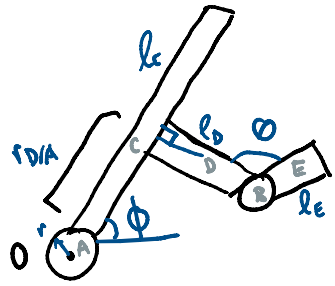
20-R-KIN-DK-12

July 22, 2020 9:46 AM

20-R-KIN-DK-12 Intermediate Radius of Gyration

Inspiration: None



An engineer puts together a form study prototype of a robotic arm to show a group of stakeholders. Specifically they want to know about its radius of gyration. Unfortunately, he forgot what material he used. If the mass moment of inertia of the arm is ***I = 15.2 kgm^2*** about point O, calculate the radius of gyration. Each component is a plate with thickness ***t = 5mm***. Assume the plates are rigidly attached to one another.
Plate A is identical to plate B, and has a radius ***r = 2w***.
Plates C, D, and E have the same width ***w = 15 cm***.
Plate C has a length ***l_C = 1.1 m***, and is angled at phi = 30 degrees with the horizontal.
Plate D is attached perpendicular to plate C at a distance ***r_D/A = 0.55 m*** from plate A, and has a length ***l_D = 0.3 m***
Plate E has a length ***l_E = 0.21 m***, and is angled theta = 105 degrees away from plate D.

Disk A: $I_{zz} = \frac{1}{2}mr^2$ $\quad m = \rho V = \rho\pi r^2 h = \rho\pi(0.3)^2(0.005)$

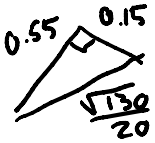
$= \frac{1}{2}\rho\pi(0.3)^4(0.005)$

Plate C: $m = \rho V = \rho(1.1 \times 0.15 \times 0.005) = 0.000825\rho$

$I_{zz} = \frac{1}{12}m(a^2+b^2)$

$I_{OC} = \frac{1}{12}(0.000825\rho)(1.1^2+0.15^2) + 0.000825\rho(0.55)^2$

Plate D:



$m = \rho V = \rho(0.3 \times 0.15 \times 0.005) = 0.000225\rho$

$I_{OD} = \frac{1}{12}(0.000225\rho)(0.3^2+0.15^2) + 0.000225\rho\left(\frac{13}{40}\right)$

Disc B: $m = \rho V = \rho\pi(0.3)^2(0.005)$

$I_{OB} = \frac{1}{2}\rho\pi(0.3)^4(0.005)$

$+\rho\pi(0.3)^2(0.005)(53/80)$

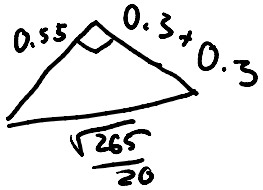
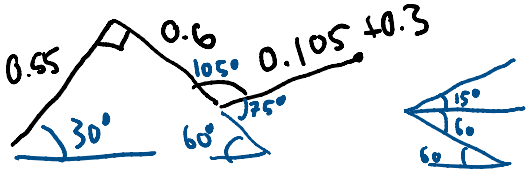


Plate E:



x: $0.55\cos30 + 0.6\cos60 + 0.405\cos15$

y: $0.55\sin30 + 0.6\sin60 + 0.405\sin60$

$d^2 = 2.674428073$

$m = \rho V = \rho(0.21 \times 0.15 \times 0.005)$

$= 0.0001575\rho$

$I_{OE} = \frac{1}{12}(0.0001575\rho)(0.21^2+0.15^2) + 0.0001575\rho(2.674428073)$

$$I = I_{OA} + I_{OB} + I_{OC} + I_{OD} + I_{OE}$$

$$15.2 = 0.0018645528\,p \qquad p = 8018.872379$$

$$m = 32.3556154 \text{ kg} \qquad k = \sqrt{\frac{I}{m}} = \boxed{0.6854}$$