



Several arms are linked to form the system shown. Link AB has a length of $l_{AB} = 0.5 \text{ m}$, link CD has a length of $l_{CD} = 0.5 \text{ m}$, and the distance from B to C is $r_{C/B} = -1 \hat{i} + 1 \hat{j} \text{ m}$. If the angle between link AB is $\theta = 45 \text{ degrees}$, determine the angular acceleration of link CD. The angular velocities of the links are given as $\omega_{AB} = -3 \text{ k rad/s}$ and $\omega_{BC} = (3\sqrt{2})/4 \text{ k rad/s}$, while the angular acceleration of AB is given as $\alpha_{AB} = -5 \text{ k rad/s}^2$.

$$\vec{r}_{B/A} = (-0.5 \cos 45 \hat{i} - 0.5 \sin 45 \hat{j})$$

$$\vec{r}_{C/B} = (-1 \hat{i} + 1 \hat{j})$$

$$\vec{r}_{D/C} = (0.5 \hat{j})$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$= -3 \hat{k} \times (-0.5 \cos 45 \hat{i} - 0.5 \sin 45 \hat{j}) = 1.5 \cos 45 \hat{j} - 1.5 \sin 45 \hat{i}$$

$$\vec{v}_C = \vec{v}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B}$$

$$= -1.5 \sin 45 \hat{i} + 1.5 \cos 45 \hat{j} + \frac{3\sqrt{2}}{4} \hat{k} \times (-1 \hat{i} + 1 \hat{j})$$

$$= -1.5 \sin 45 \hat{i} + 1.5 \cos 45 \hat{j} - \frac{3\sqrt{2}}{4} \hat{j} - \frac{3\sqrt{2}}{4} \hat{i}$$

$$= -\frac{3\sqrt{2}}{2} \hat{i}$$

$$\vec{v}_D = 0 = \vec{v}_C + \vec{\omega}_{CD} \times \vec{r}_{D/C}$$

$$0 = -\frac{3\sqrt{2}}{2} \hat{i} + \omega_{CD} \hat{k} \times (0.5 \hat{j})$$

$$\hat{i}: -\frac{3\sqrt{2}}{2} - \omega_{CD}(0.5) = 0 \quad \omega_{CD} = -3 \hat{k} \text{ rad/s}$$

$$\vec{a}_B = \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$

$$= -5 \hat{k} \times (-0.5 \cos 45 \hat{i} - 0.5 \sin 45 \hat{j}) - 9(-0.5 \cos 45 \hat{i} - 0.5 \sin 45 \hat{j})$$

$$= 2.5 \cos 45 \hat{j} - 2.5 \sin 45 \hat{i} + 4.5 \cos 45 \hat{i} + 4.5 \sin 45 \hat{j}$$

$$= 7 \sin 45 \hat{j} + 2 \cos 45 \hat{i}$$

$$\vec{a}_C = \vec{a}_B + \vec{\alpha}_{BC} \times \vec{r}_{C/B} - \omega_{BC}^2 \vec{r}_{C/B}$$

$$= 7 \sin 45 \hat{j} + 2 \cos 45 \hat{i} + \alpha_{BC} \hat{k} \times (-1 \hat{i} + 1 \hat{j}) - \frac{9}{8}(-1 \hat{i} + 1 \hat{j})$$

$$= 7 \sin 45 \hat{j} + 2 \cos 45 \hat{i} - \alpha_{BC} \hat{j} - \alpha_{BC} \hat{i} + \frac{9}{8} \hat{i} - \frac{9}{8} \hat{j}$$

$$\vec{a}_C = \vec{\alpha}_{CD} \times \vec{r}_{C/D} - \omega_{CD}^2 \vec{r}_{C/D}$$

$$= \alpha_{CD} \hat{k} \times (-0.5 \hat{j}) - 18(-0.5 \hat{j})$$

$$= 0.5 \alpha_{CD} \hat{i} + 9 \hat{j}$$

$$\hat{i}: 0.5 \alpha_{CD} = -\alpha_{BC} + \frac{9}{8} + 2 \cos 45$$

$$\hat{j}: 9 = -\alpha_{BC} - \frac{9}{8} + 7 \sin 45$$

$$\begin{aligned} \vec{\alpha}_{CD} &= -5.2721 \hat{k} \text{ rad/s}^2 \\ \vec{\alpha}_{BC} &= 5.17525 \hat{k} \text{ rad/s}^2 \end{aligned}$$