20-R-WE-DK-25

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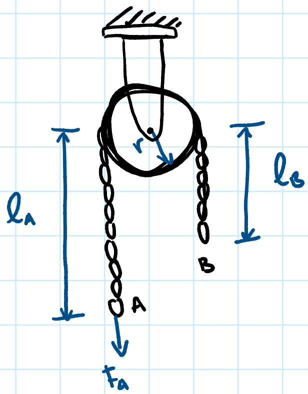
20-R-WE-DK-25 Intermediate Principle of Work and Energy

Inspiration: 18-56



For a summer, you've taken a job at your uncle's auto shop. You pull on the left side of a chain wrapped around a pulley with a force of ***F* = 50 N**. The pulley has a mass ***m* = 20 kg** and a radius ***r* = 0.2 m**. If the chain has a mass of ***3.4 kg per metre***, determine the angular velocity of the pulley after it has rotated ***theta* = 90 degrees**. There is ***l_A* = 3 m** of chain hanging off the left side and ***l_B* = 2 m** hanging off the right side of the pulley. Assume the chain does not slip and that the system was released from rest just before you pulled on it. Assume the pulley can be modelled as a disk.

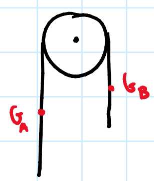
$I = \frac{1}{2}mr^2 = \frac{1}{2}(20)(0.2)^2 = 0.4$

Datum at center of pulley G_A G_B

$V_1 = -\frac{kg}{m} \times l_{A_1} g\left(\frac{l_A}{2}\right) - \frac{kg}{m} \times l_{B_1} g\left(\frac{l_B}{2}\right)$

$= -3.4(3)(9.81)\left(\frac{3}{2}\right) - 3.4(2)(9.81)(1)$

$= -216.801\ N\cdot m$



$\theta = \frac{s}{r}$ $\quad \theta = 90° \Rightarrow \frac{\pi}{2}$ $\quad r = 0.2$ $\quad \frac{\pi}{2} = \frac{s}{0.2}$ $\quad s = \frac{\pi}{10}$

when pulley rotates 90°, l_A will be extended $\frac{\pi}{10}$ and l_B will be shortened $\frac{\pi}{10}$

$V_2 = -\frac{kg}{m} l_{A_2} g\left(\frac{l_{A_2}}{2}\right) - \frac{kg}{m} \times l_{B_2} g\left(\frac{l_{B_2}}{2}\right)$

$= -3.4\left(3+\frac{\pi}{10}\right)(9.81)\left(\frac{3+\frac{\pi}{10}}{2}\right) - 3.4\left(2-\frac{\pi}{10}\right)(9.81)\left(\frac{2-\frac{\pi}{10}}{2}\right)$

$= -230.571376$

Rotate about axis and does not slip $\Rightarrow v = \omega r$

$T_1 = 0 \qquad T_2 = \frac{1}{2}I\omega^2 + \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$

$= \frac{1}{2}(0.4)\omega^2 + \frac{1}{2}\left(3.4\left(3+\frac{\pi}{10}\right)\right)(0.2\omega)^2 + \frac{1}{2}\left(3.4\left(2-\frac{\pi}{10}\right)\right)(0.2\omega)^2$

$= 0.2\omega^2 + 0.22536263\omega^2 + 0.11463717\omega^2$

$= 0.54\omega^2$

$T_1 + V_1 + \underset{\text{non-cons}}{\sum U_{1\to 2}} = T_2 + V_2$

$0 - 216.801 + (50)\left(\frac{\pi}{10}\right) = 0.54\omega^2 - 230.571376$

$\boxed{\omega = 7.368\ rad/s}$