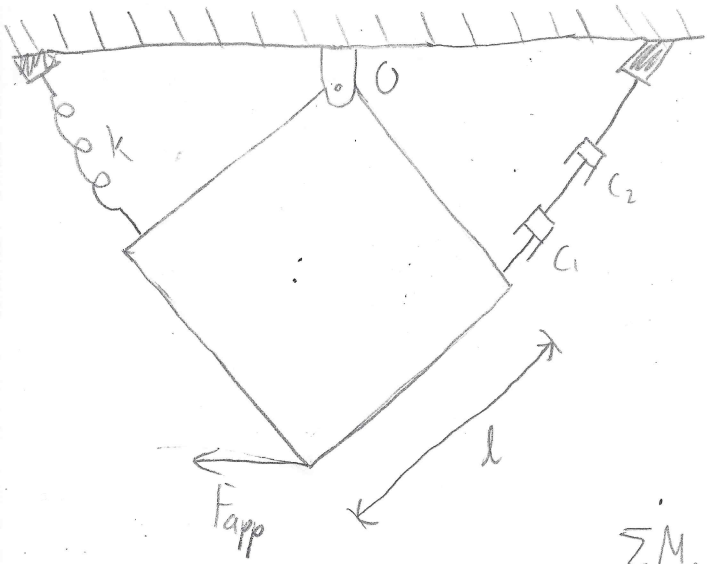
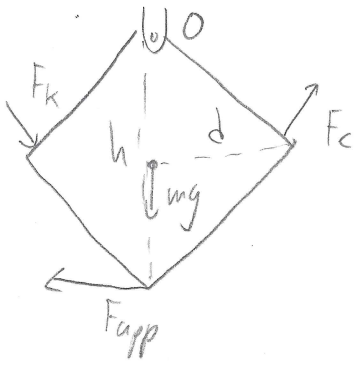
20-R-VIB-DY-46 Advanced

A thin square plate of side length $\ell = 3m$ and mass $m = 4kg$ is pinned to the ceiling by a corner. A spring ($k = 10\ N/m$) is attached to the left corner and a pair of dampers in series ($5\ Ns/m$ & $10\ Ns/m$) is installed on the right corner. If a periodic force is applied to the bottom corner horizontally, determine the equation for the force given an amplitude $D = 0.0055$ and $\phi = 0.5$.




Solution: FBD




$\Sigma M_O = -I_O \alpha$ $\quad I_O = \frac{1}{6} m\ell^2 + md^2 = m\left(\frac{1}{6}\ell^2 + d^2\right)$

$h = \sqrt{2\ell^2}$ $\quad \ell^2 = 2d^2$ $\quad d = \sqrt{\frac{\ell^2}{2}}$

$$\ell^2 k\theta + \ell^2 c\dot{\theta} + mg\theta - F_{app} h = -I_O \ddot{\theta}$$

$I_O = m = 24$

$c = \frac{c_1 c_2}{c_1 + c_2} = 3.3\ Ns/m$

$$I_O\ddot{\theta} + \theta(k\ell^2 + mg) + c\ell^2\dot{\theta} = F_{app}\sqrt{2\ell^2}$$

$c' = 30$ $\quad k' = 129.24$

$$\tan\phi = \frac{2\frac{c}{c_c}\frac{\omega_0}{\omega_n}}{1-\left(\frac{\omega_0}{\omega_n}\right)^2} \qquad 0.546 - 0.1015\omega_0^2 = 0.2732\omega_0$$

$\omega_0 = 1.443$

$c_c = \sqrt{4mk'} = 111.387\ Ns/m$

$\omega_n = \sqrt{\frac{k'}{m}} = 2.32\ rad/s$

$$D = \frac{F_0/k}{\sqrt{\left[1-\left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2\frac{c}{c_c}\frac{\omega_0}{\omega_n}\right]^2}}$$

$$F_0 = kD\sqrt{\left[1-\left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2\frac{c}{c_c}\frac{\omega_0}{\omega_n}\right]^2}$$

$= 0.316\ N$