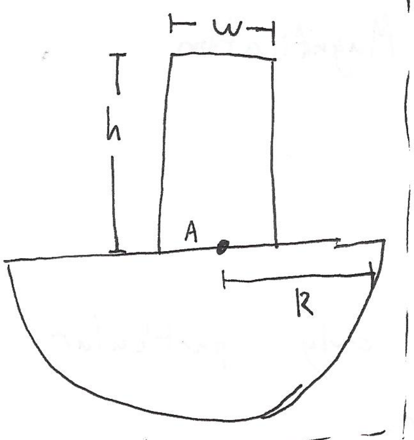
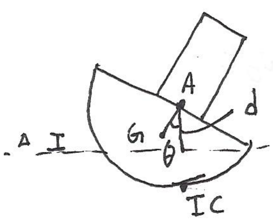
20-R-VIB-12 Intermediate

A symmetrical buoy is moved onto land for maintenance. T~~he mass of the buoy is 150 kg and the centre of mass is 0.5m off the ground~~. The tower on top of the buoy can be thought of as a rectangle with a height 0.4 and width 0.2. Given that the raidius of the bottom is 1m, find the natural period of the system. The buoy has a uniform ~~system~~ desity of 1000 kg/m².



Solution:



datum C.O.G

$\Delta = d(1-\cos\theta)$

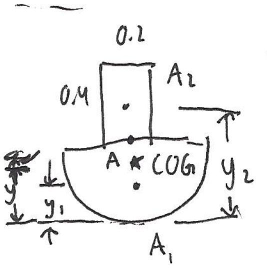
$E = T + V = \frac{1}{2} I_{IC}\dot{\theta}^2 + mgd(1-\cos\theta)$

$\dot{E} = I_{IC}\dot{\theta}\ddot{\theta} + mgd\cos\theta\,\dot{\theta} = \dot{\theta}(I_{IC}\ddot{\theta} + mgd\cos\theta)$

small θ assumption

$0 = I_{IC}\ddot{\theta} + mgd\theta$

$\omega_n = \sqrt{\frac{mgd}{I_{IC}}}$



$A_1 = \frac{\pi}{2}\ m^2$ $\quad A_2 = 0.08\ m^2$

$y_1 = \frac{4R}{3\pi} = \frac{4}{3\pi}\ m$ $\quad y_2 = 1.2\ m$

$m_1 = 200\pi\ kg$ $\quad m_2 = 80\ kg$

$Ay = A_1 y_1 + A_2 y_2 \quad y = 0.462\ m$

$\tau = 2\pi\sqrt{\frac{I_{IC}}{mgd}} \quad d = R - y$

$I_A = I_{semi} + I_{rect} = \frac{1}{2} m_1 R^2 + m_2\left(\frac{1}{12}(h^2+w^2) + 0.2^2\right)$

$= 318.69$

$\tau = 1.672\,s$

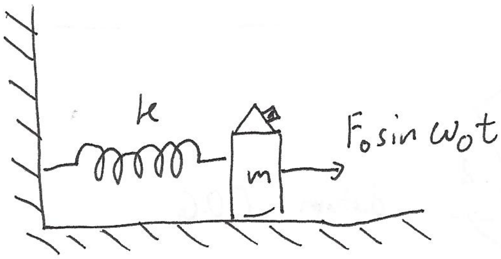
$I_A = I_{COG} + m(\underset{R-y}{y})^2 \quad I_{COG} = 113.67$

$I_{IC} = I_{COG} + m(y)^2 = 264.856$

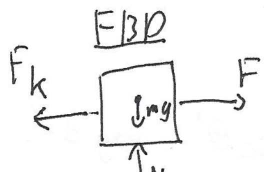
20-R-VIB-DY-13

A 2kg box of orange juice contains the instructions "Shake well". An engineering student decides to get smart and connect the juice box with a spring, which has a spring constant $k = 25\ N/m$, and applies a periodic force. The periodic force is described as $5 \sin 5t$. What is the maximum amplitude of the periodic motion at steady state?

↳ & ~~multiplication~~ factor
Magnification



Solution:



@ Steady state only particular solution matter.

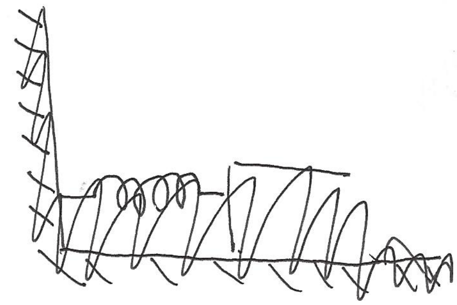
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{2}}$$

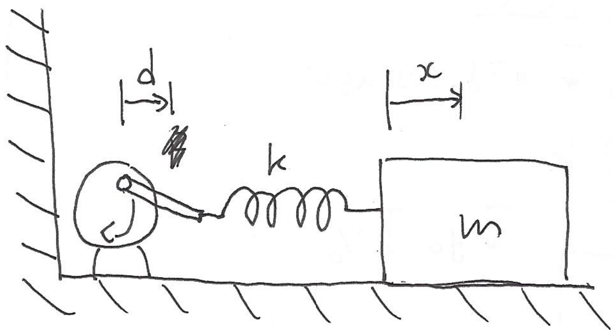
$$x_p = \underbrace{\frac{F_0/k}{1-(\omega_0/\omega_n)^2}}_{= -0.2\ m} \sin \omega_0 t$$

$$MF = \frac{1}{1-(\omega_0/\omega_n)^2} = -1$$

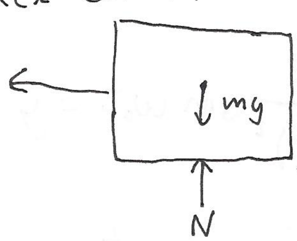
20-R-VIB-DY-14 Beginner

A~~nother~~ Shaking machine consists of a motor and a spring. The motor consists of a rotary engine which can be described by the equation $d = \cos 2t$. Given that the spring constant is 25 N/m and a load of 2kg, what is the ~~natural frequency~~ ~~resonance~~ maximum velocity and amplitude of the steady state vibration?





Solution:



$$\Sigma F_x = m a_x \qquad w_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{2}}$$

$$-k(x-\cos 2t) = m a_x$$

$$m\ddot{x} + kx = k\cos 2t$$

$$\ddot{x} + \frac{k}{m} = \frac{k}{m}\cos 2t \qquad \frac{k}{m} = \frac{F_0}{m}$$

$$k = F_0$$

~~resonance frequency = w_f~~

$$= \sqrt{\frac{25}{2}}$$

$$Amp = \frac{k/k}{1-(w_0/w_n)^2} = 1.47\text{ m}$$

$x(t) = \cos 2t$

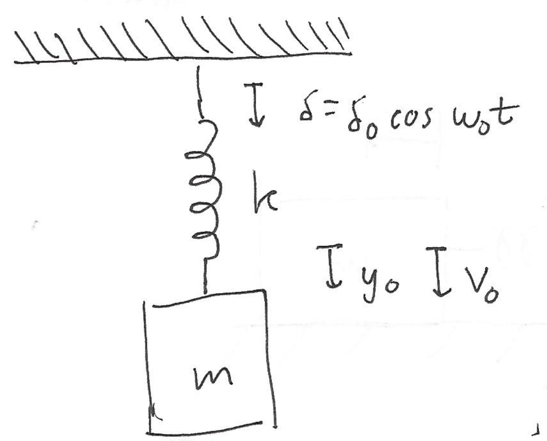
$\dot{x}(t) = 2\cos 2t$

$V_{max} = 2\text{ m/s}$

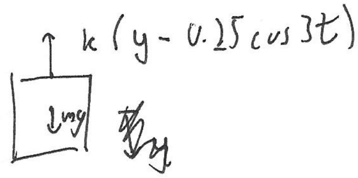
cos @ max = 1

20-R-VIB-DY-15 Beginner

A 5 kg load is hanging from the ceiling via a spring with a k value of 50 N/m. Given an initial velocity of 2 m/s and initial displacement of 0.1 m, determine the position equation of the load, if the support moves $\delta = 0.25 \cos 3t$.



FBD



Solution: $A \sin w_n t + B \cos w_n t + \dfrac{\delta_0}{1-\left(\frac{w_0}{w_n}\right)^2} \sin w_0 t = y$

$\dot{y} = v = A w_n \cos w_n t - B w_n \sin w_n t + \dfrac{\delta_0 w_0}{1-\left(\frac{w_0}{w_n}\right)^2} \cos w_0 t$

when $t=0$ $y_0 = 0.1$ $v_0 = 2\,\text{m/s}$ $w_n = \sqrt{\frac{k}{m}} = \sqrt{10}$

$y_0 = 0 + B + 0 = B = 0.1$

$v_0 = A w_n - 0 + \dfrac{\delta w_0}{1-\left(\frac{w_0}{w_n}\right)^2}$ $A = \dfrac{v_0}{w_n} - \dfrac{\delta w_0}{w_n - \frac{w_0^2}{w_n}} = -1.739$

$y = -1.739 \sin \sqrt{10}\, t + 0.1 \cos \sqrt{10}\, t + 2.5 \sin 3t$