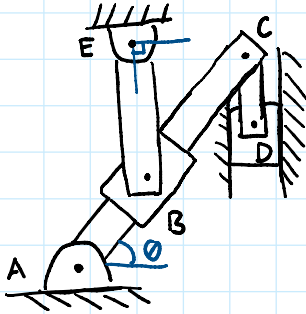


## 20-R-KM-DK-17 Intermediate Rotating Frame

Inspiration: 5.31 Example



A linkage system consists of several arms, a collar, and a block. Arm AC has a total length  $r_{AC} = 1 \text{ m}$  and, in the instant shown, is at an angle of  $\theta = 60 \text{ degrees}$  with the horizontal. The collar sits on arm AC at a distance  $r_{AB} = 0.6 \text{ m}$  and is connected to arm BE, which sits vertically and has a length of  $r_{BE} = 0.3 \text{ m}$ . If the collar slides up the arm at a relative velocity of  $v_{B/A\_rel} = 0.5 \text{ m/s}$ , determine the angular velocity and angular acceleration of arm BE.

$$V_A = 0 \quad \vec{V}_B = \vec{\Omega}_{AC} \times \vec{r}_{B/A} + (v_{B/A})_{x'y'z'} \quad \vec{V}_B = \vec{\omega}_{BE} \times \vec{r}_{B/E}$$

$$\vec{V}_B = \omega_{BE} \hat{k} \times (-0.3 \cos 30^\circ \hat{i} - 0.3 \sin 30^\circ \hat{j}) = -\omega_{BE} 0.3 \cos 30^\circ \hat{j} + 0.3 \omega_{BE} \sin 30^\circ \hat{i}$$

$$\vec{V}_B = \Omega_{AC} \hat{k} \times (0.6 \hat{i}) + 0.5 \hat{i} = 0.6 \Omega_{AC} \hat{j} + 0.5 \hat{i}$$

$$\hat{i}: 0.5 = 0.3 \omega_{BE} \sin 30^\circ \Rightarrow \boxed{\omega_{BE} = \frac{10}{3}}$$

$$\hat{j}: -\omega_{BE} 0.3 \cos 30^\circ = 0.6 \Omega_{AC} \Rightarrow \Omega_{AC} = -\frac{5\sqrt{3}}{6}$$

$$\begin{aligned} \vec{a}_B &= \vec{a}_A + \vec{\Omega}_{AC} \times \vec{r}_{B/A} + 2\vec{\Omega}_{AC} \times (\vec{v}_{B/A})_{x'y'z'} - \Omega_{AC}^2 \vec{r}_{B/A} + (\vec{a}_{B/A})_{x'y'z'} \\ &= 0 + \Omega_{AC} \hat{k} \times (0.6 \hat{i}) + 2\left(-\frac{5\sqrt{3}}{6}\right) \hat{k} \times (0.5 \hat{i}) - \left(-\frac{5\sqrt{3}}{6}\right)^2 (0.6 \hat{i}) + 0 \\ &= 0.6 \Omega_{AC} \hat{j} - 5\sqrt{3} \hat{j} - 1.25 \hat{i} \end{aligned}$$

$$\begin{aligned} \vec{a}_B &= \vec{a}_E + \alpha_{BE} \times \vec{r}_{B/E} - \omega_{BE}^2 \vec{r}_{B/E} \\ &= 0 + \alpha_{BE} \hat{k} \times (-0.3 \cos 30^\circ \hat{i} - 0.3 \sin 30^\circ \hat{j}) - \left(\frac{10}{3}\right)^2 (-0.3 \cos 30^\circ \hat{i} - 0.3 \sin 30^\circ \hat{j}) \\ &= -0.3 \alpha_{BE} \cos 30^\circ \hat{j} + 0.3 \alpha_{BE} \sin 30^\circ \hat{i} + \frac{10}{3} \cos 30^\circ \hat{i} + \frac{10}{3} \sin 30^\circ \hat{j} \end{aligned}$$

$$\hat{i}: -1.25 = 0.3 \alpha_{BE} \sin 30^\circ + \frac{10}{3} \cos 30^\circ \quad \boxed{\alpha_{BE} = -27.578 \text{ rad/s}^2}$$

$$\hat{j}: 0.6 \Omega_{AC} - 5\sqrt{3} = -0.3 \alpha_{BE} \cos 30^\circ + \frac{10}{3} \sin 30^\circ \quad \Omega_{AC} = 29.15330702 \text{ rad/s}^2$$