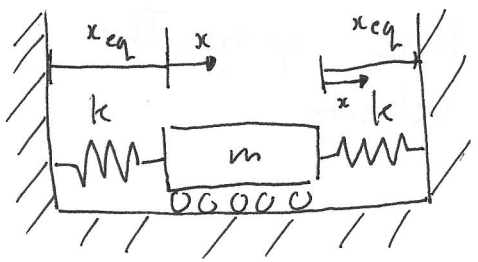
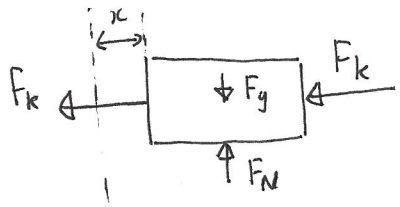
CH 22.1 - 1 Beginner / Free Undamped Vibrations



A solid rectangle, of mass 10kg, is attached to walls on it's right & left by springs. The springs have a spring constant of 50 N/m. Given an initial displacement of $x_0 =$ 0.15m and an initial velocity of 1 m/s, find the equation of motion, maximum amplitude, and natural frequency.

Solution:

FBD



ΣF_x : $-F_k = ma_x = m\ddot{x}$

$F_k = kx$

$\therefore -2kx = m\ddot{x}$

$\therefore \underline{m\ddot{x} + 2kx = 0}$ equation of motion

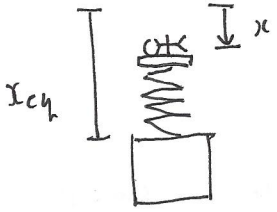
$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2k}{m}} = \underline{\sqrt{10}}$ rad/s

in this case 2k because there are 2 springs

maximum amplitude $= C = \sqrt{\left(\frac{V_0}{\omega_n}\right)^2 + x_0^2} = \sqrt{\left(\frac{1}{\sqrt{10}}\right)^2 + 0.15^2} = \underline{0.3\sqrt{}\,m}$

CH 22-2 Beginner / Free Undampened Vibration

A person tries to launch themselves into space using a large device which is comprised from a large platform and a spring. However, due to an error during the launch sequence, the person is stuck to the platform - oscillating with the spring. Given that the ~~person is 100 kg, the platform is 200 kg,~~ initial velocity is 5 m/s and the initial displacement is 10m, what is the phase angle of the spring? The man and ~~platform~~ platform weigh 300 kg together and the spring constant is 1000 N/m



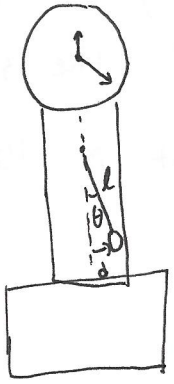
Solution: $\omega_n = \sqrt{\frac{k}{m}} = 1.8257 \text{ rad/s}$ or $\sqrt{\frac{10}{3}}$

$\tan \phi = \frac{\omega_n x_0}{V_0}$ $\quad x_0 = 10m$ $\quad V_0 = 5 m/s$

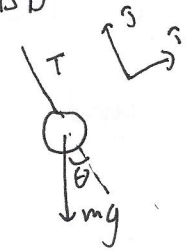
$\phi = 1.303$

CH 22-3 Intermediate / Free Undamped Vibrations

A grandfather clock contains a pendulum that swings from side to side with small angles changes. The length of the pendulum is 1.5m. What is the natural frequency of the pendulum?



Solution: FBD



ΣF_i: $-mg\sin\theta = ma$ $\quad ma + mg\sin\theta = 0$

$a + g\sin\theta = 0$

$a = \frac{d^2s}{d^2t} = \ddot{s} \rightarrow s = l\theta$

$\dot{s} = l\dot{\theta}$

$\ddot{s} = l\ddot{\theta}$

$l\ddot{\theta} + g\sin\theta = 0$ since angle is small, $\sin\theta \approx \theta$

$\ddot{\theta} + \frac{g}{l}\theta = 0$

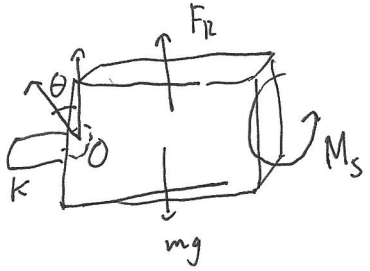
$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.81}{1.5}}$

CH22-4 Intermediate / Free Undamped Vibrations

A tall man hits his head on a store sign that is mounted horizontally from a wall. The sign is a rectangle with a heigh of 0.5m and thickness of 0.1m. The pole the sign is mounted to has a spring constant of 10N/m. Given that the man was walking at 2 m/s, What is the maximum amplitude of the sign as a result of the impact. The mass of the sign is 0.5kg. *

Solution:

FBD



$M_s = -k\theta$ $\quad \Sigma M_G = I_G \alpha$

$-k\theta = I_0 \alpha = I_0 \ddot{\theta}$

$I_0 \ddot{\theta} + k\theta(t) = 0$

$\ddot{\theta} + \frac{k}{I_0}\theta(t) = 0$ $\quad \omega_n = \sqrt{\frac{k}{I_0}}$

$= \sqrt{960}$

* Thin sign

Treat the sign as a rod

$I_0 = \frac{1}{12} M L^2$

$$C = \sqrt{\left(\frac{v_0}{\omega_n}\right)^2 + x_0^2} = \sqrt{\left(\frac{2\ m/s}{\sqrt{960}}\right)^2} = 0.0645\ m$$

($x_0 \downarrow 0$)