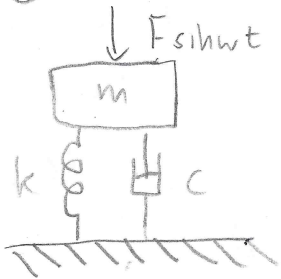


20-R-VIB-DY-47

An old spring and damper system is due for an upgrade. The mass $m = 10 \text{ kg}$ is supported mid-air by a spring, $k = 25 \text{ N/m}$, and damper, $c = 25 \text{ Ns/m}$. Determine a new damper that will decrease the amplitude by 50%.

Or a new spring that will achieve the same effect. Force is applied vertically. $F = 2 \sin 5t$.



Solution:

$$D = \frac{F_0/k}{\sqrt{\left[1 - \left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2 \frac{c}{c_c} \frac{\omega_0}{\omega_n}\right]^2}}$$

$$0.5 D = \frac{F_0/k_{\text{new}}}{\sqrt{\left[1 - \left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2 \frac{c}{c_c} \frac{\omega_0}{\omega_n}\right]^2}}$$

$$c_c = \sqrt{4mk} = \sqrt{1000}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{2.5}$$

$$k_{\text{new}} = 2k = 50 \text{ N/m}$$

$$0.5 D = \frac{F_0/k}{\sqrt{\left[1 - \left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2 \frac{c_{\text{new}} \omega_0}{c_c \omega_n}\right]^2}}$$

$$2 \sqrt{\left[1 - \left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2 \frac{c}{c_c} \frac{\omega_0}{\omega_n}\right]^2} = \sqrt{\left[1 - \left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2 \frac{c_{\text{new}} \omega_0}{c_c \omega_n}\right]^2}$$

$$4 \left(\left[1 - \left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2 \frac{c}{c_c} \frac{\omega_0}{\omega_n}\right]^2 \right) = \left[1 - \left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2 \frac{c_{\text{new}} \omega_0}{c_c \omega_n}\right]^2$$

$$3 \left[1 - \left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2 \frac{c}{c_c} \frac{\omega_0}{\omega_n}\right]^2 = \left[2 \frac{c_{\text{new}} \omega_0}{c_c \omega_n}\right]^2$$

$$c_{\text{new}} = \frac{c_c \omega_n}{2 \omega_0} \sqrt{3 \left[1 - \left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2 \frac{c}{c_c} \frac{\omega_0}{\omega_n}\right]^2} = 16.37 \text{ Ns/m}$$