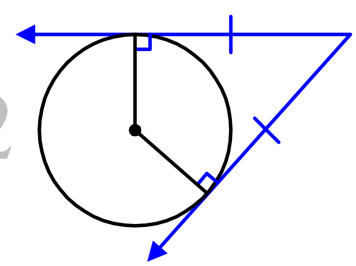

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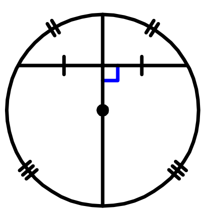

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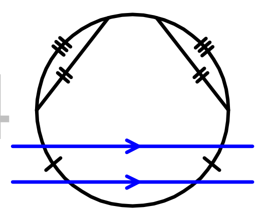

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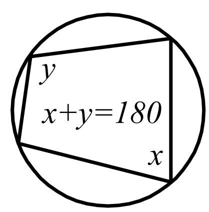

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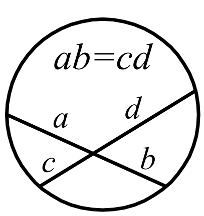

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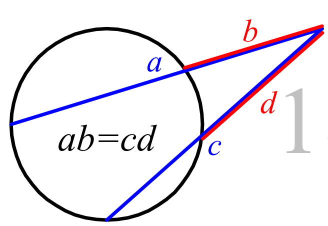

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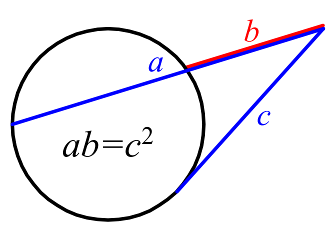


Discrete Mathematics

Important Formuli for Counting

Combinatorics

Counting principle: If a choice consists of k steps, of which the first can be made in n_1 ways, the second in n_2 ways, … , and the k^{th} in n_k ways, then the whole choice can be made in $n_1 n_2 \ldots n_k$ ways.

Factorials: $n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$

Permutations: A permutation is an arrangement of objects where order matters. (123 and 213 are considered different permutations of the digits 1, 2, and 3).

${}_nP_r$ is the number of permutations of r objects chosen from n objects.

$$_nP_r = \frac{n!}{(n-r)!}$$

Special cases: there are $n!$ ways of arranging all n objects.

Repeated objects: In an arrangement of n objects, if there are r_1 objects of type 1, r_2 objects of type 2, … r_k objects of type k, where objects of the same type are indistinguishable, then there are $\frac{n!}{r_1! r_2! \cdots r_k!}$ ways to arrange the n objects.

Circular Permutations: If n objects are arranged in a circle, there are $(n-1)!$ possible arrangements.

“Key-ring” permutations: If n objects are arranged on a key ring, there are $\frac{(n-1)!}{2}$ possible arrangements.

Combinations: In a combination, the order of objects does not matter (123 is the same as 213).

${}_nC_r$ is the number of combinations of r objects chosen from n objects.

$$_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Sets:

For sets A and B,

Union: $A \bigcup B$ is the set that contains the elements in either A, B, or both.
Intersection: $A \bigcap B$ is the set that contains only elements that are in both A and B.
Complement: A' is the set of all elements not in A.
Inclusion-Exclusion principle: If $n(S)$ is the number of elements in set S, then

$$n(A \bigcup B) = n(A) + n(B) - n(A \bigcap B).$$

This can be extended for more than two sets. (ex. For sets A, B, and C, $n(A \bigcup B \bigcup C) = n(A) + n(B) + n(C) - n(A \bigcap B) - n(B \bigcap C) - n(A \bigcap C) + n(A \bigcap B \bigcap C)$.

Probability:

If an experiment can occur in exactly n ways, and if m of these correspond to an event E, then the probability of E is given by

$$P(E) = \frac{m}{n}$$

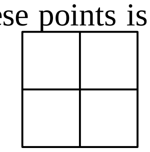
$P(A$ and $B) =$ $P(A \cap B) = P(A)P(B)$ if A and B are independent events.
$P(A$ or $B) =$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability: the conditional probability of an event E, given an event F, is denoted by $P(E/F)$ and is defined as $P(E / F) = \dfrac{P(E \cap F)}{P(F)}$.

Pigeonhole principle: If there are more than k times as many pigeons as pigeonholes, then some pigeonhole must contain at least $k+1$ pigeons. Or, if there are m pigeons and n pigeonholes, then at least one pigeonhole contains at least $\left\lfloor \dfrac{m-1}{n} \right\rfloor + 1$ pigeons.

Ex. Consider any five points P_1, P_2, P_3, P_4, and P_5 in the interior of a square S with side length 1. Denote by d_{ij} the distance between points P_I and P_j. Prove that at least one of the distances between these points is less than $\dfrac{\sqrt{2}}{2}$.

Solution: Divide S into four congruent squares. By the pigeonhole principle, two points belong to one of these squares (a point on the boundary can be claimed by both squares). The distance between these points is less than $\dfrac{\sqrt{2}}{2}$. (Problem and solution from Larson, number 2.6.2).



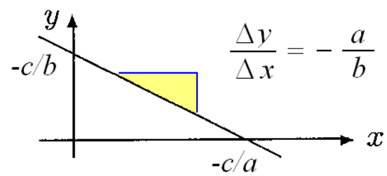
Skip for AMC

8.2 Equations of a straight line.

The graph of the equation

$$ax + by + c = 0$$

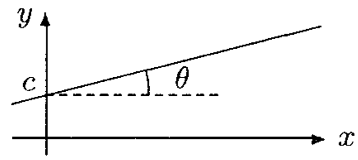
is a straight line.



Useful formulations of this equation are:

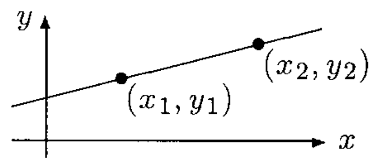
Gradient-intercept form:

$$y = mx + c. \qquad m = \frac{\Delta y}{\Delta x} = \tan\theta$$



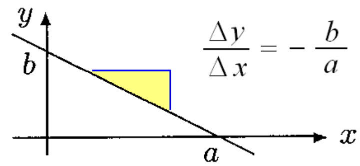
Two-point form:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}.$$



Intercept form:

$$\frac{x}{a} + \frac{y}{b} = 1.$$



8.3 The distance between two points. The distance between the points (x_1, y_1) and (x_2, y_2) is, by the Theorem of Pythagoras

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

8.4 The distance from a point to a line. The perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

8.5 The area A of the triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) is

$$A = \frac{1}{2}|(x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3) + (x_1y_2 - x_2y_1)|$$

Shoelace Formula

Blue +

Red −

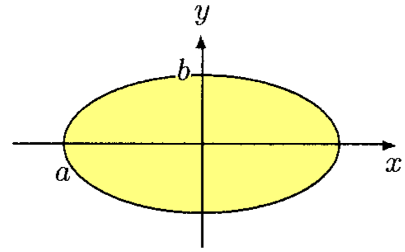
x_1	y_1		$- x_2y_1$
x_2	y_2	x_1y_2	$- x_3y_2$
x_3	y_3	x_2y_3	$- x_1y_3$
x_1	y_1	x_3y_1	

A similar shoelace formula exists for all polygons

8.8 Standard conic equations. By appropriate choice of co-ordinate axes the equations for the ellipse, parabola and hyperbola can be expressed in so-called standard form:

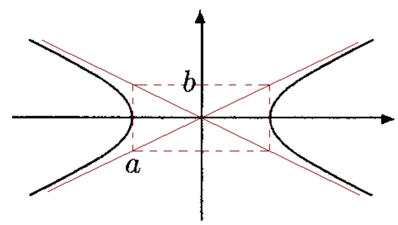
Ellipse: $$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

(When $a=b$, $x^2+y^2=a^2$, which represents a circle with radius a.)



Hyperbola: $$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$$

(When $a=b$ the hyperbola is called rectangular and $x^2-y^2=a^2$.)



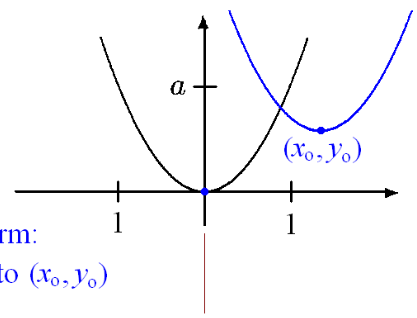
Parabola: $$y=ax^2.$$

$y = ax^2 + bx + c$

is also a parabola with the same shape factor, a, but with vertex shifted in (x, y)

Complete the square and convert to the form:

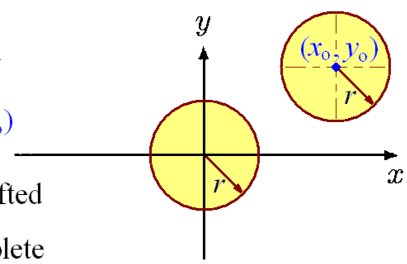
$y = a(x - x_o)^2 + y_o$ to see vertex shifted to (x_o, y_o)



Circle: $x^2 + y^2 = r^2$ centered at origin

$(x - x_o)^2 + (y - y_o)^2 = r^2$ centered at (x_o, y_o)

$x^2 + bx + y^2 + cy = d$ is also a circle shifted away from the origin, but you need to complete the square on the (x^2+bx) term and the (y^2+cy) term and convert to the second form to see the radius and the center shifted to (x_o, y_o)



Note: The line, circle and shifted circle are by far the most used coordinate geometry figures in the AMC.

The ellipse is rare in the AMC and hyperbola very rare.

The ellipse and hyperbola do show up in the AIME.