Chapter 8

Gravitation

Solutions

SECTION - A

Objective Type Questions

(Kepler's Laws)

1. According to Kepler, planets move in

   (1) Circular orbits around the sun

   (2) Elliptical orbits around the sun with sun at exact centre

   (3) Straight lines with constant velocity

   (4) Elliptical orbits around the sun with sun at one of its foci

Sol. Answer (4)

Kepler's first law,

Law of Orbits : All planets move in elliptical orbits, with the sun at one of the foci of the ellipse.

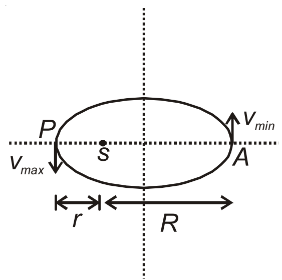
2. The minimum and maximum distances of a planet revolving around sun are r and R. If the minimum speed of planet on its trajectory is v_0, its maximum speed will be

   (1) $\frac{v_0 R}{r}$ (2) $\frac{v_0 r}{R}$ (3) $\frac{v_0 R^2}{r^2}$ (4) $\frac{v_0 r^2}{R^2}$

Sol. Answer (1)

According to Kepler's second law.

Low of Areas : The line that joins any planet to the sun sweeps out equal areas in equal intervals of time. Thus planets appear to move slower when they are farther from sun than when they are nearer.



Now, for planets moving around the sun in an elliptical orbit, Angular momentum is conserved.

$\Rightarrow \left|\overrightarrow{L_P}\right| = \left|\overrightarrow{L_A}\right|$

$mv_{max}r = mv_0R$

$v_{max} = \frac{v_0 R}{r}$

3. A planet of mass m moves around the sun of mass M in an elliptical orbit. The maximum and minimum distances of the planet from the sun are r_1 and r_2 respectively. The time period of the planet is proportional to

(1) $r_1^{3/2}$ (2) $r_2^{3/2}$ (3) $(r_1 + r_2)^{3/2}$ (4) $(r_1 - r_2)^{3/2}$

Sol. Answer (3)

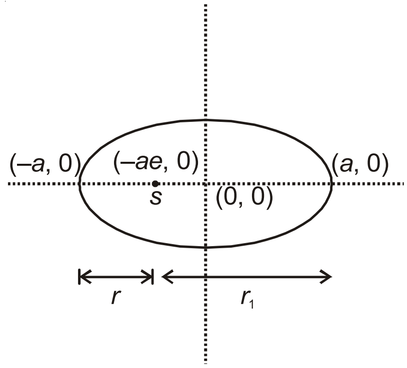
Law of periods : The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.

$T^2 \propto a^3$

where,

T = Time period of revolution of a planet.

a = Semi-major axis of the elliptical orbit traced by the planet.



$r_1 = a + ae$

$r_2 = a - ae$

$r_1 + r_2 = 2a$

$\Rightarrow \quad T^2 \propto \left(\dfrac{(r_1+r_2)}{2}\right)^3$

$\Rightarrow \quad T \,\alpha\, (r_1+r_2)^{3/2}$

Alternatively,

From figure, $r_1 + r_2 = 2a$

$\Rightarrow \quad T^2 \propto \left(\dfrac{r_1+r_2}{2}\right)^3$

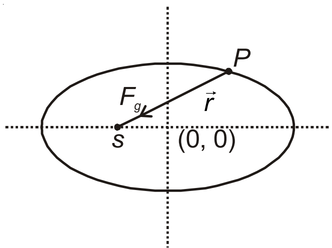
$\Rightarrow \quad T \propto (r_1 + r_2)^{3/2}$

4. The torque on a planet about the centre of sun is

(1) Zero (2) Negative

(3) Positive (4) Depend on mass of planet

Sol. Answer (1)



Force of gravity is acting on the planet,

Torque of force of gravity = $\vec{r} \times \vec{F_g} = rF_g \sin\theta$

Since θ = 180°, τ = 0

5. During motion of a planet from perihelion to aphelion the work done by gravitational force of sun on it is

(1) Zero (2) Negative

(3) Positive (4) May be positive or negative

Sol. Answer (2)

According to Kepler's Law of areas, $v_A < v_P$

v_A = speed of planet at aphelion

v_P = speed of planet at perihelion

Now, work done by gravitational force of sun $= \Delta K.E = \dfrac{1}{2}m\left(v_A^2 - v_P^2\right)$

$\Rightarrow \quad W_{\text{gravitation force}}$ is negative.

6. The time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius $4R$ is

(1) $4T$ (2) $\dfrac{T}{4}$ (3) $8T$ (4) $\dfrac{T}{8}$

Sol. Answer (3)

Using Kepler's third law,

$T^2 \propto R^3$

$\Rightarrow \dfrac{T_2}{T} = \left(\dfrac{4R}{R}\right)^{3/2}$

$\Rightarrow T_2 = T \times 2^3$

$= 8T$

(Universal Law of Gravitation)

7. Gravitation is the phenomenon of interaction between

(1) Point masses only
(2) Any arbitrary shaped masses
(3) Planets only
(4) None of these

Sol. Answer (2)

Gravitation is the phenomenon of interaction between any arbitrary shaped bodies.

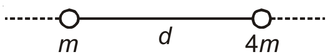
8. Force of gravitation between two masses is found to be F in vacuum. If both the masses are dipped in water at same distance then, new force will be

(1) $> F$
(2) $< F$
(3) F
(4) Cannot say

Sol. Answer (3)

Force of gravitation is independent of the medium. Force is F when masses are in vacuum. When masses are dipped in water force will be same.

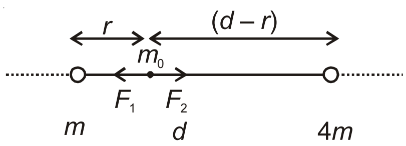
9. Two point masses m and $4m$ are separated by a distance d on a line. A third point mass m_0 is to be placed at a point on the line such that the net gravitational force on it is zero.



The distance of that point from the m mass is

(1) $\dfrac{d}{2}$
(2) $\dfrac{d}{4}$
(3) $\dfrac{d}{3}$
(4) $\dfrac{d}{5}$

Sol. Answer (3)



Force of gravitation on m_0 due to $m = \dfrac{Gmm_0}{r^2} = F_1$

Force of gravitation on m_0 due to $4m = \dfrac{G4mm_0}{(d-r)^2} = F_2$

Net force = 0

$\Rightarrow F_1 = F_2$

$\dfrac{Gmm_0}{r^2} = \dfrac{4Gmm_0}{(d-r)^2}$

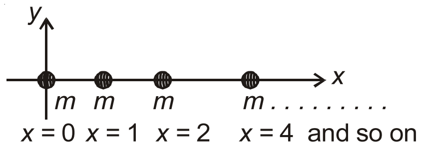
$\Rightarrow (d - r)^2 = (2r)^2$

$\Rightarrow d - r = 2r$

$\Rightarrow d = 3r$

Thus, $r = \dfrac{d}{3}$

10. A large number of identical point masses m are placed along x-axis, at x = 0, 1, 2, 4, ... The magnitude of gravitational force on mass at origin (x = 0), will be

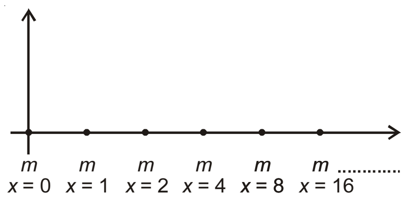


(1) Gm^2 (2) $\frac{4}{3}Gm^2$ (3) $\frac{2}{3}Gm^2$ (4) $\frac{5}{4}Gm^2$

Sol. Answer (2)

Let, F_1, F_2 , F_4, F_8 …… be the forces of gravitation due masses 'm' at x = 1, 2, 4, 8 … respectively.

$$\Rightarrow \quad F_1 = \frac{Gm^2}{1^2}$$



$$F_2 = \frac{Gm^2}{2^2}$$

$$F_4 = \frac{Gm^2}{4^2}$$

$$F_8 = \frac{Gm^2}{8^2}$$

$$F_1 + F_2 + F_4 + F_8 \ldots = \quad Gm^2 \underbrace{\left(\frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}\ldots\right)}_{\text{infinite G.P. with common ratio } = \frac{1}{4}}$$

For an infinite G.P, sum = $\left(\frac{a}{1-r}\right)$

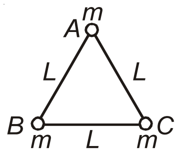
a is the first term

r is the common ratio

$$\Rightarrow \quad \text{Sum} = \frac{1}{1-\frac{1}{4}} = \left(\frac{4}{3}\right)$$

$$\Rightarrow \quad F_1 + F_2 + F_4 + F_8 \ldots\ldots = \frac{4}{3}Gm^2$$

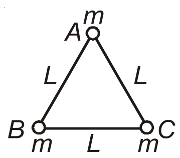
11. Three particles A, B and C each of mass m are lying at the corners of an equilateral triangle of side L. If the particle A is released keeping the particles B and C fixed, the magnitude of instantaneous acceleration of A is



(1) $\sqrt{3}\frac{Gm^2}{L^2}$ (2) $\sqrt{2}\frac{Gm^2}{L^2}$ (3) $\sqrt{2}\frac{Gm}{L^2}$ (4) $\sqrt{3}\frac{Gm}{L^2}$

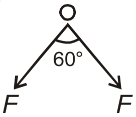
Sol. Answer (4)

At this moment,



Forces acting on particle at A can be shown,

where, $F = \dfrac{Gm^2}{L^2}$



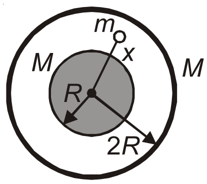
$\Rightarrow$ Net force will be resultant of both,

$$F_{\text{resultant}} = \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ} = \sqrt{3}\,F$$

$$\Rightarrow \quad F_{\text{resultant}} = \frac{\sqrt{3}\,Gm^2}{L^2}$$

$$a = \frac{F}{m} = \frac{\sqrt{3}\,Gm}{L^2}$$

12. A uniform sphere of mass M and radius R is surrounded by a concentric spherical shell of same mass but radius $2R$. A point mass m is kept at a distance x ($>R$) in the region bounded by spheres as shown in the figure. The net gravitational force on the particle is



(1) $\dfrac{GMm}{x^2}$ (2) $\dfrac{GMmx}{R^3}$ (3) $\dfrac{G(M+m)}{x^2}$ (4) Zero

Sol. Answer (1)

The gravitational force on the point mass m due to uniform sphere $= \dfrac{GMm}{x^2}$.

The gravitational force on the point mass due to the outer spherical shell is zero because gravitational force of attraction on a point mass due to various rejoins of the spherical shell cancels each other completely as their vector sum is zero.

(The Gravitational Constant, Acceleration Due to Gravity of the Earth)

13. The gravitational constant depends upon

(1) Size of the bodies (2) Gravitational mass

(3) Distance between the bodies (4) None of these

Sol. Answer (4)

Gravitational constant 'G' is independent of size of bodies, gravitational mass and distance between the bodies.

14. Two planets have same density but different radii. The acceleration due to gravity would be

(1) Same on both planets (2) Greater on the smaller planet

(3) Greater on the larger planet (4) Dependent on the distance of planet from the sun

Sol. Answer (3)

Acceleration due to gravity at the surface of a planet, $g = \frac{GM}{R^2}$, where M is the mass of planet, R is the radius of the planet,

Also, $M = pV$

$\Rightarrow \quad g = \frac{G}{R^2} \times \left(\frac{4}{3}\pi GR^3\rho\right)$

Thus, $g = \frac{4}{3}\pi GR\rho$

Thus $g \alpha$ Radius of the planet,

Thus, acceleration due to gravity would be greater on the larger planet.

15. If the radius of earth shrinks by 1.5% (mass remaining same), then the value of gravitational acceleration changes by

(1) 2% (2) –2% (3) 3% (4) –3%

Sol. Answer (3)

$g = \frac{GM}{R^2}$

$g' = \frac{GM}{(0.985\,R)^2}$

$g' = (1.0306)\frac{GM}{R^2}$

$\Rightarrow \quad g' = 1.0306\,g$

$\Rightarrow$ Acceleration changes by

$\frac{\Delta g}{g} \times 100 = +3\%$

Alternate method:

$g' = \frac{GM}{(R+\Delta R)^2}$

$g' = GM(R+\Delta R)^{-2}$

$g' = \frac{GM}{R^2}\left(1+\frac{\Delta R}{R}\right)^{-2}$

for $\frac{\Delta R}{R} \ll 1$, we can use binomial and approximately,

$g' = \frac{GM}{R^2}\left(1-\frac{2\Delta R}{R}\right)$

$\Rightarrow \quad g' = g - g\frac{2\Delta R}{R}$

$\Rightarrow \quad \frac{\Delta g}{g} = \frac{-2\Delta R}{R} = -2\times\left(\frac{-1.5}{100}\right) = \frac{+3}{100} = 3\% \quad [g' - g = \Delta g]$

16. If density of a planet is double that of the earth and the radius 1.5 times that of the earth, the acceleration due to gravity on the surface of the planet is

(1) $\frac{3}{4}$ times that on the surface of the earth (2) 3 times that on the surface of the earth

(3) $\frac{4}{3}$ times that on the surface of the earth (4) 6 times that on the surface of the earth

Sol. Answer (2)

Acceleration due to gravity on the surface of a planet is given by, $g = \frac{GM}{R^2}$

$M \rightarrow$ Mass of the planet

$R \rightarrow$ Radius of the planet

Also, $M = \frac{4}{3}\pi R^3 \times \rho$

$\Rightarrow \quad g = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\rho G \pi R$

$\rho \rightarrow$ Density of the planet.

$\Rightarrow$ Acceleration due to gravity α ρR

$\Rightarrow \quad \frac{g_{\text{planet}}}{g_{\text{earth}}} = \frac{2\rho_e \times 1.5\, R_e}{\rho_e \times R_e} = 3$

$\Rightarrow$ Acceleration due to gravity on the surface of planet is 3 times that on the surface of earth.

17. The gravitational force on a body of mass 1.5 kg situated at a point is 45 N. The gravitational field intensity at that point is

(1) 30 N/kg (2) 67.5 N/kg (3) 46.5 N/kg (4) 43.5 N/kg

Sol. Answer (1)

Gravitation force = mg

g = gravitation field intensity.

$\Rightarrow$ 45 = 1.5 × g

$\Rightarrow \quad g = \frac{45}{1.5} = 30$ N/kg

18. Two point masses having mass m and $2m$ are placed at distance d. The point on the line joining point masses, where gravitational field intensity is zero will be at distance

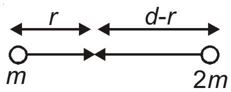
(1) $\frac{2d}{\sqrt{3}+1}$ from point mass "$2m$"

(2) $\frac{2d}{\sqrt{3}-1}$ from point mass "$2m$"

(3) $\frac{d}{1+\sqrt{2}}$ from point mass "m"

(4) $\frac{d}{1-\sqrt{2}}$ from point mass "m"

Sol. Answer (3)



Gravitational field intensity will be zero,

$\Rightarrow \quad \frac{Gm}{r^2} = \frac{2Gm}{(d-r)^2}$

$\Rightarrow \quad \frac{1}{r} = \frac{\sqrt{2}}{d-r}$

$\Rightarrow \quad d - r = \sqrt{2}\, r$

$\Rightarrow \quad r\left(1+\sqrt{2}\right) = d$

$\Rightarrow \quad r = \frac{d}{\left(1+\sqrt{2}\right)}$

(Acceleration Due to Gravity above the Surface of Earth, Acceleration Due to Gravity below the Surface of Earth)

19. At what height above the surface of earth the value of "g" decreases by 2%? [radius of the earth is 6400 km]

(1) 32 km (2) 64 km (3) 128 km (4) 1600 km

Sol. Answer (2)

Acceleration due to gravity above the surface of earth at a height h is given $g' = g\left(1 - \frac{2h}{R_e}\right)$

here, g' = 0.98 g

$\Rightarrow \quad 0.98 = 1 - \frac{2h}{R_e}$

$\Rightarrow \quad \frac{2h}{R_e} = 0.02$

$h = 0.01\ R_e$

$= 0.01 \times 6400$ km

$= 64$ km

20. During motion of a man from equator to pole of earth, its weight will (neglect the effect of change in the radius of earth)

(1) Increase by 0.34% (2) Decrease by 0.34%

(3) Increase by 0.52% (4) Decrease by 0.52%

Sol. Answer (1)

$w_{eq} = mg - m\omega^2 R_e$

$w_p = mg$

$$\frac{w_p - w_{eq}}{w_{eq}} = \frac{m\omega^2 R}{mg - m\omega^2 R}$$

$$= \frac{\omega^2 R}{g - \omega^2 R} \qquad [\omega^2 R = 0.0337 \text{ m/s}^2]$$

$$\Rightarrow \quad \frac{\Delta w}{w_{eq}} = \frac{0.0337}{9.81 - 0.0337} = 0.3447 \times 10^{-2}$$

$$\Rightarrow \quad \frac{\Delta w}{w_{eq}} \times 100 = 0.3447$$

$\Rightarrow$ Increases by 0.34%

21. If earth suddenly stop rotating, then the weight of an object of mass m at equator will [ω is angular speed of earth and R is its radius]

(1) Decrease by $m\omega^2 R$ (2) Increase by $m\omega^2 R$ (3) Decrease by $m\omega R^2$ (4) Increase by $m\omega R^2$

Sol. Answer (2)

At the equator, Apparent weight, $w' = w - m\omega^2 R$

If Earth stops rotating, w' will be equal to ω.

Thus, the weight of an object of mass m at equator will increase by $m\omega^2 R$.

22. If R is the radius of earth and g is the acceleration due to gravity on the earth's surface. Then mean density of earth is

(1) $\frac{4\pi G}{3gR}$ (2) $\frac{3\pi R}{4gG}$ (3) $\frac{3g}{4\pi RG}$ (4) $\frac{\pi Rg}{12G}$

Sol. Answer (3)

Acceleration due to gravity at earth's surface is given by,

$$g = \frac{GM}{R^2}$$

$$M = \frac{4}{3}\pi R^3 \rho$$

$M \to$ Mass of earth

$\rho \to$ Density of earth

$R \to$ Radius of earth

$$\Rightarrow \quad g = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho$$

$$\Rightarrow \quad \rho = \frac{3g}{4\,\pi GR}$$

23. The value of g at the surface of earth is 9.8 m/s^2. Then the value of 'g' at a place 480 km above the surface of the earth will be nearly (radius of the earth is 6400 km)

(1) 9.8 m/s^2 (2) 7.2 m/s^2 (3) 8.5 m/s^2 (4) 4.2 m/s^2

Sol. Answer (3)

$$g_h = g\left[\frac{R}{R+h}\right]^2$$

$$\Rightarrow \quad g_h = 9.8\left[\frac{6400}{6400+480}\right] = 8.48 \text{ m/s}^2$$

24. If the change in the value of 'g' at a height 'h' above the surface of the earth is same as at a depth x below it, then (x and h being much smaller than the radius of the earth)

(1) $x = h$ (2) $x = 2h$ (3) $x = \frac{h}{2}$ (4) $x = h^2$

Sol. Answer (2)

$$g_h = g\left(1 - \frac{2h}{R_e}\right)$$

$$g_x = g\left(1 - \frac{x}{R_e}\right)$$

According to the question,

$$g_h - g = g_x - g$$

$$\Rightarrow \quad g\left(-\frac{2h}{R_e}\right) = g\left(\frac{-x}{R_e}\right)$$

$$\Rightarrow \quad x = 2h$$

25. As we go from the equator to the poles, value of 'g'

(1) Remains the same (2) Decreases

(3) Increases (4) First increases and then decreases

Sol. Answer (3)

At Latitude λ,

$g' = g_0 - \omega^2 R \cos^2 \lambda$

at equator, $\lambda = 0$

$g' = g_0 - \omega^2 R$

at poles, $\lambda = 90°$

$g' = g_0$

$\Rightarrow$ As we g_0 from equator to the poles, value of g' increase.

26. What should be the angular speed with which the earth have to rotate on its axis so that a person on the equator would weigh $\frac{3}{5}$th as much as present?

(1) $\sqrt{\frac{2g}{5R}}$ (2) $\sqrt{\frac{2R}{5g}}$ (3) $\frac{2\sqrt{R}}{\sqrt{5g}}$ (4) $\frac{2g}{5R}$

Sol. Answer (1)

$w' = w - m\omega^2 R$

$\Rightarrow mg_e = mg - m\omega^2 R$

$mg_e = \frac{3}{5} mg$

$\Rightarrow m\omega^2 R = \frac{2}{5} mg$

$\Rightarrow \omega = \sqrt{\frac{2g}{5R}}$

27. The acceleration due to gravity on a planet is 1.96 m/s². If it is safe to jump from a height of 3 m on the earth, the corresponding height on the planet will be

(1) 3 m (2) 6 m (3) 9 m (4) 15 m

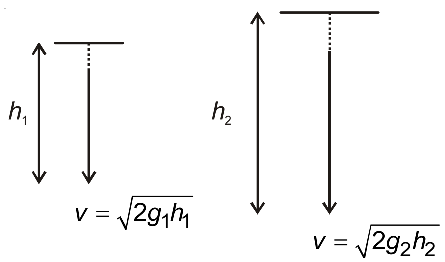
Sol. Answer (4)

It is safer to jump from a height of 3 m on earth,

$\Rightarrow$ Corresponding velocity attained = $\sqrt{2g_1h_1}$

It will be safer to jump from a height on other planet

If the velocity attained is same = $\sqrt{2g_2h_2}$



$\Rightarrow \sqrt{2g_1h_1} = \sqrt{2g_2h_2}$

$9.8 \times 3 = 1.96 \times h_2$

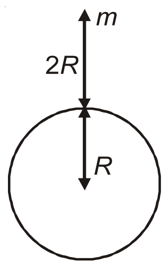
$\Rightarrow h_2 = 5 \times 3 = 15$ m

(Gravitational Potential Energy)

28. An object is taken to height $2R$ above the surface of earth, the increase in potential energy is [R is radius of earth]

(1) $\frac{mgR}{2}$ (2) $\frac{mgR}{3}$ (3) $\frac{2mgR}{3}$ (4) $2\ mgR$

Sol. Answer (3)



Potential energy at surface $= -\frac{GMm}{R}$

Potential energy at height, $2R = -\frac{GMm}{3R}$

Change in potential energy $= -\frac{GMm}{3R} + \frac{GMm}{R}$

$$= \frac{GMm}{R}\left(\frac{-1+3}{3}\right)$$

$$= \frac{2}{3}\frac{GMm}{R}$$

$$= \frac{2}{3}\left(\frac{GM}{R^2}\right)mR$$

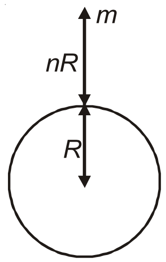
$$= \frac{2}{3}mgR$$

29. The change in potential energy when a body of mass m is raised to height nR from the earth's surface is (R is radius of earth)

(1) $mgR\left(\frac{n}{n-1}\right)$ (2) $nmgR$

(3) $mgR\left(\frac{n}{n+1}\right)$ (4) $mgR\left(\frac{n^2}{n^2+1}\right)$

Sol. Answer (3)



Potential energy at the surface $= -\frac{GMm}{R}$

Potential energy at height, $nR = -\frac{GMm}{(n+1)R}$

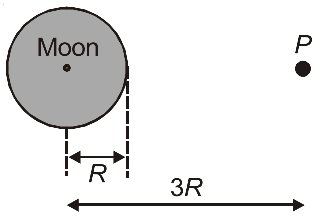
Change in potential energy $= -\frac{GMm}{(n+1)R} + \frac{GMm}{R}$

$$= \frac{GMm}{R}\left(\frac{-1+n+1}{n+1}\right)$$

$$= \left(\frac{n}{n+1}\right)\left(\frac{GM}{R^2}\right)m \times R$$

$$= mgR\left(\frac{n}{n+1}\right)$$

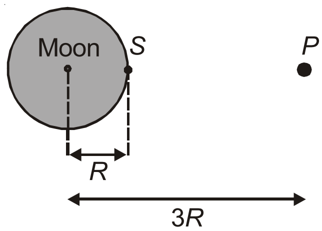
30. A stationary object is released from a point P at a distance $3R$ from the centre of the moon which has radius R and mass M. Which of the following gives the speed of the object on hitting the moon?



(1) $\left(\frac{2GM}{3R}\right)^{1/2}$ (2) $\left(\frac{4GM}{3R}\right)^{1/2}$ (3) $\left(\frac{GM}{3R}\right)^{1/2}$ (4) $\left(\frac{GM}{R}\right)^{1/2}$

Sol. Answer (2)

Conserving mechanical energy between points P and S,



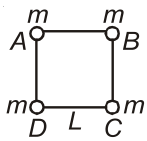
$$-\frac{GMm}{3R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\Rightarrow \frac{1}{2}mv^2 = -\frac{GMm}{3R} + \frac{GMm}{R} = \frac{GMm}{R}\left(\frac{-1}{3}+1\right)$$

$$\frac{1}{2}\cancel{m}v^2 = \frac{2GM\cancel{m}}{3R}$$

$$\Rightarrow v = \sqrt{\frac{4GM}{3R}}$$

31. Four particles A, B, C and D each of mass m are kept at the corners of a square of side L. Now the particle D is taken to infinity by an external agent keeping the other particles fixed at their respective positions. The work done by the gravitational force acting on the particle D during its movement is



(1) $2\frac{Gm^2}{L}$ (2) $-2\frac{Gm^2}{L}$ (3) $\frac{Gm^2}{L}\left(\frac{2\sqrt{2}+1}{\sqrt{2}}\right)$ (4) $-\frac{Gm^2}{L}\left(\frac{2\sqrt{2}+1}{\sqrt{2}}\right)$

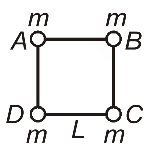
Sol. Answer (4)

Work done by the gravitational force acting on the particle D during its movement

$= -\Delta U$

$= -(U_{\text{final}} - U_{\text{initial}})$

$= U_{\text{initial}} - U_{\text{final}}$



Now, when the particle is at infinity, $U = 0$

$\Rightarrow U_{\text{final}} = 0$

$\Rightarrow$ Work done $= U_{\text{initial}}$

$$U_{\text{initial}} = -\frac{Gm^2}{L} - \frac{Gm^2}{L} - \frac{Gm^2}{\sqrt{2}L} = -\frac{Gm^2}{L}\left(2+\frac{1}{\sqrt{2}}\right) = -\frac{Gm^2}{L}\left(\frac{2\sqrt{2}+1}{\sqrt{2}}\right)$$

32. If an object is projected vertically upwards with speed, half the escape speed of earth, then the maximum height attained by it is [R is radius of earth]

(1) R (2) $\frac{R}{2}$ (3) $2R$ (4) $\frac{R}{3}$

Sol. Answer (4)

$$V_e = \sqrt{\frac{2GM}{R}}$$

$M \rightarrow$ mass of earth

$R \rightarrow$ Radius of earth

Now, conserving potential energy at the surface of earth and highest point,

$$-\frac{GMm}{R} + \frac{1}{2}m\left(\frac{1}{2}\sqrt{\frac{2GM}{R}}\right)^2 = -\frac{GMm}{r}$$

$$-\frac{GMm}{R} + \frac{GMm}{4R} = -\frac{GMm}{r}$$

$$-\frac{3GMm}{4R} = -\frac{GMm}{r}$$

$$\Rightarrow \quad r = \frac{4R}{3}$$

$$\Rightarrow \quad R + h = \frac{4R}{3}$$

$$\Rightarrow \quad h = \left(\frac{R}{3}\right)$$

33. If a satellite of mass 400 kg revolves around the earth in an orbit with speed 200 m/s then its potential energy is

(1) –1.2 MJ (2) –8.0 MJ

(3) –16 MJ (4) –2.4 MJ

Sol. Answer (3)

For a satellite,

$$\text{P.E} = -\frac{GMm}{r}$$

m = mass of satellite

r = radius of orbit

$$\text{K.E} = \frac{1}{2}mv^2 = \frac{GMm}{2r} = -\frac{\text{P.E}}{2}$$

$\Rightarrow$ P.E = – mv^2

= – 400 × 4 × 10^4

= – 16 MJ

34. An artificial satellite revolves around a planet for which gravitational force(F) varies with distance r from its centre as $F \propto r^2$. If v_0 its orbital speed, then

(1) $v_0 \propto r^{-1/2}$ (2) $v_0 \propto r^{3/2}$ (3) $v_0 \propto r^{-3/2}$ (4) $v_0 \propto r$

Sol. Answer (2)

Gravitational force (F) provides the necessary centripetal force to keep the satellite in orbit,

$$\Rightarrow \quad \frac{mv_0^2}{r} \propto F$$

$$\frac{mv_0^2}{r} \propto r^2$$

$v_0 \rightarrow$ Orbital speed

$r \rightarrow$ Radius of orbit

$$\Rightarrow \quad v_0 \propto r^{3/2}$$

35. If the gravitational potential on the surface of earth is V_0, then potential at a point at height half of the radius of earth is

(1) $\frac{V_0}{2}$ (2) $\frac{2}{3}V_0$ (3) $\frac{V_0}{3}$ (4) $\frac{3V_0}{2}$

Sol. Answer (2)

Gravitational potential on the surface,

$$V_0 = -\frac{GM_e}{R_e}$$

Gravitational potential at height h,

$$V_n = -\frac{GM_e}{\left(R_e + \frac{R_e}{2}\right)}$$

$$= -\frac{2}{3}\frac{GM_e}{R_e}$$

$$= \frac{2}{3}V_0$$

(Escape Speed)

36. The total mechanical energy of an object of mass m projected from surface of earth with escape speed is

(1) Zero (2) Infinite (3) $-\frac{GMm}{2R}$ (4) $-\frac{GMm}{3R}$

Sol. Answer (1)

Total mechanical energy = K.E + P.E

$$V_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \text{ Total mechanical energy } = \frac{1}{2}m \times \frac{2GM}{R} - \frac{GMm}{R} = 0$$

37. A body is thrown with a velocity equal to n times the escape velocity (v_e). Velocity of the body at a large distance away will be

(1) $v_e\sqrt{n^2-1}$ (2) $v_e\sqrt{n^2+1}$ (3) $v_e\sqrt{1-n^2}$ (4) None of these

Sol. Answer (1)

At large distance potential energy = 0

Conserving mechanical energy at surface of earth and large distance from earth,

$$\frac{1}{2}m(nv_e)^2 - \frac{GMm}{R} = \frac{1}{2}mv^2$$

Also, $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \frac{2GMm}{R}(n^2-1) = mv^2$$

$$\Rightarrow v = \sqrt{\frac{2GM}{R}}(n^2-1)^{1/2}$$

$$\Rightarrow v = v_e\sqrt{n^2-1}$$

38. The escape velocity of a body from earth is about 11.2 km/s. Assuming the mass and radius of the earth to be about 81 and 4 times the mass and radius of the moon, the escape velocity in km/s from the surface of the moon will be

(1) 0.54 (2) 2.48 (3) 11 (4) 49.5

Sol. Answer (2)

$$V_{\text{escape}} = \sqrt{\frac{GM}{R}}$$

$$\frac{V_{\text{escape Earth}}}{V_{\text{escape moon}}} = \sqrt{\frac{M_e}{R_e}\times\frac{R_m}{M_m}} = \sqrt{\frac{81}{4}} = \left(\frac{9}{2}\right)$$

$$\Rightarrow V_{\text{moon}} = \frac{2}{9}\times 11.2 = 2.48 \text{ km/s}$$

39. If M is mass of a planet and R is its radius then in order to become black hole [c is speed of light]

(1) $\sqrt{\frac{GM}{R}} \le c$ (2) $\sqrt{\frac{GM}{2R}} \ge c$ (3) $\sqrt{\frac{2GM}{R}} \ge c$ (4) $\sqrt{\frac{2GM}{R}} \le c$

Sol. Answer (3)

A planet can become a black hole if its mass and radius are such that it has an immense force of gravity on its surface. The force of attractum has to be so large that even light cannot escape from its surface.

Speed of light = c

$$v_e = \sqrt{\frac{2GM}{R}}$$

If $v_e \ge c$

$\Rightarrow$ Even light can't escape from the surface of such planet making it appear black.

40. The atmosphere on a planet is possible only if [where v_{rms} is root mean square speed of gas molecules on planet and v_e is escape speed on its surface]

(1) $v_{rms} = v_e$ (2) $v_{rms} > v_e$ (3) $v_{rms} \le v_e$ (4) $v_{rms} < v_e$

Sol. Answer (4)

The atmosphere on a planet is possible only if $v_{rms} < v_e$

If $v_{rms} \geq v_{escape}$ the gas molecules will leave the surroundings of the planet, i.e., will be free from gravitational pull of the planet.

41. When speed of a satellite is increased by x percentage, it will escape from its orbit, where the value of x is

(1) 11.2% (2) 41.4% (3) 27.5% (4) 34.4%

Sol. Answer (2)

For a satellite near Earth's surface,

$$v_0 = \sqrt{\frac{GM_e}{R_e}},\ v_e = \sqrt{\frac{2GM_e}{R_e}}$$

$$v_e = \sqrt{2}\,v_0$$

$\Rightarrow$ % increase, $x = \left(\frac{\sqrt{2}-1}{1}\right)\times 100 = 41.4\%$

(Earth Satellite, Energy of an Orbiting Satellite)

42. In an orbit if the time of revolution of a satellite is T, then PE is proportional to

(1) $T^{1/3}$ (2) T^3 (3) $T^{-2/3}$ (4) $T^{-4/3}$

Sol. Answer (3)

According to Kepler's third law,

$T^2 \propto r^3$

r = radius of orbit

For a satellite of mass m orbiting in an orbit of radius r around planet of mass M,

$$\text{Potential energy (PE)} = \frac{-GMm}{r}$$

$$\Rightarrow \text{PE} \propto \frac{GMm}{T^{2/3}}$$

$$\Rightarrow \text{PE} \propto T^{-2/3}$$

43. A small satellite is revolving near earth's surface. Its orbital velocity will be nearly

(1) 8 km/s (2) 11.2 km/s (3) 4 km/s (4) 6 km/s

Sol. Answer (1)

For a satellite revolving near earth's surface, $v_0 = \sqrt{\frac{GM_e}{R_e}} = \sqrt{gR_e}$

Taking $g = 9.81\ \text{m/s}^2$ and $R_e = 6400$ km

$$v_0 = \sqrt{\frac{9.8}{1000}\times 6400} = 7.92\ \text{km/s} \simeq 8\ \text{km/s}$$

44. If potential energy of a satellite is –2MJ, then the binding energy of satellite is

(1) 1 MJ (2) 2 MJ (3) 8 MJ (4) 4 MJ

Sol. Answer (1)

For a satellite of mass *m* revolving around a planet of mass in a circular orbit of radius *r*,

$$P.E = -\frac{GMm}{r}$$

$$K.E = \frac{1}{2}m\frac{GM}{r} = \frac{GMm}{2r}$$

$$T.E = -\frac{GMm}{2r}$$

$$\text{Binding energy} = |T.E| = \frac{GMm}{2r}$$

$$= \frac{|P.E|}{2} = 1\,\text{MJ}$$

Alternate method,

$$\text{Binding energy} = -\,T.E$$

$$= -\frac{P.E}{2}$$

$$= 1\text{ MJ}$$

(Geostationary and Polar Satellites)

45. The time period of polar satellites is about

(1) 24 hr (2) 100 min (3) 84.6 min (4) 6 hr

Sol. Answer (2)

Time period of polar satellites is about 100 minutes polar satellites are low altitude satellites. ($h \simeq$ 500 – 800 km)

46. The mean radius of earth is R, and its angular speed on its axis is ω. What will be the radius of orbit of a geostationary satellite?

(1) $\left(\frac{Rg}{\omega^2}\right)^{1/3}$ (2) $\left(\frac{R^2g}{\omega^2}\right)^{1/3}$ (3) $\left(\frac{R^2g}{\omega}\right)^{1/3}$ (4) $\left(\frac{R^2\omega^2}{g}\right)^{1/3}$

Sol. Answer (2)

Time period of rotation of earth $= \frac{2\pi}{\omega}$

(Duration of one day)

Geostationary satellite has same time period, $T = \frac{2\pi}{\omega}$. Let r be the radius of orbit of satellite

$\Rightarrow$ Time period of satellite $= \frac{2\pi r^{3/2}}{\sqrt{GM_e}}$

Also, $g = \frac{GM_e}{R_e^2}$

$$\Rightarrow \quad T = \frac{2\pi\, r^{3/2}}{\sqrt{g}(R_e)} = \frac{2\pi\, r^{3/2}}{R_e\sqrt{g}} = \frac{2\pi}{\omega}$$

$$\Rightarrow \quad r^{3/2} = \frac{R_e}{\omega}\sqrt{g}$$

$$\Rightarrow \quad r = \left(\frac{R_e^2}{\omega^2}g\right)^{1/3}$$

47. A satellite of the earth is revolving in a circular orbit with a uniform speed v. If the gravitational force suddenly disappears, the satellite will

(1) Continue to move with velocity v along the original orbit

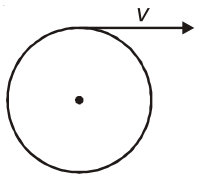
(2) Move with a velocity v, tangentially to the original orbit

(3) Fall down with increasing velocity

(4) Ultimately come to rest somewhere on the original orbit

Sol. Answer (2)

For a satellite revolving in a circular orbit, gravitational force provides the necessary centripetal force. If the gravitational force suddenly disappears, the satellite will move with a velocity v, tangentally to the original orbit.



48. The relay satellite transmits the television signals continuously from one part of the world to another because its

(1) Period is greater than the period of rotation of the earth

(2) Period is less than the period of rotation of the earth

(3) Period has no relation with the period of the earth about its axis

(4) Period is equal to the period of rotation of the earth about its axis

Sol. Answer (4)

A relay satellite transmits the television signals continuously from one part of the world to another bacause its period is equal to the period of rotation of the earth about its axis.

49. If height of a satellite from the surface of earth is increased, then its

(1) Potential energy will increase

(2) Kinetic energy will decrease

(3) Total energy will increase

(4) All of these

Sol. Answer (4)

For a satellite orbiting at height h from earth,

$$P.E = -\frac{GM_e m_s}{(R_e + h)}$$

$$K.E = \frac{GM_e m_s}{2(R_e + h)}$$

$$T.E = -\frac{GM_e m_s}{2(R_e + h)}$$

If h is increased, P.E increases (becomes less negative)

K.E decreases

T.E increases (becomes less negative)

SECTION - B

Objective Type Questions

(Kepler's Laws)

1. The ratio of kinetic energy of a planet at perigee and apogee during its motion around the sun in elliptical orbit of eccentricity e is

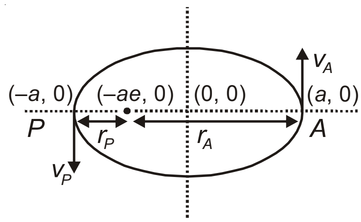
(1) $1 : e$ (2) $\frac{1+e}{1-e}$ (3) $\left(\frac{1+e}{1-e}\right)^2$ (4) $\left(\frac{1-e}{1+e}\right)^2$

Sol. Answer (3)

K.E of a planet = $\frac{1}{2}mv^2$

K.E at perigee = $\frac{1}{2}mv_P^2$

K.E at apogee = $\frac{1}{2}mv_A^2$



Using conservation of angular momentum at P and A

$\Rightarrow mv_Pr_P = mv_Ar_A$

$\Rightarrow \frac{v_P}{v_A} = \frac{r_A}{r_P} = \frac{a(1+e)}{a(1-e)}$

$\Rightarrow \frac{\text{K.E}_P}{\text{K.E}_A} = \frac{v_P^2}{v_A^2} = \left(\frac{1+e}{1-e}\right)^2$

2. An earth satellite X is revolving around earth in an orbit whose radius is one-fourth of the radius of orbit of a communication satellite. Time period of revolution of X is

(1) 3 hrs (2) 6 hrs

(3) 4 days (4) 72 days

Sol. Answer (1)

Time period of a communication satellite = 24 hours.

Using kepler's third law, $T^2 \propto r^3$

$\Rightarrow \frac{T_c}{T_x} = \left(\frac{r_c}{r_x}\right)^{3/2}$

$\Rightarrow \frac{24}{T_x} = (4)^{3/2}$

$\Rightarrow T_x = \frac{24}{8} = 3$ hrs

3. Two satellites of equal mass are revolving around earth in elliptical orbits of different semi-major axis. If their angular momenta about earth centre are in the ratio 3 : 4 then ratio of their areal velocity is

(1) $\frac{3}{4}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{4}{3}$

Sol. Answer (1)

Areal velocity, $\dfrac{\Delta A}{\Delta t} = \dfrac{|\vec{L}|}{2m} = v_A$

$\vec{L}$ is the angular momentum of satellite, m is the mass of satellite,

$\Rightarrow \dfrac{v_{A1}}{v_{A2}} = \dfrac{|\vec{L}_1|}{|\vec{L}_2|} = \left(\dfrac{3}{4}\right)$

4. When a satellite moves around the earth in a certain orbit, the quantity which remains constant is

(1) Angular velocity (2) Kinetic energy

(3) Areal velocity (4) Potential energy

Sol. Answer (3)

The path of a satellite moving around sun in a certain orbit is not exactly circular but elliptical with low value of eccentricity, e. Thus only areal velocity is constant.

5. Consider a planet moving around a star in an elliptical orbit with period T. The area of the elliptical orbit is proportional to

(1) $T^{\frac{4}{3}}$ (2) T (3) $T^{\frac{2}{3}}$ (4) $T^{\frac{1}{2}}$

Sol. Answer (1)

$A = \pi a^2$

So, $a \propto A^{1/2}$

According to Keplar's 3rd law

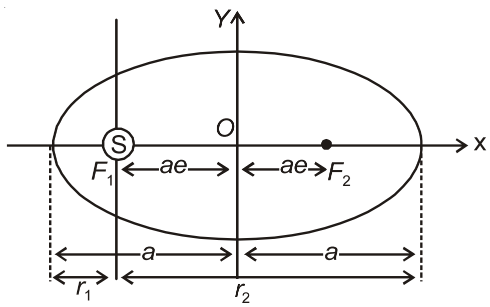
$T^2 \propto a^3$

$T^2 \propto [(A)^{1/2}]^3$

$T^2 \propto A^{3/2}$

$A \propto (T^2)^{2/3}$

$A \propto T^{4/3}$



(The Gravitational Constant, Acceleration Due to Gravity of the Earth)

6. If all objects on the equator of earth feel weightless then the duration of the day will nearly become

(1) 6.2 hr (2) 4.4 hr (3) 2.2 hr (4) 1.41 hr

Sol. Answer (4)

$W_{eq} = mg - m\omega^2 R$

$\Rightarrow mg - m\omega^2 R = 0$

$\Rightarrow \omega^2 R = g$

$\omega = \sqrt{\dfrac{g}{R}}$

$\Rightarrow$ Time period of rotation, *i.e.,* duration of the day,

$= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{R}{g}} = 1.41\,\text{hr}$

7. A satellite of mass m is revolving close to surface of a planet of density d with time period T. The value of universal gravitational constant on planet is given by

(1) $2d^2T\pi$ (2) $dT^2\pi$ (3) $\dfrac{1}{d^2T\pi}$ (4) $\dfrac{3\pi}{dT^2}$

Sol. Answer (4)

Time period of a satellite revolving close to surface,

$$T = \frac{2\pi R}{v} = \frac{2\pi R^{3/2}}{\sqrt{GM}}$$

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$M = \frac{4}{3}\pi R^3 \times d$$

$$T^2 = \frac{4\pi^2 R^3}{G\frac{4}{3}\pi R^3 d}$$

$$G = \frac{3\pi}{dT^2}$$

8. If gravitational field intensity is E at distance $R/2$ outside from then surface of a thin shell of radius R, the gravitational field intensity at distance $R/2$ from its centre is

(1) Zero (2) $2E$ (3) $\dfrac{2E}{3}$ (4) $\dfrac{3E}{2}$

Sol. Answer (1)

Gravitational field intensity at every point inside a hollow spherical shell of uniform density is zero, because gravitational field due to various regions of the spherical shell cancels each other completely as their vector sum is zero.

9. If acceleration due to gravity at distance d[< R] from the centre of earth is β, then its value at distance d above the surface of earth will be [where R is radius of earth]

(1) $\dfrac{\beta R^2}{(R+d)^3}$ (2) $\dfrac{\beta R}{2d}$ (3) $\dfrac{\beta d}{(R+d)^2}$ (4) $\dfrac{\beta R^3}{d(R+d)^2}$

Sol. Answer (4)

Here, $g_d = \dfrac{GM}{R^3}d = \beta, \quad d < R \qquad ...(1)$

$$g'_d = \frac{GM}{(R+d)^2}, \; d > R \qquad ...(2)$$

Using (1), $GM = \dfrac{\beta R^3}{d}$

$$g'_d = \frac{\beta R^3}{d(R+d)^2}$$

10. Gravitational potential in a region is given by $V = -(x + y + z)$ J/kg. Find the gravitational intensity at (2, 2, 2)

(1) $(\hat{i}+\hat{j}+\hat{k})$ N/kg (2) $2(\hat{i}+\hat{j}+\hat{k})$ N/kg (3) $3(\hat{i}+\hat{j}+\hat{k})$ N/kg (4) $4(\hat{i}+\hat{j}+\hat{k})$ N/kg

Sol. Answer (1)

Let I denote the gravitation intensity at any point,

As we know, $\vec{I} = -\left[\frac{\partial v}{\partial x}\hat{i} + \frac{\partial v}{\partial y}\hat{j} + \frac{\partial v}{\partial z}\hat{k}\right]$

$\Rightarrow$ $\vec{I}_{(2,2,2)} = -\left[-\hat{i}-\hat{j}-\hat{k}\right] = \hat{i}+\hat{j}+\hat{k}$ N/kg

11. A body weighs 72 N on surface of the earth. When it is taken to a height of $h = 2R$, where R is radius of earth, it would weigh

(1) 36 N (2) 18 N (3) 9 N (4) 8 N

Sol. Answer (4)

Weight on earth = $mg = m\times\frac{GM}{R^2} = 72$ N

Weight at height, $h = 2R$ will be

$$mg' = m\left(\frac{GM}{r^2}\right) = m\times\frac{GM}{(R+2R)^2} = \frac{GMm}{9R^2} = \frac{72}{9} = 8\text{ N}$$

(Gravitational Potential Energy)

12. A body is projected vertically upwards with a speed of $\sqrt{\frac{GM}{R}}$ (M is mass and R is radius of earth). The body will attain a height of

(1) $\frac{R}{2}$ (2) R (3) $\frac{5}{4}R$ (4) $\frac{3R}{2}$

Sol. Answer (2)

Conserving mechanical energy at earth surface and at the maximum height attained by the body,

$$\text{P.E}_i + \text{K.E}_i = \text{P.E}_j + \text{K.E}_j$$

$$-\frac{GMm}{R} + \frac{1}{2}m\left(\frac{GM}{R}\right) = -\frac{GMm}{r} + 0$$

$$\Rightarrow -\frac{GMm}{2R} = -\frac{GMm}{r}$$

$\Rightarrow$ $r = 2R$

$\Rightarrow$ $R + h = 2R$

$h = R$

13. If the gravitational potential energy of two point masses infinitely away is taken to be zero then gravitational potential energy of a galaxy is

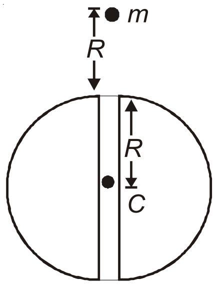
(1) Zero (2) Positive

(3) Negative (4) Can have any value

Sol. Answer (3)

A galaxy is a bounded system, for a bounded system or closed system like planet-sun, satellite Earth, electron-nucleus etc. total energy and the potential energy both are negative.

14. A particle of mass m is dropped from a height R equal to the radius of the earth above the tunnel dug through the earth as shown in the figure. Hence the correct statement is



(1) Particle will oscillate through the earth to a height $h = R$ on both sides

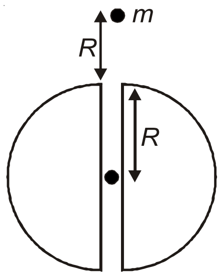
(2) Motion of the particle is periodic

(3) Motion of the particle is simple harmonic

(4) Both (1) & (2)

Sol. Answer (4)

When the particle is outside the tunnel force acting on it is $\propto \frac{1}{r^2}$ where, r is the distance from the centre of the earth.



When the particle is inside the tunnel force acting on it is $\propto \frac{1}{r}$ where, r is the distance from the centre of the earth.

In both cases, force is always directed towards the centre of the earth.

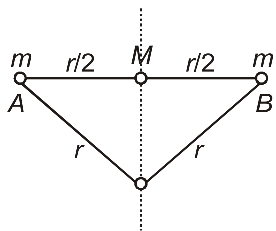
Thus motion is oscillatory and also periodic but not SHM.

15. The particles A and B of mass m each are separated by a distance r. Another particle C of mass M is placed at the midpoint of A and B. Find the work done in taking C to a point equidistant r from A and B without acceleration (G = Gravitational constant and only gravitational interaction between A, B and C is considered)

(1) $\frac{GMm}{r}$ (2) $\frac{2GMm}{r}$ (3) $\frac{3GMm}{r}$ (4) $\frac{4GMm}{r}$

Sol. Answer (2)

Since particle C is moved without any acceleration,



$\Rightarrow$ ΔK.E = 0

$\Rightarrow$ Work done by external agent + $W_{\text{gravitation}} = 0$

$\Rightarrow$ Work done by external agent = $-Wg$

$= -(-\Delta U)$

$= \Delta U$

$= U_f - U_{in}$

$$U_f = -\frac{GMm}{r} - \frac{GMm}{r} = -\frac{2GMm}{r}$$

$$U_i = -\frac{GMm}{r/2} - \frac{GMm}{r/2} = -\frac{4GMm}{r}$$

$\Rightarrow$ Work done $= \frac{2GMm}{r}$

16. The magnitude of potential energy per unit mass of an object at the surface of earth is E, then the escape velocity of the object is

(1) $\sqrt{2E}$ (2) $4E^2$ (3) $\sqrt{E}$ (4) $2E$

Sol. Answer (1)

P.E of an object on earth surface $= -\dfrac{GMm}{R}$

Magnitude of potential energy per unit mass $= \left(\dfrac{GM}{R}\right) = E$

$V_{\text{escape}} = \sqrt{\dfrac{2GM}{R}} = \sqrt{2E}$

17. If radius of an orbiting satellite is decreased, then its kinetic energy

(1) And potential energy decrease (2) And potential energy increase

(3) Decreases and potential energy increases (4) Increases and potential energy decreases

Sol. Answer (4)

$$\text{K.E} = \frac{GMm}{2r}$$

$$\text{P.E} = -\frac{GMm}{r}$$

$M \rightarrow$ mass of planet

$m \rightarrow$ mass of satellite

$r \rightarrow$ radius of orbit

When r is decreased,

Kinetic energy increases,

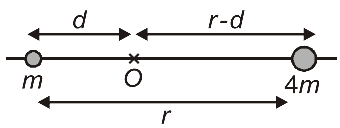
Potential energy decreases (becomes more negative).

18. Two point masses having mass M and $4M$ are placed at distance r. The gravitational potential at a point, where gravitational field intensity zero is

(1) $\dfrac{-9GM}{r}$ (2) $\dfrac{-2GM}{3r}$ (3) $\dfrac{-3GM}{r}$ (4) $\dfrac{-6GM}{5r}$

Sol. Answer (1)

Gravitational field intensity at O is zero,



$$\Rightarrow \frac{Gm}{d^2} = \frac{4Gm}{(r-d)^2}$$

$$\Rightarrow \frac{(r-d)^2}{d^2} = 4$$

$$\frac{r-d}{d} = \pm 2$$

$r - d = \pm 2d$

$\Rightarrow d = \dfrac{r}{3}, -r$ ($d = -r$, not possible)

Taking the +ve value of d,

Calculating gravitational potential at O,

$$V = \frac{-Gm}{r/3} - \frac{4Gm}{2r/3} = \frac{-3Gm}{r} - \frac{6Gm}{r} = \frac{-9Gm}{r}$$

19. If potential at the surface of earth is assigned zero value, then potential at centre of earth will be (Mass = M, Radius = R)

(1) 0 (2) $-\frac{GM}{2R}$ (3) $-\frac{3GM}{2R}$ (4) $\frac{3GM}{2R}$

Sol. Answer (2)

The concept involved here is that,

Gravitational potential difference between any two points in a gravitational field is independent of the choice of reference. When potential at the infinity is assigned zero value,

Potential at the surface $= -\frac{GM}{R} = V_s$

Potential at the centre $= -\frac{3GM}{2R} = V_c$

$$V_s - V_c = -\frac{GM}{R} + \frac{3GM}{2R} = \frac{GM}{2R}$$

Now, when potential at the surface is assigned zero value,

$$V_s - V_c = V_s' - V_c'$$

$$\Rightarrow \frac{GM}{2R} = 0 - V_c' \Rightarrow V_c' = -\frac{GM}{2R}$$

Here, V_s' and V_c' are the new values of potential at the sum and centre respectively.

20. If potential energy of a body of mass m on the surface of earth is taken as zero then its potential energy at height h above the surface of earth is [R is radius of earth and M is mass of earth]

(1) $\frac{-GMm}{R+h}$ (2) $\frac{-GMm}{h}$ (3) $\frac{GMmh}{R(R+h)}$ (4) $\frac{GMmh}{h+2R}$

Sol. Answer (3)

The concept involved here is that,

Gravitational potential energy difference between any two points in a gravitational field is independent of the choice of reference.

When potential at infinity is assigned zero value,

Potential energy of a body of mass m on the surface of earth $= \frac{-GMm}{R} = U_s$

Potential energy at height, $h = \frac{-GMm}{R+h} = U_n$

$$U_s - U_h = +GMm\left(-\frac{1}{R} + \frac{1}{R+h}\right)$$

$$= GMm\left(\frac{-R-h+R}{R(R+h)}\right)$$

$$= \frac{-GMmh}{R(R+h)}$$

Now, when potential at the surface is taken zero, Let U_s', U_h' be the new values of potential energy at the surface and height h respectively,

And, $U_s - U_h = U_s' - U_h'$

$$\Rightarrow \frac{-GMmh}{R(R+h)} = 0 - U_h'$$

$$\Rightarrow U_h' = \frac{GMmh}{R(R+h)}$$

21. A particle is projected vertically up with velocity $v = \sqrt{\frac{4gR_e}{3}}$ from earth surface. The velocity of particle at height equal to half of the maximum height reached by it

(1) $\sqrt{\frac{gR_e}{2}}$ (2) $\sqrt{\frac{gR_e}{3}}$ (3) $\sqrt{gR_e}$ (4) $\sqrt{\frac{2gR_e}{3}}$

Sol. Answer (2)

Conserving mechanical energy at the surface of earth and the maximum height attained,

$$\frac{-GMm}{R_e} + \frac{1}{2}m\frac{4GM}{3R_e^2}R_e = \frac{-GMm}{r} + 0$$

P.E$_i$ + K.E$_i$ = P.E$_f$ + K.E$_f$

$$\Rightarrow \frac{-GMm}{R_e} + \frac{2GMm}{3R^e} = \frac{-GMm}{r}$$

$$-\frac{1}{3}\frac{GMm}{R_e} = \frac{-GMm}{r}$$

$\Rightarrow r = 3R_e$

$\Rightarrow R_e + h = 3R_e$

$h = 2R_e$

Now, let us calculate the velocity of the particle at height equal to half of the maximum height *i.e* at $h = R_e$

Again using mechanical conservation of energy,

P.E$_i$ + K.E$_i$ = P.E$_j$ + K.E$_j$

$$\frac{-GMm}{R_e} + \frac{1}{2}m\frac{4}{3}\frac{GM}{R_e^2}\times R_e = \frac{-GMm}{2R} + \frac{1}{2}mv^2$$

$$\Rightarrow -\frac{1}{3}\frac{GMm}{R_e} + \frac{GMm}{2R_e} = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{GMm}{6R_e} = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{GM}{3R_e}} = \sqrt{\frac{GM}{R_e^2}\times\frac{R_e}{3}} = \sqrt{\frac{gR_e}{3}}$$

(Earth Satellite)

22. The orbital speed of a satellite revolving around a planet in a circular orbit is v_0. If its speed is increased by 10%, then

    (1) It will escape from its orbit

    (2) It will start rotating in an elliptical orbit

    (3) It will continue to move in the same orbit

    (4) It will move in a circular orbit of radius 20% more then radius of initial orbit

Sol. Answer (2)

When the orbital speed of a satellite revolving around a planet is increased by 10%, it corresponds to the case when $v_0 < v < v_e$.

23. If L is the angular momentum of a satellite revolving around earth in a circular orbit of radius r with speed v, then

    (1) $L \propto v$ (2) $L \propto r$ (3) $L \propto \sqrt{r}$ (4) $L \propto \sqrt{v}$

Sol. Answer (3)

Angular momentum of a satellite revolving around earth in a circular orbit, $L = mvr$

$m \rightarrow$ mass of satellite

$v \rightarrow$ speed of satellite

$r \rightarrow$ radius of orbit

$\Rightarrow \quad L = mvr$

Also, $v = \sqrt{\dfrac{GM}{r}}$

$$\Rightarrow \quad L = m\sqrt{\frac{GM}{r}}\,r$$

Thus, $L \propto \sqrt{r}$

24. Two satellites of mass m and 2 m are revolving in two circular orbits of radii r and $2r$ around an imaginary planet, on the surface of which gravitational force is inversely proportional to distance from its centre. The ratio of orbital speed of satellites is

    (1) 1 : 1 (2) 1 : 2 (3) 2 : 1 (4) $1:\sqrt{2}$

Sol. Answer (1)

Force of gravitation provides the necessary centripetal force,

$$\frac{mv^2}{r} = \frac{GMm}{r} \Rightarrow v = \sqrt{GM}$$

Independent of mass of satellite and radius of orbit.

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{1}{1}$$

25. When energy of a satellite-planet system is positive then satellite will

    (1) Move around planet in circular orbit (2) Move around planet in elliptical orbit

    (3) Escape out with minimum speed (4) Escape out with speed greater than escape velocity

Sol. Answer (4)

When the energy of a satellite-planet system is positive, satellite escapes away from the gravitational field of the planet with speed greater than the escape speed.

26. An object is projected horizontally with speed $\frac{1}{2}\sqrt{\frac{GM}{R}}$, from a point at height 3 R [where R is radius and M is mass of earth, then object will]

(1) Fall back on surface of earth by following parabolic path

(2) Fall back on surface of earth by following hyperbolic path

(3) Start rotating around earth in a circular orbit

(4) Escape from gravitational field of earth

Sol. Answer (3)

At height 3 R, i.e at distance 4 R from the centre of the earth, $V_{\text{orbital}} = \sqrt{\frac{GM}{r}}$

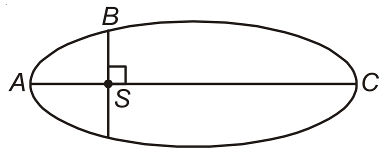
Here, $r = 4\,R \Rightarrow V_0 = \sqrt{\frac{GM}{4R}} = \frac{1}{2}\sqrt{\frac{GM}{R}}$,

Thus, an object taken to a height 3 R if projected horizontally with speed $\frac{1}{2}\sqrt{\frac{GM}{R}}$, will start rotating around earth in a circular orbit.

SECTION - C

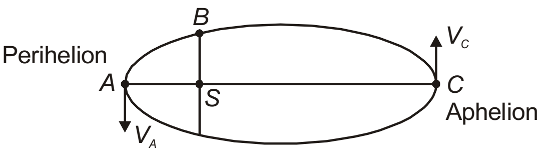
Previous Years Questions

1. The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are K_A, K_B and K_C, respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then **[NEET-2018]**



(1) $K_A < K_B < K_C$ (2) $K_A > K_B > K_C$ (3) $K_B > K_A > K_C$ (4) $K_B < K_A < K_C$

Sol. Answer (2)



Point A is perihelion and C is aphelion.

So, $V_A > V_B > V_C$

So, $K_A > K_B > K_C$

2. If the mass of the Sun were ten times smaller and the universal gravitational constant were ten times larger in magnitude, which of the following is not correct? **[NEET-2018]**

(1) Raindrops will fall faster

(2) Walking on the ground would become more difficult

(3) g on the Earth will not change

(4) Time period of a simple pendulum on the Earth would decrease

Sol. Answer (3)

If Universal Gravitational constant becomes ten times, then $G' = 10G$

So, acceleration due to gravity increases.

i.e., (3) is wrong option.

3. The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then **[NEET-2017]**

(1) $d = \frac{1}{2}$ km (2) $d = 1$ km

(3) $d = \frac{3}{2}$ km (4) $d = 2$ km

Sol. Answer (4)

Above earth surface	Below earth surface
$g' = g\left(1 - \frac{2h}{R_e}\right)$	$g' = g\left(1 - \frac{d}{R_e}\right)$
$\Delta g' = g\frac{2h}{R_e}$ ...(1)	$\Delta g = g\frac{d}{R_e}$ ...(2)

From (1) & (2)

$d = 2h \Rightarrow d = 2 \times 1$ km

4. Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will **[NEET-2017]**

(1) Keep floating at the same distance between them

(2) Move towards each other

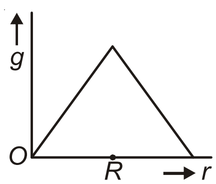
(3) Move away from each other

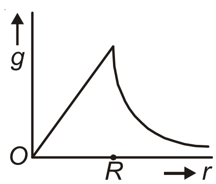
(4) Will become stationary

Sol. Answer (2)

Both the astronauts are in the condition of weightness. Gravitational force between them pulls towards each other.

5. Starting from the centre of the earth having radius R, the variation of g (acceleration due to gravity) is shown by **[NEET (Phase-2) 2016]**

(1)


(2)


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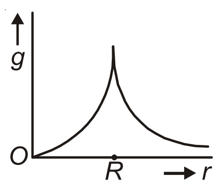

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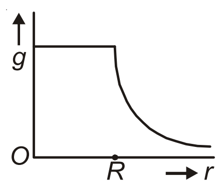
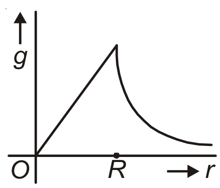


Sol. Answer (2)

$$g_{in} = \frac{GMr}{R^3} \Rightarrow g_{in} \propto r$$

$$g_{out} = \frac{GM}{r^2} \Rightarrow g_{out} \propto \frac{1}{r^2}$$



6. A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface, is **[NEET (Phase-2) 2016]**

(1) $\frac{mg_0R^2}{2(R+h)}$ (2) $-\frac{mg_0R^2}{2(R+h)}$ (3) $\frac{2mg_0R^2}{R+h}$ (4) $-\frac{2mg_0R^2}{R+h}$

Sol. Answer (2)

Total energy = $-\frac{GMm}{2r}$

Here, $r = R + h$ and $GM = g_0R^2$

$\Rightarrow \quad E = -\frac{mg_0R^2}{2(R+h)}$

7. At what height from the surface of earth the gravitation potential and the value of g are -5.4×10^7 J kg^{-2} and 6.0 ms^{-2} respectively? Take the radius of earth as 6400 km **[NEET-2016]**

(1) 2000 km (2) 2600 km

(3) 1600 km (4) 1400 km

Sol. Answer (2)

$V = -\frac{GM}{(R+h)} \qquad g' = \frac{GM}{(R+h)^2}$

$\Rightarrow \quad \frac{|V|}{g'} = R+h$

$\Rightarrow \quad \frac{5.4\times10^7}{6.0} = R+h$

$\Rightarrow \quad 9 \times 10^6 = R + h$

$\Rightarrow \quad h = (9 - 6.4) \times 10^6 = 2.6 \times 10^6 = 2600$ km

8. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is **[NEET-2016]**

(1) $1:\sqrt{2}$ (2) 1 : 2 (3) $1:2\sqrt{2}$ (4) 1 : 4

Sol. Answer (3)

$v_e = \sqrt{2qR} = R\sqrt{\frac{8}{3}\pi G\rho}$

$\Rightarrow \quad \frac{v_e}{v_p} = \frac{R\sqrt{\rho}}{R_p\sqrt{\rho}} = \frac{1}{2\sqrt{2}}$

9. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Then, **[Re-AIPMT-2015]**

(1) The acceleration of S is always directed towards the centre of the earth

(2) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant

(3) The total mechanical energy of S varies periodically with time

(4) The linear momentum of S remains constant in magnitude

Sol. Answer (1)

10. A remote-sensing satellite of earth revolves in a circular orbit at a height of 0.25×10^6 m above the surface of earth. If earth's radius is 6.38×10^6 m and $g = 9.8$ ms^{-2}, then the orbital speed of the satellite is

[Re-AIPMT-2015]

(1) 6.67 km s^{-1} (2) 7.76 km s^{-1}

(3) 8.56 km s^{-1} (4) 9.13 km s^{-1}

Sol. Answer (2)

11. Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between sun and planet, *i.e.,* $T^2 = Kr^3$, here K is constant. If the masses of sun and planet are M and m respectively then as per Newton's law of gravitation force of attraction between them is $F = \frac{GMm}{r^2}$, here G is gravitational constant. The relation between G and K is described as **[AIPMT-2015]**

(1) $K = \frac{1}{G}$ (2) $GK = 4\pi^2$

(3) $GMK = 4\pi^2$ (4) $K = G$

Sol. Answer (3)

12. A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass = 5.98×10^{24} kg) have to be compressed to be a black hole?

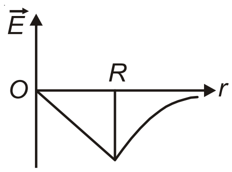
[AIPMT-2014]

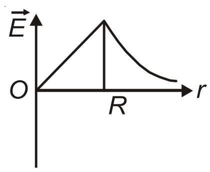
(1) 10^{-9} m (2) 10^{-6} m

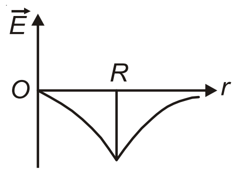
(3) 10^{-2} m (4) 100 m

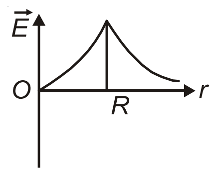
Sol. Answer (3)

13. Dependence of intensity of gravitational field ($\vec{E}$) of earth with distance (r) from centre of earth is correctly respresented by **[AIPMT-2014]**

(1) 

(2) 

(3) 

(4) 

Sol. Answer (1)

14. A body of mass m is taken from the earth's surface to the height equal to twice the radius (R) of the earth. The change in potential energy of body will be **[NEET-2013]**

(1) $\frac{2}{3}mgR$ (2) $3mgR$

(3) $\frac{1}{3}mgR$ (4) $mg2R$

Sol. Answer (1)

P.E at surface of earth $= \frac{-GMm}{R}$

$$= \frac{-GM}{R^2} \times mR$$

$$U_{in} = -mgR \qquad \left[g = \frac{GM}{R^2}\right]$$

P.E at height, $h = 2R$ $= \dfrac{-GMm}{3R}$

$$= \frac{-GM}{3R^2} \times mR$$

$$U_f = \frac{-mgR}{3}$$

$U_f - U_{in}$ = Change in P.E $= \dfrac{-mgR}{3} + mgR$

$$= \frac{2mgR}{3}$$

15. Infinite number of bodies, each of mass 2 kg are situated on x-axis at distances 1 m, 2 m, 4 m, 8 m, ..., respectively, from the origin. The resulting gravitational potential due to this system at the origin will be **[NEET-2013]**

(1) $-\dfrac{8}{3}G$ (2) $-\dfrac{4}{3}G$ (3) $-4G$ (4) $-G$

Sol. Answer (3)

16. A spherical planet has a mass M_p and diameter D_p. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity, equal to **[AIPMT (Prelims)-2012]**

(1) GM_p / D_p^2 (2) $4GM_pm / D_p^2$

(3) $4GM_p / D_p^2$ (4) GM_pm / D_p^2

Sol. Answer (3)

Acceleration due to gravity, near surface $= \dfrac{GM_p}{R_p^2} = g_p$

Here, $D_p = 2R_p$

$$\Rightarrow \quad g_p = \frac{4GM_p}{D_p^2}$$

17. A geostationary satellite is orbiting the earth at a height of $5R$ above that surface of the earth, R being the radius of the earth. The time period of another satellite in hours at a height of $2R$ from the surface of the earth is **[AIPMT (Prelims)-2012]**

(1) $6\sqrt{2}$ (2) $\dfrac{6}{\sqrt{2}}$ (3) 5 (4) 10

Sol. Answer (1)

Time period of a geostationary satellite = 24 hours.

Using Keplers third law,

$$T^2 \propto r^3$$

T_1, time period of geostationary satellite $\propto (6R)^{3/2}$ thus $T_2 \propto (3R)^{3/2}$

$$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{3R}{6R}\right)^{3/2}$$

$$\Rightarrow \quad T_2 = 24 \times \frac{1}{2^{3/2}} = \frac{24}{2\sqrt{2}} = 6\sqrt{2} \text{ hours.}$$

18. The height at which the weight of a body becomes $\frac{1}{16}$th, its weight on the surface of earth (radius R), is

[AIPMT (Prelims)-2012]

(1) $3R$ (2) $4R$ (3) $5R$ (4) $15R$

Sol. Answer (1)

Weight on surface of earth, $W = mg = m\left(\frac{GM}{R_e^2}\right)$

Weight at height h from surface, $W' = m\frac{GM}{(R_e + h)^2}$

$$\frac{W'}{W} = \frac{1}{16} = \frac{R_e^2}{(R_e + h)^2}$$

$\Rightarrow$ $R_e + h = 4R_e$

$\Rightarrow$ $h = 3R_e$

19. If v_e is escape velocity and v_o is orbital velocity of a satellite for orbit close to the earth's surface, then these are related by **[AIPMT (Mains)-2012]**

(1) $v_o = \sqrt{2}v_e$ (2) $v_o = v_e$ (3) $v_e = \sqrt{2v_o}$ (4) $v_e = \sqrt{2}v_o$

Sol. Answer (4)

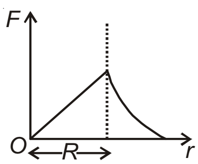
$$v_e = \sqrt{\frac{2GM}{R}}$$

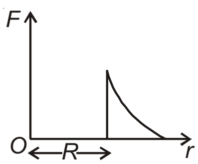
$$v_o = \sqrt{\frac{GM}{R}}$$

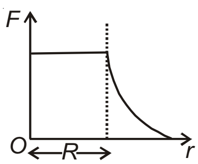
$\Rightarrow$ $v_e = \sqrt{2}v_o$

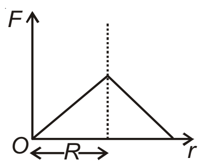
20. Which one of the following plots represents the variation of gravitational field with distance r due to a thin spherical shell of radius R? (r is measured from the centre of the spherical shell)

[AIPMT (Mains)-2012]

(1) 

(2) 

(3) 

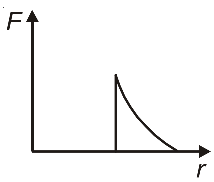
(4) 

Sol. Answer (2)

For a thin spherical shell gravitational field for $r < R$ is zero.

For a thin spherical shell gravitational field for $r > R$ is given by $F = \frac{GM}{r^2}$

Thus, most suitable plot is

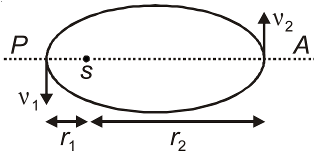


21. A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\frac{v_1}{v_2}$ is

[AIPMT (Prelims)-2011]

(1) $\frac{r_1}{r_2}$ (2) $\left(\frac{r_1}{r_2}\right)^2$ (3) $\frac{r_2}{r_1}$ (4) $\left(\frac{r_2}{r_1}\right)^2$

Sol. Answer (3)



Using conservation of angular momentum at *P* and *A*,

$\Rightarrow m\nu_1 r_1 = m\nu_2 r_2$

$\frac{\nu_1}{\nu_2} = \frac{r_2}{r_1}$

22. A particle of mass *M* is situated at the centre of a spherical shell of same mass and radius *a*. The magnitude of the gravitational potential at a point situated at $\frac{a}{2}$ distance from the centre, will be **[AIPMT (Mains)-2011]**

(1) $\frac{4GM}{a}$ (2) $\frac{GM}{a}$ (3) $\frac{2GM}{a}$ (4) $\frac{3GM}{a}$

Sol. Answer (4)

23. A particle of mass *m* is thrown upwards from the surface of the earth, with a velocity *u*. The mass and the radius of the earth are, respectively, *M* and *R*. *G* is gravitational constant and *g* is acceleration due to gravity on the surface of the earth. The minimum value of *u* so that the particle does not return back to earth, is

[AIPMT (Mains)-2011]

(1) $\sqrt{2gR^2}$ (2) $\sqrt{\frac{2GM}{R^2}}$ (3) $\sqrt{\frac{2GM}{R}}$ (4) $\sqrt{\frac{2gM}{R^2}}$

Sol. Answer (3)

Particle will not return back if it is thrown upwards with escape velocity,

$$\frac{-GMm}{R} + \frac{1}{2}mv_e^2 = 0$$

$$\Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

24. The radii of circular orbits of two satellites *A* and *B* of the earth, are 4*R* and *R*, respectively. If the speed of satellite *A* is 3*V*, then the speed of satellite *B* will be **[AIPMT (Prelims)-2010]**

(1) $\frac{3V}{4}$ (2) 6 V (3) 12 V (4) $\frac{3V}{2}$

Sol. Answer (2)

$$V_0 = \sqrt{\frac{GM_e}{r}}$$

$r \rightarrow$ radius of orbit

$$\frac{V_A}{V_B} = \sqrt{\frac{r_B}{r_A}}$$

$$\Rightarrow \quad \frac{3V}{V_B} = \sqrt{\frac{1}{4}} \quad \Rightarrow \quad V_B = 6\ V$$

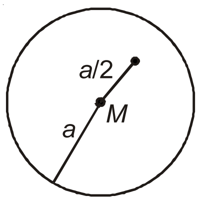
25. A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The gravitational potential at a point situated at $\frac{a}{2}$ distance from the centre, will be **[AIPMT (Prelims)-2010]**

(1) $-\frac{3GM}{a}$ (2) $-\frac{2GM}{a}$ (3) $-\frac{GM}{a}$ (4) $-\frac{4GM}{a}$

Sol. Answer (1)

Gravitational potential at a point situated at $\frac{a}{2}$ distance from the centre will be sum of potential due to spherical shell and due to mass M at the centre,

Thus, $V = V_1 + V_2$



$$V_{\text{spherical shell}} = \frac{-GM}{a} = V_1$$

$$V_{\text{mass } M} = \frac{-GM}{a/2} = \frac{-2GM}{a} = V_2$$

$$\Rightarrow \quad V_{\text{total}} = -\frac{3GM}{a}$$

26. The additional kinetic energy to be provided to a satellite of mass m revolving around a planet of mass M, to transfer it from a circular orbit of radius R_1 to another of radius R_2 ($R_2 > R_1$) is **[AIPMT (Mains)-2010]**

(1) $GmM\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)$ (2) $GmM\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (3) $2GmM\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (4) $\frac{1}{2}GmM\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Sol. Answer (4)

To find out the additional kinetic energy to be provided to a satellite of mass m,

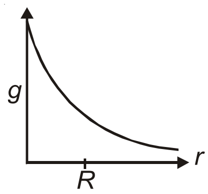
We can use conservation of mechanical energy,

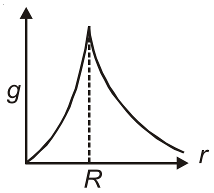
$$\text{K.E}_i + \text{P.E}_i + \text{K.E}_{\text{additional}} = \text{K.E}_f + \text{P.E}_f$$

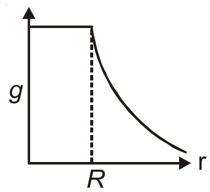
$$\frac{1}{2}\frac{GMm}{R_1} - \frac{GMm}{R_1} + \text{K.E}_{\text{additional}} = \frac{1}{2}\frac{GMm}{R_2} - \frac{GMm}{R_2}$$

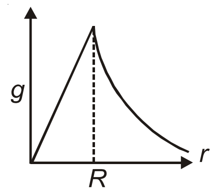
$$\Rightarrow \quad \text{K.E}_{\text{additional}} = \frac{GMm}{2}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

27. The dependence of acceleration due to gravity g on the distance r from the centre of the earth, assumed to be a sphere of radius R of uniform density is as shown in figures below

(a) 

(b) 

(c) 

(d) 

The correct figure is **[AIPMT (Mains)-2010]**

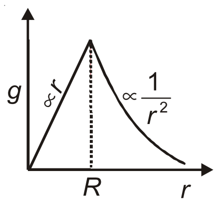
(1) (d) (2) (a) (3) (b) (4) (c)

Sol. Answer (1)

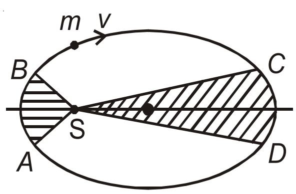
$$g_{\text{inside}} = \frac{GM}{R^3}r \quad i.e., \text{ for } r < R$$

$$g_{\text{outside}} = \frac{GM}{r^2} \quad i.e., \text{ for } r > R$$

The suitable graph is,



28. The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B then



[AIPMT (Prelims)-2009]

(1) $t_1 = 4t_2$ (2) $t_1 = 2t_2$ (3) $t_1 = t_2$ (4) $t_1 > t_2$

Sol. Answer (2)

According to Kepler's law of Areas : The line that joins any planet to the sun sweeps out equal areas in equal intervals of time

i.e $\frac{\Delta A}{\Delta t}$ is constant.

Area SCD = 2 × Area SAB

Using, $\frac{\Delta A_{SCD}}{\Delta A_{SAB}} = \frac{\Delta t_{SCD}}{\Delta t_{SAB}} = \left(\frac{t_1}{t_2}\right)$

$\Rightarrow$ $t_1 = 2t_2$

29. Two satellites of earth S_1 and S_2 are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true? **[AIPMT (Prelims)-2007]**

(1) The potential energies of earth and satellite in the two cases are equal

(2) S_1 and S_2 are moving with the same speed

(3) The kinetic energies of the two satellites are equal

(4) The time period of S_1 is four times that of S_2

Sol. Answer (2)

$v_0 = \sqrt{\frac{GM_e}{r}}$, r is the radius of the orbit.

Radius of orbit is same for both S_1 and S_2,

Thus, $v_{01} = v_{02}$

S_1 and S_2 are moving with the same speed.

30. The Earth is assumed to be a sphere of radius R. A platform is arranged at a height R from the surface of the Earth. The escape velocity of a body from this platform is fv, where v is its escape velocity from the surface of the Earth. The value of f is **[AIPMT (Prelims)-2006]**

(1) $\sqrt{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{3}$ (4) $\frac{1}{2}$

Sol. Answer (2)

Escape velocity from height, $h = R$ from earth can be evaluated using conservation of mechanical energy,

$$\frac{-GMm}{2R} + \frac{1}{2}m(v_e')^2 = 0$$

$$\Rightarrow v_e' = \sqrt{\frac{GM}{R}}$$

From surface of earth, $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow v_e' = \frac{1}{\sqrt{2}}v_e = fv$$

Thus, $f = \frac{1}{\sqrt{2}}$

31. Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g', then **[AIPMT (Prelims)-2005]**

(1) $g' = 3g$ (2) $g' = \frac{g}{9}$ (3) $g' = 9g$ (4) $g' = 27g$

Sol. Answer (1)

$$g = \frac{4\pi GR\rho}{3}$$

$$g' = \frac{4\pi G(3R)\rho}{3} = 3\times\left(\frac{4\pi GR\rho}{3}\right) = 3g$$

32. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is :

[AIPMT (Prelims)-2005]

(1) 2 (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

Sol. Answer (2)

$$\text{P.E} = \frac{-GMm}{R}$$

$$\text{K.E} = \frac{GMm}{2R}$$

$$\Rightarrow \left|\frac{\text{K.E}}{\text{P.E}}\right| = \frac{1}{2}$$

33. The radius of a planet is twice the radius of earth. Both have almost equal average mass-densities. If V_P and V_E are escape velocities of the planet and the earth, respectively, then

(1) $V_E = 1.5V_P$ (2) $V_P = 1.5V_E$ (3) $V_P = 2V_E$ (4) $V_E = 3V_P$

Sol. Answer (3)

$$V_{\text{escape}} = R\left(\sqrt{\frac{8\pi GP}{3}}\right)$$

$$\Rightarrow \frac{V_P}{V_E} = 2$$

$$V_P = 2V_E$$

34. A particle of mass 'm' is kept at rest at a height $3R$ from the surface of earth, where 'R' is radius of earth and 'M' is mass of earth. The minimum speed with which it should be projected, so that it does not return back, is (g is acceleration due to gravity on the surface of earth)

(1) $\left(\frac{GM}{R}\right)^{1/2}$ (2) $\left(\frac{GM}{2R}\right)^{1/2}$ (3) $\left(\frac{gR}{4}\right)^{1/2}$ (4) $\left(\frac{2g}{R}\right)^{1/2}$

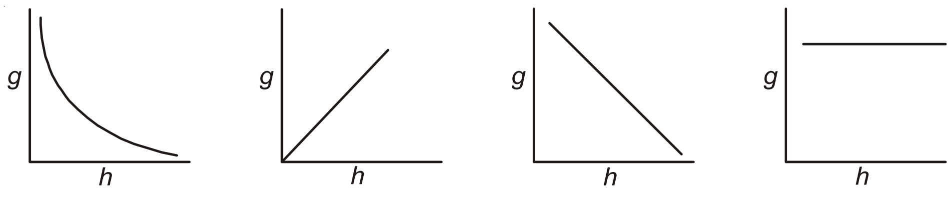
Sol. Answer (2)

The particle won't return back if it is provided speed such that its total mechanical energy is zero or positive for minimum speed, we take total energy zero,

$$\Rightarrow \frac{-GMm}{4R} + \frac{1}{2}mv^2 = 0$$

$$\Rightarrow v = \sqrt{\frac{GM}{2R}}$$

35. Which of the following graphs shows the variation of acceleration due to gravity g with depth h from the surface of the earth?



(a) (b) (c) (d)

(1) (a) (2) (b) (3) (c) (4) (d)

Sol. Answer (3)

Acceleration due to gravity at depth h from surface of earth, $g_h = g_0\left(1-\frac{h}{R}\right)$

$$g = g_0 - g_0\frac{h}{R}$$

$$g = -g_0\frac{h}{R} + g_0$$

$$g = \left(\frac{-g_0}{R}\right)^h + g_0$$

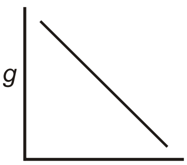
Comparing with equation of straight line,

$y = mx + c$

Slope, m is –ve

Intercept c is +ve,

Thus, most appropriate graph is



36. At what altitude (h) above the earth's surface would the acceleration due to gravity be one fourth of its value at the earth's surface?

(1) $h = R$ (2) $h = 4R$

(3) $h = 2R$ (4) $h = 16R$

Sol. Answer (1)

At altitude (h) above the earth's surface, $g_h = \frac{GM}{(R+h)^2}$

$$\Rightarrow \quad g_h = \frac{GM}{R^2}\times\frac{R^2}{(R+h)^2}$$

$$\Rightarrow \quad g_h = g\times\frac{R^2}{(R+h)^2}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{R^2}{(R+h)^2}, \pm\frac{1}{2} = \frac{R}{(R+h)}$$

Using the +ve value,

$R + h = 2R$

$\Rightarrow \quad h = R$

37. If the gravitational force between two objects were proportional to $1/R$ (and not as $1/R^2$), where R is the distance between them, then a particle in a circular path (under such a force) would have its orbital speed v, proportional to

(1) R (2) R^0 (independent of R)

(3) $\frac{1}{R^2}$ (4) $\frac{1}{R}$

Sol. Answer (2)

Gravitational force provides the necessary centripetal force for a particle to move in the circular path.

$$\Rightarrow \frac{mv^2}{R} = \frac{K}{R} \qquad \left[\text{not } \frac{K}{R^2}\right]$$

$$v = \sqrt{\frac{K}{m}}$$

Thus independent of R.

38. The distance of two planets from the sun are 10^{13} m and 10^{12} m respectively. The ratio of time periods of the planets is

(1) $\sqrt{10} : 1$ (2) $10\sqrt{10} : 1$ (3) 10 : 1 (4) 1 : 1

Sol. Answer (2)

Using Kepler's third law, $T^2 \propto r^3$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\Rightarrow \frac{T_1}{T_2} = 10^{3/2} = 10\sqrt{10}$$

39. The radius of earth is about 6400 km and that of Mars is 3200 km. The mass of the earth is about 10 times the mass of Mars. An object weighs 200 N on the surface of Earth. Its weight on the surface of mars will be

(1) 20 N (2) 8 N (3) 80 N (4) 40 N

Sol. Answer (3)

R_e = 6400 km, R_M = 3200 km

$$\frac{M_e}{M_M} = 10$$

$$W_e = m \times \frac{GM_e}{R_e^2} = mg_e$$

$$W_M = m \times \frac{GM_M}{R_M^2} = mg_M$$

$$\frac{W_e}{W_M} = \left(\frac{M_e}{M_M}\right)\left(\frac{R_M}{R_e}\right)^2$$

$$\frac{200}{W_M} = 10 \times \left(\frac{1}{2}\right)^2$$

$$\Rightarrow W_M = \frac{200 \times 4}{10} = 80 \text{ N}$$

40. The earth (mass = 6 × 10^{24} kg) revolves around the sun with an angular velocity of 2 × 10^{-7} rad/s in a circular orbit of radius 1.5 × 10^{8} km. The force exerted by the sun on the earth, in newtons, is

(1) 36 × 10^{21} (2) 27 × 10^{39}

(3) Zero (4) 18 × 10^{25}

Sol. Answer (1)

The force of gravitation exerted by sun provides the necessary centripetal force = $m\omega^2 r$

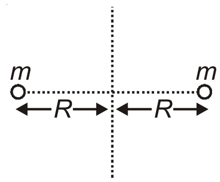
$\Rightarrow$ $F_g = 6 \times 10^{24} \times 4 \times 10^{-14} \times 1.5 \times 10^{11}$

$= 36 \times 10^{21}$ N

41. Two particles of equal mass m go around a circle of radius R under the action of their mutual gravitational attraction. The speed v of each particle is

(1) $\frac{1}{2}\sqrt{\frac{Gm}{R}}$ (2) $\sqrt{\frac{4Gm}{R}}$ (3) $\frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (4) $\sqrt{\frac{Gm}{2R}}$

Sol. Answer (1)



Gravitation force provides the necessary centripetal force,

$$\frac{Gm^2}{(2R)^2} = \frac{mv^2}{r}$$

where, r is the radius of circular path i.e R

$$\Rightarrow \quad v = \sqrt{\frac{Gm}{4R}} = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$

42. The acceleration due to gravity g and mean density of the earth ρ are related by which of the following relations? (where G is the gravitational constant and R is the radius of the earth.)

(1) $\rho = \frac{3g}{4\pi GR}$ (2) $\rho = \frac{3g}{4\pi GR^3}$ (3) $\rho = \frac{4\pi gR^2}{3G}$ (4) $\rho = \frac{4\pi gR^3}{3G}$

Sol. Answer (1)

$$g = \frac{GM}{R^2}$$

$$M = \frac{4}{3}\pi R^3 \rho \quad \Rightarrow \quad g = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho$$

Thus, $\rho = \frac{3g}{4\pi GR}$

43. What will be the formula of mass of the earth in terms of g, R and G?

(1) $G\frac{R^2}{g}$ (2) $g\frac{R^2}{G}$ (3) $g^2\frac{R}{G}$ (4) $G\frac{R}{g}$

Sol. Answer (2)

$$g = \frac{GM}{R^2}$$

$$\Rightarrow \text{ Mass of earth } = g\frac{R^2}{G}$$

44. The period of revolution of planet *A* around the sun is 8 times that of *B*. The distance of *A* from the sun is how many times greater than that of *B* from the sun?

(1) 4 (2) 5

(3) 2 (4) 3

Sol. Answer (4)

Using kepler's third law,

$$T^2 \propto r^3$$

$$\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2}$$

$$\Rightarrow (8)^{2/3} = \left(\frac{r_A}{r_B}\right)$$

$$\Rightarrow r_A = 4r_B$$

45. The escape velocity of a body on the surface of the earth is 11.2 km/s. If the earth's mass increases to twice its present value and radius of the earth becomes half, the escape velocity becomes

(1) 22.4 km/s (2) 44.8 km/s

(3) 5.6 km/s (4) 11.2 km/s

Sol. Answer (1)

$$V_e = \sqrt{\frac{2GM}{R}}$$

$$V_e' = \sqrt{\frac{2G(2M)}{R/2}} = 2\sqrt{\frac{2GM}{R}} = 22.4 \text{ km/s}$$

46. The escape velocity of a sphere of mass *m* from the surface of earth is given by (*G* = Universal gravitational constant; *M* = Mass of the earth and R_e = Radius of the earth)

(1) $\sqrt{\frac{2GMm}{R_e}}$ (2) $\sqrt{\frac{2GM}{R_e}}$

(3) $\sqrt{\frac{GM}{R_e}}$ (4) $\sqrt{\frac{2GM + R_e}{R_e}}$

Sol. Answer (2)

$$v_e = \sqrt{\frac{2GM}{R_e}}$$

$$\frac{-GMm}{R_e} + \frac{1}{2}mv_e^2 = 0$$

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM}{R_e}},$$ independent of the mass of sphere.

47. A body of weight 72 N moves from the surface of earth at a height half of the radius of earth, then gravitational force exerted on it will be

(1) 36 N (2) 32 N

(3) 144 N (4) 50 N

Sol. Answer (2)

Gravitational force on body $= mg_s = \frac{mGM}{R^2} = 72$ N

(On the surface of earth)

Gravitational force at height, $h = \frac{R}{2} = mg' = \frac{mGM}{\left(\frac{3R}{2}\right)^2}$

$$= \frac{mGM}{R^2} \times \frac{4}{9} = \frac{4}{9} \times 72 = 32 \text{ N}$$

48. A planet has mass equal to mass of the earth but radius one fourth of radius of the earth. Then escape velocity at the surface of this planet will be

(1) 11.2 km/s (2) 22.4 km/s

(3) 5.6 km/s (4) 44.8 km/s

Sol. Answer (2)

$$V_e = \sqrt{\frac{GM}{R}} = 11.2 \text{ km/s}$$

$$V_p = \sqrt{\frac{GM}{R/4}} = 2\sqrt{\frac{GM}{R}} = 22.4 \text{ km/s}$$

49. With what velocity should a particle be projected so that it attains a height equal to radius of earth?

(1) $\left(\frac{GM}{R}\right)^{1/2}$ (2) $\left(\frac{8GM}{R}\right)^{1/2}$

(3) $\left(\frac{2GM}{R}\right)^{1/2}$ (4) $\left(\frac{4GM}{R}\right)^{1/2}$

Sol. Answer (1)

Using conservation of mechanical energy at the surface of earth and at the height, $h = R$

$$P.E_i + K.E_i = P.E_j + K.E_j$$

$$\frac{-GMm}{R} + \frac{1}{2}mv^2 = \frac{-GMm}{2R}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{GMm}{R}\left(-\frac{1}{2}+1\right)$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2R}$$

$$v = \sqrt{\frac{GM}{R}}$$

50. A body of mass m is placed on earth surface which is taken from earth surface to a height of $h = 3R$, then change in gravitational potential energy is

(1) $\frac{mgR}{4}$

(2) $\frac{2}{3}mgR$

(3) $\frac{3}{4}mgR$

(4) $\frac{mgR}{2}$

Sol. Answer (3)

Potential energy of the body at earth surface

$$= \frac{-GMm}{R}$$

$$= \frac{-GM}{R^2} \times R \times m$$

$$U_i = -mgR$$

Potential energy of the body at height,

$$h = 3R = \frac{-GMm}{4R}$$

$$U_f = \frac{-mgR}{4}$$

Change in P.E $= U_f - U_i$

$$= \frac{-mgR}{4} + mgR$$

$$= \frac{3}{4}mgR$$

51. The acceleration due to gravity on a planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2 m on the surface of A. What is the height of jump by the same person on the planet B?

(1) 2/9 m (2) 18 m

(3) 6 m (4) 2/3 m

Sol. Answer (2)

Maximum height to which man jumps on A,

$$h_A = \frac{v^2}{2g_A}$$

Height to which man jumps on B,

$$h_B = \frac{v^2}{2g_B}$$

$$\frac{h_A}{h_B} = \frac{g_B}{g_A} \quad \Rightarrow \quad \frac{2}{h_B} = \frac{1}{9}$$

$\Rightarrow$ h_B = 18 m

52. Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be

(1) $3F$ (2) F

(3) F / 3 (4) F / 9

Sol. Answer (2)

Gravitational force is independent of the medium between the particles, thus force will remain unchanged.

53. The density of a newly discovered planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R, then the radius of the planet would be

(1) $2R$ (2) $4R$

(3) $\frac{1}{4}R$ (4) $\frac{1}{2}R$

Sol. Answer (4)

Acceleration due to gravity on the surface of a planet,

$$g = \frac{GM}{R^2}$$

$$= \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho = \frac{4\pi G R \rho}{3}$$

$\Rightarrow$ $R_e \rho_e = R_p \rho_p$

$R \times \rho_e = R_p \times 2\rho_e$

$$R_p = \frac{R}{2}$$

54. A ball is dropped from a spacecraft revolving around the earth at a height of 120 km. What will happen to the ball?

    (1) It will fall down to the earth gradually

    (2) It will go very far in the space

    (3) It will continue to move with the same speed along the original orbit of spacecraft

    (4) It will move with the same speed, tangentially to the spacecraft

Sol. Answer (3)

A ball dropped from a spacecraft revolving around the earth will have zero relative velocity with respect to the aircraft.

But with respect to the centre of the earth its speed will be equal to the speed of the aircraft i.e the orbital speed.

Thus, it will continue to move with same speed along the original orbit and force of gravitation of earth will provide it the necessary centripetal force.

SECTION - D

Assertion - Reason Type Questions

1. A : The gravitational force does not depend on the intervening medium.

   R : The value of G has same value anywhere in the space.

Sol. Answer (1)

Property of gravitational force:

It is independent of the medium between the particles.

2. A : The acceleration due to gravity for an object is independent from its mass.

   R : The value of 'g' depends on the mass of planet.

Sol. Answer (2)

$g = \frac{GM}{R^2}$, independent of mass of object.

$M \rightarrow$ Mass of planet.

3. A : If angular speed of the earth increases, the effective value of g will decrease at all places on earth.

   R : The value of 'g' at latitude λ is given by $g' = g - m\omega R^2 \cos\lambda$.

Sol. Answer (4)

$g' = g - m\omega R^2 \cos\lambda$ is incorrect.

$g' = g - \omega^2 R \cos^2\lambda$ is correct.

At poles $\lambda = 90°$

Thus, $g' = g$, no effect of earth's rotation.

4. A : The gravitational field intensity is zero everywhere inside a uniform spherical shell.

   R : The net force on a point mass inside a uniform spherical shell is zero everywhere.

Sol. Answer (1)

Gravitational force of attraction on a point mass due to various regions of the spherical shell cancels each other completely as their vector sum is zero.

5. A : The value of potential energy depends on the reference taken for zero potential energy.

   R : The value of change in potential energy is independent from reference level.

Sol. Answer (2)

Potential at a point depends on the choice of reference. Potential difference is independent of the choice of reference.

Potential energy is mass times the potential at the point.

6. A : When a satellite is orbiting then no energy is required to keep moving in its orbit.

   R : The total mechanical energy of a satellite is conserved.

Sol. Answer (1)

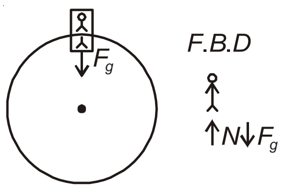
Total mechanical energy of the system is conserved, since the dissipative forces are absent or negligible.

7. A : An astronaut in a satellite may float in the free space outside and inside the satellite.

   R : An astronaut in a satellite is in weightless state.

Sol. Answer (1)

The force of gravitation provides the necessary centripetal force, for an astronant in a satellite, the *F.B.D* can be drawn,



$$F_g - N = \frac{mv_0^2}{r}$$

$$v_0 = \text{orbital speed} = \sqrt{\frac{GM}{r}}$$

$$\Rightarrow \frac{GMm}{r^2} - N = \left(\frac{m}{r}\right)\frac{GM}{r}$$

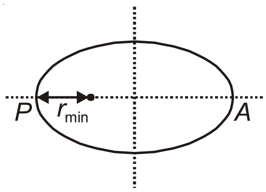
$$\Rightarrow \frac{GMm}{r^2} - N = \frac{GMm}{r^2}$$

Thus, $N = 0$, making the astronaut feel weightless.

8. A : The speed of a planet is maximum at perihelion.

R : The angular momentum of a planet about centre of sun is conserved.

Sol. Answer (1)



Angular momentum of a planet about centre of sun is conserved,

Thus, $m\vec{r}\times\vec{v} =$ constant

At perihelion r is minimum,

Thus speed of planet is maximum.

9. A : Kepler's third law of planetary motion is valid only for inverse square forces.

R : Only inverse square forces are always central.

Sol. Answer (3)

$T^2 \propto r^3$

Is valid only for inverse square forces, for a planet going in a circular orbit,

$$T = \frac{2\pi r}{v}$$

v is the orbital speed $= \sqrt{\frac{GM}{r}}$

$$\Rightarrow \quad T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = \frac{2\pi}{\sqrt{GM}} r^{3/2}$$

Also, it is not true that,

Only inverse square forces are always central.

10. A : Kepler's law cannot be used for asteroids and comets.

R : Asteroids and comets do not revolve around sun under its gravitational force.

Sol. Answer (4)

Kepler's laws can be used for asteroids and comets. All the 3 laws can be proved from the Newton's universal law of gravitation.

Asteroids and comets do revolve around sun.

11. A : During orbital motion of planet around the sun work done by the centripetal force is not zero at all points on the orbit.

R : Planet is revolving around the sun in elliptical orbit.

Sol. Answer (1)

During motion of a planet around sun, the centripetal force is not always perpendicular to the velocity of planet in an elliptical orbit. Thus work done is not zero. Although, incase of circular orbits centripetal force is always perpendicular to velocity.

12. A : Angular momentum of a satellite about a planet is constant.

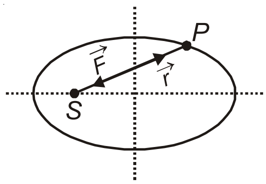
R : Gravitational force is a central force so its torque about the sun is zero.

Sol. Answer (1)

$\vec{\tau} = \vec{r} \times \vec{F}$ = $rF \sin\theta$

= $rF \sin 180°$

= 0



Also, $\vec{\tau} = \frac{d\vec{L}}{dt} = 0$, $\Rightarrow$ Angular momentum is constant.

13. A : Gravitational potential is constant everywhere inside a spherical shell.

R : Gravitational field inside a spherical shell is zero everywhere.

Sol. Answer (1)

Gravitation field, $\vec{I} = \frac{-\partial v}{\partial \vec{r}}$

Inside a spherical shell, $I = 0$

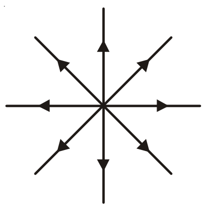
$\Rightarrow$ V is constant everywhere.

14. A : Field created by the point mass in its surroundings is a non-uniform gravitational field.

R : Since the field is $E = \frac{GM}{r^2}$ and it is dependent on r, hence Non-uniform.

Sol. Answer (1)

Field due to point mass, $E = \frac{GM}{r^2}$



Dependent on distance r from the mass, thus non-uniform.

15. A : If the force of gravitation is inversely proportional to the distance r rather than r^2 given by Newton, then orbital velocity of the satellite around the earth is independent of r.

R : $\frac{GMm}{r} = \frac{mv^2}{r}$

So, $v = \sqrt{GM}$

Hence independent of r.

Sol. Answer (1)

Force of gravitation provides the necessary centripetal force,

$$\frac{GMm}{r} = \frac{mv^2}{r}$$

$\Rightarrow$ $v = \sqrt{GM}$

Independent of *r*.

16. A : Work done by the gravitational force is positive when the two point masses are brought from infinity to any two points in space.

R : Gravitational potential energy increases during the above process.

Sol. Answer (3)

Force of gravitation is attractive, thus when masses are brought from infinity to any two points in space, displacement of masses is in the direction of force.

$\Rightarrow$ Work done is positive.

Also, $W_{\text{gravity}} = -\Delta U = U_i - U_f$

$\Rightarrow$ During this process potential energy decreases.

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