Unique Paper Code : 3122611201

Name of the Paper : Engineering Through Linear Algebra

Name of the Course : B.Tech (Information Technology & Mathematical Innovations

Semester : II

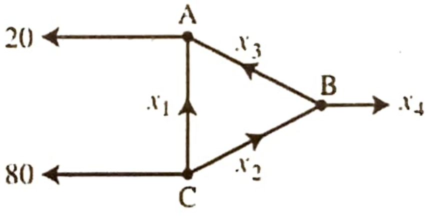
Duration : 3 hours

Maximum Marks : 90 Marks

Instructions for Candidates

1. Attempt any **SIX** questions.
2. All questions carry equal marks.

Ques. 1 (A) Find the general flow pattern of the network shown in the figure. Assuming that the flows are all nonnegative, what is the largest possible value for x_3? **[8]**



(B) Determine whether (1,1,1,1), (1,2,3,2), (2,5,6,4), (2,6,8,5) form a basis of R^4. **[7]**

Ques. 2 (A) A dietician is planning a meal that supplies certain quantities of vitamin C, calcium, and magnesium. Three foods will be used, their quantities measured in milligrams. The nutrients supplied by one unit of each food and dietary requirements are given in the table below.

Nutrient	Food 1	Food 2	Food 3	Total Required (mg)
Vitamin C	50	100	50	750
Calcium	20	60	40	450
Magnesium	10	35	30	280

Write the augmented matrix for this problem. State what variables represent, and then solve the system. **[9]**

(B) Let $A=\begin{pmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{pmatrix}$, $b=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. For what values of b, the system of equations $Ax=b$ has no solution. **[6]**

Ques. 3 (A) Verify Cayley-Hamilton theorem, find A^4 and A^{-1} when $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. **[8]**

(B) Find the 3×3 matrix that corresponds to the composite transformation of a scaling by 0.3, a rotation of 90^0 and finally a translation that adds (- 0.5, 2) to each point of a figure. **[7]**

Ques. 4 (A) The columns of $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Suppose T is a linear transformation from R^2 into R^3 such that $T(e_1) = \begin{bmatrix} 5 \\ -7 \\ 2 \end{bmatrix}$ and $T(e_2) = \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}$. Find a formula for the image of an arbitrary x in R^2. **[7]**

(B) Let $P = \{(x, y, z) \in R^3 : x + y + z = 0, 2x + 2y + 2z = 0, 3x + 3y + 3z = 0\}$. Show that W is a subspace of R^3. What is the dimension of W? **[8]**

Ques. 5 Let $A = \begin{bmatrix} 7 & -9 & -4 & 5 & 3 & -3 & -7 \\ -4 & 6 & 7 & -2 & -6 & -5 & 5 \\ 5 & -7 & -6 & 5 & -6 & 2 & 8 \\ -3 & 5 & 8 & -1 & -7 & -4 & 8 \\ 6 & -8 & -5 & 4 & 4 & 9 & 3 \end{bmatrix}$. **[15]**

I. Construct matrices C and N whose columns are bases for Col A and Nul A, respectively, and construct a matrix R whose rows form a basis for Row A.

II. Construct a matrix M whose columns form a basis for Nul A^T, form the matrices $S = [R^T\ N]$ and $T = [C\ M]$, and explain why S and T should be square. Verify that both S and T are invertible.

Ques. 6 (A) Find an equation $y = a + bx$ of the least square line that best fits the data points $(-1,0)$, $(0,1)$, $(1,2)$, $(2,4)$. [7]

(B) In old-growth forests of Douglas fir, the spotted owl dines mainly on flying squirrels. Suppose the predator–prey matrix for these two populations is $A = \begin{bmatrix} .4 & .3 \\ -p & 1.2 \end{bmatrix}$. Show that if the predation parameter p is .325, both populations grow. Estimate the long-term growth rate and the eventual ratio of owls to flying squirrels. [8]

Ques. 7 (A) Let V be the space C[-1,1] with the inner product $< f, g > = \int_{-1}^{1} f(t)\, g(t) dt$. Find an orthogonal basis for the subspace spanned by the polynomials 1, t, and t^2. [8]

(B) Orthogonally diagonalize the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ with the help of orthogonal basis.[7]