Chapter 14

Oscillations

Solutions

SECTION - A

Objective Type Questions

(Periodic and Oscillatory Motions, Period and Frequency, Displacement)

1. Identify the correct definition

   (1) If after every certain interval of time, particle repeats its motion then motion is called periodic motion

   (2) To and fro motion of a particle over the same path about its mean position in certain time interval is called oscillatory motion

   (3) Oscillatory motion described in terms of single sine and cosine functions is called simple harmonic motion

   (4) All of these

Sol. Answer (4)

All the above definition are true.

Hence answer is (4)

2. The displacement of a particle executing S.H.M. is given by $x = 0.01 \sin 100\pi(t + 0.05)$. The time period is

   (1) 0.01 s (2) 0.02 s (3) 0.1 s (4) 0.2 s

Sol. Answer (2)

$$x = 0.01 \sin 100\pi(t + 0.05)$$

Here $\omega = 100\pi$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = \frac{1}{50} = 0.02 \text{ s}$$

3. A boy is swinging in a swing. If he stands, the time period will

   (1) First decrease, then increase (2) Decrease

   (3) Increase (4) Remain same

Sol. Answer (2)

A swing is like a pendulum. So

$$T = 2\pi\sqrt{\frac{l}{g}}$$

When the boy stands the C.O.M. will become higher. Thus 'l' will become shorter and so according to the equation of time period. Time period will decrease.

4. Time period of a simple pendulum in a freely falling lift will be

(1) Finite (2) Infinite (3) Zero (4) All of these

Sol. Answer (2)

$$T = \sqrt{\frac{l}{g}}$$

In a freely falling lift $g = 0$.

Hence T = infinite.

5. If effective length of a simple pendulum is equal to radius of earth (R), time period will be

(1) $T = \pi\sqrt{\frac{R}{g}}$ (2) $T = 2\pi\sqrt{\frac{2R}{g}}$ (3) $T = 2\pi\sqrt{\frac{R}{g}}$ (4) $T = 2\pi\sqrt{\frac{R}{2g}}$

Sol. Answer (4)

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{R}+\frac{1}{l}\right)}}$$

$\because\ l = R \quad \therefore\ T = 2\pi\sqrt{\frac{R}{2g}}$

6. A body executing S.H.M. along a straight line has a velocity of 3 ms^{-1} when it is at a distance of 4 m from its mean position and 4 ms^{-1} when it is at a distance of 3 m from its mean position. Its angular frequency and amplitude are

(1) 2 rad s^{-1} & 5 m (2) 1 rad s^{-1} & 10 m (3) 2 rad s^{-1} & 10 m (4) 1 rad s^{-1} & 5 m

Sol. Answer (4)

$$v = \omega\sqrt{A^2 - x^2}$$

$v_1 = 3$ m/s $\qquad x_1 = 4$ m

$v_2 = 4$ m/s $\qquad x_2 = 3$ m

$3 = \omega\sqrt{A^2 - 4^2}$ ... (i)

$4 = \omega\sqrt{A^2 - 3^2}$ ... (ii)

Solving (i) and (ii), we get

$A = 5$ m and $\omega = 1$ rad/s

7. The frequency of oscillation of a mass m suspended by a spring is ν_1. If length of spring is cut to one third then the same mass oscillates with frequency ν_2, then

(1) $\nu_2 = 3\nu_1$ (2) $3\nu_2 = \nu_1$ (3) $\nu_2 = \sqrt{3}\,\nu_1$ (4) $\sqrt{3}\,\nu_2 = \nu_1$

Sol. Answer (3)

$$\omega_{old} = \sqrt{\frac{k_{old}}{m}}$$

When divided into 3 parts the spring constant of smaller parts

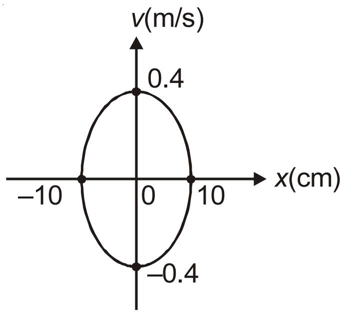
$\therefore$ $k_{final} = 3k_{old}$

$\therefore$ $\omega_{final} = \sqrt{3}\,\omega_{old}$

$\omega = 2\pi\nu$

Hence $\nu_{final} = \sqrt{3}\,\nu_{old} \Rightarrow \nu_2 = \sqrt{3}\nu_1$

8. The plot of velocity (v) versus displacement (x) of a particle executing simple harmonic motion is shown in figure. The time period of oscillation of particle is



(1) $\frac{\pi}{2}$ s (2) π s (3) 2π s (4) 3π s

Sol. Answer (1)

A = 10 cm $A\omega = 0.4$ m/s

= 0.1 m

$\therefore$ $\omega = 4$ rad/s

$$T = \frac{2\pi}{4} = \frac{\pi}{2}\text{ s}$$

9. The equation of simple harmonic motion may not be expressed as (each term has its usual meaning)

(1) $x = A\sin(\omega t + \phi)$ (2) $x = A\cos(\omega t - \phi)$

(3) $x = a\sin\omega t + b\cos\omega t$ (4) $x = A\sin(\omega t + \phi) + B\sin(2\omega t + \phi)$

Sol. Answer (4)

The fourth option is a superposition of two S.H.M.'s will different frequencies and time periods.

Hence it is not a true S.H.M.

(Simple Harmonic Motion)

10. If a particle is executing simple harmonic motion, then acceleration of particle

(1) Is uniform
(2) Varies linearly with time
(3) Is non uniform
(4) Both (2) & (3)

Sol. Answer (3)

If a particle is executing S.H.M.

$a \alpha - \omega^2 x$

Hence it is not uniform and depends on x rather than time.

Hence answer is (3).

11. What is the phase difference between acceleration and velocity of a particle executing simple harmonic motion?

(1) Zero
(2) $\frac{\pi}{2}$
(3) π
(4) 2π

Sol. Answer (2)

$v = A\omega \cos(\omega t + \phi)$

and $a = -A\omega^2 \sin(\omega t + \phi)$

$\because$ $\cos(\omega t + \phi + \pi/2) = -\sin(\omega t + \phi)$

$a = A\omega^2 \cos(\omega t + \phi + \pi/2)$

Hence velocity lags $\pi/2$ with acceleration.

12. The shape of graph plotted between velocity and position of a particle executing simple harmonic motion is

(1) A straight line
(2) An ellipse
(3) A parabola
(4) A hyperbola

Sol. Answer (2)

$$v = \omega\sqrt{A^2 - x^2} \qquad \frac{v^2}{\omega^2} + x^2 = A^2 \Rightarrow \frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$$

This is the equation of an ellipse.

Hence answer is (2)

13. If particle is executing simple harmonic motion with time period T, then the time period of its total mechanical energy is

(1) Zero
(2) $\frac{T}{2}$
(3) $2T$
(4) Infinite

Sol. Answer (4)

The total mechanical energy doesn't change in an undamped S.H.M. $\therefore$ frequency = 0.

Hence time period is infinite.

14. Select wrong statement about simple harmonic motion

(1) The body is uniformly accelerated

(2) The velocity of the body changes smoothly at all instants

(3) The amplitude of oscillation is symmetric about the equilibrium position

(4) The frequency of oscillation is independent of amplitude

Sol. Answer (1)

In S.H.M..

$$a = -\omega^2 x$$

Thus acceleration varies linearly with time.

15. A particle is executing S.H.M. with time period T. Starting from mean position, time taken by it to complete $\frac{5}{8}$ oscillations, is

(1) $\frac{T}{12}$ (2) $\frac{T}{6}$ (3) $\frac{5T}{12}$ (4) $\frac{7T}{12}$

Sol. Answer (4)

Total distance covered by particle = 4 A

For $\frac{5}{8}$ of oscillation means that it has completed $\frac{1}{2}$ the oscillation taking $\frac{T}{2}$ seconds. Now it has to cover $\frac{1}{8}$ oscillation more. The whole path may be divided into 8 parts of $\frac{A}{2}$ hence it has to travel $\frac{A}{2}$ distance from mean position.

$$\frac{A}{2} = A \sin \omega t$$

$$\frac{\pi}{6} = 8\ \omega t$$

$$t = \frac{T}{12} \qquad \left[\text{Putting } \omega = \frac{2\pi}{T}\right]$$

$$\text{Total time} = \frac{T}{2} + \frac{T}{12} = \frac{7T}{12}$$

16. A particle is executing S.H.M.. between $x = \pm A$. The time taken to go from 0 to $\frac{A}{2}$ is T_1 and to go from $\frac{A}{2}$ to A is T_2 ; then

(1) $T_1 < T_2$ (2) $T_1 > T_2$ (3) $T_1 = T_2$ (4) $T_1 = 2T_2$

Sol. Answer (1)

The velocity is greater closer to the mean position so it will take less time gains from 0 to $\frac{A}{2}$ than from $\frac{A}{2}$ to A.

17. For a particle executing simple harmonic motion, the amplitude is A and time period is T. The maximum speed will be

(1) $4AT$ (2) $\frac{2A}{T}$ (3) $2\pi\sqrt{\frac{A}{T}}$ (4) $\frac{2\pi A}{T}$

Sol. Answer (4)

Maximum speed is given by

$$v = A\omega$$

and $\omega = \frac{2\pi}{T}$

Hence $v = \frac{2\pi}{T}A$

18. A particle is executing S.H.M.. with amplitude A and has maximum velocity v_0. Its speed at displacement $\frac{3A}{4}$ will be

(1) $\frac{\sqrt{7}}{4}v_0$ (2) $\frac{v_0}{\sqrt{2}}$ (3) v_0 (4) $\frac{\sqrt{3}}{2}v_0$

Sol. Answer (1)

$$v = \omega\sqrt{A^2 - x^2}$$

$$x = \frac{3}{4}A$$

$$v = \omega\sqrt{A^2 - \frac{9A^2}{16}} = \omega A\sqrt{\frac{7}{16}}$$

or $v = v_0\sqrt{\frac{7}{4}}$ as $(v_0 = A\omega)$

(Simple Harmonic Motion and Uniform Circular Motion)

19. Two particles executing S.H.M. of same frequency, meet at $x = +A/2$, while moving in opposite directions. Phase difference between the particles is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{5\pi}{6}$ (4) $\frac{2\pi}{3}$

Sol. Answer (4)

$$x = A\sin\omega t$$

When displacement

$$x = \frac{A}{2}$$

$$\frac{A}{2} = A\sin(\omega t + \phi)$$

$$\sin^{-1}\frac{1}{2} = \omega t + \phi$$

$$\omega t + \phi = 30° \text{ or } 150°$$

When particles are in opposite direction at one lime phase is 30° and at the other 150°. So phase difference is 120°.

20. The displacements of two particles executing S.H.M. on the same line are given as $y_1 = a\sin\left(\frac{\pi}{2}t+\phi\right)$ and $y_2 = b\sin\left(\frac{2\pi}{3}t+\phi\right)$. The phase difference between them at t = 1 s is

(1) π (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

Sol. Answer (4)

Phase difference between them is just difference in the angular values.

Phase difference = $\left(\frac{2\pi}{3}+\phi\right)-\left(\frac{\pi}{2}+\phi\right)=\frac{\pi}{6}$

(Force Law for Simple Harmonic Motion)

21. For a particle showing motion under the force $F = -5(x-2)^2$, the motion is

(1) Translatory (2) Oscillatory (3) S.H.M. (4) All of these

Sol. Answer (1)

$F = -5(x-2)^2$

The motion depicts a non uniform translatory motion as the acceleration just keeps increasing in the negative direction. This is because $(x-2)$ is always positive.

22. For a particle showing motion under the force $F = -5(x-2)$, the motion is

(1) Translatory (2) Oscillatory (3) S.H.M. (4) Both (2) & (3)

Sol. Answer (4)

Force varies linearly with time with respect to $-(x-2)$

$F \propto -(x-2)$

Hence motion of the particle is an S.H.M.

23. Two identical springs have the same force constant 73.5 Nm^{-1}. The elongation produced in each spring in three cases shown in Figure-1, Figure-2 and Figure-3 are (g = 9.8 ms^{-2})

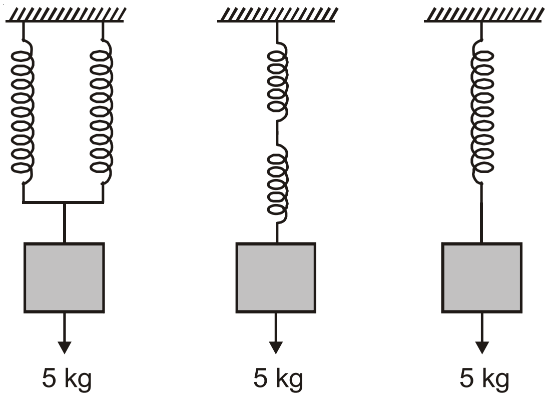


Figure 1 Figure 2 Figure 3

(1) $\frac{1}{6}$m, $\frac{2}{3}$m, $\frac{1}{3}$m (2) $\frac{1}{3}$m, $\frac{1}{3}$m, $\frac{1}{3}$m (3) $\frac{2}{3}$m, $\frac{1}{3}$m, $\frac{1}{6}$m (4) $\frac{1}{3}$m, $\frac{4}{3}$m, $\frac{2}{3}$m

Sol. Answer (4)

$k = 73.5 \text{ Nm}^{-1}$ Force = 5 × 9.8

In figure (1)

$5 \times 9.8 = (2\,k)\,x_1$

$\therefore \quad x_1 = \dfrac{5\times 9.8}{2\times 73.5} = \dfrac{1}{3}$

In figure (2)

$5 \times 9.8 = \dfrac{k\times k}{k+k}\times x_2$

or $5 \times 9.8 = \dfrac{k}{2}\times x_2$

$x_2 = \dfrac{98}{73.5} = \dfrac{4}{3}$

In figure (3)

$5 \times 9.8 = kx_3$

$x_3 = \dfrac{5\times 9.8}{73.5} = \dfrac{2}{3}$

24. A particle executes simple harmonic motion according to equation $4\dfrac{d^2x}{dt^2} + 320x = 0$. Its time period of oscillation is

(1) $\dfrac{2\pi}{5\sqrt{3}}$ s (2) $\dfrac{\pi}{3\sqrt{2}}$ s (3) $\dfrac{\pi}{2\sqrt{5}}$ s (4) $\dfrac{2\pi}{\sqrt{3}}$ s

Sol. Answer (3)

$4\dfrac{d^2x}{dt^2} + 320x = 0$

$4\,a = -\,320\,x$

$a = -\,80\,x$

Since $a = -\,\omega^2\,x$ in S.H.M.

$80 = \omega^2$

$\sqrt{16\times 5} = \omega$

or $\omega = 4\sqrt{5}$

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4\sqrt{3}} = \dfrac{\pi}{2\sqrt{5}}$ s

25. A particle of mass 10 g is undergoing S.H.M. of amplitude 10 cm and period 0.1 s. The maximum value of force on particle is about

(1) 5.6 N (2) 2.75 N (3) 3.5 N (4) 4 N

Sol. Answer (4)

$T = 0.1$ $\qquad m = 0.01$ kg

$\therefore \quad \omega = 20\,\pi$ rad/s

Amplitude $A = 0.1$ m

$a = -\,\omega^2 x$

Maximum acceleration = $-\,\omega^2 A$

Maximum force = $-\,m\omega^2 A$

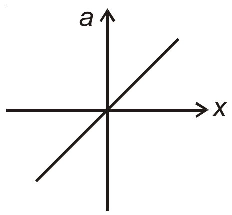
$F_{max} = -\,0.01 \times (20\,\pi)^2 \times 0.1$

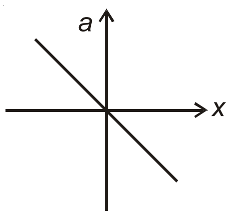
$F_{max} = -\,0.001 \times 400\,\pi^2$

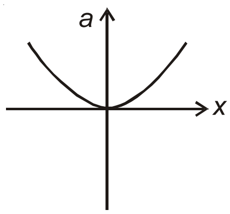
$= -3.95$ N

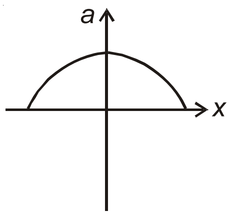
or -4 N approximately

26. Which of the following graphs best represents the variation of acceleration '*a*' with displacement *x*?

(1)


(2)


(3)


(4)


Sol. Answer (2)

$a = -\,kx$ is an S.H.M.

Hence it will be a straight line with negative slope as in option (2)

27. A block is resting on a piston which executes simple harmonic motion with a period 2.0 s. The maximum velocity of the piston, at an amplitude just sufficient for the block to separate from the piston is

(1) 1.57 ms^{-1} (2) 3.12 ms^{-1} (3) 2.0 ms^{-1} (4) 6.42 ms^{-1}

Sol. Answer (2)

Period (T) = 2 s

$$\omega = \frac{2\pi}{2} = \pi \text{ rad/s}$$

When block just represent from a piston, maximum acceleration must be equal to *g*.

$g = -\,\omega^2 x$

Acceleration is maximum when $x = A$

$g = -\,\omega^2 A$

or $A = \dfrac{9.8}{\pi^2}$

Maximum velocity = $A\omega$

$$= \frac{9.8}{\pi^2}\times\pi = \frac{9.8}{\pi}\text{ m/s} = 3.119\text{ m/s} = 3.12\text{ m/s}$$

28. Two masses m_1 = 1 kg and m_2 = 0.5 kg are suspended together by a massless spring of spring constant 12.5 Nm^{-1}. When masses are in equilibrium m_1 is removed without disturbing the system. New amplitude of oscillation will be

(1) 30 cm (2) 50 cm (3) 80 cm (4) 60 cm

Sol. Answer (3)

Points of equilibrium of the spring will be when no force acts on it.

or $kx = (m_1 + m_2)g$

$$x = \frac{(m_1+m_2)g}{k}$$

The new equilibrium position which will be the mean position of S.H.M. will be simply $\frac{m_2g}{k}$

New amplitude will be maximum displacement from $\frac{m_2g}{k}$ which is :

$$A = \frac{(m_1+m_2)g}{k} - \frac{m_2g}{k}$$

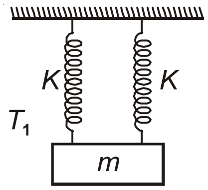
or $A = \frac{m_1g}{k}$

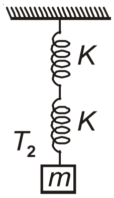
or $A = \frac{1\times10}{12.5}$

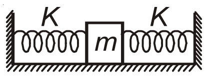
or $A = \frac{4}{5}m$

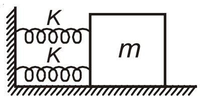
$\therefore$ A = 0.8 m or 80 cm

29. A mass m is attached to two springs of same force constant K, as shown in following four arrangements. If T_1, T_2, T_3 and T_4 respectively be the time periods of oscillation in the following arrangements, in which case time period is maximum?

(a) 

(b) 

(c) 

(d) 

(1) (a) (2) (b) (3) (c) (4) (d)

Sol. Answer (2)

$$T = 2\pi\sqrt{\frac{m}{K}}$$

Time period is maximum when K is minimum.

In (a), (c) and (d) the spring constants are in parallel so the $K_{eq} = 2K$.

Only in case (b) springs are in series.

So, $K_{eq} = \frac{K}{2}$

Hence time period in this case will be maximum.

(Energy in Simple Harmonic Motion)

30. A particle is executing S.H.M. with time period T. If time period of its total mechanical energy is T' then $\frac{T'}{T}$ is

(1) 2 (2) $\frac{1}{2}$ (3) Zero (4) Infinite

Sol. Answer (4)

Total mechanical energy will never change so $T' = \infty \therefore \frac{T'}{T} = \infty$

31. A body executes S.H.M. with an amplitude A. At what displacement from the mean position, is the potential energy of the body one-fourth of its total energy?

(1) $\frac{A}{4}$ (2) $\frac{A}{2}$ (3) $\frac{3A}{4}$ (4) Some other fraction of A

Sol. Answer (2)

Potential energy at displacement x from mean position is given by

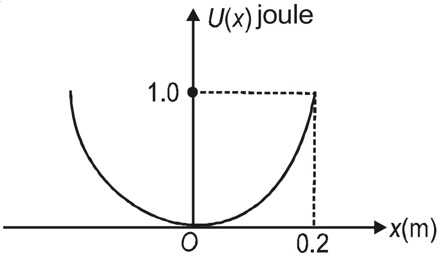
$$\text{P.E.} = \frac{1}{2}kx^2$$

Let total energy be $E = \frac{1}{2}kA^2$

$$\frac{1}{2}kx^2 = \frac{E}{4} = \frac{1}{8}kA^2$$

$$x = \frac{A}{2}$$

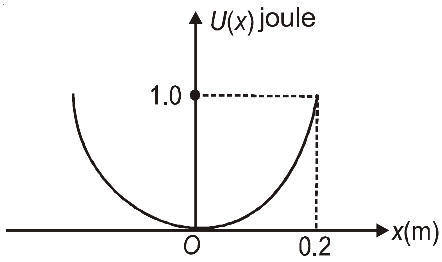
32. A particle of mass 4 kg moves simple harmonically such that its $PE(U)$ varies with position x, as shown. The period of oscillations is



(1) $\frac{2\pi}{25}$ s (2) $\frac{\pi\sqrt{2}}{5}$ s (3) $\frac{4\pi}{5}$ s (4) $\frac{2\pi\sqrt{2}}{5}$ s

Sol. Answer (4)

Mass = 4 kg



Maximum P.E. = $\frac{1}{2}kA^2$

$$1 = \frac{1}{2} \times k \times (0.2)^2$$

$$\frac{2}{0.04} = k$$

$k = 50$ N/m

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{4}{50}} = \frac{2\sqrt{2}\pi}{5}\text{s}$$

33. The kinetic energy and potential energy of a particle executing S.H.M. are equal, when displacement in terms of amplitude 'A' is

(1) $\frac{A}{2}$ (2) $\frac{A}{\sqrt{2}}$ (3) $\frac{A\sqrt{2}}{3}$ (4) $A\sqrt{2}$

Sol. Answer (2)

Total energy = $\frac{1}{2}kA^2$

When P.E. is half of total energy P.E. = K.E.

$$\Rightarrow \frac{1}{2}kx^2 = \frac{1}{2}k(A^2 - x^2)$$

$$x^2 = A^2 - x^2 \Rightarrow 2x^2 = A^2$$

$$x = \frac{A}{\sqrt{2}}$$

(Some Systems Executing, Simple Harmonic Motion)

34. Two identical pendulums oscillate with a constant phase difference $\frac{\pi}{4}$ and same amplitude. If the maximum velocity of one is v, the maximum velocity of the other will be

(1) v (2) $\sqrt{2}v$ (3) $2v$ (4) $\frac{v}{\sqrt{2}}$

Sol. Answer (1)

If the phase difference is constant, they are moving with same frequency and ω.

Since maximum velocity $A\omega = v$ (given)

Maximum velocity of other will still be v.

35. A simple pendulum suspended from the ceiling of a stationary lift has period T_0. When the lift descends at steady speed, the period is T_1, and when it descends with constant downward acceleration, the period is T_2. Which one of the following is true?

(1) $T_0 = T_1 = T_2$ (2) $T_0 = T_1 < T_2$ (3) $T_0 = T_1 > T_2$ (4) $T_0 < T_1 < T_2$

Sol. Answer (2)

Pseudo force only when there is an acceleration.

Hence $T_0 = T_1$ as there is uniform downward motion.

When it moves downward with a steady acceleration then pseudo force acts upwards, reducing net 'g'

Since $T_2 = 2\pi\sqrt{\frac{l}{g_{\text{net}}}}$

When g reduces time period T_2 increases

$$T_0 = T_1 < T_2$$

36. If a Second's pendulum is moved to a planet where acceleration due to gravity is 4 times, the length of the second's pendulum on the planet should be made

(1) 2 times (2) 4 times (3) 8 times (4) 15 times

Sol. Answer (2)

Time period of a pendulum

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ or } T_2\sqrt{\frac{l}{g}}$$

If g becomes 4 times. l must also be increased by 4 times to keep T constant.

37. A simple pendulum with a metallic bob has a time period T. The bob is now immersed in a non-viscous liquid and oscillated. If the density of the liquid is 1/4 that of metal, the time period of the same pendulum will be

(1) $\frac{T}{\sqrt{3}}$ (2) $\frac{2T}{\sqrt{3}}$ (3) $\frac{4}{3}T$ (4) $\frac{2}{3}T$

Sol. Answer (2)

Normal time period $T = 2\pi\sqrt{\frac{l}{g}}$

When immersed in a liquid. It experiences an upthrust.

$$\text{Upthrust} = \frac{\rho}{4} \times \text{volume } g$$

Upward acceleration = Upward force/mass of ball = $\frac{g}{4}$

$$T' = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$$g_{\text{eff}} = g - \frac{g}{4} = \frac{3}{4}g$$

$$T' = 2\pi = \sqrt{\frac{l}{3g} \times 4} = \frac{2T}{\sqrt{3}}$$

38. Two pendulums of length 1.21 m and 1.0 m start vibrating. At some instant, the two are in the mean position in same phase. After how many vibrations of the longer pendulum, the two will be in phase?

(1) 10 (2) 11 (3) 20 (4) 21

Sol. Answer (1)

Lengths l_1 = 1.21 m l_2 = 1 m

$$T_1 = 2\pi\sqrt{\frac{l_1}{g}} \qquad T_2 = 2\pi\sqrt{\frac{l_2}{g}}$$

$$T_1 = \frac{11}{10}T_2$$

or $10\,T_1 = 11T_2$

Hence it oscillations of longer pendulum is equal to 11 oscillation of shorter one.

Hence the will be in phase again after 10 oscillations of longer pendulum.

39. The time period of oscillations of a simple pendulum is 1 minute. If its length is increased by 44%. then its new time period of oscillation will be

(1) 96 s (2) 58 s (3) 82 s (4) 72 s

Sol. Answer (4)

Let initial length be l_1

Final length $l_2 = l_1 \times \frac{144}{100}$

$$T_1 = 2\pi\sqrt{\frac{l_1}{g}}$$

$$T_2 = 2\pi\sqrt{\frac{l_1}{g}\times\frac{144}{100}}$$

or $T_2 = 1.2\ T_1$

$T_1 = 60$ s

So $T_2 = 72$ s

40. If the length of a clock pendulum increases by 0.2% due to atmospheric temperature rise, then the loss in time of clock per day is

(1) 86.4 s (2) 43.2 s (3) 72.5 s (4) 32.5 s

Sol. Answer (1)

Time period $= 2\pi\sqrt{\frac{l}{g}}$

$$T \propto \sqrt{l}$$

$$\frac{T'}{T} \propto \sqrt{\frac{l'}{l}}$$

$$T' = T\sqrt{\frac{l + l\alpha\Delta\theta}{l}}$$

$$T' = T\left(1+\frac{1}{2}\alpha\Delta\theta\right)\ [\alpha\Delta\theta = 0.002]$$

$$\Delta T = T' - T = \frac{1}{2}T\alpha\Delta\theta = T\times 0.001$$

Time lost in time t is

$\Delta T = \frac{1}{2}$ t = 1 day = 24 × 3600 s = 86400 s

$$\Delta T = \left(\frac{\Delta T}{T}\right)\times t$$

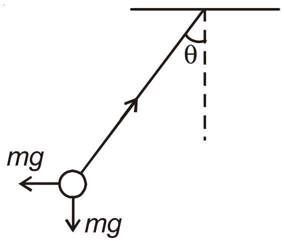
$\Delta T = 0.001 \times 86400$

$\Delta T = 86.4$ s

41. A simple pendulum is oscillating in a trolley moving on a horizontal straight road with constant acceleration a. If direction of motion of trolley is taken as positive x direction and vertical upward direction as positive y direction then the mean position of pendulum makes an angle

(1) $\tan^{-1}\left(\frac{g}{a}\right)$ with y axis in $+x$ direction
(2) $\tan^{-1}\left(\frac{a}{g}\right)$ with y axis in $-x$ direction
(3) $\tan^{-1}\left(\frac{a}{g}\right)$ with y axis in $+x$ direction
(4) $\tan^{-1}\left(\frac{g}{a}\right)$ with y axis in $-x$ direction

Sol. Answer (2)



$T\sin\theta = ma$ ... (i)

$T\cos\theta = mg$ ... (ii)

Dividing (i) and (ii)

$$T\cos\theta = \frac{a}{g}$$

$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

42. The time period of oscillations of a second's pendulum on the surface of a planet having mass and radius double those of earth is

(1) 4 s
(2) 1 s
(3) $\sqrt{2}$ s
(4) $2\sqrt{2}$ s

Sol. Answer (4)

$$g_1 = \frac{Gm}{R^2}$$

$$g_2 = \frac{G\times 2m}{4R^2} = \frac{g_1}{2}$$

$$T_1 = 2\pi\sqrt{\frac{l}{g_1}}$$

$$T_2 = 2\pi\sqrt{\frac{l}{g_2}}$$

$$T_2 = \sqrt{2}T_1$$

Since T_1 is time period of seconds pendulum $T_1 = 2$.

Hence $T_2 = 2\sqrt{2}$

43. The shape of graph between time period of a simple pendulum and its length is

(1) Straight line (2) Parabolic

(3) Hyperbolic (4) Elliptical

Sol. Answer (2)

44. A hollow metal sphere is filled with water through a small hole in it. It is hung by a long thread and is made to oscillate. Water slowly flows out of the hole at the bottom. Select the correct variation of its time period

(1) The period will go on increasing till the sphere is empty

(2) The period will go on decreasing till the sphere is empty

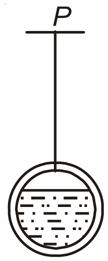
(3) The period will not be affected at all

(4) The period will increase first, then decrease to initial value till the sphere is empty

Sol. Answer (4)

As the water level goes down, the distance of C.O.M.

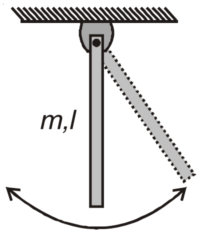
From point of oscillation P keeps increasing.



Since $T = 2\pi\sqrt{\frac{l}{g}}$

The time period of pendulum keep increasing.

45. A uniform rod of mass m and length l is suspended about its end. Time period of small angular oscillations is



(1) $2\pi\sqrt{\frac{l}{g}}$ (2) $2\pi\sqrt{\frac{2l}{g}}$ (3) $2\pi\sqrt{\frac{2l}{3g}}$ (4) $2\pi\sqrt{\frac{l}{3g}}$

Sol. Answer (3)

This is the case of a physical pendulum.

$$T = 2\pi\sqrt{\frac{I_{com}}{mg\,L_{com}}}$$

$$L_{com} = \frac{L}{2} \qquad I_{com} = \frac{mL^2}{3}$$

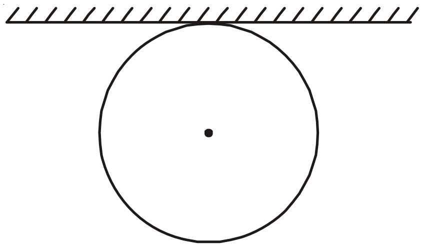
$$T = 2\pi\sqrt{\frac{2l}{3g}}$$

46. A uniform disc of mass M and radius R is suspended in vertical plane from a point on its periphery. Its time period of oscillation is

(1) $2\pi\sqrt{\frac{3R}{g}}$ (2) $2\pi\sqrt{\frac{R}{3g}}$ (3) $2\pi\sqrt{\frac{2R}{3g}}$ (4) $2\pi\sqrt{\frac{3R}{2g}}$

Sol. Answer (4)

It is the case of a physical pendulum.



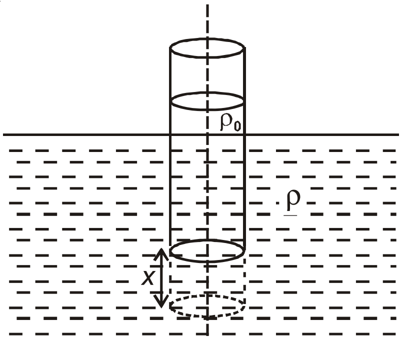
$$T = 2\pi\sqrt{\frac{I_{\text{c.o.m.}}}{mg\,L_{\text{com}}}}$$

$$I_{\text{com}} = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

$$L_{\text{com}} = R$$

$$T = 2\pi\sqrt{\frac{3R}{2g}}$$

47. A solid cylinder of density ρ_0 , cross-section area A and length l floats in a liquid of density ρ (> ρ_0) with its axis vertical, as shown. If it is slightly displaced downward and released, the time period will be

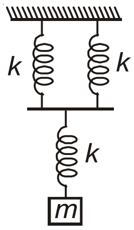


(1) $2\pi\sqrt{\frac{l}{g}}$ (2) $2\pi\sqrt{\frac{\rho_0 l}{\rho g}}$ (3) $2\pi\sqrt{\frac{\rho l}{\rho_0 g}}$ (4) $2\pi\sqrt{\frac{l}{2g}}$

Sol. Answer (2)

The time period of a floating uniform cylinder is simply given as $R = l$.

48. A block of mass m hangs from three springs having same spring constant k. If the mass is slightly displaced downwards, the time period of oscillation will be



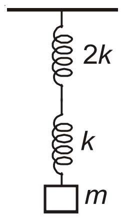
(1) $2\pi\sqrt{\frac{m}{3k}}$ (2) $2\pi\sqrt{\frac{3m}{2k}}$ (3) $2\pi\sqrt{\frac{2m}{3k}}$ (4) $2\pi\sqrt{\frac{3k}{m}}$

Sol. Answer (2)

The first two springs are in parallel.

So, k_{eq} of 1st 2 will be = $2k$

Then it becomes



The springs $2k$ and k are in series.

So, $k_{eq} = \dfrac{2k \times k}{2k + k}$

$= \dfrac{2k \times k}{3k} = \dfrac{2}{3}k$

$T = 2\pi\sqrt{\dfrac{m}{k_{eq}}}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{3m}{2k}}$

49. A clock S is based on oscillations of a spring and a clock P is based on pendulum motion. Both clocks run at the same rate on earth. On a planet having same density as earth but twice the radius then

(1) S will run faster than P

(2) P will run faster than S

(3) Both run at same rate

(4) Both run at same rate but different than earth

Sol. Answer (2)

Time period of spring = $2\pi\sqrt{\dfrac{k}{m}}$

Time period of pendulum = $2\pi\sqrt{\dfrac{l}{g}}$

Time period of spring will not be affected by gravitational acceleration.

Let mass of earth be m

Mass of new planet = $\rho \times \dfrac{4}{3}\pi(2R)^3$ = 8 m

$g_2 = \dfrac{GM_2}{(R_2)^2} = \dfrac{G \times 8M}{(2R)^2}$ = 2 g

$T_2 = 2\pi\sqrt{\dfrac{l}{2g}}$

$T_2 = \dfrac{T}{\sqrt{2}}$

Hence P will move faster.

50. A 100 g mass stretches a particular spring by 9.8 cm, when suspended vertically from it. How large a mass must be attached to the spring if the period of vibration is to be 6.28 s?

(1) 1000 g

(2) 10^5 g

(3) 10^7 g

(4) 10^4 g

Sol. Answer (4)

At point of equilibrium $kx = mg$

$k \times 9.8 \times 10^{-2} = 100 \times 10^{-3} \times 9.8$

$k = 100 \times 10^{-1}$

$k = 10$ N/m

Period of vibration needed = 6.28 s

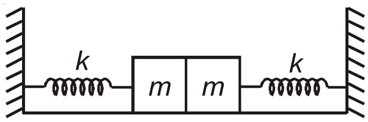
$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$6.28 = 2 \times 3.14\sqrt{\frac{m}{10}}$$

$$1 = \frac{m}{10}$$

$m = 10$ kg or 10^4 g

51. An assembly of identical spring-mass systems is placed on a smooth horizontal surface as shown. At this instant, the springs are relaxed. The left mass is displaced to the left and the right mass is displaced to the right by same distance and released. The resulting collision is elastic. The time period of the oscillations of system is



(1) $2\pi\sqrt{\frac{2m}{k}}$ (2) $2\pi\sqrt{\frac{m}{2k}}$ (3) $\pi\sqrt{\frac{m}{k}}$ (4) $2\pi\sqrt{\frac{m}{k}}$

Sol. Answer (3)

If there was no collision each spring will oscillate with period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Because of collisions the springs are only compressed but cannot extend beyond their natural length. Hence the perform only half oscillation.

Hence $T = 2\pi\sqrt{\frac{m}{k}} \div 2$

or $T = \pi\sqrt{\frac{m}{k}}$

52. A spring block system in horizontal oscillation has a time-period T. Now the spring is cut into four equal parts and the block is re-connected with one of the parts. The new time period of vertical oscillation will be

(1) $\frac{T}{\sqrt{2}}$ (2) $2T$ (3) $\frac{T}{2}$ (4) $\frac{T}{2\sqrt{2}}$

Sol. Answer (3)

When spring is cut into 4 parts. The spring constant of each part will become 4k.

$$T_2 = 2\pi\sqrt{\frac{m}{4k}}$$

$$T_2 = \frac{T}{2}$$

53. A block of mass m is suspended separately by two different springs have time period t_1 and t_2. If same mass is connected to parallel combination of both springs, then its time period is given by

(1) $\frac{t_1 t_2}{t_1 + t_2}$ (2) $\frac{t_1 t_2}{\sqrt{t_1^2 + t_2^2}}$ (3) $\sqrt{\frac{t_1 t_2}{t_1 + t_2}}$ (4) $t_1 + t_2$

Sol. Answer (2)

$$t_1 = 2\pi\sqrt{\frac{m}{k_1}} \quad , \quad t_2 = 2\pi\sqrt{\frac{m}{k_a}} \quad , \quad t_{eq} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

Let $2\pi\sqrt{m}$ be any constant c.

$$t_1 = \frac{c}{\sqrt{k_1}} \quad , \quad t_2 = \frac{c}{\sqrt{k_2}}$$

$$k_1 = \frac{c^2}{t_1^2} \quad , \quad k_2 = \frac{c}{t_2^2}$$

$$t_{eq} = 2\pi\sqrt{\frac{m}{c^2/t_1^2 + c^2/t_2^2}}$$

$$t_{eq} = 2\pi\sqrt{\frac{m\ t_1^2 + t_2^2}{c^2 t_2^2 + c^2\ t_1^2}}$$

$$t_{eq} = \frac{t_1 t_2}{\sqrt{t_1^2 + t_2^2}}$$

(Damped Simple Harmonic Motion, Forced Oscillations and Resonance)

54. In damped oscillations, damping force is directly proportional to speed of oscillator. If amplitude becomes half of its maximum value in 1 s, then after 2 s amplitude will be (Initial amplitude = A_0)

(1) $\frac{1}{4}A_0$ (2) $\frac{1}{2}A_0$ (3) A_0 (4) $\frac{\sqrt{3}A_0}{2}$

Sol. Answer (1)

$$A = A_0\ e^{-bt}$$

Amplitude becomes half hence

$$\frac{A_0}{2} A_0\ e^{-bt}\ [t = 1]$$

$\therefore \quad e^{-b} = \frac{1}{2}$

$\therefore$ In two seconds

$$A = A_0\left(\frac{1}{2}\right)^2$$

$$A = \frac{A_0}{4}$$

55. In forced oscillations, a particle oscillates simple harmonically with a frequency equal to

(1) Frequency of driving force

(2) Natural frequency of body

(3) Difference of frequency of driving force and natural frequency

(4) Mean of frequency of driving force and natural frequency

Sol. Answer (1)

In forced oscillations a particle oscillator simple harmonically with a frequency equal to driving frequency.

56. Resonance is a special case of

(1) Forced oscillations (2) Damped oscillations

(3) Undamped oscillations (4) Coupled oscillations

Sol. Answer (1)

Resonance is a special case of force oscillation due to which oscillation to place with greater amplitude.

57. The S.H.M. of a particle is given by the equations = 2 sin ωt + 4 cos ωt. Its amplitude of oscillation is

(1) 4 units (2) 2 units

(3) 6 units (4) $2\sqrt{5}$ units

Sol. Answer (4)

$x = 2 \sin \omega t + 4 \cos \omega t$

It can also be written as

$$x = \sqrt{2^2+4^2}\left(\frac{2}{\sqrt{2^2+4^2}}\sin\omega t \frac{4}{\sqrt{2^2+4^2}}\cos\omega t\right)$$

$$x = \sqrt{20}\sin(\omega t+\phi)$$

$\sqrt{20}$ = Amplitude

or Amplitude = $2\sqrt{5}$

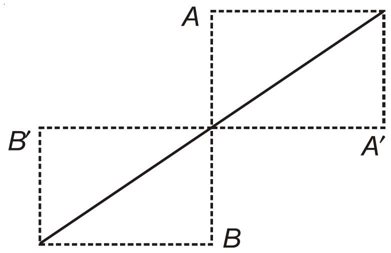
58. If two mutually perpendicular simple harmonic motion of same amplitude, frequency and having zero phase difference superimpose on a particle, then its resultant path will be

(1) A straight line (2) A circle

(3) An ellipse (4) A hyperbola

Sol. Answer (1)

If $A - B$ is the path followed by one particle superimpose, then the result will be as shown and the particle will oscillate diagonally.



59. Which of the following represents a S.H.M.?

(1) $\sin \omega t - \cos \omega t$ (2) $\sin \omega t + \cos \omega t$

(3) $\sin \omega t + 2 \cos \omega t$ (4) All of these

Sol. Answer (4)

All of them are superposition of two S.H.M. in the same phase and hence they all represent S.H.M's.

SECTION - B

Objective Type Questions

(Periodic and Oscillatory Motions)

1. The circular motion of a particle with constant speed is

(1) Periodic but not simple harmonic (2) Simple harmonic but not periodic

(3) Period and simple harmonic (4) Neither periodic nor simple harmonic

Sol. Answer (1)

The motion repeats itself after same intervals hence it is periodic. Put since acceleration is not proportional to $-x$, the motion is periodic but not simple harmonic.

(Period and Frequency)

2. A 1.00×10^{-20} kg particle is vibrating under simple harmonic motion with a period of 1.00×10^{-5} s and with a maximum speed of 1.00×10^{3} m/s. The maximum displacement of particle from mean position is

(1) 1.59 mm (2) 1.00 m (3) 10 m (4) 3.18 mm

Sol. Answer (1)

$m = 1 \times 10^{-20}$ kg $\qquad T = 1 \times 10^{-5}$ kg

Maximum speed $= A\omega = 1 \times 10^{3}$ m/s ... (i)

$$\omega = \frac{2\pi}{T} = 2\pi \times 10^{5} \text{ rad/s}$$

Putting value of ω in (i)

$$A \times 2\pi \times 10^{5} = 1 \times 10^{3}$$

$$A = \frac{1}{2\pi \times 10^{2}} = 1.59 \text{ mm}$$

3. The equation of an S.H.M. with amplitude A and angular frequency ω in which all the distances are measured from one extreme position and time is taken to be zero at the other extreme position is

(1) $x = A \sin \omega t$
(2) $x = A (\cos \omega t + \sin \omega t)$
(3) $x = A - A \cos \omega t$
(4) $x = A + A \cos \omega t$

Sol. Answer (4)

At $t = 0$ the distance from 1 extreme is $2A$

At $\omega t = \pi$ $\qquad x = 0$

Hence by resulting values we can get equation for S.H.M.. from S.H.M..

4. A body oscillates with S.H.M. according to the equation $x = (5.0 \text{ m}) \cos [(2\pi \text{ rad s}^{-1})t + \pi/4]$

At $t = 1.5$ s, its acceleration is

(1) – 139.56 m/s^2
(2) 139.56 m/s^2
(3) 69.78 m/s^2
(4) – 69.78 m/s^2

Sol. Answer (2)

$$x = 5 \cos (2\pi t + \pi/4)$$

$$t = \frac{3}{2} \text{ s}$$

$$x = 5 \cos (3\pi + \pi/4)$$

$$x = 5 \cos \left(\frac{13\pi}{4}\right)$$

$$x = -5 \cos \frac{\pi}{4} = -\frac{5}{\sqrt{2}}$$

Acceleration

$$a = -\omega^2 x$$

$$a = -4\pi^2 x - \frac{5}{\sqrt{2}}$$

$$a \simeq 139.56 \text{ m/s}^2$$

5. The time period of a particle executing S.H.M. is 8 s. At $t = 0$ it is at the mean position. The ratio of distance covered by the particle in 1st second to the 2nd second is

(1) $(\sqrt{2}-1)$
(2) $\sqrt{2}$
(3) $(\sqrt{2}+1)$
(4) $\frac{1}{\sqrt{2}}$

Sol. Answer (3)

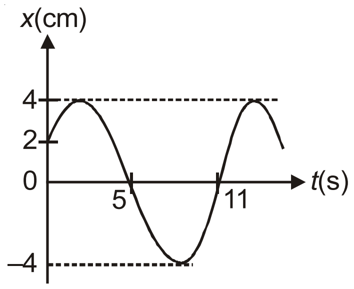
$$T = 8 \text{ s} \qquad \omega = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$x_1 = A \sin \frac{\pi}{4} = \frac{A}{\sqrt{2}}$$

$$x_2 = A \sin \frac{\pi}{4} \times 2 - A \sin \frac{\pi}{4} = A - \frac{A}{\sqrt{2}} = \frac{A}{\sqrt{2}}(\sqrt{2}-1)$$

$$\frac{x_1}{x_2} = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \sqrt{2}+1$$

6. Figure shows the position-time graph of an object in S.H.M. The correct equation representing this motion is



(1) $2\sin\left(\frac{2\pi}{5}t+\frac{\pi}{6}\right)$ (2) $4\sin\left(\frac{\pi}{5}t+\frac{\pi}{6}\right)$ (3) $4\sin\left(\frac{\pi}{6}t+\frac{\pi}{3}\right)$ (4) $4\sin\left(\frac{\pi}{6}t+\frac{\pi}{6}\right)$

Sol. Answer (4)

Time period is 12 s from diagram.

$$\omega = \frac{2\pi}{12}=\frac{\pi}{6}$$

Amplitude A = 4

Initial phase is determined by putting known values in the equation.

$$2 = 4\sin\left(\frac{\pi}{6}t+\phi\right)$$

$$\sin^{-1}\frac{1}{2} = \phi \ [t = 0]$$

$$\frac{\pi}{6}=\phi$$

Hence equation is $x = \left(\frac{\pi}{6}t+\frac{\pi}{6}\right)$

7. A particle executes S.H.M. and its position varies with time as $x = A\sin\omega t$. Its average speed during its motion from mean position to mid-point of mean and extreme position is

(1) Zero (2) $\frac{3A\omega}{\pi}$ (3) $\frac{A\omega}{2\pi}$ (4) $\frac{2A\omega}{\pi}$

Sol. Answer (2)

Phase at mean position = 0

Phase at mid point

$$\frac{A}{2} = A\sin\phi$$

$$\phi = \frac{\pi}{6}$$

Time it takes to travel a phase difference of ϕ

$$t = \frac{2\pi}{\omega}\times\frac{\phi}{2\pi}$$

or $t = \dfrac{\phi}{\omega}$

or $t = \dfrac{\pi}{6\omega}$

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Time taken}}$$

$$= \frac{A/2}{\pi/6\omega} = \frac{3A\omega}{\pi}$$

8. A particle is executing S.H.M. and its velocity v is related to its position (x) as $v^2 + ax^2 = b$, where a and b are positive constants. The frequency of oscillation of particle is

(1) $\dfrac{1}{2\pi}\sqrt{\dfrac{b}{a}}$ (2) $\dfrac{\sqrt{a}}{2\pi}$ (3) $\dfrac{\sqrt{b}}{2\pi}$ (4) $\dfrac{1}{2\pi}\sqrt{\dfrac{a}{b}}$

Sol. Answer (2)

$v^2 + ax^2 = b$

$v^2 = b - ax^2$

$$v^2 = a\left(\frac{b}{a} - x^2\right)$$

Comparing it to equation

$v^2 = \omega^2 (A^2 - x^2)$

$\omega = \sqrt{a}$

$$f = \frac{\omega}{2\pi} = \frac{\sqrt{a}}{2\pi}$$

(Simple Harmonic Motion)

9. A particle executes S.H.M. according to equation $x = 10\ (\text{cm})\cos\left[2\pi t + \dfrac{\pi}{2}\right]$, where t is in second. The magnitude of the velocity of the particle at $t = \dfrac{1}{6}$ s will be

(1) 24.7 cm/s (2) 20.5 cm/s (3) 28.3 cm/s (4) 31.4 cm/s

Sol. Answer (4)

$$x = 10\cos\left[2\pi t + \frac{\pi}{2}\right]$$

At $t = \frac{1}{6}$ s

$$x = 10\cos\left[\frac{\pi}{2} + \frac{\pi}{3}\right]$$

$$x = -10\sin\frac{\pi}{3}$$

$$x = -5\sqrt{3}$$

$$v = \omega\sqrt{A^2 - x^2}$$

$$v = 2\pi\sqrt{100 - 75}$$

$$v = 10\pi$$

or $v = 31.4$ cm/s

(Simple Harmonic Motion and Uniform Circular Motion)

10. Two particle executing S.H.M. of same amplitude of 20 cm with same period along the same line about same equilibrium position. The maximum distance between the two is 20 cm. Their phase difference in radian is equal to

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$ (3) $\frac{2\pi}{3}$ (4) $\frac{4\pi}{5}$

Sol. Answer (1)

$$x_1 = A\sin(\omega t + \phi_1)$$

$$x_2 = A\sin(\omega t + \phi_2)$$

$$x_1 - x_2 = A\sin(\omega t + \phi_1) - A\sin(\omega t + \phi_2)$$

$$20 = 2 \times 20\sin\left(\frac{\phi_1 - \phi_2}{2}\right)\cdot\cos\left[\omega t + \left(\frac{\phi_1 + \phi_2}{2}\right)\right]$$

$$\frac{1}{2} = \sin\left(\frac{\phi_1 - \phi_2}{2}\right)\cdot\cos\left(\omega t + \left(\frac{\phi_1 + \phi_2}{2}\right)\right) \text{ for maximum value. } \Rightarrow \frac{\phi_1 - \phi_2}{2} = \frac{\pi}{6} \Rightarrow \phi_1 - \phi_2 = \frac{\pi}{3}$$

11. A particle execute S.H.M. along a straight line. The amplitude of oscillation is 2 cm. When displacement of particle from the mean position is 1 cm, the magnitude of its acceleration is equal to magnitude of its velocity. The time period of oscillation is

(1) $\frac{2\pi}{\sqrt{2}}$ (2) $\frac{\sqrt{2}}{2\pi}$ (3) $\frac{2\pi}{\sqrt{3}}$ (4) $\frac{\sqrt{3}}{2\pi}$

Sol. Answer (3)

$$A = 2 \text{ cm} = 2 \times 10^{-2}$$

$$a = -\omega^2 x$$

and $v = \omega\sqrt{A^2 - x^2}$

$$\omega^2 \times 1 \times 10^{-2} = \omega\sqrt{(4-1)10^{-4}}$$

$$\omega \times 1 \times 10^{-2} = \sqrt{3} \times 10^{-2}$$

$$\omega = \sqrt{3}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}$$

12. When a mass m is attached to a spring it oscillates with period 4 s. When an additional mass of 2 kg is attached to a spring, time period increases by 1 s. The value of m is

(1) 3.5 kg (2) 8.2 kg (3) 4.7 kg (4) 2.6 kg

Sol. Answer (1)

$$\omega_1 = \sqrt{\frac{k}{m}}$$

$$\omega_2 = \sqrt{\frac{k}{m+2}}$$

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{m+2}{m}}$$

Since $\omega = \frac{2\pi}{T}$

$$\frac{T_2}{T_1} = \sqrt{\frac{m+2}{m}}$$

$$\left(\frac{5}{4}\right)^2 = \frac{m+2}{m}$$

$$\frac{25}{16} = \frac{m+2}{m}$$

$$25\,m = 16\,m + 32$$

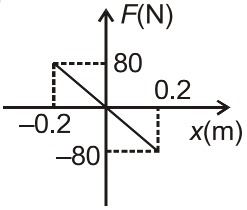
$$9\,m = 32$$

$$m = 3\frac{5}{9} \text{ kg}$$

$$m \simeq 3.5 \text{ kg}$$

(Force Law for Simple Harmonic Motion)

13. A body of mass 0.01 kg executes simple harmonic motion about $x = 0$ under the influence of a force as shown in figure. The time period of S.H.M. is



(1) 1.05 s (2) 0.52 s (3) 0.25 s (4) 0.03 s

Sol. Answer (4)

Maximum restoring force on particle

$F = 80$ N

$x = -0.2$

Since $F = -kx$

$80 = k \times 0.2$

$400 = k$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.01}{400}} \text{ kg}$$

$$= 2\pi\sqrt{\frac{1}{40000}} = 2\pi\sqrt{\frac{1}{2\times10^2}}$$

$$= \frac{1}{2\times10^2} \approx 0.03 \text{ s}$$

14. A particle of mass m in a unidirectional potential field have potential energy $U(x) = \alpha + 2\beta x^2$, where α and β are positive constants. Find its time period of oscillation.

(1) $2\pi\sqrt{\frac{2\beta}{m}}$ (2) $2\pi\sqrt{\frac{m}{2\beta}}$ (3) $\pi\sqrt{\frac{m}{\beta}}$ (4) $\pi\sqrt{\frac{\beta}{m}}$

Sol. Answer (3)

$U(x) = \alpha + 2\beta x^2$

$$F = -\frac{dU(x)}{dx}$$

$F = -4\beta x$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{4\beta}} \qquad \text{cos } [k = \beta]$$

$$T = \pi\sqrt{\frac{m}{\beta}}$$

15. A loaded vertical spring executes S.H.M. with a time period of 4 s. The difference between the kinetic energy and potential energy of this system varies with a period of

(1) 2 s (2) 1 s (3) 8 s (4) 4 s

Sol. Answer (1)

$T = 4$ s $\omega = \frac{2\pi}{T} = \frac{\pi}{2}$ rad/s

$x = A \sin \omega t$ $v = A\omega \cos \omega t$

$$\text{K.E.} = \frac{1}{2}mv^2 \qquad \frac{1}{2}mA^2\omega^2 \cos^2\omega t \qquad \text{... (i)}$$

$$\text{P.E.} = \frac{1}{2}kx^2 \qquad \frac{1}{2}kA^2 \sin^2\omega t \qquad \text{... (ii)}$$

$$\omega = \sqrt{\frac{k}{m}}$$

or $k = m\omega^2$ putting this value in (ii).

$$\text{K.E.} - \text{P.E.} = \frac{1}{2}m\omega^2A^2(\cos^2\omega t - \sin^2\omega t)$$

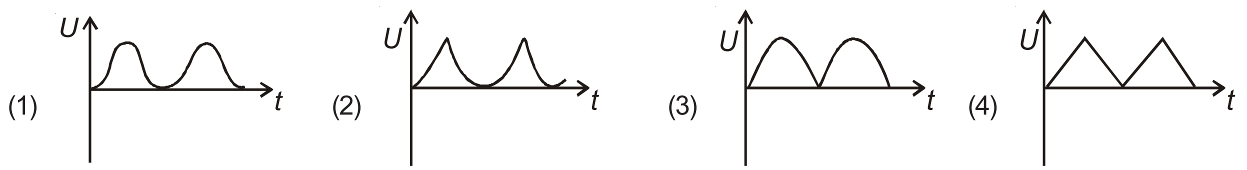
$$\text{K.E.} - \text{P.E.} = E_{\text{max.}}(\cos 2\omega t)$$

Hence time period of difference of K.E. and P.E. is

$$T = \frac{2\pi}{2\omega}$$

or $T = 2$ s

16. As a body performs S.H.M., its potential energy U varies with time t as indicated in



Sol. Answer (1)

$$\text{Potential energy} = \frac{1}{2}kx^2 = \frac{1}{2}A^2\omega^2\sin^2(\omega t + \phi)$$

The graph for $\sin^2(\omega t + \phi)$ is given by (1).

17. A particle is performing S.H.M. with energy of vibration 90 J and amplitude 6 cm. When the particle reaches at distance 4 cm from mean position, it is stopped for a moment and then released. The new energy of vibration will be

(1) 40 J (2) 50 J (3) 90 J (4) 60 J

Sol. Answer (1)

Energy = 90 J Amplitude = 6 cm

Maximum energy = $\frac{1}{2}mA^2\omega^2 = 90$

or $m\omega^2 = \frac{180}{36\times10^{-4}}$

$\therefore$ $m\omega^2 = 30 \times 10^2$

When particle is stopped the point where it is stopped is the new amplitude but angular velocity will remain same.

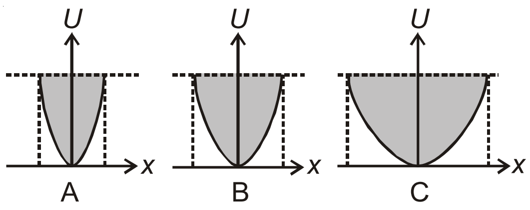
$E = \frac{1}{2}mA_2^2\omega^2$

or $E = 3000\ A_2^2$

$A_2 = 4 \times 10^{-2}$

$A_2 = 3000$

18. The variations of potential energy (U) with position x for three simple harmonic oscillators A, B and C are shown in figure. The oscillators have same mass. The time period of oscillation is greatest for



(1) A (2) B (3) C (4) Same for all

Sol. Answer (3)

$U = \frac{1}{2}kx^2$

$x^2 = \frac{2U}{k}$

or $x \propto \frac{1}{k}$ (Since U is constant)

Also $T = 2\pi\sqrt{\frac{m}{k}}$

or $T \propto \frac{1}{\sqrt{k}}$

Therefore $x \propto T$

Hence the oscillation with maximum x will have the maximum time period.

19. If the particle repeats its motion after a fixed time interval of 8 s then after how much time its maximum value of PE will be attained after attaining its minimum value?

(1) 2 s (2) 4 s (3) 8 s (4) 1 s

Sol. Answer (1)

$T = 8$ s

Maximum value of potential energy is reached two times per oscillation which is $\frac{T}{4}$ time away from mean position which has minimum value at position.

20. A particle is executing S.H.M. with total mechanical energy 90 J and amplitude 6 cm. If its energy is somehow decreased to 40 J then its amplitude will become

(1) 2 cm (2) 4 cm (3) $\frac{8}{3}$ cm (4) $\frac{4}{3}$ cm

Sol. Answer (2)

$$\frac{1}{2}mA^2\omega^2 = 90 \text{ J}$$

m and ω remaining same energy is reduced to 40 J.

$$\frac{A_1^2}{A_2^2} = \frac{9}{4}$$

or $\frac{A_1}{A_2} = \frac{3}{2}$

or $A_2 = 4$ cm

21. A linear harmonic oscillator of force constant 6×10^5 N/m and amplitude 4 cm, has a total energy 600 J. Select the correct statement.

(1) Maximum potential energy is 600 J (2) Maximum kinetic energy is 480 J

(3) Minimum potential energy is 120 J (4) All of these

Sol. Answer (4)

$k = 6 \times 10^5$ N/m

Amplitude = 4 cm = 4×10^{-2} m

$$\frac{1}{2}kx^2 = \frac{1}{2} \times 6 \times 10^5 \times (4 \times 10^{-2})^2$$

or $E = 480$ J

Since energy of S.H.M. is 480 J and there is 600 J provided to the oscillation there must be 600 – 120 = 480 J stored as energy.

22. A simple pendulum of mass m executes S.H.M. with total energy E. If at an instant it is at one of extreme positions, then its linear momentum after a phase shift of $\frac{\pi}{3}$ rad will be

(1) $\sqrt{2mE}$ (2) $\sqrt{\frac{3mE}{2}}$ (3) $2\sqrt{mE}$ (4) $\sqrt{\frac{2mE}{3}}$

Sol. Answer (2)

Energy = E_0

After a phase shift of $\frac{\pi}{3}$

$$E = E_0 \cos^2 \frac{\pi}{3}$$

$$\frac{P^2}{2m} = \frac{E_0 3}{4}$$

$$P = \sqrt{\frac{3mE}{2}}$$

23. A flat horizontal board moves up and down under S.H.M. vertically with amplitude A. The shortest permissible time period of the vibration such that an object placed on the board may not lose contact with the board is

(1) $2\pi\sqrt{\frac{g}{A}}$ (2) $2\pi\sqrt{\frac{A}{g}}$ (3) $2\pi\sqrt{\frac{2A}{g}}$ (4) $\frac{\pi}{2}\sqrt{\frac{A}{g}}$

Sol. Answer (2)

Maximum acceleration of the system (a_{max}) = $-\omega^2 A$

For a block to escape the board the acceleration must be equal to 9 at the top-most point.

$$g = \omega^2 A$$

$$\omega = \sqrt{\frac{g}{A}}$$

Time period = $\frac{2\pi}{\omega} = \sqrt{\frac{A}{g}}$

(Some Systems Executing Simple Harmonic Motion)

24. A second's pendulum is mounted in a rocket. Its period of oscillation will decrease when rocket is

(1) Moving down with uniform acceleration

(2) Moving around the earth in geostationary orbit

(3) Moving up with uniform velocity

(4) Moving up with uniform acceleration

Sol. Answer (4)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

When the rocket is moving up with acceleration Pseudo force is acting downwards increasing effective gravitational acceleration (g).

25. The curve between square of frequency of oscillation and length of the simple pendulum is

(1) Straight line (2) Parabola (3) Ellipse (4) Hyperbola

Sol. Answer (4)

$$l = kT^2$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

or $f^2 = \dfrac{k}{l}$

This is the equation of hyperbola between f^2 and l.

26. A small iron ball of mass m is suspended with the help of a massless rod of length L and is free to oscillate in vertical plane. Its time period of oscillation is

(1) $2\pi\sqrt{\dfrac{mL}{2g}}$ (2) $2\pi\sqrt{\dfrac{mL}{g}}$ (3) $2\pi\sqrt{\dfrac{L}{g}}$ (4) $2\pi\sqrt{\dfrac{m}{gL}}$

Sol. Answer (3)

Since this is the case of a massless rod, the condition is same as that of a pendulum.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

27. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium it undergoes oscillations with a time period T, then

(1) $T \propto \dfrac{1}{\sqrt{m}}$ (2) $T \propto \sqrt{\rho}$ (3) $T \propto \dfrac{1}{\sqrt{A}}$ (4) $T \propto \dfrac{1}{\rho}$

Sol. Answer (3)

Time period of a floating cylinder is given by

$$T = 2\pi\sqrt{\frac{L\rho_s}{\rho_L g}}$$

When ρ_s is density of limit

$$T = 2\pi\sqrt{\frac{L^3\rho_s}{L^2\rho_L g}} = 2\pi\sqrt{\frac{M_s}{A\rho_L g}}$$

$$T \propto \frac{1}{\sqrt{A}}$$

28. A body of mass 5 kg hangs from a spring and oscillates with a time period of 2π second. If the body is removed, the length of the spring will decrease by

(1) g/k metre (2) k/g metre (3) 2π metre (4) g metre

Sol. Answer (4)

$T = 2\pi$ sec Mass = 5 kg

Spring constant = k

$\omega = 1$ rad/sec

Now $\omega = \sqrt{\dfrac{k}{m}}$

So, $k = 5$

Equilibrium position when it is oscillating is at

$k\Delta x = mg$

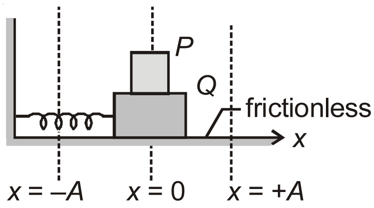
or $\Delta x = \dfrac{mg}{k}$

When the mass is removed the spring will return to its natural length, which is Δx upwards.

Since $m = 5$ and $k = 5$

$\Delta x = g$ metre

29. In the figure shown, there is friction between the blocks P and Q but the contact between the block Q and lower surface is frictionless. Initially the block Q with block P over it lies at $x = 0$, with spring at its natural length. The block Q is pulled to right and then released. As the spring - blocks system undergoes S.H.M. with amplitude A, the block P tends to slip over Q. P is more likely to slip at

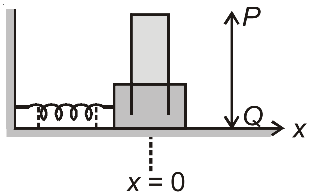


(1) $x = 0$ (2) $x = +A$ (3) $x = +\dfrac{A}{2}$ (4) $x = +\dfrac{A}{\sqrt{2}}$

Sol. Answer (2)

The block is most likely to slip when there is maximum acceleration.

This happens when the blocks are at the extremities where the displacement is either $+A$ or $-A$.



30. A simple pendulum with iron bob has a time period T. The bob is now immersed in a non-viscous liquid and oscillated. If the density of liquid is $\frac{1}{12}$th that of iron, then new time period will be

(1) $T\sqrt{\frac{8}{7}}$ (2) $T\sqrt{\frac{12}{13}}$ (3) $T\sqrt{\frac{12}{11}}$ (4) $T\sqrt{\frac{6}{5}}$

Sol. Answer (3)

When bob is inserted, in liquid effective of is reduced because of the force of upthrust.

ρ_S is density of solid.

ρ_L is density of liquid.

V is volume of solid

and $T = 2\pi\sqrt{\frac{l}{g}}$

$$T_{\text{new}} = 2\pi\sqrt{\frac{l}{g_{\text{new}}}}$$

$$T_{\text{new}} = 2\pi\sqrt{\frac{l}{g - \frac{\rho_L}{\rho_S}g}} = 2\pi\sqrt{\frac{l \times 12}{g \times 11}}$$

or $T_{\text{new}} = \sqrt{\frac{12}{11}}T$

SECTION - C

Previous Years Questions

1. A pendulum is hung from the roof of a sufficiently high building and is moving freely to and fro like a simple harmonic oscillator. The acceleration of the bob of the pendulum is 20 m/s^2 at a distance of 5 m from the mean position. The time period of oscillation is **[NEET-2018**

(1) 2π s (2) π s (3) 1 s (4) 2 s

Sol. Answer (2)

$|a| = \omega^2 y$

$\Rightarrow 20 = \omega^2(5)$

$\Rightarrow \omega = 2$ rad/s

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ s

2. A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is **[NEET-2017]**

(1) $\frac{\sqrt{5}}{\pi}$ (2) $\frac{\sqrt{5}}{2\pi}$ (3) $\frac{4\pi}{\sqrt{5}}$ (4) $\frac{2\pi}{\sqrt{3}}$

Sol. Answer (3)

$$v = \omega\sqrt{A^2 - x^2}$$

$$a = x\omega^2$$

$$v = a$$

$$\omega\sqrt{A^2 - x^2} = x\omega^2$$

$$\sqrt{(3)^2 - (2)^2} = 2\left(\frac{2\pi}{T}\right)$$

$$\sqrt{5} = \frac{4\pi}{T}$$

$$T = \frac{4\pi}{\sqrt{5}}$$

3. A body of mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3 s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5 s. The value of m in kg is

[NEET(Phase-2)-2016]

(1) $\frac{3}{4}$ (2) $\frac{4}{3}$ (3) $\frac{16}{9}$ (4) $\frac{9}{16}$

Sol. Answer (4)

$$T_1 = 3 = 2\pi\sqrt{\frac{m}{K}}$$

Then, $T_2 = 5 = 2\pi\sqrt{\frac{m+1}{K}}$

Dividing, $\frac{3}{5} = \sqrt{\frac{m}{m+1}}$

$$\frac{9}{25} = \frac{m}{m+1}$$

$$9m + 9 = 25m$$

$$16m = 9$$

$$m = \frac{9}{16}$$

4. A particle is executing a simple harmonic motion. Its maximum acceleration is α and maximum velocity is β. Then, its time period of vibration will be **[Re-AIPMT - 2015]**

(1) $\frac{2\pi\beta}{\alpha}$ (2) $\frac{\beta^2}{\alpha^2}$ (3) $\frac{\alpha}{\beta}$ (4) $\frac{\beta^2}{\alpha}$

Sol. Answer (1)

We know acceleration

$$a = \omega^2 X$$

So, $\alpha = \omega^2 A$ ...(i)

And, $v = \omega X$

So, $\beta = \omega A$ ...(ii)

(i) ÷ (ii)

$$\frac{\alpha}{\beta} = \omega = \frac{2\pi}{T}$$

$$\boxed{T = \frac{2\pi\beta}{\alpha}}$$

5. A particle is executing SHM along a straight line. Its velocities at distances x_1 and x_2 from the mean position are V_1 and V_2 respectively. Its time period is **[AIPMT - 2015]**

(1) $2\pi\sqrt{\frac{V_1^2 - V_2^2}{x_1^2 - x_2^2}}$ (2) $2\pi\sqrt{\frac{x_1^2 + x_2^2}{V_1^2 + V_2^2}}$ (3) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$ (4) $2\pi\sqrt{\frac{V_1^2 + V_2^2}{x_1^2 + x_2^2}}$

Sol. Answer (3)

We know,

$$V = \omega\sqrt{A^2 - y^2}$$

So, $V^2 = \omega^2 A^2 - \omega^2 y^2$

If position is x_1 and velocity is V_1, then

$$V_1^2 = \omega^2 A^2 - \omega^2 x_1^2 \quad \text{...(i)}$$

For position x_2 and velocity is V_2

$$V_2^2 = \omega^2 A^2 - \omega^2 x_2^2 \quad \text{...(ii)}$$

(i) – (ii)

So, $V_1^2 - V_2^2 = \omega^2\left[x_2^2 - x_1^2\right]$

$$\omega^2 = \frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}$$

$$\omega = \sqrt{\frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}} = \frac{2\pi}{T}$$

$$\boxed{T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}}$$

6. When two displacements represented by $y_1 = a\sin(\omega t)$ and $y_2 = b\cos(\omega t)$ are superimposed the motion is **[AIPMT - 2015]**

(1) Simple harmonic with amplitude $\frac{(a+b)}{2}$

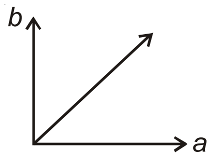
(2) Not a simple harmonic

(3) Simple harmonic with amplitude $\frac{a}{b}$

(4) Simple harmonic with amplitude $\sqrt{a^2 + b^2}$

Sol. Answer (4)

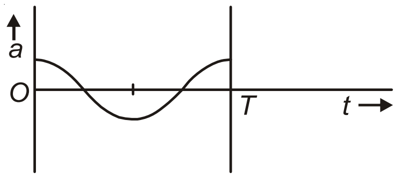
Angle between a and b can be given 90º, so their resultant is

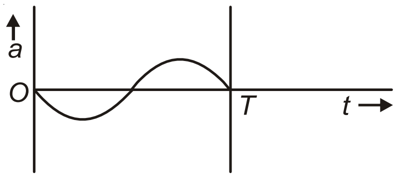


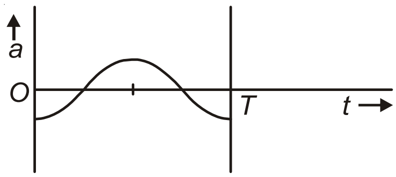
$$R = \sqrt{a^2 + b^2}$$

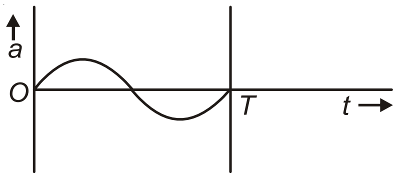
Frequency of y_1 and y_2 are similar so motion of particle is S.H.M.

7. The oscillation of a body on a smooth horizontal surface is represented by the equation, $x = A\cos(\omega t)$, where x is displacement at time t and ω is frequency of oscillation. Which one of the following graphs shows correctly the variation a with t? (Here a = acceleration at time t and T = time period) **[AIPMT - 2014]**

(1) 

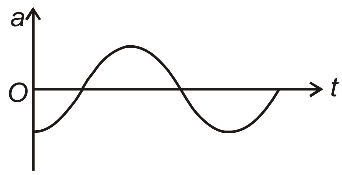
(2) 

(3) 

(4) 

Sol. Answer (3)

$X = A\cos\omega t$



$$v = \frac{dx}{dt} = -A\omega \sin \omega t$$

$$a = \frac{d^2x}{dt^2} = -A\omega^2 \cos \omega t$$

8. Out of the following functions representing motion of a particle which represents SHM.

(A) $y = \sin \omega t - \cos \omega t$

(B) $y = \sin^3 \omega t$

(C) $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$

(D) $y = 1 + \omega t + \omega^2 t^2$ **[AIPMT (Prelims)-2011]**

(1) Only (A) and (B)
(2) Only (A)
(3) Only (D) does not represent SHM
(4) Only (A) and (C)

Sol. Answer (4)

Only (A) and (C) are of the form

$x = A \sin (\omega t + \phi)$

Hence they are the only ones which represents an S.H.M.

9. Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other, moving in opposite directions when their displacement is half of the amplitude. The mean positions of the two particles lie on a straight line perpendicular to the paths of the two particles. The phase difference is **[AIPMT (Mains)-2011]**

(1) π
(2) $\frac{\pi}{6}$
(3) Zero
(4) $\frac{2\pi}{3}$

Sol. Answer (4)

$v_1 = A\omega \cos \omega t$ ... (i)

$v_2 = A\omega \cos \omega t + \phi$ ... (ii)

According to equation $v_1 = -v_2$ when

$$x = \frac{A}{2}$$

$$\frac{A}{2} = A \sin \omega t$$

$$\omega t = \frac{\pi}{6} \text{ when } x = \frac{A}{2}$$

$\cos \omega t = -\cos(\omega t + \phi)$ equating (i) and (ii)

$$\cos^{-1} - \frac{\sqrt{3}}{2} = \frac{\pi}{6} + \phi$$

$$\frac{5\pi}{6} - \frac{\pi}{6} = \phi$$

$$\phi = \frac{2\pi}{3}$$

10. The displacement of a particle along the x-axis is given by $x = a \sin^2 \omega t$. The motion of the particle corresponds to **[AIPMT (Prelims)-2010]**

(1) Simple harmonic motion of frequency $\frac{\omega}{\pi}$

(2) Simple harmonic motion of frequency $\frac{3\omega}{2\pi}$

(3) Non simple harmonic motion

(4) Simple harmonic motion of frequency $\frac{\omega}{2\pi}$

Sol. Answer (1)

$x = a \sin^2 \omega t$

$$x = \frac{a(1-\cos 2\omega t)}{2}$$

$2x - a = -\cos 2\omega t$

or $\cos 2\omega t = a - 2x$

The period of this function is $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

Hence frequency $= \frac{1}{T} = \frac{\omega}{\pi}$

11. The period of oscillation of a mass M suspended from a spring of negligible mass is T. If along with it another mass M is also suspended, the period of oscillation will now be **[AIPMT (Prelims)-2010]**

(1) T

(2) $\frac{T}{\sqrt{2}}$

(3) $2T$

(4) $\sqrt{2}T$

Sol. Answer (4)

$$T \propto \sqrt{m} \quad \frac{T'}{T} = \sqrt{\frac{2M}{M}} \Rightarrow T' = \sqrt{2}T$$

12. A simple pendulum performs simple harmonic motion about $x = 0$ with an amplitude a and time period T. The speed of the pendulum at $x = \frac{a}{2}$ will be **[AIPMT (Prelims)-2009]**

(1) $\frac{\pi a}{T}$ (2) $\frac{3\pi^2 a}{T}$ (3) $\frac{\pi a\sqrt{3}}{T}$ (4) $\frac{\pi a\sqrt{3}}{2T}$

Sol. Answer (3)

$$v = A\omega \cos \omega t$$

$$x = \frac{A}{2}$$

$$\omega\frac{A}{2} = A \sin \omega t$$

$$\frac{\pi}{6} = \omega t$$

$$v = A\omega \cos \frac{\pi}{6}$$

$$v = A\omega\frac{\sqrt{3}}{2} = \frac{2\pi}{T}\frac{\sqrt{3}}{2} = \frac{\pi\sqrt{3}}{T}A$$

13. Which one of the following equations of motion represents simple harmonic motion? (Where k, k_0, k_1 and a are all positive.) **[AIPMT (Prelims)-2009]**

(1) Acceleration = $-k(x + a)$ (2) Acceleration = $k(x + a)$

(3) Acceleration = kx (4) Acceleration = $-k_0x + k_1x^2$

Sol. Answer (1)

According to equation of S.H.M..

$$a = -\omega^2 x.$$

The only option of the same form is the third one k_0.

Acceleration = $-k(x + a)$

Hence answer is (1)

14. Two simple harmonic motions of angular frequency 100 and 1000 rad s^{-1} have the same displacement amplitude. The ratio of their maximum accelerations is **[AIPMT (Prelims)-2008]**

(1) $1 : 10^4$ (2) $1 : 10$

(3) $1 : 10^2$ (4) $1 : 10^3$

Sol. Answer (3)

Maximum acceleration occurs at the extreme points of an S.H.M.. motion.

$$a = -\omega^2 x$$

At $x = A$ $\quad a = -\omega^2 A$

$$a_1 = (100)^2 A$$

$$a_2 = (1000)^2 A$$

$$\frac{a_1}{a_2} = \frac{1}{100}$$

$$a_1 : a_2 = 1 : 10^2$$

15. A point performs simple harmonic oscillation of period T and the equation of motion is given by $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$. After the elapse of what fraction of the time period the velocity of the point will be equal to half of its maximum velocity? **[AIPMT (Prelims)-2008]**

(1) $\frac{T}{12}$ (2) $\frac{T}{8}$ (3) $\frac{T}{6}$ (4) $\frac{T}{3}$

Sol. Answer (1)

Maximum velocity = $a\omega$

$$v = a\omega \cos \omega t + \frac{\pi}{6}$$

Let time where $v = \frac{a\omega}{2}$

Let $\frac{a\omega}{2} = a\omega\cos\omega t + \frac{\pi}{6}$

$$\frac{\pi}{3} = \omega t + \frac{\pi}{6}$$

or $\omega t = \frac{\pi}{6}$

$$\frac{2\pi}{T} = \frac{\pi}{6} \qquad \left[\omega = \frac{2\pi}{T}\right]$$

$$t = \frac{T}{12}$$

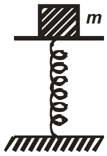
16. A mass of 2.0 kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible. When pressed slightly and released the mass executes a simple harmonic motion. The spring constant is 200 N/m. What should be the minimum amplitude of the motion so that the mass gets detached from the pan? [Take g = 10 m/s^2] **[AIPMT (Prelims)-2007]**

(1) 10.0 cm

(2) Any value less than 12.0 cm

(3) 4.0 cm

(4) 8.0 cm



Sol. Answer (1)

When it disconnects from plates acceleration is maximum for minimum amplitude. Acceleration is maximum at the extremities.

When block leaves from $a = g$

$$g = -\omega^2 x$$

$$g = -\omega^2 a$$

$$g = -\frac{k}{m}a$$

$$-g\frac{m}{k} = a$$

$$|a| = \frac{10 \times 2}{200}$$

$$a = 10 \text{ cm}$$

17. The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is **[AIPMT (Prelims)-2007]**

(1) Zero (2) $0.5\,\pi$ (3) π (4) $0.707\,\pi$

Sol. Answer (2)

$$v = A\omega\cos(\omega + \phi)$$

$$a = -A\omega^2\sin(\omega t + \phi)$$

Now $a = +A\omega^2\cos\left(\omega t + \phi + \frac{\pi}{2}\right)$

Hence $A\phi = \left(\omega t + \phi + \frac{\pi}{2}\right) - (\omega t + \phi) = \frac{\pi}{2}$

18. The particle executing simple harmonic motion has a kinetic energy $K_0\cos^2\omega t$. The maximum values of the potential energy and the total energy are respectively **[AIPMT (Prelims)-2007]**

(1) K_0 and K_0 (2) 0 and $2K_0$ (3) $\frac{K_0}{2}$ and K_0 (4) K_0 and $2K_0$

Sol. Answer (1)

Let kinetic energy at 4 point be $K = K_0\cos^2\omega t$

At maximum value of K $\cos^2\omega t = 1$ and $K = K_0$

Maximum value of K.E. = Maximum value of P.E. = Total mechanical energy as mechanical energy is converted in S.H.M.

19. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is

[AIPMT (Prelims)-2007]

(1) $\frac{T}{2}$ (2) $\frac{T}{4}$ (3) $\frac{T}{8}$ (4) $\frac{T}{12}$

Sol. Answer (4)

$$x = a\sin\omega t$$

$$\frac{a}{2} = a\sin\omega t$$

$$\therefore\quad \omega t = \frac{\pi}{6}$$

$$t = \frac{\pi}{6}\times\frac{1}{\omega} = \frac{\pi}{6}\times\frac{T}{2\pi}$$

$$t = \frac{T}{12}$$

20. A rectangular block of mass m and area of cross-section A floats in a liquid of density ñ. If it is given a small vertical displacement from equilibrium it undergoes oscillation with a time period T. Then

[AIPMT (Prelims)-2006]

(1) $T \propto \sqrt{\rho}$ (2) $T \propto \frac{1}{\sqrt{A}}$ (3) $T \propto \frac{1}{\rho}$ (4) $T \propto \frac{1}{\sqrt{m}}$

Sol. Answer (2)

21. The circular motion of a particle with constant speed is: **[AIPMT (Prelims)-2005]**

(1) Simple harmonic but not periodic (2) Periodic and simple harmonic

(3) Neither periodic nor simple harmonic (4) Periodic but not simple harmonic

Sol. Answer (4)

22. A particle executing simple harmonic motion of amplitude 5 cm has maximum speed of 31.4 cm/s. The frequency of its oscillation is **[AIPMT (Prelims)-2005]**

(1) 3 Hz (2) 2 Hz (3) 4 Hz (4) 1 Hz

Sol. Answer (4)

$A = 5$ cm

Maximum speed ($A\omega$) = 31.4

$$\omega = \frac{31.4}{5}$$

or $2\pi f = 31.4$

$$f = \frac{31.4}{10 \times 3.14}$$

$f = 1$ Hz

23. Which of the following is simple harmonic motion?

(1) Particle moving in a circle with uniform speed

(2) Wave moving through a string fixed at both ends

(3) Earth spinning about its axis

(4) Ball bouncing between two rigid vertical walls

Sol. Answer (2)

A wave on a string is an example of simple harmonic motion as the displacement of particles in the motion may be described by

$x = A \sin \omega t$

24. A particle executes S.H.M.. along x-axis. The force acting on it is given by

(1) $A \cos (kx)$ (2) Ae^{-kx} (3) kx (4) $-kx$

Sol. Answer (4)

Force acts along $-kx$ according to theory of S.H.M..

25. Which one of the following statements is true for the speed v and the acceleration a of a particle executing simple harmonic motion?

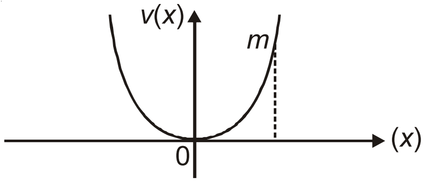
(1) When v is maximum, a is maximum (2) Value of a is zero, whatever may be the value of v

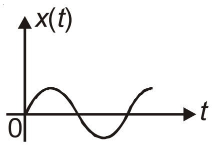
(3) When v is zero, a is zero (4) When v is maximum, a is zero

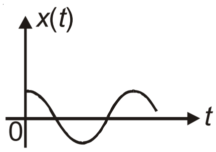
Sol. Answer (4)

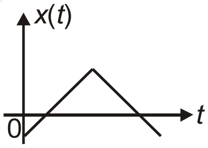
Acceleration (a) is zero is the mean position where velocity is maximum.

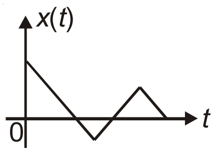
26. A particle of mass m is released from rest and follows a parabolic path as shown. Assuming that the displacement of the mass from the origin is small. Which graph correctly depicts the position of the particle as a function of time?



(1) 

(2) 

(3) 

(4) 

Sol. Answer (2)

Particle of mass m is released in a parabolic path. It will perform S.H.M.. just like a pendulum.'

The particle is released from amplitude.

Option (2) represents graph of an S.H.M.. starting from amplitude position.

27. In a simple harmonic motion, when the displacement is one-half the amplitude, what fraction of the total energy is kinetic?

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) Zero (4) $\frac{1}{4}$

Sol. Answer (2)

K.E. of half of amplitude = $\left(x = \frac{A}{2}\right)$

K.E. of $\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}k\left(A^2 - \frac{A^2}{4}\right)$

K.E. = $\frac{1}{2}k\left(\frac{3A^2}{4}\right)$

Fraction of the total energy is kinetic energy = $\frac{K.E.}{T.E.} = \frac{\frac{1}{2}k\left(\frac{3A^2}{4}\right)}{\frac{1}{2}kA^2} = \frac{3}{4}$

28. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its

(1) Maximum P.E. is 160 J (2) Maximum P.E. is zero

(3) Maximum P.E. is 100 J (4) Maximum P.E. is 120 J

Sol. Answer (1)

At maximum potential energy all mechanical energy is stored as potential energy.

Hence maximum P.E. = Total mechanical energy

= 160 J

29. Displacement between maximum potential energy position and maximum kinetic energy position for a particle executing simple harmonic motion is

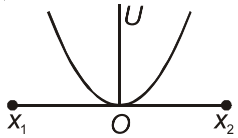
(1) $\pm \frac{a}{2}$ (2) $+ a$ (3) $\pm a$ (4) -1

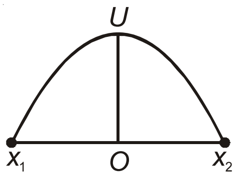
Sol. Answer (3)

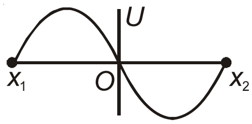
Maximum P.E. is at extremities and maximum K.E. is at mean position.

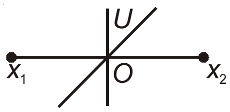
Hence the difference in between the two position is ± a.

30. A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2, the equilibrium position being O. Its potential energy U is plotted. It will be as given below in the graph

(1) 

(2) 

(3) 

(4) 

Sol. Answer (1)

In S.H.M.. potential energy is minimum at mean position and maximum at the extremities. Also graph will be parabolic as

$$U = \frac{1}{2}kx^2$$

Hence answer is (1).

31. The potential energy of a simple harmonic oscillator when the particle is half way to its end point is

(1) $\frac{2}{3}E$ (2) $\frac{1}{8}E$

(3) $\frac{1}{4}E$ (4) $\frac{1}{2}E$

Sol. Answer (3)

$$E = \frac{1}{2}kA^2 \text{ [where } E \text{ is total energy]}$$

When $x = \frac{A}{2}$

$$E_x = \frac{1}{2}kx^2 = \frac{1}{2}\frac{kA^2}{4} = \frac{kA^2}{8}$$

or $E_x = \frac{E}{4}$

32. If the length of a simple pendulum is increased by 2%, then the time period

(1) Increases by 1% (2) Decreases by 1%

(3) Increases by 2% (4) Decreases by 2%

Sol. Answer (1)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

If length is increased by 2%

$$T_2 = 2\pi\sqrt{\frac{l}{g}\times\frac{10^2}{100}}$$

or $T_2 = 2\pi\sqrt{\frac{l}{g}(1+0.02)}$

or $T_2 = 2\pi\sqrt{\frac{l}{g}}\times 1.01$ [By binomial theorem]

or $T_2 = T + \frac{T}{100}$

$\therefore$ T_2 is 1% more than T.

33. Two simple pendulums of length 5 m and 20 m respectively are given small linear displacements in one direction at the same time. They will again be in the same phase when the pendulum of shorter length has completed ______ oscillations.

(1) 2 (2) 1

(3) 5 (4) 3

Sol. Answer (1)

$l_1 = 5$ m $l_2 = 20$ m

$T_1 = 2\pi\sqrt{\frac{5}{g}}$ $T_2 = 2\pi\sqrt{\frac{20}{g}}$

$$\frac{T_1}{T_2} = \frac{1}{2}$$

$$2T_1 = T_2$$

Hence they will be in phase again when shorter one has completed 2 oscillation.

34. Two masses M_A and M_B are hung from two strings of length l_A and l_B respectively. They are executing S.H.M. with frequency relation $f_A = 2f_B$, then relation

(1) $l_A = \frac{l_B}{4}$, does not depend on mass

(2) $l_A = 4l_B$, does not depend on mass

(3) $l_A = 2l_B$ and $M_A = 2M_B$

(4) $l_A = \frac{l_B}{2}$ and $M_A = \frac{M_B}{2}$

Sol. Answer (1)

Mass M_A and M_B

Length l_A and l_B

If $f_A = 2f_B$

$T_B = 2T_A$ [as $f = \frac{2\pi}{T}$ given]

$$2\pi\sqrt{\frac{l_A}{g}} = 4\pi\sqrt{\frac{l_B}{g}}$$

$$\frac{l_A}{g} = 4l_B$$

or $l_B = \frac{l_A}{4}$ which does not depend on mass.

35. A mass m is vertically suspended from a spring of negligible mass, the system oscillates with a frequency n. What will be the frequency of the system, if a mass $4m$ is suspended from the same spring?

(1) $\frac{n}{2}$ (2) $4n$ (3) $\frac{n}{4}$ (4) $2n$

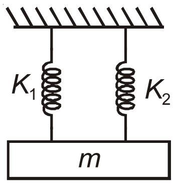
Sol. Answer (1)

$$\omega = 2\pi n = \sqrt{\frac{k}{m}}$$

$$= n \propto \frac{1}{\sqrt{m}}$$

When m becomes 4 m hence n is halved $\frac{n}{2}$

36. A mass is suspended separately by two different springs in successive order then time periods is t_1 and t_2 respectively. If it is connected by both the springs as shown in figure then time period is t_0, the correct relation is



(1) $t_0^2 = t_1^2 + t_2^2$ (2) $t_0^{-2} = t_1^{-2} + t_2^{-2}$

(3) $t_0^{-1} = t_1^{-1} + t_2^{-1}$ (4) $t_0 = t_1 + t_2$

Sol. Answer (2)

$$t_1 = 2\pi\sqrt{\frac{m}{k_1}}$$

$$t_2 = 2\pi\sqrt{\frac{m}{k_2}}$$

The springs in parallel have $k_{eq} = k_1 + k_2$

$$t_0 = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

$$t_0 = 2\pi\sqrt{\frac{m}{4\pi^2\left(\frac{m}{t_1^2} + \frac{m}{t_2^2}\right)}} = \sqrt{\frac{t_1^2 t_2^2}{t_1^2 + t_2^2}}$$

$$\frac{1}{t_0^2} = \frac{t_1^2 + t_2^2}{t_1^2 + t_2^2}$$

$$\frac{1}{t_0^2} = \frac{1}{t_2^2} + \frac{1}{t_1^2}$$

37. The time period of mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be

(1) $\frac{T}{4}$ (2) T (3) $\frac{T}{2}$ (4) $2T$

Sol. Answer (3)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The k of spring becomes 4 k when cut.

$$T' = 2\pi\sqrt{\frac{m}{4k}} \quad \text{or } T' = \frac{T}{2}$$

38. A particle, with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force F sinωt. If the amplitude of the particle is maximum for $\omega = \omega_1$, and the energy of the particle maximum for $\omega = \omega_2$, then (where ω_0 is natural frequency of oscillation of particle)

(1) $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_0$ (2) $\omega_1 = \omega_0$ and $\omega_2 = \omega_0$

(3) $\omega_1 = \omega_0$ and $\omega_2 \neq \omega_0$ (4) $\omega_1 \neq \omega_0$ and $\omega_2 \neq \omega_0$

Sol. Answer (2)

$F = F_0 \sin \omega t$ is the equation of forces vibration.

In the case of resonance ω is the resonating frequency.

At the same ω amplitude and energy is maximum.

Hence $\omega_1 = \omega_2 = \omega$

39. When an oscillator completes 100 oscillations its amplitude reduced to $\frac{1}{3}$ of initial value. What will be its amplitude, when it completes 200 oscillations?

(1) $\frac{1}{8}$ (2) $\frac{2}{3}$ (3) $\frac{1}{6}$ (4) $\frac{1}{9}$

Sol. Answer (4)

$A = A_0 e^{-bt}$

When t = 200 T $\qquad A = \frac{A_0}{3}$

$$\frac{A_0}{3} = A_0 e^{-b\times 100\text{ T}}$$

$$e^{-100b\text{T}} = \frac{1}{3}$$

$$A' = A_0(e^{-b\times 200\text{ T}})$$

$$A' = A_0(e^{-b\times 100\text{ T}})^2$$

$$A' = A_0\left(\frac{1}{3}\right)^2 = \frac{A_0}{9}$$

40. In case of a forced vibrations, the resonance wave becomes very sharp when the

(1) Damping force is small (2) Restoring force is small

(3) Applied periodic force is small (4) Quality factor is small

Sol. Answer (1)

When restoring force is very small there is very little dissipation of energy and the driving force can deliver maximum amplitude.

41. Two S.H.M.'s with same amplitude and time period, when acting together in perpendicular directions with a phase difference of $\frac{\pi}{2}$, give rise to

(1) Straight motion (2) Elliptical motion

(3) Circular motion (4) None of these

Sol. Answer (3)

Let x be $x = a \sin \omega t$

and y be $y = a \sin (\omega t + \delta)$ or $y = a \cos \omega t$

If they are perpendicular $x^2 + y^2 = a^2$ which is the equation of circle.

42. The equations of two S.H.M.'s is given as $x = a \cos (\omega t + \delta)$ and $y = a \cos (\omega t + \alpha)$, where $\delta = \alpha + \frac{\pi}{2}$, the resultant of the two S.H.M.'s represents

(1) A hyperbola (2) A circle

(3) An ellipse (4) None of these

Sol. Answer (2)

$x = a \cos (\omega t + \delta)$

$y = a \cos (\omega t + \delta)$ ... (i)

$\delta = \alpha + \frac{\pi}{2}$

$a - a \sin (\omega t + \alpha)$ ... (ii)

Squaring both (i) and (ii) it is of the form.

$x^2 + y^2 = a^2$

Hence it represents a circle.

43. The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are

(1) kg ms^{-1} (2) kg ms^{-2} (3) kg s^{-1} (4) kg s

Sol. Answer (3)

$F = kv$

$k = \dfrac{F}{v}$

$[k] = [F]\ [v]^{-1}$

$[k] = [\text{kg ms}^{-2}\ \text{m}^{-1}\text{s}]$

$[k] = \text{kg s}^{-1}$

44. A wave has S.H.M. whose period is 4 s while another wave which also possess S.H.M. has its period 3 s. If both are combined, then the resultant wave will have the period equal to

(1) 4 s (2) 5 s (3) 12 s (4) 3 s

Sol. Answer (3)

$T_1 = 4$ s

$T_2 = 3$ s

The resultant wave will have a time period equal to LCM of the two waves.

LCM of 4 and 3 is 12

Hence $T = 12$ s

SECTION - D

Assertion - Reason Type Questions

1. A : Simple harmonic motion is not a uniform motion.

R : Simple harmonic motion can be regarded as the projection of uniform circular motion.

Sol. Answer (2)

Simple harmonic motion is not a uniform motion as acceleration velocity and displacement are all variable with time simple harmonic motion can be regarded as a **project ion** of uniform circular motion but this is not the correct explanation of assertion.

2. A : In simple harmonic motion, the velocity is maximum when the displacement is minimum.

R : Displacement and velocity of S.H.M. differ in phase by $\pi/2$.

Sol. Answer (1)

The assertion is true and the reason is the correct explanation of the given assertion.

3. A : In reality the amplitude of a freely oscillating pendulum decreases gradually with time.

R : The frequency of the pendulum decreases with time.

Sol. Answer (3)

Assertion is true as damping and dissipative forces are present reducing the energy and hence amplitude of the pendulum.

The frequency positive remains constant.

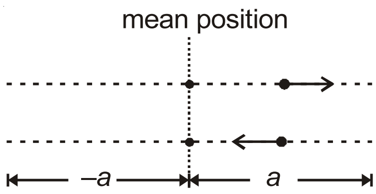
4. A : The graph of velocity as a function of displacement for a particle executing S.H.M. is an ellipse.

R : The velocity and displacement are related as $v = \omega\sqrt{A^2 - x^2}$

Sol. Answer (1)

The assertion is true and the explanation gives the equation of velocity v_s displacement (x) which is in the form of an ellipse.

5. A : The phase difference between the two particles shown below is π. (Assuming both particles have same time periods and same amplitudes).



R : If the particles cross each other while they move in the opposite direction, they have a phase difference of π radian.

Sol. Answer (4)

The phase difference between two particles is π only when they pass the **mean point** with opposite velocities at the same point.

Both the assertion and reason are hence false.

6. A : All trigonometric functions are periodic.

R : All trigonometric functions can represent S.H.M.

Sol. Answer (3)

All trigonometric function may be periodic but not all can represent S.H.M. Examples of exception include tan, cot, cosec etc. Hence answer is (3)

7. A : In a S.H.M. both kinetic energy and potential energy oscillates with double the frequency of S.H.M.

R : Frequency of oscillation of total energy in S.H.M. is infinite

Sol. Answer (3)

$$\text{K.E.} = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi)$$

$$\text{P.E.} = \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \phi)$$

$\cos^2\phi$ and $\sin^2\phi$ may both be written in the form of doubles angles. Hence they represent S.H.M. with angular frequency = 2ω.

Total energy of an S.H.M. is always constant. Constant values are said to have infinite time period and zero frequency.

8. A : If a clock based on simple pendulum is taken to hill it will become slower.

R : With increase of height above surface of earth g decreases so T will increase.

Sol. Answer (1)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The assertion is the correct explanation of the reason.

9. A : If a spring block system, oscillating in a vertical plane is made to oscillate on a horizontal surface, the time period will remain same.

R : The time period of spring block system does not depend on g.

Sol. Answer (2)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Gives the time period of spring block system any time period remains the same in both cases.

Time period of spring block system doesn't depend on g but that explanation alone is not enough to explain the non-variation time period of S.H.M.

10. A : In a forced oscillator the energy transferred from driving force to damped oscillator is maximum in resonance state.

R : The amplitude of forced oscillator depends on the frequency of external force.

Sol. Answer (2)

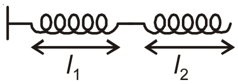
The assertion and reason are both true. But the reason does not exsplain why maximum energy transfer occurs in that state. It merely says that the amplitude depends on frequency of external force.

11. A : If length of a spring is halved, then its force constant becomes double.

R : The spring constant is inversely proportional to length of spring.

Sol. Answer (1)

Suppose a spring of natural length l into made of two parts l_1 and l_2.



The entire spring is displaced by a distance Δx.

$$\Delta x = \Delta x_1 + \Delta x_2$$

Now $\Delta x_1 : \Delta x_2 = l_1 : l_2$

$$\frac{F}{k_1} : \frac{F}{k_2} : = l_1 : l_2$$

$$\frac{k_2}{k_1} = \frac{l_1}{l_2}$$

$$k \propto \frac{1}{l}$$

So if length is halved k_{eq} is doubled.

12. A : When soldier cross a bridge, they are asked to break steps.

R : If they do not break steps, then they will apply large force on bridge simultaneously.

Sol. Answer (3)

When soldiers cross a bridge, they are asked to break steps. So the assertion is true.

They are told to do this to avoid forced vibration to be created on the bridge. If it is the same as natural frequency of the bridge, it may cause resonance. The bridge may oscillate with a higher amplitude and the bridge may fall.

13. A : In S.H.M. the change in velocity is not uniform.

R : In S.H.M. the acceleration of body varies linearly with its displacement.

Sol. Answer (1)

Both the assertion and reason are correct and reason the correct explanation of assertion.

$$a = -\omega^2 x$$

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