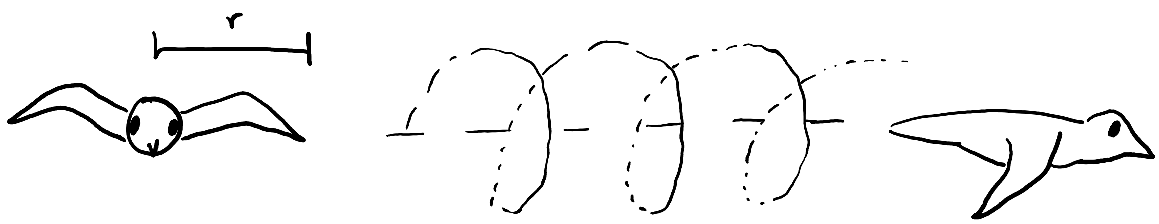
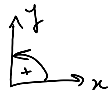
20-R-AM-JH-003-005

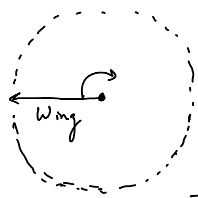
Friday, June 19, 2020 5:06 PM

20-R-AM-JH-003) A swallow with wingspan of 30cm is flying horizontally with a velocity of 10 m/s. The swallow decides to barrel roll, rotating about its central axis with minimal change in body posture and maintaining its horizontal velocity. If it completes 3 clockwise barrel rolls in 6 seconds, what is the average speed of its wingtips with respect to the ground?



a) Model:





1 roll every 2 seconds $\left\}\begin{array}{l} \vec{\alpha} = 0 \ \text{rad/s}^2 \\ \vec{\theta}_0 = 0 \ \text{rad} \\ \vec{\theta}_f = -2\pi \\ t = 2\ \text{s} \end{array}\right.$

$$\vec{\theta}_f = \vec{\theta}_0 + \vec{\omega}_0 t + \frac{1}{2}\vec{\alpha}t^2$$

$$2\pi = \vec{\omega}_0 (2)$$

$$\vec{\omega}_0 = \pi \hat{k} \ \frac{\text{rad}}{\text{s}}$$

$$\vec{v} = \vec{\omega}_0 \times \vec{r}$$

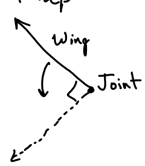
$$= \pi \hat{k} \times 0.15(-\hat{i})$$

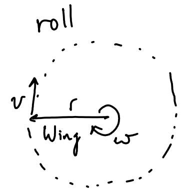
$$= 0.47 \frac{\text{m}}{\text{s}} (-\hat{j})$$

$$\therefore |\vec{v}| = 0.47 \frac{\text{m}}{\text{s}}$$

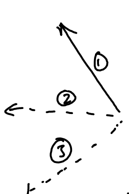
20-R-AM-JH-004) Now the swallow flaps its wings during the barrel rolls. Each flap cycle sweeps a total of 90 degrees symmetrically with respect to the horizontal. If the swallow completes three flap cycles (each cycle defined as 90 degrees down, 90 degrees up) over all three barrel rolls, what is its maximum wingtip velocity? Assume the wing rotates about a fixed joint at the centroid of the swallow and that its wingspan remains constant throughout. Each flap starts and ends at 0 rad/s.

b) Model: flap + roll





Contribution from flap:



① $\theta_1 = 0$ ② $\theta_2 = \frac{\pi}{4}$ ③ $\theta_3 = \frac{\pi}{2}$

$\omega_1 = 0$ $\omega_2 = ?$ $\omega_3 = 0$

3 flaps over 6s.

$\therefore$ ① → ③ in 1s

Assume symmetric flap

$t_{1-2} = t_{2-3} \therefore ① \rightarrow ② \text{ in } \frac{1}{2}s$.

$\omega_2 = \omega_1 + \alpha t_{1-2}$

(i) $\omega_2 = 0 + \alpha\left(\frac{1}{2}\right)$

$\omega_2^2 = \omega_1^2 + 2\alpha(\theta_2 - \theta_1)$

(ii) $\omega_2^2 = 0 + 2\alpha\left(\frac{\pi}{4} - 0\right)$

i→ii) $\left(\frac{1}{2}\right)^2\alpha^2 = 2\alpha\left(\frac{\pi}{4}\right)$

$\alpha = 2\pi \frac{rad}{s^2}$

$\therefore \omega_2 = \frac{1}{2}(2\pi) = \pi \frac{rad}{s}$

$V_2 = \omega_2 \times r = \pi \hat{k} \times 0.15(-\hat{i})$

$= 0.47(-\hat{j}) \frac{m}{s}$

Contribution from Roll:

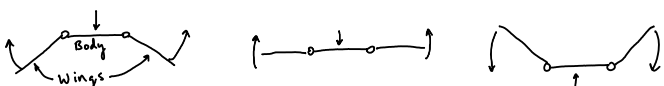
from a) $V_R = 0.47 \frac{m}{s}$

Max when flap & roll are in same direction

$\therefore V_{max} = 0.47 + 0.47 = 0.94 \frac{m}{s}$

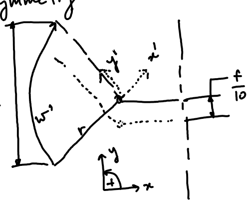
20-R-AM-JH-005) More realistically, the swallow's body reacts with each wing flap by moving in the opposite vertical direction. Now just traveling in a straight line with no rolling, the bird's body moves in the opposite direction of its wingflaps at 10% the total vertical distance spanned by each flap. If we model the bird as a central beam with pinned joints at each end. When both wings and body reach horizontal positions during an upwards flap, what is the wingtip velocity with respect to ground? Assume each flap occurs with an angular acceleration of 5 rad/s^2.

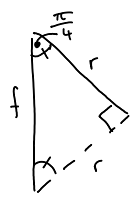
Model:



FRONT VIEWS

Symmetry section:

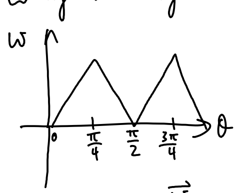


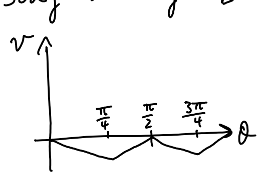


$f = \frac{r}{\sin\frac{\pi}{4}} = \sqrt{2}r = 0.283m$

$\therefore \frac{f}{10} = 0.0283m$

Wing velocity "v_w" Body Velocity "v_B"





Strategy: ① $\vec{V}_{tip/bod}$ $(\vec{v}_{t/b})$ ② $\vec{V}_{bod}$ ③ $\vec{V}_T = \vec{v}_{bod} + \vec{v}_{t/b}$

① $\vec{v}_{t/b} = \vec{\omega}' \times \vec{r}$

$\vec{\omega}'^2 = \vec{\omega}_0^2 + 2\alpha(\theta - \theta_0)$

For upward flap to horizontal position:

$f = 0.283m$

$r = 0.2m(\hat{i})$

$\vec{\omega}_0 = 0 \frac{rad}{s}$

$\vec{\theta}_0 =$

$\vec{\theta} =$

$\vec{\alpha} = 5\ rad/s^2\ (-\hat{k})$

$\therefore\ \vec{\omega}'^2 = 0^2 + 2(-5)\left(\frac{3\pi}{4} - \pi\right)$

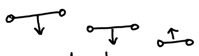
$= \sqrt{\frac{5\pi}{2}}\ \frac{rad}{s}\ (-\hat{k})$

$\vec{v}_{t/b} = \sqrt{\frac{5\pi}{2}}\ (-\hat{k}) \times 0.2\ m\ (\hat{i})$

② $\vec{v}_b$ at horizontal:

Rectilinear motion ($\vec{a}$ constant)

$\vec{v}^2 = \vec{v}_0^2 + 2a(s - s_0)$



at midpoint:

$s_0 = 0\ m$

$s = \frac{1}{2}\ \frac{f}{10} = \frac{f}{20} = 0.01415 m$

$v_0 = 0$

$a = ?$

From ①:

$t = \frac{\omega - \omega_0}{\alpha} =$

Then $s = \frac{1}{2} a t^2 + \vec{v}_0 t + s_0$

$a = \frac{2(s - s_0 - \vec{v}_0 t)}{t^2}$