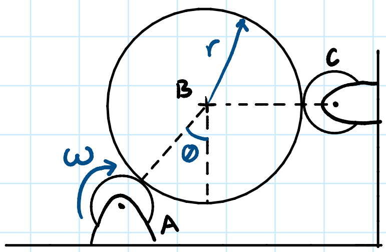
20-R-KIN-DK-40

July 22, 2020 9:39 AM

20-R-KIN-DK-40 Advanced Rotation (RBK)

Inspiration: 17-74



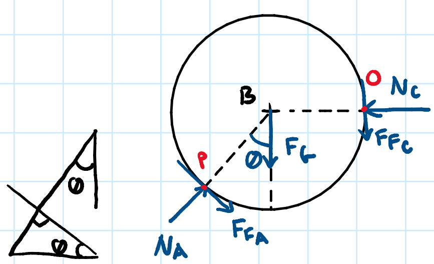
The disk B has a mass of ***m = 5*** and is initially at rest when it is placed into contact with rotor A and roller C. If disk B has a radius ***r_disk = 1 m*** and rotor A spins at a constant ***omega = 3 rad/s***, determine the angular acceleration of disk B at the instant contact is made. The point of tangency between A and B is at an angle of ***theta = 30 degrees*** with the vertical. The coefficients of kinetic and static friction are given as ***mu_k = 0.25*** and ***mu_s = 0.3*** between all contacting surfaces respectively. Roller C starts from rest and rotor A was spinning before contact. Both the rotor and the roller have a radius ***r = 0.2 m***.

$$I_B = \frac{1}{2} m r^2 = \frac{1}{2}(5)(1)^2 = 2.5$$

$\sum F_x$: $N_A \sin\theta + F_{FA}\cos\theta - N_C = 0$

$\sum F_y$: $N_A \cos\theta - F_{FA}\sin\theta - F_G - F_{FC} = 0$

$\sum M_B$: $I_G \alpha = F_{FA} r - N_C r$



There will be slipping at point P (between A and B) as they initially have different velocities $\Rightarrow$ impossible for it to be rolling without slipping $F_{FA} = \mu_K N_A$

Assume rolling without slipping at point O first (between B and C)

$F_{FC} \le \mu_S N_C$

$v_O = 0$ $\quad \vec{a}_O = \vec{a}_C + \vec{\alpha}_C \times \vec{r}_{O/C} - \omega_C^2 \vec{r}_{O/C}$

$= 0 + 0 - 0 = 0$

$\vec{a}_B = \vec{a}_O + \vec{\alpha}_B \times \vec{r}_{B/O} - \omega_B^2 \vec{r}_{B/O}$

$= 0 + \alpha_B \hat{k} \times (-1\hat{i}) - 0$

$0 = -\alpha_B \hat{j}$ $\quad \alpha_B = 0$

$0 = 0.25 N_A (1) - N_C (1)$ $\quad N_C = 0.25 N_A$

$N_A \sin 30 + 0.25 N_A \cos 30 - 0.25 N_A = 0$ $\quad N_A = 0$

$N_A \cos 30 - 0.25 N_A \sin 30 - (5)(9.81) = F_{FC}$

$F_{FC} = -49.05$ $\quad 49.05 \le 0$ ✗

$\therefore$ slipping at P and slipping at O

$\sum F_x$: $N_A \sin 30 + 0.25 N_A \cos 30 - N_C = 0$

$\sum F_y$: $N_A \cos 30 - 0.25 N_A \sin 30 - (5)(9.81) - 0.25 N_C = 0$

$\sum M_B$: $2.5\alpha = 0.25 N_A (1) - 0.25 N_C (1)$

$N_C = \frac{4+\sqrt{3}}{8} N_A$

$N_A \cos 30 - 0.25 N_A \sin 30 - 0.25 \left(\frac{4+\sqrt{3}}{4} N_A\right) = (5)(9.81)$

$N_A = 87.293296 \text{ N} \qquad N_C = 62.5462$

$\boxed{\alpha = 2.4747 \text{ rad/s}^2}$