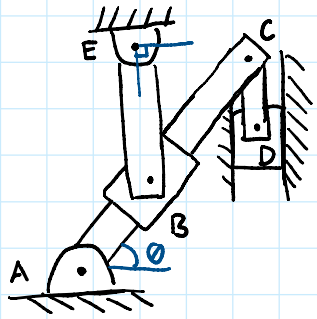
20-R-KM-DK-17

July 31, 2020 11:26 AM

20-R-KM-DK-17 Intermediate Rotating Frame

Inspiration: 5.31 Example



A linkage system consists of several arms, a collar, and a block. Arm AC has a total length ***r_AC = 1 m*** and, in the instant shown, is at an angle of ***theta = 60 degrees*** with the horizontal. The collar sits on arm AC at a distance ***r_AB = 0.6 m*** and is connected to arm BE, which sits vertically and has a length of ***r_BE = 0.3 m***. If the collar slides up the arm at a constant rate with a relative velocity of ***v_B/A_rel = 0.5 m/s***, determine the angular velocity and angular acceleration of arm BE.

$V_A = 0 \qquad \vec{V}_B = \vec{\Omega}_{AC} \times \vec{r}_{B/A} + (\vec{v}_{B/A})_{x'y'z'} \qquad \vec{V}_B = \vec{\omega}_{BE} \times \vec{r}_{B/E}$

$\vec{V}_B = \omega_{BE}\hat{k} \times (-0.3\cos30\,\hat{i} - 0.3\sin30\,\hat{j}) = -\omega_{BE}\,0.3\cos30\,\hat{j} + 0.3\omega_{BE}\sin30\,\hat{i}$

$\vec{V}_B = \Omega_{AC}\hat{k} \times (0.6\hat{i}) + 0.5\hat{i} = 0.6\Omega_{AC}\,\hat{j} + 0.5\hat{i}$

$\hat{i}: \; 0.5 = 0.3\omega_{BE}\sin30 \Rightarrow \boxed{\vec{\omega}_{BE} = \frac{10}{3}\hat{k}\ \text{rad/s}}$

$\hat{j}: \; -\omega_{BE}\,0.3\cos30 = 0.6\,\Omega_{AC} \Rightarrow \Omega_{AC} = -\frac{5\sqrt{3}}{6}$

$\vec{a}_B = \vec{a}_A + \dot{\vec{\Omega}}_{AC} \times \vec{r}_{B/A} + 2\vec{\Omega}_{AC} \times (\vec{v}_{B/A})_{x'y'z'} - \Omega_{AC}^2\,\vec{r}_{B/A} + (\vec{a}_{B/A})_{x'y'z'}$

$= 0 + \dot{\Omega}_{AC}\hat{k} \times (0.6\hat{i}) + 2\left(-\frac{5\sqrt{3}}{6}\hat{k}\right) \times (0.5\hat{i}) - \left(-\frac{5\sqrt{3}}{6}\right)^2(0.6\hat{i}) + 0$

$= 0.6\,\dot{\Omega}_{AC}\,\hat{j} - 5\sqrt{3}\,\hat{j} - 1.25\hat{i}$

$\vec{a}_B = \vec{a}_E + \vec{\alpha}_{BE} \times \vec{r}_{B/E} - \omega_{BE}^2\,\vec{r}_{B/E}$

$= 0 + \alpha_{BE}\hat{k} \times (-0.3\cos30\,\hat{i} - 0.3\sin30\,\hat{j}) - \left(\frac{10}{3}\right)^2(-0.3\cos30\,\hat{i} - 0.3\sin30\,\hat{j})$

$= -0.3\,\alpha_{BE}\cos30\,\hat{j} + 0.3\,\alpha_{BE}\sin30\,\hat{i} + \frac{10}{3}\cos30\,\hat{i} + \frac{10}{3}\sin30\,\hat{j}$

$\hat{i}: \; -1.25 = 0.3\,\alpha_{BE}\sin30 + \frac{10}{3}\cos30 \qquad \boxed{\vec{\alpha}_{BE} = -27.576\ \text{rad/s}^2}$

$\hat{j}: \; 0.6\,\dot{\Omega}_{AC} - 5\sqrt{3} = -0.3\,\alpha_{BE}\cos30 + \frac{10}{3}\sin30 \qquad \dot{\Omega}_{AC} = 29.15330702\ \text{rad/s}^2$