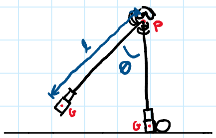
20-R-IM-DK-21

July 15, 2020 10:04 AM

Intermediate Eccentric Impact

Inspiration: 19-49 Hibbeler



You and your friends are having a great time at mini golf. You are about to take a very light shot at a golf ball with a putter, so you let the putter fall under its own weight. If the putter consists of a head with mass ***m_H = 0.3 kg*** and a radius of gyration ***k_G = 0.05***, and a slender rod that extends from point P to G with a length ***l = 0.9 m*** and mass ***m_r = 0.1 kg***, determine the velocity of the golf ball and the angular velocity of the putter right after impact. The center of gravity of the putter is located in its head. The coefficient of restitution is ***e = 0.9*** and the golf ball has mass ***m_b = 0.05 kg***. The putter is released from rest when ***theta = 45 degrees***.

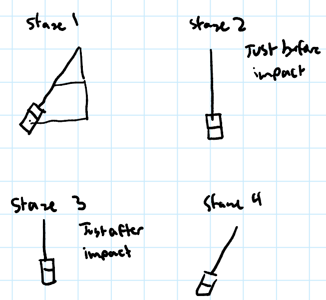
Notes for artist:

These weird things around P are hands.

The length is from P to the middle of the putter head

The middle of the putter head is in line with the middle of the golf ball

This is like a before and after picture; starts at an angle and ends perpendicular to the ground



Set datum to be at P

$$I_{PH} = m_H k_G^2 + m_H l^2$$
$$= (0.3)(0.05)^2 + (0.3)(0.9)^2$$
$$= 0.24375$$

$$I_{Pr} = \frac{1}{3} m_r l^2$$
$$= \frac{1}{3}(0.1)(0.9)^2$$
$$= 0.027$$

$$T_1 = 0 \qquad T_2 = \frac{1}{2} I_{PH} w_2^2 + \frac{1}{2} m_H V_{H_2}^2 + \frac{1}{2} I_{Pr} w_2^2 + \frac{1}{2} m_r V_{r_2}^2$$

Putter is pinned at P $\Rightarrow$ $V_{r_2} = w_2 \frac{l}{2}$ $\qquad V_H = w_2 l_2$

$$T_2 = \frac{1}{2}(0.24375) w_2^2 + \frac{1}{2}(0.3)(0.9^2) w_2^2 + \frac{1}{2}(0.027) w_2^2 + \frac{1}{2}(0.1)\left(\frac{0.9^2}{4}\right) w_2^2$$
$$= 0.267 w_2^2$$

$$T_1 + V_1 = T_2 + V_2$$
$$0 - m_r g \frac{l}{2}\cos\theta - m_H g\, l\cos\theta = T_2 - m_r g \frac{l}{2} - m_H g\, l$$

$$-0.1(9.81)\frac{0.9}{2}\cos 45 - 0.3(9.81)(0.9)\cos 45 = 0.267 w_2^2 - 0.1(9.81)\left(\frac{0.9}{2}\right) - 0.3(9.81)(0.9)$$
$$w_2 = 1.841148 \text{ rad/s}$$

The angular momentum of the system is conserved about point P

$$(H_P)_2 = (H_O)_3$$

$$I_{Pr} w_2 + \frac{l}{2} m_r V_{r_2} + I_{PH} w_2 + l m_H V_{H_2} = I_{Pb} w_3 + l m_b V_{b_3} - I_{Pr} w_3 - \frac{l}{2} m_r V_{r_3} - I_{PH} w_3 - l m_H V_{H_3}$$

$$0.027(1.441148) + \frac{0.9}{2}(0.1)(1.441148)\left(\frac{0.9}{2}\right) + (0.24375)(1.841148) + 0.9(0.3)(1.841148)(0.9)$$
$$= 0.983173$$

$$0.983173 = 0 + (0.9)(0.05) V_{b_3} - 0.027 w_3 - \frac{0.9}{2}(0.1) w_3 \left(\frac{0.9}{2}\right) - 0.24375 w_3 - 0.9(0.3)(0.9) w_3$$

$$0.983173 = 0.045 V_{b_3} - 0.534 w_3$$

$$e = \frac{V_{b_3} - V_{H_3}}{V_{H_2} - V_{b_2}} \qquad 0.9 = \frac{V_{b_3} - (-w_3(0.9))}{1.841148(0.9) - 0}$$

$$1.49132988 = V_{b_3} + 0.9 w_3$$

$$0.983173 = 0.045(1.49132984 - 0.9 w_3) - 0.534 w_3$$

$0.916063155 = -0.5745\,\omega_3$ $\omega_3 = -1.594539$ rad/s

$\vec{\omega_3} = 1.594539$

$\boxed{V_{b_3} = 2.9264 \text{ m/s}}$