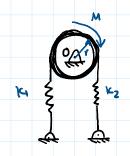20-R-WE-DK-8 **Intermediate**   Principle of Work and Energy

Inspiration: Example 18.2

Students are working on a self-righting balance system consisting of a disk and two springs with spring constants $k\_1 = 10$ N/m and $k\_2 = 5$ N/m. If the disk with mass $m = 15$ kg and radius $r = 0.4$ m is subjected to a constant couple moment $M = 5$ Nm, determine the angle through which the disk must rotate to achieve an angular velocity of $omega = 1$ rad/s. Both springs are initially unstretched.

$T_1 = 0 \quad V_1 = 0 \quad$ Unstretched and not moving

$T_2 = \frac{1}{2} I_0 w^2 = \frac{1}{2}\left(\frac{1}{2}(15)(0.4)^2\right)(1^2) = 0.6$

$V_2 = \frac{1}{2}k_1 s^2 + \frac{1}{2}k_2 s^2 \qquad S = r\theta = 0.4\theta$

$\quad = \frac{1}{2}(10)(0.4\theta)^2 + \frac{1}{2}(5)(0.4\theta)^2$

$T_1 + V_1 + \sum_{non\ cons} U_{1\to 2} = T_2 + V_2 \qquad U_M = M\theta$

$0 + 0 + U_M = T_2 + V_2 \qquad U_M - V_2 = T_2$

$5\theta - 0.8\theta^2 - 0.4\theta^2 = 0.6 \qquad -1.2\theta^2 + 5\theta - 0.6 = 0$

$\dfrac{-5 \pm \sqrt{25 - 4(-1.2)(-0.6)}}{2(-1.2)} \qquad \dfrac{-5 \pm \sqrt{55.3}}{-2.4}$

$\boxed{\theta = 0.12367}, \ 4.042996003$