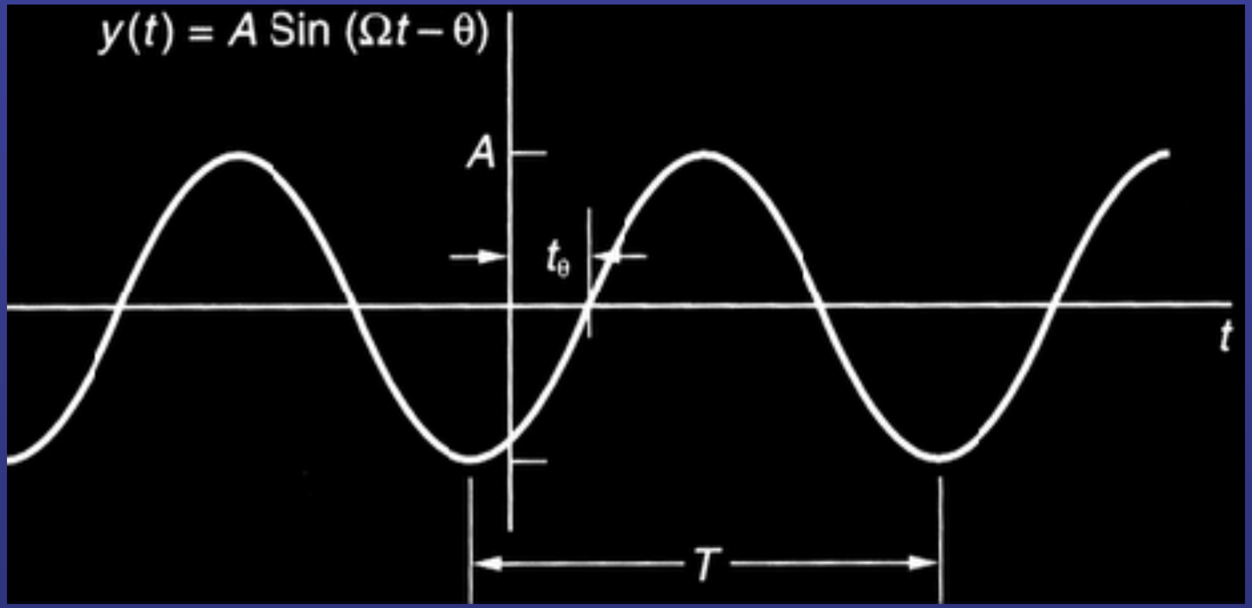
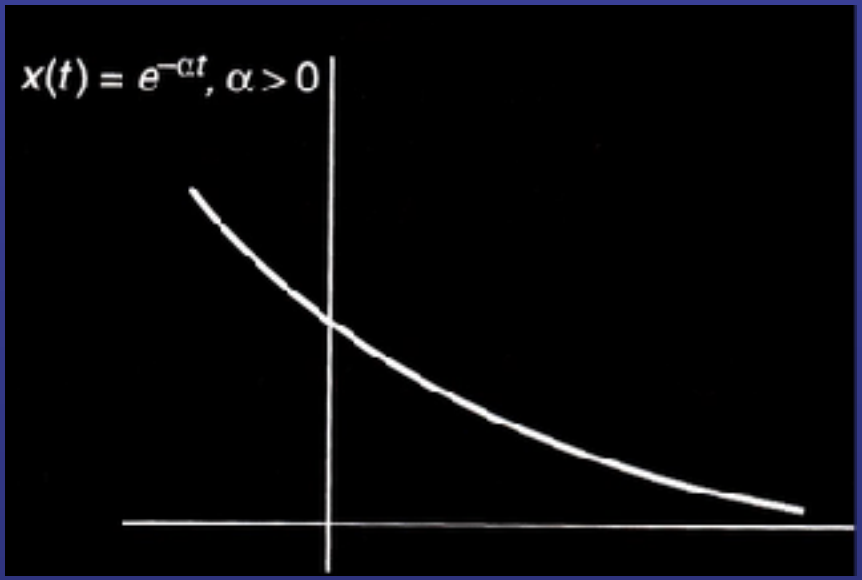
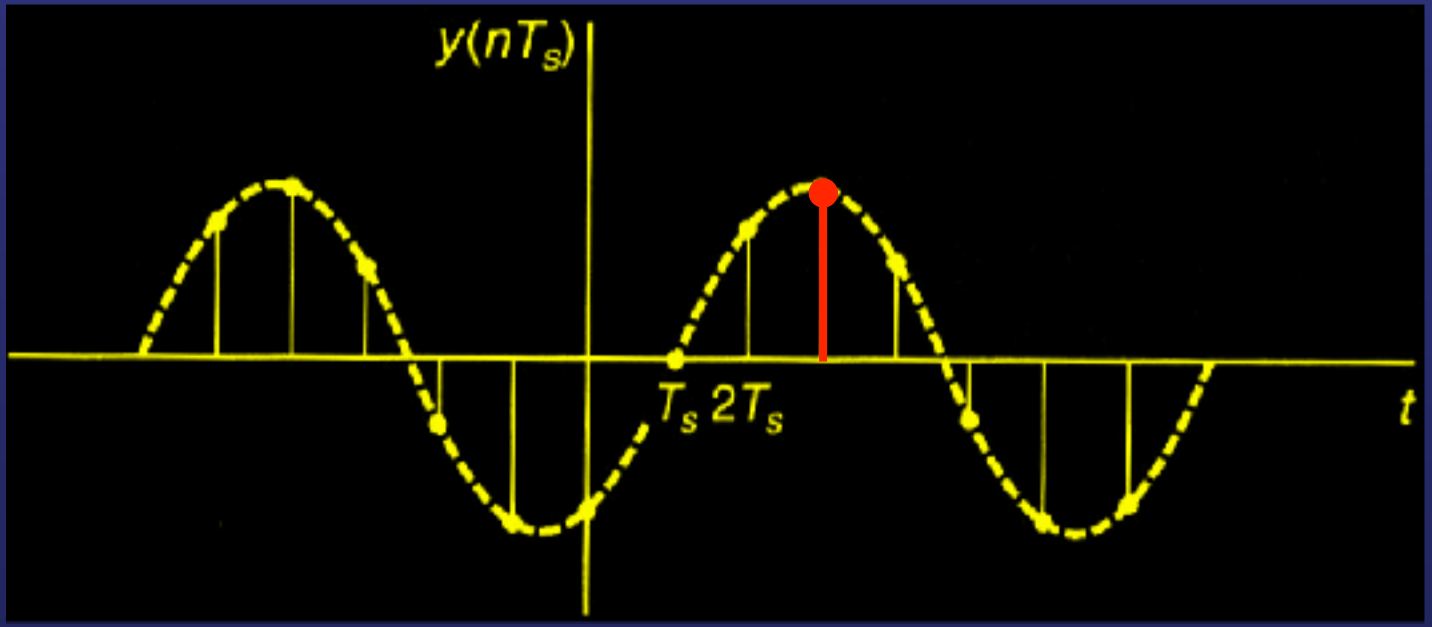
Fourier Transform

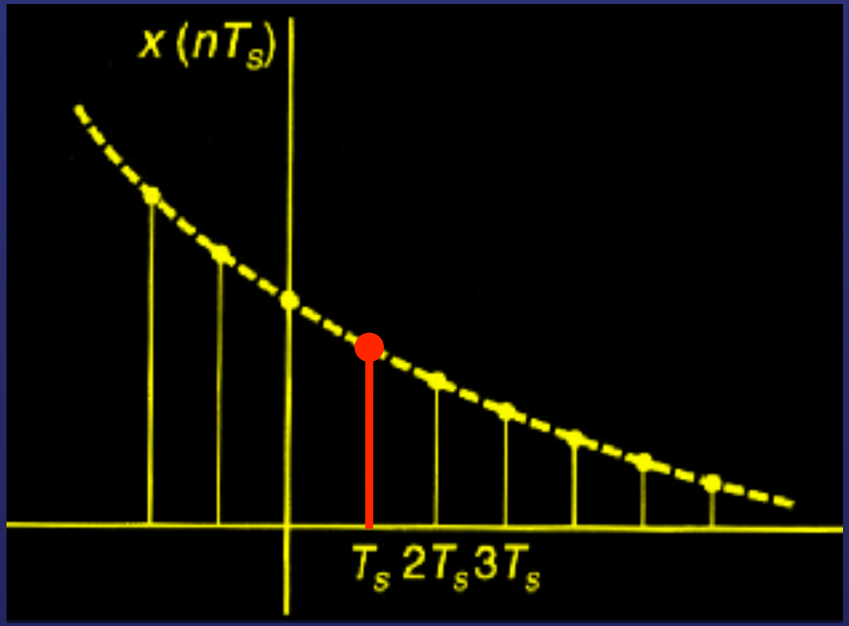
柯正雯

Continuous vs. Discrete Signals







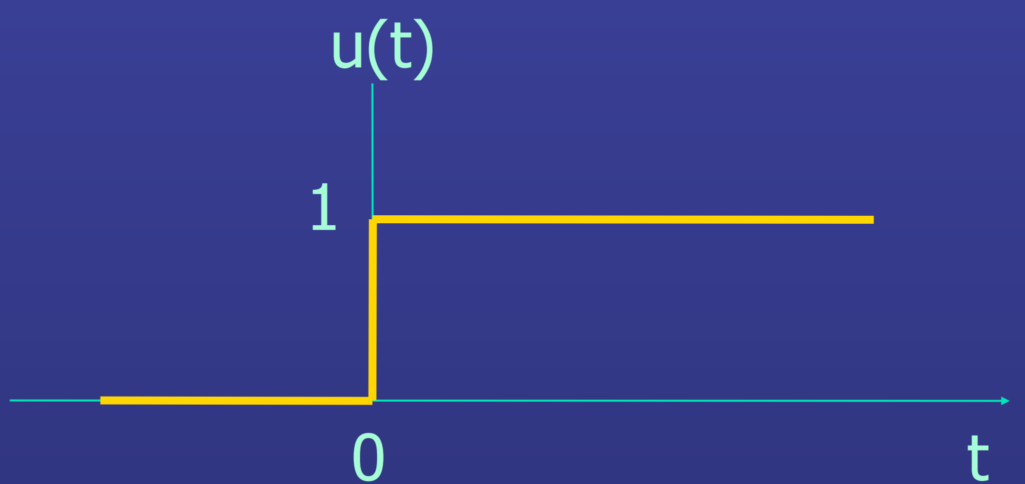


Basic Functions

Impulse & Step

Unit Step Func.

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

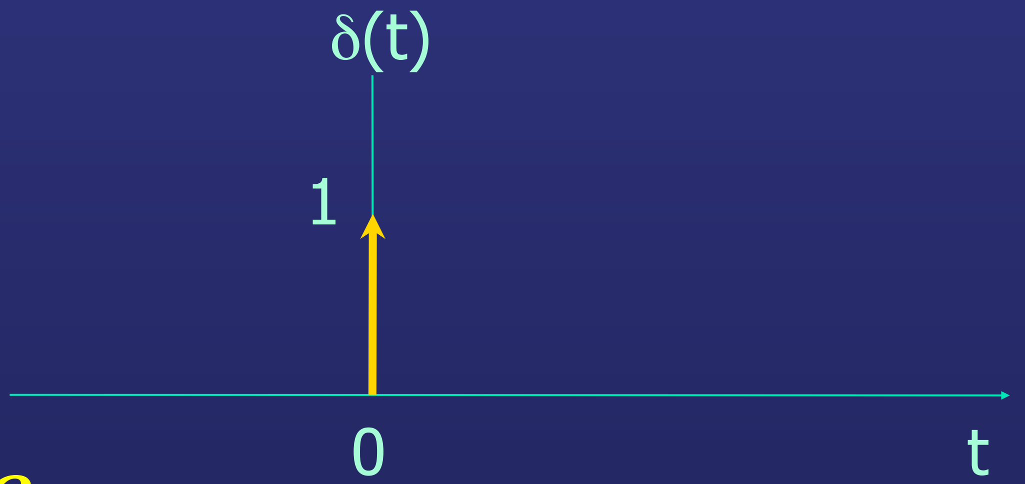


$$\delta(t) = \frac{du(t)}{dt}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

Unit Impulse

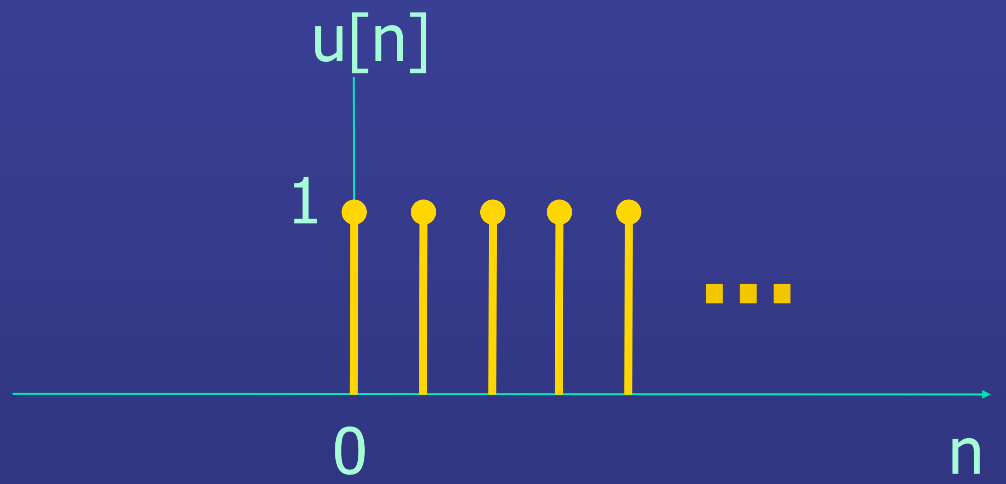
$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & otherwise \end{cases}$$



Impulse & Step

Unit Step Func.

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$



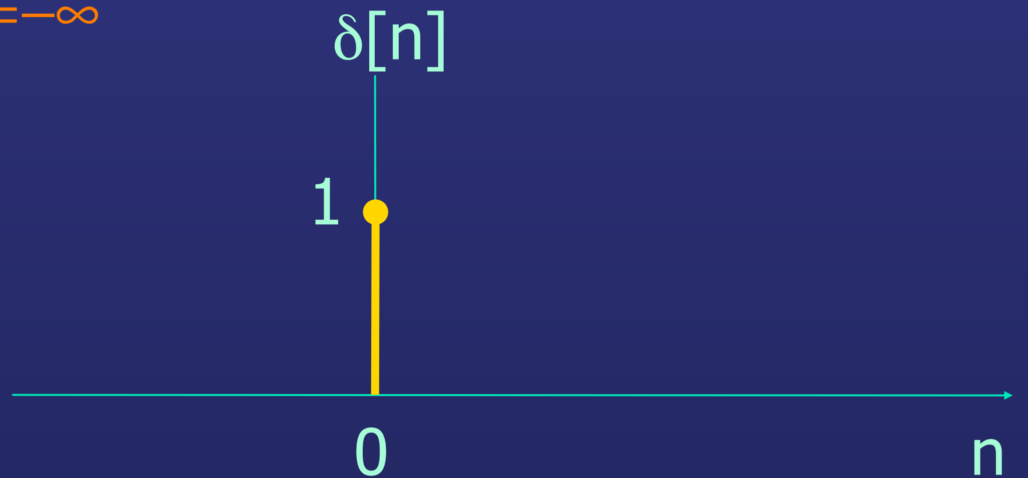
$\delta[n] =$

$u[n] - u[n-1]$

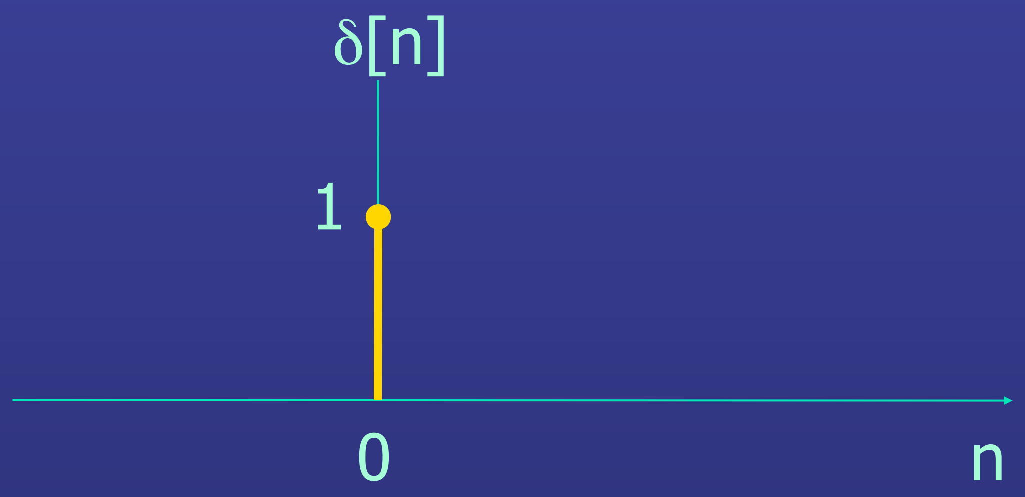
$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

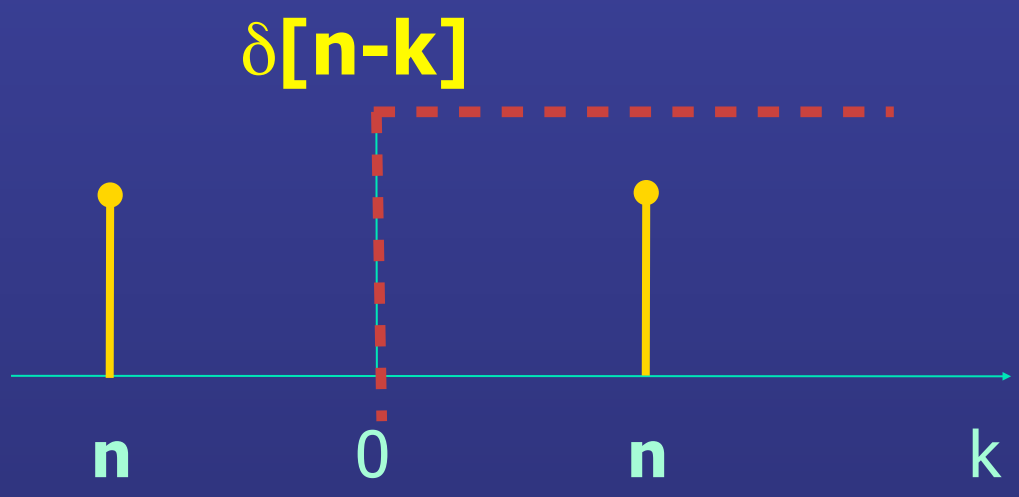
Unit Impulse

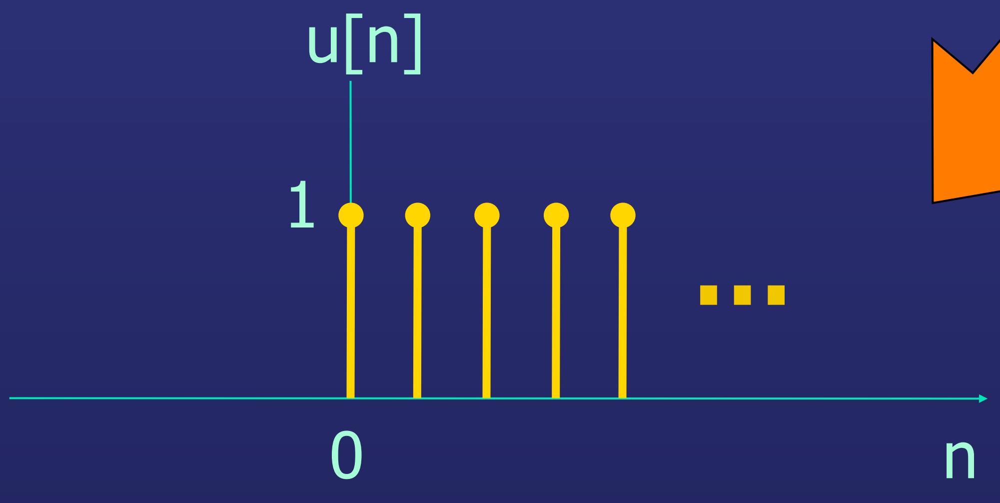
$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



Impulse & Step







$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

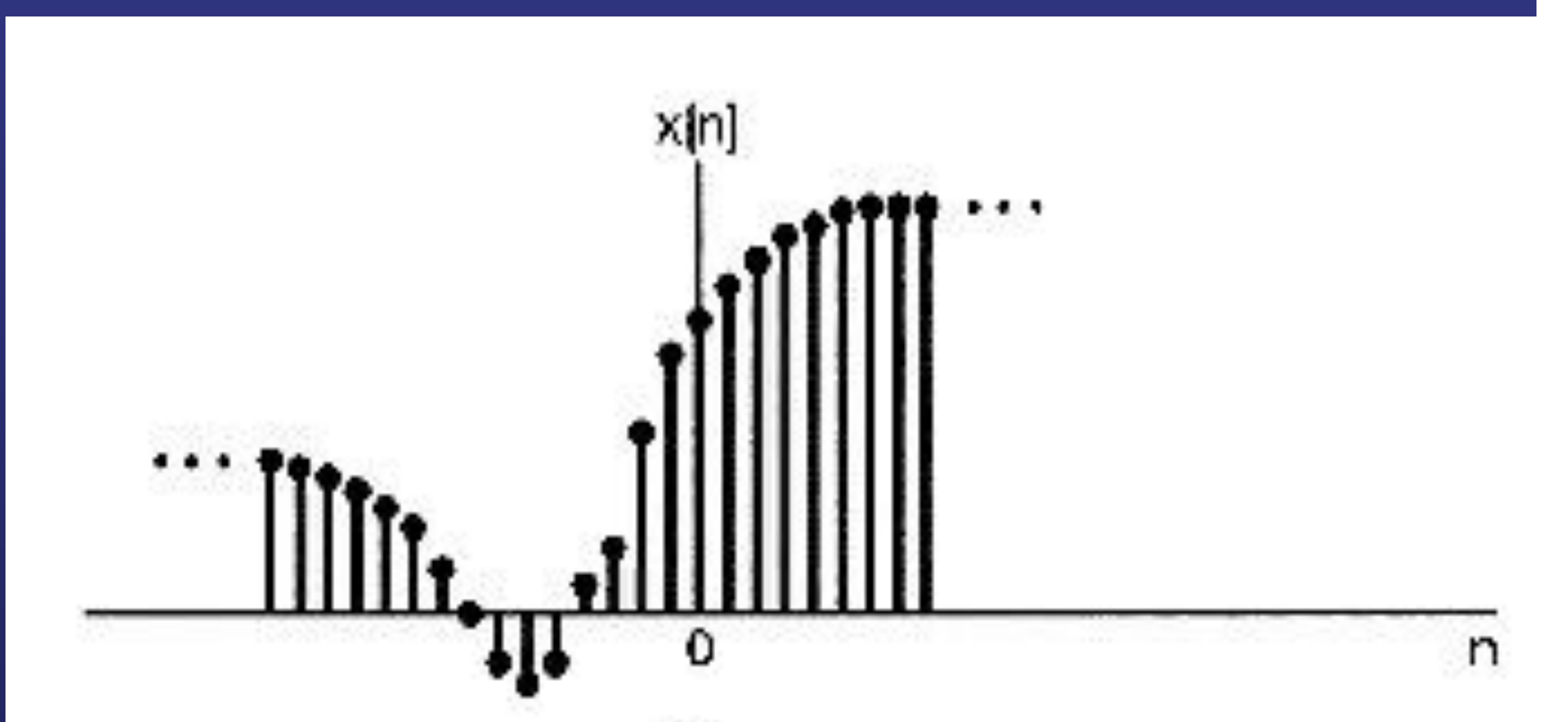
$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

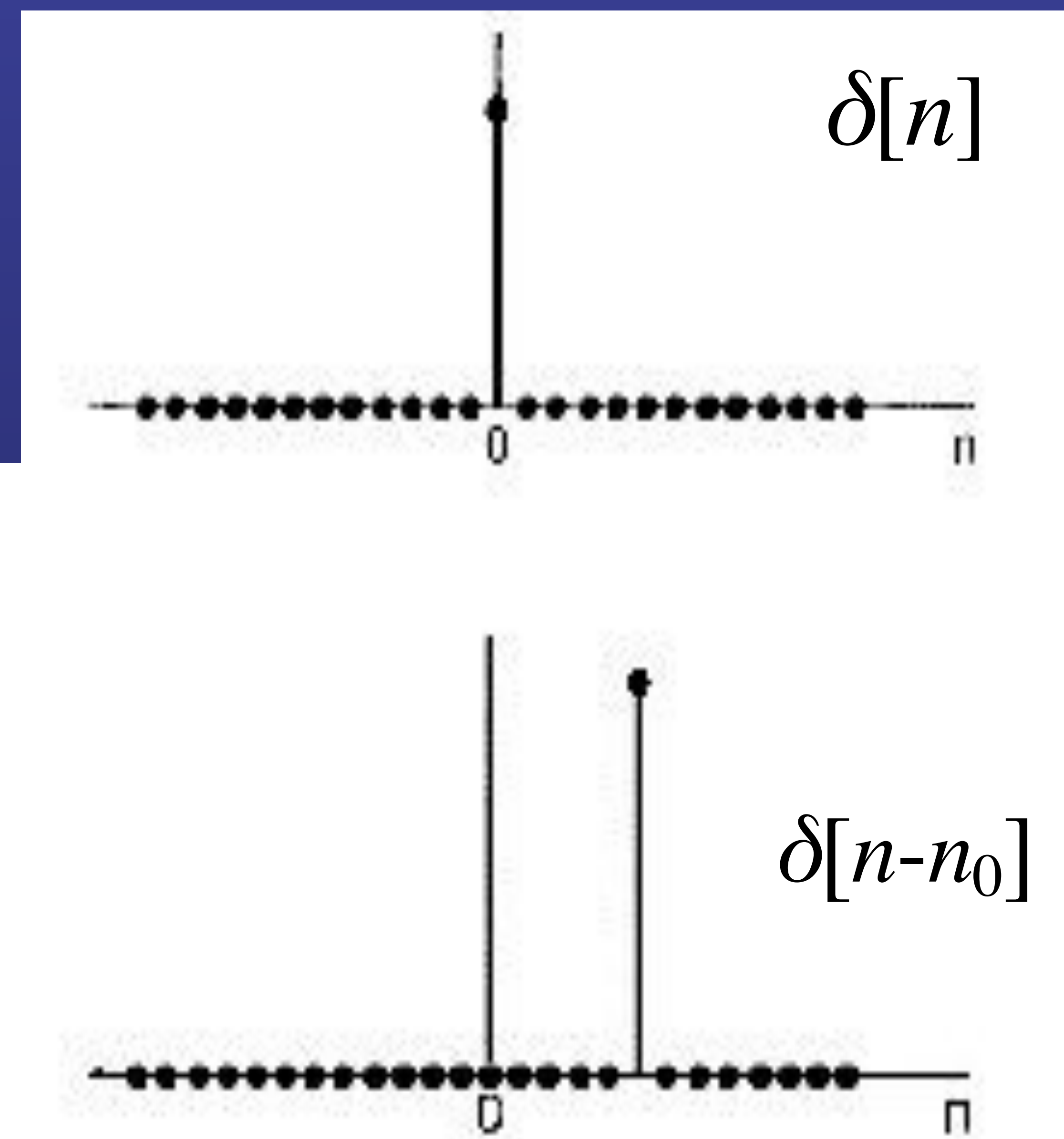
m = n-k

Sample by Unit Impulse

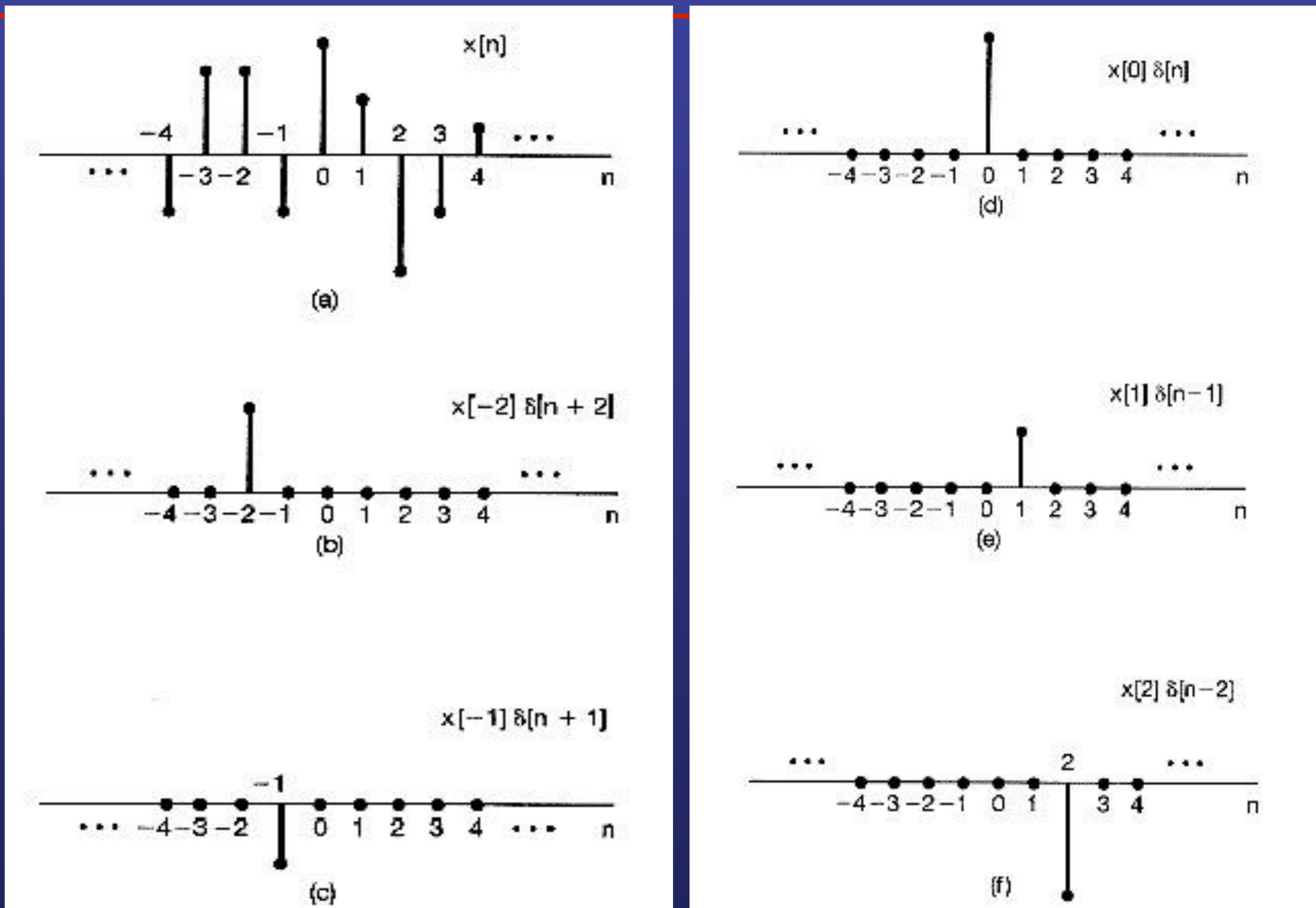
$$x[n]\delta[n]=x[0]\delta[n]$$

$$x[n]\delta[n-n_0]=x[n_0]\delta[n-n_0]$$





Representation of DT Signals by Impulses



Representation of DT Signals by Impulses

- Representing an arbitrary signal as a sequence of unit impulses

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \underline{\delta[n-k]}$$

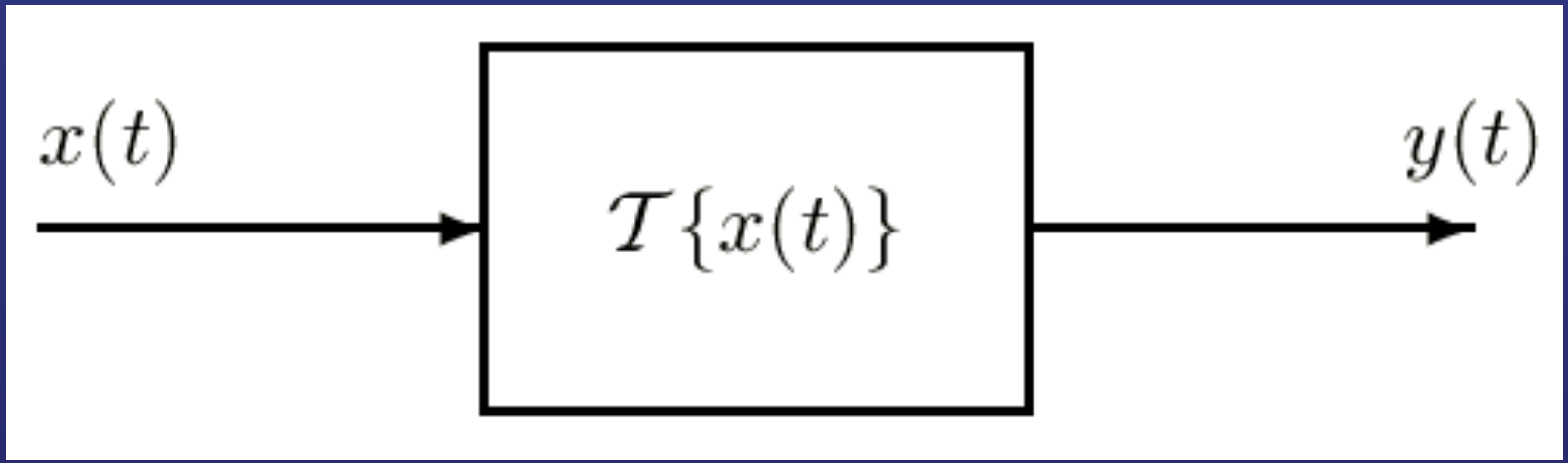
an unit impulse located at n = k on the index n

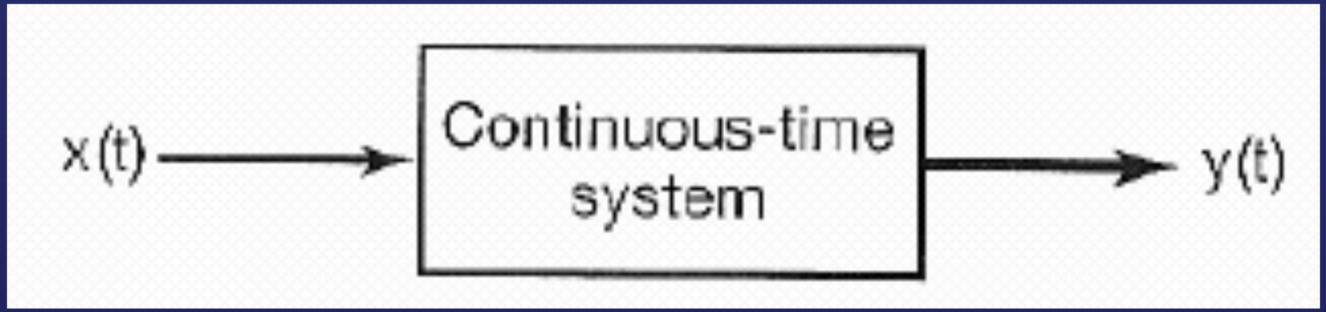
a special case:

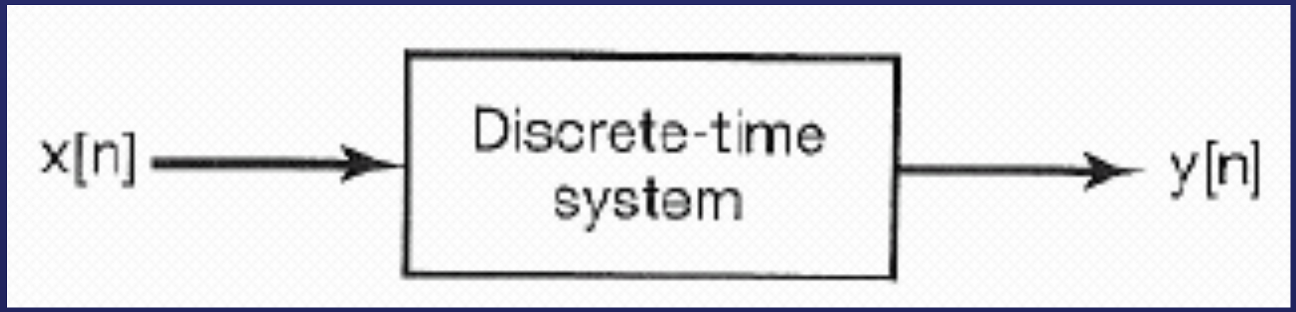
$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

Systems

A system can be viewed as a **process** in which **input** signals are transformed by the system or cause the system to respond in some way, resulting in other signals or **outputs**.







System: Time Invariance

- Time invariant:
  behavior and characteristic of the system are fixed over time
- A time shift in the input signal results in an identical time shift in the output signal

$$x[n] \rightarrow y[n] \Longleftrightarrow x[n-n_0] \rightarrow y[n-n_0]$$

Linear Systems

If an input consists of the weighted sum of several signals, then the output is the superposition of the responses of the system to each of those signals

$$x_1[n] \rightarrow y_1[n]$$

$$x_2[n] \rightarrow y_2[n]$$

IF (1) $x_1[n] + x_2[n] \rightarrow y_1[n] + y_2[n]$ (additivity)

(2) $ax_1[n] \rightarrow ay_1[n]$ (scaling or homogeneity)

a: any complex constant

THEN, the system is linear

Linear Systems

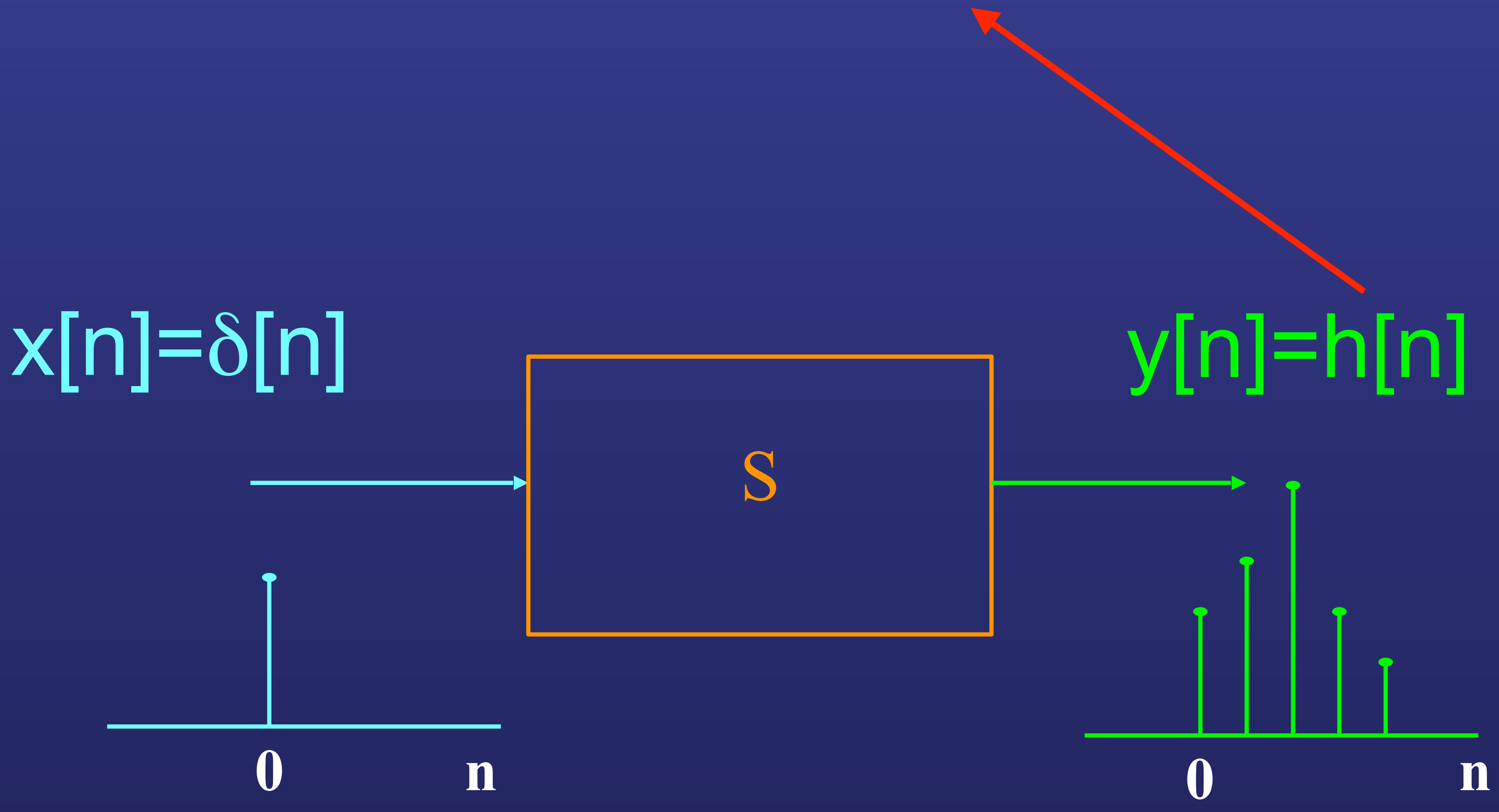
- Superposition property

$$x_k[n] \rightarrow y_k[n]$$

$$\sum_k a_k x_k[n] \rightarrow \sum_k a_k y_k[n]$$

Impulse Response

- Defining the output for an unit impulse input as the **Unit Impulse Response**



DT Unit Impulse Response & Convolution Sum

$$x[n] \rightarrow \boxed{\text{Linear System}} \rightarrow y[n]$$

$$\delta[n] \rightarrow \boxed{\text{Linear System}} \rightarrow h_0[n]$$

$$\delta[n-1] \rightarrow \boxed{\text{Linear System}} \rightarrow h_1[n]$$

$$\delta[n-2] \rightarrow \boxed{\text{Linear System}} \rightarrow h_2[n]$$

$$\delta[n-k] \rightarrow \boxed{\text{Linear System}} \rightarrow h_k[n]$$

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] \Longrightarrow y[n] = \sum_{k=-\infty}^{+\infty} x[k]h_k[n]$$

DT Unit Impulse Response & Convolution Sum

- If the linear system is also time-invariant

$$h_k[n] = h_0[n-k] = h[-k]$$

- for an LTI system,

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

Convolution of discrete-time signals

- Convolution sum

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

$$y[n] = x[n] * h[n]$$

- Known as the convolution of x[n] & h[n]

Convolution of continuous signals

- Convolution integral

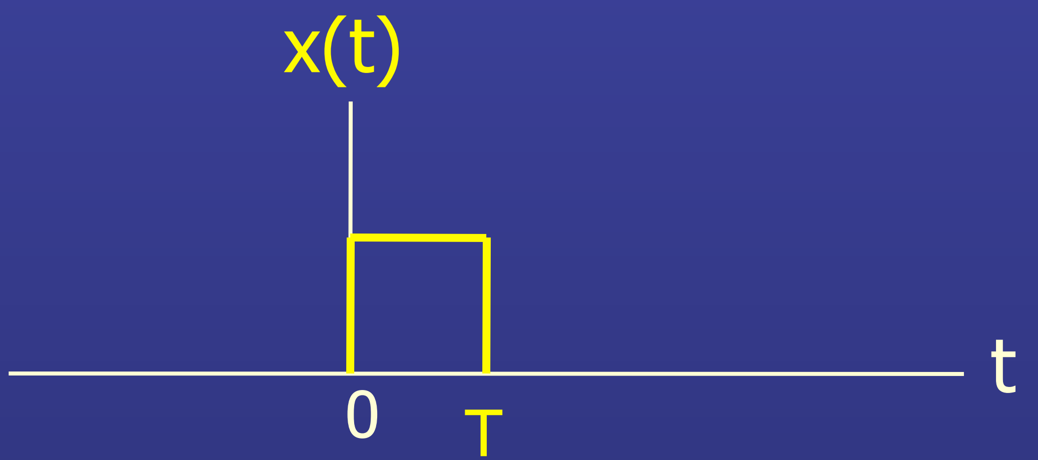
$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

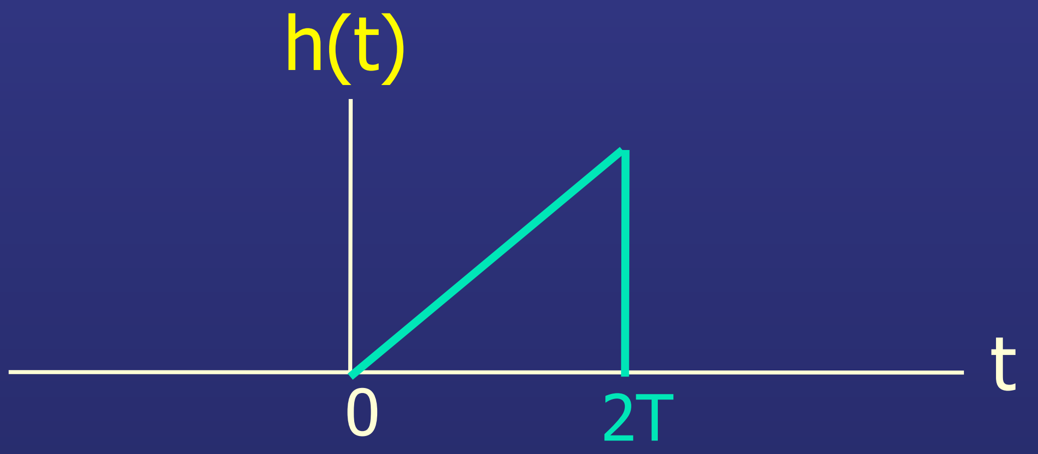
$$y(t) = x(t) * h(t)$$

Convolution Integral

$$x(t) = \begin{cases} 1, & 0 < t < T \\ 0, & otherwise \end{cases}$$

$$h(t) = \begin{cases} t, & 0 < t < 2T \\ 0, & otherwise \end{cases}$$

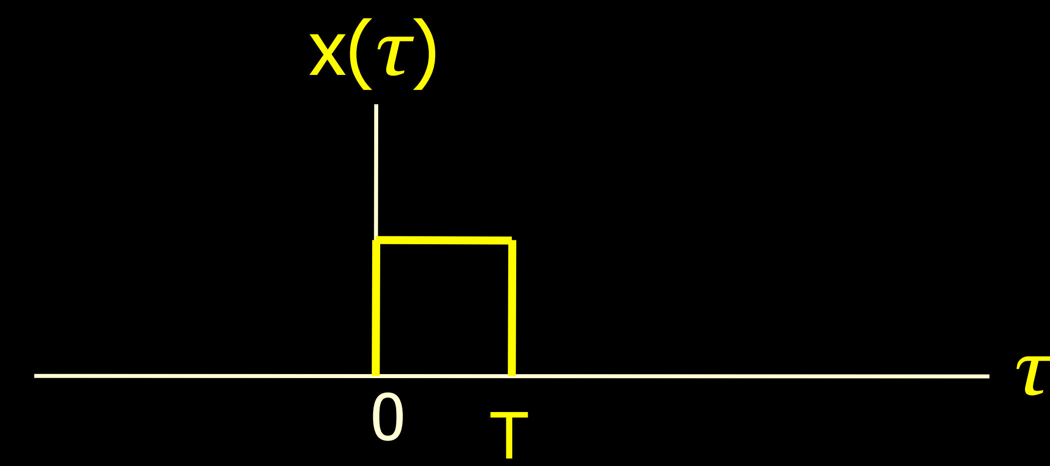


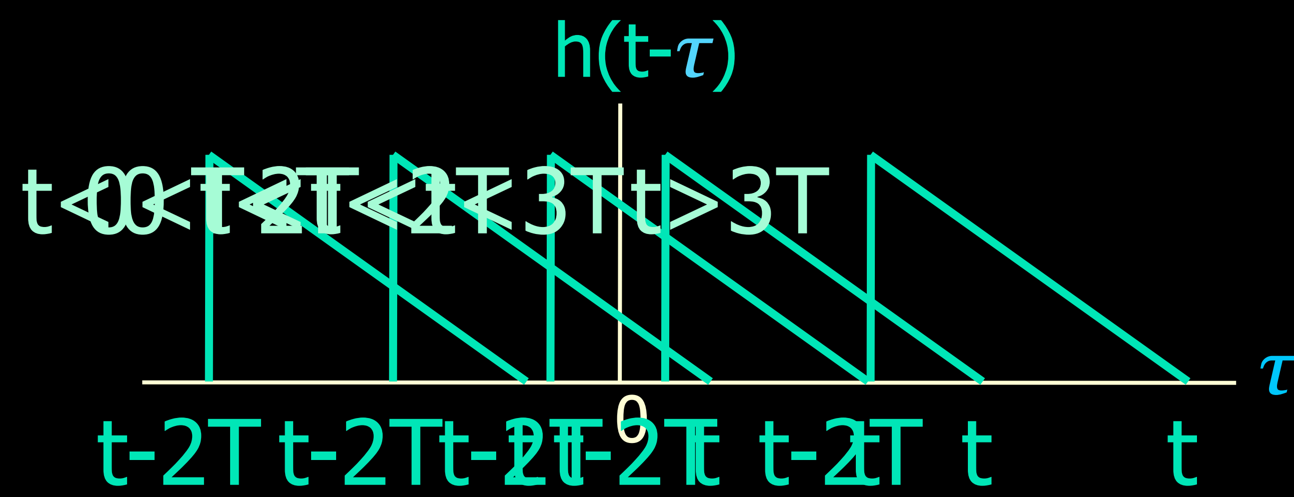


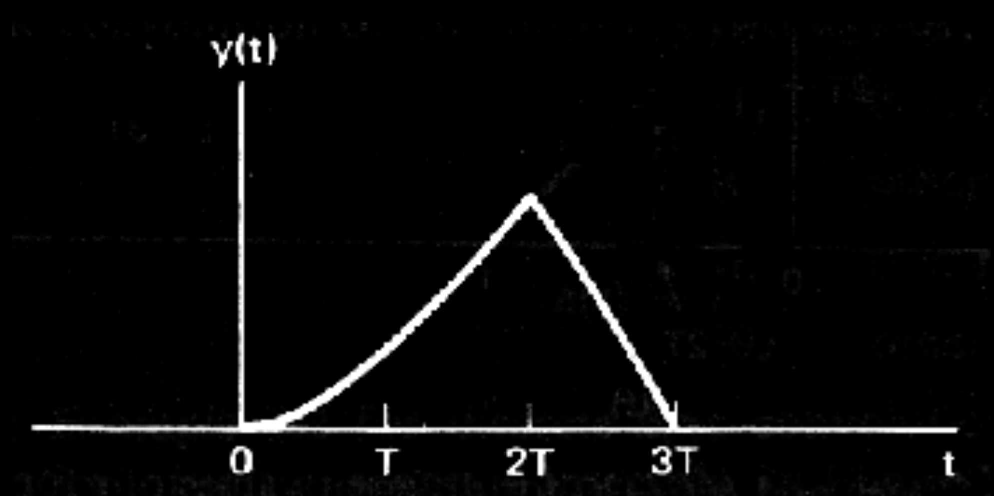
$$\boxed{y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau}$$

$$y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau$$

$$y(t)=\begin{cases}0, & t<0\\ \frac{1}{2}t^2, & 0<t<T\\ Tt-\frac{1}{2}T^2, & T<t<2T\\ -\frac{1}{2}t^2+Tt+\frac{3}{2}T^2, & 2T<t<3T\\ 0, & 3T<t\end{cases}$$







Convolution of discrete-time signals

- Convolution sum

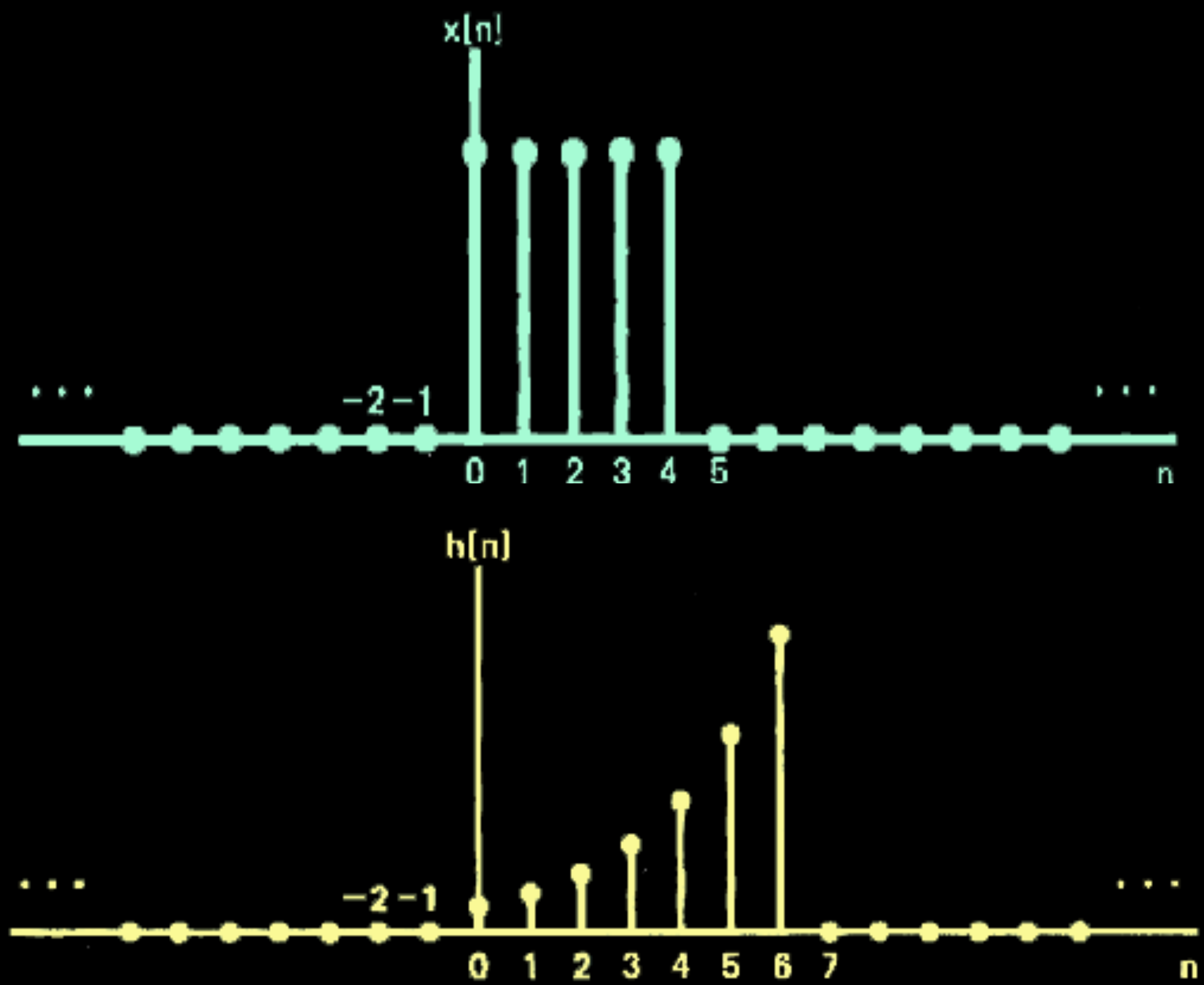
$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

$$y[n] = x[n] * h[n]$$

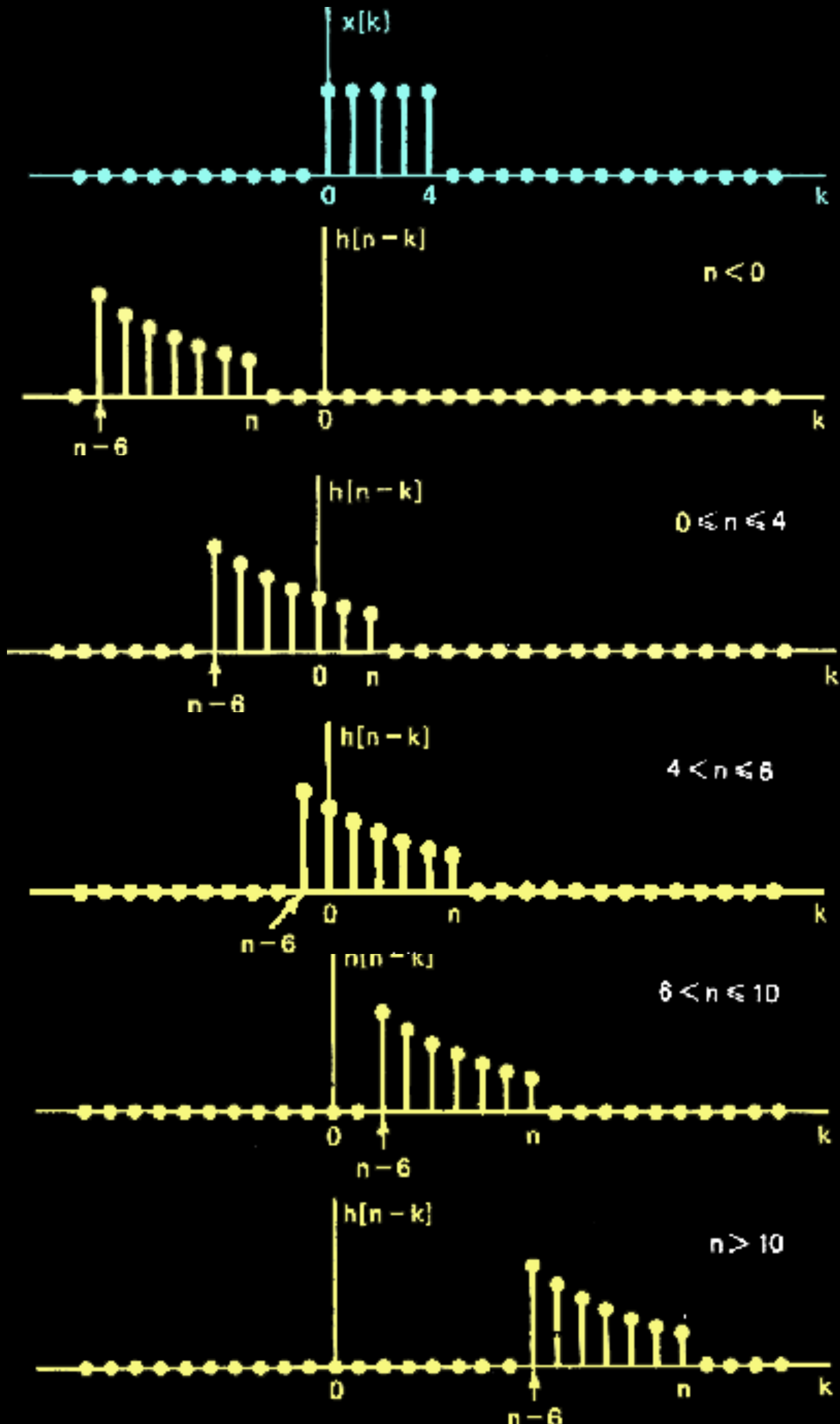
Convolution Sum

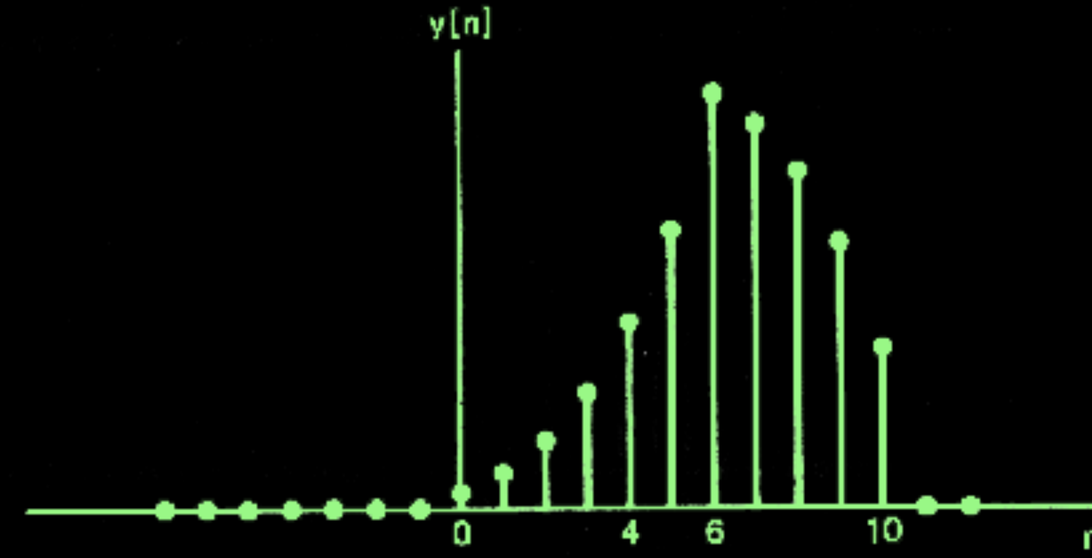
$$x[n]=\begin{cases}1, & 0\le n\le 4\\ 0, & otherwise\end{cases}$$

$$h[n]=\begin{cases}\alpha^n, & 0\le n\le 6\\ 0, & otherwise\end{cases}$$



$$\boxed{y[n]=\sum_{k=-\infty}^{+\infty}x[k]h[n-k]}$$





$$y[n]=\begin{cases} 0, & n<0 \\ \dfrac{1-\alpha^{n+1}}{1-\alpha}, & 0\le n\le 4 \\ \dfrac{\alpha^{n-4}-\alpha^{n+1}}{1-\alpha}, & 4<n\le 6 \\ \dfrac{\alpha^{n-4}-\alpha^{7}}{1-\alpha}, & 6<n\le 10 \\ 0, & 10<n \end{cases}$$

Properties

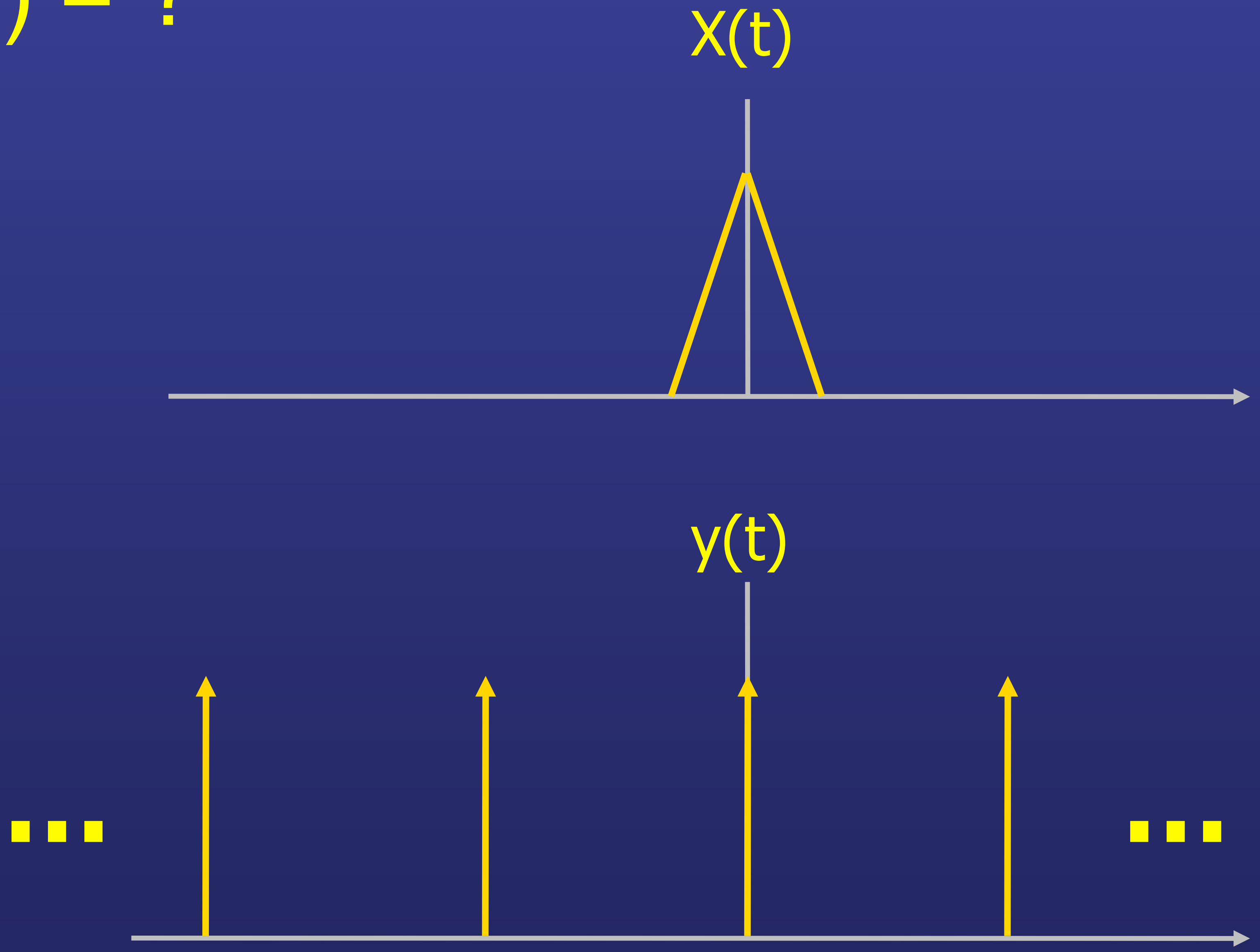
$$x(t)*h(t)=h(t)*x(t)$$

$$x(t)*[h_1(t)*h_2(t)]=[x(t)*h_1(t)]*h_2(t)$$

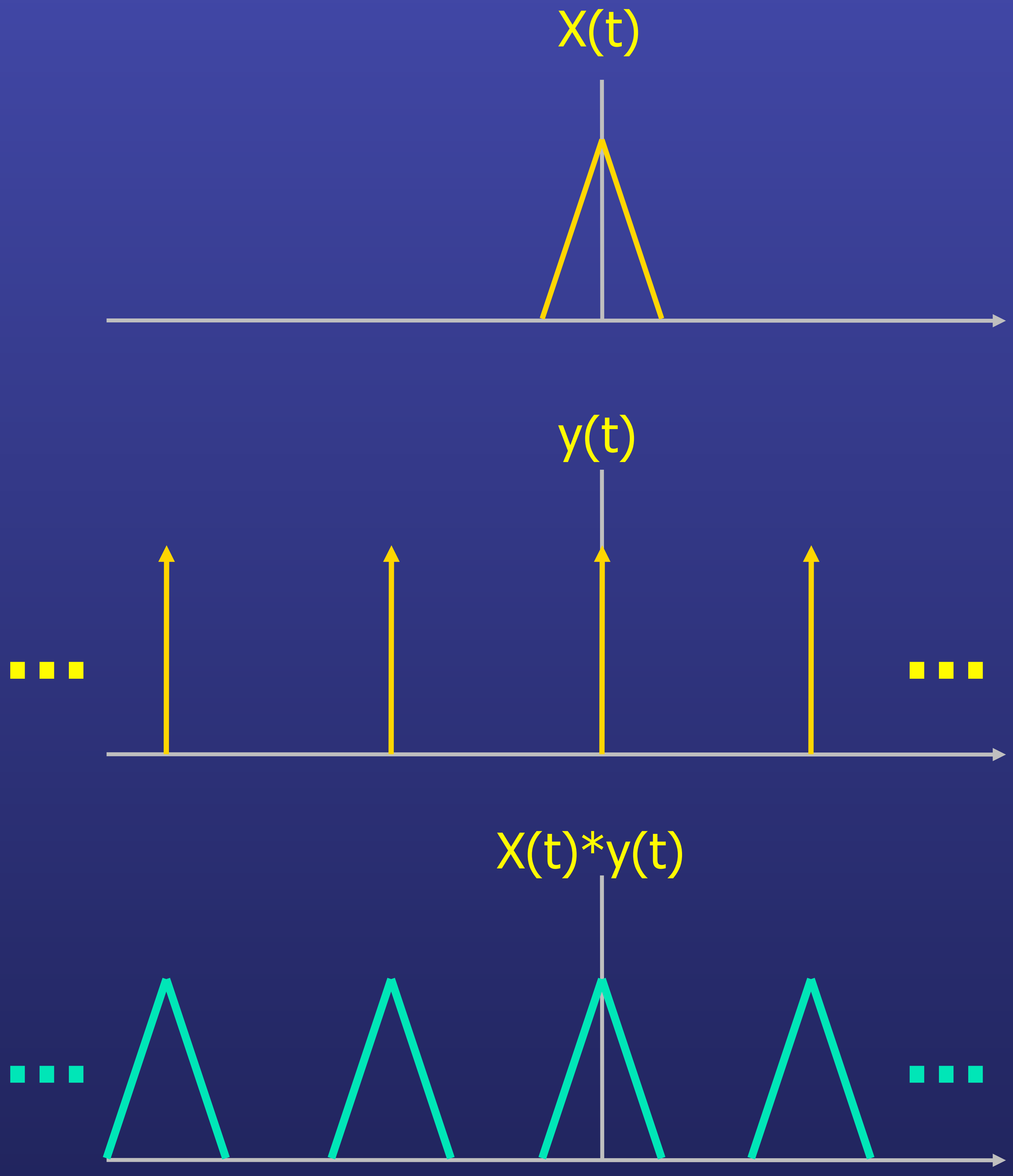
$$x(t)*[h_1(t)+h_2(t)]=[x(t)*h_1(t)]+[x(t)*h_2(t)]$$

Quiz

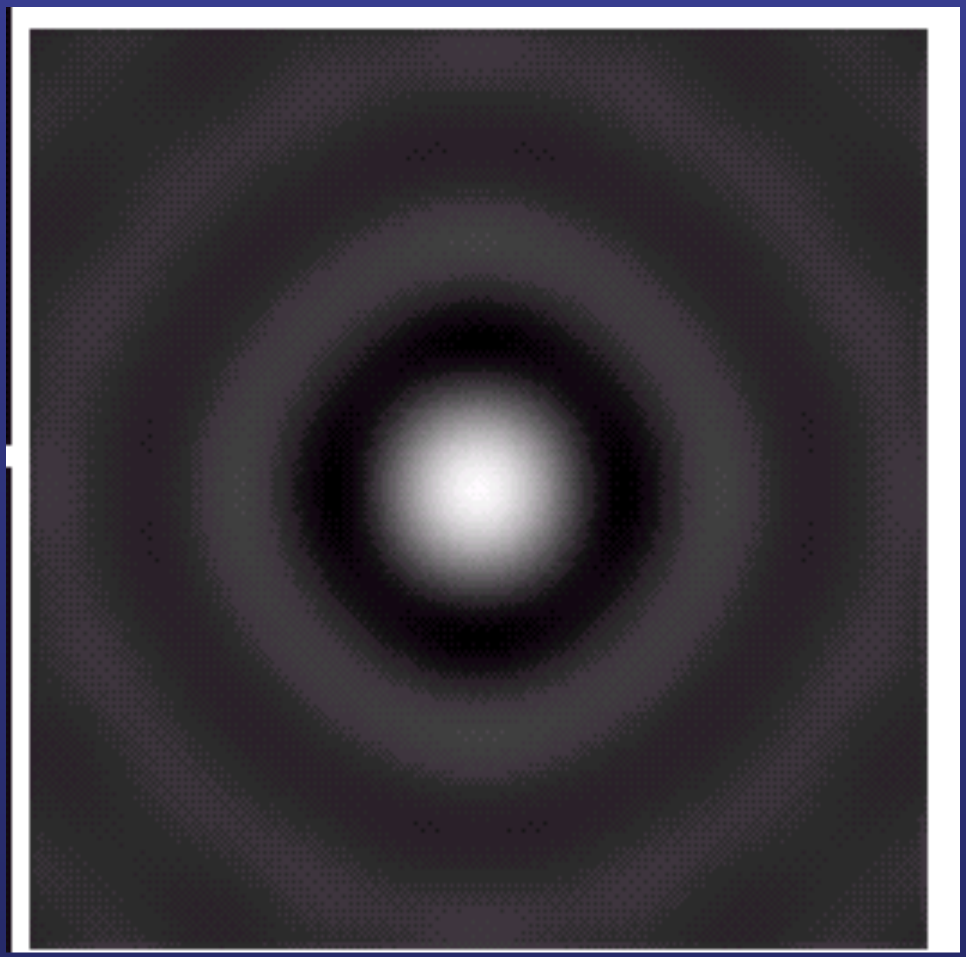
- $x(t) * y(t) = ?$



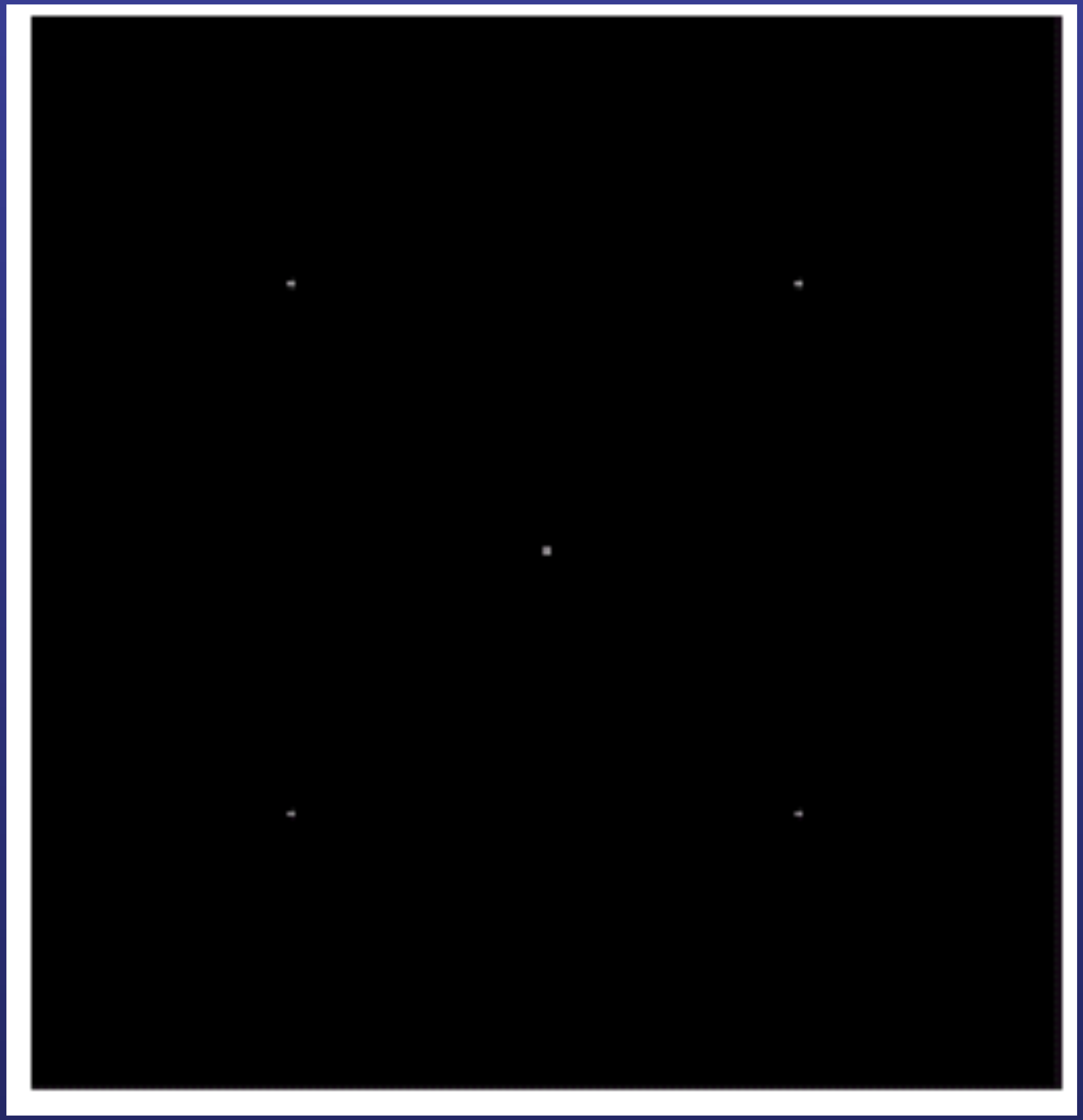
Quiz



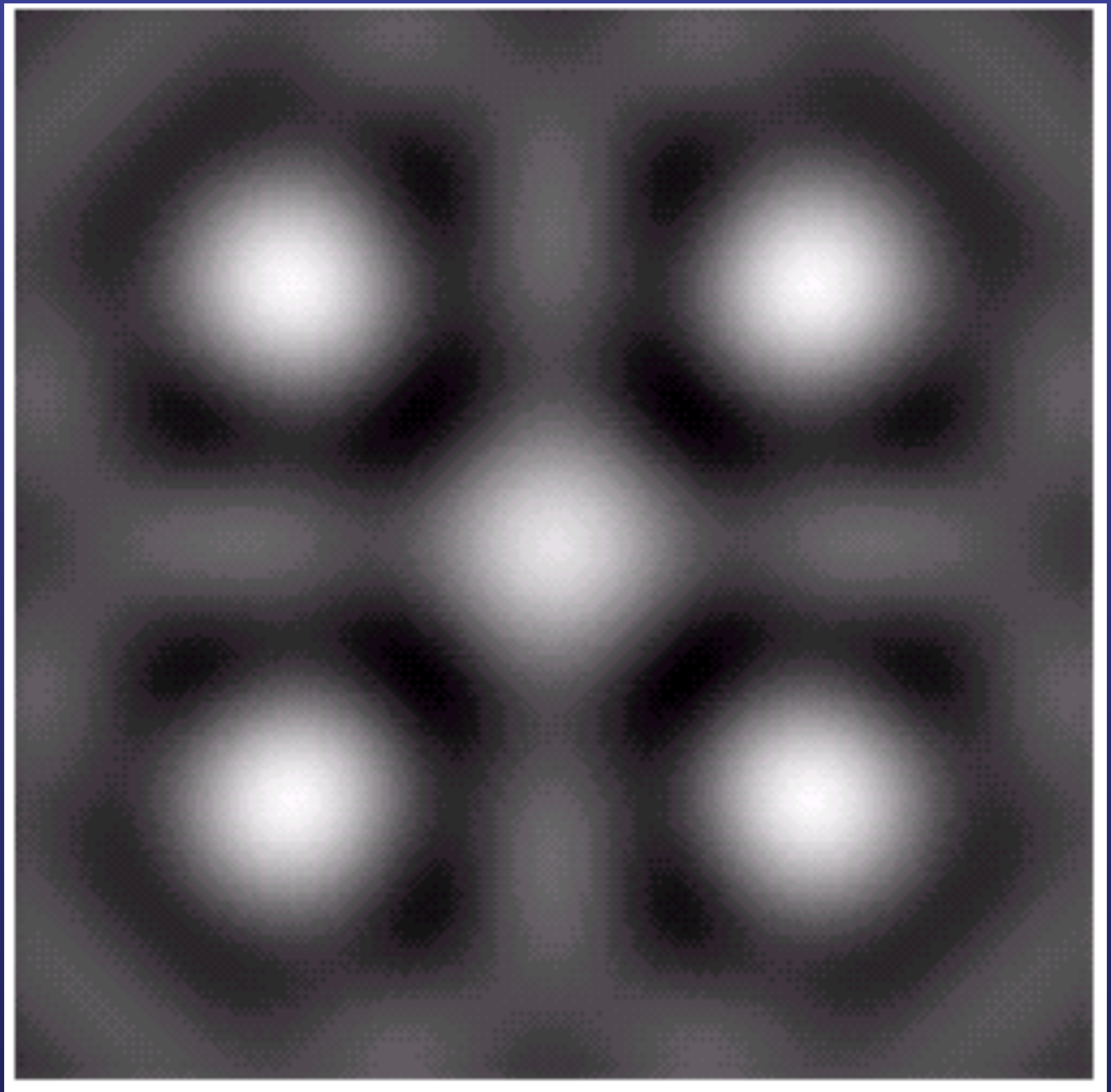
2D convolution

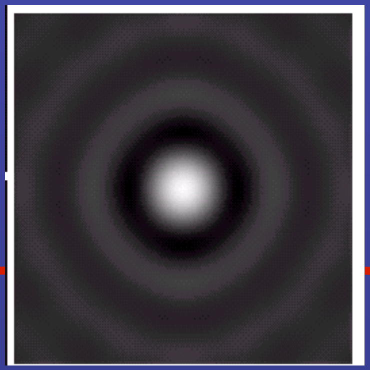


$*$



2D convolution





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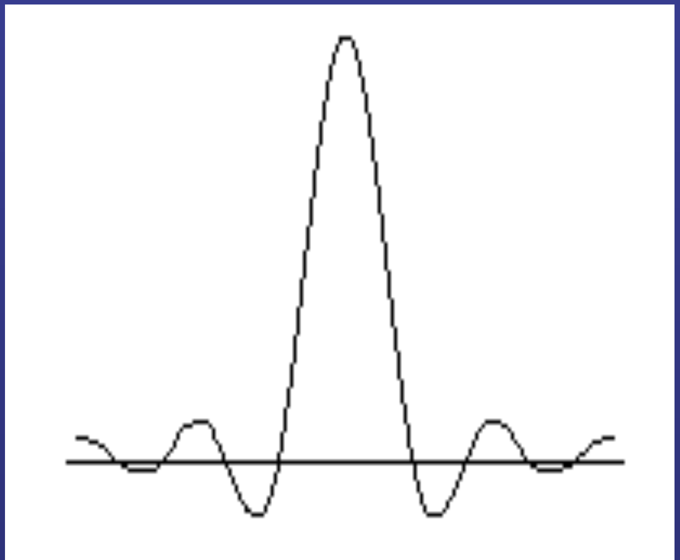
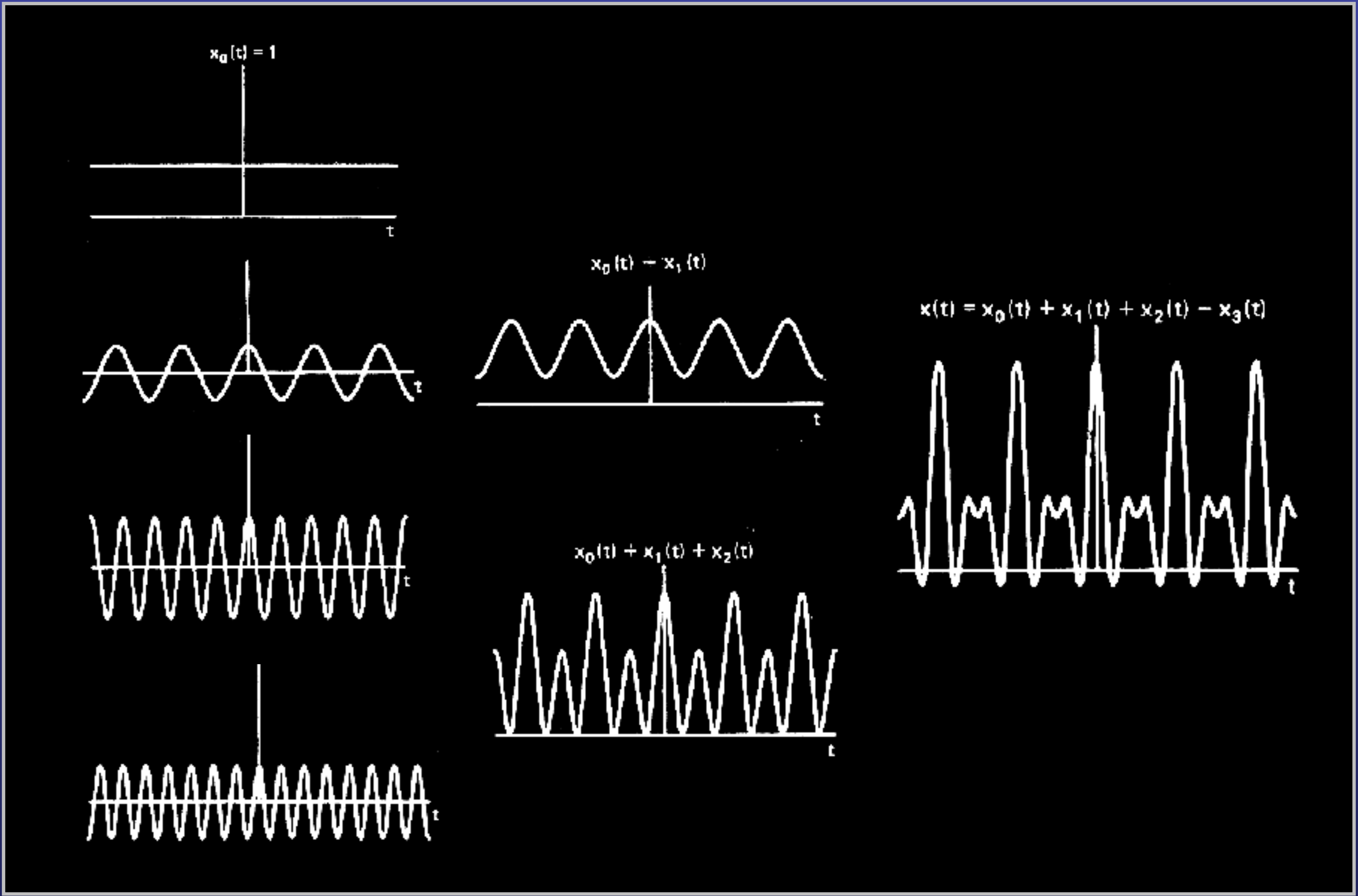


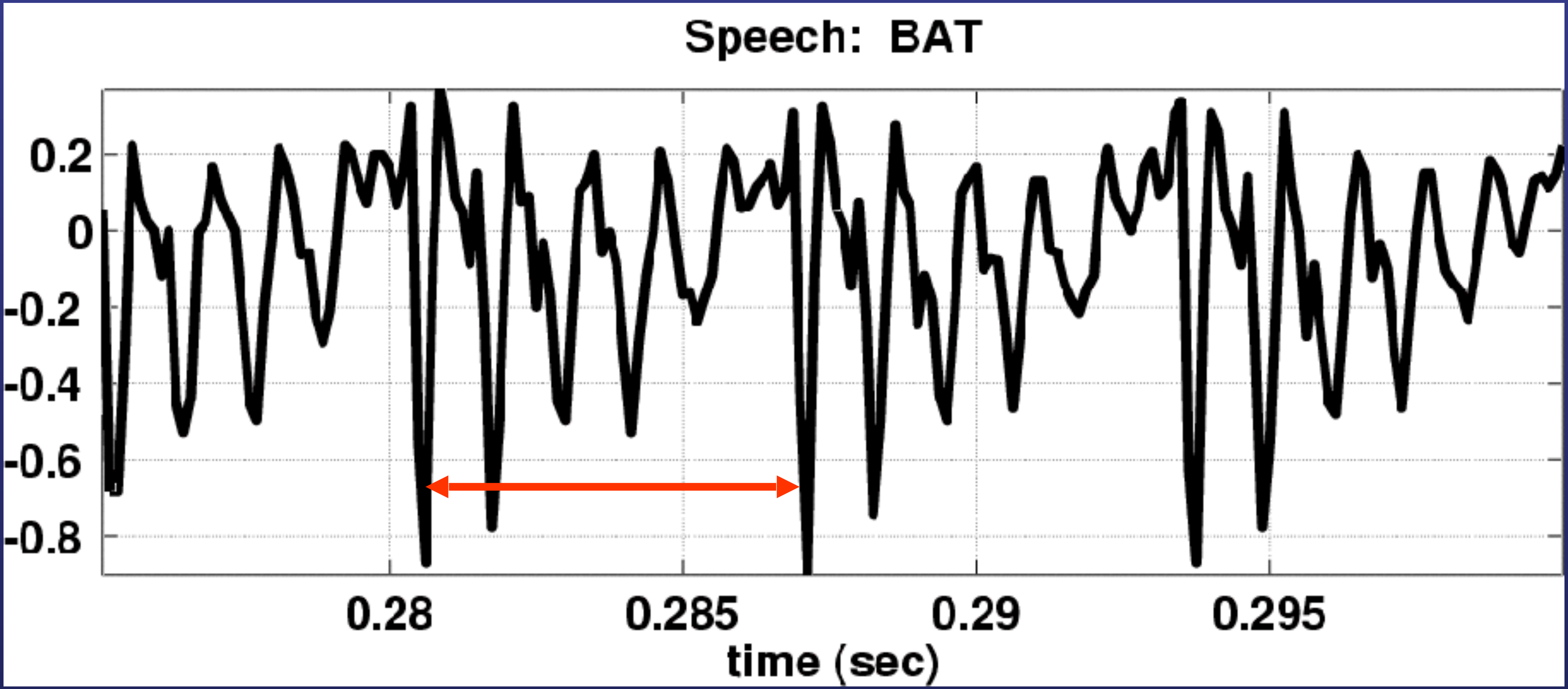
Signal representation



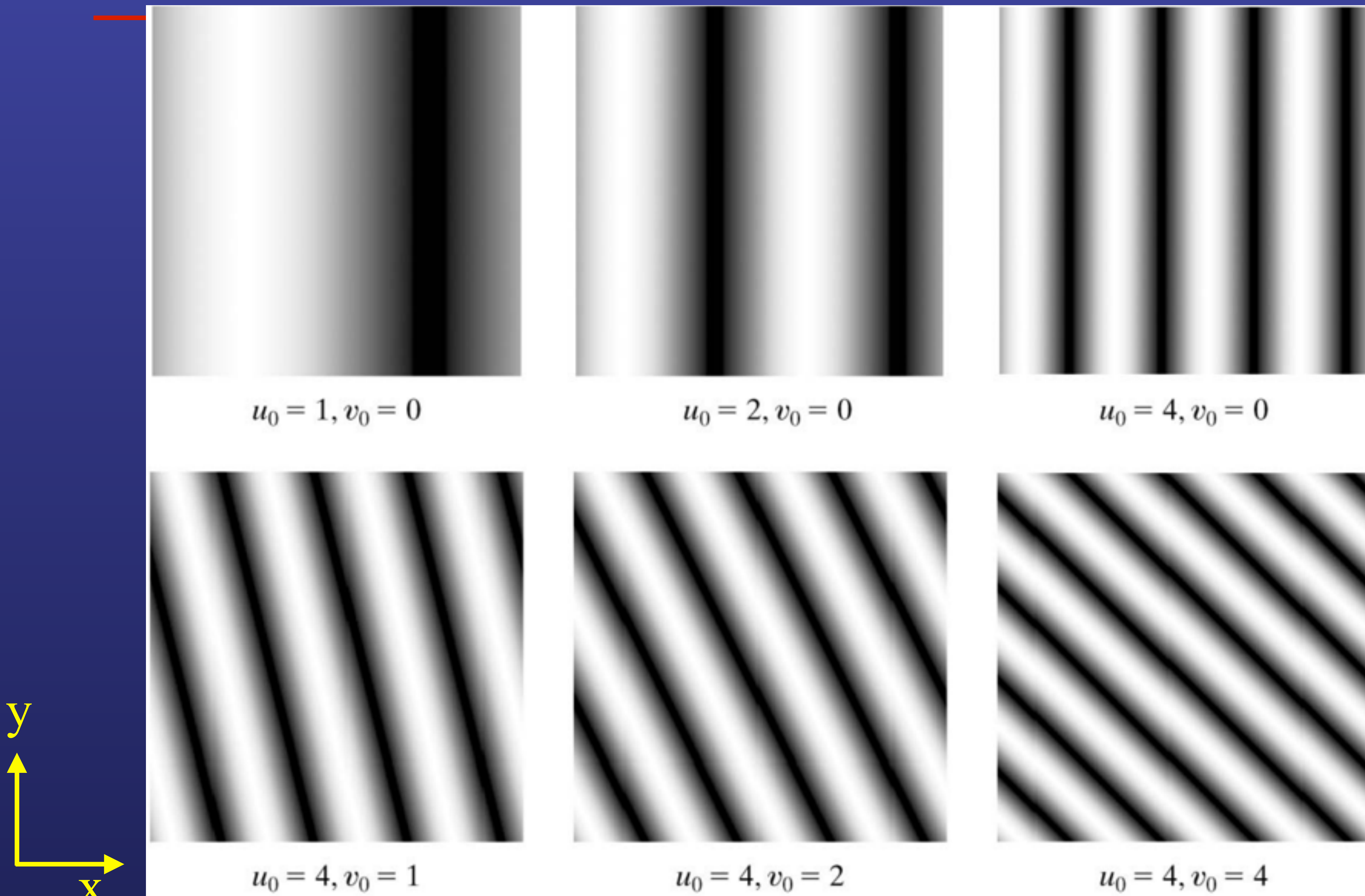
Sinusoidal and exponential sequences play an important role in representing discrete-time signals

Speech Signal: BAT

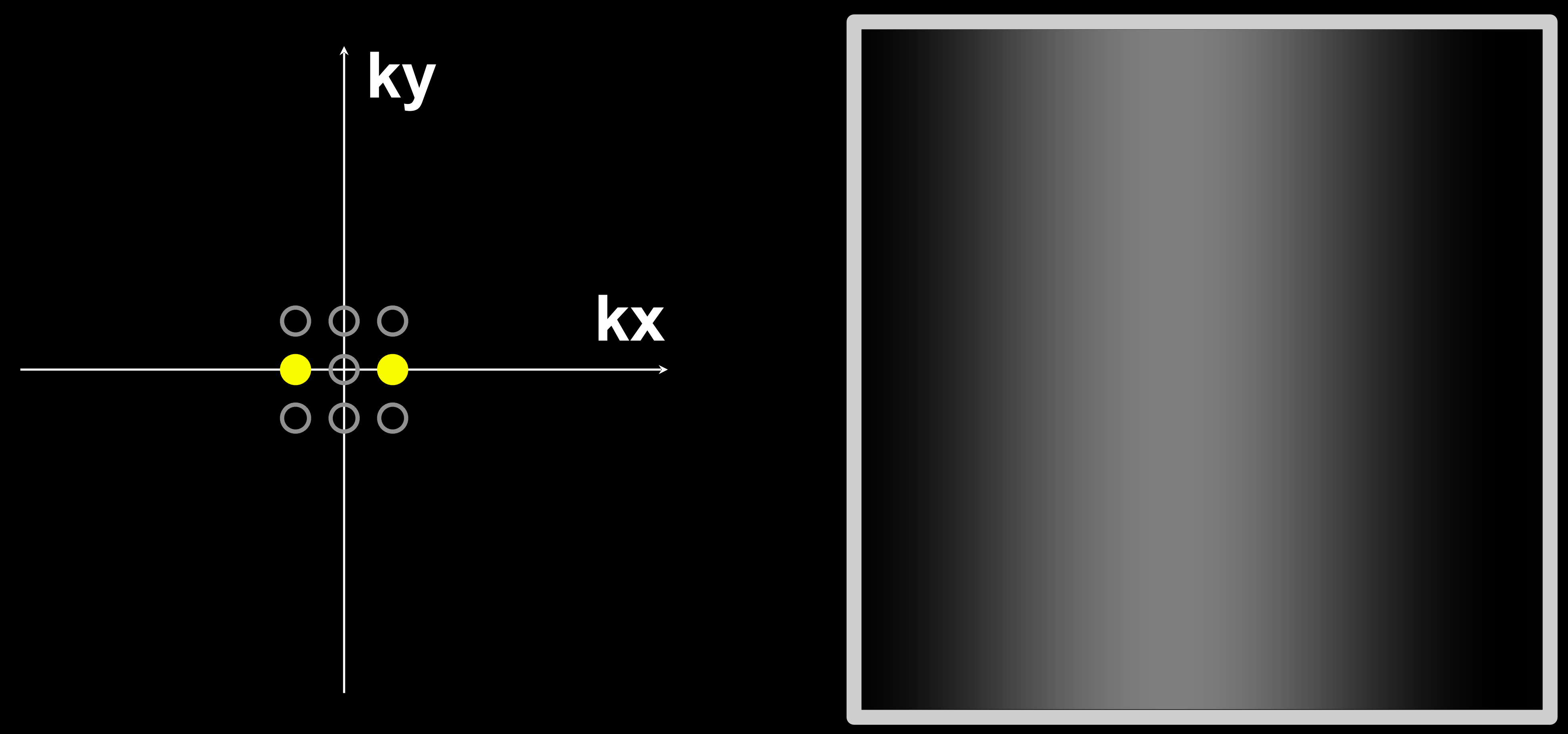
- Nearly **Periodic** in Vowel Region
  - Period is (Approximately) T = 0.0065 sec



$$s(x,y)=\sin[2\pi(u_0x+v_0y)]$$

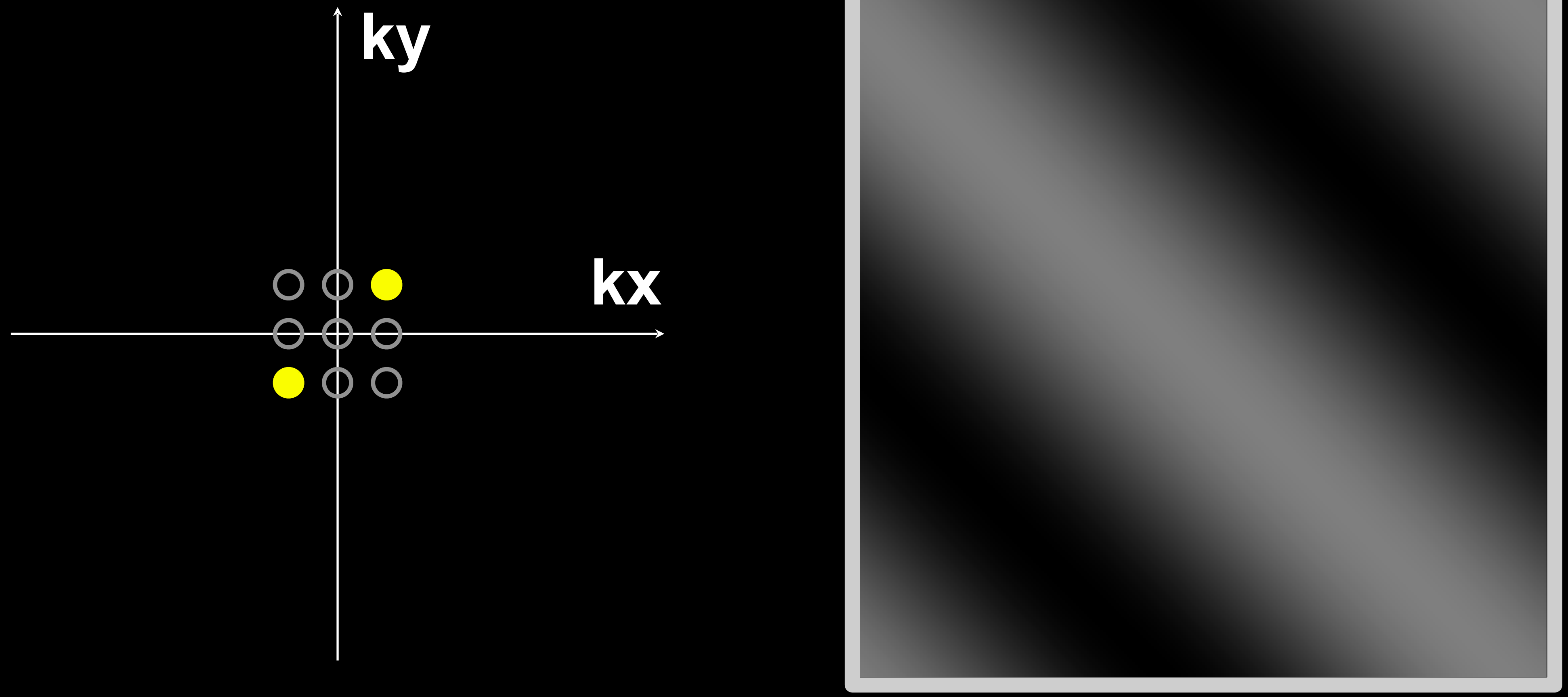


kx = +/- 1, ky = 0 (cycles/FOV)



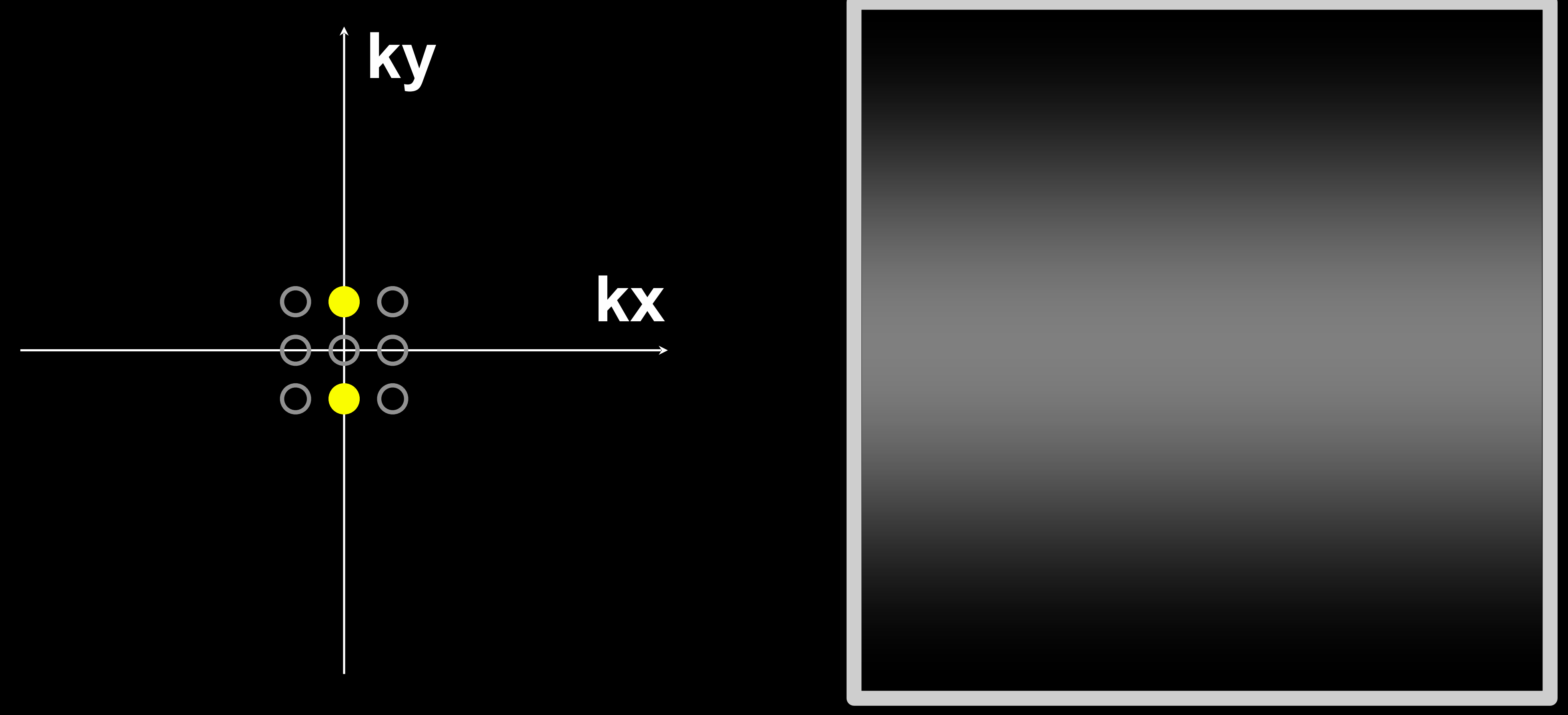
x 方向變化 1 個週期　　y 方向變化 0 個週期

kx = +/- 1, ky = +/- 1



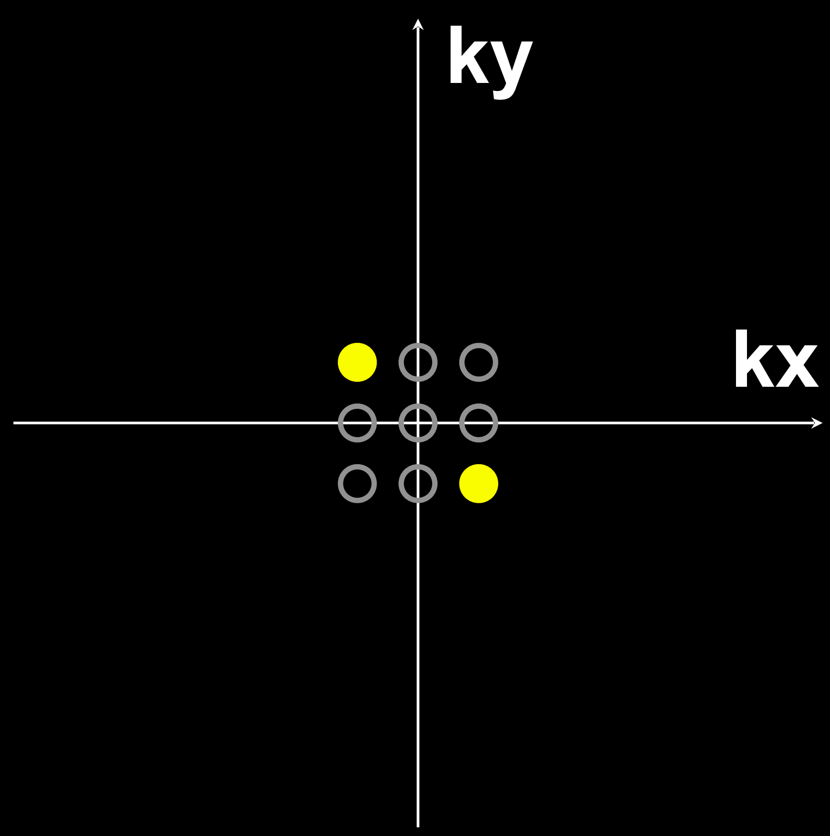
x 方向變化 1 個週期　　y 方向變化 1 個週期

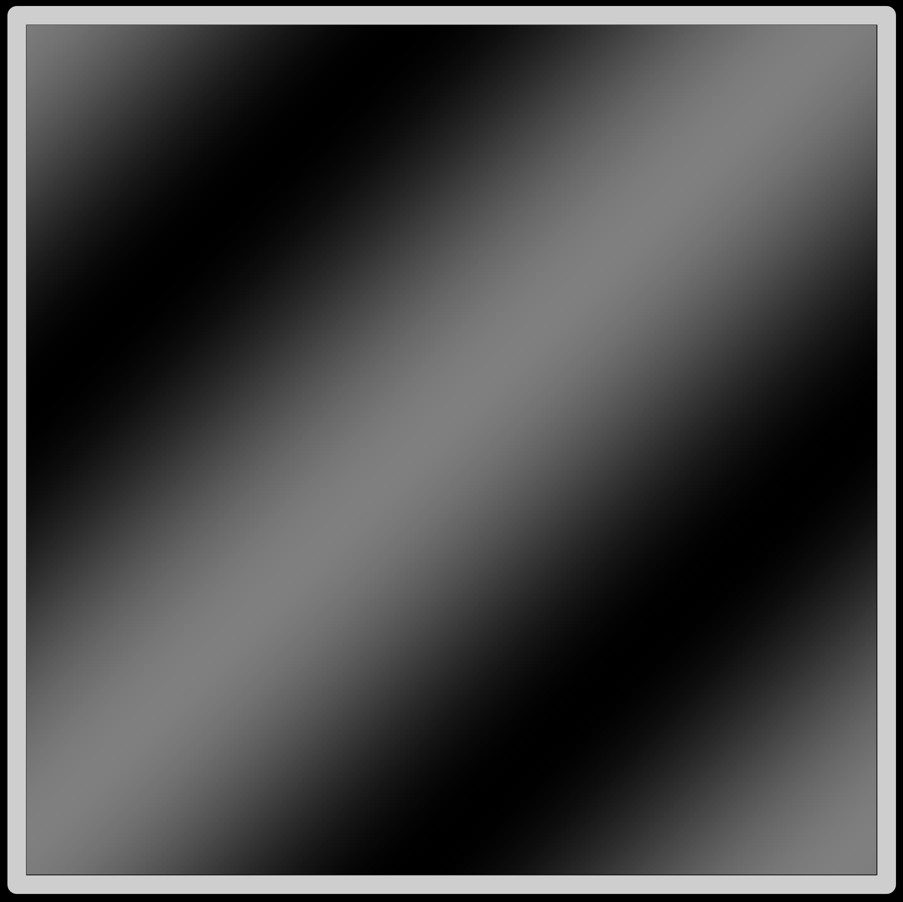
kx = 0 , ky = +/- 1



x 方向變化 0 個週期　　y 方向變化 1 個週期

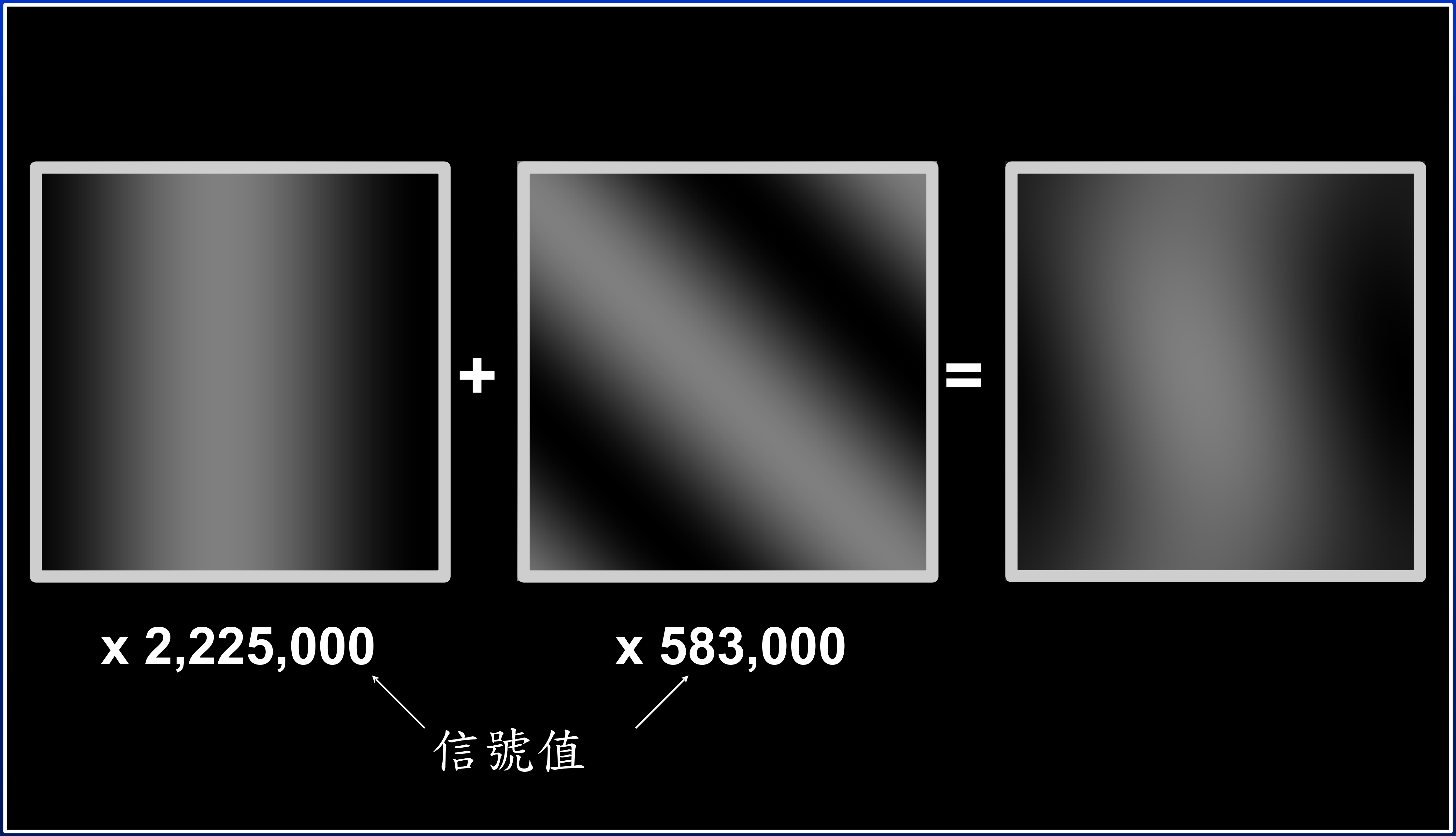
kx = +/- 1, ky = -/+ 1



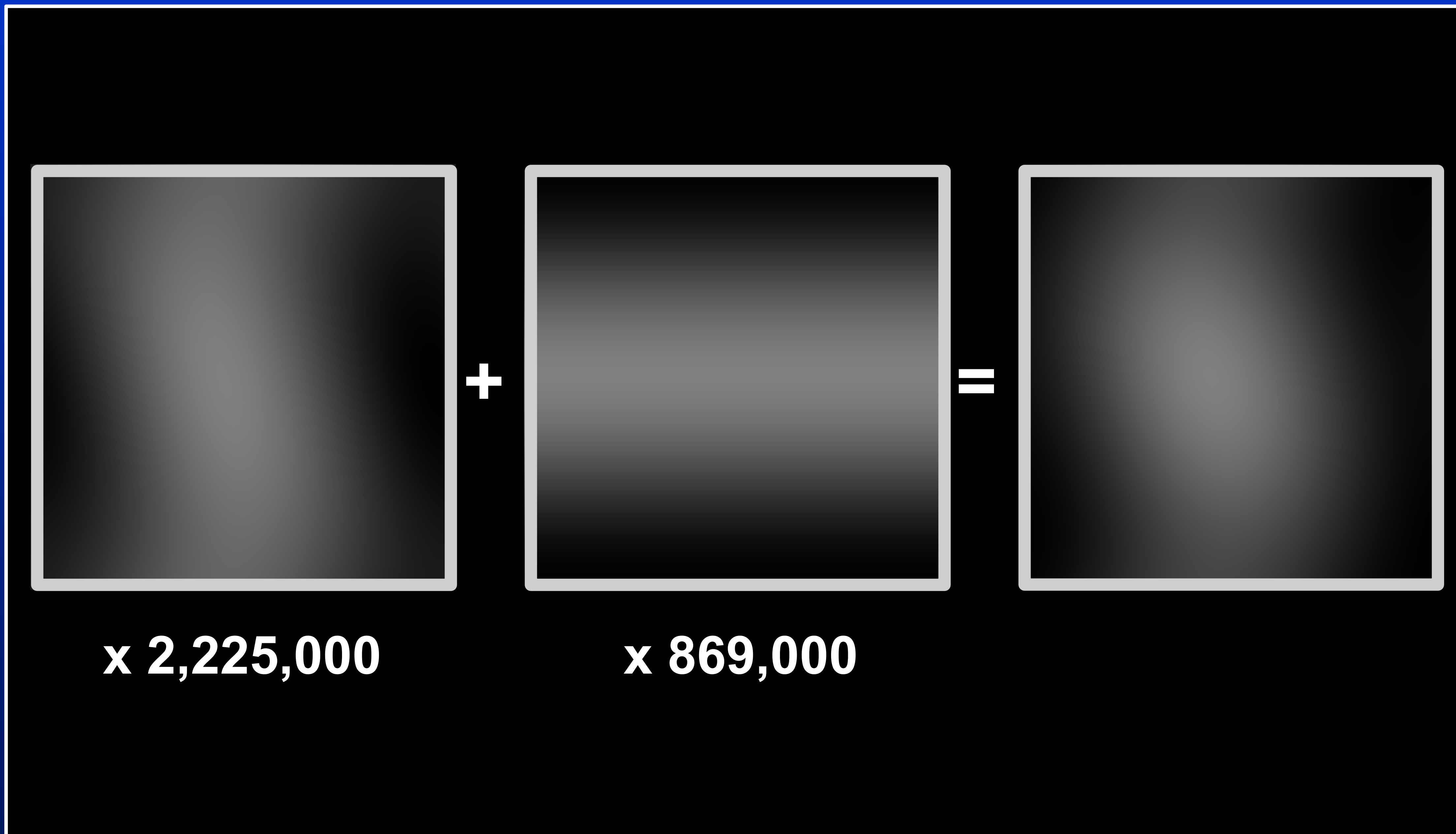


x 方向變化 1 個週期　　y 方向變化 -1 個週期

影像 = 各波形成份的加權總和



影像 = 各波形成份的加權總和



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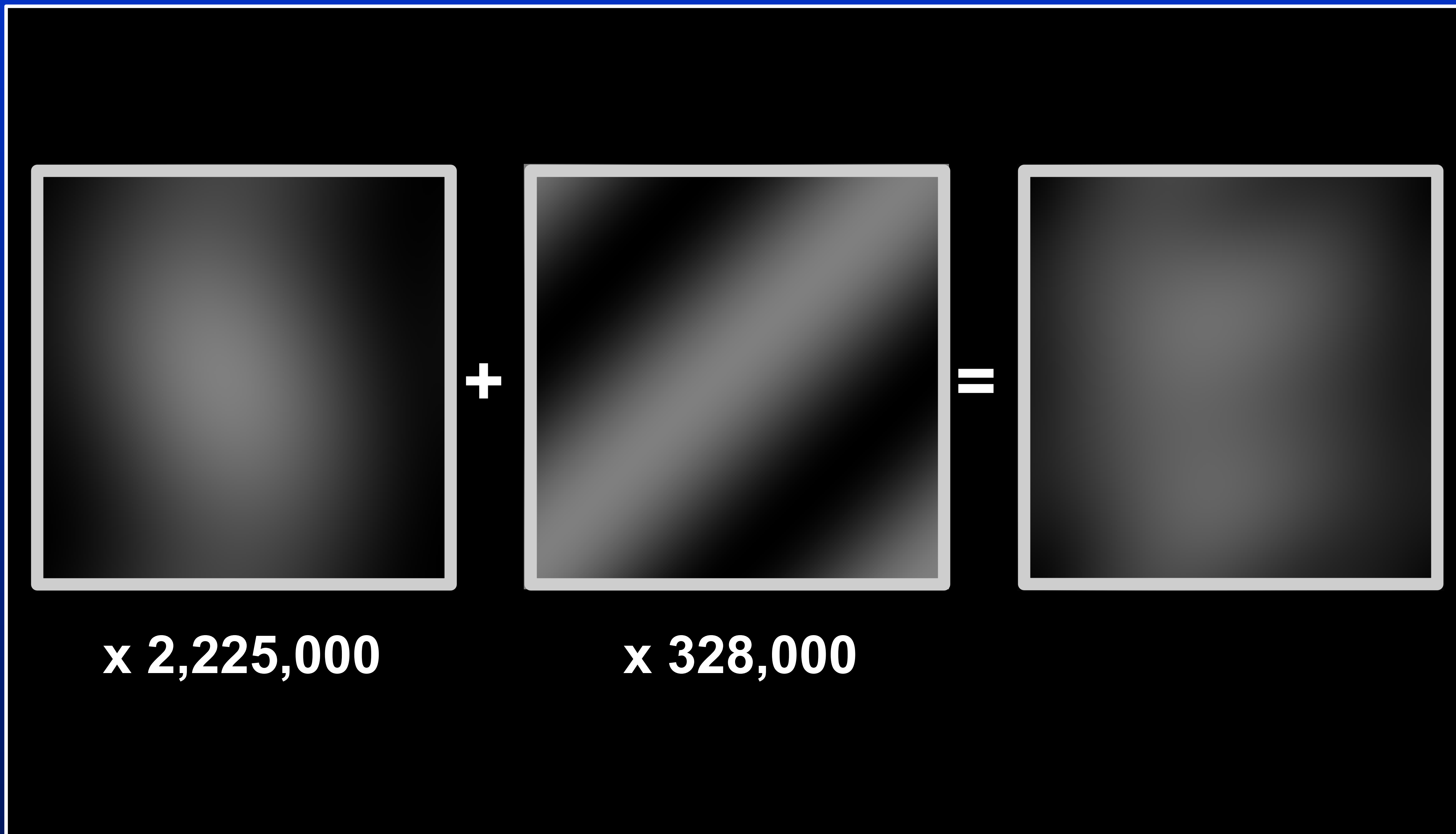
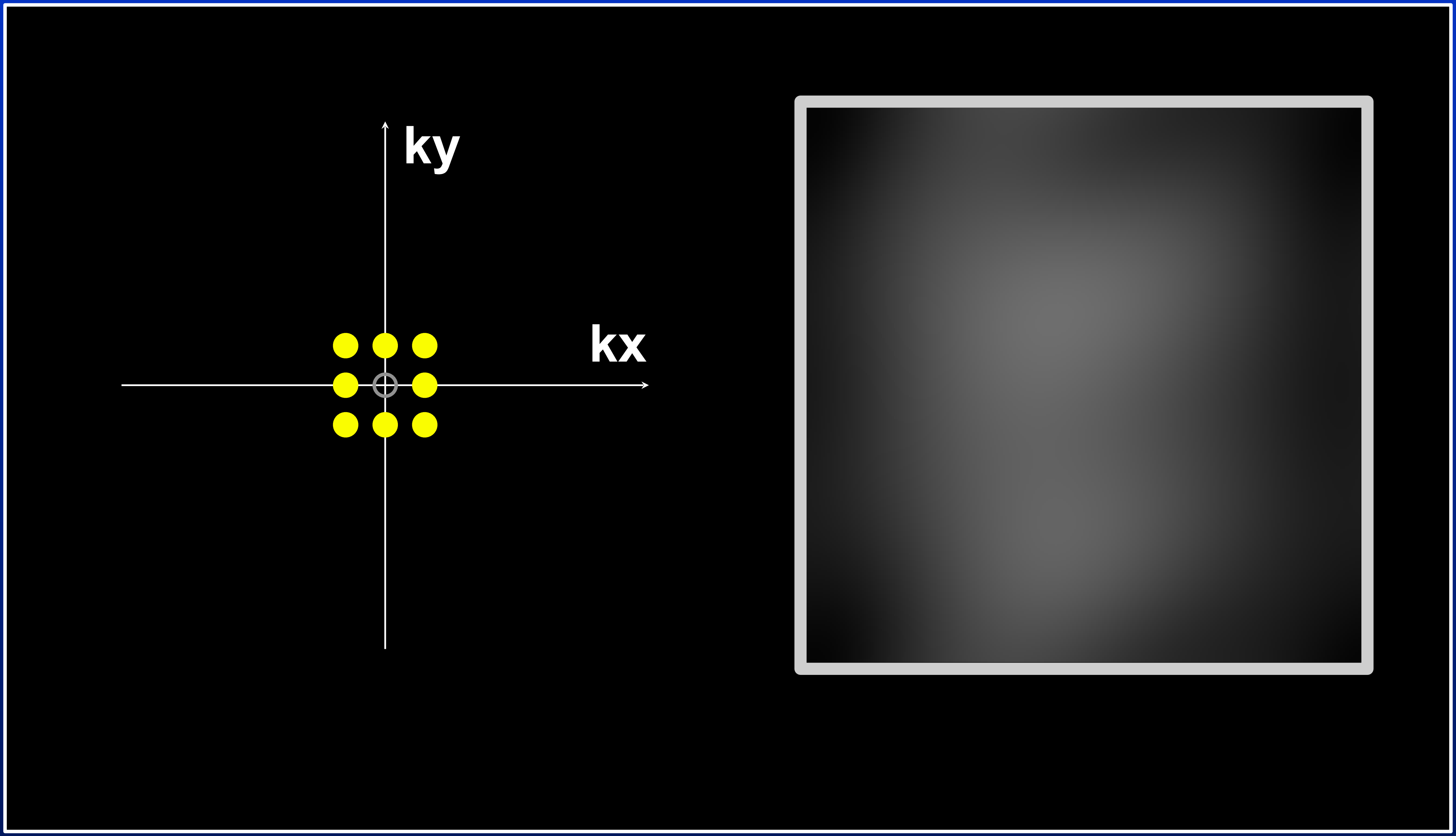
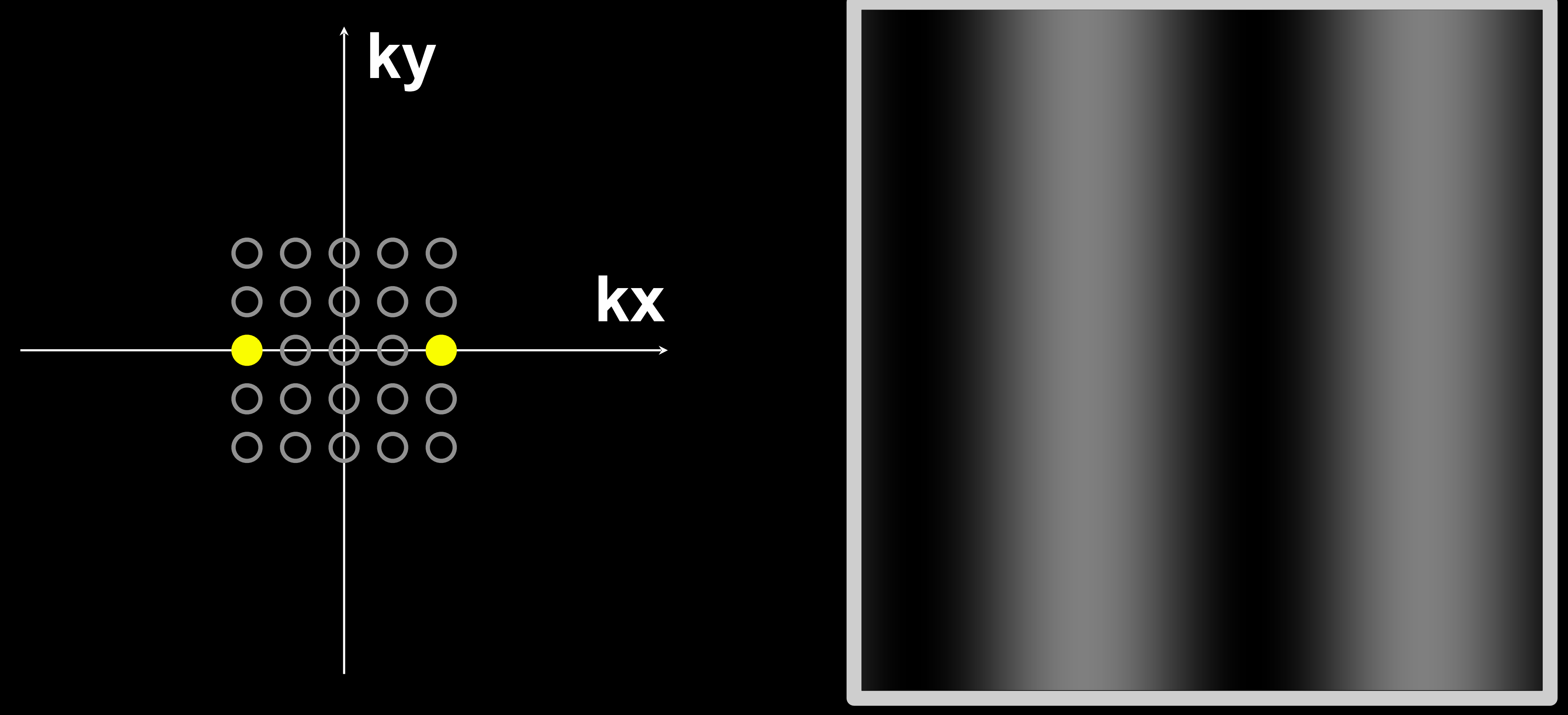


由以上八個數據所組合而成的影像

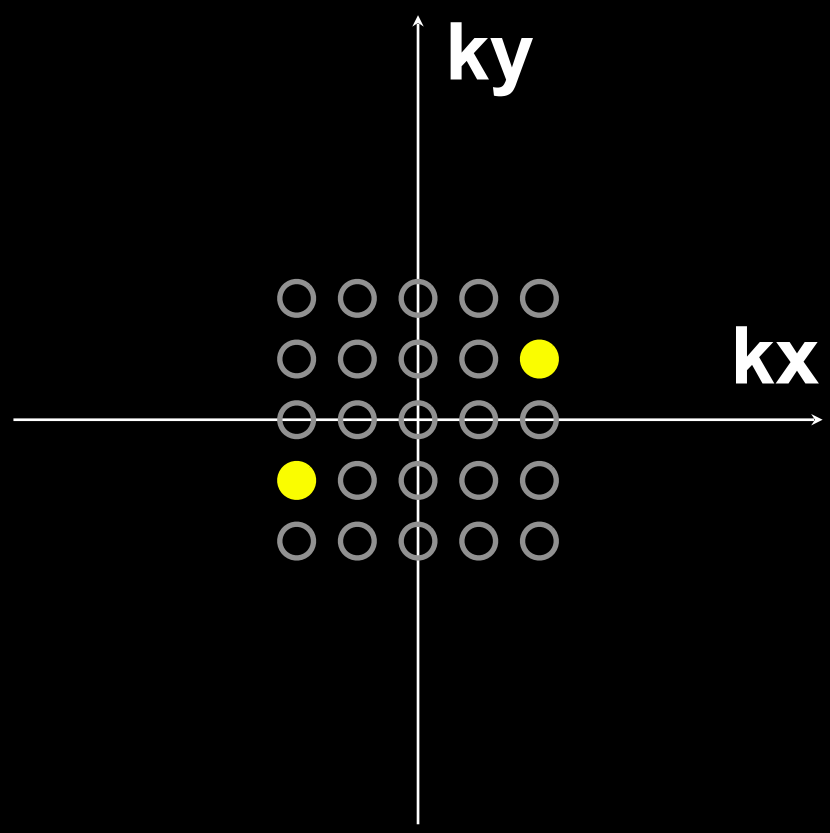


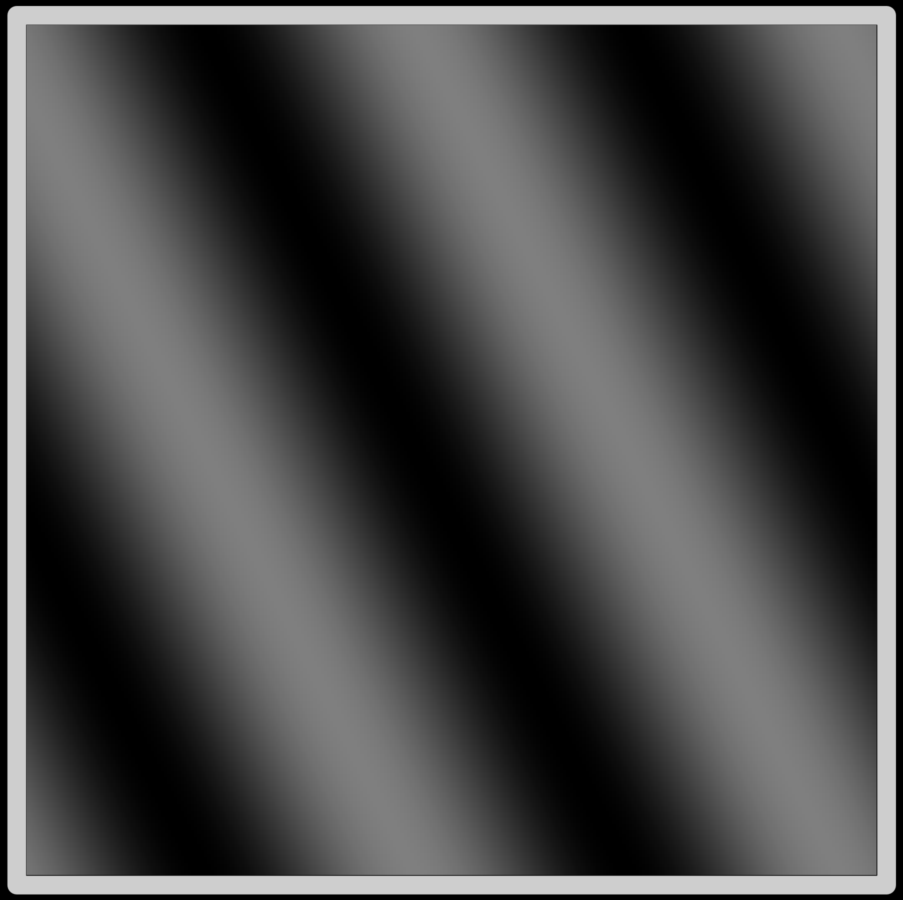
kx = +/- 2 , ky = 0



x 方向變化 2 個週期　　y 方向變化 0 個週期

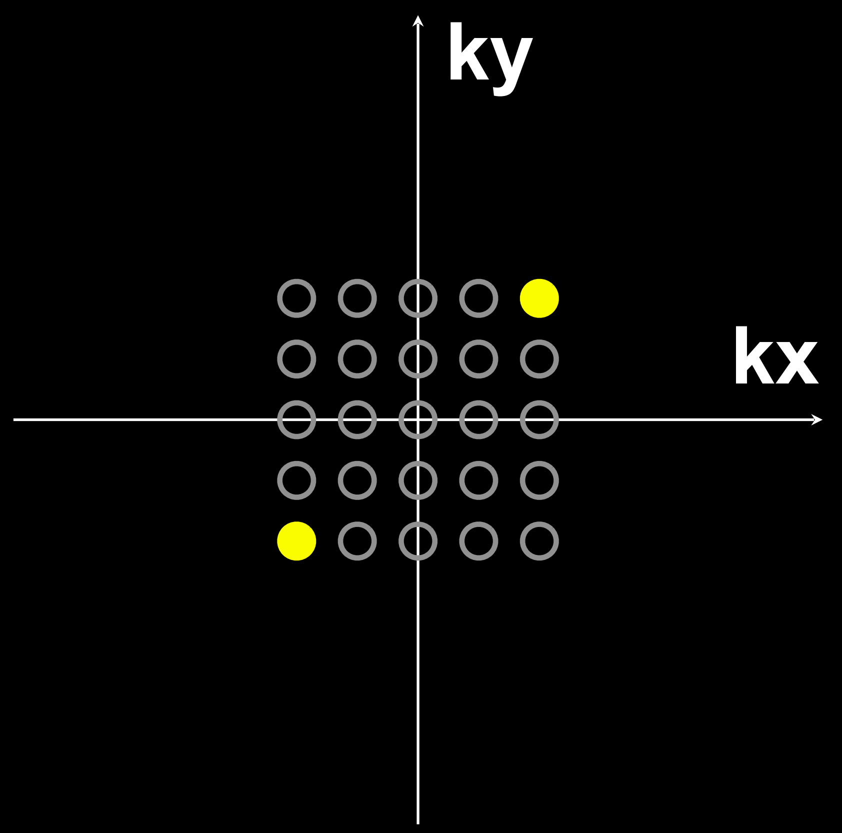
kx = +/- 2 , ky = +/- 1

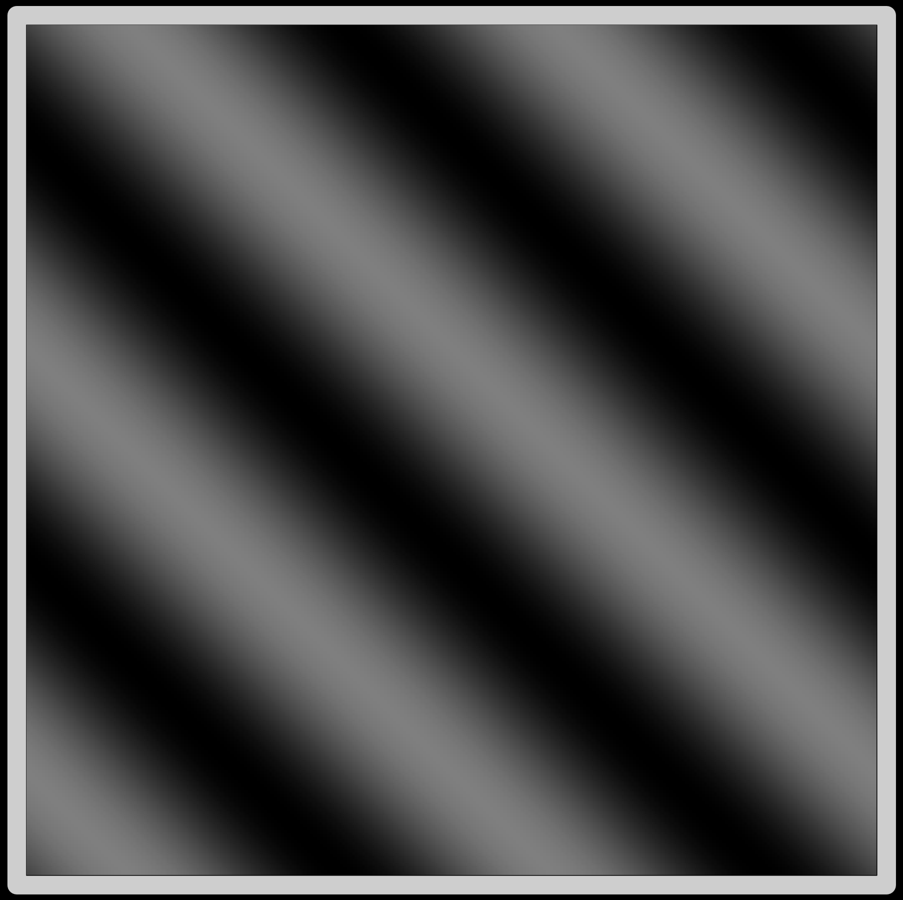




x 方向變化 2 個週期　　y 方向變化 1 個週期

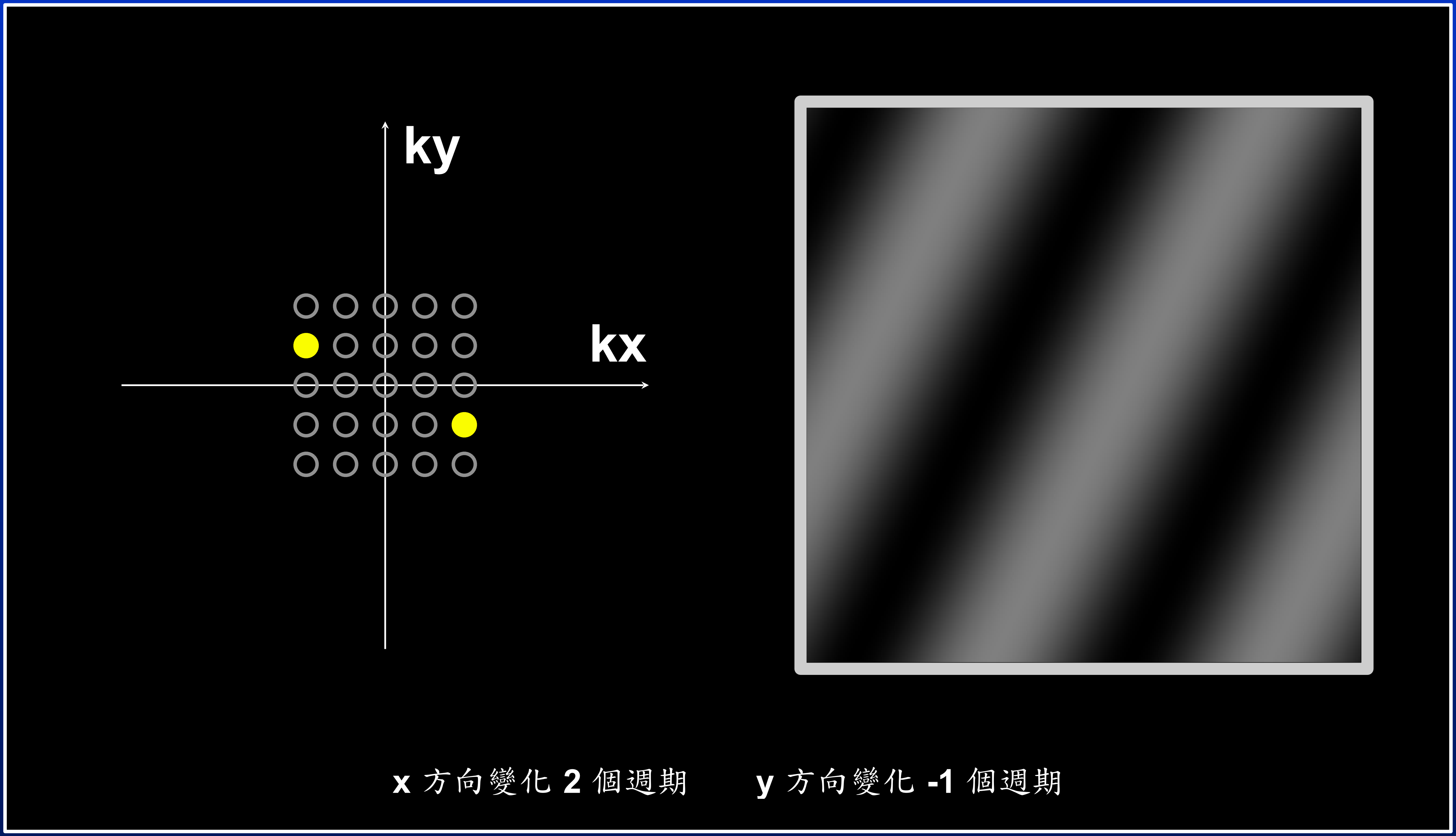
kx = +/- 2 , ky = +/- 2



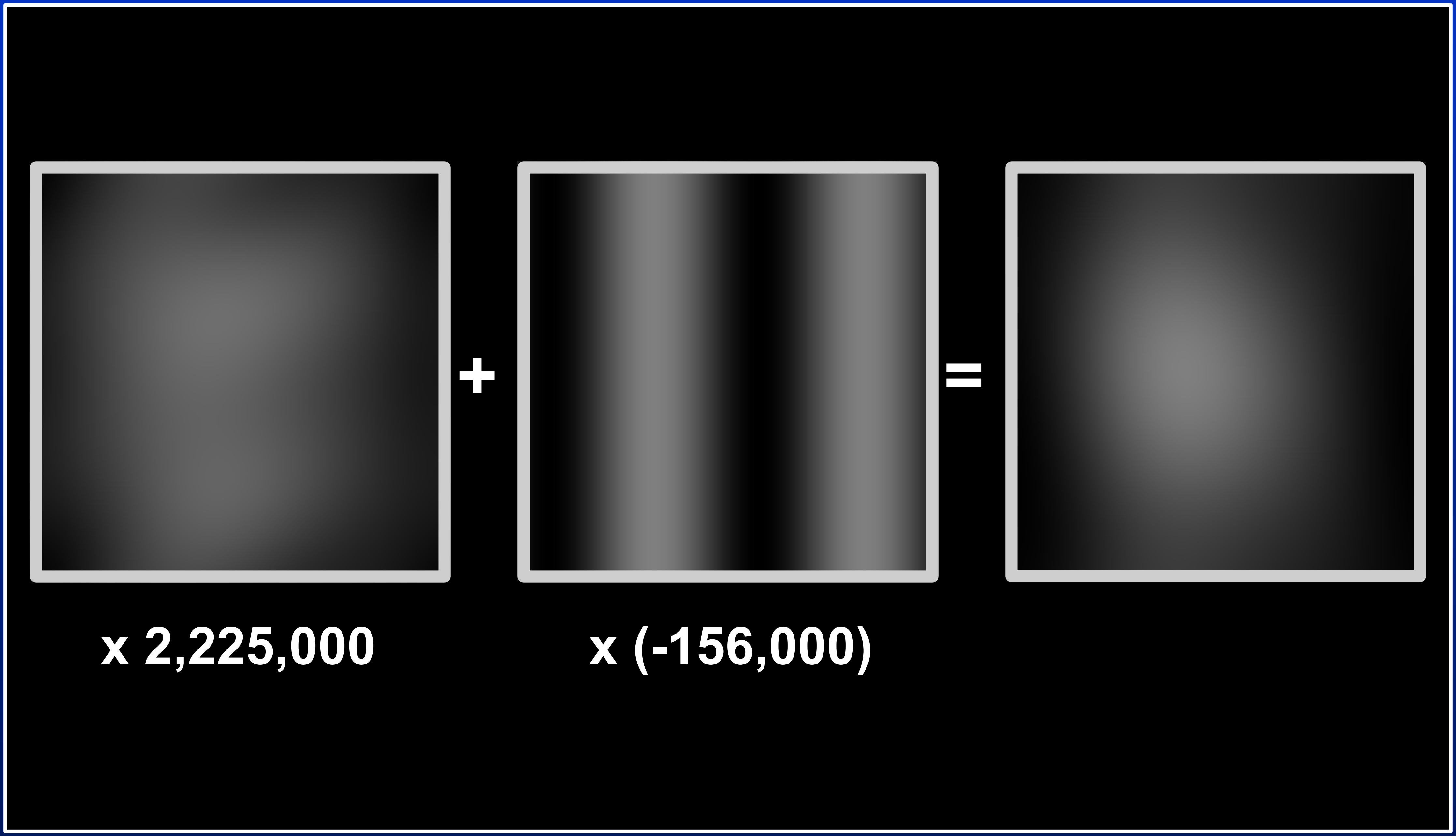


x 方向變化 2 個週期　　y 方向變化 2 個週期

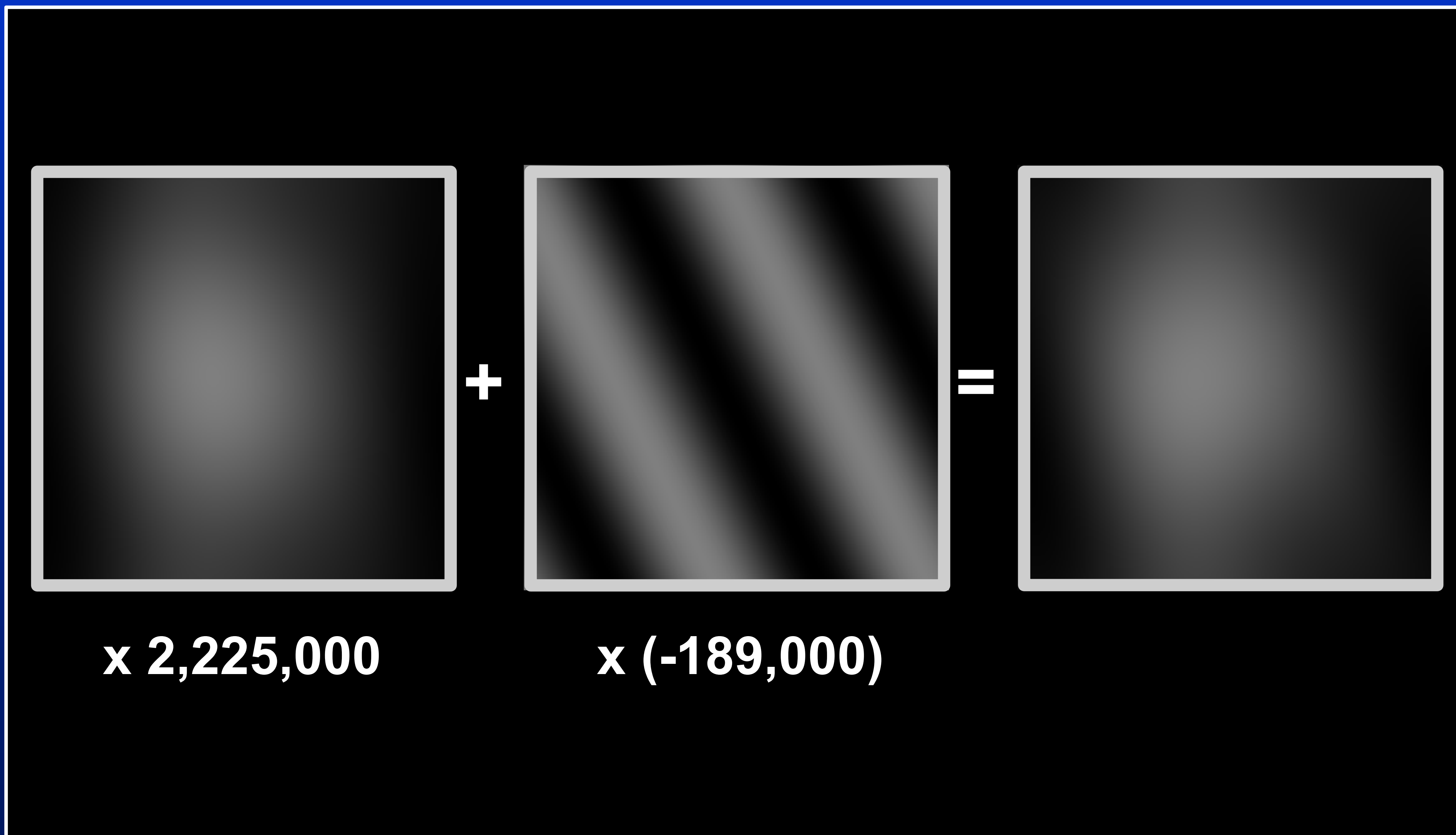
kx = +/- 2 , ky = -/+ 1



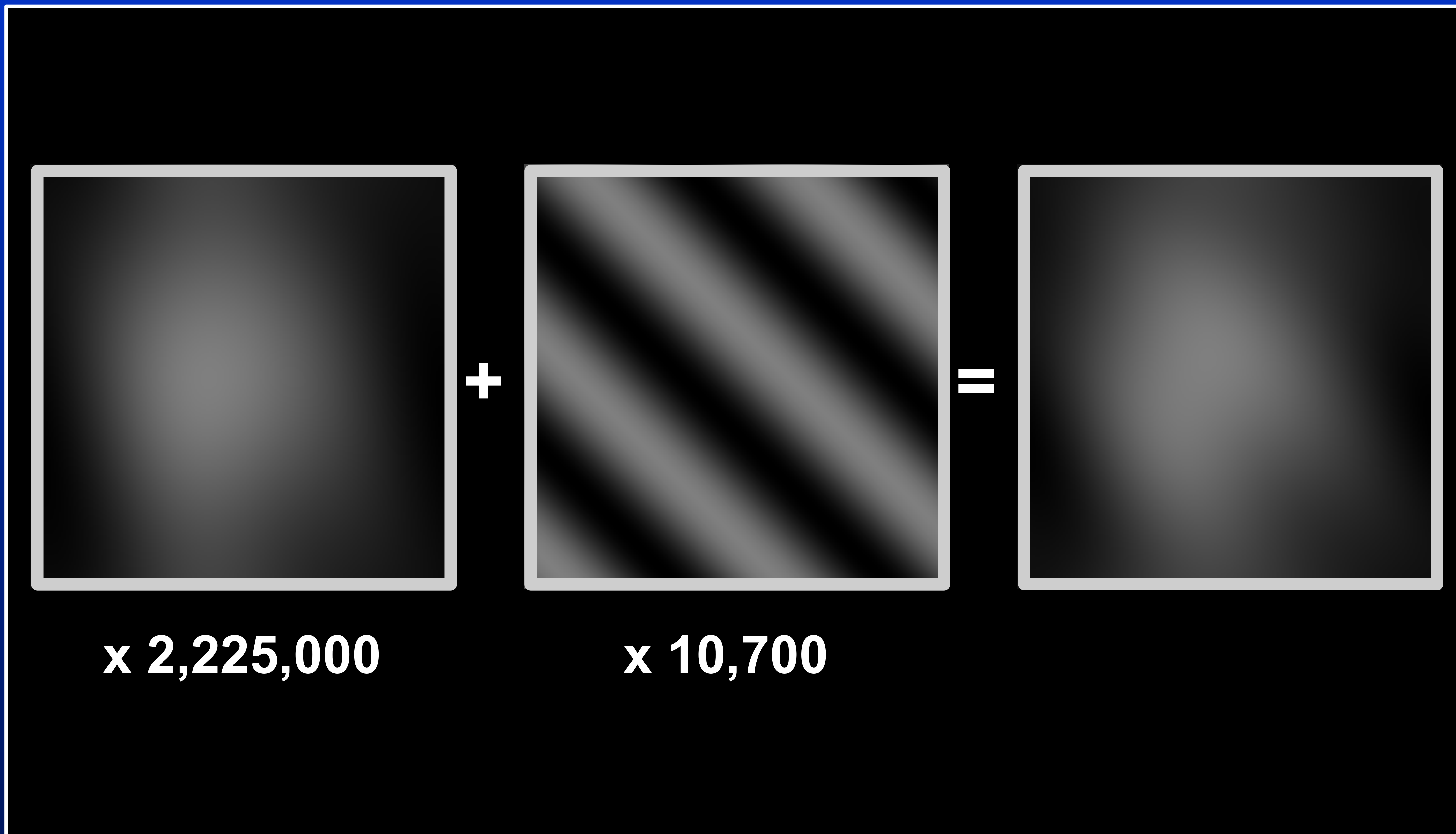
影像 = 各波形成份的加權總和



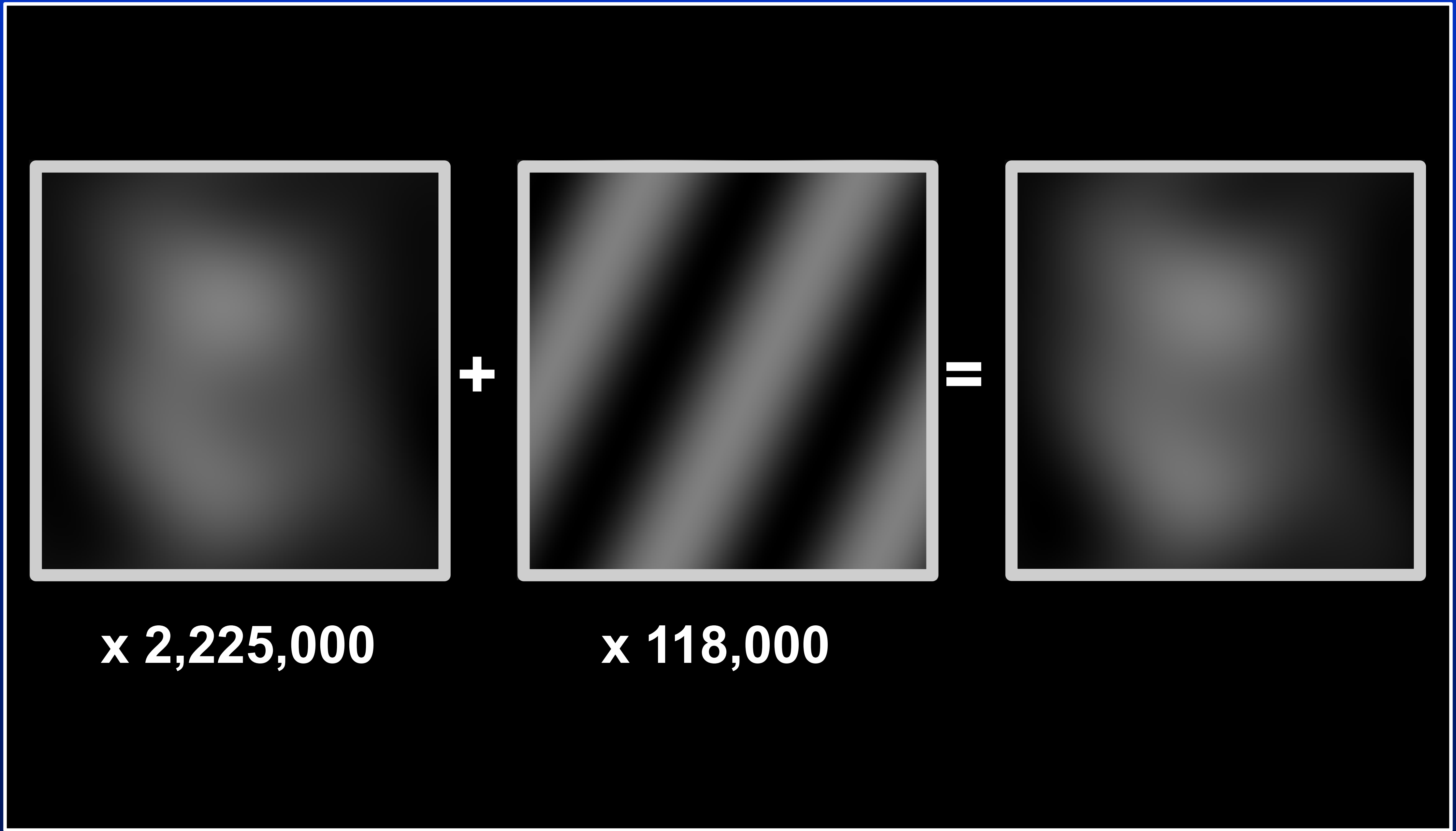
影像 = 各波形成份的加權總和



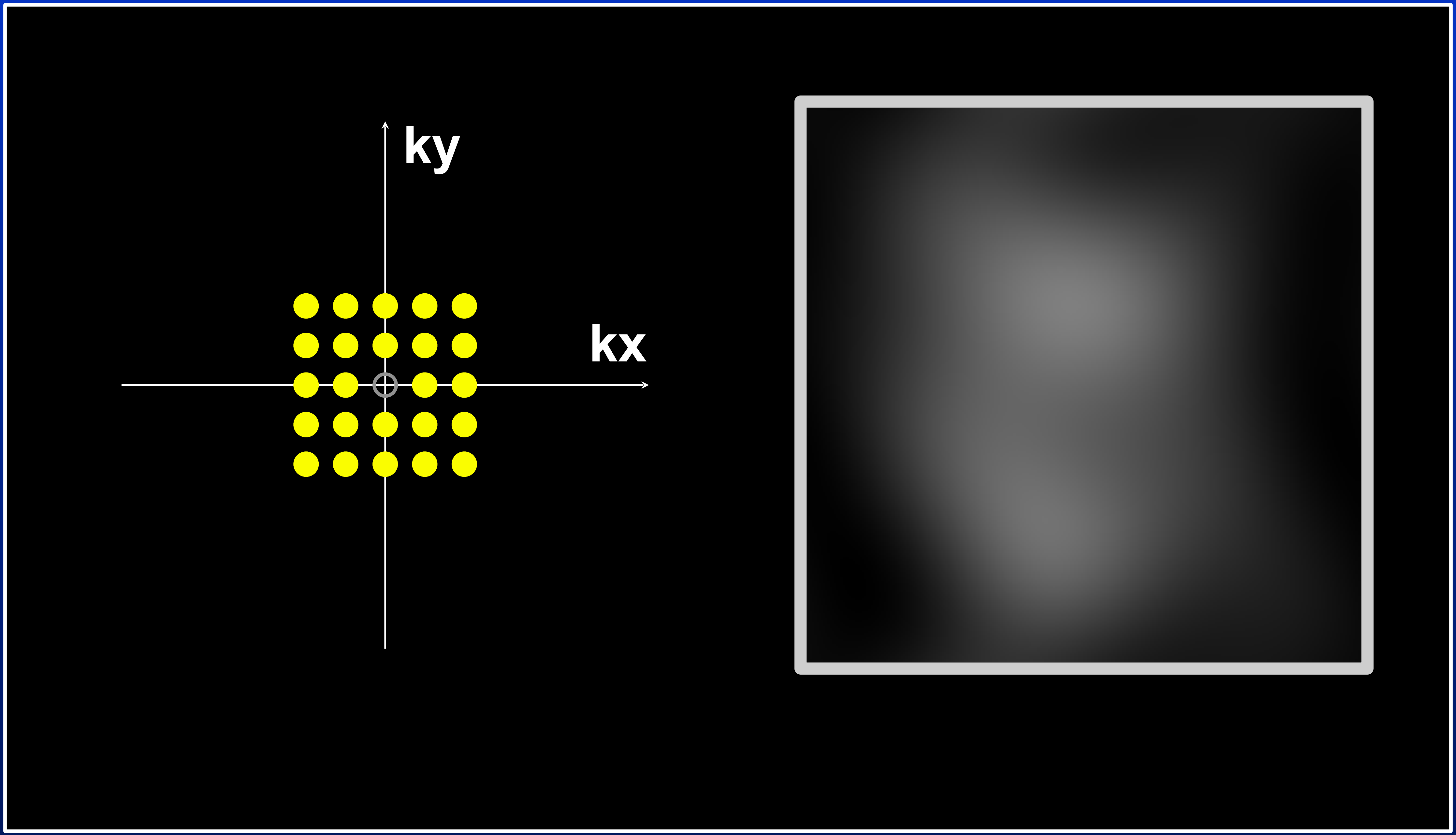
影像 = 各波形成份的加權總和



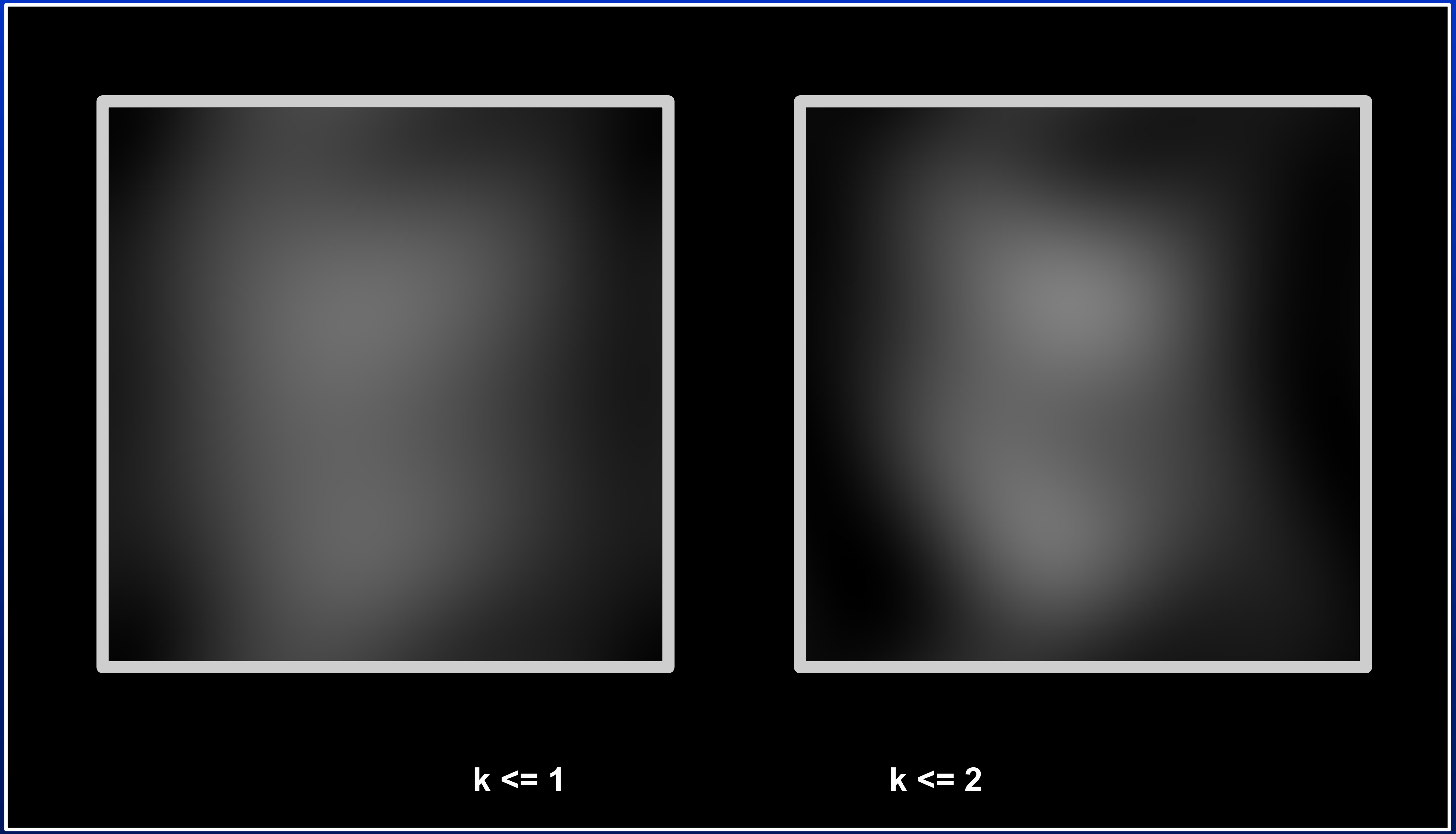
影像 = 各波形成份的加權總和



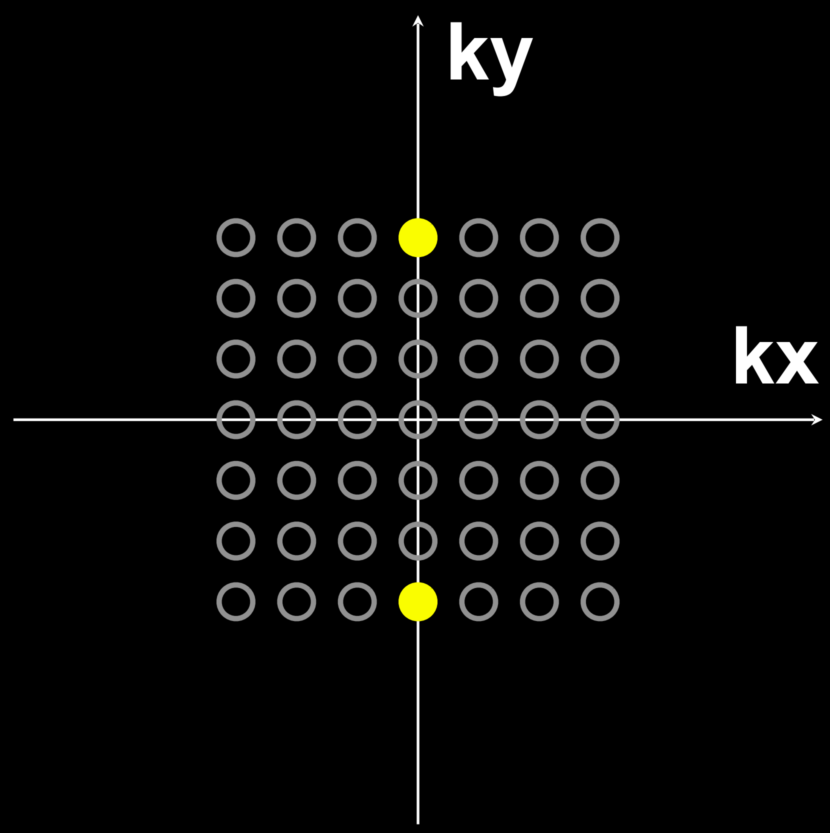
由以上共 24 個數據組合而成的影像

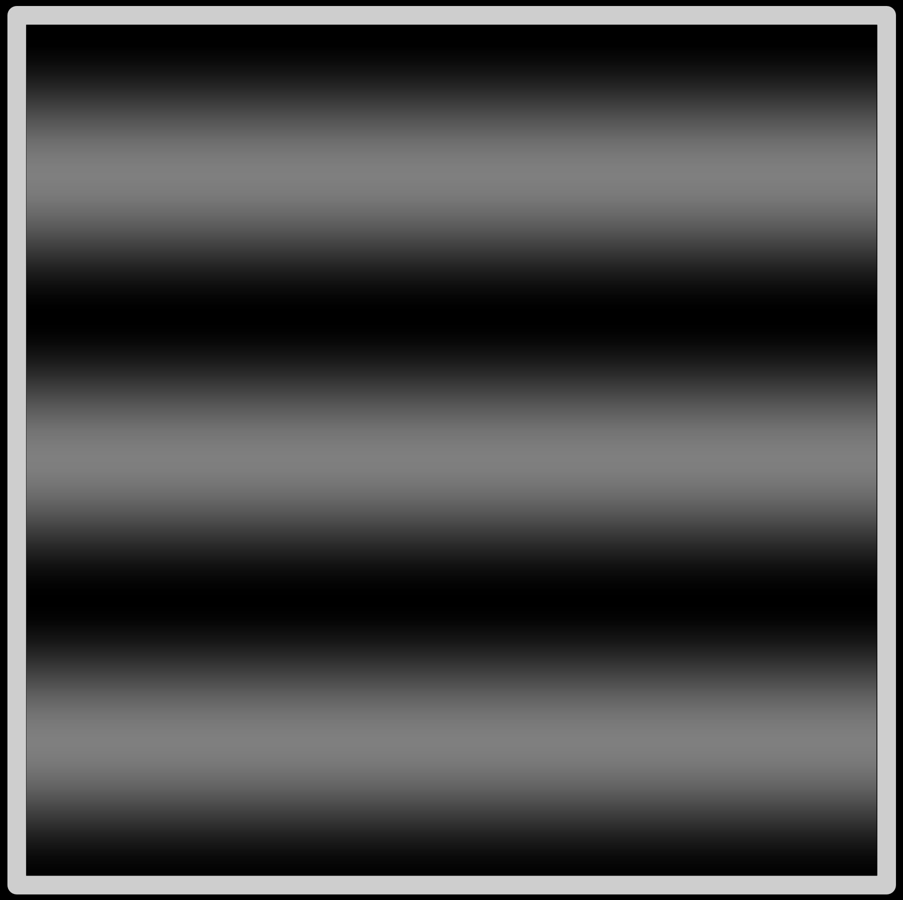


影像比較



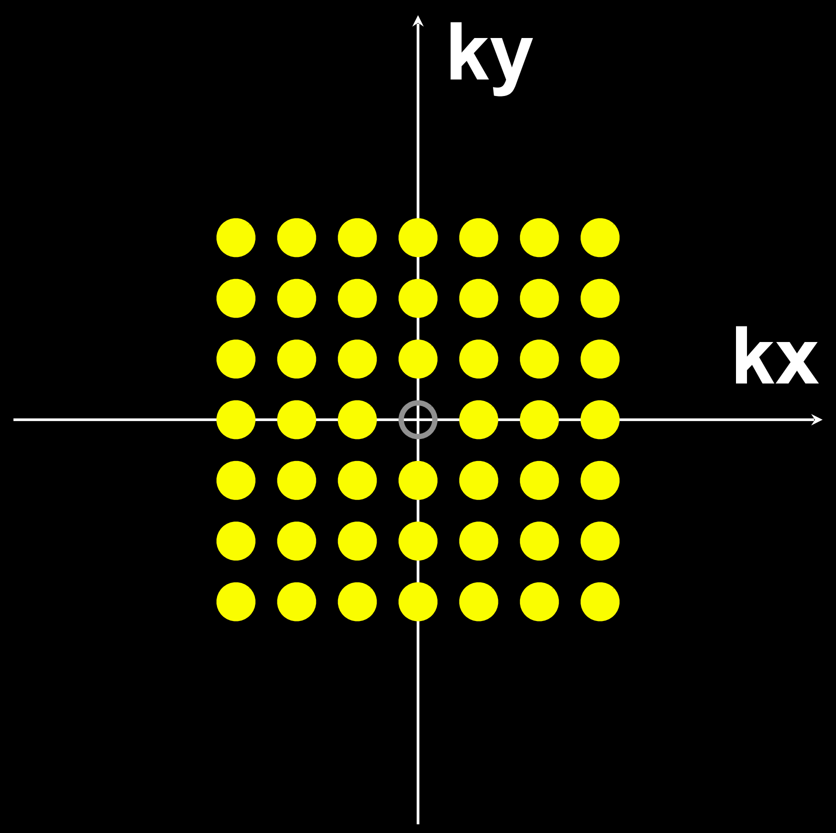
kx = 0 , ky = +/- 3



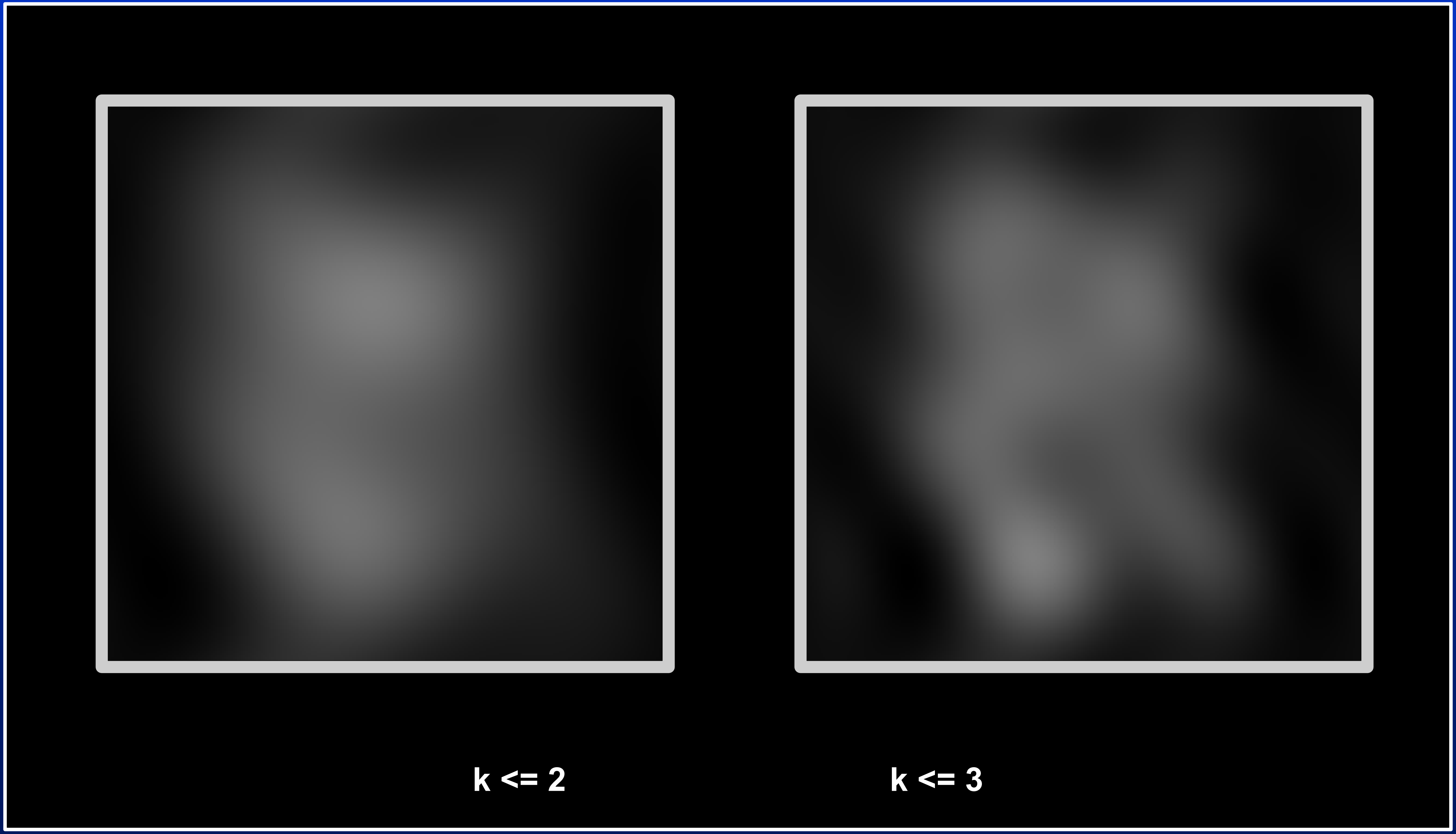


x 或 y 方向變化 3 個 cosine 週期 ...

由以上共 48 個數據組合而成的影像



影像比較



k <= 4 (最多 4 個 sine 週期 / FOV)

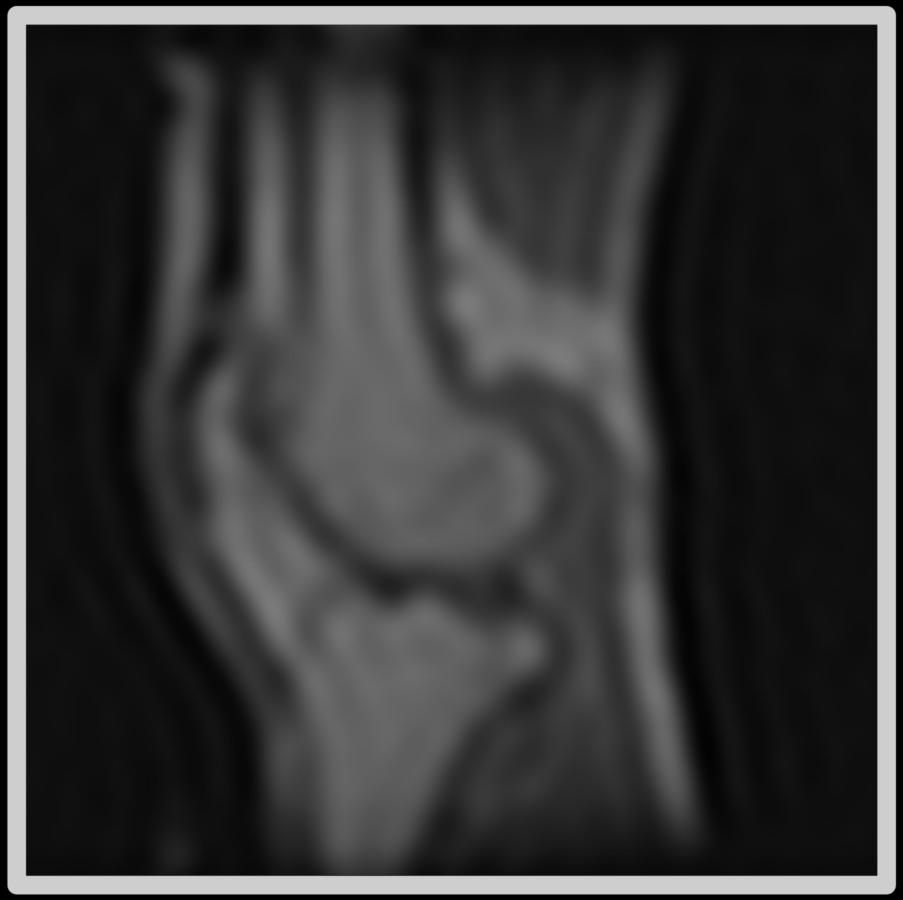
k <= 5 (最多 5 個 sine 週期 / FOV)

k <= 6 (最多 6 個 sine 週期 / FOV)

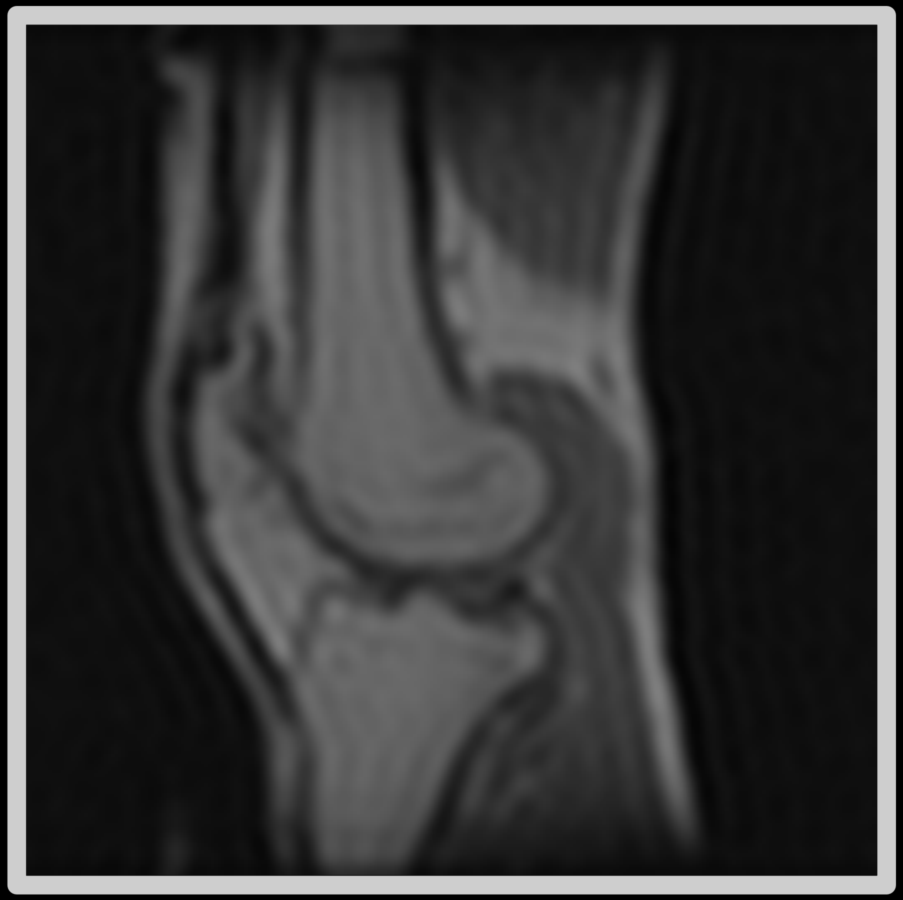
k <= 8 (最多 8 個 sine 週期 / FOV)

k <= 12 (24x24 影像)

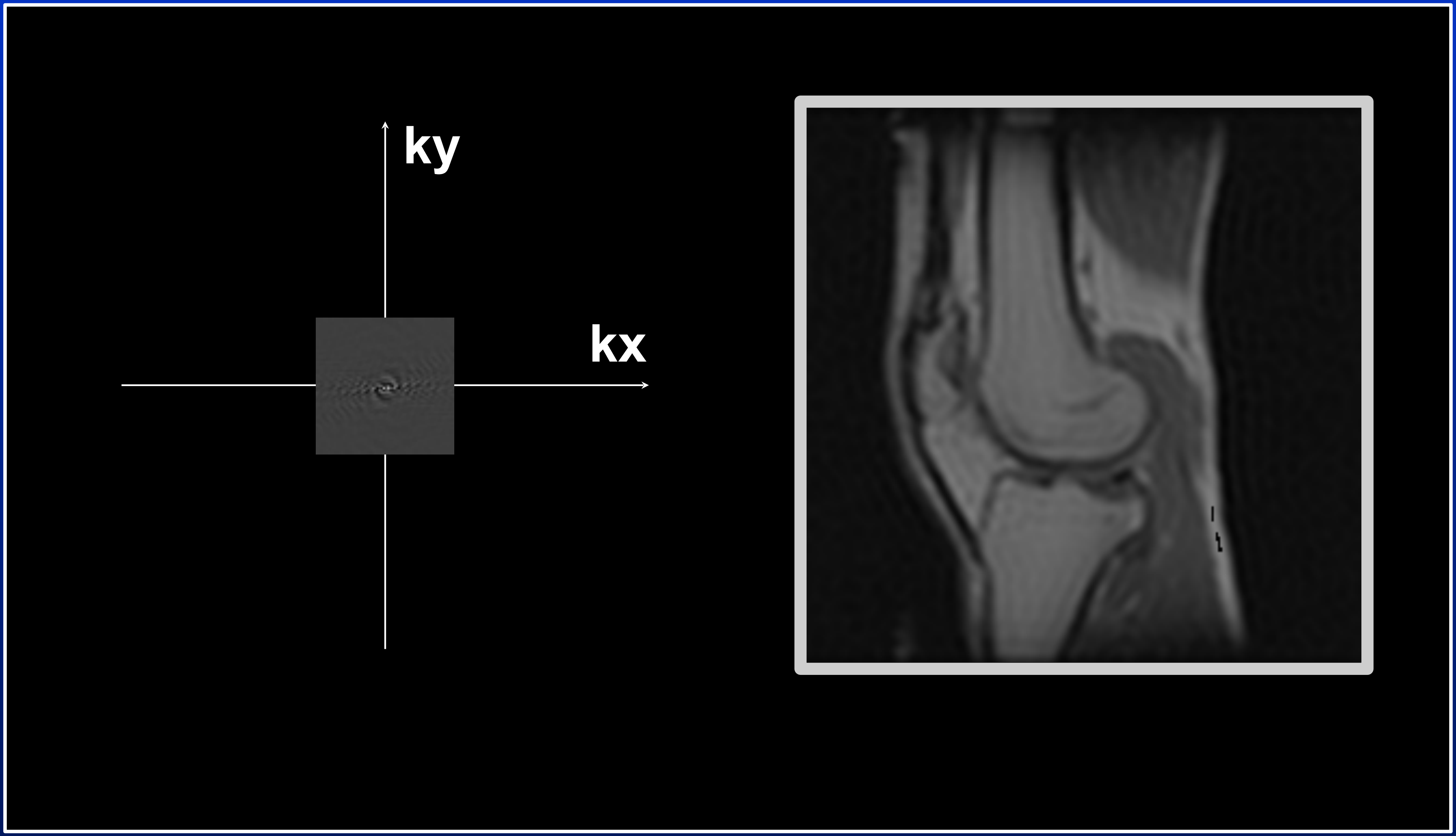
k <= 16 (32x32 影像)



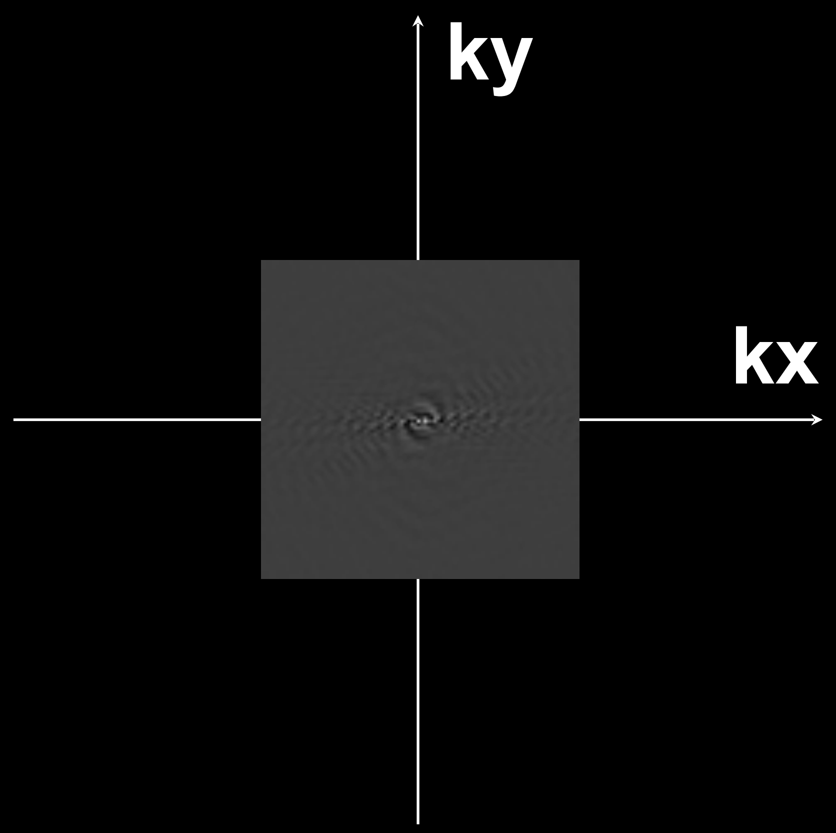
k <= 24 (48x48 影像)



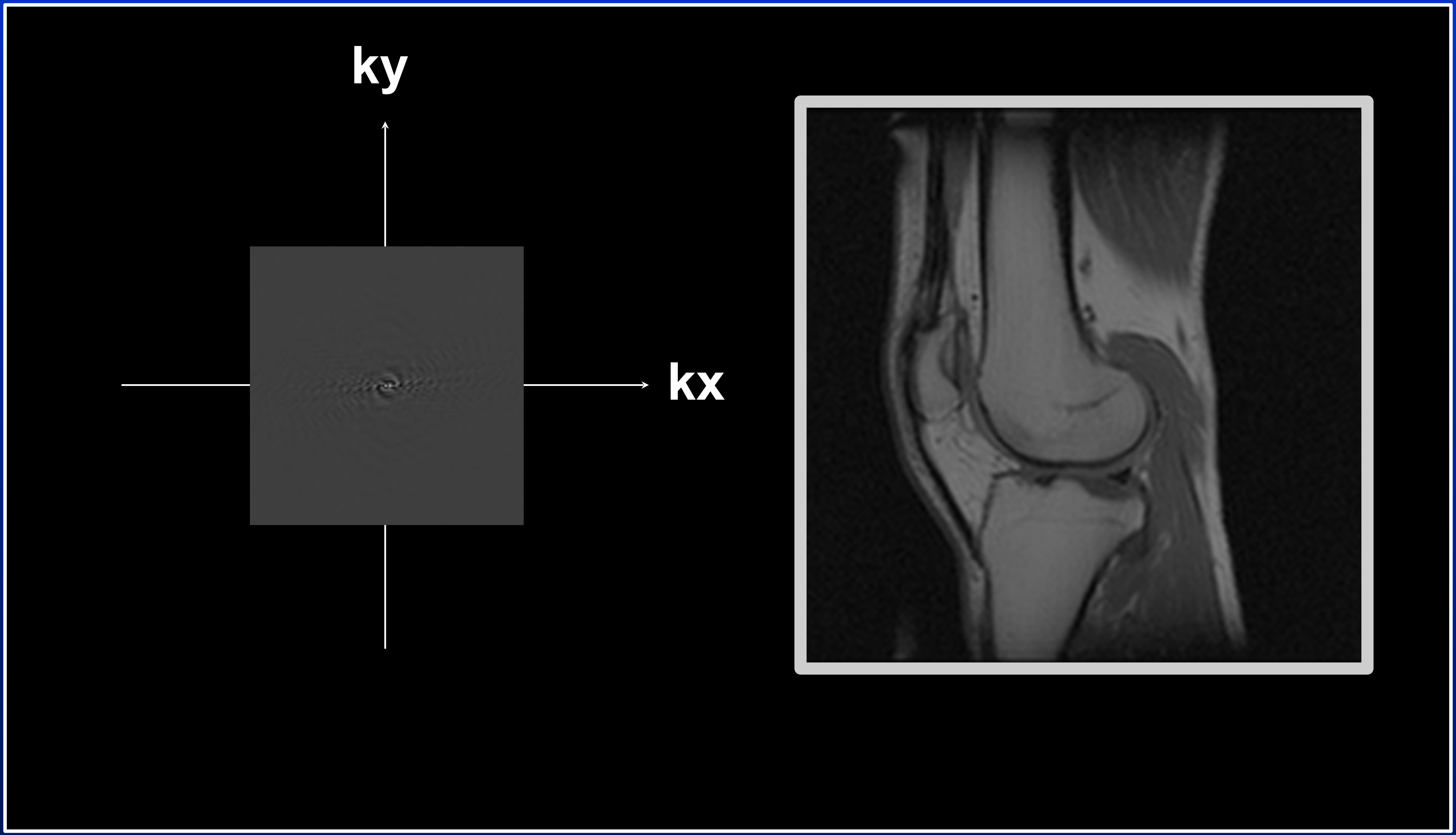
k <= 32 (64x64 影像)



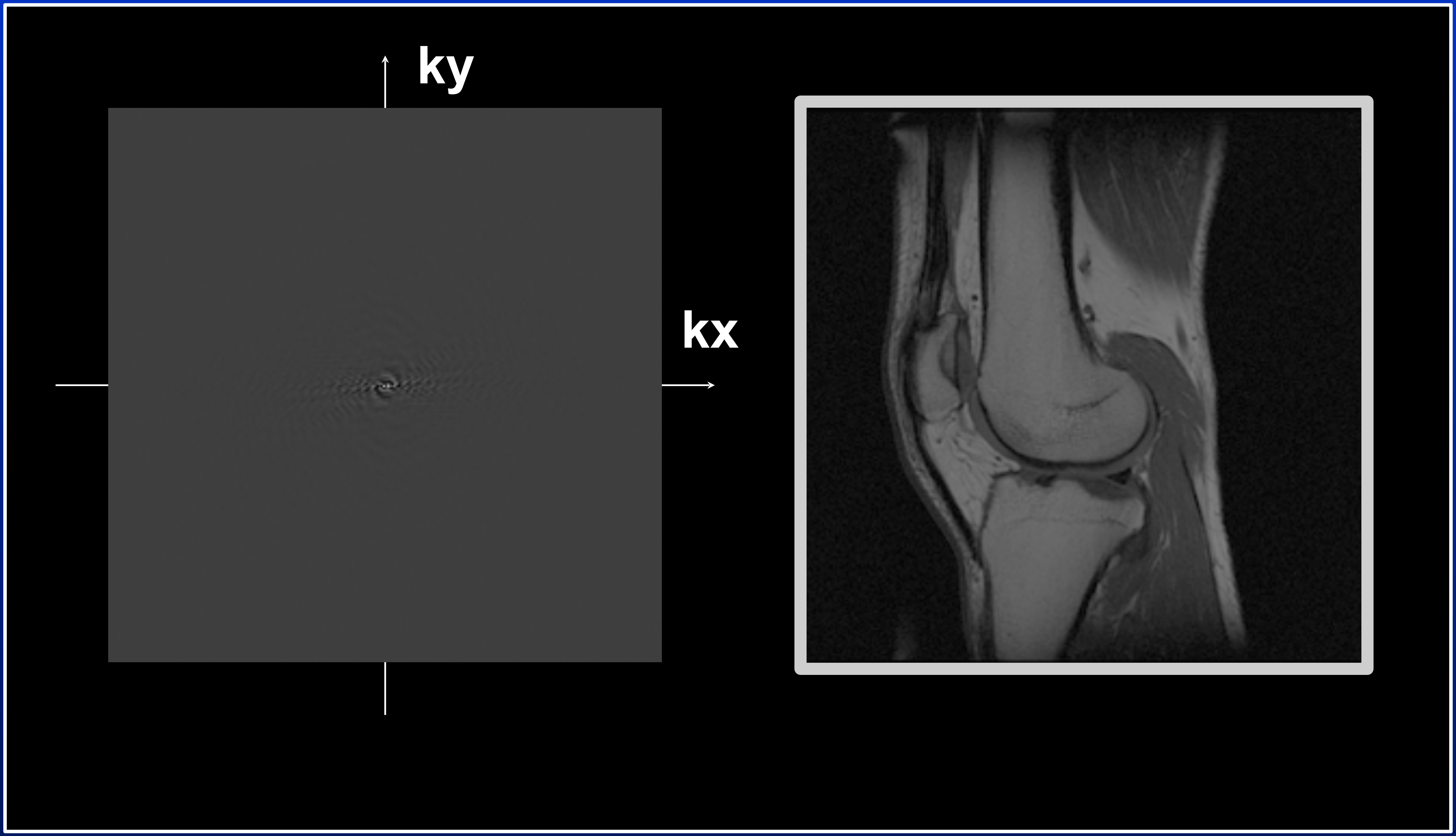
k <= 48 (96x96 影像)



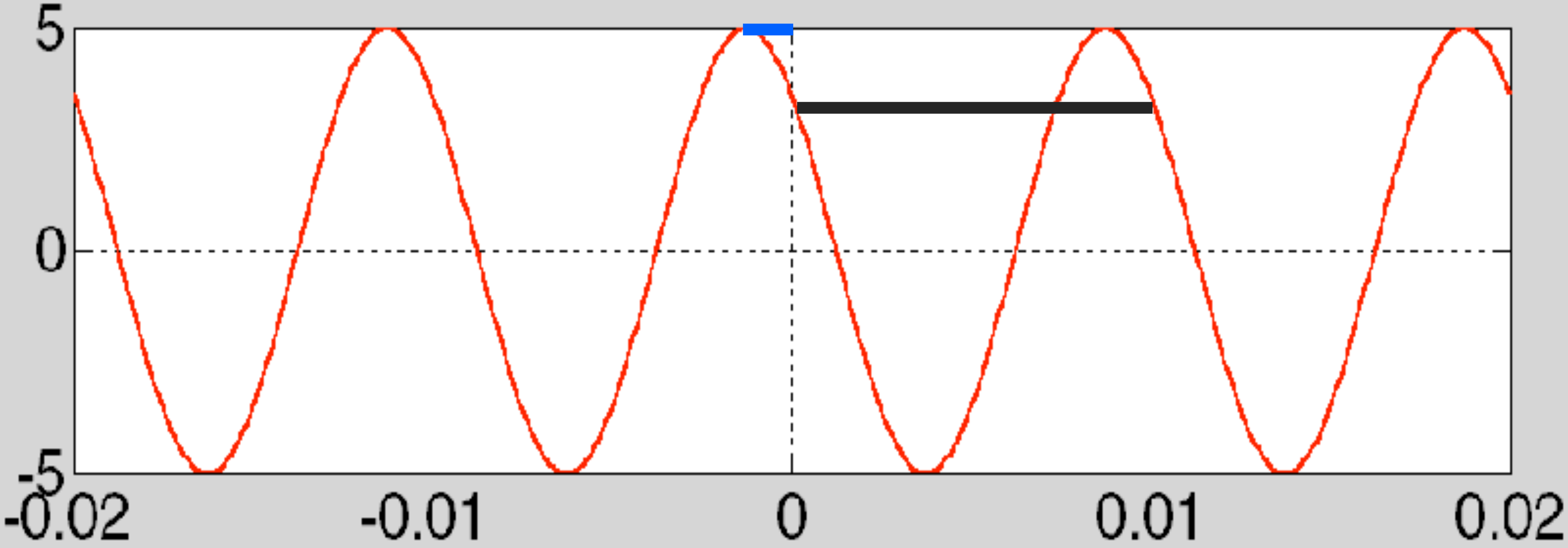
k <= 64 (128x128 影像)



最後的 256x256 影像



$\underline{A}\cos(\underline{\omega}t + \underline{\varphi})$



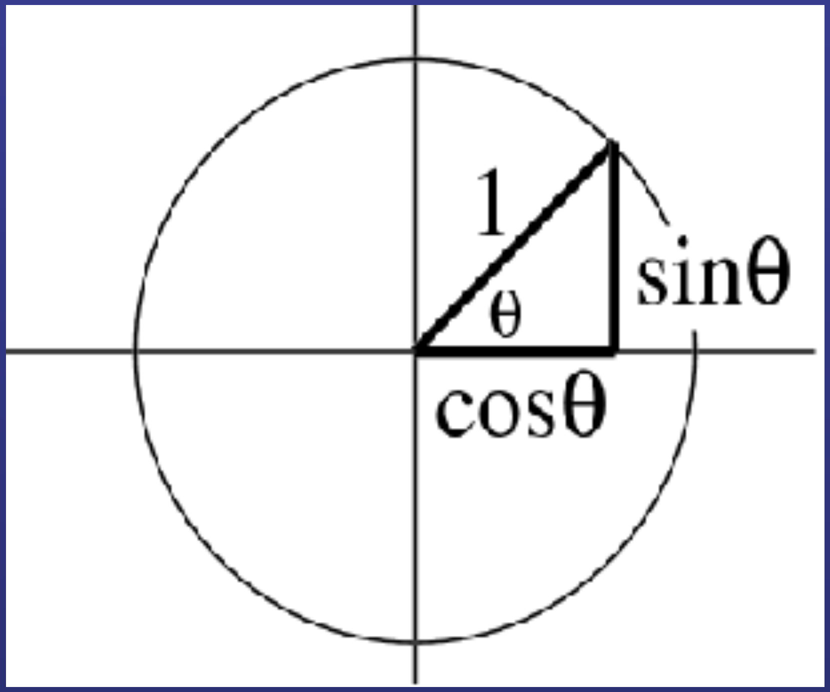
$$T = \frac{0.01\,\text{sec}}{1\ \text{period}} = \frac{1}{100}$$ → $$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.01} = 200\pi$$

$$t_m = -0.00125\,\text{sec}$$ → $$\varphi = -\omega t_m = -(200\pi)(t_m) = 0.25\pi$$

Euler's formula

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\boxed{re^{j\theta} = r\cos(\theta) + jr\sin(\theta)}$$



$$\boxed{e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)}$$

Euler's formula

$$x(t) = A\cos(\omega t + \varphi)$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$X = |X|e^{j(\omega t + \varphi)} = |X|e^{j\varphi}e^{j\omega t}$$

AVOID Trigonometry

- Algebra, even complex, is EASIER !!!
- Can you recall $\cos(\theta_1+\theta_2)$?
- Use: real part of $e^{j(\theta 1+\theta 2)} = \cos(\theta_1+\theta_2)$

$$= (\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)$$

$$= (\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + j(\ldots)$$

cos = REAL PART

Real Part of Euler's

$$\cos(\omega t) = \Re e\{e^{j\omega t}\}$$

General Sinusoid

$$x(t) = A\cos(\omega t + \varphi)$$

So,

$$A\cos(\omega t + \varphi) = \Re e\{Ae^{j(\omega t + \varphi)}\}$$
$$= \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

Fourier Transform

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

Fourier Analysis $x(t) \xrightarrow{F} X(\omega)$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt$$

Fourier Synthesis $X(\omega) \xrightarrow{F^{-1}} x(t)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{j\omega t}\,d\omega$$

Properties

$$ax(t)+by(t)\longleftrightarrow aX(\omega)+bY(\omega)$$

$$x(-t)\longleftrightarrow X(-\omega)$$

$$x(at)\longleftrightarrow \frac{1}{|a|}X(\frac{\omega}{a})$$

$$x(t)*y(t)\longleftrightarrow X(\omega)Y(\omega)$$

$$x(t)y(t)\longleftrightarrow \frac{1}{2\pi}X(\omega)*Y(\omega)$$

$$\frac{d}{dt}x(t)\longleftrightarrow j\omega X(\omega)$$

$$\int_{-\infty}^{t}x(t)dt\longleftrightarrow \frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$$

Properties

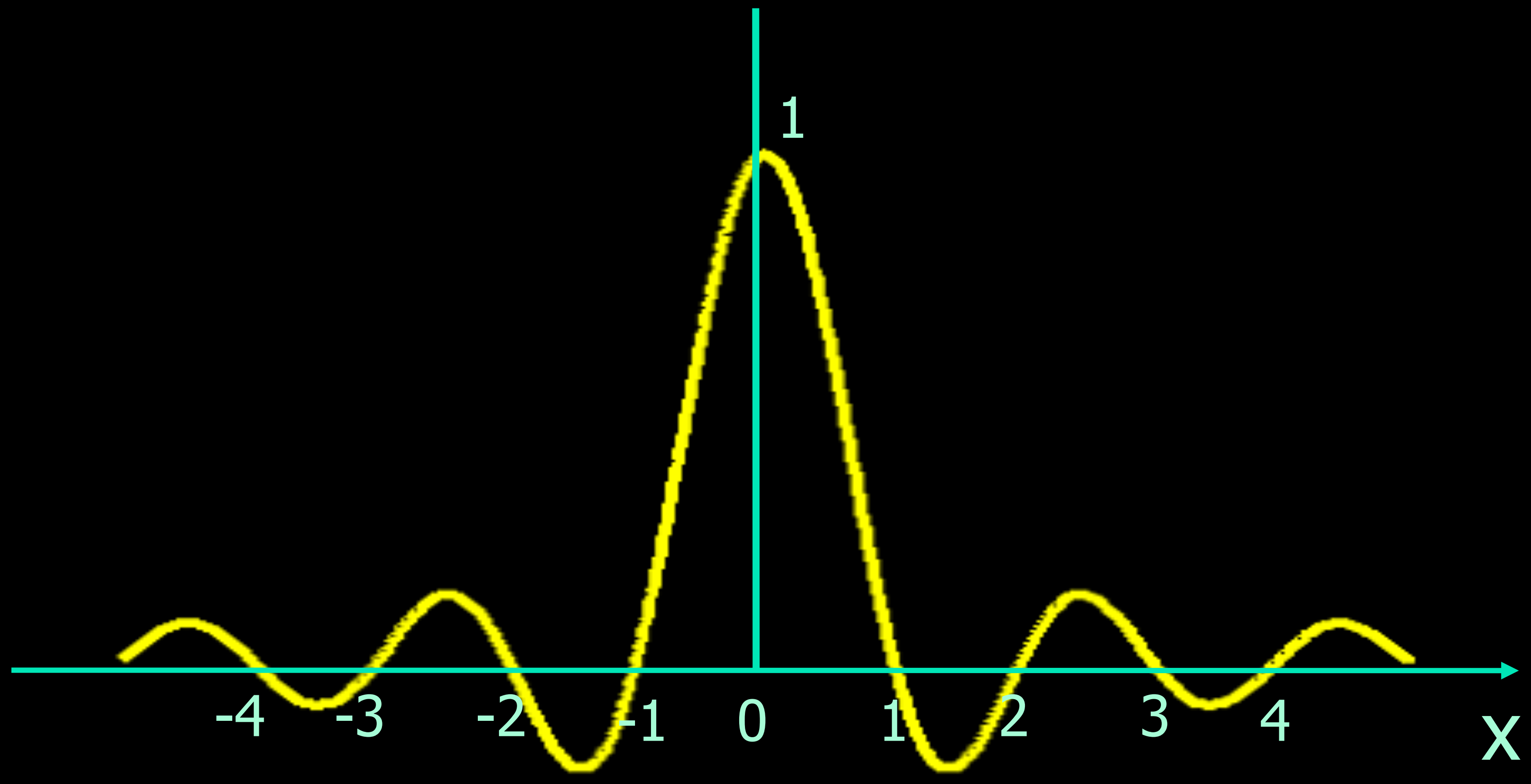
- Duality

$$f(u) = \int_{-\infty}^{+\infty} g(v) e^{-juv} dv$$

$$\boxed{\begin{aligned} f(\omega) &= \mathcal{F}\{g(t)\} \\ g(-\omega) &= \frac{1}{2\pi}\mathcal{F}\{f(t)\} \end{aligned}}$$

Sinc function

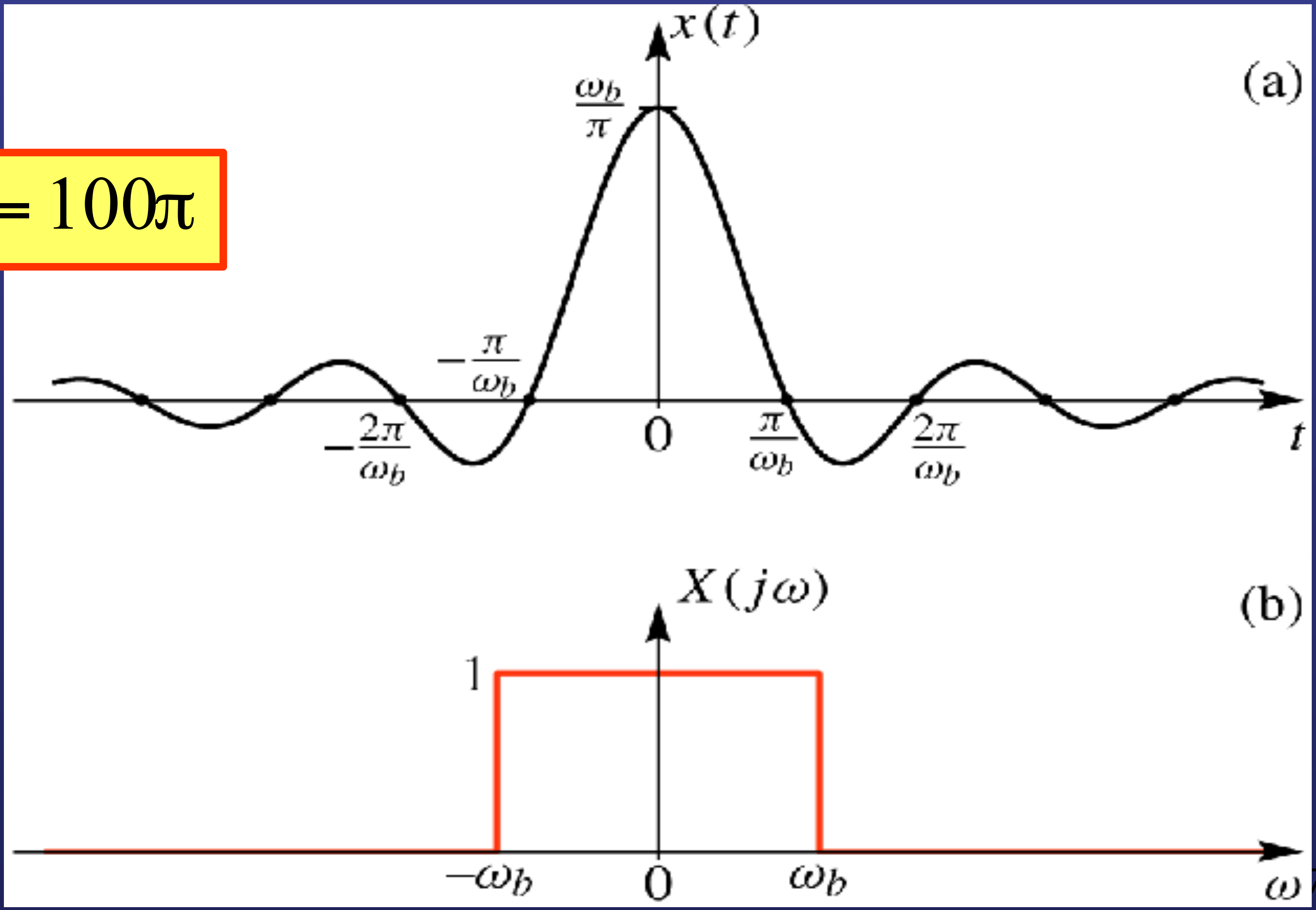
$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$



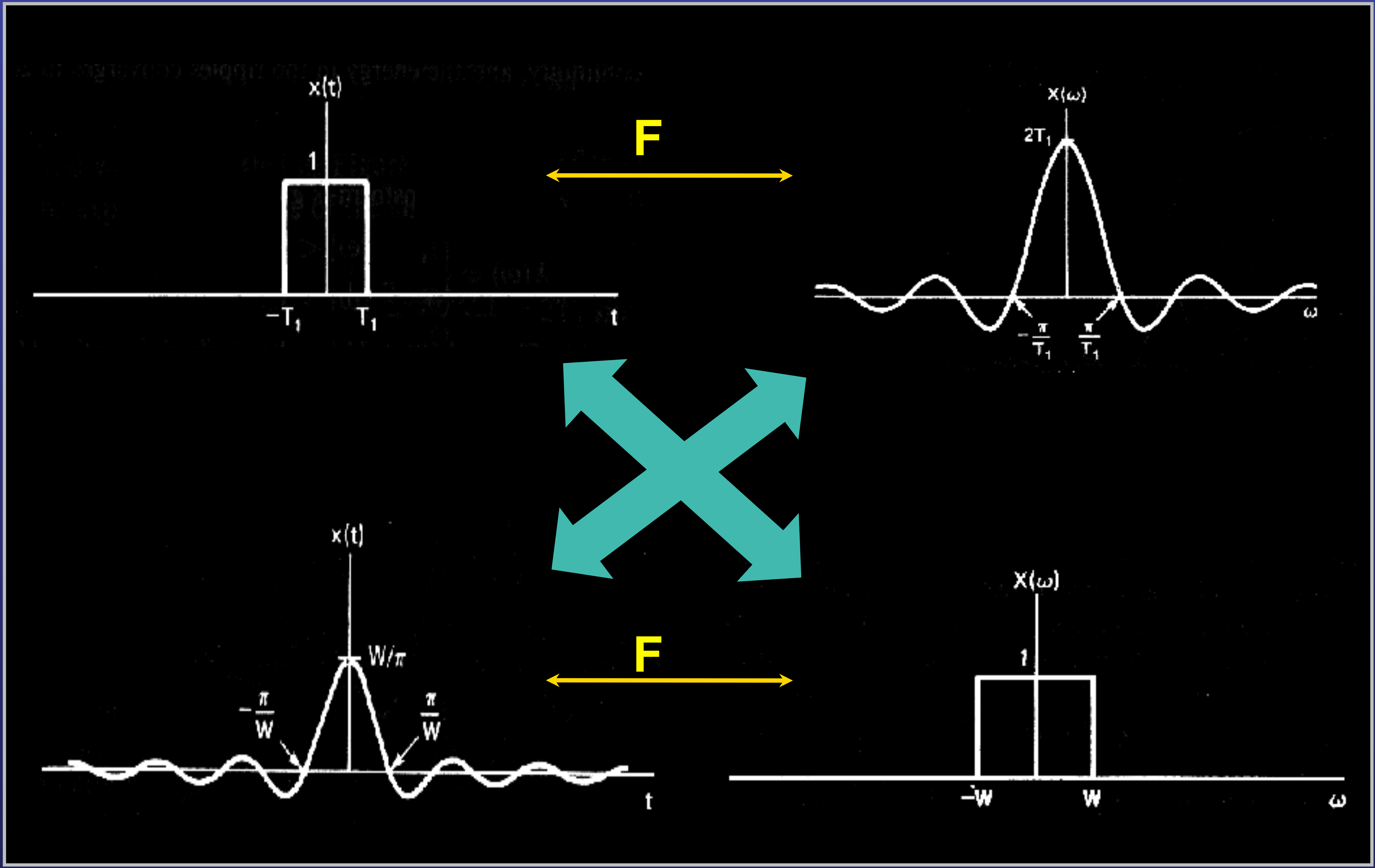
Ideally Bandlimited Signal

$$x(t) = \frac{\sin(100\pi t)}{\pi t} \quad \Leftrightarrow \quad X(j\omega) = \begin{cases} 1 & |\omega| < 100\pi \\ 0 & |\omega| > 100\pi \end{cases}$$

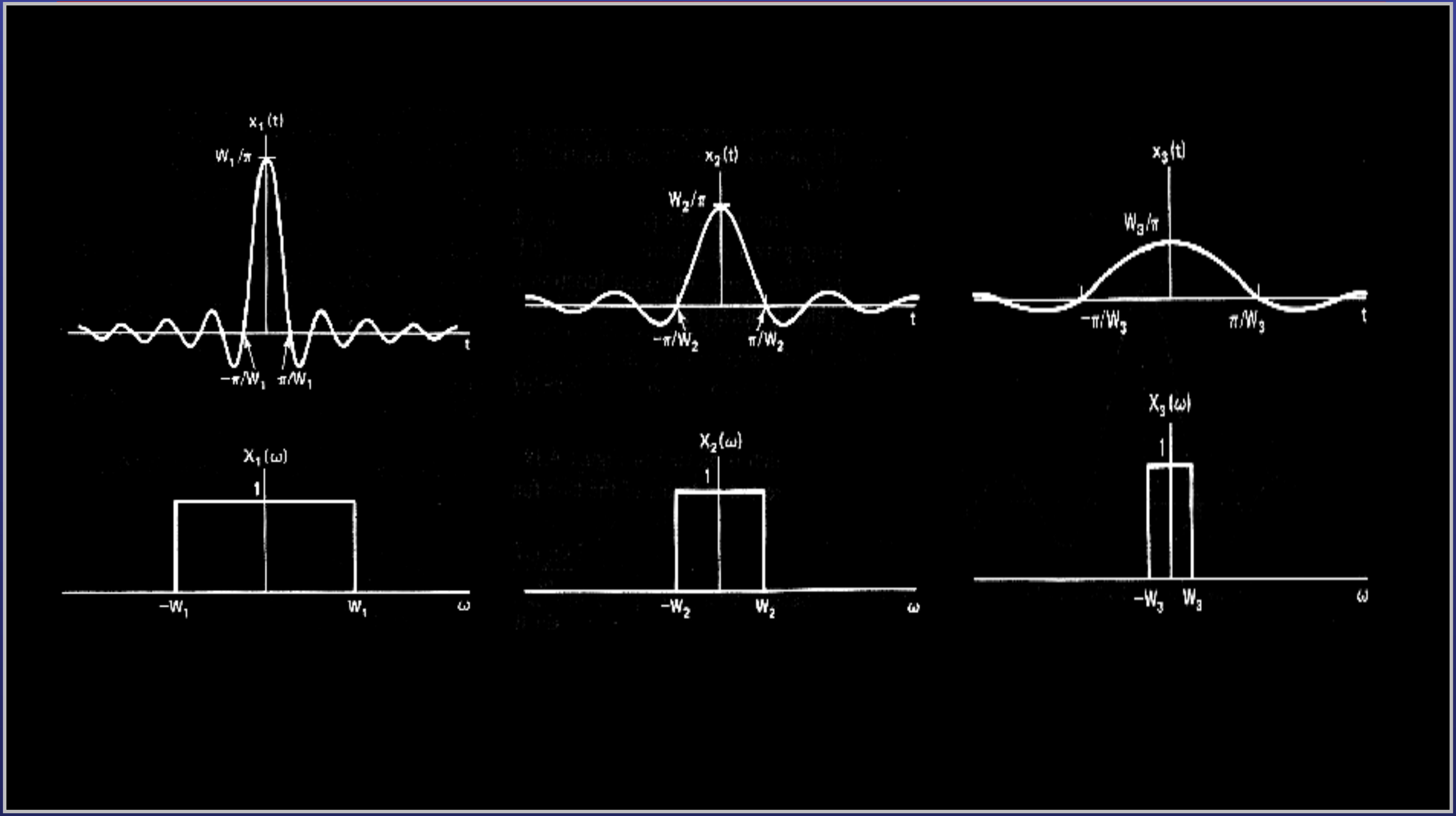
$$\omega_b = 100\pi$$



Sinc v.s. Duality



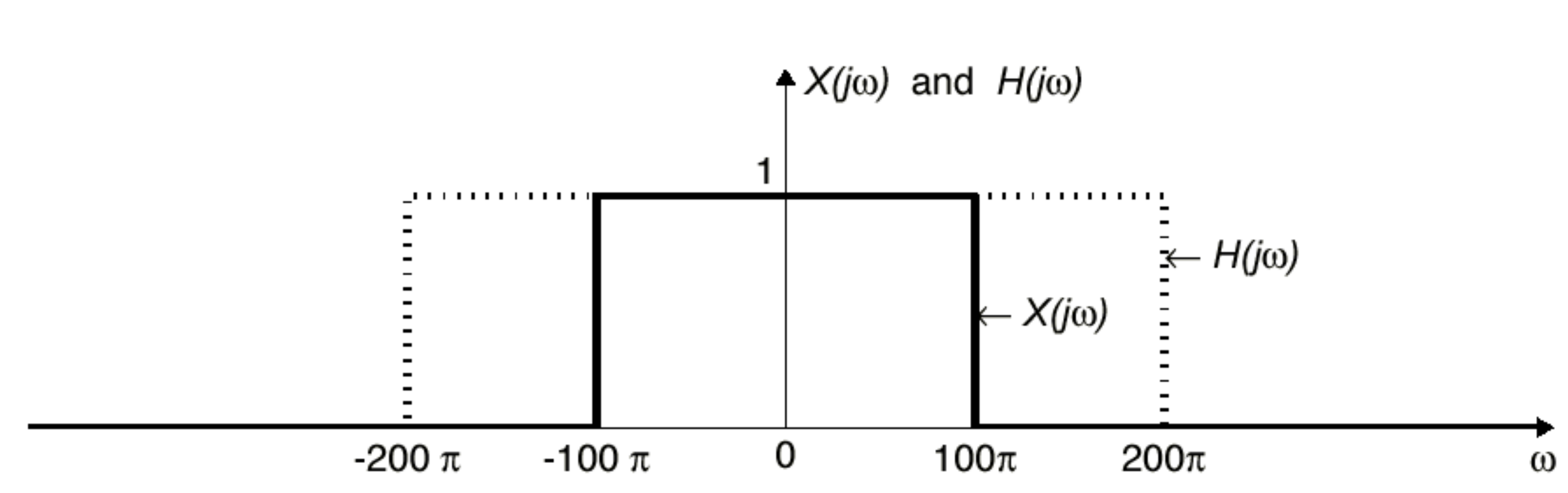
Sinc with different W



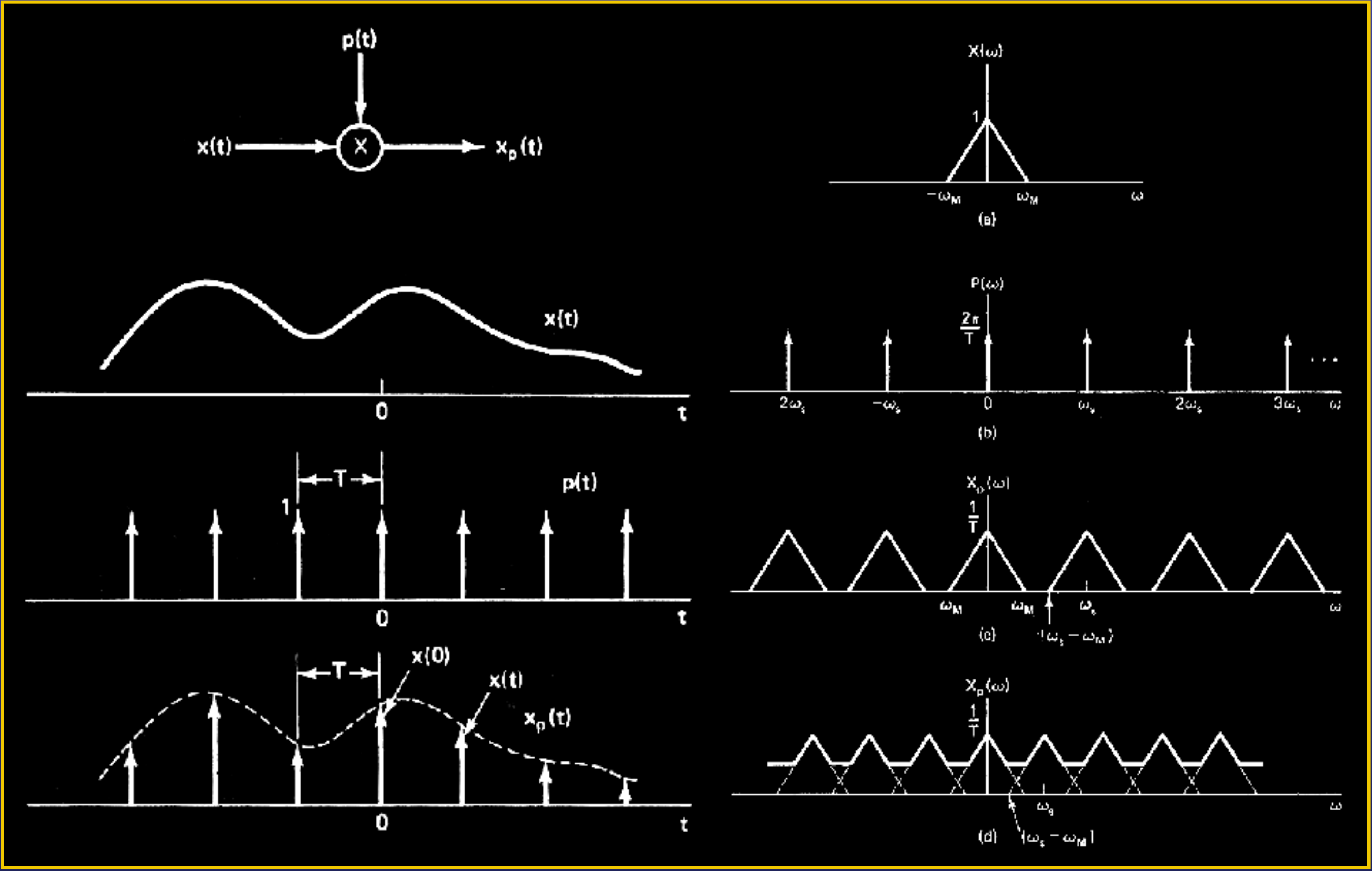
Convolution Example

$$x(t) * h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

$$\frac{\sin(100\pi t)}{\pi t} * \frac{\sin(200\pi t)}{\pi t} = \frac{\sin(100\pi t)}{\pi t}$$



Sampling

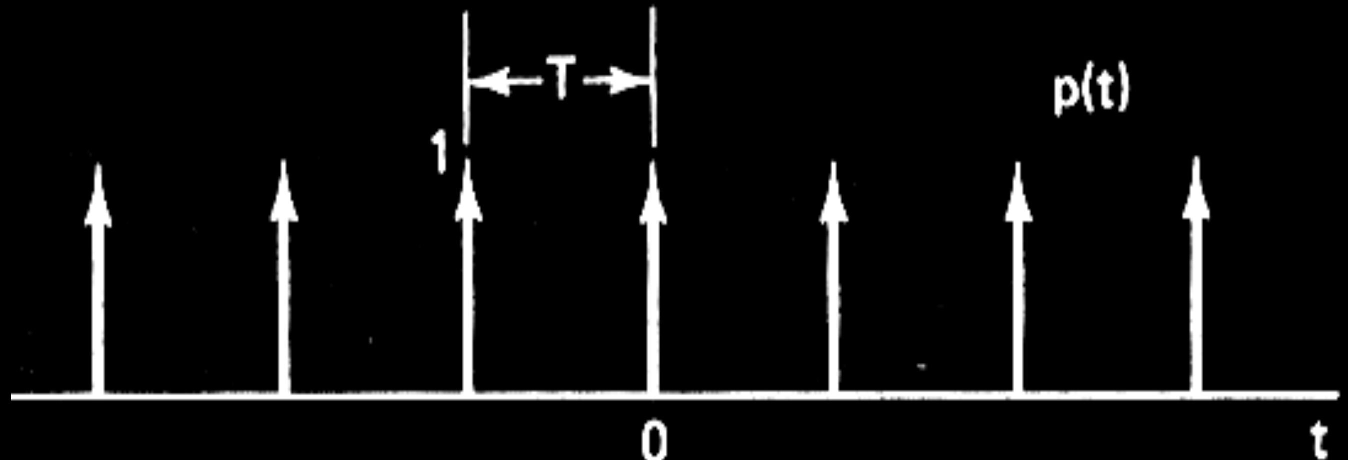


p(t)
x(t)
X
$x_p(t)$
x(t)
0
t
T
p(t)
1
0
t
T
x(0)
x(t)
$x_p(t)$
0
t
$X(\omega)$
1
$-\omega_M$
ω_M
ω
(a)
$P(\omega)$
$\frac{2\pi}{T}$
$2\omega_s$
$-\omega_s$
0
ω_s
$2\omega_s$
$3\omega_s$
ω
(b)
$X_p(\omega)$
$\frac{1}{T}$
ω_M
ω_M
ω_s
ω
(c)
$(\omega_s - \omega_M)$
$X_p(\omega)$
$\frac{1}{T}$
ω_s
ω
(d)
$(\omega_s - \omega_M)$

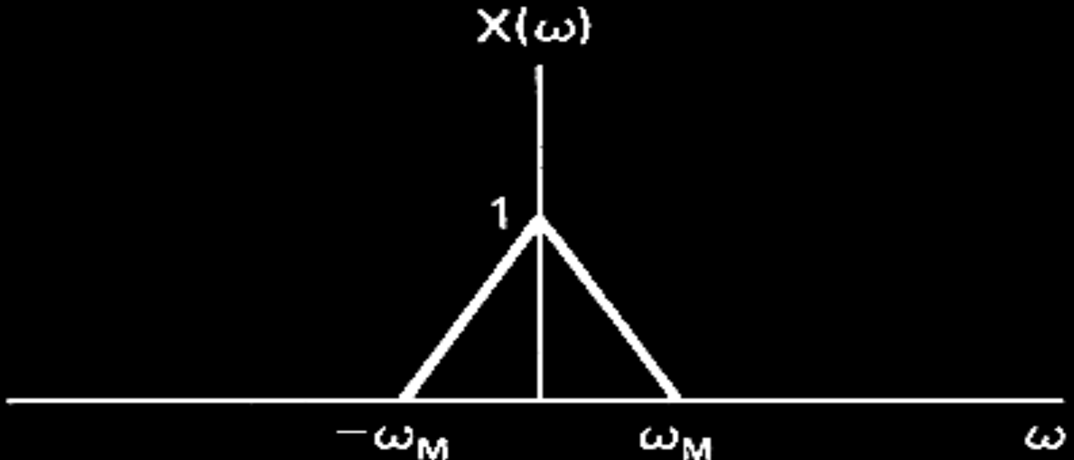
Sampling

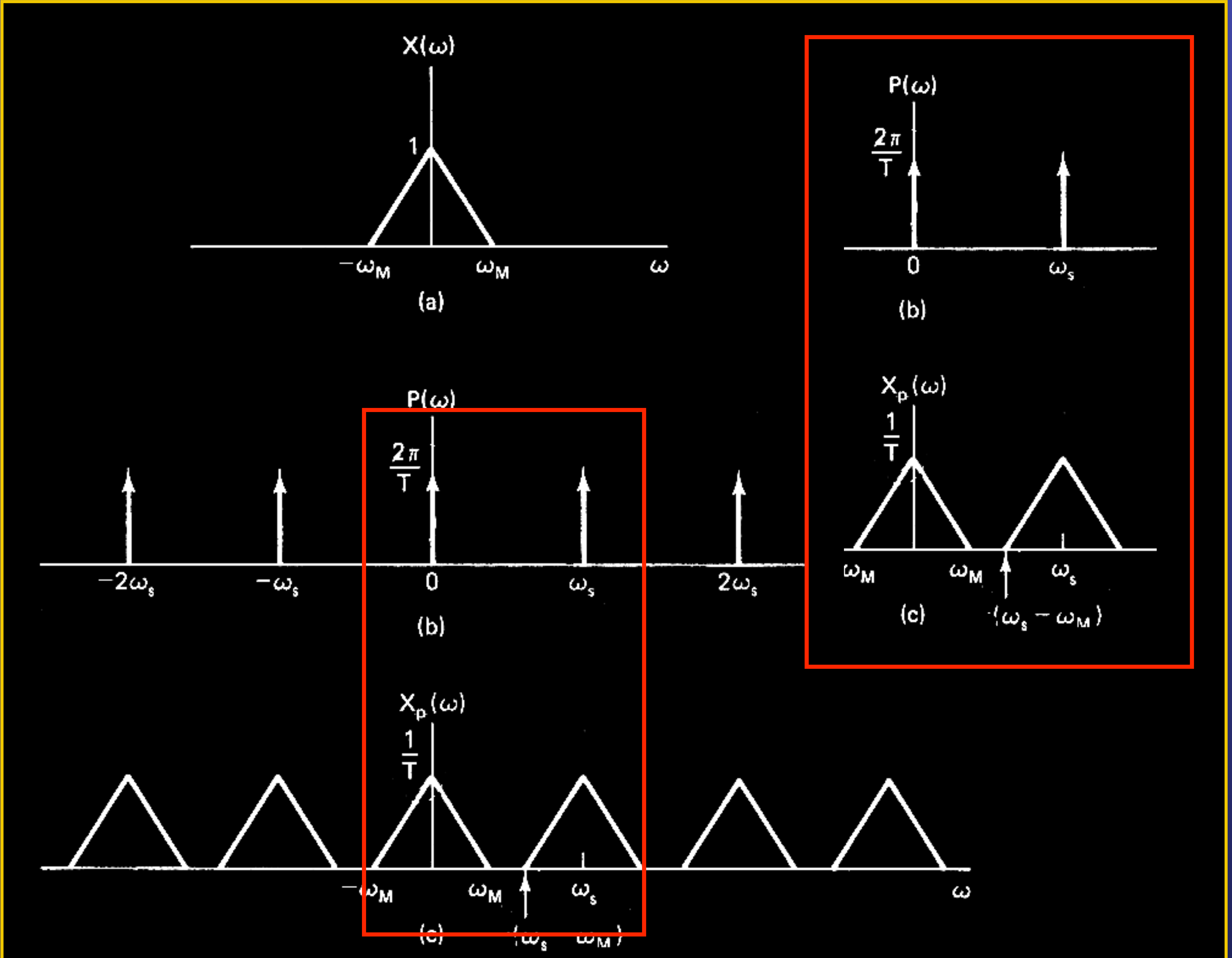
- Sampling period: T
- Sampling frequency: ω_s

$$\omega_s = \frac{2\pi}{T}$$



$$\boxed{\omega_s \;\; ?? \;\; \omega_M}$$





X(ω)
1
−ω_M
ω_M
ω
(a)
P(ω)
2π/T
−2ω_s
−ω_s
0
ω_s
2ω_s
(b)
X_p(ω)
1/T
−ω_M
ω_M
ω_s
ω
(c)
(ω_s − ω_M)

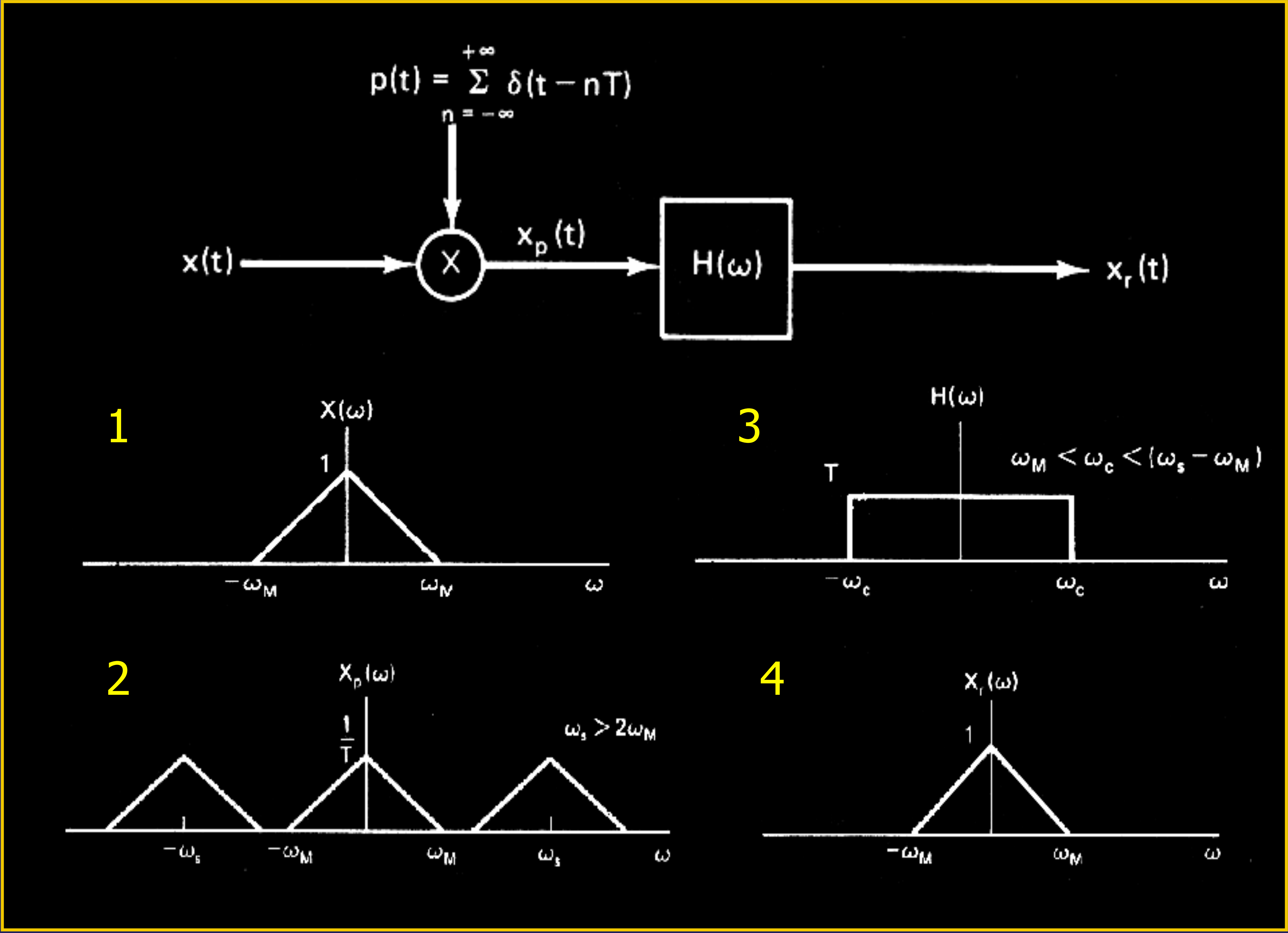
Sampling Theorem

- Let x(t) be a bandlimited signal with $X(\omega)=0$ for $|\omega| > \omega_M$
  then x(t) is uniquely determined by its samples x(nT), n=0, ±1, ± 2,…, if

$$\boxed{\omega_s > 2\omega_M}$$

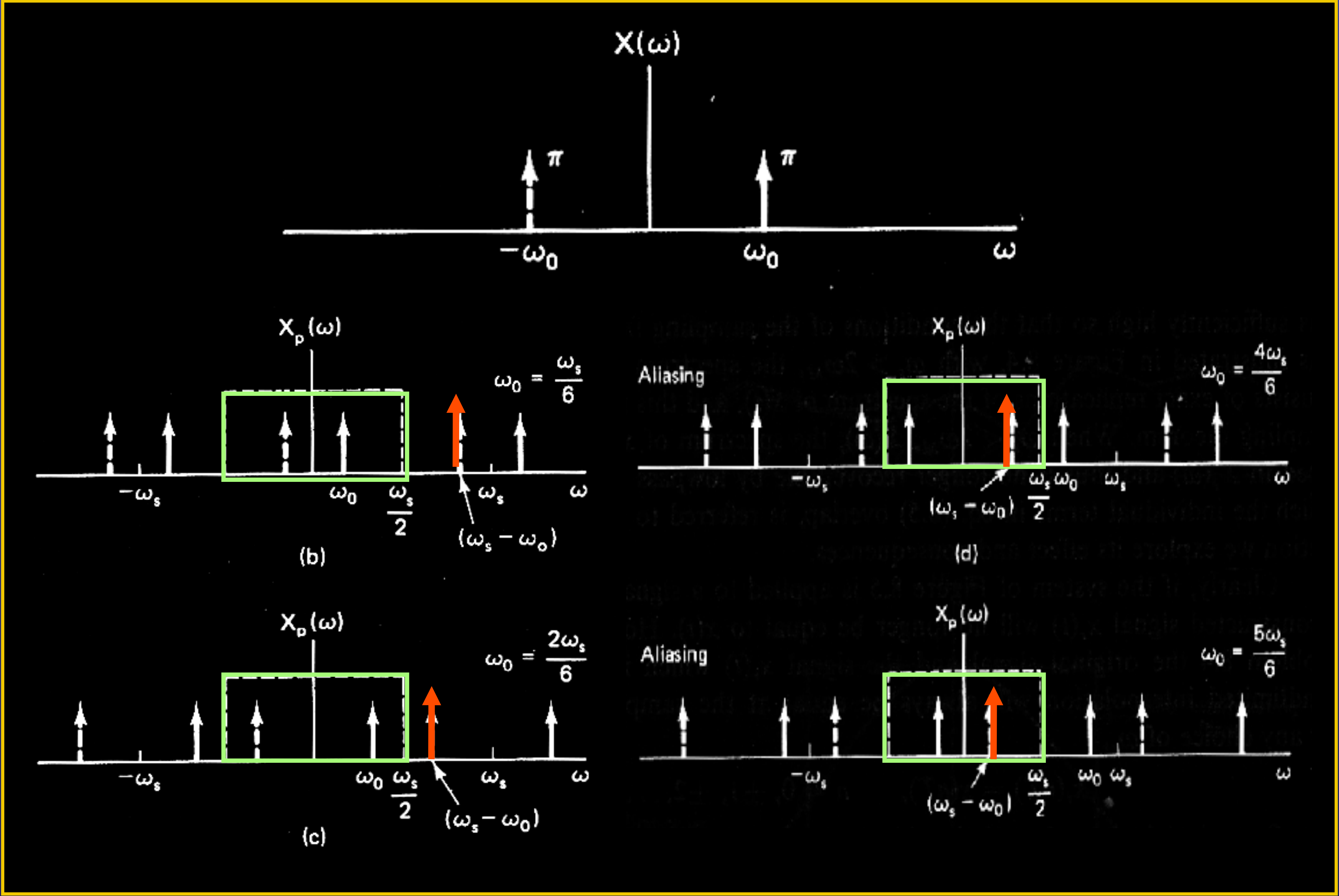
where

$$\omega_s = \frac{2\pi}{T}$$

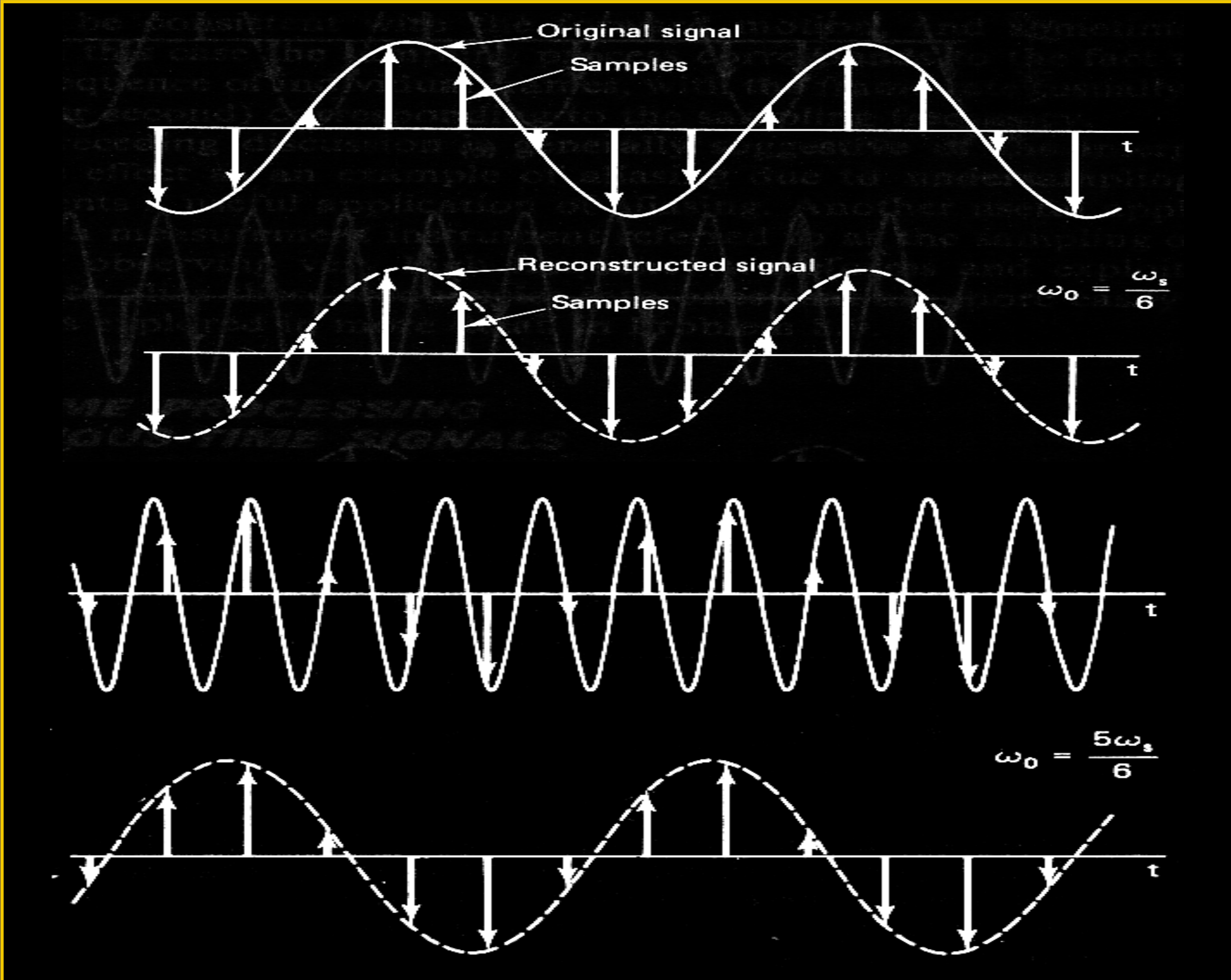


$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$
$x(t)$
$x_p(t)$
$H(\omega)$
$x_r(t)$
1
$X(\omega)$
1
$-\omega_M$
ω_M
ω
3
$H(\omega)$
T
$\omega_M < \omega_c < (\omega_s - \omega_M)$
$-\omega_c$
ω_c
ω
2
$X_p(\omega)$
$\frac{1}{T}$
$\omega_s > 2\omega_M$
$-\omega_s$
$-\omega_M$
ω_M
ω_s
ω
4
$X_r(\omega)$
1
$-\omega_M$
ω_M
ω

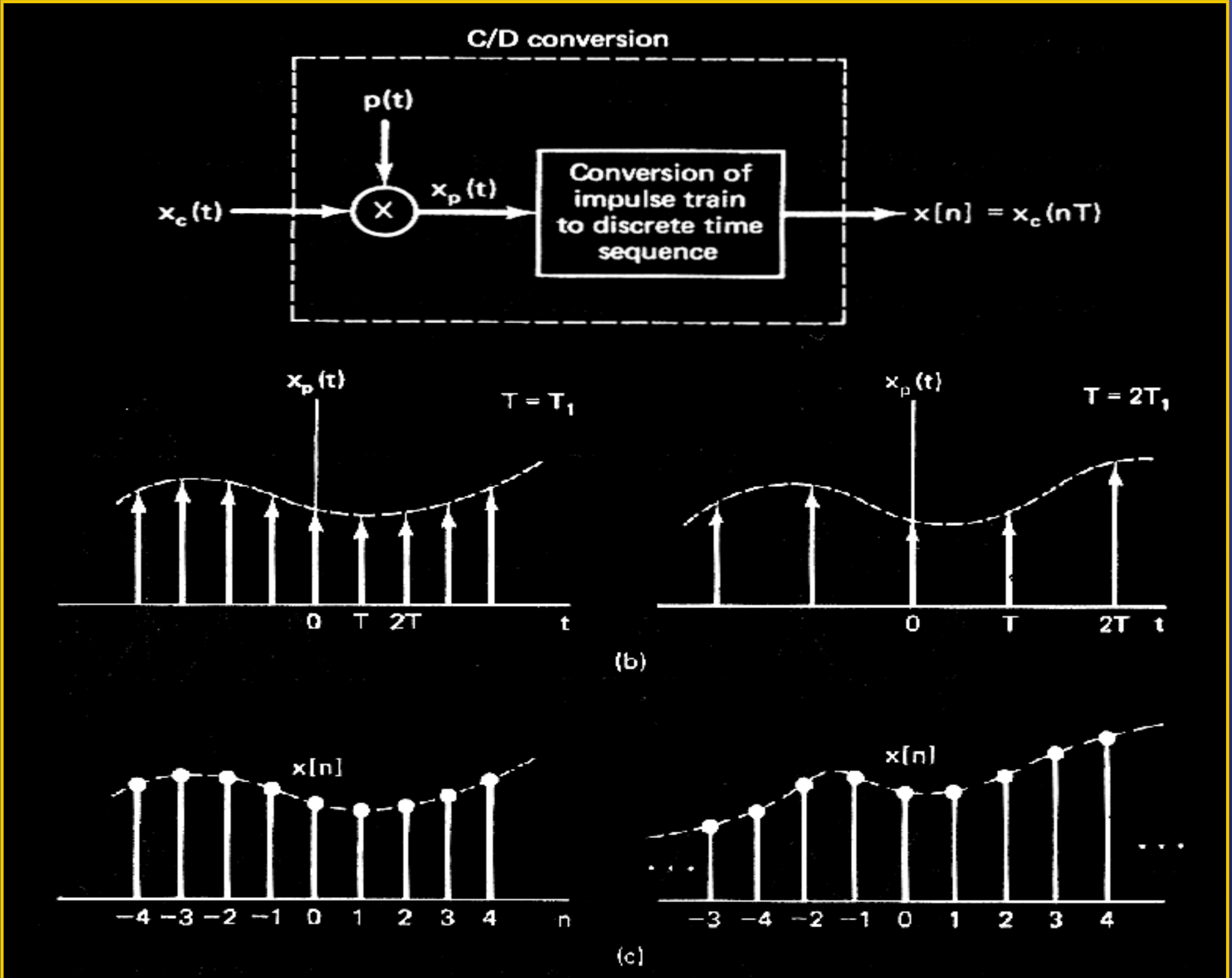
Aliasing



X(ω)
π
π
−ω0
ω0
ω
Xp(ω)
ω0 = ωs/6
−ωs
ω0
ωs/2
ωs
ω
(ωs − ωo)
(b)
Xp(ω)
ω0 = 2ωs/6
−ωs
ω0
ωs/2
ωs
ω
(ωs − ω0)
(c)
Xp(ω)
Aliasing
ω0 = 4ωs/6
−ωs
ωs/2
ω0
ωs
ω
(ωs − ω0)
(d)
Xp(ω)
Aliasing
ω0 = 5ωs/6
−ωs
ωs/2
ω0
ωs
ω
(ωs − ω0)



Original signal
Samples
t
Reconstructed signal
Samples
$\omega_0 = \frac{\omega_s}{6}$
t
t
$\omega_0 = \frac{5\omega_s}{6}$
t



C/D conversion
p(t)
$x_c(t)$
$x_p(t)$
Conversion of impulse train to discrete time sequence
$x[n] = x_c(nT)$
$x_p(t)$
$T = T_1$
0 T 2T t
$x_p(t)$
$T = 2T_1$
0 T 2T t
(b)
x[n]
−4 −3 −2 −1 0 1 2 3 4 n
x[n]
−3 −4 −2 −1 0 1 2 3 4
(c)

Fourier Transform

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

Fourier Analysis $x(t) \xrightarrow{F} X(\omega)$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

Fourier Synthesis $X(\omega) \xrightarrow{F^{-1}} x(t)$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega$$

Discrete-time Fourier Transform

$$X(\Omega) = \sum_{-\infty}^{+\infty} x[n] e^{-j\Omega n}$$

- Discrete in time domain
- Continuous in frequency domain

How to create the discrete form in the frequency domain?

$$X(\Omega) = \sum_{-\infty}^{+\infty} x[n] e^{-j\Omega n}$$

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk(2\pi/N)n}, k = 0,1,\ldots,N-1$$

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{jk(2\pi/N)n}, n = 0,1,\ldots,N-1$$

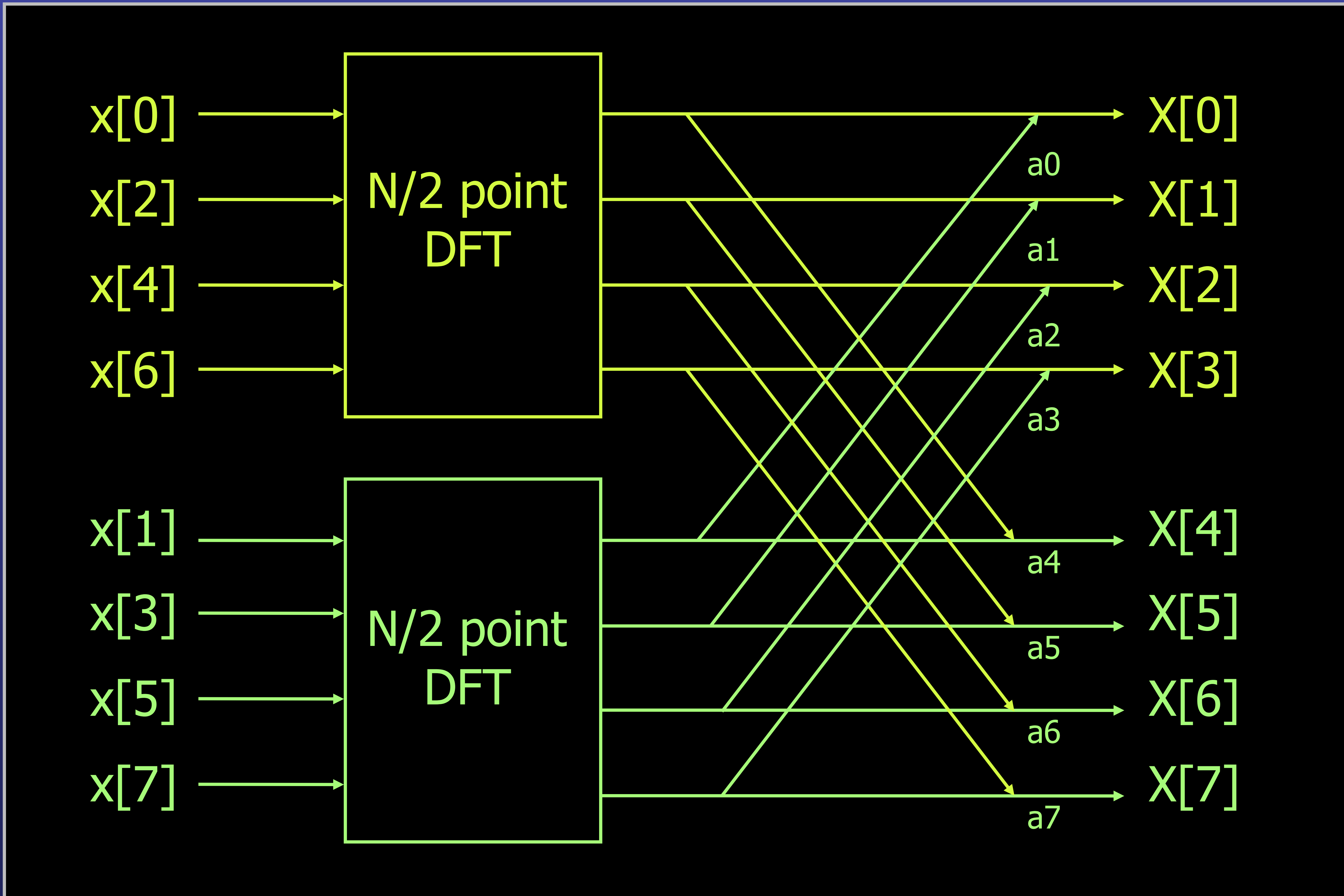
Discrete Fourier Transform

$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk(2\pi/N)n}, k = 0,1,\ldots,N-1$$

$$x[n] = \sum_{k=0}^{N-1} X[k]e^{jk(2\pi/N)n}, n = 0,1,\ldots,N-1$$

→ $O(N^2)$

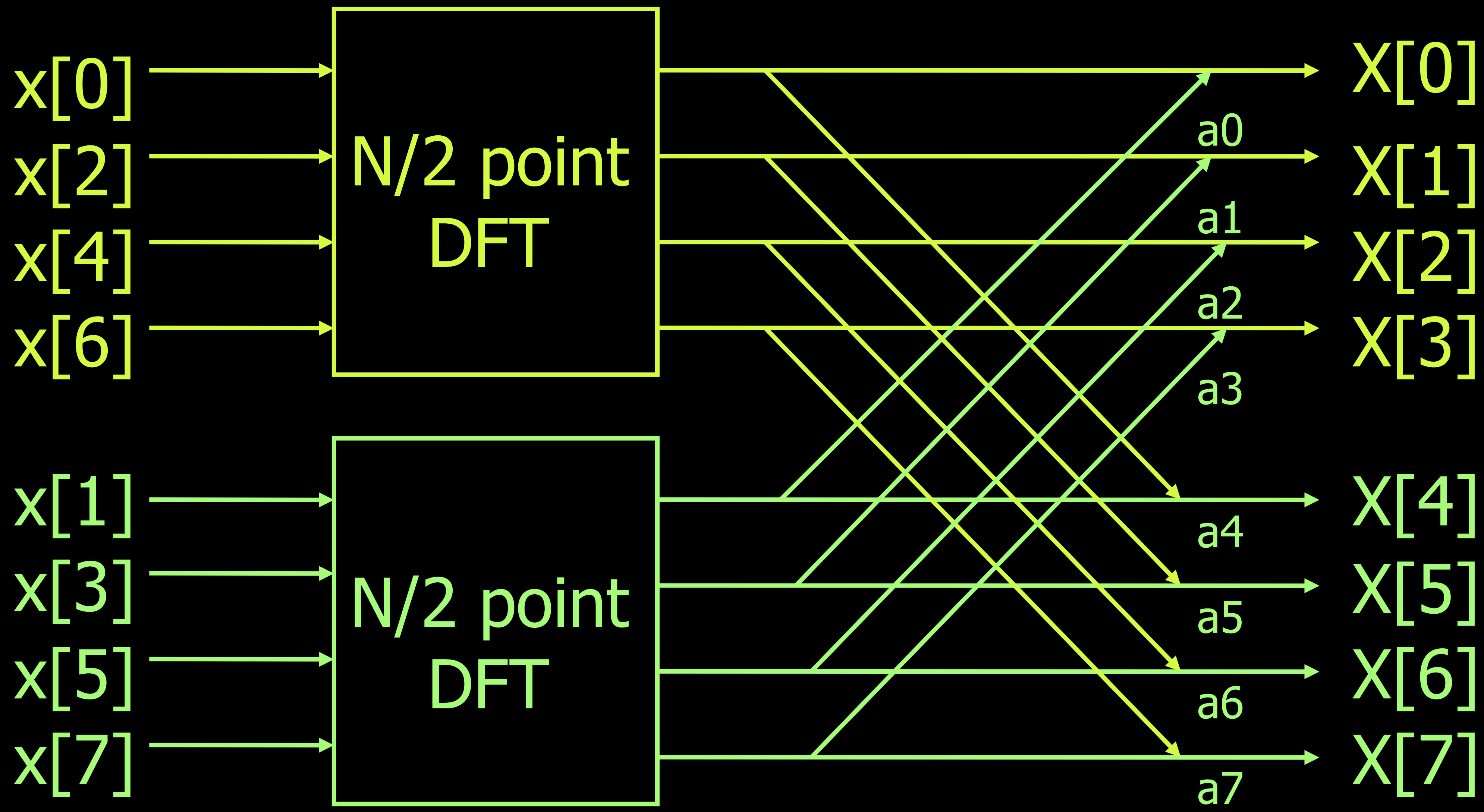
Fast Fourier Transform (FFT)



Fast Fourier Transform (FFT)

For N-point signals, if N is the power of 2, i.e., $v=\log_2 N$,

$\rightarrow N+2(N/2)^2$



Fast Fourier Transform (FFT)

For N-point signals, if N is the power of 2, i.e., $v=\log_2 N$,

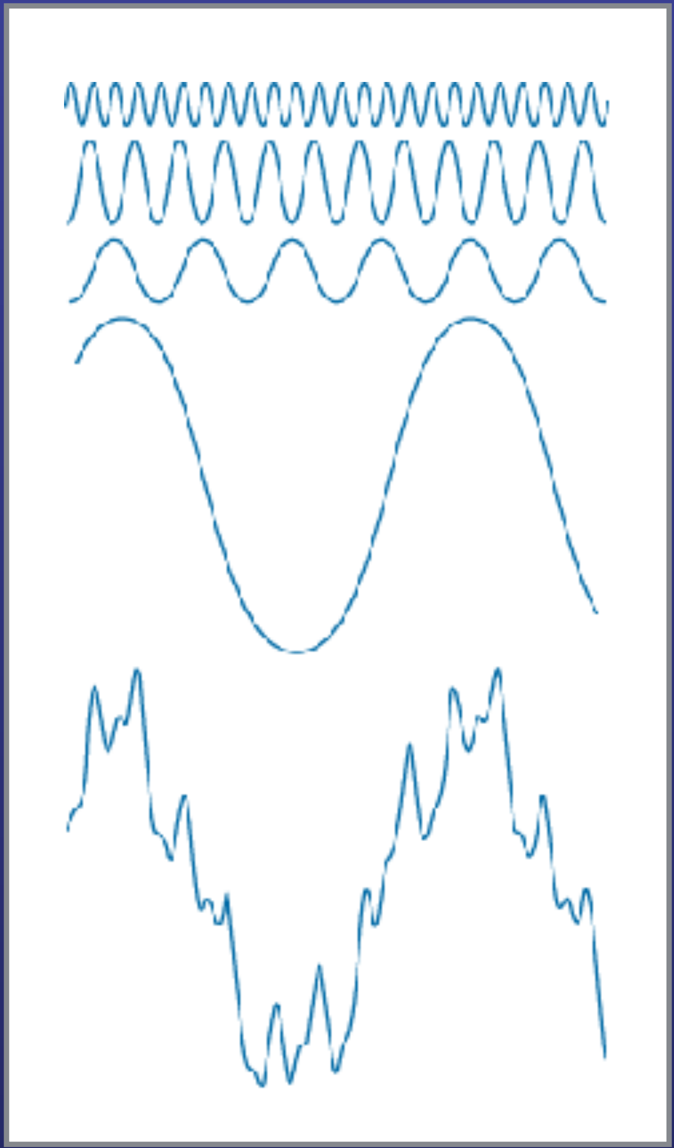
$\rightarrow N+2(N/2)^2$

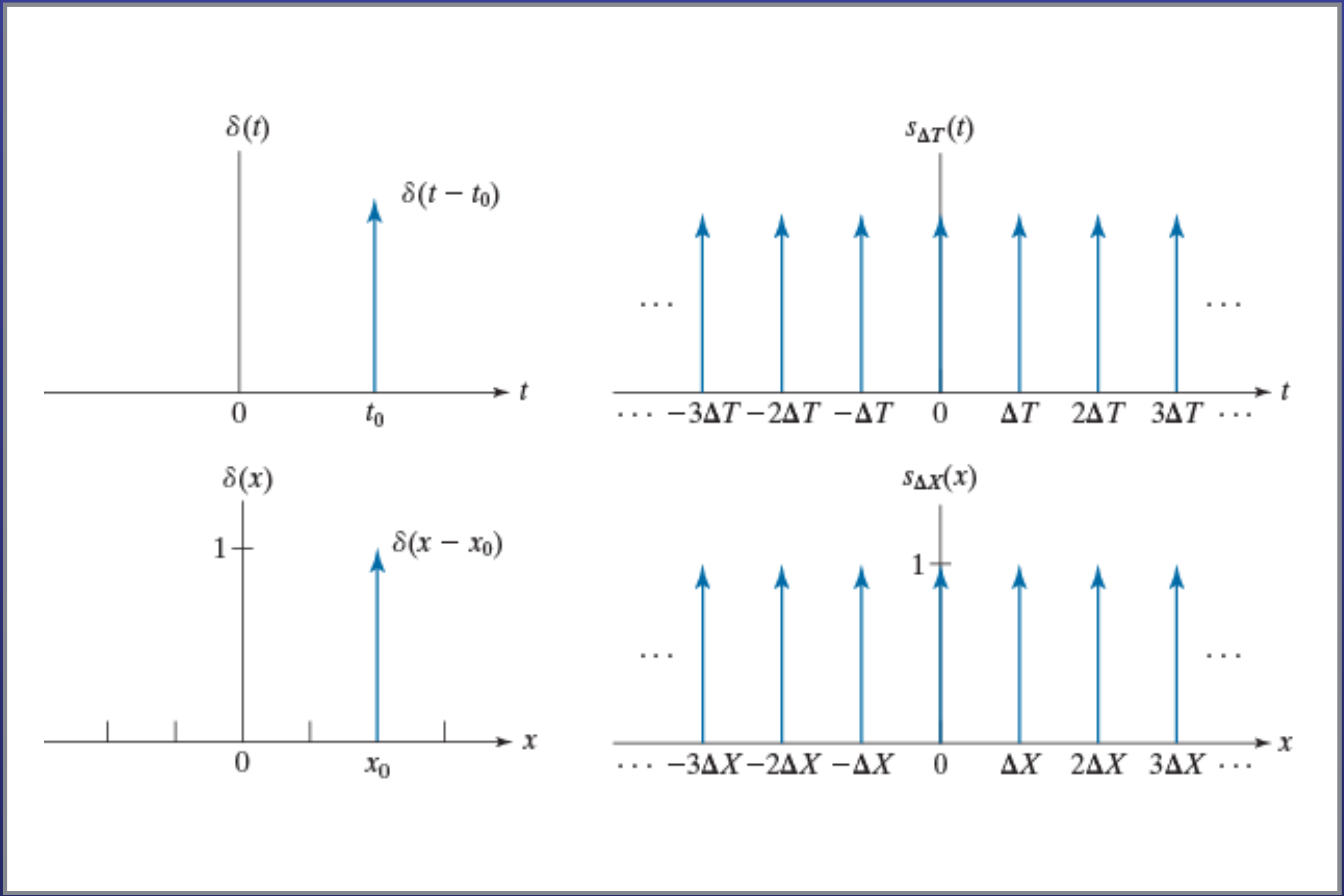
$\rightarrow N+2[N/2+2(N/4)^2] \rightarrow N+N+4(N/4)^2$

$\vdots$

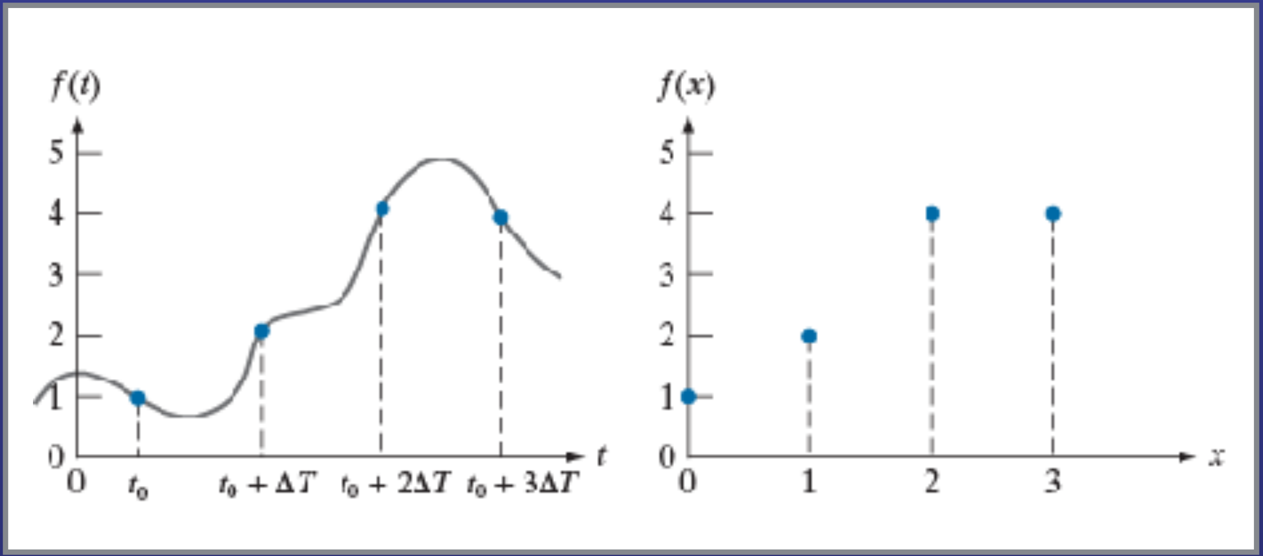
$\rightarrow N \log_2 N$

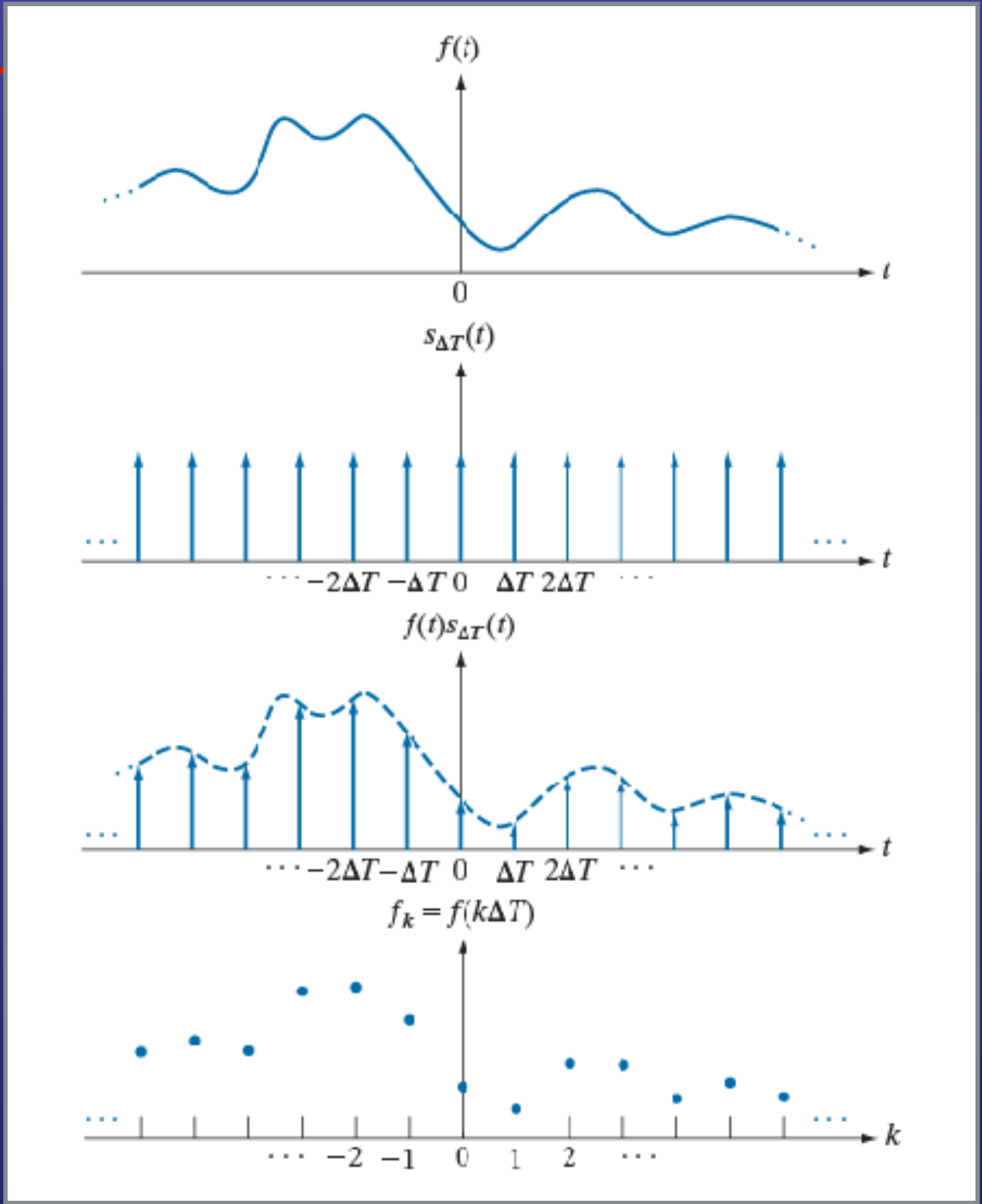
Review: Fourier Transform



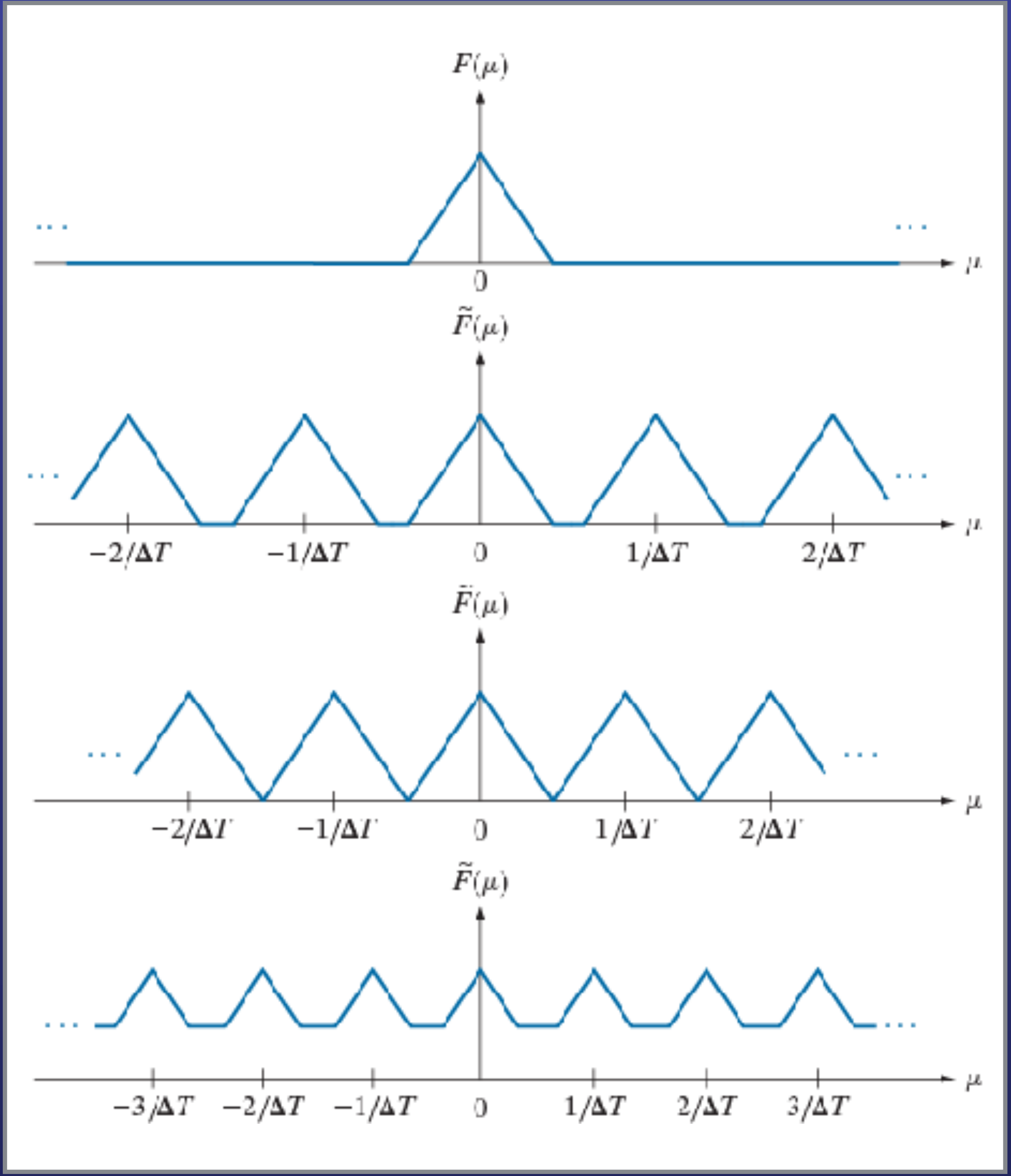


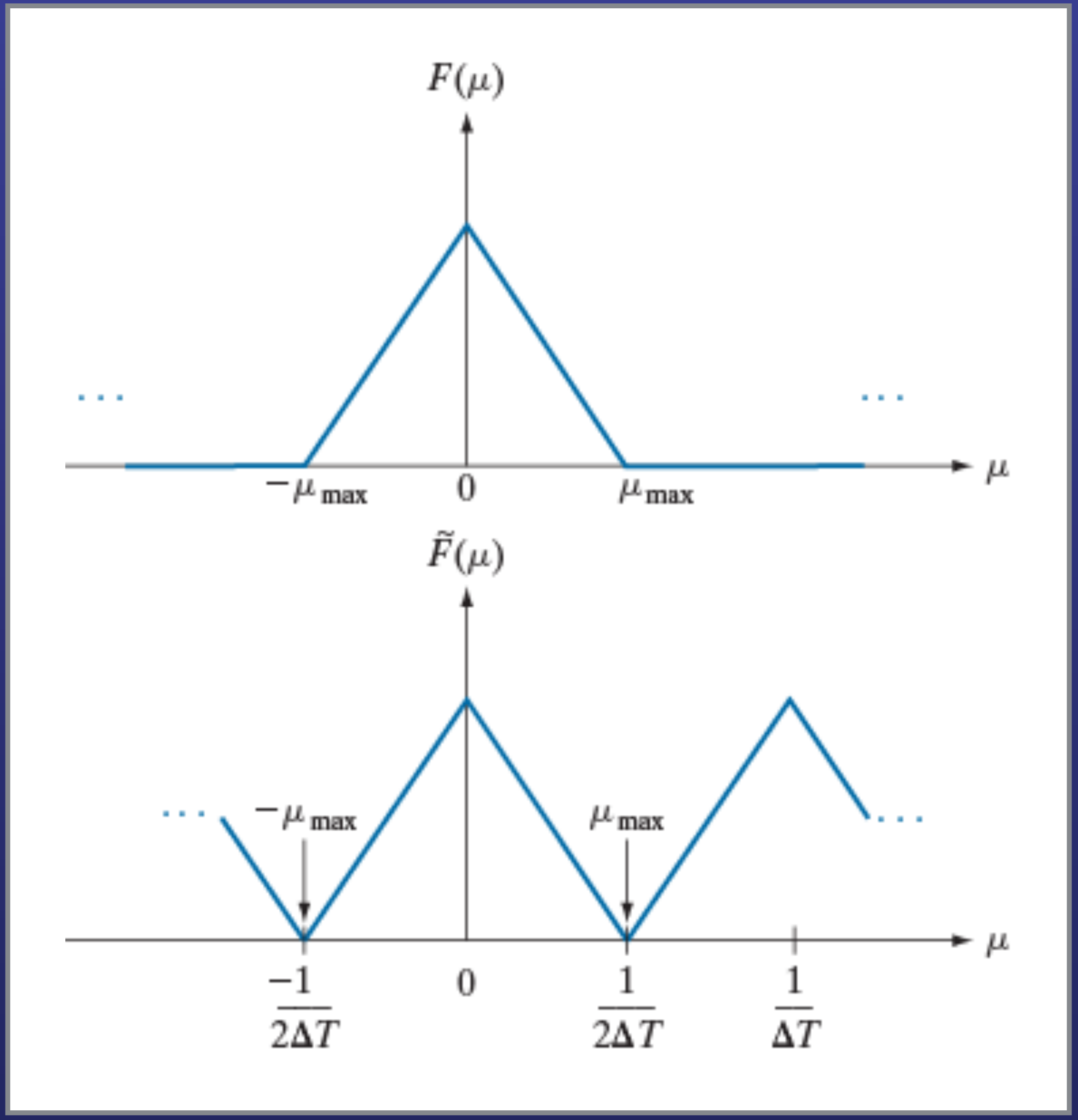
Review: Fourier Transform



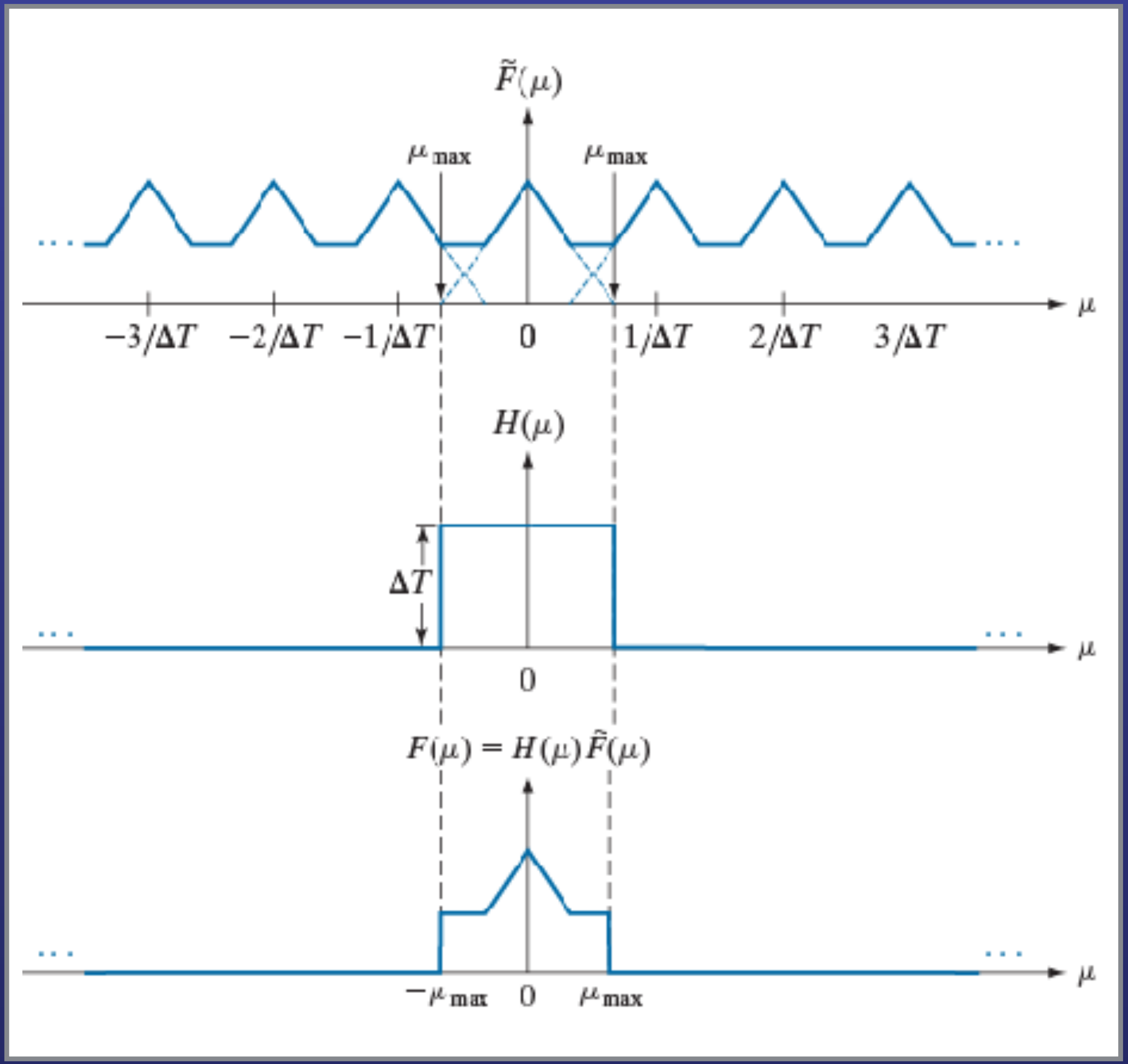


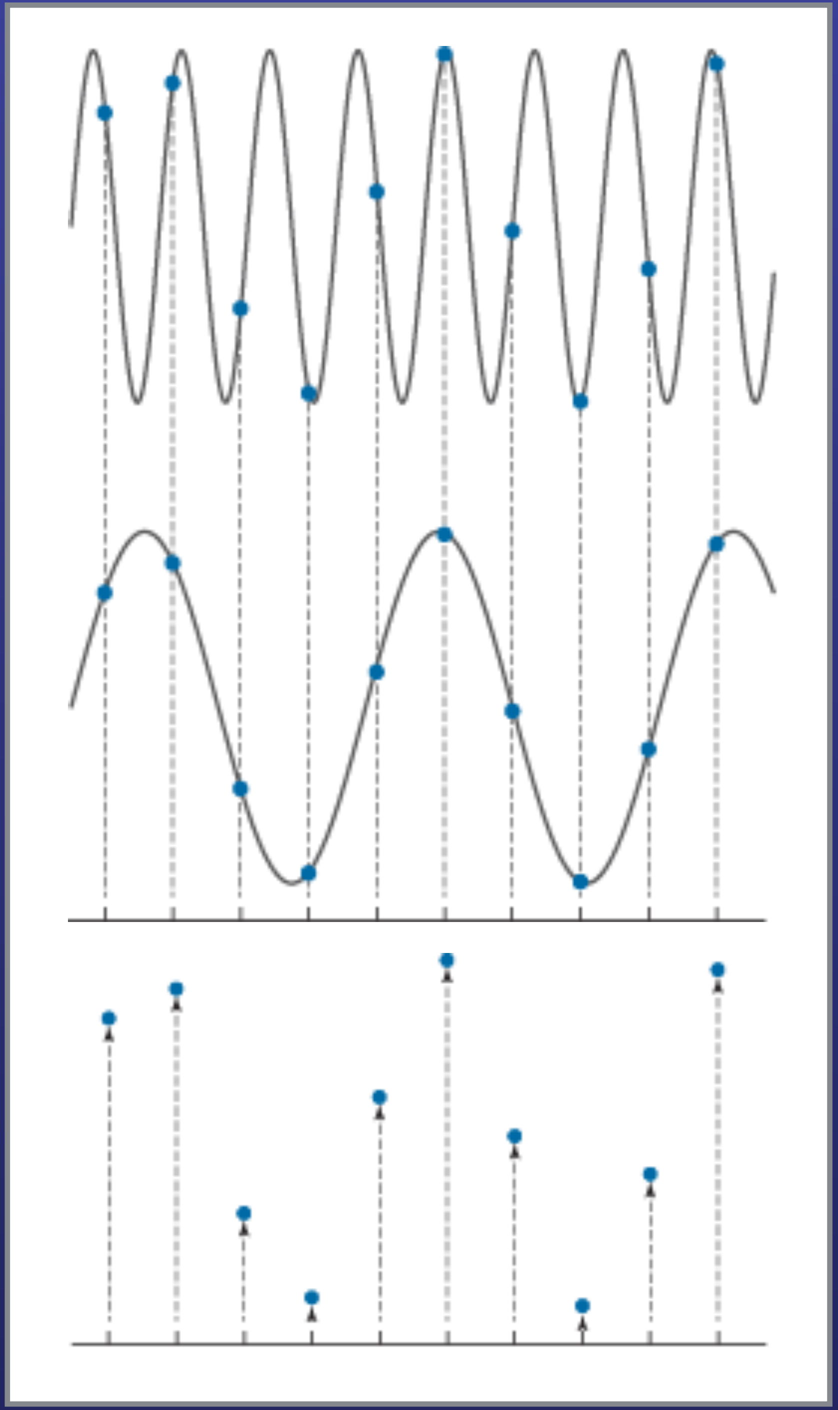
Review: Fourier Transform



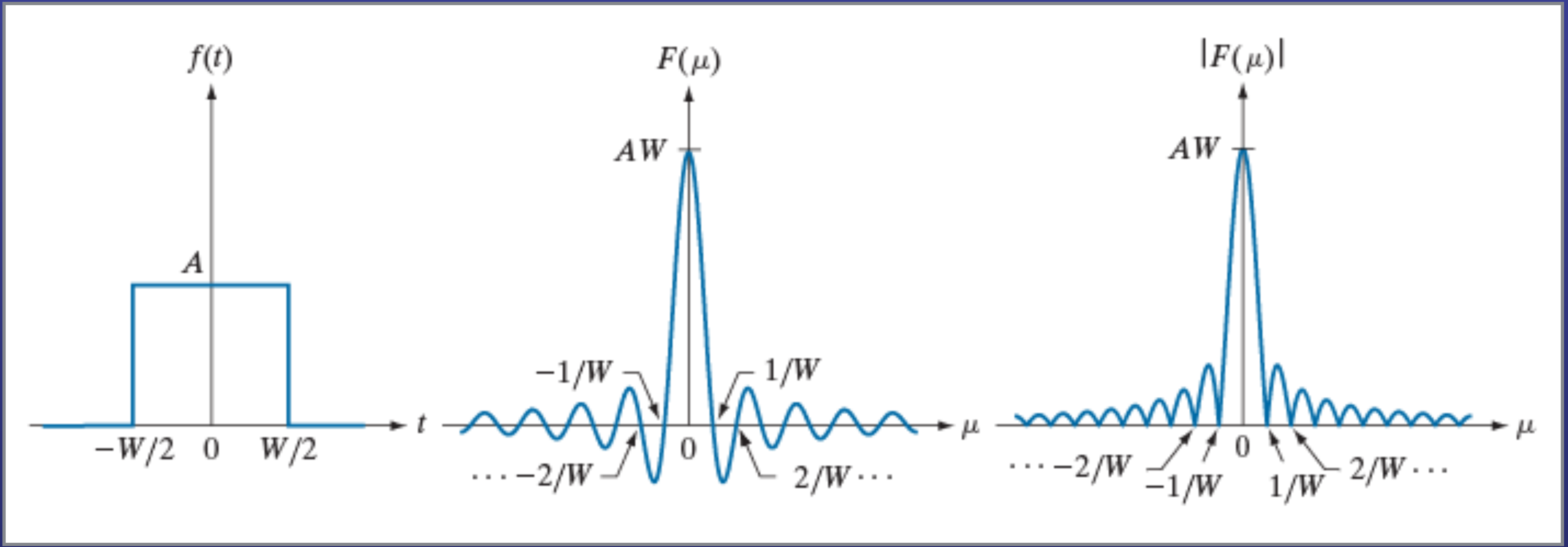


Review: Fourier Transform





Review: Fourier Transform



2D - Fourier Transform

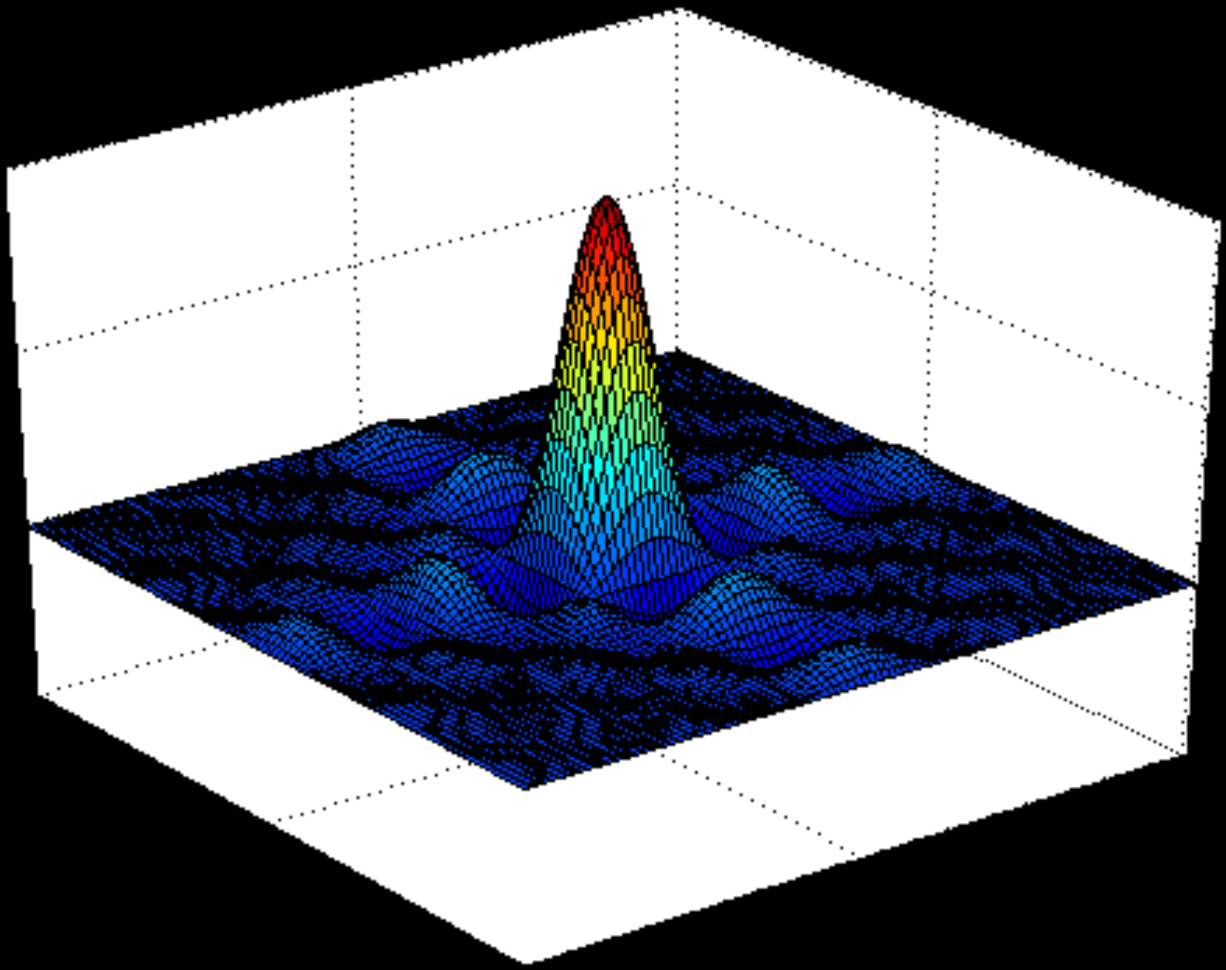
$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

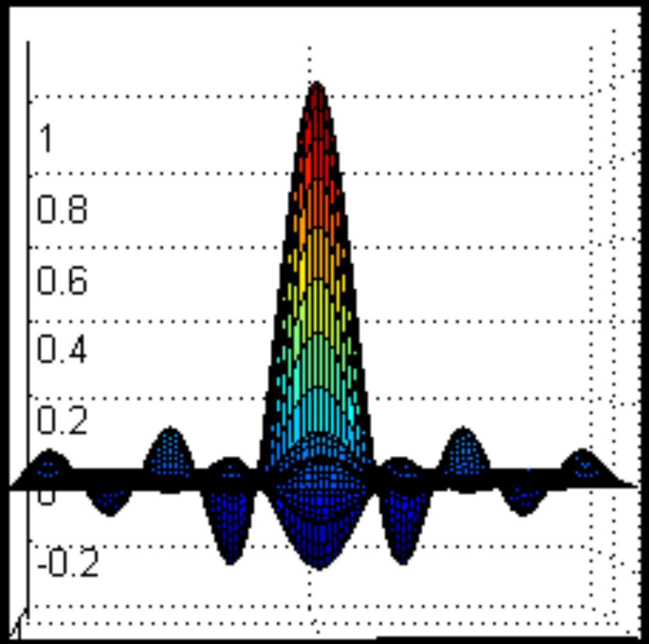
x,y

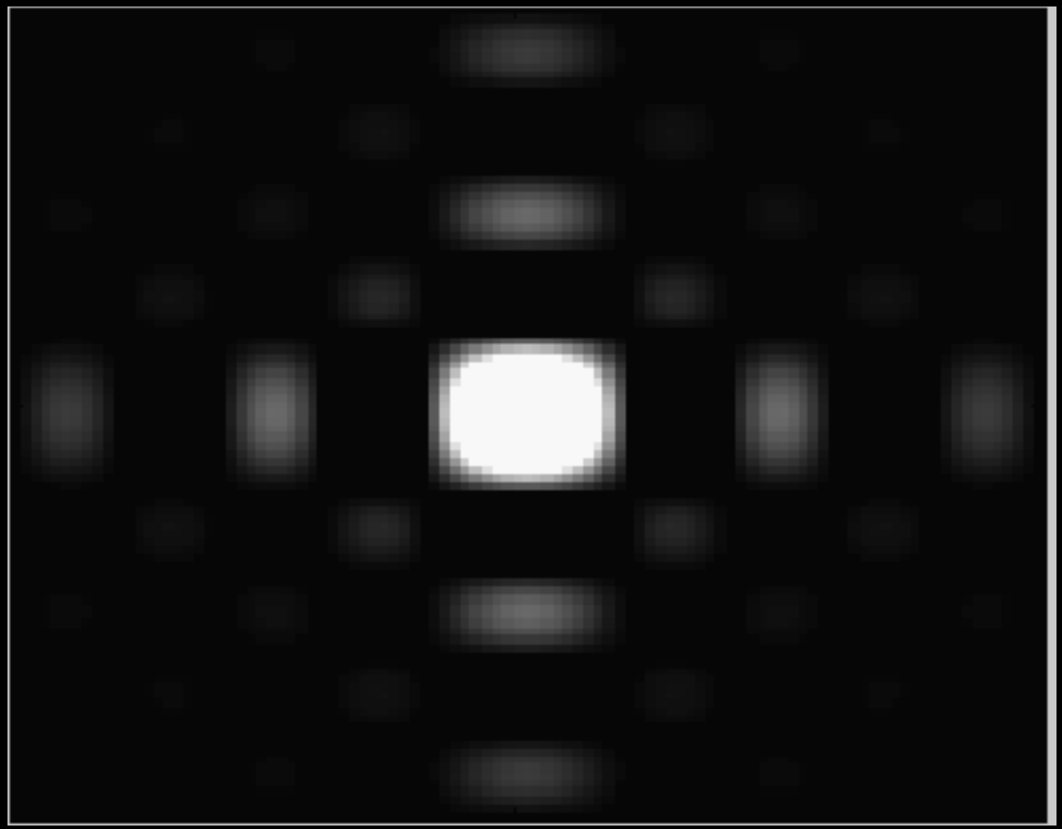
ω_x, ω_y f(x,y)

$$F(\omega_x, \omega_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,y) e^{-j(\omega_x x + \omega_y y)} dxdy$$

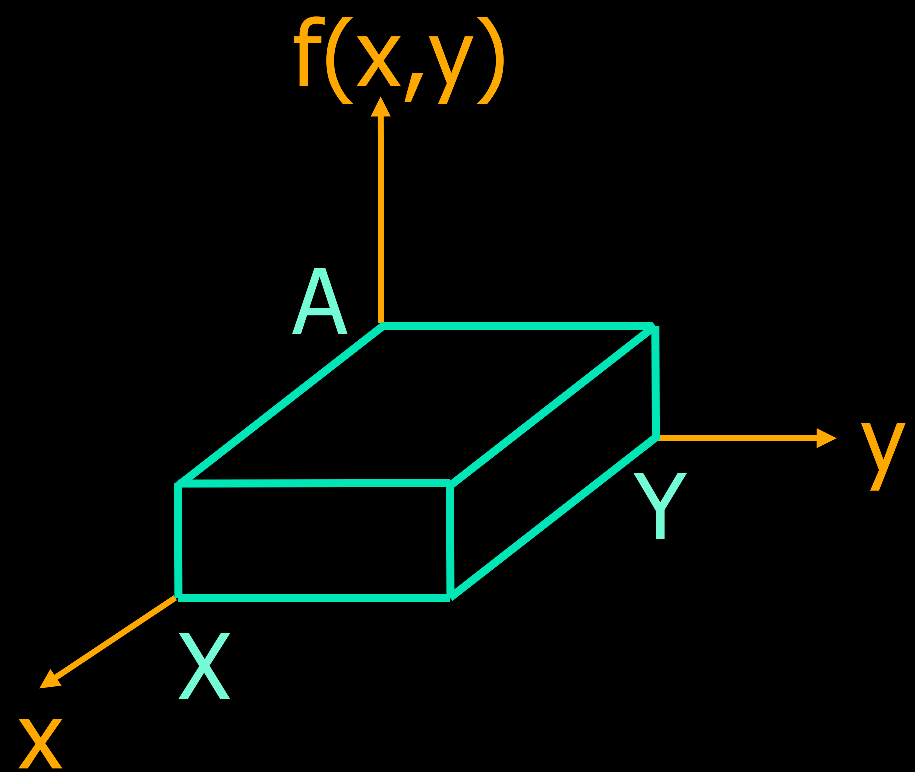
$$F(u,v) = \frac{\sin(\pi u)}{\pi u} \frac{\sin(\pi v)}{\pi v}$$

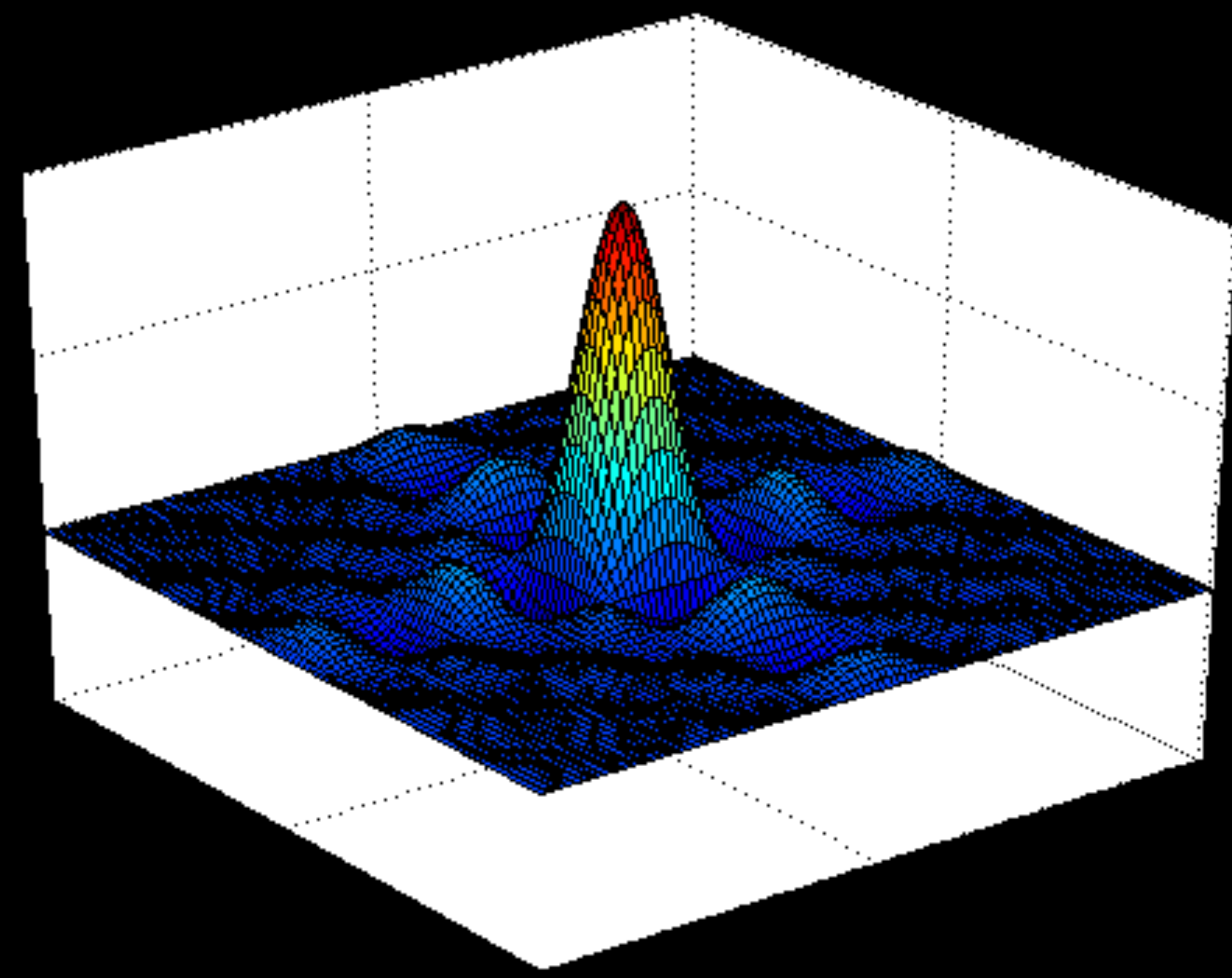






$$\left|F(u,v)\right| = AXY\left[\frac{\sin(\pi uX)}{\pi uX}\right]\left[\frac{\sin(\pi vY)}{\pi vY}\right]$$





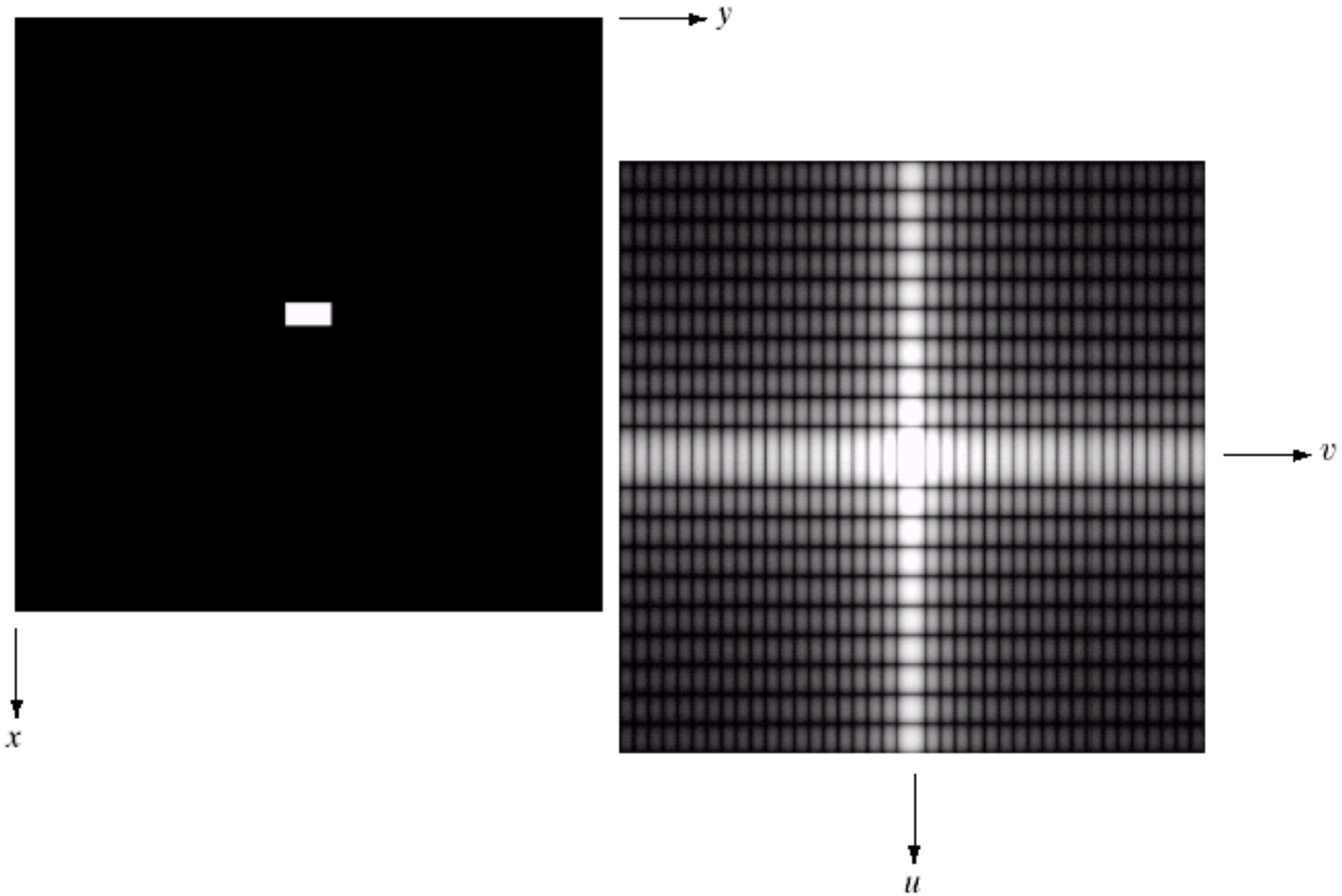
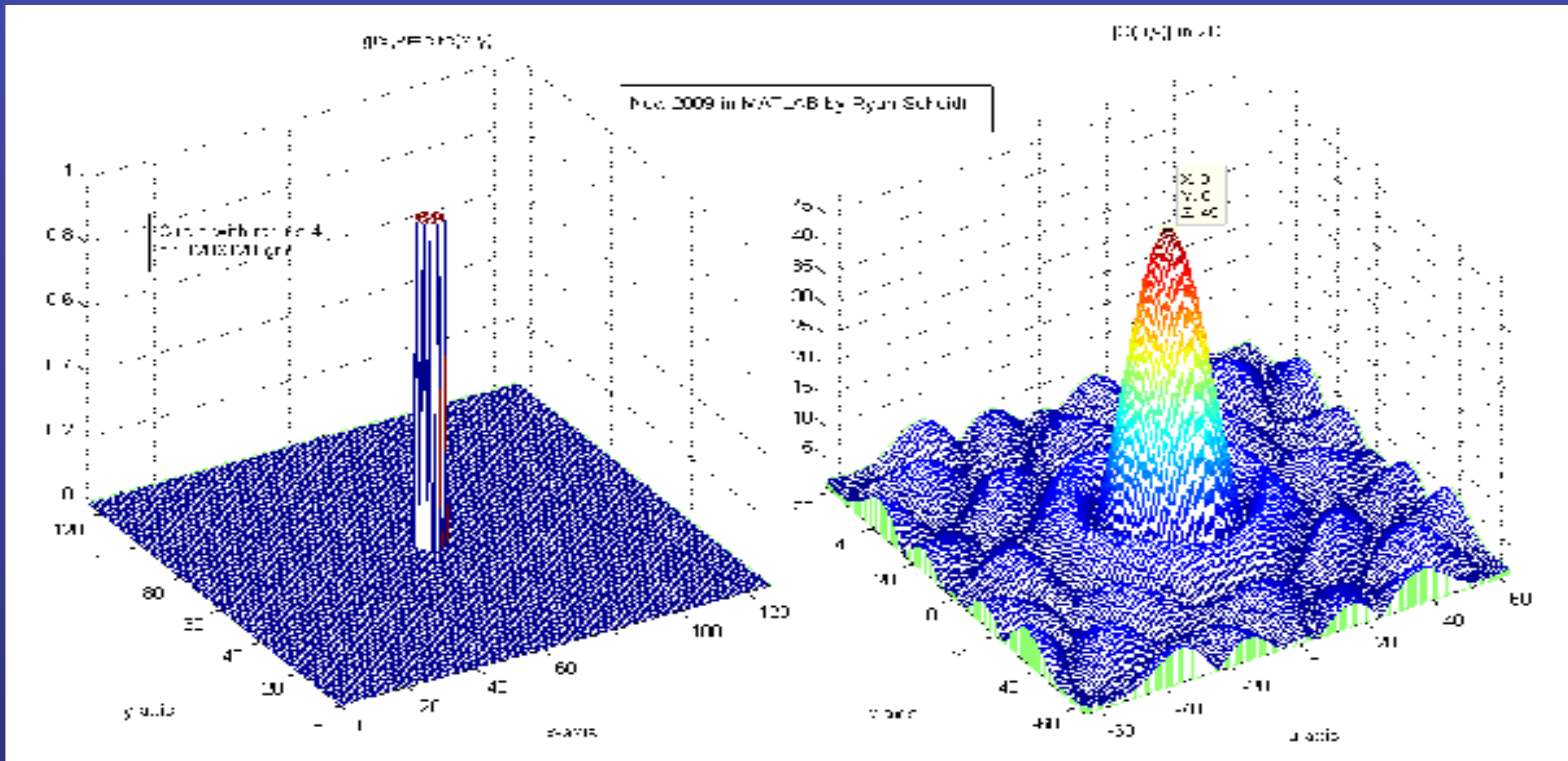
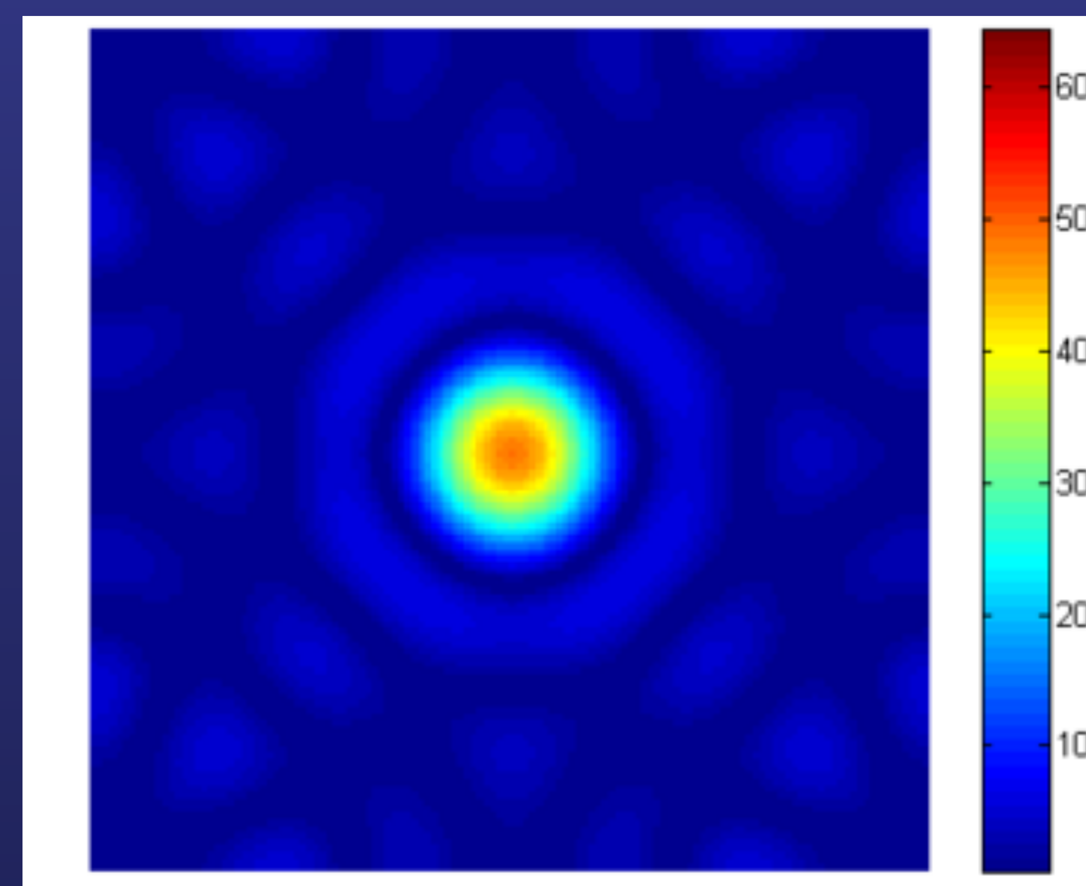


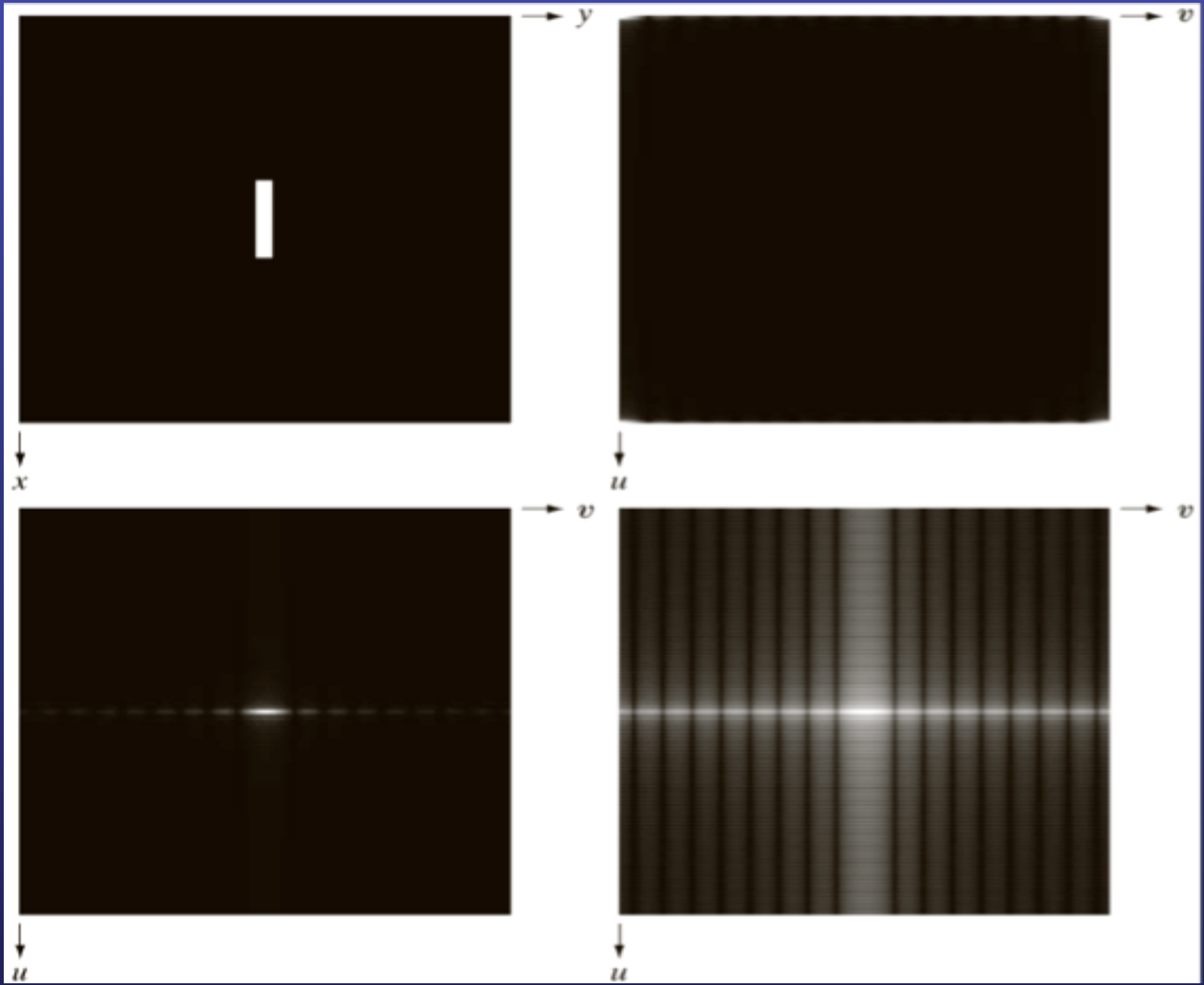
FIGURE 4.3 (a) Image of a 20×40 white rectangle on a black background of size 512×512 pixels. (b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.



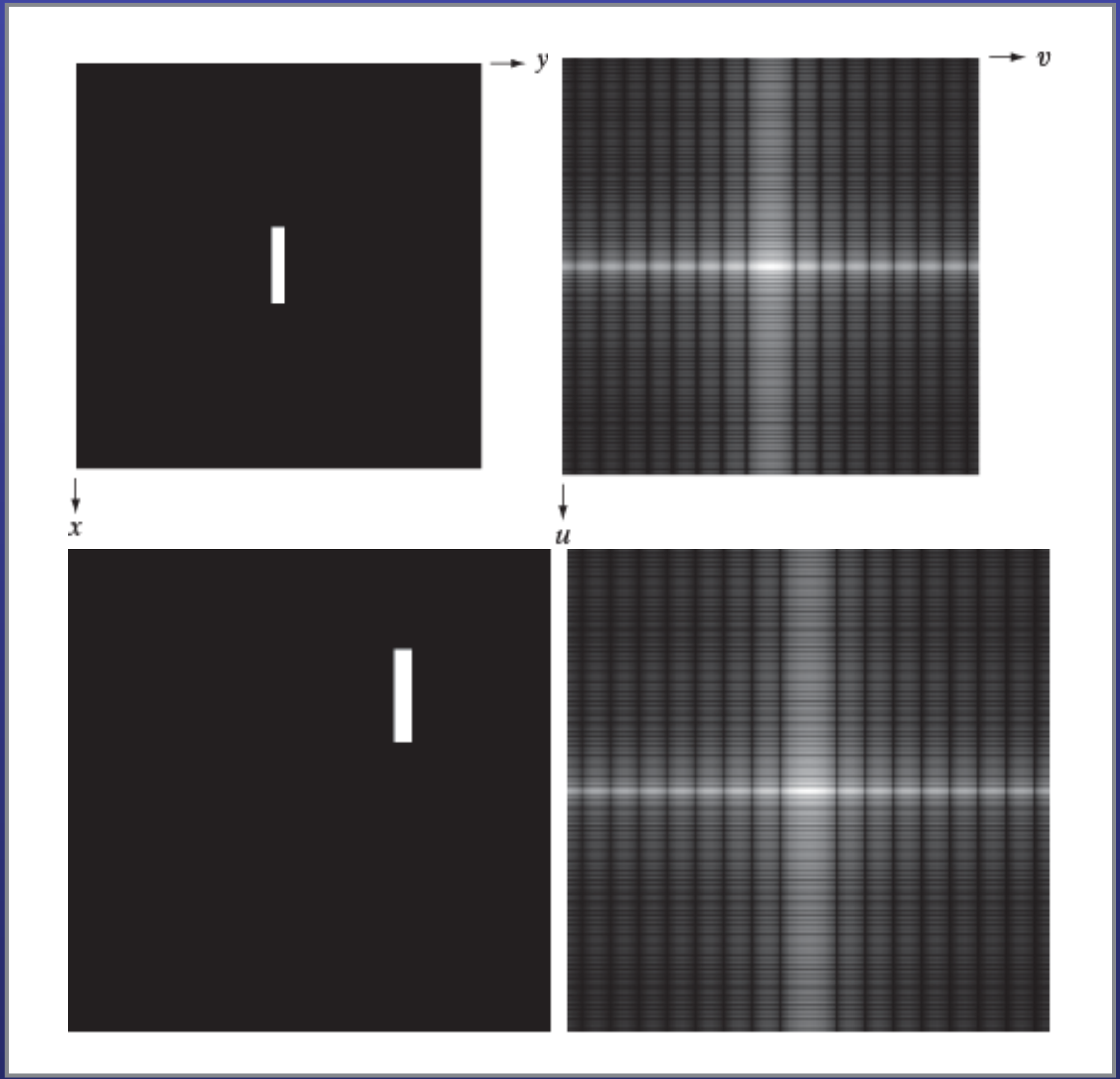
120
100
60
40
20



60
50
40
30
20
10

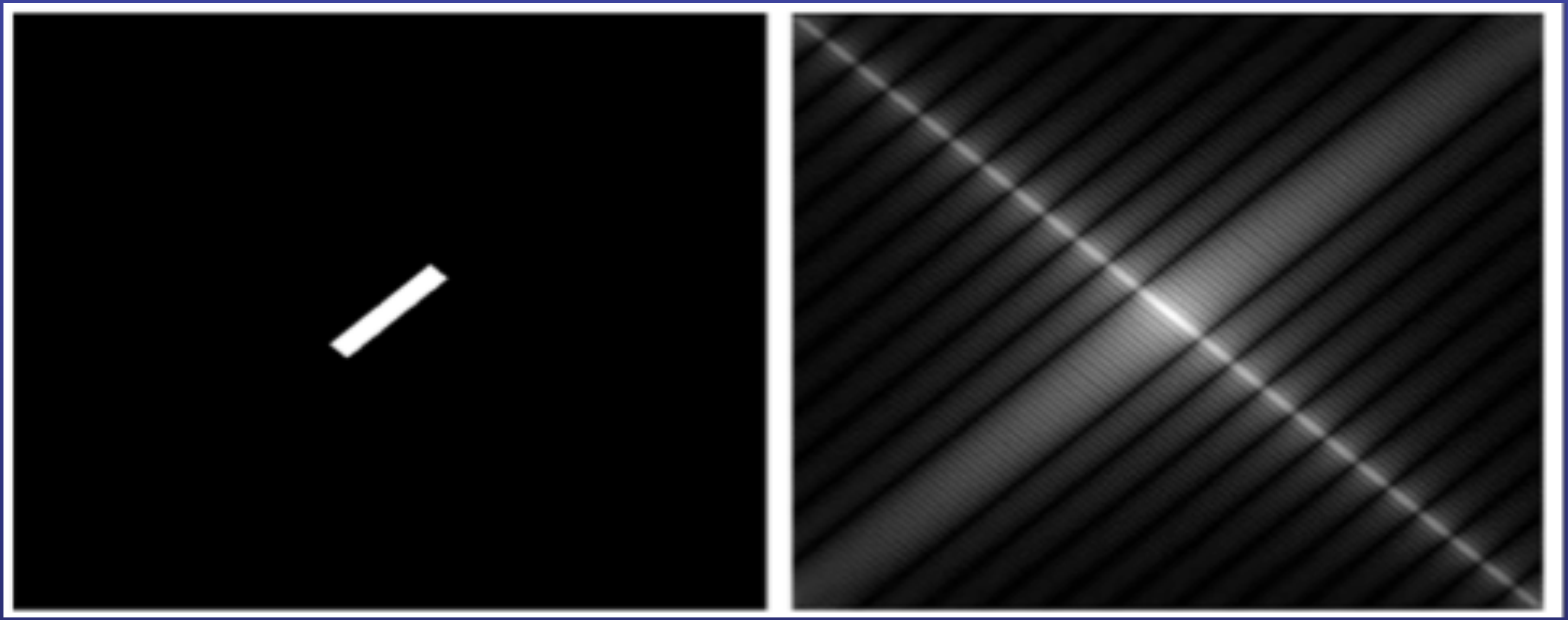


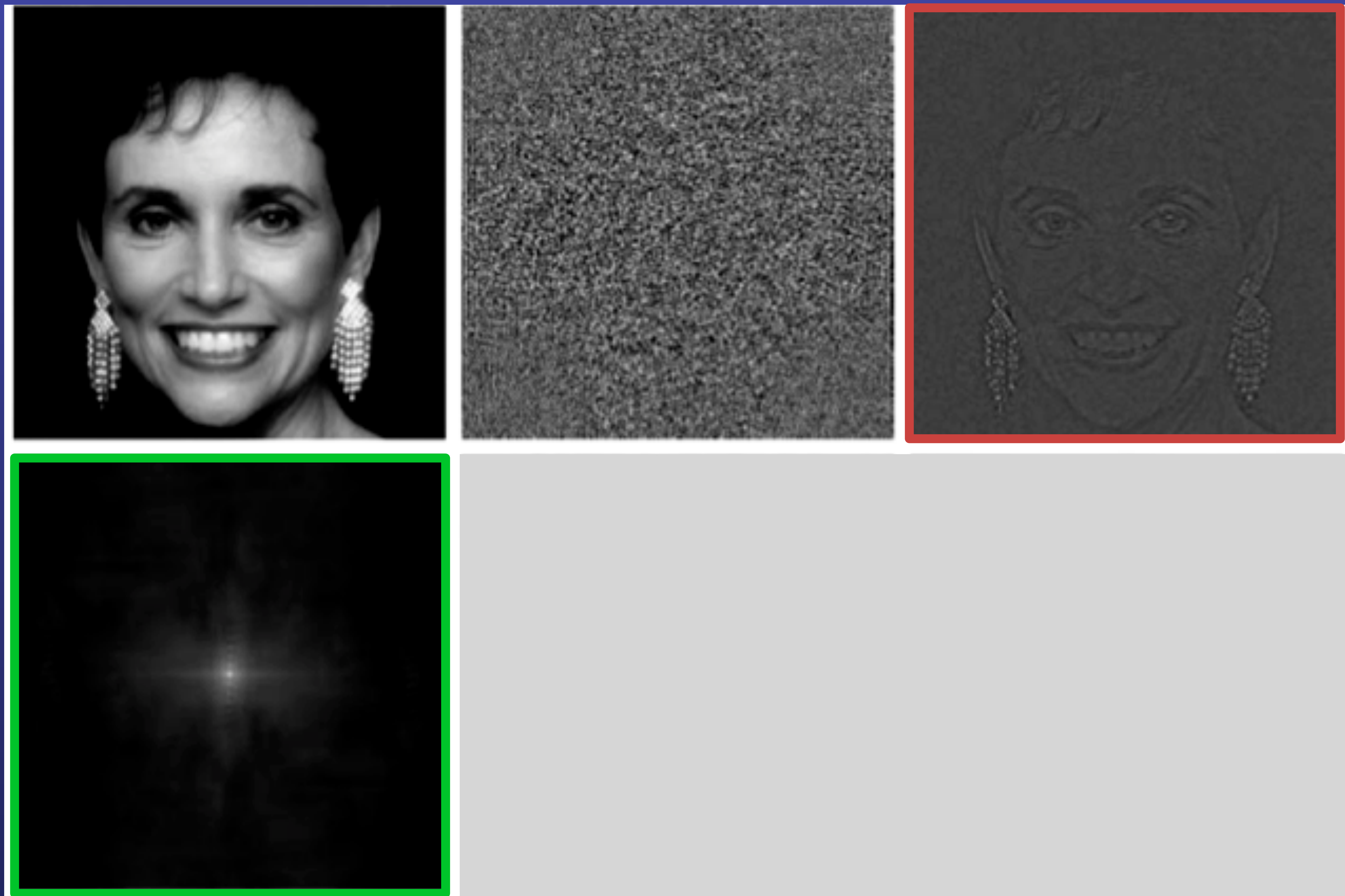
y
x
v
u
v
u
v
u



$$|F(u,v)|$$

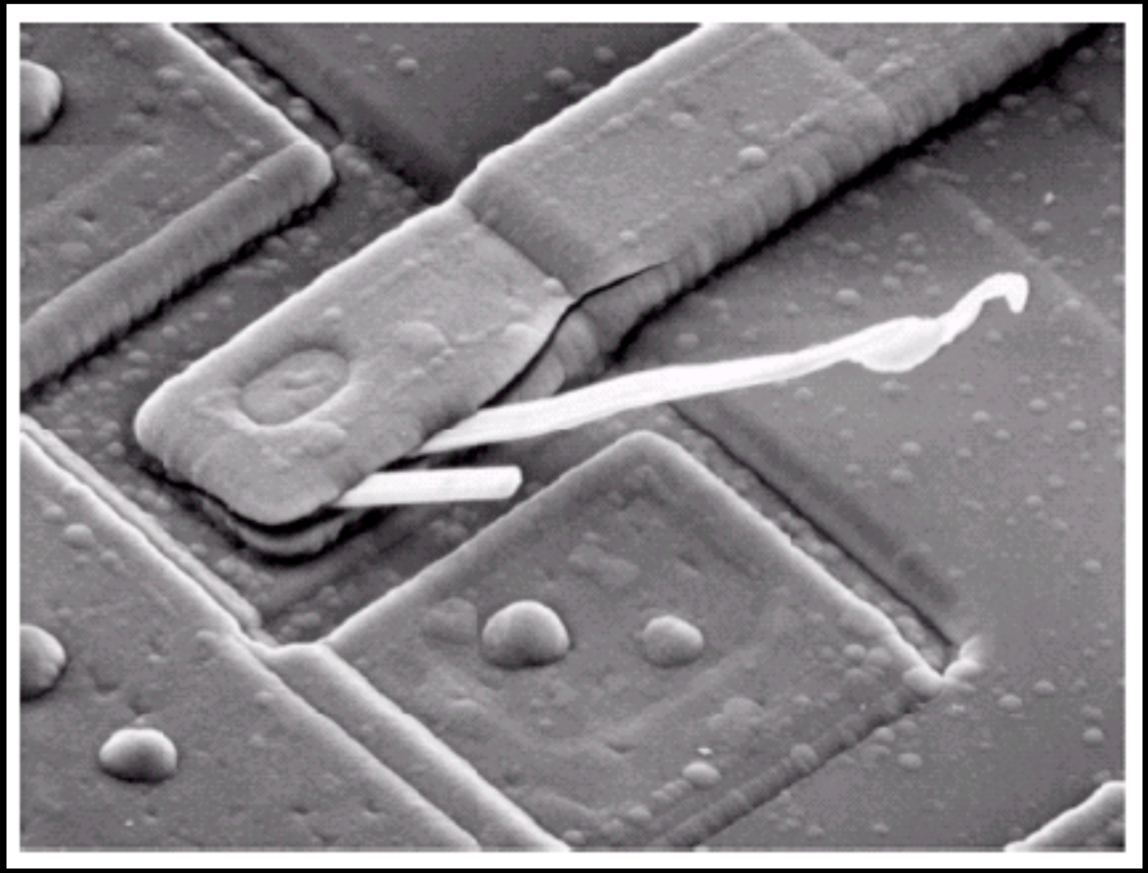
The same

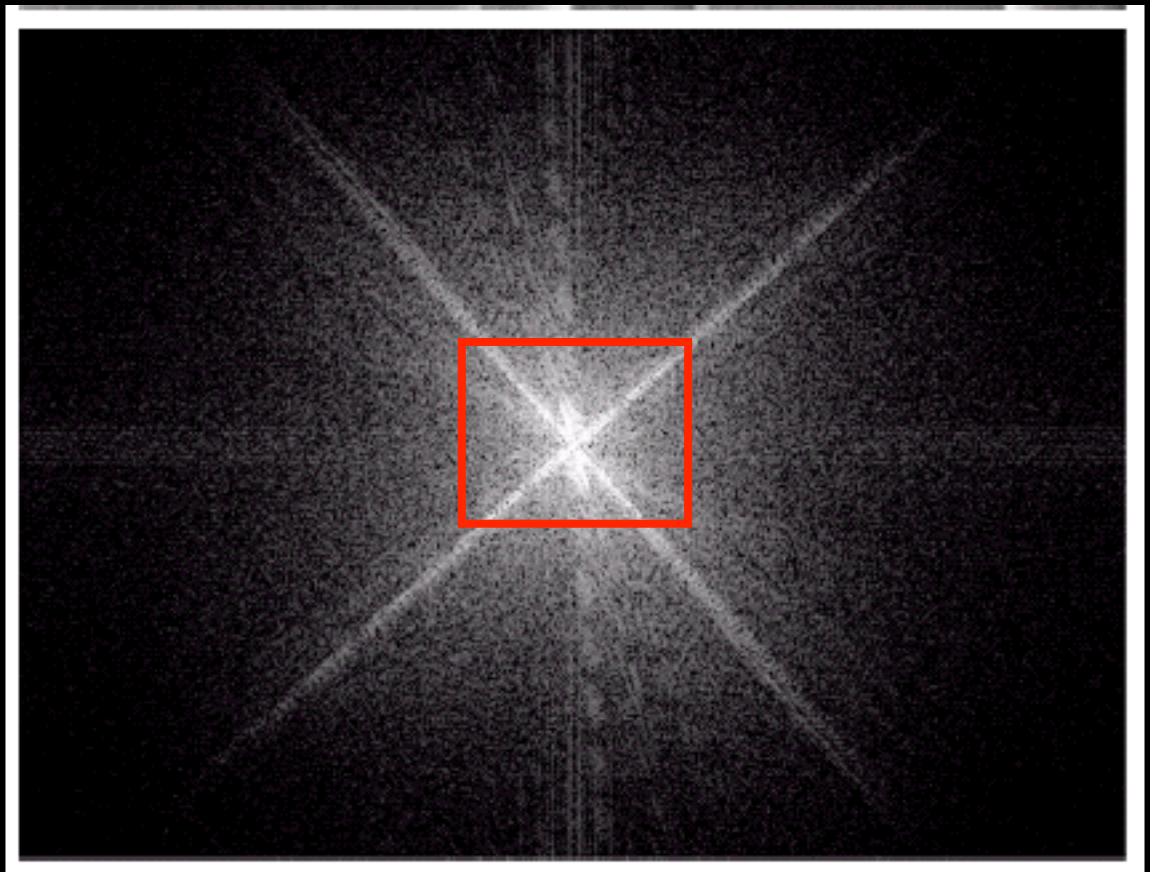




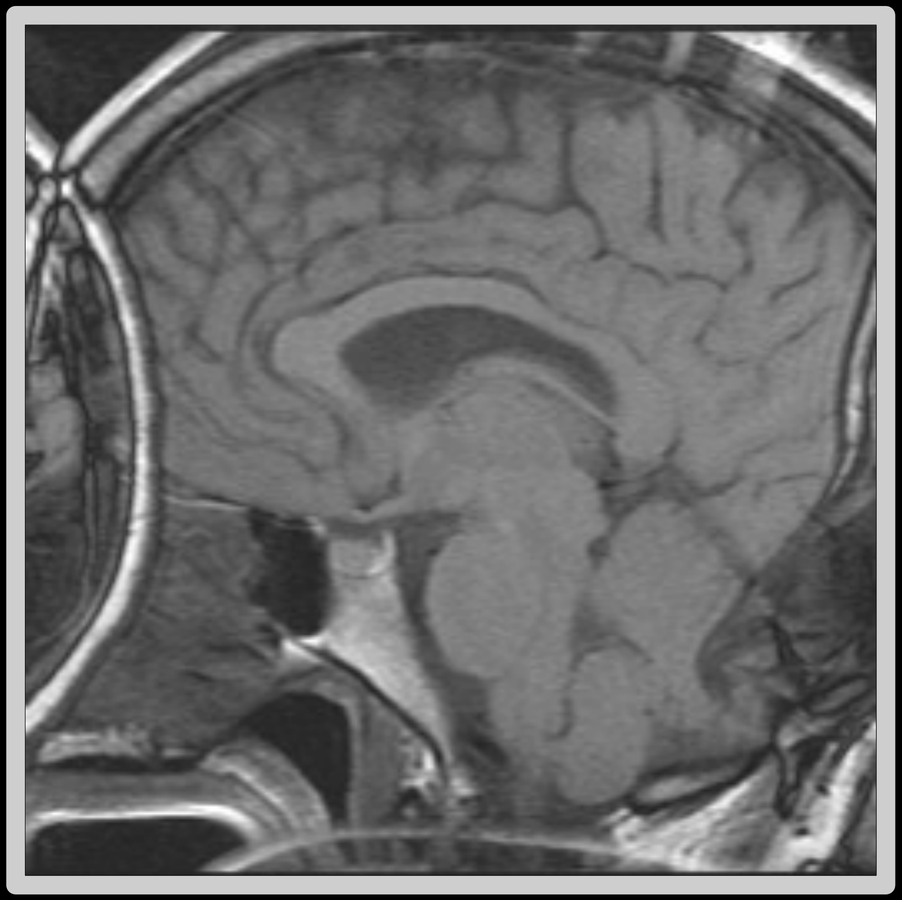
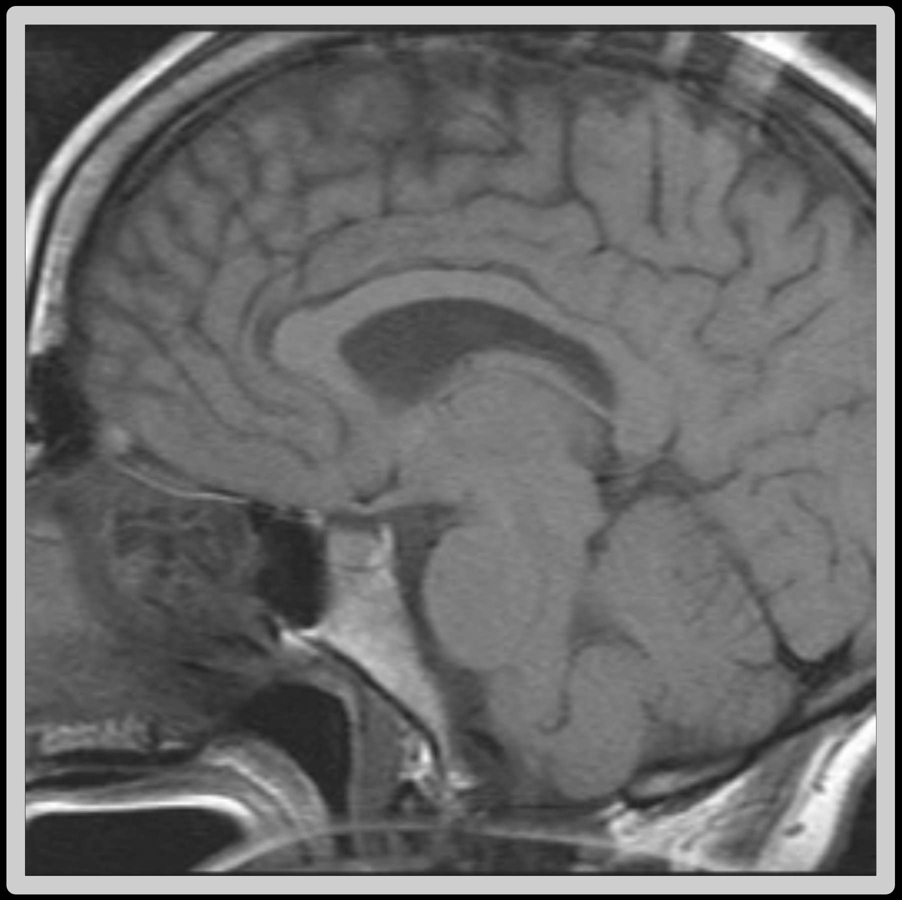
Low-freq. → slowly varying

High-freq. → edges, or noise

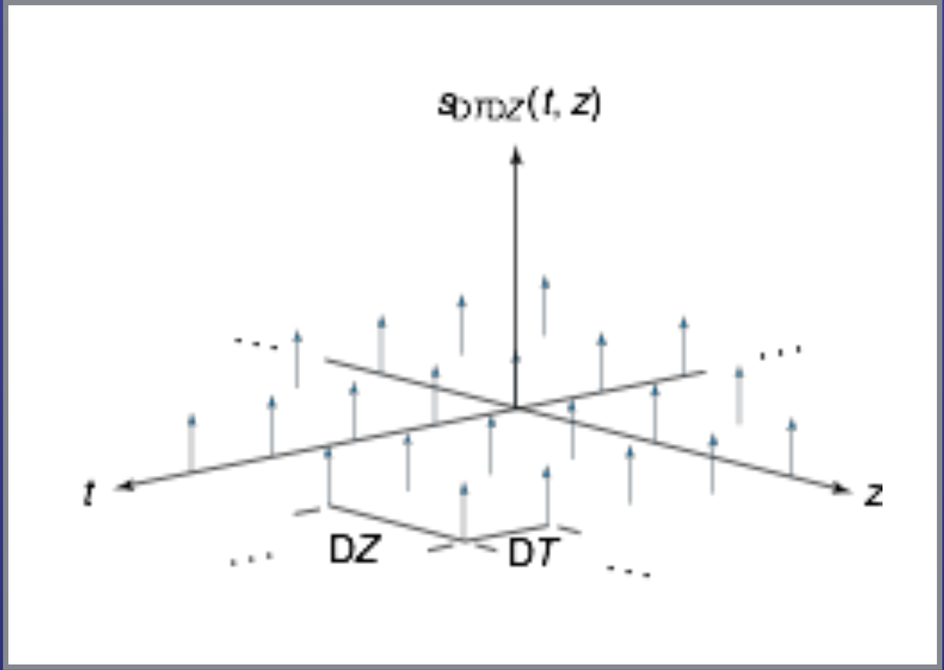


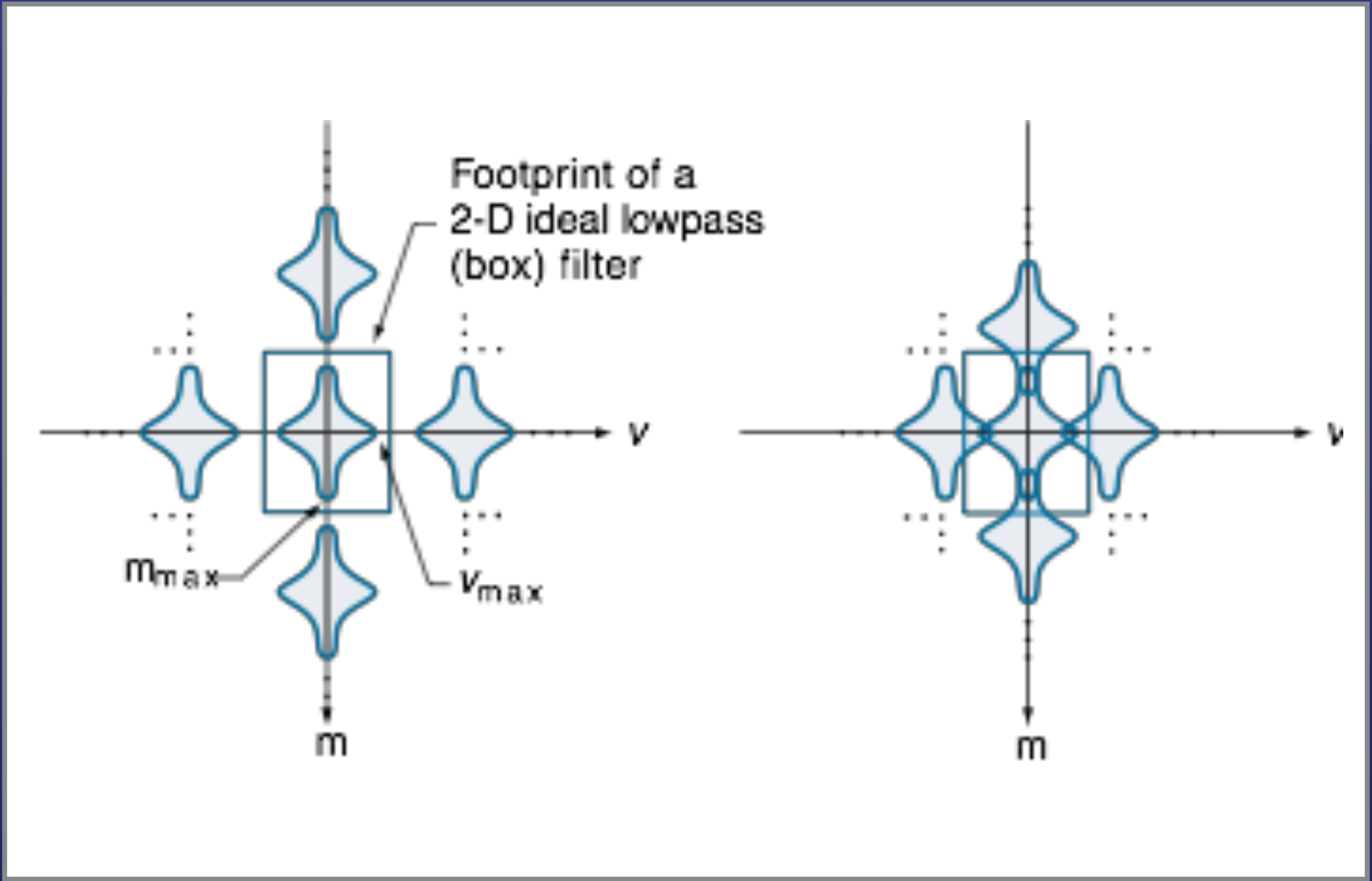


Aliasing 假影

2D Sampling & Aliasing



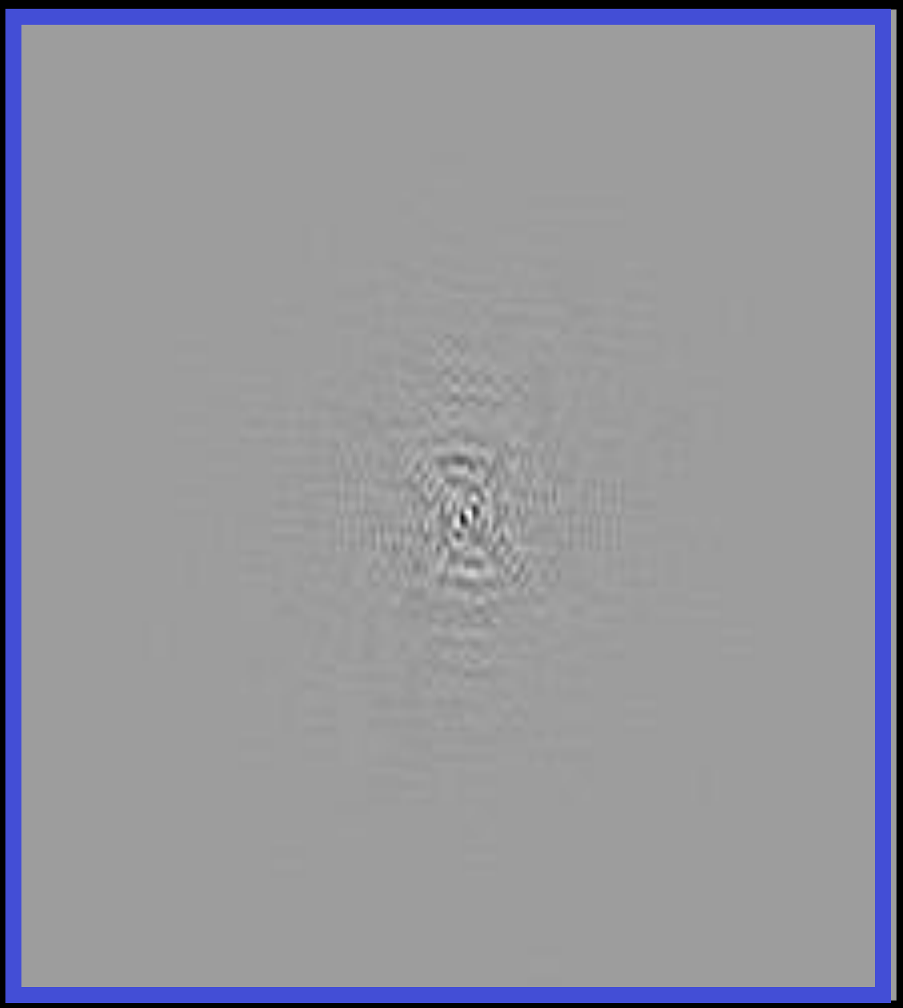


2D Sampling & Aliasing

Some applications of FFT

Fourier Transform 重建 CT 影像

原始物體影像



二維傅立葉轉換

二維傅立葉轉換法

- 影像與傅立葉轉換一對一對應
- 只要能取得傅氏轉換數據，就能由 inverse FT 計算到影像
- CT 投影與傅氏轉換的關係？

投影定理

- Projection theorem
  - 也稱為 central slice theorem
- 影像沿著某一方向的投影，經過 1D FT 之後，對應 2D FT 平面的一條線

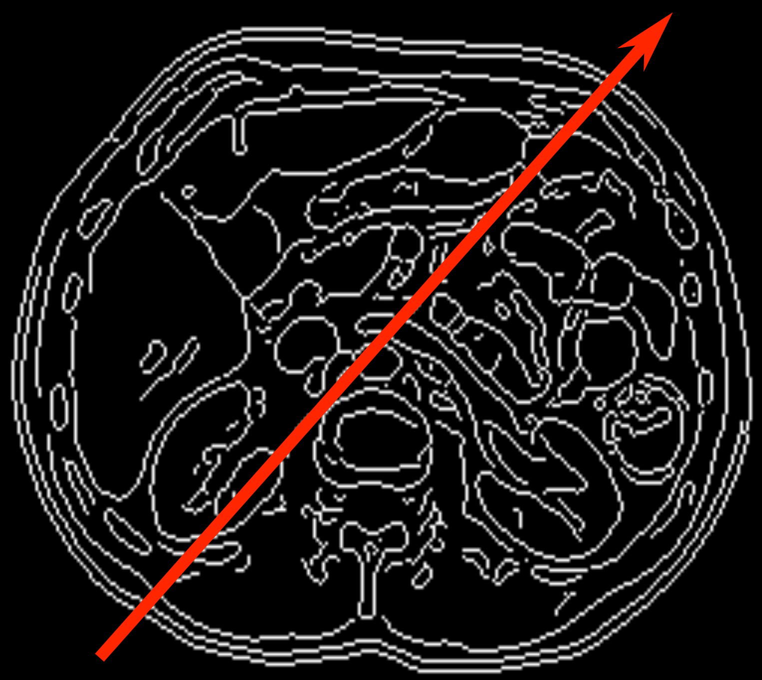
數學證明

- $F(u,v) = \iint f(x,y)\, e^{-j2\pi(ux+vy)}\, dxdy$
- $F(0,v) = \iint f(x,y)\, e^{-j2\pi vy}\, dxdy$
- $F(0,v) = \iint f(x,y) dx\, e^{-j2\pi vy}\, dy$
- $F(0,v) = x$ 方向投影的 FT

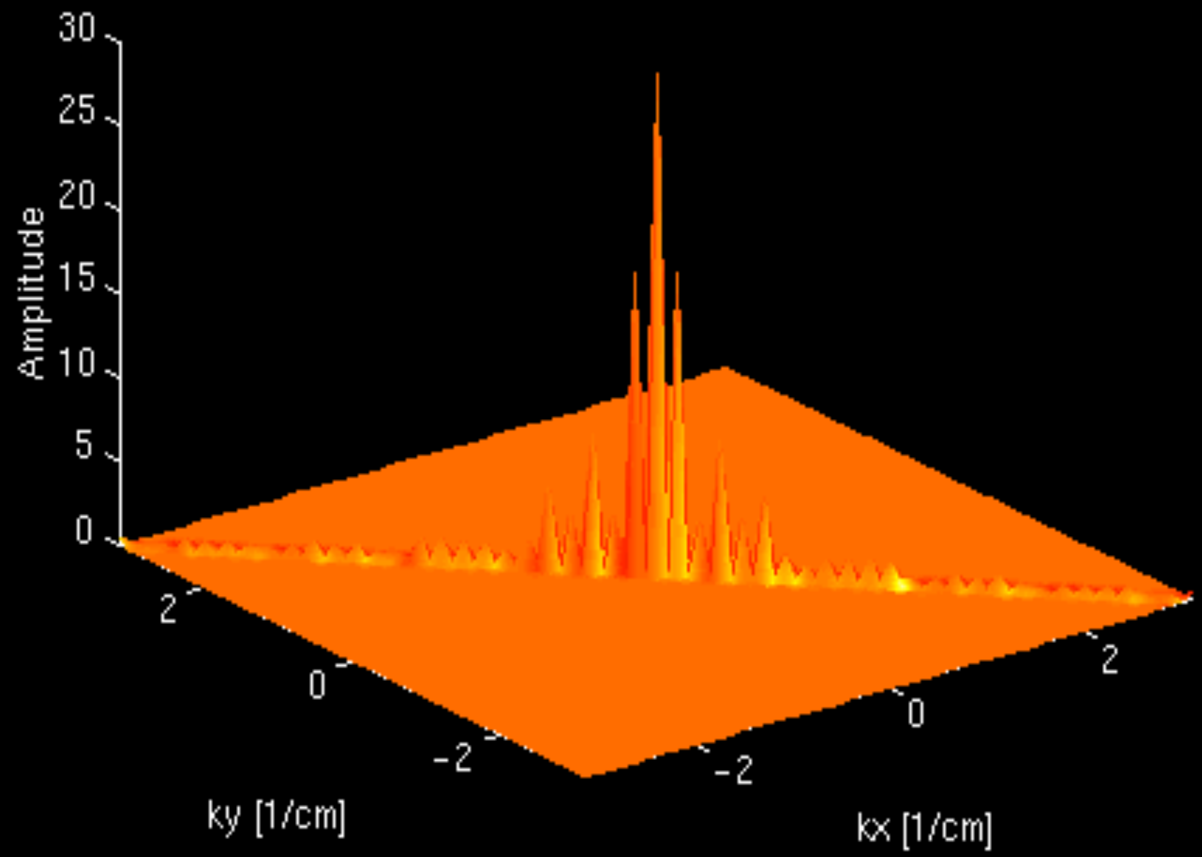
那就容易了啊

- CT 掃瞄過程 = 收集各角度投影
- 每一投影都計算 1D FT
- 彙整 = 極座標 2D FT 平面
- Inverse 2D FT 算回影像

2D Fourier Transform 過程

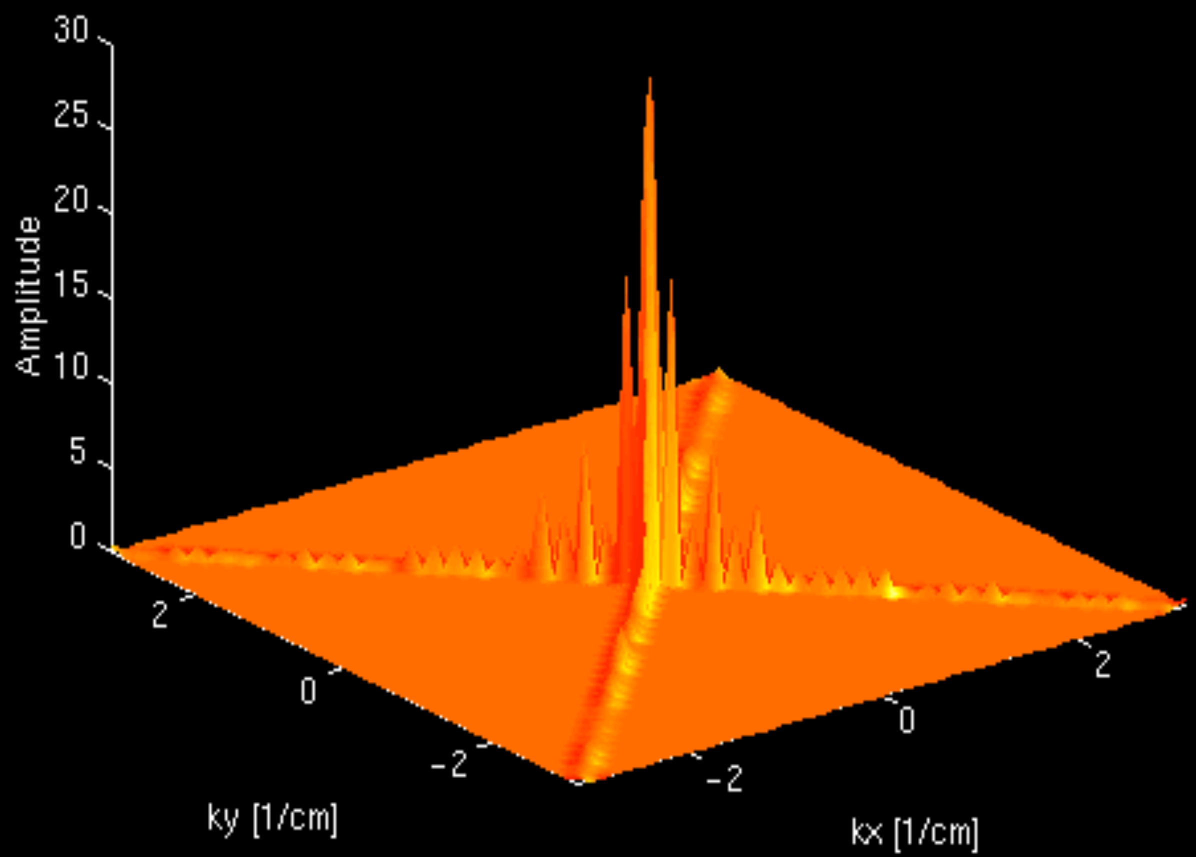


1 acquisition



Fourier space

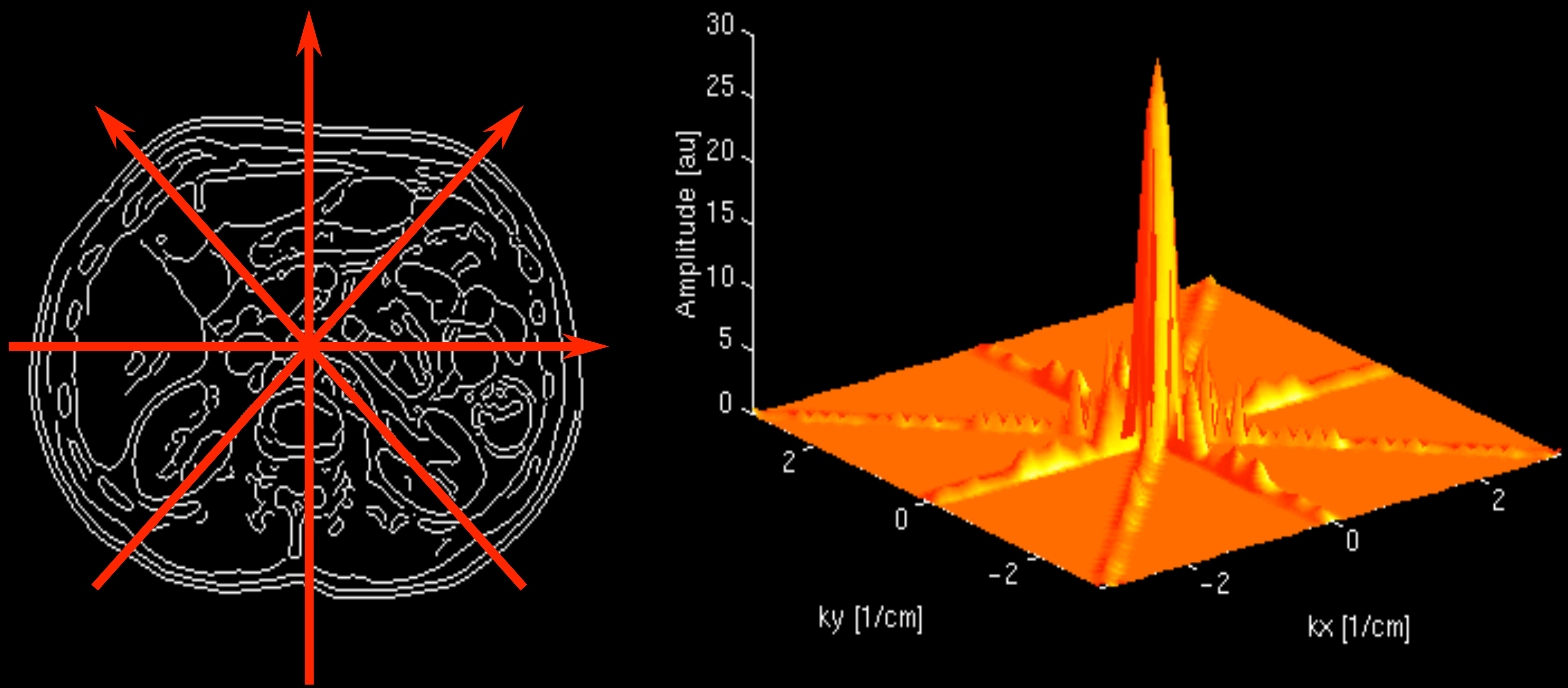
2D Fourier Transform 過程



2 acquisitions

Fourier space

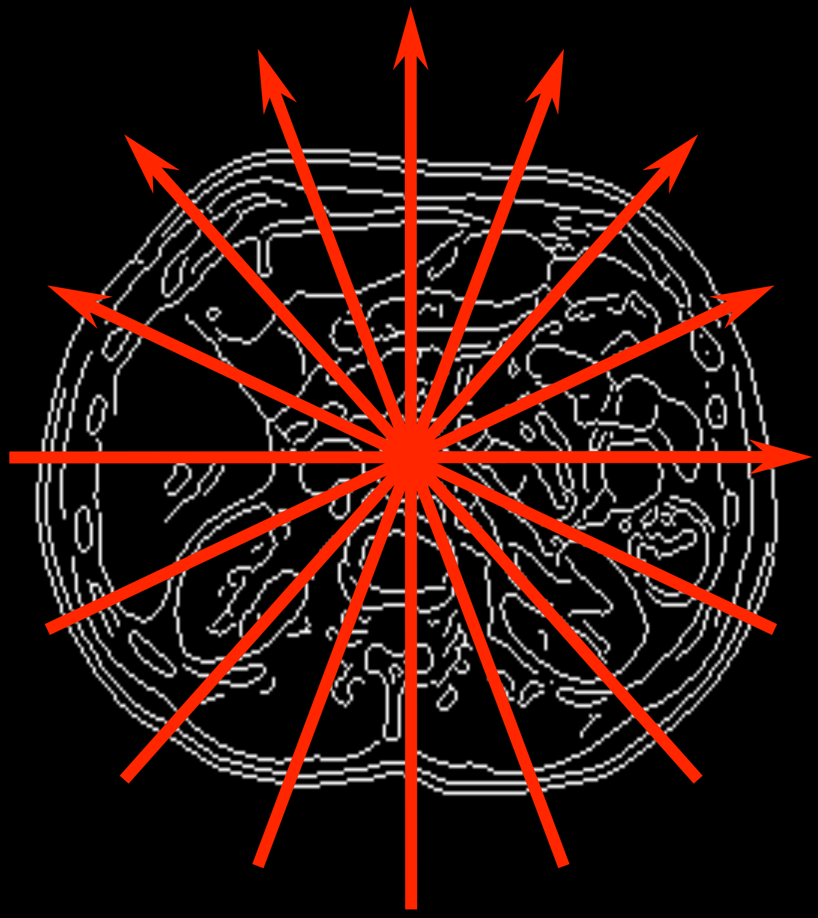
2D Fourier Transform 過程



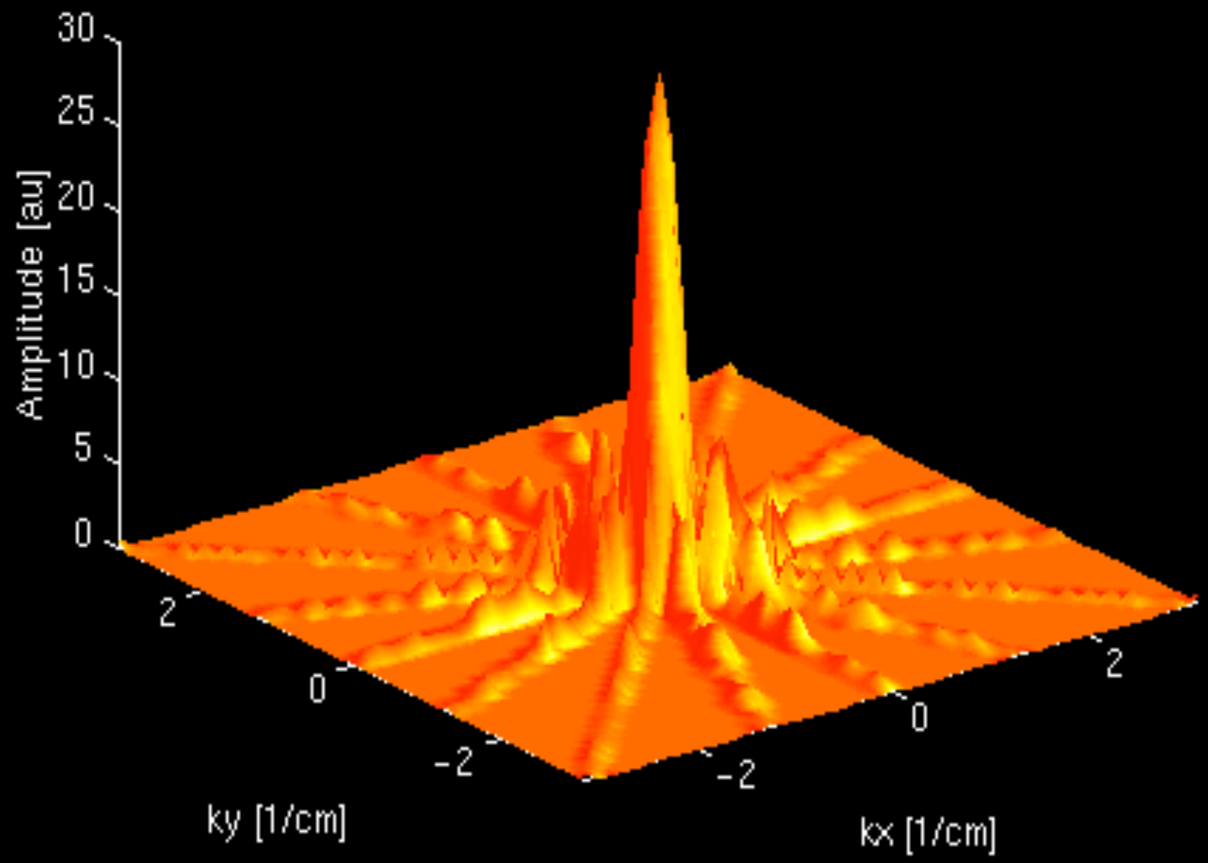
4 acquisitions

Fourier space

2D Fourier Transform 過程



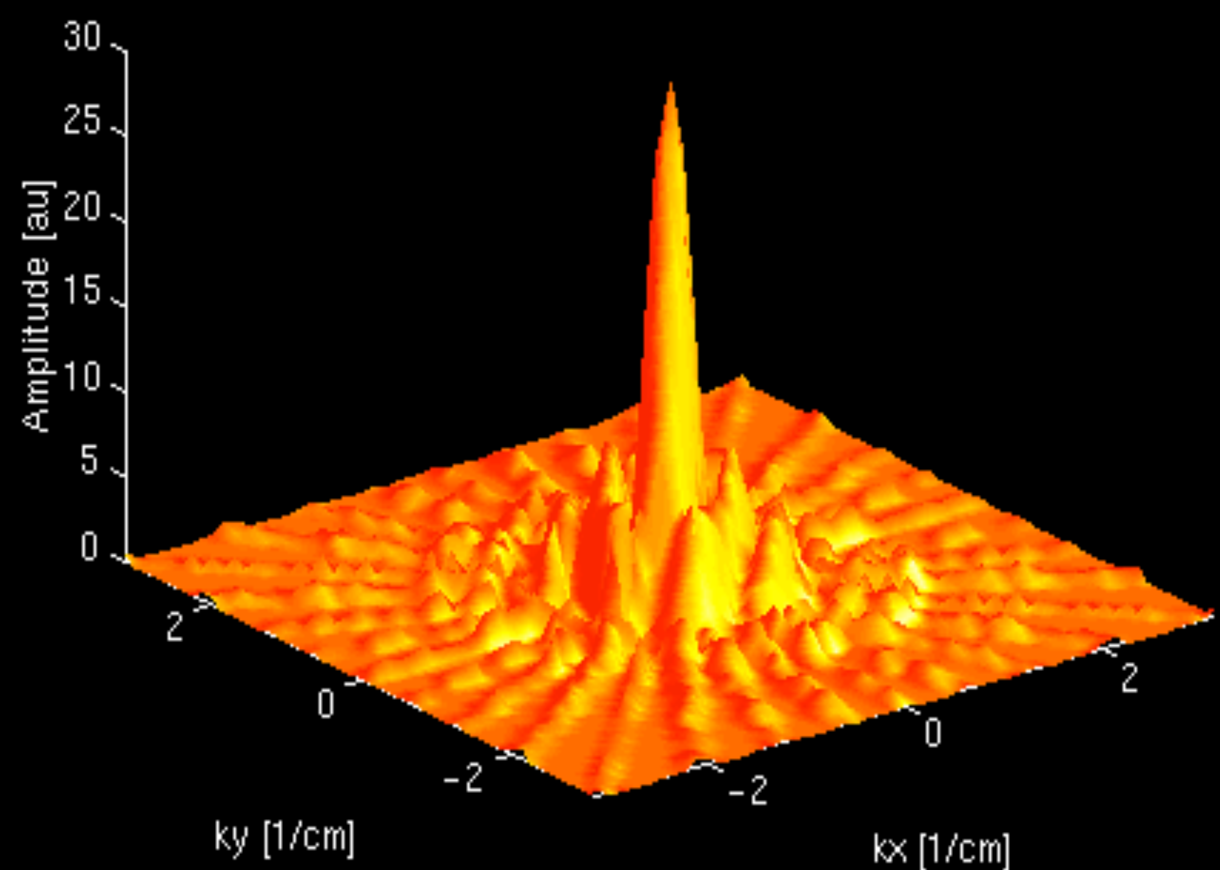
8 acquisitions



Fourier space

2D Fourier Transform 過程

畫不下了 …

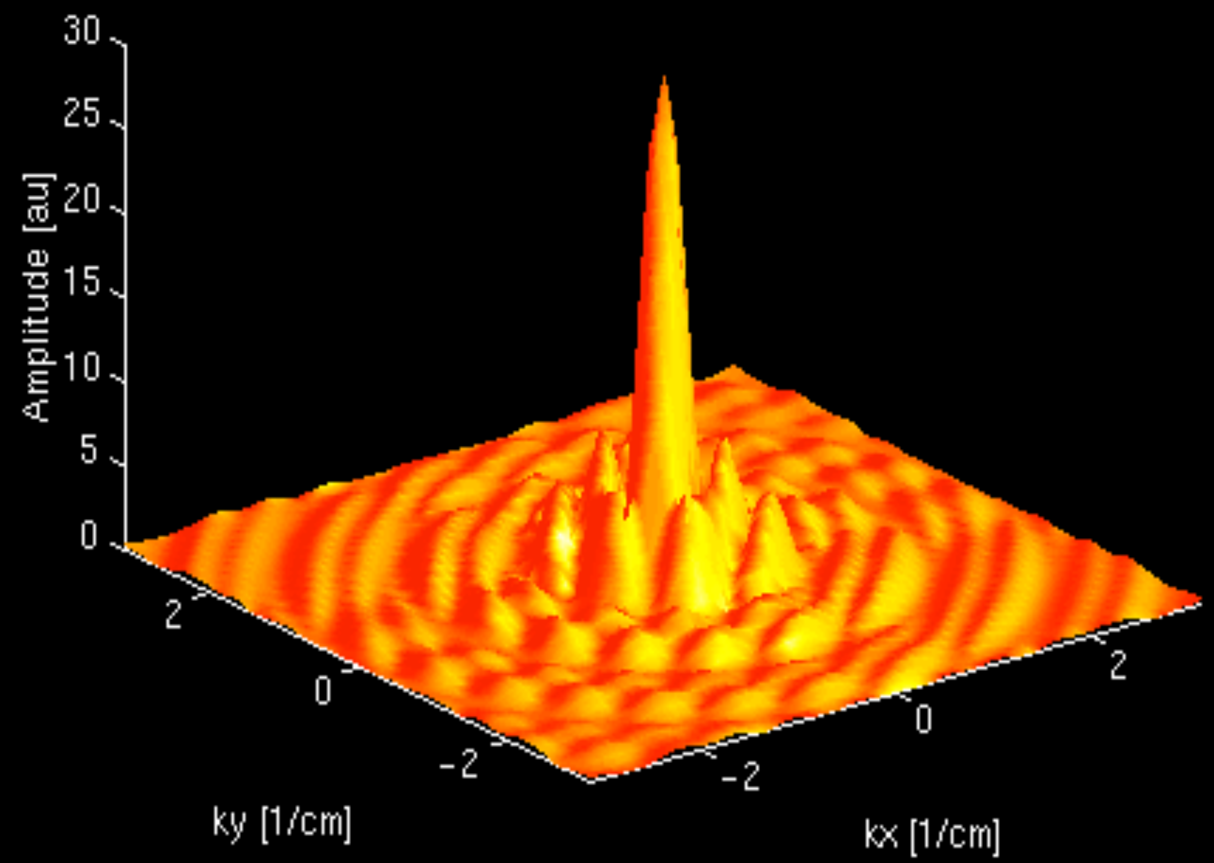


16 acquisitions

Fourier space

2D Fourier Transform 過程

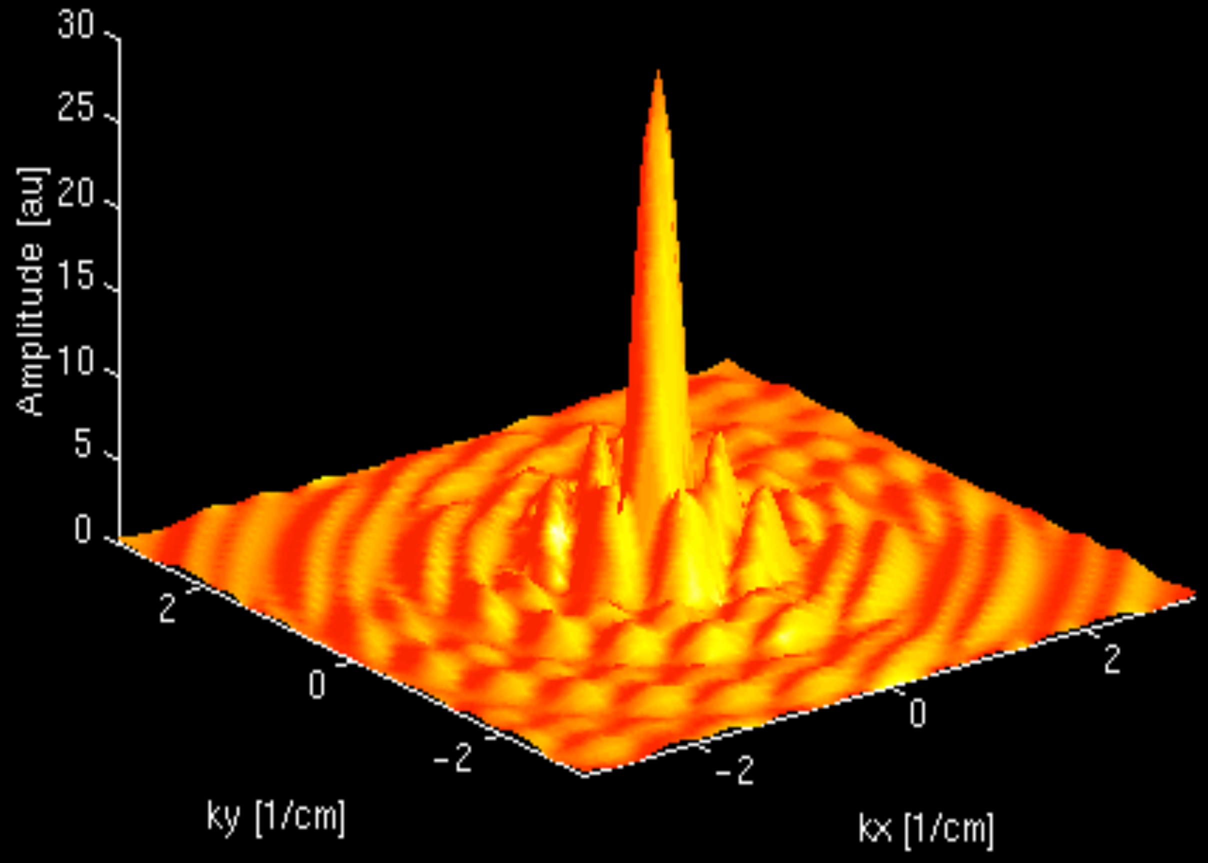
反正就是繞完了一圈



64 acquisitions

Fourier space

2D Fourier Transform 過程



計算回影像　←←←Fourier space

看似容易，其實不然

- 2D FT 在 Fourier space 中也是典型的 Cartesian 座標系統
  - 和影像一樣的規則格子點 (grid)
- 但是極座標投影是斜向 → re-grid