DOE
Flipped Assignment 08
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Ans to the Ques 01

Given, I = fluid type (populations) = 4

J = No. of observations in each population = 6

$\therefore$ There are $(I-1)$ or $(4-1) = 3$ degrees of freedom in treatments.

Ans to the Ques NO: 02

We have the ANOVA table

Source	d.f	~~SS~~ SS	~~MS~~ MS	F-stat
Treatment	$I-1$	SST_r	MST_r	$\frac{MST_r}{MSE}$
Error				
Total				

Now we have 3 degree's of freedom in treatments. So for I treatments, there may be a set of $(I-1)$ orthogonal contrasts that would completely partition the SST_r. In our case, we have $\boxed{\text{3 sets}}$ of orthogonal contrasts that would completely partition the SST_r.

Now the ANOVA table

Source	df	SS	MS	F-stat
Treatment	$3\ (I-1)$	SST_r	MST_r	
partitioned into				
C_1	1	SSC_1	MSC_1	$\frac{MSC_1}{MSE} \sim F_{1, I(J-1)}$
C_2	1	SSC_2	MSC_2	$\frac{MSC_2}{MSE} \sim F_{1, I(J-1)}$
C_3	1	SSC_3	MSC_3	$\frac{MSC_3}{MSE} \sim F_{1, I(J-1)}$

Ans to the Ques 03

Contrast Test

$$C = (\mu_1 + \mu_2) - (\mu_3 + \mu_4)$$

$$= 18.65 + 17.95 - (20.95 + \cancel{18}\ 18.82)$$

$$= -3.17$$

Hypothesis

$H_0: (\mu_1+\mu_2)-(\mu_3+\mu_4)=0$

$H_a: (\mu_1+\mu_2)-(\mu_3+\mu_4)\neq 0$

we know

$$Z=\frac{\sum c_i \bar{y}_{i.}}{\sqrt{\frac{\sigma^2}{n}\sum c_i^2}} \Rightarrow t_r=\frac{\sum c_i \bar{y}_{i.}}{\sqrt{\frac{MSE}{n}\sum c_i^2}}$$

where $MSE=\frac{SSE}{I(J-1)}=\frac{63.99\text{ (given)}}{4(6-1)}$

$=3.1995$

we have $n=$ total no. of observations from all populations $=24$

$\sum c_i^2=(1)^2+(1)^2+(-1)^2+(-1)^2=4$

and $\sum c_i \bar{y}_{i.}=1(18.65)+1(17.95)+(-1)20.95+(-1)(18.82)$

$=-3.17$

where, $\bar{y}_{i.}=\hat{\mu}_i$

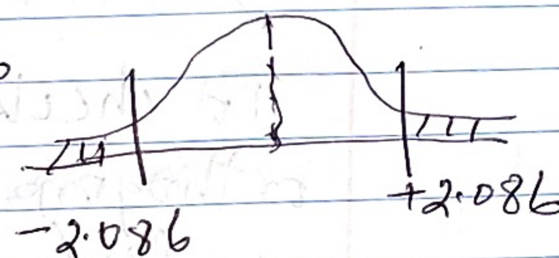
$\therefore$ T-stat $= T_{n-i} = \dfrac{-3.17}{\sqrt{\dfrac{3.1995}{24} \times 4}} = \dfrac{-3.17}{0.7302}$

$= -4.341$

$\sim T_{24-4} \sim T_{20}$

Using R, critical value at $\alpha = 0.05$

$T_{critical}|_{\alpha=0.05} = \pm 2.086$



Since $T_{stat} < T_{critical}$

or $-4.341 < -2.086$

$\therefore$ we reject Null hypothesis H_0.

$\therefore (\mu_1 + \mu_2) - (\mu_3 + \mu_4) \neq 0$

Means of fluid 1 & 2 is smaller than fluid 3 & 4.

Ans to the Ques 4

We have

$$c_1 = (\mu_1 + \mu_4) - (\mu_2 + \mu_3)$$

and the contrast in 3 is

$$c_2 = (\mu_1 + \mu_2) - (\mu_3 + \mu_4)$$

To check whether the contrasts are orthogonal or not we need to satisfy $\sum c_i d_i = 0$

$\therefore \sum c_1 d_1 = 1 \times 1 = 1$

$\sum c_2 d_2 = -1 \times 1 = -1$

$\sum c_3 d_3 = -1 \times -1 = 1$

$\sum c_4 d_4 = 1 \times -1 = -1$

$\therefore \sum c_i d_i = 1 - 1 + 1 - 1 = 0$

$\therefore$ The Contrast's are orthogonal.

Ans to the Ques 5

LSD test with $\alpha = 0.05$

for population 1 & population 2

LSD, $$\frac{\bar{X}_{i.} - \bar{X}_{j.}}{\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}} \sim t_{IJ-I}$$

$$\Rightarrow \bar{X}_{i.} - \bar{X}_{j.} = t_{IJ-I} \times \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

$$\Rightarrow 18.65 - 17.95 = t_{20} \times \sqrt{3.1995\left(\frac{1}{6} + \frac{1}{6}\right)}$$

$$\Rightarrow 0.7 = t_{20} \times 1.033$$

$$\Rightarrow 0.7 = \underbrace{2.086}_{\text{critical value using R}} \times 1.033 = 2.155$$

$\therefore$ $0.7 < 2.155$ (fail to reject H_0)

Similarly for population 1 & population 3

-2.3 or $+2.3 > 2.155$ (reject H_0)

because -2.3 (diffrence in mean of ① & (3)) will be same as (3) & (1) but positive.

Similarly for population (1) & population (4)

$-0.17 < 2.155$ (fail to reject H_0)

Hence

Difference in means for pop^n (1) & (2) → fail to reject H_0

" " " " pop^n (1) & (3) → reject H_0

" " " " pop^n (1) & (4) → fail to reject H_0

∴ Mean difference between population (1) & population (3) pairs of fluids significantly differ in mean lifetime.