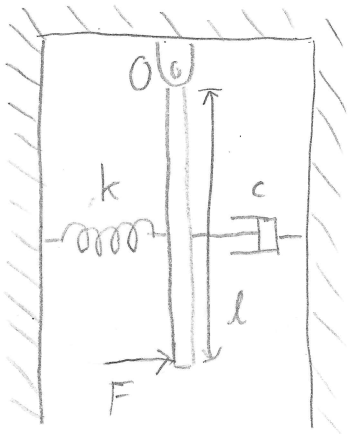
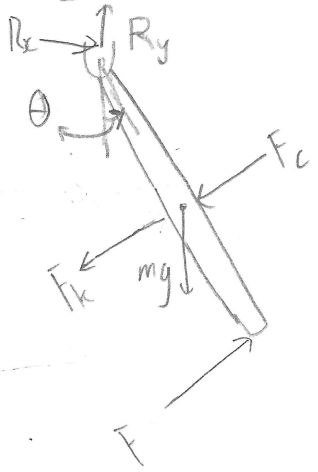
20-R-VIB-DY-37 Beginner

A $\ell = 5m$ long bar of mass $m = 10kg$ is pinned to the ceiling. A damper, $c = 50\ Ns/m$, and spring, $k = 75\ N/m$, are attached halfway down the bar. If a force $f = 20\sin 2.5t$ is applied to the end of the bar, what is the particular solution of the vibration.



Solution: FBD



$$\Sigma M_o : I_o \alpha$$

$$\ell F - \frac{\ell}{2}\left(ks + c\dot{s} + mg\sin\theta\right) = \frac{1}{3}m\ell^2\ddot{\theta}$$

small angle $\quad s = r\theta \quad \dot{s} = r\dot{\theta} \quad \sin\theta = \theta$

$$\ell F = \underbrace{\frac{1}{3}m\ell^2}_{m'}\ddot{\theta} + \underbrace{c\left(\frac{\ell}{2}\right)^2}_{c'}\dot{\theta} + \underbrace{\frac{\ell}{2}\left(\frac{k\ell}{2} + mg\right)}_{k'}\theta$$

$\omega_n = \sqrt{\frac{k'}{m'}} = \sqrt{\frac{714}{833}} = 2.927$ $\quad \theta_p(t) = D\sin\omega t$ $\quad$ no initial conditions

$c_c = \sqrt{4mk'} = 487.85$

$\omega = 2.5$

$F_o = 20$

$c' = 312.5$

$$D = \frac{F_o/k'}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\frac{c'}{c_c}\frac{\omega}{\omega_n}\right]^2}} = 0.000385$$

$$\theta_p(t) = 0.000385\sin 2.5t$$