Flipped Assignment (2/15)

a) Model Equation

$$\boxed{Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon}$$

where

$$Y = \underbrace{\begin{bmatrix} 7.5 \\ 15 \\ 22 \\ 28.6 \\ 31.6 \\ 34.0 \\ 35.0 \\ \vdots \\ 52.5 \\ 34.4 \\ 46.5 \\ 50 \\ 51.9 \end{bmatrix}}_{36 \times 1}, \quad X_1 = \underbrace{\begin{bmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ \vdots \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \end{bmatrix}}_{36 \times 1}, \quad X_2 = \underbrace{\begin{bmatrix} 10 \\ 50 \\ 85 \\ 110 \\ 140 \\ \vdots \\ 250 \\ 60 \\ 90 \\ 120 \\ 150 \end{bmatrix}}_{36 \times 1}$$

$$\underline{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}_{3\times 1}, \quad \underline{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_{35} \\ \varepsilon_{36} \end{bmatrix}_{36\times 1}$$

$$\therefore \underline{Y} = \underline{X} \cdot \underline{\beta} + \underline{\varepsilon}$$

where $\underline{X}$ is a matrix & $\underline{Y}$, $\underline{\beta}$ & $\underline{\varepsilon}$ are vectors

$$\underline{X} = \begin{bmatrix} 1 & 0.00 & 10 \\ 1 & 0.00 & 50 \\ 1 & 0.00 & 85 \\ \vdots & \vdots & \vdots \\ 1 & 0.05 & 150 \end{bmatrix}_{36\times 3}$$

$$\therefore \underset{36\times 1}{\underline{Y}} = \underbrace{\underset{36\times 3}{\underline{X}} \cdot \underset{3\times 1}{\underline{\beta}}}_{36\times 1} + \underset{36\times 1}{\underline{\varepsilon}}$$

So the dimensionality works out in this matrix expression for the linear model.

b) $\boxed{\begin{array}{l}\text{Dimensionality of } Y \text{ is } 36\times 1 \\ \text{Dimentimality of } X \text{ is } 36\times 3\end{array}}$

c) Least squares estimates of the regression parameters

$$\boxed{\hat{\beta} = (X^T X)^{-1} X^T Y}$$

where $\hat{\beta}$ dimentionality is 3×1

X^T	"	"	3×36
X	"	"	36×3
Y	"	"	36×1
$\therefore X^T X$	"	"	3×3
$X^T Y$	"	"	3×1
$\therefore (X^T X)^{-1} X^T Y$	"	"	3×1

Also the dimentionality works out in this model too.

$$\therefore \hat{\beta} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} = \begin{bmatrix} 11.09 \\ 350.12 \\ 0.11 \end{bmatrix}$$

(from the R script)

$\therefore$ Model Equation

$$\boxed{Y = 11.09 + 350.12 X_1 + 0.11 X_2 + \varepsilon}$$