

Cluster Innovation Centre, University of Delhi, Delhi-110007

Examination : End Semester Examination – December 2024

Unique Paper Code : 3122611101

Name of the Paper : Single and Multivariable Calculus

Name of the Course : B.Tech (IT & MI)

Semester : I

Duration : 3 Hours

Maximum Marks : 90 Marks

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
 2. Attempt any **Five** questions. All questions carry equal marks.
-

✓Q.1 (a) Examine the convergence or divergence of the series [5]

$$\sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \cdot \dots \cdot (2n+1)}{n^2 \cdot 2^n} x^{n+1}$$

(b) A hot air balloon rising straight up from a level field is tracked by a range finder 500 ft from the liftoff point. At the moment the range finder's elevation angle is $\frac{\pi}{4}$, the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment? [5]

(c) Sketch a graph of the function $f = x^4 - 2x^2$ using the following steps. [8]

I. Identify where the extrema of f occur.

II. Find the intervals on which f is increasing and the intervals on which f is decreasing.

III. Find where the graph of f is concave up and where it is concave down.

IV. Plot some specific points, such as local maximum and minimum points, point of inflection and intercepts. Then sketch the curve.

✓Q.2 (a) On January 15, 2010, the longest annular solar eclipse until 3040 occurred over Africa and the Indian Ocean (in an annular eclipse, the sun is partially obscured by the moon and looks like a ring). The path of the full eclipse on the earth's surface is modeled by

$$y(x) = 0.0125x^2 - 1.157x + 22.864, \quad 15 < x < 90,$$

where x is the number of degrees of longitude east of the prime meridian and $y(x)$ is the number of degrees of latitude north (positive) or south (negative) of the equator. Find the longitude and latitude of the southernmost point at which the full eclipse could be viewed. [8]

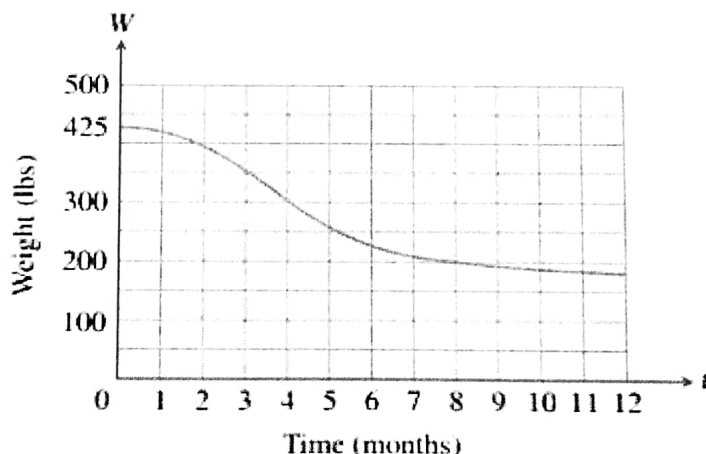
(b) A registrar's office finds that the number of inkjet cartridges, I , required each year for its copiers and printers varies directly with the number of students enrolled, s . [6]

I. Find an equation of variation that expresses I as a function of s , if the office requires 16 cartridges when 2800 students enroll.

II. How many cartridges would be required if 3100 students enrolled?

(c) Determine a rational function f which has a horizontal asymptote at $y = 0$ and vertical asymptotes at $x = -2$ and $x = 3$ and $f(1) = 1$. [4]

Q.3 (a) Jared Fogle, also known as the "Subway Sandwich Guy," weighed 425 lb in 1997 before losing more than 240 lb in 12 months. A chart showing his possible dramatic weight loss is given in the accompanying figure. [4]



I. When does Jared lose weight most rapidly and what is this rate of weight loss?

II. Graph the derivative of weight W .

(b) For what values of a and b is $g(x) = \begin{cases} ax + 2b, & x \leq 0 \\ x^2 + 3a - b, & 0 < x \leq 2 \\ 3x - 5, & x > 2 \end{cases}$ continuous at every x ? [4]

x ?

(c) Kim likes to create stylish tee shirts, one style with a script x on the front and another with a script p on the front. She sells them to her math students as a fundraiser for her favorite charity. Kim determines that her weekly revenue is modeled by the two-variable function

$$P(x, p) = -x^2 - p^2 - xp + 20x + 22p - 25$$

where x is the number of x -shirts sold, and p is the number of p -shirts sold. Kim spends 2 hr working on each x -shirt and 4 hr working on each p -shirt, and she works no more than 40 hr per week on this project. How many of each style should she produce in order to maximize her weekly revenue? Assume $x \geq 0$ and $p \geq 0$; in other words, she cannot produce negative quantities of the tee shirts. [10]

Q.4 (a) Find the volume of the solid generated by revolving the region bounded by $y = 0$, $y = 2 - x$ and $y = \sqrt{x}$ about the x -axis. [6]

(b) A volleyball is hit when it is 4 ft above the ground and 12 ft from a 6-ft-high net. It leaves the point of impact with an initial velocity of 35 ft/sec at an angle of 27° and slips by the opposing team untouched. [12]

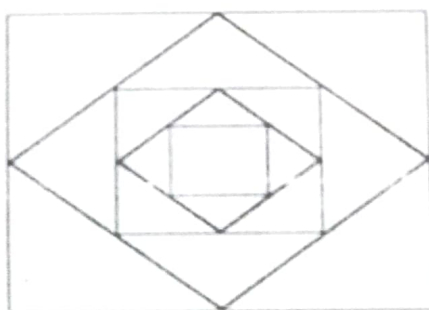
I. Find a vector equation for the path of the volleyball.

II. How high does the volleyball go, and when does it reach maximum height?

III. Find its range and flight time.

IV. When is the volleyball 7 ft above the ground? How far (ground distance) is the volleyball from where it will land?

Q.5 (a) The following figure shows the first five of a sequence of squares. The outermost square has an area of each of the other squares is obtained by joining the midpoints of the sides of the squares before it. Find the sum of the areas of all the squares. [6]



(b) A stereo manufacturer determines that in order to sell x units of a new stereo, the price per unit, in dollars, must be $P(x) = 1000 - x$. The manufacturer also determines that the total cost of producing x units is given by $C(x) = 3000 + 20x$. [12]

I. Find the total revenue $R(x)$. Find the total profit $P(x)$.

II. How many units must the company produce and sell in order to maximize profit?

III. What is the maximum profit?

IV. What price per unit must be charged in order to make this maximum profit?

✓ **Q.6 (a)** Define $f(0,0)$ in a way that extends $f(x,y)=xy\frac{x^2-y^2}{x^2+y^2}$ to be continuous at the origin. [5]

(b) Antonio's score on a test is given by $s(t) = t^2$, $0 \leq t \leq 10$, where $s(t)$ is his score after t hours of studying. Bonnie's score on the same test is given by $S(t) = 10t$, $0 \leq t \leq 10$, where $S(t)$ is her score after t hours of studying. [8]

I. For $0 < t < 10$ who will have the higher test score?

II. Find the average value of $s(t)$ over the interval $[7,10]$, and explain what it represents.

III. Find the average value of $S(t)$ over the interval $[7,10]$ and explain what it represents.

IV. Assuming that both students have the same study habits and are equally likely to study for any number of hours, t , in $[0,10]$, on average, how far apart will their test scores be?

(c) Evaluate $\int_0^1 (\int_{x^2}^{2-x} xy dy) dx$ by changing the order of integration. [5]