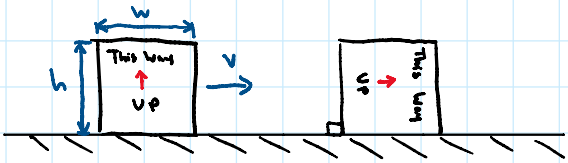
20-R-IM-DK-20

July 15, 2020 9:18 AM

Intermediate Eccentric Impact

Inspiration: 19-45 Hibbeler

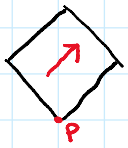


You are helping your friend move into their new place. While sliding a box full of fragile things on a frictionless floor, the box gets caught on a small bar in the doorway and completely flips over. Oops. If the box has a mass $m = 20$ kg, a height of $h = 0.4$ m, and a width of $w = 0.4$ m, determine how fast the box had to be moving in order for this to happen. Assume the weight of the box during the impact was non-impulsive and that the size of the bar is negligible.

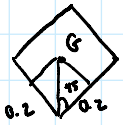
Stage 1
Just before impact

Stage 2
Just after impact

Stage 3



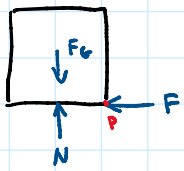
$$\bar{I}_P = \frac{1}{12}m(a^2+b^2) + m d^2$$
$$= \frac{1}{12}(20)(0.4^2+0.4^2) + (20)\left(\sqrt{0.2^2+0.2^2}\right)^2$$
$$= \frac{32}{15}$$



$\frac{0.2}{h_3} = \sin 45$ $\quad h_3 = \frac{0.2}{\sin 45} = \frac{\sqrt{2}}{5}$

Set datum as the ground

$$T_2 + V_2 = T_3 + V_3$$
$$\frac{1}{2}\bar{I}_O \omega_2^2 + mgh_2 = 0 + mgh_3$$
$$\frac{1}{2}\left(\frac{32}{15}\right)\omega_2^2 + (20)(9.81)(0.2) = (20)(9.81)\left(\frac{\sqrt{2}}{5}\right)$$
$$\omega_2 = 3.90357$$



$\int F_G\,dt$ and $\int N\,dt$ are negligible
Angular momentum conserved about P

$$(H_P)_1 = (H_P)_2$$
$$\bar{I}_G\vec{\omega}_1 + \left(-\tfrac{w}{2}\hat{i} + \tfrac{h}{2}\hat{j}\right) \times (m)(v_G\hat{i}) = \bar{I}_G\vec{\omega}_2 + r_{G/O} \times \vec{L}_2$$
$$0.2(20)v = \frac{32}{15}(3.90357)$$

$$v = 2.0819 \text{ m/s}$$