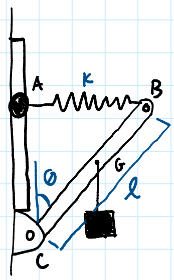
20-R-WE-DK-16

July 30, 2020 10:54 AM

20-R-WE-DK-16 Intermediate Principle of Work and Energy

Inspiration: 18-28



A hardworking engineering student is designing a lever system that will slowly lower the lever and its load. The ***10 kg*** slender rod BC has a mass ***m = 5 kg*** attached at the rod's center of gravity G, and a length ***l = 0.6 m***. If the rod is released from rest when the spring is unstretched at ***theta = 30 degrees***, determine the spring constant k needed to obtain an angular velocity of ***omega = 0.5 rad/s*** at the instant ***theta = 60 degrees***. As the rod rotates, the spring always remains horizontal because of the roller support at A.

Variable ranges: 5–20 (10 kg) m: 1–10 (5 kg) ℓ: 0.3–0.9 (0.6)

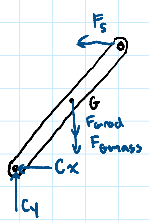
$\theta_1 = 15-45$ (30) $\theta_2 = \theta_1 + 30°$ ω: 0.2–0.9 rad/s (0.5)

$I_C = \frac{1}{12} m\ell^2 + md^2 = \frac{1}{3} m\ell^2 = \frac{1}{3}(10)(0.6)^2 = 1.2 \text{ kg m}^2$

$T_1 = 0$ $V_1 = m_{rod}gh + m_{mass}gh + \frac{1}{2}ks^2 = 0$ Take datum to be the respective original positions

$T_2 = \frac{1}{2} I_C \omega^2 + \frac{1}{2} m v^2$ $V_2 = m_{rod}gh + m_{mass}gh + \frac{1}{2}ks^2$ difference in height is the same

$= (10)(9.81)(0.6\cos 60 - 0.6\cos 30) + (5)(9.81)(0.6\cos 60 - 0.6\cos 30) + \frac{1}{2}k(0.6\sin 60 - 0.6\sin 30)^2$



$T_1 + V_1 + \sum_{\text{non cons}} U_{1\to 2} = T_2 + V_2 \Rightarrow 0 = \frac{1}{2} I_C \omega^2 + \frac{1}{2} m v^2 + m_{rod}gh + m_{mass}gh + \frac{1}{2}ks^2$

$-(10)(9.81)(0.6\cos 60 - 0.6\cos 30) - (5)(9.81)(0.6\cos 60 - 0.6\cos 30) - \frac{1}{2}k(0.6\sin 60 - 0.6\sin 30)^2 = \frac{1}{2}(1.2)(0.5)^2 + \frac{1}{2}(5)v^2$

$\vec{V}_G = \vec{V}_C + \vec{\omega} \times \vec{r}_{G/C} = 0 + (-0.5\hat{k}) \times (0.6\sin 60\,\hat{i} + 0.6\cos 60\,\hat{j})$
$= -0.3\sin 60\,\hat{j} + 0.3\cos 60\,\hat{i}$

$V_G = \sqrt{(0.3\sin 60)^2 + (0.3\cos 60)^2} = 0.3$

$32.3163829 - \frac{1}{2}(0.6\sin 60 - 0.6\sin 30)^2 k = 0.15 + 0.225$

$\boxed{k = 1324.520709 \text{ N/m}}$