

Examination	:	End-Semester Examination – May/June 2025
Name of the Course	:	B.Tech. (IT & Mathematical Innovations)
Name of the Paper	:	Applied Probability and Statistics
Unique Paper Code	:	3122612401
Semester	:	IV
Duration	:	3 hours
Maximum Marks	:	90

Instruction to students:

1. Read the question paper carefully. This question paper contains two sections, attempt specified number of questions from each section.
 2. Use of scientific calculator is allowed in the exam. Use provided Z-tables, t-tables and χ^2 -tables for calculations.
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Section-A

Attempt any TEN questions from this section. Each carries equal 6 marks. (6 × 10 = 60)

1. Given a random permutation of the integers in the set {1, 2, 3, 4, 5}, let X equal the number of integers that are in their natural position. The moment-generating function of X is

$$M(t) = \frac{44}{120} + \frac{45}{120}e^t + \frac{20}{120}e^{2t} + \frac{10}{120}e^{3t} + \frac{1}{120}e^{5t} \quad (3 \times 2 = 6)$$

A. Find the mean and variance of X.

B. Find the probability that at least one integer is in its natural position.

2. A bakery sells rolls in units of a dozen. The demand X (in 1000 units) for rolls has a gamma distribution with parameters $\alpha = 3$, $\theta = 0.5$ where θ is in units of days per 1000 units of rolls. It costs \$2 to make a unit that sells for \$5 on the first day when the rolls are fresh. Any leftover units are sold on the second day for \$1. How many units should be made to maximise the expected value of the profit?

3. The probability that a company's work force has no accidents in a given month is 0.7. The numbers of accidents from month to month are independent. What is the probability that the third month in a year is the first month that at least one accident occurs?

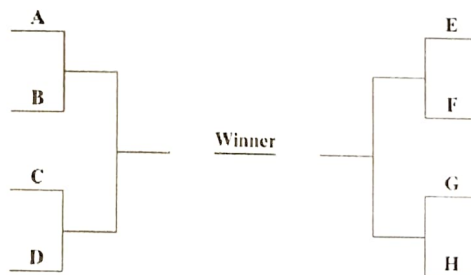
4. The "fill" problem is important in many industries, such as those making cereal, toothpaste, beer, and so on. If an industry claims that it is selling 12 ounces of its product in a container, it must have a mean greater than 12 ounces, or else the FDA will crack down, although the FDA will allow a very small percentage of the containers to have less than 12 ounces. (3 × 2 = 6)

A. If the content X of a container has a $N(12.1, \sigma^2)$ distribution, find σ so that $P(X < 12) = 0.01$.

B. If $\sigma = 0.05$, find μ so that $P(X < 12) = 0.01$.

5. A particle starts at (0, 0) and moves in one-unit independent steps with equal probabilities of 1/4 in each of the four directions: north, south, east, and west. Let S equal the east-west position and T the north-south position after n steps. (3 × 2 = 6)

- A. Define the joint pmf of S and T with $n = 3$. On a two-dimensional graph, give the probabilities of the joint pmf and the marginal pmfs.
- B. What are the marginal distributions of X and Y?
6. The heights of a population of women are normally distributed with mean μ cm and standard deviation σ cm. It is known that 30% of the women are taller than 172 cm and 5% are shorter than 154 cm. (3 \times 2 = 6)
- A. Sketch a diagram to show the distribution of heights represented by this information.
- B. Show that $\mu = 154 + 1.645\sigma$
7. Let X equal the maximal oxygen intake of a human on a treadmill, where the measurements are in milliliters of oxygen per minute per kilogram of weight. Assume that, for a particular population, the mean of X is $\mu = 54.030$ and the standard deviation is $\sigma = 5.8$. Let \bar{X} be the sample mean of a random sample of size $n = 47$. Find $P(52.761 \leq \bar{X} \leq 54.453)$, approximately.
8. Let $f(x; \theta) = \frac{1}{\theta} x^{\frac{(1-\theta)}{\theta}}$, $1 < x < \infty$, $0 < \theta < 1$. (3 \times 2 = 6)
- A. Show that the maximum likelihood estimator of θ is $\hat{\theta} = - (1/n) \sum_{i=1}^n \ln X_i$
- B. Show that $E(\hat{\theta}) = \theta$ and thus is an unbiased estimator of θ .
9. Let T_1 and T_2 be random times for a company to complete two steps in a certain process. Say T_1 and T_2 are measured in days and they have the joint pdf that is uniform over the space $1 < t_1 < 10$, $2 < t_2 < 6$, $t_1 + 2t_2 < 14$. What is $P(T_1 + T_2 > 10)$?
10. An eight-team single-elimination tournament is set up as follows:



For example, eight students (called A–H) set up a tournament among themselves. The top listed student in each bracket calls heads or tails when his or her opponent flips a coin. If the call is correct, the student moves on to the next bracket. (3 \times 2 = 6)

- A. How many coin flips are required to determine the tournament winner?
- B. What is the probability that you can predict all of the winners?

11. A fast food restaurant has one drive in window. It is estimated that cars arrive according to a Poisson distribution at the rate of 2 every 5 minutes and that there is enough space to accommodate a

line of 10 cars. Other arriving cars can wait outside this space ,if necessary. It takes 15 minutes on the average to fill an order, but the service time actually varies according to an exponential distribution . Determine the following; (3 × 2 = 6)

- A. The probability that the facility is idle.
- B. The expected number of customers waiting to be served.

SETION-B

Attempt any THREE questions from this section. Each carries equal 10 marks. (10 × 3 = 30)

1. An automotive supplier is considering changing its electrical wire harness to save money. The idea is to replace a current 20-gauge wire with a 22-gauge wire. Since not all wires in the harness can be changed, the new wire must work with the current wire splice process. To determine whether the new wire is compatible, random samples were selected and measured with a pull test. A pull test measures the force required to pull the spliced wires apart. The minimum pull force required by the customer is 20 pounds. Twenty observations of the forces needed for the current wire are

28.8	24.4	30.1	25.6	26.4	23.9	22.1	22.5	27.6	28.1
20.8	27.7	24.4	25.1	24.6	26.3	28.2	22.2	26.3	24.4

Twenty observations of the forces needed for the new wire are

14.1	12.2	14.0	14.6	8.5	12.6	13.7	14.8	14.1	13.2
12.1	11.4	10.1	14.2	13.6	13.1	11.9	14.8	11.1	13.5

- A. Does the current wire meet the customer’s specifications? (3)
- B. Find a 90% confidence interval for the difference of the means for these two sets of wire. (4)
- C. What is your recommendation for this company? (3)

2. Some nurses in county public health conducted a survey of women who had received inadequate prenatal care. They used information from birth certificates to select mothers for the survey. The mothers selected were divided into two groups: 14 mothers who said they had five or fewer prenatal visits and 14 mothers who said they had six or more prenatal visits. Let X and Y equal the respective birth weights of the babies from these two sets of mothers, and assume that the distribution of X is $N(\mu_X, \sigma^2)$ and the distribution of Y is $N(\mu_Y, \sigma^2)$.

- A. Define the test statistic and critical region for testing $H_0 : \mu_X - \mu_Y = 0$ against $H_1 : \mu_X - \mu_Y < 0$. Let $\alpha = 0.05$. (3)
- B. Given that the observations of X were

48	108	110	82	93	114	134
114	96	52	101	114	120	116

and the observations of Y were

133	108	93	119	119	98	106
131	87	153	116	129	97	110

calculate the value of the test statistic and state your conclusion. (4)

C. Approximate the p-value. (3)

3. The joint pmf of X and Y is $f(x, y) = 1/6$, $0 \leq x + y \leq 2$, where x and y are nonnegative integers.

(2 × 5 = 10)

A. Sketch the support of X and Y.

B. Record the marginal pmfs $f_X(x)$ and $f_Y(y)$ in the “margins.”

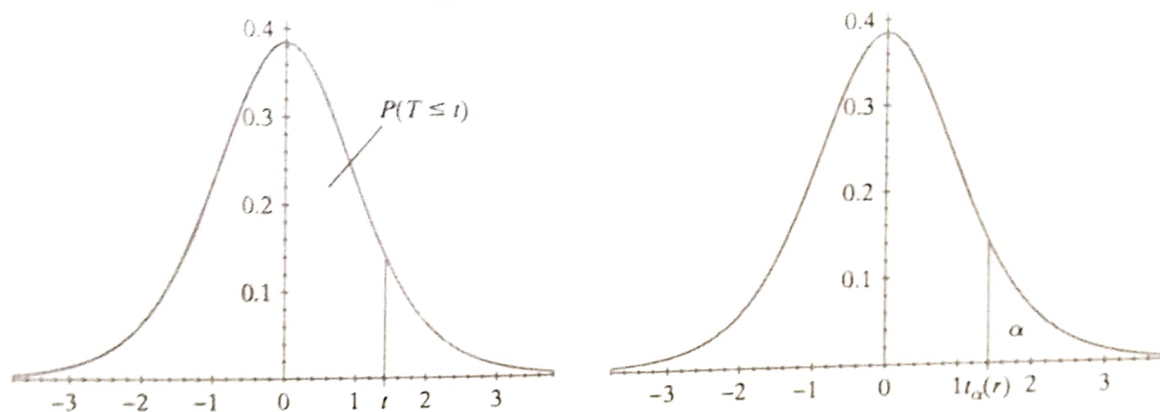
C. Compute $\text{Cov}(X, Y)$.

D. Determine ρ , the correlation coefficient.

E. Find the best-fitting line and draw it on your figure.

4. A. Find the moment-generating function for the gamma distribution with parameters α and θ . (5)

B. Find moment generating function(M.G.F) for Negative Binomial Distribution. Derive mean and variance for ‘Negative Binomial distribution’ using its M.G.F. (5)

Table VI The t Distribution

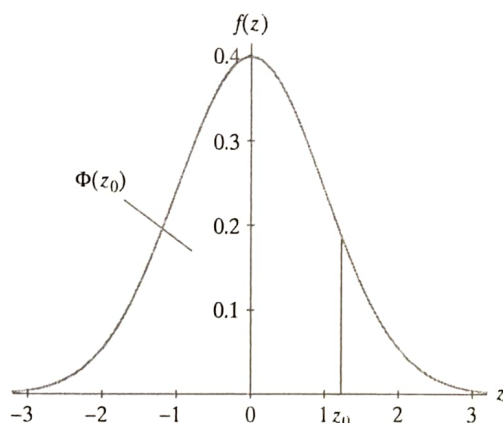
$$P(T \leq t) = \int_{-\infty}^t \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r} \Gamma(r/2) (1 + w^2/r)^{(r+1)/2}} dw$$

$$P(T \leq -t) = 1 - P(T \leq t)$$

	$P(T \leq t)$						
	0.60	0.75	0.90	0.95	0.975	0.99	0.995
r	$t_{0.40}(r)$	$t_{0.25}(r)$	$t_{0.10}(r)$	$t_{0.05}(r)$	$t_{0.025}(r)$	$t_{0.01}(r)$	$t_{0.005}(r)$
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012
14	0.258	0.692	1.345	1.761	2.145	2.624	2.997
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576

This table is taken from Table III of Fisher and Yates, *Statistical Tables for Biological, Agricultural, and Medical Research*, published by Longman Group Ltd., London (previously published by Oliver and Boyd, Edinburgh).

Table Va The Standard Normal Distribution Function

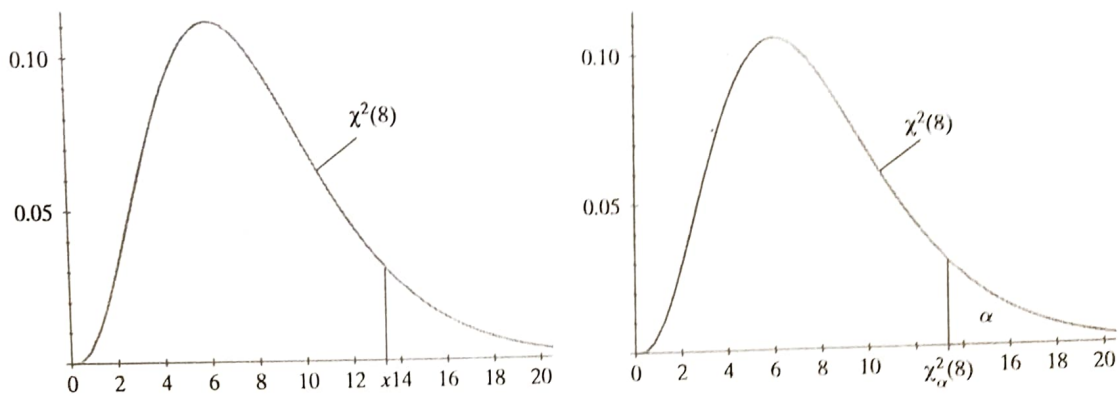


$$P(Z \leq z) = \Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-w^2/2} dw$$

$$\Phi(-z) = 1 - \Phi(z)$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
α	0.400	0.300	0.200	0.100	0.050	0.025	0.020	0.010	0.005	0.001
z _α	0.253	0.524	0.842	1.282	1.645	1.960	2.054	2.326	2.576	3.090
z _{α/2}	0.842	1.036	1.282	1.645	1.960	2.240	2.326	2.576	2.807	3.291

Table IV The Chi-Square Distribution



$$P(X \leq x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2} dw$$

	$P(X \leq x)$							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
r	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	13.36	15.51	17.54	20.09
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58
16	5.812	6.908	7.962	9.312	23.54	26.30	28.84	32.00
17	6.408	7.564	8.672	10.08	24.77	27.59	30.19	33.41
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.80
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	35.56	38.88	41.92	45.64
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89
40	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
60	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38
70	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4
80	53.34	57.15	60.39	64.28	96.58	101.9	106.6	112.3

This table is abridged and adapted from Table III in *Biometrika Tables for Statisticians*, edited by E.S.Pearson and H.O.Hartley.