BMS 3: Battery State of Charge (SOC) Estimation

Week 1: Importance of a good SOC estimation.

When BMS is turned off, the algorithm store their status information in a non volatile memory for use when the battery pack is activated.

BMS must be able to ~~measure~~ estimate 2 fundamentally types of non-measurable battery pack quantity

1. **States**: quantities that change relatively quickly. (eg SOC, diffusion voltage, hysteresis voltage)
2. **Parameters**: change very slowly; change due cell degrading (eg cell capacities, resistances, aging effects).

Having an estimate of SOC for every cell is important because SOC is an input to an algorithm to know how to balance cells, power limit, and energy consumption.

SOC - State

SOH - Parameter estimation

It is compared to a dashboard fuel guage in an ICEV that reports a value from "Empty (0%)" to "Full (100%)".

While there exist sensors to measure a gasoline level in a tank accurately, there is presently no sensor available to measure SOC. Hence, SOC estimation is required

#1 Advantage of a good Estimator

i) **Longevity**: If a fuel tank is overfilled or emptied, no damage occurs. However, over-charging or over-discharging of a battery cell may cause permanent damage or reduced lifetime.

ii) **Performance**: Without good SOC estimator, one must be overly conservative when using battery pack to avoid over/undercharge due to trusting poor estimate.

iii) **Reliability**: a good SOC estimator is consistent and dependable for any driving profile enhancing overall power-system reliability.

iv) **Density**: Accurate battery state estimates allow battery pack to be used within design limits.

v) **Economy**: Smaller battery systems cost less. Warranty service on reliable systems costs less.

Summary

BMS must estimate battery cell States & Parameters

#2 : How do we define SOC carefully?

Estimate the SOC using measurement of only cell terminal voltage and cell current. While cell OCV is closely related to SOC, the terminal voltage is a poor predictor of OCV unless the cell is in electrochemical equilibrium (and hysteresis is negligible).

N.B The only time $V_t = OCV$ is when the cell has rested a long time and is in electrochemical equilibrium.

Using current only also is a poor predictor. Hence, combine voltage and current!

* How to know/validate good SOC estimator

→ Some definitions can calibrate lab tests.

- A cell is fully charged when its OCV equals fixed value ($V_h(T)$) specified by the battery-cell manufacturer (temperature)

  e.g $V_h(25°C) = 4.2V$ for LMO
  $V_h(25°C) = 3.6V$ for LFP

- A cell is fully discharged when OCV equal $V_L(T)$ - minimum voltage specified by the manufacturer.

  e.g $V_L(25°C) = 3.0V$ for LMO
  $V_L(25°C) = 2.0V$ for LFP

- Total capacity: quantity of charge removed ~~from~~ as cell ~~that~~ is brought from fully charged state to fully discharged state.
  Q

  → S.I unit for charge is coulombs (C), but in battery world, much more to use ampere hours (Ah) or milliampere hours (mAh)

  Q Not a function of temperature or rate at which the cell is charging or discharging but slowly change overtime as a cell ages.

- Discharge Capacity (Q_{rate}): is quantity of charge removed as cell discharged at constant rate from fully charged state until terminal voltage V_t reaches $V_L(T)$ - minimum voltage

  → it based on a loaded terminal voltage instead of an OCV at the lower end. Dependent on cell's internal resistance, rate, & temp.

- It is the rate at which a battery discharges; If discharge at relatively low rate, we can get almost the entire total capacity out of the battery cell before encountering a voltage limit.

  Unless $i(t) \to 0$, discharge capacity is less than total capacity

- If discharged at a faster rate, less capacity will be gotten out of the battery cell before encountering voltage limit because of the ohmic voltage drop across the resistance of the cell.

- Nominal capacity: manufacturer's specified quantity intended to be representative
Q_{nom} of 1C-rate discharge capacity Q_{1C} of a particular manufactured lot of cells at room temperature, 25°C.

It is a constant value. $Q_{nom} \neq Q_{1C}$ — discharge (representative of a single individual cell)

representative of lot of cells

$Q_{nom} \neq Q$

→ representative of a discharge capacity, not a total capacity.

- Residual capacity: amount of charge presently in a battery cell that could be removed from cell if it were brought from its present state to a fully discharged state

- State of charge: the ratio of residual capacity to total capacity

$$\boxed{\begin{aligned} z(t) &= z(0) - \frac{1}{Q}\int_0^t \eta(t)\,i(t)\,dt \\ z_{k+1} &= z_k - \eta_k i_k \Delta t / Q \end{aligned}}$$

#4 Approach to Estimating Battery cell SOC

- One method would be to measure cell terminal voltage, V_t, and look up on "SOC vs OCV" curve → method works very poorly.

  - Recall $V_t \neq OCV$, hence the method ignores polarization effect, including ohmic losses, diffusion voltages and hysteresis on V_t
  - Some battery chemistry has OCV that is very flat (on graph) curve dilute of estimate.
  - To improve it

$$V_t = OCV(z(t)) - i(t)R_0$$

→ if ohmic losses is known.

find $OCV \Rightarrow V_t + i(t)R_0$; then look up on "SOC vs OCV" curve.

- Another: Coulomb counting — keeps track of charge in/out of cell

$$\boxed{\hat{z}(t) = \hat{z}(0) - \frac{1}{Q}\int_0^t \eta(\tau)\,i_{meas}(\tau)\,d\tau}$$

$$i_{meas}(t) = i_{true}(t) + i_{noise}(t) + i_{bias}(t) + i_{nonlin}(t) + i_{sd}(t) + i_{leakage}(t)$$

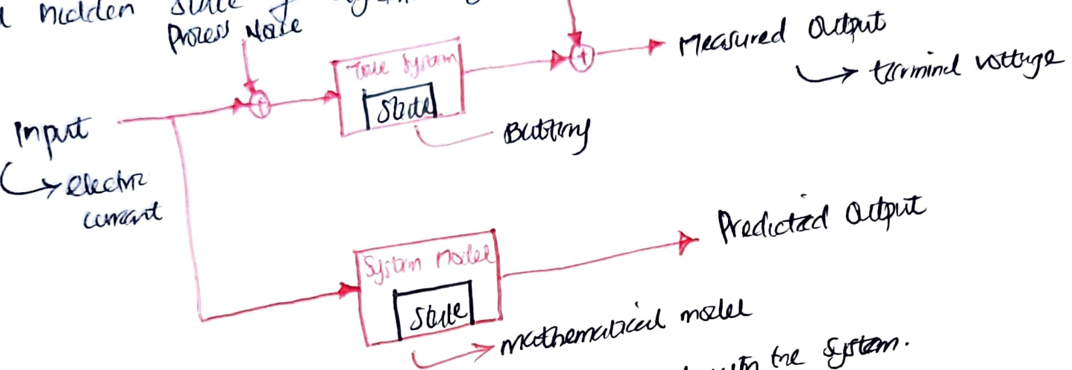
Coulomb works well if initial condition are known well or can be reset frequently.

↳ the method is subject to drift due to current sensor's fluctuations, current-sensor bias, incorrect capacity estimates, other losses

- Another: Model-based state estimation

An alternative to a voltage-only method or a current-only method is somehow to combine the approaches.

Model-based estimators implement algorithms that use sensed measurements to infer internal hidden state of dynamic system.



operating in parallel with the system.

Same input propagated through true system, model, measured, and predicted outputs compared; error used to update model's state estimate

↳ process noise: unmeasured noise as per the influence of the process state input to the battery; not measured but impact the battery

↳ Sensor noise: appear on the voltage measurement but not affect the true system state

The difference between the predicted and measured output (voltage) might be due to having an incorrect estimate of: state, measurement, or model errors

→ Linear kalman filter

Under specific conditions, kalman filter (KF, special case of sequential probabilistic inference) gives optimal state estimate.

Assume a general, possibly nonlinear, state-space model:

$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1})$$

$$y_k = h(x_k, u_k, v_k)$$

u_k – known (measured) input signal

w_k – process-noise random input

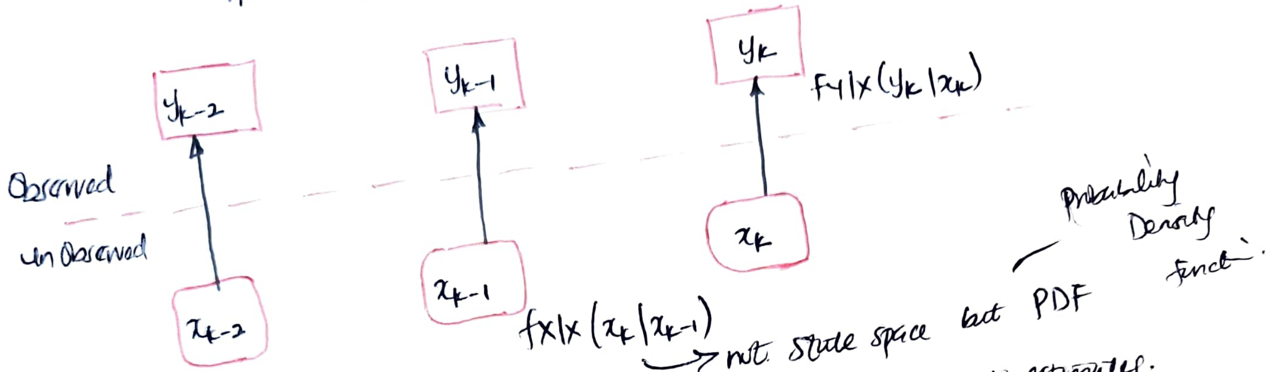
v_k – sensor-noise random input

function $f()$ and $h()$ may be time-varying.

→ Sequential Probabilistic inference

Kalman filter is a special case of broader category of filter known as sequential probabilistic inference (SPI). The goal is to estimate the present state of dynamic systems using all measurements made from the system.

$$\mathbb{Y}_k = \{y_0, y_1, \ldots, y_k\}$$



↳ Process and sensor noise randomness will always cause imperfect estimates.

Summary

i) Direct lookup of terminal voltage in OCV versus SOC table gives very poor estimates of cell SOC

ii) Modifying lookup to account for ohmic resistance helps, but not enough

iii) Coulomb counting also isn't sufficient & enough.

iv) model-based state estimators combine voltage and current measurements, using a cell model to do so, to yield better estimates.

v) Sequential probabilistic inference (SPI) is the general framework that describes model-based state estimators of interest.

#5: Understanding uncertainty via mean and covariance

Sequential probabilistic inference seek to find the best state estimate in the presence of process and sensor noises on measurement.

Noise is not deterministic - it is random in some sense. To discuss impact of noise on the system dynamics, "random variables" (RVs) must be understood.

- each time RV is measured, exact value can be predicted
- but different possible measurements of RVs can be characterized using "probability density function (pdf)"

* Random vectors

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}, \quad x_0 = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

X - random vector
x_0 - sample vector.

- $f_X(x_0)$ means $f_X(X_1 = x_1, X_2 = x_2, \dots X_n = x_n)$
- $f_X(x_0)\, dx_1 dx_2 \dots dx_n$ is probability that X is between x_0 and $x_0 + dx$
- $f_X(x_0)$ is scaled probability or "likelihood" of measuring sample vector x_0

* Properties of joint pdf of random vector

1. PDF is a non-negative function (0 or +ve)

$$f_X(x) \geq 0 \quad \forall x$$

2. Volume underneath PDF is equal to 1

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} f_X(x)\, dx_1 dx_2 \dots dx_n = 1$$

3. $$\bar{x} = E[X] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} x f_X(x)\, dx_1 dx_2 \dots dx_n$$

↳ expected value

4. Random variables have moments - The second moment of random variable = Expected of that variable squared

Correlation Matrix

$$\Sigma_X = E[XX^T] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} xx^T f_X(x)\, dx_1 dx_2 \dots dx_n$$

5. For ~~scalar~~ vector random variable; Define $\tilde{X} = X - \bar{x}$. Then,

Covariance matrix:

$$\Sigma_{\tilde{X}} = E[(X - \bar{x})(X - \bar{x})^T] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} (x - \bar{x})(x - \bar{x})^T f_X(x)\, dx_1 dx_2 \dots dx_n$$

Scalar RVs - variance

* Properties of correlation and covariance matrices

- Covariance matrix $\Sigma_{\tilde{X}}$ is always a square symmetric matrix. It has a mathematical property called positive-semi-definite (psd) i.e

$$y^T \Sigma_{\tilde{x}} y \geq 0 \quad \forall y$$

$y^T \times$ covariance matrix $\times y$ gives a scalar result which is non-negative

- PSD - a matrix is strictly positive-semi-definite if all of its eigen values are strictly positive

- Correlation matrix of a random variable = covariance matrix of the vector has zero-mean.

- Entries in covariance matrix has specific meanings

$$(\Sigma_{\tilde{x}})_{ii} = \sigma^2_{x_i}$$

$$(\Sigma_{\tilde{x}})_{ij} = \rho_{ij}\sigma_{x_i}\sigma_{x_j} = (\Sigma_{\tilde{x}})_{ji}$$

* Multivariable Gaussian PDF

There are infinite variety in PDFs; However, for KF, multivariable gaussian pdf is assumed

- All noises and the state vector itself are assumed to be Gaussian random vectors.

- Normal PDF

$\hookrightarrow x \sim \mathcal{N}(\bar{x}, \Sigma_{\tilde{x}})$

Multi-variable PDF

$$f_x(x) = \frac{1}{(2\pi)^{n/2}|\Sigma_{\tilde{x}}|^{1/2}} \exp\left(-\frac{1}{2}(x-\bar{x})^T \Sigma_{\tilde{x}}^{-1}(x-\bar{x})\right)$$

$|\Sigma_{\tilde{x}}| = \det(\Sigma_{\tilde{x}})$, $\Sigma_{\tilde{x}}^{-1}$ requires positive-definite $\Sigma_{\tilde{x}}$

Summary

- To develop sequential-probabilistic-inference solution, one must have background understanding of random variables (RVs)
- RVs are described by PDFs (probability density functions)
- For all Random Variables, multivariable Gaussian (or normal) distribution is assumed.

#6 Independence & Uncorrelation - Understanding joint uncertainty of 2 unknown quantities

Independence: If jointly-distributed RVs are independent, then

$$f_X(x_1, x_2, \dots x_n) = f_{X_1}(x_1) f_{X_2}(x_2) \dots f_{X_n}(x_n)$$

When independence is true, the particular value of the random variable X_1 has no impact on value obtained for the random variable X_2. Hence, no nonlinear or linear relationship to predict one variable from the other.

Uncorrelated: Two jointly-distributed RVs X_1 and X_2 are uncorrelated if

$$\text{cov}(X_1, X_2) = E[(X_1 - \bar{x}_1)(X_2 - \bar{x}_2)^T] = 0$$

no linear relationship between RVs

Uncorrelated variable means there is no linear relationship between RVs

- In general, the condition for RVs to be uncorrelated is much weaker than for them to be independent
  - ↪ If jointly-distributed RVs X_1 and X_2 are independent they must also be uncorrelated: independence implies uncorrelation
  - ↪ Uncorrelated RVs are not necessarily independent.
- If jointly normally (Gaussian) distributed RVs are uncorrelated, are also independent. → Very special case. Reason why it is assumed that all PDFs in the sequential probabilistic inference solution are Gaussian

* Conditional Probability

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

- joint pdf
- marginal pdf

N.B! Marginal pdf $f_Y(y)$ may be calculated as:

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$$

* Baye's' rule

It relates posterior probability to prior probability and it forms a key step in Kalman filter (KF) derivation

$$f_{X|Y}(x|y) = \frac{f_{Y|X}(y|x) f_X(x)}{f_Y(y)}$$

* Conditional Expectation

$$E[X = x \mid Y = y] = E[X \mid Y] = \int_{-\infty}^{\infty} x f_{X|Y}(x|y)\,dx$$

Conditional expectation is critical. KF is an algorithm to compute $E[x_k \mid y_k]$
conditional expectation by computing the expected state of a dynamic system (x_k) given
complete set of measurements made from the system.

* Central Limit Theorem

if $Y = \sum_i X_i$ X_i are independent, and identically distributed and have finite mean and variance

random variable

then Y is approximately normally distributed and approximation improves as more RVs are summed.

Assumption for KF derivation:

i Assume state x_k, process noise w_k, sensor noise v_k are normally distributed RVs (Gaussian)

ii Assume w_k and v_k are uncorrelated with each other.

Even when these assumptions are broken in practice, KF works quite well.

Summary

- If two random variables are independent, joint pdf equals product of marginal pdfs and knowing value of one RV cannot be used to predict value of other.
- If two random variables are uncorrelated, expected value of product equals product of expected values and knowing value of one RV cannot be used with linear equation to ~~help~~ predict other value.
- In most cases, RVs are correlated, so conditional expectation will help to predict value of one RV given others.
- Central limit theorem justifies assumption that RVs are Gaussian

#7: Understanding time-varying uncertain quantities

→ Stochastic processes

A stochastic or random process is a family of random vectors indexed by a parameter set ("time")

Usually assumed that random processes have a property called "stationarity"

↳ The statistics (i.e. pdf) on the RV are time-shift invariant.

$$\mathbb{E}[X_k] = \bar{x} \text{ for all } k$$

$$\mathbb{E}[X_{k_1} X_{k_2}^T] = R_X(k_1 - k_2)$$

↳ Autocorrelation is a function of time shift denoted with τ

$$R_X(k_1, k_2) = \mathbb{E}[X_{k_1} X_{k_2}^T]. \text{ If stationary,}$$

$$R_X(\tau) = \mathbb{E}[X_k X_{k+\tau}^T]$$

↳ Autocovariance :

$$C_X(k_1, k_2) = \mathbb{E}[(X_{k_1} - \mathbb{E}[X_{k_1}])(X_{k_2} - \mathbb{E}[X_{k_2}])^T]. \text{ If}$$

$$\text{stationary} \quad C_X(\tau) = \mathbb{E}[(X_k - \bar{x})(X_{k+\tau} - \bar{x})^T]$$

→ White noise

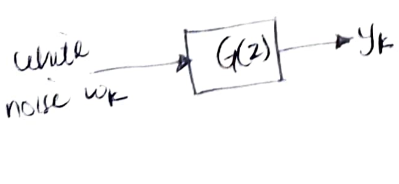
It is a random process and it has a unique autocorrelation function!

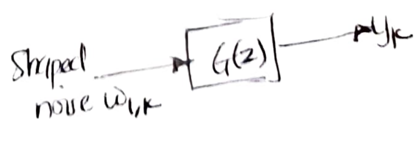
- The function itself has zero mean — spectral density
- $R_X(\tau) = \mathbb{E}[X_k X_{k+\tau}^T] = S_X \delta(\tau)$ where $\delta(\tau)$ is the Dirac delta

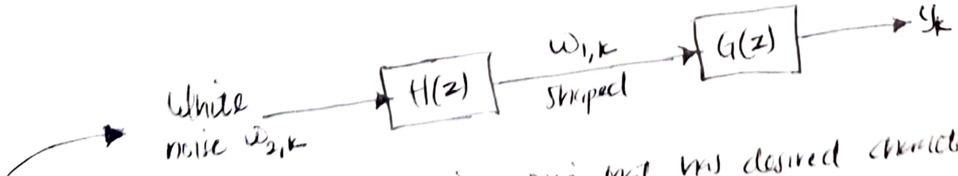
$$\delta(\tau) = 0 \ \forall \ \tau \neq 0 \quad \text{(white noise function)}$$

- The process is uncorrelated in time

• In dynamic systems, it is assumed that the noise inputs are white







Drive the linear system with noise that has desired characteristics by introducing shaping filter $H(z)$ that itself is driven by white noise

Summary

- Random process is a family of RVs indexed by time
- Autocorrelation and autocovariance measure self predictability of a signal at different time offsets
- White noise is zero mean signal, completely uncorrelated with self ("completely random")
  - ↪ white noise is an abstraction, but a very useful one

BMS 3, Week 1 Summary

- Limitation of simple voltage-based and current-based (Coulomb counting) methods for SOC estimation..
  - Better estimates with model-based ~~simulation~~ estimation