20-R-KM-DK-12

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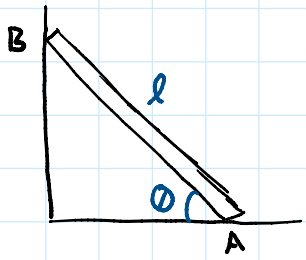
20-R-KM-DK-12 Beginner Acceleration (Relative Motion)



A handyman has left a ladder leaning on a wall which begins to slip. At a given instant, the top of the ladder has an acceleration ***a_B = 3m/s^2*** and a velocity of ***v_B = 5 m/s***, both acting downward. Determine the acceleration of the bottom of the ladder, A, and the ladder's angular acceleration at this instant. The length of the ladder is ***l = 10 m*** and forms an angle of ***theta = 30 degrees*** with the ground at A.

$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$ $\quad -5\hat{j} = v_A\hat{i} + \omega\hat{k} \times (-10\cos 30\,\hat{i} + 10\sin 30\,\hat{j})$

$-5\hat{j} = v_A\hat{i} - \omega(10\cos 30)\hat{j} - \omega(10\sin 30)\hat{i}$

$\hat{j}: \; -5 = -\omega(10\cos 30)$

$\omega = \frac{5}{10\cos 30} = \frac{\sqrt{3}}{3}$

$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$

$a_A\hat{i} = -3\hat{j} + \alpha\hat{k} \times (10\cos 30\,\hat{i} - 10\sin 30\,\hat{j}) - \left(\frac{\sqrt{3}}{3}\right)^2 (10\cos 30\,\hat{i} - 10\sin 30\,\hat{j})$

$a_A\hat{i} = -3\hat{j} + \alpha(10\cos 30)\hat{j} + \alpha(10\sin 30)\hat{i} - \frac{1}{3}(10\cos 30)\hat{i} + \frac{1}{3}(10\sin 30)\hat{j}$

$\hat{i}: \; a_A = \alpha(10\sin 30) - \frac{1}{3}(10\cos 30)$

$\hat{j}: \; 0 = -3 + \alpha(10\cos 30) + \frac{1}{3}(10\sin 30) \qquad \alpha = \frac{4\sqrt{3}}{45}$

$$\boxed{a_A = -\frac{11\sqrt{3}}{9}\ m/s^2}$$