Derivada segunda com erro de ordem 3 e filosofia backward

1. $f(xi-\Delta x)=f(xi)-\frac{1}{1!}\frac{df(xi)}{dx}(\Delta x)^1+\frac{1}{2!}\frac{d^2f(xi)}{dx^2}(\Delta x)^2-\frac{1}{3!}\frac{d^3f(xi)}{dx^3}(\Delta x)^3+\frac{1}{4!}\frac{d^4f(xi)}{dx^4}(\Delta x)^4-\frac{1}{5!}\frac{d^5f(xi)}{dx^5}(\Delta x)^5$
2. $f(xi-2\Delta x)\alpha=f(xi)\alpha-\frac{1}{1!}\frac{df(xi)}{dx}(2\Delta x)^1\alpha+\frac{1}{2!}\frac{d^2f(xi)}{dx^2}(2\Delta x)^2\alpha-\frac{1}{3!}\frac{d^3f(xi)}{dx^3}(2\Delta x)^3\alpha+\frac{1}{4!}\frac{d^4f(xi)}{dx^4}(2\Delta x)^4\alpha-\frac{1}{5!}\frac{d^5f(xi)}{dx^5}(2\Delta x)^5\alpha$
3. $f(xi-3\Delta x)\beta=f(xi)\beta-\frac{1}{1!}\frac{df(xi)}{dx}(3\Delta x)^1\beta+\frac{1}{2!}\frac{d^2f(xi)}{dx^2}(3\Delta x)^2\beta-\frac{1}{3!}\frac{d^3f(xi)}{dx^3}(3\Delta x)^3\beta+\frac{1}{4!}\frac{d^4f(xi)}{dx^4}(3\Delta x)^4\beta-\frac{1}{5!}\frac{d^5f(xi)}{dx^5}(3\Delta x)^5\beta$
4. $f(xi-4\Delta x)\gamma=f(xi)\gamma-\frac{1}{1!}\frac{df(xi)}{dx}(4\Delta x)^1\gamma+\frac{1}{2!}\frac{d^2f(xi)}{dx^2}(4\Delta x)^2\gamma-\frac{1}{3!}\frac{d^3f(xi)}{dx^3}(4\Delta x)^3\gamma+\frac{1}{4!}\frac{d^4f(xi)}{dx^4}(4\Delta x)^4\gamma-\frac{1}{5!}\frac{d^5f(xi)}{dx^5}(4\Delta x)^5\gamma$

$$\frac{1}{1!}\frac{df(xi)}{dx}(\Delta x)(-1-2\alpha-3\beta-4\gamma)=0$$

$$\frac{1}{3!}\frac{d^3f(xi)}{dx^3}(\Delta x)^3(-1-8\alpha-27\beta-64\gamma)=0$$

$$\frac{1}{4!}\frac{d^4f(xi)}{dx^4}(\Delta x)^4(1+16\alpha+81\beta+256\gamma)=0$$

$$\begin{bmatrix}2 & 3 & 4\\ 8 & 27 & 64\\ 16 & 81 & 256\end{bmatrix}\cdot\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix}=\begin{bmatrix}-1\\ -1\\ -1\end{bmatrix}$$

$$\alpha=\frac{-57}{52};\ \beta=\frac{7}{13};\ \gamma=\frac{-11}{104}$$

$$\frac{-3}{13}\frac{\frac{d^2f_i}{dx^2}(\Delta x)^2}{2!}=f_{i-1}-f_i-\frac{57}{52}(f_{i-2}-f_i)+\frac{7}{13}(f_{i-3}-f_i)-\frac{11}{104}(f_{i-4}-f_i)+\frac{\frac{d^5f_i}{dx^5}(\Delta x)^5}{5!}(1-\frac{57}{52}\cdot 32+\frac{7}{13}\cdot 243-\frac{11}{104}\cdot 1024)$$

$$\frac{d^2fi}{dx^2}=\frac{1}{(\Delta x)^2}\left(\frac{35}{12}f_i-\frac{26}{3}f_{i-1}+\frac{57}{6}f_{i-2}-\frac{14}{3}f_{i-3}+\frac{11}{12}f_{i-4}\right)+\frac{13\cdot\frac{d^5f_i}{dx^5}\cdot(\Delta x)^3}{180}$$