20-R-KM-DK-12

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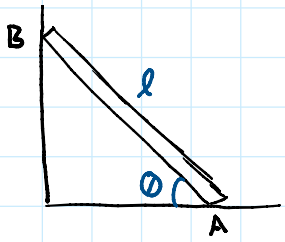
20-R-KM-DK-12 Beginner Acceleration (Relative Motion)



A handyman has left a ladder leaning on a wall which begins to slip. At a given instant, the top of the ladder has an acceleration ***a_B = 3m/s^2*** and a velocity of ***v_B = 5 m/s***, both acting downward. Determine the magnitude of acceleration of the bottom of the ladder, A, and the magnitude of the ladder's angular acceleration at this instant. The length of the ladder is ***l = 10 m*** and forms an angle of ***theta = 30 degrees*** with the ground at A.

$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$ $\quad -5\hat{j} = v_A\hat{i} + \omega\hat{k} \times (-10\cos30\,\hat{i} + 10\sin30\,\hat{j})$

$-5\hat{j} = v_A\hat{i} - \omega(10\cos30)\hat{j} - \omega(10\sin30)\hat{i}$

$\hat{j}: \; -5 = -\omega(10\cos30)$

$\omega = \frac{5}{10\cos30} = \frac{\sqrt{3}}{3}$

$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$

$a_A\hat{i} = -3\hat{j} + \alpha\hat{k} \times (10\cos30\,\hat{i} - 10\sin30\,\hat{j}) - \left(\frac{\sqrt{3}}{3}\right)^2 (10\cos30\,\hat{i} - 10\sin30\,\hat{j})$

$a_A\hat{i} = -3\hat{j} + \alpha(10\cos30)\hat{j} + \alpha(10\sin30)\hat{i} - \frac{1}{3}(10\cos30)\hat{i} + \frac{1}{3}(10\sin30)\hat{j}$

$\hat{i}: \; a_A = \alpha(10\sin30) - \frac{1}{3}(10\cos30)$

$\hat{j}: \; 0 = -3 + \alpha(10\cos30) + \frac{1}{3}(10\sin30) \qquad \alpha = \frac{4\sqrt{3}}{45}\hat{k} \qquad \vec{a}_A = -\frac{11\sqrt{3}}{9}\hat{i}\ m/s^2$

$\|\vec{a}_A\| = \frac{11\sqrt{3}}{9}\ m/s^2$

$\alpha = \frac{4\sqrt{3}}{45}\ rad/s^2$