Course 2: Equivalent Circuit Cell Model Simulation

In the course, you will learn:

1. How to write equations that describe how a cell's voltage responds to a change in its input current
2. The laboratory tests needed to gather data required to find unknown parameter value in these equations
3. How to analyze data in the lab to compute value of these parameters
4. How to write code to simulate battery cells and packs in operation
5. How to simulate a representative battery load

After completing the course, you'll be able to:

1. State purpose for each component in an equivalent-circuit model
2. Compute approximate parameter values for a circuit model using data from a simple lab test on a physical cell.
3. Determine coulombic efficiency of a cell from lab data
4. Use provided Octave/MATLAB scripts to compute open-circuit-voltage and optimized values for dynamic parameters in model.
5. Simulate battery packs to understand and predict behaviors when there is cell-to-cell variation in parameter values.
6. Simulate an electric vehicle to yield estimates of range and to specify drivetrain components

Week 1: Defining an ECM of an Li-ion cell

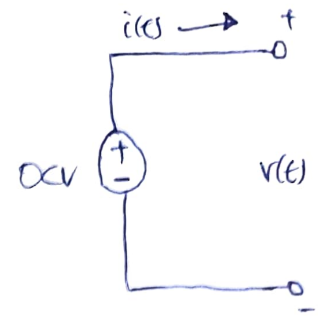
#2 Open Circuit Voltage and State of Charge

Models are set of equations that describe something physical. In this context, we are building a model of certain equations that describe the behavior of Lithium ion battery cells. The models are called "equivalent circuit" models. they have some desirable properties:

- Help give feeling for how cell respond to different usage & scenarios
- Basis for the BMS algorithms

* Open-Circuit Voltage

Battery operation is compared to an ideal voltage source. The cell voltage is considered constant



$$v(t) = OCV$$

- voltage is not a function of current
- voltage is not a function of past usage
- voltage is constant (no matter what!)

This model is inadequate, but provides a starting point of how a battery cell works

- Battery do supply a voltage to a load
- When the cell is unloaded and in complete equilibrium (i.e. open circuit), the voltage is fairly predictable
- An ideal voltage source will be part of ECM

* State of charge and total capacity

When a cell is fully charged, its open-circuit voltage is higher than when it is discharged. State of charge (SOC) $z(t)$ of cell:

- $z = 100\%$ when the cell is fully charged
- $z = 0\%$ when the cell is fully discharged

Total capacity Q is the total amount of charge removed when discharging from $z = 100\%$ to $z = 0\%$. Q is usually measured in Ah or mAh.

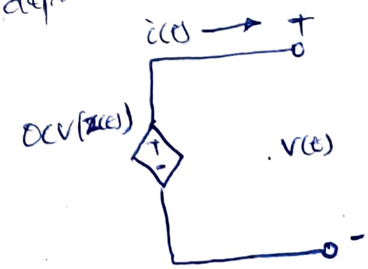
* Modeling State of Charge

$$\dot{z}(t) = \frac{-i(t)}{Q}$$

$$z(t) = z(t_0) - \frac{1}{Q}\int_{t_0}^{t} i(\tau)\,d\tau$$

} continuous time

dependent voltage source



where $\dot{z}(t) = dz/dt$; sign of $i(t)$ is +ve on discharge

$$v(t) = OCV(z(t))$$

- In discrete time, if we assume that current is constant over sampling interval Δt, in BMS, almost all algorithms operate in discrete time or sampled time

$$z[k+1] = z[k] \pm \frac{\Delta t}{Q} i[k]$$

$$v[k] = OCV(z[k])$$

$$z[k+1] = z[k] - \frac{\Delta t}{Q} i[k]$$

* Coulombic Efficiency

Not all the charge put into battery cell is used to raise the level of SOC. Hence there are some "charges loss"

$$\dot{z}(t) = \frac{-i(t)\,\eta(t)}{Q}$$

η = Coulombic efficiency
= 1 in ideal scenario

$$z[k+1] = z[k] - \frac{i[k]\,\eta[k]\,\Delta t}{Q}$$

- $\eta[k] \le 1$ on charge, as some charge is typically lost due to unwanted side reactions.
- $\eta[k] = 1$ on discharge

Coulombic (or charge) efficiency ≠ energy efficiency

- amount of charge that can be withdrawn from battery cell

$$= \frac{\text{Charge out}}{\text{Charge in}}$$; often around 99% in Li-ion

$$\text{Energy Efficiency} = \frac{\text{Energy out}}{\text{Energy in}}$$; often closer to 95%

Energy is lost in resistive heating but charge is not

* Open-Circuit Voltage

- It is the function of state of charge (SOC). OCV vs SOC depends on the battery cell chemistries.
- OCV is also function of temperature $OCV(z(t), T(t))$
- DOD is the inverse of SOC

$DOD = 1 - SOC$ if expressed as fraction

Sometimes is expressed in Ah: $DOD = Q(1 - SOC)$

* Summary

- Model has state-of-charge and voltage equations

$$\dot{z}(t) = \frac{-i(t)\,\eta(t)}{Q} \quad \text{or} \quad z[k+1] = z[k] - \frac{i[k]\,\eta[k]\,\Delta t}{Q}$$

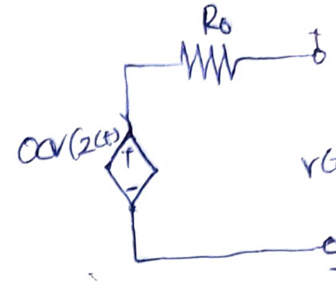
$$v(t) = OCV(z(t)) \qquad v[k] = OCV(z[k])$$

- Capacity Q is the amount of charge stored between z = 0% and z = 100%
- Coulombic efficiency $0 < \eta \le 1$, different from energy efficiency
- Open-circuit (OCV) is at-rest voltage of cell at different SOCs, will depend on chemistry of cell being modeled.

#3 How to model voltage polarization

Polarization is a term that refers to any difference between the measured terminal voltage of a cell and cell's internal open circuit voltage due to the passage of electrical current through that cell.

Cell's voltage drops when it is under load. This can be modelled as a "resistance in series" with the ideal voltage source (Rint model)



$$v(t) = OCV(z(t)) - i(t) R_0$$

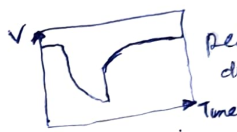
Rint ↳ internal resistance

$v(t) > OCV(z(t))$ on charge

$v(t) < OCV(z(t))$ on discharge

$v(t)$ – terminal voltage

Power loss through a resistor $= I^2R$. So having a non-zero resistance means there's power lost due to heat. Hence, energy efficiency of a battery is not perfect.



response to discharge pulse

* Diffusion voltage

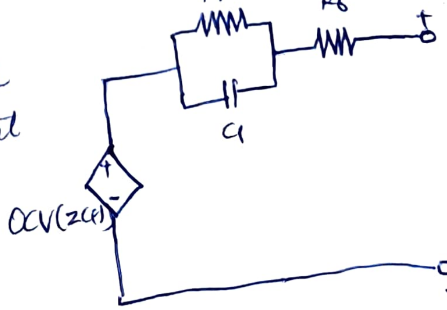
"Rint" model suffices for simple electronics designs for circuit that include battery cells but when building BMS for advanced consumer electronics application or xEV or grid storage application, it is not accurate enough.

- $i(t) \times R_0$ models the instantaneous voltage response to a change on the input current. In practice, we observe a non instantaneous responses to change in input current. Non instantaneous responses are **dynamic responses**

- When cell rest, voltage doesn't immediately return to OCV; it relaxes gradually. Slow relaxation of voltage is caused by slow internal diffusion of lithium from one part of the battery cell to another where concentration gradients build up while the cells was being discharged and have to slowly relax back to their equilibrium values.

* Thevenin Model cell voltage

To include diffusion voltages, add a parallel resistor capacitor branch in series with the current part of the battery cell



Cell voltage in "Thevenin model" is

$$v(t) = OCV(z(t)) - v_{C_1}(t) - i(t)R_0$$

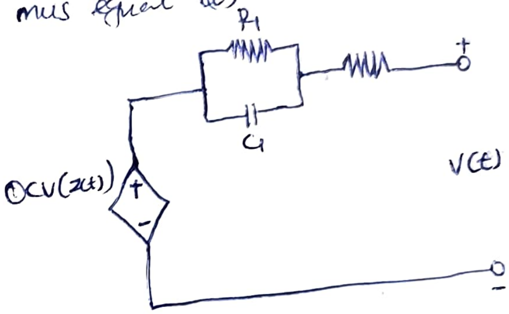
Process to identify parameter values from test data simpler if we write voltage in terms of element current instead:

$$v(t) = OCV(z(t)) - R_1 i_{R_1} - R_0 i(t)$$

R_0 carries the full cell load current; R_1 doesn't carry the full cell current at every point in time

* Thevenin Model Resistor R_1 current

To find an expression for $i_{R_1}(t)$, recognise that current through C_1 must equal $i(t)$ current through R_1 plus the current through C_1



$i_{C_1}(t) = C_1 \dot{v}_{C_1}(t)$ which gives:

$$i_{R_1}(t) + C_1 \dot{v}_{C_1}(t) = i(t)$$

capacitance

time derivative

Since $v_{C_1}(t) = R_1 i_{R_1}(t)$

$$i_{R_1}(t) + R_1 C_1 \frac{di_{R_1}(t)}{dt} = i(t)$$

total current

Re-arranging into standard ODE form

$$\frac{di_{R_1}(t)}{dt} = -\frac{1}{R_1 C_1} i_{R_1}(t) + \frac{1}{R_1 C_1} i(t)$$

* Summary

- "Rint" model adds equivalent-series resistance (ESR) term R_0. It describes only instantaneous changes to voltage when input current changed
- Thevenin model adds resistor-capacitor sub-circuit(s) to model diffusion processes in the lithium-ion battery
- Continuous-time model has two "state" equations and one "output" equation

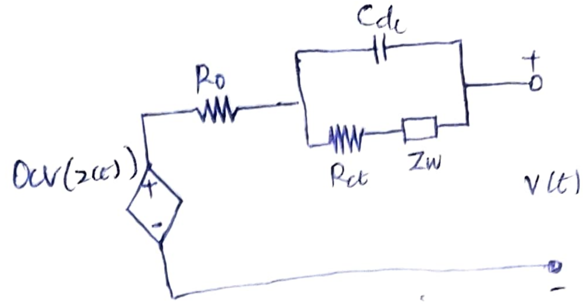
$$\frac{dz(t)}{dt} = -\frac{\eta(t)}{Q} i(t)$$

$$\frac{di_{R_1}(t)}{dt} = -\frac{1}{R_1 C_1} i_{R_1}(t) + \frac{1}{R_1 C_1} i(t)$$

$$v(t) = OCV(z(t)) - R_1 i_{R_1}(t) - R_0 i(t)$$

#4 Warburg Impedance

In the literature, ECMs is given containing a "Warburg impedance" element, Z_W



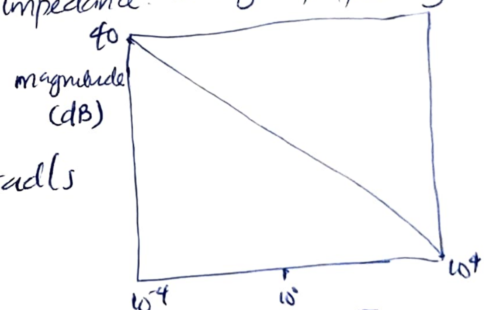
Randles circuit

- Randles circuit based on electrochemistry
  - R_0 models the electrolyte resistance
  - R_{ct} is charge-transfer resistance, models the voltage drop over the electrode-electrolyte interface due to a load.
  - C_{dl} is double-layer capacitance, models the effect of charges building up in the electrolytes at electrode surface
  - Z_W is a Warburg impedance, models slow diffusion processes instead of resistance-capacitance branch

Impedance is similar concept with resistance. Impedance describes complex resistance of an element. Resistance is some constant value. For example, capacitive has high impedance to DC. ~~but low impedance~~ Impedance is a funct of frequency. Inductor has low impedance to DC but high impedance to high frequency instead.

$$Z_W = \frac{A_W}{\sqrt{j\omega}}$$

ω = frequency rad/s

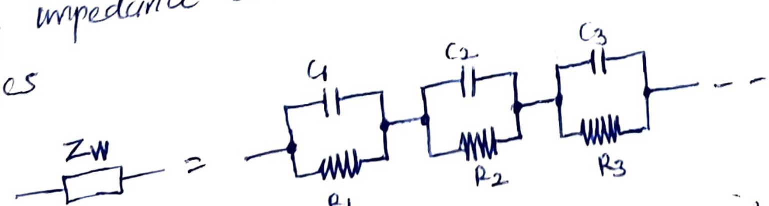


Magnitude response of Z_W

- No simple differential equation represents a Warburg impedance, which makes precise circuit simulation intractable

* Approximating a Warburg Impedance

Warburg impedance can be ~~used to~~ reproduced using multiple resistor-capacitor networks in series



For an exact equivalence, an infinite number of resistor-capacitor networks are needed. but, the circuit can often be modeled very well over some frequency range using a small number of R-C pairs.

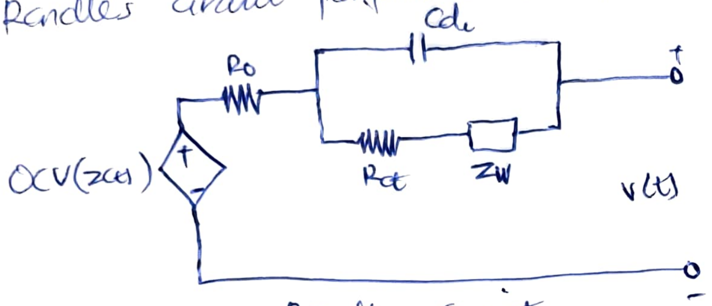
* Approximating with R-C pairs

R-C pairs can provide magnitude response segments with slope −20dB decade and 0dB decade

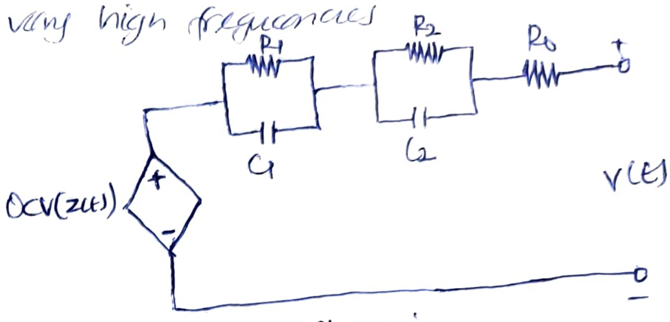
Phase response of capacitor −90° & phase response of resistor 0°

* Resulting model

Double-layer capacitance are often omitted because it has little impact on Randles circuit performance except at very high frequencies



Randle circuit



Thevenin

When C_{dl} removed from the circuit, R_0 and R_{ct} combined, and Warburg impedance replaced by a small finite number of R-C circuits, model collapses to Thevenin model with additional R-C pairs.

* Summary

- Randles circuit is an electrochemically inspired model
- It include warburg impedance which cannot be modeled perfectly with a finite-order differential equation
- But, it can be approximated well for frequencies of interest using a relatively small number of R-C pairs
- So, electrochemically inspired model reduces to Thevenin model, which gives us confidence that the Thevenin model is a reasonable description of cell dynamics.

#5 Converting Continuous-time model to a discrete-time model

* Converting to discrete (ODEs)

General formula:

$$\dot{x}(t) = ax(t) + bu(t)$$

into an equivalent discrete-time

$$x[k+1] = a_d x[k] + b_d u[k]$$

* How to:

i) step) Solve differential equation

$$\dot{x}(t) = ax(t) + bu(t)$$

$$x(t) = e^{at}x(0) + \underbrace{\int_0^t e^{a(t-\tau)} bu(\tau)\,d\tau}_{\text{convolution}}$$

$$\dot{x}(t) = ax(t) + bu(t)$$

$$\dot{x}(t) - ax(t) = bu(t)$$

multiply both sides of the equation by e^{-at}

$$e^{-at}[\dot{x}(t) - ax(t)] = \frac{d}{dt}[e^{-at}x(t)] = e^{-at}bu(t)$$

⟶ product rule

$-ae^{-at}x(t) + e^{-at}\dot{x}(t)$ which is the same as

$e^{-at}[\dot{x}(t) - ax(t)]$

$$\frac{d}{dt}[e^{-at}x(t)] = e^{-at}bu(t)$$

integral of both side

LHS $$\int_0^t \frac{d}{d\tau}[e^{-a\tau}x(\tau)]\,d\tau = e^{-at}x(t) - x(0)$$

RHS $$\int_0^t e^{-a\tau}bu(\tau)\,d\tau$$

Rearrange & multiply through by e^{at}

$$e^{-at}x(t) - x(0) = \int_0^t e^{-a\tau}bu(\tau)\,d\tau$$

$$x(t) = e^{at}x(0) + \int_0^t e^{a(t-\tau)}bu(\tau)\,d\tau$$

Step ii: Factor out $x[k]$

We wish to evaluate $x(t)$ at discrete time $x[k] \triangleq x(k\Delta t)$

$$x[k+1] = x((k+1)\Delta t)$$

Recall

$$x(t) = e^{at}x(0) + \int_0^t e^{a(t-\tau)} bu(\tau)\,d\tau$$

$$x[k+1] = e^{a(k+1)\Delta t} x(0) + \int_0^{(k+1)\Delta t} e^{a((k+1)\Delta t-\tau)} bu(\tau)\,d\tau$$

Break both the exponential and integral into two pieces so; $x[k+1]$ is

$$= e^{a\Delta t}e^{ak\Delta t}x(0) + \int_0^{k\Delta t} e^{a((k+1)\Delta t-\tau)} bu(\tau)\,d\tau + \int_{k\Delta t}^{(k+1)\Delta t} e^{a((k+1)\Delta t-\tau)} bu(\tau)\,d\tau$$

$$= e^{a\Delta t}e^{ak\Delta t}x(0) + \int_0^{k\Delta t} e^{a\Delta t}e^{a(k\Delta t-\tau)} bu(\tau)\,d\tau + \int_{k\Delta t}^{(k+1)\Delta t} e^{a((k+1)\Delta t-\tau)} bu(\tau)\,d\tau$$

$$= e^{a\Delta t}\left[e^{ak\Delta t}x(0) + \int_0^{k\Delta t} e^{a(k\Delta t-\tau)} bu(\tau)\,d\tau\right] + \int_{k\Delta t}^{(k+1)\Delta t} e^{a((k+1)\Delta t-\tau)} bu(\tau)\,d\tau$$

$$= e^{a\Delta t}x[k] + \int_{k\Delta t}^{(k+1)\Delta t} e^{a((k+1)\Delta t-\tau)} bu(\tau)\,d\tau$$

where $x[k] = e^{ak\Delta t}x(0) + \int_0^{k\Delta t} e^{a(k\Delta t-\tau)} bu(\tau)\,d\tau$

Step iii: For $a \neq 0$

$$x[k+1] = e^{a\Delta t}x[k] + \int_{k\Delta t}^{(k+1)\Delta t} e^{a((k+1)\Delta t-\tau)} bu(\tau)\,d\tau$$

Assume $u(\tau)$ is constant from $k\Delta t$ to $(k+1)\Delta t$ and equal to $u(k\Delta t)$

$$x[k+1] = e^{a\Delta t}x[k] + e^{a(k+1)\Delta t}\left(\int_{k\Delta t}^{(k+1)\Delta t} e^{-a\tau}\,d\tau\right) bu[k]$$

$$= e^{a\Delta t}x[k] + e^{a(k+1)\Delta t}\left(-\frac{1}{a}e^{-a\tau}\Big|_{k\Delta t}^{(k+1)\Delta t}\right) bu[k]$$

$$= e^{a\Delta t}x[k] + \frac{1}{a}e^{a(k+1)\Delta t}\left(e^{-ak\Delta t} - e^{-a(k+1)\Delta t}\right) bu[k]$$

$$x[k+1] = e^{a\Delta t}x[k] + \frac{1}{a}\left(e^{a\Delta t} - 1\right) bu[k]$$

Compare with

$$x[k+1] = a_d x[k] + b_d u[k]$$

$$a_d = e^{a\Delta t}$$
$$b_d = \frac{1}{a}\left(e^{a\Delta t} - 1\right)b$$

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* Application to the R-C equation

- So, we can convert $\dot{x}(t) = ax(t) + bu(t)$ into

$$x[k+1] = e^{a\Delta t} x[k] + \frac{1}{a}(e^{a\Delta t} - 1) bu[k]$$

- To use this result for the ODE describing the R-C circuit ($\tau_1 = R_1 C_1$)

$$\frac{di_{R_1}(t)}{dt} = \left(\frac{-1}{\tau_1}\right) i_{R_1}(t) + \left(\frac{1}{\tau_1}\right) i(t)$$

$$a = -1/\tau_1 \quad b = 1/\tau_1, \quad x[k] = i_{R_1}[k] \text{ and } u[k] = i[k].$$

- Substituting these values into the generic result, we get

$$i_{R_1}[k+1] = \exp\left(-\frac{\Delta t}{\tau_1}\right) i_R[k] + (-\tau_1)\left(\exp\left(\frac{-\Delta t}{\tau_1}\right) - 1\right)\left(\frac{1}{\tau_1}\right) i[k]$$

$$= \exp\left(-\frac{\Delta t}{\tau_1}\right) i_{R_1}[k] + \left(1 - \exp\left(\frac{-\Delta t}{\tau_1}\right)\right) i[k]$$

Step iv: For $a = 0$ and the SOC equation

Recall: $$x[k+1] = e^{a\Delta t} x[k] + \int_{k\Delta t}^{(k+1)\Delta t} e^{a((k+1)\Delta t - \tau)} bu(\tau)\, d\tau$$

If $a = 0$ and $u(\tau)$ is constant from $k\Delta t$ to $(k+1)\Delta t$ and equal to $u(k\Delta t)$

$$x[k+1] = x[k] + \left(\int_{k\Delta t}^{(k+1)\Delta t} 1\, d\tau\right) bu[k] = x[k] + (\Delta t)\, bu[k]$$

$$x[k+1] = x[k] + (\Delta t)\, bu[k]$$

To use this result for the ODE describing SOC, $\dot{z}(t) = \left(-\frac{\eta(t)}{Q}\right) i(t)$, we have

$a = 0$, $b = -\eta[k]/Q$, $x[k] = z[k]$ and $u[k] = i[k]$

Hence

$$z[k+1] = z[k] - \frac{\eta[k]\Delta t}{Q} i[k]$$

* Discrete-time model

$$\frac{dz(t)}{dt} = -\frac{\eta(t)}{Q} i(t)$$

$$\frac{di_{R_1}(t)}{dt} = -\frac{1}{R_1C_1} i_{R_1}(t) + \frac{1}{R_1C_1} i(t)$$

$$v(t) = OCV(z(t)) - R_1 i_{R_1}(t) - R_0 i(t)$$

$\longrightarrow$

$$z[k+1] = z[k] - \frac{\eta[k]\Delta t}{Q} i[k]$$

$$i_{R_1}[k+1] = \exp\left(-\frac{\Delta t}{R_1C_1}\right) i_{R_1}[k] + \left(1 - \exp\left(-\frac{\Delta t}{R_1C_1}\right)\right) i[k]$$

$$v[k] = OCV(z[k]) - R_1 i_{R_1}[k] - R_0 i[k]$$

* Summary

- Generically (except when $a=0$)

$$\dot{x}(t) = ax(t) + bu(t) \longrightarrow x[k+1] = e^{a\Delta t} x[k] + \frac{1}{a}\left(e^{a\Delta t} - 1\right) b u[k]$$

- In the special case when $a=0$

$$\dot{x}(t) = ax(t) + bu(t) \longrightarrow x[k+1] = x[k] + (b\Delta t) u[k]$$

#6 Approximate Model Parameter values

* Ball parking SOC parameters

$$z[k+1] = z[k] - \frac{\eta[k]\Delta t}{Q} i[k]$$

$$i_{R_1}[k+1] = \exp\left(-\frac{\Delta t}{R_1C_1}\right) i_{R_1}[k] + \left(1 - \exp\left(-\frac{\Delta t}{R_1C_1}\right)\right) i[k]$$

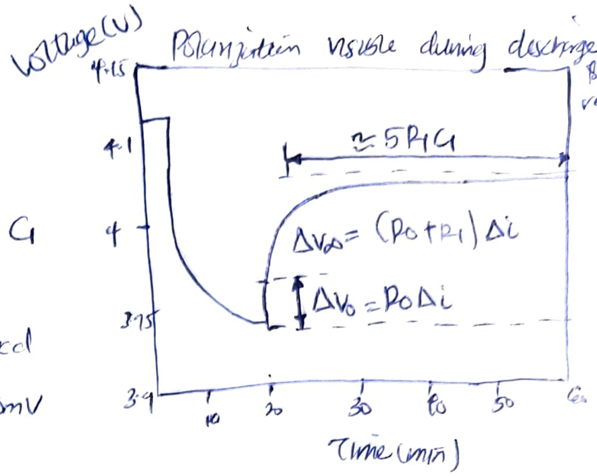
$$v[k] = OCV(z[k]) - R_1 i_{R_1}[k] - R_0 i[k]$$

- Can compute Q by discharging cell slowly from 100% to 0% SOC and recording ampere hours removed.
- Can find η by charging from 0% to 100% SOC, computing discharge capacity divided by charge capacity
- Can average dis/charge voltages at every SOC to find $OCV(z[k])$

* Ballparking R_0 parameter

Conduct a cell running test on a physical battery cell. Once R_0 is known, we can find the approximate of R_1 from the same data test.

$|\Delta v_\infty| = (R_0+R_1)|\Delta i|$, from which we can deduce R_1



* Backpacking C_1 parameter

consider the duration of time required for the voltage to decay into a steady-state to find C_1

Example: for cell test conducted to gather plotted data $|\Delta i| = 5A$, $|\Delta v_0| = 41mV$, and $|\Delta v_\infty| = 120mV$

$R_0 \approx 8.2m\Omega$ $\qquad \frac{41}{5} = 8.2m\Omega$

$R_1 = \frac{120-41}{5} = 15.8m\Omega$

ii) Time to steady state is
$60 - 20min = 40min$
$= 2400s$

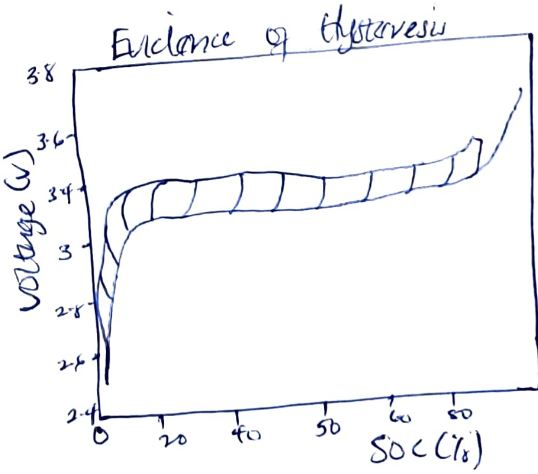
so! $5R_1C_1 \approx 2400s$

$C_1 = 480/R_1 \approx \frac{480}{15.8} \approx 30kF$

#7 Hysteresis Voltage

If the battery cell rest long enough, the diffusion of voltages decay to zero so the model voltage decays to OCV. In a real cell, this doesn't happen. Instead, for every SOC, we find a range of possible stable OCV values.

- Hysteresis (pronounce History)

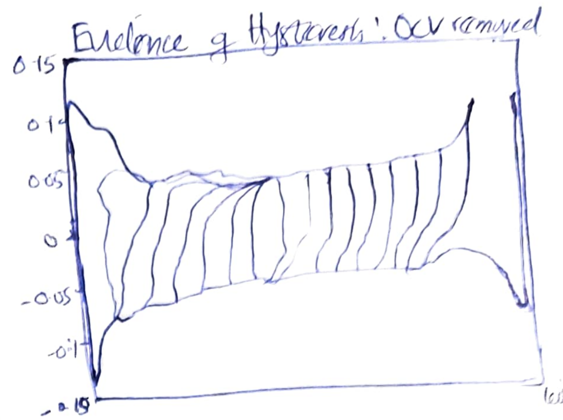


Diffusion voltage Vs Hysteresis
↳ will always cause the cell voltage to decay to some constant value overtime if the cell is allowed to rest.

- Diffusion voltage changes simply as a function of time.
- Hysteresis voltage doesn't change over time when the cell is at rest. It changes only as a function of state-of-charge

N.B: Diffusion voltages change directly with time but hysteresis voltages change when SOC changes.

* Examining Nature of Hysteresis



Appears there is a maximum plus/minus hysteresis may be SOC dependent: $M(z)$

Amount is possitive if cell is presently charging; otherwise negative: $M(z, \dot{z})$

* Simple differential equation in z

$$\frac{dh(z,t)}{dz} = \gamma \operatorname{sgn}(\dot{z})\left(M(z,\dot{z}) - h(z,t)\right)$$

actual amount of hysteresis

Max hysteresis $M(z,\dot{z})$ positive for charge ($\dot{z} > 0$), negative for discharge ($\dot{z} < 0$)

- $M(z,\dot{z}) - h(z,t)$ term causes hysteresis rate-of-change to be proportional to the distance away from major hysteresis loop
- Positive γ tunes rate of decay, and $\operatorname{sgn}(\dot{z})$ forces stability for both dis/charge.

* Simple differential equation in t

To fit differential equation for $h(z,t)$ into cell model, we must manipulate it to be a differential equation in time, not in SOC

To achieve this, multiple both side of hysteresis equation by dz/dt

$$\frac{dh(z,t)}{dz} \cdot \frac{dz}{dt} = \gamma \operatorname{sgn}(\dot{z})\left(M(z,\dot{z}) - h(z,t)\right)\frac{dz}{dt}$$

NB: $\dot{z}\operatorname{sgn}(\dot{z}) = |\dot{z}|$

LHS becomes $\dot{h}(t)$; on RHS, $\dot{z}(t) = \dfrac{-\eta(t)i(t)}{Q}$

$$\dot{h}(t) = -\left|\frac{\eta(t)i(t)\gamma}{Q}\right|h(t) + \left|\frac{\eta(t)i(t)\gamma}{Q}\right|M(z,\dot{z})$$

* Convert to discrete time

Convert to discrete time using method from (#5), assuming $i(t)$ and $M(z,\dot{z})$ are constant over sample period.

$$h[k+1] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)h[k] + \left(1 - \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)\right)M(z,\dot{z})$$

- Linear-time-varying system as factors multiplying state and input change with $i[k]$

Simplest form of when $M(z,\dot{z}) = -M \,\mathrm{sgn}(i[k])$, when

$$h[k+1] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)h[k] - \left(1-\exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)\right) M\,\mathrm{sgn}(i[k]).$$

Hysteresis values is always between $-M \le h[k] \le M$ at all times, and $h[k]$ has units of volts.

* Unitless hysteresis state

When optimizing model parameter values, it helps to rewrite in equivalent but slightly different representation, which has unitless hysteresis state $-1 \le h[k] \le 1$

$$h[k+1] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)h[k] - \left(1-\exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)\right)\mathrm{sgn}(i[k])$$

$$v_h[k] = M h[k]$$

M has been moved from the input part of the equation to output equation. M appears linearly in output equation, makes estimating M from lab-test data easier.

* Instantaneous Hysteresis

Dynamic hysteresis changes as SOC changes but instantaneous hysteresis changes when sign of $i[k]$ changes

$$s[k] = \begin{cases} \mathrm{sgn}(i[k]), & |i[k]| > 0; \\ s[k-1], & \text{otherwise}. \end{cases}$$

Instantaneous hysteresis is modeled as

$$h_i[k] = M_0 s[k]$$

and overall hysteresis is

$$v_h[k] = M_0 s[k] + M h[k]$$

* Summary

- Hysteresis is a path-dependent voltage that does not decay to zero when the cell rests, unlike diffusion voltage
- Laboratory evidence indicates presence of both instantaneous and dynamic elements to hysteresis
- Simple model of hysteresis; though not fantastic but better than no model at all even if constant parameters γ, M, and M_0 are used.
- Hysteresis model can be improved if SOC-dependent values are used.

#8 Summarizing an Equivalent Circuit Model of a Li-ion cell

We can now summarize a cell model that combines

i SOC-dependent open circuit voltage

ii Ohmic resistance and diffusion voltages

iii Hysteresis

The combined model of all above equations is known as **Enhanced Self Correcting (ESC)** cell model.

- Enhanced: Model includes (poor) description of hysteresis (better than none)
- Self-correcting: Transient behavior model may be imperfect, but steady-state is correct
  - Voltage converges to OCV + hysteresis on rest
  - Converges to OCV + hysteresis $- i\sum R$ on constant-current event

*** Using Multiple R-C pairs**

Enhanced self-correcting model can contain more than a single parallel R-C pair. It can define vector valued

$$\begin{bmatrix} i_{R_1}[k+1] \\ i_{R_2}[k+1] \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} \exp\left(\frac{-\Delta t}{R_1C_1}\right) & 0 & \cdots \\ 0 & \exp\left(\frac{-\Delta t}{R_2C_2}\right) & \\ \vdots & & \ddots \end{bmatrix}}_{A_{RC}} \begin{bmatrix} i_{R_1}[k] \\ i_{R_2}[k] \\ \vdots \end{bmatrix} + \underbrace{\begin{bmatrix} \left(1-\exp\left(\frac{-\Delta t}{R_1C_1}\right)\right) \\ \left(1-\exp\left(\frac{-\Delta t}{R_2C_2}\right)\right) \\ \vdots \end{bmatrix}}_{B_{RC}} i[k]$$

$$i_R[k+1] = A_{RC}\, i_R[k] + B_{RC}\, i[k]$$

*** State and output equations of ESC model**

If we define $A_H[k] = \exp\left(-\left|\frac{\eta[k]\, i[k]\, \gamma\, \Delta t}{Q}\right|\right)$, then

$$\underbrace{\begin{bmatrix} z[k+1] \\ i_R[k+1] \\ h[k+1] \end{bmatrix}}_{x[k+1]} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & A_{RC} & 0 \\ 0 & 0 & A_H[k] \end{bmatrix}}_{A[k]} \underbrace{\begin{bmatrix} z[k] \\ i_R[k] \\ h[k] \end{bmatrix}}_{x[k]} + \underbrace{\begin{bmatrix} -\eta[k]\Delta t/Q & 0 \\ B_{RC} & 0 \\ 0 & A_H[k]-1 \end{bmatrix}}_{B[k]} \underbrace{\begin{bmatrix} i[k] \\ \mathrm{sgn}(i[k]) \end{bmatrix}}_{u[k]}$$

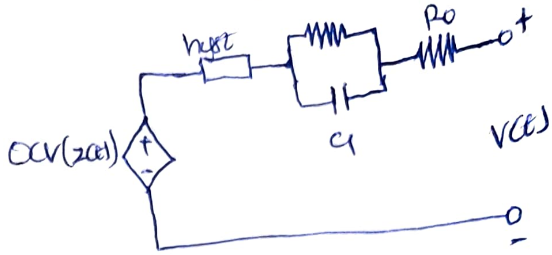
- ESC "state equation" $x[k+1] = A[k]x[k] + B[k]u[k]$ describes all dynamic effects
- ESC "output equation" computes voltage

$$v[k] = OCV(z[k], T[k]) + M_0 s[k] + M h[k] - \sum_j R_j i_{R_j}[k] - R_0 i[k]$$

- Can be write as $v[k] = f(x[k], u[k])$ if we augment state $x[k]$ with $s[k]$

N.B: All model parameter values must be non-negative

Model can be visualized as an equivalent circuit, where mathematically it comprises two coupled equations:



$$x[k+1] = A[k]x[k] + B[k]u[k]$$

$$v[k] = f(x[k], u[k])$$