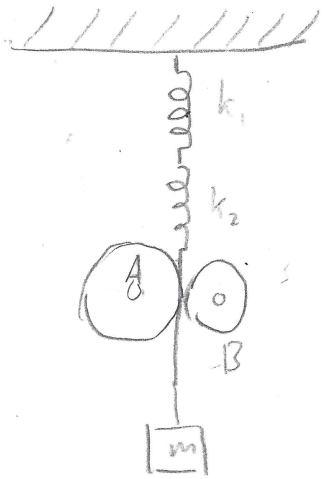
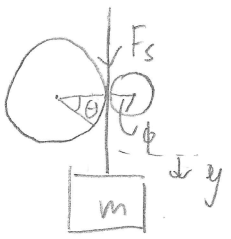
20-R-VIB-DY-24

There is a 5kg mass suspended vertically from the ceiling via rope and two springs in series with spring constants 10 N/m & 25 N/m. To ensure straightness, the rope is held between two rollers of radius r = 0.5 & r = 0.25 m and mass m = 5 kg & m = 2 kg respectively. What is the natural frequency of the mass?

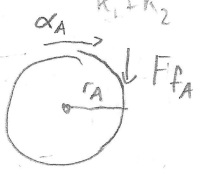


Solution:



$F_s = k_{eq} y$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$



$$\Sigma M_A: -F_{f_A} r_A = -I_A \alpha_A$$

$$F_{f_A} = \frac{I_A \alpha_A}{r_A} = \frac{I_A}{r_A^2} \ddot{y}$$

$$\Sigma M_B: F_{f_B} r_B = I_B \alpha_B$$

$$F_{f_B} = \frac{I_B \alpha_B}{r_B} = \frac{I_B}{r_B^2} \ddot{y}$$

$$\Sigma F_y: -ky - F_{f_B} - F_{f_A} = m\ddot{y}$$

$$ky + \left(m + \frac{I_B}{r_B^2} + \frac{I_A}{r_A^2}\right)\ddot{y} = 0$$

$$\omega_n = \sqrt{\frac{\frac{k_1 k_2}{k_1 + k_2}}{\left(m + \frac{I_B}{r_B^2} + \frac{I_A}{r_A^2}\right)}}$$