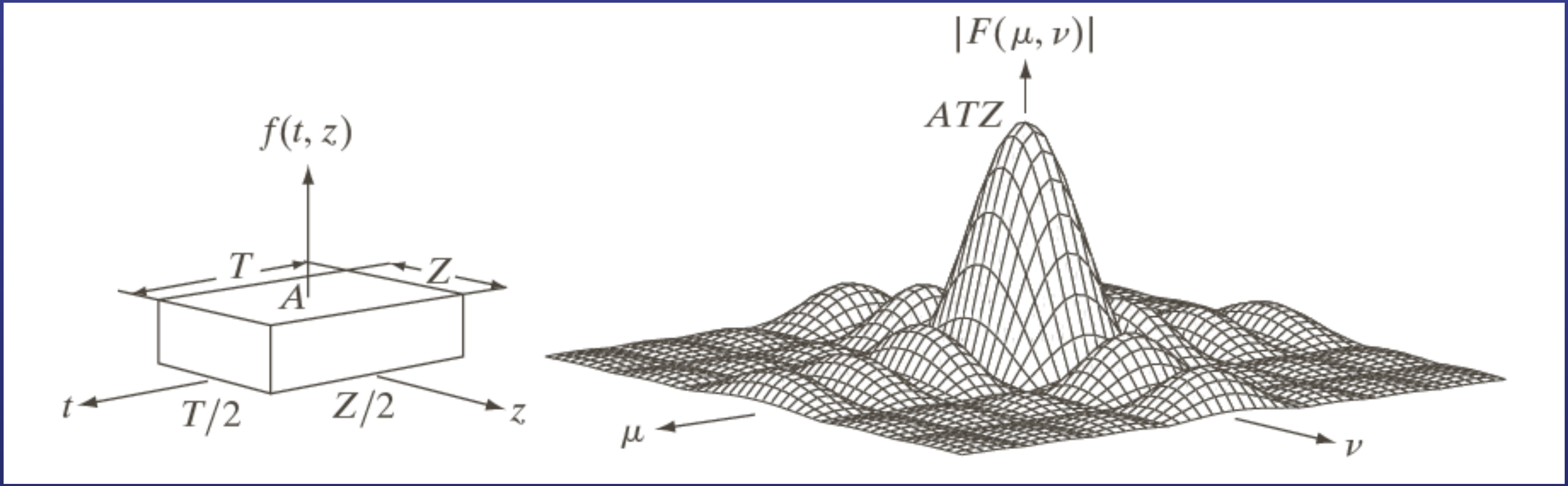
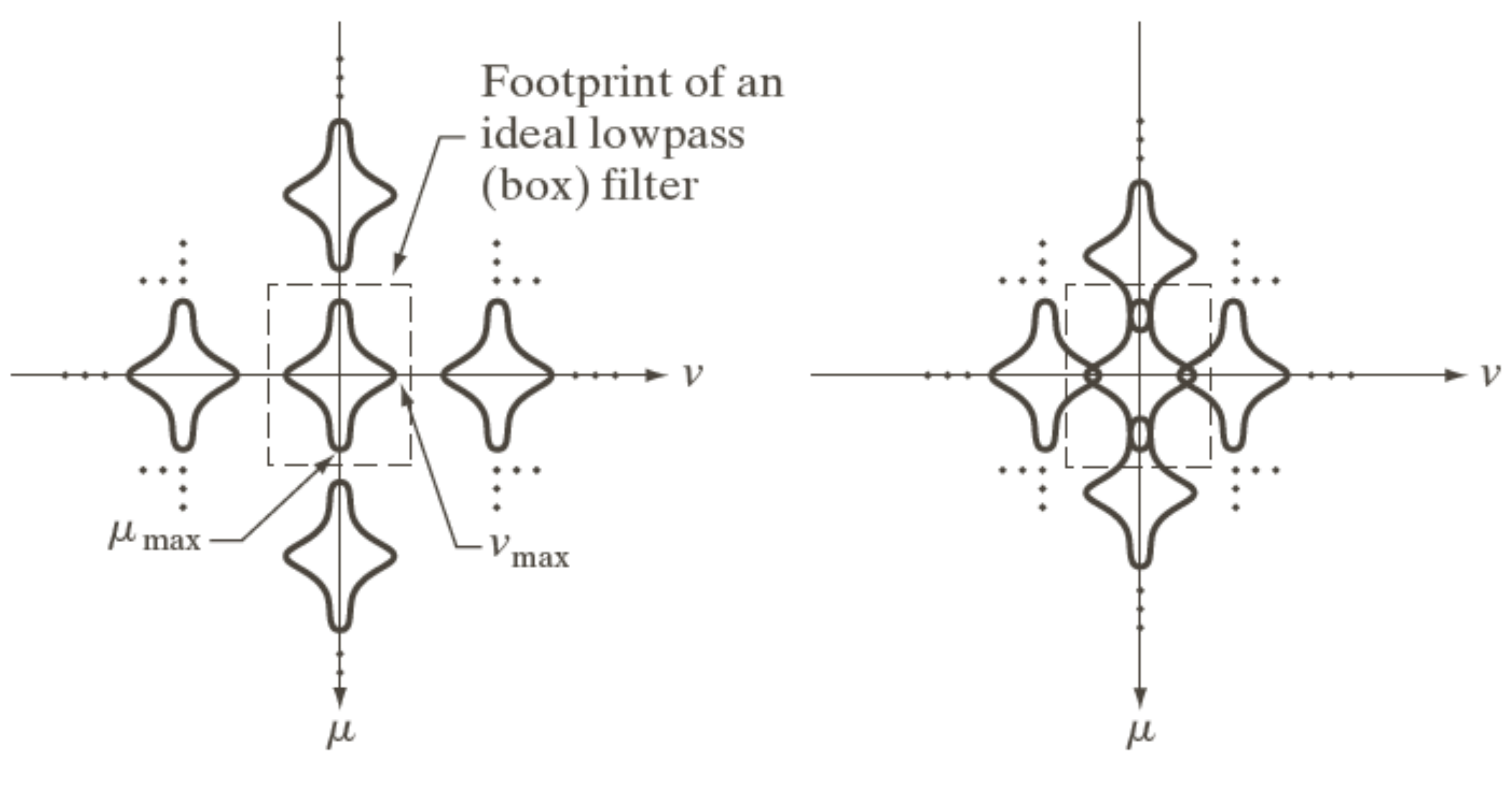
Image Enhancement in the Frequency Domain

柯正雯

2D Sinc function

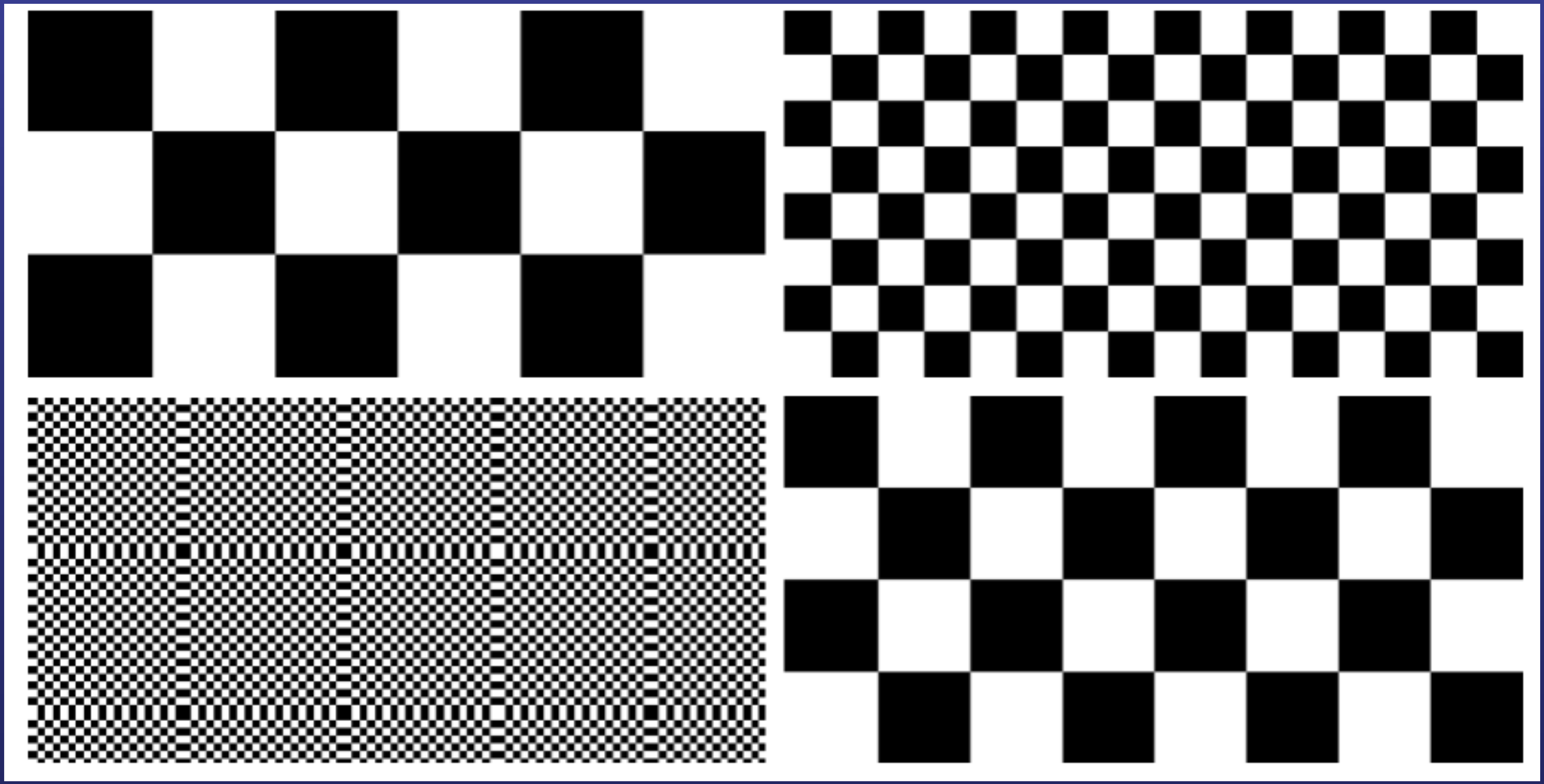


over sampled vs. under sampled



Aliasing

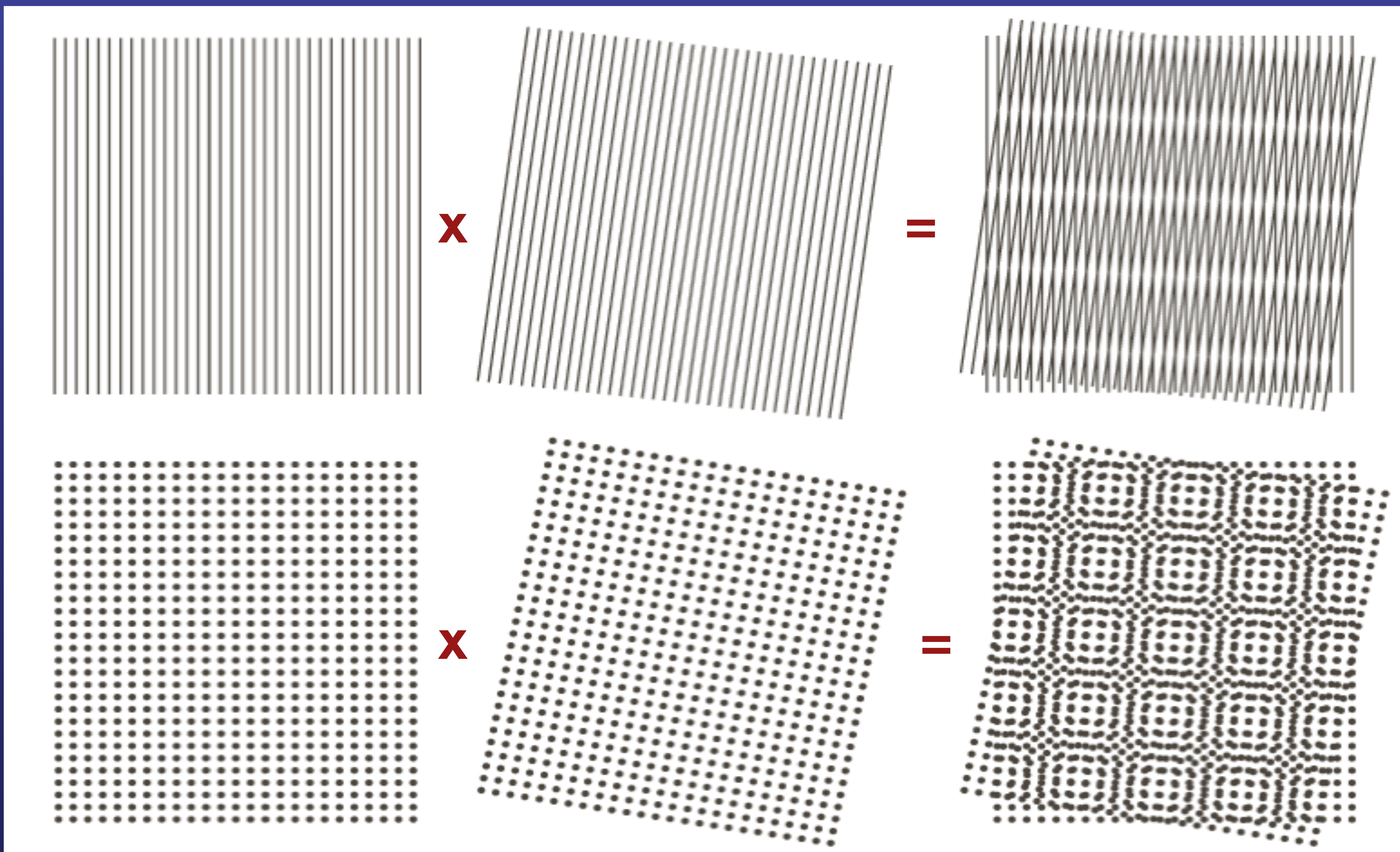
Normal Images



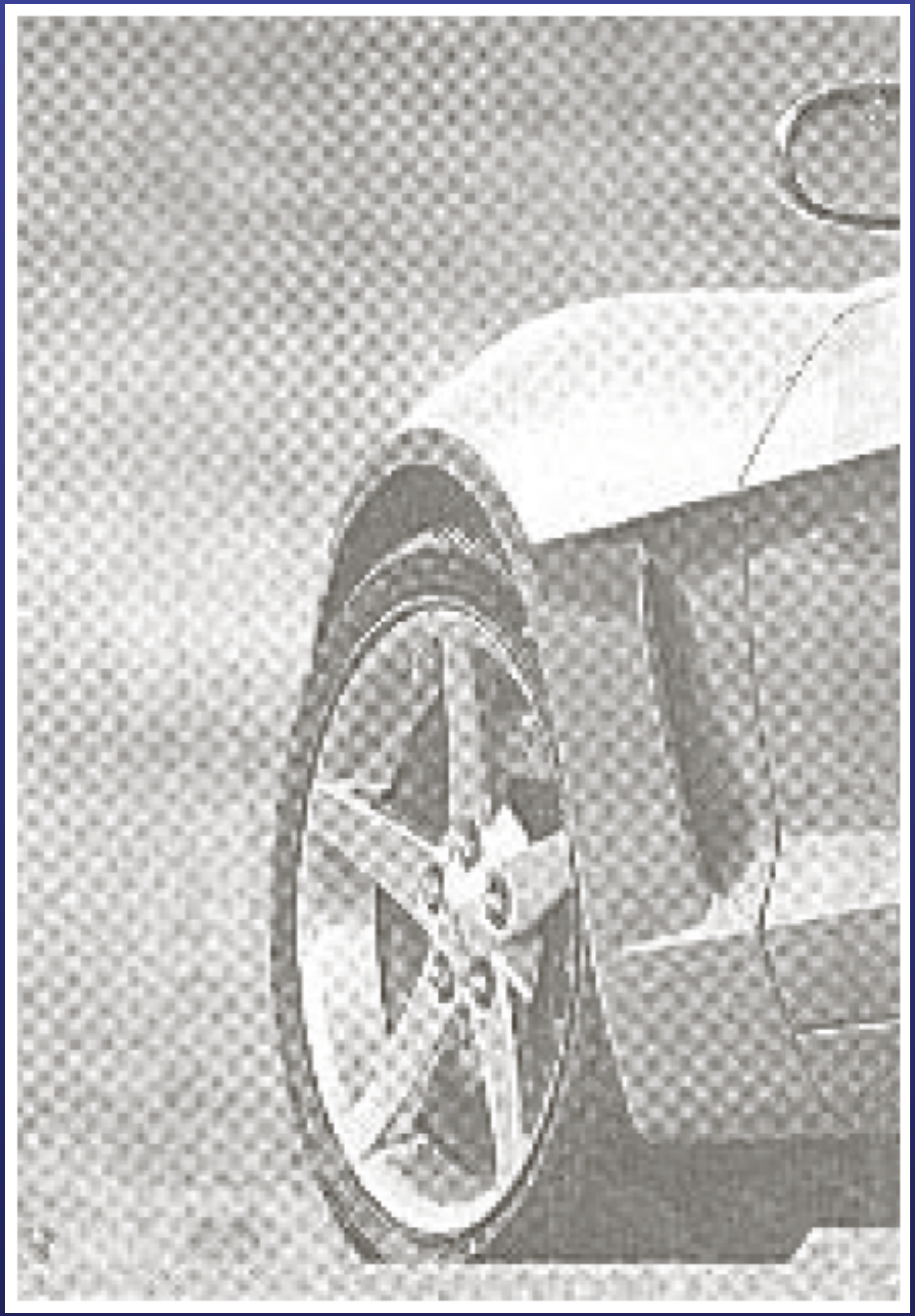
Aliased Images

2D Sampling & Aliasing

Moiré Patterns

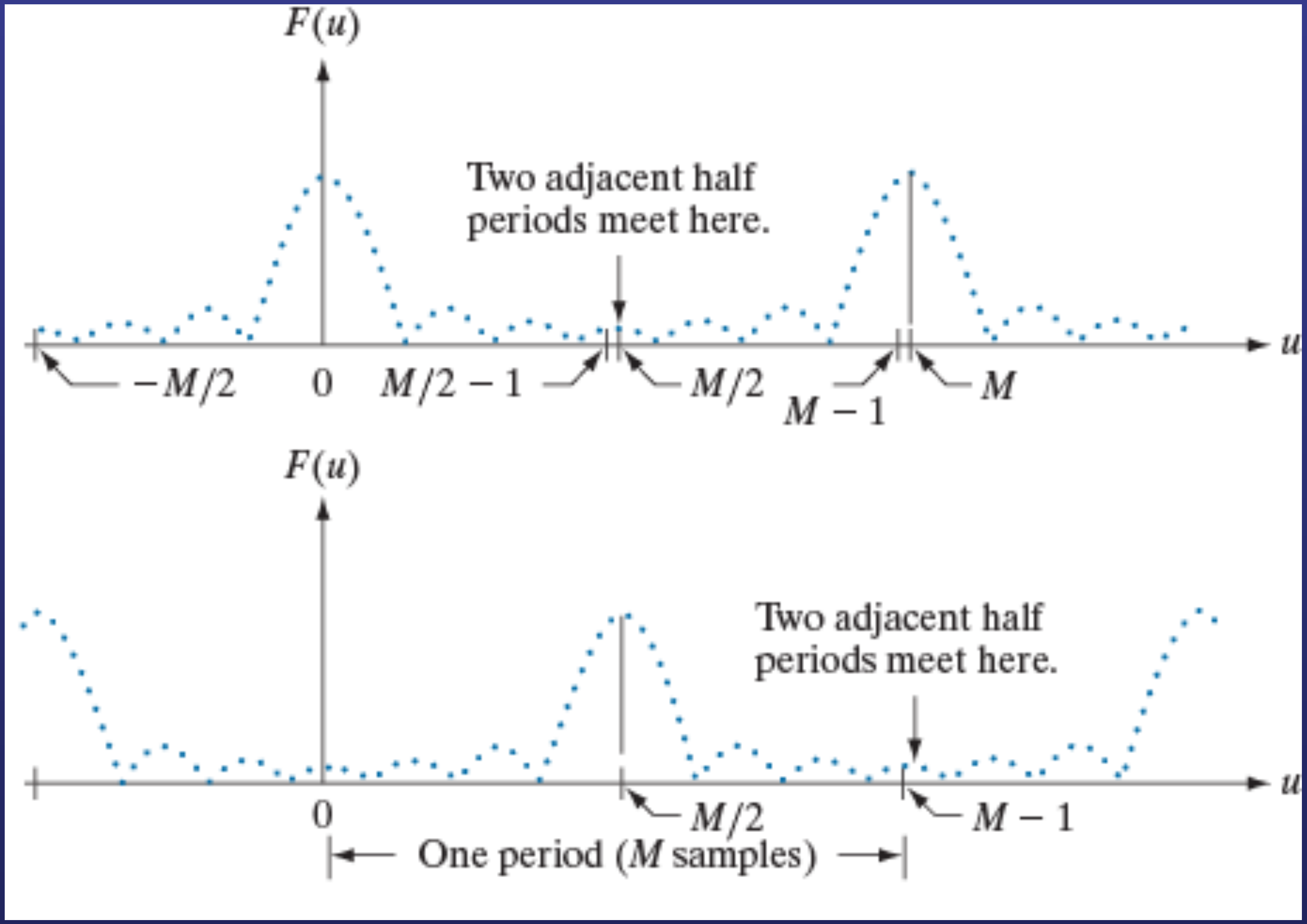


Moiré Patterns



Periodicity

$$f(x)\, e^{j2\pi(u_0 x/M)} \Leftrightarrow F(u-u_0)$$



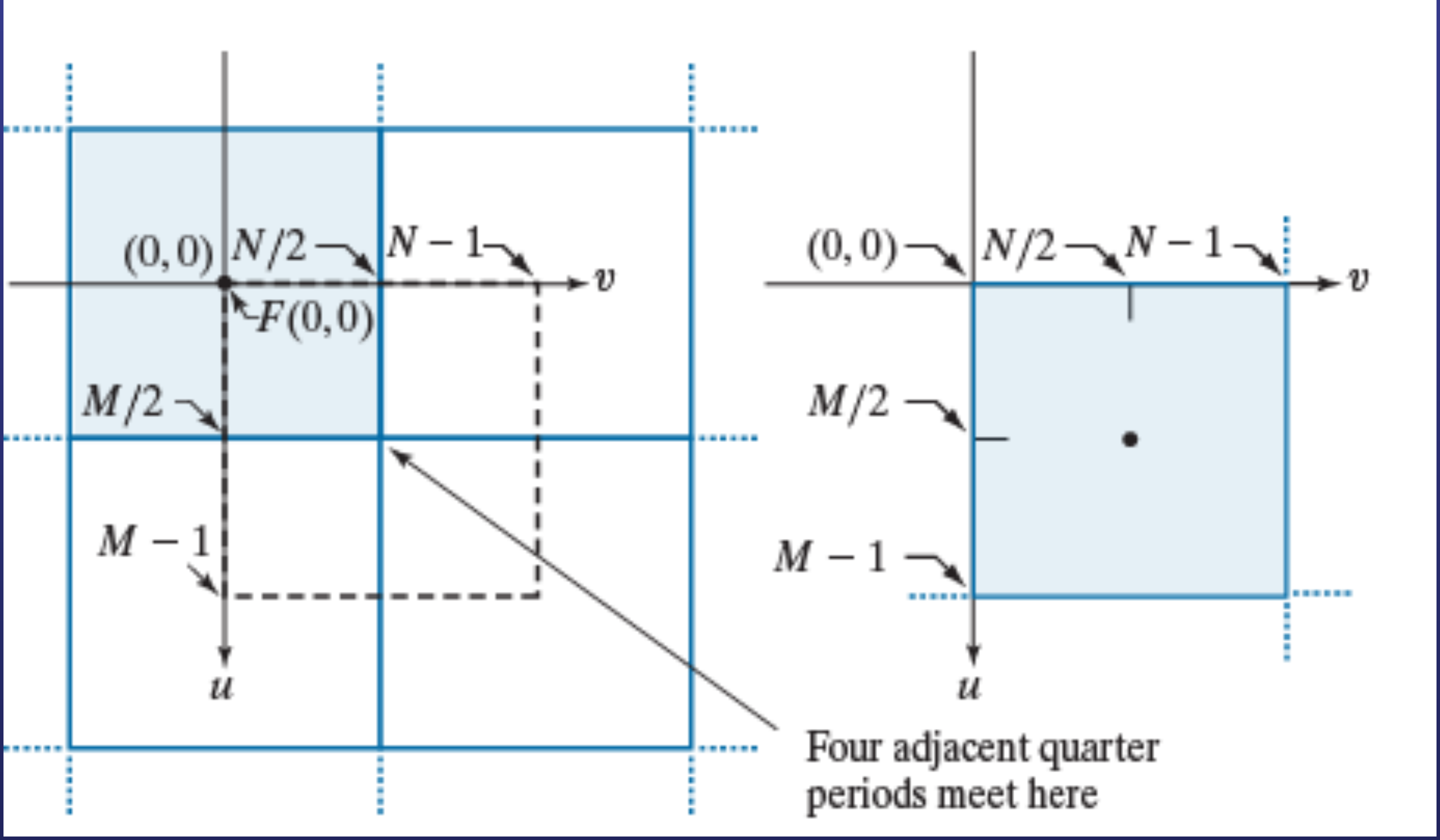
$$u_0 = M/2$$

$$f(x)\, e^{j\pi x}$$

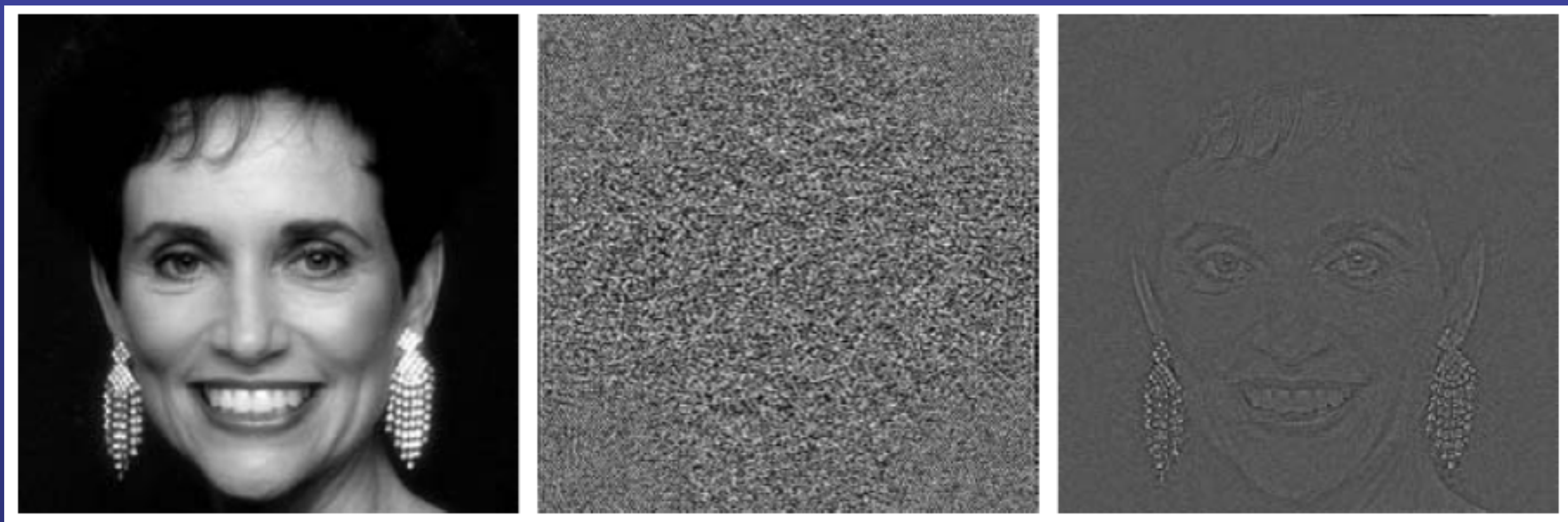
$$= f(x)\,(-1)^x$$

Periodicity

$$f(x,y)(-1)^{x+y} \Leftrightarrow F(u - M/2, v - N/2)$$



Phase Images

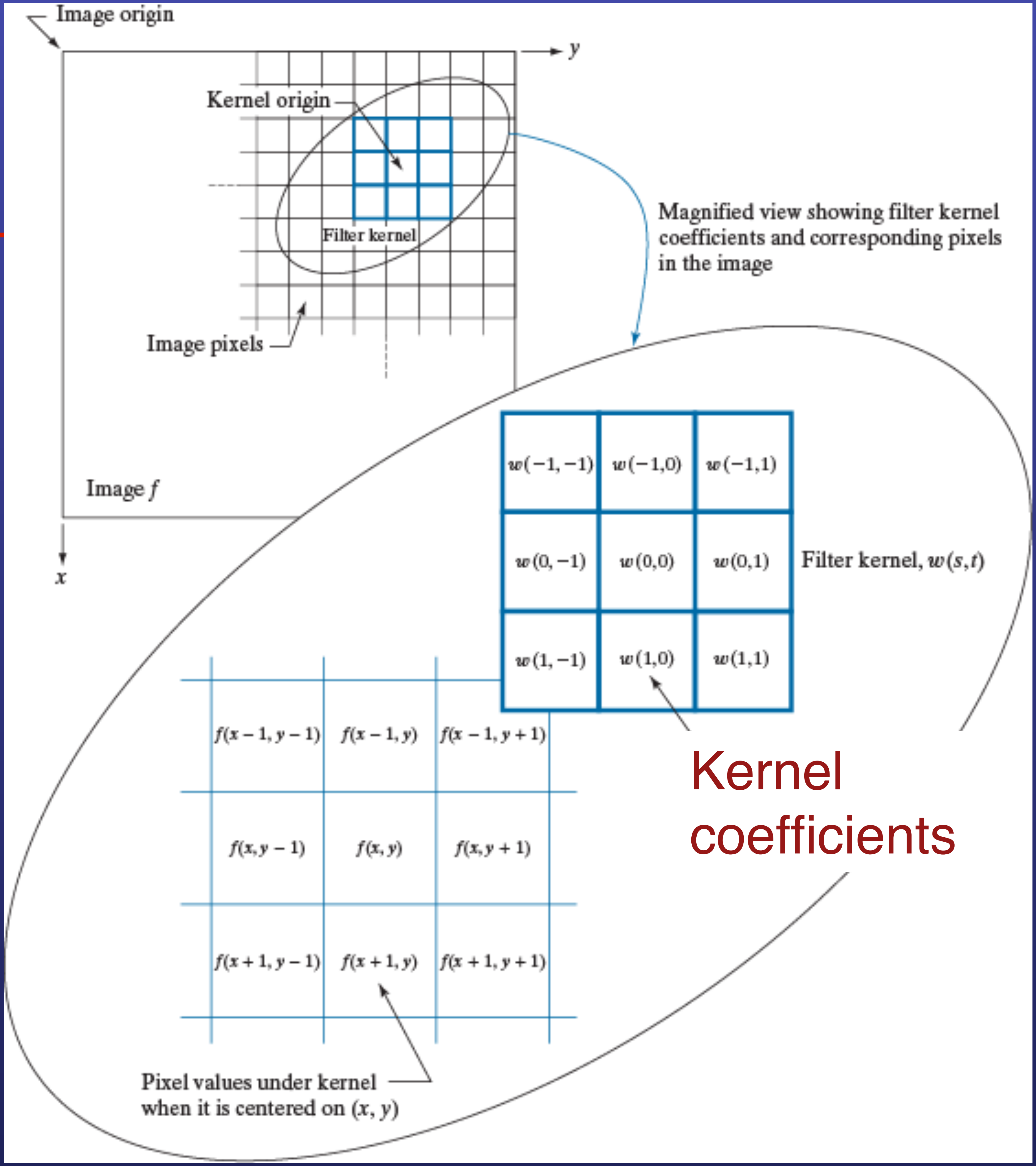


$f(x,y)$ $\qquad\qquad$ $\phi(u,v)$ $\qquad\qquad$ $g(x,y)$

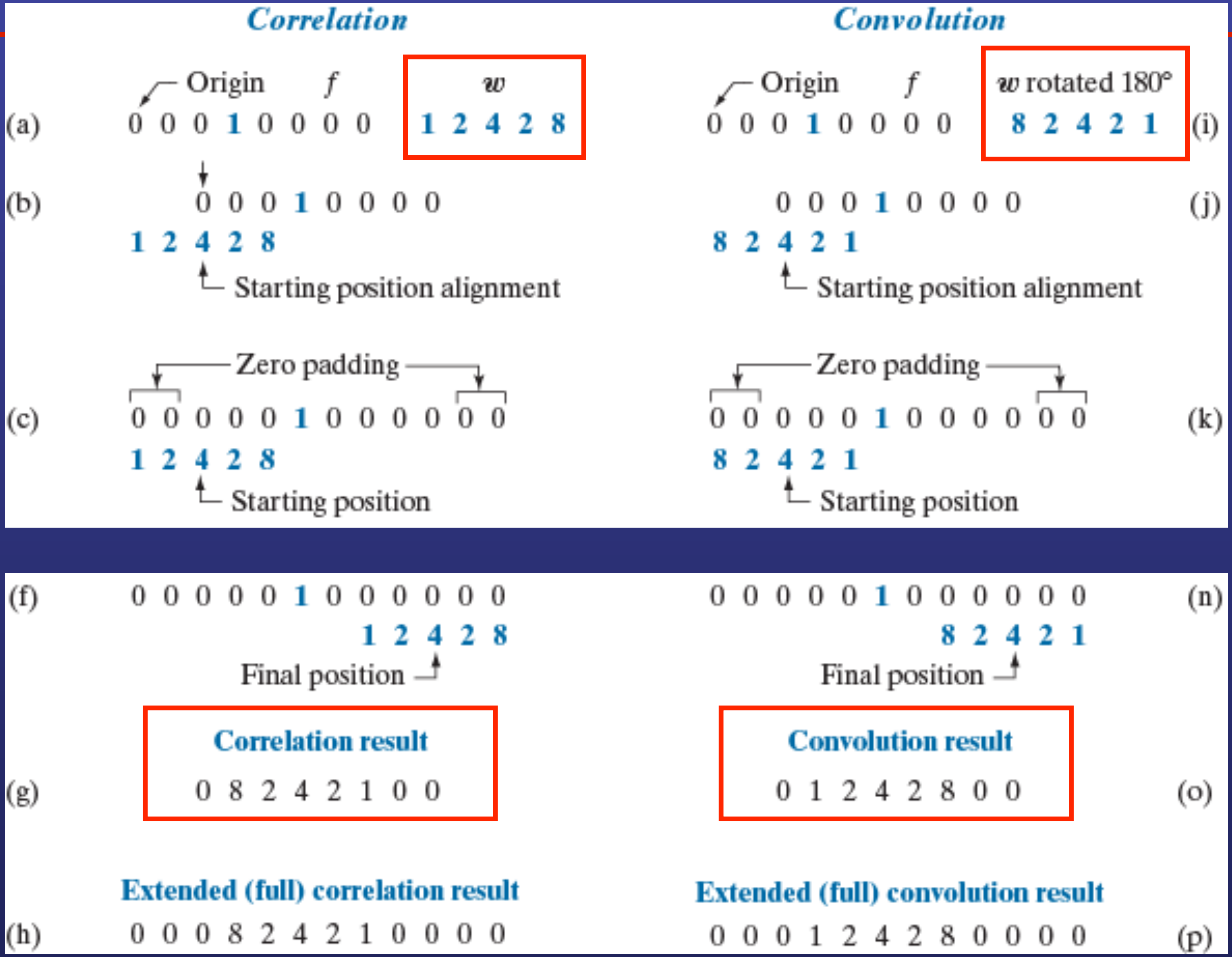
$$F(u,v)$$
$$= |F(u,v)|\, e^{j\,\phi(u,v)}$$

$$G(u,v) = 1\, e^{j\,\phi(u,v)}$$

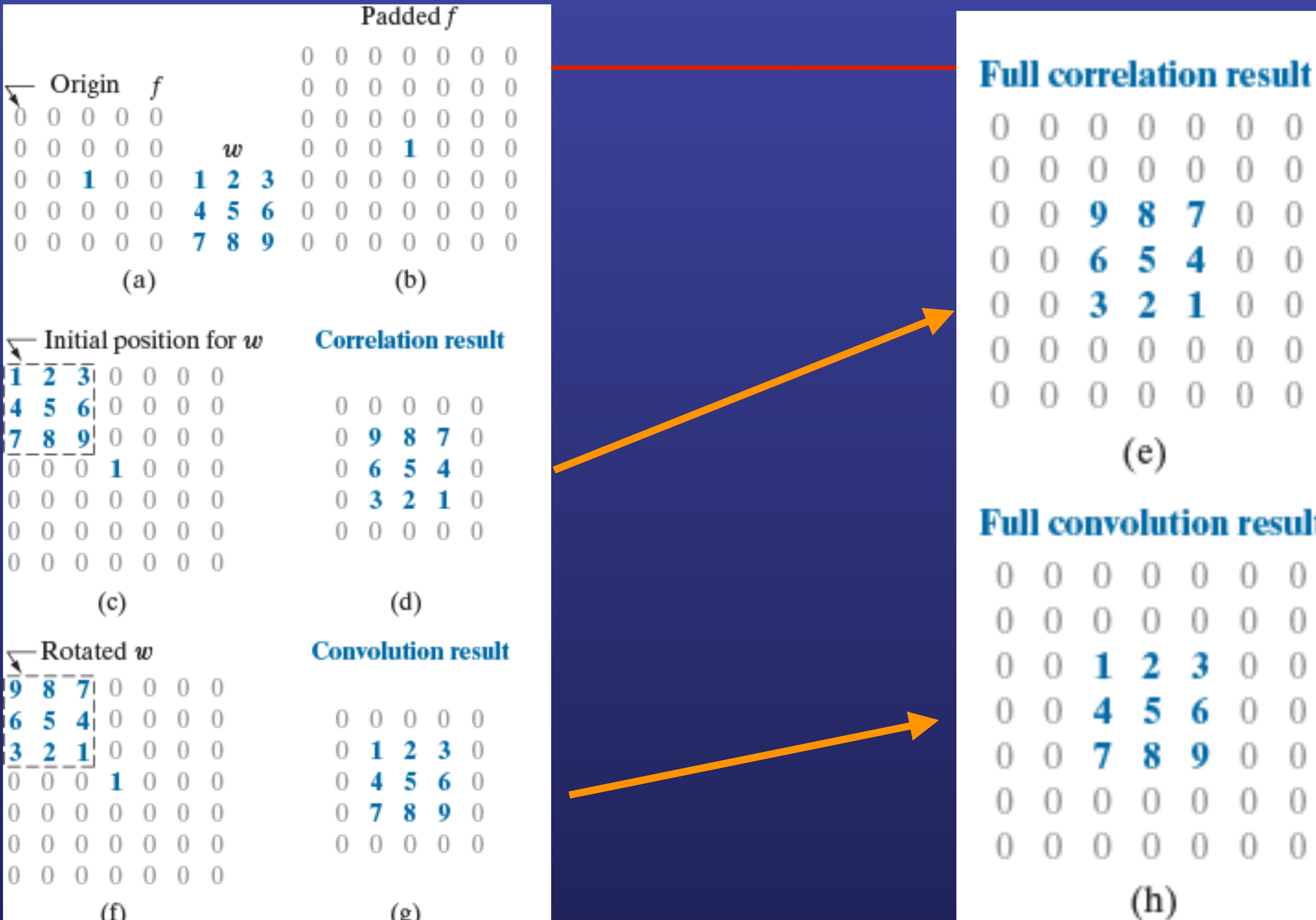
Spatial Convolution



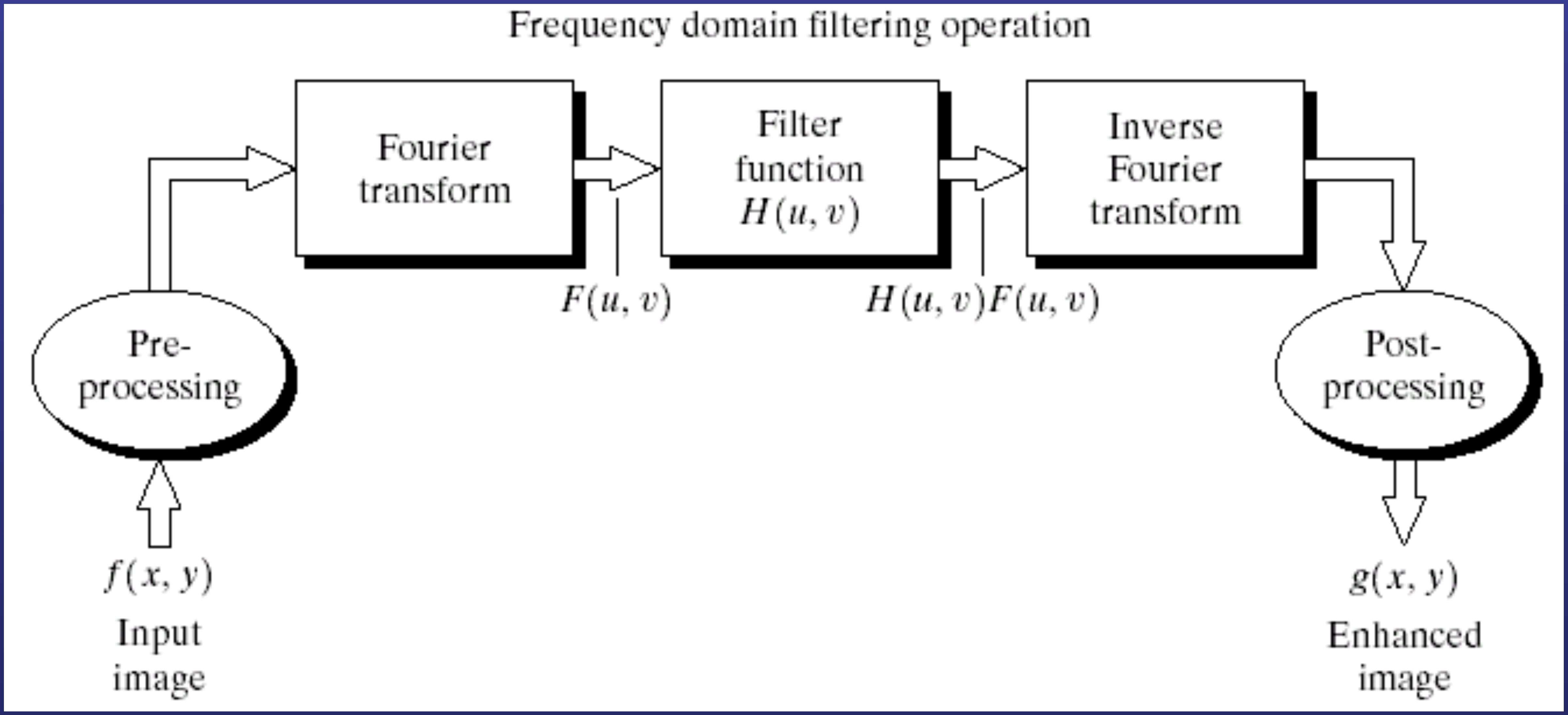
Spatial Convolution



Spatial Convolution

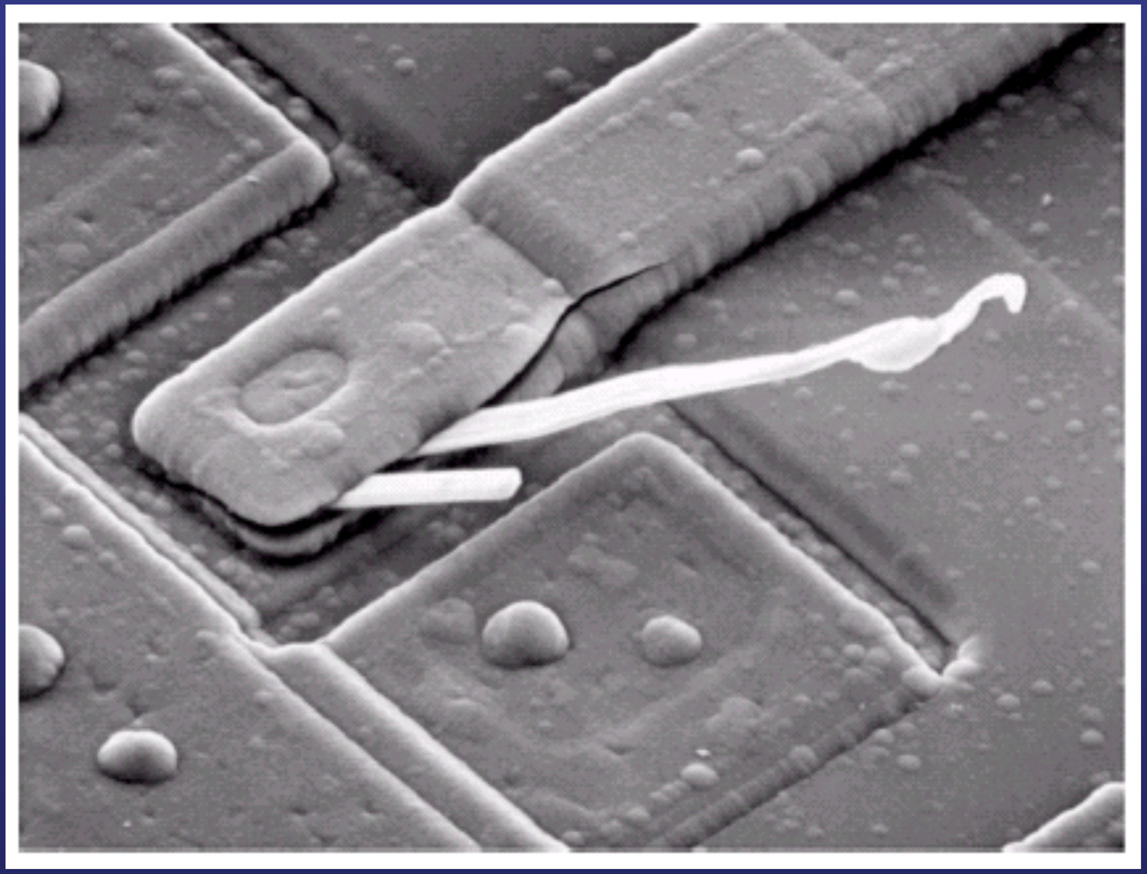


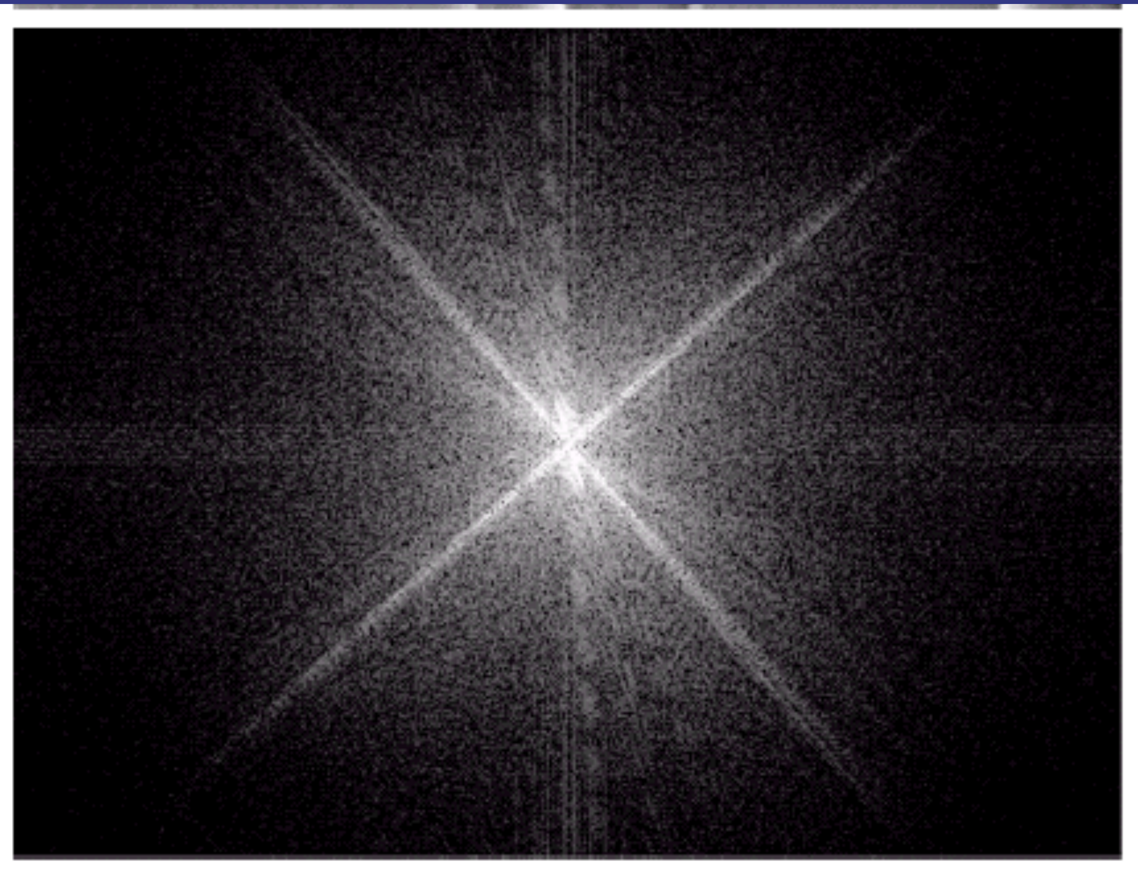
Basic scheme of filtering



Basic filter

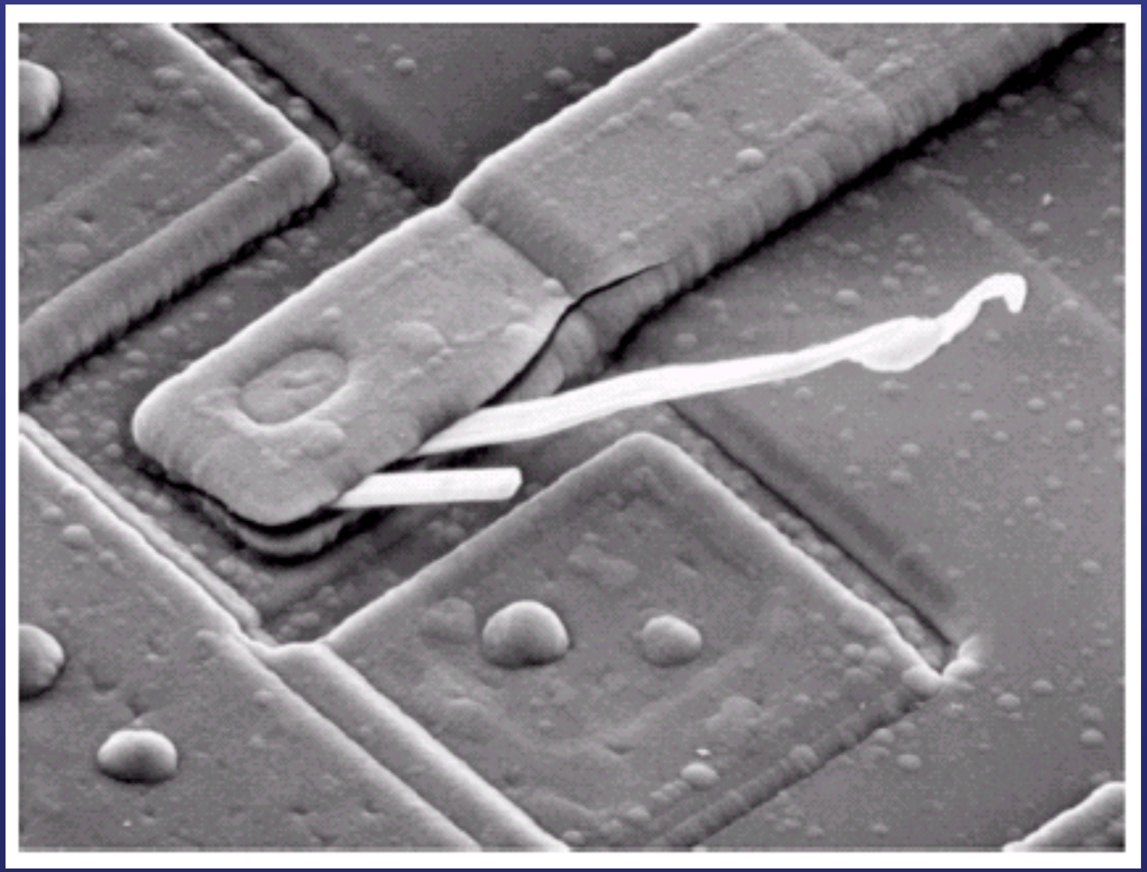
$$H(u,v)=\begin{cases}0, & if\,(u,v)=(M/2,N/2)\\ 1, & otherwise.\end{cases}$$

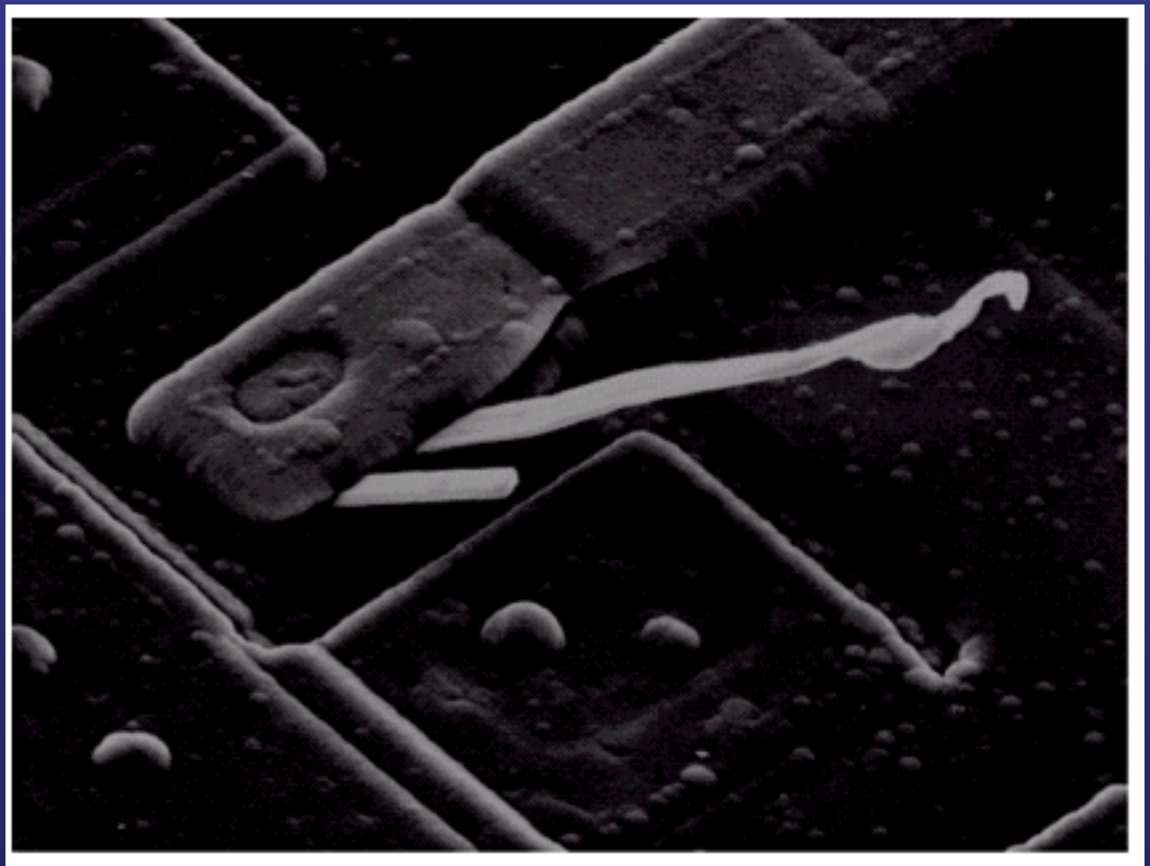




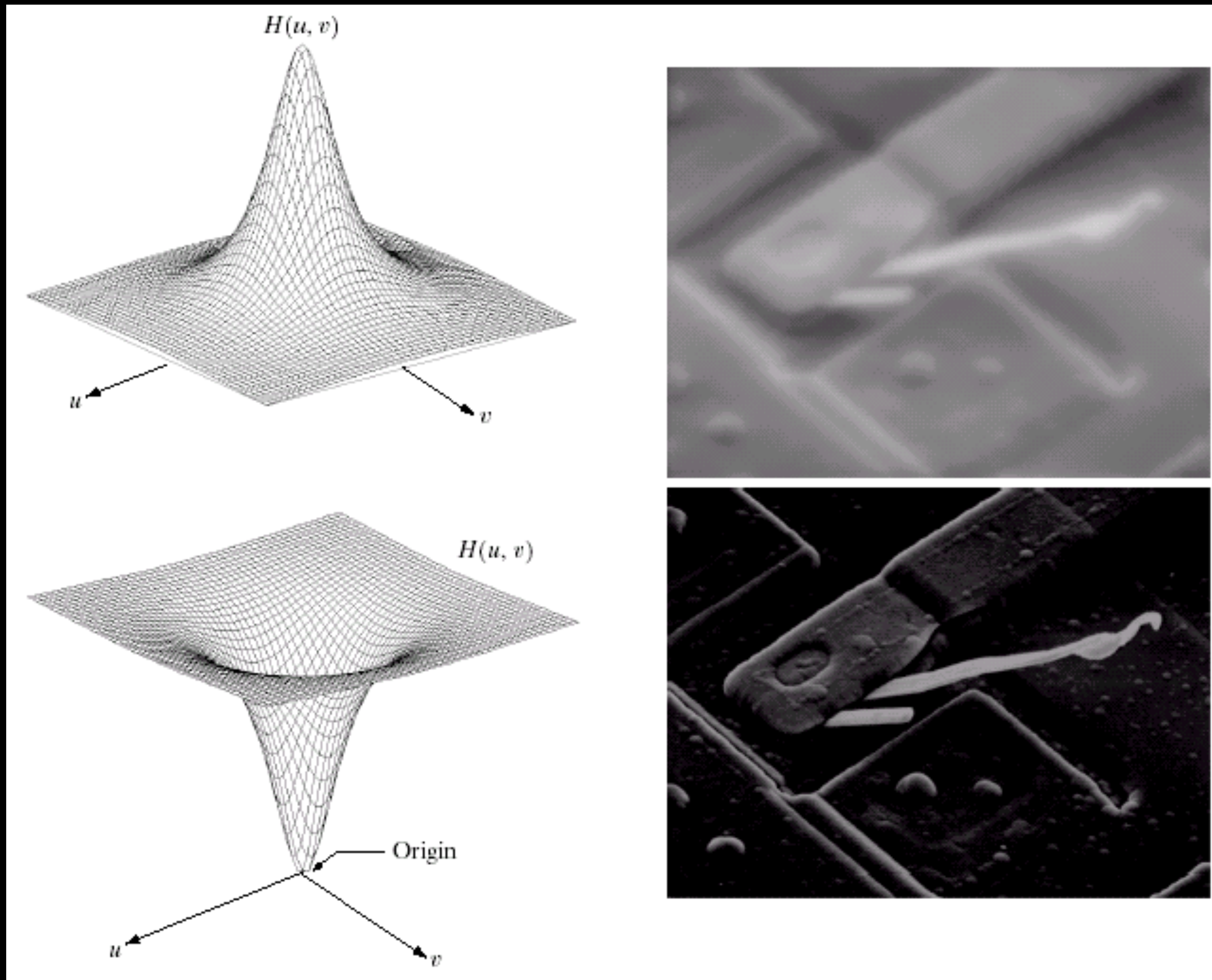
Basic filter

$$H(u,v) = \begin{cases} 0, & if\,(u,v) = (M/2, N/2) \\ 1, & otherwise. \end{cases}$$



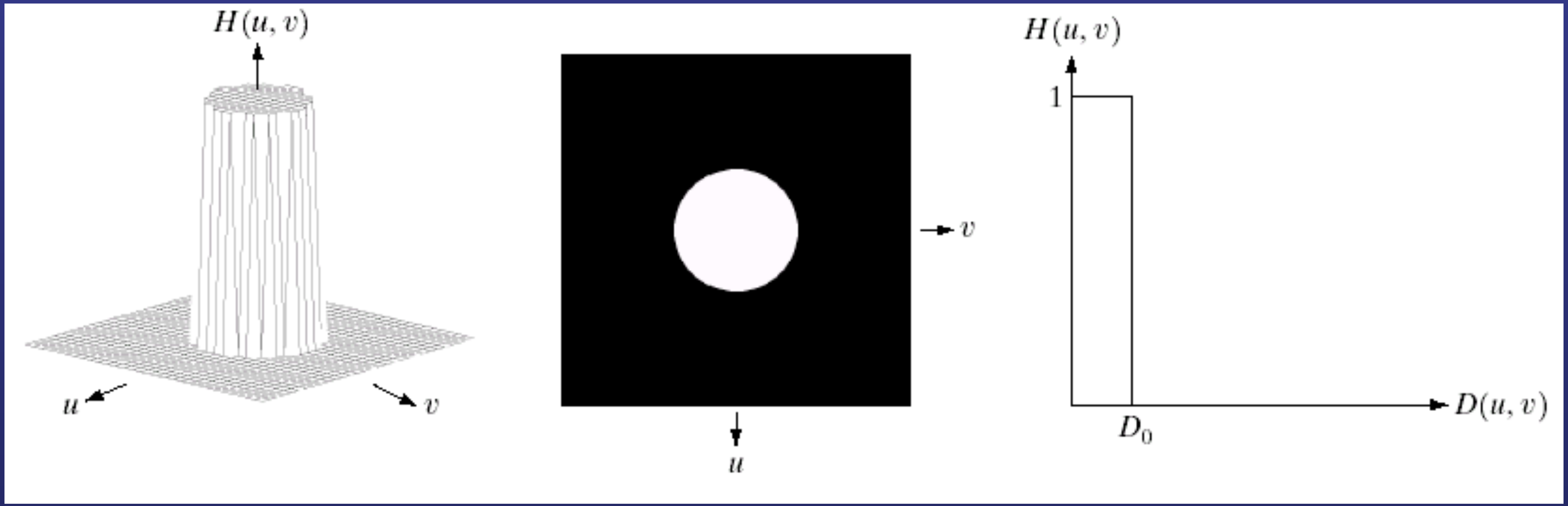


Filtering



Ideal Low-pass Filter

$$H(u,v)=\begin{cases}1, & if \quad D(u,v)\le D_0\\ 0, & if \quad D(u,v) > D_0\end{cases}$$

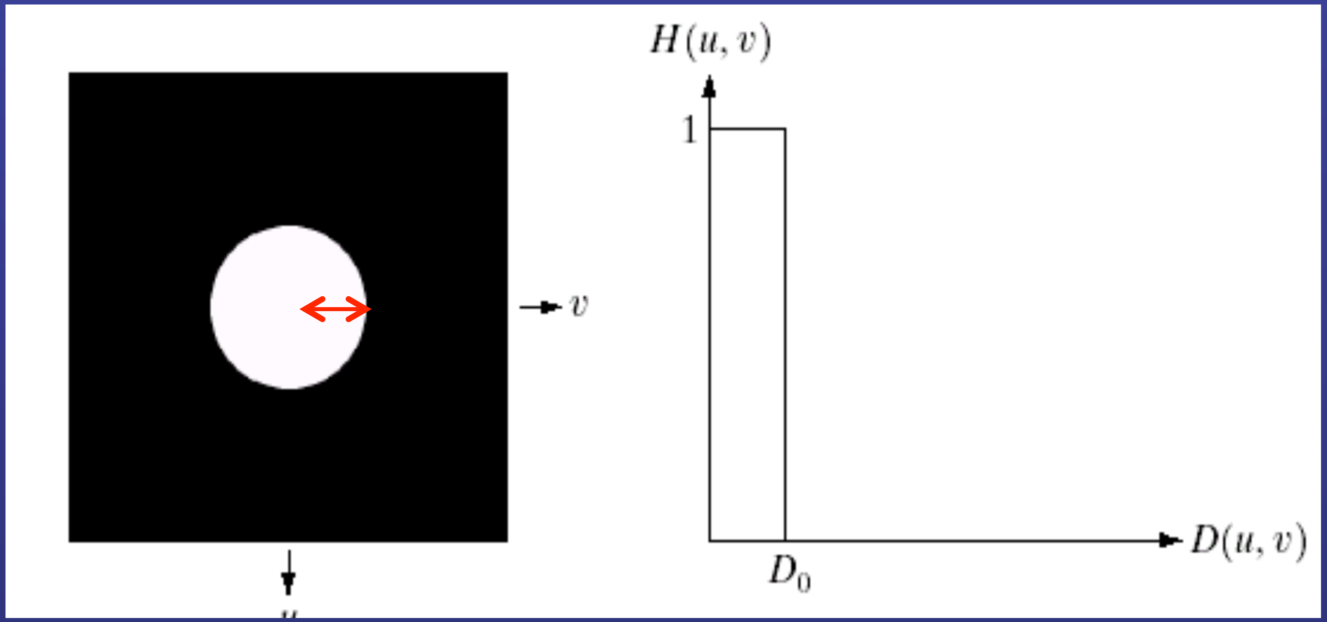


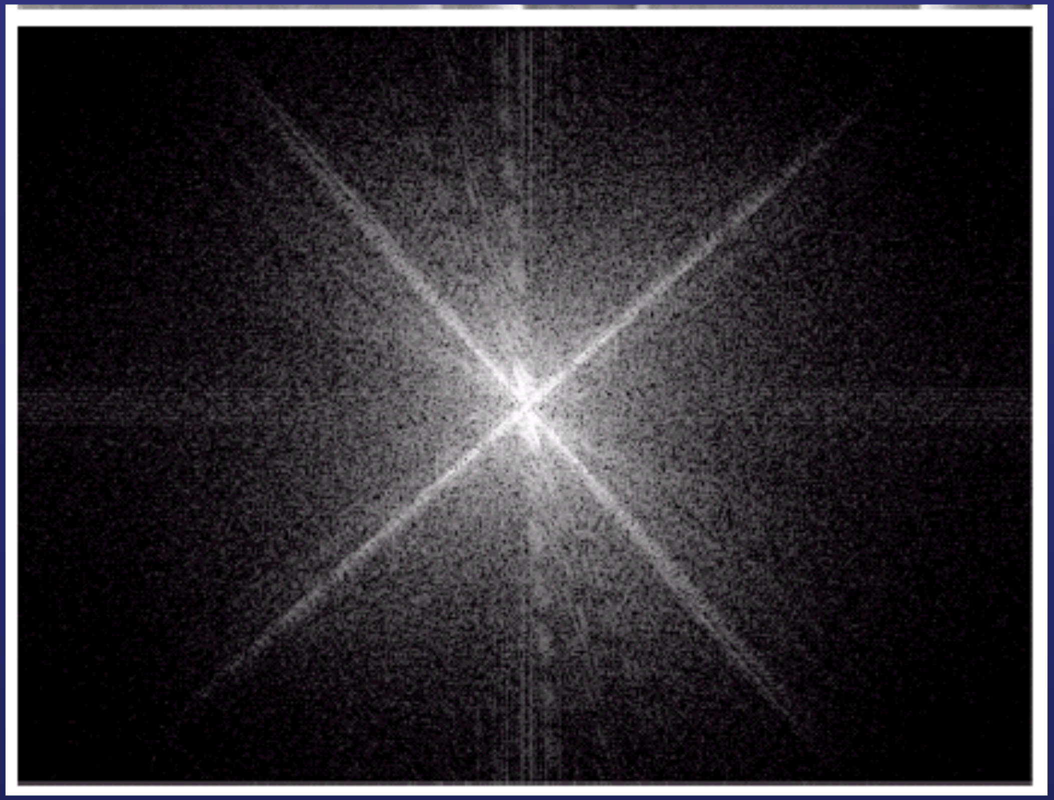
$$D(u,v)=[(u-M/2)^2+(v-N/2)^2]^{1/2}$$

Percentage of Power

$D_0 \rightarrow$ cutoff frequency

$$\alpha = 100\left[\sum_u \sum_v P(u,v) / P_T\right]$$

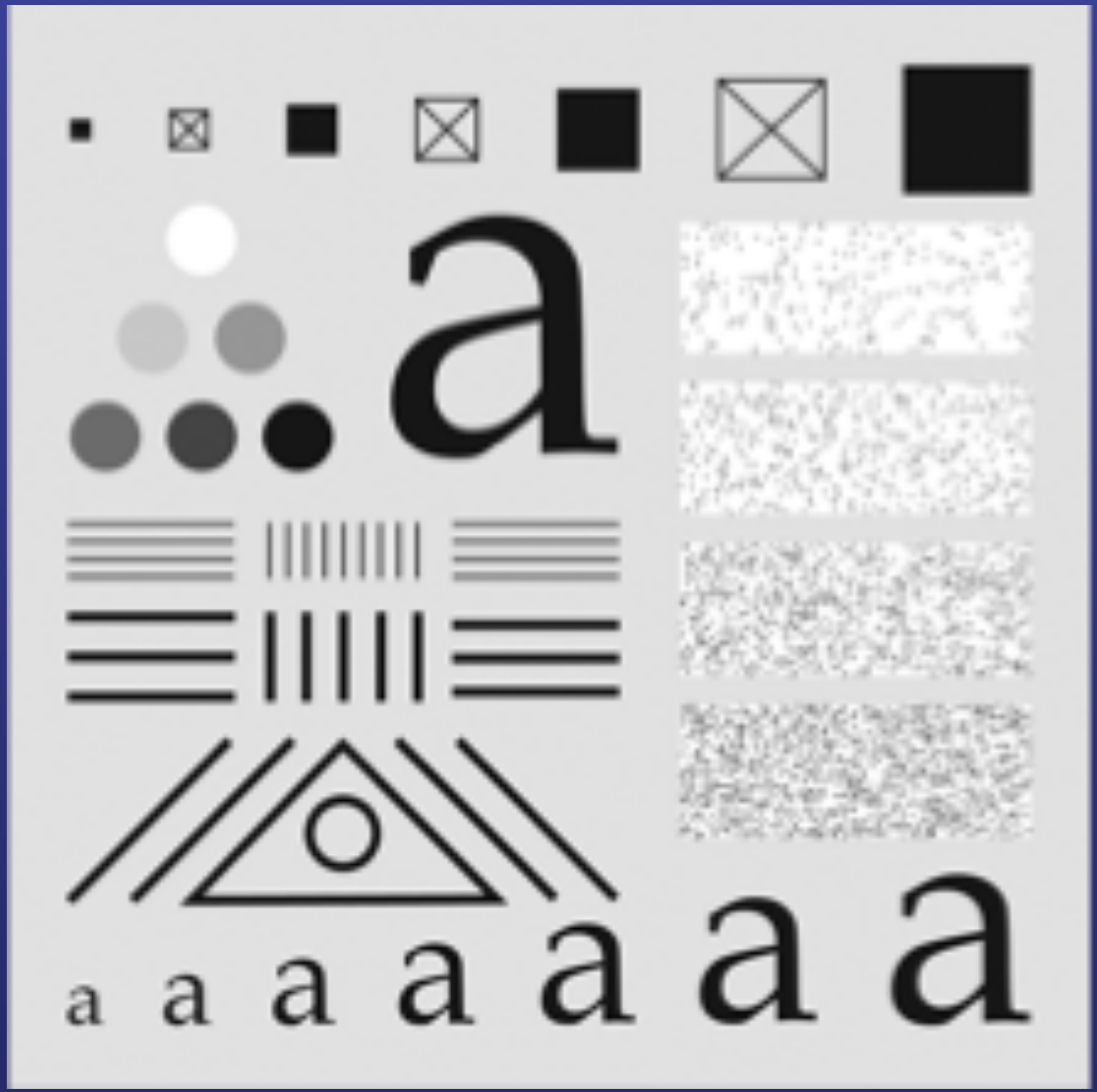


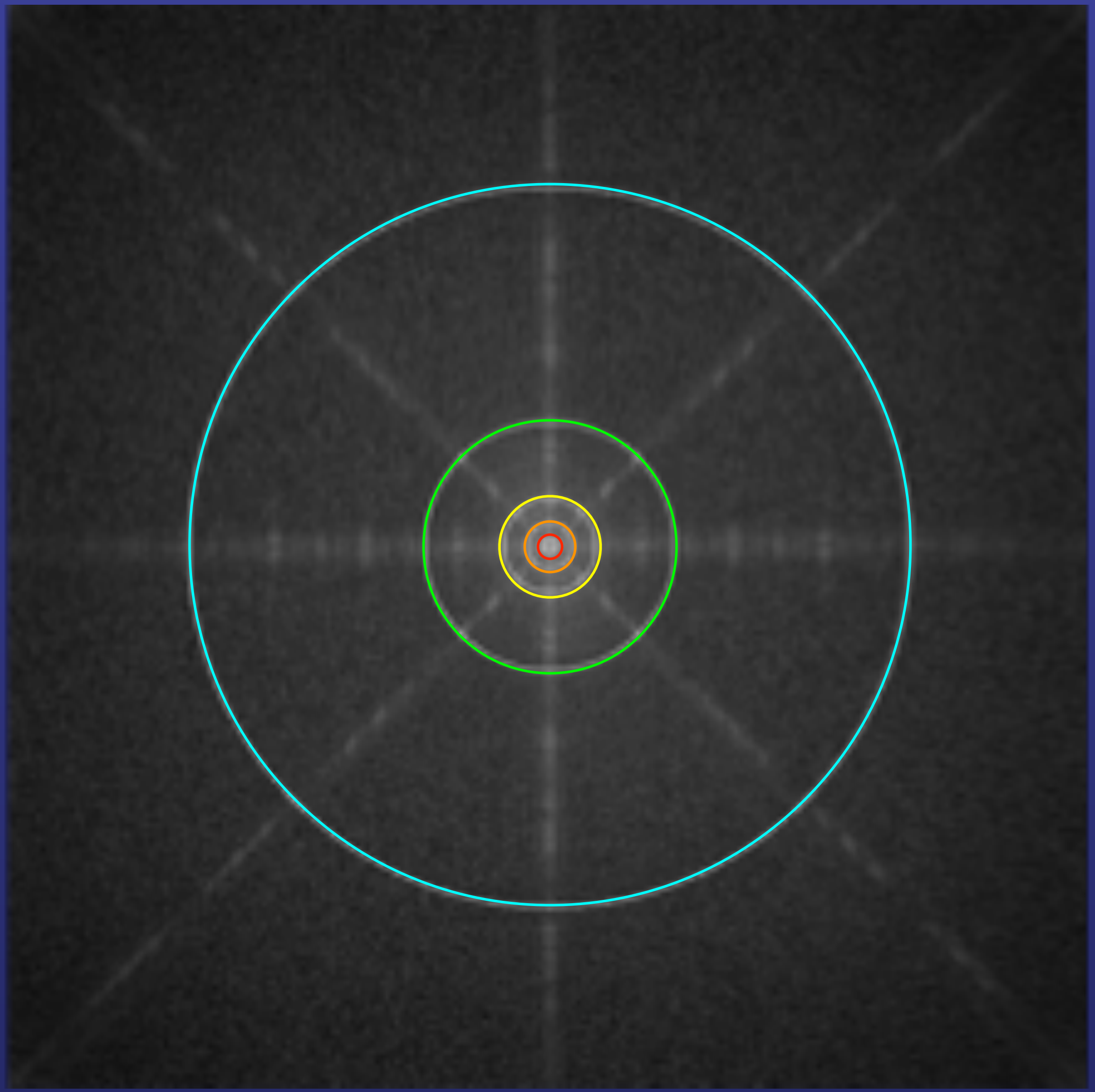


$$P_T = \sum_{u=0}^{P-1} \sum_{v=0}^{Q-1} P(u,v)$$

$$P(u,v) = |F(u,v)|^2$$

Percentage of Power

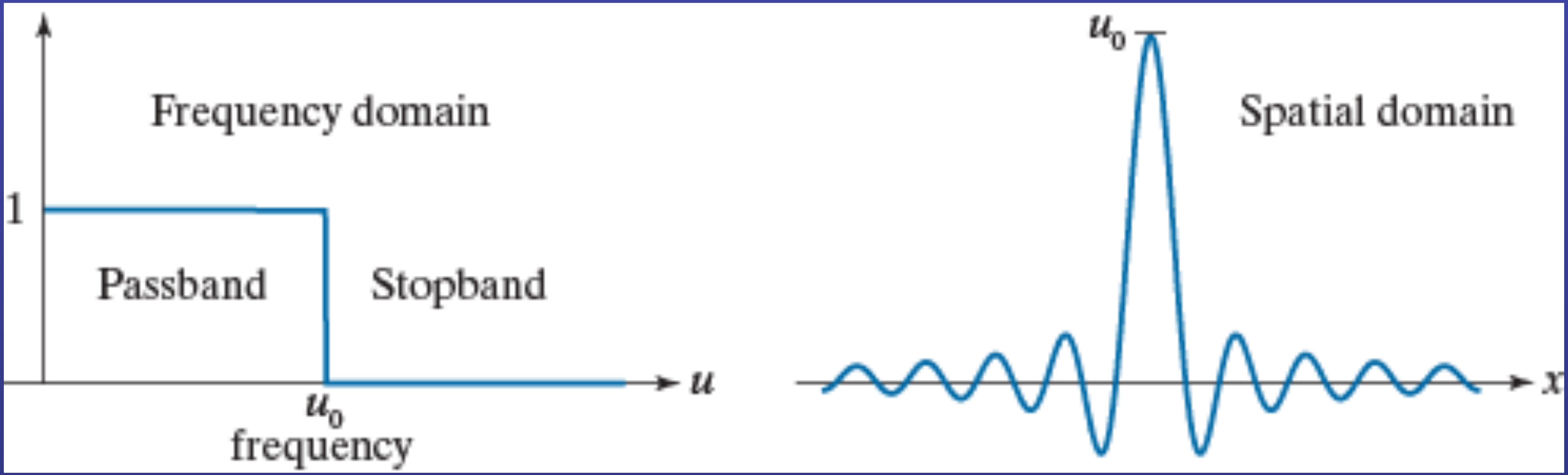




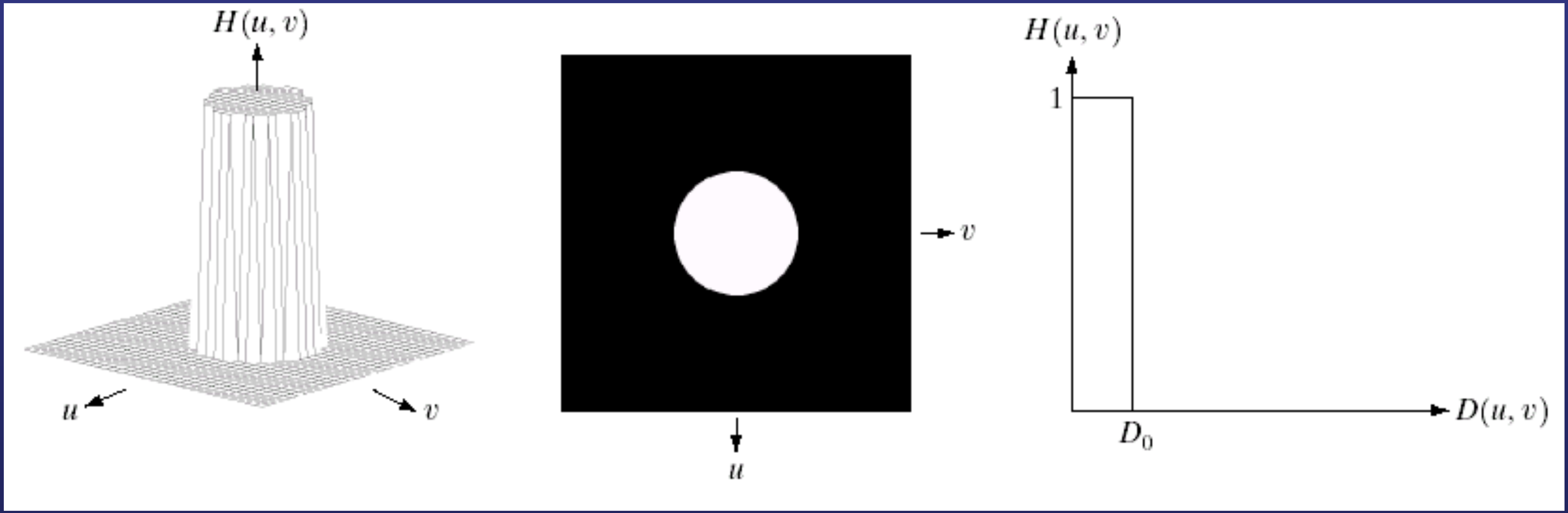
Radii: 10, 30, 60, 160, 460

Power: 86.9%, 92.8%, 95.1%, 97.6%, 99.4%

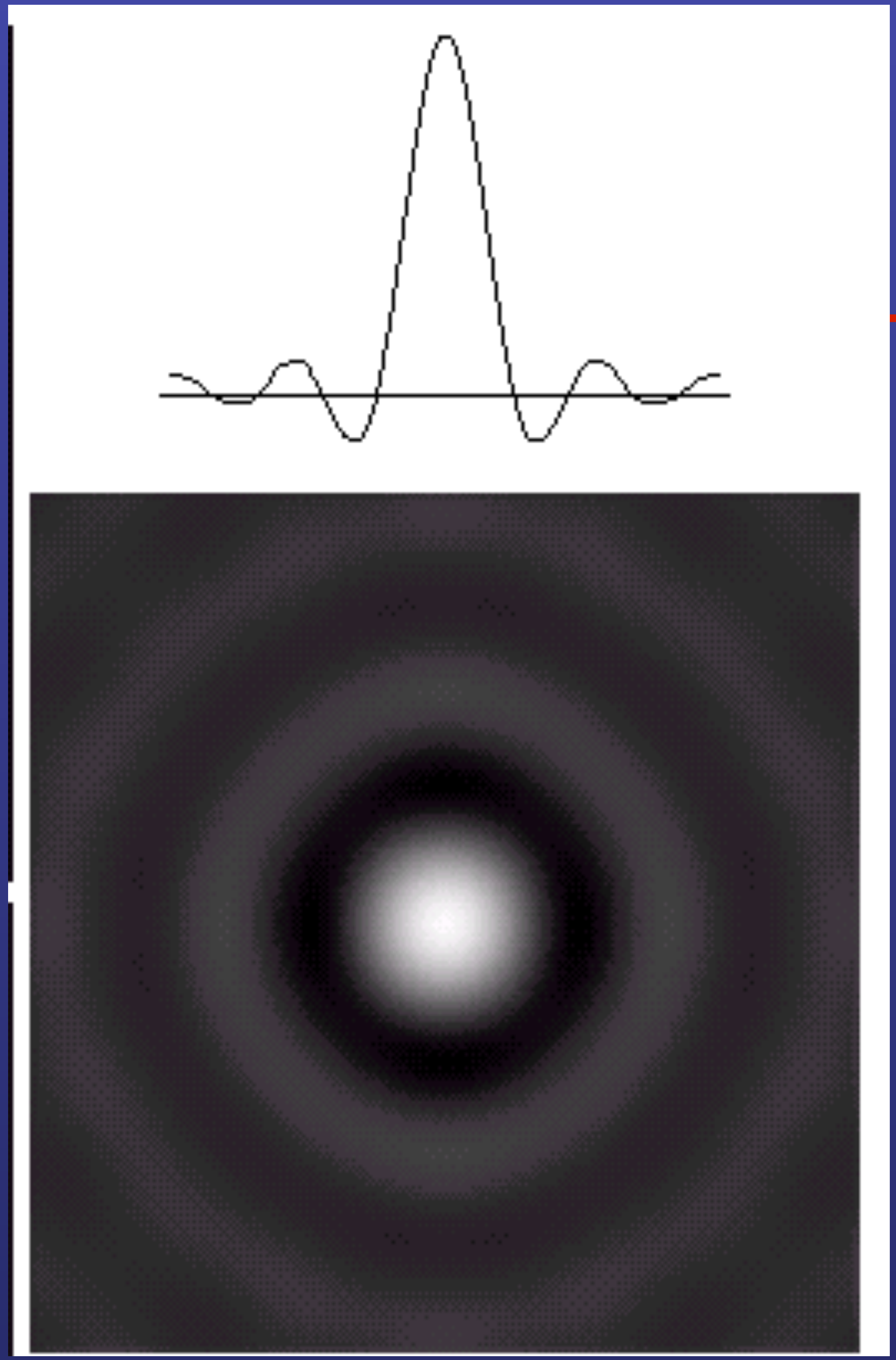
Notice the “ringing” in b-f



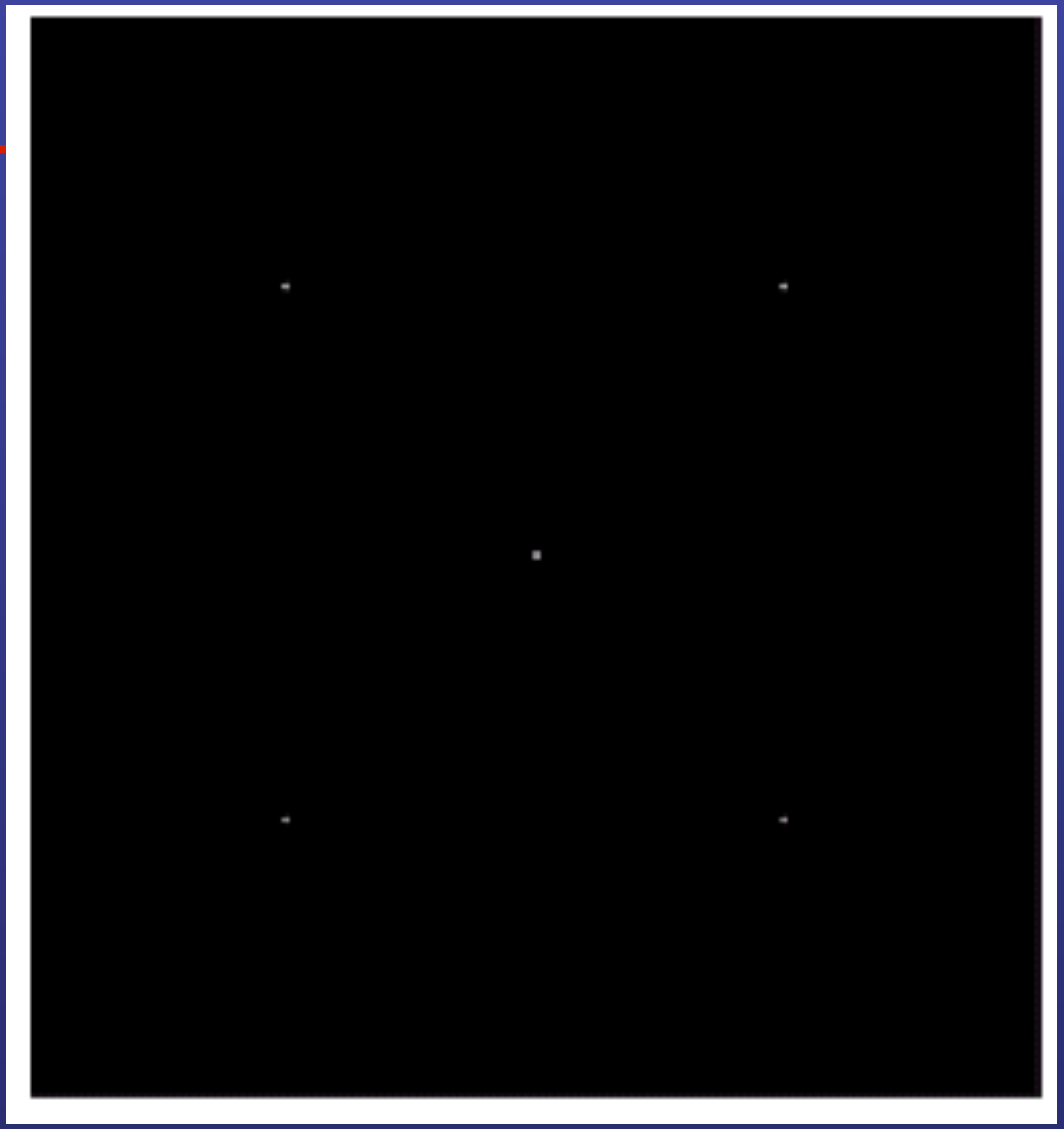
Frequency domain
1
Passband
Stopband
u
u0
frequency
u0
Spatial domain
x

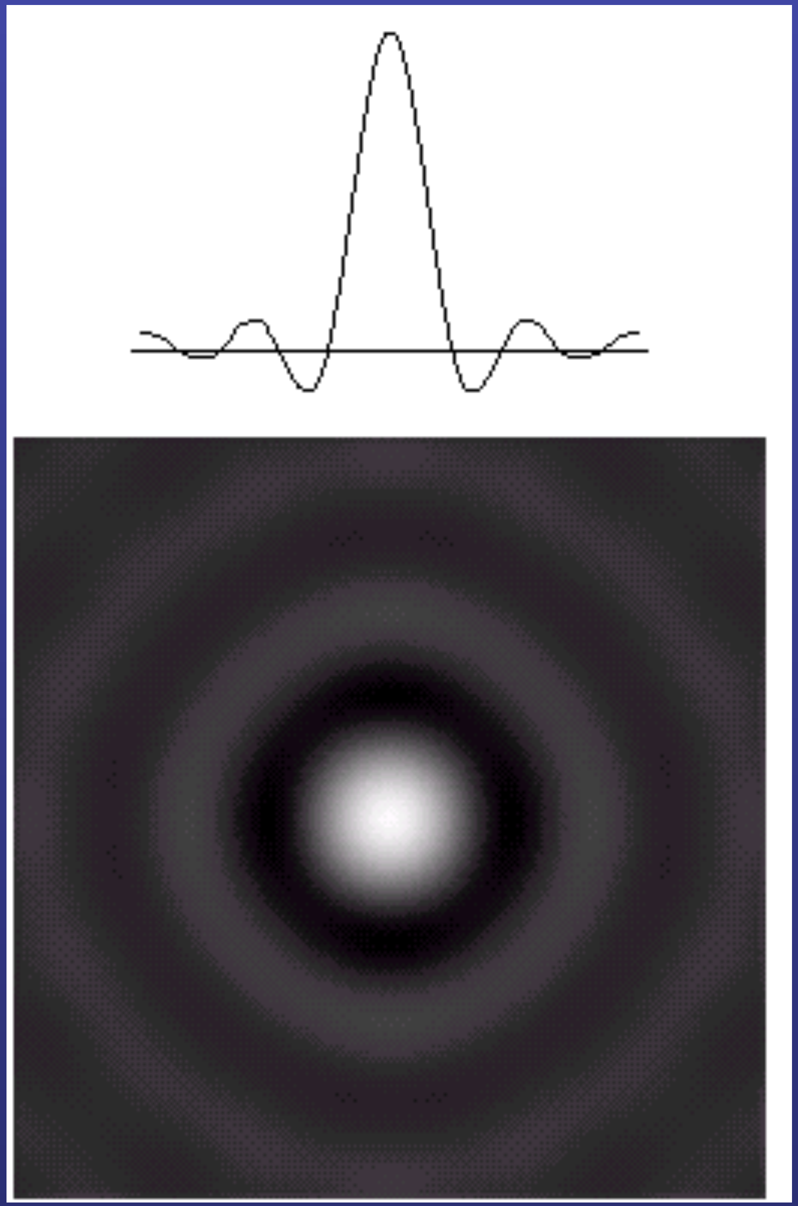


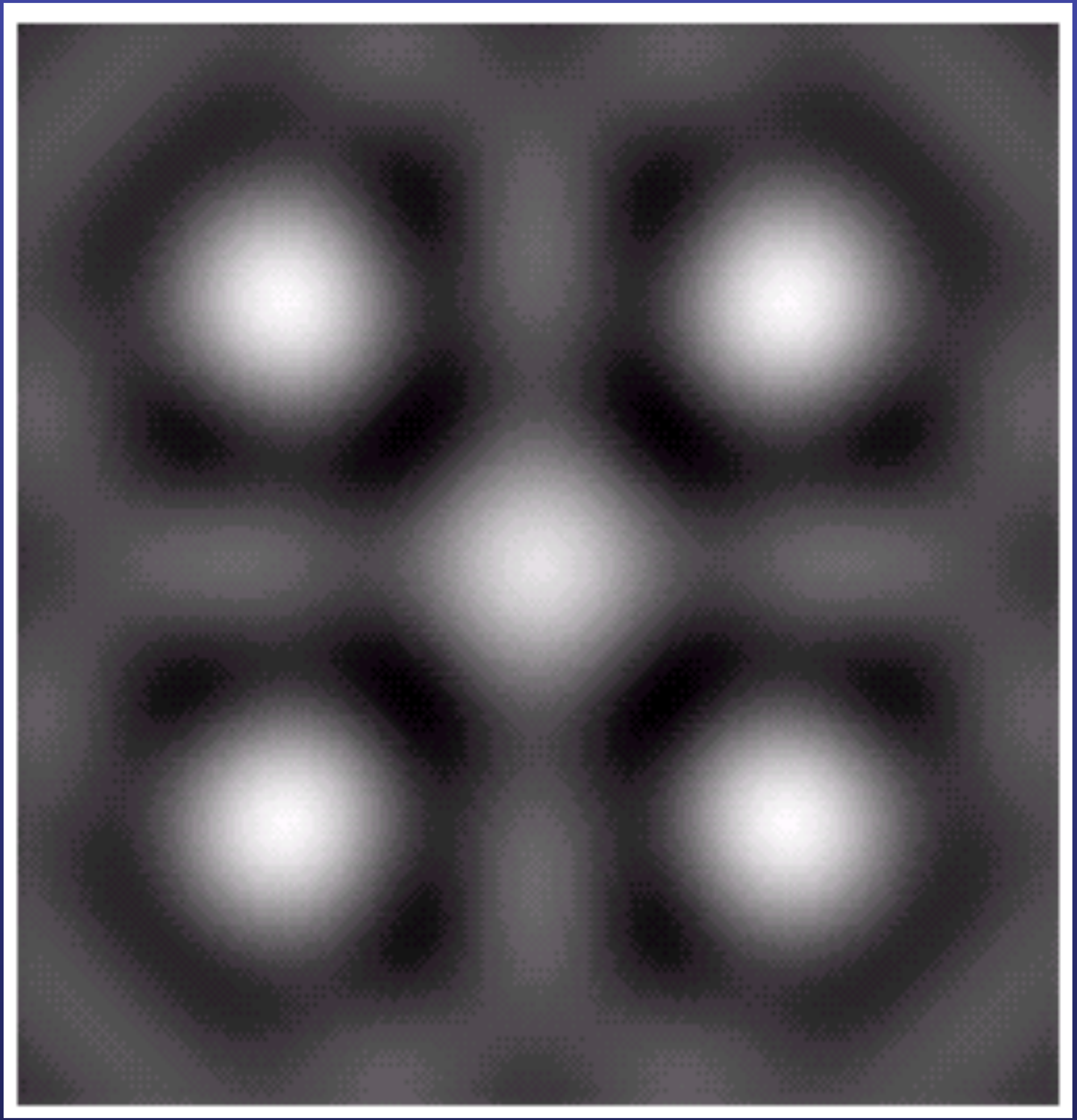
H(u, v)
u
v
v
u
H(u, v)
1
D(u, v)
D0



*





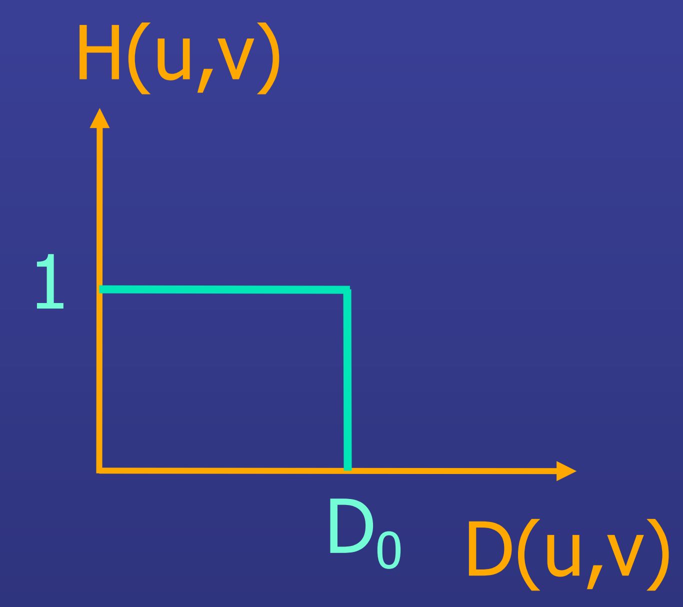


Butterworth Lowpass filter

- ILPF

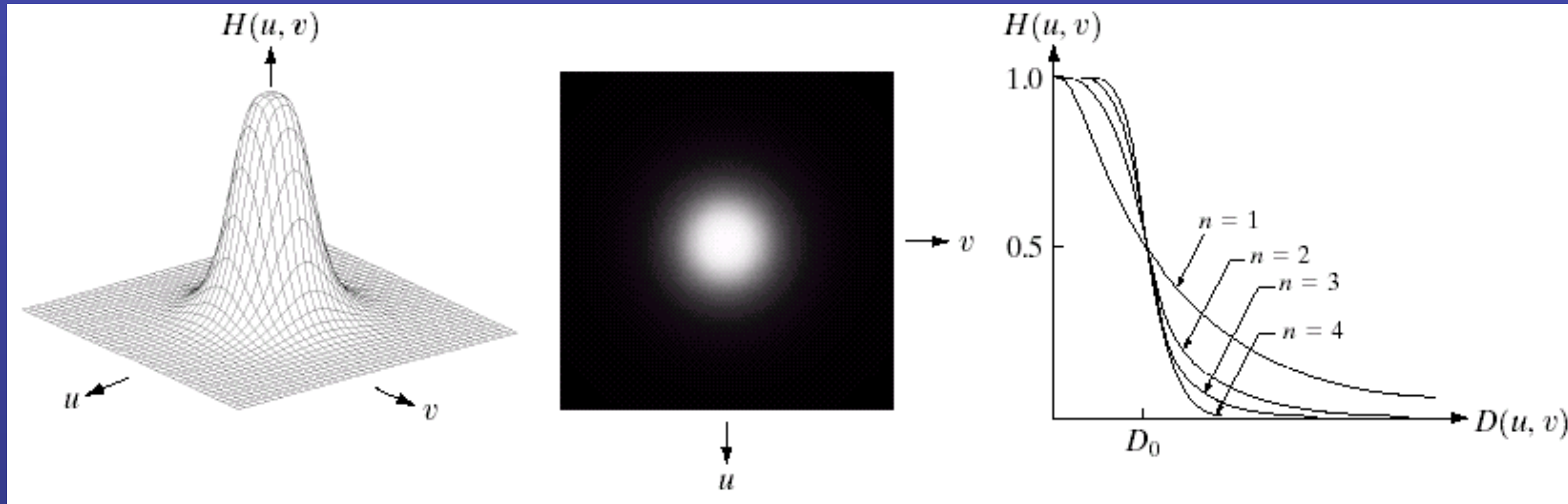
$$G(u,v) = H(u,v)F(u,v)$$

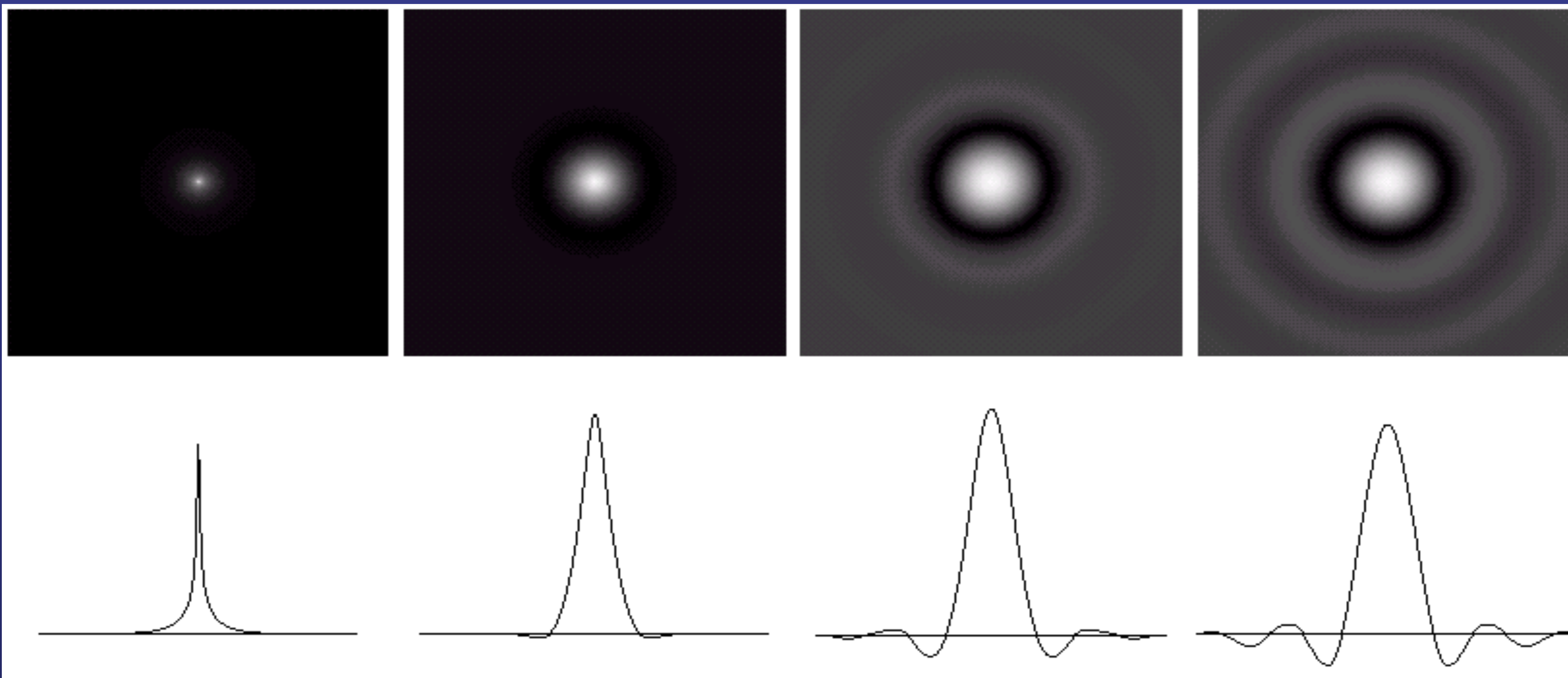
$$H(u,v) = \begin{cases} 1, & if \quad D(u,v) \le D_0 \\ 0, & if \quad D(u,v) > D_0 \end{cases}$$



- Butterworth

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$



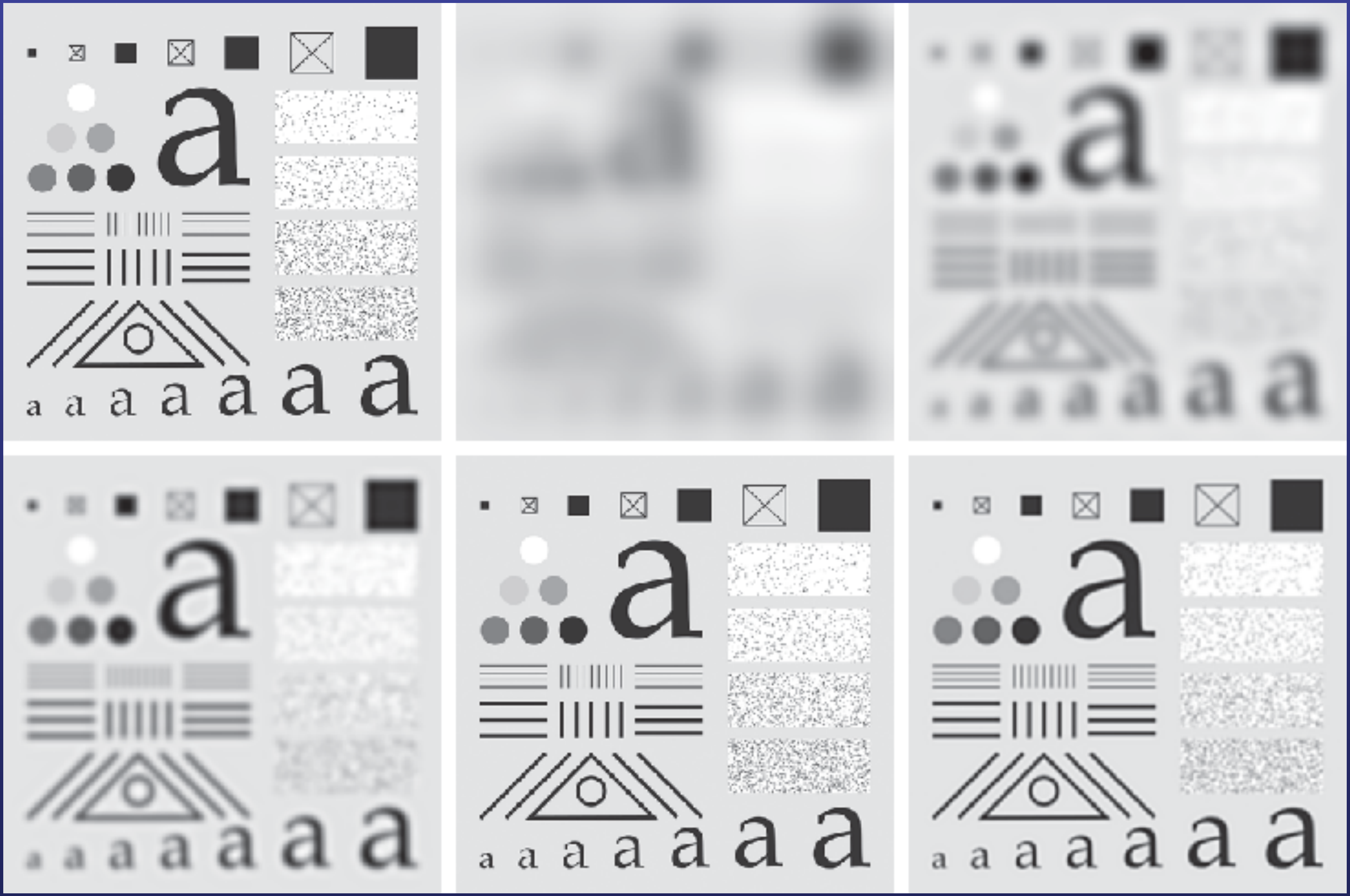


a b c d

FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

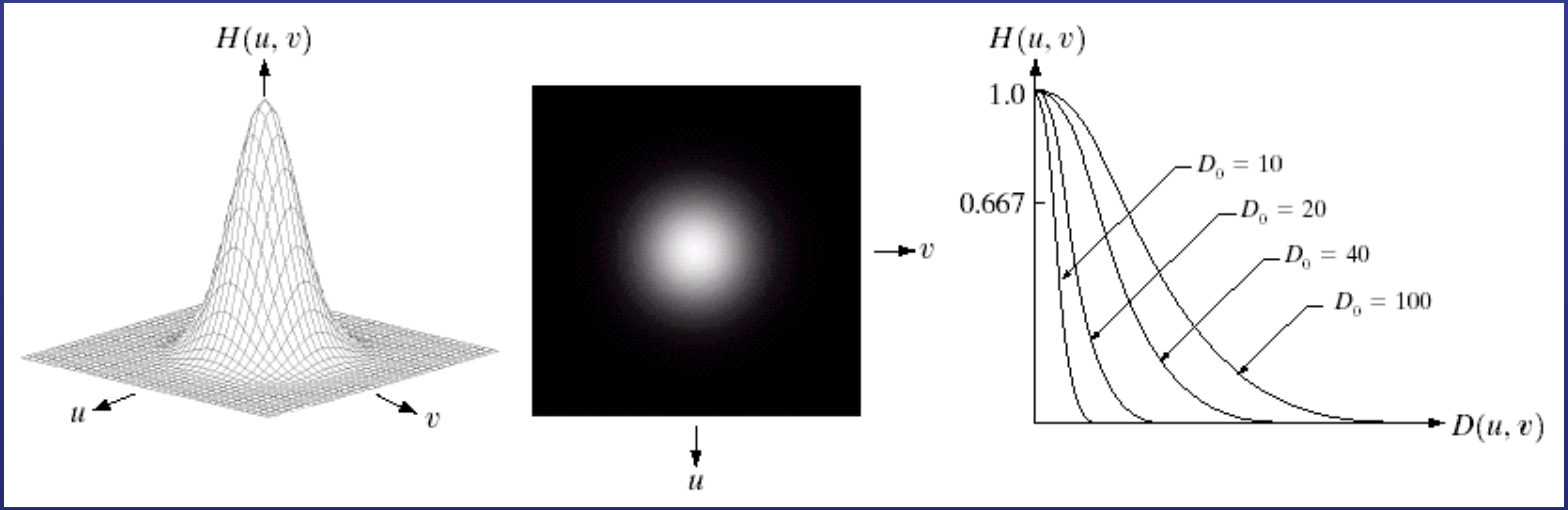
Butterworth Lowpass filter

n=2.25



Gaussian Lowpass Filter

$$H(u,v) = \exp(-D^2(u,v)/2D_o^{\,2})$$

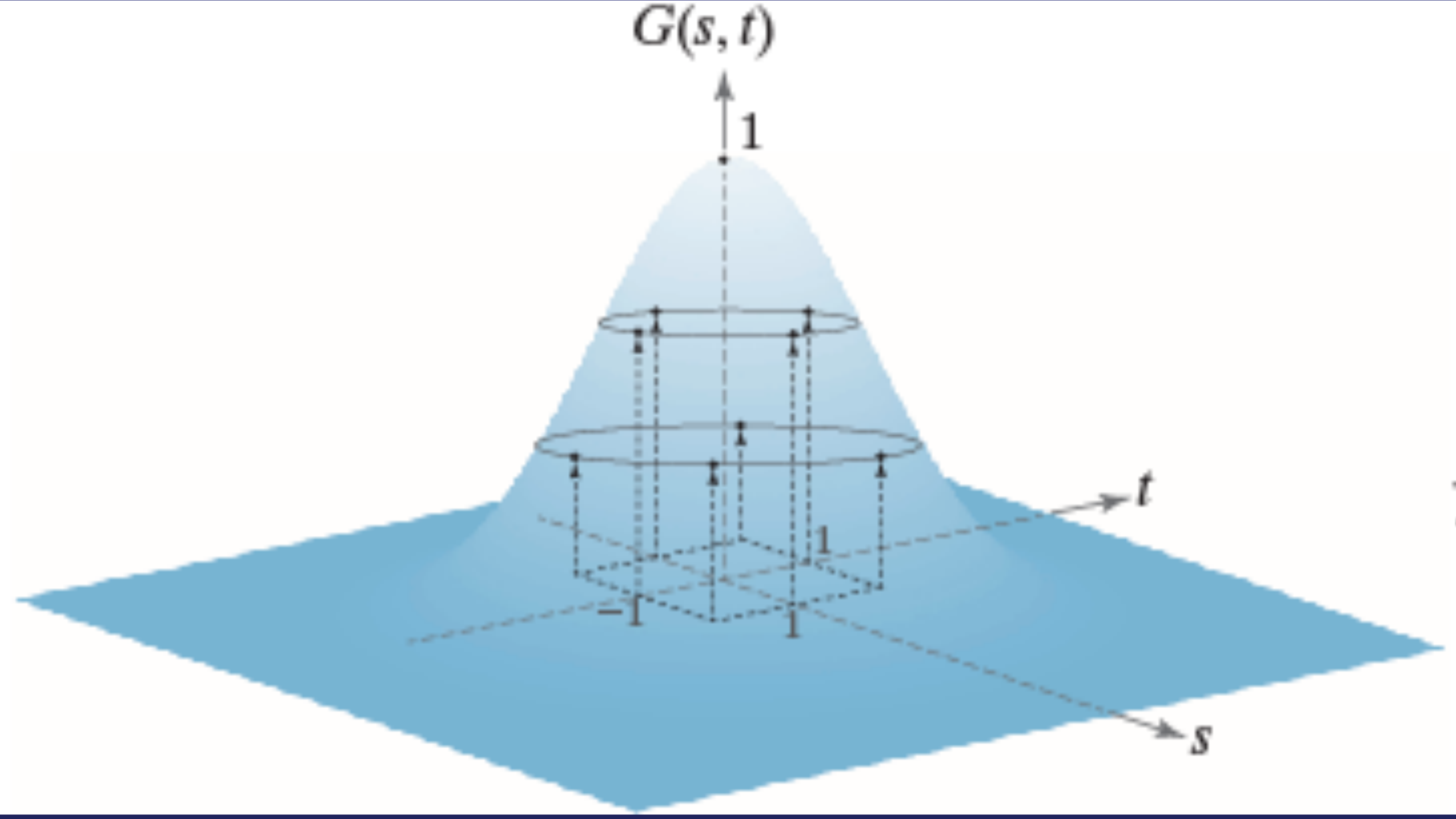


$$Gaussian \xrightarrow{\mathscr{F}} Gaussian$$

Gaussian Lowpass Filter

$$H(u,v) = \exp(-D^2(u,v)/2D_o^2)$$

Gaussian kernel in spatial domain

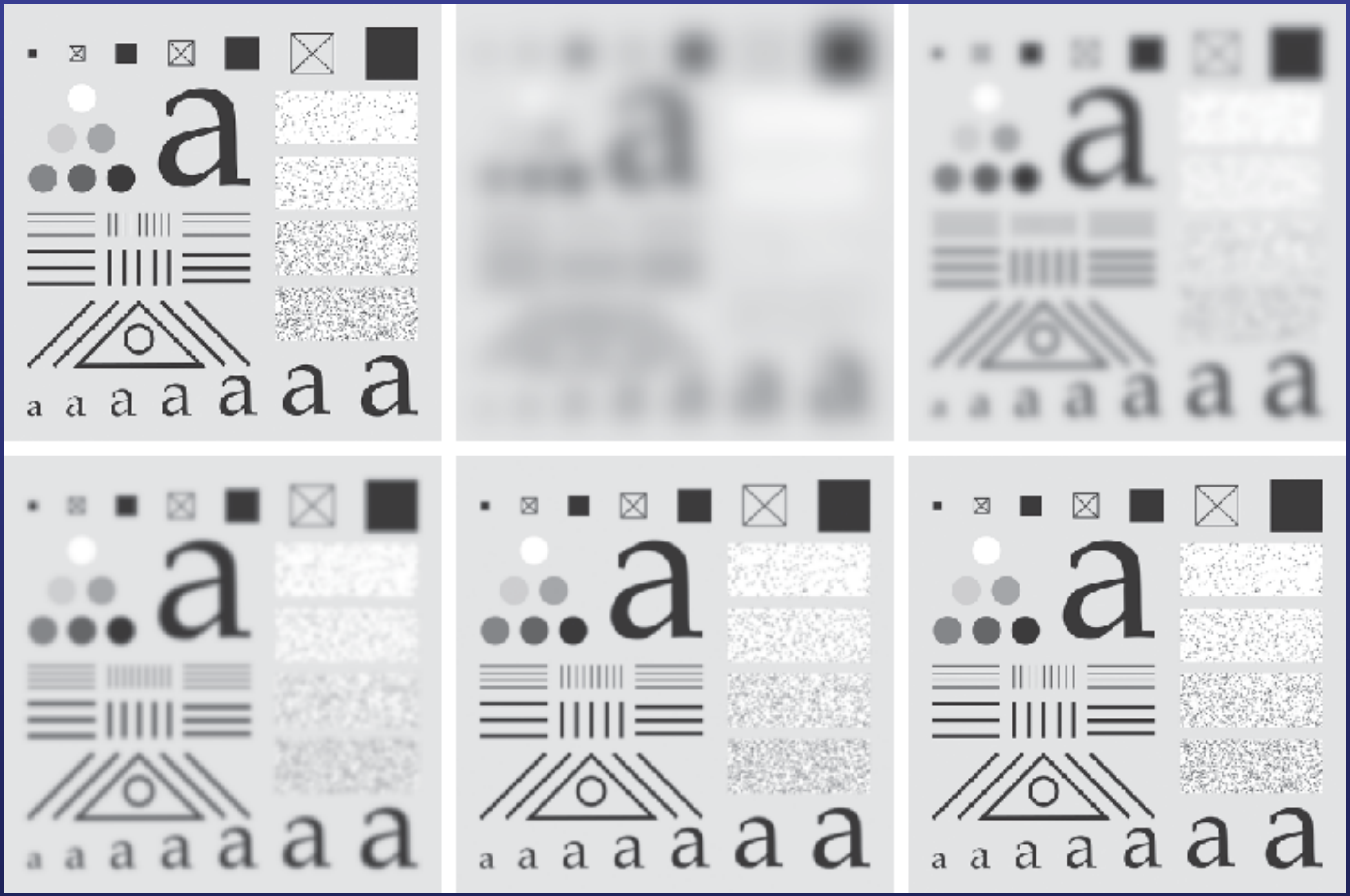


$\frac{1}{4.8976} \times$

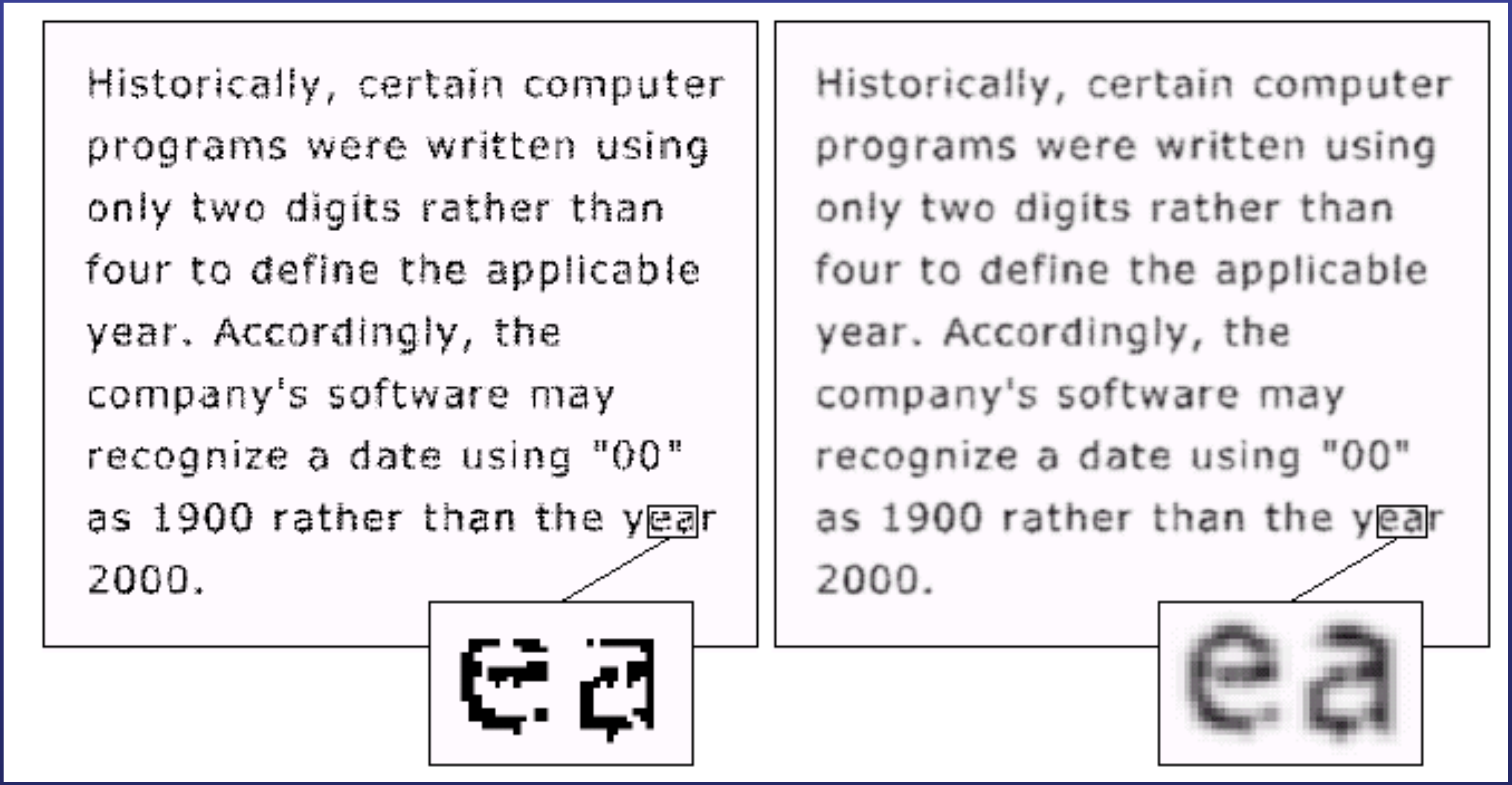
0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

Gaussian Lowpass Filter

- No ringing
- Good for medical imaging



Another examples



Another examples

High Pass Filters

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

- IHPF

$$H(u,v) = \begin{cases} 0, & if \quad D(u,v) \le D_0 \\ 1, & if \quad D(u,v) > D_0 \end{cases}$$

- Butterworth Filter

$$H(u,v) = \frac{1}{1 + \left[D_0 / D(u,v)\right]^{2n}}$$

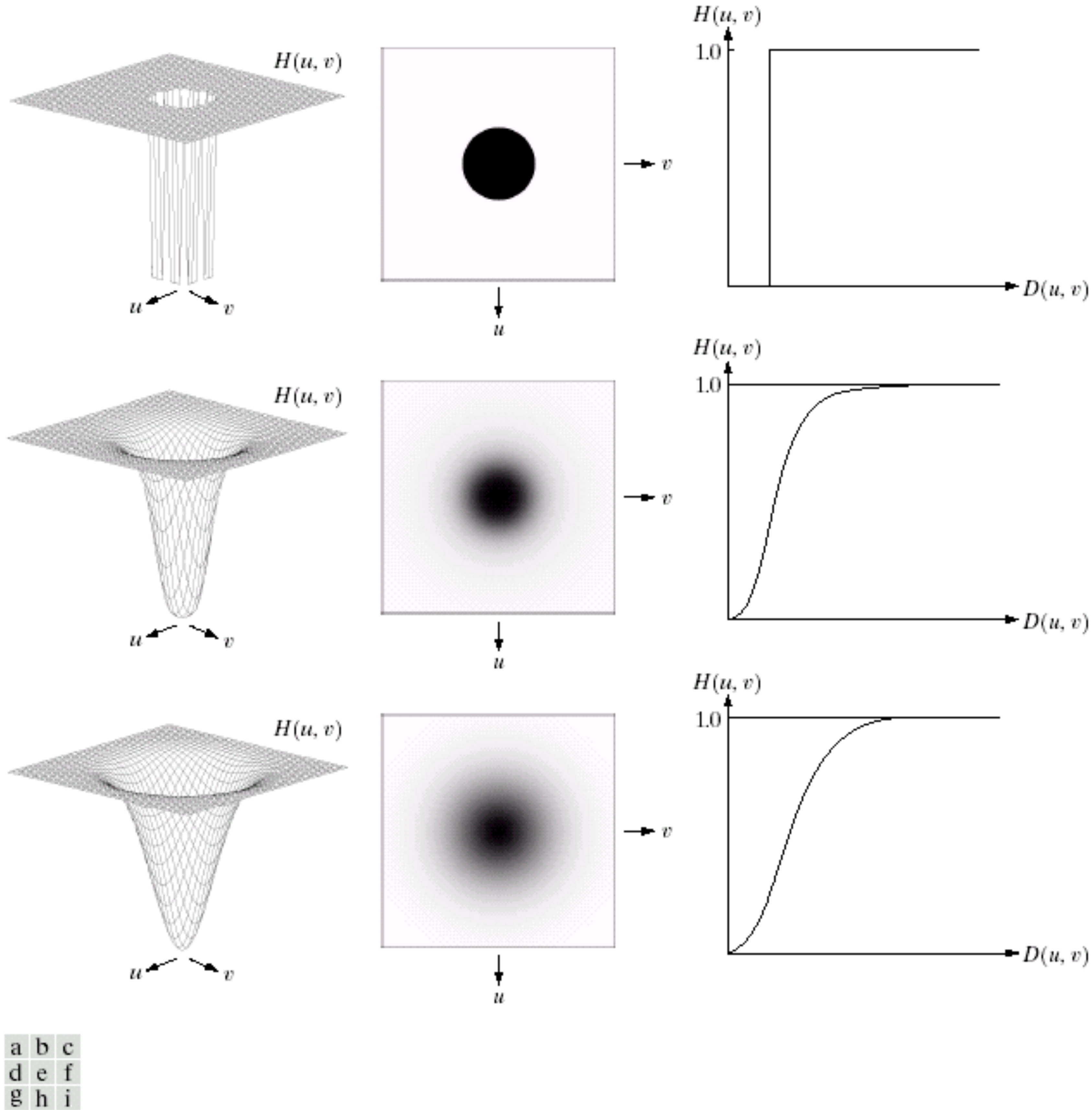
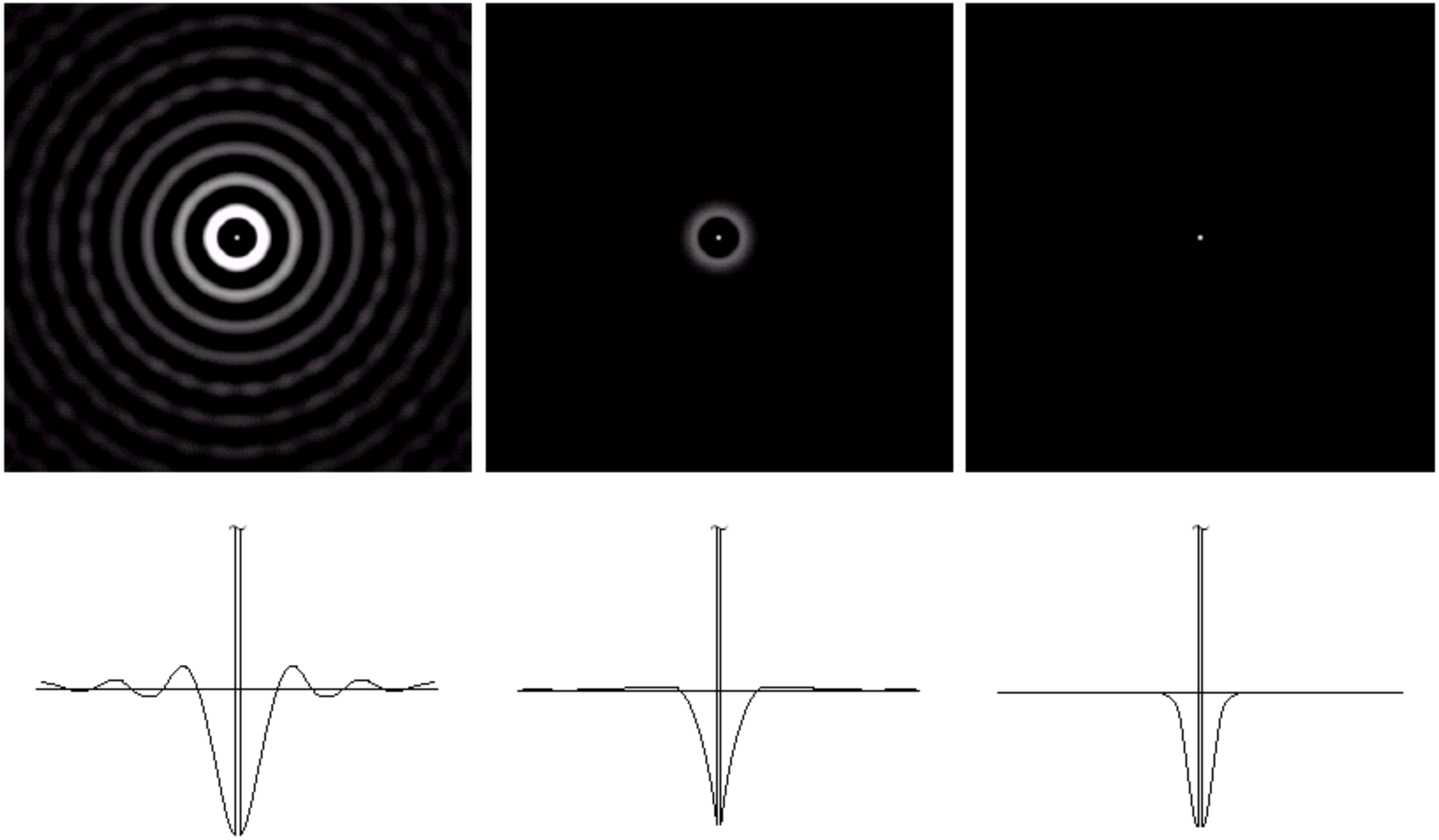


FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.



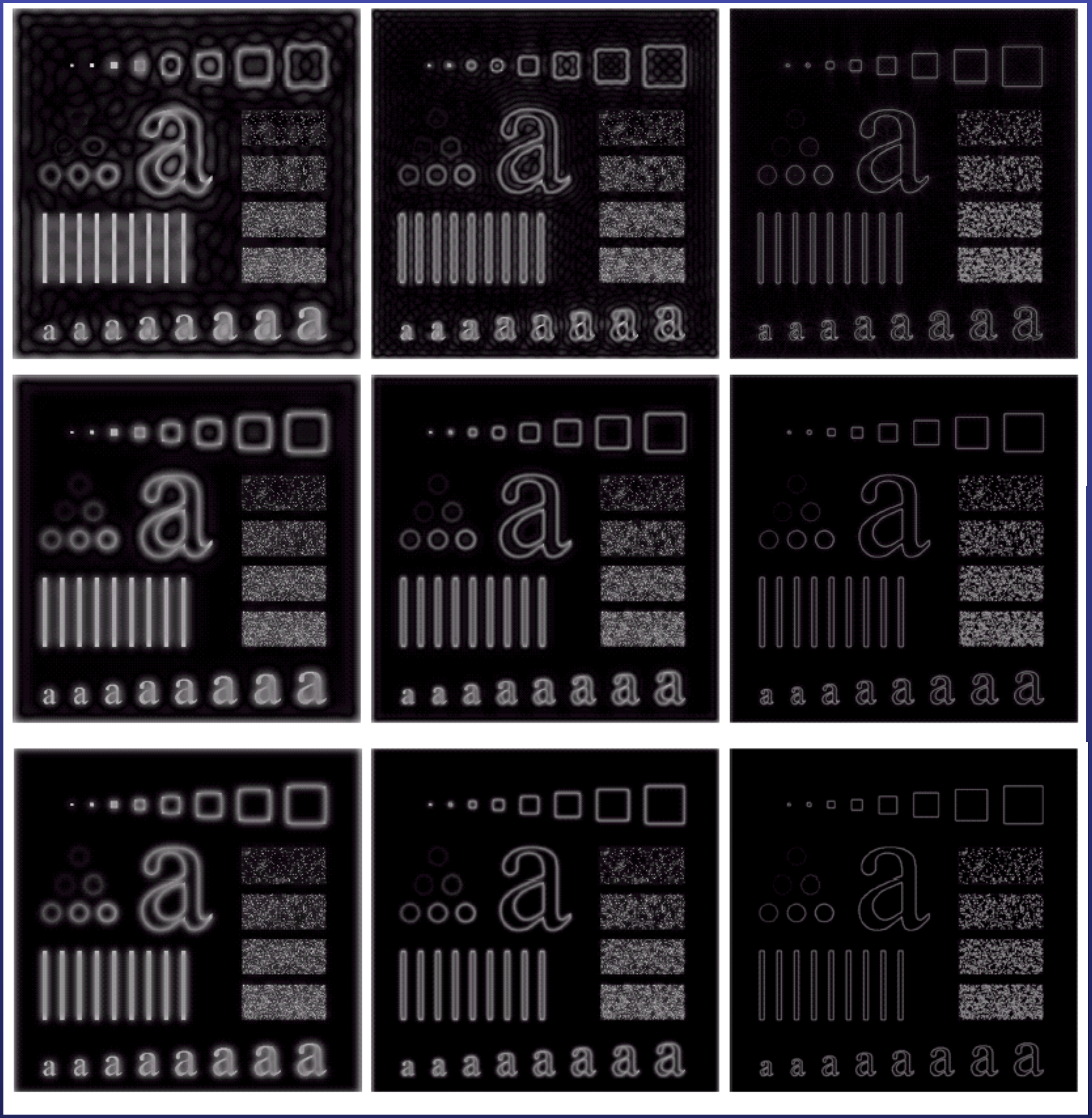
a b c

FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

IHPF

BHPF

GHPF



Homomorphic Filtering

$$f(x,y) = i(x,y)r(x,y)$$

$$0 < i(x,y) < \infty$$ illumination

$$0 < r(x,y) < 1$$ reflectance

$$F\{f(x,y)\} \neq F\{i(x,y)\}F\{r(x,y)\}$$

Homomorphic Filtering

$$let \quad z(x,y) = \ln f(x,y)$$
$$= \ln i(x,y) + \ln r(x,y)$$

$$\therefore Z(u,v) = I_{\ln} + R_{\ln}$$

$$S(u,v) = H(u,v)Z(u,v)$$
$$= HI_{\ln} + HR_{\ln}$$
$$\therefore s(x,y) = F^{-1}\{HI_{\ln}\} + F^{-1}\{HR_{\ln}\}$$
$$= i'(x,y) + r'(x,y)$$

Homomorphic Filtering

$$g(x,y) = e^{s(x,y)} = e^{i'(x,y)} \times e^{r'(x,y)}$$

$$= i_o(x,y) r_o(x,y)$$

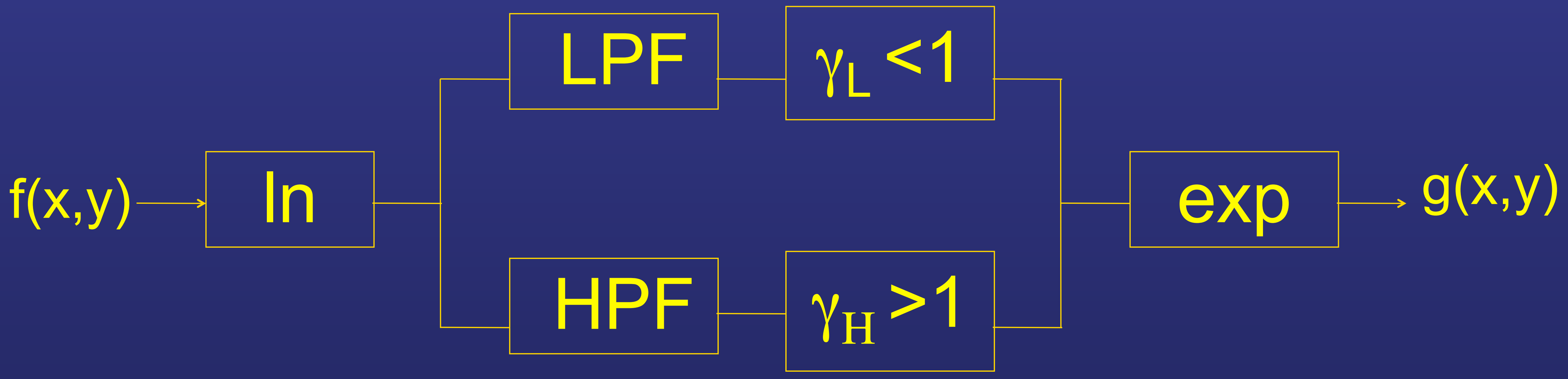
$$i_o(x,y) = e^{i'(x,y)}$$

$$i'(x,y) \to F^{-1}\{H(u,v) I_{ln}(u,v)\}$$

$$I_{ln}(u,v) \to F\{\ln i(x,y)\}$$

Homomorphic Filtering

- In practice: i → slow varying → LPF
  r → fast varying → HPF



Homomorphic Filtering