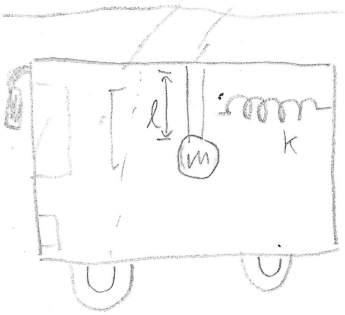
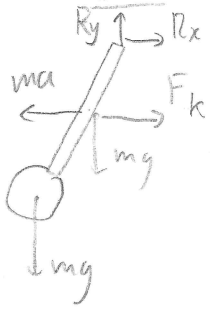
20-R-VIB-DY-34 Advanced

A bus has handles for passengers to use while standing up. The handle consists of a bar, sphere at the end, and a spring ($k = 10$ N/m) placed at the midpoint of the bar. The bar is 2 m long and 5 kg while the sphere has a mass $m = 5$ kg. While driving, the transmission breaks down causing a jerking motion and acceleration $a = \sin 2t$. Assume small angle.

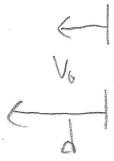


Solution: FBD — find equation of motion.



$a = \sin 2t$ $\omega = 2$

$$\ell_{COG} = \frac{m_{bar}\frac{1}{2}\ell + \ell m_{sphere}}{m_{tot}} = 1.5\text{ m}$$



$$k\left(\frac{\ell}{2}\right)^2 \sin\theta - \ell_{COG} m_{tot} a_{sys} + \sin\theta\left(m_{bar}g\frac{\ell}{2} + m_{sphere}g\ell\right) = I_o\ddot{\theta}$$

small angle

$$I_o = \frac{1}{3}m_{bar}\ell^2 + m_{sphere}\ell^2$$

$$= \ell^2\left(\frac{1}{3}m_{bar} + m_{sphere}\right)$$

$$\ddot{\theta} + \theta\left(\frac{m_{bar}g\frac{\ell}{2} + m_{sphere}g\ell + k\left(\frac{\ell}{2}\right)^2}{I_o}\right) = \frac{\ell_{COG} m_{tot} a_{sys}}{I_o}$$

$\theta_p = D\sin\omega_o t$

$\ddot{\theta}_p = -D\omega^2\sin\omega_o t$

$$\theta(t) = \frac{F_o/k}{1-\left(\frac{\omega_o}{\omega_n}\right)^2}\sin\omega t$$

$\omega_n = \sqrt{\frac{157.15}{26.67}} = 2.427$ rad/s

$\theta(t) = 0.397\sin 2t$

$F_o = \ell_{COG} m_{tot} a_{sys} = 20.$

$k = m_{bar}g\frac{\ell}{2} + m_{sphere}g\ell + k\left(\frac{\ell}{2}\right)^2 = 157.15$