Chapter 9

Mechanical Properties of Solids

Solutions

SECTION - A

Objective Type Questions

(Elastic Behaviour of Solids)

1. Select the correct alternative(s)

   (1) Elastic forces are not always conservative

   (2) Elastic forces are always conservative

   (3) Elastic forces are conservative only when Hooke's law is obeyed

   (4) Elastic forces are not conservative

Sol. Answer (1)

Since at every value of force material is not able to gain its shape. Therefore elastic forces are not always conservative.

2. Which of the following affects the elasticity of a substance?

   (1) Change in temperature (2) Impurity in substance

   (3) Hammering (4) All of these

Sol. Answer (4)

Elasticity is hampered by change in temperature as it changes the structure of grains of the material. Impurity also changes elasticity.

By hammering also grain shape gets changes and effects elasticity.

3. Select the wrong definition

   (1) Deforming Force – force that changes configuration of body

   (2) Elasticity – property of regaining original configuration

   (3) Plastic body – which can be easily melted

   (4) Elastic limit – beyond which material begins to flow

Sol. Answer (3)

Plastic body is defined as a body which cannot regain its shape and size after deforming force is removed.

(Stress and Strain)

4. The shear strain is possible in

(1) Solids (2) Liquids

(3) Gases (4) All of these

Sol. Answer (1)

Shear strain is possible in solids only, as only solids have a definite surface.

5. The ratio of radii of two wires of same material is 2 : 1. If these wires are stretched by equal force, the ratio of stresses produced in them is

(1) 2 : 1 (2) 1 : 2

(3) 1 : 4 (4) 4 : 1

Sol. Answer (3)

We know,

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} \qquad \begin{Bmatrix} S = \text{Stress} \\ F = \text{Force} \\ A = \text{Area} \\ r = \text{radius} \end{Bmatrix}$$

So, Stress × Area = Force

$S \times A = F$

∵ (Since) Force applied on the wires is equal we can relate two conditions as

$$S_1A_1 = S_2A_2$$

$$\frac{S_1}{S_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2}$$

$$\frac{S_1}{S_2} = \frac{r^2}{(2r)^2} = \frac{r^2}{4r^2} = \frac{1}{4}$$

$$\begin{Bmatrix} \text{Where} \\ S_1 - \text{Stress in } 1^{st} \text{ wire} \\ A_1 - \text{Area of } 1^{st} \text{ wire} \\ r_1 - \text{Radius of } 1^{st} \text{ wire} \\ S_2 - \text{Stress in } 2^{nd} \text{ wire} \\ A_2 - \text{Area of } 2^{nd} \text{ wire} \\ r_2 - \text{Radius of } 2^{nd} \text{ wire} \end{Bmatrix}$$

6. A steel wire of diameter 2 mm has a breaking strength of 4×10^5 N. What is the breaking force of similar steel wire of diameter 1.5 mm?

(1) 2.3×10^5 N (2) 2.6×10^5 N

(3) 3×10^5 N (4) 1.5×10^5 N

Sol. Answer (1)

We know,

Breaking force = Breaking stress × area

As breaking stress is dependent on material.

So we can use

$$\frac{F_1}{F_2} = \frac{d_1^2}{d_2^2} \qquad \begin{Bmatrix} F_1 = 4\times10^5 \text{ N} \\ d_1 = 2 \text{ mm} \\ F_2 = ? \\ d_2 = 1.5 \text{ mm} \end{Bmatrix}$$

Substituting values

$$\frac{4\times10^5}{F_2} = \frac{(2)^2}{(1.5)^2}$$

$F_2 = 2.3 \times 10^5$ N

7. A steel wire is 1 m long and 1 mm^2 in area of cross-section. If it takes 200 N to stretch this wire by 1 mm, how much force will be required to stretch a wire of the same material as well as diameter from its normal length of 10 m to a length of 1002 cm?

(1) 1000 N

(2) 200 N

(3) 400 N

(4) 2000 N

Sol. Answer (3)

$$\frac{FL}{AY} = \Delta x$$

Since A, Y remain constant in given case

We can say

$FL \propto \Delta x$

or $\frac{F_1L_1}{F_2L_2} = \frac{\Delta x_1}{\Delta x_2}$

$$\left\{\begin{array}{l} F_1 = 200 \text{ N} \\ \Delta x_1 = 1\text{ mm} \\ \Delta x_2 = 10.02 \text{ m} - 10 \text{ m} = 0.02 \text{ m} = 20 \text{ mm} \\ L_1 = 1\text{ m} \\ L_2 = 10 \text{ m} \end{array}\right.$$

Substitute the values

$$\frac{200\times 1}{F_2\times 10} = \frac{1\text{ mm}}{20\text{ mm}}$$

F_2 = 400 N

8. What is the percentage increase in length of a wire of diameter 2.5 mm, stretched by a force of 100 kg wt? Young's modulus of elasticity of wire = 12.5 × 10^{11} dyne/cm^2

(1) 0.16%

(2) 0.32%

(3) 0.08%

(4) 0.12%

Sol. Answer (1)

$$Y = \frac{FL}{A\Delta L} \quad \Rightarrow \quad \text{Percentage increase } \frac{\Delta L}{L}\times 100 = \frac{F}{AY}\times 100$$

Diameter = 2.5 mm $\quad d = \frac{2.5}{1000}\text{m}$

Area $= \frac{\pi d^2}{4} = \frac{\pi}{4}\left(\frac{2.5}{1000}\right)^2 \text{m}^2$ $\quad Y$ = 12.5 × 10^{11} dyne/cm^2 $\quad \left\{\frac{1\text{ dyne}}{\text{cm}^2} = \frac{0.1\text{ N}}{\text{m}^2}\right\}$

$\Rightarrow$ F = 100 × 10 = 1000 N

$$\Rightarrow \frac{1000\times 100}{\frac{3.14\times(2.5)^2}{4\times(1000)^2}\times 12.5\times 10^{11}\times 0.1} = \frac{\Delta L}{L}\times 100$$

= 0.16%

9. The Poisson's ratio of a material is 0.5. If a force is applied to a wire of this material, there is a decrease in the cross-sectional area by 4%. The percentage increase in the length is

(1) 1% (2) 2% (3) 2.5% (4) 4%

Sol. Answer (4)

$$\frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \eta$$

$$-\left(\frac{\Delta r / r}{\Delta l / l}\right) = 0.5$$

Substitute $\Delta r/r$ = (–2/100)

$$\frac{\Delta l}{l} = \frac{4}{100}$$

$\because\ A \propto r^2$

So $\dfrac{\Delta A}{A} = \dfrac{2\Delta r}{r}$

$$-\frac{4}{100} = 2\times\frac{\Delta r}{r}$$

$$-\frac{2}{100} = \frac{\Delta r}{r}$$

$\therefore$ % increase $= \dfrac{\Delta l}{l}\times 100 = 4\%$

10. If the temperature of a wire of length 2 m and area of cross-section 1 cm^2 is increased from 0° C to 80°C and is not allowed to increase in length, then force required for it is $\{Y = 10^{10}\ \text{N/m}^2,\ \alpha = 10^{-6}/°\text{C}\}$

(1) 80 N (2) 160 N (3) 400 N (4) 120 N

Sol. Answer (1)

Thermal expansion would be = $L \propto \Delta T$

Where L = original length

α = coefficient of linear expansion

ΔT = Change in temperature

So substituting values

$\Delta L = 2 \times 10^{-6} \times 80$

$\Delta L = 1.6 \times 10^{-4}$ m

Now $\Delta L = \dfrac{FL}{AY}$

$$\frac{\Delta L \times AY}{L} = F$$

Substitute values

$$\frac{1.6\times10^{-4}\times10^{10}\times1}{2\times10000} = F$$

80 N = F

11. A rod of length l and radius r is held between two rigid walls so that it is not allowed to expand. If its temperature is increased, then the force developed in it is proportional to

(1) L (2) $1/L$ (3) r^2 (4) r^{-2}

Sol. Answer (3)

(ΔL) Thermal expansion = $L \propto \Delta T$

Where L = Length original

$\propto$ = Coefficient of linear expansion

ΔT = Change in temperature

Or we can say

$\Delta L \propto L$

And force required to produce similar elongation can be calculated by

$$F = AY \cdot \frac{\Delta L}{L} \qquad [\because Y \text{ is constant}]$$

So $F \propto r^2 \cdot \dfrac{\Delta L}{L}$

Also $\Delta L \propto L$

So F only proportional to r^2

12. A uniform cubical block is subjected to volumetric compression, which decreases its each side by 2%. The Bulk strain produced in it is

(1) 0.03 (2) 0.02

(3) 0.06 (4) 0.12

Sol. Answer (3)

Volume = (side)^3

$V = (a)^3$

So $\dfrac{\Delta V}{V} = \dfrac{3\Delta a}{a}$ $\left\{\text{given } \dfrac{\Delta a}{a} = -2\%\right\}$

$\therefore \dfrac{\Delta V}{V} = 3 \times -2$ Side decreases so we used (–)ve sign

= –6%

So bulk strain produced is 0.06

13. The Poisson's ratio cannot have a value of

(1) 0.7 (2) 0.2

(3) 0.1 (4) 0.5

Sol. Answer (1)

Poisson's ratios value can't be practically more than 1/2 so only value above 1/2 is 0.7

14. A material has Poisson's ratio 0.5. If a uniform rod of it suffers a longitudinal strain of 3×10^{-3}, what will be percentage increase in volume?

(1) 2% (2) 3% (3) 5% (4) 0%

Sol. Answer (4)

$$\frac{\Delta V}{V} = (1-2\sigma)\frac{\Delta L}{L}$$

As $\sigma = 0.5$; $\dfrac{\Delta V}{V} = 0$

(Hooke's Law)

15. Hooke's law is applicable for

(1) Elastic materials only (2) Plastic materials only

(3) Elastomers only (4) All of these

Sol. Answer (1)

Hooke's law is applicable only for elastic materials as only they follow the stress-strain proportionality.

16. When a load of 10 kg is suspended on a metallic wire, its length increase by 2 mm. The force constant of the wire is

(1) 3×10^4 N/m (2) 2.5×10^3 N/m (3) 5×10^4 N/m (4) 7.5×10^3 N/m

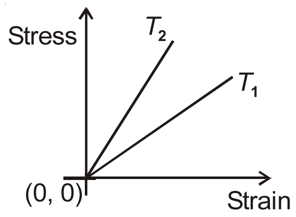
Sol. Answer (3)

Force constant $(K) = \dfrac{\text{Force}}{\text{Elongation}} = \dfrac{F}{\Delta x}$ $\quad \begin{cases} F = 10 \text{ kg} = 100 \text{ N} \\ \Delta x = 2 \text{ mm} = 0.002 \text{ m} \end{cases}$

Substituting values

$$K = \frac{100}{0.002} = 5\times10^4 \text{ N/m}$$

17. Figure shows graph between stress and strain for a uniform wire at two different temperatures. Then



(1) $T_1 > T_2$ (2) $T_2 > T_1$ (3) $T_1 = T_2$ (4) None of these

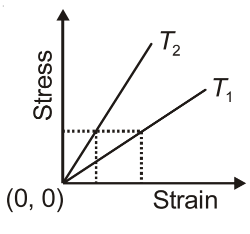
Sol. Answer (1)

From the graph we can see young's modulus is less for T_1 as compared to T_2

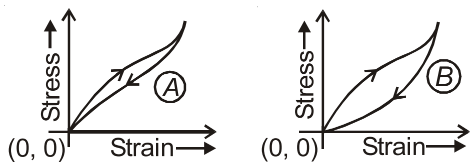
(Y = slope of stress-strain curve)

As T increases Y decreases

So $T_1 > T_2$



18. Two different types of rubber are found to have the stress-strain curves as shown. Then



(1) A is suitable for shock absorber
(2) B is suitable for shock absorber
(3) B is suitable for car tyres
(4) None of these

Sol. Answer (2)

One with higher hysterysis loss suitable for shock absorber because high hysterysis loss will lead to dampen shocks in a easy manner.

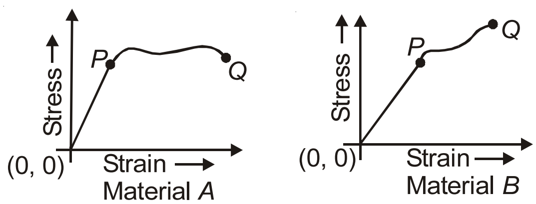
One with lower hysterysis loss suitable for types because it will have lesser energy dissipated into heat.

Area between loop gives amount of hysterysis loss. More area more loss, less area less loss.

Therefore, B is suitable for shock absorber and A for types.

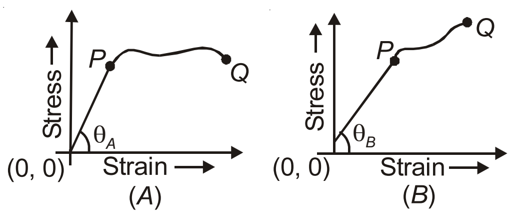
(Stress-Strain Curve)

19. The stress strain graphs for two materials A and B are shown in figure. The graphs are drawn to the same scale. Select the correct statement



(1) Material A has greater Young's Modulus
(2) Material A is ductile
(3) Material B is brittle
(4) All of these

Sol. Answer (4)



Slope of stress strain curve (tan θ) gives the value of young's modulus for given material

$\Rightarrow$ $\tan\theta = Y$

And from the graph we can clearly see

$\tan\theta_A > \tan\theta_B$

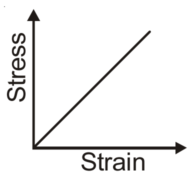
So material A has greater young's modulus

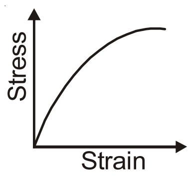
P to Q distance in material A is greater than P to Q distance in material B

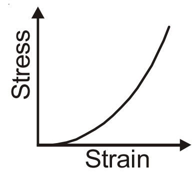
Which implies more deformation is possible in A as compared to B

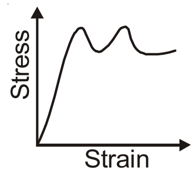
Hence we can say A is ductile, B is brittle.

20. Which of the following is the graph showing stress-strain variation for elastomers?

(1) 

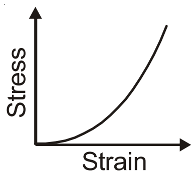
(2) 

(3) 

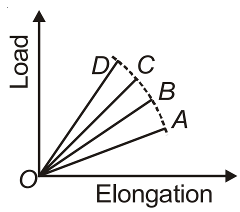
(4) 

Sol. Answer (3)

In elastomers stress varies exponentially with strain e.g. Rubber



21. The load versus elongation graph for four wires of same length and the same material is shown in figure. The thinnest wire is represented by line



(1) *OC* (2) *OD*

(3) *OA* (4) *OB*

Sol. Answer (3)

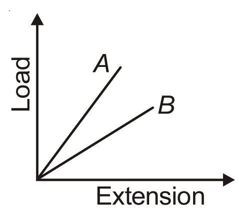
For the same load wire with maximum elongation has minimum cross-section area

As $\dfrac{FL}{AY} = \Delta x$

F, L, Y are fixed so $\dfrac{1}{A} \propto \Delta x$

$\Rightarrow$ *OA* is the thinnest.

22. In the given figure, if the dimensions of the two wires are same but materials are different, then Young's modulus is

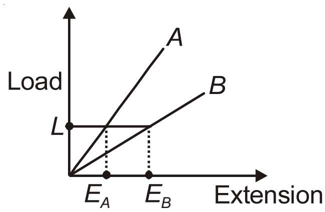


(1) More for *A* than *B* (2) More for *B* than *A*

(3) Equal for *A* and *B* (4) None of these

Sol. Answer (1)

At same value of load



A has less elongation than B ...(1)

$$\frac{FL}{AY} = \Delta L \qquad \left\{\begin{matrix} \because L, A \text{ are same} \\ F - \text{Load is also taken same} \end{matrix}\right\}$$

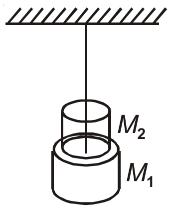
So $\frac{1}{Y} \propto \Delta L$ ...(2)

Using conditions (1) and (2)

We can say

$Y_A > Y_B$ {Young's modulus of A greater than B}

23. The length of wire, when M_1 is hung from it, is l_1 and is l_2 with both M_1 and M_2 hanging. The natural length of wire is



(1) $\frac{M_1}{M_2}(l_1 - l_2) + l_1$ (2) $\frac{M_2 l_1 - M_1 l_2}{M_1 + M_2}$ (3) $\frac{l_1 + l_2}{2}$ (4) $\sqrt{l_1 l_2}$

Sol. Answer (1)

Let the natural length of wire be = l

When only M_1 hanging

Using $\Delta l = \frac{FL}{AY}$

$$(l_1 - l) = \frac{M_1 g \cdot l}{AY} \quad ...(1)$$

When both M_1, M_2 hanging

$$(l_2 - l) = \frac{(M_1 + M_2)\, g \cdot l}{AY} \quad ...(2)$$

Dividing (1) by (2)

$$\frac{l_1 - l}{l_2 - l} = \frac{M_1}{M_1 + M_2}$$

Solving this we get

$$l = \frac{M_1}{M_2}(l_1 - l_2) + l_1$$

24. The substances having very short plastic region are

(1) Ductile (2) Brittle

(3) Malleable (4) All of these

Sol. Answer (2)

Substances with short plastic region are brittle because less amount of permanent deformation could be done in them.

(Elastic Moduli)

25. Due to addition of impurities, the modulus of elasticity

(1) Decreases (2) Increases

(3) Remains constant (4) May increase or decrease

Sol. Answer (4)

It depends on the elastic property of impurities if they themselves more elastic, elasticity will increase. If they are less elastic, elasticity will decrease.

26. A load of 2 kg produces an extension of 1 mm in a wire of 3 m in length and 1 mm in diameter. The Young's modulus of wire will be

(1) 3.25×10^{10} Nm^{-2} (2) 7.48×10^{12} Nm^{2}

(3) 7.48×10^{10} Nm^{-2} (4) 7.48×10^{-10} Nm^{-2}

Sol. Answer (3)

We know

$$\frac{\text{Force} \times \text{Length}}{\text{Area of cross-section} \times \text{elongation}} = \text{Young's Modulus}$$

$$\frac{F \times L}{A \times \Delta L} = Y \qquad \begin{Bmatrix} F = 2 \times 10 \text{ N}, \ A = \pi \times (1/2)^2 \times 10^{-6} \text{ m}^2 \\ L = 3 \text{ m} \quad , \Delta L = 1 \times 10^{-3} \text{ m} \end{Bmatrix}$$

Substituting values

$$\frac{20 \times 3}{\pi \times \frac{1}{4} \times 10^{-6} \times 1 \times 10^{-3}} = Y$$

$$\frac{20 \times 3 \times 4}{3.14 \times 10^{-9}} = Y$$

7.48×10^{10} Nm^{-2} = Y

27. Young's modulus depends upon

(1) Stress applied on material (2) Strain produced in material

(3) Temperature of material (4) All of these

Sol. Answer (3)

Young's modulus is a material property and it also depends on temperature of material.

28. The value of Young's modulus for a perfectly rigid body is

(1) 1 (2) Less than 1 (3) Zero (4) Infinite

Sol. Answer (4)

For perfectly rigid body the condition is that there should not be any elognation (ΔL = 0) for any value of force

So from the formulae we know $\frac{FL}{A \cdot \Delta L} = Y$

If we put ΔL = 0

We get Y as ∞

29. A spherical ball contracts in volume by 0.01% when subjected to a normal uniform pressure of 100 atm. The Bulk modulus of its material is

(1) 1.01×10^{11} Nm^{-2} (2) 1.01×10^{12} Nm^{-2} (3) 1.01×10^{10} Nm^{-2} (4) 1.0×10^{13} Nm^{-2}

Sol. Answer (1)

We know $\dfrac{\Delta V}{V} = -\dfrac{P}{B}$

Substituting values

$$\frac{-\dfrac{0.01}{100}\times V}{V} = \frac{-100}{B}\times 1.01\times 10^5 \qquad \{1 \text{ atm} = 1.01 \times 10^5 \text{ Pa or Nm}^{-2}\}$$

$B = 1.01 \times 10^{11}$ Nm^{-2}

30. A metallic rod of length l and cross-sectional area A is made of a material of Young's modulus Y. If the rod is elongated by an amount y, then the work done is proportional to

(1) y (2) $\dfrac{1}{y}$ (3) y^2 (4) $\dfrac{1}{y^2}$

Sol. Answer (3)

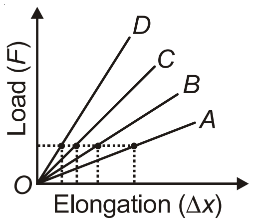
Work done = energy stored

$$W = \frac{1}{2} \times \text{force} \times \text{elongation} \qquad \left\{\text{Force} = \frac{\Delta L}{L}\cdot AY\right.$$

$$W = \frac{1}{2}\times\frac{\Delta L}{L}\times A\times Y\times \Delta L$$

$$W = \frac{1}{2}\frac{AY}{L}\times \Delta L^2$$

$W \propto \Delta L^2$



31. If the Bulk modulus of lead is 8.0×10^9 N/m^2 and the initial density of the lead is 11.4 g/cc, then under the pressure of 2.0×10^8 N/m^2, the density of the lead is

(1) 11.3 g/cc (2) 11.5 g/cc (3) 11.6 g/cc (4) 11.7 g/cc

Sol. Answer (4)

We know,

$$\rho_2 = \frac{\rho_1 B}{(B-P)}$$

Substituting the values

We get

ρ_2 = 11.7 g/cc

$$\left\{\begin{array}{l}\text{Where}\\ P = 2\times 10^8 \text{ N/m}^2\\ B = 8\times 10^9 \text{ N/m}^2\\ \rho_1 = 11.4 \text{ g/cc}\\ \rho_2 = ?\end{array}\right.$$

32. For a given material, the Young's modulus is 2.4 times its modulus of rigidity. Its Poisson's ratio is

(1) 0.2 (2) 0.4 (3) 1.2 (4) 2.4

Sol. Answer (1)

$Y = 2\eta\,[1 + \sigma]$

$\Rightarrow\ 2.4\eta = 2\eta\,[1 + \sigma]$

$\Rightarrow\ 0.2 = \sigma$

$$\left\{\begin{array}{l}\text{Where}\\ Y = \text{Young's modulus}\\ \sigma = \text{Poisson's ratio}\\ \eta = \text{Modulus of rigidity}\end{array}\right.$$

33. When the temperature of a gas is constant at 20°C and pressure is changed from $P_1 = 1.01 \times 10^5$ Pa to $P_2 = 1.165 \times 10^5$ Pa, then the volume changes by 10%. The Bulk modulus of the gas is

(1) 1.55×10^5 Pa (2) 1.01×10^5 Pa (3) 1.4×10^5 Pa (4) 0.115×10^5 Pa

Sol. Answer (1)

$$\frac{\Delta V}{V} = \frac{-\Delta P}{B}$$

Substituting the values

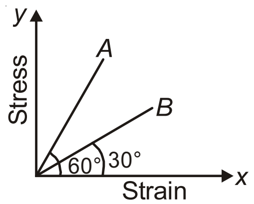
$$\frac{-10}{100} = \frac{-(1.165\times10^6 - 1.01\times10^6)}{B}$$

$$\frac{1}{10} = \frac{.155\times10^5}{B}$$

$B = 1.55 \times 10^5$ Pa

$$\left\{\begin{array}{l}\Delta V = 10\%\text{ of }V\ (\because\text{ Pressure increases volume must}\\ \qquad\qquad\text{decreases by 10\% so we will use a +ve sign)}\\ \text{If } V = 100\text{ cc}\\ \Rightarrow \Delta V = -10\text{ cc}\\ \Delta P = P_2 - P_1\\ \quad = 1.165\times10^5 - 1.01\times10^5\end{array}\right.$$

34. The stress versus strain graph for wires of two materials A and B are as shown in the figure. If Y_A and Y_B are the Young's moduli of the materials, then



(1) $Y_B = 2Y_A$ (2) $Y_A = 3Y_B$ (3) $Y_B = 3Y_A$ (4) $Y_A = Y_B$

Sol. Answer (2)

$Y = \tan\theta$

$Y_A = \tan 60°$, $\quad Y_B = \tan 30°$

$\quad = \sqrt{3}$ $\qquad = 1/\sqrt{3}$

$\Rightarrow\ Y_A = 3Y_B$

35. The ratio of adiabatic to isothermal elasticity of a diatomic gas is

(1) 1.67 (2) 1.4 (3) 1.33 (4) 1.27

Sol. Answer (2)

$K_{\text{isothermal}}$ = Pressure of gas (P)

$K_{\text{adiabatic}} = \gamma \times$ pressure of gas ($\gamma \cdot P$)

$$\text{Ratio} = \frac{\gamma P}{P} = \gamma$$

$$\gamma\text{ of diatomic gas} = \frac{7}{5} = 1.4$$

36. When a rubber ball is taken to the bottom of a sea of depth 1400 m, its volume decreases by 2%. The Bulk modulus of rubber ball is [density of water is 1 g cc and g = 10 m/s^2]

(1) 7×10^8 N/m^2 (2) 6×10^8 N/m^2 (3) 14×10^8 N/m^2 (4) 9×10^8 N/m^2

Sol. Answer (1)

Pressure at the bottom of sea = $\rho_w gh$ = 1000 kg/m^3 × 10 m/s^2 × 1400 m = 14000000 N/m^2

Also we know

$$\frac{\Delta V}{V} = -\frac{P}{B} \qquad \left\{\frac{\Delta V}{V} = \frac{-2}{100}\right\}$$

$$\Rightarrow \frac{-2}{100} = \frac{-14000000}{B} \Rightarrow B = 7 \times 10^8 \text{ N/m}^2$$

37. A spherical ball contracts in volume by 0.02%, when subjected to a normal uniform pressure of 50 atmosphere. The Bulk modulus of its material is

(1) 1×10^{11} N/m^2 (2) 2×10^{10} N/m^2 (3) 2.5×10^{10} N/m^2 (4) 1×10^{13} N/m^2

Sol. Answer (3)

$$\frac{\Delta V}{V} = -\frac{P}{B} \qquad \left\{\frac{\Delta V}{V} = \frac{-0.02}{100}\right\}$$

P = 50 atm = 50 × 1.01 × 10^5 Pa or N/m^2

So $B = 50 \times 1.01 \times 10^5 \times \frac{100}{0.02} = 2.5 \times 10^{10}$ N/m^2

38. For an elastic material

(1) $Y > \eta$ (2) $Y < \eta$ (3) $Y\eta = 1$ (4) $Y = \eta$

Sol. Answer (1)

Y = Young's modulus

η = modulus of rigidity

We have a formulae

$Y = 2\eta\,[1 + \sigma]$

$0 < \sigma \le 0.5$

$$\begin{bmatrix}\text{Where } \sigma = \text{poisson's ratio practical} \\ \text{value of } \sigma \text{ lies between 0 to 0.5}\end{bmatrix}$$

Using maximum value of σ

$Y = 3\eta$

$\Rightarrow Y > \eta$

39. Correct pair is

(1) Change in shape – Longitudinal strain (2) Change in volume – Shear strain

(3) Change in length – Bulk strain (4) Reciprocal of Bulk modulus – Compressibility

Sol. Answer (4)

B = bulk modulus and $\frac{1}{B}$ is defined as compressibility

(Applications of Elastic Behaviour of Materials)

40. The breaking stress of aluminium is 7.5×10^7 Nm^{-2}. The greatest length of aluminium wire that can hang vertically without breaking is (Density of aluminium is 2.7×10^3 kg m^{-3})

(1) 283×10^3 m (2) 28.3×10^3 m (3) 2.83×10^3 m (4) 0.283×10^3 m

Sol. Answer (3)

Breaking stress = $\rho \times g \times L_{max}$

Substitute values from the question

Breaking stress = 7.5×10^7 Nm^{-2}

$\rho = 2.7 \times 10^3$ kg m^{-3}

$g = 9.8$ m/s

$$\left\{\begin{array}{l}\rho = \text{Density of material}\\ g = \text{Acceleration due to gravity}\\ L_{max} = \text{Length of wire that can hang without breaking}\end{array}\right\}$$

$7.5 \times 10^7 = 2.7 \times 10^3 \times 9.8 \times L_{max}$

$$\frac{7.5\times10^7}{2.7\times10^3\times9.8} = L_{max}$$

$2.83 \times 10^3 \text{ m} = L_{max}$

41. A wire 2 m in length suspended vertically stretches by 10 mm when mass of 10 kg is attached to the lower end. The elastic potential energy gain by the wire is (take g = 10 m/s^2)

(1) 0.5 J (2) 5 J (3) 50 J (4) 500 J

Sol. Answer (1)

Change in potential energy , $\Delta U = \frac{1}{2}\cdot F \cdot \Delta L$

Substituting values $\left\{\begin{array}{l}F = 10\times10 \text{ N}\\ \Delta L = 10 \text{ mm} = 10\times10^{-3} \text{ m}\end{array}\right.$

$$\Delta U = \frac{1}{2}\times100\times\frac{10}{1000}$$

$\Delta U = 0.5$ J

42. A wire of length L and cross-sectional area A is made of material of Young's modulus Y. The work done in stretching the wire by an amount x is

(1) $\frac{YAx^2}{L}$ (2) $\frac{YAx^2}{2L}$ (3) $\frac{2YAx^2}{L}$ (4) $\frac{4YAx^2}{L}$

Sol. Answer (2)

$$\because \quad W = \frac{1}{2}Fx$$

$$\text{and } Y = \frac{FL}{Ax}$$

$$\therefore \quad F = \frac{YAx}{L}$$

$$W = \frac{1}{2}\left(\frac{YAx}{L}\right)x$$

$$W = \frac{1}{2}\frac{YAx^2}{L}$$

43. Two exactly similar wires of steel and copper are stretched by equal forces. If the total elongation is 2 cm, then how much is the elongation in steel and copper wire respectively? Given, Y_{steel} = 20 × 10^{11} dyne/cm^2, Y_{copper} = 12 × 10^{11} dyne/cm^2.

(1) 1.25 cm; 0.75 cm (2) 0.75 cm; 1.25 cm (3) 1.15 cm; 0.85 cm (4) 0.85 cm; 1.15 cm

Sol. Answer (2)

Let us say that elongation in copper = x

Than elongation in steel = 2 – x

We know

$$\frac{FL}{AY} = \Delta x$$

$\because$ F, A, L are same only material is different

We can say

$$\frac{1}{Y} \propto \Delta x$$

$$\frac{Y_2}{Y_1} = \frac{\Delta x_1}{\Delta x_2}$$

$$\left\{\begin{array}{l}\text{Where}\\ Y_2 = Y_{steel}\\ Y_1 = Y_{copper}\\ \Delta x_1 = \text{elongation in copper} = x\\ \Delta x_2 = \text{elongation in steel} = 2 - x\end{array}\right.$$

Substituting values

$$\frac{20\times 10^{11}}{12\times 10^{11}} = \frac{x}{2-x}$$

$\Rightarrow$ x = 1.25 cm

So Δx_{copper} = 1.25 cm, Δx_{steel} = 0.75 cm

44. A steel rod has a radius 10 mm and a length of 1.0 m. A force stretches it along its length and produces a strain of 0.32%. Young's modulus of the steel is 2.0 × 10^{11} Nm^{-2}. What is the magnitude of the force stretching the rod?

(1) 100.5 kN (2) 201 kN (3) 78 kN (4) 150 kN

Sol. Answer (2)

Strain = 0.32%

$$\Rightarrow \frac{\Delta L}{L}\times 100 = 0.32$$

$$\Rightarrow \frac{\Delta L}{L} = \frac{0.32}{100}$$

$$A = \pi r^2 = 3.14\times\left(\frac{10}{1000}\right)^2$$

Y = 2 × 10^{11} Nm^2

We know

$$\frac{FL}{AY} = \Delta L$$

$$F = \left(\frac{\Delta L}{L}\right)\times A\times Y$$

Substituting values

$$F = \frac{0.32}{100}\times 3.14\times\left(\frac{10}{1000}\right)^2\times 2\times 10^{11}$$

F = 201 kN

45. The proportional limit of steel is 8 × 10^8 N/m^2 and its Young's modulus is 2 × 10^{11} N/m^2. The maximum elongation, a one metre long steel wire can be given without exceeding the proportional limit is

(1) 2 mm (2) 4 mm

(3) 1 mm (4) 8 mm

Sol. Answer (2)

At proportional limit

Stress $\propto$ strain

Stress = $Y \times$ strain {Y = Young's Modulus}

Stress = $Y \times \dfrac{\Delta L}{L}$

$$\begin{cases} \text{Stress} = 8\times10^{8}\ \text{N/m}^2 \\ Y = 2\times10^{11}\ \text{N/m}^2 \\ L = 1\ \text{m} \end{cases}$$

Substituting values

$$\frac{8\times10^{8}\times1}{2\times10^{11}} = \Delta L$$

$$4\ \text{mm} = \Delta L$$

46. In a series combination of copper and steel wires of same length and same diameter, a force is applied at one of their ends while the other end is kept fixed. The combined length is increased by 2 cm. The wires will have

(1) Same stress and same strain

(2) Different stress and different strain

(3) Different stress and same strain

(4) Same stress and different strain

Sol. Answer (4)

Stress = $\dfrac{F}{A}$

Force is same, A is same

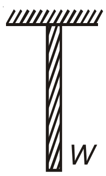
So same stress

Strain = $\dfrac{\Delta L}{L}$

L is same, but due to different young's modulus

ΔL would be different so strain is different

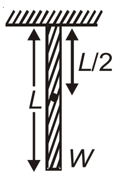
47. A rod of uniform cross-sectional area A and length L has a weight W. It is suspended vertically from a fixed support. If Young's modulus for rod is Y, then elongation produced in rod is



(1) $\dfrac{WL}{YA}$ (2) $\dfrac{WL}{2YA}$ (3) $\dfrac{WL}{4YA}$ (4) $\dfrac{3WL}{4YA}$

Sol. Answer (2)

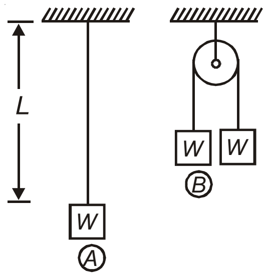
Center of mass is at $\dfrac{L}{2}$ distance from top so it can be assumed for easy calculation that W weight is hanged to a $\dfrac{L}{2}$ length string



Now use $\dfrac{FL}{AY}\cdot\Delta L$

$$\Delta L = \frac{W\times L}{2AY}$$

48. If in case A, elongation in wire of length L is l, then for same wire elongation in case B will be



(1) $4l$ (2) $2l$ (3) l (4) $l/2$

Sol. Answer (3)

Since tension in both cases is same and all other parametrs (Y, A, L) are also same

$\Rightarrow$ Elongation will be same in both cases.

49. Two wires A and B of same material have radii in the ratio 2 : 1 and lengths in the ratio 4 : 1. The ratio of the normal forces required to produce the same change in the lengths of these two wires is

(1) 1 : 1 (2) 2 : 1 (3) 1 : 2 (4) 1 : 4

Sol. Answer (1)

From $\dfrac{FL}{AY} = \Delta x$ $\{\because\ \Delta x,\ Y \text{ same}\}$

We using $F \propto \dfrac{L}{A} \propto \dfrac{L}{r^2}$

So $\dfrac{F_1}{F_2} = \dfrac{L_1}{r_1^2} \times \dfrac{r_2^2}{L_2} = \left(\dfrac{L_1}{L_2}\right) \times \left(\dfrac{r_2}{r_1}\right)^2$

Substitute the ratio's

We get $\dfrac{F_1}{F_2} = \dfrac{1}{1}$ or $F_1 : F_2 :: 1 : 1$

50. Energy stored per unit volume in a stretched wire having Young's modulus Y and stress 'S' is

(1) $\dfrac{YS}{2}$ (2) $\dfrac{S^2Y}{2}$ (3) $\dfrac{S^2}{2Y}$ (4) $\dfrac{S}{2Y}$

Sol. Answer (3)

$$\Delta U = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$= \frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y} \quad \{\because\ \text{Stress} = Y \cdot \text{strain}\}$$

$$= \frac{1}{2} \times \frac{S^2}{Y}$$

51. A wire suspended vertically from one end is stretched by attaching a weight 200 N to the lower end. The weight stretches the wire by 1 mm. The elastic potential energy gained by the wire is

(1) 0.1 J (2) 0.2 J (3) 0.4 J (4) 10 J

Sol. Answer (1)

Elastic potential energy = $\frac{1}{2}$ × force × elongation

$$= \frac{1}{2} \times 200 \times \frac{1}{1000} = 0.1\text{ J}$$

52. Work done by restoring force in a wire within elastic limit is –10 J. Maximum amount of heat produced in the wire is

(1) 10 J (2) 20 J (3) 5 J (4) 15 J

Sol. Answer (1)

Work done by external constant force = heat produced + potential energy

$20\text{ J} = \Delta H + 10\text{ J}$

$\Rightarrow \Delta H = 10\text{ J}$

53. The work done per unit volume to stretch the length of area of cross-section 2 mm^2 by 2% will be [$Y = 8 \times 10^{10}$ N/m^2]

(1) 40 MJ/m^3 (2) 16 MJ/m^3 (3) 64 MJ/m^3 (4) 32 MJ/m^3

Sol. Answer (2)

Work done per unit volume in stretching

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} Y \times (\text{Strain})^2$$

$$= \frac{1}{2} \times 8 \times 10^{10} \times \left(\frac{2}{100}\right)^2$$

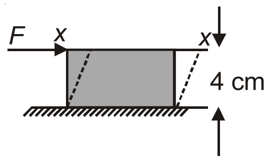
= 16 MJ/m^3

Work done per unit volume

Substitute values $= 8 \times 10^{10} \cdot \left(\frac{2}{100}\right)^2 \times \frac{1}{2}$

= 16 MJ/m^3

54. A steel plate of face area 1 cm^2 and thickness 4 cm is fixed rigidly at the lower surface. A tangential force F = 10 kN is applied on the upper surface as shown in the figure. The lateral displacement x of upper surface w.r.t. the lower surface is (Modulus of rigidity for steel is 8×10^{11} N/m^2)



(1) 5×10^{-5} m (2) 5×10^{-6} m

(3) 2.5×10^{-3} m (4) 2.5×10^{-4} m

Sol. Answer (2)

Modulus of rigidity (G) $= \dfrac{\text{Force} \times \text{Length}}{\text{Area} \times \text{Lateral displacement}} = \dfrac{FL}{A \times \Delta x}$

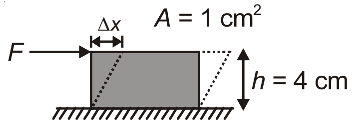
F = 10 kN = 10 × 10^3 N

L = 4 cm = 0.04 m

A = 1 cm^2 = 1 × 10^{-4} m^2

G = 8 × 10^{11} N/m^2

Substituting values



$$8 \times 10^{11} = \frac{10 \times 10^3 \times 0.04}{1 \times 10^{-4} \times \Delta x}$$

$$\Delta x = \frac{10 \times 10^3 \times 0.04}{1 \times 10^{-4} \times 8 \times 10^{11}} = 5 \times 10^{-6}\ \text{m}$$

55. When a uniform metallic wire is stretched the lateral strain produced in it is β. If σ and Y are the Poisson's ratio and Young's modulus for wire, then elastic potential energy density of wire is

(1) $\dfrac{Y\beta^2}{2}$

(2) $\dfrac{Y\beta^2}{2\sigma^2}$

(3) $\dfrac{Y\sigma\beta^2}{2}$

(4) $\dfrac{Y\sigma^2}{2\beta}$

Sol. Answer (2)

β = Strain (lateral)

σ = Poisson's ratio

Y = Young's modulus

Elastic potential energy density = $\dfrac{1}{2}$ × Y × (strain longitudinal)^2 ...(1)

Also $\dfrac{\text{Lateral strain}}{\text{Longitudinal strain}} = \text{Poisson's ratio}$

$$\frac{\beta}{\text{Longitudinal strain}} = \sigma$$

$\Rightarrow$ Longitudinal strain $= \dfrac{\beta}{\sigma}$

Substituting the value in equation (1)

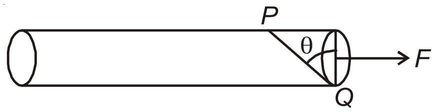
$$E.P.E = \frac{1}{2} Y \times \left(\frac{\beta}{\sigma}\right)^2 = \frac{1Y\beta^2}{2\sigma^2}$$

SECTION - B

Objective Type Questions

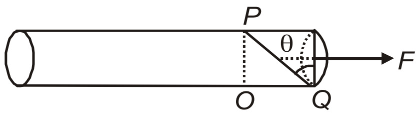
(Stress and Strain)

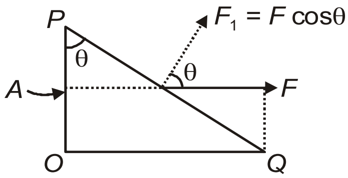
1. A force F is applied along a rod of transverse sectional area A. The normal stress to a section PQ inclined θ to transverse section is



(1) $\dfrac{F\sin\theta}{A}$ (2) $\dfrac{F}{A}\cos\theta$ (3) $\dfrac{F}{2A}\sin 2\theta$ (4) $\dfrac{F}{A}\cos^2\theta$

Sol. Answer (4)





$$\text{Stress} = \frac{F_{\text{normal}}}{\text{Area}} = \frac{F_1}{\text{Area}}$$

$$= \frac{F\cos\theta}{A/\cos\theta}$$

$$= \frac{F}{A}\cos^2\theta$$

Cross-sectional area of $PO = A$

Cross-sectional area of $PQ = \dfrac{PO}{\cos\theta} = \dfrac{A}{\cos\theta}$

2. A vertical hanging bar of length l and mass m per unit length carries a load of mass M at lower end, its upper end is clamped to a rigid support. The tensile stress a distance x from support is ($A \rightarrow$ area of cross-section of bar)

(1) $\dfrac{Mg + mg(l - x)}{A}$ (2) $\dfrac{Mg}{A}$ (3) $\dfrac{Mg + mgl}{A}$ (4) $\dfrac{(M + m)gx}{Al}$

Sol. Answer (1)

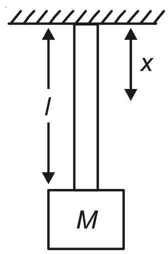
Tensile stress = $\dfrac{\text{Tension at point}}{\text{Area}}$

Tension at distance x from top would be the amount of force acting due to all the weight below it

= Mass per unit length of rod × length of rod + Mg

= $m \times (l - x)\, g + Mg$

So Tensile stress $= \dfrac{m(l - x)g + Mg}{A}$



3. A wire of length 5 m is twisted through 30° at free end. If the radius of wire is 1 mm, the shearing strain in the wire is

(1) 30° (2) 0.36′

(3) 1° (4) 0.18°

Sol. Answer (2)

$$\theta = \frac{r}{L}\phi$$

$$\theta = \frac{1\times10^{-3}\times30°}{5}$$

$$\theta = 6 \times 10^{-3}$$

$= 0.36'$

$$\begin{cases}\text{Where} \\ \theta = \text{Angle of shear} \\ \phi = \text{Angle of twist} \\ r = \text{Radius of rod} \\ l = \text{length or rod}\end{cases}$$

4. One end of uniform wire of length L and of weight W is attached rigidly to a point in roof and a weight W_1 is suspended from the lower end. If A is area of cross-section of the wire, the stress in the wire at a height $\frac{3L}{4}$ from its lower end is

(1) $\frac{W_1}{A}$ (2) $\frac{\left(W_1+\frac{W}{4}\right)}{A}$ (3) $\frac{\left(W_1+\frac{3W}{4}\right)}{A}$ (4) $\frac{W_1+W}{A}$

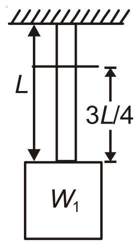
Sol. Answer (3)

$$\text{Stress} = \frac{\text{Tension at point}}{\text{Area of cross-section}}$$

Tension = force due to weight hanging below the choosen point

That is $\left(\frac{3W}{4}+W_1\right)$

$$\text{Stress} = \frac{3W/4+W_1}{A}$$



5. What is called the ratio of the breaking stress and the working stress?

(1) Elastic fatigue (2) Elastic after effect (3) Yield point (4) Factor of safety

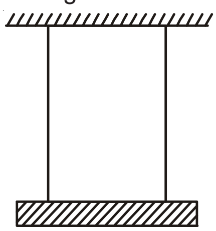
Sol. Answer (4)

$$\frac{\text{Breaking stress}}{\text{Working stress}} = n$$

n = Factor of safely

(Elastic Moduli)

6. Two wires of equal length and cross-sectional area are suspended as shown in figure. Their Young's modulii are Y_1 and Y_2 respectively. The equivalent Young's modulii will be

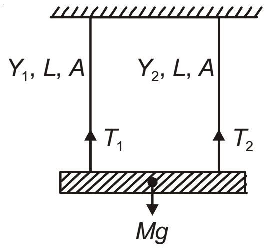


(1) $Y_1 + Y_2$ (2) $\frac{Y_1Y_2}{Y_1+Y_2}$ (3) $\frac{Y_1+Y_2}{2}$ (4) $\sqrt{Y_1Y_2}$

Sol. Answer (3)

Forces acting on both wires would be equal to T_1 and T_2 respectively by free body diagram

Let equivalent force constant of wire = K



$$K_1 + K_2 = K \qquad \begin{bmatrix}\text{Crossectional area will double} \\ \text{when both wires taken together}\end{bmatrix}$$

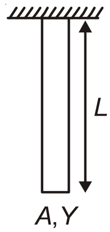
$$\frac{AY_1}{L} + \frac{AY_2}{L} = \frac{2AY}{L} \Rightarrow Y = \frac{Y_1 + Y_2}{2}$$

7. A uniform rod of length L has a mass per unit length λ and area of cross-section A. If the Young's modulus of the rod is Y. Then elongation in the rod due to its own weight is

(1) $\frac{2\lambda gL^2}{AY}$ (2) $\frac{\lambda gL^2}{AY}$ (3) $\frac{\lambda gL^2}{4AY}$ (4) $\frac{\lambda gL^2}{2AY}$

Sol. Answer (4)

Total mass can be assumed to be concentrated at center of mass at distance $\frac{L}{2}$ from top



$$\frac{M}{L} = \lambda$$

$$M = \lambda L$$

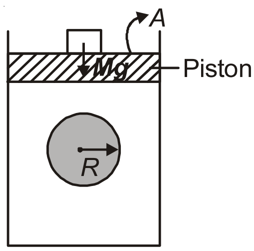
$$\Delta x = \frac{FL/2}{AY} = \frac{1}{2} \times \frac{\lambda L^2 g}{AY}$$

8. A solid sphere of radius R made of a material of bulk modulus B surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. Find the fractional decrease in the radius of the sphere $\left(\frac{\Delta R}{R}\right)$ when a mass M is placed on the piston to compress the liquid

(1) $\frac{Mg}{AB}$ (2) $\frac{Mg}{4AB}$ (3) $\frac{Mg}{3AB}$ (4) $\frac{Mg}{2AB}$

Sol. Answer (3)

Pressure increased to weight M



$$P = \frac{\text{Force}}{\text{Area}} = \frac{Mg}{A} \qquad ...(1)$$

And we know,

$$\frac{-\Delta V}{V} = \frac{P}{B}$$

$$\frac{-\Delta V}{V} = \frac{Mg}{AB} \qquad \left[\because P = \frac{Mg}{A}\right] \qquad ...(2)$$

$$\left\{\begin{array}{l}\text{Where} -\frac{\Delta V}{V} = \text{Fractional decrease in volume} \\ P = \text{Pressure increased} \\ B = \text{Bulk modulus}\end{array}\right.$$

Volume of a sphere is $V = \frac{4}{3}\pi R^3$ $\begin{Bmatrix}\text{Where} \\ r \text{ is radius}\end{Bmatrix}$

$$\Rightarrow \frac{\Delta V}{V} = \frac{3\Delta R}{R}$$

Using (2)

$$-\frac{Mg}{3AB} = \frac{\Delta R}{R}$$ [(–)ive sign indicates decrease]

$\therefore$ Fractional decrease in radius is $\frac{Mg}{3AB}$

9. A sphere contracts in volume by 0.01% when taken to the bottom of sea 1 km deep. Find Bulk modulus of the material of sphere

(1) 9.8×10^6 N/m^2 (2) 1.2×10^{10} N/m^2 (3) 9.8×10^{10} N/m^2 (4) 9.8×10^{11} N/m^2

Sol. Answer (3)

Pressure at bottom of sea = $\rho_w gh$

$\rho_w = 1000$ kg/m^3 = 1 g/cc, $g = 9.8$ m/s^2, $h = 1000$ m

$P = 10^3 \times 9.8 \times 1000$ N/m^2

Now $\frac{-\Delta V}{V} = \frac{P}{B}$ $\left\{\frac{-\Delta V}{V} = \frac{0.01}{100}\right.$ (given)

$$\frac{0.01}{100} = \frac{10^3 \times 9.8 \times 1000}{B}$$

$B = 9.8 \times 10^{10}$ N/m^2

10. A solid cube of copper of edge 10 cm subjected to a hydraulic pressure of 7×10^6 pascal. If Bulk modulus of copper is 140 GPa, then contraction in its volume will be

(1) 5×10^{-8} m^3 (2) 4×10^{-8} m^3

(3) 2×10^{-8} m^3 (4) 10^8 m^3

Sol. Answer (1)

Initial volume V = (side)^3 = $(10 \times 10^{-2})^3 = 10^{-3}$ m^3

$P = 7 \times 10^6$ Pa

$B = 140 \times 10^9$ Pa

We know

$$\frac{-\Delta V}{V} = \frac{P}{B}$$ {$-\Delta V$ = Contraction in volume}

$$\frac{-\Delta V}{10^{-3}} = \frac{7 \times 10^6}{140 \times 10^9}$$

$-\Delta V = 5 \times 10^{-8}$ m^3

11. Three bars having length l, $2l$ and $3l$ and area of cross-section A, $2A$ and $3A$ are joined rigidly end to end. Compound rod is subjected to a stretching force F. The increase in length of rod is (Young's modulus of material is Y and bars are massless)

(1) $\frac{13Fl}{2AY}$ (2) $\frac{Fl}{AY}$

(3) $\frac{9Fl}{AY}$ (4) $\frac{3Fl}{AY}$

Sol. Answer (4)

If extension of rod = x

$x = x_1 + x_2 + x_3$ Where x_1, x_2, x_3 are individual extensions in rod 1, 2, 3

$$x_1 = \frac{Fl}{AY},\ x_2 = \frac{2Fl}{2AY},\ x_3 = \frac{3Fl}{3AY}$$

So $x = \dfrac{3Fl}{AY}$

12. An ideal gas has adiabatic exponent γ. It contracts according to the law $PV = \alpha$, where α is a positive constant. For this process, the Bulk modulus of the gas is

(1) P

(2) $\dfrac{P}{\alpha}$

(3) αP

(4) $(1 - \alpha)P$

Sol. Answer (1)

From $PV = \alpha$

$P\Delta V + V\Delta P = 0$

$$\Rightarrow\ P = -\frac{\Delta P}{\frac{\Delta V}{V}}$$

$\Rightarrow\ P = B$

So $B = P$

13. Two wire A and B are stretched by same force. If, for A and B, $Y_A : Y_B = 1 : 2$, $r_A : r_B = 3 : 1$ and $l_A : l_B = 4 : 1$, then ratio of their extension $\left(\dfrac{\Delta l_A}{\Delta l_B}\right)$ will be

(1) 10 : 13

(2) 11 : 7

(3) 8 : 9

(4) 6 : 5

Sol. Answer (3)

$$\Delta x = \frac{FL}{AY}$$

For wire A

$$\Delta L_A = \frac{F \cdot L_A}{\pi r_A^2 \cdot Y_A} \quad ...(1)$$

For wire B

$$\Delta L_B = \frac{F \cdot L_B}{\pi r_B^2 \cdot Y_B} \quad ...(2)$$

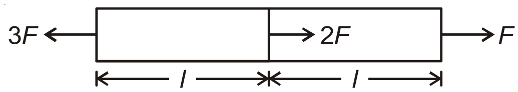
Divide (1) by (2)

$$\frac{\Delta L_A}{\Delta L_B} = \frac{F \cdot L_A}{\pi r_A^2 \cdot Y_A} \times \frac{\pi r_B^2 \cdot Y_B}{F \times L_B} = \frac{L_A}{L_B} \times \left(\frac{r_B}{r_A}\right)^2 \times \frac{Y_B}{Y_A}$$

Substituting the value of ratio's

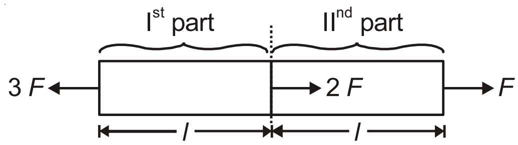
$$\frac{\Delta L_A}{\Delta L_B} = \frac{4}{1} \times \left(\frac{1}{3}\right)^2 \times \frac{2}{1} = \frac{8}{9}$$

14. A bar is subjected to axial forces as shown. If E is the modulus of elasticity of the bar and A is its cross-section area. Its elongation will be

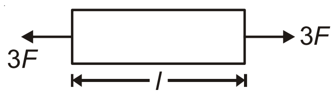


(1) $\frac{Fl}{AE}$ (2) $\frac{2Fl}{AE}$ (3) $\frac{3Fl}{AE}$ (4) $\frac{4Fl}{AE}$

Sol. Answer (4)

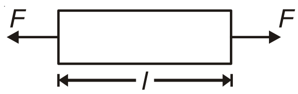


Elongation in Ist part,



$$\Delta x_1 = \frac{3Fl}{AE}$$

Elongation in IInd part,



$$\Delta x_2 = \frac{Fl}{AE}$$

So, Net elongation, $\Delta x = \Delta x_1 + \Delta x_2 = \frac{3Fl}{AE} + \frac{FL}{AE} = \frac{4Fl}{AE}$

15. A metal ring of initial radius r and cross-sectional area A is fitted onto a wooden disc of radius $R > r$. If Young's modulus of metal is Y then tension in the ring is

(1) $\frac{AYR}{r}$ (2) $\frac{AY(R-r)}{r}$ (3) $\frac{Y}{A}\left(\frac{R-r}{r}\right)$ (4) $\frac{Yr}{AR}$

Sol. Answer (2)

r – radius of metal ring

R – radius of wooden disc

Given, $R > r$

So $2\pi R > 2\pi r$

To get the metal ring fitted on wooden disc the circumfrence should be increased by $(2\pi R - 2\pi r)$ of metal ring

$\therefore$ $\Delta L = 2\pi(R - r)$

F = tension developed in ring

$\therefore$ $2\pi(R-r) = \frac{T(2\pi r)}{AY}$ $\left(\Delta L = \frac{FL}{AY}\right)$

$$\frac{AY(R-r)}{r} = T$$

16. Two wires A and B of same length and of same material have radii r_1 and r_2 respectively. Their one end is fixed with a rigid support and at other end equal twisting couple is applied. Then ratio of the angle of twist at the end of A and the angle of twist at the end of B will be

(1) $\frac{r_1^2}{r_2^2}$ (2) $\frac{r_2^2}{r_1^2}$

(3) $\frac{r_2^4}{r_1^4}$ (4) $\frac{r_1^4}{r_2^4}$

Sol. Answer (3)

$$r_1^4\,\phi_A = r_2^4\,\phi_B$$

$$\frac{\phi_A}{\phi_B} = \frac{(r_B)^4}{(r_A)^4} = \left(\frac{r_2}{r_1}\right)^4$$

(Applications of Elastic Behaviour of Materials)

17. When a small mass m is suspended at lower end of an elastic wire having upper end fixed with ceiling. There is loss in gravitational potential energy, let it be x, due to extension of wire, mark correct option

(1) The lost energy can be recovered (2) The lost energy is irrecoverable

(3) Only $\frac{x}{2}$ amount of energy is recoverable (4) Only $\frac{x}{3}$ amount of energy is recoverable

Sol. Answer (3)

ΔU (loss in gravitational potential energy) = $mg \times \Delta l$

$\Delta U = x$ (given)

So $x = mg \times \Delta l$

$$\begin{cases} \text{Where} \\ m = \text{mass suspended} \\ \Delta l = \text{elongation in wire} \end{cases}$$

$$\text{Elastic potential energy gained} = \frac{1}{2}\times\text{Force}\times\text{Elongation}$$

$$= \frac{1}{2}\times Mg\times\Delta l$$

$$= \frac{1}{2}Mg\times\Delta l \qquad [\because\ Mg\Delta l = x]$$

$$= \frac{1}{2}x$$

So only $\frac{x}{2}$ amount of energy is recoverable which is stored as elastic potential energy in wire.

18. A mild steel wire of length $2l$ meter cross-sectional area A m^2 is fixed horizontally between two pillars. A small mass m kg is suspended from the mid point of the wire. If extension in wire are within elastic limit. Then depression at the mid point of wire will be

(1) $\left(\frac{Mg}{YA}\right)^{1/3}$ (2) $\left(\frac{Mg}{lA}\right)^{1/3}$ (3) $\left(\frac{Mgl^3}{YA}\right)^{1/3}$ (4) $\frac{Mg}{2YA}$

Sol. Answer (3)

Let $OC = x$ [depression]

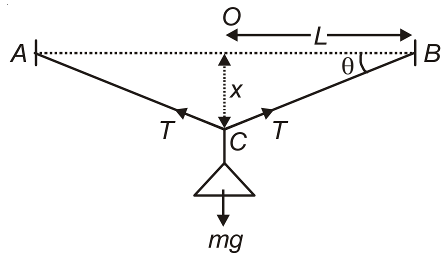
and θ be small angle

$\because$ x is small

ΔL (Extension in OB part of wire) = $BC - OB$

$BC = (L^2 + x^2)^{1/2}$ and $OB = L$

$\Delta L = \{(L^2 + x^2)^{1/2} - L\}$



We know $F = \dfrac{YA\,\Delta L}{L}$

So $F = \dfrac{YA}{L}\left\{(L^2 + x^2)^{1/2} - L\right\}$

$$= YA\left\{\left(1 + \frac{x^2}{L^2}\right)^{1/2} - 1\right\}$$

$$= YA\left\{1 + \frac{x^2}{2L^2} - 1\right\}$$

$\left\{\begin{array}{l}\text{Using binomial expansion} \\ \dfrac{x^2}{L^2} << 1\end{array}\right.$

$$F = \frac{YAx^2}{2L^2}$$

So $\left(1 + \dfrac{x^2}{L^2}\right)^{1/2} \simeq 1 + \dfrac{x^2}{2L^2}$

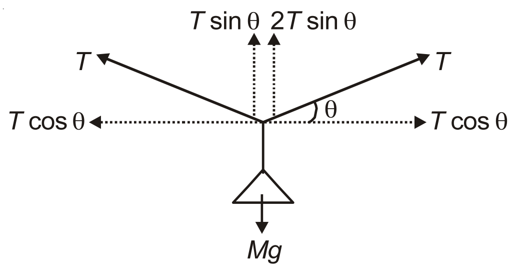
Tension in each part of wire will be equal to 'F'

By vertical equilibrium

$Mg = 2T\sin\theta$

$= 2T\,\theta$ {If θ is small. So $\sin\theta \simeq \theta$}

$\because$ $T = F$



$$Mg = 2 \times \frac{YAx^2}{2L^2} \times \theta$$

$\left\{\begin{array}{l}\text{in } \Delta\, OBC \\ \dfrac{x}{L} = \tan\theta\end{array}\right.$

$$Mg = 2 \times \frac{YAx^2}{2L^2} \times \frac{x}{L}$$

$\because$ θ is small, $\tan\theta \simeq \sin\theta \simeq \theta$

We get,

$$\left(\frac{MgL^3}{YA}\right)^{1/3} = x$$

19. A rigid bar of mass 15 kg is supported symmetrically by three wire each of 2 m long. These at each end are of copper and middle one is of steel. Young's modulus of elasticity for copper and steel are 110×10^9 N/m^2 and 190×10^9 N/m^2 respectively. If each wire is to have same tension, ratio of their diameters will be

(1) $\sqrt{\dfrac{11}{19}}$ (2) $\sqrt{\dfrac{19}{11}}$ (3) $\sqrt{\dfrac{30}{11}}$ (4) $\sqrt{\dfrac{11}{30}}$

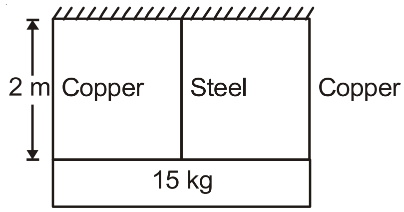
Sol. Answer (2)

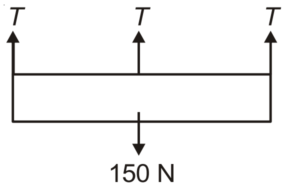
Tension is same (given)

From free body diagram

$3T = 150$ N

$T = 50$ N





Since the bar has to be supported symmetrically

Therefore extension in each wire will be same

We know $\Delta x = \dfrac{FL}{AY}$

Compare 1 copper wire with another steel wire

$$\frac{FL}{A_C Y_C} = \frac{FL}{A_S Y_S}$$

$$\Rightarrow \frac{A_S}{A_C} = \frac{Y_C}{Y_S}$$

Where,
A_C – Area of copper wire
Y_C – Young's modulus copper
A_S – Area of steel wire
Y_S – Young's modulus steel

Substitutuing value of Y_C and Y_s

$$\frac{d_S^2}{d_C^2} = \frac{110\times10^9}{190\times10^9}$$

$$\frac{d_S}{d_C} = \sqrt{\frac{11}{19}}$$

d_S – diameter of steel wire
d_C – diameter of copper wire

$$\frac{d_C}{d_S} = \sqrt{\frac{19}{11}}$$

20. The strain energy stored in a body of volume V due to shear strain ϕ is (shear modulus is η)

(1) $\dfrac{\phi^2 V}{2\eta}$ (2) $\dfrac{\phi V^2}{2\eta}$

(3) $\dfrac{\phi^2 V}{\eta}$ (4) $\dfrac{1}{2}\eta\phi^2 V$

Sol. Answer (4)

$$\text{Shear modulus} = \frac{\text{Shear stress}}{\text{Shear stress}}$$

$$\eta = \frac{\text{Shear stress}}{\phi}$$

$\eta\phi$ = Shear stress

Strain energy per unit volume = $\dfrac{1}{2}$ × shear stress × shear strain

$$\Rightarrow \frac{\text{Strain energy}}{\text{Volume}} = \frac{1}{2}\times\eta\phi\times\phi \quad \text{(Cross multiply volume)}$$

$$\text{Strain energy} = \frac{1}{2}\eta\phi^2 V$$

21. A metal wire having Poisson's ratio 1/4 and Young's modulus 8×10^{10} N/m^2 is stretched by a force, which produces a lateral strain of 0.02% in it. The elastic potential energy stored per unit volume in wire is [in J/m^3]

(1) 2.56×10^4 (2) 1.78×10^6

(3) 3.72×10^2 (4) 2.18×10^5

Sol. Answer (1)

$$\frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \text{Poisson's ratio}$$

$$\begin{cases} Y = \text{(Young's modulus)} \\ \quad = 8 \times 10^{10} \quad \text{(given)} \\ \text{Poission's ratio} = \dfrac{1}{4} \quad \text{(given)} \\ \text{Lateral strain} = 0.02\% \quad \text{(given)} \end{cases}$$

$$\frac{0.02/100}{\Delta l/l} = \frac{1}{4}$$

$$\frac{\Delta l}{l} = \frac{0.08}{100}$$

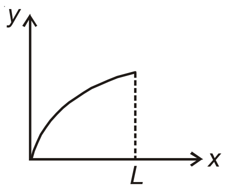
ΔU (Elastic potential energy per unit volume = $\dfrac{1}{2} \times Y \times (\text{Longitudinal strain})^2$

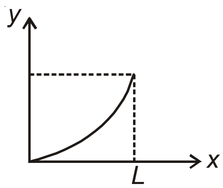
Substituting values

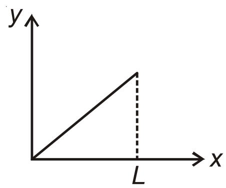
$$\Delta U = \frac{1}{2} \times 8 \times 10^{10} \times \left(\frac{0.08}{100}\right)^2$$

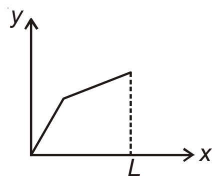
$\Delta U = 2.56 \times 10^4$ J/m^3

22. Which of the following curve represents the correctly distribution of elongation (y) along heavy rod under its own weight $L \rightarrow$ length of rod, $x \rightarrow$ distance of point from lower end?

(1)


(2)


(3)


(4)


Sol. Answer (2)

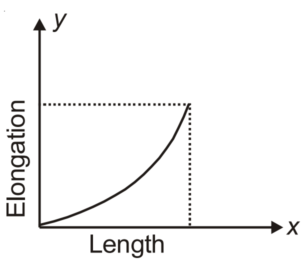
For elongation of rod under its own weight

We know $\Delta x = \dfrac{\rho g x^2}{2Y}$

We can clearly see that elongation $\propto (x^2)$

So graph of Δx *vs* x should be a upward parabola.

$$\begin{cases} \text{Where,} \\ \Delta x = \text{Elongation} \\ \rho = \text{Density of rod} \\ Y = \text{Young's modulus} \\ L = \text{Length} \\ g = \text{Acceleration due to gravity} \\ x = \text{Distance of point from lower end} \end{cases}$$



23. The length of a metal wire is l_1, when tension in it is T_1 and l_2 when its tension is T_2. The natural length of the wire is

(1) $\sqrt{l_1 l_2}$ (2) $\frac{l_1T_2 - l_2T_1}{T_2 - T_1}$ (3) $\frac{l_2T_2 - l_1T_1}{T_1 + T_2}$ (4) $\frac{l_1 + l_2}{2}$

Sol. Answer (2)

Let natural length of wire = l

Case I : when tension in wire is T_1

$$l_1 - l = \frac{T_1 l}{AY} \quad ...(1) \qquad \left\{\Delta l = \frac{FL}{AY}\right\}$$

Case II : when tension in wire is T_2

$$l_2 - l = \frac{T_2 l}{AY} \quad ...(2)$$

Dividing (2) by (1)

$$\frac{l_2 - l}{l_1 - l} = \frac{T_2}{T_1}$$

Solving this we get

$$l = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}$$

24. A wire can sustain a weight of 15 kg. If it cut into four equal parts, then each part can sustain a weight

(1) 5 kg (2) 45 kg

(3) 15 kg (4) 30 kg

Sol. Answer (3)

$$\text{Stress} = \frac{F}{A}$$

So $\text{Stress} \propto \frac{1}{A}$

Since, we are not reducing the crossectional area of the wire. Therefore each part can still sustain same force i.e, 15 kg weight.

25. The normal density of gold is ρ and its modulus is B. The increase in density of piece of gold when pressure P is applied uniformly from all sides

(1) $\frac{\rho P}{2B}$ (2) $\frac{\rho B}{2P}$

(3) $\frac{\rho P}{B - P}$ (4) $\frac{\rho B}{B - P}$

Sol. Answer (3)

We know

$$\frac{\Delta V}{V} = \frac{P}{B} \quad ...(1) \qquad \begin{cases} \Delta V - \text{Change in volume} \\ P - \text{Pressure applied} \\ B - \text{Bulk modulus} \end{cases}$$

And $\rho = \dfrac{M}{V}$ ...(2) $\qquad \begin{cases} \rho = \text{Density} \\ M = \text{Mass} \\ V = \text{Volume} \end{cases}$

From (2)

$$\Delta\rho = \frac{M}{V - \Delta V} - \frac{M}{V}$$

$$\Delta\rho = \frac{M}{V} \times \frac{\Delta V}{V - \Delta V}$$

$$\Delta\rho = \rho \times \frac{1}{\frac{V}{\Delta V} - 1} \qquad \text{[From eq. (2)]}$$

$$\Delta\rho = \rho \times \frac{1}{\frac{B}{P} - 1} \qquad \text{[From eq. (1)]}$$

$$\Delta\rho = \frac{\rho P}{B - P}$$

26. A uniform wire of length L and radius r is twisted by an angle α. If modulus of rigidity of the wire is η, then the elastic potential energy stored in wire, is

(1) $\dfrac{\pi\eta r^4\alpha}{2L^2}$

(2) $\dfrac{\pi\eta r^4\alpha^2}{4L}$

(3) $\dfrac{\pi\eta r^4\alpha}{4L^2}$

(4) $\dfrac{\pi\eta r^4\alpha^2}{2L}$

Sol. Answer (2)

U = Work done = $\dfrac{1}{2}C\phi^2$, where $C = \dfrac{\pi\eta r^4}{2L}$ (torsional constant)

$= \dfrac{\pi\eta r^4\phi^2}{4L}$ (ϕ = angle of twist = α, η = Modulus of rigidity)

Substituting values, $U = \dfrac{\pi\eta r^4\alpha^2}{4L}$

27. If δ is the depression produced in a beam of length L, breadth b and thickness d, when a load is placed at the mid point, then

(1) $\delta \propto L^3$

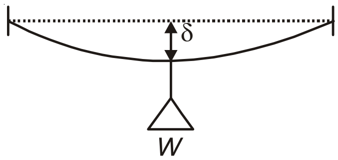
(2) $\delta \propto \dfrac{1}{b^3}$

(3) $\delta \propto \dfrac{1}{d}$

(4) All of these

Sol. Answer (1)

Since we know



$$\delta = \frac{WL^3}{4Y\,bd^3}$$

So we can say $\delta \propto L^3$

SECTION - C

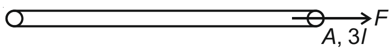
Previous Years Questions

1. Two wires are made of the same material and have the same volume. The first wire has cross-sectional area A and the second wire has cross-sectional area $3A$. If the length of the first wire is increased by Δl on applying a force F, how much force is needed to stretch the socond wire by the same amount? **[NEET-2018]**

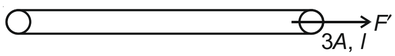
(1) $9F$ (2) $6F$ (3) F (4) $4F$

Sol. Answer (1)

Wire 1 :



Wire 2 :



For wire 1, $\Delta l = \left(\frac{F}{AY}\right)3l$ ...(i)

For wire 2, $\frac{F'}{3A} = Y\frac{\Delta l}{l}$

$\Rightarrow \Delta l = \left(\frac{F'}{3AY}\right)l$ ...(ii)

From equation (i) & (ii),

$$\Delta l = \left(\frac{F}{AY}\right)3l = \left(\frac{F'}{3AY}\right)l \Rightarrow \boxed{F' = 9F}$$

2. The bulk modulus of a spherical object is B. If it is subjected to uniform pressure p, the fractional decrease in radius is **[NEET-2017]**

(1) $\frac{p}{B}$ (2) $\frac{B}{3p}$ (3) $\frac{3p}{B}$ (4) $\frac{p}{3B}$

Sol. Answer (4)

$$B = \frac{p}{\left(\frac{\Delta V}{V}\right)}$$

$$\frac{\Delta V}{V} = \frac{p}{B}$$

$$3\frac{\Delta r}{r} = \frac{p}{B}$$

$$\frac{\Delta r}{r} = \frac{p}{3B}$$

3. The Young's modulus of steel is twice that of brass. Two wires of same length and of same area of cross section, one of steel and another of brass are suspended from the same roof. If we want the lower ends of the wires to be at the same level, then the weights added to the steel and brass wires must be in the ratio of **[Re-AIPMT-2015]**

(1) 1 : 1 (2) 1 : 2 (3) 2 : 1 (4) 4 : 1

Sol. Answer (3)

4. The approximate depth of an ocean is 2700 m. The compressibility of water is 45.4×10^{-11} Pa^{-1} and density of water is 10^3 kg/m^3. What fractional compression of water will be obtained at the bottom of the ocean?

[AIPMT-2015]

(1) 1.4×10^{-2} (2) 0.8×10^{-2} (3) 1.0×10^{-2} (4) 1.2×10^{-2}

Sol. Answer (4)

$$B = \frac{P}{\Delta V / V} \Rightarrow \frac{\Delta V}{V} = \frac{P}{B}$$

$$\frac{\Delta V}{V} = \frac{hdg}{B}$$

$$= \frac{2700 \times 10^3 \times 10}{\dfrac{1}{45.4 \times 10^{-11}}}$$

$$= 1.2 \times 10^{-2}$$

5. Copper of fixed volume V is drawn into wire of length l. When this wire is subjected to a constant force F, the extension produced in the wire is Δl. Which of the following graphs is a straight line? **[AIPMT-2014]**

(1) Δl versus $\frac{1}{l}$ (2) Δl versus l^2

(3) Δl versus $\frac{1}{l^2}$ (4) Δl versus l

Sol. Answer (2)

$$V = A \cdot L$$

$$Y = \frac{FL}{A\Delta L} \Rightarrow \Delta L = \frac{FL}{\frac{V}{L}Y}$$

$$\Delta L = \frac{FL^2}{VY}$$

$$\Rightarrow \Delta L \propto L^2$$

Thus, ΔL versus L^2 is straight line.

6. The following four wires of length L and radius r are made of the same material. Which of these will have the largest extension, when the same tension is applied? **[NEET-2013]**

(1) L = 400 cm, r = 0.8 mm (2) L = 300 cm, r = 0.6 mm

(3) L = 200 cm, r = 0.4 mm (4) L = 100 cm, r = 0.2 mm

Sol. Answer (4)

We know, $\Delta x = \frac{FL}{AY} = \frac{FL}{\pi r^2 Y}$

$\Rightarrow \quad \Delta x \propto \frac{L}{r^2}$

Δx directly proportional to L

And Δx inversely proportional to r^2

For option (1), $\frac{L}{r^2} = \frac{400 \times 10}{(0.8)^2} = 6250$

For option (2), $\frac{L}{r^2} = \frac{300 \times 10}{(0.6)^2} = 8333.33$

For option (3), $\frac{L}{r^2} = \frac{200 \times 10}{(0.4)^2} = 12,500$

For option (4), $\frac{L}{r^2} = \frac{100 \times 10}{(0.2)^2} = 25,000$

For option (4) we are getting maximum value of $\frac{L}{r^2}$

$\Rightarrow$ Δx also maximum for L = 100 cm and r = 0.2 mm

7. A rope 1 cm in diameter breaks, if the tension in it exceeds 500 N. The maximum tension that may be given to similar rope of diameter 3 cm is

(1) 500 N (2) 3000 N

(3) 4500 N (4) 2000 N

Sol. Answer (3)

Tension $\propto$ (radius)^2 $\left[\Delta x = \frac{FL}{\pi r^2 Y}\right]$

$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^2$ So $T \propto r^2$

Substituting values

Let $T_2 = x$ $\left.\begin{matrix} T_1 = 500 \text{ N} \\ r_1 = 1 \text{ cm} \\ r_2 = 3 \text{ cm} \end{matrix}\right\}$ given

$\frac{500}{x} = \frac{1^2}{3^2}$

$x = 4500$ N

$\Rightarrow \quad T_2 = 4500$ N

8. A wire of length L and radius r fixed at one end and a force F applied to the other end produces an extension l. The extension produced in another wire of the same material of length $2L$ and radius $2r$ by a force $2F$, is

(1) l (2) $2l$ (3) $4l$ (4) $\frac{l}{2}$

Sol. Answer (1)

$$L = \frac{FL}{\pi r^2 Y} \quad ...(1) \qquad \left[\Delta x = \frac{FL}{AY}\right]$$

Now, new parameters

$F = 2F$

$L = 2L$

$r = 2r$

Substituting new parameters in eq. (1)

$$L' = \frac{2F \times 2L}{\pi (2r)^2 Y}$$

$$= \frac{FL}{\pi r^2 Y} \qquad \left\{\because \frac{FL}{\pi r^2 Y} = l \text{ from equation 1}\right\}$$

$L' = L$

9. The increase in pressure required to decrease the 200 L volume of a liquid by 0.008% in kPa is (Bulk modulus of the liquid = 2100 MPa is)

(1) 8.4 (2) 84 (3) 92.4 (4) 168

Sol. Answer (4)

$V = 200$ L

$\Delta V = -0.008\%$ of 200 L (Decrease in volume so we use (–)ive sign)

$$= \frac{0.008}{100} \times 200 = -0.016 \text{ L}$$

$B = 2100$ MPa $= 21 \times 10^8$ Pa

We know

$$\frac{-\Delta V}{V} = \frac{\Delta P}{B}$$

$$\frac{0.016}{200} = \frac{\Delta P}{21 \times 10^8}$$

168×10^3 Pa $= \Delta P$

168 kPa $= \Delta P$

10. Which of the following relations is true?

(1) $Y = 2\eta(1 - 2\sigma)$ (2) $Y = 2\eta(1 + 2\sigma)$

(3) $Y = 2\eta(1 - \sigma)$ (4) $(1 + \sigma)2\eta = Y$

Sol. Answer (4)

$Y = 2\eta(1 + \sigma)$

$$\left\{\begin{array}{l}\text{Where,} \\ Y = \text{Young's modulus} \\ \eta = \text{Shear modulus} \\ \sigma = \text{Poisson's ratio}\end{array}\right.$$

11. A 5 m long aluminium wire ($Y = 7 \times 10^{10}$ Nm^{-2}) of diameter 3 mm supports a 40 kg mass. In order to have the same elongation in the copper wire ($Y = 12 \times 10^{10}$ Nm^{-2}) of the same length under the same weight, the diameter should now be (in mm)

(1) 1.75 (2) 1.5 (3) 2.3 (4) 5.0

Sol. Answer (3)

For aluminium wire

$$\Delta x_1 = \frac{FL}{AY} = \frac{4FL}{\pi d^2 Y}$$

$$\begin{cases} F = 400 \text{ N} \\ L = 5 \text{ m} \\ d = 3 \text{ mm} \\ Y = 7\times 10^{10} \text{ Nm}^{-2} \end{cases}$$

Substituting values

$$\Delta x_1 = \frac{4\times 400\times 5}{\pi\times (3)^2\times 7\times 10^{10}} \quad ...(1)$$

For copper wire

Using same formulae

$$\Delta x_2 = \frac{4FL}{\pi d^2 Y}$$

$$\begin{cases} F = 400 \text{ N} \\ L = 5 \text{ m} \\ Y = 12\times 10^{10} \\ d = ? \end{cases}$$

Let diameter be = d

$$\Delta x_2 = \frac{4\times 400\times 5}{\pi d^2\times 12\times 10^{10}} \quad ...(2)$$

Equating (1) & (2)

Because $\Delta x_1 = \Delta x_2$ [given condition]

$$\frac{4\times 400\times 5}{\pi\times (3)^2\times 7\times 10^{10}} = \frac{4\times 400\times 5}{\pi d^2\times 12\times 10^{10}}$$

Solving this we get

$$d = \frac{\sqrt{21}}{2} \simeq 2.3 \text{ mm}$$

12. Two wires of same material and radius have their lengths in ratio 1 : 2. If these wires are stretched by the same force, the strain produced in the two wires will be in the ratio

(1) 2 : 1 (2) 1 : 1

(3) 1 : 2 (4) 1 : 4

Sol. Answer (2)

$$\text{Strain} = \frac{\Delta l}{l}$$

We know

$$\Delta l = \frac{FL}{AY}$$

$$\frac{\Delta l}{l} = \frac{F}{AY} = \frac{F}{\pi r^2 Y}$$

For wire 1

$$S_1 = \text{strain} = \frac{\Delta l_1}{L} = \frac{F}{\pi r^2 Y} \quad ...(1)$$

For wire 2

$$S_2 = \text{strain} = \frac{\Delta l_2}{2L} = \frac{F}{\pi r^2 Y} \quad ...(2)$$

Therefore,

$$\text{Ratio of strains } = \frac{S_1}{S_2} = \frac{F \times \pi r^2 Y}{\pi r^2 Y \times F} = \frac{1}{1}$$

1 : 1

13. A steel wire of cross-sectional area 3×10^{-6} m^2 can withstand a maximum strain of 10^{-3}. Young's modulus of steel is 2×10^{11} Nm^{-2}. The maximum mass the wire can hold is (take g = 10 ms^{-2})

(1) 40 kg (2) 60 kg

(3) 80 kg (4) 100 kg

Sol. Answer (2)

$$\text{Strain} = \frac{\Delta l}{l} = \frac{F}{AY}$$

Substituting values

$$10^{-3} = \frac{F}{3 \times 10^{-6} \times 2 \times 10^{11}}$$

$$\left\{\begin{array}{l}\text{Given,} \\ \dfrac{\Delta l}{l} = 10^{-3} \\ A = 3 \times 10^{-6} \text{ m}^2 \\ Y = 2 \times 10^{11} \text{ Nm}^{-2}\end{array}\right.$$

600 N = F

$$\text{Therefore maximum mass } = \frac{F}{g} = \frac{600}{10} = 60 \text{ kg}$$

14. The hollow shaft is ...... than a solid shaft of same mass, material and length.

(1) Less stiff (2) More stiff (3) Equally stiff (4) None of these

Sol. Answer (2)

Let C' = restoring couple per unit twist for hollow cylinder

$$\text{So } C' = \frac{\pi S(r_2^4 - r_1^4)}{2L} \qquad \left\{\begin{array}{l}\text{Where,} \\ r_2 \text{ and } r_1 \text{ are outer and inner radii}\end{array}\right\}$$

And

C = restoring couple per unit twist for solid cylinder

$$C = \frac{\pi S r^4}{2L}$$

$$\Rightarrow \frac{C'}{C} = \frac{r_2^4 - r_1^4}{r^4} = \frac{(r_2^2 - r_1^2)(r_2^2 + r_1^2)}{r^4}$$

$$\left\{\begin{array}{l}\text{If mass of both cylinder same than} \\ \pi r^2 L\rho = \pi(r_2^2 - r_1^2)L\rho \\ \text{or } r^2 = r_2^2 - r_1^2\end{array}\right.$$

$$\therefore \frac{C'}{C} = \frac{r_2^2 + r_1^2}{r_2^2 - r_1^2} > 1$$

Hence hollow cylinder more stronger than solid one.

15. The Bulk modulus for an incompressible liquid is

(1) Zero (2) Unity

(3) Infinity (4) Between 0 and 1

Sol. Answer (3)

We know

$$\frac{-\Delta V}{V} = \frac{P}{B}$$

$$B = \frac{-P \cdot V}{\Delta V}$$

For incompressible liquid

ΔV (Change in volume) = 0

For every value of pressure applied

Put $\Delta V = 0$

$\Rightarrow$ $B = \infty$ (Infinity)

16. A copper rod length L and radius r is suspended from the ceilling by one of its ends. What will be elongation of the rod due to its own weight when ρ and Y are the density and Young's modulus of the copper respectively?

(1) $\frac{\rho^2 g L^2}{2Y}$ (2) $\frac{\rho g L^2}{2Y}$ (3) $\frac{\rho^2 g^2 L^2}{2Y}$ (4) $\frac{\rho g L}{2Y}$

Sol. Answer (2)

Let W be the total weight acting downwards

Let the centre of mass

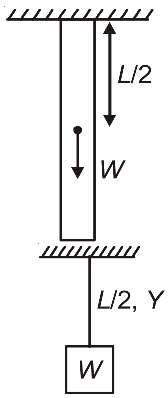
which is at the distance of $\frac{L}{2}$ for the top

So it can be assumed that a mass W is hung by a massless

wire of length $\frac{L}{2}$, Young's modulus Y,

ρ = density of wire, $W = Mg = \rho \times \pi r^2 L \times g$

Using $\Delta L = \frac{FL}{AY} = \frac{\rho \pi r^2 L g}{\pi r^2} \times \frac{L}{2} \times \frac{1}{Y} = \frac{\rho g L^2}{2Y}$



17. Which of the following substances has the highest elasticity?

(1) Steel (2) Copper

(3) Rubber (4) Sponge

Sol. Answer (1)

Substance which requires more force for per unit elongation have more elasticity

OR

Less stretchable means more elastic

So, steel is least stretchable

$\Rightarrow$ Most elastic.

18. When a wire of length 10 m is subjected to a force of 100 N along its length, the lateral strain produced is 0.01×10^{-3} m. The Poisson's ratio was found to be 0.4. If the area of cross-section of wire is 0.025 m^2, its Young's modulus is

(1) 1.6×10^8 Nm^{-2}
(2) 2.5×10^{10} Nm^{-2}
(3) 1.25×10^{11} Nm^{-2}
(4) 16×10^9 Nm^{-2}

Sol. Answer (1)

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\underbrace{\frac{\Delta l}{l}}_{\text{Longitudinal strain}} = \frac{F}{AY}$$

So $\eta = \dfrac{\text{Lateral strain}}{F/AY}$

Therefore

$$Y = \frac{\eta \times F}{\text{Lateral strain} \times A}$$

$$\left\{\begin{array}{l}\text{Given,}\\ F = 100\text{ N}\\ \text{Lateral strain} = 0.01 \times 10^{-3}\text{ m}\\ \eta = 0.4\\ A = 0.025\text{ m}^2\end{array}\right.$$

Substituting values

$$Y = \frac{0.4 \times 100}{0.01 \times 10^{-3} \times 0.025} = 1.6 \times 10^8\ \text{Nm}^{-2}$$

19. Two wires of length l, radius r and length $2l$, radius $2r$ respectively having same Young's modulus are hung with a weight mg. Net elongation is

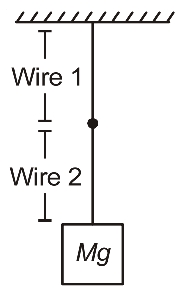
(1) $\dfrac{3\,mgl}{\pi r^2 Y}$
(2) $\dfrac{2\,mgl}{3\pi r^2 Y}$
(3) $\dfrac{3\,mgl}{2\pi r^2 Y}$
(4) $\dfrac{3\,mgl}{4\pi r^2 Y}$

Sol. Answer (3)

Tension in both wires will be same

Let elongation in wire 1 be = Δl_1

$$\Delta l_1 = \frac{mgl}{\pi r^2 Y} \qquad \left[\Delta x = \frac{FL}{AY}\right]$$



Let elongation in wire 2 be = Δl^2

$$\Delta l_2 = \frac{mg \times 2l}{\pi(2r^2)Y}$$

Net elongation = $\Delta l_1 + \Delta l_2$

$$= \frac{mgl}{\pi r^2 Y} + \frac{mgl}{2\pi r^2 Y}$$

$$= \frac{3mgl}{2\pi r^2 Y}$$

20. A cube of side 40 mm has its upper face displaced by 0.1 mm by a tangential force of 8 kN. The shearing modulus of cube is

(1) 2×10^9 Nm^{-2} (2) 4×10^9 Nm^{-2}

(3) 8×10^9 Nm^{-2} (4) 16×10^9 Nm^{-2}

Sol. Answer (1)

Shear modulus $= \dfrac{F \cdot h}{A \cdot x}$

Substituting values

$$= \frac{8000 \times 40 \times 10^{-3}}{1600 \times 10^{-6} \times 0.1 \times 10^{-3}}$$

$= 2 \times 10^9$ Nm^{-2}

$$\left\{\begin{array}{l}\text{Given,}\\ F = 8\text{ kN} = 8000\text{ N}\\ A = 40 \times 40 = 1600 \times 10^{-6}\text{ m}^2\\ x = 0.1\text{ mm} = 0.1 \times 10^{-3}\text{ m}\\ h = 40 \times 10^{-3}\text{ m}\end{array}\right.$$

21. A rod of length l and radius r is joined to a rod of length $\dfrac{l}{2}$ and radius $\dfrac{r}{2}$ of same material. The free end of small rod is fixed to a rigid base and the free end of larger rod is given a twist of $\theta°$, the twist angle at the joint will be

(1) $\dfrac{\theta}{4}$ (2) $\dfrac{\theta}{2}$ (3) $\dfrac{5\theta}{6}$ (4) $\dfrac{8\theta}{9}$

Sol. Answer (4)

Torque will be same

$\Rightarrow$ ϕ × Restoring couple per unit twist will be same for both the rods

$$\frac{\pi S r^4}{2L} \times \phi = \tau$$

We can use

$$\frac{\pi S r_1^4}{2L_1}\phi_1 = \frac{\pi S r_2^4}{2L_2}\phi_2$$

$$\frac{r_1^4}{L_1} \times \phi_1 = \frac{r_2^4}{L_2}\phi_2$$

$$\left\{\begin{array}{l}\text{given}\\ r_1 = r\\ r_2 = r/2\\ l_l = l\\ l_2 = l/2\end{array}\right.$$

Substituting values

$$\frac{r^4}{L} \times \phi_1 = \frac{(r/2)^4}{l/2} \times \phi_2$$

$$\phi_1 = \frac{\phi_2}{8}$$

Also $\phi_1 + \phi_2 = \theta$ (given)

$$\frac{\phi_2}{8} + \phi_2 = \theta$$

$$\phi_2 = \frac{8\theta}{9}$$

22. The Young's modulus of the material of a wire is 2×10^{10} Nm^{-2}. If the elongation strain is 1%, then the energy stored in the wire per unit volume is Jm^{-3} is

(1) 10^6

(2) 10^8

(3) 2×10^6

(4) 2×10^8

Sol. Answer (1)

$U = \frac{1}{2}Y(\text{strain})^2$ (given)

Substituting values $\left(\frac{\Delta l}{l} = \frac{1}{100}\right)$

$$U = \frac{1}{2} \times 2 \times 10^{10} \times \left(\frac{1}{100}\right)^2$$

$U = 10^6$

23. A wire is stretched under a force. If the wire suddenly snaps, the temperature of the wire

(1) Remains the same (2) Decreases

(3) Increases (4) First decreases then increases

Sol. Answer (3)

We know

$$\Delta L = \frac{FL}{AY} \text{ and } \Delta L = \alpha L \Delta\theta$$

Equating both

$$\frac{FL}{AY} = \alpha L \Delta\theta \Rightarrow F \propto \Delta\theta$$

$\left\{\begin{array}{l}\text{Where,} \\ \Delta\theta - \text{Change in temperature} \\ \alpha - \text{Coefficient of linear expansion} \\ \Delta L - \text{Elongation} \\ L - \text{Original length}\end{array}\right.$

So whenever stretching is there $\Delta\theta$ will be positive hence temperature will increase.

24. A wire of natural length l, Young's modulus Y and area of cross-section A is extended by x. Then the energy stored in the wires is given by

(1) $\frac{1}{2}\frac{YA}{l}x^2$ (2) $\frac{1}{3}\frac{YA}{l}x^2$ (3) $\frac{1}{2}\frac{Yl}{A}x^2$ (4) $\frac{1}{2}\frac{YA}{l^2}x^2$

Sol. Answer (1)

Energy density per unit volume $= \frac{1}{2} \times (\text{strain})^2 \times Y$

Volume = length × area of cross-section

$\therefore$ Energy (total) $= \frac{1}{2} \times (\text{strain})^2 \times Y \times L \times A$

$$= \frac{1}{2}\frac{x^2}{L^2} Y L A$$

$$E = \frac{1}{2}\frac{YA}{L}x^2$$

$\left\{\begin{array}{l}\text{Where,} \\ \text{Strain} = \frac{x}{l} \\ Y = \text{Young's modulus}\end{array}\right.$

25. When a force is applied on a wire of uniform cross-sectional area 3×10^{-6} m^2 and length 4 m, the increase in length is 1 mm. Energy stored in it will be ($Y = 2 \times 10^{11}$ N/m^2)

(1) 6250 J (2) 0.177 J (3) 0.075 J (4) 0.150 J

Sol. Answer (3)

Energy stored $= \frac{1}{2} \times$ work done

$= \frac{1}{2} \times F \times \Delta x$

$= \frac{1}{2} \times \frac{YA}{L} \Delta x \cdot \Delta x$ $\left[F = \frac{YA\Delta x}{L}\right]$

Substituting values

$$E = \frac{1}{2} \times \frac{2 \times 10^{11} \times 3 \times 10^{-6} \times 1 \times 10^{-3} \times 10^{-3}}{4}$$

$E = 0.075$ J

26. If in a wire of Young's modulus Y, longitudinal strain X is produced then the potential energy stored in its unit volume will be

(1) 0.5 YX^2 (2) 0.5 Y^2X (3) 2 YX^2 (4) YX^2

Sol. Answer (1)

Potential energy per unit volume $= \frac{1}{2} \times$ (strain)^2 × Young's modulus

Substituting data from question

We get, $U = \frac{1}{2} X^2 Y$

27. A uniform metal rod of 2 mm^2 cross-section is heated from 0°C to 20°C. The coefficient of the linear expansion of the rod is 12×10^{-6}/°C. Its Young's modulus of elasticity is 10^{11} Nm^{-2}. The energy stored per unit volume of the rod is

(1) 1440 Jm^{-3} (2) 15750 Jm^{-3} (3) 1500 Jm^{-3} (4) 2880 Jm^{-3}

Sol. Answer (4)

We know

$\Delta L = \alpha L \Delta\theta$

$\Rightarrow \frac{\Delta L}{L} = \alpha \Delta\theta$

Also $\frac{\Delta L}{L} =$ strain

$\Rightarrow$ Strain $= \alpha\Delta\theta$ ...(i)

Energy stored per unit volume $= \frac{1}{2}(\text{strain})^2 Y$

$= \frac{1}{2} \times \alpha^2 \Delta\theta^2 \times Y$ [Using eqn. (i)]

Substituting values $= \frac{1}{2} \times (12 \times 10^{-6})^2 \times (20)^2 \times 10^{11} = 2880$ Jm^{-3}

28. A material has Poisson's ratio 0.50. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, then the percentage change in volume is

(1) 0.6 (2) 0.4 (3) 0.2 (4) Zero

Sol. Answer (4)

Poisson's ratio = 0.50

$$\frac{-\text{Lateral strain}}{\text{Longitudinal strain}} = 0.50$$

$$\Rightarrow \frac{-\Delta r / r}{\Delta L / L} = \frac{1}{2}$$

$$\frac{-2\Delta r}{r} = \frac{\Delta L}{L} \quad \text{...(1)}$$

Volume = area × length

$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta A}{A} + \frac{\Delta L}{L}$$

$$\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta L}{L} \qquad \begin{bmatrix} A \propto r^2 \\ \dfrac{\Delta A}{A} = \dfrac{2\Delta r}{r} \end{bmatrix}$$

Using equation (1)

We get

$$\frac{\Delta V}{V} = 0$$

So $\dfrac{\Delta V}{V} \times 100 = 0\%$

29. There is no change in the volume of a wire due to the change in its length on stretching. The Poisson's ratio of the material of the wire is

(1) $+\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $+\frac{1}{4}$ (4) $-\frac{1}{4}$

Sol. Answer (1)

$V = A \times L$

$V = \pi r^2 L$

$$\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta L}{L}$$

$$\frac{\Delta V}{V} = 0 \qquad \text{[given]}$$

$$\frac{-2\Delta r}{r} = \frac{\Delta L}{L} \quad \text{...(1)}$$

Poisson's ratio $= \dfrac{-\Delta r}{r} \Big/ \dfrac{\Delta L}{L}$ ...(2)

Using equation (1) in (2)

$$\sigma = \frac{-\Delta r}{r} \Big/ 2\left(\frac{-\Delta r}{r}\right) \Rightarrow \sigma = \frac{1}{2}$$

30. If Young's modulus of elasticity Y for a material is one and half times its rigidity coefficient η, the Poisson's ratio σ will be

(1) $+\frac{2}{3}$ (2) $-\frac{1}{4}$ (3) $+\frac{1}{4}$ (4) $-\frac{2}{3}$

Sol. Answer (2)

$Y = \frac{3}{2}\eta$ [given]

$\sigma = ?$

And we know, $Y = 2\eta(1 + \sigma)$

$$\frac{3\eta}{2} = 2\eta(1+\sigma)$$

Solving we get, $\sigma = -\frac{1}{4}$

SECTION - D

Assertion - Reason Type Questions

1. A : Hooke's law is obeyed only for small values of strain.

   R : The deformation beyond elastic limit is called plasticity.

Sol. Answer (2)

Statement (A) is true

Statement (R) is also true

But (R) is not the correct explaination of (A)

Because correct reason is Hooke's law is obeyed in elastic limit only.

2. A : Strain is a dimensionless quantity.

   R : Strain is internal force per unit area of a body.

Sol. Answer (3)

(A) Is true

Strain = $\frac{\Delta L}{L}$

(R) Is false because strain is change in dimension by original dimension.

3. A : Diamond is more elastic than rubber.

   R : When same deforming force is applied diamond deforms less than rubber.

Sol. Answer (1)

(A) Is true because modulus of elasticity is more for diamond so less deformation in diamond than rubber when same deforming force applied

(R) Is true and correct explanation.

4. A : Bulk modulus for a perfectly plastic body is zero.

   R : For perfect plastic material, there is no restoring force.

Sol. Answer (1)

(A) Is true because a perfectly plastic body cannot regain its shape even when the deforming forces are removed because restoring forces are absent

(R) Is true and correct explanation for (A)

5. A : The railway bridges are declared unfit after their use for a long period.

R : Due to repeated strain the elasticity of material decreases.

Sol. Answer (1)

(A) Is true because after a long use the material weakens and shows dangerous deformation when load is applied because its elasticity has decreased gradually over the time.

(R) Is true and correct explanation for (A)

6. A : Spring balances show wrong readings after they have been used for a long time.

R : Spring in spring balance temporary losses elasticity due to repeated alternating deforming force.

Sol. Answer (1)

(A) Is true because after a long use elasticity decreases and small temporary deformation remains these which in turn tend to be the reason of wrong readings.

(R) Is true and correct explanation for (A)

7. A : Modulus of elasticity is independent of dimensions of the body.

R : Modulus of elasticity depends on the material of the body.

Sol. Answer (2)

(A) True because modulus of elasticity is a material property

(R) True

But (R) is not the correct explanation because no where it reasons why modulus of elasticity is independent of dimensions of the body.

8. A : Adiabatic elasticity of a gas is greater than isothermal elasticity.

R : $\frac{E_{\text{adiabatic}}}{E_{\text{isothermal}}} = \gamma$.

Sol. Answer (1)

(A) True

Because $\frac{E_{\text{adiabatic}}}{E_{\text{isothermal}}} = \gamma$ and γ always greater than 1

So $E_{\text{adiabatic}}$ is always greater than $E_{\text{isothermal}}$

(R) True and also correct explanation.

9. A : When a beam is bent only tensile strain is produced.

R : The depression produced in a rectangular beam is directly proportional to its width.

Sol. Answer (4)

(A) Is false because strain is there so stress will also be present

$\because$ Stress $\propto$ strain

(R) False depression $\propto \frac{1}{\text{width}}$

10. A : To minimise the depression in a beam, it is designed as ‘I’ shape girder.

R : The ‘I’ shape girders have large load bearing surface, which decreases the stress.

Sol. Answer (1)

(A) Is true because having more surface area means less force per unit area i.e. less stress

(R) Is true and correct explanation of (A)

11. A : Iron is more elastic than copper.

R : Under a given deforming force, Iron is deformed less than copper.

Sol. Answer (1)

(A) Is true because less deformation under a similar deforming force means more elasticity

(R) Is true and correct explanation of (A)

12. A : Lateral strain is directly proportional to the longitudinal strain within the elastic limit.

R : Poisson's ratio for a given material at a constant temperature is constant.

Sol. Answer (1)

(A) Is true because,

$$\frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \sigma$$

$\Rightarrow$ Lateral strain $\propto$ longitudinal strain

As σ is constant

(R) Is true and correct explanation of (A)

13. A : Equal amount of work is done when two identical springs of steel and copper are equally stretched.

R : Both springs have same spring constant.

Sol. Answer (4)

(A) Is wrong because amount of work done is not same because the spring constants are different.

(R) Is wrong.

14. A : Increase in temperature of a substance decreases the modulus of elasticity.

R : The graph between potential energy of molecules and separation between them is asymmetric.

Sol. Answer (1)

(A) Is true because when we increase the temperature the average distance between the molecules tend to increase hence decreasing the modulus of elasticity.

(R) Is true and correct explanation.

15. A : It is the breaking stress and not the breaking strength which depends on the material.

R : Breaking strength = $\frac{\text{Breaking stress}}{\text{Area}}$.

Sol. Answer (4)

(A) Is wrong both depend on the material because breaking strength is maximum stress a body can take

(R) Is wrong

Breaking strength = breaking stress × area

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