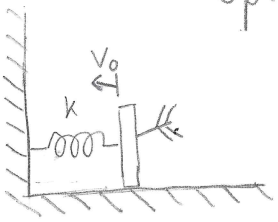
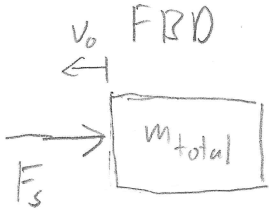
20-R-VIB-DY-49 Beginner

A target, with $m = 5\,kg$, in an archery range is attached to a wall with a spring, $k = 50\,N/m$. An archer shoots a $m = 1\,kg$ arrow which sticks into the target. As a result of an impact with the arrow, the target has a horizontal initial velocity $v_0 = 5\,m/s$. Determine the maximum amplitude and the displacement at $t = 10s$.



Solution:



$$m_{total} = m_{target} + m_{arrow} = 6kg$$

$$\Sigma F_x = -ma_x$$

$$kx + m\ddot{x} = 0$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$x = A\sin\omega_n t + B\cos\omega_n t$$

$$\dot{x} = A\omega_n\cos\omega_n t - B\omega_n\sin\omega_n t$$

$$\omega_n = \sqrt{\frac{k}{m}} = 2.887$$

@ $t = 0$ $\quad x_0 = B$ $\quad v_0 = A\omega_n$

$$A = \frac{V_0}{\omega_n}$$

$\downarrow$

max amplitude $= 1.732\,m$

$$x(t) = \frac{V_0}{\omega_n}\cos\omega_n t$$

@ $t = 10$

$= -1.436$