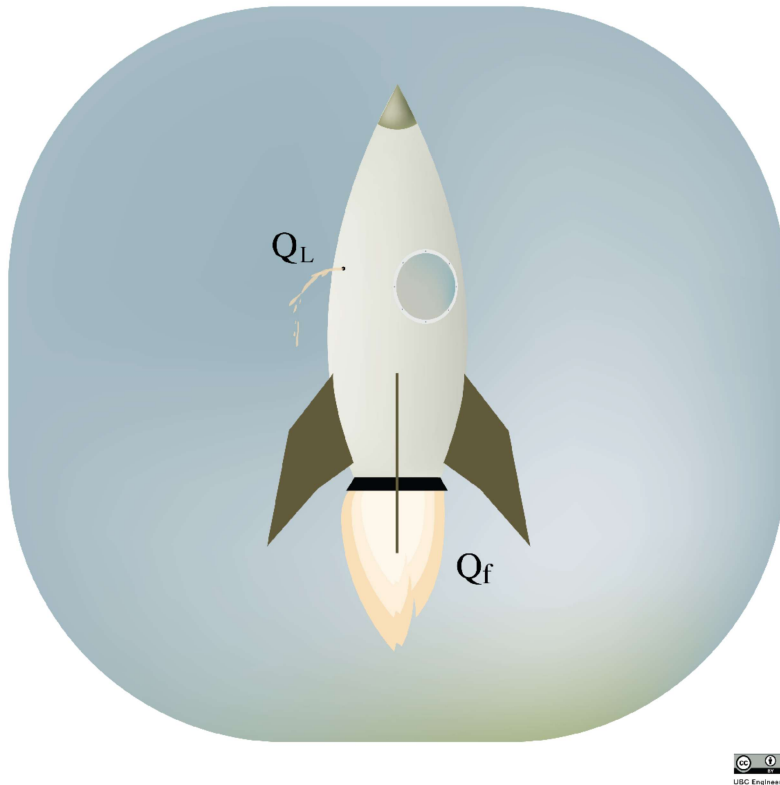


20-P-MOM-DY-41

A rocket with mass (including fuel) m_0 kg has m_f kg of its mass dedicated to fuel. During flight, it is discovered that there is a puncture that allows Q_L kg/s of fuel to leak from the system at v_L m/s relative to the rocket. The rocket engine ejects fuel at Q_f kg/s with velocity v_f m/s relative to the rocket. Assuming up to be positive, determine the magnitude of the maximum velocity of the rocket if the rocket begins at rest. Ignore the effect of altitude on the weight of the rocket, and drag forces. Assume the propulsion of the leaking fuel is negligible towards the system.



Solution:

$$\text{Recall: } \Sigma F_{cv} = m \frac{dv}{dt} - v_{D/e} \frac{dm_i}{dt}$$

$$\rightarrow -W = m \frac{dv}{dt} - Q_f v_f$$

In order to solve for v_{max} , we rearrange the equation and integrate, using the information that $v(0) = 0$:

$$\int_0^v dv' = \int_0^t \left(\frac{Q_f v_f}{m} - g \right) d\tau$$

Since $m = (m_0 - Q_t t)$, where $Q_t = Q_f + Q_L$, we get $\int_0^v dv' = \int_0^t \left(\frac{Q_f v_f}{m_0 - Q_t t} - g \right) d\tau$

$$\rightarrow v(t) = \left[-\frac{Q_f v_f}{Q_t} \ln(m_0 - Q_t \tau) - g\tau \right]_0^t$$

$$\rightarrow v(t) = \frac{Q_f v_f}{Q_t} \ln\left(\frac{m_0}{m_0 - Q_t t}\right) - gt$$

The time t' needed to consume all the fuel is $t' = \frac{m_f}{Q_t}$, so:

$$v_{max} = v(t') = \frac{Q_f v_f}{Q_t} \ln\left(\frac{m_0}{m_0 - m_f}\right) - \frac{gm_f}{Q_t} = \frac{Q_f v_f}{Q_f + Q_L} \ln\left(\frac{m_0}{m_0 - m_f}\right) - \frac{gm_f}{Q_f + Q_L}$$