1

Design of Experiment

Homework Week 04

Name: Md Ariful Haque Miah
R# 11636945

Answer to the Ques No:- 3.7(a)

3.7(a) Here, I = No. of Mixing Technique
$I = 4$ (which is the population)
J = No. of observations = 4 (from each population)

Hypothesis

Null $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
Alternative H_a: at least one of the μ_i differs
$i = 1, 2, 3, 4$

μ_1 = mean tensile strength of Mixing Technique 1
μ_2 = " " " " " 2
μ_3 = " " " " " 3
μ_4 = " " " " " 4

$$\bar{y}_{i.} = \frac{\sum_{j=1}^{J} y_{ij}}{J} \Rightarrow \bar{y}_{1.} = \frac{y_{11}+y_{12}+y_{13}+y_{14}}{4} = 2971 = \mu_1$$

similarly, $\bar{y}_{2.} = \mu_2 = \frac{y_{21}+y_{22}+y_{23}+y_{24}}{4} = 3156.25$

$\bar{y}_{3.} = \mu_3 = \frac{y_{31}+ \cdots +y_{34}}{4} = 2933.75$

$\bar{y}_{4.} = \mu_4 = \frac{y_{41}+ \cdots +y_{44}}{4} = 2666.25$

2

$\therefore$ Grandmean, $\bar{y}_{..} = \dfrac{\sum_{i=1}^{I} \bar{y}_{i.}}{I} = \dfrac{\bar{y}_{1.} + \bar{y}_{2.} + \bar{y}_{3.} + \bar{y}_{4.}}{I}$

$= 2931.81$

Now, $SSE = \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2$

$= \sum_i \Big[\{(y_{11} - \bar{y}_{1.})^2 + \cdots + (y_{14} - \bar{y}_{1.})^2 + (y_{21} - \bar{y}_{2.})^2 + \cdots + (y_{24} - \bar{y}_{2.})^2$
$+ (y_{31} - \bar{y}_{3.})^2 + \cdots + (y_{34} - \bar{y}_{3.})^2 + (y_{41} - \bar{y}_{4.})^2 + \cdots$
$\cdots + (y_{44} - \bar{y}_{4.})^2\} \Big]$

$= 153908.25$ (using excel)

$\therefore MSE = \dfrac{SSE}{I(J-1)} = 12825.69$

$SST_r = J \times \sum_i (\bar{y}_{i.} - \bar{y}_{..})^2 \quad i = 1, 2, 3, 4$

$= J \times [(\bar{y}_{1.} - \bar{y}_{..})^2 + \cdots + (\bar{y}_{4.} - \bar{y}_{..})^2]$

$= 489740.19$ (using excel)

$MST_r = \dfrac{SST_r}{I-1} = 163246.73$

$MST_r > MSE$, we reject H_0 but still we will check F distn to determine which one is larger

$F = \dfrac{MST_r}{MSE} = 12.73$, critical value at $\alpha = 0.05$ level of significance $= 3.49$ (using R)

since F > critical value (or $12.73 > 3.49$)
we reject Null hypothesis H_0.

$\therefore$ Mixing Technique affect the strength of the cement.

Ans to the Ques :- 3.10(a)

3.10(a) Here, I = No. of cotton weight ($y_{.}$) = 5 (population)

J = No. of observations in each population = 5

Hypothesis Null: $H_0: \mu_1 = \mu_2 = \cdots = \mu_5 = 0$

Alternative H_a: at least one of the μ_i differs

$i = 1, 2, \cdots, 5$

μ_1 = mean obs. of cotton weight ($y_{.}$) 15 $= \bar{y}_{1.}$

μ_2 = " " " " " 20 $= \bar{y}_{2.}$

μ_3 = " " " " " 25 $= \bar{y}_{3.}$

μ_4 = " " " " " 30 $= \bar{y}_{4.}$

μ_5 = " " " " " 35 $= \bar{y}_{5.}$

$\bar{y}_{1.} = \dfrac{\sum_{j=1}^{J} y_{ij}}{J} \Rightarrow \bar{y}_{1.} = \dfrac{y_{11} + \cdots + y_{15}}{5} = 9.80 = \mu_1$

Similarly, $\bar{y}_{2.} = \dfrac{y_{21} + \cdots + y_{25}}{5} = 15.40 = \mu_2$

$\bar{y}_{3.} = \dfrac{y_{31} + \cdots + y_{35}}{5} = 17.60 = \mu_3$

$\bar{y}_{4.} = \dfrac{y_{41} + \cdots + y_{45}}{5} = 21.60 = \mu_4$

$\bar{y}_{5.} = \dfrac{y_{51} + \cdots + y_{55}}{5} = 10.80 = \mu_5$

Grand Mean, $\bar{y}_{..} = \dfrac{\sum_{i=1}^{I} \bar{y}_{i.}}{I} = \dfrac{\bar{y}_{1.} + \bar{y}_{2.} + \cdots + \bar{y}_{5.}}{5} = 15.04$

$SSE = \sum_{i} \sum_{j} (y_{ij} - \bar{y}_{i.})^2$

$= \sum_{i} [(y_{11} - \bar{y}_{1.})^2 + \cdots (y_{15} - \bar{y}_{1.})^2 + (y_{21} - \bar{y}_{2.})^2 + \cdots (y_{25} - \bar{y}_{2.})^2$
$+ \cdots + (y_{51} - \bar{y}_{5.})^2 + \cdots + (y_{55} - \bar{y}_{5.})^2]$

$= 161.20$ (using excel)

$MSE = \frac{SSE}{I(J-1)} = 8.06$

$SST_r = J \times \sum_i (\bar{y}_{i.} - \bar{y}_{..})^2, \quad i = 1, 2, \cdots, 5$

$= J \times [(\bar{y}_{1.} - \bar{y}_{..})^2 + \cdots + (\bar{y}_{5.} - \bar{y}_{..})^2]$

$= 475.76$ (using excel)

$MST_r = \frac{SST_r}{I-1} = 118.94$

$MST_r > MSE$, reject Null hypothesis H_0" we will check f distn to determine which one is larger

$F = \frac{MST_r}{MSE} = 14.76$,

critical value at $\alpha = 0.05$ level of significance $= 2.866$ (using R)

Since $f >$ critical value (or, $14.76 > 2.866$)

we reject Null hypothesis H_0.

$\therefore$ The Cotton content affects the mean tensile strength.

Ans to the Ques 3.20

3.20(a) Here, I = no. of Rodd. level = 4 (population)

J = No. of observation in each population = 3

Hypothesis

Null $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

Alternative H_a: at least one of the μ_i differs

$i = 1, 2, 3, 4$

μ_1 = mean compress strength of Rodd. level 10 $= \bar{y}_{1.}$
μ_2 = " " " " " 15 $= \bar{y}_{2.}$
μ_3 = " " " " " 20 $= \bar{y}_{3.}$
μ_4 = " " " " " 25 $= \bar{y}_{4.}$

$$\bar{y}_{i.} = \frac{\sum_{j=1}^{J} y_{ij}}{J} \Rightarrow \bar{y}_{1.} = \frac{y_{11}+y_{12}+y_{13}}{3} = 1500$$

$$\bar{y}_{2.} = \frac{y_{21}+y_{22}+y_{23}}{3} = 1586.67$$

$$\bar{y}_{3.} = \frac{y_{31}+y_{32}+y_{33}}{3} = 1606.67$$

$$\bar{y}_{4.} = \frac{y_{41}+y_{42}+y_{43}}{3} = 1500$$

Grand mean, $\bar{y}_{..} = \frac{\sum_{i=1}^{I} \bar{y}_{i.}}{I} = \frac{\bar{y}_{1.}+\cdots+\bar{y}_{4.}}{4} = 1548.33$

$$SSE = \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2$$
$$= \sum_i [(y_{11} - \bar{y}_{1.})^2 + \cdots + (y_{13} - \bar{y}_{1.})^2 + \cdots + (y_{41} - \bar{y}_{4.})^2 + \cdots + (y_{43} - \bar{y}_{4.})^2]$$

= 40933.33 (using excel)

$$MSE = \frac{SSE}{I(J-1)} = 5116.67$$

$$SSTr = J \times \sum_i (\bar{y}_{i.} - \bar{y}_{..})^2, \quad i = 1,2,3,4$$
$$= J \times [(\bar{y}_{1.} - \bar{y}_{..})^2 + \cdots + (\bar{y}_{4.} - \bar{y}_{..})^2]$$

= 28633.33 (using excel)

6

$$MST_r = \frac{SST_r}{I-1} = 9544.44$$

$MST_r > MSE$, reject H_0. Now we will F-dist? to determine which one is larger.

$$F = \frac{MST_r}{MSE} = 1.87$$

critical value at $\alpha = 0.05$ level of significance $= 4.066$ (using R)

Since $F <$ critical value (or $1.87 < 4.066$)

we fail to reject Null hypothesis H_0.

$\therefore$ There is no difference in compressive strength due to the rodding level.

3.20 (b) The p-value for the F-statistics in part (a) is $\boxed{0.214}$ (see the attached ".R" script)