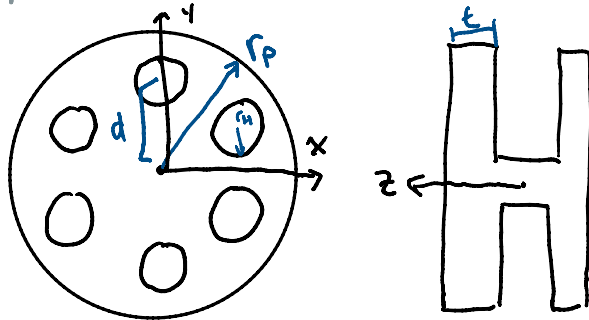


20-R-KIN-DK-5

05-26-5 Intermediate Composite Bodies Homework

Inspiration: None



A film reel consists of two circular plates connected by a cylindrical core. The core has a radius of $r = 4.5 \text{ cm}$ and a height of $h = 8 \text{ cm}$, while the plates have a radius of $r_P = 20 \text{ cm}$. Each plate has 6 holes punched into it, each with a radius of $r_H = 4 \text{ cm}$ placed $d = 12 \text{ cm}$ away from the center of the plate. Calculate the moment of inertia of the film reel if it rotates about the z axis from its center. Take the density of the material to be $\rho = 3000 \text{ kg/m}^3$ and the thickness of each plate as $t = 3 \text{ cm}$.

Moment of inertia of cylinder: $I_{zz} = \frac{1}{2} m r^2$

Masses

Plate: $m = \rho V = 3000 (\pi (0.2)^2 (0.03)) = \frac{18}{5} \pi$

Core: $m = \rho V = 3000 (\pi (0.045)^2 (0.08)) = \frac{243}{500} \pi$

Hole: $m = \rho V = 3000 (\pi (0.04)^2 (0.03)) = \frac{18}{125} \pi$

Inertia

Plate: $I_{zz} = \frac{1}{2} \left(\frac{18}{5} \pi \right) (0.2)^2 = \frac{9}{125} \pi$

Core: $I_{zz} = \frac{1}{2} \left(\frac{243}{500} \pi \right) (0.045)^2 = 0.001545899$

Hole: $I = \frac{1}{2} m r^2 + m d^2 = \frac{1}{2} \left(\frac{18}{125} \pi \right) (0.04)^2 + \left(\frac{18}{125} \pi \right) (0.12)^2 = 0.054648632$

$I = 2I_{\text{plate}} + I_{\text{core}} - 6I_{\text{hole}} = 0.126043446$