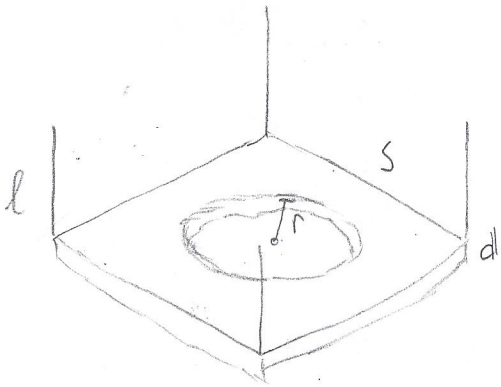
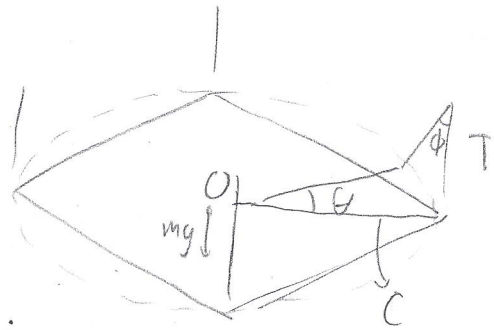
20-R-VIB-DY-23

A $\rho = 1000$, $s \times s \times d$ square plate is supported by ropes, length $l = \ell$, on each of its four corners. The plate has a large circular cut-out in the center of radius $r = r$. Given that the plate is given a small rotation about a vertical axis at its center, what is the natural frequency of vibration.



Solution: FBD



$$l\phi = C\theta$$

$$\phi = \frac{C\theta}{l}$$

$$C = \frac{\sqrt{2s^2}}{2}$$

$$m_p = \rho \times s \times s \times d$$

$$m_c = \rho \times d \times \pi \times r^2$$

$$\Sigma M_o = I_o \alpha \qquad I_o = \left(\frac{1}{12} m_p (2s^2) - \frac{1}{2} m_c r^2\right)$$

$$-4T\sin\phi\, R = I_o \alpha \qquad \text{small angle } \sin\phi = \phi$$

$$I_o \ddot{\theta} + 4T\phi R = 0 \qquad I_o \ddot{\theta} + \frac{4CTR}{l}\theta = 0$$

$$\ddot{\theta} + \frac{4CTR}{I_o l}\theta = 0$$

$$\omega_n = \sqrt{\frac{4CTR}{I_o l}}$$