20-R-KM-DK-11

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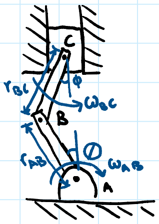

20-R-KM-DK-11 Intermediate Acceleration (Relative Motion)

Inspiration: Example 16.7 Hibbeler

Students are attempting to create a lift to raise their model car. The lift is assembled with two linkages, link AB and link BC, as seen in the picture shown. If the links have length ***l_AB = 0.2 m*** and ***l_BC = 0.4 m***, determine the velocity and acceleration of the lift at the instant where the angular velocity of AB is ***omega_AB = 5 rad/s*** and the angular acceleration of AB is ***alpha_AB = 7 rad/s^2***. Take the angles to be ***theta_AB = 30 degree*** and ***phi_BC = 20 degrees***.

$$\vec{r}_{B/A} = -0.2\sin 30\,\hat{i} + 0.2\cos 30\,\hat{j} \text{ m}$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$
$$= 0 + (-5\hat{k}) \times (-0.2\sin 30\,\hat{i} + 0.2\cos 30\,\hat{j})$$
$$= 0.5\hat{j} + \frac{\sqrt{3}}{2}\hat{i}$$

$$\vec{v}_C = \vec{v}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B} \qquad \vec{v}_C = v_C\hat{j}$$
$$= 0.5\hat{j} + \frac{\sqrt{3}}{2}\hat{i} + \omega_{BC}\hat{k} \times (0.4\sin 20\,\hat{i} + 0.4\cos 20\,\hat{j})$$

$\hat{i}$: $0 = \frac{\sqrt{3}}{2} - \omega_{BC}(0.4\cos 20)$ $\qquad \omega_{BC} = 2.304$

$\hat{j}$: $v_C = 0.5 + 2.304(0.4\sin 20)$ $\qquad v_C = 0.815207\,\hat{j}$ m/s

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2\,\vec{r}_{B/A}$$
$$= 0 + (-7\hat{k}) \times (-0.2\sin 30\,\hat{i} + 0.2\cos 30\,\hat{j}) - 25(-0.2\sin 30\,\hat{i} + 0.2\cos 30\,\hat{j})$$
$$= 0.7\hat{j} + \frac{7\sqrt{3}}{10}\hat{i} + 2.5\hat{i} - \frac{5\sqrt{3}}{2}\hat{j}$$

$$\vec{a}_C = \vec{a}_B + \vec{\alpha}_{BC} \times \vec{r}_{C/B} - \omega_{BC}^2\,\vec{r}_{C/B}$$
$$= 0.7\hat{j} + \frac{7\sqrt{3}}{10}\hat{i} + 2.5\hat{i} - \frac{5\sqrt{3}}{2}\hat{j} + \alpha_{BC}\hat{k} \times (0.4\sin 20\,\hat{i} + 0.4\cos 20\,\hat{j}) - 2.304^2(0.4\sin 20\,\hat{i} + 0.4\cos 20\,\hat{j})$$
$$a_C\hat{j} = 0.7\hat{j} + \frac{7\sqrt{3}}{10}\hat{i} + 2.5\hat{i} - \frac{5\sqrt{3}}{2}\hat{j} + \alpha_{BC}(0.4\sin 20)\hat{j} - \alpha_{BC}(0.4\cos 20)\hat{i} - (2.304^2)(0.4\sin 20)\hat{i} - (2.304^2)(0.4\cos 20)\hat{j}$$

$\hat{i}$: $0 = \frac{7\sqrt{3}}{10} + 2.5 - \alpha_{BC}(0.4\cos 20) - 2.304^2(0.4\sin 20)$ $\qquad \alpha_{BC} = 7.94462311$ rad/s²

$\hat{j}$: $a_C = 0.7 - \frac{5\sqrt{3}}{2} + \alpha_{BC}(0.4\sin 20) - 2.304^2(0.4\cos 20)$ $\qquad \boxed{a_C = -4.53855 \text{ m/s}^2}$