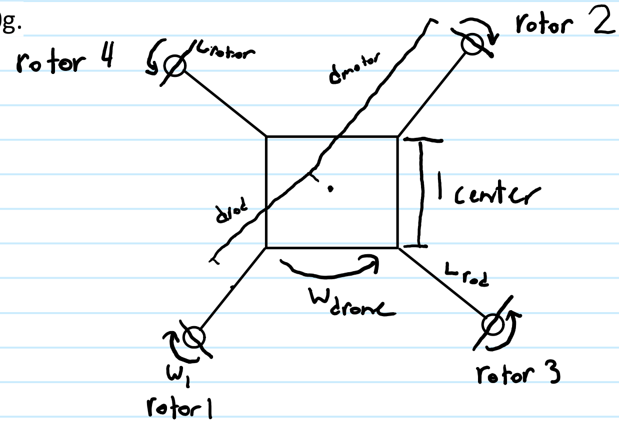
20-R-IM-PT-9

July 22, 2020 11:34 PM

A 4 rotor drone is hovering above the ground with no change in position. Drone rotors 1 and 2 rotate with an angular velocity of 785rad/s in the clockwise direction, and rotors 3 and 4 rotate with the same angular speed in the opposite direction. What is the new
angular velocity of the drone if rotors 3 and 4 stop working?

Assume that the drone consists of a square shaped center, with a mass of 500g and a
side length of 15cm. The center is connected to each motor and propeller through rods with a length of 25cm
and a mass of 30g. The motor can be treated as a point mass with a mass of 70g, and the propellers have a length of 15 cm and a mass of 20g.



Solution

$$(H_{sys})_1 = (H_{sys})_2$$

$$\cancel{L_{Drone}}_0 + L_{r_1} + L_{r_2} + L_{r_3} + L_{r_4} = L'_{Drone} + L'_{r_1} + L'_{r_2} + \cancel{L'_{r_3}}_0 + \cancel{L'_{r_4}}_0$$

$$L_{r_1} = L_{r_2} = L'_{r_1} = L'_{r_2}$$
$$L_{r_3} = L_{r_4}$$
$$L_{Drone} = 0 = L'_{r_3} = L'_{r_4}$$

$$2L_{r_1} + 2L_{r_3} = L'_{Drone} + 2L_{r_1}$$

$$2L_{r_3} = L'_{Drone}$$

$$2(I_{r_3} w_{r_3}) = I_{Drone} w'^{2}_{Drone}$$

$$I_3 = \frac{1}{12} m_r L_r^2$$
$$= \frac{1}{12}(0.07)(0.15)^2$$
$$= 0.00013125$$

$m_r = 70\,g = 0.07\,Kg$
$L_r = 15\,cm = 0.15\,m$

$$I_{Drone} = I_{center} + I_{rods} + I_{motors}$$

$$I_{center} = \frac{0.15^2}{6} \cdot 0.5 = 0.001875$$

$$d_{rod} = \frac{0.25}{2} + \frac{\sqrt{(0.15)^2 + (0.15)^2}}{2} = 0.231\,m$$

$$I_{rods} = 4 \cdot \left(\frac{1}{12} m L^2 + m d_{rod}^2\right)$$
$$I_{rods} = 4 \cdot \left(\frac{1}{12} \cdot (0.030) \cdot (0.25)^2 + 0.030 \cdot 0.231^2\right) = 0.001757$$

$$I_{motors/rotors} = (m_{motors} + m_{rotors}) \cdot (d^2_{motors})$$
$$= (0.1 + 0.07)(0.356^2) = 0.02155$$

$$d_{motors} = 0.25 + \frac{\sqrt{(0.15)^2 + (0.15)^2}}{2} = 0.356$$

$$I_{Drone} = 0.001875 + 0.001752 + 0.002155 = 0.0252$$

$$w'_{Drone} = \frac{I_{r_3} \cdot w_{r_3} \cdot 2}{I_{Drone}} \qquad w_{r_3} = 785 \text{ rad/s}$$

$$w'_{Drone} = \frac{(0.00013125) \cdot (785 \text{ rad/s}) \cdot 2}{(0.0252)}$$

$$w'_{Drone} = 8.18 \text{ rad/s}$$

$$\boxed{\text{The angular speed of the drone is } 8.18 \text{ rad/s}}$$