Fechada com polinômio de substituição de grau 3

$$I = \int_a^b f(x)dx = \int_0^3 f(x(s))\frac{dx(s)}{ds}ds = h\int_0^3 g(s)ds$$

Com

$$g(s) = \sum_{k=0}^{3}\binom{s}{k}\Delta^k f = f_0 + s(f_1 - f_0) + \frac{1}{2}(s^2 - s)(f_2 - 2f_1 + f_0) + \frac{1}{6}(s^3 - 3s^2 + 2s)(f_3 - 3f_2 + 3f_1 - f_0)$$

Onde

$f_k = f\big(x(k)\big)$ e $x(s) = a + hs$

Logo

$g(0) = f\big(x(0)\big) = f(a)$
$g(1) = f\big(x(1)\big) = f(a+h)$
$g(2) = f\big(x(2)\big) = f(a+2h)$
$g(3) = f\big(x(3)\big) = f(b)$

Com

$$h = \frac{b-a}{3}$$

Portanto

$$I \approx \int_0^3 g(s)\,ds = h(I_1 + I_2 + I_3 + I_4)$$

$$I_1 = \int_0^3 f_0\,ds = f_0\int_0^3 ds = 3f_0$$

$$I_2 = \int_0^3 s(f_1 - f_0)\;ds = (f_1 - f_0)\int_0^3 s\,ds = \frac{9}{2}(f_1 - f_0)$$

$$I_3 = \int_0^3 \frac{1}{2}(s^2 - s)(f_2 - 2f_1 + f_0)\;ds = \frac{1}{2}(f_2 - 2f_1 + f_0)\left[\int_0^3 s^2\,ds - \int_0^3 s\,ds\right]$$

$$= \frac{1}{2}(f_2 - 2f_1 + f_0)\left[9 - \frac{9}{2}\right] = \frac{9}{4}(f_2 - 2f_1 + f_0)$$

$$I_4 = \int_0^3 \frac{1}{6}(s^3 - 3s^2 + 2s)(f_3 - 3f_2 + 3f_1 - f_0)\ ds$$

$$= \frac{1}{6}(f_3 - 3f_2 + 3f_1 - f_0)\left[\int_0^3 s^3\ ds - 3\int_0^3 s^2\ ds + 2\int_0^3 s\ ds\right]$$

$$= \frac{9}{24}(f_3 - 3f_2 + 3f_1 - f_0)$$

$$I = h(3f_0 + \frac{9}{2}(f_1 - f_0) + \frac{9}{4}(f_2 - 2f_1 + f_0) + \frac{9}{24}(f_3 - 3f_2 + 3f_1 - f_0))$$

$$= h(\frac{9}{24}f_0 + \frac{9}{8}f_1 - \frac{9}{4}f_2 + \frac{9}{24}f_3)$$

$$= \frac{3}{8}h(f_0 + 3f_1 + 6f_2 + f_3)$$