20-R-KM-DK-13

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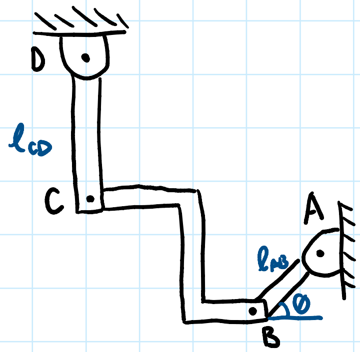

20-R-KM-DK-13 Intermediate Acceleration (Relative Motion)

Several arms are linked to form the system shown. Link AB has a length of ***l_AB = 0.5 m***, link CD has a length of ***l_CD = 0.5 m***, and the distance from B to C is ***r_C/B = -1 i + 1 j m***. If the angle between link AB is ***theta = 45 degrees***, determine the angular acceleration of link CD. The angular velocities of the links are given as ***omega_AB = -3 rad/s*** and ***omega_BC = (3sqrt2)/4 rad/s***, while the angular acceleration of AB is given as ***alpha_AB = -5 rad/s^2***.

$$\vec{r}_{B/A} = (-0.5\cos 45\,\hat{i} - 0.5\sin 45\,\hat{j})$$

$$\vec{r}_{C/B} = (-1\,\hat{i} + 1\,\hat{j})$$

$$\vec{r}_{D/C} = (0.5\,\hat{j})$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$= -3\,\hat{k} \times (-0.5\cos 45\,\hat{i} - 0.5\sin 45\,\hat{j}) = 1.5\cos 45\,\hat{j} - 1.5\sin 45\,\hat{i}$$

$$\vec{v}_C = \vec{v}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B}$$

$$= -1.5\sin 45\,\hat{i} + 1.5\cos 45\,\hat{j} + \frac{3\sqrt{2}}{4}\hat{k} \times (-1\,\hat{i} + 1\,\hat{j})$$

$$= -1.5\sin 45\,\hat{i} + 1.5\cos 45\,\hat{j} - \frac{3\sqrt{2}}{4}\hat{j} - \frac{3\sqrt{2}}{4}\hat{i}$$

$$= -\frac{3\sqrt{2}}{2}\hat{i}$$

$$\vec{v}_D = 0 = \vec{v}_C + \vec{\omega}_{CD} \times \vec{r}_{D/C}$$

$$0 = -\frac{3\sqrt{2}}{2}\hat{i} + \omega_{CD}\,\hat{k} \times (0.5\,\hat{j})$$

$$\hat{i}: -\frac{3\sqrt{2}}{2} - \omega_{CD}(0.5) = 0 \qquad \boxed{\omega_{CD} = -3\sqrt{2}}$$

$$\vec{a}_B = \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2\,\vec{r}_{B/A}$$

$$= -5\,\hat{k} \times (-0.5\cos 45\,\hat{i} - 0.5\sin 45\,\hat{j}) - 9(-0.5\cos 45\,\hat{i} - 0.5\sin 45)$$

$$= 2.5\cos 45\,\hat{j} - 2.5\sin 45\,\hat{i} + 4.5\cos 45\,\hat{i} + 4.5\sin 45\,\hat{j}$$

$$= 7\sin 45\,\hat{j} + 2\cos 45\,\hat{i}$$

$$\vec{a}_C = \vec{\alpha}_{BC} \times \vec{r}_{C/B} - \omega_{BC}^2\,\vec{r}_{C/B}$$

$$= \alpha_{BC}\,\hat{k} \times (-1\,\hat{i} + 1\,\hat{j}) - \frac{9}{8}(-1\,\hat{i} + 1\,\hat{j})$$

$$= -\alpha_{BC}\,\hat{j} - \alpha_{BC}\,\hat{i} + \frac{9}{8}\hat{i} - \frac{9}{8}\hat{j}$$

$$\vec{a}_C = \vec{\alpha}_{CD} \times \vec{r}_{C/D} - \omega_{CD}^2\,\vec{r}_{C/D}$$

$$= \alpha_{CD}\,\hat{k} \times (-0.5\,\hat{j}) - 18(-0.5\,\hat{j})$$

$$= 0.5\,\alpha_{CD}\,\hat{i} + 9\,\hat{j}$$

$$\hat{i}: \; 0.5\,\alpha_{CD} = -\alpha_{BC} + \frac{9}{8}$$

$$\hat{j}: \; 9 = -\alpha_{BC} - \frac{9}{8}$$

$$\boxed{\begin{aligned} \alpha_{BC} &= -\frac{81}{8} \\ \alpha_{CD} &= 22.5 \end{aligned}}$$