Derivada terceira com erro de ordem 4 e filosofia forward

1. $f(xi+\Delta x)=f(xi)+\frac{1}{1!}\frac{df(xi)}{dx}(\Delta x)^1+\frac{1}{2!}\frac{d^2f(xi)}{dx^2}(\Delta x)^2+\cdots+\frac{1}{7!}\frac{d^7f(xi)}{dx^7}(\Delta x)^7$

2. $f(xi+2\Delta x)\alpha=f(xi)\alpha+\frac{1}{1!}\frac{df(xi)}{dx}(2\Delta x)^1\alpha+\frac{1}{2!}\frac{d^2f(xi)}{dx^2}(2\Delta x)^2\alpha+\cdots+\frac{1}{7!}\frac{d^7f(xi)}{dx^7}(2\Delta x)^7\alpha$

3. $f(xi+3\Delta x)\beta=f(xi)\beta+\frac{1}{1!}\frac{df(xi)}{dx}(3\Delta x)^1\beta+\frac{1}{2!}\frac{d^2f(xi)}{dx^2}(3\Delta x)^2\beta+\cdots+\frac{1}{7!}\frac{d^7f(xi)}{dx^7}(3\Delta x)^7\beta$

4. $f(xi+4\Delta x)\gamma=f(xi)\gamma+\frac{1}{1!}\frac{df(xi)}{dx}(4\Delta x)^1\gamma+\frac{1}{2!}\frac{d^2f(xi)}{dx^2}(4\Delta x)^2\gamma+\cdots+\frac{1}{7!}\frac{d^7f(xi)}{dx^7}(4\Delta x)^7\gamma$

5. $f(xi+5\Delta x)\theta=f(xi)\theta+\frac{1}{1!}\frac{df(xi)}{dx}(5\Delta x)^1\theta+\frac{1}{2!}\frac{d^2f(xi)}{dx^2}(5\Delta x)^2\theta+\cdots+\frac{1}{7!}\frac{d^7f(xi)}{dx^7}(5\Delta x)^7\theta$

6. $f(xi+6\Delta x)\mu=f(xi)\mu+\frac{1}{1!}\frac{df(xi)}{dx}(6\Delta x)^1\mu+\frac{1}{2!}\frac{d^2f(xi)}{dx^2}(6\Delta x)^2\mu+\cdots+\frac{1}{7!}\frac{d^7f(xi)}{dx^7}(6\Delta x)^7\mu$

$$\frac{1}{1!}\frac{df(xi)}{dx}(\Delta \mathrm{x})(1+2\alpha+3\beta+4\gamma+5\theta+6\mu)=0$$

$$\frac{1}{2!}\frac{d^2f(xi)}{dx^2}(\Delta x)^2(1+4\alpha+9\beta+16\gamma+35\theta+36\mu)=0$$

$$\frac{1}{4!}\frac{d^4f(xi)}{dx^4}(\Delta x)^4(1+16\alpha+81\beta+256\gamma+625\theta+1296\mu)=0$$

$$\frac{1}{5!}\frac{d^5f(xi)}{dx^5}(\Delta x)^5(1+32\alpha+243\beta+1024\gamma+3125\theta+7776\mu)=0$$

$$\frac{1}{6!}\frac{d^6f(xi)}{dx^6}(\Delta x)^6(1+64\alpha+729\beta+4096\gamma+15625\theta+46656\mu)=0$$

$$\begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 4 & 9 & 16 & 25 & 36 \\ 16 & 81 & 256 & 625 & 1296 \\ 32 & 243 & 1024 & 3125 & 7776 \\ 64 & 729 & 4096 & 15625 & 46656 \end{bmatrix}\cdot\begin{bmatrix}\alpha\\ \beta\\ \gamma\\ \theta\\ \mu\end{bmatrix}=\begin{bmatrix}-1\\ -1\\ -1\\ -1\\ -1\end{bmatrix}$$

$$\alpha=\frac{-461}{232};\ \beta=\frac{62}{29};\ \gamma=\frac{-307}{232};\ \theta=\frac{13}{29};\ \mu=\frac{-15}{232}$$

$$\frac{\frac{d^3f_i}{dx^3}(\Delta x)^3}{29}=\left[f_{i+1}-f_i-\frac{461}{232}(f_{i+2}-f_i)+\frac{62}{29}(f_{i+3}-f_i)-\frac{307}{232}(f_{i+4}-f_i)+\frac{13}{29}(f_{i+5}-f_i)-\frac{15}{232}(f_{i+6}-f_i)\right]$$
$$-\frac{\frac{d^7f_i}{dx^7}(\Delta x)^7}{7!}\left(1-\frac{461}{232}\cdot 128+\frac{62}{29}\cdot 2187-\frac{307}{232}\cdot 16384+\frac{13}{29}\cdot 78125-\frac{15}{232}\cdot 279936\right)$$

$$\frac{d^3fi}{dx^3}=\frac{29}{(\Delta x)^3}\left(-\frac{49}{232}f_i+f_{i+1}-\frac{461}{232}f_{i+2}+\frac{62}{29}f_{i+3}-\frac{307}{232}f_{i+4}+\frac{13}{29}f_{i+5}-\frac{15}{232}f_{i+6}\right)+\frac{29\cdot\frac{d^7f_i}{dx^7}\cdot(\Delta x)^4}{15}$$