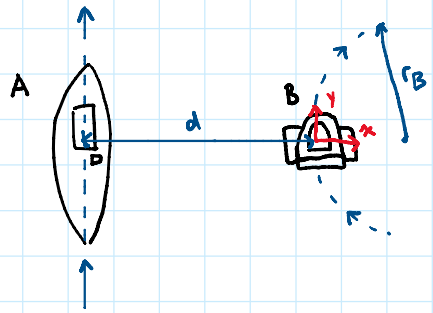
20-R-KM-DK-22

July 31, 2020 11:27 AM

20-R-KM-DK-22 Beginner Rotating Frame

Inspiration: None



Two boats are sailing on the high seas. At one instant, the boats are a distance ***d = 20 m*** apart and have parallel velocities. Boat A is heading straight while boat B sails in a circular path with radius ***r_B = 10 m***. Boat A has a velocity ***v_A = 30 m/s*** and an acceleration of ***a_A = 15 m/s^2*** in the same direction. Boat B has a velocity ***v_B = 5 m/s*** and a tangential acceleration of ***a_B_tangential = 20 m/s^2***. Determine the velocity and acceleration of boat A as seen from boat B.

$\vec{V}_A = 30\ m/s\ \hat{j}$ $\quad \vec{V}_B = 5\ m/s\ \hat{j}$

$\vec{a}_A = 15\ m/s^2\ \hat{j}$ $\quad \vec{a}_{B\,Tan.} = 20\ m/s^2\ \hat{j}$

$\omega_B = \frac{V_B}{r} = \frac{5}{10} = 0.5$ $\quad \vec{\omega}_B = -0.5\ \hat{k}\ rad/s$

$\vec{V}_A = \vec{V}_B + \vec{\Omega}_B \times \vec{r}_{A/B} + (V_{A/B})_{x'y'z'}$

$30\hat{j} = 5\hat{j} + (-0.5\hat{k}) \times (-20\hat{i}) + (V_{A/B})_{x'y'z'}$

$30\hat{j} = 5\hat{j} + 10\hat{j} + (V_{A/B})_{x'y'z'}$ $\quad \boxed{(V_{A/B})_{x'y'z'} = 15\hat{j}\ m/s}$

$\vec{a}_B = \vec{\alpha}_B \times \vec{r}_B - \omega^2 \vec{r}_B$ $\quad \vec{a}_{B\,Tan} = \vec{\alpha}_B \times \vec{r}_B$ $\quad \vec{a}_{B\,Norm} = -\omega^2 \vec{r}_B$

$20\hat{j} = \alpha\hat{k} \times (-10\hat{i})$

$\alpha = -2\hat{k}\ rad/s^2$

$\vec{a}_A = \vec{a}_B + \dot{\vec{\Omega}} \times \vec{r}_{A/B} + 2\vec{\Omega} \times (\vec{V}_{A/B})_{x'y'z'} - \Omega^2 \vec{r}_{A/B} + (\vec{a}_{A/B})_{x'y'z'}$

$15\hat{j} = 2.5\hat{i} + 20\hat{j} + (-2\hat{k}) \times (-20\hat{i}) + 2(-0.5\hat{k}) \times (15\hat{j}) - (0.5)^2(-20\hat{i}) + (\vec{a}_{A/B})_{x'y'z'}$

$15\hat{j} = 2.5\hat{i} + 20\hat{j} - 40\hat{j} + 15\hat{i} + 5\hat{i} + (\vec{a}_{A/B})_{x'y'z'}$

$\hat{i}$: $0 = 2.5 + 15 + 5 + (a_{A/B})_{x'y'z'}\hat{i}$

$\hat{j}$: $15 = 20 - 40 + (a_{A/B})_{x'y'z'}\hat{j}$

$\boxed{a_{A/B} = -22.5\hat{i} + 35\hat{j}\ m/s^2}$