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Flipped Assignment 02
R#11636945

x	y	xy	x^2	$(x-\bar{x})^2$
1	1.5	1.5	1	3.648
2	2	4	4	0.828
3	3	9	9	0.008
3	4	12	9	0.008
4	4.5	18	16	1.188
4.5	6	27	20.25	2.528
$\sum =$ 17.5	21	71.5	59.25	8.208

$n = 6$

$\bar{x} = \frac{17.5}{6} = \cancel{2.90}\ 2.91$

$\bar{y} = 3.5$

(a) $b_1 = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} = \frac{71.5 - \frac{17.5 \times 21}{6}}{59.25 - \frac{(17.5)^2}{6}}$

$= \frac{10.250}{8.208}$

$= 1.249$

$b_0 = \bar{y} - b_1\bar{x} = 3.5 - (1.249) \times 2.91$

$= -0.135$

$\therefore$ Linear regression model,

$\hat{y} = -0.135 + 1.249x$ (Ans)

(b) $E[Y|X] = b_0 + b_1 x$

$= -0.135 + 1.249x$

(c) $Var[Y|X] = Var[Y] = \sigma^2$

we know that

$$\sigma^2 = MSE = \frac{SSE}{n-2} = \frac{\sum(y_i - \hat{y}_i)^2}{n-2}$$

Now

x	y	$\hat{y}$	$(y-\hat{y})^2$
1	1.5	1.11	0.152
2	2	2.36	0.130
3	3	3.61	0.372
3	4	3.61	0.152
4	4.5	4.86	0.130
4.5	6	5.49	0.260
			$\sum = 1.196$

$\therefore \sigma^2 = \frac{1.196}{n-2} = \frac{1.196}{6-2} = \frac{1.196}{4} = 0.299$

$\therefore Var[Y|X] = Var[Y] = \sigma^2 = 0.299$ (Ans)

(d) here, $df = n-2 = 6-2 = 4$

So $t = \frac{b_1 - 0}{\sqrt{\frac{MSE}{\sum(x_i-\bar{x})^2}}} = \frac{1.249}{\sqrt{\frac{0.299}{8.208}}} = \frac{1.249}{0.191}$

$= 6.539$

from R studio's calculation, we get

$P = 0.003$

Since $P < 0.05$ so we can reject the hypothesis that $\beta_1 = 0$.

(e) Since $P < 0.05$, the regression is significant.

(f) when $x=3$

$T_{n-2,\alpha/2} = T_{4,0.025} = 2.78$ (Using t-table)

~~Confidence~~

95% confidence Interval

Upper bound =

$$-0.135+1.249\times 3+2.78\sqrt{0.299\left(\frac{1}{6}+\frac{(3-2.91)^2}{8.208}\right)}$$

$$\boxed{=4.234}$$

Lower bound =

$$-0.135+1.249\times 3-2.78\sqrt{0.299\left(\frac{1}{6}+\frac{(3-2.91)^2}{8.208}\right)}$$

$$\boxed{=2.99}$$

(g) 95% prediction Interval

Upper bound =

$$-0.135+(1.249\times 3)+2.78\times\sqrt{0.299\left(1+\frac{1}{6}+\frac{(3-2.91)^2}{8.208}\right)}$$

$$\boxed{=5.255}$$

Lower bound

$$= \cancel{2.01}$$

$$-0.135 + (1.249 \times 3) - 2.78 \times \sqrt{0.299 \times \left(1 + \frac{1}{6} + \frac{(3-2.91)^2}{8.208}\right)}$$

$$\boxed{= 1.969}$$