

1 Itô integral (I)

$$\begin{aligned} E \left[\sum_{i=1}^n X_{t_{i-1}} \Delta W_{t_{i-1}} \cdot \sum_{i=1}^n Y_{t_{i-1}} \Delta W_{t_{i-1}} \right] &= \sum_{i=1}^n E \left[X_{t_{i-1}} Y_{t_{i-1}} \Delta W_{t_{i-1}}^2 \right] \\ &= \sum_{i=1}^n E \left[X_{t_{i-1}} Y_{t_{i-1}} E[\Delta W_{t_{i-1}}^2 | \mathcal{F}_t] \right] = \sum_{i=1}^n E \left[X_{t_{i-1}} Y_{t_{i-1}} \Delta t \right] \end{aligned}$$

2 Itô integral (II)

$$\begin{aligned} \text{Var}\left(\int_0^t X_s dW_s\right) &= E\left[\left(\int_0^t X_s dW_s\right)^2\right] = E\left[\int_0^t X_s dW_s \cdot \int_0^t X_s dW_s\right] \\ &= \int_0^t E\left(X_s^2 dW_s^2\right) = \int_0^t E\left[X_s^2 E\left(dW_s^2|\mathcal{F}_s\right)\right] = \int_0^t E\left(X_s^2\right) dS = X_t^2 \end{aligned}$$