Chapter 4

Motion in a Plane

Solutions

SECTION - A

Objective Type Questions

(Scalars and Vectors, Multiplication of Vectors by Real Numbers, Addition and Subtraction of Vectors - Graphical Methods, Resolution of Vectors, Vector Addition - Analytical Method)

1. Which of the following is a vector?

   (1) Current (2) Time (3) Acceleration (4) Volume

Sol. Answer (3)

Acceleration is a vector quantity.

2. The change in a vector may occur due to

   (1) Rotation of frame of reference (2) Translation of frame of reference

   (3) Rotation of vector (4) Both (1) & (3)

Sol. Answer (3)

Change in a vector may occur due to rotation of vector and not due to rotation of frame of reference.

3. Which one of the following pair cannot be the rectangular components of force vector of 10 N?

   (1) 6 N & 8 N (2) 7 N & $\sqrt{51}$ N (3) $6\sqrt{2}$ N & $2\sqrt{7}$ N (4) 9 N & 1 N

Sol. Answer (4)

The vector magnitude = $\sqrt{A_{x^2} + A_{y^2}}$

Vector magnitude = 10

But (4) option gives the magnitude

$\Rightarrow \sqrt{9^2+1^2} = \sqrt{82} \neq 10$ [by trial method check options]

4. The resultant of two vectors at an angle 150° is 10 units and is perpendicular to one vector. The magnitude of the smaller vector is

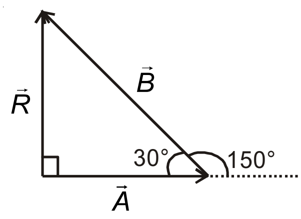
   (1) 10 units (2) $10\sqrt{3}$ units (3) $10\sqrt{2}$ units (4) $5\sqrt{3}$ units

Sol. Answer (2)

$\Rightarrow R^2 + A^2 = B^2$ .....(1)

$R = 10$

Also $\tan 30° = \dfrac{\text{Perpendicular}}{\text{Base}}$



$\dfrac{1}{\sqrt{3}} = \dfrac{R}{A}$

From equation (1) $A = 10\sqrt{3}$

$(10)^2 + (10\sqrt{3})^2 = B^2$

$B = 20$

5. Two vectors, each of magnitude A have a resultant of same magnitude A. The angle between the two vectors is

(1) 30° (2) 60° (3) 120° (4) 150°

Sol. Answer (3)

$|\vec{A}| = |\vec{B}| = |\vec{R}|$

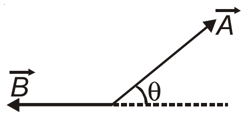
$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

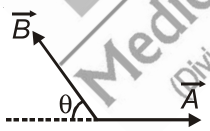
$A^2 = A^2 + A^2 + 2A^2\cos\theta$

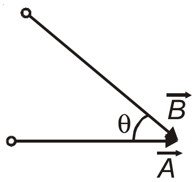
$-A^2 = 2A^2\cos\theta$

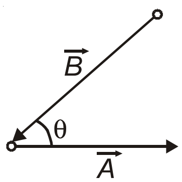
$\cos\theta = -\dfrac{1}{2} \Rightarrow \theta = 120°$

6. Let θ be the angle between vectors $\vec{A}$ and $\vec{B}$. Which of the following figures correctly represents the angle θ?

(1) 

(2) 

(3) 

(4) 

Sol. Answer (3)

To find angle between vectors, they will be joined either head to head or tail to tail.

7. $\vec{A}$ is a vector of magnitude 2.7 units due east. What is the magnitude and direction of vector $4\vec{A}$?

(1) 4 units due east (2) 4 units due west (3) 2.7 units due east (4) 10.8 units due east

Sol. Answer (4)

$\vec{A} = 2.7\,\hat{i}$

Vector $4\vec{A}$

$\Rightarrow 4(2.7\hat{i}) = 10.8\hat{i}$ or 10.8 units due east.

8. Two forces of magnitude 8 N and 15 N respectively act at a point. If the resultant force is 17 N, the angle between the forces has to be

(1) 60° (2) 45° (3) 90° (4) 30°

Sol. Answer (3)

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$A = 8$, $B = 15$, $R = 17$

$17^2 = 8^2 + 15^2 + 2 \times 8 \times 15 \times \cos\theta$

$289 = 64 + 225 + 240 \cos\theta$

$\Rightarrow 289 = 289 + 24\cos\theta$

$24\cos\theta = 0$

$\cos\theta = 0 \Rightarrow \theta = 90^\circ$

9. Two forces of 10 N and 6 N act upon a body. The direction of the forces are unknown. The resultant force on the body may be

(1) 15 N (2) 3 N (3) 17 N (4) 2 N

Sol. Answer (1)

The resultant of two vectors always lie between $(A + B)$ & $(A - B)$.

So the resultant of 10 N & 6 N should lie between 16 N & 4 N.

So answer is 15 N.

10. The vector $\overrightarrow{OA}$ where O is origin is given by $\overrightarrow{OA} = 2\hat{i} + 2\hat{j}$. Now it is rotated by 45° anticlockwise about O. What will be the new vector?

(1) $2\sqrt{2}\,\hat{j}$ (2) $2\,\hat{j}$ (3) $2\,\hat{i}$ (4) $2\sqrt{2}\,\hat{i}$

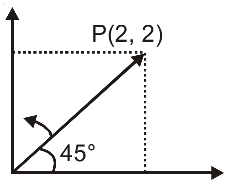
Sol. Answer (1)

$\overrightarrow{OA} = 2\hat{i} + 2\hat{j}$

$|\overrightarrow{OA}| = \sqrt{4+4} \Rightarrow 2\sqrt{2}$

On rotating by an angle of 45° anticlockwise it will lie along y-axis.

So $\vec{A} = 2\sqrt{2}\,\hat{j}$



11. If the sum of two unit vectors is also a unit vector, then magnitude of their difference and angle between the two given unit vectors is

(1) $\sqrt{3}, 60^\circ$ (2) $\sqrt{3}, 120^\circ$ (3) $\sqrt{2}, 60^\circ$ (4) $\sqrt{2}, 120^\circ$

Sol. Answer (2)

$|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$|\vec{A}| = |\vec{B}| = |\vec{R}| = 1$

$1 = 1 + 1 + 2 \times 1 \times 1 \times \cos\theta$

$\cos\theta = -\dfrac{1}{2} \Rightarrow \theta = 120^\circ$

$|\vec{R}| = |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos 120^\circ}$

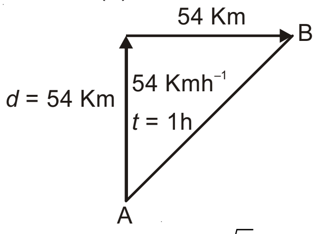
$= \sqrt{1^2 + 1^2 - 2 \times 1 \times 1 \times \left(-\dfrac{1}{2}\right)} = \sqrt{3} = |\vec{A} - \vec{B}|$

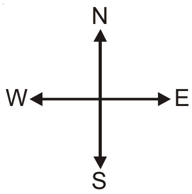
(Motion in a Plane, Motion in a Plane with Constant Acceleration, Relative Velocity in Two Dimensions)

12. A car moves towards north at a speed of 54 km/h for 1 h. Then it moves eastward with same speed for same duration. The average speed and velocity of car for complete journey is

(1) 54 km/h, 0 (2) 15 m/s, $\frac{15}{\sqrt{2}}$ m/s (3) 0, 0 (4) 0, $\frac{54}{\sqrt{2}}$ km/h

Sol. Answer (2)





$\text{Displacement} = \frac{54\sqrt{2}}{\text{Km}}$

Distance = 2 × 54 = 108 Km

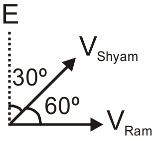
$\text{Average speed} = \frac{108}{2} = 54\ \text{Kmh}^{-1} \times \frac{5}{18} = 15\ \text{ms}^{-1}$

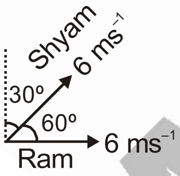
$\text{Average velocity} = \frac{\text{disp.}}{\text{time}} = \frac{54\sqrt{2}}{2} \Rightarrow 27\sqrt{2} \times \frac{5}{18} \Rightarrow \frac{15}{\sqrt{2}}\ \text{m/s}$

13. Ram moves in east direction at a speed of 6 m/s and Shyam moves 30° east of north at a speed of 6 m/s. The magnitude of their relative velocity is

(1) 3 m/s (2) 6 m/s (3) $6\sqrt{3}$ m/s (4) $6\sqrt{2}$ m/s

Sol. Answer (2)

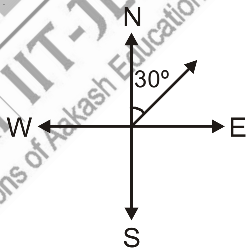




$\left|\vec{V}_{RS}\right| = \sqrt{V_R^2 + V_S^2 - 2V_R V_S \cos\theta}$

$= \sqrt{6^2 + 6^2 - 2 \times 6^2 \times \frac{1}{2}}$

= 6 ms^{-1}



14. A train is running at a constant speed of 90 km/h on a straight track. A person standing at the top of a boggey moves in the direction of motion of the train such that he covers 1 meters on the train each second. The speed of the person with respect to ground is

(1) 25 m/s (2) 91 km/h (3) 26 km/h (4) 26 m/s

Sol. Answer (4)

$V_T = 90\ \text{Kmh}^{-1} = 90 \times \frac{5}{18} = 25\ \text{ms}^{-1}$

V_m = ?

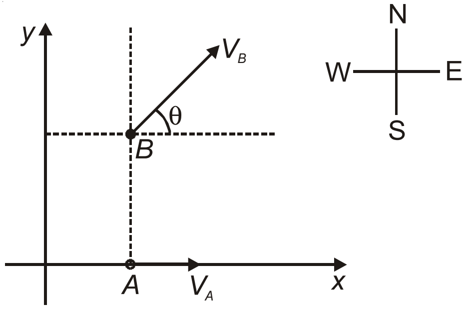
d = speed × time

$d_{\text{net}} = V_{\text{net}} \times t$

$1 = (V_m - 25) \times 1$

$V_m = 26\ \text{ms}^{-1}$

15 . Figure shows two ships moving in *x*-*y* plane with velocities V_A and V_B. The ships move such that B always remains north of *A*. The ratio $\frac{V_A}{V_B}$ is equal to



(1) cosθ (2) sinθ (3) secθ (4) cosecθ

Sol. Answer (1)

If ship *B* is always north of ship *A* then, their horizontal component should be equal, so,

$V_A = V_B \cos\theta$

$\Rightarrow \frac{V_A}{V_B} = \cos\theta$

16. Four persons *P*, *Q*, *R* and *S* are initially at the four corners of a square of side *d*. Each person now moves with a constant speed *v* in such a way that *P* always moves directly towards *Q*, *Q* towards *R*, *R* towards *S*, and *S* towards *P*. The four persons will meet after time

(1) $\frac{d}{2v}$ (2) $\frac{d}{v}$ (3) $\frac{3d}{2v}$ (4) They will never meet

Sol. Answer (2)

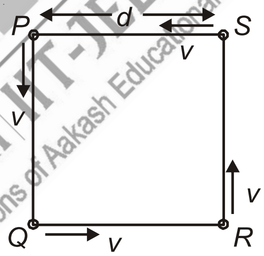
$T = \frac{d}{v_{rel}}$

$v_{rel} = v - v\cos 90^\circ$

$= v - 0$

$= v$

$T = \frac{d}{v}$



17. A person, reaches a point directly opposite on the other bank of a flowing river, while swimming at a speed of 5 m/s at an angle of 120° with the flow. The speed of the flow must be

(1) 2.5 m/s (2) 3 m/s (3) 4 m/s (4) 1.5 m/s

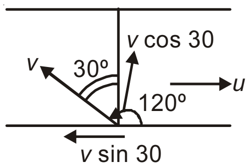
Sol. Answer (1)

For drift to be zero

$u = v \sin 30^\circ$

$= 5 \times \frac{1}{2}$

$= 2.5\ ms^{-1}$



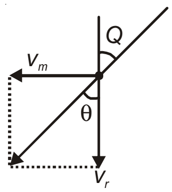
18. A car with a vertical windshield moves in a rain storm at a speed of 40 km/hr. The rain drops fall vertically with constant speed of 20 m/s. The angle at which rain drops strike the windshield is

(1) $\tan^{-1}\frac{5}{9}$ (2) $\tan^{-1}\frac{9}{5}$ (3) $\tan^{-1}\frac{3}{2}$ (4) $\tan^{-1}\frac{2}{3}$

Sol. Answer (1)

$$\tan\theta = \frac{v_m}{v_r} = \frac{\frac{20\times 5}{9}}{20}$$

$$\theta = \tan^{-1}\left(\frac{5}{9}\right)$$



(Projectile Motion)

19. A particle projected from origin moves in *x*-*y* plane with a velocity $\vec{v} = 3\hat{i} + 6x\hat{j}$, where $\hat{i}$ and $\hat{j}$ are the unit vectors along *x* and *y* axis. Find the equation of path followed by the particle

(1) $y = x^2$ (2) $y = \frac{1}{x^2}$ (3) $y = 2x^2$ (4) $y = \frac{1}{x}$

Sol. Answer (1)

Method 1:

$$\vec{V} = 3\hat{i} + 6x\hat{j}$$

also $\vec{V} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$

$$\Rightarrow \frac{dx}{dt} = 3,$$

$$\int dx = \int 3dt$$

$$x = 3t$$

$$\frac{dy}{dt} = 6x$$

$$dy = 6x \times dt$$

$$\int dy = \int 6\times 3t\,dt$$

$$= 18\int t\,dt \Rightarrow 18\times\frac{t^2}{2}$$

$$y = 9t^2$$

$$= 9\times\frac{x^2}{9}$$

$$y = x^2$$

Method 2:

$$V_x\hat{i} + V_y\hat{j} = \vec{V}$$

$$V_x = 3$$

$$V_y = 6x$$

We know

$$\frac{d_y}{d_x} = \tan\theta = \frac{V_y}{V_x}$$

$$\frac{d_y}{d_x} = \frac{6x}{3x}$$

$$\int_0 dy = \int_0 2x\,dx$$

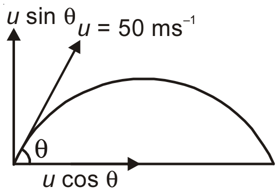
$$y = x^2$$

20. A body of mass 1 kg is projected from ground at an angle 30º with horizontal on a level ground at a speed 50 m/s. The magnitude of change in momentum of the body during its flight is (g = 10 m/s^2)

(1) 50 kg ms^{-1} (2) 100 kg ms^{-1} (3) 25 kg ms^{-1} (4) Zero

Sol. Answer (1)

$\Rightarrow$ The change in momentum = $-2mu\sin\theta\hat{j}$



$|\Delta\vec{p}| = 2mu\sin\theta$

= 2 × 1 × 50 × sin 30º

$|\Delta\vec{p}|$ = 50 Kg ms^{-1}

21. Two projectiles are projected at angles $\left(\frac{\pi}{4}+\theta\right)$ and $\left(\frac{\pi}{4}-\theta\right)$ with the horizontal, where $\theta < \frac{\pi}{4}$, with same speed. The ratio of horizontal ranges described by them is

(1) tan θ : 1 (2) 1 : tan^2 θ (3) 1 : 1 (4) $1:\sqrt{3}$

Sol. Answer (3)

The horizontal range is same when the angles of projection are complimentary to each other.

22. A shell is fired vertically upwards with a velocity v_1 from a trolley moving horizontally with velocity v_2. A person on the ground observes the motion of the shell as a parabola, whose horizontal range is

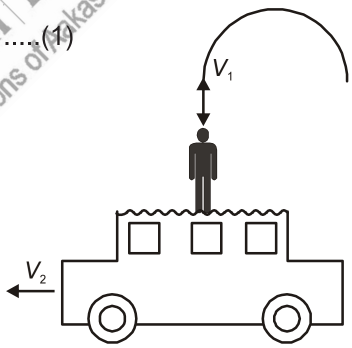
(1) $\frac{2v_1^2v_2}{g}$ (2) $\frac{2v_1^2}{g}$ (3) $\frac{2v_2^2}{g}$ (4) $\frac{2v_1v_2}{g}$

Sol. Answer (4)

There is no acceleration in the horizontal direction.

$S_x = U_xT + \frac{1}{2}a_0 \times T^2$

$R = U_xT$ .....(1)



$S_y = U_yT + \frac{1}{2}g_yT^2$

$O = V_1T - \frac{1}{2}gT^2$

$\Rightarrow V_1T = \frac{1}{2}gT$

$T = \frac{2V_1}{g}$

We know,

(R) range = (Horizontal velocity 4x) × flight + time (T)

i.e., R = 4x × T

$R = V_2 \times \frac{2V_1}{g} \Rightarrow \frac{2V_1V_2}{g}$

23. The position coordinates of a projectile projected from ground on a certain planet (with no atmosphere) are given by $y = (4t - 2t^2)$m and $x = (3t)$ metre, where t is in second and point of projection is taken as origin. The angle of projection of projectile with vertical is

(1) 30° (2) 37° (3) 45° (4) 60°

Sol. Answer (2)

$y = 4t - 2t^2$

$x = 3t$

$V = V_{x\hat{i}} + V_{y\hat{j}}$

$V_x = \dfrac{dx}{dt},\ V_y = \dfrac{dy}{dt}$

$V_x = 3,\ V_y = 4 - 4t$

for $t = 0$, $V_y = 4$

$\tan\theta = \dfrac{V_y}{V_x} = \dfrac{4}{3}$

$\theta = 53^o$ with horizontal

With vertical

$\theta = 37^o$

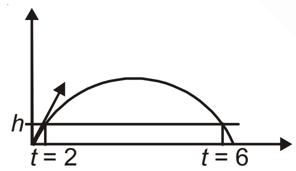
24. A particle is projected from ground with speed 80 m/s at an angle 30° with horizontal from ground. The magnitude of average velocity of particle in time interval t = 2 s to t = 6 s is [Take g = 10 m/s^2]

(1) $40\sqrt{2}$ m/s (2) 40 m/s (3) Zero (4) $40\sqrt{3}$ m/s

Sol. Answer (4)

Average velocity of the projectile when it is at the same vertical height is : $u\cos\theta$.

$\Rightarrow$ 80 × cos 30° $\Rightarrow$ $40\sqrt{3}$ m/s .



25. A stone projected from ground with certain speed at an angle θ with horizontal attains maximum height h_1. When it is projected with same speed at an angle θ with vertical attains height h_2. The horizontal range of projectile is

(1) $\dfrac{h_1 + h_2}{2}$ (2) $2h_1h_2$ (3) $4\sqrt{h_1h_2}$ (4) $h_1 + h_2$

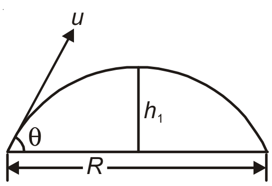
Sol. Answer (3)

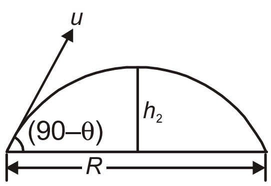
When the angles are complimentary the range is same,

$h_1 = \dfrac{u^2\sin^2\theta}{2g}$,

$h_2 = \dfrac{u^2\sin^2(90-\theta)}{2g}$

$h_1 = \dfrac{u^2\sin^2\theta}{2g}$





$$h_2 = \frac{u^2\cos^2\theta}{2g}$$

$$h_1h_2 = \frac{u^4\sin^2\theta\cos^2\theta}{4g^2} \Rightarrow \left(\frac{2u\sin\theta\cos\theta}{g}\right)^2 \times \frac{1}{4g} \times \frac{1}{4}$$

$$h_1h_2 = R^2\,\frac{1}{16} \;\Rightarrow\; R^2 = 16\,h_1h_2$$

$$R = 4(\sqrt{h_1h_2})$$

26. Two objects are thrown up at angles of 45° and 60° respectively, with the horizontal. If both objects attain same vertical height, then the ratio of magnitude of velocities with which these are projected is

(1) $\sqrt{\frac{5}{3}}$ (2) $\sqrt{\frac{3}{5}}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\sqrt{\frac{3}{2}}$

Sol. Answer (4)

$h_1 = h_2$

$$\frac{u_1^2\sin^2 45^\circ}{2g} = \frac{V_2^2\sin^2 60^\circ}{2g}$$

$$\frac{u_1^2}{V_2^2} = \frac{\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{3}{2}$$

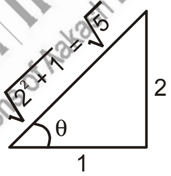
$$\frac{V_1}{V_2} = \sqrt{\frac{3}{2}}$$

27. For an object projected from ground with speed u horizontal range is two times the maximum height attained by it. The horizontal range of object is

(1) $\frac{2u^2}{3g}$ (2) $\frac{3u^2}{4g}$ (3) $\frac{3u^2}{2g}$ (4) $\frac{4u^2}{5g}$

Sol. Answer (4)

$R = 24$ also, $\frac{H}{R} = \frac{1}{4}\tan\theta$



$$\frac{H}{R} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{1}{4}\tan\theta$$

$$\tan\theta = 2 = \frac{P}{B}$$

$$R = \frac{2u^2\sin\theta\cos\theta}{g}$$

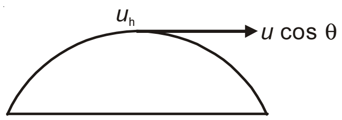
$$R = \frac{2u^2}{g}.\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{5}}$$

$$R = \frac{4u^2}{5g}$$

28. The velocity at the maximum height of a projectile is $\frac{\sqrt{3}}{2}$ times its initial velocity of projection (u). Its range on the horizontal plane is

(1) $\frac{\sqrt{3}u^2}{2g}$ (2) $\frac{3u^2}{2g}$ (3) $\frac{3u^2}{g}$ (4) $\frac{u^2}{2g}$

Sol. Answer (1)



$u_h = u\cos\theta$

$\frac{\sqrt{3}}{2}u = u\cos\theta$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 30^\circ$

$R = \frac{u^2 \sin 2\theta}{g}$

$= \frac{u^2 \sin 60^\circ}{g} \Rightarrow \frac{\sqrt{3}u^2}{2g} = R$

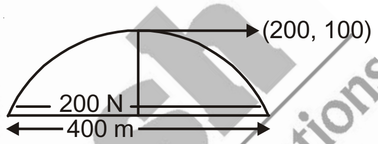
29. A projectile is thrown into space so as to have a maximum possible horizontal range of 400 metres. Taking the point of projection as the origin, the co-ordinates of the point where the velocity of the projectile is minimum are

(1) (400, 100) (2) (200, 100) (3) (400, 200) (4) (200, 200)

Sol. Answer (2)

R_{max} = 400 m

The velocity is minimum at the highest point



$\Rightarrow H \rightarrow \frac{R}{2}$

$R = 4H$

$400 = 4 \times H$

$H = 100$ m

30. If the time of flight of a bullet over a horizontal range R is T, then the angle of projection with horizontal is

(1) $\tan^{-1}\left(\frac{gT^2}{2R}\right)$ (2) $\tan^{-1}\left(\frac{2R^2}{gT}\right)$ (3) $\tan^{-1}\left(\frac{2R}{g^2T}\right)$ (4) $\tan^{-1}\left(\frac{2R}{gT}\right)$

Sol. Answer (1)

$T = \frac{2u\sin\theta}{g} \Rightarrow u = \frac{gT}{2\sin\theta}$

$R = \frac{2u^2 \sin\theta\cos\theta}{g}$

$R = \frac{2u\sin\theta}{g} \times u\cos\theta$

$R = T \times u \cos\theta$

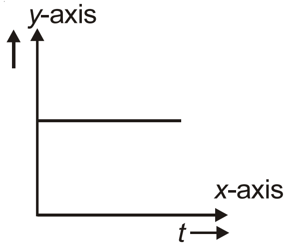
$R = T \times \frac{gT\cos\theta}{2\sin\theta}$

$R = \frac{gT^2}{2}\frac{1}{\tan\theta}$

$\tan\theta = \frac{gT^2}{2R}$

$\theta = \tan^{-1}\left(\frac{gT^2}{2R}\right)$

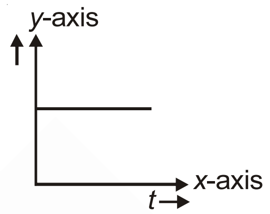
31. In the graph shown in figure, which quantity associated with projectile motion is plotted along y-axis?



(1) Kinetic energy (2) Momentum (3) Horizontal velocity (4) None of these

Sol. Answer (3)

It is the horizontal component of velocity that remains constant throughout the motion as there is no acceleration in that direction $a_x = 0$, u_x = constant



32. The equation of a projectile is $y = ax - bx^2$. Its horizontal range is

(1) $\frac{a}{b}$ (2) $\frac{b}{a}$ (3) $a + b$ (4) $b - a$

Sol. Answer (1)

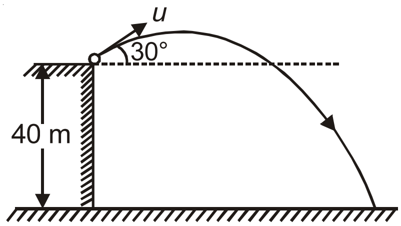
$y = ax - bx^2$

When the body lands then y = 0, $x = R$, $0 = aR - bR^2$

$aR = bR$

$R = \frac{a}{b}$

33. Figure shows a projectile thrown with speed u = 20 m/s at an angle 30° with horizontal from the top of a building 40 m high. Then the horizontal range of projectile is



(1) $20\sqrt{3}$ m (2) $40\sqrt{3}$ m

(3) 40 m (4) 20 m

Sol. Answer (2)

$S_y = u_y T + \frac{1}{2} g_y T^2$

$-40 = 4 \sin 30 T - \frac{1}{2} g T^2$

$-40 = 20 \times \frac{1}{2} T - 5T^2$

$-8 = 2T - T^2$

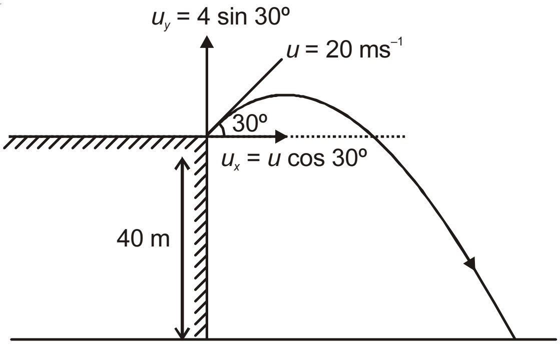
$T^2 - 2T - 8 = 0$

$T^2 - 4T + 2T - 8 = 0$

$T = -2, 4$

$R = u \cos\theta T = 20 \times \frac{\sqrt{3}}{2} \times 4$

$R = 40\sqrt{3}$ m



34. When a particle is projected at some angle to the horizontal, it has a range R and time of flight t_1. If the same particle is projected with the same speed at some other angle to have the same range, its time of flight is t_2, then

(1) $t_1 + t_2 = \dfrac{2R}{g}$ (2) $t_1 - t_2 = \dfrac{R}{g}$ (3) $t_1 t_2 = \dfrac{2R}{g}$ (4) $t_1 t_2 = \dfrac{R}{g}$

Sol. Answer (3)

The angles has to be complimentary *i.e.,* if $\theta_1 \to \theta,\ \theta_2 \to (90 - \theta)$

$$t_1 = \frac{2u\sin\theta}{g},\ t_2 = \frac{2u\sin(90-\theta)}{g}$$

$$t_2 = \frac{2u\cos\theta}{g}$$

$$t_1 t_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g}$$

$$t_1 t_2 = \frac{2R}{g}$$

35. A projectile is thrown with velocity v at an angle θ with horizontal. When the projectile is at a height equal to half of the maximum height, the vertical component of the velocity of projectile is

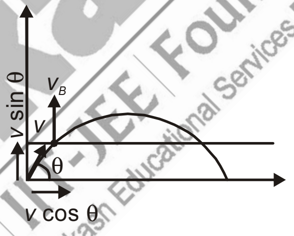
(1) $v \sin\theta \times 3$ (2) $\dfrac{v\sin\theta}{3}$ (3) $\dfrac{v\sin\theta}{\sqrt{2}}$ (4) $\dfrac{v\sin\theta}{\sqrt{3}}$

Sol. Answer (3)

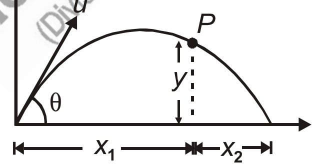
$$v_B^2 = v^2\sin^2\theta - \frac{2g}{2}\left(\frac{u^2\sin^2\theta}{2g}\right)$$

$$v_B^2 = \frac{v^2\sin^2\theta}{2}$$

$$v_B = \frac{v\sin\theta}{\sqrt{2}}$$



36. In the given figure for a projectile



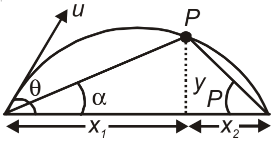
(1) $y = \left[\dfrac{x_1x_2}{x_1 - x_2}\right]\tan\theta$ (2) $y = \left[\dfrac{x_1x_2}{x_1 + x_2}\right]\tan\theta$ (3) $y = \left[\dfrac{2x_1x_2}{x_1 + x_2}\right]\cos\theta$ (4) $y = \left[\dfrac{2x_1x_2}{x_1 + x_2}\right]\tan\theta$

Sol. Answer (2)

The equation of trajectory for point 'P' can be written as :

$$y = x\tan\theta\left(1 - \frac{x}{R}\right) = x_1\tan\theta\left(1 - \frac{x_1}{x_1 + x_2}\right) = x_1\tan\theta\left(\frac{x_1 + x_2 - x_1}{x_1 + x_2}\right)$$

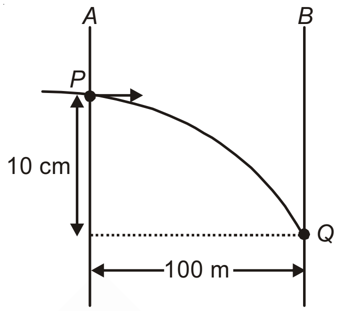
$$y = \frac{x_1x_2}{x_1 + x_2}\tan\theta$$



37. Two paper screens *A* and *B* are separated by distance 100 m. A bullet penetrates *A* and *B*, at points *P* and *Q* respectively, where *Q* is 10 cm below *P*. If bullet is travelling horizontally at the time of hitting *A*, the velocity of bullet at *A* is nearly

(1) 100 m/s (2) 200 m/s (3) 600 m/s (4) 700 m/s

Sol. Answer (4)



10 cm $\Rightarrow$ 10 × 10^{-2} m $\Rightarrow$ 10^{-1} $\Rightarrow$ 0.1 m

It is a case of horizontal projectile.

So, $a_x = 0$, $u_x = 4$, $u_y = 0$, $a_y = -g$

$R = 100\text{m}$, $T = \sqrt{\dfrac{2H}{g}} \Rightarrow$ Time of flight

$R = u_x T$

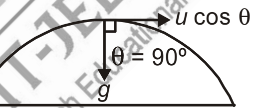
$$100 = u\sqrt{\frac{2\times 0.1}{100}} \Rightarrow \frac{u\sqrt{2}}{10} = 100$$

$$u = \frac{1000}{\sqrt{2}} \approx 707\ \text{ms}^{-1}$$

38. An object is projected from ground with speed *u* at angle θ with horizontal. the radius of curvature of its trajectory at maximum height from ground is

(1) $\dfrac{u^2 \sin 2\theta}{g}$ (2) $\dfrac{u^2\cos^2\theta}{g}$ (3) $\dfrac{u^2\sin^2\theta}{g}$ (4) $\dfrac{u^2\sin^2\theta}{2g}$

Sol. Answer (2)



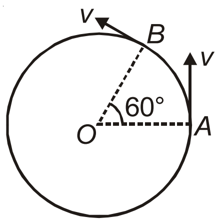
$$a_c = \frac{v^2}{r}$$

$$r = \frac{v^2}{a_c}, \frac{u^2\cos^2\theta}{g}$$

$$r = \frac{u^2\cos^2\theta}{g}$$

(Uniform Circular Motion)

39. A particle is moving in a circle of radius *r* having centre at *O*, with a constant speed *v*. The magnitude of change in velocity in moving from *A* to *B* is



(1) 2*v* (2) 0 (3) $\sqrt{3}\,v$ (4) *v*

Sol. Answer (4)

$$[\Delta\vec{V}] = 2V\sin\frac{\theta}{2} = 2\times V\times \sin\left(\frac{60^\circ}{2}\right) = 2\times V\times\frac{1}{2} \Rightarrow V = |\Delta\vec{V}|$$

40. A car is going round a circle of radius R_1 with constant speed. Another car is going round a circle of radius R_2 with constant speed. If both of them take same time to complete the circles, the ratio of their angular speeds and linear speeds will be

(1) $\sqrt{\frac{R_1}{R_2}}, \frac{R_1}{R_2}$ (2) 1, 1 (3) $1, \frac{R_1}{R_2}$ (4) $\frac{R_1}{R_2}, 1$

Sol. Answer (3)

The angular speed is given by

$$\omega = \frac{2\pi}{T}$$

$$\omega \propto \frac{1}{T} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

if $T_1 = T_2 \Rightarrow \omega_1 = \omega_2$

So, ratio $\Rightarrow$ 1 : 1

and linear speed $v = R\omega$

$$V \propto R$$

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

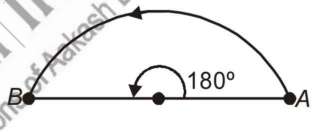
41. A body revolves with constant speed v in a circular path of radius r. The magnitude of its average acceleration during motion between two points in diametrically opposite direction is

(1) Zero (2) $\frac{v^2}{r}$ (3) $\frac{2v^2}{\pi r}$ (4) $\frac{v^2}{2r}$

Sol. Answer (3)

$$a_{avg} = \frac{2v\sin\left(\frac{\theta}{2}\right)}{\left(\frac{r\theta}{v}\right)}$$

$$a_{avg} = \frac{2v^2\sin\left(\frac{\theta}{2}\right)}{r\theta}$$



Here, $\theta = \pi$ rad

$$a_{avg} = \frac{2v^2\sin\left(\frac{\pi}{2}\right)}{r\times\pi}$$

$$a_{avg} = \frac{2v^2}{\pi r}$$

42. An object of mass m moves with constant speed in a circular path of radius R under the action of a force of constant magnitude F. The kinetic energy of object is

(1) $\frac{1}{2}FR$ (2) FR (3) $2FR$ (4) $\frac{1}{4}FR$

Sol. Answer (1)

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{F}{a}\times v^2 = \frac{1}{2}\frac{F\times v^2}{\left(\frac{v^2}{R}\right)} = \frac{1}{2}FR$$

43. The angular speed of earth around its own axis is

(1) $\frac{\pi}{43200}$rad/s (2) $\frac{\pi}{3600}$rad/s (3) $\frac{\pi}{86400}$rad/s (4) $\frac{\pi}{1800}$rad/s

Sol. Answer (1)

Angular speed = $\frac{2\pi}{T}$

$T \rightarrow$ Time period of earth = 24 h

$\omega = \frac{2\pi}{24\times60\times60} = \frac{\pi}{43200}$ rad s^{-1}

44. A particle moves in a circle of radius 25 cm at two revolutions per second. The acceleration of the particle is (in m/s^2)

(1) π^2 (2) $8\pi^2$ (3) $4\pi^2$ (4) $2\pi^2$

Sol. Answer (3)

$a = r\omega^2$

$a = \frac{25}{100}(2\times2\pi)^2$

$a = 4\pi^2$ m/s^2

45. A particle is revolving in a circular path of radius 25 m with constant angular speed 12 rev/min. Then the angular acceleration of particle is

(1) $2\pi^2$ rad/s^2 (2) $4\pi^2$ rad/s^2 (3) π^2 rad/s^2 (4) Zero

Sol. Answer (4)

Angular acceleration is the rate of change of angular speed or angular velocity if $\vec{\omega}$ remains constant then $\alpha = 0$

46. Two particles are moving in circular paths of radii r_1 and r_2 with same angular speeds. Then the ratio of their centripetal acceleration is

(1) 1 : 1 (2) $r_1 : r_2$ (3) $r_2 : r_1$ (4) $r_2^2 : r_1^2$

Sol. Answer (2)

Centripetal acceleration is given by

$a = \frac{v^2}{r} = r\omega^2$

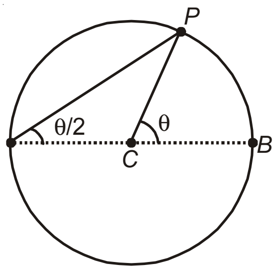
For same 'ω'

$a_c \propto r \Rightarrow \frac{a_1}{a_2} = \frac{r_1}{r_2}$

47. A particle P is moving in a circle of radius r with uniform speed v. C is the centre of the circle and AB is diameter. The angular velocity of P about A and C is in the ratio

(1) 4 : 1 (2) 2 : 1 (3) 1 : 2 (4) 1 : 1

Sol. Answer (3)



$$\omega_{P/C} = \frac{d\theta}{dt}$$

$$\omega_{P/A} = \frac{1}{2}\frac{d\theta}{dt}$$

$$\omega_{P/A} = \frac{1}{2}\omega_{P/C}$$

$$\frac{\omega_{P/A}}{\omega_{P/C}} = \frac{1}{2} = 1:2$$

48. A car is moving at a speed of 40 m/s on a circular track of radius 400 m. This speed is increasing at the rate of 3 m/s^2. The acceleration of car is

(1) 4 m/s^2 (2) 7 m/s^2 (3) 5 m/s^2 (4) 3 m/s^2

Sol. Answer (3)

v = 40 ms^{-1}

r = 400 m

a_T = 3 ms^{-2}

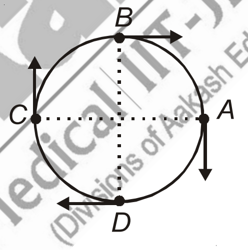
$$a_c = \frac{V^2}{r} = \frac{40\times 40}{400} = 4\text{ ms}^{-2}$$

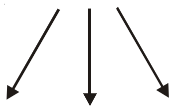
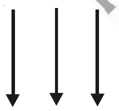
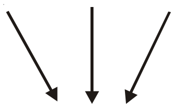
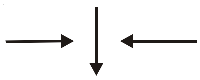
$$a = \sqrt{a_C^2 + a_T^2}$$

$$a = \sqrt{4^2+3^2} = 5\text{ ms}^{-2}$$

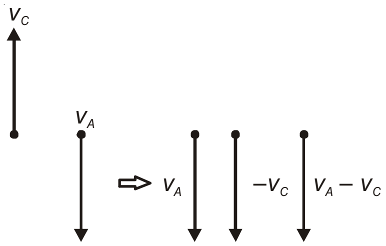
a = 5 ms^{-2}

49. Four particles A, B, C and D are moving with constant speed v each. At the instant shown relative velocity of A with respect to B, C and D are in directions

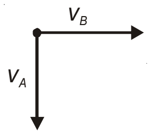


(1)  (2)  (3)  (4) 

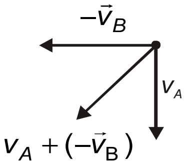
Sol. Answer (1)

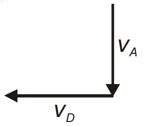
(i) 

$\vec{v}_{AC} = \vec{v}_A - \vec{v}_C \Rightarrow$  $\vec{v}_A - \vec{v}_C$

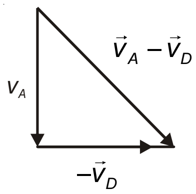
(ii) 

$\Rightarrow \vec{v}_{AB} = \vec{v}_A - \vec{v}_B \Rightarrow \vec{v}_A + (-\vec{v}_A)$



(iii) 

$\Rightarrow \vec{v}_{AD} = \vec{v}_A - \vec{v}_D = \vec{v}_A + (-\vec{v}_D)$

$\Rightarrow$ 

50. The ratio of angular speeds of minute hand and hour hand of a watch is

(1) 6 : 1 (2) 12 : 1 (3) 60 : 1 (4) 1 : 60

Sol. Answer (2)

ω_{mh} = Angular speed of minute hand

ω_{hh} = Angular speed of hour hand

$$\omega_{mh} = \frac{2\pi}{60\,\text{m}} = \frac{2\pi}{60\times 60}\text{rad s}^{-1}$$

$$\omega_{hh} = \frac{2\pi}{12\text{ h}} = \frac{2\pi}{12\times 60\times 60}\text{rad s}^{-1}$$

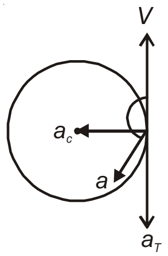
$$\frac{\omega_{mh}}{\omega_{hh}} = \frac{\dfrac{2\pi}{60\times 60}}{\dfrac{2\pi}{12\times 60\times 60}} = \frac{1}{1}\times\frac{12}{1}$$

$\omega_{mh} : \omega_{hh} \Rightarrow 12 : 1$

51. If θ is angle between the velocity and acceleration of a particle moving on a circular path with decreasing speed, then

(1) $\theta = 90°$ (2) $0° < \theta < 90°$ (3) $90° < \theta < 180°$ (4) $0° \le \theta \le 180°$

Sol. Answer (3)



θ between v & Q is

$90° < \theta < 180°$

52. If speed of an object revolving in a circular path is doubled and angular speed is reduced to half of original value, then centripetal acceleration will become/remain

(1) Same (2) Double (3) Half (4) Quadruple

Sol. Answer (1)

$a_c = r\omega^2 = (r\omega)(\omega)$

$a_c = v\omega$

$a_c = (2v)\left(\dfrac{\omega}{2}\right) = v\omega = a_c$

SECTION - B

Objective Type Questions

(Motion in a Plane, Motion in a Plane with Constant Acceleration, Relative Velocity in Two Dimensions)

1. Two particles *A* and *B* start moving with velocities 20 m/s and $30\sqrt{2}$ m/s along x-axis and at an angle 45º with x-axis respectively in xy-plane from origin. The relative velocity of *B* w.r.t. *A*

(1) $(10\hat{i} + 30\hat{j})$ m/s (2) $(30\hat{i} + 10\hat{j})$ m/s

(3) $(30\hat{i} - 20\sqrt{2}\hat{j})$ m/s (4) $(30\sqrt{2}\hat{i} + 10\sqrt{2}\hat{j})$ m/s

Sol. Answer (1)

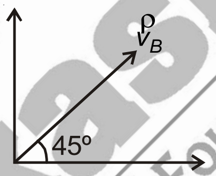
$v_A = 20$ m/s

$v_B = 30\sqrt{2}$ m/s along 45º with *x*-axis

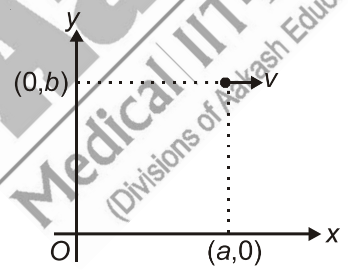
$\vec{v}_B = v_B \cos 45^\circ \hat{i} + v_B \sin 45^\circ \hat{j} = 30\hat{i} + 30\hat{j}$

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 30\hat{i} + 30\hat{j} - 20\hat{i}$

$\boxed{\vec{v}_{BA} = 10\hat{i} + 30\hat{j}}$



2. A particle is moving with constant speed *v* in *xy* plane as shown in figure. The magnitude of its angular velocity about point *O* is



(1) $\dfrac{v}{\sqrt{a^2+b^2}}$ (2) $\dfrac{v}{b}$ (3) $\dfrac{vb}{(a^2+b^2)}$ (4) $\dfrac{v}{a}$

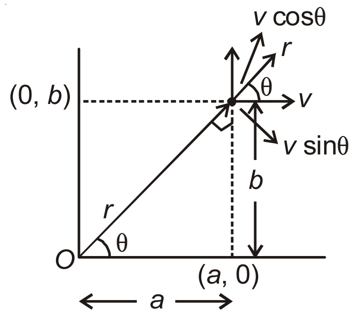
Sol. Answer (3)

$v \sin\theta = r\omega$

$v \sin\theta = \sqrt{a^2+b^2} \times \omega$

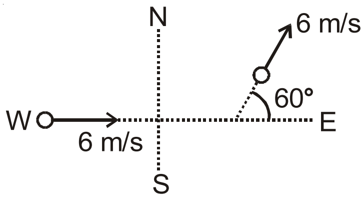
$\dfrac{v}{\sqrt{a^2+b^2}} \dfrac{b}{\sqrt{a^2+b^2}} = \omega$

$\boxed{\dfrac{vb}{(a^2+b^2)} = \omega}$



$v_\perp = r\omega$

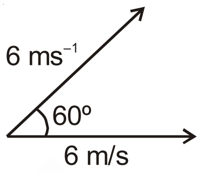
3. A particle is moving eastwards with a speed of 6 m/s. After 6 s, the particle is found to be moving with same speed in a direction 60° north of east. The magnitude of average acceleration in this interval of time is



(1) 6 m/s² (2) 3 m/s² (3) 1 m/s² (4) Zero

Sol. Answer (3)

$$|\Delta \vec{v}| = 2v\sin\frac{\theta}{2} = 2v\sin\left(\frac{60}{2}\right) = 2v\sin 30° = v$$



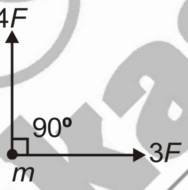
$$a_{av} = \frac{(\Delta \vec{v})}{\Delta t}$$

$|\Delta \vec{v}| = 6\ \text{ms}^{-1}$

$\Delta t = 6\ \text{s}$

so, $a_{av} = \frac{6}{6} = 1\ \text{ms}^{-2}$

4. When a force F acts on a particle of mass m, the acceleration of particle becomes a. Now if two forces of magnitude $3F$ and $4F$ acts on the particle simultaneously as shown in figure, then the acceleration of the particle is



(1) a (2) $2a$ (3) $5a$ (4) $8a$

Sol. Answer (3)

$F_{net} = \sqrt{3^2 + 4^2} = 5\ F$

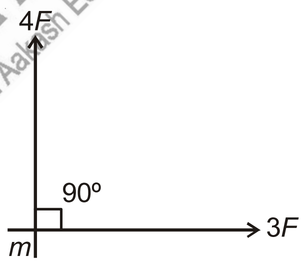
So, $F_{net} = ma$

$5F = ma'$

$\Rightarrow\ a' = \frac{5F}{m}$

$\boxed{a' = 5a}$

$\frac{F}{m} = a$



5. Out of the two cars A and B, car A is moving towards east with a velocity of 10 m/s whereas B is moving towards north with a velocity 20 m/s, then velocity of A w.r.t. B is (nearly)

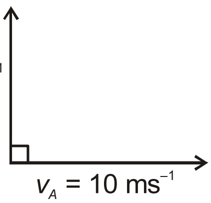
(1) 30 m/s (2) 10 m/s (3) 22 m/s (4) 42 m/s

Sol. Answer (3)

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$v_{AB} = \sqrt{v_A^2 + v_B^2}$

$|\vec{v}_{AB}| = \sqrt{10^2 + 20^2} = \sqrt{100 + 400} = \sqrt{500} \simeq 22\ \text{ms}^{-1}$



6. A man moves in an open field such that after moving 10 m on a straight line, he makes a sharp turn of 60º to his left. The total displacement just at the start of 8th turn is equal to

(1) 12 m (2) 15 m (3) 17.32 m (4) 14.14 m

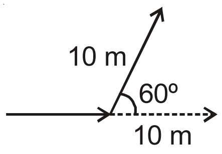
Sol. Answer (3)

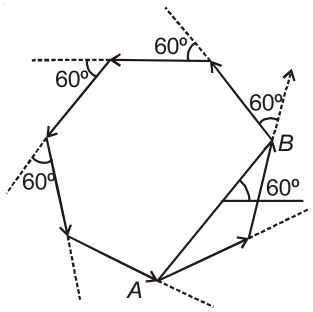
After 8 such turns object is at '*B*'.

Displacement = *AB*

Two vectors are at 60º

$$\sqrt{10^2+10^2+2\times 10^2\times\frac{1}{2}} = 10\sqrt{3}\text{ m}$$

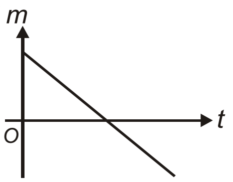


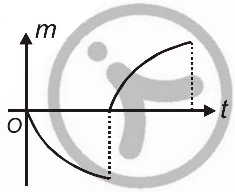


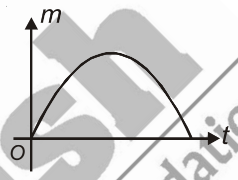
$\Rightarrow$ $\boxed{17.32\text{ m} = AB}$

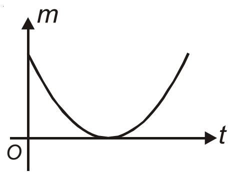
(Projectile Motion)

7. A particle is projected at angle θ with horizontal from ground. The slop (m) of the trajectory of the particle varies with time (t) as

(1) 

(2) 

(3) 

(4) 

Sol. Answer (1)

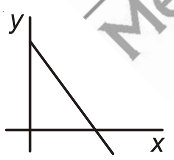
Slope of trajectory

$$\tan\phi = \frac{u\sin\theta - gt}{u\cos\theta}$$

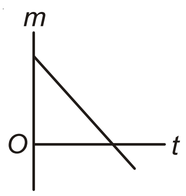
So, $m = \dfrac{u\sin\theta}{u\cos\theta} - \dfrac{gt}{u\cos\theta}$

$$m = \tan\theta - \frac{g}{u\cos\theta}t$$

$\Rightarrow$ $y = a - bx$



Therefore,

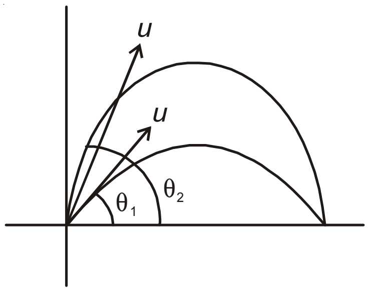


8. If H_1 and H_2 be the greatest heights of a projectile in two paths for a given value of range, then the horizontal range of projectile is given by

(1) $\dfrac{H_1+H_2}{2}$ (2) $\dfrac{H_1+H_2}{4}$ (3) $4\sqrt{H_1 H_2}$ (4) $4[H_1+H_2]$

Sol. Answer (3)

$\theta_1 + \theta_2 = 90^\circ$



$$H_1 = \frac{u^2 \sin^2 \omega_1}{2g}$$

$$H_2 = \frac{u^2 \sin(90^\circ - \theta_1)}{2g}$$

$$H_1 H_2 = \frac{R^2}{16} \qquad \because\ R = \frac{u^2 \sin^2 \theta_1}{g}$$

$$\boxed{R = 4\sqrt{H_1 H_2}}$$

9. If R and H are the horizontal range and maximum height attained by a projectile, than its speed of projection is

(1) $\sqrt{2gR + \frac{4R^2}{gH}}$ (2) $\sqrt{2gH + \frac{R^2 g}{8H}}$ (3) $\sqrt{2gH + \frac{8H}{Rg}}$ (4) $\sqrt{2gH + \frac{R^2}{H}}$

Sol. Answer (2)

$$H = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow \sin\theta = \sqrt{\frac{2gH}{u^2}}$$

$$R = \frac{2u^2 \sin\theta \cos\theta}{g}$$

$$R = \frac{2u^2}{g}\sqrt{\frac{2gH}{u^2}} \times \sqrt{1 - \frac{2gH}{u^2}}$$

$$R = \frac{2u^2}{g}\sqrt{\frac{2gH}{u^2}} \times \sqrt{\frac{u^2 - 2gH}{u^2}}$$

$$\frac{gR}{2\sqrt{2gH}} = \sqrt{u^2 - 2gH}$$

Squaring both the sides,

$$\frac{gR^2}{4 \times 2gH} = u^2 - 2gH$$

$$\Rightarrow \quad u^2 = 2gH + \frac{9R^2}{8H}$$

$$\boxed{u = \sqrt{2gH + \frac{gR^2}{8H}}}$$

10. A particle projected from ground moves at angle 45º with horizontal one second after projection and speed is minimum two seconds after the projection. The angle of projection of particle is [Neglect the effect of air resistance]

(1) $\tan^{-1}(3)$ (2) $\tan^{-1}(2)$ (3) $\tan^{-1}(\sqrt{2})$ (4) $\tan^{-1}(4)$

Sol. Answer (2)

$\theta = 45^\circ$, $t = 1$ s

$$\tan\phi = \frac{V_y}{U_y} = \frac{u\sin\theta - gt}{u\cos\theta}$$

$$\tan 45^\circ = \frac{u\sin\theta - g\times 1}{u\cos\theta} \Rightarrow u\cos\theta = u\sin\theta - g$$

also, V_y = 0, after 1st (as speed is minimum)

$$u\sin\theta - g\times 2 = 0 \Rightarrow \boxed{u\sin\theta = 2g} \quad \text{...(i)}$$

so, $u\cos\theta = 2g - g$

$$\boxed{u\cos\theta = g} \quad \text{...(ii)}$$

so, $\dfrac{\text{(i)}}{\text{(ii)}} = \dfrac{u\sin\theta}{u\cos\theta} = \dfrac{2g}{g}$

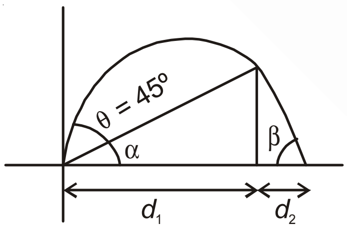
$\Rightarrow \quad \tan\theta = 2$

$$\boxed{\theta = \tan^{-1}(2)}$$

11. A ball is projected from ground at an angle 45º with horizontal from distance d_1 from the foot of a pole and just after touching the top of pole it the falls on ground at distance d_2 from pole on other side, the height of pole is

(1) $2\sqrt{d_1 d_2}$ (2) $\dfrac{d_1 + d_2}{4}$ (3) $\dfrac{2d_1 d_2}{d_1 + d_2}$ (4) $\dfrac{d_1 d_2}{d_1 + d_2}$

Sol. Answer (4)



$$\tan\theta + \tan\beta = \tan\theta$$

$$\frac{y}{d_1} + \frac{y}{d_2} = \tan 45^\circ$$

$$y = \left(\frac{d_1 d_2}{d_1 + d_2}\right)$$

12. A particle is projected with speed u at angle θ with horizontal from ground. If it is at same height from ground at time t_1 and t_2, then its average velocity in time interval t_1 to t_2 is

(1) Zero (2) $u\sin\theta$ (3) $u\cos\theta$ (4) $\dfrac{1}{2}[u\cos\theta]$

Sol. Answer (3)

When projectile is at same height, average velocity = $u\cos\theta$.

13. A particle is projected from ground at an angle θ with horizontal with speed u. The ratio of radius of curvature of its trajectory at point of projection to radius of curvature at maximum height is

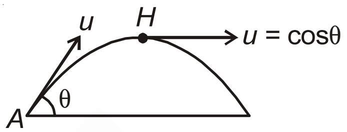
(1) $\dfrac{1}{\sin^2\theta\cos\theta}$ (2) $\cos^2\theta$ (3) $\dfrac{1}{\sin^3\theta}$ (4) $\dfrac{1}{\cos^3\theta}$

Sol. Answer (4)

At the point of projection

$$r_A = \frac{u^2}{g\cos\theta}$$

$$r_H = \frac{u^2\cos^2\theta}{g}$$



Ratio, $\dfrac{r_A}{r_H} = \dfrac{\dfrac{u^2}{g\cos\theta}}{\dfrac{u^2\cos^2\theta}{g}} = \boxed{\dfrac{1}{\cos^3\theta} = \dfrac{r_A}{r_H}}$

14. An object of mass 10 kg is projected from ground with speed 40 m/s at an angle 60º with horizontal. The rate of change of momentum of object one second after projection in SI unit is [Take g = 9.8 m/s^2]

(1) 73 (2) 98 (3) 176 (4) 140

Sol. Answer (2)

Force = $\dfrac{\Delta p}{\Delta t}$, force remains constant = mg

$\Rightarrow$ 10 × 9.8 $\Rightarrow$ 98 N

At t = 1, particle is at its maximum height.

15. An object is projected from ground with speed 20 m/s at angle 30º with horizontal. Its centripetal acceleration one second after the projection is [Take g = 10 m/s^2]

(1) 10 m/s^2 (2) Zero (3) 5 m/s^2 (4) 12 m/s^2

Sol. Answer (1)

Centripetal acceleration = $\dfrac{v^2}{r} = g = 10 \text{ ms}^2$

16. A particle is moving on a circular path with constant speed v. It moves between two points A and B, which subtends an angle 60º at the centre of circle. The magnitude of change in its velocity and change in magnitude of its velocity during motion from A to B are respectively

(1) Zero, Zero (2) v, 0 (3) 0, v (4) 2v, v

Sol. Answer (2)

$$\Delta v = 2v\sin\frac{\theta}{2}$$

$$= 2v \times \sin\left(\frac{60}{2}\right)$$

$$\boxed{|\Delta v| = v}$$

Change in magnitude of velocity = 0

17. What is the path followed by a moving body, on which a constant force acts in a direction other than initial velocity (*i.e.* excluding parallel and antiparallel direction)?

(1) Straight line (2) Parabolic (3) Circular (4) Elliptical

Sol. Answer (2)

The path will be parabolic.

18. Two stones are thrown with same speed u at different angles from ground in air. If both stones have same range and height attained by them are h_1 and h_2, then $h_1 + h_2$ is equal to

(1) $\frac{u^2}{g}$ (2) $\frac{u^2}{2g}$ (3) $\frac{u^2}{3g}$ (4) $\frac{u^2}{4g}$

Sol. Answer (2)

If range is same then, one angle is θ and other angle is $(90 - \theta)$

$$\Rightarrow \quad h_1 = \frac{u^2 \sin^2\theta}{2g}, \quad h_2 = \frac{u^2 \sin^2(90-\theta)}{2g}$$

$$h_1 = \frac{u^2 \sin^2\theta}{2g}, \quad h_2 = \frac{u^2 \cos^2\theta}{2g}$$

So, $h_1 + h_2 \Rightarrow \frac{u^2 \sin^2\theta}{2g} + \frac{u^2 \cos^2\theta}{2g} = \frac{u^2}{2g}(\sin^2\theta + \cos^2\theta)$

$$\boxed{h_1 + h_2 = \frac{u^2}{2g}}$$

19. A projectile is projected with speed u at an angle θ with the horizontal. The average velocity of the projectile between the instants it crosses the same level is

(1) $u \cos\theta$ (2) $u \sin\theta$ (3) $u \cot\theta$ (4) $u \tan\theta$

Sol. Answer (1)

20. A ball is thrown at an angle θ with the horizontal. Its horizontal range is equal to its maximum height. This is possible only when the value of $\tan\theta$ is

(1) 4 (2) 2 (3) 1 (4) 0.5

Sol. Answer (1)

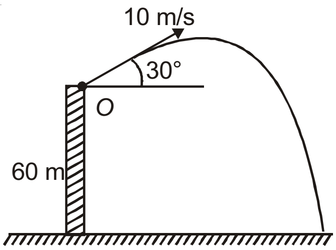
$$\frac{H}{R} = \frac{1}{4}\tan\theta$$

$\Rightarrow$ $H = R$, given,

$$\boxed{\tan\theta = 4}$$

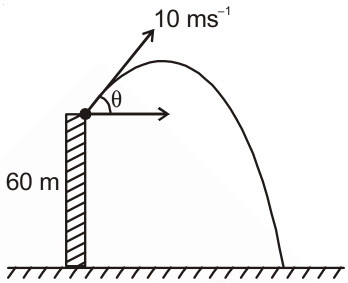
$$\Rightarrow \boxed{\theta = \tan^{-1}(4)}$$

21. A ball is projected from a point O as shown in figure. It will strike the ground after (g = 10 m/s^2)



(1) 4 s (2) 3 s (3) 2 s (4) 5 s

Sol. Answer (1)



$$s_y = u_x T + \frac{1}{2} a_y T^2$$

$$-60 = 10\sin 30°\cdot T - \frac{1}{2} g T^2$$

$$-60 = 5T - 5T^2$$

$$T^2 - T - 2 = 0$$

$$\boxed{T = 4\text{ s}}$$

22. A particle is thrown with a velocity of u m/s. It passes A and B as shown in figure at time t_1 = 1 s and t_2 = 3 s. The value of u is (g = 10 m/s^2)

(1) 20 m/s (2) 10 m/s (3) 40 m/s (4) 5 m/s

Sol. Answer (3)

$$t_1 + t_2 = \frac{2u\sin\theta}{g}$$

$$1 + 3 = \frac{2u \times \sin 30°}{10}$$

20 × 2 = u

$$\Rightarrow \boxed{u = 40\text{ ms}^{-1}}$$

23. Which one of the following statements is not true about the motion of a projectile?

(1) The time of flight of a projectile is proportional to the speed with which it is projected at a given angle of projection

(2) The horizontal range of a projectile is proportional to the square root of the speed with which it is projected

(3) For a given speed of projection, the angle of projection for maximum range is 45°

(4) At maximum height, the acceleration due to gravity is perpendicular to the velocity of the projectile

Sol. Answer (2)

$$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow \boxed{R \propto u^2}$$

24. A projectile is thrown with speed 40 ms^{-1} at angle θ from horizontal. It is found that projectile is at same height at 1 s and 3 s. What is the angle of projection?

(1) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (2) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (3) $\tan^{-1}\left(\sqrt{3}\right)$ (4) $\tan^{-1}\left(\sqrt{2}\right)$

Sol. Answer (2)

$$\tan\theta = \frac{v_y}{v_x}$$

Also, $t_1 + t_2 = \dfrac{2u\sin\theta}{g}$

$$4 = \frac{2\times 40\times \sin\theta}{10}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

So, $\tan\theta = \tan 30° \Rightarrow \dfrac{1}{\sqrt{3}}$

$$\boxed{\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)}$$

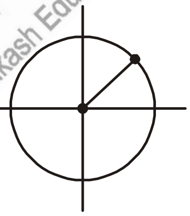
(Uniform Circular Motion)

25. A particle is moving in xy-plane in a circular path with centre at origin. If at an instant the position of particle is given by $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$, then velocity of particle is along

(1) $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$ (2) $\frac{1}{\sqrt{2}}(\hat{j}-\hat{i})$ (3) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (4) Either (1) or (2)

Sol. Answer (4)

$$\vec{r} = \frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$



$\vec{v}\cdot\vec{r} = 0$ as velocity is always tangential to the path.

$$(v_x\hat{i} + v_y\hat{j})\cdot\frac{1}{2}(\hat{i}+\hat{j}) = 0$$

$v_x + v_y = 0 \quad \Rightarrow \quad \boxed{v_x = -v_y}$ or $\boxed{v_y = -v_x}$

$v = \sqrt{v_x^2 + v_y^2} \quad \Rightarrow \quad \sqrt{2}v_x = v \quad \Rightarrow \quad \boxed{v_x = \frac{v}{\sqrt{2}}}$

$$v_y = -\frac{v}{\sqrt{2}}$$

or $v_x = -\dfrac{v}{\sqrt{2}},\ v_y = \dfrac{v}{\sqrt{2}}$

So, possible value of $v \Rightarrow v_x\hat{i} + v_y\hat{j} \Rightarrow \dfrac{v}{\sqrt{2}}\hat{i} - \dfrac{v}{\sqrt{2}}\hat{j}$ or $\dfrac{-v}{\sqrt{2}}\hat{i} + \dfrac{v}{\sqrt{2}}\hat{j}$

26. Consider the two statements related to circular motion in usual notations

A. In uniform circular motion $\vec{\omega}$, $\vec{v}$ and $\vec{a}$ are always mutually perpendicular

B. In non-uniform circular motion, $\vec{\omega}$, $\vec{v}$ and $\vec{a}$ are always mutually perpendicular

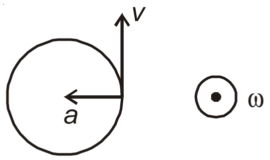
(1) Both A and B are true

(2) Both A and B are false

(3) A is true but B is false

(4) A is false but B is true

Sol. Answer (3)



Only first statement is correct.

$\omega \to$ mutually perpendicular to v and a.

27. Which of the following quantities remains constant during uniform circular motion?

(1) Centripetal acceleration (2) Velocity

(3) Momentum (4) Speed

Sol. Answer (4)

Speed remains constant.

SECTION - C

Previous Years Questions

1. The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in meters and t in seconds. The acceleration of the particle at $t = 2$ s is **[NEET-2017]**

(1) 0

(2) 5 m/s^2

(3) –4 m/s^2

(4) –8 m/s^2

Sol. Answer (3)

$x = 5t - 2t^2$ $\qquad$ $y = 10t$

$\frac{dx}{dt} = 5 - 4t$ $\qquad$ $\frac{dy}{dt} = 10$

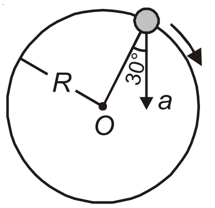
$v_x = 5 - 4t$ $\qquad$ $v_y = 10$

$\frac{dv}{dt}x = -4$ $\qquad$ $\frac{dv}{dt}y = 10$

$a_x = -4$ $\qquad$ $a_y = 0$

Acceleration of particle at $t = 2$ s is = –4 m/s^2

2. In the given figure, $a = 15$ m/s^2 represents the total acceleration of a particle moving in the clockwise direction in a circle of radius $R = 2.5$ m at a given instant of time. The speed of the particle is **[NEET (Phase-2) 2016]**



(1) 4.5 m/s (2) 5.0 m/s (3) 5.7 m/s (4) 6.2 m/s

Sol. Answer (3)

$$a\cos 30^\circ = \frac{v^2}{r} \Rightarrow 15\frac{\sqrt{3}}{2} = \frac{v^2}{2.5} \Rightarrow v = 5.7 \text{ m/s}$$

3. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is **[NEET-2016]**

(1) 180° (2) 0° (3) 90° (4) 45°

Sol. Answer (3)

$$|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}| \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$$

4. A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$, where ω is a constant. Which of the following is true? **[NEET-2016]**

(1) Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin

(2) Velocity and acceleration both are perpendicular to $\vec{r}$

(3) Velocity and acceleration both are parallel to $\vec{r}$

(4) Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin

Sol. Answer (4)

$$\vec{r} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y},$$

$$\vec{v} = \frac{d\vec{r}}{dt} = -\omega\sin\omega t\,\hat{x} + \omega\cos\omega t\,\hat{y}$$

$$\Rightarrow \vec{v}.\vec{r} = 0$$

$$\vec{a} = -\omega^2\cos\omega t\,\hat{x} - \omega^2\sin\omega t\,\hat{y} = -\omega^2\vec{r}$$

5. A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion? **[NEET-2016]**

(1) 0.2 m/s^2 (2) 0.1 m/s^2 (3) 0.15 m/s^2 (4) 0.18 m/s^2

Sol. Answer (2)

$m = 0.01$ kg, $r = 6.4$ cm

$$\frac{1}{2}mv^2 = 8\times10^{-4} \text{ J}$$

$$v^2 = \frac{16\times10^{-4}}{0.01} = 16\times10^{-2}$$

Speed $v^2 = 2a_t s$

$$v^2 = 2a_t 4\pi r \Rightarrow a_t = \frac{v^2}{8\pi r} = \frac{16\times10^{-2}}{8\times3.14\times6.4\times10^{-2}} = 0.1 \text{ m/s}^2$$

6. The position vector of a particle $\vec{R}$ as a function of time is given by $\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$, where R is in meters, t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x-and y-directions, respectively. Which one of the following statements is wrong for the motion of particle? **[Re-AIPMT-2015]**

(1) Path of the particle is a circle of radius 4 meter

(2) Acceleration vector is along $-\vec{R}$

(3) Magnitude of acceleration vector is $\frac{v^2}{R}$, where v is the velocity of particle

(4) Magnitude of the velocity of particle is 8 meter/second

Sol. Answer (4)

$\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j} \ = x\hat{i} + y\hat{j}$

Now, $x^2 + y^2 = 4^2$ which is equation of circle of radius R.

So, the motion is UCM with speed

$V = 8\pi\sqrt{2}$ m/s

7. Two particles A and B, move with constant velocities $\vec{v}_1$ and $\vec{v}_2$. At the initial moment their position vectors are $\vec{r}_1$ and $\vec{r}_2$ respectively. The condition for particles A and B for their collision is **[Re-AIPMT-2015]**

(1) $\vec{r}_1 - \vec{r}_2 = \vec{v}_1 - \vec{v}_2$ (2) $\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$ (3) $\vec{r}_1 \cdot \vec{v}_1 = \vec{r}_2 \cdot \vec{v}_2$ (4) $\vec{r}_1 \times \vec{v}_1 = \vec{r}_2 \times \vec{v}_2$

Sol. Answer (2)

For collision final positions should be equal

$\Rightarrow \quad \vec{r}_1 + \vec{v}_1 t = \vec{r}_2 + \vec{v}_2 t$

$\Rightarrow \quad \vec{r}_1 - \vec{r}_2 = (\vec{v}_2 - \vec{v}_1)t$

$\Rightarrow \quad \vec{r}_1 - \vec{r}_2 = (\vec{v}_2 - \vec{v}_1)\frac{|\vec{r}_1 - \vec{r}_2|}{|\vec{v}_2 - \vec{v}_1|}$

8. A ship A is moving Westwards with a speed of 10 km h^{-1} and a ship B 100 km South of A, is moving Northwards with a speed of 10 km h^{-1}. The time after which the distance between them becomes shortest, is **[AIPMT-2015]**

(1) $10\sqrt{2}$ h (2) 0 h (3) 5 h (4) $5\sqrt{2}$ h

Sol. Answer (3)

9. A projectile is fired from the surface of the earth with a velocity of 5 ms^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 ms^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is (Given g = 9.8 ms^{-2}) **[AIPMT-2014]**

(1) 3.5 (2) 5.9 (3) 16.3 (4) 110.8

Sol. Answer (1)

Since trajectory is same, so range and maximum height both will be identical from earth and planet. So equating maximum height (Answer can be obtained by equating range also)

$$\frac{u_e^2 \sin^2\theta}{2g_e} = \frac{u_p^2 \sin^2\theta}{2g_p}$$

$$\frac{2.5}{9.8} = \frac{9}{g_p} \Rightarrow g_p = 3.5 \text{ m/s}^2$$

10. A particle is moving such that its position coordinates (x, y) are

(2 m, 3 m) at time $t = 0$,

(6 m, 7 m) at time $t = 2$ s and

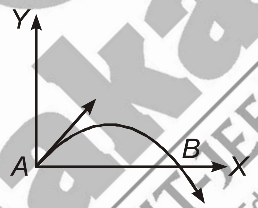
(13 m, 14 m) at time $t = 5$ s.

Average velocity vector $(\vec{v}_{\mathbf{av}})$ from $t = 0$ to $t = 5$ s is **[AIPMT-2014]**

(1) $\frac{1}{5}(13\hat{i} + 14\hat{j})$ (2) $\frac{7}{3}(\hat{i} + \hat{j})$ (3) $2(\hat{i} + \hat{j})$ (4) $\frac{11}{5}(\hat{i} + \hat{j})$

Sol. Answer (4)

11. The velocity of a projectile at the initial point A is $(2\hat{i} + 3\hat{j})$ m/s. Its velocity (in m/s) at point B is **[NEET-2013]**



(1) $-2\hat{i} + 3\hat{j}$ (2) $2\hat{i} - 3\hat{j}$ (3) $2\hat{i} + 3\hat{j}$ (4) $-2\hat{i} - 3\hat{j}$

Sol. Answer (2)

The change is only in the y-component

So, $\boxed{v_f = 2\hat{i} - 3\hat{j}}$ $\boxed{\because a_x = 0}$

12. The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is **[AIPMT (Prelims)-2012]**

(1) $\theta = \tan^{-1}(2)$ (2) $\theta = 45°$ (3) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ (4) $\theta = \tan^{-1}(4)$

Sol. Answer (4)

$H = R$

$\tan\theta = 4$ $\left(\because \frac{H}{R} = \frac{1}{4}\tan\theta\right)$

$\boxed{\theta = \tan^{-1}(4)}$

13. A particle has initial velocity $\left(2\vec{i}+3\vec{j}\right)$ and acceleration $\left(0.3\vec{i}+0.2\vec{j}\right)$. The magnitude of velocity after 10 s will be **[AIPMT (Prelims)-2012]**

(1) 5 units (2) 9 units (3) $9\sqrt{2}$ units (4) $5\sqrt{2}$ units

Sol. Answer (4)

$\vec{u}=3\hat{i}+3\hat{j}$, $\vec{a}=0.3\hat{i}+0.2\hat{j}$

Δt = 10 s

$\vec{v}=\vec{u}+\vec{a}t$

$\vec{v}=2\hat{i}+3\hat{j}+(0.3\hat{i}+0.2\hat{j})\times 10$

$=2\hat{i}+3\hat{j}+3\hat{i}+2\hat{j}$

$\vec{v}=5\hat{i}+5\hat{j}$

$v=\sqrt{5^2+5^2}=\sqrt{50}$

$\boxed{v \Rightarrow 5\sqrt{2}\ \text{ms}^{-1}}$

14. A particle moves in a circle of radius 5 cm with constant speed and time period 0.2 π s. The acceleration of the particle is **[AIPMT (Prelims)-2011]**

(1) 5 m/s^2 (2) 15 m/s^2 (3) 25 m/s^2 (4) 36 m/s^2

Sol. Answer (1)

r = 5 cm, v = ?, T = 0.2 π s

$T=\dfrac{2\pi}{\omega} \Rightarrow \omega=\dfrac{20\pi}{0.2\pi}=10\ \text{rad s}^{-1}$

$a=r\omega^2=5\times 10^{-2}\times 100$

$\boxed{a=5\ \text{ms}^{-2}}$

15. A body is moving with velocity 30 m/s towards east. After 10 s its velocity becomes 40 m/s towards north. The average acceleration of the body is **[AIPMT (Prelims)-2011]**

(1) 5 m/s^2 (2) 1 m/s^2

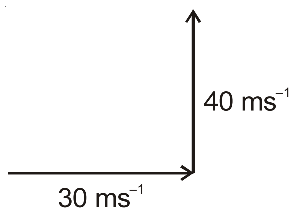
(3) 7 m/s^2 (4) $\sqrt{7}$ m/s^2

Sol. Answer (1)

$a_{av}=\dfrac{|\Delta\vec{v}|}{\Delta t}$

$\Delta v=(\vec{v}_2-\vec{v}_1)=\sqrt{v_2^2+v_1^2}$ $(\because\ \theta=90^\circ)$

$=5\sqrt{30^2+40^2}=50\ \text{ms}^{-1}$



so, $a_{av}=\dfrac{50}{10} \Rightarrow \boxed{5\ ms^{-2}=a_{av}}$

16. A missile is fired for maximum range with an initial velocity of 20 m/s. If g = 10 m/s^2, the range of the missile is **[AIPMT (Prelims)-2011]**

(1) 20 m (2) 40 m (3) 50 m (4) 60 m

Sol. Answer (2)

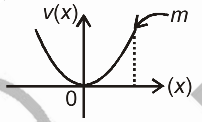
For maximum range $\theta = 45°$

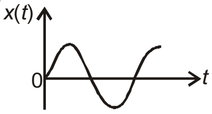
$v = 20 \text{ ms}^{-1}$

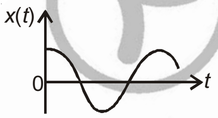
$$R = \frac{u^2}{g} = \frac{20 \times 20}{10} \qquad [\because \theta = 45°]$$

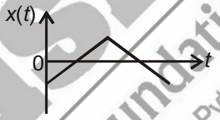
$$\boxed{R = 40 \text{ m}}$$

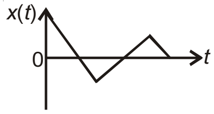
17. A particle of mass m is released from rest and follows a parabolic path as shown. Assuming that the displacement of the mass from the origin is small, which graph correctly depicts the position of the particle as a function of time? **[AIPMT (Prelims)-2011]**



(1) 

(2) 

(3) 

(4) 

Sol. Answer (2)

18. A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection is **[AIPMT (Mains)-2011]**

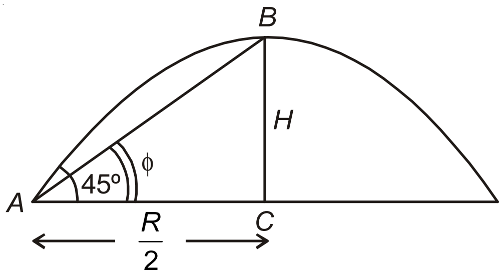
(1) $\tan^{-}\left(\frac{\sqrt{3}}{2}\right)$ (2) 45°

(3) 60° (4) $\tan^{-1}\frac{1}{2}$

Sol. Answer (4)

$$\frac{1}{4}\tan\theta = \frac{H}{R}$$

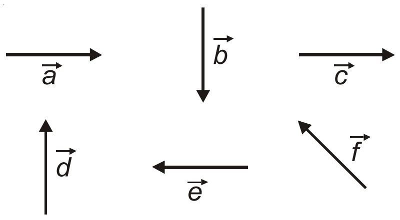
$$4H = R \qquad ...(i)$$



In ΔABC, $\tan\phi = \dfrac{H}{\frac{R}{2}} = \dfrac{2H}{R}$

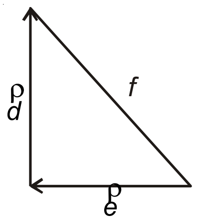
$$\tan\phi = \frac{2H}{4H} = \frac{1}{2} \Rightarrow \phi = \tan^{-1}\left(\frac{1}{2}\right)$$

19. Six vectors, $\hat{a}$ through $\hat{f}$ have the magnitudes and directions indicated in the figure. Which of the following statements is true? **[AIPMT (Prelims)-2010]**



(1) $\vec{b}+\vec{c}=\vec{f}$ (2) $\vec{d}+\vec{c}=\vec{f}$ (3) $\vec{d}+\vec{e}=\vec{f}$ (4) $\vec{b}+\vec{e}=\vec{f}$

Sol. Answer (3)



$\vec{d}+\vec{e}=\vec{f}$

20. The speed of a projectile at its maximum height is half of its initial speed. The angle of projection is

[AIPMT (Mains)-2010]

(1) 60º (2) 15º (3) 30º (4) 45º

Sol. Answer (1)

$u_H = u\cos\theta$

$\frac{u}{2} = u\cdot\cos\theta \Rightarrow \cos\theta = \frac{1}{2}$

$\boxed{\theta = 60^\circ}$

21. A particle moves in *x*-*y* plane according to rule $x = a \sin \omega t$ and $y = a \cos\omega t$. The particle follows

[AIPMT (Mains)-2010]

(1) An elliptical path

(2) A circular path

(3) A parabolic path

(4) A straight line path inclined equally to *x* and *y*-axes

Sol. Answer (2)

$x = a\sin\omega t \Rightarrow x^2 = a^2\sin^2\omega t$

$y = a\cos\omega t \Rightarrow y^2 = a^2\cos^2\omega t$

$x^2+y^2 = a^2(\sin^2\omega t+\cos^2\omega t)$

$\boxed{x^2+y^2=a^2}$ $\rightarrow$ equation of circle.

22. A particle has initial velocity $(3\hat{i} + 4\hat{j})$ and has acceleration $(0.4\hat{i} + 0.3\hat{j})$. Its speed after 10 s is

[AIPMT (Prelims)-2010]

(1) 7 units (2) $7\sqrt{2}$ units (3) 8.5 units (4) 10 units

Sol. Answer (2)

23. A particle of mass *m* is projected with velocity *v* making an angle of 45° with the horizontal. When the particle lands on the level ground the magnitude of the change in its momentum will be **[AIPMT (Prelims)-2008]**

(1) Zero (2) 2 *mv* (3) $\frac{mv}{\sqrt{2}}$ (4) $mv\sqrt{2}$

Sol. Answer (4)

$$\Delta p = -2mv\sin\theta\hat{j}$$

$$|\Delta\vec{p}| = 2mv\sin\theta = 2mv \times \frac{1}{\sqrt{2}}$$

$$\boxed{|\Delta\vec{p}| = \sqrt{2}\ mv}$$

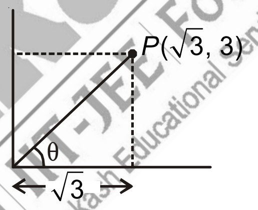
24. A particle starting from the origin (0, 0) moves in a straight line in the (*x, y*) plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the *x*-axis an angle of **[AIPMT (Prelims)-2007]**

(1) 0° (2) 30° (3) 45° (4) 60°

Sol. Answer (4)

$$\tan\theta = \frac{P}{B} = \frac{3}{\sqrt{3}}$$

$$\boxed{\theta = 60°}$$



25. $\vec{A}$ and $\vec{B}$ are two vectors and θ is the angle between them, if $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$, the value of θ is

[AIPMT (Prelims)-2007]

(1) 90° (2) 60° (3) 45° (4) 30°

Sol. Answer (2)

26. For angles of projection of a projectile at angles (45°– θ) and (45° + θ), the horizontal ranges described by the projectile are in the ratio of **[AIPMT (Prelims)-2006]**

(1) 1 : 1

(2) 2 : 3

(3) 1 : 2

(4) 2 : 1

Sol. Answer (1)

27. A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 s for every circular lap. The average velocity and average speed for each circular lap respectively is **[AIPMT (Prelims)-2006]**

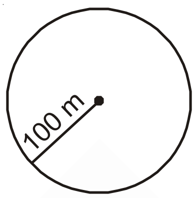
(1) 0, 0
(2) 0, 10 m/s
(3) 10 m/s, 10 m/s
(4) 10 m/s, 0

Sol. Answer (2)

T = 62.8 s

r = 100 m

$T = \frac{2\pi}{\omega}$



$\omega = \frac{2\pi}{T} = \frac{2\times 3.14\times 10}{62.8\times 100}$

$\boxed{\omega = 0.1 \text{ rad s}^{-1}}$

$v = r\omega$

$v = 100 \times 0.1$

$\boxed{v = 10 \text{ ms}^{-1}}$

Average velocity = 0
Average speed = 10 ms^{-1}

28. The vectors $\vec{A}$ and $\vec{B}$ are such that: $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$. The angle between the two vectors is **[AIPMT (Prelims)-2006]**

(1) 90°
(2) 60°
(3) 75°
(4) 45°

Sol. Answer (1)

$|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

$\sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{A^2+B^2-2AB\cos\theta}$

Squaring both the sides,

$A^2+B^2+2AB\cos\theta = A^2+B^2-2AB\cos\theta$

$4AB\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \boxed{\theta = 90^\circ}$

29. If a vector $2\hat{i}+3\hat{j}+8\hat{k}$ is perpendicular to the vector $4\hat{j}-4\hat{i}+\alpha\hat{k}$, then the value of α is

[AIPMT (Prelims)-2005]

(1) –1
(2) $\frac{1}{2}$
(3) $-\frac{1}{2}$
(4) 1

Sol. Answer (3)

30. A stone tied to the end of a string of 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 s, what is the magnitude and direction of acceleration of the stone?

[AIPMT (Prelims)-2005]

(1) $\frac{\pi^2}{4}$ ms^{-2} and direction along the radius towards the centre

(2) π^2 ms^{-2} and direction along the radius away from centre

(3) π^2 ms^{-2} and direction along the radius towards the centre

(4) π^2 ms^{-2} and direction along the tangent to the circle

Sol. Answer (3)

$\omega = \frac{22 \times 2\pi}{44 \text{ s}} \Rightarrow \pi \text{ rad s}^{-1}$

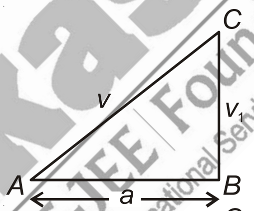
Centripetal acceleration, $a = r\omega^2$

$\boxed{a = 1 \times \pi^2}$ ms^{-2} along the radius towards the centre

31. Two boys are standing at the ends A and B of a ground, where $AB = a$. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other boy in a time t, where t is **[AIPMT (Prelims)-2005]**

(1) $\frac{a}{\sqrt{v^2 + v_1^2}}$ (2) $\sqrt{\frac{a^2}{v^2 - v_1^2}}$ (3) $\frac{a}{(v - v_1)}$ (4) $\frac{a}{(v + v_1)}$

Sol. Answer (2)

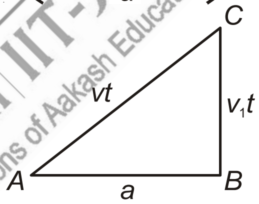


The distance travelled by body at B,

= Speed × t

= $v_1 t$

So, $BC = v_1 t$, similarly, $AC = vt$



Applying pythagoras in ΔABC,

$v^2 t^2 = v_1^2 t^2 + a^2$

$(v^2 - v_1^2)t^2 = a^2$

$t^2 = \frac{a^2}{v^2 - v_1^2}$

$\boxed{t = \frac{a}{\sqrt{v^2 - v_1^2}}}$

32. If the angle between the vectors $\vec{A}$ and $\vec{B}$ is θ, the value of the product $(\vec{B} \times \vec{A}) \cdot \vec{A}$ is equal to

[AIPMT (Prelims)-2005]

(1) $BA^2 \cos\theta$ (2) $BA^2 \sin\theta$

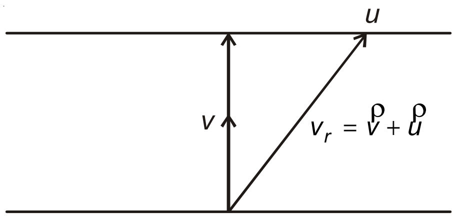
(3) $BA^2 \sin\theta \cos\theta$ (4) Zero

Sol. Answer (4)

33. A boat is sent across a river with a velocity of 8 km/h. If the resultant velocity of the boat is 10 km/h, then velocity of the river is

(1) 8 km/h (2) 10 km/h (3) 12.8 km/h (4) 6 km/h

Sol. Answer (4)



$v_r = \sqrt{v^2 + u^2}$

$v_r = 10\ \text{kmh}^{-1}$, $v = 8\ \text{kmh}^{-1}$

$u = ?$

$100 = 8^2 + u_R^2$

$\Rightarrow \quad u_R^2 = 36 \Rightarrow \boxed{u_R = 6\ \text{km/h}}$

34. Which of the following is correct relation between an arbitrary vector $\vec{A}$ and null vector $\vec{0}$?

(1) $\vec{A}+\vec{0}+\vec{A}\times\vec{0}=\vec{A}$ (2) $\vec{A}+\vec{0}+\vec{A}\times\vec{0}\neq\vec{A}$ (3) $\vec{A}+\vec{0}+\vec{A}\times\vec{0}=\vec{0}$ (4) None of these

Sol. Answer (1)

Knowledge based.

35. An object is being thrown at a speed of 20 m/s in a direction 45° above the horizontal. The time taken by the object to return to the same level is

(1) $20/g$ (2) $20\,g$ (3) $20\sqrt{2}/g$ (4) $20\sqrt{2}g$

Sol. Answer (3)

$u = 20\ \text{ms}^{-1}$

$\theta = 45°$

$$T = \frac{2u\sin\theta}{g}$$

$$T = \frac{2u}{g}\times\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{10}u$$

$$T = \frac{2\times 20}{g}\,\frac{1}{\sqrt{2}} \Rightarrow \boxed{\frac{20\sqrt{2}}{g} = T}$$

36. A body is whirled in a horizontal circle of radius 20 cm. It has an angular velocity of 10 rad/s. What is its linear velocity at any point on circular path?

(1) 20 m/s (2) $\sqrt{2}$ m/s (3) 10 m/s (4) 2 m/s

Sol. Answer (4)

$v = r\omega = 20\times10^{-2}\times10$

$\boxed{v = 2\ \text{ms}^{-1}}$

37. Identify the vector quantity among the following.

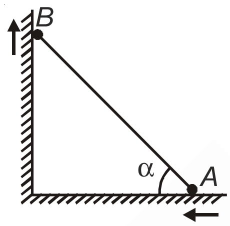
(1) Distance (2) Angular momentum

(3) Heat (4) Energy

Sol. Answer (2)

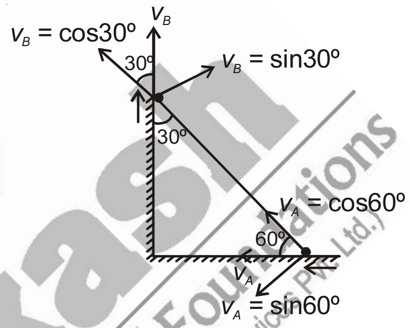
Angular momentum is an axial vector.

38. Two particles *A* and *B* are connected by a rigid rod *AB*. The rod slides along perpendicular rails as shown here. The velocity of *A* to the left is 10 m/s. What is the velocity of *B* when angle α = 60°?



(1) 10 m/s (2) 9.8 m/s (3) 5.8 m/s (4) 17.3 m/s

Sol. Answer (3)



$$v_A \cos 60° = v_B \cos 30°$$

$$10 \times \frac{1}{2} = v_B \times \frac{\sqrt{3}}{2}$$

$$\boxed{v_B = \frac{10}{\sqrt{3}}}$$

39. The speed of a boat is 5 km/h in still water. It crosses a river of width 1.0 km along the shortest possible path in 15 minutes. The velocity of the river water (in km/h) is

(1) 3 (2) 1 (3) 4 (4) 5

Sol. Answer (1)

v = 5 kmh^{-1}

d = 1.0 km

t = 15 min

40. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time *t*. The ratio of the angular speeds of the first to the second car is

(1) $r_1 : r_2$

(2) $m_1 : m_2$

(3) 1 : 1

(4) $m_1 m_2 : r_1 r_2$

Sol. Answer (3)

If time is same then,

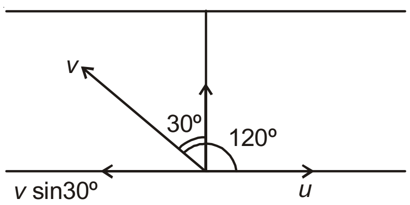
$\omega_1 : \omega_2 \Rightarrow 1 : 1$ $\left[\because \omega = \frac{2\pi}{T}\right]$

41. A person aiming to reach exactly opposite point on the bank of a stream is swimming with a speed of 0.5 m/s at an angle of 120° with the direction of flow of water. The speed of water in the stream is

(1) 0.25 m/s (2) 0.5 m/s

(3) 1.0 m/s (4) 0.433 m/s

Sol. Answer (1)



$v \sin 30° = u$

$$0.5 \times \frac{1}{2} = u \Rightarrow \boxed{u = 0.25 \text{ ms}^{-1}}$$

42. Two projectiles of same mass and with same velocity are thrown at an angle 60° and 30° with the horizontal, then which will remain same

(1) Time of flight (2) Range of projectile

(3) Maximum height acquired (4) All of these

Sol. Answer (2)

Range is same for complimentary angles.

43. Two particles having mass M and m are moving in a circular path having radius R and r. If their time periods are same, then the ratio of their angular velocities will be

(1) $\frac{r}{R}$ (2) $\frac{R}{r}$ (3) 1 (4) $\sqrt{\frac{R}{r}}$

Sol. Answer (3)

44. If $|\vec{A}+\vec{B}| = |\vec{A}| = |\vec{B}|$ then angle between A and B will be

(1) 90° (2) 120°

(3) 0° (4) 60°

Sol. Answer (2)

$$|\vec{A}+\vec{B}| = |\vec{A}| = |\vec{B}|$$

$$|\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$A^2 = A^2 + A^2 + 2A^2\cos\theta$$

$$-\frac{1}{2} = \cos\theta \Rightarrow \boxed{\theta = 120°}$$

45. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is

(1) 40 m/s^2

(2) 640π m/s^2

(3) 160π m/s^2

(4) 40π m/s^2

Sol. Answer (1)

$r = \frac{20}{\pi}$ m

$a_T \to$ constant

v = 80 ms^{-1}, θ = 4π rad

$v = r\omega$

$80 = \frac{20}{\pi} \cdot \omega \;\Rightarrow\; \boxed{\omega = 4\pi \text{ rad s}^{-1}}$

ω = 0,

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$4\pi \times 4\pi = 2 \times \alpha \times 4\pi$

$\Rightarrow \boxed{\alpha = 2\pi \text{ rad s}^{-2}}$

$a = r\alpha = \frac{20}{\pi} \times 2\pi$

$\boxed{a = 40 \text{ ms}^{-2}}$

46. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces

(1) Are equal to each other

(2) Are equal to each other in magnitude

(3) Are not equal to each other in magnitude

(4) Cannot be predicted

Sol. Answer (2)

$(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$

$A^2 - B^2 - AB + BA = 0$

$A^2 = B^2 \;\Rightarrow\; \boxed{A = B}$

so, $\boxed{|\vec{A}| = |\vec{B}|}$

47. A wheel has angular acceleration of 3.0 rad/s^2 and an initial angular speed of 2.00 rad/s. In a time of 2 s it has rotated through an angle (in radian) of

(1) 10 (2) 12

(3) 4 (4) 6

Sol. Answer (1)

α = 3 rad s^{-2}

ω_0 = 2 rad s^{-1}

t = 2 s

$\omega = \omega_0 + \alpha t$

$\omega = 2 + 3 \times 2$

$\boxed{\omega = 8 \text{ rad s}^{-1}}$

$\omega^2 - \omega_0^2 = 2 \times \theta$

$64 - 4 = 2 \times 3 \times \theta$

$\frac{60}{6} = \theta \Rightarrow \boxed{\theta = 10 \text{ rad s}^{-1}}$

48. A particle is moving such that its position coordinates (x, y) are

(2 m, 3 m) at time t = 0,

(6 m, 7 m) at time t = 2 s and

(13 m, 14 m) at time t = 5 s

Average velocity vector $(\vec{V}_{av})$ from t = 0 to t = 5 s is

(1) $\frac{1}{5}(13\hat{i} + 14\hat{j})$ (2) $\frac{7}{3}(\hat{i} + \hat{j})$ (3) $2(\hat{i} + \hat{j})$ (4) $\frac{11}{5}(\hat{i} + \hat{j})$

Sol. Answer (4)

$$V_{av} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t} = \frac{(13-2)\hat{i} + (14-3)\hat{j}}{(5-0)} = \frac{11}{5}(\hat{i} + \hat{j})$$

SECTION - D

Assertion - Reason Type Questions

1. A : If $\vec{A} \perp \vec{B}$, then $|\vec{A} + \vec{B}| = |A - \vec{B}|$.

R : If $\vec{A} \perp \vec{B}$, then $(\vec{A} + \vec{B})$ is perpendicular to $\vec{A} - \vec{B}$.

Sol. Answer (3)

2. A : The addition of two vectors $\vec{P}$ and $\vec{Q}$ is commutative.

R : By triangle law of vector addition we can prove $\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$.

Sol. Answer (1)

3. A : A vector cannot be divided by other vector.

R : A vector can be divided by a scalar.

Sol. Answer (2)

4. A : At the highest point the velocity of projectile is zero.

R : At maximum height projectile comes to rest.

Sol. Answer (4)

5. A : Horizontal range of a projectile is always same for angle of projection θ with horizontal or θ with vertical.

R : Horizontal range depends only on angle of projection.

Sol. Answer (4)

6. A : Horizontal motion of projectile without effect of air is uniform motion.

R : Without air effect the horizontal acceleration of projectile is zero.

Sol. Answer (1)

7. A : Path of a projectile with respect to another projectile is straight line.

R : Acceleration of a projectile with respect to another projectile is zero.

Sol. Answer (1)

8. A : In the case of ground to ground projection of a projectile from ground the angle of projection with horizontal is $\theta = 30°$. There is no point on its path such that instantaneous velocity is normal to the initial velocity.

R : Maximum deviation of the projectile is $2\theta = 60°$.

Sol. Answer (1)

9. A : Three vectors having magnitudes 10, 10 and 25 cannot produce zero resultant.

R : If three vectors are producing zero resultant, then sum of magnitude of any two is more than or equal to magnitude of third and difference is less than or equal to the magnitude of third.

Sol. Answer (1)

10. A : Uniform circular motion is accelerated motion still speed remains unchanged.

R : Instantaneous velocity is always normal to instantaneous acceleration in uniform circular motion.

Sol. Answer (1)

11. A : When a body moves on a curved path with increasing speed, then angle between instantaneous velocity and acceleration is acute angle.

R : When the speed is increasing, its tangential acceleration is in the direction of instantaneous velocity.

Sol. Answer (1)

12. A : A uniform circular motion have non uniform acceleration.

R : The direction of acceleration of a particle in uniform circular motion changes continuously.

Sol. Answer (1)

13. A : Angular displacement is vector quantity only for small values.

R : The direction of angular displacement is perpendicular to plane of rotation of object.

Sol. Answer (2)

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