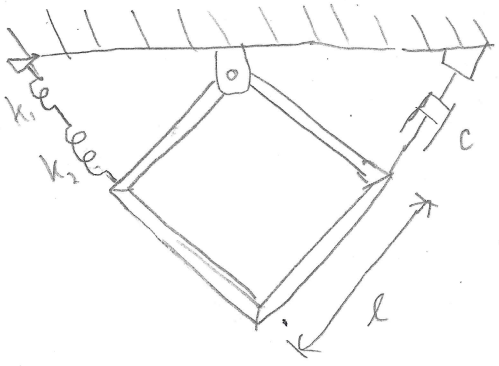
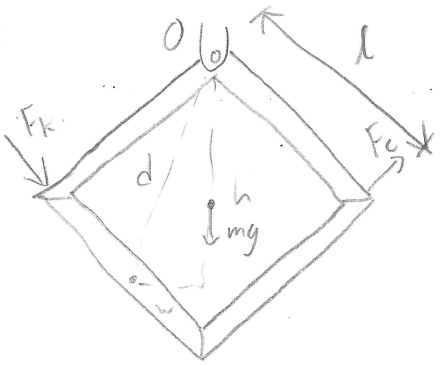
20-R-VIB-DY-43 Advanced

A square frame is made up of 4 individual bars with length $\ell = 2m$ and mass $m = 2kg$. The frame is pinned to the ceiling at a corner. A pair of springs ($k = 25$ N/m each) in series is attached to the left corner and a damper is installed on the right corner. Determine the damping constant that will make the system critically damped.



Solution:



$\Sigma M_o = -I_o \alpha \qquad I_o = 2\left(\frac{1}{3}m\ell^2\right) + 2\left(\frac{1}{12}m\ell^2 + md^2\right)$

$k = \frac{k_1 k_2}{k_1 + k_2} = 12.5\ N/m \qquad h = \frac{\ell}{2}\sin 45° \qquad \omega = \frac{\ell}{2}\sin 45° \qquad d = \sqrt{h^2 + \omega^2} = \sqrt{5}$

$ks + c\dot{s} + mg\sin\theta = -I_o\ddot{\theta}$ small angle assumption

$\underbrace{(k\ell + mg)}_{k'}\theta + \underbrace{c\ell}_{c'}\dot{\theta} + \underbrace{I_o}_{m'}\ddot{\theta} = 0 \qquad I_o = 26.67 = m' \qquad k' = 103.48$

$\omega_n = \sqrt{\frac{k'}{m'}} = 1.97$ rad/s $\qquad c_c = 2m'\omega_n = 105.07 = c\ell$

$c = 52.53\ Ns/m$