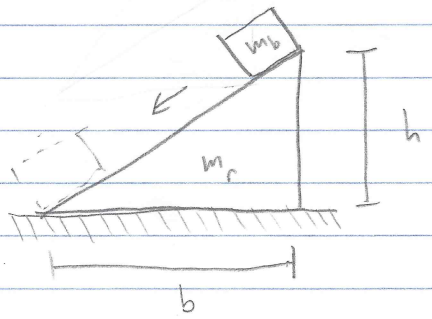
20-P-MOM-DY-10

A box slides down a frictionless ramp. The box have a mass $m_b = 5kg$ and the ramp $m_r = 50kg$. Determine the final velocity of the box when it reaches the end of the ramp. The ramp has a height of 2m and base of 3m.



Solution:

$$T_1 + V_1 = T_2 + V_2$$
$$0 + mgh = \frac{1}{2}(m_b)v_b^2 + \frac{1}{2}m_r v_r^2$$
$$98.1 = 2.5\,v_b^2 + 25\,v_r^2$$
$$\theta = \text{atan}\left(\frac{h}{b}\right) = 33.69°$$

$$v_b = v_r + v_{b/r}$$
$$= v_r\hat{i} + (-v_{b/r}\cos\theta\,\hat{i} - v_{b/r}\sin\theta\,\hat{j})$$
$$= (v_r - v_{b/r}\cos\theta)\,\hat{i} - v_{b/r}\sin\theta\,\hat{j}$$
$$\bar{v}_b = \sqrt{(v_r - v_{b/r}\cos\theta)^2 + (v_{b/r}\sin\theta)^2}$$
$$= \sqrt{v_r^2 - 2v_r v_{b/r}\cos\theta + v_{b/r}^2\cos^2\theta + v_{b/r}^2\sin^2\theta}$$
$$= \sqrt{v_r^2 + v_{b/r}^2(\cos^2\theta + \sin^2\theta) - 2v_r v_{b/r}\cos\theta}$$
$$\downarrow 1$$

$$0 = m_r v_r + m_b(v_b)_x$$
$$= 50v_r + 5(v_r - v_{b/r}\cos\theta)$$
$$0 = 55v_r - 5v_{b/r}\cos\theta$$

$$13.22\,v_r = v_{b/r}$$
$$\sqrt{3.924 - 0.1v_b^2} = v_r$$

$$v_b = [(3.924 - 0.1v_b^2) + 174.77v_r^2 - (86.324 - 2.2v_b^2)]^{1/2}$$
$$= [3.924 - 0.1v_b^2 + 685.797 - 17.477v_b^2 - 86.324 + 2.2v_b^2]^{1/2}$$
$$v_b^2 = 603.397 - 15.377v_b^2$$

$$v_b = 6.07\ m/s$$
$$v_r = 0.489\ m/s$$
$$v_{b/r} = 6.4645\ m/s$$

$$V_b = -4.89\,\hat{i} - 3.59\,\hat{j}\ m/s$$