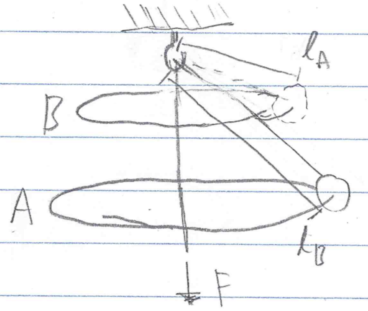
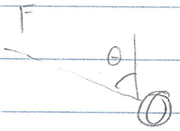
20-P-MOM-DY-26

A $m = 5$ kg mass is suspended by a rope and hook. The mass is given a horizontal speed $v = 3\,m/s$ such that it rotates around a circular path A. The downward force on the cord, F, increases so that the mass rises and follows the circular path B instead. The radius of the paths at A and B are $l_A = 0.75\,m$ and $l_B = 0.5\,m$ respectively. Determine the speed of the mass after it rises.





Solution:

$\Sigma F_y : F\cos\theta = mg$

$\Sigma F_n : F\sin\theta = m\left(\frac{v^2}{r_A}\right)$

$F = \frac{mg}{\cos\theta}$

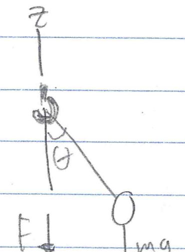
$F = m\frac{v^2}{\sin\theta\, r_A}$

$r_A = l_A \sin\theta$

$\frac{g}{\cos\theta} = \frac{v^2}{l_A \sin^2\theta}$ $\qquad \frac{1-\cos^2\theta}{\cos\theta} = \frac{v^2}{l_A g}$

$\cos^2\theta + \underbrace{\frac{v^2}{l_A g}}_{1.223}\cos\theta - 1 = 0$

$\theta = 55.9$



$(H_z)_1 = (H_z)_2$

$r_A m v_1 = r_B m v_2$ $\qquad v_2 = \frac{r_A m v_1}{r_B m}$ $\qquad r_B = l_B \sin\theta_2$

$v_2 = \frac{l_A \sin\theta\, v_1}{l_B \sin\theta_2}$ $\qquad v_2 \sin\theta_2 = \frac{l_A \sin\theta}{l_B} v_1 = 1.24$

$\frac{1-\cos^2\theta_2}{\cos\theta_2} = \frac{v_2^2}{l_B g} = \frac{1-\cos^2\theta_2}{\cos\theta_2} = \frac{0.313}{\sin^2\theta_2}$ $\qquad \frac{\sin^4\theta_2}{\cos\theta_2} = 0.313$

$1 - 2\cos^2\theta_2 + \cos^4\theta_2 - 0.313\cos\theta_2 = 0$ $\qquad \cos\theta_2 = 1.27764,\ 0.7239$

✗ ✓

$\theta_2 = 43.6°$

$v_2 = \frac{l_A \sin\theta\, v_1}{l_B \sin\theta_2} = 5.4\,m/s$