HW Week 12

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Problem 7.12

Since each replicate would be considered as a Block so No. of replicates = No. of Blocks = 7.

2^4 factorial design, with 7 replications so total no. of observations are 112. There will be 16 corner points in each Block and each Block is replicated 7 times.

So Each Block will contain 16 corner points as follow: [(1) a b ab c ac bc abc d ad bd abd cd acd bcd abcd] These corner points are randomized within Block and Full replication within each Block. Model Equation:

$Y_{ijklmn} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \gamma_k + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + \delta_l + \ldots\ldots\ldots + (\alpha\beta\gamma\delta)_{ijkl} + \tau_m + \epsilon_{ijklmn}$

Where, τ_m = Block effect and n = replicate

ANOVA with Blocking

```
library(GAD)
```

```
## Loading required package: matrixStats
```

```
## Loading required package: R.methodsS3
```

```
## R.methodsS3 v1.8.2 (2022-06-13 22:00:14 UTC) successfully loaded. See ?R.methodsS3 for help.
```

```
A <- c(rep(-1,7),rep(1,7))
B <- c(rep(-1,14),rep(1,14))
C <- c(rep(-1,28),rep(1,28))
D <- c(rep(-1,56),rep(1,56))
Block <- c(rep(seq(1,7),16))
obs <- c(10,18,14,12.5,19,16,18.5,0,16.5,4.5,17.5,20.5,17.5,33,4,6,1,14.5,12,14,5,0,10,34,11,25.5,21.5,(
         5,20.5,18,20,29.5,19,10,6.5,18.5,7.5,6,0,10,0,16.5,4.5,0,23.5,8,8,8,4.5,18,14.5,10,0,17.5,6,19
         36,15,16,8.5,0,0.5,9,3,41.5,39,6.5,3.5,7,8.5,36,8,4.5,6.5,10,13,41,14,21.5,10.5,6.5,0,15.5,24,1
         5,7,10,32.5,18.5,8)
A <- as.fixed(A)
B <- as.fixed(B)
C <- as.fixed(C)
D <- as.fixed(D)
Block <- as.fixed(Block)
dat <- data.frame(A,B,C,D,Block,obs)
dat
```

```
##      A  B  C  D Block  obs
## 1   -1 -1 -1 -1     1 10.0
## 2   -1 -1 -1 -1     2 18.0
## 3   -1 -1 -1 -1     3 14.0
## 4   -1 -1 -1 -1     4 12.5
## 5   -1 -1 -1 -1     5 19.0
## 6   -1 -1 -1 -1     6 16.0
```

```
## 7   -1 -1 -1 -1     7 18.5
## 8    1 -1 -1 -1     1  0.0
## 9    1 -1 -1 -1     2 16.5
## 10   1 -1 -1 -1     3  4.5
## 11   1 -1 -1 -1     4 17.5
## 12   1 -1 -1 -1     5 20.5
## 13   1 -1 -1 -1     6 17.5
## 14   1 -1 -1 -1     7 33.0
## 15  -1  1 -1 -1     1  4.0
## 16  -1  1 -1 -1     2  6.0
## 17  -1  1 -1 -1     3  1.0
## 18  -1  1 -1 -1     4 14.5
## 19  -1  1 -1 -1     5 12.0
## 20  -1  1 -1 -1     6 14.0
## 21  -1  1 -1 -1     7  5.0
## 22   1  1 -1 -1     1  0.0
## 23   1  1 -1 -1     2 10.0
## 24   1  1 -1 -1     3 34.0
## 25   1  1 -1 -1     4 11.0
## 26   1  1 -1 -1     5 25.5
## 27   1  1 -1 -1     6 21.5
## 28   1  1 -1 -1     7  0.0
## 29  -1 -1  1 -1     1  0.0
## 30  -1 -1  1 -1     2  0.0
## 31  -1 -1  1 -1     3 18.5
## 32  -1 -1  1 -1     4 19.5
## 33  -1 -1  1 -1     5 16.0
## 34  -1 -1  1 -1     6 15.0
## 35  -1 -1  1 -1     7 11.0
## 36   1 -1  1 -1     1  5.0
## 37   1 -1  1 -1     2 20.5
## 38   1 -1  1 -1     3 18.0
## 39   1 -1  1 -1     4 20.0
## 40   1 -1  1 -1     5 29.5
## 41   1 -1  1 -1     6 19.0
## 42   1 -1  1 -1     7 10.0
## 43  -1  1  1 -1     1  6.5
## 44  -1  1  1 -1     2 18.5
## 45  -1  1  1 -1     3  7.5
## 46  -1  1  1 -1     4  6.0
## 47  -1  1  1 -1     5  0.0
## 48  -1  1  1 -1     6 10.0
## 49  -1  1  1 -1     7  0.0
## 50   1  1  1 -1     1 16.5
## 51   1  1  1 -1     2  4.5
## 52   1  1  1 -1     3  0.0
## 53   1  1  1 -1     4 23.5
## 54   1  1  1 -1     5  8.0
## 55   1  1  1 -1     6  8.0
## 56   1  1  1 -1     7  8.0
## 57  -1 -1 -1  1     1  4.5
## 58  -1 -1 -1  1     2 18.0
## 59  -1 -1 -1  1     3 14.5
## 60  -1 -1 -1  1     4 10.0
```

```
## 61  -1 -1 -1  1     5  0.0
## 62  -1 -1 -1  1     6 17.5
## 63  -1 -1 -1  1     7  6.0
## 64   1 -1 -1  1     1 19.5
## 65   1 -1 -1  1     2 18.0
## 66   1 -1 -1  1     3 16.0
## 67   1 -1 -1  1     4  5.5
## 68   1 -1 -1  1     5 10.0
## 69   1 -1 -1  1     6  7.0
## 70   1 -1 -1  1     7 36.0
## 71  -1  1 -1  1     1 15.0
## 72  -1  1 -1  1     2 16.0
## 73  -1  1 -1  1     3  8.5
## 74  -1  1 -1  1     4  0.0
## 75  -1  1 -1  1     5  0.5
## 76  -1  1 -1  1     6  9.0
## 77  -1  1 -1  1     7  3.0
## 78   1  1 -1  1     1 41.5
## 79   1  1 -1  1     2 39.0
## 80   1  1 -1  1     3  6.5
## 81   1  1 -1  1     4  3.5
## 82   1  1 -1  1     5  7.0
## 83   1  1 -1  1     6  8.5
## 84   1  1 -1  1     7 36.0
## 85  -1 -1  1  1     1  8.0
## 86  -1 -1  1  1     2  4.5
## 87  -1 -1  1  1     3  6.5
## 88  -1 -1  1  1     4 10.0
## 89  -1 -1  1  1     5 13.0
## 90  -1 -1  1  1     6 41.0
## 91  -1 -1  1  1     7 14.0
## 92   1 -1  1  1     1 21.5
## 93   1 -1  1  1     2 10.5
## 94   1 -1  1  1     3  6.5
## 95   1 -1  1  1     4  0.0
## 96   1 -1  1  1     5 15.5
## 97   1 -1  1  1     6 24.0
## 98   1 -1  1  1     7 16.0
## 99  -1  1  1  1     1  0.0
## 100 -1  1  1  1     2  0.0
## 101 -1  1  1  1     3  0.0
## 102 -1  1  1  1     4  4.5
## 103 -1  1  1  1     5  1.0
## 104 -1  1  1  1     6  4.0
## 105 -1  1  1  1     7  6.5
## 106  1  1  1  1     1 18.0
## 107  1  1  1  1     2  5.0
## 108  1  1  1  1     3  7.0
## 109  1  1  1  1     4 10.0
## 110  1  1  1  1     5 32.5
## 111  1  1  1  1     6 18.5
## 112  1  1  1  1     7  8.0
```

```
model <- aov(obs~(A*B*C*D)+Block,data=dat)
summary(model)
```

```
##             Df Sum Sq Mean Sq F value  Pr(>F)
## A            1    917   917.1  10.396 0.00176 **
## B            1    388   388.1   4.400 0.03875 *
## C            1    145   145.1   1.645 0.20290
## D            1      1     1.4   0.016 0.90021
## Block        6    376    62.7   0.710 0.64202
## A:B          1    219   218.7   2.479 0.11890
## A:C          1     12    11.9   0.135 0.71433
## B:C          1    115   115.0   1.304 0.25655
## A:D          1     94    93.8   1.063 0.30522
## B:D          1     56    56.4   0.640 0.42594
## C:D          1      2     1.6   0.018 0.89227
## A:B:C        1      7     7.3   0.082 0.77499
## A:B:D        1    113   113.0   1.281 0.26073
## A:C:D        1     39    39.5   0.448 0.50520
## B:C:D        1     34    33.8   0.383 0.53767
## A:B:C:D      1     96    95.6   1.084 0.30055
## Residuals   90   7940    88.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

When considering Block, P value's of Factor A and Factor B are less than $\alpha = 0.05$ level of significance and hence the length of Putt (Factor A) and the type of putter (Factor B) are significant. All other remaining factors, their interaction terms and the Block are not significant based on their respective P value's.

ANOVA without Blocking

```
library(GAD)
A <- c(rep(-1,7),rep(1,7))
B <- c(rep(-1,14),rep(1,14))
C <- c(rep(-1,28),rep(1,28))
D <- c(rep(-1,56),rep(1,56))
Block <- c(rep(seq(1,7),16))
obs <- c(10,18,14,12.5,19,16,18.5,0,16.5,4.5,17.5,20.5,17.5,33,4,6,1,14.5,12,14,5,0,10,34,11,25.5,21.5,
         5,20.5,18,20,29.5,19,10,6.5,18.5,7.5,6,0,10,0,16.5,4.5,0,23.5,8,8,8,4.5,18,14.5,10,0,17.5,6,19
         36,15,16,8.5,0,0.5,9,3,41.5,39,6.5,3.5,7,8.5,36,8,4.5,6.5,10,13,41,14,21.5,10.5,6.5,0,15.5,24,
         5,7,10,32.5,18.5,8)
A <- as.fixed(A)
B <- as.fixed(B)
C <- as.fixed(C)
D <- as.fixed(D)
Block <- as.fixed(Block)
dat <- data.frame(A,B,C,D,Block,obs)
dat
```

```
##      A  B  C  D Block  obs
## 1   -1 -1 -1 -1     1 10.0
## 2   -1 -1 -1 -1     2 18.0
## 3   -1 -1 -1 -1     3 14.0
## 4   -1 -1 -1 -1     4 12.5
## 5   -1 -1 -1 -1     5 19.0
## 6   -1 -1 -1 -1     6 16.0
## 7   -1 -1 -1 -1     7 18.5
```

```
## 8    1 -1 -1 -1     1  0.0
## 9    1 -1 -1 -1     2 16.5
## 10   1 -1 -1 -1     3  4.5
## 11   1 -1 -1 -1     4 17.5
## 12   1 -1 -1 -1     5 20.5
## 13   1 -1 -1 -1     6 17.5
## 14   1 -1 -1 -1     7 33.0
## 15  -1  1 -1 -1     1  4.0
## 16  -1  1 -1 -1     2  6.0
## 17  -1  1 -1 -1     3  1.0
## 18  -1  1 -1 -1     4 14.5
## 19  -1  1 -1 -1     5 12.0
## 20  -1  1 -1 -1     6 14.0
## 21  -1  1 -1 -1     7  5.0
## 22   1  1 -1 -1     1  0.0
## 23   1  1 -1 -1     2 10.0
## 24   1  1 -1 -1     3 34.0
## 25   1  1 -1 -1     4 11.0
## 26   1  1 -1 -1     5 25.5
## 27   1  1 -1 -1     6 21.5
## 28   1  1 -1 -1     7  0.0
## 29  -1 -1  1 -1     1  0.0
## 30  -1 -1  1 -1     2  0.0
## 31  -1 -1  1 -1     3 18.5
## 32  -1 -1  1 -1     4 19.5
## 33  -1 -1  1 -1     5 16.0
## 34  -1 -1  1 -1     6 15.0
## 35  -1 -1  1 -1     7 11.0
## 36   1 -1  1 -1     1  5.0
## 37   1 -1  1 -1     2 20.5
## 38   1 -1  1 -1     3 18.0
## 39   1 -1  1 -1     4 20.0
## 40   1 -1  1 -1     5 29.5
## 41   1 -1  1 -1     6 19.0
## 42   1 -1  1 -1     7 10.0
## 43  -1  1  1 -1     1  6.5
## 44  -1  1  1 -1     2 18.5
## 45  -1  1  1 -1     3  7.5
## 46  -1  1  1 -1     4  6.0
## 47  -1  1  1 -1     5  0.0
## 48  -1  1  1 -1     6 10.0
## 49  -1  1  1 -1     7  0.0
## 50   1  1  1 -1     1 16.5
## 51   1  1  1 -1     2  4.5
## 52   1  1  1 -1     3  0.0
## 53   1  1  1 -1     4 23.5
## 54   1  1  1 -1     5  8.0
## 55   1  1  1 -1     6  8.0
## 56   1  1  1 -1     7  8.0
## 57  -1 -1 -1  1     1  4.5
## 58  -1 -1 -1  1     2 18.0
## 59  -1 -1 -1  1     3 14.5
## 60  -1 -1 -1  1     4 10.0
## 61  -1 -1 -1  1     5  0.0
```

```
## 62  -1 -1 -1  1     6 17.5
## 63  -1 -1 -1  1     7  6.0
## 64   1 -1 -1  1     1 19.5
## 65   1 -1 -1  1     2 18.0
## 66   1 -1 -1  1     3 16.0
## 67   1 -1 -1  1     4  5.5
## 68   1 -1 -1  1     5 10.0
## 69   1 -1 -1  1     6  7.0
## 70   1 -1 -1  1     7 36.0
## 71  -1  1 -1  1     1 15.0
## 72  -1  1 -1  1     2 16.0
## 73  -1  1 -1  1     3  8.5
## 74  -1  1 -1  1     4  0.0
## 75  -1  1 -1  1     5  0.5
## 76  -1  1 -1  1     6  9.0
## 77  -1  1 -1  1     7  3.0
## 78   1  1 -1  1     1 41.5
## 79   1  1 -1  1     2 39.0
## 80   1  1 -1  1     3  6.5
## 81   1  1 -1  1     4  3.5
## 82   1  1 -1  1     5  7.0
## 83   1  1 -1  1     6  8.5
## 84   1  1 -1  1     7 36.0
## 85  -1 -1  1  1     1  8.0
## 86  -1 -1  1  1     2  4.5
## 87  -1 -1  1  1     3  6.5
## 88  -1 -1  1  1     4 10.0
## 89  -1 -1  1  1     5 13.0
## 90  -1 -1  1  1     6 41.0
## 91  -1 -1  1  1     7 14.0
## 92   1 -1  1  1     1 21.5
## 93   1 -1  1  1     2 10.5
## 94   1 -1  1  1     3  6.5
## 95   1 -1  1  1     4  0.0
## 96   1 -1  1  1     5 15.5
## 97   1 -1  1  1     6 24.0
## 98   1 -1  1  1     7 16.0
## 99  -1  1  1  1     1  0.0
## 100 -1  1  1  1     2  0.0
## 101 -1  1  1  1     3  0.0
## 102 -1  1  1  1     4  4.5
## 103 -1  1  1  1     5  1.0
## 104 -1  1  1  1     6  4.0
## 105 -1  1  1  1     7  6.5
## 106  1  1  1  1     1 18.0
## 107  1  1  1  1     2  5.0
## 108  1  1  1  1     3  7.0
## 109  1  1  1  1     4 10.0
## 110  1  1  1  1     5 32.5
## 111  1  1  1  1     6 18.5
## 112  1  1  1  1     7  8.0
```

```
model <- aov(obs~A*B*C*D,data=dat)
summary(model)
```

```
##             Df Sum Sq Mean Sq F value  Pr(>F)
## A            1    917   917.1  10.588 0.00157 **
## B            1    388   388.1   4.481 0.03686 *
## C            1    145   145.1   1.676 0.19862
## D            1      1     1.4   0.016 0.89928
## A:B          1    219   218.7   2.525 0.11538
## A:C          1     12    11.9   0.137 0.71178
## B:C          1    115   115.0   1.328 0.25205
## A:D          1     94    93.8   1.083 0.30066
## B:D          1     56    56.4   0.651 0.42159
## C:D          1      2     1.6   0.019 0.89127
## A:B:C        1      7     7.3   0.084 0.77294
## A:B:D        1    113   113.0   1.305 0.25623
## A:C:D        1     39    39.5   0.456 0.50121
## B:C:D        1     34    33.8   0.390 0.53386
## A:B:C:D      1     96    95.6   1.104 0.29599
## Residuals   96   8316    86.6
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

When considering not to Block, P value's of Factor A and Factor B are less than $\alpha = 0.05$ level of significance and hence the length of Putt (Factor A) and the type of putter (Factor B) are significant. All other remaining factors and their interaction terms are not significant based on their respective P value's.

If Comparing between Block and not to Block- On both cases, the length of Putt (Factor A) and the type of putter (Factor B) are significant. Also P-value's on both cases (Block/not to Block) are pretty much similar. Also, when Blocking, value of SSE is smaller than the value of SSE (when not Blocking).