20-R-IM-PT-7(Solution)

July 20, 2020 10:42 PM

Two students are sitting on spinning chairs with their arms full extended, when they fully retract their arms. Assume that their legs are retracted too, having no effect on their rotation speed.

Student 1 weighs of 60kg and holds one weight that is 10kg in each arm. Assume that, when extended, student's 1 body on the chair is a cylinder of radius 0.22m, and the weights are negligible, and their arms are thin rods with a weight of 7 kg each and a length of 0.7m. When their arms are retracted, the system can be considered as a cylinder of 0.25m radius. The initial speed is 0.7 rad/s

Student 2 weighs of 55kg and holds two weights that are 12kg each. Assume, when extended, student's 2 body on the chair is a cylinder of radius 0.19m, and the weights are negligible size, and their arms are thin rods with a weight of 5 kg each and a length of 0.65m. When their arms are retracted, the system can be considered as a cylinder of 0.21m radius. The initial speed is 0.6 rad/s

Neglecting the frictional forces, and effects due to mass of the chairs, who completes 35 revolutions first, and in how much time does that student complete it?

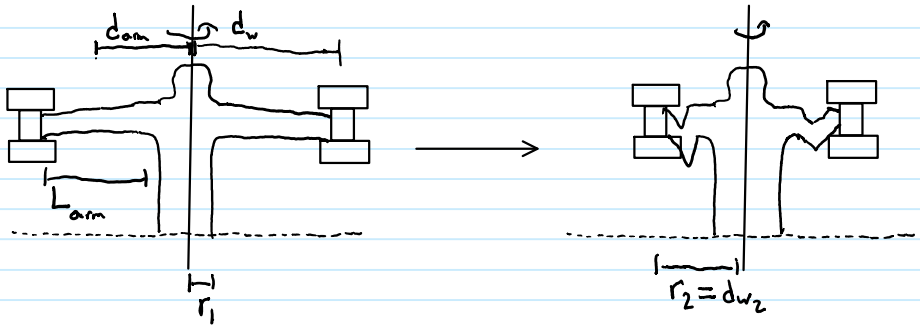

Image depicts upper section view of rotation

Both students:

$I_1 \omega_1 = I_2 \omega_2$

When Extended:

$I_1 = I_{weights} + I_{arms} + I_{body}$

$I_{weights} = 2 \cdot m_w \cdot d_w^2$

$I_{arms} = 2 \cdot \frac{1}{12} \cdot m_{arm} \cdot L_{arm}^2 + 2 \cdot m_{arm} \cdot d_{arm}^2$

$I_{body} = \frac{1}{2} m_{body} r_1^2$

$I_1 = 2 \cdot m_w \cdot d_w^2 + 2 \cdot \frac{1}{12} \cdot m_{arm} \cdot L_{arm}^2 + 2 \cdot m_{arm} \cdot d_w^2 + \frac{1}{2} m_{body} r_1^2$

When retracted:

$I_2 = I_{weight} + I_{2\,Body} = 2 \cdot m_w \cdot d_{w_2}^2 + \frac{1}{2} (m_{body})(r_2^2)$

Student 1 - I_1

$m_w = 10kg$ $d_w = 0.92m$ $m_{arm} = 7kg$ $L_{arm} = 0.7m$

$d_{arm} = 0.57m$ $m_{body} = 60kg$ $r_1 = 0.22m$

$I_1 = 2 \cdot m_{weights} \cdot d_{weights}^2 + 2 \cdot \frac{1}{12} \cdot m_{arm} \cdot L_{arm}^2 + 2 \cdot m_{arm} \cdot d_w^2 + \frac{1}{2} m_{body} r^2$

$I_1 = 2 \cdot 10 \cdot (0.92)^2 + 2\left(\frac{1}{12}(7)(0.7)^2\right) + 2(7)(0.57)^2 + \frac{1}{2}(60)(0.22)^2$

$= 23.5$

I_2:

$m_w = 10kg$ $d_{w_2} = 0.25m$

$m_{body} = 60kg$ $r_2 = 0.25m$

$I_2 = 2 \cdot m_w \cdot d_{w_2}^2 + \frac{1}{2}(m_{body}) \cdot (r_2^2)$

$I_2 = 2 \cdot 10 \cdot (0.25)^2 + \frac{1}{2}(60)(0.25^2)$

$I_2 = 3.125$

$I_1 \omega_1 = I_2 \omega_2$

$\omega_2 = \frac{I_1 \omega_1}{I_2} = \frac{23.5}{3.125} \cdot 0.7 = 5.264$ rad/s

Student 2 - I_1

$m_w = 12$ kg $\quad d_w = 0.84$ m $\quad m_{arm} = 5$ kg $\quad L_{arm} = 0.67$ m

$d_{arm} = 0.515 \quad m_{body} = 55$ kg $\quad r_1 = 0.19$

$I_1 = 2 \cdot m_{weights} \cdot d^2_{weights} + 2 \cdot \frac{1}{12} \cdot m_{arm} \cdot L_{arm}^2 + 2 \cdot m_{arm} \cdot d_w^2 + \frac{1}{2} m_{body} r^2$

$I_1 = 2 \cdot 12 \cdot (0.84)^2 + 2\left(\frac{1}{12}(5)(0.67)^2\right) + 2(5)(0.515)^2 + \frac{1}{2}(55)(0.19)^2$

$= 20.9$

I_2:

$m_w = 12$kg $\quad d_{w_2} = 0.21$m

$m_{body} = 55$kg $\quad r_2 = 0.21$m

$I_2 = 2 \cdot m_w \cdot d^2_{w_2} + \frac{1}{2}(m_{body}) \cdot (r_2^2)$

$I_2 = 2 \cdot 12 (0.21)^2 + \frac{1}{2}(55)(0.21^2)$

$I_2 = 2.2684$

$I_1 \omega_1 = I_2 \omega_2$

$\omega_2 = \frac{I_1 \omega_1}{I_2} = \frac{20.9}{2.2684} \cdot 0.6 = 5.54$ rad/s

$\omega_{Student\,2} > \omega_{Student\,1}$

$\frac{5.54 \text{ rad/s}}{2\pi} = 0.8817$ rev/s

$t = \frac{35 \text{ rev}}{0.8817 \text{ rev/s}} = 39.695 = 39.7$ s

Student 2 is faster and completes 35 revolutions in 39.7 seconds