Chapter 10

Mechanical Properties of Fluids

Solutions

SECTION - A

Objective Type Questions

(Pressure)

1. The term 'fluid' is used for

   (1) Liquids only (2) Gases only

   (3) A mixture of liquid and gas only (4) Both liquids and gases

Sol. Answer (4)

Substances that can flow is called fluid. Thus both liquids and gases are fluids.

2. Select wrong statement about pressure

   (1) Pressure is a scalar quantity (2) Pressure is always compressive in nature

   (3) Pressure at a point is same in all directions (4) None of these

Sol. Answer (4)

Pressure is scalar as it is not added vectorially. Pressure is compressive in nature, it is same in all the directions at a point.

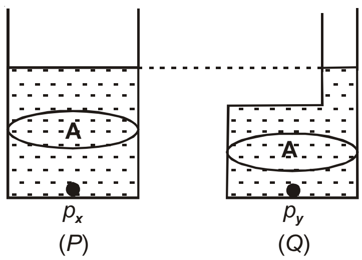
3. Gauge pressure

   (1) May be positive (2) May be negative (3) May be zero (4) All of these

Sol. Answer (4)

Gauge pressure depends on the reference chosen, it can be positive, negative or zero.

4. Figure shows two containers P and Q with same base area A and each filled upto same height with same liquid. Select the correct alternative



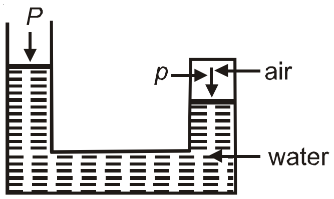
   (1) $p_x = p_y$ (2) $p_x > p_y$ (3) $p_y > p_x$ (4) Cannot say

Sol. Answer (1)

The level of water above both points is same so by hydrostatic paradox.

$p_x = p_y$

5. The pressure of confined air is p. If the atmospheric pressure is P, then



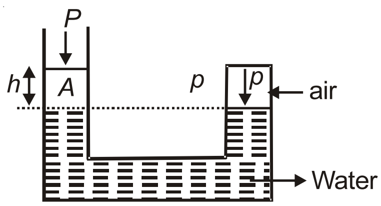
(1) P is equal to p

(2) P is less than p

(3) P is greater than p

(4) P may be less or greater than p depending on the mass of the confined air

Sol. Answer (2)



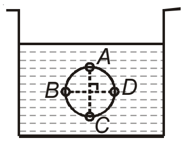
Pressure at point $A = P$ (Hydrostatic paradox)

and $P + \rho_w gh$ = pressure at $A = p$

$\Rightarrow \quad P = p - \rho_w gh$

So, $P < p$

6. Figure shows a container filled with a liquid of density ρ. Four points A, B, C and D lie on the diametrically opposite points of a circle as shown. Points A and C lie on vertical line and points B and D lie on horizontal line. The incorrect statement is (p_A, p_B, p_C, p_D are absolute pressure at the respective points)

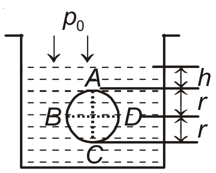


(1) $p_D = p_B$ (2) $p_A < p_B = p_D < p_C$ (3) $p_D = p_B = \dfrac{p_C - p_A}{2}$ (4) $p_D = p_B = \dfrac{p_C + p_A}{2}$

Sol. Answer (3)

Points at same height have same pressure, points with height difference say 'h' will have difference of ρgh.

Let radius of circle is r



$p_A = p_0 + h\rho g$

$p_B = p_D = p_0 + (h + r)\rho g$

$p_C = p_0 + (h + 2r)\rho g$

Then,

$p_C \approx p_A = [p_0 + (h + 2r)\rho g] + [p_0 + h\rho g]$

$= p_0 + h\rho g + 2r\rho g + p_0 + h\rho g$

$= 2[p_0 + (h + r)\rho g]$

$$\frac{p_C + p_A}{2} = p_0 + (h + r)\rho g$$

$$\frac{p_C + p_A}{2} = p_B = p_D$$

i.e., option (1), (2) and (4) gives correct statement but incorrect statement is (3)

7. The volume of an air bubble is doubled as it rises from the bottom of lake to its surface. The atmospheric pressure is 75 cm of mercury. The ratio of density of mercury to that of lake water is $\frac{40}{3}$. The depth of the lake in metre is

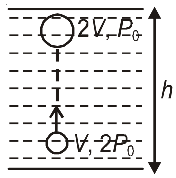
(1) 10 (2) 15 (3) 20 (4) 25

Sol. Answer (1)

$2P_0 = P_0 + \rho gh$

$\Rightarrow P_0 = \rho gh$

$\Rightarrow P_0 = 75$ cm mercury [Atmospheric pressure]



$\Rightarrow \rho_{\text{mercury}} \times g \times \frac{75}{100} = \rho_{\text{water}} \times g \times h$

$\Rightarrow \frac{\rho_m}{\rho_w} \times \frac{75}{100} = h$

$\Rightarrow \frac{40}{3} \times \frac{75}{100} = h \quad \left[\because \frac{\rho_m}{\rho_w} = \frac{40}{3} \text{ (given)}\right]$

$\Rightarrow h = 10$ m

8. A beaker containing a liquid of density ρ moves up with an acceleration 'a'. The pressure due to the liquid at a depth h below free surface of the liquid is

(1) $h\rho g$ (2) $h\rho (g - a)$

(3) $h\rho (g + a)$ (4) $2h\rho g \left(\frac{g+a}{g-a}\right)$

Sol. Answer (3)

Due to upward acceleration pseudo force will act downwards so value of acceleration due to gravity will increase by 'a'

$\therefore g' = (g + a)$

$P = \rho g' h$

$\Rightarrow P = \rho (g + a)h$ (Substitute g')

9. A barometer kept in an elevator reads 76 cm when it is at rest. If the elevator goes up with some acceleration, the reading will be

(1) 76 cm (2) > 76 cm (3) < 76 cm (4) Zero

Sol. Answer (3)

When elevator goes up with some acceleration upward, due to pseudo force acting downwards. Value of g increases to g'.

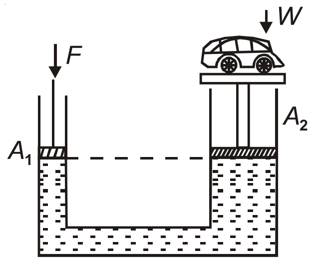
If g increases to g', *i.e.*, $g' = g + a$

$\because P = \rho gh = \rho(g + a)h' =$ constant

Then, $h' < h$

i.e., $h' < 76$ cm

10. In a hydraulic jack as shown, mass of the car $W = 800$ kg, $A_1 = 10\ cm^2$, $A_2 = 10\ m^2$. The minimum force F required to lift the car is



(1) 1 N (2) 0.8 N (3) 8 N (4) 16 N

Sol. Answer (2)

Pressure in a liquid is divided equally so we can say pressure at both the pistons should be same

$$\Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Substituting values,

$$\frac{F}{10} = \frac{8000}{10\times10^4}$$

$$\Rightarrow F = 0.8\text{ N}$$

$$\left\{\begin{array}{l}\text{Where,}\\ F_1 = F\\ A_1 = 10\text{ cm}^2\\ A_2 = 10\text{ m}^2 = 10\times10^4\text{ cm}^2\\ F_2 = 8000\text{ N}\end{array}\right.$$

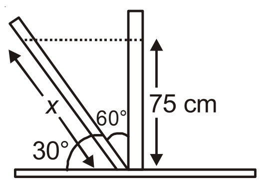
$$\left\{\begin{array}{l}\text{Take}\\ g = 10\text{ m/s}^2\end{array}\right\}$$

11. A barometer tube reads 75 cm of Hg. If tube is gradually inclined at an angle of 30° with horizontal, keeping the open end in the mercury container, then find the length of mercury column in the barometer tube

(1) 86.7 cm (2) 150 cm (3) 75 cm (4) 92.5 cm

Sol. Answer (2)

$\because$ We are not changing the atmospheric pressure, so height of Hg from the surface should not change.



$$\frac{75}{x} = \cos 60°$$

$$\Rightarrow x = 150\text{ cm}$$

(Archimedes' principle)

12. A wooden cube just floats inside water with a 200 gm mass placed on it. When the mass is removed, the cube floats with its top surface 2 cm above the water level. What is the side of the cube ?

(1) 6 cm (2) 8 cm (3) 10 cm (4) 12 cm

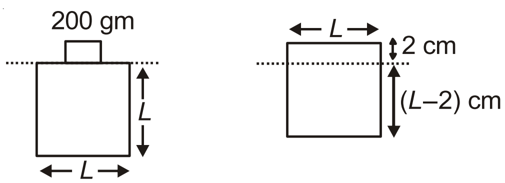
Sol. Answer (3)

Mass × g = Volume of part of cube × ρ × g

$\Rightarrow$ 200 × g = L^2 (2 × ρ_w × g)

$\Rightarrow$ 100 = L^2 $\{\because \rho_w = 1\}$

$\Rightarrow$ 10 cm = L



From the two figures we can see that the 200 gm block is provided with required buoyant force but a part of cube which is afloat in 2nd figure.

13. A block of steel of size $5 \times 5 \times 5$ cm^3 is weighed in water. If relative density of steel is 7, its apparent weight is

(1) $6 \times 5 \times 5 \times 5$ g wt (2) $4 \times 4 \times 4 \times 7$ g wt

(3) $5 \times 5 \times 5 \times 7$ g wt (4) $4 \times 4 \times 4 \times 6$ g wt

Sol. Answer (1)

Apparent weight $= \rho_s v_b g - \rho_w v_b g$

$= v_b g (\rho_s - \rho_w)$

$= 5 \times 5 \times 5 \times g \times (7 - 1)$ $\left[\begin{array}{l}\rho_w = 1 \\ \rho_s = 7 \text{ (given)}\end{array}\right.$

$= 6 \times 5 \times 5 \times 5 \times g$ wt

$\left\{\begin{array}{l}\text{Where,} \\ \rho_s - \text{density of steel} \\ \rho_w - \text{density of water} \\ v_b - \text{volume of block} \\ \quad (\text{side} \times \text{side} \times \text{side})\end{array}\right.$

14. A block of wood floats in water with $\frac{4}{5}$th of its volume submerged, but it just floats in another liquid. The density of liquid is (in kg/m^3)

(1) 750 (2) 800 (3) 1000 (4) 1250

Sol. Answer (2)

$$\frac{4}{5} v_b \times \rho_w \times g = v_b \times \rho_b \times g$$

$\left\{\begin{array}{l}\text{Where,} \\ v_b = \text{volume of block} \\ \rho_w = \text{density of water} = 1000 \text{ kg/m}^3 \\ \rho_b = \text{density of block}\end{array}\right.$

$$\Rightarrow \frac{\rho_w}{\rho_b} = \frac{5}{4}$$

$$\rho_b = \frac{4}{5} \times 1000 = 800 \text{ kg/m}^3$$

And when block is put in liquid of density ρ_l it just floats

So, $v_b \times \rho_b \times g = v_b \times \rho_l \times g$

$\Rightarrow \rho_b = \rho_l$

So, $\rho_l = 800$ kg/m^3

15. A cubical block is floating in a liquid with one fourth of its volume immersed in the liquid. If whole of the system accelerates upward with acceleration $g/4$, the fraction of volume immersed in the liquid will be

(1) 1/4 (2) 1/2 (3) 3/4 (4) 2/3

Sol. Answer (1)

Upward acceleration just causes the acceleration due to gravity increases by some value, but since the term of 'g' gets cancelled out in the buoyancy equation.

Volume immersed $\times \rho_w \times g$ = Total volume $\times \rho_{cube} \times g$

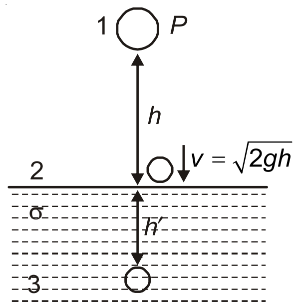
So, increasing it will not have any effect on the immersed volume.

16. A body of density ρ is dropped from rest from a height h into a lake of density σ, where $\sigma > \rho$. Neglecting all dissipative forces, the maximum depth to which the body sinks before returning to float on surface

(1) $\frac{h}{\sigma - \rho}$ (2) $\frac{h\rho}{\sigma}$ (3) $\frac{h\rho}{\sigma - \rho}$ (4) $\frac{h\sigma}{\sigma - \rho}$

Sol. Answer (3)

Between point 2 and 3



$$P_0 + \rho g h' + \frac{1}{2}\rho v^2 = \sigma g h' + P_0 \qquad [P_0 = \text{atmospheric pressure}]$$

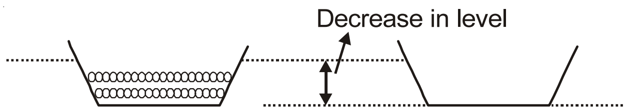
$$\Rightarrow \rho g h' + \rho g h = \sigma g h' \qquad \left[\because v = \sqrt{2gh}\right]$$

So, $h' = \dfrac{h\rho}{\sigma - \rho}$

17. A boat carrying a number of stones is floating in a water tank. If the stones are unloaded into water, the water level in the tank will
    (1) Remain unchanged
    (2) Rise
    (3) Fall
    (4) Rise or fall depends on the number of stones unloaded

Sol. Answer (3)

Previously when stones are on the boat they are increasing the weight on the boat and to balance this weight boat needs to generate buoyancy force by displacing more water, but when stones are removed the boat starts displacing less amount of water hence the level of water in tank falls.



18. A block of ice is floating in a liquid of specific gravity 1.2 contained in a beaker. When the ice melts completely, the level of liquid in the vessel
    (1) Increases (2) Decreases
    (3) Remain unchanged (4) First increases then decreases

Sol. Answer (1)

Density of ice is less than water and density of liquid is more than water. So even when ice melts the level will rise.

If $\rho_{liquid} > \rho_{water}$ then level (liquid + water) will rise.

19. Two liquids having densities d_1 and d_2 are mixed in such a way that both have same mass. The density of the mixture is
    (1) $\dfrac{d_1 + d_2}{2}$ (2) $\dfrac{d_1 + d_2}{d_1 d_2}$ (3) $\dfrac{d_1 d_2}{d_1 + d_2}$ (4) $\dfrac{2d_1 d_2}{d_1 + d_2}$

Sol. Answer (4)

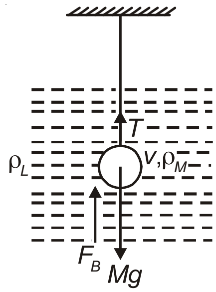
Let each have mass = M and densities d_1 and d_2

$$d_{mix} = \frac{M_{mix}}{V_{mix}} = \frac{M + M}{\left(\dfrac{M}{d_1}\right) + \left(\dfrac{M}{d_2}\right)} = \frac{2d_1 d_2}{d_1 + d_2}$$

20. A metallic sphere weighing 3 kg in air is held by a string so as to be completely immersed in a liquid of relative density 0.8. The relative density of metallic is 10. The tension in the string is

(1) 18.7 N (2) 42.5 N (3) 32.7 N (4) 27.6 N

Sol. Answer (4)



Where,
F_B – Force of Buoyancy
ρ_L – Density of liquid
ρ_M – Density of metal
M – Mass of sphere
T – Tension in string

$T = Mg$ – Buoyant Force

$\Rightarrow T = \rho_M Vg - \rho_L Vg \quad \left(\rho_M = \frac{M}{V}\right)$

$= (\rho_M - \rho_L)\, Vg$

$= (10 - 0.8) \times \frac{3}{10} \times 10$

$\Rightarrow T = 9.2 \times 3 = 27.6$ N

21. A rectangular block is 10 cm × 10 cm × 15 cm in size is floating in water with 10 cm side vertical. If it floats with 15 cm side vertical, then the level of water will

(1) Rise (2) Fall

(3) Remain same (4) Change according to density of block

Sol. Answer (3)

Mass of block remains same, volume displaced of water will also remain same so level of water will not change.

22. Two cubical blocks identical in dimensions float in water in such a way that 1st block floats with half part immersed in water and second block floats with 3/4 of its volume inside the water. The ratio of densities of blocks is

(1) 2 : 3 (2) 3 : 4 (3) 1 : 3 (4) 1 : 4

Sol. Answer (1)

$$\frac{3}{4} V \times \rho_2 \times g = \frac{1}{2} V \times \rho_1 \times g$$

$$\Rightarrow \frac{\rho_2}{\rho_1} = \frac{2}{3}$$

Where,
V = Volume of block
ρ_1 = density of liquid 1
ρ_2 = density of liquid 2

(Streamline Flow, Bernoulli's Principle)

23. Which of the following is not the property of an ideal fluid?

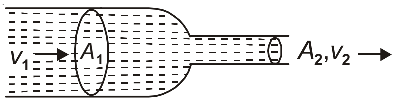
(1) Fluid flow is irrotational (2) Fluid flow is streamline

(3) Fluid is incompressible (4) Fluid is viscous

Sol. Answer (4)

An ideal fluid is not viscous.

24. A liquid flows in the tube from left to right as shown in figure. A_1 and A_2 are the cross-sections of the portions of the tube as shown. The ratio of speed $\frac{v_1}{v_2}$ will be



(1) $\frac{A_1}{A_2}$ (2) $\frac{A_2}{A_1}$ (3) $\sqrt{\frac{A_2}{A_1}}$ (4) $\sqrt{\frac{A_1}{A_2}}$

Sol. Answer (2)

By equation of continuity

$v_1A_1 = v_2A_2$

So, $\frac{v_1}{v_2} = \frac{A_2}{A_1}$

25. Water (ρ = 1000 kg/m^3) and kerosene (σ = 800kg/m^3) are filled in two identical cylindrical vessels. Both vessels have small holes at their bottom. The speed of the water and kerosene coming out of their holes are v_1 and v_2 respectively. Select the correct alternative

(1) $v_1 = v_2$ (2) $v_1 = 0.8\ v_2$ (3) $0.8\ v_1 = v_2$ (4) $v_1 = \sqrt{0.8}\, v_2$

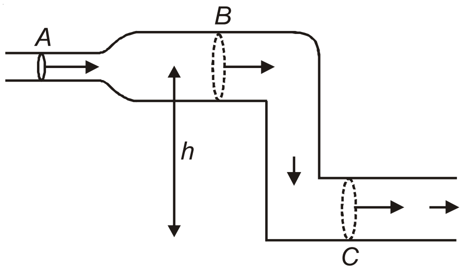
Sol. Answer (1)

Velocity of efflux for small holes $= \sqrt{2gh}$

Which clearly is independent of 'ρ' (density)

So, $v_1 = v_2$

26. Water is flowing through a channel (lying in a vertical plane) as shown in the figure. Three sections A, B and C are shown. Sections B and C have equal area of cross section. If P_A, P_B and P_C are the pressures at A, B and C respectively then



(1) $P_A > P_B = P_C$ (2) $P_A < P_B < P_C$

(3) $P_A < P_B = P_C$ (4) $P_A > P_B > P_C$

Sol. Answer (2)

Solution by using Bernoulli's principle and equation of continuity

Comparing points A and B

$A_Av_A = A_Bv_B$ {equation of continuity}

$\because$ $A_A < A_B$

$v_A > v_B$

$P_A + \frac{1}{2}\rho V_A^2 + \rho gh = P_B + \frac{1}{2}\rho V_B^2 + \rho gh$ {Bernoulli's equation}

$\because$ $v_A > v_B$

$\Rightarrow$ $\frac{1}{2}\rho V_A^2 > \frac{1}{2}\rho V_B^2$

$\therefore$ $P_A < P_B$ ...(1)

Now comparing C and B

$A_B = A_C \Rightarrow v_B = v_C$ [equation of continuity]

$$P_B + \frac{1}{2}\rho V^2 + \rho g h_B = P_C + \frac{1}{2}\rho V^2 + \rho g h_C$$

$\Rightarrow$ $P_B + \rho g h_B = P_C + \rho g h_C$

$\because$ $h_B > h_C$ then ...(2)

$P_B < P_C$

Using (1) and (2)

We can say, $P_A < P_B < P_C$

(Viscosity)

27. A flat plate of area 0.1 m^2 is placed on a flat surface and is separated from it by a film of oil 10^{-5} m thick whose coefficient of viscosity is 1.5 N sm^{-2}. The force required to cause the plate to slide on the surface at constant speed of 1 mm s^{-1} is

(1) 10 N (2) 15 N (3) 20 N (4) 25 N

Sol. Answer (2)

$$F = \eta A \frac{v}{l}$$

Substituting values,

$$= 1.5 \times 0.1 \times \frac{1 \times 10^{-3}}{10^{-5}}$$

= 15 N

28. A ball of density σ and radius r is dropped on the surface of a liquid of density ρ from certain height. If speed of ball does not change even on entering in liquid and viscosity of liquid is η, then the height from which ball dropped is

(1) $2g\left[\frac{(\sigma-\rho)r}{9\eta}\right]^2$ (2) $\frac{2g(\sigma-\rho)^2 r^2}{9\eta}$ (3) $\frac{2(\sigma-\rho)gr^2}{9\eta}$ (4) $2g\left[\frac{(\sigma-\rho)r^2}{9\eta}\right]^2$

Sol. Answer (4)

The ball has already reached the magnitude of velocity which is equal to its terminal velocity in fluid.

$v = \sqrt{2gh}$ {Velocity of body fallen form height $h = \sqrt{2gh}$ }

$$v_{\text{Terminal}} = \frac{2r^2}{9\eta}(\sigma-\rho)g$$

Equating both

$$\sqrt{2gh} = \frac{2r^2}{9\eta}(\sigma-\rho)g$$

$$\Rightarrow \quad h = 2g\left[\frac{(\sigma-\rho)r^2}{9\eta}\right]^2$$

29. Viscous drag force depends on

(1) Size of body (2) Velocity with which it moves

(3) Viscosity of fluid (4) All of these

Sol. Answer (4)

$$F = \eta A \frac{V}{d}$$

$$\begin{cases} \text{Where,} \\ F = \text{Drag Force} \\ \eta = \text{Viscosity of fluids} \\ A = \text{Area} \propto \text{size of body} \\ V = \text{Velocity} \end{cases}$$

30. The terminal velocity of a small sized spherical body of radius *r* falling vertically in a viscous liquid is given by the proportionality

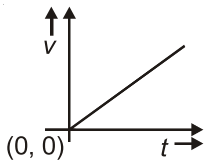
(1) $v \propto \frac{1}{r^2}$ (2) $v \propto r^2$ (3) $v \propto \frac{1}{r}$ (4) $v \propto r$

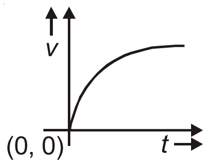
Sol. Answer (2)

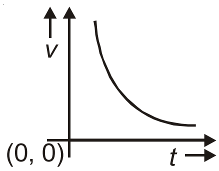
$$v_T = \frac{2r^2}{9\eta}[\sigma - \rho]g$$

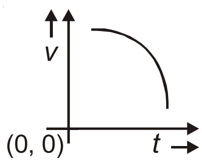
So, $v_T \propto r^2$

31. A spherical ball is dropped in a long column of viscous liquid. The speed *v* of the ball varies as function of time as

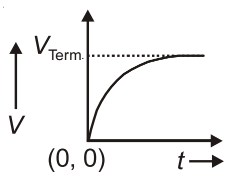
(1) 

(2) 

(3) 

(4) 

Sol. Answer (2)



Velocity does not increases after terminal speed is achieved.

(Surface Tension)

32. On putting a capillary tube in a pot filled with water, the level of water rises upto a height of 4 cm in the tube. If a tube of half the diameter is used instead, the water will rise to a height of nearly

(1) 2 cm (2) 4 cm (3) 8 cm (4) 11 cm

Sol. Answer (3)

For capillary tube

$$h = \frac{2T}{r\rho g}$$

We can say

$$h \propto \frac{1}{r} \text{ or } h \propto \frac{1}{d}$$

So, $\frac{h_1}{h_2} = \frac{d_2}{d_1}$

$\Rightarrow \frac{4}{x} = \frac{d}{2d}$

$\Rightarrow x = 8$ cm

33. Soap helps in cleaning clothes, because

(1) It attracts the dirt particles

(2) It decreases the surface tension of water

(3) It increases the cohesive force between water molecules

(4) It increases the angle of contact

Sol. Answer (2)

Soap helps cleaning clothes because, it decreases the surface tension of water thus water molecules penetrate easily into dirt and oil.

34. On increasing temperature of a liquid, its surface tension generally

(1) Increases (2) Decreases

(3) Remains constant (4) First increases and then decreases

Sol. Answer (2)

On increasing the temperature energy increases hence surface tension decreases. Because surface tension is nothing but some extra energy required by surface molecules to stay at the place.

35. The raincoats are made water proof by coating it with a material, which

(1) Absorb water (2) Increase surface tension of water

(3) Increase the angle of contact (4) Decreases the density of water

Sol. Answer (3)

Raincoats are coated with material which increase the angle of contact, so water does not penetrates inside the layer.

36. An iron needle slowly placed on the surface of water floats because

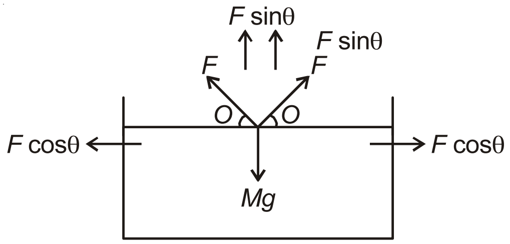
(1) It displaces water more than its weight

(2) The density of material of needle is less than that of water

(3) Of surface tension

(4) Of its shape

Sol. Answer (3)



Needle floats due to surface tension of water which balances the weight of needle.

In equilibrium $2F \sin\theta = mg$

37. Two water droplets merge with each other to form a larger droplet. In this process

(1) Energy is liberated (2) Energy is absorbed

(3) Energy is neither liberated nor absorbed (4) Some mass is converted into energy

Sol. Answer (1)

Work is done when we break a drop into 'n' drops equal to $4\pi R^2\sigma(n^{1/3} - 1)$

So energy will be liberated if we merge back those drops.

38. The radius of a soap bubble is r. The surface tension of soap solution is 'S'. Keeping temperature constant, the radius of the soap bubble is doubled. The energy necessary for this will be

(1) $24\ \pi r^2S$ (2) $8\ \pi r^2S$ (3) $16\ \pi r^2S$ (4) $12\ \pi r^2S$

Sol. Answer (1)

Work done in making a soap bubble of radius $r = 4\ \pi r^2S \times 2 = 8\ \pi r^2S$ $\left\{\begin{array}{l}\text{Multiply by 2 due}\\ \text{to two free surface}\end{array}\right\}$

$\therefore$ Energy of bubble $= 8\ \pi r^2S = E_r$

Work done in making a $2r$ radius soap bubble $= 4\ \pi(2r)^2\ S \times 2 = 32\ \pi r^2S$

$\therefore$ Energy of bubble $= 32\ \pi r^2S = E_{2r}$

So energy required to expand a bubble from r to $2r$ will be equal to $E_{2r} - E_r$

Substituting values

We get,

$$32\ \pi r^2S - 8\ \pi r^2S = 24\ \pi r^2S$$

39. The surface tension of a liquid is 5 N/m. If a film is held on a ring of area 0.02 m^2, its surface energy is about

(1) 5×10^{-2} J (2) 2.5×10^{-2} J (3) 2×10^{-1} J (4) 3×10^{-1} J

Sol. Answer (3)

Surface energy = surface tension × area of film × number of free surface

$= 5 \times 0.02 \times 2$

$= 2 \times 10^{-1}$ J

40. Two soap bubbles having radii 3 cm and 4 cm in vacuum, coalesce under isothermal conditions. The radius of the new bubble is

(1) 1 cm (2) 5 cm (3) 7 cm (4) 3.5 cm

Sol. Answer (2)

Energy initial = Energy final

$\Rightarrow$ $8\pi(3)^2S + 8\pi(4)^2S = 8\pi(r)^2S$ {Surface tension remains constant throughout process

$\Rightarrow$ $(3)^2 + (4)^2 = (r)^2$

$\Rightarrow$ $5 \text{ cm} = r$

41. The excess pressure in a soap bubble is double that in other one. The ratio of their volume is

(1) 1 : 2 (2) 1 : 8 (3) 1 : 4 (4) 1 : 1

Sol. Answer (2)

Excess pressure in soap bubble $= \dfrac{4S}{R}$

Let for first bubble,

$$P = \frac{4S}{R} \qquad \begin{cases} S - \text{Surface tension} \\ R - \text{Radius} \end{cases}$$

For second bubble,

$$2P = \frac{4S}{x} \qquad \begin{cases} S - \text{Surface tension} \\ x - \text{Radius} \end{cases}$$

Substitute value of P

$$2 \times \frac{4S}{R} = \frac{4S}{x}$$

$$\Rightarrow \quad x = \frac{R}{2}$$

Ratio of Radii $= \dfrac{R/2}{R} = \dfrac{1}{2}$

So, Ratio of volume = (Ratio of Radii)^3

$$= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

42. The work done to break a spherical drop of radius R in n drops of equal size is proportional to

(1) $\dfrac{1}{n^{2/3}} - 1$ (2) $\dfrac{1}{n^{1/3}} - 1$ (3) $n^{1/3} - 1$ (4) $n^{4/3} - 1$

Sol. Answer (3)

$\because$ Volume = constant

$$i.e., \ \frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$$

Radius of each new droplet $= \dfrac{R}{n^{1/3}}$

Work done to break into 'n' drops $= S\,[n \times 4\pi r^2 - 4\pi R^3]$

$= S \times 4\pi R^2\,[n^{1/3} - 1]$

43. The kerosene oil rises up in the wick of a lamp

(1) Due to high surface tension of oil (2) Because the wick attract s the oil

(3) Because wick decreases the surface tension of oil (4) Due to capillaries formed in the wick

Sol. Answer (4)

Capillary action is responsible.

Wick has a lot of capillaries which help the oil rise.

44. Ploughing help to retain water by soil

(1) By creating capillaries (2) By breaking capillaries

(3) By turning the soil upside down (4) None of these

Sol. Answer (2)

By breaking capillaries as they do not allow water to seep inside.

45. A capillary tube of radius *r* is immersed in a liquid and mass of liquid, which rises up in it is *M*. If the radius of tube is doubled, then the mass of liquid which will rise in capillary tube will be

(1) 2 *M* (2) *M* (3) *M*/2 (4) *M*/4

Sol. Answer (1)

$M = \rho v$

$\Rightarrow$ Mass $\propto$ volume of tube

Mass $\propto$ height × Area

$\Rightarrow$ Mass $\propto \frac{1}{r} \times r^2$

$\left\{\begin{array}{l} \text{Area} \propto r^2 \\ \text{Height} \propto \dfrac{1}{r} \\ \text{Where } r = \text{radius of cube} \end{array}\right.$ $\left(h = \dfrac{2S}{rgh}\right)$

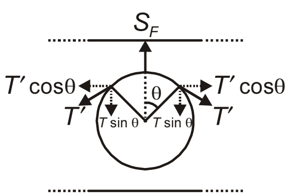
Mass $\propto r$

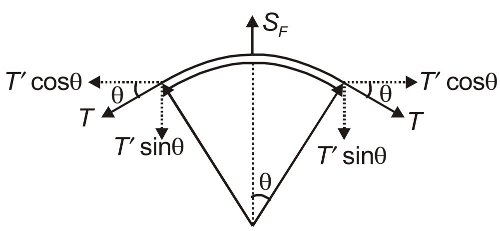
$\therefore$ If radius doubles, mass of liquid that rises up also doubles.

46. A massless inextensible string in the form of a loop is placed on a horizontal film of soap solution of surface tension *T*. If film is pierced inside the loop and it convert into a circular loop of diameter *d*, then the tension produced in string is

(1) Td (2) πTd (3) πd^2T (4) $\frac{\pi d^2 T}{4}$

Sol. Answer (1)





By force balancing in vertical direction

$S_F = 2T' \sin\theta$

$S_F = 2T'\theta$ $\left\{\begin{array}{l}\because\ \theta \text{ is small}\\ \sin\theta \simeq \theta\end{array}\right.$

$S \times 2r \times 2\theta = 2 \times T' \times \theta$

$S \times 2r = \text{Tension}$ $\left\{\begin{array}{l}r - \text{radius}\\ d - \text{diameter}\end{array}\right.$

$S \times d = \text{Tension}$

$\because\ S = T$

So, Tension = Td

$\left\{\begin{array}{l}\text{Where,}\\ S_F = \text{Force due to surface tension}\\ T' = \text{Tension in string}\\ \theta = \text{Small angle}\\ S \text{ or } T = \text{Surface tension}\end{array}\right.$

(Miscellaneous)

47. A tank is filled with water to a height H. A hole is made in one of the walls at a depth D below the water surface. The distance x from the foot of the wall at which the stream of water coming out of the tank strikes the ground is given by

(1) $x = 2\,[D\,(H - D)]^{1/2}$

(2) $x = 2\,(gD)^{1/2}$

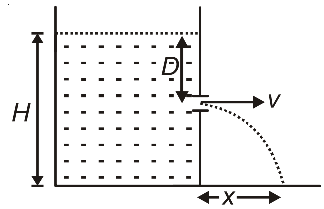
(3) $x = 2\,[D\,(H + D)]^{1/2}$

(4) None of these

Sol. Answer (1)

Velocity of efflux $= \sqrt{2gD} = v$

Say time taken by water to travel the vertical distance of $(H - D)$ = 't'



Using $s = ut + \frac{1}{2}at^2$ $\left\{\begin{array}{l}\text{Where,}\\ s = H - D\\ u = 0\\ a = g\end{array}\right.$

We get,

$$'t' = \sqrt{\frac{2(H-D)}{g}}$$

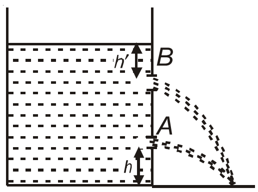
Now, $x = v \times t$

Substituting the values

$$x = \sqrt{2gD} \times \sqrt{\frac{2(H-D)}{g}}$$

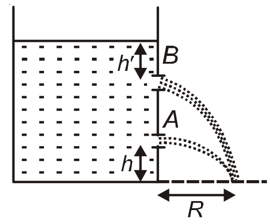
$\Rightarrow\ x = 2[D\,(H - D)]^{1/2}$

48. A tank is filled with water and two holes A and B are made in it. For getting same range, ratio of h'/h is



(1) 2 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) 1

Sol. Answer (4)



For hole 'A'

Velocity of efflux $= \sqrt{2g(x+h')}$

$R = 2[(x + h')h]^{1/2}$ ...(1)

For hole 'B'

Velocity of efflux $= \sqrt{2gh'}$

$R = 2[h'(x + h)]^{1/2}$ ...(2)

Equating (1) and (2)

We get

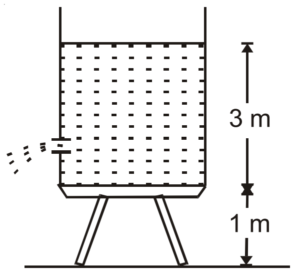
$2[(x + h')h]^{1/2} = 2[h'(x + h)]^{1/2}$

$\Rightarrow$ $(x + h')h = h'(x + h)$

$\Rightarrow$ $h = h'$

$\Rightarrow$ $\dfrac{h'}{h} = 1$

49. Water is filled in a tank upto 3 m height. The base of the tank is at height 1 m above the ground. What should be the height of a hole made in it, so that water can be sprayed upto maximum horizontal distance on ground?



(1) 3 m from ground (2) 1.5 m from ground

(3) 1.5 m from base of tank (4) 2 m from ground

Sol. Answer (4)

Let height of hole from the base of container be h

Velocity of efflux $= \sqrt{2g(3-h)}$

$R = 2[(h + 1)(3 - h)]^{1/2}$ [R = Range proved in Q. 47]

$\Rightarrow$ $R = 2(-h^2 + 2h + 3)^{1/2}$

$\dfrac{dR}{dh} = -2h + 2$

If $\dfrac{dR}{dh} = 0,$ then range would be more for corresponding height

So, $0 = -2h + 2$ $\Rightarrow$ $h = 1$

$\therefore$ Height from the ground = 1 + 1 = 2 m

SECTION - B

Objective Type Questions

(Pressure)

1. The atmospheric pressure at a place is 10^5 Pa. If tribromomethane (specific gravity = 2.9) be employed as the barometric liquid, the barometric height is

(1) 3.52 m (2) 1.52 m (3) 4.52 m (4) 2.52 m

Sol. Answer (1)

1 atm $\simeq$ 10^5 Pa = 76 cm of Hg

Density of Hg = 13.6

$\rho_{Hg} \times g \times h_{Hg} = \rho_{TBM} \times g \times h_{TBM}$

Substituting values

$13.6 \times 76 = 2.9 \times h$

$3.52 \text{ m} = h$

$$\left\{\begin{aligned} \rho_{Hg} &= \text{density of Hg} \\ h_{Hg} &= \text{height of Hg} \\ \rho_{TBM} &= \text{density of tribromomethane} \\ h_{TBM} &= \text{height of tribromomethane} \end{aligned}\right.$$

2. A large vessel of height H, is filled with a liquid of density ρ, upto the brim . A small hole of radius r is made at the side vertical face, close to the base. The horizontal force is required to stop the gushing of liquid is

(1) $(\rho gH)\pi r^2$ (2) ρgH (3) $\rho gH\pi r$ (4) $\rho g\pi r^2$

Sol. Answer (1)

Pressure close to the base = ρgH

Force required = pressure × area of hole = $\rho gH(\pi r^2)$

3. A vertical U-tube of uniform cross-section contains water in both the arms. A 10 cm glycerine column (R.D. = 1.2) is added to one of the limbs. The level difference between the two free surfaces in the two limbs will be

(1) 4 cm (2) 2 cm (3) 6 cm (4) 8 cm

Sol. Answer (2)

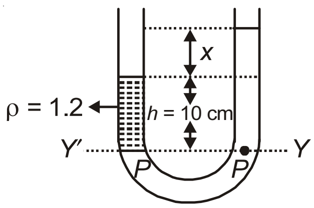
Let the difference between 2 limbs be x

Pressure on the line $Y\,Y'$ should be same below both limbs, so

$\rho_{glycerine} \times g \times h = \rho_{water} \times g \times (h + x)$

$\Rightarrow$ $1.2 \times 10 = 1 \times (h + x)$

$\Rightarrow$ $2 \text{ cm} = x$



4. The pressure at the bottom of a water tank is 4 P, where P is atmospheric pressure. If water is drawn out till the water level decreases by $\frac{3}{5}$th, then pressure at the bottom of the tank is

(1) $\frac{3P}{8}$ (2) $\frac{7P}{6}$ (3) $\frac{11P}{5}$ (4) $\frac{9P}{4}$

Sol. Answer (3)

Let height of water in tank be h

So, $4P - P = \rho_w gh$ ...(1)

$\because$ $\frac{3}{5}$ water taken out $\frac{2}{5}$th water is left to exert pressure

$$P' = P + \frac{2}{5}\rho_w gh$$

$$\Rightarrow P' = P + \frac{2}{5}\times 3P \qquad \text{[From eq. (1)]}$$

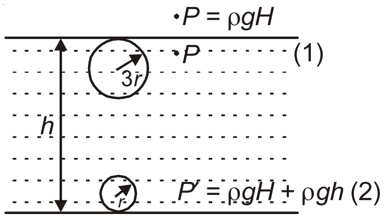
$$\Rightarrow P' = \frac{11P}{5}$$

5. A air bubble rises from bottom of a lake to surface. If its radius increases by 200% and atmospheric pressure is equal to water coloumn of height H, then depth of lake is

(1) 21 H (2) 8 H (3) 9 H (4) 26 H

Sol. Answer (4)

Let initial radius be = r



Final radius = r + 200% of r

= $3r$

Atmospheric pressure = ρgH

Let depth of the lake be h

So, pressure at the bottom of lake = $\rho gH + \rho gh$

Using $P_1V_1 = P_2V_2$

$$\rho gH \times \frac{4}{3}\pi(3r)^3 = (\rho gH + \rho gh)\times\frac{4}{3}\pi r^3$$

$$\rho gH \times \frac{4}{3}\pi \times 27r^3 = (\rho gH)\frac{4}{3}\pi r^3 + \rho gh \times \frac{4}{3}\pi r^3$$

Solving this equation we get

$26\,H = h$

6. A liquid mixture of volume V, has two liquids as its ingredients with densities α and β. If density of the mixture is σ, then mass of the first liquid in mixture is

(1) $\frac{\alpha V[\sigma\beta + 1]}{\beta[\alpha + \sigma]}$ (2) $\frac{\alpha V[\sigma - \beta]}{[\sigma + \beta]}$ (3) $\frac{\alpha V(\beta - \sigma)}{\beta - \alpha}$ (4) $\frac{\alpha V[1 - \sigma\alpha]}{\beta[\alpha - \sigma]}$

Sol. Answer (3)

Let mass of liquid with density $\alpha = M_1$

Let mass of liquid with density $\beta = M_2$

Total volume = V

Net density of mixture = σ

Now,

Total mass = $M_1 + M_2$

$$\Rightarrow V\sigma = M_1 + M_2 \qquad \left[\because \frac{\text{Total mass}}{V} = \sigma\right]$$

$$\Rightarrow M_2 = V\sigma - M_1 \qquad ...(1)$$

Now,

$$\sigma = \frac{\text{Total mass}}{\text{Total volume}} = \frac{(M_1 + M_2)}{\left(\frac{M_1}{\alpha}\right) + \left(\frac{M_2}{\beta}\right)}$$

Substituting value of M_2 from equation (1)

$$\sigma = \frac{M_1 + (V\sigma - M_1)}{\frac{M_1}{\alpha} + \frac{(V\sigma - M_1)}{\beta}}$$

Solving this we get

$$M_1 = \frac{\alpha V(\beta - \sigma)}{\beta - \alpha}$$

Archimedes' principle

7. A piece of gold weighs 10 g in air and 9 g in water. What is the volume of cavity?
(Density of gold = 19.3 g cm^{-3})
(1) 0.182 cc (2) 0.282 cc (3) 0.382 cc (4) 0.482 cc

Sol. Answer (4)

When dipped in water

$W_{app} = W_{air} - F_B$

$\Rightarrow$ 9 gm × g = 10 gm × g – F_B

$\Rightarrow$ 1 × g = F_B

Now (total volume displaced) × ρ_w × g = 1 × g

$(V_c + V_g) \times 1 = 1$

$$V_c = 1 - \frac{\text{Mass of gold in air}}{\rho_g} = 1 - \frac{10}{19.3} = 0.482 \text{ cc}$$

Where,
V_c = volume of cavity
V_g = volume of gold
W_{app} = 9 gm
W_{air} = 10 gm
F_B = force of buoyancy
ρ_w = density of water = 1
ρ_g = density of gold = 19.3

8. A block of ice floats in an oil in a vessel when the ice melts, the level of oil will
(1) Go up (2) Go down
(3) Remain same (4) Go up or down depending on quantity of ice

Sol. Answer (2)
Since block of ice is displacing some oils to stay afloat when the ice block melts level of oil will go down.

9. An object suspended by a wire stretches it by 10 mm. When object is immersed in a liquid the elongation in wire reduces by $\frac{10}{3}$ mm. The ratio of relative densities of the object and liquid is
(1) 3 : 1 (2) 1 : 3 (3) 1 : 2 (4) 2 : 1

Sol. Answer (1)

$$\Delta L = \frac{FL}{AY}$$

$\Rightarrow$ Elongation $\propto$ force and force is due to weight

So elongation $\propto$ weight

Let density of liquid = ρ
Let density of object = σ
Mass of object = M

$\Delta L_1 \propto$ weight ...(1) {When not submerged in liquid}

$\Delta L_2 \propto$ apparent weight ...(2) {When submerged in liquid}

Dividing (1) by (2)

$$\frac{10}{10-\frac{10}{3}} = \frac{Mg}{Mg - \frac{Mg\rho}{\sigma}}$$

$$\Rightarrow \quad \frac{1}{1-\frac{1}{3}} = \frac{1}{1-\frac{\rho}{\sigma}}$$

Solving this we get

$$\frac{\rho}{\sigma} = \frac{1}{3}$$

So relative densities of object (σ) and liquid (ρ) is 3 : 1

10. A spring balance reads 200 gF when carrying a lump of lead in air. If the lead is now immersed with half of its volume in brine solution, what will be the new reading of the spring balance? specific gravity of lead and brine are 11.4 and 1.1 respectively

(1) 190.4 gF (2) 180.4 gF (3) 210 gF (4) 170.4 gF

Sol. Answer (1)

$$W' = W - F_B$$

$$= v\sigma g - \frac{v}{2}\rho g$$

$$= v\sigma g\left(1-\frac{\rho}{2\sigma}\right)$$

$$\begin{cases} \text{Where,} \\ W' = \text{apparent weight} \\ W = \text{read weight = actual weight of body in vaccum} \\ \rho = \text{density of solution (1.1)} \\ \sigma = \text{density of material (11.4)} \end{cases}$$

$$W' = 200\left(1-\frac{1.1}{11.4\times 2}\right)$$

$$= 190.35 \text{ gF}$$

(Streamline Flow, Bernoulli's Principle and Viscosity)

11. Water flows in a stream line manner through a capillary tube of radius a. The pressure difference being P and the rate of flow is Q. If the radius is reduced to $\frac{a}{4}$ and the pressure is increased to $4P$, then the rate of flow becomes

(1) $4Q$ (2) $\frac{Q}{2}$ (3) Q (4) $\frac{Q}{64}$

Sol. Answer (4)

Rate of flow $\propto$ pressure difference × (radius)^4

$$Q \propto P \times a^4 \qquad \left\{\because\ Q = \frac{\pi P r^4}{8\eta L}\right\}$$

$$\text{So, } \frac{Q_1}{Q_2} = \frac{P_1 a_1^4}{P_2 a_2^4}$$

$$\frac{Q_1}{Q_2} = \frac{P\times a^4}{4P\times\left(\frac{a}{4}\right)^4} = \frac{64}{1}$$

$$\therefore \quad Q_2 = \frac{Q_1}{64} = \frac{Q}{64}$$

12. Three capillaries of length L, $\frac{L}{2}$ and $\frac{L}{3}$ are connected in series. Their radii are r, $\frac{r}{2}$ and $\frac{r}{3}$ respectively. Then if stream-line flow is to be maintained and the pressure across the first capillary is P, then

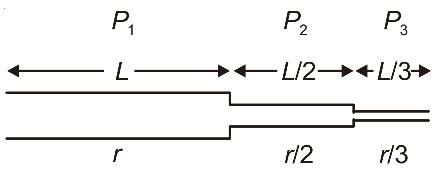
    (1) The pressure difference across the ends of second capillary is $8P$

    (2) The pressure difference across the third capillary is $43P$

    (3) The pressure difference across the ends of second capillary is $16P$

    (4) The pressure difference across the third capillary is $59P$

Sol. Answer (1)



$P_1 = P$ (given)

$\because$ Rate of flow will be same across all pipes

So, pressure across the pipe $\propto \frac{\text{length}}{(\text{radius})^4}$

$\left\{\begin{array}{l}\text{rate of flow of liquid } (Q) \\ Q = \dfrac{\pi P r^4}{8\eta L}\end{array}\right.$

$$\frac{P_1}{P_2} = \frac{\left(L/r^4\right)}{\left(\dfrac{L/2}{(r/2)^4}\right)} = \frac{1}{8}$$

Then $P_2 = 8P_1$

13. Air streams horizontally past an air plane. The speed over the top surface is 60 m/s and that under the bottom surface is 45 m/s. The density of air is 1.293 kg/m^3, then the difference in pressure is

    (1) 1018 N/m^2 (2) 516 N/m^2 (3) 1140 N/m^2 (4) 2250 N/m^2

Sol. Answer (1)

Applying Bernoullis equation

$$P_1 + \rho g h + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2} \times \rho\, [v_1^2 - v_2^2] = P_2 - P_1 = \Delta P$$

$$\Rightarrow \frac{1}{2} \times 1.293\, [(60)^2 - (45)^2] = \Delta P$$

$$\Rightarrow 1018 \text{ N/m}^2 \simeq \Delta P$$

14. Two water pipes P and Q having diameter 2×10^{-2} m and 4×10^{-2} m respectively are joined in series with the main supply line of water. The velocity of water flowing in pipe P is

    (1) Four times that of Q (2) Two times that of Q

    (3) $\frac{1}{2}$ times that of Q (4) $\frac{1}{4}$ times that of Q

Sol. Answer (1)

Rate of flow through both pipes will be same

i.e., $Q_1 = Q_2$

$$\frac{V_1}{t} = \frac{V_2}{t}$$

$$\frac{\pi r_1^2 l_1}{t} = \frac{\pi r_2^2 l_2}{t}$$

$$\left(\text{Where } \frac{l_1}{t} = V_P \text{ and } \frac{l_2}{t} = V_Q\right)$$

$$\Rightarrow \frac{\pi d_1^2}{4} V_P = \frac{\pi d_2^2}{4} \times V_Q$$

$$\Rightarrow V_P = \left(\frac{d_2}{d_1}\right)^2 V_Q$$

$$\Rightarrow V_P = \left(\frac{4\times 10^{-2}}{2\times 10^{-2}}\right)^2 V_Q$$

$\Rightarrow V_P = 4V_Q$

15. At what speed, the velocity head of water is equal to pressure head of 40 cm of mercury?

(1) 2.8 m/s (2) 10.32 m/s (3) 5.6 m/s (4) 8.4 m/s

Sol. Answer (2)

$$\frac{1}{2}\rho_{\text{water}} V^2 = \rho_{\text{mercury}}\, gh$$

$$V = \sqrt{2\times\frac{\rho_{\text{mercury}}}{\rho_{\text{water}}}\times g\times h}$$

$$= \sqrt{2\times 13.6\times 9.8\times\frac{40}{100}}$$

$\Rightarrow V = 10.32$ m/s

16. If the terminal speed of a sphere of gold (density 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid.

(1) 0.2 m/s (2) 0.4 m/s (3) 0.1 m/s (4) 0.133 m/s

Sol. Answer (3)

$$V_{\text{terminal}} = \frac{2a^2}{9\eta}(\rho - \sigma)g \qquad \left\{\begin{array}{l}\text{Where}\\ \rho = \text{density of material}\\ \sigma = \text{density of liquid}\end{array}\right.$$

$\Rightarrow V_T \propto (\rho - \sigma)$

$$\Rightarrow \frac{V_{T_1}}{V_{T_2}} = \frac{\rho_{gold} - \sigma_{liquid}}{\rho_{silver} - \sigma_{liquid}}$$

$$\Rightarrow \frac{0.2}{V} = \frac{19.5 - 1.5}{10.5 - 1.5}$$

$\Rightarrow V = 0.1$ m/s

$$\begin{cases} \text{Given,} \\ V_{T_1} = 0.2 \text{ m/s} \\ V_{T_2} = V = \quad ? \\ \rho_{gold} = 19.5 \text{ kg/m}^3 \\ \sigma_{liquid} = 1.5 \text{ kg/m}^3 \\ \rho_{silver} = 10.5 \text{ kg/m}^3 \end{cases}$$

(Surface Tension)

17. If T is the surface tension of a fluid, then the energy needed to break a liquid drop of radius R into 64 equal drops is

(1) $6\pi R^2T$ (2) πR^2T (3) $12\pi R^2T$ (4) $8\pi R^2T$

Sol. Answer (3)

Work done = surface tension × change in area

Since volume will remain equal

Let us assume radius of new drop = r each

$$\Rightarrow \frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{R}{4} = r$$

$$W = T \cdot \Delta A$$

$$= T[n \times 4\pi r^2 - 4\pi R^2]$$

$$= T\left[64 \times 4\pi\left(\frac{R}{4}\right)^2 - 4\pi R^2\right] = 12\pi R^2 T$$

18. The excess pressure inside a spherical drop of water is four times that of another drop. Then their respective mass ratio is

(1) 1 : 16 (2) 1 : 64 (3) 1 : 4 (4) 1 : 8

Sol. Answer (2)

$$\Delta P = \frac{2T}{R}$$

Pressure $\propto \frac{1}{\text{Radius}}$

$$\begin{cases} \text{Where,} \\ P_0 = \text{Excess pressure} \\ T = \text{Surface tension} \\ R = \text{Radius} \end{cases}$$

$$\Rightarrow \frac{\Delta P_1}{\Delta P_2} = \frac{R_2}{R_1}$$

$$\Rightarrow \frac{4P}{P} = \frac{R_2}{R_1}$$

$\Rightarrow$ $4R_1 = R_2$

$$\Rightarrow \frac{R_1}{R_2} = \frac{1}{4}$$

$M = V \times \rho$

And $V \propto R^3 \Rightarrow M \propto \rho R^3$

ρ is same for both

$M \propto R^3$

$$\begin{cases} \text{Where,} \\ M = \text{Mass} \\ V = \text{Volume} \\ \rho = \text{Density} \end{cases}$$

So, $\dfrac{M_1}{M_2} = \left(\dfrac{R_1}{R_2}\right)^3 = \left(\dfrac{1}{4}\right)^3 = \dfrac{1}{64}$

19. The work done in blowing a soap bubble of 10 cm radius is (surface tension of soap solution is 0.03 N/m).

(1) 37.68×10^{-4} J (2) 75.36×10^{-4} J (3) 126.82×10^{-4} J (4) 75.36×10^{-3} J

Sol. Answer (2)

Work done = surface tension × change in area × number of free surfaces = $S \times \Delta A \times 2$

$= 0.03 \times 4\pi \times (10 \times 10^{-2})^2 \times 2$

$= 75.36 \times 10^{-4}$ J

20. A glass capillary tube of inner diameter 0.28 mm is lowered vertically into water in a vessel. The pressure to be applied on the water in the capillary tube so that water level in the tube is same as that in the vessel is (surface tension of water = 0.07 N/m and atmospheric pressure = 10^5 N/m^2).

(1) 10^3 (2) 99×10^3 (3) 100×10^3 (4) 101×10^3

Sol. Answer (4)

Height of liquid in capillary $= \dfrac{2T}{r\rho g} = h$

Pressure we need to apply = $\rho gh + P_0$

Substitute value of h

$$P = \rho g \times \frac{2T}{r\rho g} + P_0 = \frac{2T}{r} + P_0 = \frac{4T}{d} + P_0$$

$$\begin{cases} \text{Where,} \\ T = \text{Surface tension} \\ r = \text{Radius of capillary} \\ \rho = \text{Density of liquid} \\ P_0 = \text{Atmospheric pressure} \end{cases}$$

$\Rightarrow$ $P = \dfrac{4 \times 0.07}{(0.28 \times 10^{-3})} + P_0$ = 1000 Nm^{-2} + 10^5 Nm^{-2}

$\Rightarrow$ $P = (10^3 + 10^5)$ Nm^{-2} = 101×10^3 Nm^{-2}

$$\begin{cases} \text{Given,} \\ T = 0.07 \text{ N/m} \\ d = 0.28 \text{ mm} \end{cases}$$

21. Water rises to a height of 10 cm in a capillary tube and mercury falls to a depth of 3.42 cm in the same capillary tube. If the density of mercury is 13.6 kg/m^3 and angle of contact is 135°, the ratio of surface tension for water and mercury is (angle of contact for water and glass is 0°).

(1) 1 : 0.5 (2) 1 : 3 (3) 1 : 6.5 (4) 1.5 : 1

Sol. Answer (3)

$$h = \frac{2T\cos\theta}{r\rho g}$$

For water,

$$10\text{ cm} = \frac{2\times T_w \times \cos 0^\circ}{r\times 1\times g} \quad ...(1) \qquad \{T_w - \text{Surface tension of water}$$

For mercury,

$$-3.42\text{ cm} = \frac{2\times T_M \times \cos 135^\circ}{r\times 13.6\times g} \quad ...(2) \qquad \{T_M - \text{Surface tension of mercury}$$

Dividing Eq^n (1) by (2)

$$\frac{10}{-3.42} = \frac{2\times T_w \times 1\times r\times 13.6\times g}{r\times 1\times g\times 2\times T_M \times \frac{-1}{\sqrt{2}}}$$

$$\Rightarrow \frac{10}{3.42} = \sqrt{2}\times 13.6\times \frac{T_w}{T_M}$$

$$\Rightarrow \frac{10}{3.42\times 1.41\times 13.6} = \frac{T_w}{T_M}$$

$$\Rightarrow \frac{1}{6.5} = \frac{T_w}{T_M}$$

22. A spherical drop of water has 1 mm radius. If the surface tension of water is 75 × 10^{-3} N/m, then difference of pressure between inside and outside of the drop is

(1) 35 N/m^2 (2) 70 N/m^2 (3) 140 N/m^2 (4) 150 N/m^2

Sol. Answer (4)

$$\text{Excess pressure } = \frac{2T}{R} \qquad \begin{cases} T = \text{surface tension} \\ R = \text{radius} \end{cases}$$

$$= \frac{2\times 75\times 10^{-3}}{1\times 10^{-3}}$$

$$= 150\text{ N/m}^2$$

23. A capillary tube is dipped in water and it is 20 cm outside water. The water rises upto 8 cm. If the entire arrangement is put in freely falling elevator the length of water column in the capillary tube will be

(1) 20 cm (2) 4 cm (3) 10 cm (4) 8 cm

Sol. Answer (1)

If entire arrangement is in free fall then the weight of water in capillary will be balanced by pseudo force which would be equal to the weight of water.

Hence, surface tension has no weight to balance so full capillary will be filled with water.

24. If the excess pressure inside a soap bubble is balanced by an oil column of height 2 mm, then the surface tension of soap solution will be (r = 1 cm, density of oil = 0.8 g/cm^3)

(1) 3.9 N/m (2) 3.9 × 10^{-2} N/m (3) 3.9 × 10^{-3} N/m (4) 3.9 × 10^{-1} N/m

Sol. Answer (2)

Pressure due to oil column = ρ_{oil} × g × h_{oil} $= \frac{0.08\times 10^{-3}}{(10^{-2})^3}\times 9.8\times 2\times 10^{-3} = 15.68$

Now, excess pressure = pressure due to oil column

$\Rightarrow \dfrac{4T}{R} = 15.68$

$\Rightarrow \dfrac{4 \times T}{1 \times 10^{-2}} = 15.68$

$\Rightarrow T = 3.92 \times 10^{-2}$ N/m

25. There is small hole in a hollow sphere. The water enters in it when it is taken to a depth of 40 cm under water. The surface tension of water is 0.07 N/m. The diameter of hole is

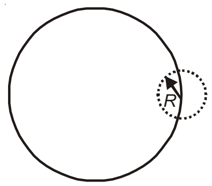
(1) 7 mm (2) 0.07 mm (3) 0.0007 mm (4) 0.7 m

Sol. Answer (2)

Let take g = 10 m/s^2

For water to enter the sphere, pressure required is = ρgh

$= 1 \times 10 \times \dfrac{40}{100} \times 1000$ (ρ = 1000 kg/m^3)



= 4000 $\dfrac{\text{N}}{\text{m}^2}$ = excess pressure

Let the hole have radius = R

Excess pressure $= \dfrac{2T}{R}$ [One surface air, one surface water]

$\Rightarrow 4000 = \dfrac{2 \times 0.07}{R}$

$\Rightarrow 2R = 0.07 \times 10^{-3}$ m

$\Rightarrow d = 0.07$ mm

26. Two equal drops are falling through air with a steady velocity of 5 cm/second. If two drops coalesce, then new terminal velocity will be

(1) $5 \times (4)^{1/3}$ cm/s (2) $5\sqrt{2}$ cm/s (3) $\dfrac{5}{\sqrt{2}}$ cm/s (4) 5×2 cm/s

Sol. Answer (1)

$V_{\text{Terminal}} \propto r^2$

If initial radius = r, let new radius = R

Then $2 \times \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3$

$\Rightarrow (2)^{1/3} r = R$

$\Rightarrow V_T \propto R^2$

$\propto (2)^{2/3} r^2$ (For bigger drops)

$\dfrac{V_{T\text{ smaller drop}}}{V_{T\text{ bigger drop}}} = \dfrac{r^2}{(2)^{2/3} r^2}$

$\Rightarrow \dfrac{5}{x} = \dfrac{1}{(2)^{2/3}}$

$\Rightarrow 5 \times (2)^{2/3} = x$

$\Rightarrow 5 \times (4)^{1/3}$ cm/s = x

27. A small drop of water falls from rest through a large height h in air; the final velocity is

(1) Proportional to $\sqrt{h}$ (2) Proportional to h

(3) Inversely proportional to h (4) Almost independent of h

Sol. Answer (4)

Since drop is falling from a large height it achieves its terminal velocity and then there is no further increase in velocity so v is independent of 'h' if 'h' is very large.

(Miscellaneous)

28. A vessel contain a liquid has a constant acceleration 19.6 m/s^2 in horizontal direction. The free surface of water get sloped with horizontal at angle

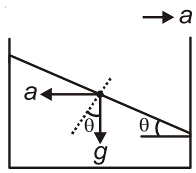
(1) $\tan^{-1}\left[\frac{1}{2}\right]$ (2) $\sin^{-1}\left[\frac{1}{\sqrt{3}}\right]$ (3) $\tan^{-1}\left[\sqrt{2}\right]$ (4) $\sin^{-1}\left[\frac{2}{\sqrt{5}}\right]$

Sol. Answer (4)

$$\tan\theta = \frac{a}{g} = \frac{19.6}{9.8} = 2$$

$\tan\theta = 2$

$$\Rightarrow \sin\theta = \frac{2}{\sqrt{5}} \Rightarrow \theta = \sin^{-1}\left[\frac{2}{\sqrt{5}}\right]$$



29. A cylinder containing water, stands on a table of height H. A small hole is punched in the side of cylinder at its base. The stream of water strikes the ground at a horizontal distance R from the table. Then the depth of water in the cylinder is

(1) H (2) R (3) $\sqrt{RH}$ (4) $R^2/4H$

Sol. Answer (4)

Let depth of water in cylinder be x

So velocity (v) of efflux $= \sqrt{2gx}$

Time taken (t) by water to travel vertical distance of H

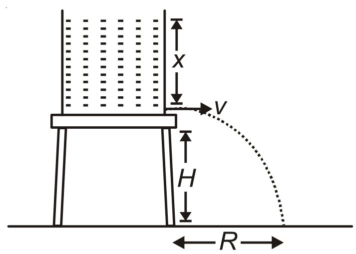
$$= \sqrt{\frac{2H}{g}}$$

$\Rightarrow$ Range = $v \times t$

$$R = \sqrt{2gx} \times \sqrt{\frac{2H}{g}}$$

Solving this we get

$$\frac{R^2}{4H} = x$$



30. A large open tank has two holes in its wall. One is a square of side a at a depth x from the top and the other is a circular hole of radius r at depth $4x$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then r is equal to

(1) $2\pi a$ (2) a (3) $\frac{a}{\sqrt{2\pi}}$ (4) $\frac{a}{\pi}$

Sol. Answer (3)

Since quantities of water flowing out of both holes is same

$\Rightarrow$ Area of hole × velocity of efflux = constant

So, $A_1 \times V_1 = A_2 \times V_2$

Substituting values.

$$a^2 \times \sqrt{2gx} = \pi r^2 \times \sqrt{8gx}$$

$\Rightarrow$ $a^2 = 2\pi r^2$

$$\Rightarrow \quad \frac{a}{\sqrt{2\pi}} = r$$

$$\begin{cases} A_1 = \text{Area of square hole} \\ V_1 = \text{Velocity of efflux from square hole} = \sqrt{2gx} \\ A_2 = \text{Area of circular hole} \\ V_2 = \text{Velocity of efflux from} \\ \quad\quad \text{circular hole} = \sqrt{2g(4x)} \end{cases}$$

SECTION - C

Previous Years Questions

1. A small sphere of radius r falls from rest in a viscous liquid. As a result, heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity, is proportional to

[NEET- 2018]

(1) r^3 (2) r^2 (3) r^4 (4) r^5

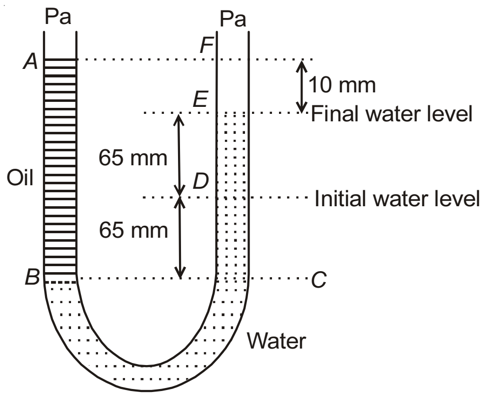
Sol. Answer (4)

Power = $6\pi\eta r V_T \cdot V_T = 6\pi\eta r V_T^2$

$V_T \propto r^2 \Rightarrow \text{Power} \propto r^5$

2. A U tube with both ends open to the atmosphere, is partially filled with water. Oil, which is immiscible with water, is poured into one side until it stands at a distance of 10 mm above the water level on the other side. Meanwhile the water rises by 65 mm from its original level (see diagram). The density of the oil is

[NEET-2017]



(1) 650 kg m^{-3} (2) 425 kg m^{-3} (3) 800 kg m^{-3} (4) 928 kg m^{-3}

Sol. Answer (4)

$h_{oil}\, \rho_{oil}\, g = h_{water}\, \rho_{water}\, g$

$140 \times \rho_{oil} = 130 \times \rho_{water}$

$$\rho_{oil} = \frac{13}{14} \times 1000 \text{ kg/m}^3$$

$\rho_{oil} = 928$ kg m^{-3}

3. A rectangular film of liquid is extended from (4 cm × 2 cm) to (5 cm × 4 cm). If the work done is 3×10^{-4} J, the value of the surface tension of the liquid is **[NEET (Phase-2) - 2016]**

(1) 0.250 Nm^{-1} (2) 0.125 Nm^{-1}

(3) 0.2 Nm^{-1} (4) 8.0 Nm^{-1}

Sol. Answer (2)

$W = 2(A_f - A_i)T$

$$\Rightarrow T = \frac{W}{(A_f - A_i) \times 2}$$

$$= \frac{3 \times 10^{-4} \text{ J}}{2[5 \times 4 \times 10^{-4} - 4 \times 2 \times 10^{-4}]} = 0.125 \text{ Nm}^{-1}$$

4. Three liquids of densities ρ_1, ρ_2 and ρ_3 (with $\rho_1 > \rho_2 > \rho_3$), having the same value of surface tension T, rise to the same height in three identical capillaries. The angles of contact θ_1, θ_2 and θ_3 obey

[NEET (Phase-2) - 2016]

(1) $\frac{\pi}{2} > \theta_1 > \theta_2 > \theta_3 \geq 0$ (2) $0 \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$

(3) $\frac{\pi}{2} < \theta_1 < \theta_2 < \theta_3 < \pi$ (4) $\pi > \theta_1 > \theta_2 > \theta_3 > \frac{\pi}{2}$

Sol. Answer (2)

$$h = \frac{2T \cos\theta}{r\rho g}$$

$\Rightarrow r \propto \cos\theta$ (as T, h and r are constants)

$\rho \uparrow \;\Rightarrow\; \theta \downarrow$

$\theta_1 < \theta_2 < \theta_3$

Its rise so $0 \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$

5. Two non-mixing liquids of densities ρ and $n\rho$ ($n > 1$) are put in a container. The height of each liquid is h. A solid cylinder of length L and density d is put in this container. The cylinder floats with its axis vertical and length pL ($p < 1$) in the denser liquid. The density d is equal to **[NEET-2016]**

(1) $\{1 + (n - 1)p\}\rho$

(2) $\{1 + (n + 1)p\}\rho$

(3) $\{2 + (n + 1)p\}\rho$

(4) $\{2 + (n - 1)p\}\rho$

Sol. Answer (1)

Weight of cylinder = $Th_1 + Th_2$

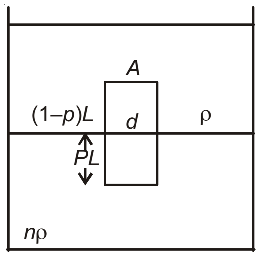
$ALdg = (1 - P)\, LA\rho g + (PLA)\, n\rho g$

$\Rightarrow d = (1 - P)\,\rho + Pn\rho$

$\Rightarrow \quad = \rho - P\rho + nP\rho$

$= \rho + (n - 1)P\rho$

$= \rho\,[1 + (n - 1)P]$



6. The cylindrical tube of a spray pump has radius *R*, one end of which has *n* fine holes, each of radius *r*. If the speed of the liquid in the tube is *V*, the speed of the ejection of the liquid through the holes is

[Re-AIPMT-2015]

(1) $\frac{V^2R}{nr}$ (2) $\frac{VR^2}{n^2r^2}$ (3) $\frac{VR^2}{nr^2}$ (4) $\frac{VR^2}{n^3r^2}$

Sol. Answer (3)

7. Water rises to a height *h* in capillary tube. If the length of capillary tube above the surface of water is made less than *h*, then **[Re-AIPMT-2015]**

(1) Water does not rise at all

(2) Water rises upto the tip of capillary tube and then starts overflowing like a fountain

(3) Water rises upto the top of capillary tube and stays there without overflowing

(4) Water rises upto a point a little below the top and stays there

Sol. Answer (3)

8. A wind with speed 40 m/s blows parallel to the roof of a house. The area of the roof is 250 m^2. Assuming that the pressure inside the house is atmospheric pressure, the force exerted by the wind on the roof and the direction of the force will be (P_{air} = 1.2 kg/m^3) **[AIPMT-2015]**

(1) 2.4 × 10^5 N, downwards (2) 4.8 × 10^5 N, downwards

(3) 4.8 × 10^5 N, upwards (4) 2.4 × 10^5 N, upwards

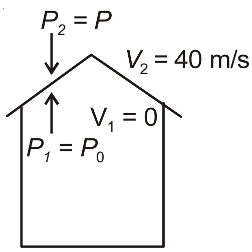
Sol. Answer (4)

$$P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho\left(v_2^2-v_1^2\right)=P_1-P_2$$

$$\frac{1}{2}\rho\left(v_2^2-v_1^2\right)=\frac{F}{A}$$

$$\frac{1}{2}\times1.2\times\left(40^2-0^2\right)=\frac{F}{250}$$



F = 2.4 × 10^5 N, upward (because $P_1 > P_2$)

9. A certain number of spherical drops of a liquid of radius *r* coalesce to form a single drop of radius *R* and volume *V*. If *T* is the surface tension of the liquid, then **[AIPMT-2014]**

(1) Energy = $4VT\left(\frac{1}{r}-\frac{1}{R}\right)$ is released (2) Energy = $3VT\left(\frac{1}{r}+\frac{1}{R}\right)$ is absorbed

(3) Energy = $3VT\left(\frac{1}{r}-\frac{1}{R}\right)$ is released (4) Energy is neither released nor absorbed

Sol. Answer (3)

Let *n* drops of radius *r* coaslece to form a big drop of radius *R*

$$n\times\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3=V$$

$n \times 4\pi r^3 = 4\pi R^3 = 3V$ ...(1)

Energy = $T.\Delta A$

$= T\,[n \times 4\pi r^2 - 4\pi R^2]$

$$= T\left[\frac{n\times 4\pi r^3}{r} - \frac{4\pi R^3}{R}\right]$$

$$= T\left[\frac{3V}{r} - \frac{3V}{R}\right]$$

$$= 3VT\left[\frac{1}{r} - \frac{1}{R}\right]$$

10. The wettability of a surface by a liquid depends primarily on : **[NEET-2013]**

(1) Surface tension

(2) Density

(3) Angle of contact between the surface and the liquid

(4) Viscosity

Sol. Answer (3)

11. The neck and bottom of a bottle are 3 cm and 15 cm in radius respectively. If the cork is pressed with a force 12 N in the neck of the bottle, then force exerted on the bottom of the bottle is

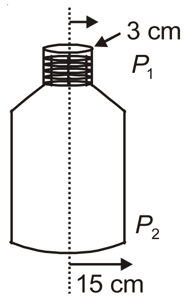
(1) 30 N

(2) 150 N

(3) 300 N

(4) 600 N

Sol. Answer (3)



Pressure applied on 1 point in a liquid spreads equally

So let P_1 be pressure at neck, P_2 be pressure at bottom

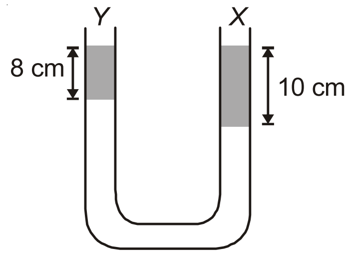
$P_1 = P_2$

$$\Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad \left[\because\ P = \frac{F}{A}\right]$$

$$\Rightarrow \frac{12}{\pi\times 9} = \frac{F_2}{\pi\times 225}$$

$\Rightarrow$ 300 N = F_2

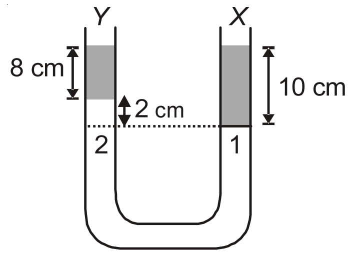
12. A liquid X of density 3.36 g cm^{-3} is poured in a U-tube, which contains Hg. Another liquid Y is poured in left arm with height 8 cm, upper levels of X and Y are same what is density of Y?



(1) 0.8 gcc^{-1} (2) 1.2 gcc^{-1} (3) 1.4 gcc^{-1} (4) 1.6 gcc^{-1}

Sol. Answer (1)

Pressure at 1 and 2 will be same



$\rho_X gH_X = \rho_Y gH_Y + \rho_{Hg} g \times 2$

$\Rightarrow$ $3.36 \times 10 = \rho_Y \times 8 + 13.6 \times 2$

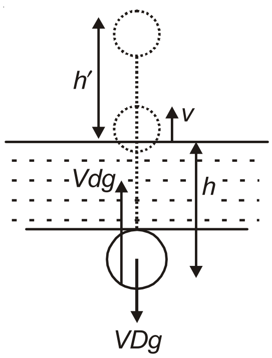
Solving this we get

$\rho_Y = 0.8$ g cc^{-1}

13. A wooden ball of density D is immersed in water of density d to a depth h below the surface of water and then released. Upto what height will the ball jump out of water?

(1) $\frac{d}{D}h$ (2) $\left(\frac{d}{D}-1\right)h$ (3) h (4) Zero

Sol. Answer (2)



Force acting on ball at depth 'h' (*i.e.* apparent weight)

$F = Vg\,[d - D]$

Acceleration $(a) = \dfrac{Vg[d-D]}{VD}$ [Mass = volume × density]

Velocity $= \sqrt{2ah} = v$ [Using $v^2 - u^2 = 2as$]

h' (height above water) $= \dfrac{v^2}{2g} = \dfrac{2 \times Vg[d-D]h}{2 \times gVD} = \left[\dfrac{d}{D}-1\right]h$

14. A piece of solid weighs 120 g in air, 80 g in water and 60 g in a liquid. The relative density of the solid and that of the liquid are respectively

(1) 3, 2

(2) $2, \frac{3}{4}$

(3) $\frac{3}{2}, 2$

(4) $3, \frac{3}{2}$

Sol. Answer (4)

$$w' = w\left[1 - \frac{\rho}{\sigma}\right] \qquad \begin{cases} \text{where,} \\ \rho = \text{density of liquid} \\ \sigma = \text{density of body} \end{cases}$$

Inside water

$$\Rightarrow \quad 80 = 120\left[1 - \frac{\rho_{water}}{\rho_{solid}}\right]$$

$\Rightarrow \quad \rho_{solid} = 3 \qquad [\because \quad \rho_{water} = 1]$

Inside liquid

$$60 = 120\left[1 - \frac{\rho_{liquid}}{\rho_{solid}}\right]$$

Using $\rho_{solid} = 3$

We get $\rho_{liquid} = \frac{3}{2}$

15. A solid sphere of volume V and density ρ floats at the interface of two immiscible liquids of densities ρ_1 and ρ_2 respectively. If $\rho_1 < \rho < \rho_2$, then the ratio of volume of the parts of the sphere in upper and lower liquids is

(1) $\frac{\rho_2 - \rho}{\rho - \rho_1}$

(2) $\frac{\rho + \rho_1}{\rho + \rho_2}$

(3) $\frac{\rho + \rho_2}{\rho + \rho_1}$

(4) $\frac{\sqrt{\rho_1 \rho_2}}{\rho}$

Sol. Answer (1)

$\rho_1 < \rho < \rho_2$ (given)

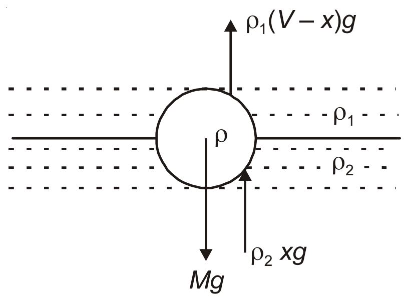
Let volume of sphere in lower liquid = x

Force of buoyancy by lower liquid = $\rho_2 x g$

Force of buoyancy by upper liquid = $\rho_1(V - x)g$

Force of gravity on sphere = Mg = $V\rho g$

Balancing all the forces for vertical equilibrium



We get

$$V\rho g = \rho_1(V - x)\, g + \rho_2 x g$$

Solving this we get

$$x = \frac{V(\rho - \rho_1)}{(\rho_2 - \rho_1)}$$

$$\text{So} \quad \frac{V - x}{x} = \frac{\rho_2 - \rho}{\rho - \rho_1} \qquad \begin{cases} \text{Where,} \\ V - x = \text{volume of sphere in upper liquid} \\ x = \text{volume of sphere in lower liquid} \end{cases}$$

16. Ice pieces are floating in a beaker *A* containing water and also in a beaker *B* containing miscible liquid of specific gravity 1.2. When ice melts, the level of

(1) Water increases in *A*
(2) Water decreases in *A*
(3) Liquid in *B* decreases
(4) Liquid in *B* increases

Sol. Answer (4)

For beaker '*A*'

Ice is floating in water

$$\rho_{ice}\, v_{ice}\, g = \rho_{water}\, v_{water\ displaced}\, g$$

$\because$ $\rho_{ice} \simeq \rho_{water}$

So we can say

$$v_{ice} \simeq v_{water\ displaced}$$

So after the ice melts the level of water will not change.

For beaker '*B*'

Ice is floating in liquid with density 1.2

clearly $\rho_{liquid} > \rho_{ice}$

So from above analogy

$$v_{ice} > v_{liquid\ displaced}$$

So when ice melts the level in beaker '*B*' increases.

17. A vessel contains oil (density 0.8 g cm^{-3}) over mercury (density 13.6 g cm^{-3}). A homogenous sphere floats with half volume immersed in mercury and the other half in oil. The density of the material of the sphere in g cm^{-3} is

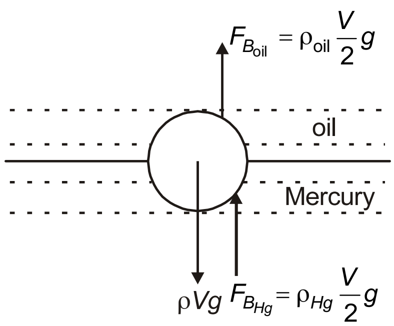
(1) 12.8
(2) 7.2
(3) 6.4
(4) 3.3

Sol. Answer (2)

Let density of sphere be ρ

And volume be v

Balancing forces for vertical equilibrium



$$\rho Vg = \frac{\rho_{Hg} Vg}{2} + \frac{\rho_{oil} Vg}{2}$$

$$\Rightarrow \quad \rho = \frac{13.6}{2} + \frac{0.8}{2}$$

$$\Rightarrow \quad \rho = 7.2 \text{ g cm}^{-3}$$

18. Two solid pieces, one of steel and the other of aluminium when immersed completely in water have equal weights. When the solid pieces are weighed in air

(1) The weight of aluminium is half the weight of steel

(2) Steel peice will weigh more

(3) They have the same weight

(4) Aluminium piece will weigh more

Sol. Answer (4)

Apparent weight = weight in air – $F_{Buoyancy}$

$\because$ Apparent weight of steel and aluminium is same

So weight of aluminium – F_B on Aluminium = weight of steel – F_B on steel ...(1)

$\rho_{Al} V_{Al} g - \rho_{water} V_{Al} g = \rho_{steel} V_{steel} g - \rho_{water} V_{steel} g$

$\rho_{steel} > \rho_{Al}$ and $\rho_{water} = 1$

So $(\rho_{Al} - 1) V_{Al} = (\rho_{steel} - 1) V_{steel}$

$\because$ $\rho_{steel} > \rho_{Al}$

$(\rho_{steel} - 1) > (\rho_{Al} - 1)$

So $V_{Al} > V_{steel}$

Also $\rho_{water} V_{Al} g > \rho_{water} V_{steel} g$

$\Rightarrow$ Force of buoyancy on Aluminium > Force of buoyancy on steel.

Using this condition in equation (1)

We get,

weight of Aluminium – weight of steel > 0

$\Rightarrow$ weight of Aluminium > weight of steel

19. A piece of wood is floating in water. When the temperature of water rises, the apparent weight of the wood will

(1) Increase (2) Decrease

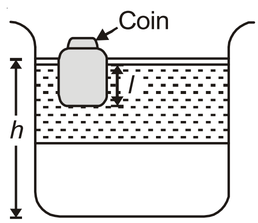
(3) May increase or decrease (4) Remain same

Sol. Answer (3)

When temperature of water is raised from 0 to 4°C, its density increases and after 4°C density decreases and apparent weight $\propto F_{Buoyancy} \propto$ density of water

So apparent weight may increase or decrease.

20. A wooden block, with a coin placed on its top, floats in water as shown in the figure. The distances *h* and *l* are shown there. After some time, the coin falls into the water, then



(1) Both *l* and *h* increase (2) Both *l* and *h* decrease

(3) *l* decreases and *h* increases (4) *l* increases and *h* decreases

Sol. Answer (2)

When coin falls into water block has to displace lesser volume to stay afloat.

Implies that block will go up and water will go down.

Hence both *l* and *h* will decrease.

21. An iceberg is floating in water. The density of ice in the iceberg is 917 kg m^{-3} and the density of water is 1024 kg m^{-3}. What percentage fraction of the iceberg would be visible?

(1) 5% (2) 10% (3) 12% (4) 8%

Sol. Answer (2)

ρ_{ice} × volume of ice × g = ρ_{water} × volume of ice inside water × g

917 × volume of ice = 1024 × volume of ice inside water

Let volume of ice = V

$$\therefore \quad \%\text{ Volume visible } = \frac{V - \text{volume inside water}}{V} \times 100$$

$$= \left(\frac{V - \dfrac{917V}{1024}}{V}\right) \times 100$$

$$= \frac{\left(\dfrac{1024-917}{1024}\right)V}{V} \times 100$$

= 10%

22. A piece of wax wieghs 18.03 g in air. A piece of metal is found to weigh 17.3 g in water. It is tied to the wax and both together weigh 15.23 g in water. Then, the specific gravity of wax is

(1) $\dfrac{18.03}{17.03}$ (2) $\dfrac{17.03}{18.03}$ (3) $\dfrac{18.03}{19.83}$ (4) $\dfrac{15.03}{17.03}$

Sol. Answer (3)

Weight of wax in air = 18.03 g

Apparent weight of metal in water = 17.3 g

Apparent weight = weight in air – ρ_{water} V_{metal} g

So weight of metal in air = apparent weight + V_{metal} g $[\because \ \rho_{water} = 1]$

= 17.3 + V_{metal} × g

When wax and metal are tied together

Total weight in air = 18.03 + 17.3 + V_{metal} × g

And apparent weight in water = 15.23 = weight in air – ρ_{water} V_{wax} g – ρ_{water} V_{metal} g

$\Rightarrow$ 15.23 = 18.03 + 17.3 + V_{metal} g – V_{wax} g – V_{metal} g

$\Rightarrow$ V_{wax} g = 20.1

$$\Rightarrow \frac{\text{Mass of wax}}{\text{density}} \times g = 20.1 \qquad \left[\because \ \rho = \frac{M}{V}\right]$$

$$\Rightarrow \frac{18.03}{g \times \rho} \times g = 20.1 \qquad \left[\text{Mass} = \frac{\text{weight}}{g}\right]$$

$$\text{So specific gravity of wax } = \frac{18.03}{20.1} = 0.897 \sim 0.9$$

$$\Rightarrow \left(\frac{18.03}{19.83} = 0.9\right)$$

23. Eight equal drops of water are falling through air with a steady velocity of 10 cm^{-1}. If the drops combine to form a single drop big in size, then the terminal velocity of this big drop is

(1) 80 cms^{-1} (2) 30 cms^{-1} (3) 10 cms^{-1} (4) 40 cms^{-1}

Sol. Answer (4)

Let radius of smaller drops be *r*, and bigger be *R*

When 8 such drops combine to form a bigger drop the total volume of water remains same

So, $8\times\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$\therefore \quad 2r = R$

And we know,

$V_{\text{Terminal}} \propto r^2$

$\therefore \quad \frac{V_{T\,\text{smaller}}}{V_{T\,\text{bigger}}} = \frac{r^2}{R^2} = \frac{r^2}{4r^2}$

$\Rightarrow \quad \frac{10}{V_{T\,\text{bigger}}} = \frac{1}{4}$ [$\because$ $V_{T\,\text{smaller}}$ = 10 cms^{-1} (given)]

$\Rightarrow \quad V_{T\,\text{bigger}}$ = 40 cms^{-1}

24. A small spherical ball falling through a viscous medium of negligible density has terminal velocity ν. Another ball of the same mass but of radius twice that of the earlier falling through the same viscous medium will have terminal velocity

(1) ν (2) $\frac{\nu}{4}$ (3) $\frac{\nu}{2}$ (4) 4ν

Sol. Answer (4)

$v_{\text{Terminal}} \propto r^2$

$\frac{v_{T_1}}{v_{T_2}} = \frac{r_1^2}{r_2^2}$

Where, $v_{T_1} = \nu$, $r_1 = r$, $r_2 = 2r$, $v_{T_2} = ?$

Substituting values

$\frac{\nu}{v_{T_2}} = \frac{r^2}{4r^2}$

$\Rightarrow \quad v_{T_2} = 4\nu$

25. Streamline flow is more likely for liquid with

(1) High density and low viscosity (2) Low density and high viscosity

(3) High density and high viscosity (4) Low density and low viscosity

Sol. Answer (2)

Streamline flow is more likely for liquid with low density and high viscosity.

26. An air bubble of radius 10^{-2} m is rising up at a steady rate of 2×10^{-3} ms^{-1} through a liquid of density 1.5×10^3 kg m^{-3}, the coefficient of viscosity neglecting the density of air, will be (g = 10 ms^{-2})

(1) 23.2 units (2) 83.5 units (3) 334 units (4) 167 units

Sol. Answer (4)

$$V_T = \frac{2a^2}{9\eta}g(\rho - \sigma)$$

Substituting values

$$2\times10^{-3} = \frac{2\times10^{-4}\times10\times1.5\times10^{3}}{9\eta}$$

$\Rightarrow$ $\eta \sim 167$ units

$$\left\{\begin{array}{l}\text{Where given is} \\ V_T = 2\times10^{-3}\ \text{ms}^{-1} \\ a = 10^{-2}\ \text{m} \\ \rho = 1.5\times10^{3}\ \text{kg m}^{-3} \\ \sigma \sim 0 \\ g = 10\ \text{ms}^{-2}\end{array}\right.$$

27. The flow of liquid is laminar or streamline is determined by

(1) Rate of flow of liquid (2) Density of fluid

(3) Radius of the tube (4) Coefficient of viscosity of liquid

Sol. Answer (1)

It is decided by rate of flow of liquid

Given by Reynolds number $= \dfrac{\rho v d}{n}$

28. A boat carrying a number of large stones is floating in a water tank. What would happen to the water level, if a few stones are unloaded into water?

(1) Rises

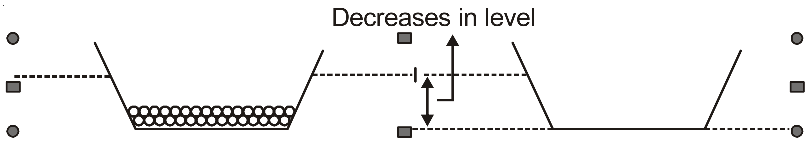
(2) Falls

(3) Remains unchanged

(4) Rises till half the number of stones are unloaded and then begins to fall

Sol. Answer (2)

Previously when stones are on the boat they are increasing the weight on the boat and to balance this weight boat needs to generate buoyancy force by displacing more water, but when stones are removed the boat starts displacing less amount of water hence the level of water in tank falls.



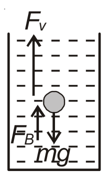
29. The velocity of a small ball of mass M and density d_1 when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is d_2, the viscous force acting on the ball is

(1) $Mg\left(1-\dfrac{d_2}{d_1}\right)$ (2) $Mg\dfrac{d_1}{d_2}$

(3) $mg(d_1 - d_2)$ (4) mgd_1d_2

Sol. Answer (1)

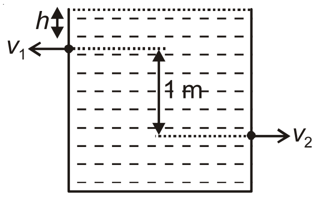
$$F_v = mg - F_v = vd_1g - vd_2g = vd_1g\left(1-\frac{d_2}{d_1}\right) = mg\left(1-\frac{d_2}{d_1}\right)$$



30. There are two holes one each along the opposite sides of a wide rectangular tank. The cross-section of each hole is 0.01 m^2 and the vertical distance between the holes is one metre. The tank is filled with water. The net force on the tank in newton when the water flows out of the holes is (density of water = 1000 kgm^{-3})

(1) 100 (2) 200 (3) 300 (4) 400

Sol. Answer (2)



Net force = $F_2 - F_1$

$= \rho A v_2^2 - \rho A v_1^2$

$$F = ma = v\rho \times \left(\frac{v}{t}\right) = Ah\rho\left(\frac{v}{t}\right) = A\rho v^2$$

$= 2\rho g\ (h + 1)\ A - 2\rho ghA \qquad \left[v = \sqrt{2gx}\right]$

$= 2\rho gA$

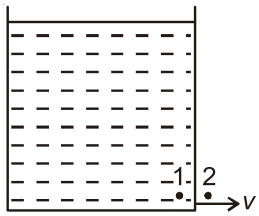
$= 1000 \times 10 \times 0.01 \times 2$

$= 200$ N

31. A hole is made at the bottom of the tank filled with water (density 1000 kg/m^3). If the total pressure at the bottom of the tank is 3 atm (1 atm = 10^5 N/m^2), then the velocity of efflux is

(1) $\sqrt{200}$ m/s (2) $\sqrt{400}$ m/s (3) $\sqrt{500}$ m/s (4) $\sqrt{800}$ m/s

Sol. Answer (2)



Apply Bernoulli's theorem

$$\underbrace{P + \rho gH}_{\text{Total pressure}} + \frac{1}{2}\rho v^2 = \text{constant} \qquad \begin{cases}\text{Given} \\ \text{Total pressure} = 3 \text{ atm}\end{cases}$$

At point 1, 3 atm + 0 = constant ...(i)

At point 2, 1 atm + $\frac{1}{2}\rho v^2$ = constant ...(ii)

Equate (i) and (ii)

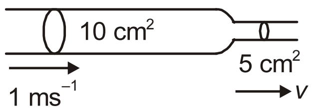
$3 = 1 + \frac{1}{2}\rho v^2$ [Use ρ = 1000 and 1 atm = 10^5 N/m^2]

We get, $v = \sqrt{400}$ m/s

32. A horizontal pipe line carries water in stremline flow. At a point where the cross-sectional area is 10 cm^2 the water velocity is 1 ms^{-1} and pressure is 2000 Pa. The pressure of water at another point where the cross-sectional area is 5 cm^2, is

(1) 200 Pa (2) 400 Pa (3) 500 Pa (4) 800 Pa

Sol. Answer (3)



P = 2000 Pa

$A_1V_1 = A_2V_2$ (equation of continuity)

$10 \times 1 = 5 \times v$

So, $v = 2\ \text{ms}^{-1}$

Apply Bernoulli theorem at both the points,

$$2000 + \frac{1}{2} \times 1000 \times 1^2 = P + \frac{1}{2} \times 1000 \times 4 \Rightarrow P = 500\ \text{Pa}$$

33. A rectangular vessel when full of water, takes 10 min to be emptied through an orifice in its bottom. How much time will it take to be emptied when half filled with water?

(1) 9 min (2) 7 min

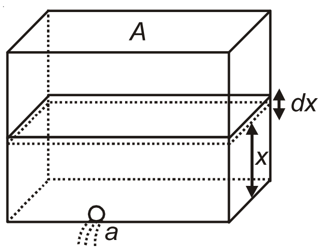
(3) 5 min (4) 3 min

Sol. Answer (2)

Let time taken by height 'x' to get reduced by $dx = dt$

$$\therefore \quad dt = \frac{\text{volume}}{\text{efflux speed}} = \frac{A \times dx}{\sqrt{2gx}} \qquad \{A \text{ is area of cross-section}\}$$

$$\int_0^T dt = \int_0^h \frac{A}{\sqrt{2g}} \frac{dx}{\sqrt{x}}$$



$$\Rightarrow \quad T = \frac{A}{a}\sqrt{\frac{2h}{g}}$$

$$\Rightarrow \quad T \propto \sqrt{h}$$

So we can use

$$\frac{T_1}{T_2} = \frac{\sqrt{h_1}}{\sqrt{h_2}}$$

$$\frac{10\ \text{min}}{t\ \text{min}} = \frac{\sqrt{h}}{\sqrt{h/2}}$$

$$\Rightarrow \quad t = \frac{10}{\sqrt{2}} \sim 7\ \text{min}$$

34. A metal plate of area $10^3\ \text{cm}^2$ rests on a layer of oil 6 mm thick. A tangential force 10^{-2} N is applied on it to move it with a constant velocity of $6\ \text{cms}^{-1}$. The coefficient of viscosity of the liquid is

(1) 0.1 poise (2) 0.5 poise

(3) 0.7 poise (4) 0.9 poise

Sol. Answer (1)

$$F = \eta A \frac{v}{d}$$

$$\Rightarrow \quad 10^{-2} = \eta \times (10^3 \times 10^{-4}) \times \frac{6\times10^{-2}}{6\times10^{-3}} = 0.01\,\text{poiseuille}$$

$$= 0.1 \text{ poise}$$

$$\begin{cases} \text{Where,} \\ F = \text{Force} \\ \eta = \text{Coefficient of viscosity} \\ A = \text{Area} \\ v = \text{Velocity} \\ d = \text{Thickness of layer} \end{cases}$$

35. With an increase in temperature, surface tension of liquid (except molten copper and cadmium)

(1) Increases (2) Remain same

(3) Decreases (4) First decreases then increases

Sol. Answer (3)

When we increase the temperature, we are providing energy to the molecules. This increase in potential energy causes the surface energy to drop and become less negative hence decreasing surface tension because surface tension is nothing but surface energy per unit area.

36. Determine the energy stored in the surface of a soap bubble of radius 2.1 cm if its tension is 4.5×10^{-2} Nm^{-1}.

(1) 8 mJ (2) 2.46 mJ (3) 4.93×10^{-4} J (4) None of these

Sol. Answer (3)

Energy = surface tension × surface area × number of free surfaces

$= (4.5 \times 10^{-2}) \times 4\pi \times (2.1 \times 10^{-2}) \times 2 = 4.98 \times 10^{-4}$ J

37. A mercury drop of radius 1.0 cm is sprayed into 10^6 droplets of equal sizes. The energy expended in this process is (surface tension of mercury is equal to 32×10^{-2} Nm^{-1})

(1) 3.98×10^{-4} J (2) 8.46×10^{-4} J (3) 3.98×10^{-2} J (4) 8.46×10^{-2} J

Sol. Answer (3)

Energy expended = surface tension × increase in area (Formulae)

So, volume initially = volume of 10^6 drops

$$\Rightarrow \quad \frac{4}{3}\pi\left(\frac{1}{100}\right)^3 = 10^6 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad \left[\left(\frac{1}{100}\right)^3 \times \frac{1}{10^6}\right]^{1/3} = r \qquad \text{[Let radius of small drops = } r\text{]}$$

$\Rightarrow \quad 10^{-4}$ m = r

So increase in surface area $= 4\pi\left[\left(\frac{1}{10000}\right)^2 \times 10^6 - \left(\frac{1}{100}\right)^2\right] = 4\pi\left[\frac{1}{100} - \frac{1}{10000}\right]$

$$\Rightarrow \quad \Delta A = \frac{4\pi \times 0.99}{100}$$

Using this value in formulae

Energy $= 32 \times 10^{-2} \times \frac{4\pi \times 0.99}{100}$ [$\because$ Surface tension = 32×10^{-2} (given)]

$= 3.98 \times 10^{-2}$ J

38. When a glass capillary tube of radius 0.015 cm is dipped in water, the water rises to a height of 15 cm within it. Assuming contact angle between water and glass to be 0°, the surface tension of water is [ρ_{water} = 1000 kg m^{-3}, g = 9.81 ms^{-2}]

(1) 0.11 Nm^{-1} (2) 0.7 Nm^{-1} (3) 0.072 Nm^{-1} (4) None of these

Sol. Answer (1)

$$h = \frac{2\,S\cos\theta}{\rho r g}$$

Substituting values

$$\frac{15}{100} = \frac{2\times S\times 1\times 100}{1000\times 0.015\times 9.81}$$

$\therefore$ S = 0.11 Nm^{-1}

$$\left\{\begin{array}{l}\text{Where,}\\ S = \text{surface tension} = ?\\ h = \text{height of water in capillary} = 15\text{ cm}\\ r = \text{radius of capillary} = 0.015\text{ cm}\\ \theta = \text{angle of contact} = 0^\circ\\ g = 9.8\text{ ms}^{-2}\end{array}\right.$$

39. A liquid does not wet the sides of a solid, if the angle of contact is

(1) Obtuse (2) 90° (3) Acute (4) Zero

Sol. Answer (1)

Solid will not get wet if the liquid has high surface tension (example mercury) and liquids with high surface tension have obtuse angle of contact.

40. Two drops of equal radius coalesce to form a bigger drop. What is ratio of surface energy of bigger drop to a smaller one?

(1) $2^{1/2}$: 1 (2) 1 : 1

(3) $2^{2/3}$: 1 (4) None of these

Sol. Answer (3)

Surface energy = surface tension × surface area

Let the radius of smaller drops be r

And that of bigger drop be R

Then ratio of surface energies = ratio of surface area [$\because$ Surface tension is same for both]

= $4\pi R^2 : 4\pi r^2$

= $R^2 : r^2$ ...(1)

$\because$ 2 smaller drops are forming 1 big drop so $2\times\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

So, $2^{1/3}r = R$ ...(2)

$\Rightarrow$ Using 1 and 2 we can say that ratio of surface energies = $2^{2/3}r^2 : r^2 = 2^{2/3} : 1$

41. The excess pressure inside a spherical drop of water is frour times that of another drop. Then their respective mass ratio is

(1) 1 : 16 (2) 8 : 1

(3) 1 : 4 (4) 1 : 64

Sol. Answer (4)

Excess pressure $= \dfrac{2T}{r}$ {Where, r = radius of drop

$$\Delta P \propto \frac{1}{r}$$

$$\frac{\Delta P_1}{\Delta P_2} = \frac{r_2}{r_1} = \frac{4}{1} \qquad \left[\because \ \frac{\Delta P_1}{\Delta P_2} = \frac{1}{4}\right]$$

$$V \propto r^3$$

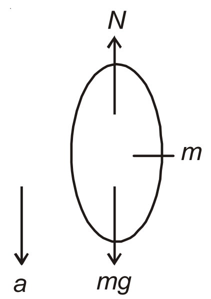
$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

$\therefore$ $M = V \times \rho$ then $\dfrac{M_1}{M_2} = \dfrac{V_1}{V_2} = \dfrac{1}{64}$

42. A balloon with mass m is descending down with an acceleration a (where $a < g$). How much mass should be removed from it so that it starts moving up with an acceleration a?

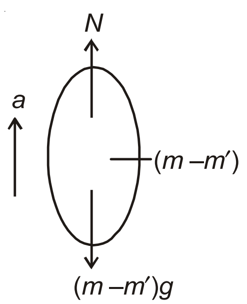
(1) $\dfrac{2ma}{g+a}$ (2) $\dfrac{2ma}{g-a}$ (3) $\dfrac{ma}{g+a}$ (4) $\dfrac{ma}{g-a}$

Sol. Answer (1)



$mg - N = ma$ ..... (1)

Let m mass remove from ballon



$N - (m - m')g = (m - m')a$ ..... (2)

After addition of equation (1) & (2), then

$$\boxed{m' = \frac{2ma}{g+a}}$$

SECTION - D

Assertion - Reason Type Questions

1. A : Hydraulic lift is based on Pascal's Law.

   R : Hydrostatic pressure is a scalar quantity

Sol. Answer (2)

A : is true

R : is true

But reason is not the correct explanation. Correct explanation is, change in pressure is transferred, undiminished from one point to the other.

2. A : The apparent weight of a body floating on the surface of a liquid is zero.

   R : The net force on a body floating on the surface of a liquid is zero.

Sol. Answer (1)

A : is true

R : is true

And reason is also the correct explanation.

3. A : It is better to wash cloths in hot water than cold water.

   R : On increasing temperature surface tension of water decreases.

Sol. Answer (1)

A : is true

R : is true and correct explanation.

4. A : The impurities added to water may increase or decrease surface tension.

   R : The change in surface tension depends on the nature of impurities.

Sol. Answer (1)

A is true, R is true and correct explanation.

5. A : On increasing temperature the angle of contact generally decreases.

   R : With rise in temperature, the surface tension of liquid increases.

Sol. Answer (4)

A : is false : Angle of contact increases with increase in temperature.

R : is false : Surface tension decreases with rise in temperature.

6. A : If air blows over the roof of a house, the force on the roof is upwards.

   R : When air blows over the roof, the pressure over it from out side decreases.

Sol. Answer (1)

A is true, R is true and correct explanation.

7. A : When rain drops fall through air some distance, they attain a constant velocity.

   R : The viscous drag of air just balances the weight of rain drops.

Sol. Answer (1)

A : is true

R : is true and correct explanation.

8. A : Bernoulli's theorem holds good only for non-viscous and incompressible liquid.

   R : Bernoulli's theorem is based on the conservation of energy.

Sol. Answer (2)

A : is true

R : is true

Correct explanation of 'A' is Bernoulli's equation does not take into account the elastic energy of the fluids.

9. A : At high altitudes (mountains), it is very difficult to stop bleeding from a cut in the body.

   R : At high altitude the atmospheric pressure is less than the blood pressure inside the body.

Sol. Answer (1)

A : is true

R : is true and correct explanation.

10. A : When liquid drops merge into each other to form a large drop, energy is released.

    R : When liquid drops merge to form large drop surface tension decreases.

Sol. Answer (3)

A : is true

R : is false, because when large drop is formed, surface area gets reduced. Hence surface energy gets reduced due to reduction in surface area not the surface tension.

11. A : Excess pressure inside a soap bubble is $\frac{4T}{r}$ (symbols have their usual meanings).

    R : The pressure difference across a curved surface of radius of curvature r is $\frac{2T}{r}$. There are two surfaces in a soap bubble.

Sol. Answer (1)

A : is true

R : is true and correct explanation.

12. A : Buoyant force is always vertically upward.

    R : Buoyant force is always opposite to the direction of acceleration due to gravity.

Sol. Answer (4)

A : is wrong, Buoyant force is not always vertically upwards

R : is wrong, Buoyant force is always opposite to the direction of effective acceleration.

13. A : Equation of continuity is $A_1v_1\rho_1 = A_2v_2\rho_2$ (symbols have their usual meanings).

    R : Equation of continuity is valid only for incompressible liquids.

Sol. Answer (3)

A : is true

R : is false,

14. A : Atomizer is based on the principle of Bernoulli's theorem.

    R : Bernoulli's theorem is based on the conservation of energy.

Sol. Answer (2)

A : is true

R : is true, but not the correct explanation

Correct explanation is, decrease in pressure forces the water to move up the tube and get sprayed.

15. A : The spiders and insects can run on the surface of water.

R : Buoyant force balances the weight of insects.

Sol. Answer (3)

A : true

R : is false, surface tension balances the weight of insect.

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