



## 20-R-KM-DK-11 Intermediate Acceleration (Relative Motion)

## Inspiration: Example 16.7 Hibbeler

Students are attempting to create a lift to raise their model car. The lift is assembled with two linkages, link AB and link BC, as seen in the picture shown. If the links have length  $L_{AB} = 0.2 \text{ m}$  and  $L_{BC} = 0.4 \text{ m}$ , determine the velocity and acceleration of the lift at the instant where the angular velocity of AB is  $\omega_{AB} = -5 \text{ rad/s}$  and the angular acceleration of AB is  $\alpha_{AB} = -7 \text{ rad/s}^2$ . Take the angles to be  $\theta_{AB} = 30 \text{ degrees}$  and  $\phi_{BC} = 20 \text{ degrees}$ .

$$\vec{r}_{B/A} = -0.2 \sin 30 \hat{i} + 0.2 \cos 30 \hat{j} \text{ m}$$

$$\begin{aligned} \vec{v}_B &= \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A} \\ &= 0 + (-5 \hat{k}) \times (-0.2 \sin 30 \hat{i} + 0.2 \cos 30 \hat{j}) \\ &= 0.5 \hat{j} + \frac{\sqrt{3}}{2} \hat{i} \end{aligned}$$

$$\begin{aligned} \vec{v}_C &= \vec{v}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B} \quad \vec{v}_C = v_C \hat{j} \\ &= 0.5 \hat{j} + \frac{\sqrt{3}}{2} \hat{i} + \omega_{BC} \hat{k} \times (0.4 \sin 20 \hat{i} + 0.4 \cos 20 \hat{j}) \\ \hat{i}: 0 &= \frac{\sqrt{3}}{2} - \omega_{BC} (0.4 \cos 20) \quad \omega_{BC} = 2.304 \\ \hat{j}: v_C &= 0.5 + 2.304 (0.4 \sin 20) \quad v_C = 0.915207 \hat{j} \text{ m/s} \end{aligned}$$

$$\begin{aligned} \vec{a}_B &= \vec{a}_A + \alpha_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A} \\ &= 0 + (-7 \hat{k}) \times (-0.2 \sin 30 \hat{i} + 0.2 \cos 30 \hat{j}) - 25(-0.2 \sin 30 \hat{i} + 0.2 \cos 30 \hat{j}) \\ &= 0.7 \hat{j} + \frac{7\sqrt{3}}{10} \hat{i} + 2.5 \hat{i} - \frac{5\sqrt{3}}{2} \hat{j} \end{aligned}$$

$$\begin{aligned} \vec{a}_C &= \vec{a}_B + \alpha_{BC} \times \vec{r}_{C/B} - \omega_{BC}^2 \vec{r}_{C/B} \quad \downarrow \text{Purposely chose wrong} \\ &= 0.7 \hat{j} + \frac{7\sqrt{3}}{10} \hat{i} + 2.5 \hat{i} - \frac{5\sqrt{3}}{2} \hat{j} + \alpha_{BC} \hat{k} \times (0.4 \sin 20 \hat{i} + 0.4 \cos 20 \hat{j}) - 2.304^2 (0.4 \sin 20 \hat{i} + 0.4 \cos 20 \hat{j}) \\ a_C \hat{j} &= 0.7 \hat{j} + \frac{7\sqrt{3}}{10} \hat{i} + 2.5 \hat{i} - \frac{5\sqrt{3}}{2} \hat{j} + \alpha_{BC} (0.4 \sin 20) \hat{j} - \alpha_{BC} (0.4 \cos 20) \hat{i} - (2.304^2)(0.4 \sin 20) \hat{i} - (2.304^2)(0.4 \cos 20) \hat{j} \end{aligned}$$

$$\hat{i}: 0 = \frac{7\sqrt{3}}{10} + 2.5 - \alpha_{BC} (0.4 \cos 20) - 2.304^2 (0.4 \sin 20)$$

$$\hat{j}: a_C = 0.7 - \frac{5\sqrt{3}}{2} + \alpha_{BC} (0.4 \sin 20) - 2.304^2 (0.4 \cos 20)$$

$$\alpha_{BC} = 7.94462311 \text{ rad/s}^2$$

$$a_C = -4.53655 \text{ m/s}^2$$