20-R-AM-JH-001V2 and -002V2

Friday, June 19, 2020 4:09 PM

For 221 Dynamics Chapters 1 and 2

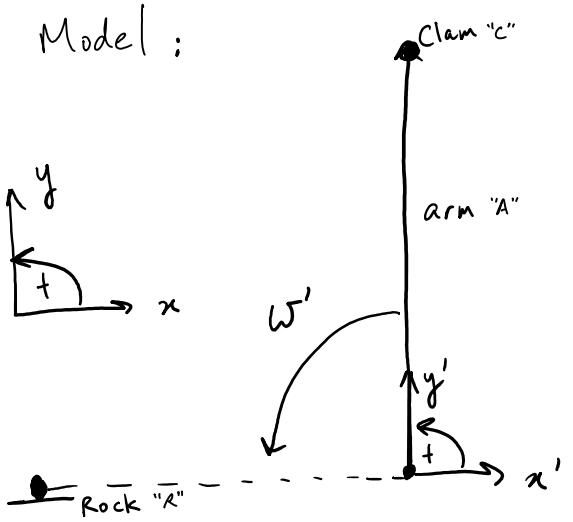
Louise the otter is enjoying a meal of clams by smashing them against a rock that is placed on her belly. She is floating along the shore with a current of 3.6 km/hr. The clams are thick, and only break if impacting the rock at 5m/s

a) If Louise's hands start vertically and move in a perfectly circular motion before reaching the rock, what acceleration must be generated to break the clams? The center of the clam to Louise's shoulder measures 20cm and remains constant throughout.

a) Condition: $v \geq 5\ m/s$

Assume: - Rigid body motion (no otter belly or buoyancy effects)
- Single rotating joint at shoulder

Model:



$\vec{r} = -0.2\ (\hat{i})$ $\vec{\theta}_0 = \frac{\pi}{2}$ rads $(\hat{k})$

$\vec{v}_0' = 0 \frac{m}{s}$ $\quad \vec{\theta} = \pi$ rads $(\hat{k})$

$\vec{v}' = -5 \frac{m}{s}(\hat{j})$ $\quad \vec{\alpha}' = ? \left[\frac{rads}{s^2}\right]$

Relationships:

① $\vec{v} = \vec{\omega} \times \vec{r}$

No time given, so use...

② $\vec{\omega}'^2 = \vec{\omega}_0'^2 + 2\vec{\alpha}'(\vec{\theta} - \vec{\theta}_0)$

$\vec{\alpha}' = \frac{\vec{\omega}'^2 - \vec{\omega}_0'^2}{2(\vec{\theta} - \vec{\theta}_0)}$

$\vec{v}' = \vec{\omega}' \times \vec{r}$

$-5 \frac{m}{s}(\hat{j}) = \vec{\omega}' \times -0.2\,(\hat{i})$

$-5(\hat{j}) = \begin{vmatrix} i & j & k \\ \omega_1' & \omega_2' & \omega_3' \\ -0.2 & 0 & 0 \end{vmatrix}$

$= \hat{i}(0-0) - \hat{j}(0 - \omega_3(-0.2)) + \hat{k}(0 - \omega_2(-0.2))$

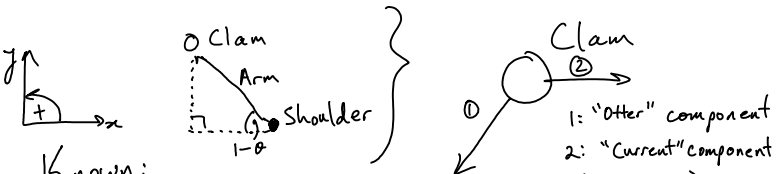
$-5\hat{j} = -0.2\omega_3'\hat{j} + 0.2\omega_2'\hat{k}$

$\therefore \vec{\omega}' = 10 \frac{rad}{s}(\hat{k})$

$\vec{\alpha}' = \frac{\vec{\omega}'^2 - \vec{\omega}_0'^2}{2(\vec{\theta} - \vec{\theta}_0)} = \frac{10^2 - 0^2}{2(\pi - \frac{\pi}{2})} = \frac{100}{\pi} = 31.83 \frac{rad}{s^2}(\hat{k})$

b) Assume an angular acceleration of 5 rad/s^2. When Louise's arm reaches 30 degrees from horizontal, what velocity is the clam experiencing with respect to a person standing on shore?

b) Model:



Known:

$\vec{\alpha} = 5(\hat{k}) \frac{rad}{s^2}$

$\vec{v}_0 = 0 \frac{rad}{s}$

$\vec{\theta}_0 = 0$ rad

$\vec{\theta} = \frac{5\pi}{6}(\hat{k})$ rad

Relationships:

② $\vec{\omega}^2 = \vec{\omega}_0^2 + 2\vec{\alpha}(\vec{\theta} - \vec{\theta}_0)$

$\vec{\omega}^2 = 0 + 2(5)(\frac{\pi}{6} - 0)$

$\vec{\omega} = \sqrt{\frac{10\pi}{6}} = 2.29 \frac{rad}{s}(\hat{k})$

$\vec{v}_1 = \vec{\omega}_f \times \vec{r} = 2.29(\hat{k}) \times [0.2\cos\theta(\hat{i}) + 0.2\sin\theta(\hat{j})]$

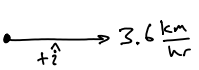
$= \begin{vmatrix} i & j & k \\ 0 & 0 & 2.29 \\ -0.17 & 0.1 & 0 \end{vmatrix} = -(2.29)(0.1)\hat{i} - (-2.29)(-0.17)\hat{j} + 0\hat{k}$

$= -0.229\hat{i} - 0.389\hat{j} \left[\frac{m}{s}\right]$

$\vec{v}_2$:

$\begin{vmatrix} -0.17 & 0.9 & 0 \\ \end{vmatrix} = -0.229\,\hat{i} - 0.389\,\hat{j}\left[\frac{m}{s}\right]$

$\vec{v}_2$:



$3.6\frac{km}{hr}\left(\frac{1000\,m}{1\,km}\right)\left(\frac{1\,hr}{3600\,s}\right) = 1\frac{m}{s}(\hat{i})$

$\vec{v}_T = \vec{v}_1 + \vec{v}_2 = -0.229\hat{i} - 0.389\hat{j} + 1(\hat{i})$

$= 0.771\,\hat{i} - 0.389\,\hat{j}\left[\frac{m}{s}\right]$