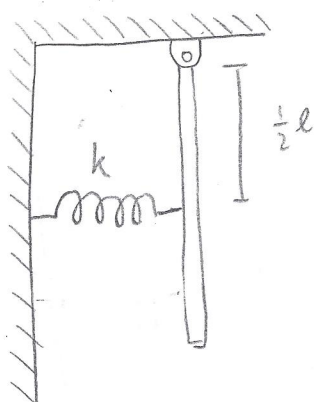
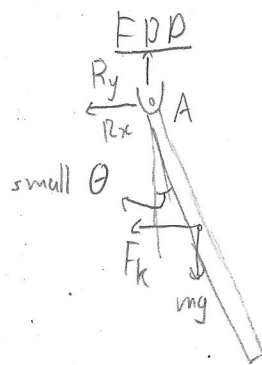


20-R-VIB-8 Intermediate

A $\frac{2m}{V}$ rod is pinned to the ceiling. Halfway down the length it is connected to a spring horizontally. The spring has a spring constant $k = 25 \text{ N/m}$. Given that the rod end is displaced a small angle, what is the natural frequency of the vibration.

rod $m = 5 \text{ kg}$

Solution:



$$\sum M_A = I_A \alpha$$

$$-F_k \frac{l}{2} - mg \frac{l}{2} \sin \theta = \frac{1}{3} m l^2 \ddot{\theta}$$

$$F_k = kx \quad x \approx r\theta \quad \sin \theta \approx \theta$$

$$\approx \frac{l}{2} \theta$$

$$k \frac{l}{2} \theta + \frac{mg}{2} \theta + \frac{1}{3} m l^2 \ddot{\theta} = 0$$

$$\frac{1}{3} m l^2 \ddot{\theta} + \frac{l}{2} (k + mg) \theta = 0$$

$$\ddot{\theta} + \frac{3}{2l} \left(\frac{k}{m} + g \right) \theta = 0$$

$$\omega_n = \sqrt{\left(\frac{3k}{2ml} + \frac{3g}{2l} \right)} = 3.77 \text{ rad/s}$$