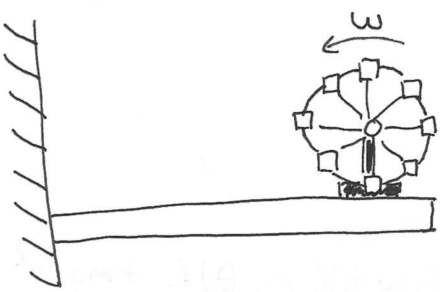
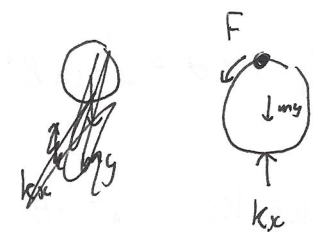
20-R-VIB-DY-16 Intermediate

The world's smallest Ferris ~~wheel~~ wheel ($m = 20kg$) is mounted on the end of a horizontal beam for no apparent reason. The wheel is mounted eccentrically in such a way that the eccentricity is equivalent to a mass of 5kg located 0.15m away from the axis of rotation. The static weight of the Ferris wheel causes a deflection of 20mm in the beam. Given that the wheel spins at a rate of 15 rad/s, find the steady-state amplitude of vibration.



Solution:



$$k = \frac{F}{\Delta y} = \frac{mg}{\Delta y} = \frac{20(9.81)}{0.02} = 9810 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{9810}{20}} = 22.147 \text{ rad/s}$$

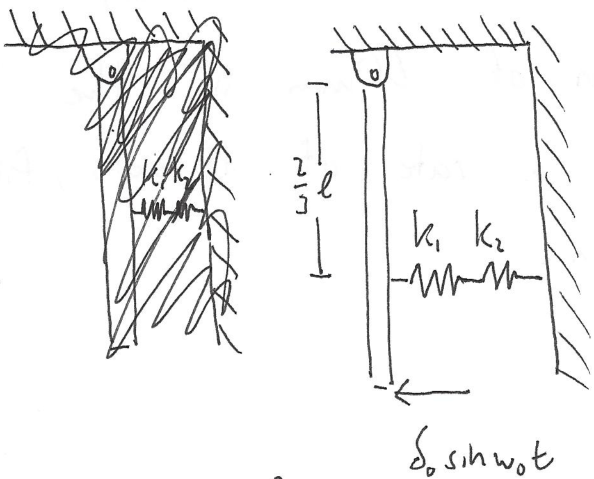
$$F_0 = mr\omega^2 = (5)(0.15)(15)^2 = 168.75 \text{ N}$$

$$x_{p_{amp}} = \left| \frac{F_0/k}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right| = \frac{168.75/9810}{1 - \left(\frac{15}{22.147}\right)^2}$$

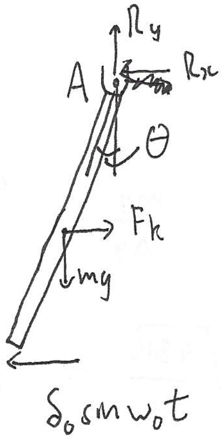
$$= 0.0318 \text{ m}$$

20-R-VIB-DY-17 Intermediate

A rod is pinned to the ceiling. 2/3 of the length down, it is connected to a series of springs horizontally. The springs have a spring constant of 50 N/m & 75 N/m. Given that the rod end is periodical distrurbed $\delta = \delta_0 \sin w_0 t$, find the steady-state vibration amplitude.



Solution:



$$\Sigma M_A = I_A \alpha \quad \left(-\tfrac{2}{3}\delta_0 k \sin w_0 t\right)(\cos\theta)l + mg\tfrac{2}{3}l\sin\theta + F_k\cos\theta = -\tfrac{1}{3}ml^2\ddot{\theta}$$

$$k = \frac{k_1 k_2}{k_1 + k_2} \qquad F_k = ks \qquad s = r\theta = \tfrac{2}{3}l\theta$$

small angle assumption

$$\delta_0 \rightarrow \tfrac{2}{3}k\delta_0 = F$$

$$\cancel{\tfrac{2}{3}\delta_0 k \sin w_0 t + k\tfrac{2}{3}l\theta = -\tfrac{1}{3}ml^2\ddot{\theta}}$$

~~mg~~

$$\ddot{\theta} + 2\theta\left(\frac{g}{l} + \frac{2k}{m}\right) = \frac{2\delta_0 k}{lm}\sin w_0 t \quad \text{after reducing}$$

$$\theta_p = C\sin wt$$

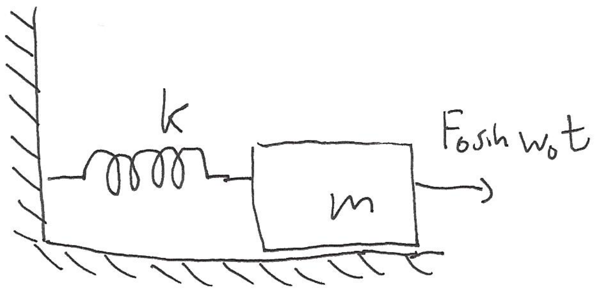
$$-Cw^2\sin wt + 2(C\sin wt)\left(\frac{g}{l} + \frac{2k}{m}\right) = \frac{2\delta_0 k}{lm}\sin wt$$

$$\ddot{\theta}_p = -Cw^2\sin wt$$

$$C = \frac{2\delta_0 k}{lm\left[2\left(\frac{g}{l} + \frac{2k}{m}\right) - w^2\right]}$$

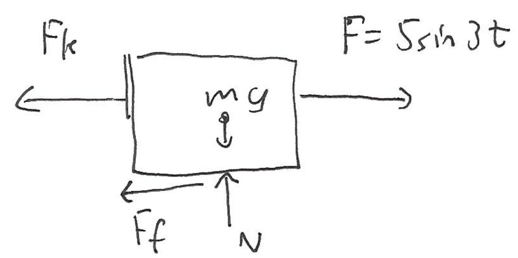
20-R-VIB-DY-18 Intermediate

A periodic force $F = 5\sin 3t$ is applied to a 5kg load, which is connected to a 10N/m spring. Given that the floor's coefficient of friction is $\mu = 0.5$, what is the max amplitude of the steady-state function.



Solution:

FBD



$$\Sigma F_x: \quad F_0 \sin w_0 t - F_k - F_f = m a_x$$

$$F_0 \sin w_0 t - kx - \mu m g = m\ddot{x}$$

$$F_0 \sin w_0 t = m\ddot{x} + kx + \mu m g$$

$$x_p = C \sin w_0 t$$

$$\ddot{x}_p = -C w_0^2 \sin w_0 t$$

$$F_0 \sin w_0 t = -m C w_0^2 \sin w_0 t + k C \sin w_0 t + \mu m g$$

$$C [k - m w_0^2] = F_0 - \frac{\mu m g}{\sin w_0 t}$$

$$C = \frac{F_0 - \frac{\mu m g}{\sin w_0 t}}{[k - m w_0^2]} \underset{\sin\theta = 0}{\overset{@\ max}{=}} \frac{F_0}{[k - m w_0^2]}$$

$$= -0.143 \text{ m}$$