[This question paper contains 8 printed pages.]

Your Roll No. 5004

Sr. No. of Question Paper : 1910 **G**

Unique Paper Code : 3122611102

Name of the Paper : Discrete Mathematics and its Applications

Name of the Course : **B. Tech. (Information Technology & Mathematical Innovation)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper contains **seven** questions, out of which any **five** are to be attempted.
3. Each question carries equal marks. Marks for each question are indicated.

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1. (a) The police have three suspects for the murder of Mr. Cooper: Mr. Smith, Mr. Jones, and Mr. Williams. Smith, Jones, and Williams each declare that they did not kill Cooper. Smith also states that Cooper was a friend of Jones and that Williams disliked him. Jones also states that he did not know Cooper and that he was out of town the day Cooper was killed. Williams also states that he saw both Smith and Jones with Cooper the day of the killing and that either Smith or Jones must have killed him. Can you determine who the murderer was if:

   (i) One of the three men is guilty, the two innocent men are telling the truth, but the statements of the guilty man may or may not be true?

   (ii) Innocent men do not lie? (9)

   (b) Suppose that each pair of a genetically engineered species of rabbits left on an island produces two new pairs of rabbits at the age of 1 month and six new pairs of rabbits at the age of 2 months and every month afterward. None of the rabbits ever die or leave the island. Find a recurrence relation for the number of pairs of rabbits on the island n months after one newborn pair is left on the island.

By solving the recurrence relation, determine the number of pairs of rabbits on the island n months after one pair is left on the island. (9)

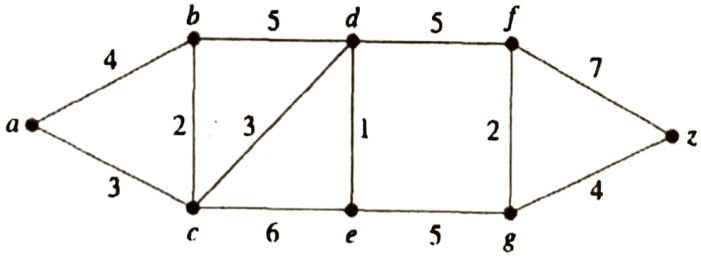
2. (a) During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games. (9)

(b) Suppose that a weapons inspector must inspect each of five different sites twice, visiting one site per day. The inspector is free to select the order in which to visit these sites, but cannot visit site X, the most suspicious site, on two consecutive days. In how many different orders can the inspector visit these sites? (9)

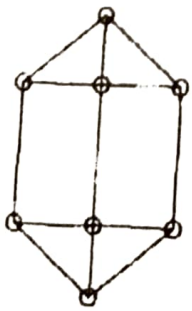
3. (a) You and your buddy return home after a semester at college and are greeted at the airport by your mothers and your buddy's two sisters. Not uncharacteristically, there is a certain amount of hugging! Later, the other five people tell you the number of hugs they got and, curiously, these numbers are all different. Assume that you and your buddy did not hug each other, your mothers

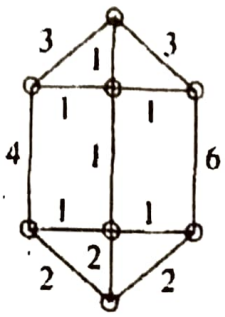
did not hug each other, and your buddy's sisters did not hug each other. Assume also that the same two people hugged at most once. How many people did you hug? How many people hugged your buddy? (9)

(b) The diagram below shows roads connecting towns. The numbers on each arc represent the time, in minutes, required to travel along each road. Peter is delivering books from his base at a to z. Find the minimum time for Peter's journey. (9)



4. (a) Solve the Chinese Postman Problem for each graph (9)

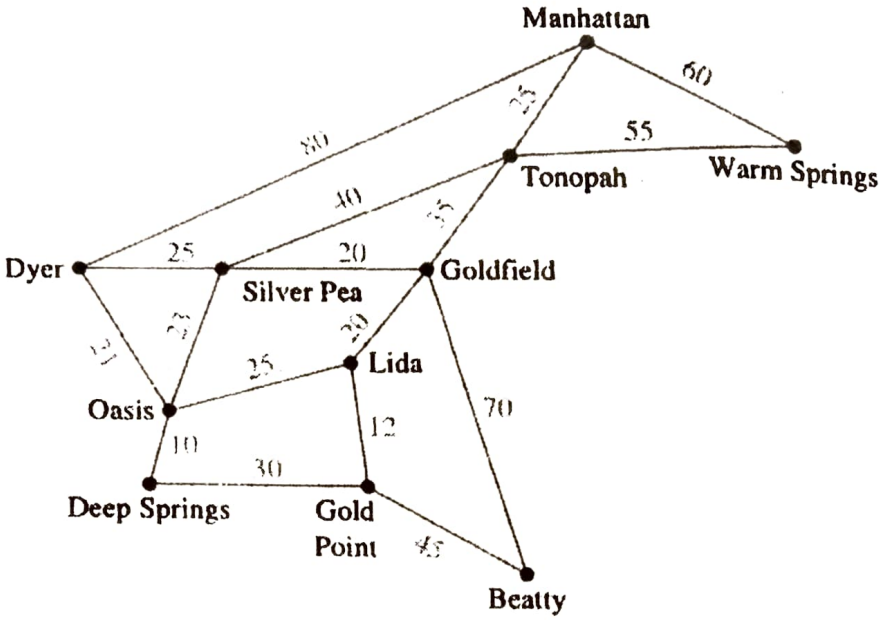




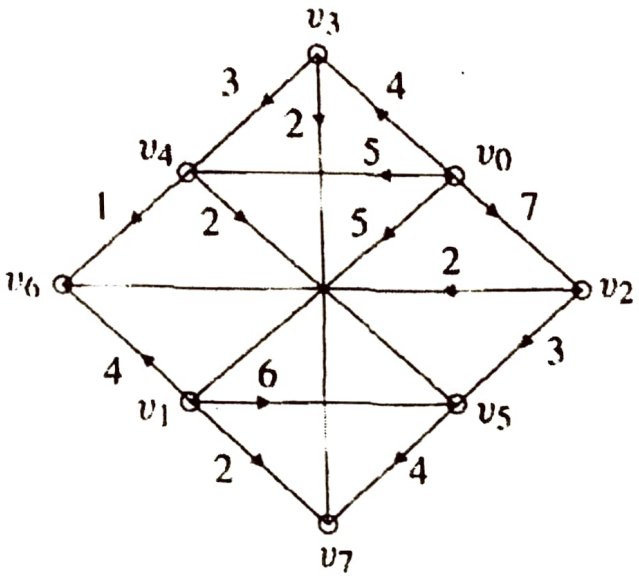
(b) To complete her Master's thesis, a geology student must perform field work (F) and laboratory analysis (L), conduct a library search (S) of the literature, create a database of relevant articles (D), and write the thesis (W). The write-up cannot begin until all tasks except for the laboratory analysis are complete. It takes three units of time if it is the last task and otherwise five units of time. The field work, which takes four units of time, must precede both the laboratory analysis and the database creation. The library search takes two units of time. The laboratory analysis takes two units of time if it is undertaken after the library search and otherwise three units of time. Creation of the database takes one unit of time if it is delayed until the library search is complete and otherwise four units of time. What type of scheduling problem is this (I or II)? Draw the appropriate directed network. What is the shortest possible time in which this student can complete her thesis? In what order should she perform her thesis-related tasks in order to achieve this minimum time? (9)

X (a) The roads represented by this graph are all unpaved. The lengths of the roads between pairs of towns are represented by edge weights. Which roads should be paved so that there is a path of

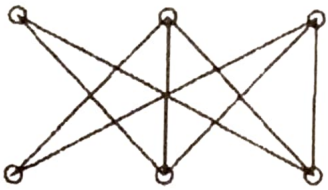
paved roads between each pair of towns so that a minimum road length is paved? (9)

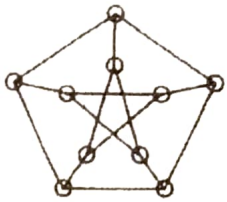


(b) The following digraph is acyclic, and in canonical labeling. Apply Bellman's algorithm to the following digraph in order to find the lengths of shortest paths from v_0 to each other vertex. (9)



6. (a) Use Kuratowski theorem to determine whether the given graphs are planar or not. (9)





(b) Television channels are to be assigned to stations based in nine cities A, B ..... I. Broadcasting regulations require that cities within 150 km of each other be assigned different channels. What is the least number of channels required if the distances between the cities are as given in the table. (9)

	A	B	C	D	E	F	G	H	I
B	85								
C	137	165							
D	123	39	205						
E	164	132	117	171					
F	105	75	235	92	201				
G	134	191	252	223	298	177			
H	114	77	113	117	54	147	247		
I	132	174	22	213	138	237	245	120	

7. Suppose that there are five young women and six young men on an island. Each woman is willing to

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marry some of the men on the island and each man is willing to marry any woman who is willing to marry him. Suppose that Anna is willing to marry Jason, Larry, and Matt; Barbara is willing to marry Kevin and Larry; Carol is willing to marry Jason, Nick, and Oscar; Diane is willing to marry Jason, Larry, Nick, and Oscar; and Elizabeth is willing to marry Jason and Matt.

(a) Model the possible marriages on the island using a bipartite graph. (6)

(b) Find a matching of the young women and the young men on the island such that each young woman is matched with a young man whom she is willing to marry. (6)

(c) Is the matching you found in part (b) a complete matching? Is it a maximum matching? (6)