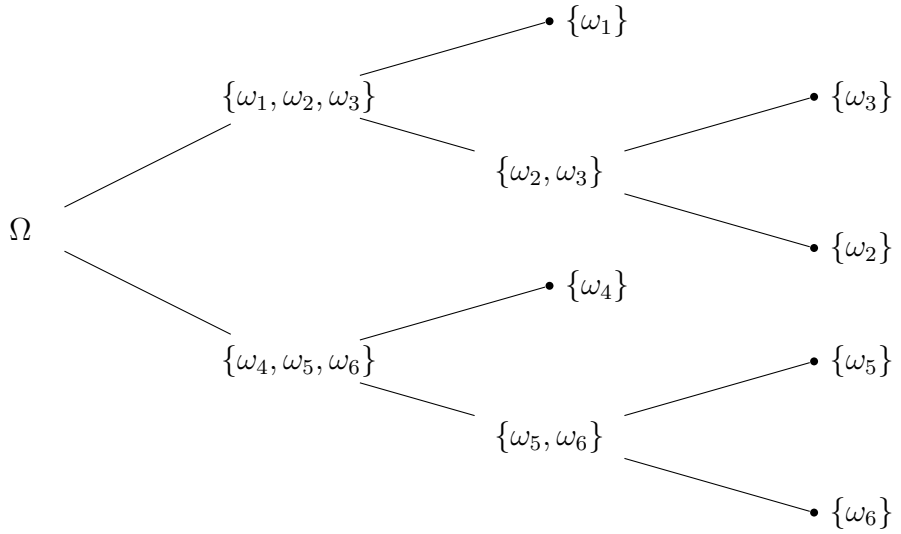
1 σ-field



$$\mathcal{F}_0, \mathcal{F}_1 = \sigma(X_0, X_1) = \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4, \omega_5, \omega_6\}, \phi, \Omega\}$$
$$\begin{aligned}\mathcal{F}_2 = \sigma(X_0, X_1, X_2) = \{&\omega_1, \omega_4, \{\omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}, \{\omega_1, \omega_2, \omega_3, \omega_5, \omega_6\}\{\omega_1, \omega_2, \omega_3\}, \{\omega_4, \omega_5, \omega_6\},\\ &\{\omega_2, \omega_3, \omega_5, \omega_6\}, \{\omega_1, \omega_4\}, \{\omega_1, \omega_4, \omega_5, \omega_6\}, \{\omega_2, \omega_3\}, \{\omega_5, \omega_6\}, \{\omega_1, \omega_2, \omega_3, \omega_4\},\\ &\{\omega_1, \omega_5, \omega_6\}, \{\omega_2, \omega_3, \omega_4\}\phi, \Omega\}\end{aligned}$$
$$\mathcal{F}_3 = \sigma(X_0, X_1, X_2, X_3) = \{\omega_1, \omega_2, \cdots, \phi, \Omega\}$$

2 Stopping time

$$\tau_1(\omega_i) = \begin{cases} 1, & i = 1, 2, 3 \\ 2, & i = 4 \\ 3, & i = 5, 6 \end{cases} \qquad \tau_2(\omega_i) = \begin{cases} 1, & i = 1 \\ 2, & i = 2, 3 \\ 3, & i = 4, 5, 6 \end{cases}$$

$[\tau \le t] = \{\omega \in \Omega | \tau(\omega) \le t\} \in \mathcal{F}_t$
$[\tau_1 \le 1] = \{\omega \in \Omega | \tau_1(\omega) \le 1\} = \{\omega_1, \omega_2, \omega_3\} \in \mathcal{F}_1$
$[\tau_1 \le 2] = \{\omega \in \Omega | \tau_1(\omega) \le 2\} = \{\omega_1, \omega_2, \omega_3, \omega_4\} \in \mathcal{F}_2$
$[\tau_1 \le 3] = \{\omega \in \Omega | \tau_1(\omega) \le 3\} = \Omega \in \mathcal{F}_3$

$[\tau_2 \le 1] = \{\omega \in \Omega | \tau_2(\omega) \le 1\} = \{\omega_1\} \notin \mathcal{F}_1$

Solution

τ_1 is stopping time and τ_2 isn't