20-R-KM-DK-21   Beginner   Rotating Frame
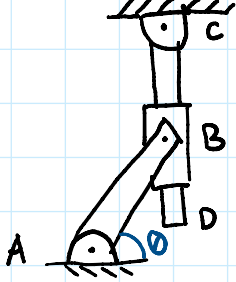

A linkage system consists of two arms and a collar. Arm AB has a length $r\_AB = 0.5\ m$ and, in the instant shown, is at an angle of **theta = 45 degrees** with the horizontal. The collar sits on arm CD and is at a distance $r\_BC = 0.8\ m$ from point C. If the collar slides up arm CD at a relative velocity of **$v\_B/C\_rel = 1.1$ m/s** , determine the angular velocity and angular acceleration of both arms.

$$\vec{V}_B = \vec{V}_c + \vec{\Omega}_{co} \times \vec{r}_{B/c} + (v_{B/c})_{x'y'z'}$$

$$\vec{V}_B = \vec{W}_{AB} \times \vec{r}_{B/A}$$

$$\vec{r}_{B/c} = (-0.8\,\hat{i}) \qquad \vec{r}_{B/A} = (0.5\cos45\,\hat{i} - 0.5\sin45\,\hat{j})$$

$$W_{AB}\,\hat{k} \times (0.5\cos45\,\hat{i} - 0.5\sin45\,\hat{j}) = \Omega_{co}\,\hat{k} \times (-0.8\,\hat{i}) + (1.1\,\hat{i})$$

$$0.5\,W_{AD}\cos45\,\hat{j} + 0.5\,W_{AB}\sin45\,\hat{i} = -0.8\,\Omega_{co}\,\hat{j} + 1.1\,\hat{i}$$

$$1.1 = 0.5\,W_{AB}\sin45 \qquad \boxed{\vec{W}_{AB} = \tfrac{11\sqrt{2}}{5}\,\hat{k}} \qquad \boxed{\vec{\Omega}_{co} = -\tfrac{11}{8}\,\hat{k}}\ \text{rad/s}$$

$$\vec{a}_B = \dot{\vec{\Omega}}_{co} \times \vec{r}_{B/c} + 2\,\vec{\Omega}_{co} \times (\vec{v}_{B/c})_{x'y'z'} - \Omega^2\,\vec{r}_{B/c}$$

$$= \dot{\Omega}_{co}\,\hat{k} \times (-0.8\,\hat{i}) + 2(-\tfrac{11}{8}\,\hat{k}) \times (1.1\,\hat{i}) - (-\tfrac{11}{8})^2(-0.8\,\hat{i})$$

$$= -0.8\,\dot{\Omega}_{co}\,\hat{j} - \tfrac{121}{40}\,\hat{j} + \tfrac{121}{80}\,\hat{i}$$

$$\vec{a}_B = \alpha_{AB}\,\hat{k} \times (0.5\cos45\,\hat{i} - 0.5\sin45\,\hat{j}) - (\tfrac{11\sqrt{2}}{5})^2(0.5\cos45\,\hat{i} - 0.5\sin45\,\hat{j})$$

$$= 0.5\,\alpha_{AB}\cos45\,\hat{j} + 0.5\,\alpha_{AB}\sin45\,\hat{i} - \tfrac{121}{25}\cos45\,\hat{i} + \tfrac{121}{25}\sin45\,\hat{j}$$

$$\hat{i}:\ 0.5\,\alpha_{AB}\sin45 - \tfrac{121}{25}\cos45 = \tfrac{121}{80} \qquad \boxed{\alpha_{AB} = 13.95799603}\ \hat{k}\ \text{rad/s}^2$$

$$\hat{j}:\ -0.8\,\dot{\Omega}_{co} - \tfrac{121}{40} = 0.5\,\alpha_{AB}\cos45 + \tfrac{121}{25}\sin45 \qquad \boxed{\dot{\Omega}_{co} = -14.227667}\ \hat{k}\ \text{rad/s}^2$$