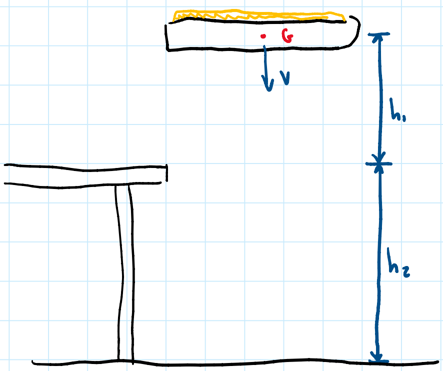
20-R-IM-DK-23

July 15, 2020 3:24 PM

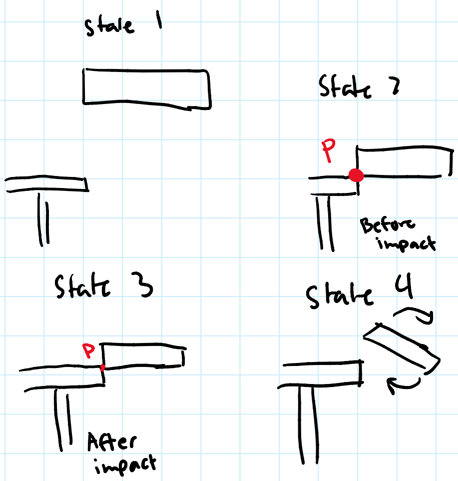
Intermediate Eccentric Impact

Inspiration: None

A mechanical engineering student woke up late and prepares his breakfast in a hurry. Right as he finishes buttering up his toast, he drops it. As the ***50 g*** toast falls, it hits the counter right along the edge of the toast. If the toast starts at a height ***h_1 = 20 cm*** from the top of the counter and falls an additional ***h_2 = 75 cm***, how far will the toast rotate? If it starts butter-side up, will it land butter-side down? Assume the toast can be treated as a 12cm x 12 cm thin plate and the counter is frictionless. Take the toast to start from rest. The coefficient of restitution is ***e = 0.8***.



Drawing not to scale



$T_1 + V_1 = T_2 + V_2$ Set counter as datum

$0 + mgh_1 = \frac{1}{2}mv_2^2 + 0$

$(0.05)(9.81)(0.2) = \frac{1}{2}(0.05)v_2^2$

$v_2 = 1.9809$ m/s

$e = \dfrac{V_{toast_3} - V_{counter_3}}{V_{counter_2} - V_{toast_2}}$ $\quad 0.8 = \dfrac{V_{toast3}}{-(-1.9809)}$ $\quad V_{toast_3} = 1.584727$

$(H_P)_2 = (H_P)_3$ Angular momentum is conserved about P only

$I_G\vec{\omega}_2 + \vec{r}_{G/P} \times m\vec{v}_{G2} = I_G\vec{\omega}_3 + \vec{r}_{G/P} \times m\vec{v}_{G3}$

$0 + (0.06\hat{i}) \times (0.05)(-1.9809\hat{j}) = \frac{1}{12}(0.05)(0.12)^2\omega_3\hat{k} + (0.06\hat{i}) \times (0.05)V_{G3}\hat{j}$

$-0.0059427\hat{k} = 0.00006\omega_3\hat{k} + 0.003(1.584727)\hat{k}$

$\omega_3 = -178.28135\hat{k}$ rad/s

$d = v_0 t + \frac{1}{2}at^2$ $\quad 0.75 = 1.584727t + 0.5(9.81)t^2$

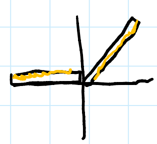
$t = -0.585, 0.262$

$\theta = \omega_0 t + \frac{1}{2}at^2$ $\quad \theta = (178.28135)(0.262) + 0$

$= \boxed{46.7097137 \text{ rad}}$

Toast rotates 7.434 revolutions

0.434 revs ⇒ 2.727 rad ⇒ 156.264



The toast will land butter-side down :(