Homework-Week 03

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Subject: DOE (IE 5342)

Answer to the problem 2.32

(a)

Inspector	Caliper 1	Caliper 2	difference (d_i)	(difference)^2 (d_i)^2
1	0.265	0.264	0.001	0.000001
2	0.265	0.265	0	0
3	0.266	0.264	0.002	0.000004
4	0.267	0.266	0.001	0.000001
5	0.267	0.267	0	0
6	0.265	0.268	-0.003	0.000009
7	0.267	0.264	0.003	0.000009
8	0.267	0.265	0.002	0.000004
9	0.265	0.265	0	0
10	0.268	0.267	0.001	0.000001
11	0.268	0.268	0	0
12	0.265	0.269	-0.004	0.000016
			$\sum = 0.003$	$\sum = 0.000045$

~~From the given table, we see that, paired T-test is suitable here.~~

we have $n = 12$, $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i = \frac{d_1 + d_2 + \cdots + d_{12}}{n}$

significance level, $\alpha = 0.05$

$= \frac{0.003}{12} = 0.00025$

$$s_d = \sqrt{\frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n} d_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} d_i\right)^2}{n-1}}$$

$$= \sqrt{\frac{0.000045 - 0.00000075}{11}} = 0.002005673$$

∴ Test statistic: $\frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{0.00025}{0.000579} = 0.432$

∴ $t_{\alpha/2, n-1} = t_{0.05/2, 11} = 2.201$ (from t-distribution table)

∴ $|t_0| = 0.432 < 2.201$

Hypothesis: $H_0: \mu_1 = \mu_2$ or $H_0: D = 0$

$H_a: \mu_1 \neq \mu_2$ or $H_a: D \neq 0$

Since $|t_0| < t_{\alpha/2, n-1}$ so we fail to reject H_0. so there is no significant difference between the means of the populations of the measurements from which the two samples were st selected.

(b) from the R script, the value of p is

$\boxed{P = 0.67}$ → (see the attached pdf file for details)

(c) 95% Confidence interval on the difference in mean is

$$\bar{d} - t_{\alpha/2, n-1}\frac{s_d}{\sqrt{n}} \leq \mu_1 - \mu_2 \leq \bar{d} + t_{\alpha/2, n-1}\frac{s_d}{\sqrt{n}}$$

$$= 0.00025 - 2.201\frac{0.002}{\sqrt{12}} \leq \mu_1 - \mu_2 \leq 0.00025 + 2.201\frac{0.002}{\sqrt{12}}$$

$$= \boxed{-0.00102 \leq \mu_1 - \mu_2 \leq 0.00152}$$

Answer to the problem 2.34

(a)

Girder	Karlsruhe Method	Lehigh method	diff (d_i)	diff^2 (d_i^2)
S1/1	1.186	1.061	0.125	0.015625
S2/1	1.151	0.992	0.159	0.025281
S3/1	1.322	1.063	0.259	0.067081
S4/1	1.339	1.062	0.277	0.076729
S5/1	1.200	1.065	0.135	0.018225
S2/1	1.402	1.178	0.224	0.050176
S2/2	1.365	1.037	0.328	0.107584
S2/3	1.537	1.086	0.451	0.203401
S2/4	1.559	1.052	0.507	0.257049
			$\Sigma = 2.465$	$\Sigma = 0.821151$

$n=9,\ \bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i = 0.274,\ \alpha = 0.05$

$$S_d = \sqrt{\frac{\sum_{i=1}^{n} d_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} d_i\right)^2}{n-1}} = \sqrt{\frac{0.821151 - 0.675136}{8}}$$

$= 0.1351$

$\therefore$ Test statistics, $t_0 = \frac{\bar{d}-0}{S_d/\sqrt{n}} = \frac{0.274}{0.1351/\sqrt{9}} = \frac{0.274}{0.0450}$

$= 6.089$

$t_{\alpha/2, n-1} = t_{0.025, 8} = 2.306$ (from t-distn table)

$\therefore |t_0| = 6.089 > 2.306$

$H_0: \mu_1 - \mu_2 = 0$ or $D = 0$

$H_a: \mu_1 - \mu_2 \neq 0$ or $D \neq 0$

Since $|t_0| > t_{\alpha/2, n-1}$ so we reject null hypothesis H_0. So there is evidence to support that there is a difference in mean performance between the two methods.

(b) From the R script, p-value is $\boxed{0.0002953}$ (see details in the attached pdf. file)

(c) 95% confidence interval for the mean difference

$$\bar{d} - t_{\alpha/2, n-1} \frac{s_d}{\sqrt{n}} \le \mu_1 - \mu_2 \le \bar{d} + t_{\alpha/2, n-1} \frac{s_d}{\sqrt{n}}$$

$$= 0.274 - 2.306 \frac{0.1351}{\sqrt{9}} \le \mu_1 - \mu_2 \le 0.274 + 2.306 \frac{0.1351}{\sqrt{9}}$$

$$\boxed{= 0.1702 \le \mu_1 - \mu_2 \le 0.3777}$$

(d) Approximately Normally distributed (Normality assumptions for both samples) please see the attached pdf file to see the Investigation of Normality.

(e) Normality assumptions for the difference in ratios. Approximately normally distributed. please further see the attached pdf for more details.

(f) see the attached pdf for more details
for paired t-test, Normality assumption applies for the difference distribution but on individuals normality assumptions are little of importance. (Role of Normality assumption in the paired t-test)

Ans to the problem 2.29

(e) Approximately normally distributed. No significant deviations being found. please see the attached pdf file to see the investigation of the Normality.

(f) $\text{Effect} = \dfrac{\text{abs(mean difference)}}{\text{Standard deviation (S.D)}} = \dfrac{2.5}{\text{S.D}}$

$$S.D = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(8-1)(4.9)+(8-1)(2.09)}{8+8-2}}$$

$$= 1.86$$

$\therefore \text{Effect} = \dfrac{2.5}{1.86} = 1.34$

From the R, see the attached pdf for more details.

n = 8
d = 1.34
sig. level = 0.05
type = two.sample
Alternative = two.sided

$\boxed{\therefore \text{power} = 0.703}$

Answer to the problem 2.27

(a)

obs	groups	order	Rank	
2.6	125	1	~~1.5~~	1
2.7	125	2	~~1.5~~	2
2.9	200	3	~~3~~	3
3.0	125	4	~~4.5~~	4
3.2	125	5	~~4.5~~	5
3.4	200	6	~~6.5~~	6
3.5	200	7	~~6.5~~	7
3.8	125	8	~~8~~	8
4.1	200	9	~~9~~	9
4.6	125	10	~~10~~	10.5
4.6	200	11		10.5
5.1	200	12		12

Sum of rank corresponding group 125 = 30.5

" " " " 200 = 47.5

group 125 < group 200

$T = 30.5,\quad \mu = n\cdot\bar{R} = 6 \times 6.5 = 39$ $\quad \bar{R} = \frac{6+6}{2} = 6 \approx 6.5$ (continuity correction)

$\sigma = \sqrt{\frac{n_1 n_2}{(n_1+n_2)}} \times \cancel{3.59+1.73} \times s_R\,(3.59)$ (from R)

$= 6.23$

Z - statistics, $Z = \frac{30.5 - 39}{\cancel{1.73}\ 6.23} = \cancel{-4.91}\ -1.36$

from Z table $P = 0.08 > \alpha\ (0.05)$

Hypothesis $H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$

since $P > \alpha$ so, we don't reject H_0

so C_2F_6 flow doesn't have affect average etch uniformity.