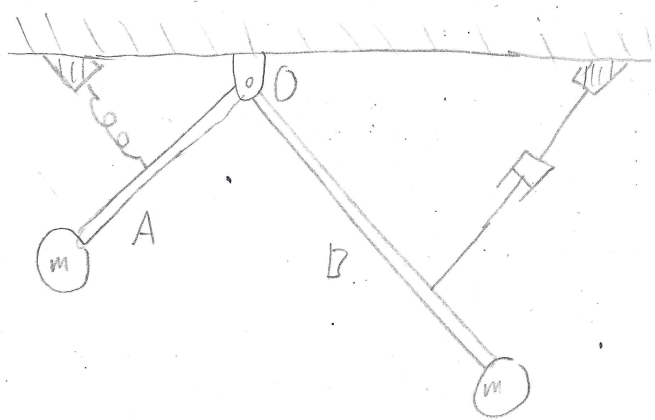
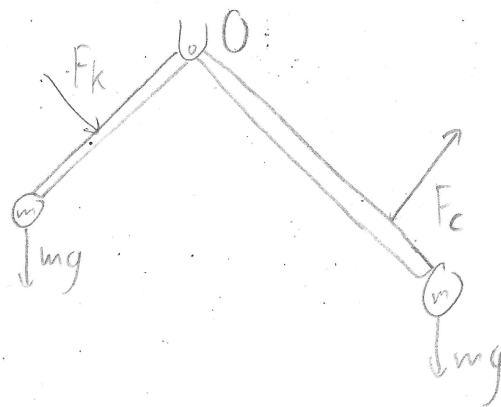


20-R-VIB-DY-45 Advanced

A L-shaped bar of negligible mass is pinned to the ceiling by the corner at point O. Arm A, length $l_A = 2\text{ m}$, has a spring $k_A = 10\text{ N/m}$ attached halfway and arm B, length $l_B = 3\text{ m}$, has a damper $c_B = 20\text{ Ns/m}$ attached $2/3^{\text{rd}}$ of the length down the bar. Determine the mass of the balls at the end if the system is critically damped.



Solution: FBD



$$\Sigma M_O = I \alpha = 0 = \left(\frac{l_A}{2}\right)^2 k \theta + \left(\frac{2l_B}{3}\right)^2 c \dot{\theta} + l_A m g \theta + l_B m g \theta + m l_A^2 \ddot{\theta} + m l_B^2 \ddot{\theta}$$

$$\ddot{\theta} \underbrace{m(l_A^2 + l_B^2)}_{m'} + \underbrace{\left(\frac{2l_B}{3}\right)^2 c}_{c'} \dot{\theta} + \underbrace{\theta(l_A m g + l_B m g + \frac{l_A^2}{4} k)}_{k'} = 0$$

$$c_c = \sqrt{4m'k'} = c' = 80$$

$$1600 = m(13) + m(5g) + 10$$

$$1600 = 965m + 130m \quad m = \frac{-2600 \pm \sqrt{1639144000}}{25506}$$

$$m = 1.485 \text{ kg}$$