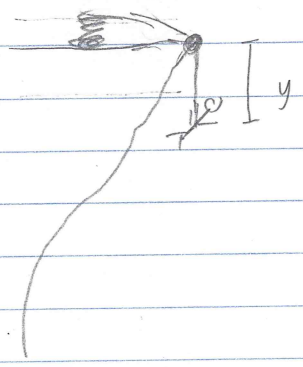
20-MOM-DY-40

A $m_c = 70kg$ rock climber descends from the top of a cliff after a hard day of climbing. The rope has a mass per length $\rho = 1 kg/m$. Determine the velocity of the climber as a function of y.



Solution: $\sum F = m\frac{dv}{dt} + v\frac{dm}{dt}$

@ time t $m = m_c + \rho y$ $\quad \frac{dm}{dt} = \rho\frac{dy}{dt} = \rho v$

$$(m_c + \rho y)g = (m_c + \rho y)\frac{dv}{dt} + v^2\rho \qquad v = \frac{dy}{dt} \qquad dt = \frac{dy}{v}$$

$$(m_c + \rho y)g = v(m_c + \rho y)\frac{dv}{dy} + v^2\rho$$

$$(m_c + \rho y)g\,dy = v(m_c + \rho y)dv + v^2\rho dy$$

$$\int[(m_c + \rho y)g - v^2\rho]dy = \int v(m_c + \rho y)dv$$

$$m_c y + \frac{\rho y^2 g}{2} - v^2\rho y = \frac{v^2(m_c + \rho y)}{2} + C$$

$v = 0$ @ $y = 0$ $\qquad C = 0$

$$v = \sqrt{\frac{m_c y + \frac{\rho y^2 g}{2}}{\rho y + \frac{(m_c + \rho y)}{2}}}$$