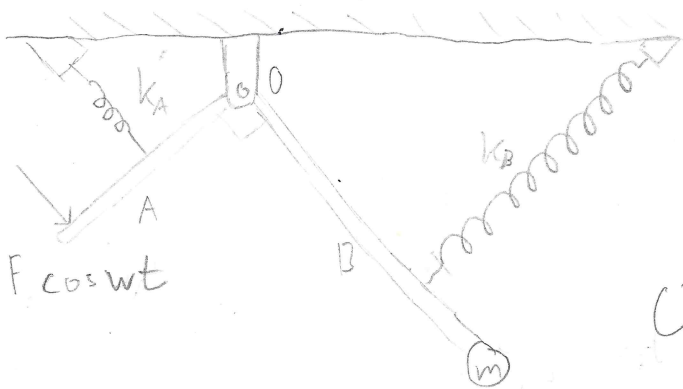20-R-VIB-DY-32   Advanced

A L-shaped bar of negligible mass is pinned to the
ceiling by its at point O. Arm A, length $\ell = 2m$, has
a spring $k_A = 10$ N/m attached and a force $F = 2\cos 3t$
applied at the end of the bar. Arm B, length 3m, has a
spring $k_B = 20$ N/m attached $\frac{2}{3}$ of the length down the bar
and a 5kg mass at the end.   Given
that initially the bar is at rest, find the angle at $t = 10s$.

Solution: $F_k = ks$ $s = r\theta$

$$\Sigma M_o = I_o \alpha \quad I_o = m\ell_B^2$$

$$0 = (F \cos w_s t)\ell_A - k_A \left(\frac{\ell_A}{2}\right)^2 \theta - k_B \left(\frac{2\ell_B}{3}\right)^2 \theta - \ell_B^2 m \ddot{\theta}$$

$$\theta_p = A \cos wt \quad \ddot{\theta}_p = -Aw^2 \cos wt$$

$$F \cos wt = \left[k_A \left(\frac{\ell_A}{2}\right)^2 + k_B \left(\frac{2\ell_B}{3}\right)^2\right] A \cos wt - (\ell_B^2 m) Aw^2 \cos wt$$

$$A = \frac{F}{\left[k_A \left(\frac{\ell_A}{2}\right)^2 + k_B \left(\frac{2\ell_B}{3}\right)^2 - \ell_B^2 m w^2\right]} = -0.00635$$

$$\theta(t) = A \sin wt = -0.00635 \sin 3t \qquad @ \ t=10 \quad \underline{\theta = 0.00627}$$