# HW Week 12

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#### Problem 7.12

Since each replicate would be considered as a Block so No. of replicates = No. of Blocks = 7.

 $2^4$  factorial design, with 7 replications so total no. of observations are 112. There will be 16 corner points in each Block and each Block is replicated 7 times.

So Each Block will contain 16 corner points as follow: [(1) a b ab c ac bc abc d ad bd abd cd acd bcd abcd] These corner points are randomized within Block and Full replication within each Block. Model Equation:

```
Y_{ijklmn} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \gamma_k + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + \delta_l + \dots + (\alpha\beta\gamma\delta)_{ijkl} + \tau_m + \epsilon_{ijklmn}
```

Where,  $\tau_m$  = Block effect and n = replicate

ANOVA with Blocking

```
library(GAD)
```

```
## Loading required package: matrixStats
```

## Loading required package: R.methodsS3

## R.methodsS3 v1.8.2 (2022-06-13 22:00:14 UTC) successfully loaded. See ?R.methodsS3 for help.

```
A \leftarrow c(rep(-1,7), rep(1,7))
B \leftarrow c(rep(-1,14), rep(1,14))
C \leftarrow c(rep(-1,28), rep(1,28))
D \leftarrow c(rep(-1,56), rep(1,56))
Block \leftarrow c(rep(seq(1,7),16))
obs \leftarrow c(10,18,14,12.5,19,16,18.5,0,16.5,4.5,17.5,20.5,17.5,33,4,6,1,14.5,12,14,5,0,10,34,11,25.5,21.5,
          5,20.5,18,20,29.5,19,10,6.5,18.5,7.5,6,0,10,0,16.5,4.5,0,23.5,8,8,8,4.5,18,14.5,10,0,17.5,6,19
          36,15,16,8.5,0,0.5,9,3,41.5,39,6.5,3.5,7,8.5,36,8,4.5,6.5,10,13,41,14,21.5,10.5,6.5,0,15.5,24,
          5,7,10,32.5,18.5,8)
A <- as.fixed(A)
B <- as.fixed(B)
C <- as.fixed(C)
D <- as.fixed(D)
Block <- as.fixed(Block)</pre>
dat <- data.frame(A,B,C,D,Block,obs)</pre>
dat
```

```
##
       A B C D Block obs
                       1 10.0
## 1
      -1 -1 -1 -1
      -1 -1 -1 -1
                       2 18.0
## 3
      -1 -1 -1 -1
                       3 14.0
      -1 -1 -1 -1
                       4 12.5
## 5
      -1 -1 -1 -1
                       5 19.0
      -1 -1 -1 -1
## 6
                       6 16.0
```

```
7 18.5
## 7
       -1 -1 -1 -1
## 8
        1 -1 -1 -1
                           0.0
                         1
## 9
        1 -1 -1 -1
                         2 16.5
        1 -1 -1 -1
                           4.5
## 10
                         3
## 11
        1 -1 -1 -1
                         4 17.5
## 12
        1 -1 -1 -1
                         5 20.5
## 13
        1 -1 -1 -1
                         6 17.5
        1 -1 -1 -1
                         7 33.0
## 14
## 15
       -1 1 -1 -1
                         1
                            4.0
## 16
           1 -1 -1
                            6.0
       -1
                         2
## 17
       -1
           1 -1 -1
                         3
                           1.0
## 18
            1 -1 -1
                         4 14.5
       -1
## 19
       -1
            1 -1 -1
                         5 12.0
           1 -1 -1
## 20
                         6 14.0
       -1
       -1
## 21
            1 -1 -1
                         7
                            5.0
## 22
        1
            1 -1 -1
                         1
                            0.0
## 23
           1 -1 -1
                         2 10.0
        1
## 24
        1
           1 -1 -1
                         3 34.0
## 25
           1 -1 -1
                         4 11.0
        1
## 26
        1
            1 -1 -1
                         5 25.5
## 27
        1
          1 -1 -1
                         6 21.5
## 28
        1
          1 -1 -1
                         7
                            0.0
## 29
       -1 -1
               1 -1
                            0.0
                         1
## 30
       -1 -1
               1 -1
                         2
                            0.0
## 31
       -1 -1
                         3 18.5
               1 -1
## 32
       -1 -1
               1 -1
                         4 19.5
## 33
       -1 -1
               1 -1
                         5 16.0
## 34
       -1 -1
               1 -1
                         6 15.0
## 35
       -1 -1
               1 -1
                         7 11.0
## 36
        1 -1
               1 -1
                         1
                            5.0
## 37
        1 -1
               1 -1
                         2 20.5
## 38
        1 -1
               1 -1
                         3 18.0
                         4 20.0
## 39
        1 -1
               1 -1
## 40
                         5 29.5
        1 -1
               1 -1
## 41
        1 -1
               1 -1
                         6 19.0
        1 -1
## 42
                         7 10.0
               1 -1
## 43
       -1
           1
               1 -1
                            6.5
## 44
       -1
            1
               1 -1
                         2 18.5
## 45
       -1
            1
               1 -1
                         3
                            7.5
                            6.0
## 46
       -1
            1
                         4
               1 -1
## 47
       -1
            1
               1 -1
                            0.0
                         5
## 48
            1
               1 -1
                         6 10.0
       -1
## 49
           1
               1 -1
                         7
                            0.0
       -1
                         1 16.5
## 50
        1
            1
               1 -1
## 51
           1
               1 -1
                         2
                            4.5
        1
                            0.0
## 52
            1
               1 -1
                         3
        1
## 53
           1
                         4 23.5
        1
               1 -1
## 54
        1 1
               1 -1
                         5
                            8.0
## 55
        1
           1
              1 -1
                         6
                            8.0
                         7
                            8.0
## 56
        1
           1
               1
                 -1
## 57
                            4.5
       -1 -1 -1
                         1
                  1
                         2 18.0
## 58
       -1 -1 -1
## 59
       -1 -1 -1
                  1
                         3 14.5
## 60
       -1 -1 -1 1
                         4 10.0
```

```
## 61 -1 -1 1
                        5 0.0
## 62
       -1 -1 -1
                        6 17.5
                 1
       -1 -1 -1
## 63
                 1
                        7
                           6.0
        1 -1 -1
                        1 19.5
## 64
                  1
## 65
        1 -1 -1
                  1
                        2 18.0
## 66
        1 -1 -1
                        3 16.0
                 1
## 67
        1 -1 -1
                        4 5.5
                  1
        1 -1 -1
                        5 10.0
## 68
                  1
## 69
        1 -1 -1
                 1
                        6
                           7.0
## 70
        1 -1 -1
                        7 36.0
                 1
## 71
       -1
           1 -1
                  1
                        1 15.0
                        2 16.0
## 72
           1 -1
       -1
                  1
                           8.5
## 73
       -1
           1 -1
                  1
                        3
## 74
                           0.0
       -1
           1 -1
                         4
## 75
       -1
           1 -1
                        5
                           0.5
                  1
## 76
       -1
           1 -1
                  1
                        6
                           9.0
## 77
       -1
           1 -1
                        7
                           3.0
                 1
## 78
        1
           1 -1
                  1
                        1 41.5
           1 -1
## 79
                        2 39.0
                  1
        1
## 80
        1
           1 -1
                  1
                        3
                           6.5
## 81
        1
           1 -1
                 1
                        4
                           3.5
## 82
        1
           1 -1
                  1
                        5
                           7.0
## 83
           1 -1
                        6
                           8.5
        1
                  1
## 84
        1 1 -1
                  1
                        7 36.0
## 85
       -1 -1
              1
                           8.0
                 1
                        1
## 86
       -1 -1
              1
                  1
                        2
                           4.5
## 87
       -1 -1
               1
                        3
                           6.5
                  1
## 88
       -1 -1
                        4 10.0
               1
                 1
## 89
       -1 -1
               1
                        5 13.0
                 1
## 90
       -1 -1
               1
                  1
                        6 41.0
## 91
       -1 -1
               1
                  1
                        7 14.0
## 92
        1 -1
               1
                 1
                        1 21.5
## 93
                        2 10.5
        1 -1
               1
                 1
## 94
                           6.5
        1 -1
               1
                        3
                  1
## 95
        1 -1
               1
                  1
                        4
                           0.0
        1 -1
## 96
               1
                        5 15.5
                 1
## 97
        1 -1
               1
                        6 24.0
## 98
        1 -1
               1
                        7 16.0
                  1
## 99
       -1
           1
               1
                  1
                        1
                           0.0
## 100 -1
                           0.0
           1
               1
                        2
                 1
## 101 -1
           1
               1
                        3
                           0.0
                  1
## 102 -1
           1
               1
                        4
                           4.5
                  1
## 103 -1
           1
                        5
                           1.0
               1
                  1
## 104 -1
                           4.0
           1
               1
                        6
                  1
## 105 -1
           1
                        7
                           6.5
               1
                  1
## 106
        1
           1
               1
                        1 18.0
                  1
           1
                        2
                           5.0
## 107
        1
              1
                 1
## 108
        1
           1
               1
                           7.0
                        3
## 109
        1
           1
               1
                  1
                        4 10.0
                        5 32.5
## 110
        1
           1
               1
                  1
## 111
           1
                        6 18.5
        1
               1
                 1
## 112 1
          1
              1
                        7 8.0
```

```
model <- aov(obs~(A*B*C*D)+Block,data=dat)</pre>
summary(model)
```

```
##
               Df Sum Sq Mean Sq F value Pr(>F)
## A
                1
                      917
                            917.1 10.396 0.00176 **
## B
                1
                      388
                            388.1
                                    4.400 0.03875 *
## C
                      145
                            145.1
                                    1.645 0.20290
                1
## D
                1
                        1
                              1.4
                                    0.016 0.90021
                     376
                             62.7
## Block
                6
                                    0.710 0.64202
## A:B
                1
                      219
                            218.7
                                    2.479 0.11890
## A:C
                1
                      12
                             11.9
                                    0.135 0.71433
## B:C
                      115
                            115.0
                                    1.304 0.25655
                1
## A:D
                             93.8
                      94
                                    1.063 0.30522
                1
## B:D
                       56
                             56.4
                                    0.640 0.42594
                1
                        2
## C:D
                                    0.018 0.89227
                1
                              1.6
## A:B:C
                1
                        7
                              7.3
                                    0.082 0.77499
## A:B:D
                            113.0
                                    1.281 0.26073
                1
                      113
## A:C:D
                1
                      39
                             39.5
                                    0.448 0.50520
## B:C:D
                       34
                             33.8
                                    0.383 0.53767
                1
## A:B:C:D
                       96
                             95.6
                                    1.084 0.30055
                1
## Residuals
               90
                    7940
                             88.2
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

When considering Block, P value's of Factor A and Factor B are less than  $\alpha = 0.05$  level of significance and hence the length of Putt (Factor A) and the type of putter (Factor B) are significant. All other remaining factors, their interaction terms and the Block are not significant based on their respective P value's.

### ANOVA without Blocking

```
library(GAD)
A \leftarrow c(rep(-1,7), rep(1,7))
B \leftarrow c(rep(-1,14),rep(1,14))
C \leftarrow c(rep(-1,28), rep(1,28))
D \leftarrow c(rep(-1,56), rep(1,56))
Block \leftarrow c(rep(seq(1,7),16))
obs \leftarrow c(10,18,14,12.5,19,16,18.5,0,16.5,4.5,17.5,20.5,17.5,33,4,6,1,14.5,12,14,5,0,10,34,11,25.5,21.5,
          5,20.5,18,20,29.5,19,10,6.5,18.5,7.5,6,0,10,0,16.5,4.5,0,23.5,8,8,8,4.5,18,14.5,10,0,17.5,6,19
          36,15,16,8.5,0,0.5,9,3,41.5,39,6.5,3.5,7,8.5,36,8,4.5,6.5,10,13,41,14,21.5,10.5,6.5,0,15.5,24,
          5,7,10,32.5,18.5,8)
A <- as.fixed(A)
B <- as.fixed(B)
C <- as.fixed(C)</pre>
D <- as.fixed(D)
Block <- as.fixed(Block)</pre>
dat <- data.frame(A,B,C,D,Block,obs)</pre>
dat
##
        A B
              C
                 D Block obs
```

```
## 8
        1 -1 -1 -1
                         1 0.0
## 9
        1 -1 -1 -1
                         2 16.5
## 10
        1 -1 -1 -1
                         3
                           4.5
        1 -1 -1 -1
                         4 17.5
## 11
## 12
        1 -1 -1 -1
                         5 20.5
## 13
        1 -1 -1 -1
                         6 17.5
## 14
        1 -1 -1 -1
                         7 33.0
       -1 1 -1 -1
                            4.0
## 15
                         1
## 16
       -1
           1 -1 -1
                         2
                            6.0
## 17
           1 -1 -1
                           1.0
       -1
                         3
## 18
       -1
           1 -1 -1
                         4 14.5
## 19
           1 -1 -1
                         5 12.0
       -1
## 20
       -1
           1 -1 -1
                         6 14.0
           1 -1 -1
## 21
                         7
                            5.0
       -1
## 22
           1 -1 -1
                            0.0
        1
                         1
## 23
        1
           1 -1 -1
                         2 10.0
## 24
           1 -1 -1
                         3 34.0
        1
## 25
        1 1 -1 -1
                         4 11.0
## 26
        1 1 -1 -1
                         5 25.5
## 27
        1
           1 -1 -1
                         6 21.5
## 28
        1
          1 -1 -1
                         7
                            0.0
## 29
       -1 -1
               1 -1
                         1
                            0.0
## 30
       -1 -1
               1 -1
                         2
                            0.0
## 31
       -1 -1
               1 -1
                         3 18.5
## 32
       -1 -1
                         4 19.5
               1 -1
## 33
       -1 -1
               1 -1
                         5 16.0
## 34
       -1 -1
               1 -1
                         6 15.0
## 35
       -1 -1
               1 -1
                         7 11.0
## 36
        1 -1
               1 -1
                           5.0
                         1
## 37
        1 -1
               1 -1
                         2 20.5
## 38
        1 -1
               1 -1
                         3 18.0
## 39
        1 -1
               1 -1
                         4 20.0
## 40
        1 -1
               1 -1
                         5 29.5
## 41
                         6 19.0
        1 -1
               1 -1
## 42
        1 -1
               1 -1
                         7 10.0
## 43
           1
                           6.5
       -1
               1 -1
                         1
## 44
       -1
           1
               1 -1
                         2 18.5
## 45
       -1
           1
               1 -1
                         3
                           7.5
## 46
       -1
           1
               1 -1
                         4
                            6.0
                           0.0
## 47
       -1
           1
               1 -1
                         5
## 48
       -1
           1
               1 -1
                         6 10.0
## 49
       -1
           1
               1 -1
                         7
                            0.0
## 50
           1
               1 -1
                         1 16.5
        1
                           4.5
## 51
        1
           1
               1 -1
                         2
## 52
        1 1
                         3
                            0.0
               1 -1
                         4 23.5
## 53
           1
               1 -1
        1
        1 1
                            8.0
## 54
               1 -1
                         5
## 55
        1 1
              1 -1
                            8.0
                         6
## 56
        1
           1
              1 -1
                         7
                            8.0
## 57
       -1 -1 -1
                  1
                         1
                            4.5
## 58
       -1 -1 -1
                         2 18.0
                 1
## 59
       -1 -1 -1
                  1
                         3 14.5
## 60
       -1 -1 -1
                 1
                         4 10.0
## 61
      -1 -1 -1 1
                         5 0.0
```

```
6 17.5
## 62
       -1 -1 -1
                 1
## 63
        -1 -1 -1
                         7
                            6.0
                  1
##
   64
         1 -1 -1
                         1 19.5
##
   65
                         2 18.0
         1 -1 -1
                   1
##
   66
         1 -1
              -1
                   1
                         3 16.0
##
   67
         1 -1 -1
                            5.5
                  1
##
   68
         1 -1 -1
                   1
                         5 10.0
## 69
         1 -1 -1
                            7.0
                   1
                         6
##
   70
         1 -1 -1
                  1
                         7 36.0
##
   71
            1 -1
                         1 15.0
       -1
                  1
##
   72
       -1
            1 -1
                   1
                         2 16.0
                             8.5
##
   73
            1 -1
                         3
        -1
                   1
##
                             0.0
   74
       -1
            1 -1
                         4
                   1
##
   75
                         5
                             0.5
            1 -1
##
   76
       -1
            1 -1
                         6
                             9.0
                   1
##
   77
        -1
            1
              -1
                         7
                             3.0
##
   78
            1 -1
                         1 41.5
         1
                  1
##
   79
         1
            1 -1
                         2 39.0
            1 -1
##
   80
                         3
                             6.5
                   1
         1
## 81
            1
              -1
                         4
                             3.5
##
   82
         1
            1 -1
                   1
                         5
                             7.0
## 83
         1
            1
                         6
                             8.5
                         7 36.0
## 84
         1
            1
              -1
                   1
## 85
        -1 -1
               1
                   1
                         1
                             8.0
## 86
                         2
                             4.5
       -1 -1
               1
                   1
##
   87
        -1 -1
               1
                   1
                         3
                             6.5
## 88
       -1 -1
               1
                          4 10.0
                   1
##
   89
        -1 -1
                         5
                           13.0
               1
                   1
##
   90
       -1 -1
               1
                         6 41.0
                   1
## 91
                         7 14.0
        -1 -1
               1
                  1
## 92
         1 -1
               1
                   1
                         1 21.5
##
   93
         1 -1
               1
                  1
                         2 10.5
##
   94
                             6.5
         1 -1
               1
##
   95
                             0.0
         1 -1
               1
                          4
                   1
##
   96
         1 -1
               1
                   1
                         5 15.5
##
   97
         1 -1
                         6 24.0
               1
                  1
## 98
               1
                         7 16.0
## 99
       -1
            1
               1
                             0.0
                   1
                         1
## 100 -1
            1
               1
                   1
                         2
                             0.0
## 101 -1
                             0.0
            1
                         3
               1
                  1
## 102 -1
            1
               1
                             4.5
## 103 -1
            1
               1
                         5
                             1.0
                   1
## 104 -1
            1
                         6
                             4.0
               1
                  1
## 105 -1
                             6.5
            1
               1
                  1
                         7
## 106
        1
            1
                         1 18.0
               1
                   1
                         2
                             5.0
## 107
        1
            1
               1
                   1
## 108
        1
            1
                         3
                             7.0
               1
                  1
##
   109
         1
            1
                          4 10.0
               1
                         5 32.5
## 110
        1
            1
               1
                  1
                         6 18.5
## 111
        1
            1
               1
## 112
        1
           1
               1
                         7
                             8.0
                  1
model <- aov(obs~A*B*C*D,data=dat)</pre>
summary(model)
```

```
##
                Df Sum Sq Mean Sq F value Pr(>F)
## A
                 1
                      917
                             917.1
                                    10.588 0.00157 **
                                      4.481 0.03686
## B
                 1
                      388
                             388.1
## C
                      145
                             145.1
                                      1.676 0.19862
                 1
## D
                 1
                        1
                               1.4
                                      0.016 0.89928
                      219
## A:B
                 1
                             218.7
                                      2.525 0.11538
## A:C
                 1
                        12
                              11.9
                                      0.137 0.71178
## B:C
                 1
                      115
                             115.0
                                      1.328 0.25205
## A:D
                 1
                       94
                              93.8
                                      1.083 0.30066
                        56
## B:D
                 1
                              56.4
                                      0.651 0.42159
## C:D
                 1
                         2
                               1.6
                                      0.019 0.89127
                        7
                 1
                               7.3
## A:B:C
                                      0.084 0.77294
## A:B:D
                 1
                      113
                             113.0
                                      1.305 0.25623
                                      0.456 0.50121
## A:C:D
                 1
                        39
                              39.5
## B:C:D
                        34
                              33.8
                 1
                                      0.390 0.53386
## A:B:C:D
                 1
                        96
                              95.6
                                      1.104 0.29599
                96
                              86.6
## Residuals
                     8316
## ---
                    0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
```

When considering not to Block, P value's of Factor A and Factor B are less than  $\alpha = 0.05$  level of significance and hence the length of Putt (Factor A) and the type of putter (Factor B) are significant. All other remaining factors and their interaction terms are not significant based on their respective P value's.

If Comparing between Block and not to Block- On both cases, the length of Putt (Factor A) and the type of putter (Factor B) are significant. Also P-value's on both cases (Block/not to Block) are pretty much similar. Also, when Blocking, value of SSE is smaller than the value of SSE (when not Blocking).