Fechada com polinômio de substituição de grau 4

$$I = \int_a^b f(x)dx = \int_0^4 f(x(s))\frac{dx(s)}{ds}ds = h\int_0^4 g(s)ds$$

Com

$$g(s) = \sum_{k=0}^{4} \binom{s}{k} \Delta^k f$$

$$= f_0 + s(f_1 - f_0) + \frac{1}{2}(s^2 - s)(f_2 - 2f_1 + f_0) + \frac{1}{6}(s^3 - 3s^2 + 2s)(f_3 - 3f_2 + 3f_1 - f_0) + \frac{1}{24}(s^4 - 6s^3 + 5s^2 - 6s)(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)$$

Onde
$f_k = f\big(x(k)\big)$ e $x(s) = a + hs$

Logo

$$g(0) = f\big(x(0)\big) = f(a)$$
$$g(1) = f\big(x(1)\big) = f(a+h)$$
$$g(2) = f\big(x(2)\big) = f(a+2h)$$
$$g(3) = f\big(x(3)\big) = f(a+3h)$$
$$g(4) = f\big(x(4)\big) = f(b)$$

Com
$$h = \frac{b-a}{4}$$

Portanto
$$I \approx \int_0^4 g(s)\,ds = h(I_1 + I_2 + I_3 + I_4 + I_5)$$

$$I_1 = \int_0^4 f_0\,ds = f_0\int_0^4 ds = 4f_0$$

$$I_2 = \int_0^4 s(f_1 - f_0)\ ds = (f_1 - f_0)\int_0^4 s\,ds = 8(f_1 - f_0)$$

$$I_3 = \int_0^4 \frac{1}{2}(s^2 - s)(f_2 - 2f_1 + f_0)\ ds = \frac{1}{2}(f_2 - 2f_1 + f_0)\left[\int_0^4 s^2\,ds - \int_0^4 s\,ds\right]$$

$$= \frac{1}{2}(f_2 - 2f_1 + f_0)\left[\frac{64}{3} - 8\right] = \frac{40}{6}(f_2 - 2f_1 + f_0)$$

$$I_4 = \int_0^4 \frac{1}{6}(s^3 - 3s^2 + 2s)(f_3 - 3f_2 + 3f_1 - f_0)\, ds$$

$$= \frac{1}{6}(f_3 - 3f_2 + 3f_1 - f_0)\left[\int_0^4 s^3\, ds - 3\int_0^4 s^2\, ds + 2\int_0^4 s\, ds\right]$$

$$= \frac{16}{6}(f_3 - 3f_2 + 3f_1 - f_0)$$

$$I_5 = \int_0^4 \frac{1}{24}(s^4 - 6s^3 + 5s^2 - 6s)(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)\, ds$$

$$= \frac{1}{24}(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)\left[\int_0^4 s^4\, ds - 6\int_0^4 s^3\, ds + 5\int_0^4 s^2\, ds - 6\int_0^4 s\, ds\right]$$

$$= -\frac{226}{45}(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)$$

$$I = h(4f_0 + 8(f_1 - f_0) + \frac{40}{6}(f_2 - 2f_1 + f_0) + \frac{16}{6}(f_3 - 3f_2 + 3f_1 - f_0) - \frac{226}{45}(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0))$$

$$= h(-\frac{752}{90}f_0 + \frac{2048}{90}f_1 - \frac{2832}{90}f_2 + \frac{2048}{90}f_3 - \frac{452}{90}f_4$$

$$= \frac{16}{90}h(47f_0 + 128f_1 - 177f_2 + 128f_3 - \frac{113}{4}f_4)$$