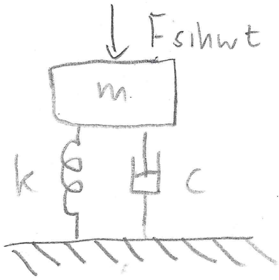
20-R-VIB-DY-47

An old spring and damper system is due for an upgrade. The mass $m = 10$ kg is supported mid-air by a spring, $k = 25$ N/m, and damper, $c = 25$ Ns/m. Determine a new damper that will decrease the amplitude by 50% and a new spring that will achieve the same effect. Force is applied vertically. $F = 2\sin 5t$.



Solution:

$$D = \frac{F_0/k}{\sqrt{\left[1-\left(\frac{w_0}{w_n}\right)^2\right]^2 + \left[2\frac{c}{c_c}\frac{w_0}{w_n}\right]^2}}$$

$$0.5D = \frac{F_0/k_{new}}{\sqrt{\left[1-\left(\frac{w_0}{w_n}\right)^2\right]^2 + \left[2\frac{c}{c_c}\frac{w_0}{w_n}\right]^2}}$$

$c_c = \sqrt{4mk} = \sqrt{1000}$

$w_n = \sqrt{\frac{k}{m}} = \sqrt{2.5}$

$$k_{new} = 2k = 50 \text{ N/m}$$

$$0.5D = \frac{F_0/k}{\sqrt{\left[1-\left(\frac{w_0}{w_n}\right)^2\right]^2 + \left[2\frac{c_{new}}{c_c}\frac{w_0}{w_n}\right]^2}}$$

$$2\sqrt{\left[1-\left(\frac{w_0}{w_n}\right)^2\right]^2 + \left[2\frac{c}{c_c}\frac{w_0}{w_n}\right]^2} = \sqrt{\left[1-\left(\frac{w_0}{w_n}\right)^2\right]^2 + \left[2\frac{c_{new}}{c_c}\frac{w_0}{w_n}\right]^2}$$

$$4\left(\left[1-\left(\frac{w_0}{w_n}\right)^2\right]^2 + \left[2\frac{c}{c_c}\frac{w_0}{w_n}\right]^2\right) = \left[1-\left(\frac{w_0}{w_n}\right)^2\right]^2 + \left[2\frac{c_{new}}{c_c}\frac{w_0}{w_n}\right]^2$$

$$3\left[1-\left(\frac{w_0}{w_n}\right)^2\right]^2 + \left[2\frac{c}{c_c}\frac{w_0}{w_n}\right]^2 = \left[2\frac{c_{new}}{c_c}\frac{w_0}{w_n}\right]^2$$

$$c_{new} = \frac{c_c w_n}{2w_0}\sqrt{3\left[1-\left(\frac{w_0}{w_n}\right)^2\right]^2 + \left[2\frac{c}{c_c}\frac{w_0}{w_n}\right]^2} = 16.37 \text{ Ns/m}$$