CH4. Multi-Variable Distribution

Conditional pdf

Given X, Y's joint pdf is $f_{XY}(xy)$

$$f(x|y) = \frac{f(x,y)}{f(y)} \tag{1}$$

$$f(y|x) = \frac{f(x,y)}{f(x)} \tag{2}$$

Use $f(y|x)$ as examples:

1. for x, we don't need to concern the function relationship between x and y
2. for y, we need to concern the function relationship between x and y

If the condition is a point:

$$P(X < a|Y = y) = \int_{-\infty}^{a} f(x|y)dx = \int_{-\infty}^{a} \frac{f(x,y)}{f(y)}dx \tag{3}$$

If the condition is a range:

$$P(X < a|Y < b) = \frac{P(X < a \cap Y < b)}{P(Y < b)} \tag{4}$$

Moments

Given X and Y are bivariate random variables, their joint pdf is $f(x, y)$, then the expected value of $g(x, y)$ is:

$$E[g(X,Y)] = \int_y \int_x g(x,y)f(x,y)dxdy = \int_x \int_y g(x,y)f(x,y)dydx \tag{5}$$

Covariance and correlation coefficient

$$\sigma_{XY} = Cov(X,Y) = E\{[X - E(X)][Y - E(Y)]\} = E(XY) - E(X)E(Y) \tag{6}$$

$$\rho_{XY} = Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} \tag{7}$$

Properties:

1. $E(aX + bY) = E(aX) + E(bY) = aE(x) + bE(y)$
2. $Cov(X, X) = Var(X)$
3. $Cov(X, c) = 0$
4. $Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$
5. $Cov(aX + b, cY + d) = acCov(X, Y)$
6. $Corr(aX + b, cY + d) = \frac{ac}{|ac|}Corr(X, Y) = sign(ac)Corr(X, Y)$

Given $X_1, X_2, \cdots, X_n$, then:

$$E(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} E(X_i) \tag{8}$$

$$Var(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} Var(X_i) + \sum_{i=1}^{n} \sum_{j \neq i} Cov(X_i, X_j) \tag{9}$$

$$E(\sum_{i=1}^{n} a_i X_i) = \sum_{i=1}^{n} a_i E(X_i) \tag{10}$$

$$Var(\sum_{i=1}^{n} a_i X_i) = \sum_{i=1}^{n} a_i^2 Var(X_i) + \sum_{i=1}^{n} \sum_{j \neq i} a_i a_j Cov(X_i, X_j) \tag{11}$$