20-R-WE-DK-23

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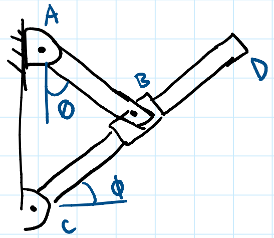
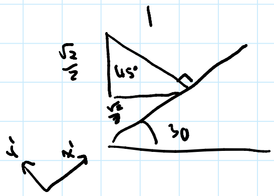
20-R-WE-DK-23 Intermediate Kinetic Energy

Inspiration: None

If collar B moves along the bar CD towards D at a constant rate of ***1 m/s***, find the kinetic energy of the entire mechanical system. Assume the collar has negligible mass and each bar can be treated as a slender rod. The links have lengths ***l_AB = 1 m, l_CD = 3 m***, and the angles are given as *theta = 45 degrees* and ***phi = 30 degrees*** at this instant. Each link has a mass of ***1 kg***.



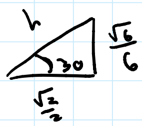
$\cos 45 = \frac{x}{1}$ $x = \frac{\sqrt{2}}{2}$ $\cos 30 = \frac{\frac{\sqrt{2}}{2}}{h}$ $h = \frac{\sqrt{6}}{3}$



$V_{B/C}(\text{rel.}) = 1\,\text{m/s}\ \hat{i}' = \cos 30\ \hat{i} + \sin 30\ \hat{j}$

$\vec{V}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A} = \omega_{AB}\hat{k} \times \left(\frac{\sqrt{2}}{2}\hat{i} - \frac{\sqrt{2}}{2}\hat{j}\right)$
$= \frac{\sqrt{2}}{2}\omega_{AB}\ \hat{j} + \frac{\sqrt{2}}{2}\omega_{AB}\ \hat{i}$

$\vec{V}_B = \vec{V}_C + \vec{\Omega}_{CD} \times \vec{r}_{B/C} + \vec{V}_{B/C}(\text{rel.})$
$= 0 + \Omega_{CD}\hat{k} \times \left(\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{6}}{6}\hat{j}\right) + \cos 30\ \hat{i} + \sin 30\ \hat{j}$
$= \frac{\sqrt{2}}{2}\Omega_{CD}\ \hat{j} - \frac{\sqrt{6}}{6}\Omega_{CD}\ \hat{i} + \cos 30\ \hat{i} + \sin 30\ \hat{j}$



Equate $\vec{V}_B$:
$\hat{i}:\ \frac{\sqrt{2}}{2}\omega_{AB} = -\frac{\sqrt{6}}{6}\Omega_{CD} + \cos 30\ \hat{i}$
$\hat{j}:\ \frac{\sqrt{2}}{2}\omega_{AB} = \frac{\sqrt{2}}{2}\Omega_{CD} + \sin 30$

$-\frac{\sqrt{6}}{6}\Omega_{CD} + \cos 30 = \frac{\sqrt{2}}{2}\Omega_{CD} + \sin 30$
$\Omega_{CD} = 0.328169399$
$\omega_{AB} = 1.03527618$

$T_{tot} = T_{AB} + T_{CD}$

$T_{AB} = \frac{1}{2} I_A \omega_{AB}^2 = \frac{1}{2}\left(\frac{1}{3} m_{AB} l_{AB}^2\right)\omega_{AB}^2$
$= \frac{1}{2}\left(\frac{1}{3}(1)(1)\right)(1.03527618)^2$
$= 0.178632795$

$T_{CD} = \frac{1}{2} I_C \Omega_{CD}^2 = \frac{1}{2}\left(\frac{1}{3}(1)(3^2)\right)(0.328169399)^2$
$= 0.161542731$

$\boxed{T_{tot} = 0.340175526}$