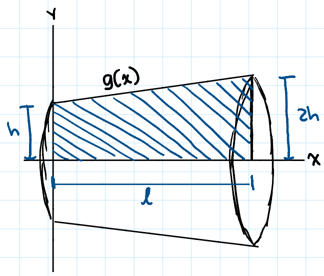
20-R-KIN-DK-43

July 22, 2020 9:40 AM

20-R-KIN-DK-43 Intermediate Radius of Gyration

Inspiration: 17-7

An engineering student has modelled a truncated cone in CAD software by rotating the coloured area about the x-axis. If the y-coordinate can be described by the equation ***g(x) = 1/3x + 1*** and the cup has constant density ***rho = 600 kg/m^3***, determine its radius of gyration about the x-axis. The cone has dimensions ***h = 1m*** and ***l = 3m***.



$dm = \rho\, dV$ dV depends on dx and the value of y at that instance

$= \rho\, \underbrace{\pi y^2}_{\text{area}}\, \underbrace{dx}_{\text{height}} = \rho\pi\left(\frac{1}{3}x+1\right)^2 dx = \rho\pi\left(\frac{1}{9}x^2 + \frac{2}{3}x + 1\right)dx$

Rotating about x-axis $\Rightarrow$ much like ring or disk $I_x = \frac{1}{2}mr^2$ ← radius is y in this case

$dI_x = \frac{1}{2}dm\, y^2 = \frac{1}{2}\rho\pi\left(\frac{1}{9}x^2 + \frac{2}{3}x+1\right)\left(\frac{1}{9}x^2 + \frac{2}{3}x+1\right)dx$

$= \frac{1}{2}\rho\pi\left(\frac{1}{81}x^4 + \frac{2}{27}x^3 + \frac{1}{9}x^2 + \frac{2}{27}x^3 + \frac{4}{9}x^2 + \frac{2}{3}x + \frac{1}{9}x^2 + \frac{2}{3}x + 1\right)dx$

$= 300\pi\left(\frac{1}{81}x^4 + \frac{4}{27}x^3 + \frac{2}{3}x^2 + \frac{4}{3}x + 1\right)dx$

$I_x = \int dI_x = 300\pi\int_0^3 \frac{1}{81}x^4 + \frac{4}{27}x^3 + \frac{2}{3}x^2 + \frac{4}{3}x + 1\, dx$

$= 300\pi(18.6) = 5580\pi = 17530.08701$

$m = 600\pi\int_0^3 \frac{1}{9}x^2 + \frac{2}{3}x + 1\, dx = 600\pi(7) = 4200\pi = 6597.344573$

$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{5580\pi}{4200\pi}} = \boxed{1.152636}$