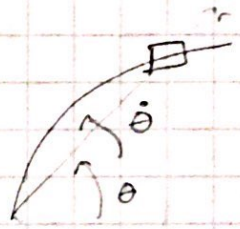
20-P-FA-AF-012

E o M Cylindrical Components: Advanced

Q: The smooth rod follows a path described as $r = 5/(1-\sin\theta)$, where θ is in radians and r is in m. The collar has a weight of W N. Determine the forces when $\theta = \pi/6$, and $\dot{\theta} = 2$?

A: $r = \frac{5}{(1-\sin\theta)}$, $\dot{r} = \frac{5\cos\theta\,\dot{\theta}}{(1-\sin\theta)^2}$, $\ddot{r} = \frac{5\cos\theta\,\ddot{\theta}}{(1-\sin\theta)^2} - \frac{5\sin\theta\,\dot{\theta}^2}{(1-\sin\theta)^2} + \frac{10\cos^2\theta\,\dot{\theta}^2}{(1-\sin\theta)^3}$

$\theta = \pi/3$, $\dot{\theta} = 2$, $\ddot{\theta} = 0$



$r = 10$, $\dot{r} = \frac{5\,\frac{\sqrt{3}}{2}\,2}{(1/2)^2} = 5\sqrt{3}\,4 = 20\sqrt{3}$

$\ddot{r} = 0 - \frac{5\cdot 1/2\cdot 4}{(1/2)^2} + \frac{10\,(\sqrt{3}/2)^2\,4}{(1/2)^3} = -40 + 240 = 200$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 200 - 10\cdot 2^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2\cdot 20\sqrt{3}\cdot 2$

$dr/d\theta = 5\cos(\theta)/(1-\sin(\theta))^2 = 5\sqrt{3}/4 / (1/2)^2 = 10\sqrt{3}$

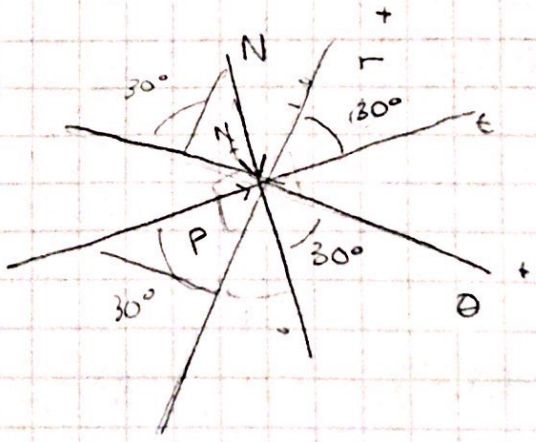
$\tan\psi = \frac{r}{(dr/d\theta)} = \frac{10}{10\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \psi = \pi/6 = 30°$

$+\uparrow \Sigma F_r = ma_r = N\sin(30°) + P\cos(30°)$

$\nearrow \Sigma F_\theta = ma_\theta = -N\cos(30°) + P\sin(30°)$

$N = \left(\frac{W}{9.8}a_r - P\cos(30°)\right)\frac{1}{\sin(30°)}$

$N = \left(P\sin(30) - ma_\theta\right)\frac{1}{\cos(30)}$



$$\frac{1}{\sin(30)}\left[\frac{W}{9.80}a_r - P\cos(30)\right] = \frac{1}{\cos(30)}\left[P\sin(30) - \frac{W}{9.8}a_\theta\right]$$

$$\left[ma_r - P\cos(30)\right] = \tan(30)\left[P\sin(30) - m a_\theta\right]$$

$$m\left[a_r - \tan(30)\,a_\theta\right] = \tan(30)\,P\sin(30) + P\cos(30)$$

$$m\left[a_r - \tan(30)a_\theta\right] = P\left[\tan(30)\sin(30) + \cos(30)\right]$$

$$P = \frac{m\left[a_r - \tan(30)\,a_\theta\right]}{\left[\tan 30 \cdot \sin(30) + \cos(30)\right]}$$

↳ just use P in an eqn for N

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labelling