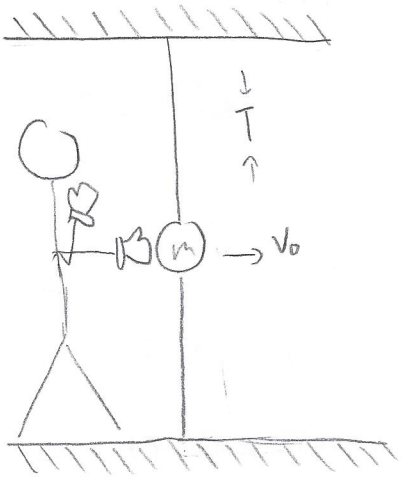
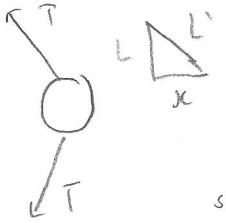
20-R-VIB-DY-3 Intermediate

A new type of punching ball consists of a 5kg ball suspended mid-air by two 2m taut ropes, each with a tension of 25 N. One rope connects the ball to the ceiling and the other to the ground. If an impact results in an initial velocity $2\,m/s\,\hat{i}$, find the equation of motion.



Solution:



$L' \simeq L$

$\sin\theta = \frac{x}{L'} = \frac{x}{L}$

$\sum F_x = ma_x$

$-2T\sin\theta = ma_x$

$-2T\frac{x}{L} = m\ddot{x}$

$m\ddot{x} + 2T\frac{x}{L} = 0$

$\ddot{x} + \frac{2T}{mL}x = 0 \qquad \omega_n = \sqrt{\frac{2T}{mL}} = \sqrt{5}$

$x = A\sin\omega_n t + B\cos\omega_n t$

$\dot{x} = A\omega_n\cos\omega_n t - B\omega_n\sin\omega_n t$

$x(0) = B = 0$

$\dot{x}(0) = A\omega_n = 2\,m/s \qquad A = \frac{2}{\sqrt{5}}$

$x = \frac{2}{\sqrt{5}}\sin\sqrt{5}\,t$