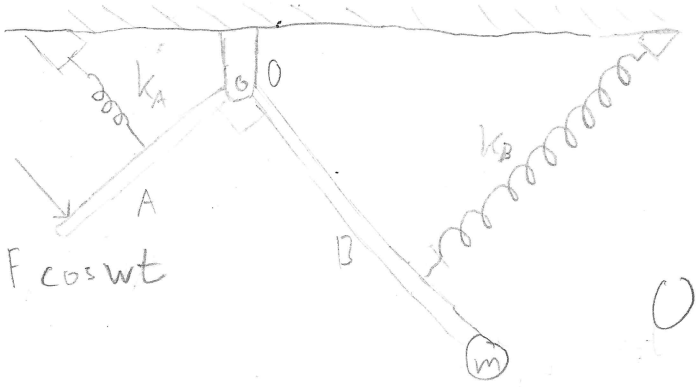
20-R-VIB-DY-32 Advanced

A L-shaped bar of negligible mass is pinned to the ceiling by its at point O. Arm A, length $\ell = 2m$, has a spring $k_A = 10$ N/m attached and a force $F = 2\cos 3t$ applied at the end of the bar. Arm B, length 3m, has a spring $k_B = 20$ N/m attached $\frac{2}{3}$ of the length down the bar and a 5kg mass at the end. Given that initially the bar is at rest, find the angle at $t = 10s$.



Solution: $F_k = ks \quad s = r\theta$

$\Sigma M_O = I_O \alpha \quad I_O = 0$

$$0 = (F\cos\omega t)\ell_A - k_A\left(\frac{\ell_A}{2}\right)^2\theta - k_B\left(\frac{2\ell_B}{3}\right)^2\theta - \ell_B^2 m\ddot{\theta}$$

$\theta_p = A\cos\omega t \quad \ddot{\theta}_p = -A\omega^2\cos\omega t$

$$F\cos\omega t = -\left[k_A\left(\frac{\ell_A}{2}\right)^2 + k_B\left(\frac{2\ell_B}{3}\right)^2\right]A\cos\omega t - (\ell_B^2 m)A\omega^2\cos\omega t$$

$$A = \frac{F}{\left[k_A\left(\frac{\ell_A}{2}\right)^2 + k_B\left(\frac{2\ell_B}{3}\right)^2 - \ell_B^2 m\omega^2\right]} = -0.00635$$

$\theta(t) = A\sin\omega t = -0.00635\sin 3t \quad @\ t = 10 \quad \underline{\theta = 0.00627}$