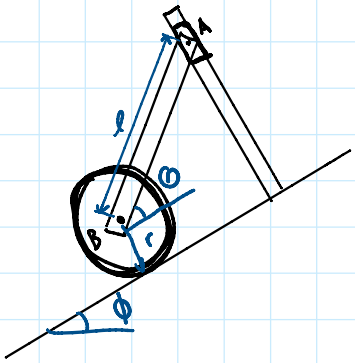
20-R-WE-DK-24

July 30, 2020 10:54 AM

20-R-WE-DK-24 Advanced Principle of Work and Energy

Inspiration: 18-66 Hibbeler

For an experiment, your professor assembles a system as shown, consisting of a ***30 kg*** disk, a ***12 kg*** slender rod, and a ***5 kg*** smooth collar. The goal of the experiment is to find information on the collar at given intervals. If the disk rolls without slipping, determine the velocity of the collar at the instant where ***theta = 30 degrees***. Assume the system is released from rest at ***theta = 45 degrees***. The hill has an incline of ***phi = 30 degrees***, the rod has length ***l = 2 m***, and the radius of the disk is ***r = 0.5 m***. Assume the system is frictionless.



Released from rest $\rightarrow T_1 = 0$

State 2

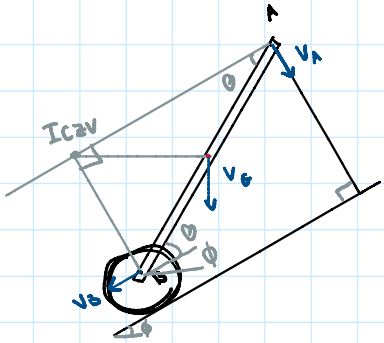
$r_{A/IC} = l\cos\theta = 2\cos 30 = \sqrt{3}$

$r_{B/IC} = l\sin\theta = 2\sin 30 = 1$

$r_{G/IC} = \sqrt{(l\sin\theta)^2 + (\frac{l}{2})^2 - 2(l\sin\theta)(\frac{l}{2})\cos(90-\theta)}$

$= \sqrt{1^2 + 1^2 - 2(1)(1)\cos 60} = 1$

State 2 @ $\theta = 30°$



$v_{A_2} = \omega r_{A/IC} = \omega\sqrt{3}$ $\quad \omega = \frac{v_{A_2}}{\sqrt{3}}$

$v_{B_2} = \omega r_{B/IC} = \frac{v_{A_2}}{\sqrt{3}}(1)$ $\quad v_{B_2} = \frac{v_{A_2}}{\sqrt{3}}$

$v_{G_2} = \omega r_{G/IC} = \frac{v_{A_2}}{\sqrt{3}}(1)$ $\quad v_{G_2} = \frac{v_{A_2}}{\sqrt{3}}$

Rolling without slipping: $v_{B_2} = \omega_{disk} r_{disk} \Rightarrow v_{B_2} = \frac{v_{A_2}}{\sqrt{3}} = \omega_{disk}(0.5)$

$\frac{2v_{A_2}}{\sqrt{3}} = \omega_{disk}$

$T_2 = \frac{1}{2}m v_G^2 + \frac{1}{2}I_G\omega^2 + \frac{1}{2}m_{Disk}v_{B_2}^2 + \frac{1}{2}I_B\omega_{disk}^2 + \frac{1}{2}m_{collar}v_{A_2}^2$

$= \frac{1}{2}(12)(\frac{v_{A_2}}{\sqrt{3}})^2 + \frac{1}{2}(\frac{1}{12}(12)(2^2))(\frac{v_{A_2}}{\sqrt{3}})^2 + \frac{1}{2}(30)(\frac{v_{A_2}}{\sqrt{3}})^2 + \frac{1}{2}(\frac{1}{2}(30)(0.5^2))(\frac{2v_{A_2}}{\sqrt{3}})^2 + \frac{1}{2}(5)v_{A_2}^2$

$= 12.\overline{666}\, v_{A_2}^2$

Set datum to be where the disk is at state 2

$s = l\cos\theta_2 - l\cos\theta_1 = 2\cos 30 - 2\cos 45 = \sqrt{3} - \sqrt{2} \approx 0.3178$

$h_{B_1} = s\sin\phi = (\sqrt{3}-\sqrt{2})\sin 30 = 0.1589$

$h_{B_2} = 0$

$h_{G_1} = s\sin\phi + \frac{l}{2}\sin(\phi+\theta_1) = (\sqrt{3}-\sqrt{2})\sin 30 + 1\sin 75 = 1.1248$

$h_{G_2} = \frac{l}{2}\sin(\phi+\theta_2) = 1\sin 60 = 0.8660$

$h_{A_1} = s\sin\phi + l\sin(\phi+\theta_1) = (\sqrt{3}-\sqrt{2})\sin 30 + 2\sin 75 = 2.0408$

$h_{A_2} = l\sin(\phi+\theta_2) = 2\sin 60 = 1.7321$

$V_{B_1} = m_{disk}g h_{B_1} = 30(9.81)(0.1589) = 46.77$

$V_{B_2} = m_{disk}g h_{B_2} = 0$

$V_{G_1} = m_{rod}g h_{G_1} = (12)(9.81)(1.1248) = 132.42$

$V_{G_2} = m_{rod}g h_{G_2} = (12)(9.81)(0.8660) = 101.95$

$V_{A_1} = m_{collar}g h_{A_1} = (5)(9.81)(2.0408) = 102.55$

$V_{G2} = m_{rod}\, g\, h_{G2} = (12)(9.81)(0.6660) = 101.45$

$V_{A1} = m_{roller}\, g\, h_{A1} = (5)(9.81)(2.0904) = 102.55$

$V_{A2} = m_{roller}\, g\, h_{A2} = (5)(9.81)(1.7321) = 84.96$

$T_1 + V_1 = T_2 + V_2$

$0 + 46.77 + 132.42 + 102.55 = 12.\overline{66}\, v_{A_2}^2 + 101.45 + 84.96$

$v_{A_2} = 2.7362 \text{ m/s}$