[This question paper contains 8 printed pages.]

Your Roll No.5004

Sr. No. of Question Paper : 1908 **G**

Unique Paper Code : 3122611101

Name of the Paper : Single and Multivariable Calculus

Name of the Course : **B. Tech. (IT and Mathematical Innovations)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Five** questions.
3. **All** questions carry equal marks.

1. (a) Sketch a graph of the function $f(x)=\dfrac{2x^2}{x^2-1}$ using the following steps. (12)

P.T.O.

(i) Identify where the extrema of f occur.

(ii) Find the intervals on which f is increasing and the intervals on which f is decreasing.

(iii) Find where the graph of f is concave up and where it is concave down.

(iv) Sketch the general shape of the graph for f.

(v) Plot some specific points, such as extrema points, point of inflection and intercepts. Then sketch the curve.

(b) If resistors of R_1, R_2 and R_3 ohms are connected in parallel to make an R -ohm resistor, the value of R can be found from the equation $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1=30$, $R_2=45$ and $R_3=90$ ohms. (6)

2. (a) Determine a rational function f (in lowest terms) whose graph has vertical asymptotes at x=–5 and x=2 and a horizontal asymptote at y=2 and for which f(1) = 3. (6)

(b) The figure 1 shows the velocity $v = \frac{ds}{dt} = f(t)$ (m/sec) of a body moving along a coordinate line. (12)

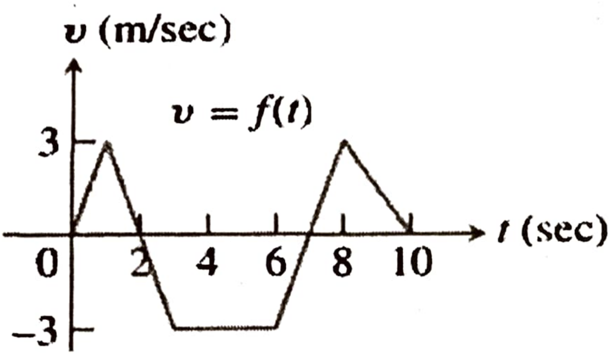

Figure 1

(i) When does the body reverse direction?

(ii) When (approximately) is the body moving at a constant speed?

(iii) Graph the body's speed for $0 \le t \le 10$.

(iv) Graph the acceleration, where defined.

Give reasons for your answers.

3. (a) Find the values of constants a, b and c so that the graph of $y = ax^3 + bx^2 + cx$ has a local maximum at $x = 3$, local minimum at $x = -1$ and inflection point at (1, 11). (6)

(b) Find the Taylor series generated by

$$f(x)=\frac{1}{3}(2x+x\cos(x)) \text{ at } x=0.$$ (6)

(c) Examine the convergence or divergence of the series (6)

$$\sum_{n=1}^{\infty}\frac{1\cdot 3\cdot \ldots \cdot(2n-1)}{4^n 2^n n!}$$

4. (a) Graph the function

$$f(x)=\begin{cases}\sqrt{1-x^2}, & \text{for } 0\le x<1\\ 1, & \text{for } 1\le x<2\\ 2, & \text{for } x=2\end{cases}.$$

Then answer the following questions. (8)

(i) What are the domain and range of f?

(ii) At what points c, if any, does $\lim_{x \to c} f(x)$ exist?

(iii) At what points does only the left-hand limit exist?

(iv) At what points does only the right-hand limit exist?

(b) Does the sequence whose n^{th} term is $\dfrac{3^n \cdot 6^n}{n! \cdot 2^{-n}}$ converge? If so, find the limit of the sequence. (5)

(c) Let $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ for $(x,y) \neq (0,0)$. Is it possible to define $f(0,0)$ in a way that makes f continuous at the origin? Why? (5)

5. (a) Let $T = f(x,y)$ be the temperature at the point (x,y) on the circle $x = \cos(t)$, $y = \sin(t)$, $0 \le t \le 2\pi$ and suppose that $\frac{\partial T}{\partial x} = 8x - 4y$, $\frac{\partial T}{\partial y} = 8y - 4x$. Find where the maximum and minimum temperatures on the circle occur by examining the derivatives $\frac{dT}{dt}$ and $\frac{d^2T}{dt^2}$. (10)

(b) Find the area of the propeller-shaped region enclosed by the curve $x - y^3 = 0$ and the line $x - y = 0$. (8)

6. (a) Find parametric equations and a parameter interval for the motion of a particle starting at the point (2, 0) and tracing the top half of the circle $x^2 + y^2 = 4$ four times. (6)

(b) A baseball is hit when it is 3 ft above the ground. It leaves the bat with initial speed of 152 ft/sec, making an angle of 20° with the horizontal. At the instant the ball is hit, an instantaneous gust of wind blows in the horizontal direction directly opposite the direction the ball is taking toward the outfield, adding a component of $-8.8\mathbf{i}$ (ft/sec) to the ball's initial velocity (8.8 ft/sec = 6 mph). Find a vector equation (position vector) for the path of the baseball. How high does the baseball go, and when does it reach maximum height? Assuming that the ball is not caught, find its range and flight time. (12)

7. (a) A delivery company accepts only rectangular boxes the sum of whose length and girth (perimeter of a cross-section) does not exceed 108 in. Find the dimensions of an acceptable box of largest volume. (8)

(b) If $f(x,y)=\dfrac{10000e^{y}}{\left(1+\frac{|x|}{2}\right)}$ represents the "population density" of a certain bacterium on the xy-plane, where x and y are measured in centimeters, find the total population of bacteria within the rectangle $-5 \le x \le 5$ and $-2 \le y \le 0$. (5)

(c) Find a curve with a positive derivative through the point (1,1) whose the length integral is $\int_1^4 \left(1+\frac{1}{4x}\right)^{\frac{1}{2}} dx$. How many such curves are there? Give reasons your answer. (5)