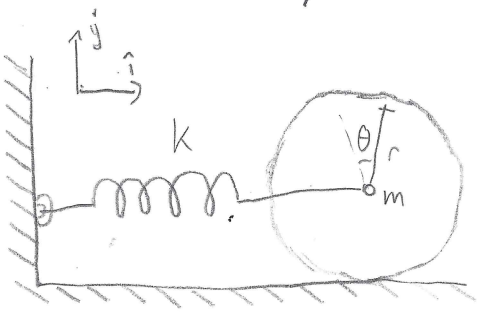
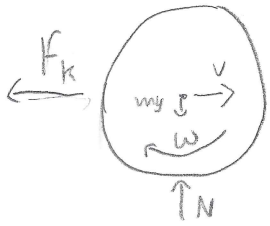
20-R-VIB-04-22 Intermediate

A new pizza cutter consists of a spring ($k = 10$ N/m) and a lightweight ($m = 0.5$ kg) circular blade with the radius $r = 0.25$m. Assuming that the blade does not slip with the ground, if the blade is given an initial velocity of $2\frac{m}{s}\hat{i}$ find the equation that describes the angle of the blade.



Solution:



$\theta_p = A\sin\omega_n t + B\cos\omega_n t$

$\dot{\theta}_p = A\omega_n\cos\omega_n t - B\omega_n\sin\omega_n t$

$\theta_p(0) = B = \theta_0 = 0$

$V_0 = \omega_0 r \quad \omega_0 = 8$ rad/s

$\theta_p(0) = A\omega_n = \omega_0$

$A = 8\sqrt{\frac{3}{40}}$

$\theta(t) = 8\sqrt{\frac{3}{40}}\sin\sqrt{\frac{40}{3}}t$

$I_G = \frac{1}{2}mr^2 = 0.015625$

$T = \frac{1}{2}I_G\omega^2 + \frac{1}{2}mv^2 =$

$V = \omega r = 0.25\omega$

$= 0.0234375\omega^2$

$V = \frac{1}{2}kx^2 = \frac{1}{2}(10)(0.25\theta)^2 = 0.3125\theta^2$

$x = r\theta$

$E = T + V = 0.0234375\dot{\theta}^2 + 0.3125\theta^2$

$\dot{E} = 0.046875\ddot{\theta}\dot{\theta} + 0.625\dot{\theta}\theta$

$= \dot{\theta}(0.046875\ddot{\theta} + 0.625\theta)$

$\ddot{\theta} + \frac{40}{3}\theta = 0 \qquad \omega_n = \sqrt{\frac{40}{3}}$