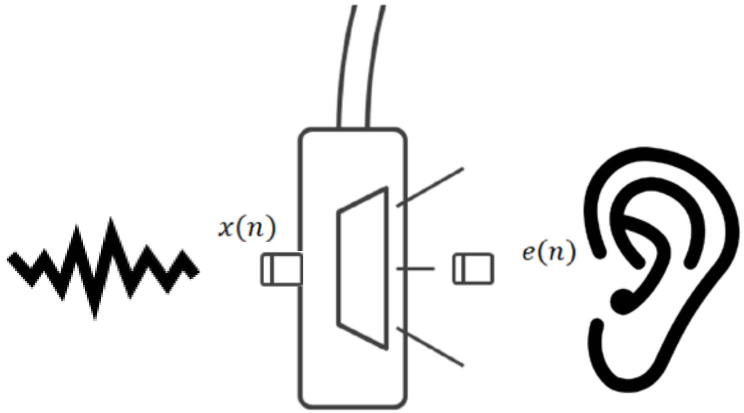
上海科技大学
ShanghaiTech University

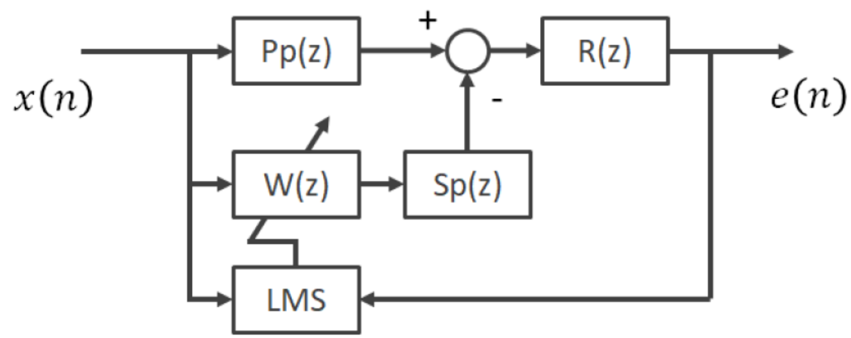
Kalman Filter for Active Noise Control

—— A system identification viewpoint @EE 160

Zhiqiang Xie
SIST@ShanghaiTech University

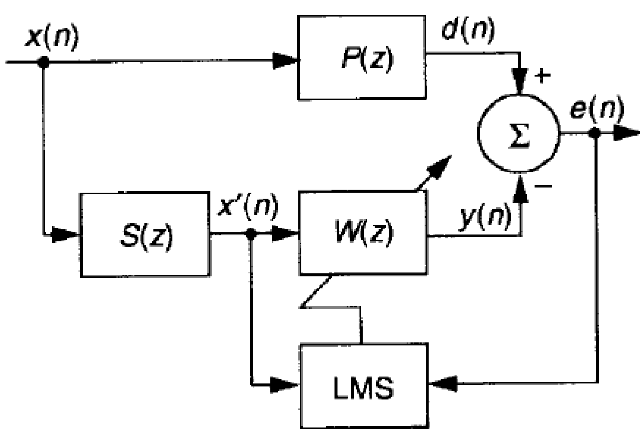
Active Noise Control Problem





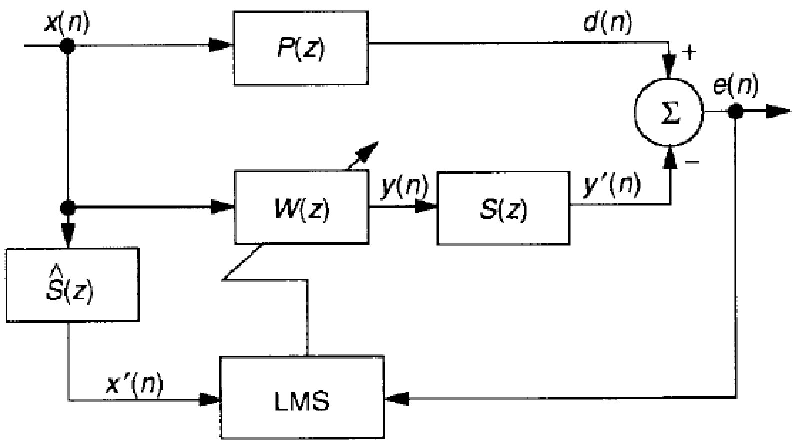
From a system identification view, all we want is to find a $W(z)$ to minimize the error signal.

$$
\begin{aligned}
e(n) &= d(n) - s(n) * [w^T(n)x(n)] \\
&\approx d(n) - w^T(n)[s(n) * x(n)]
\end{aligned}
$$

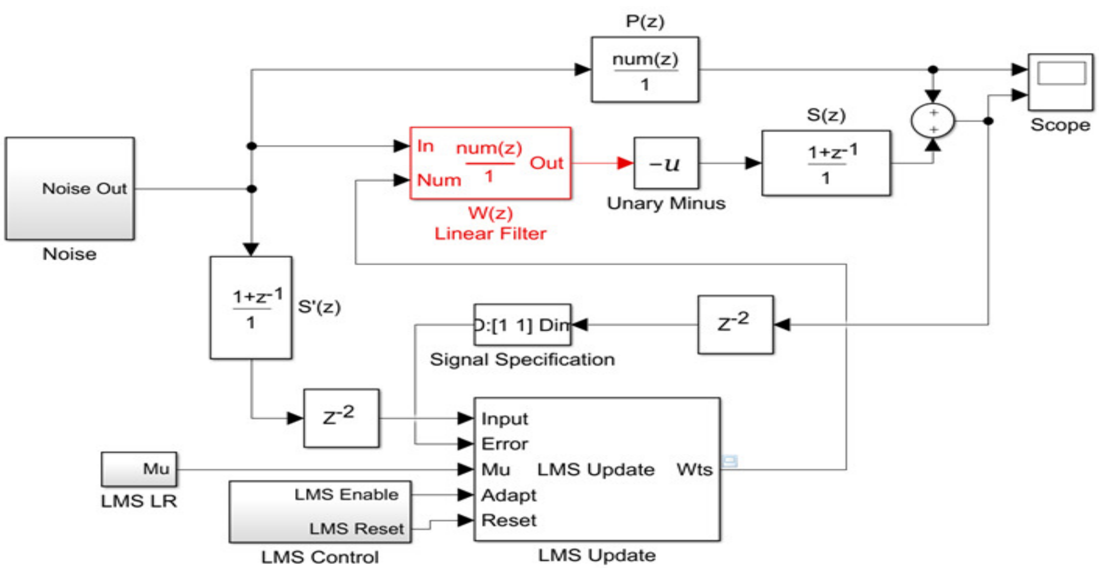


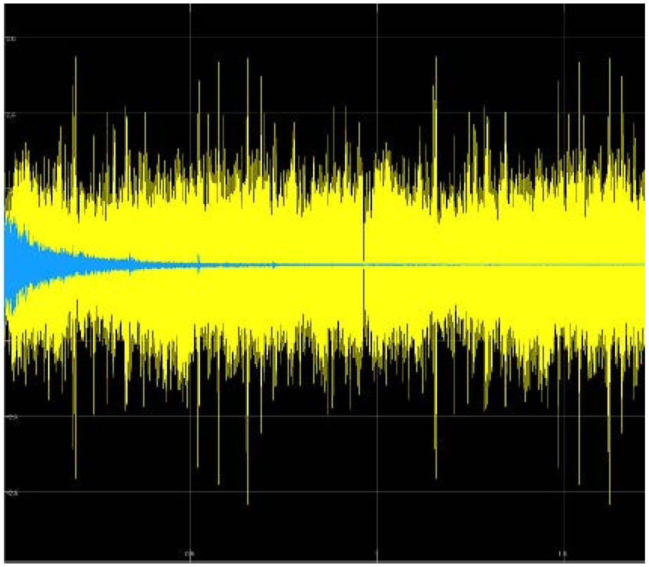
Filtered-X structure

Widrow, Shur and Shaffer, 1981



The filtered-X structure is introduced and it's usually argued as allowable due to slowly changing filter coefficients.

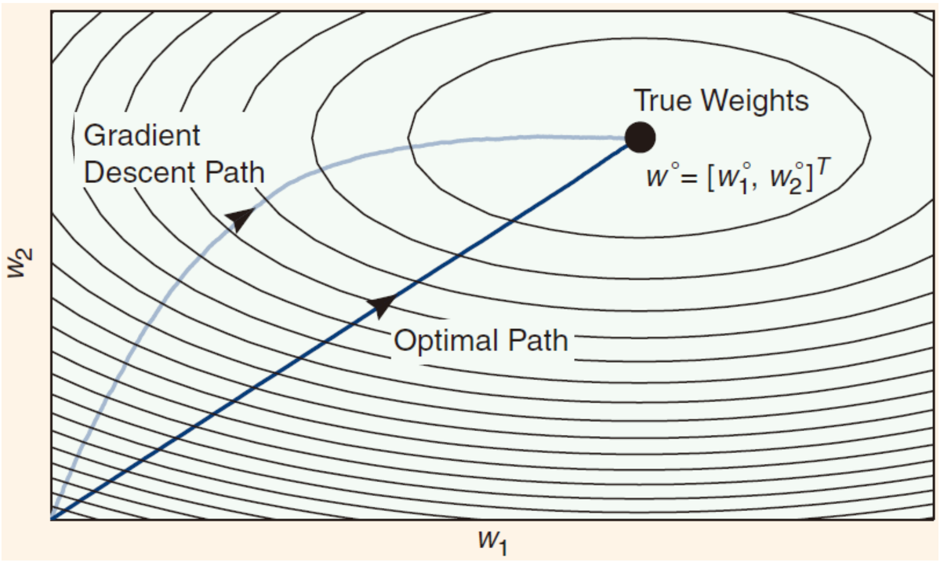




The blue part is the residual error noise.

Kalman filter for system identification

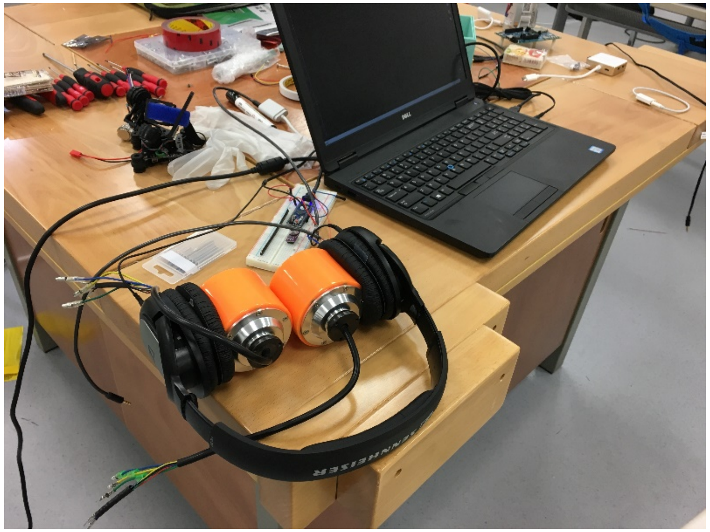
Mandic, Kanna and Constantinides, 2015



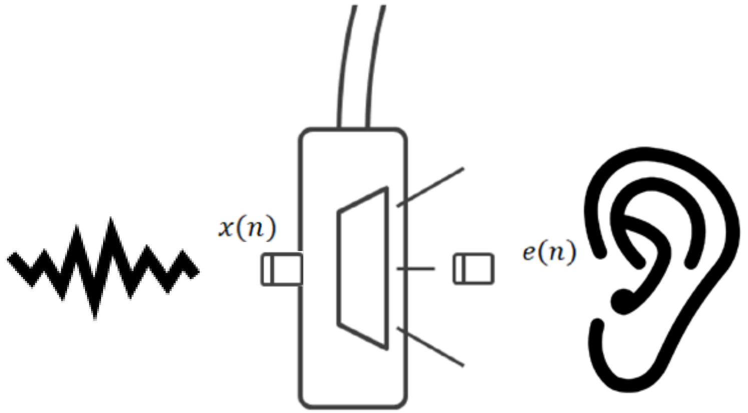
LMS filter based on gradient descent is a generic and robust algorithm. It's locally optimal but globally slower converging than the optimal path.

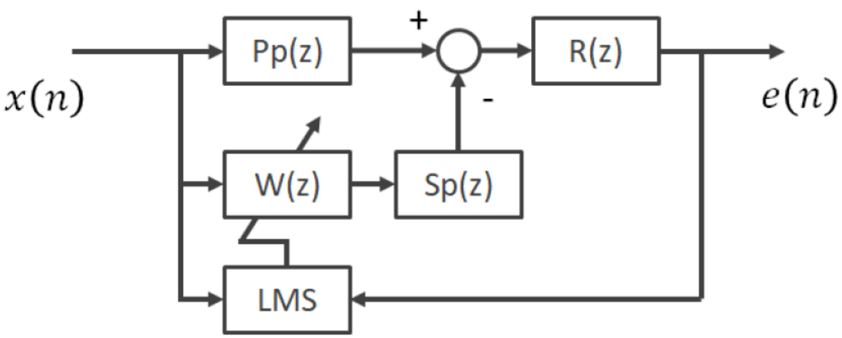
Besides, the Kalman filter can be interpreted as an LMS algorithm with optimal variable step size.

From a sequential Bayesian Estimator view: with some simplification and assumption (prior knowledge), we can present a practical system identification solution by Kalman filter.



Simplification & Assumption





1. The secondary path is modeled offline.
2. Simplify the system: using multiplication instead of convolution.
3. No time delay is taken into account.
4. The measurement noise is a white Gaussian noise.

$$w_{k+1} = w_k$$

$$d_k = x_k^T w^o + n_k = e_k + x_k^T w_{k-1}$$

$$e_k = x_k^T \tilde{w}_{k-1} + n_k$$

$$\tilde{w}_{k-1} = w^o - w_{k-1}$$

Where $\tilde{w}$ is the innovation

$$P_k = E\{\tilde{w}_k \tilde{w}_k^T\}$$

Procedures and simulation

Define $x_k = [x_{k1}, x_{k2}, \ldots, x_{k10}]^T, w^o = [w_1^o, w_2^o, \ldots, w_{10}^o]^T$

1) Compute the Kalman gain (optimal learning gain)

$$g_k = P_{k-1} x_k \left(x_k^T P_{k-1} x_k + \sigma_n^2 \right)^{-1}$$

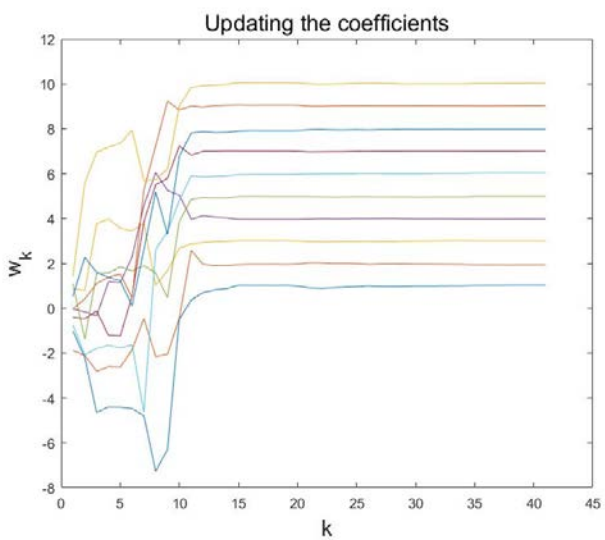
2) Update the coefficient estimate:

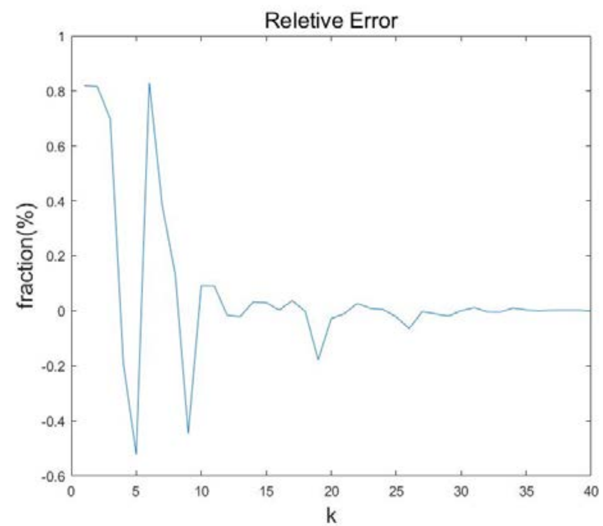
$$w_k = w_{k-1} + g_k e_k$$

3) Update the innovation covariance matrix (predict error):

$$P_k = P_{k-1} - g_k x_k^T P_{k-1}$$

Pick $P_0 = I, \sigma_n = 1$
Randomly initialize w_0
with ten $w_i \sim N(0,1)$

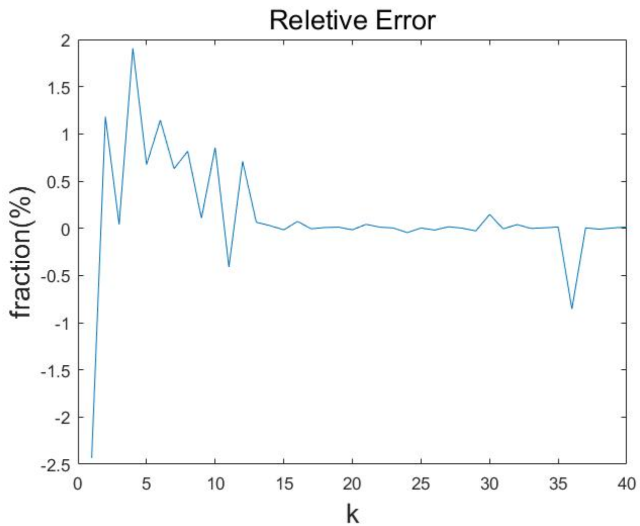


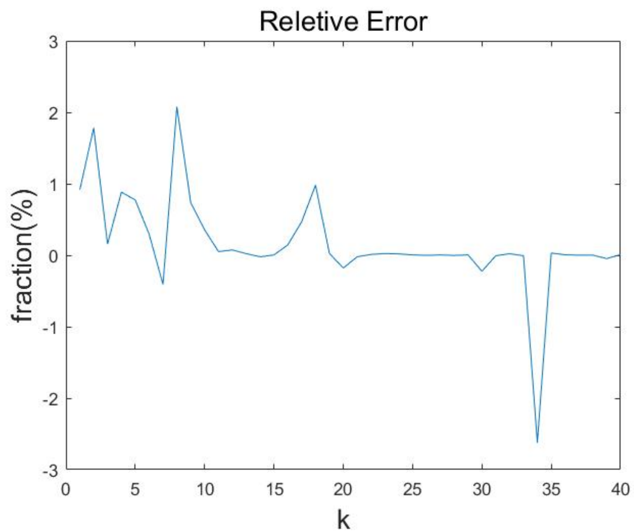


Conclusion and related work

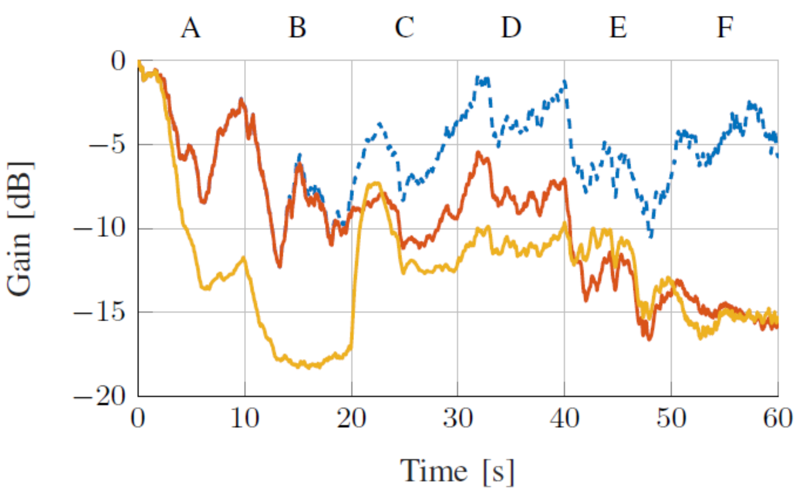
Kalman filter is fast and powerful, but it's not reliable without good prior knowledge.

2 times (left) and 4 times (right) deviation of prior knowledge are introduced.





Recently, a purely data-driven Kalman-like (no prior knowledge but estimating online) algorithm is proposed.



Liebich, Fabry, Jax and Vary EUSIPCO 2017