20-R-IM-PT-9

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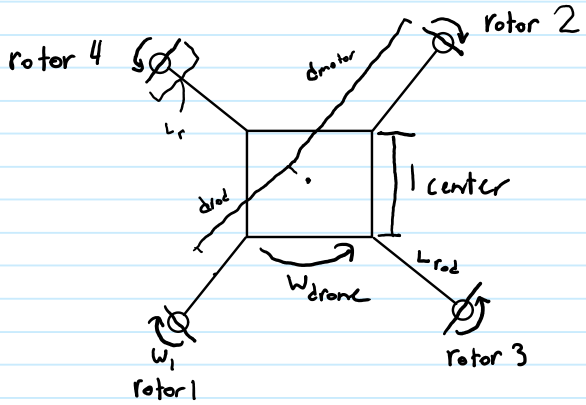
A 4 rotor drone is hovering above the ground with no change in position or rotation. Drone rotors 1 and 2 rotate with an angular velocity of 785rad/s in the clockwise direction, and rotors 3 and 4 rotate with the same angular speed in the opposite direction. What is the new angular velocity of the drone if rotors 3 and 4 stop working?

Assume that the drone consists of a square shaped center, with a mass of 500g and a side length of 15cm. The center is connected to each motor and propeller through rods with a length of 25cm and a mass of 30g each. The motor can be treated as a point mass with a mass of 70g, and the rotors have a length of 15 cm and a mass of 20g each.



Solution

$$(H_{sys})_1 = (H_{sys})_2$$

$$H_{Drone_0} + H_{r_1} + H_{r_2} + H_{r_3} + H_{r_4} = H'_{Drone} + H'_{r_1} + H'_{r_2} + H'_{r_3} + H_{r_4})_0$$

$H_{r_1} = H_{r_2} = H'_{r_1} = H'_{r_2}$
$H_{r_3} = H_{r_4}$
$H_{Drone} = 0 = H'_{r_3} = H'_{r_4}$

$$2H_{r_1} + 2H_{r_3} = H'_{Drone} + 2H_{r_1}$$

$$2H_{r_3} = H'_{Drone}$$

$$2(I_{r_3} w_{r_3}) = I_{Drone} w'^{2}_{Drone}$$

$$I_3 = \frac{1}{12} m_r L_r^2$$

$m_r = 20g = 0.02Kg$
$L_r = 15cm = 0.15m$

$$= \frac{1}{12}(0.02)(0.15)^2$$

$$= 0.0000375$$

$$I_{Drone} = I_{center} + I_{rods} + I_{motors}$$

$$I_{center} = \frac{0.15^2}{6} \cdot 0.5 = 0.001875$$

$$d_{rod} = \frac{0.25}{2} + \frac{\sqrt{(0.15)^2 + (0.15)^2}}{2} = 0.231\,m$$

$$I_{rods} = 4 \cdot \left(\tfrac{1}{12} m L^2 + m d_{rod}^2\right)$$

$$I_{rods} = 4 \cdot \left(\frac{1}{12} \cdot (0.030) \cdot (0.25)^2 + 0.030 \cdot 0.231^2\right) = 0.001757$$

$$I_{motors/rotors} = 4 \cdot \left((m_{motors} + m_r) \cdot (d^2_{motors}) + 1/12\, m_r L_r^2\right)$$

$$= \left((0.07 + 0.02)(0.356^2) + 1/_{12}(0.02)(0.15)^2\right) \cdot 4$$

$$= 0.04577496 = 0.0458$$

$$d_{motors} = 0.25 + \frac{\sqrt{(0.15)^2 + (0.15)^2}}{2} = 0.356$$

$$I_{Drone} = 0.001875 + 0.001752 + 0.0458 = 0.0494$$

$$w'_{Drone} = \frac{I_{r_3} \cdot w_{r_3} \cdot 2}{I_{Drone}}$$

$w_{r_3} = 785\,rad/s$

$$\omega'_{Drone} = \frac{(0.0000375) \cdot (785\ rad/s) \cdot 2}{(0.0494)}$$

$$\omega'_{Drone} = 1.1916\ rad/s$$

$\boxed{\text{The angular speed of the drone is } 1.92\ rad/s}$