Chapter 3

Motion in a Straight Line

Solutions

SECTION - A

Objective Type Questions

(Position, Path length and Displacement, Average Velocity and Average Speed)

1. A particle is moving along a circle such that it completes one revolution in 40 seconds. In 2 minutes 20 seconds, the ratio $\frac{|\text{displacement}|}{\text{distance}}$ is

   (1) 0 (2) $\frac{1}{7}$ (3) $\frac{2}{7}$ (4) $\frac{1}{11}$

Sol. Answer (4)

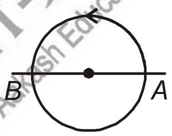
T = 40 s

If t = 2 minute 20 second

= 2 × 60 + 20

= 140 s

So, it has completed $3\frac{1}{2}$ revolution.



Distance travelled = 3 × 2πR + πR

= $7\pi R$

Displacement = $2R$

$$\frac{|\text{Displacement}|}{\text{Distance}} = \frac{2R}{7\pi R} = \frac{2}{7\times\frac{22}{7}} = \frac{1}{11}$$

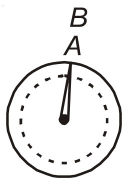
2. Consider the motion of the tip of the second hand of a clock. In one minute (R be the length of second hand), its

   (1) Displacement is $2\pi R$ (2) Distance covered is $2R$

   (3) Displacement is zero (4) Distance covered is zero

Sol. Answer (3)

The second hand of the clock in minute covers an angle of 360º and the initial and final positions are same.



So, $\boxed{\text{Displacement} = 0}$

3. The position of a body moving along x-axis at time t is given by $x = (t^2 - 4t + 6)$ m. The distance travelled by body in time interval $t = 0$ to $t = 3$ s is

(1) 5 m (2) 7 m (3) 4 m (4) 3 m

Sol. Answer (1)

$x = t^2 - 4t + 6$

$$\frac{dx}{dt} = 2t - 4$$

At $t = 2$ s, particle is at rest and reverses its position so,

$$\left.\begin{array}{l} x|_{t=0} = 6 \text{ m} \\ x|_{t=2\text{ s}} = 2 \text{ m} \end{array}\right] 4 \text{ m}$$

$$\left.\begin{array}{l} x|_{t=2\text{ s}} = 2 \text{ m} \\ x|_{t=3\text{ s}} = 3 \text{ m} \end{array}\right] 1 \text{ m}$$

Distance = (4 + 1) m = 5 m

Displacement = 3 m

4. A particle moves along x-axis with speed 6 m/s for the first half distance of a journey and the second half distance with a speed 3 m/s. The average speed in the total journey is

(1) 5 m/s (2) 4.5 m/s (3) 4 m/s (4) 2 m/s

Sol. Answer (3)

If a body travels equal distance with speed v_1 and v_2 then average speed is given by

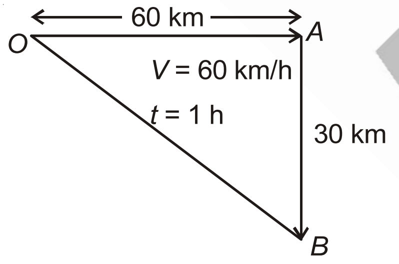
$$v_{av} = \frac{2v_1v_2}{v_1 + v_2} = \frac{2 \times 6 \times 3}{6 + 3} = 4 \text{ ms}^{-1}$$

5. A car moves with speed 60 km/h for 1 hour in east direction and with same speed for 30 min in south direction. The displacement of car from initial position is

(1) 60 km (2) $30\sqrt{3}$ km (3) $30\sqrt{5}$ km (4) $60\sqrt{2}$ km

Sol. Answer (3)

Displacement of car = $\sqrt{60^2 + 30^2} = 30\sqrt{5}$ km



$$\left[\begin{array}{l} \text{Distance } OA = \text{Speed} \times \text{Time} \\ \Rightarrow 60 \times 1 \text{ h} = 60 \text{ km} \end{array}\right]$$

6. A person travels along a straight road for the first $\frac{t}{3}$ time with a speed v_1 and for next $\frac{2t}{3}$ time with a speed v_2. Then the mean speed v is given by

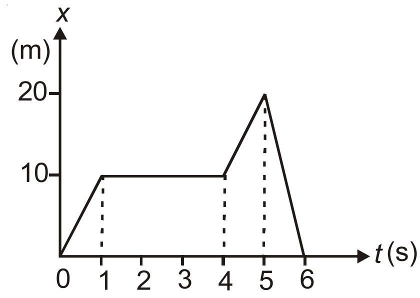
(1) $v = \frac{v_1 + 2v_2}{3}$ (2) $\frac{1}{v} = \frac{1}{3v_1} + \frac{2}{3v_2}$ (3) $v = \frac{1}{3}\sqrt{2v_1v_2}$ (4) $v = \sqrt{\frac{3v_2}{2v_1}}$

Sol. Answer (1)

$$v_{av} = \frac{\text{Distance}}{\text{Time}} = \frac{\text{Speed} \times \text{Time}}{\text{Time}} = \frac{v_1 \times \frac{t}{3} + v_2 \times \frac{2t}{3}}{\frac{t}{3} + \frac{2t}{3}}$$

$$\Rightarrow \quad v_{av} = \frac{\frac{v_1}{3} + \frac{2v_2}{3}}{1} = \frac{v_1 + 2v_2}{3} \quad \Rightarrow \quad \boxed{v_{av} = \frac{v_1 + 2v_2}{3}}$$

7. Figure shows the graph of x-coordinate of a particle moving along x-axis as a function of time. Average velocity during t = 0 to 6 s and instantaneous velocity at t = 3 s respectively, will be



(1) 10 m/s, 0 (2) 60 m/s, 0

(3) 0, 0 (4) 0, 10 m/s

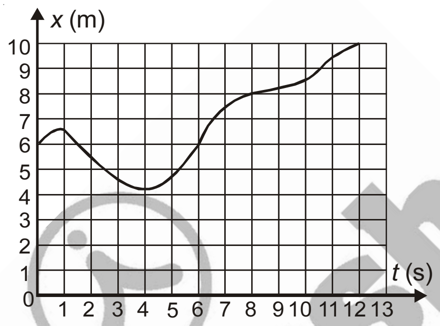
Sol. Answer (3)

From 0 to 6 s $\rightarrow$ Displacement = 0

so, average velocity = 0

at t = 3 s, the displacement = 0, so v = 0

8. Position-time graph for a particle is shown in figure. Starting from t = 0, at what time t, the average velocity is zero?



(1) 1 s (2) 3 s (3) 6 s (4) 7 s

Sol. Answer (3)

If we look at the graph very carefully at t = 0, x = 6 m

The average velocity will be zero if it comes back to the initial position.

It is evident that at t = 6 s, x = 6 m

So, v_{av} at t = 6 s is zero.

(Instantaneous Velocity and Speed, Acceleration)

9. A body in one dimensional motion has zero speed at an instant. At that instant, it must have

(1) Zero velocity (2) Zero acceleration

(3) Non-zero velocity (4) Non-zero acceleration

Sol. Answer (1)

Magnitude of velocity = Speed

So, if the speed is zero then it must have zero velocity also.

10. If a particle is moving along straight line with increasing speed, then

(1) Its acceleration is negative (2) Its acceleration may be decreasing

(3) Its acceleration is positive (4) Both (2) & (3)

Sol. Answer (2)

If the speed of body is increasing then acceleration is in the direction of velocity.

It may be positive or negative.

If acceleration is in negative direction then acceleration is increasing but in negative side, so it will be called as decreasing.

11. At any instant, the velocity and acceleration of a particle moving along a straight line are v and a. The speed of the particle is increasing if

(1) $v > 0, a > 0$ (2) $v < 0, a > 0$ (3) $v > 0, a < 0$ (4) $v > 0, a = 0$

Sol. Answer (1)

For increasing speed both velocity (v) and acceleration (a) are in the same direction.

12. If magnitude of average speed and average velocity over a time interval are same, then

(1) The particle must move with zero acceleration

(2) The particle must move with non-zero acceleration

(3) The particle must be at rest

(4) The particle must move in a straight line without turning back

Sol. Answer (4)

The magnitude of average speed and average velocity can only be equal if object moves in a straight line without turning back. In that condition distance will be equal to displacement.

13. If v is the velocity of a body moving along x-axis, then acceleration of body is

(1) $\dfrac{dv}{dx}$ (2) $v\dfrac{dv}{dx}$ (3) $x\dfrac{du}{dx}$ (4) $v\dfrac{dx}{dv}$

Sol. Answer (2)

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} \qquad \left(\frac{dx}{dt} = \text{velocity}\right)$$

$$\boxed{a = \frac{vdv}{dx}}$$

14. If a body is moving with constant speed, then its acceleration

(1) Must be zero (2) May be variable (3) May be uniform (4) Both (2) & (3)

Sol. Answer (2)

Acceleration is the rate of change of velocity. The magnitude of velocity (*i.e.,* speed) is constant but it may change in direction. So, acceleration may be variable due to change in direction.

15. When the velocity of body is variable, then

(1) Its speed may be constant (2) Its acceleration may be constant

(3) Its average acceleration may be constant (4) All of these

Sol. Answer (4)

If velocity is changing they may change in magnitude or direction or both.

(i) So, if velocity is changing in direction only the magnitude is constant so speed is constant.

(ii) If only direction of velocity is changing and magnitude is constant then acceleration will also be constant in magnitude (in case of uniform circular motion).

(iii) Average acceleration may be constant.

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1}$$

16. An object is moving with variable speed, then

(1) Its velocity may be zero
(2) Its velocity must be variable
(3) Its acceleration may be zero
(4) Its velocity may be constant

Sol. Answer (2)

If speed is changing then velocity must change.

17. The position of a particle moving along x-axis is given by $x = 10t - 2t^2$. Then the time (t) at which it will momently come to rest is

(1) 0
(2) 2.5 s
(3) 5 s
(4) 10 s

Sol. Answer (2)

$x = 10t - 2t^2$

$$v = \frac{dx}{dt} = 10 - 4t$$

$v = 0$, at the time of coming to rest, so

$10 - 4t = 0$

$\boxed{t = 2.5 \text{ s}}$

18. If the displacement of a particle varies with time as $\sqrt{x} = t + 7$, then

(1) Velocity of the particle is inversely proportional to t
(2) Velocity of the particle is proportional to t^2
(3) Velocity of the particle is proportional to $\sqrt{t}$
(4) The particle moves with constant acceleration

Sol. Answer (4)

$\sqrt{x} = t + 7$

$\Rightarrow \quad x = (t+7)^2$

$= t^2 + 49 + 14t$ (squaring)

$$\frac{dx}{dt} = 2t + 14$$

$\boxed{v = 2t + 14} \Rightarrow \boxed{v \propto t}$

Acceleration :

$$a = \frac{dv}{dt}$$

$\boxed{a = 2 \text{ ms}^{-2}} \rightarrow$ constant

19. The initial velocity of a particle is u (at $t = 0$) and the acceleration a is given by $\alpha t^{3/2}$. Which of the following relations is valid?

(1) $v = u + \alpha t^{3/2}$
(2) $v = u + \dfrac{3\alpha t^3}{2}$
(3) $v = u + \dfrac{2}{5}\alpha t^{5/2}$
(4) $v = u + \alpha t^{5/2}$

Sol. Answer (3)

$a = \alpha t^{3/2}$ (acceleration is a function of time)

$$\int_u^v dv = \int_0^t a\,dt$$

$$\Rightarrow \int_u^v dv = \int_0^t \alpha t^{3/2} dt$$

$$\Rightarrow v\Big|_u^v = \alpha \left.\frac{t^{3/2}+1}{\frac{3}{2}+1}\right|_0^t$$

$$\Rightarrow (v-u) = \alpha \times \frac{2}{5} \quad (t^{5/2}-0)$$

$$\Rightarrow v-u = \frac{2}{5}\alpha t^{5/2}$$

$$\Rightarrow \boxed{v = u + \frac{2}{5}\alpha t^{5/2}}$$

Note : *The equations of kinematics are valid only for constant acceleration, here a is a function of 't' so we didn't apply those equations.*

20. The position x of particle moving along x-axis varies with time t as $x = A\sin(\omega t)$ where A and ω are positive constants. The acceleration a of particle varies with its position (x) as

(1) $a = Ax$ (2) $a = -\omega^2 x$ (3) $a = A\,\omega\,x$ (4) $a = \omega^2\,x\,A$

Sol. Answer (2)

$x = A\sin \omega t$

$$\frac{dx}{dt} = A\omega\cos\omega t$$

$$\Rightarrow \frac{d^2x}{dt^2} = -A\omega^2 \sin\omega t$$

$$\Rightarrow \boxed{a = -\omega^2 x} \quad (\because\ A\sin\omega t = x)$$

21. A particle moves in a straight line and its position x at time t is given by $x^2 = 2 + t$. Its acceleration is given by

(1) $\frac{-2}{x^3}$ (2) $-\frac{1}{4x^3}$ (3) $-\frac{1}{4x^2}$ (4) $\frac{1}{x^2}$

Sol. Answer (2)

$$x^2 = t + 2 \Rightarrow \boxed{\frac{1}{x^2} = \frac{1}{t+2}} \quad ....(i)$$

$$\Rightarrow x = \sqrt{t+2}$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{2}(t+2)^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{2}(t+2)^{-\frac{1}{2}}$$

$$\Rightarrow \frac{d^2x}{dt^2} = \frac{1}{2}\left(-\frac{1}{2}\right)(t+2)^{-\frac{1}{2}-1}$$

$$\Rightarrow a = -\frac{1}{4}(t+2)^{-\frac{3}{2}} = -\frac{1}{4(t+2)} \times \frac{1}{(t+2)^{\frac{1}{2}}} = -\frac{1}{4} \times \frac{1}{x^2} \times \frac{1}{x}$$

$$\Rightarrow \boxed{a = -\frac{1}{4x^3}}$$

22. A body is moving with variable acceleration (a) along a straight line. The average acceleration of body in time interval t_1 to t_2 is

(1) $\frac{a[t_2 + t_1]}{2}$ (2) $\frac{a[t_2 - t_1]}{2}$ (3) $\frac{\int_{t_1}^{t_2} a\,dt}{t_2 + t_1}$ (4) $\frac{\int_{t_1}^{t_2} a\,dt}{t_2 - t_1}$

Sol. Answer (4)

$$\text{Average acceleration} = \frac{\text{Change in velocity}}{\text{Time}} \Rightarrow \boxed{a_{av} = \frac{\int_{t_1}^{t_2} a\,dt}{t_2 - t_1}}$$

23. The position of a particle moving along x-axis given by $x = (-2t^3 + 3t^2 + 5)$m. The acceleration of particle at the instant its velocity becomes zero is

(1) 12 m/s^2 (2) –12 m/s^2 (3) –6 m/s^2 (4) Zero

Sol. Answer (3)

$$x = (-2t^3 + 3t^2 + 5) \text{ m}$$

$$\Rightarrow \frac{dx}{dt} = -6t^2 + 6t = v$$

$$\Rightarrow \frac{d^2x}{dt^2} = -12t + 6 \qquad (\text{for } v = 0,\ 6t = 6t^2 \Rightarrow t = 1 \text{ s})$$

$$a|_{t=1\text{ s}} = -12 + 6 = -6 \text{ ms}^{-2}$$

24. A particle move with velocity v_1 for time t_1 and v_2 for time t_2 along a straight line. The magnitude of its average acceleration is

(1) $\frac{v_2 - v_1}{t_1 - t_2}$ (2) $\frac{v_2 - v_1}{t_1 + t_2}$ (3) $\frac{v_2 - v_1}{t_2 - t_1}$ (4) $\frac{v_1 + v_2}{t_1 - t_2}$

Sol. Answer (2)

$$\boxed{a_{avg} = \frac{v_2 - v_1}{t_1 + t_2}} = \frac{\text{Change in velocity}}{\text{Time interval}}$$

(Kinematic Equations for Uniformly Accelerated Motion)

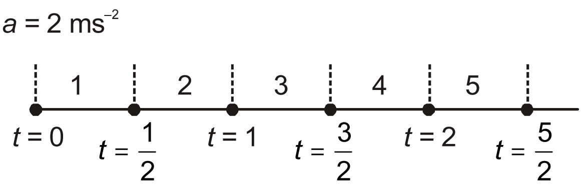
25. A particle starts moving with acceleration 2 m/s^2. Distance travelled by it in 5th half second is

(1) 1.25 m (2) 2.25 m (3) 6.25 m (4) 30.25 m

Sol. Answer (2)

$S_{2.5} - S_2 = ?$ (distance travelled in 5th half second)

$$S_{2.5} = ut + \frac{1}{2}at^2$$



$$\Rightarrow \quad S_{2.5} = \frac{1}{2}\times 2\times(2.5)^2 = 6.25 \text{ m} \quad (\because \; u = 0)$$

$$S_2 = \frac{1}{2}\times 2\times 4 = 4 \text{ m}$$

So, $\boxed{S_{2.5} - S_2 = 2.25 \text{ m}}$

26. The two ends of a train moving with constant acceleration pass a certain point with velocities u and $3u$. The velocity with which the middle point of the train passes the same point is

(1) $2u$ (2) $\frac{3}{2}u$ (3) $\sqrt{5}\,u$ (4) $\sqrt{10}\,u$

Sol. Answer (3)

Final velocity

Initial velocity

Velocity at the mid-point = $\sqrt{\dfrac{v^2+u^2}{2}}$

(When acceleration is constant)

Given, $v = 3u$, $u = u$

So, $v_{\text{mid}} = \sqrt{\dfrac{9u^2+u^2}{2}} = \sqrt{\dfrac{10u^2}{2}}$

$v_{\text{mid}} = \sqrt{5u^2} = \boxed{\sqrt{5}u = v_{\text{mid}}}$

27. A train starts from rest from a station with acceleration 0.2 m/s^2 on a straight track and then comes to rest after attaining maximum speed on another station due to retardation 0.4 m/s^2. If total time spent is half an hour, then distance between two stations is [Neglect length of train]

(1) 216 km (2) 512 km (3) 728 km (4) 1296 km

Sol. Answer (1)

Shortcut : $S = \dfrac{1}{2}\dfrac{\alpha\beta}{\alpha+\beta}T^2$

$\alpha \to$ Acceleration

$\beta \to$ Deceleration (magnitude only)

$T \to$ Time of journey

$S \to$ Distance travelled

Given, $\alpha = 0.2\ \text{ms}^{-2}$

$\beta = 0.4\ \text{ms}^{-2}$

T = half an hour = 30 × 60 s = 1800 s

$$S = \frac{1}{2} \times \left(\frac{0.2 \times 0.4}{0.2 + 0.4}\right) \times (1800)^2$$

$\Rightarrow$ S = 216000 m

$\Rightarrow$ $\boxed{S = 216\ \text{km}}$

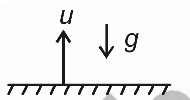
28. A body is projected vertically upward direction from the surface of earth. If upward direction is taken as positive, then acceleration of body during its upward and downward journey are respectively

(1) Positive, negative (2) Negative, negative (3) Positive, positive (4) Negative, positive

Sol. Answer (2)

Whether body move upwards or downwards the earth tries to pull it downwards only. Hence during both the motion g will negative.

So, negative, negative



29. A particle start moving from rest state along a straight line under the action of a constant force and travel distance x in first 5 seconds. The distance travelled by it in next five seconds will be

(1) x (2) $2x$ (3) $3x$ (4) $4x$

Sol. Answer (3)

Body starts from rest and moves with a constant acceleration, then the distance travelled in equal time intervals will be in the ratio of odd number. (Galileo's law of odd number)

$x_1 : x_2 \Rightarrow 1 : 3$

$x : x_2 \Rightarrow 1 : 3$

$\Rightarrow \quad \frac{x}{x_2} = \frac{1}{3}$

$\Rightarrow \quad \boxed{x_2 = 3x}$

30. A body is projected vertically upward with speed 40 m/s. The distance travelled by body in the last second of upward journey is [take g = 9.8 m/s^2 and neglect effect of air resistance]

(1) 4.9 m (2) 9.8 m (3) 12.4 m (4) 19.6 m

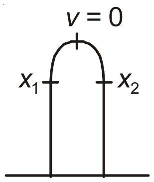
Sol. Answer (1)

As the motion under gravity is symmetric, so distance travelled in last second of ascent is equal to first second of descent.

t = 1 s (1$^{\text{st}}$ second)

$$-x_2 = ut - \frac{1}{2} g \times 1^2$$

$$x_2 = \frac{1}{2} \times 9.8 \times 1^2 \quad (\because\ u = 0)$$



$\Rightarrow \quad \boxed{x_2 = 4.9\ \text{m}}$

This distance is constant for every body thrown with any speed.

31. A body is projected vertically upward with speed 10 m/s and other at same time with same speed in downward direction from the top of a tower. The magnitude of acceleration of first body w.r.t. second is {take g = 10 m/s^2}

(1) Zero (2) 10 m/s^2 (3) 5 m/s^2 (4) 20 m/s^2

Sol. Answer (1)

The acceleration of first body

a_1 = 10 ms^{-2}

a_2 = 10 ms^{-2}

$a_{rel} = a_1 - a_2$ = 10 ms^{-2} – 10 ms^{-2} = 0

32. A car travelling at a speed of 30 km/h is brought to rest in a distance of 8 m by applying brakes. If the same car is moving at a speed of 60 km/h then it can be brought to rest with same brakes in

(1) 64 m (2) 32 m (3) 16 m (4) 4 m

Sol. Answer (2)

$$d_s = \frac{u^2}{2a} \Rightarrow d_s \propto u^2$$

$$u' = 2u$$

$$\frac{d'}{d} = \frac{(2u)^2}{u^2}$$

$$\Rightarrow \frac{d'}{8} = 4$$

$$\Rightarrow d' = 32$$

33. A particle is thrown with any velocity vertically upward, the distance travelled by the particle in first second of its decent is

(1) g (2) $\frac{g}{2}$ (3) $\frac{g}{4}$ (4) Cannot be calculated

Sol. Answer (2)

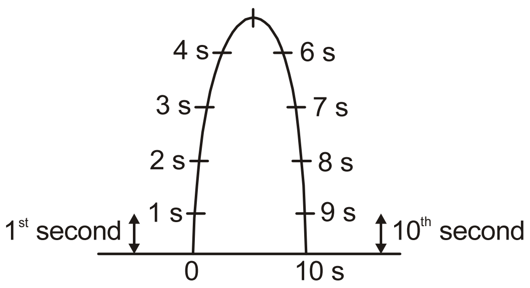
$$s = \frac{1}{2}g \times 1^2 \Rightarrow \boxed{s = \frac{g}{2}}$$

34. A body is thrown vertically upwards and takes 5 seconds to reach maximum height. The distance travelled by the body will be same in

(1) 1st and 10th second (2) 2nd and 8th second (3) 4th and 6th second (4) Both (2) & (3)

Sol. Answer (1)

The motion under gravity is a symmetric motion and the time taken to go up is same as time taken to come back to the initial position.



So, clearly the distance travelled in 1st second is same as that travelled in 10th second.

35. A ball is dropped from a bridge of 122.5 metre above a river. After the ball has been falling for two seconds, a second ball is thrown straight down after it. Initial velocity of second ball so that both hit the water at the same time is

(1) 49 m/s (2) 55.5 m/s (3) 26.1 m/s (4) 9.8 m/s

Sol. Answer (3)

$-h = -\frac{1}{2}gt^2$ 1st ball

$\Rightarrow \quad 122.5 = \frac{1}{2} \times 9.8 t^2$

$\Rightarrow \; t^2 = 25 \; \Rightarrow \; \boxed{t = 5\text{ s}}$

Another ball is dropped after 2 second so it took only (5 – 2) = 3 s

$-122.5 = -u(3) - \frac{1}{2} \times 9.8 \times 3^2$

$\Rightarrow \quad 122.5 = 3u + 4.9 \times 9$

$\Rightarrow \quad 3u = 78.4$

$\Rightarrow \quad \boxed{u = 26.1\text{ s}}$

36. A balloon starts rising from ground from rest with an upward acceleration 2 m/s². Just after 1 s, a stone is dropped from it. The time taken by stone to strike the ground is nearly

(1) 0.3 s (2) 0.7 s (3) 1 s (4) 1.4 s

Sol. Answer (2)

$u = 0,\; a = 2\text{ ms}^{-2}$

The velocity of object after one second

$v = u + at$

$\Rightarrow \quad \boxed{v = 2\text{ ms}^{-1}}$

$s = \frac{1}{2} \times 2 \times 1^2 = 1\text{ m}$

Now after separating from the balloon it will move under the effect of gravity alone.

$-h = vt - \frac{1}{2} \times 9.8 \times t^2$

$\Rightarrow \quad -1 = 2t - 4.9t^2$

$\Rightarrow \quad 4.9t^2 - 2t - 1 = 0$

$\Rightarrow \quad \boxed{t = 0.7\text{ s}}$

37. A boy throws balls into air at regular interval of 2 second. The next ball is thrown when the velocity of first ball is zero. How high do the ball rise above his hand? [Take g = 9.8 m/s²]

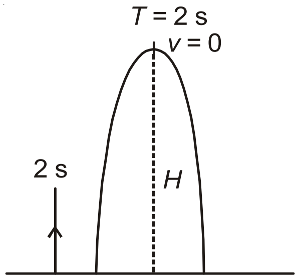
(1) 4.9 m (2) 9.8 m (3) 19.6 m (4) 29.4 m

Sol. Answer (3)

$2T = \frac{2u}{g} \Rightarrow 2 = \frac{u}{9.8} \Rightarrow u = 19.6$

$H = \frac{u^2}{2g} = \frac{19.6 \times 19.6}{2 \times 9.8}$

$\Rightarrow \quad \boxed{H = 19.6\text{ m}}$



38. A ball projected from ground vertically upward is at same height at time t_1 and t_2. The speed of projection of ball is [Neglect the effect of air resistance]

(1) $g[t_2 - t_1]$ (2) $\frac{g[t_1 + t_2]}{2}$ (3) $\frac{g[t_2 - t_1]}{2}$ (4) $g[t_1 + t_2]$

Sol. Answer (2)

$t_1 + t_2$ = total time of flight

$t_1 + t_2 = 2T$

$T = \frac{t_1 + t_2}{2}$, also $T = \frac{u}{g}$

$$\frac{u}{g} = \frac{t_1 + t_2}{2} \Rightarrow \boxed{u = \frac{1}{2}g(t_1 + t_2)}$$

39. Two balls are projected upward simultaneously with speeds 40 m/s and 60 m/s. Relative position (x) of second ball w.r.t. first ball at time t = 5 s is [Neglect air resistance].

(1) 20 m (2) 80 m (3) 100 m (4) 120 m

Sol. Answer (3)

$$S_{\text{rel}} = U_{\text{rel}}\,t + \frac{1}{2}a_{\text{rel}}\,t^2$$

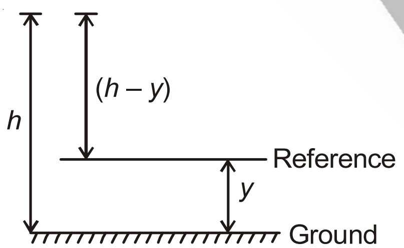
$\Rightarrow$ $S_{\text{rel}} = (60 - 40)\ 5$ ($a_{\text{rel}} = 0$)

$\Rightarrow$ $\boxed{S_{\text{rel}} = 100 \text{ m}}$

40. A ball is dropped from a height h above ground. Neglect the air resistance, its velocity (v) varies with its height above the ground as

(1) $\sqrt{2g(h-y)}$ (2) $\sqrt{2gh}$ (3) $\sqrt{2gy}$ (4) $\sqrt{2g(h+y)}$

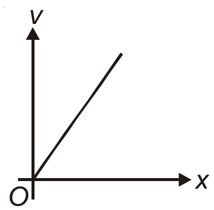
Sol. Answer (1)

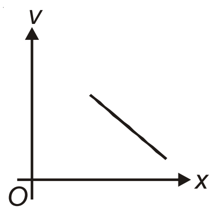


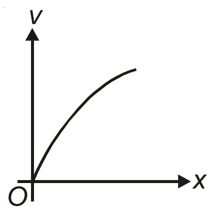
$\boxed{v = \sqrt{2g(h-y)}}$

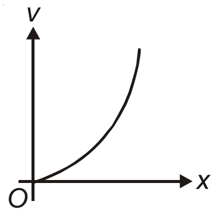
(Graphs)

41. For a body moving with uniform acceleration along straight line, the variation of its velocity (v) with position (x) is best represented by

(1) 

(2) 

(3) 

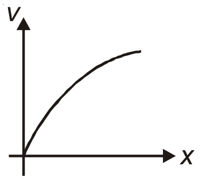
(4) 

Sol. Answer (3)

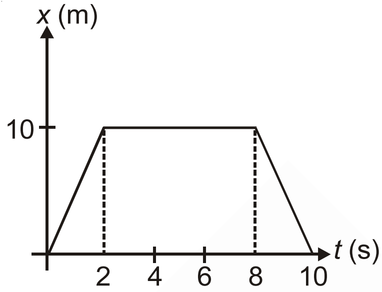
For uniform acceleration, $a \rightarrow$ constant

$v^2 = u^2 + 2as$

$\Rightarrow \boxed{v^2 \propto x}$ $(\because\ u = \text{rest})$



42. The position-time graph for a particle moving along a straight line is shown in figure. The total distance travelled by it in time t = 0 to t = 10 s is



(1) Zero (2) 10 m (3) 20 m (4) 80 m

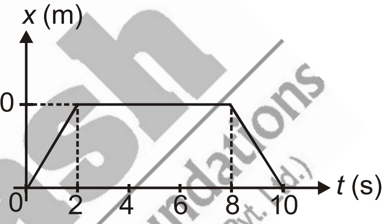
Sol. Answer (3)

The total distance travelled from 0 to 2 s is 10 m

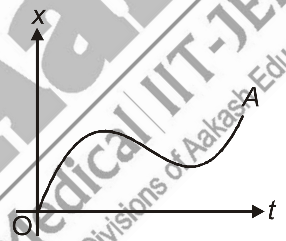
2 s to 8 s $\rightarrow$ Zero distance

and from 8 s to 10 s $\rightarrow$ 10 m

So, distance = 10 + 0 + 10 = 20 m



43. The position-time graph for a body moving along a straight line between O and A is shown in figure. During its motion between O and A, how many times body comes to rest?

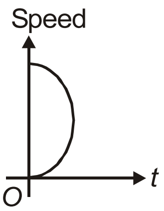


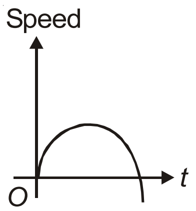
(1) Zero (2) 1 time (3) 2 times (4) 3 times

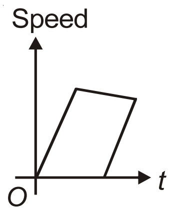
Sol. Answer (3)

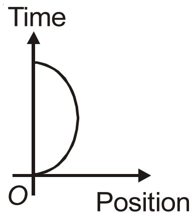
As there are two extremes in the graph one is maxima and other is minima. At both maxima and minima the slope is zero. So, it comes to rest twice.

44. Which one of the following graph for a body moving along a straight line is possible?

(1)


(2)


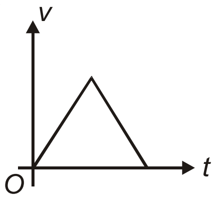
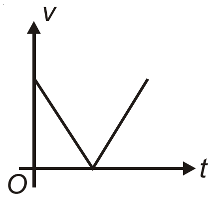
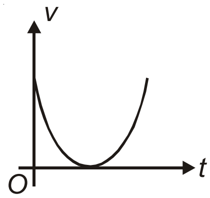
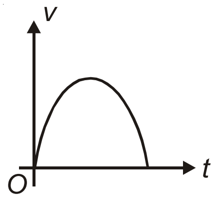
(3)


(4)


Sol. Answer (4)

This graph is possible.

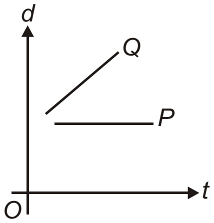
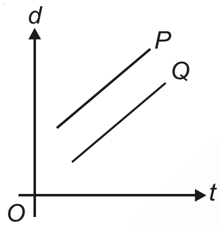
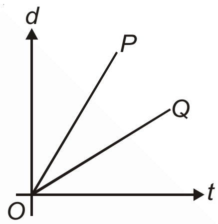
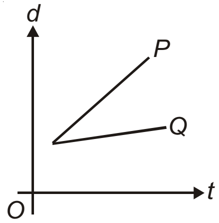
45. A body is projected vertically upward from ground. If we neglect the effect of air, then which one of the following is the best representation of variation of speed (v) with time (t)?

(1) 
(2) 
(3) 
(4) 

Sol. Answer (2)

The speed of an object is directly proportional to time $\boxed{v \propto t}$.

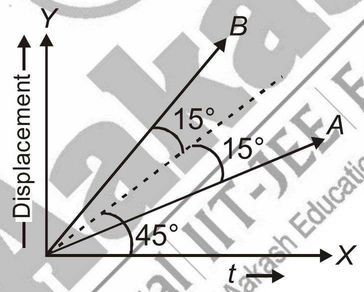
46. Which one of the following time-displacement graph represents two moving objects P and Q with zero relative velocity?

(1) 
(2) 
(3) 
(4) 

Sol. Answer (2)

Zero relative velocity means that both of them have same slope.

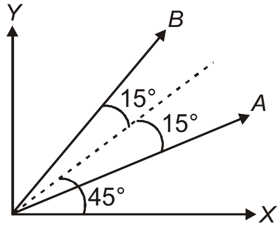
47. The displacement-time graph for two particles A and B is as follows. The ratio $\frac{v_A}{v_B}$ is



(1) 1 : 2 (2) $1:\sqrt{3}$ (3) $\sqrt{3}:1$ (4) 1 : 3

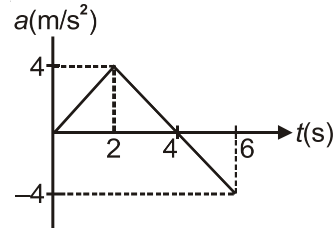
Sol. Answer (4)

The slope of line A is tan30° and B = tan60°



$$\frac{v_A}{v_B} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3} \Rightarrow \boxed{v_A : v_B = 1:3}$$

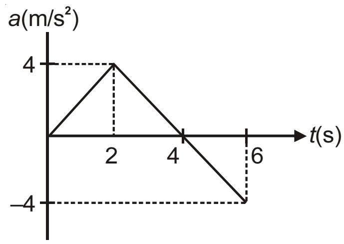
48. For the acceleration-time (a-t) graph shown in figure, the change in velocity of particle from t = 0 to t = 6 s is



(1) 10 m/s (2) 4 m/s (3) 12 m/s (4) 8 m/s

Sol. Answer (2)

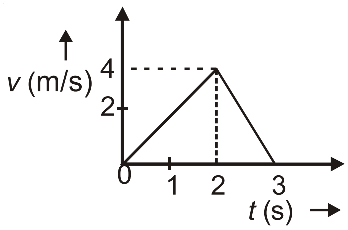
Area under *a*-*t* graph gives change in velocity.



So, $\Delta v = \frac{1}{2}\times 4\times 4 - \frac{1}{2}\times 2\times 4 = 8-4$

$\boxed{\Delta v = 4\ \text{ms}^{-1}}$

49. The velocity versus time graph of a body moving in a straight line is as shown in the figure below



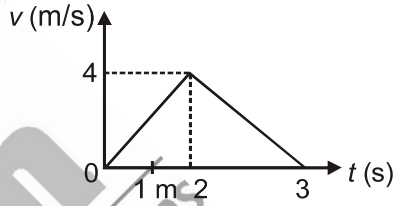
(1) The distance covered by the body in 0 to 2 s is 8 m

(2) The acceleration of the body in 0 to 2 s is 4 ms^{-2}

(3) The acceleration of the body in 2 to 3 s is 4 ms^{-2}

(4) The distance moved by the body during 0 to 3 s is 6 m

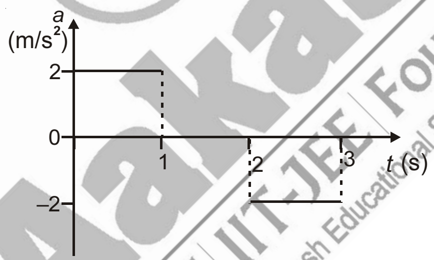
Sol. Answer (4)



Distance covered = Area under *v*-*t* graph = $\frac{1}{2}\times 3\times 4 = 6$ m

$\text{Acceleration}\big|_{t=0\text{ to }2\text{ s}} = \frac{4-0}{2} = 2\ \text{ms}^{-2}$

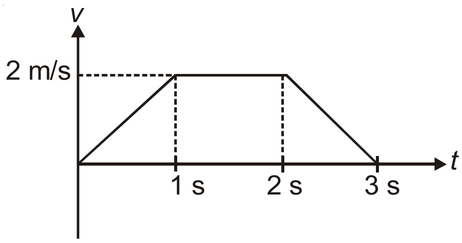
50. Acceleration-time graph for a particle is given in figure. If it starts motion at $t = 0$, distance travelled in 3 s will be



(1) 4 m (2) 2 m (3) 0 (4) 6 m

Sol. Answer (1)

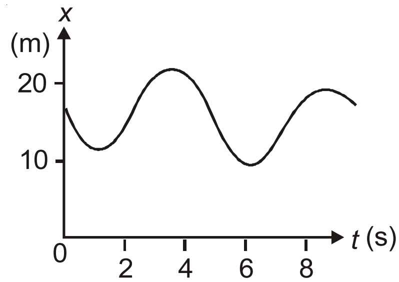
Draw the *v*-*t* graph from *a*-*t* graph.



Area under *v*-*t* graph = $\frac{1}{2}\times 2\times(3+1)$

= 4 m

51. Figure shows the position of a particle moving on the *x*-axis as a function of time



(1) The particle has come to rest 4 times

(2) The velocity at t = 8 s is negative

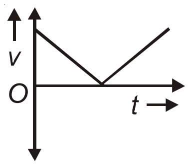
(3) The velocity remains positive for t = 2 s to t = 6 s

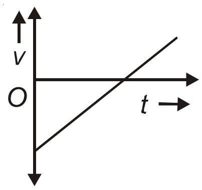
(4) The particle moves with a constant velocity

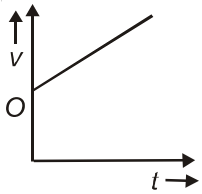
Sol. Answer (1)

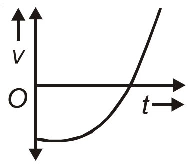
The particle has come to rest four times.

52. A particle moves along x-axis in such a way that its x-co-ordinate varies with time according to the equation $x = 4 - 2t + t^2$. The speed of the particle will vary with time as

(1) 

(2) 

(3) 

(4) 

Sol. Answer (1)

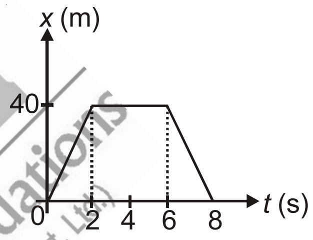
$x = 4 - 2t + t^2 \Rightarrow \dfrac{dx}{dt} = -2 + 2t$

$v = 2t - 2 \rightarrow$ Straight line

Slope $\rightarrow$ Positive

Intercept $\rightarrow$ Negative

53. The position (x) of a particle moving along x-axis varies with time (t) as shown in figure. The average acceleration of particle in time interval $t = 0$ to $t = 8$ s is



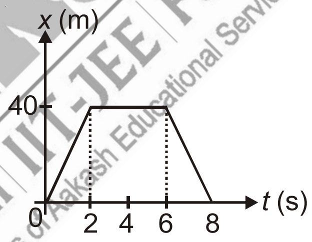
(1) 3 m/s^2 (2) –5 m/s^2

(3) – 4 m/s^2 (4) 2.5 m/s^2

Sol. Answer (2)

$t = 0$ to $t = 2$ $\quad$ $t = 6$ to $t = 8$

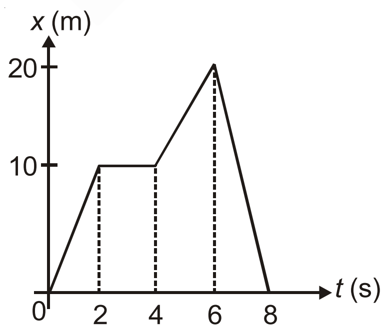
$v = 20$ m/s $\quad$ $v = -20$ m/s



$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{-20 - 20}{8} = \frac{-40}{8} = -5 \text{ ms}^{-2}$$

$$\boxed{a_{avg} = -5 \text{ ms}^{-2}}$$

54. The position (x)-time (t) graph for a particle moving along a straight line is shown in figure. The average speed of particle in time interval $t = 0$ to $t = 8$ s is



(1) Zero (2) 5 m/s (3) 7.5 m/s (4) 9.7 m/s

Sol. Answer (2)

$$v = \frac{\text{Distance}}{\text{Time}} = \frac{40}{8} = 5 \text{ ms}^{-1}$$

(Relative Motion)

55. A boat covers certain distance between two spots in a river taking t_1 hrs going downstream and t_2 hrs going upstream. What time will be taken by boat to cover same distance in still water?

(1) $\frac{t_1+t_2}{2}$ (2) $2(t_2-t_1)$ (3) $\frac{2t_1t_2}{t_1+t_2}$ (4) $\sqrt{t_1t_2}$

Sol. Answer (3)

For upstream, Speed $\Rightarrow v-u$ (where $v \to$ man and $u \to$ water)

For downstream, Speed $\Rightarrow v+u$

$$t_{up}=\frac{d}{v-u}$$

$$t_2=\frac{d}{v-u}$$

$$\Rightarrow \quad d=(v-u)t_2 \quad \text{...(i)}$$

$$t_{down}=\frac{d}{v+u}$$

$$t_1=\frac{d}{v+u}$$

$$\Rightarrow \quad d=(v+u)t_1 \quad \text{...(ii)}$$

$$t_{still}=\frac{d}{v}$$

$$\boxed{t_{still}=\frac{2t_1t_2}{t_1+t_2}}$$

On equating (i) and (ii)

$(v-u)\,t_2=(v+u)\,t_1$

$\Rightarrow vt_2-ut_2=vt_1+ut_1$

$\Rightarrow v(t_2-t_1)=u(t_1+t_2)$

$$\Rightarrow \quad u=\frac{v(t_2-t_1)}{t_2+t_1}$$

$$\text{So, } d=\left(v-\frac{v(t_2-t_1)}{t_1+t_2}\right)t_2=vt_2\left(\frac{t_1+t_2-t_2+t_1}{t_1+t_2}\right)$$

$\boxed{\frac{d}{v}=\frac{2t_1t_2}{t_1+t_2}}$ $\to$ **Remember as shortcut**

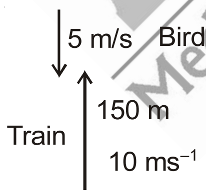
56. A train of 150 m length is going towards North at a speed of 10 m/s. A bird is flying at 5 m/s parallel to the track towards South. The time taken by the bird to cross the train is

(1) 10 s (2) 15 s (3) 30 s (4) 12 s

Sol. Answer (1)

$$\text{Time}=\frac{150}{10+5}=\frac{150}{15}$$

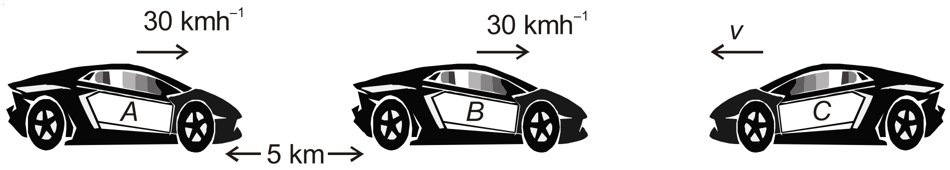
$$\Rightarrow \quad \boxed{T=10\text{ s}}$$



57. Two cars are moving in the same direction with a speed of 30 km/h. They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 minutes. The speed of the third car is

(1) 30 km/h (2) 25 km/h (3) 40 km/h (4) 45 km/h

Sol. Answer (4)



The distance of 5 km is in between A and B is covered by C in 4 minute with relative velocity (v + 30).

So, $d_{\text{rel}} = v_{\text{rel}} \times t$

$\Rightarrow$ $5 \text{ km} = (v+30) \times \dfrac{4}{60}$

$\Rightarrow$ $75 \text{ kmh}^{-1} = v + 30$

$\Rightarrow$ $\boxed{v = 45 \text{ kmh}^{-1}}$

58. Two cars *A* and *B* are moving in same direction with velocities 30 m/s and 20 m/s. When car *A* is at a distance *d* behind the car *B*, the driver of the car *A* applies brakes producing uniform retardation of 2 m/s^2. There will be no collision when

(1) $d < 2.5$ m (2) $d > 125$ m (3) $d > 25$ m (4) $d < 125$ m

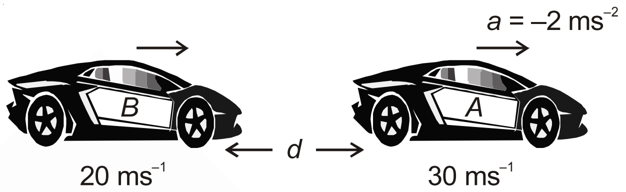
Sol. Answer (3)

$v^2 = u^2 + 2ad$

$\Rightarrow$ $0 = (10)^2 - 2 \times 2 \times d_{\text{rel}}$

$\Rightarrow$ $\dfrac{100}{4} \leq d_{\text{rel}}$

$\Rightarrow$ $\boxed{d_{\text{rel}} \geq 25 \text{ m}}$



59. Two trains each of length 100 m moving parallel towards each other at speed 72 km/h and 36 km/h respectively. In how much time will they cross each other?

(1) 4.5 s (2) 6.67 s (3) 3.5 s (4) 7.25 s

Sol. Answer (2)

When two trains are moving in opposite direction then

$v_{\text{rel}} = (20+10) = 30 \text{ ms}^{-1}$

$t = \dfrac{200}{30} = 6.67 \text{ s}$

60. A ball is dropped from the top of a building of height 80 m. At same instant another ball is thrown upwards with speed 50 m/s from the bottom of the building. The time at which balls will meet is

(1) 1.6 s (2) 5 s (3) 8 s (4) 10 s

Sol. Answer (1)

$t = \dfrac{h}{v_{\text{rel}}} = \dfrac{80}{50}$

$\Rightarrow$ $\boxed{t = 1.6 \text{ s}}$

SECTION - B

Objective Type Questions

(Position, Path length and Displacement, Average Velocity and Average Speed)

1. If average velocity of particle moving on a straight line is zero in a time interval, then

(1) Acceleration of particle may be zero

(2) Velocity of particle must be zero at an instant

(3) Velocity of particle may be never zero in the interval

(4) Average speed of particle may be zero in the interval

Sol. Answer (2)

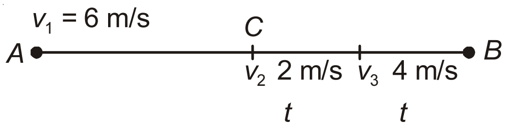
If average velocity = zero, then displacement is zero it means particle takes a turn in the opposite direction and at the time of turning back velocity has to be zero.

2. A particle travels half of the distance of a straight journey with a speed 6 m/s. The remaining part of the distance is covered with speed 2 m/s for half of the time of remaining journey and with speed 4 m/s for the other half of time. The average speed of the particle is

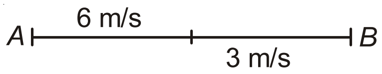
(1) 3 m/s (2) 4 m/s (3) 3/4 m/s (4) 5 m/s

Sol. Answer (2)

From C to B the time interval of travelling is same.



So, $v_{av} = \dfrac{v_2 + v_3}{2} = \dfrac{2+4}{2} = 3$ m/s

Now, 

Now, first half is covered with 6 ms^{-1} and second half with 3 ms^{-1}. So when distances are same.

$$v_{av} = \frac{2v_1v_2}{v_1 + v_2} = \frac{2\times 6\times 3}{6+3} = 4 \text{ ms}^{-1}$$

$$\boxed{v_{av} = 4 \text{ ms}^{-1}}$$

3. If magnitude of average speed and average velocity over an interval of time are same, then

(1) Particle must move with zero acceleration

(2) Particle must move with uniform acceleration

(3) Particle must be at rest

(4) Particle must move in a straight line without turning back

Sol. Answer (4)

Particle should have same distance and displacement in order to have final average speed and average velocity which is only possible only in case of an object moving on a straight line without turning back.

(Instantaneous Velocity and Speed, Acceleration)

4. The initial velocity of a particle moving along x-axis is u (at t = 0 and x = 0) and its acceleration a is given by $a = kx$. Which of the following equation is correct between its velocity (v) and position (x)?

(1) $v^2 - u^2 = 2kx$ (2) $v^2 = u^2 + 2kx^2$ (3) $v^2 = u^2 + kx^2$ (4) $v^2 + u^2 = 2kx$

Sol. Answer (3)

$a = kx$ and $\dfrac{vdv}{dx} = a$

$$\Rightarrow \int_u^v vdv = \int_0^x adx = \int_0^x kxdx$$

$$\Rightarrow \left.\frac{v^2}{2}\right|_u^v = \left.\frac{kx^2}{2}\right|_0^x$$

$$\Rightarrow v^2 - u^2 = kx^2 \Rightarrow \boxed{v^2 = u^2 + kx^2}$$

5. The velocity v of a body moving along a straight line varies with time t as $v = 2t^2 e^{-t}$, where v is in m/s and t is in second. The acceleration of body is zero at t =

(1) 0 (2) 2 s (3) 3 (4) Both (1) & (2)

Sol. Answer (4)

$v = 2t^2 e^{-t}$

$$a = \frac{dv}{dt} = 2[t^2e^{-t}\times(-1) + e^{-t}\times 2t]$$

Put, $a = 0$,

$-2t^2e^{-t} + 4te^{-t} = 0$

$\Rightarrow \quad -2t^2 + 4t = 0 \Rightarrow t\,(t - 2) = 0 \Rightarrow t = 0$ and $t = 2$

6. The velocity of a body depends on time according to the equation $v = \dfrac{t^2}{10} + 20$. The body is undergoing

(1) Uniform acceleration
(2) Uniform retardation
(3) Non-uniform acceleration
(4) Zero acceleration

Sol. Answer (3)

$$v = \frac{t^2}{10} + 20$$

To find acceleration find $\dfrac{dv}{dt}$

So, $a = \dfrac{dv}{dt} = \dfrac{2t}{10} + 0$

$\Rightarrow \quad a = \dfrac{t}{5} \Rightarrow \boxed{a \propto t}$

$\because$ a is a function of time so it is not constant, rather it is non-uniform.

7. A body starts from origin and moves along x-axis so that its position at any instant is $x = 4t^2 - 12t$ where t is in second and v in m/s. What is the acceleration of particle?

(1) 4 m/s^2
(2) 8 m/s^2
(3) 24 m/s^2
(4) 0 m/s^2

Sol. Answer (2)

$$x = 4t^2 - 12t$$

$$v = \frac{dx}{dt} = 8t - 12$$

$$a = \frac{d^2x}{dt^2} = 8 \Rightarrow \boxed{a = 8\ \text{ms}^{-2}}$$

(Kinematic Equations for Uniformly Accelerated Motion)

8. A car moving with speed v on a straight road can be stopped with in distance d on applying brakes. If same car is moving with speed $3v$ and brakes provide half retardation, then car will stop after travelling distance

(1) 6 d
(2) 3 d
(3) 9 d
(4) 18 d

Sol. Answer (4)

$$d_s = \frac{u^2}{2a}$$

$$d_s \propto \frac{u^2}{a} \Rightarrow \frac{d_s}{d'_s} - \frac{u^2}{u'^2} \times \frac{a'}{a}$$

$u' = 3v$, $a' = a / 2$

$u = v$

So, $\dfrac{d_s}{d'_s} = \dfrac{v^2}{9v^2} \times \dfrac{(a/2)}{a}$

$\Rightarrow \quad d'_s = 18d_s$

$\Rightarrow \quad \boxed{d'_s = 18d}$

9. The relation between position (x) and time (t) are given below for a particle moving along a straight line. Which of the following equation represents uniformly accelerated motion? [where α and β are positive constants]

(1) $\beta x = \alpha t + \alpha\beta$ (2) $\alpha x = \beta + t$ (3) $xt = \alpha\beta$ (4) $\alpha t = \sqrt{\beta + x}$

Sol. Answer (4)

For uniformly accelerated motion,

$$\underbrace{v^2 = u^2 + 2as}_{\text{Constant}} \quad \text{or} \quad \underbrace{s = ut + \frac{1}{2}at^2}_{\text{Constant}}$$

$$x = \frac{1}{2}at^2 + ut$$

Or the maximum power of t has to be two.

So, $\boxed{4}$.

10. A ball is dropped from an elevator moving upward with acceleration 'a' by a boy standing in it. The acceleration of ball with respect to [Take upward direction positive]

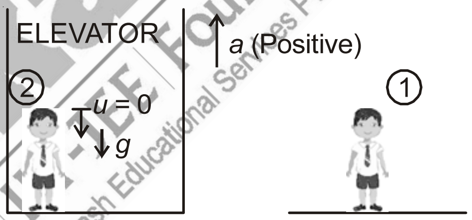
(1) Boy is – g (2) Boy is – (g + a) (3) Ground is – g (4) Both (2) & (3)

Sol. Answer (4)

Upward direction $\rightarrow$ Positive

Negative direction $\rightarrow$ Negative

If a person is observing from ground then, for him the acceleration of ball is in the downward direction.



$$a_{\text{ball }G} = a_{\text{ball}} - a_{\text{ground}} = -g - 0$$

$$\boxed{a_{bG} = -g}$$

a_{bG} = Acceleration of ball w.r.t. ground.

$$a_{\text{ball boy}} = a_{\text{ball}} - a_{\text{boy}} = -g - a$$

$\boxed{a_{bb} = -(g + a)}$, a_{bb} = Acceleration of ball w.r.t. boy.

11. A ball is thrown upward with speed 10 m/s from the top of the tower reaches the ground with a speed 20 m/s. The height of the tower is [Take g = 10 m/s^2]

(1) 10 m (2) 15 m (3) 20 m (4) 25 m

Sol. Answer (2)

$$v = \sqrt{u^2 + 2gh}$$

$$\Rightarrow (-20)^2 = 10^2 + 2 \times 10 \times h$$

$$\Rightarrow \frac{300}{2 \times 10} = h \Rightarrow \boxed{h = 15 \text{ m}}$$

12. A ball dropped from the top of tower falls first half height of tower in 10 s. The total time spend by ball in air is [Take g = 10 m/s^2]

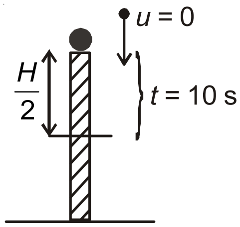
(1) 14.14 s (2) 15.25 s (3) 12.36 s (4) 17.36 s

Sol. Answer (1)

$$\frac{-H}{2} = ut - \frac{1}{2}g \times 10^2$$

$\Rightarrow$ $H = g \times 10^2$

$\Rightarrow$ $-H = -\frac{1}{2}gt^2$ (Full journey)



$$g \times 10^2 = \frac{1}{2}gt^2$$

$\Rightarrow$ $t^2 = 200$

$\Rightarrow$ $t = 10\sqrt{2}$ s

$\Rightarrow$ $t = 10 \times 1.414$ s

$= \boxed{14.14 \text{ s} = t}$

13. An object thrown vertically up from the ground passes the height 5 m twice in an interval of 10 s. What is its time of flight?

(1) $\sqrt{28}$ s (2) $\sqrt{86}$ s (3) $\sqrt{104}$ s (4) $\sqrt{72}$ s

Sol. Answer (3)

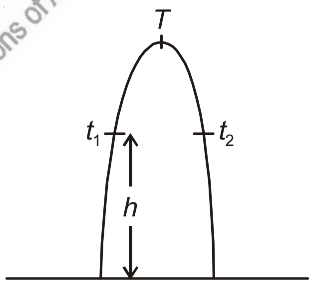
h = 5 m (given)

$t_2 - t_1$ = 10 s

$T \rightarrow$ Time taken to reach the highest point.

$$t_1 = T - \sqrt{T^2 - \frac{2h}{g}}, \quad t_2 = T + \sqrt{T^2 - \frac{2h}{g}}$$

$$t_2 - t_1 = T + \sqrt{T^2 - \frac{2h}{g}} - T + \sqrt{T^2 - \frac{2h}{g}}$$



$\Rightarrow$ $10 = 2\sqrt{T^2 - \frac{2 \times 5}{10}}$

$\Rightarrow$ $5 = \sqrt{T^2 - 1}$ $\Rightarrow$ $25 = T^2 - 1$

$T^2 = 26$

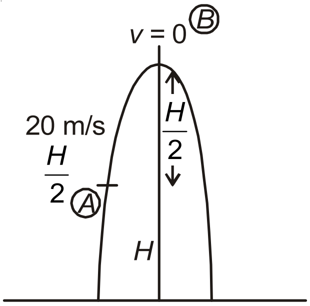
$\Rightarrow$ $\boxed{T = \sqrt{26}}$

Total time of flight $\Rightarrow$ $2T = 2\sqrt{26} = \sqrt{4 \times 26} = \sqrt{104}$ s

14. A ball is projected vertically upwards. Its speed at half of maximum height is 20 m/s. The maximum height attained by it is [Take g = 10 ms^2]

(1) 35 m (2) 15 m (3) 25 m (4) 40 m

Sol. Answer (4)



$$v_B^2 - v_A^2 = -2g\left(\frac{H}{2}\right)$$

$$\Rightarrow \quad 0 - 400 = -2 \times 10 \times \frac{H}{2}$$

$$\Rightarrow \quad \boxed{40 \text{ m} = H}$$

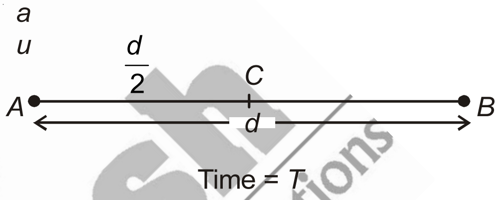
15. A particle starts with initial speed u and retardation a to come to rest in time T. The time taken to cover first half of the total path travelled is

(1) $\frac{T}{\sqrt{2}}$ (2) $T\left(1-\frac{1}{\sqrt{2}}\right)$ (3) $\frac{T}{2}$ (4) $\frac{3T}{4}$

Sol. Answer (2)

Retardation $\rightarrow a$

Initial velocity $\rightarrow u$



(I) **For total journey**

$$v = u + at$$

$$0 = u - aT$$

$$\Rightarrow \quad u = aT \quad \text{...(i)}$$

$$d = uT - \frac{1}{2}aT^2$$

Dividing by 2 on both sides

$$\frac{d}{2} = \frac{uT}{2} - \frac{1}{2}\frac{aT^2}{2} \quad \text{....(ii)}$$

On comparing equation (i) & (iii)

$$\frac{uT}{2} - \frac{1}{2}\frac{aT^2}{2} = ut - \frac{1}{2}at^2$$

Put $u = aT$

$$\Rightarrow \quad \frac{aT^2}{2} - \frac{aT^2}{4} = aTt - \frac{1}{2}at^2$$

$$\Rightarrow \quad \frac{T^2}{4} = Tt - \frac{t^2}{2}$$

Multiplying by 4 on both sides

$$T^2 = 4Tt - 2t^2 \Rightarrow 2t^2 - 4Tt + T^2 = 0$$

On solving this quadratic equation,

$$t = T - \frac{T}{\sqrt{2}} \Rightarrow \boxed{t = T\left(1-\frac{1}{\sqrt{2}}\right)}$$

(II) **For half journey**

$$\frac{d}{2} = ut - \frac{1}{2}at^2 \quad \text{...(iii)}$$

16. A body thrown vertically up with initial velocity 52 m/s from the ground passes twice a point at h height above at an interval of 10 s. The height h is (g = 10 m/s²)

(1) 22 m (2) 10.2 m (3) 11.2 m (4) 15 m

Sol. Answer (2)

Given, $t_2 - t_1 = 10$ s

$$t_2 + t_1 = \frac{2u}{g} = \frac{2\times 52}{10} = 10.4$$

$\Rightarrow$ $2t_2 = 20.4$

$\Rightarrow$ $t_2 = 10.2$ s

$t_1 = 0.2$ s

So, $t_1 t_2 = \dfrac{2h}{g}$

$$0.2\times 10.2 = \frac{2\times h}{10}$$

$\Rightarrow$ $1 \times 10.2 = h \Rightarrow \boxed{10.2\text{ m} = h}$

17. When a particle is thrown vertically upwards, its velocity at one third of its maximum height is $10\sqrt{2}$ m/s. The maximum height attained by it is

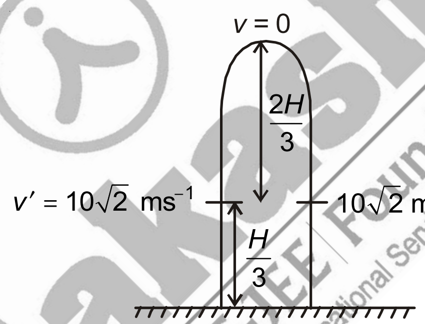
(1) $20\sqrt{2}$ m (2) 30 m (3) 15 m (4) 12.8 m

Sol. Answer (3)

$$v^2 - u^2 = -2g\times\frac{2H}{3}$$

$$\Rightarrow \quad -100\times 2 = -2\times 10\times\frac{2H}{3}$$

$$\Rightarrow \quad \boxed{H = 15\text{ m}}$$



18. A body is dropped from a height H. The time taken to cover second half of the journey is

(1) $2\sqrt{\dfrac{2H}{g}}$ (2) $\sqrt{\dfrac{H}{g}}$ (3) $\sqrt{\dfrac{H}{g}}\left(\sqrt{2}-1\right)$ (4) $\sqrt{\dfrac{2H}{g}}\times\dfrac{1}{(\sqrt{2}-1)}$

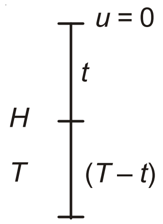
Sol. Answer (3)

The total time of journey

$$-s = ut - \frac{1}{2}gt^2$$

$$\Rightarrow \quad H = \frac{1}{2}gT^2 \qquad \text{...(i)}$$

$$\frac{-H}{2} = ut - \frac{1}{2}gt^2 \;\Rightarrow\; T = \sqrt{\frac{2H}{g}}$$



$$\Rightarrow \quad \frac{H}{2} = \frac{1}{2}gt^2$$

$$\Rightarrow \quad \frac{1}{2}gT^2 = gt^2 \qquad (\because\; ut = 0)$$

$$\Rightarrow \quad t = \frac{T}{\sqrt{2}}$$

$\Rightarrow$ Second half time = $T - t = T - \dfrac{T}{\sqrt{2}} = T\left(1 - \dfrac{1}{\sqrt{2}}\right) = \sqrt{\dfrac{2H}{g}}\left(1 - \dfrac{1}{\sqrt{2}}\right) = \sqrt{\dfrac{H}{g}}\left(\sqrt{2}-1\right)$

19. A stone dropped from the top of a tower is found to travel $\left(\frac{5}{9}\right)$ of the height of the tower during the last second of its fall. The time of fall is

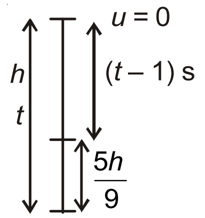
(1) 2 s (2) 3 s (3) 4 s (4) 5 s

Sol. Answer (2)

Let the total height of tower = H

Total time of journey = t

Time taken to cover the $\frac{5h}{9}$ is = last second



So, $s_t - s_{t-1} = \frac{5h}{9}$

$$\Rightarrow \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2 = \frac{5}{9}\times\frac{1}{2}gt^2 \qquad \left[\because h = \frac{1}{2}gt^2\right]$$

$$\Rightarrow \frac{1}{2}g(t^2 - t^2 - 1 + 2t) = \frac{1}{2}gt^2 \times \frac{5}{9}$$

$$\Rightarrow (2t-1) = \frac{5}{9}t^2$$

$\Rightarrow$ $18t - 9 = 5t^2$

$\Rightarrow$ $5t^2 - 18t + 9 = 0$

$\Rightarrow$ $5t^2 - 15t - 3t + 9 = 0$

$\Rightarrow$ $5t(t-3) - 3(t-3) = 0$

$\Rightarrow$ $(5t-3)(t-3) = 0$

$t = \frac{3}{5}$, $\boxed{t = 3\text{ s}}$ ($t = \frac{3}{5}$, doesn't satisfy the given criterion, so we neglect it)

20. A stone thrown upward with a speed u from the top of a tower reaches the ground with a velocity $4u$. The height of the tower is

(1) $\frac{15u^2}{2g}$ (2) $\frac{7u^2}{2g}$ (3) $\frac{16u^2}{g}$ (4) Zero

Sol. Answer (1)

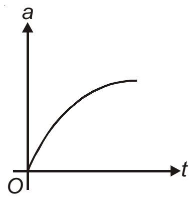
$$v = \sqrt{u^2 + 2gh}$$

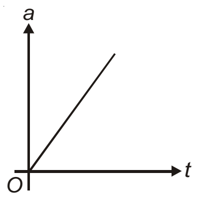
$$(4u)^2 = u^2 + 2gh$$

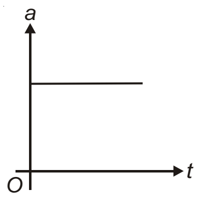
$$\frac{16u^2 - u^2}{2g} = h \Rightarrow \boxed{h = \frac{15u^2}{2g}}$$

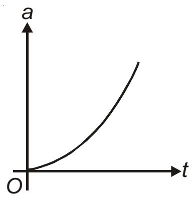
(Graphs)

21. The velocity v of a particle moving along x-axis varies with its position (x) as $v = \alpha\sqrt{x}$; where α is a constant. Which of the following graph represents the variation of its acceleration (a) with time (t)?

(1)


(2)


(3)


(4)


Sol. Answer (3)

$v = \alpha\sqrt{x}$

Squaring both sides $v^2 = \alpha^2 x$

Comparing above equation with 3rd equation of kinematics.

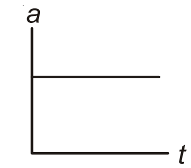
$$v^2 = u^2 + 2ax$$

$\alpha^2 x = 2ax$

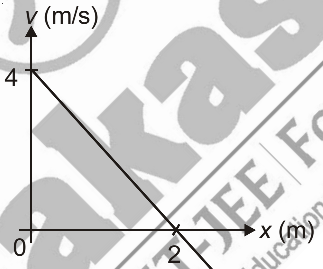
$\Rightarrow \boxed{a = \frac{\alpha^2}{2}}$

Constant $\rightarrow$ not a function of time

So,



22. The velocity (v) of a particle moving along x-axis varies with its position x as shown in figure. The acceleration (a) of particle varies with position (x) as



(1) $a^2 = x + 3$ (2) $a = 2x^2 + 4$ (3) $2a = 3x + 5$ (4) $a = 4x - 8$

Sol. Answer (4)

$a = \frac{vdv}{dx}$

$\frac{dv}{dx} \rightarrow$ slope

So, $\frac{-4}{2} = -2$

Intercept = + 4

$a \rightarrow$ Negative

So, $a = \frac{vdv}{dx}$

Relation between v and x

$\Rightarrow \frac{v-4}{x-0} = \frac{0-4}{2-0}$

$\Rightarrow \frac{v-u}{x} = \frac{-4}{2}$

$\Rightarrow v - 4 = -2x$

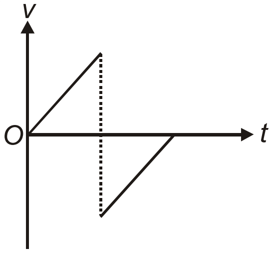
$\Rightarrow v = -2x + 4$

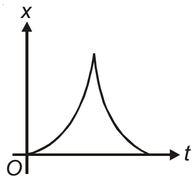
$\Rightarrow \frac{dv}{dx} = -2$

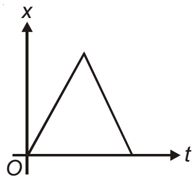
$\Rightarrow a = \frac{vdv}{dx} = (-2x+4)(-2)$

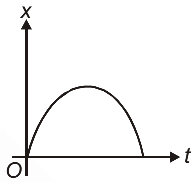
$\Rightarrow \boxed{a = 4x - 8}$

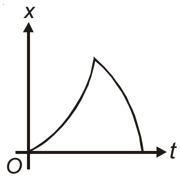
23. The velocity (v)-time (t) graph for a particle moving along x-axis is shown in the figure. The corresponding position (x)- time (t) is best represented by



(1) 

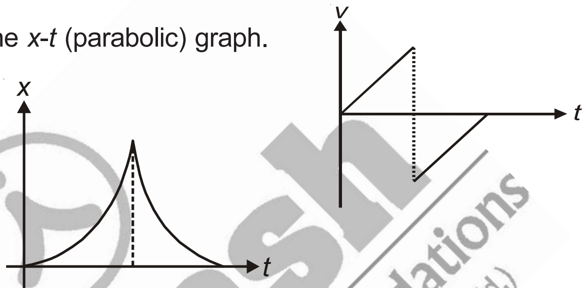
(2) 

(3) 

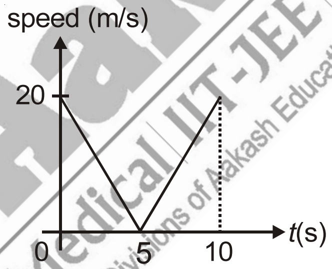
(4) 

Sol. Answer (1)

The graph of v-t can be converted into the x-t (parabolic) graph.



24. The speed-time graph for a body moving along a straight line is shown in figure. The average acceleration of body may be



(1) 0 (2) 4 m/s^2 (3) – 4 m/s^2 (4) All of these

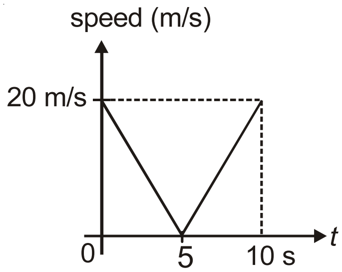
Sol. Answer (4)

The acceleration from zero to 5 s is

$$a = \frac{0-20}{5-0} = \frac{-20}{5} = -4 \text{ ms}^{-2}$$

From 5 s to 10 s

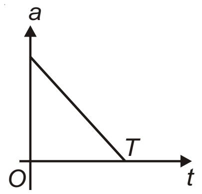
$$a = \frac{20-0}{10-5} = 4 \text{ ms}^{-2}$$

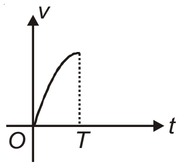


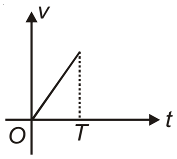
$$a = \frac{\text{Total change in velocity}}{\text{Time}}$$

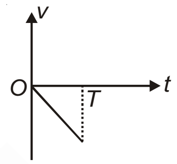
$$= \frac{20-20}{10-0} = 0 \text{ ms}^{-2}$$

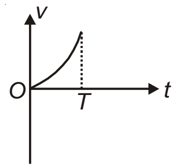
25. The acceleration (a)-time (t) graph for a particle moving along a straight starting from rest is shown in figure. Which of the following graph is the best representation of variation of its velocity (v) with time (t)?



(1) 

(2) 

(3) 

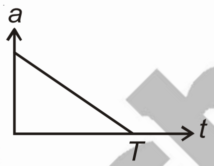
(4) 

Sol. Answer (1)

From the graph it is evident that the acceleration is decreasing with time.

Also, $a \propto -t$

$\Rightarrow \quad a = -kt$ (decreasing with time)



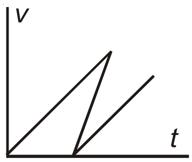
To find velocity,

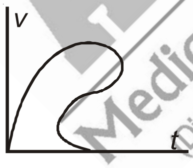
$$\frac{dv}{dt} = -kt$$

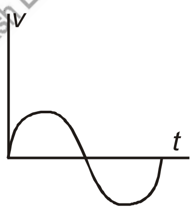
$$\int dv = \int -kt\,dt$$

$v \propto -t^2$ or graph of velocity should be parabolic with a decreasing slope.

26. Which of the following speed-time (v-t) graphs is physically not possible?

(1) 

(2) 

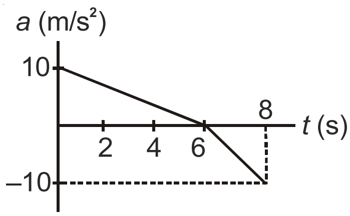
(3) 

(4) All of these

Sol. Answer (4)

None of the graph is physically possible.

27. The acceleration-time graph for a particle moving along x-axis is shown in figure. If the initial velocity of particle is –5 m/s, the velocity at t = 8 s is



(1) +15 m/s (2) +20 m/s (3) –15 m/s (4) –20 m/s

Sol. Answer (1)

The area under *a-t* graph gives change in velocity.

Given, $u = -5$ m/s

$\Rightarrow$ Area on positive side = $\frac{1}{2}\times 6\times 10$ = 30 ms^{-1}

$\Rightarrow$ Area on negative side = $\frac{1}{2}\times 2\times 10$ = 10 ms^{-1}

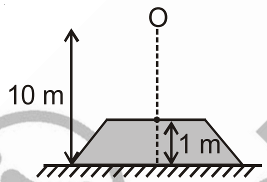
Net area = 30 – 10 = 20 ms^{-1}

Δv = Area

$v - (-5) = 20$

$\Rightarrow$ $\boxed{v = 15 \text{ ms}^{-1}}$

28. A body falling from a vertical height of 10 m pierces through a distance of 1 m in sand. It faces an average retardation in sand equal to (g = acceleration due to gravity)



(1) g (2) $9g$ (3) $100g$ (4) $1000g$

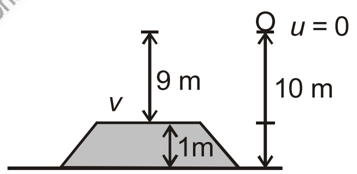
Sol. Answer (2)

If the ball is dropped then $x = 0$, the velocity with which it will hit the sand will be given by

$v^2 - u^2 = 2(-g)(-9)$

$v^2 - 0 = 18g$

$\boxed{v^2 = 18g}$ ...(i)



Now on striking sand, the body penetrates into sand for 1 m and comes to rest. So, $v \to$ initial for sand and final velocity = 0

$v'^2 - v^2 = 2(a)\times(-1)$

$\Rightarrow$ $-18g = -2a$

$\Rightarrow$ $\boxed{a = 9g}$

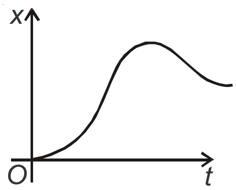
29. The displacement (x) - time (t) graph of a particle is shown in figure. Which of the following is correct?

(1) Particle starts with zero velocity and variable acceleration

(2) Particle starts with non-zero velocity and variable acceleration

(3) Particle starts with zero velocity and uniform acceleration

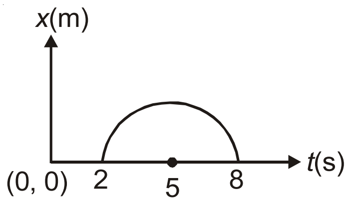
(4) Particle starts with non-zero velocity and uniform acceleration



Sol. Answer (1)

From the graph it is clear that the x is a function of time and speed/velocity is also changing. So, if velocity is changing then definitely the acceleration also changes with time. So, at $t = 0$, $x = 0$, so $v = 0$ but it is function of time and hence non-uniform.

30. Position time graph of a particle moving along straight line is shown which is in the form of semicircle starting from t = 2 to t = 8 s. Select correct statement



(1) Velocity of particle between t = 0 to t = 2 s is positive

(2) Velocity of particle is opposite to acceleration between t = 2 to t = 5 s

(3) Velocity of particle is opposite to acceleration between t = 5 to t = 8 s

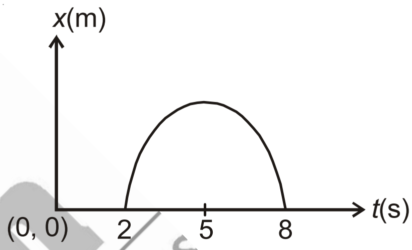
(4) Acceleration of particle is positive between t_1 = 2 s to t_2 = 5 s while it is negative between t_1 = 5 s to t_2 = 8 s

Sol. Answer (2)

(i) From 0 to 2 s the velocity = 0 as displacement is zero.

(ii) From 2 to 5 s velocity is decreasing but nature is positive, but acceleration is negative.

So, v and a have opposite nature.



(Relative Motion)

31. A body is dropped from a certain height h (h is very large) and second body is thrown downward with velocity of 5 m/s simultaneouly. What will be difference in heights of the two bodies after 3 s?

(1) 5 m (2) 10 m (3) 15 m (4) 20 m

Sol. Answer (3)

$u_{rel} = u_1 - u_2 = 0 - (-5) = 5\ \text{ms}^{-1}$

t = 3 s

$a_{rel} = a_1 - a_2 = -g - (-g) = 0\ \text{ms}^{-2}$

$s_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2$

$\Rightarrow \quad s_{rel} = 5 \times 3 = 15\text{ m} \qquad (\because\ a_{rel} = 0)$

So, $\boxed{s_{rel} = 15\text{ m}}$

32. Two bodies starts moving from same point along a straight line with velocities v_1 = 6 m/s and v_2 = 10 m/s, simultaneously. After what time their separation becomes 40 m?

(1) 6 s (2) 8 s (3) 12 s (4) 10 s

Sol. Answer (4)

$s_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2$

a_{rel} = 0

$\Rightarrow \quad 40 = (10 - 6) \times t$

$\Rightarrow \quad \frac{40}{4} = t \ \Rightarrow\ \boxed{t = 10\text{ s}}$

33. Ball A is thrown up vertically with speed 10 m/s. At the same instant another ball B is released from rest at height h. At time t, the speed of A relative to B is

(1) 10 m/s (2) $10 - 2\,gt$ (3) $\sqrt{10^2 - 2gh}$ (4) $10 - gt$

Sol. Answer (1)

$$v_A = 10\text{ ms}^{-1} - 10t$$

$$v_B = 0 - 10t$$

$$v_{AB} = v_A - v_B = 10 - (10t) - (-10t) = 10 - 10t + 10t \ = 0$$

$$\Rightarrow \quad \boxed{v_{AB} = 10\text{ ms}^{-1}}$$

SECTION - C

Previous Years Questions

1. Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be **[NEET-2017]**

(1) $\dfrac{t_1 + t_2}{2}$ (2) $\dfrac{t_1 t_2}{t_2 - t_1}$ (3) $\dfrac{t_1 t_2}{t_2 + t_1}$ (4) $t_1 - t_2$

Sol. Answer (3)

Velocity of girl w.r.t. elevator $= \dfrac{d}{t_1} = v_{ge}$

Velocity of elevator w.r.t. ground $v_{eG} = \dfrac{d}{t_2}$ then velocity of girl w.r.t. ground

$$\vec{v}_{gG} = \vec{v}_{ge} + \vec{v}_{eG}$$

i.e, $v_{gG} = v_{ge} + v_{eG}$

$$\frac{d}{t} = \frac{d}{t_1} + \frac{d}{t_2}$$

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$$

$$t = \frac{t_1 t_2}{(t_1 + t_2)}$$

2. Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $x_P(t) = at + bt^2$ and $x_Q(t) = ft - t^2$. At what time do the cars have the same velocity?

[NEET (Phase-2) 2016]

(1) $\dfrac{a - f}{1 + b}$ (2) $\dfrac{a + f}{2(b - 1)}$ (3) $\dfrac{a + f}{2(1 + b)}$ (4) $\dfrac{f - a}{2(1 + b)}$

Sol. Answer (4)

$$v_P = \frac{dx_P}{dt} = a + 2bt$$

$$v_Q = \frac{dx_Q}{dt} = f - 2t$$

$v_P = v_Q$

$\Rightarrow \; a + 2bt = f - 2t$

$2t + 2bt = f - a \;\Rightarrow\; t = \dfrac{f-a}{2(b+1)}$

3. If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between 1 s and 2 s is **[NEET-2016]**

(1) $\dfrac{A}{2}+\dfrac{B}{3}$ (2) $\dfrac{3}{2}A+4B$ (3) $3A + 7B$ (4) $\dfrac{3}{2}A+\dfrac{7}{3}B$

Sol. Answer (4)

$v = At + Bt^2$

$\Rightarrow \; \dfrac{dx}{dt} = At + Bt^2$

$\Rightarrow \; dx = (At + Bt^2)dt$

$$\Rightarrow \; x = \left[\frac{At^2}{2}+\frac{Bt^3}{3}\right]_1^2 = \frac{A}{2}(4-1)+\frac{B}{3}(8-1) = \frac{3}{2}A+\frac{7}{3}B$$

4. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = \beta x^{-2n}$, where β and n are constants and x is the position of the particle. The acceleration of the particle as a function of x, is given by **[AIPMT-2015]**

(1) $-2n\beta^2 e^{-4n+1}$ (2) $-2n\beta^2 x^{-2n-1}$ (3) $-2n\beta^2 x^{-4n-1}$ (4) $-2\beta^2 x^{-2n+1}$

Sol. Answer (3)

5. A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respectively. The relation between h_1, h_2 and h_3 is **[NEET-2013]**

(1) $h_1 = \dfrac{h_2}{3} = \dfrac{h_3}{5}$ (2) $h_2 = 3h_1$ and $h_3 = 3h_2$ (3) $h_1 = h_2 = h_3$ (4) $h_1 = 2h_2 = 3h_3$

Sol. Answer (1)

When a body starts from rest and under the effect of constant acceleration then the distance travelled by the body in final time intervals is in the ratio of odd number *i.e.*, 1 : 3 : 5 : 7

So, $h_1 : h_2 : h_3 \Rightarrow 1 : 3 : 5$

$\dfrac{h_1}{h_2} = \dfrac{1}{3}, \; \dfrac{h_1}{h_3} = \dfrac{1}{5}$

$\Rightarrow \; h_1 = \dfrac{h_2}{3}, \; h_1 = \dfrac{h_3}{5}$

So, $h_1 = \dfrac{h_2}{3} = \dfrac{h_3}{5}$

6. The motion of a particle along a straight line is described by equation $x = 8 + 12t - t^3$ where x is in metre and t in second. The retardation of the particle when its velocity becomes zero, is **[AIPMT (Prelims)-2012]**

(1) 6 ms^{-2} (2) 12 ms^{-2} (3) 24 ms^{-2} (4) Zero

Sol. Answer (2)

$x = 8 + 12t - t^3$

$$\frac{dx}{dt} = 12 - 3t^2$$

If $v = 0$, then $12 - 3t^2 = 0$

$\Rightarrow 4 = t^2$

$\Rightarrow \boxed{t = 2\text{ s}}$

$$a = \frac{d^2x}{dt^2} = -6t$$

$$a|_{t=2\text{ s}} \Rightarrow -12\text{ ms}^{-2}$$

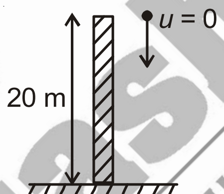
$$\boxed{|a| = 12\text{ ms}^{-2}}$$

7. A boy standing at the top of a tower of 20 m height drops a stone. Assuming $g = 10\text{ ms}^{-2}$, the velocity with which it hits the ground is **[AIPMT (Prelims)-2011]**

(1) 5.0 m/s (2) 10.0 m/s (3) 20.0 m/s (4) 40.0 m/s

Sol. Answer (3)

$$-s = ut - \frac{1}{2}gt^2$$



$$\Rightarrow \quad -20 = -\frac{1}{2} \times 10 \times t^2 \qquad (\because\ u = 0)$$

$\Rightarrow$ $40 = 10t^2$

$\Rightarrow$ $\boxed{t = 2\text{ s}}$

$v = u - gt$

$\Rightarrow$ $\boxed{v = -20\text{ ms}^{-1}}$ $(\because\ u = 0)$

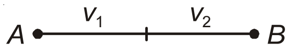
$\Rightarrow$ $\boxed{|v| = 20\text{ ms}^{-1}}$

8. A particle covers half of its total distance with speed v_1 and the rest half distance with speed v_2. Its average speed during the complete journey is **[AIPMT (Mains)-2011]**

(1) $\dfrac{v_1^2 v_2^2}{v_1^2 + v_2^2}$ (2) $\dfrac{v_1 + v_2}{2}$ (3) $\dfrac{v_1 v_2}{v_1 + v_2}$ (4) $\dfrac{2v_1 v_2}{v_1 + v_2}$

Sol. Answer (4)

As the distances are same so,

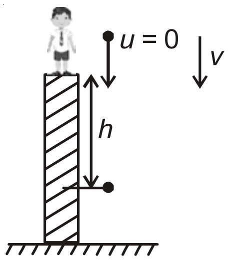


$$\boxed{v_{av} = \frac{2v_1 v_2}{v_1 + v_2}}$$

9. A ball is dropped from a high rise platform at $t = 0$ starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v. The two balls meet at $t = 18$ s. What is the value of v? (Take $g = 10\text{ m/s}^2$) **[AIPMT (Prelims)-2010]**

(1) 60 m/s (2) 75 m/s (3) 55 m/s (4) 40 m/s

Sol. Answer (2)



As the ball meet at t = 18 s

So, it means both of them covered the same distance 'h'.

But the time of travel is different

1st body $\rightarrow t$

2nd body $\rightarrow (t - 6) \rightarrow$ as theorem after 6 s.

1st body	**2nd body**
$-h = -\frac{1}{2}gt^2$	$-h = -v(t-6) - \frac{1}{2}g(t-6)^2$
$h = \frac{1}{2}gt^2$ ...(i)	$h = v(t-2) + \frac{1}{2}(t-2)^2$ ...(ii)

Equating (i) and (ii), we get

v = 75 m/s

For fitst body, t = 18 s

For second body, t = (18 – 6) = 12 s

$$h = \frac{1}{2} \times 10 \times (18)^2 = 5 \times 324$$

h = 1620 m

For second body

$$1600 = v \times (18 - 6) + \frac{1}{2} \times 10\ (18 - 6)^2$$

$$1620 = v \times 12 + 5 \times 144$$

$$\frac{1620 - 720}{12} = v$$

$$\frac{900}{12} = v$$

$$\Rightarrow \boxed{v = 75 \text{ ms}^{-1}}$$

10. A particle moves a distance x in time t according to equation $x = (t + 5)^{-1}$. The acceleration of particle is proportional to **[AIPMT (Prelims)-2010]**

(1) (Velocity)$^{3/2}$ (2) (Distance)^2 (3) (Distance)$^{-2}$ (4) (Velocity)$^{2/3}$

Sol. Answer (1)

$x = (t + 5)^{-1}$

$$v = \frac{dx}{dt} = (-1)(t+5)^{-2} \qquad \left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

$v = -(t + 5)^{-2}$

$$a = \frac{dv}{dt} = (-1)(-2)(t+5)^{-3}$$

$$a = 2(t+5)^{-3} - 2(t+5)^{-2} \times (t+5)^{-1} \qquad \left[\because\ v \propto \frac{1}{(t+5)^2} \Rightarrow v^{\frac{1}{2}} \propto \frac{1}{t+5}\right]$$

$$\propto 2(v) \times v^{\frac{1}{2}}$$

$$a \propto 2v^{\frac{3}{2}}$$

$$\boxed{a \propto (\text{velocity})^{\frac{3}{2}}}$$

11. A bus is moving with a speed of 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus? **[AIPMT (Prelims)-2009]**

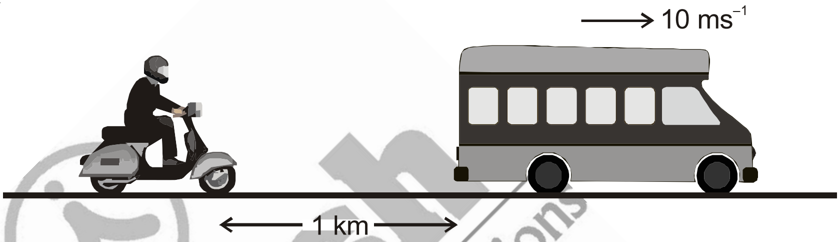
(1) 40 ms^{-1} (2) 25 ms^{-1} (3) 10 ms^{-1} (4) 20 ms^{-1}

Sol. Answer (4)

T = 100 s

S_{rel} = 1000 m

$S_{rel} = U_{rel}\, t \qquad (\because\ a_{rel} = 0)$



1000 = (v – 10) × 100

$$\boxed{v = 20\ \text{ms}^{-1}}$$

12. A particle starts its motion from rest under the action of a constant force. If the distance covered in first 10 seconds is S_1 and that covered in the first 20 seconds is S_2, then **[AIPMT (Prelims)-2009]**

(1) $S_2 = 3S_1$ (2) $S_2 = 4S_1$ (3) $S_2 = S_1$ (4) $S_2 = 2S_1$

Sol. Answer (2)

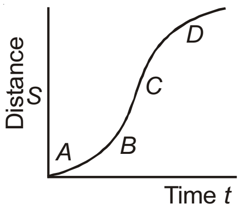
$u = 0$, $a \rightarrow$ Constant

$$S_1 = \frac{1}{2}a(10)^2,\ S_2 = \frac{1}{2}a(20)^2$$

$$\frac{S_1}{S_2} = \frac{10^2}{(20)^2} = \frac{100}{400}$$

$$\boxed{S_2 = 4S_1}$$

13. A particle shows distance-time curve as given in this figure. The maximum instantaneous velocity of the particle is around the point **[AIPMT (Prelims)-2008]**



(1) A (2) B (3) C (4) D

Sol. Answer (3)

Maximum instantaneous velocity will be at that point which has maximum slope.

As clear from the graph 'C' has maximum slope.

14. A particle moves in a straight line with a constant acceleration. It changes its velocity from 10 ms^{-1} to 20 ms^{-1} while passing through a distance 135 m in t second. The value of t is **[AIPMT (Prelims)-2008]**

(1) 9 (2) 10 (3) 1.8 (4) 12

Sol. Answer (1)

Using 3rd equation, we first find acceleration,

$v^2 - u^2 = 2as$

$20^2 - 10^2 = 2a \times 135$

$\Rightarrow \dfrac{300}{2 \times 135} = a$

$\Rightarrow a = \dfrac{20}{18}$

$\Rightarrow \boxed{\dfrac{10}{9} \text{ ms}^{-2} = a}$

$\Rightarrow v = u + at$

$\Rightarrow 20 = 10 + \dfrac{10}{9} \times t$

$\Rightarrow 10 = \dfrac{10}{9} t \Rightarrow \boxed{t = 9 \text{ s}}$

15. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}$ ms^{-2}, in the third second is **[AIPMT (Prelims)-2008]**

(1) $\dfrac{19}{3}$m (2) 6 m

(3) 4 m (4) $\dfrac{10}{3}$m

Sol. Answer (4)

$S_{n^{\text{th}}} = u + \dfrac{a}{2}(2n-1)$

$n = 3$, (given), $a = \dfrac{4}{3}$ ms^{-2}

$S_{n^{\text{th}}} = u + \dfrac{a}{2}(2n-1)$

$\Rightarrow S_{n^{\text{th}}} = 0 + \dfrac{4}{3} \times \dfrac{1}{2}(2 \times 3 - 1) = \dfrac{2}{3} \times 5$

$\Rightarrow \boxed{\dfrac{10}{3} \text{ m} = S_{3^{\text{rd}}}}$

16. A particle moving along x-axis has acceleration f, at time t, given $f = f_0\left(1 - \dfrac{t}{T}\right)$, where f_0 and T are constants. The particle at t = 0 has zero velocity. When f = 0, the particle's velocity (v_x) is **[AIPMT (Prelims)-2007]**

(1) $\dfrac{1}{2}f_0T$ (2) f_0T

(3) $\dfrac{1}{2}f_0T^2$ (4) f_0T^2

Sol. Answer (1)

17. A car moves from x to y with a uniform speed v_u and returns to y with a uniform speed v_d. The average speed for this round trip is **[AIPMT (Prelims)-2007]**

(1) $\dfrac{v_u + v_d}{2}$ (2) $\dfrac{2v_u v_d}{v_d + v_u}$ (3) $\sqrt{v_u v_d}$ (4) $\dfrac{v_d + v_u}{v_d + v_u}$

Sol. Answer (2)

18. The position x of a particle with respect to time t along x-axis is given by $x = 9t^2 - t^3$, where x is in metres and t in seconds. What will be the position of this particle when it achieves maximum speed along the positive x-direction? **[AIPMT (Prelims)-2007]**

(1) 24 m (2) 32 m (3) 54 m (4) 81 m

Sol. Answer (3)

$x = 9t^2 - t^3$

$\dfrac{dx}{dt} = 18t - 3t^2 \Rightarrow v = 18t - 3t^2$

To find the maxima of speed,

$\dfrac{dv}{dt} = 18 - 6t$

Put, $\dfrac{dv}{dt} = 0 \quad \Rightarrow \quad 18 - 6t = 0$

$\Rightarrow \quad \boxed{t = 3\text{ s}}$

So, the positions of particle at $t = 3 = ?$

$x|_{t=3\text{ s}} = 9(3^2) - 3^3$

$\boxed{x = 54\text{ m}}$

19. A particle moves along a straight line OX. At a time t (in seconds) the distance x (in metres) of the particle from O is given by $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest?

[AIPMT (Prelims)-2006]

(1) 24 m (2) 40 m (3) 56 m (4) 16 m

Sol. Answer (4)

$\boxed{x = 40 + 12t - t^3}$

The particle will come to rest when $v = 0$,

$v = \dfrac{dx}{dt} = 12 - 3t^2$

$\Rightarrow \quad v = 0 \ \Rightarrow 12 = 3t^2 \Rightarrow t^2 = 4 \ \Rightarrow \boxed{t = 2\text{ s}}$

So, the distance travelled by object is 2 s.

$x|_{t=0} = 40\text{ m}$,

$x|_{t=2\text{s}} = 40 + 12 \times 2 - 8$

$= 40 + 24 - 8$

$= 40 + 16$

$= 56\text{ m}$

Distance travelled = (56 – 40) = 16 m

20. Two bodies, A (of mass 1 kg) and B (of mass 3 kg) are dropped from heights of 16 m and 25 m, respectively. The ratio of the time taken by them to reach the ground is **[AIPMT (Prelims)-2006]**

(1) $\frac{5}{4}$ (2) $\frac{12}{5}$

(3) $\frac{5}{12}$ (4) $\frac{4}{5}$

Sol. Answer (4)

$$T = \sqrt{\frac{2H}{g}} \Rightarrow T \propto \sqrt{H}$$

$$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{H_1}{H_2}}$$

$$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$ (Given, H_1 = 16 m, H_2 = 25 m)

$$\Rightarrow \boxed{\frac{T_1}{T_2} = \frac{4}{5}}$$

21. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will **[AIPMT (Prelims)-2005]**

(1) Go on decreasing with time (2) Be independent of β

(3) Drop to zero when α and β (4) Go on increasing with time

Sol. Answer (4)

$$x = ae^{-\alpha t} + be^{\beta t}$$

$$\frac{dx}{dt} = a(-\alpha)e^{-\alpha t} + b(\beta)e^{\beta t}$$

$$v = b\beta e^{\beta t} - a\alpha e^{-\alpha t}$$

As we increase time $e^{\beta t}$ increases and $e^{-\alpha t}$ decreases.

So, v keeps on increasing with time.

22. A ball is thrown vertically upward. It has a speed of 10 m/s when it has reached one half of its maximum height. How high does the ball rise? (Taking g = 10 m/s^2) **[AIPMT (Prelims)-2005]**

(1) 15 m (2) 10 m

(3) 20 m (4) 5 m

Sol. Answer (2)

23. The displacement 'x' (in meter) of a particle of mass 'm' (in kg) moving in one dimension under the action of a force, is related to time 't' (in sec) by $t = \sqrt{x} + 3$. The displacement of the particle when its velocity is zero, will be

(1) 2 m (2) 4 m

(3) Zero (4) 6 m

Sol. Answer (3)

$$t = \sqrt{x} + 3$$

$$(t-3) = \sqrt{x}$$

$\Rightarrow \quad x = (t-3)^2 = t^2 + 9 - 6t$

$\Rightarrow \quad v = \dfrac{dx}{dt} = 2t - 6$

If $v = 0$, $2t - 6 = 0$

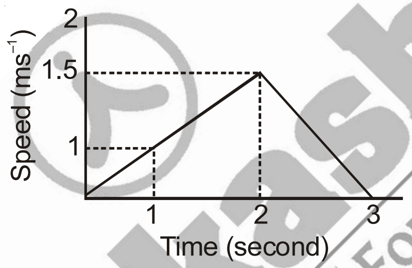
$\Rightarrow \quad \boxed{t = 3\text{ s}}$

At, $t = 3$ s, $x = ?$

$$x = (t-3)^2 = (3-3)^2$$

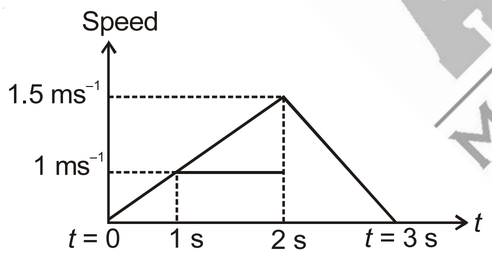
$\Rightarrow \quad \boxed{x = 0}$

24. The speed-time graph of a particle moving along a solid curve is shown below. The distance traversed by the particle from $t = 0$ to $t = 3$ is



(1) $\dfrac{9}{2}$ m (2) $\dfrac{9}{4}$ m (3) $\dfrac{10}{3}$ m (4) $\dfrac{10}{5}$ m

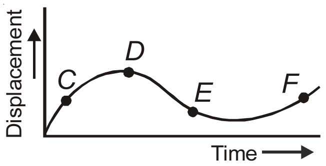
Sol. Answer (2)



Area under the speed-time graph gives distance.

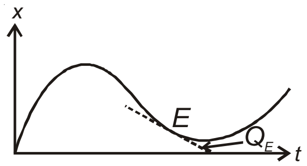
Area = $\dfrac{1}{2} \times 3 \times 1.5 \Rightarrow \dfrac{9}{4}$ m

25. The displacement-time graph of a moving particle is shown below. The instantaneous velocity of the particle is negative at the point



(1) E (2) F (3) C (4) D

Sol. Answer (1)



The angle made by the tangent at point '*C*' is obtuse hence

tan Q_E = negative, so slope = negative

hence, velocity is also negative.

26. Two bodies *A* (of mass 1 kg) and *B* (of mass 3 kg) are dropped from heights of 16 m and 25 m, respectively. The ratio of the time taken by them to reach the ground is

(1) $\frac{4}{5}$ (2) $\frac{5}{4}$ (3) $\frac{12}{5}$ (4) $\frac{5}{12}$

Sol. Answer (1)

27. A particle moving along *x*-axis has acceleration *f* at time *t* given by $f = f_0\left(1-\frac{t}{T}\right)$, where f_0 and *T* are constants. The particle at *t* = 0 has zero velocity. In the time interval between *t* = 0 and the instant when *f* = 0, the particle's velocity (v_x) is

(1) $\frac{1}{2}f_0T^2$ (2) f_0T^2 (3) $\frac{1}{2}f_0T$ (4) f_0T

Sol. Answer (3)

$f = f_0\left(1-\frac{t}{T}\right)$

$f \rightarrow$ Acceleration

$f_0 \rightarrow$ Initial acceleration

Initial/lower limit of time = 0, u = 0

Upper limit of time = T, v = ?

$$a = \frac{dv}{dt}$$

$$\Rightarrow \int_0^{v_x} dv = \int_0^t a\,dt$$

$$\int_0^{v_x} dv = \int_0^T f_0\left(1-\frac{t}{T}\right)dt$$

$$v\Big|_0^{v_x} = f_0\,t\Big|_0^T - \frac{f_0}{T}\frac{t^2}{2}\Big|_0^T$$

$$v_x - 0 = f_0(T-0) - \frac{f_0}{2T}(T^2-0)$$

$$\Rightarrow v_x = f_0T - \frac{1}{2}f_0T$$

$$\Rightarrow \boxed{v_x = \frac{1}{2}f_0T}$$

28. A ball is dropped from a high rise platform at t = 0 starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v. The two balls meet at t = 18 s. What is the value of v? (Take g = 10 m/s^2)

(1) 60 m/s (2) 75 m/s (3) 55 m/s (4) 40 m/s

Sol. Answer (2)

29. The velocity of train increases uniformly from 20 km/h to 60 km/h in 4 hour. The distance travelled by the train during this period is

(1) 160 km (2) 180 km (3) 100 km (4) 120 km

Sol. Answer (1)

$v^2 - u^2 = 2as$

$v = u + at$

$60 = 20 + a \times 4$

$40 = 4a$

$\boxed{a = 10 \text{ km/h}^{-2}}$

$60^2 - 20^2 = 2 \times 10 \times s$

$\frac{3600 - 400}{20} = s$

$\Rightarrow \boxed{s = 160 \text{ km}}$

30. A particle moves along a straight line such that its displacement at any time t is given by $s = (t^3 - 6t^2 - 3t + 4)$ metres. The velocity when the acceleration is zero is

(1) 3 m/s (2) 42 m/s (3) –9 m/s (4) –15 m/s

Sol. Answer (4)

$s = t^3 - 6t - 3t + 4$

$v = \frac{ds}{dt} = 3t^2 - 12t - 3$

$a = \frac{dv}{dt} = 6t - 12$

Put $a = 0 \Rightarrow 6t - 12 = 0$

$\boxed{t = 2 \text{ s}}$

$v|_{t=2\text{ s}} = 3(2)^2 - 12(2) - 3$

$= 12 - 24 - 3$

$= -12 - 3$

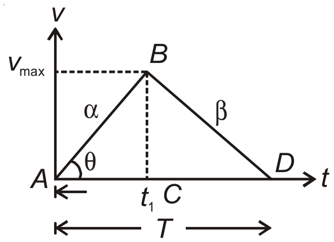
$\boxed{v = -15 \text{ ms}^{-1}}$

31. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β and comes to rest. If total time elapsed is t, then maximum velocity acquired by car will be

(1) $\frac{(\alpha^2 - \beta^2)t}{\alpha\beta}$ (2) $\frac{(\alpha^2 + \beta^2)t}{\alpha\beta}$ (3) $\frac{(\alpha + \beta)t}{\alpha\beta}$ (4) $\frac{\alpha\beta t}{\alpha + \beta}$

Sol. Answer (4)

$$\boxed{v_{\max} = \frac{\alpha\beta t}{\alpha+\beta}}$$



In ΔABC, $\tan\theta = \text{slope} = \dfrac{v_{\max}}{t_1}$

In ΔBCD, $1-\beta = \dfrac{-v_{\max}}{T-t_1}$

$$\alpha t_1 = \beta T - \beta t_1$$

$$\Rightarrow \quad t_1 = \frac{\beta T}{\alpha+\beta}$$

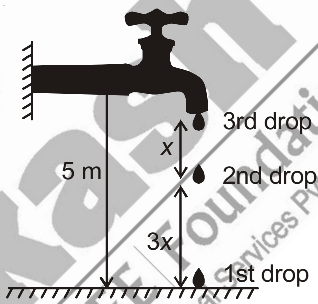
$$v_{\max} = \alpha \times t_1$$

$$\boxed{v_{\max} = \frac{\alpha\beta T}{\alpha+\beta}}$$

32. The water drops fall at regular intervals from a tap 5 m above the ground. The third drop is leaving the tap at instant the first drop touches the ground. How far above the ground is the second drop at that instant? (Take g = 10 ms^{-2})

(1) 3.75 m (2) 4.00 m (3) 1.25 m (4) 2.50 m

Sol. Answer (1)



$x = 3x = 5$ m

$\Rightarrow \quad 4x = 5$ m

$\boxed{x = 1.25 \text{ m}}$

So, second drop is at $3x$

$\Rightarrow 3 \times 1.25 = 3.75$ m above ground.

33. The acceleration of a particle is increasing linearly with time t as bt. The particle starts from origin with an initial velocity v_0. The distance travelled by the particle in time t will be

(1) $v_0 t + \frac{1}{3}bt^2$ (2) $v_0 t + \frac{1}{2}bt^2$ (3) $v_0 t + \frac{1}{6}bt^3$ (4) $v_0 t + \frac{1}{3}bt^3$

Sol. Answer (3)

$a = bt$

$u = v_0$

$$a = \frac{dv}{dt}$$

$$\int_{v_0}^{v} dv = \int_0^t a\,dt$$

$$\int_{v_0}^{v} dv = \int bt\,dt$$

$$v - v_0 = \left.\frac{bt^2}{2}\right|_0^t$$

$$v - v_0 = \frac{b}{2}(t^2 - 0)$$

$v = v_0 + \frac{1}{2}bt^2$

Now, $v = \frac{dx}{dt}$

$\Rightarrow \int_0^x dx = \int_0^t v dt$

$\int_0^x dx = \int_0^t \left(v_0 + \frac{1}{2}bt^2\right) dt$

$\boxed{x = v_0 t + \frac{1}{6}bt^3}$

34. If a car at rest accelerates uniformly to a speed of144 km/h in 20 s, it covers a distance of

(1) 1440 cm
(2) 2980 cm
(3) 20 m
(4) 400 m

Sol. Answer (4)

$u = 0$, $a \rightarrow$ constant

v = 144 km/h^{-1} = $144 \times \frac{5}{18}$ = 40 ms^{-1}

t = 20 s

$v = u + at$

$40 = a \times 20$

$\boxed{a = 2 \text{ ms}^2}$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times 400$

$\boxed{s = 400 \text{ m}}$

35. The position x of a particle varies with time, (t) as $x = at^2 - bt^3$. The acceleration will be zero at time t equal to

(1) $\frac{a}{3b}$
(2) Zero
(3) $\frac{2a}{3b}$
(4) $\frac{a}{b}$

Sol. Answer (1)

$x = at^2 - bt^3$

$v = \frac{dx}{dt} = 2at - 3bt^2$

$a = \frac{dv}{dt} = 2a - 6bt$

Put a = 0, to find 't'

$2a = 6bt \Rightarrow \boxed{t = \frac{a}{3b}}$

36. Motion of a particle is given by equation, $s = (3t^3 + 7t^2 + 14t + 8)$ m. The value of acceleration of the particle at $t = 1$ s is

(1) 10 m/s^2 (2) 32 m/s^2

(3) 23 m/s^2 (4) 16 m/s^2

Sol. Answer (2)

$s = 3t^3 + 7t^2 + 14t + 8$

$$v = \frac{ds}{dt} = 9t^2 + 14t + 14$$

$$a = \frac{d^2s}{dt^2} = 18t + 14$$

$$a|_{t=1\text{ s}} = 18 + 14$$

$$\boxed{a|_{t=1\text{ s}} = 32\text{ ms}^{-2}}$$

37. If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

(1) ut (2) $\frac{1}{2}gt^2$

(3) $ut - \frac{1}{2}gt^2$ (4) $(u + gt)t$

Sol. Answer (2)

As the motion is symmetric the distances covered during the last t seconds of ascent is same as that travelled during 1st t seconds of descent.

At highest point, $v = 0$

$$-s = -\frac{1}{2}gt^2$$

$$\Rightarrow \boxed{s = \frac{1}{2}gt^2}$$

38. A man throws balls with the same speed vertically upwards one after the other at an interval of 2 second. What should be the speed of the throw so that more than two balls are in the sky at any time?

(Given $g = 9.8$ m/s^2)

(1) More than 19.6 m/s (2) At least 9.8 m/s

(3) Any speed less than 19.6 m/s (4) Only with speed 19.6 m/s

Sol. Answer (1)

For move than two ball in air, time of flight should be

Total time of flight $\leq \frac{2u}{g}$

$$4 \leq \frac{2u}{g}$$

$$2 \times 9.8 \leq u$$

$$\boxed{u \geq 19.6\text{ ms}^{-1}}$$

SECTION - D

Assertion - Reason Type Questions

1. A : It is not possible to have constant velocity and variable acceleration.

   R : Accelerated body cannot have constant velocity.

Sol. Answer (1)

2. A : The direction of velocity of an object can be reversed with constant acceleration.

   R : A ball projected upward reverse its direction under the effect of gravity.

Sol. Answer (2)

3. A : When the velocity of an object is zero at an instant, the acceleration need not be zero at that instant.

   R : In motion under gravity, the velocity of body is zero at the top-most point.

Sol. Answer (4)

4. A : A body moving with decreasing speed may have increasing acceleration.

   R : The speed of body decreases, when acceleration of body is opposite to velocity.

Sol. Answer (1)

5. A : For a moving particle distance can never be negative or zero.

   R : Distance is a scalar quantity and never decreases with time for moving object.

Sol. Answer (1)

6. A : If speed of a particle is never zero than it may have zero average speed.

   R : The average speed of a moving object in a closed path is zero.

Sol. Answer (4)

7. A : The magnitude of average velocity in an interval can never be greater than average speed in that interval.

   R : For a moving object distance travelled $\geq$ | Displacement |

Sol. Answer (1)

8. A : The area under acceleration-time graph is equal to velocity of object.

   R : For an object moving with constant acceleration, position-time graph is a straight line.

Sol. Answer (4)

9. A : The motion of body projected under the effect of gravity without air resistance is uniformly accelerated motion.

   R : If a body is projected upwards or downwards, then the direction of acceleration is downward.

Sol. Answer (2)

10. A : The relative acceleration of two objects moving under the effect of gravity ,only is always zero, irrespective of direction of motion.

    R : The acceleration of object moving under the effect of gravity have acceleration always in downward direction and is independent from size and mass of object.

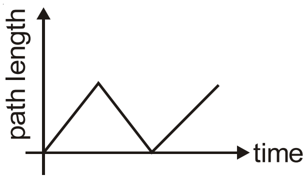
Sol. Answer (1)

11. A : In the presence of air resistance, if the ball is thrown vertically upwards then time of ascent is less than the time of descent.

    R : Force due to air friction always acts opposite to the motion of the body.

Sol. Answer (1)

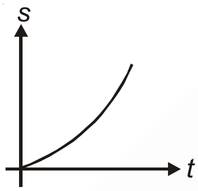
12. A : The following graph can't exist actually



R : Total path length never decreases with time.

Sol. Answer (1)

13. A : The displacement (s) time graph shown in the figure represents an accelerated motion.



R : Slope of graph increases with time.

Sol. Answer (1)

14. A : Average velocity can be zero, but average speed of a moving body can not be zero in any finite time interval.

R : For a moving body displacement can be zero but distance can never be zero.

Sol. Answer (1)

15. A : For a particle moving in a straight line, its acceleration must be either parallel or antiparallel to velocity.

R : A body moving along a curved path may have constant acceleration.

Sol. Answer (2)

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