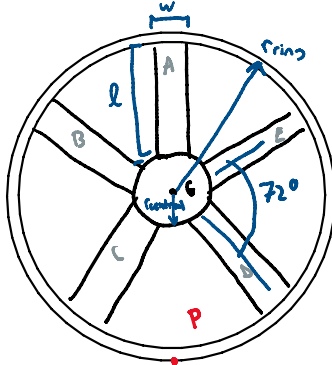


20-R-KIN-DK-10

Intermediate Composite Bodies

Inspiration: None



A student on UBC Formula creates a prototype wheel cover, consisting of a thin ring, five rectangular plates, and a central circular plate. Each plate has a mass of $m_{\text{plate}} = 0.5 \text{ kg}$ while the ring has a mass of $m_{\text{ring}} = 1 \text{ kg}$. What is the mass moment of inertia if the wheel cover rotates about point P? Assume the thickness of the ring is negligible. The ring has a radius $r_{\text{ring}} = 25 \text{ cm}$ while the central plate has a radius $r_{\text{central}} = 7.5 \text{ cm}$. Each plate has a length $l = 17.5 \text{ cm}$ and width $w = 6 \text{ cm}$. Each plate is spaced 72 degrees apart from one another.

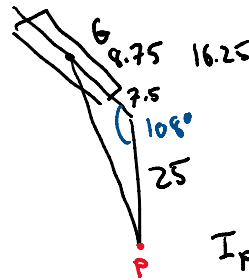
$$\text{Ring: } I_p = mr^2 + md^2 \\ = 1(0.25)^2 + 1(0.25)^2 \\ = 0.125$$

$$\text{Circular Plate: } I_p = \frac{1}{2}mr^2 + md^2 = \frac{1}{2}(0.5)(0.075)^2 + 0.5(0.25)^2 \\ = \frac{209}{6400}$$

$$\begin{matrix} 8.75 \\ 7.5 + 25 \end{matrix} \left[\begin{matrix} 41.25 \\ \cdot \end{matrix} \right]$$

$$\text{Plate A: } I_{p_A} = \frac{1}{12}m(a^2 + b^2) + md^2 = \frac{1}{12}(0.5)(0.175^2 + 0.06^2) + 0.5(0.4125)^2 \\ = 0.086504166$$

Plate B, E:



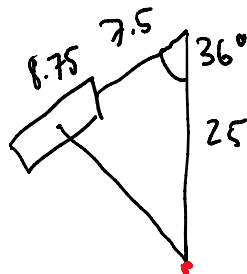
$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 16.25^2 + 25^2 - 2(16.25)(25) \cos 108$$

$$c = 33.76594154$$

$$I_{p_B} = \frac{1}{12}(0.5)(0.175^2 + 0.06^2) + 0.5(0.3376594154)^2 \\ = 0.054432982$$

Plate C, D:



$$c^2 = 16.25^2 + 25^2 - 2(16.25)(25) \cos 36$$

$$c = 15.22284383$$

$$I_{p_C} = \frac{1}{12}(0.5)(0.175^2 + 0.06^2) + 0.5(0.1522284383)^2 \\ = 0.013012951$$

$$I = I_{p_p} + I_{p_A} + 2I_{p_B} + 2I_{p_C} = \boxed{0.262052082}$$