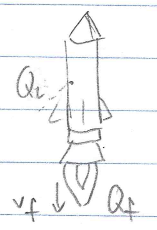
20-P-MOM-DY-41

A m = 1500 kg rocket has 1000kg of its mass dedicated to fuel. During flight, it is discovered that there is a puncture that allows the fuel to leak from the system at -2 m/s. relative to the rocket. Determine if the rocket has enough fuel to still escape earth's atmosphere

$\dot{Q}_f = 50000 \, kg/s$ $\dot{Q}_L = 100 \, kg/s$ $v_{f/r} = 15 \, km/s$



Solution: $\uparrow \Sigma F = m\frac{dv}{dt} - v_{f/r}\frac{dm}{dt} \Rightarrow -W = m\frac{dv}{dt} - v_{f/r}\frac{dm_f}{dt} - v_{L/r}\frac{dm_L}{dt}$

$m = m_0 - \left(\frac{dm_f}{dt}\right)t - \left(\frac{dm_L}{dt}\right)t = m_0 - ft - Lt$

$-(m_0 - ft - Lt)g = (m_0 - ft - Lt)\frac{dv}{dt} - v_{f/r}f - v_{L/r}L$

$$\frac{v_{f/r}f + v_{L/r}L - (m_0 - ft - Lt)g}{(m_0 - ft - Lt)}\,dt = dv$$

at $t = 0$ $v = 0$

$$\int_0^t \left(\frac{v_{f/r}f + v_{L/r}L}{m_0 - ft - Lt} - g\right)dt = \int_0^v dv$$

$m_0 - (f+L)t$

$$v = \frac{(v_{f/r}f + v_{L/r}L)}{(f+L)} \ln\left(\frac{m_0}{m_0 - (f+L)t}\right) - gt$$

$m_f = \frac{dm_f}{dt}t = ft$ $t = \frac{m_f}{f+L}$

$$v = \left(\frac{v_{f/r}f + v_{L/r}L}{(f+L)}\right) \ln\left(\frac{m_0}{m_0 - m_f}\right) - g\frac{m_f}{f+L}$$

$v = 16\,446 \, m/s$