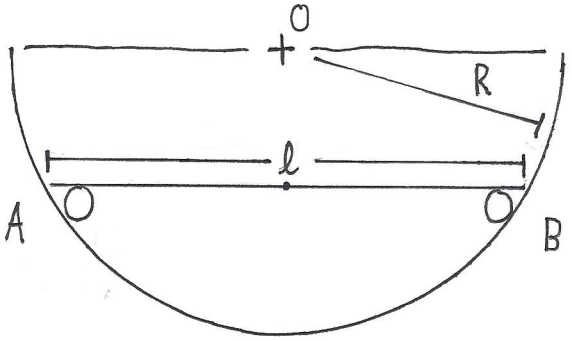
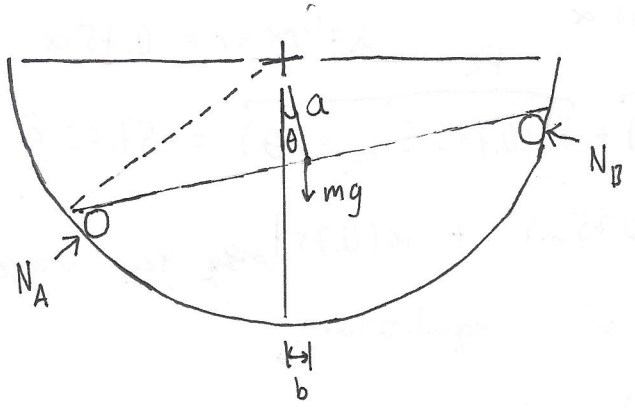
20-R-VIB 8 Intermediate

A 1.5m skateboard is balanced in the middle of a half pipe as shown in the figure. A gust of wind causes a small displacement. Given that the half pipe's radius is 1 m, what is the natural frequency of the vibration?



Solution: FBD



$$a^2 + \left(\frac{\ell}{2}\right)^2 = R^2$$

$$a = \sqrt{R^2 - \frac{\ell^2}{4}}$$

small θ so $\quad b = a\theta = \theta\sqrt{R^2 - \frac{\ell^2}{4}}$

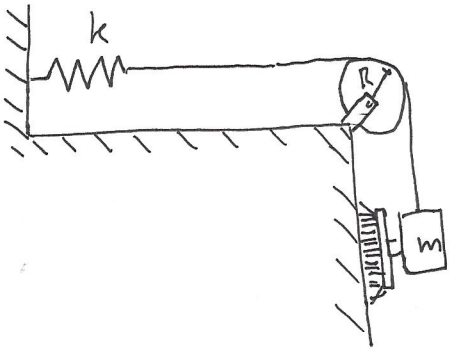
$$I_O = \frac{1}{12}m\ell^2 + m(a)^2 = m\left(R^2 - \frac{\ell^2}{4} + \frac{1}{12}\ell^2\right) = m\left(R^2 - \frac{1}{6}\ell^2\right)$$

$$\circlearrowleft \Sigma M_O = I_O\alpha \quad -mg\left(\theta\sqrt{R^2 - \frac{\ell^2}{4}}\right) = m\left(R^2 - \frac{1}{6}\ell^2\right)\ddot{\theta} \quad \ddot{\theta} + \frac{3g\sqrt{4R^2 - \ell^2}}{6R^2 - \ell^2}\theta = 0$$

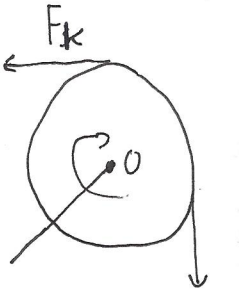
$$\omega_n = \sqrt{\frac{3g\sqrt{4R^2 - \ell^2}}{6R^2 - \ell^2}} = \sqrt{\frac{3g\sqrt{8}}{12.5}} = 2.58 \text{ Hz}$$

20-R-VIB-9 Intermediate

A window cleaner is hanging over the side of a building with a pulley system and a spring. Given that the radius of the pulley is 0.75m and the weight of the cleaner is 75 kg, what is the spring constant if the natural vibration of the system is 1.66 Hz? The pulley's weight is 5kg.



Solution:



$\circlearrowleft \sum M_0 = -ma \cdot r - I_0 \alpha$ $\qquad I_0 = \frac{1}{2} m_0 r^2 = 1.4$

$= -57.65\alpha$ $\qquad a = \alpha \times r = 0.75\alpha$

$$\overbrace{-75(9.81)(0.75m)}^{mg} + \overbrace{k(0.75)(y_{eq} + r\theta)}^{Fk} = -57.65\ddot{\theta}$$

$-75(9.81)(0.75m) + k(0.75)y_{eq}$ y_{eq} cancel

because of equilibrium

$$k(0.75)(r\theta) = -57.65\ddot{\theta}$$

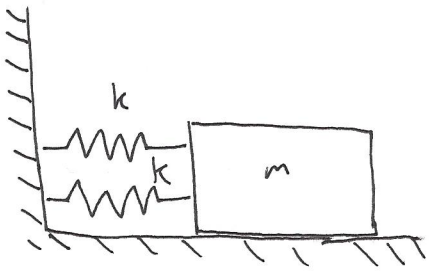
$$\ddot{\theta} + \frac{k0.5625}{57.65}\theta = 0$$

$$\omega_n = \sqrt{\frac{k\ 0.5625}{57.65}} = 1.66\ Hz$$

$$k = 282.4\ N/m$$

20-R-VIB-10 Beginner

Two springs, with spring constants of 50 N/m each, are used to connect a 10kg cargo load to a wall. Assuming the floor is frictionless, determine the ~~potential energy~~ energy equation (?)



Solution: $T + V = \text{const}$

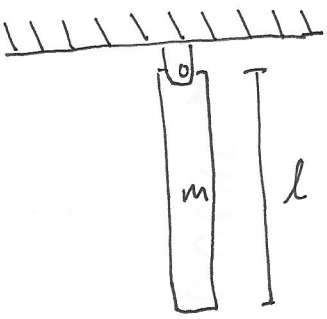
$$T = \frac{1}{2}(10)\dot{x}^2$$

$$V = \frac{1}{2}(50)x^2 + \frac{1}{2}(50)x^2$$

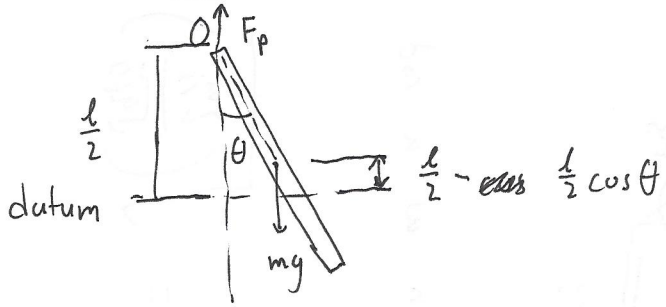
$$T + V = 5\dot{x}^2 + 50x^2 = 0$$

20-R-VIB Beginner 11

A 1.5m bar is pinned by one end to the ceiling. Given that the mass of the bar is 2kg, what is the natural frequency of the system?



Solution: FBD



$I_o = \frac{1}{3}ml^2 = 1.5 \ kg \cdot m^2$

$T = \frac{1}{2}(I_o)(\dot{\theta})^2 = 0.75\dot{\theta}^2$

$V = mg\left(\frac{l}{2}\right)(1-\cos\theta)$

$= 14.715 - 14.715\cos\theta$

$T+V = 0.75\dot{\theta}^2 + 14.715 - 14.715\cos\theta$

take derivative

$0 = 1.5\ddot{\theta}\dot{\theta} + 14.715(\sin\theta)\dot{\theta}$

for small θ, $\sin\theta \approx \theta$

$0 = \dot{\theta}(1.5\ddot{\theta} + 14.715\theta)$

$\omega_n = \sqrt{14.715} = 3.84 \ Hz$