20-R-IM-PT-6

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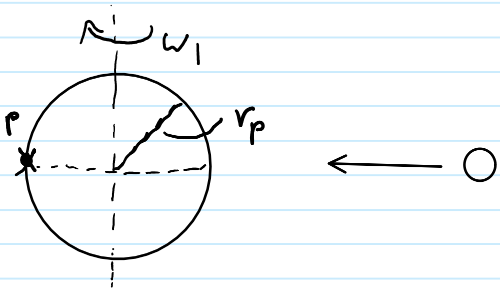
An uninhabited rogue planet is spinning in space with a mass of 11944*10^24 kg, and a radius of 12742km. It is spinning with an angular velocity of 5.402*10^-5 rad/s clockwise, when an asteroid collides and sticks with the edge of the planet. The mass of the asteroid is 2.9*10^21 kg, and collides with a speed of 33,528 m/s. The asteroid collides with the planet along its equator, in the same direction as its rotation. What is the new angular velocity of the planet?

Assume the asteroid is a point mass.



$$(H_{sys})_1 = (H_{sys})_2$$

$m_a = 2.9 \cdot 10^{21}\ kg$
$V_a = 33528\ m/s$
$r_p = 12742\ km = 12742000\ m$
$\omega_1 = 5.402 \times 10^{-5}\ rad/s$
$m_p = 11.944 \times 10^{24}\ kg$

$$I_{p_1} \omega_1 + (m_a \cdot V_a \cdot r_p) = I_{p_2} \omega_2$$

$$I_{p_1} = \frac{2}{5} m_p r_p^2 = \tfrac{2}{5}(11.944 \times 10^{24})(12742000\,m)^2$$
$$= 7.757 \times 10^{38}$$

$$I_{p_2} = \frac{2}{5}(m_p + m_a)(r_p^2) = \tfrac{2}{5}(2.9 \times 10^{21} + 11.944 \times 10^{24}) \cdot (12742000)^2$$
$$= 7.759 \times 10^{38}$$

$$\omega_2 = \frac{I_{p_1} \omega_1 + (m_a \cdot v_a \cdot r_p)}{I_{p_2}}$$

$$= \frac{7.757 \times 10^{38} \cdot 5.402 \times 10^{-5} + 2.9 \times 10^{21} \cdot 33528 \cdot 12742000}{7.759 \times 10^{38}}$$

$$\omega_2 = 5.56\ rad/s$$

The new angular velocity is 5.56 rad/s in the clockwise direction