20-P-KM-AF-019

Curvilinear Motion : Intermediate

Q: Traffic circles are very common and are often expressed by equations. If a circle follows the eqⁿ $4x^2 + y^2 = 4$ and the max velocity is C m/s. What are minimum and maximum acceleration felt by the car.

A: $4x^2 + y^2 = 4$

$y = \sqrt{4 - 4x^2}$

$dy/dx = \dfrac{-4x}{\sqrt{4 - 4x^2}}$

$d^2y/dx^2 = \dfrac{-16}{(4 - 4x^2)^{3/2}}$

$\rho = \dfrac{\left[1 + \left(-4x/\sqrt{4-4x^2}\right)^2\right]^{3/2}}{\dfrac{-16}{(4 - 4x^2)^{3/2}}}$

@ $(0, 2)$

$\rho = \dfrac{\left[1 + \left(0/\sqrt{4}\right)^2\right]^{3/2}}{\left|\dfrac{-16}{4^{3/2}}\right|} = 1/2$

$a = v^2/\rho = C^2/\rho = C^2/{}^1/_2 = 2C^2$

↑
max

$x = \sqrt{1 - y^2/4}$

$dx/dy = \dfrac{-y}{4\sqrt{1 - y^2/4}}$

$d^2x/dy^2 = -{}^1/_4 (1 - y^2/4)^{3/2}$

$\rho = \dfrac{\left[1 + \left(-y/4\sqrt{1 - y^2/4}\right)^2\right]^{3/2}}{\dfrac{-1}{4(1 - y^2/4)^{3/2}}}$

@ $(1, 0)$

$\rho = \dfrac{\left[1 + \left[0/4\sqrt{1}\right]^2\right]^{3/2}}{\left|\dfrac{-1}{4(1)^{3/2}}\right|} = 4$

$a = v^2/\rho = C^2/4$

↑
min