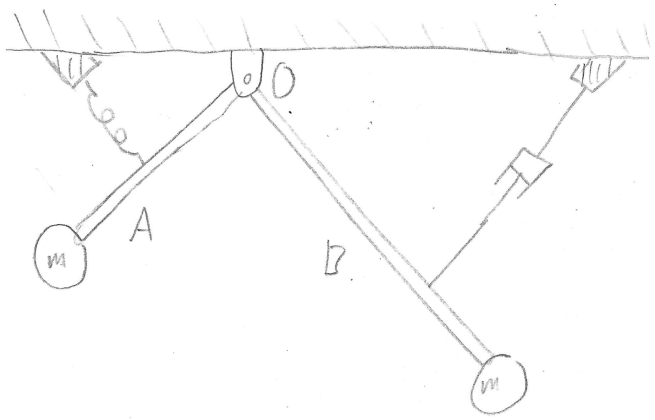
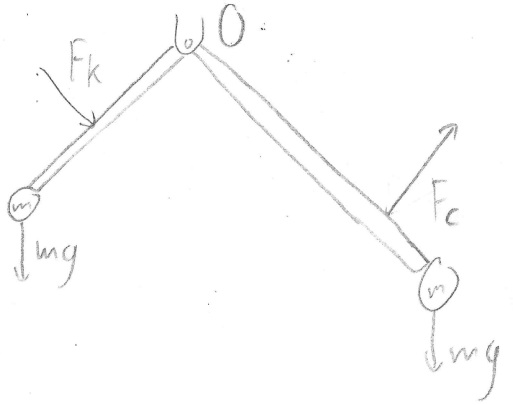
20-R-VIB-DY-45 Advanced

A L-shaped bar of negligible mass is pinned to the ceiling by the corner at point O. Arm A, length $\ell_A = 2m$, has a spring $k_A = 10$ N/m attached halfway and arm B, length $\ell_B = 3m$, has a damper $c_B = 20$ Ns/m attached 2/3rd of the length down the bar. Determine the mass of the balls at the end if the system is critically damped.



Solution: FBD



$$\Sigma M_O = I\alpha = 0 = \left(\frac{\ell_A}{2}\right)^2 k\theta + \left(\frac{2\ell_B}{3}\right)^2 c\dot{\theta} - \ell_A mg\theta + \ell_B mg\theta + m\ell_A^2\ddot{\theta} + m\ell_B^2\ddot{\theta}$$

$$\ddot{\theta}\underbrace{m(\ell_A^2+\ell_B^2)}_{m'} + \underbrace{\left(\frac{2\ell_B}{3}\right)^2 c}_{c'}\dot{\theta} + \theta\underbrace{\left(\ell_B mg - \ell_A mg + \frac{\ell_A^2}{4}k\right)}_{k'} = 0$$

$$c_c = \sqrt{4m'k'} = c' = 80$$

$$1600 = m(13) + m(-g) + 10)$$

$$1600 = g13m^2 + 130m \qquad m = \frac{-130 \pm \sqrt{130^2 + 4(13)(1600)}}{26}$$

$$13gm^2 + 130m - 1600 = 0$$

$$m = 17.169 \text{ kg}$$