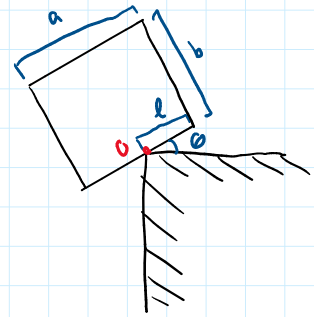
20-R-WE-DK-21

July 30, 2020 10:54 AM

# 20-R-WE-DK-21 Intermediate Principle of Work and Energy

Inspiration: 10.11.2 Example 2 Mech Notes

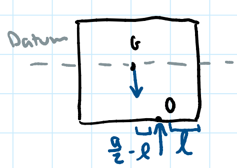



A thin plate with dimensions ***a* = 1 m**, ***b* = 0.8 m**, has a mass ***m* = 2 kg**. If the plate is at rest when ***theta* = 0 degrees**, determine the angle ***theta*** at which it begins to slip. The point of contact O between the plate and the ledge is located a length ***l* = 0.3 m** from one side of the plate. Take the coefficient of static friction to be ***mu_s* = 0.3**.

State 1

$T_1 = 0 \quad V_1 = V_{G1} = 0 \quad \omega_1 = 0$

State 1




state 2

$T_2 = \frac{1}{2} I_O \omega^2$ as O acts as a pin

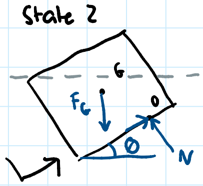
$= \frac{1}{2} I_G \omega^2 + \frac{1}{2} m v_G^2$

$I_O = \frac{1}{12} m(a^2+b^2) + m\left(\left(\frac{b}{2}\right)^2 + \left(\frac{a}{2} - l\right)^2\right)$

$= \frac{1}{12}(2)(1+0.8^2) + 2\left((0.4)^2 + (0.5-0.3)^2\right)$

$= \frac{101}{150}$

State 2




$V_2 = V_{G2} = -mg\left(\frac{a}{2} - l\right)\sin\theta$

$T_1 + V_1 + \sum_{\text{non-cons}} U_{1\to 2} = T_2 + V_2$

$\sum_{\text{non-cons}} U_{1\to 2} = 0$ as friction exists, but if it is not slipping, friction does no work

$0 = \frac{1}{2}\left(\frac{101}{150}\right)\omega^2 - (2)(9.81)(0.5-0.3)\sin\theta$

$\omega^2 = \frac{5886}{505}\sin\theta$

state 2

$\sum F_x: \; F_F - mg\sin\theta = ma_{Gx}$ $\qquad F_F = \mu_s N$

$\sum F_y: \; N - mg\cos\theta = ma_{Gy}$

$\sum M_O: \; mg\sin\theta\left(\frac{b}{2}\right) + mg\cos\theta\left(\frac{a}{2} - l\right) = I_O\alpha$

O acts like a pin $\vec{a}_O = 0$ $\qquad \vec{a}_G = \vec{a}_O + \vec{\alpha}\times\vec{r}_{G/O} - \omega^2\vec{r}_{G/O}$ $\qquad \vec{r}_{G/O} = \left(-\left(\frac{a}{2} - l\right)\hat{i} + \frac{b}{2}\hat{j}\right)$

$\vec{a}_G = \alpha\hat{k}\times(-0.2\hat{i} + 0.4\hat{j}) - \omega^2(-0.2\hat{i} + 0.4\hat{j})$

$= -0.2\alpha\hat{j} - 0.4\alpha\hat{i} + 0.2\omega^2\hat{i} - 0.4\omega^2\hat{j}$

$a_{Gx} = -0.4\alpha + 0.2\omega^2 \qquad a_{Gy} = -0.2\alpha - 0.4\omega^2$

$0.3N - (2)(9.81)\sin\theta = 2(-0.4\alpha + 0.2\omega^2)$

$N - (2)(9.81)\cos\theta = 2(-0.2\alpha - 0.4\omega^2)$

$(2)(9.81)\sin\theta(0.4) + (2)(9.81)\cos\theta(0.2) = \frac{101}{150}\alpha$

$0.3N - 19.62\sin\theta = -0.8\alpha + 0.4\omega^2$

$N - 19.62\cos\theta = -0.4\alpha - 0.8\omega^2$

$7.848\sin\theta + 3.924\cos\theta = \frac{101}{150}\alpha$

$\frac{5886}{505}\sin\theta + \frac{2943}{505}\cos\theta = \alpha$

$0.3N - 19.62\sin\theta = \frac{11772}{2525}\sin\theta - \frac{23544}{2525}\sin\theta - \frac{11772}{2525}\cos\theta$

$N = \frac{25179}{505}\sin\theta - \frac{7848}{505}\cos\theta$

$N - 19.62\cos\theta = -\frac{11772}{2525}\sin\theta - \frac{5886}{2525}\cos\theta - \frac{23544}{2525}\sin\theta$ $\qquad N = -\frac{34716}{2525}\sin\theta + \frac{87309}{5050}\cos\theta$

$\frac{25179}{505}\sin\theta - \frac{7848}{505}\cos\theta = -\frac{34716}{2525}\sin\theta + \frac{87309}{5050}\cos\theta$

$$\frac{25179}{505}\sin\phi - \frac{7844}{505}\cos\phi = -\frac{34716}{2525}\sin\phi + \frac{67309}{5050}\cos\phi$$

$$63.60831683\sin\phi = 32.82950495\cos\phi \qquad \tan\phi = 0.518565436$$

$$\boxed{\phi = 27.4097^\circ}$$