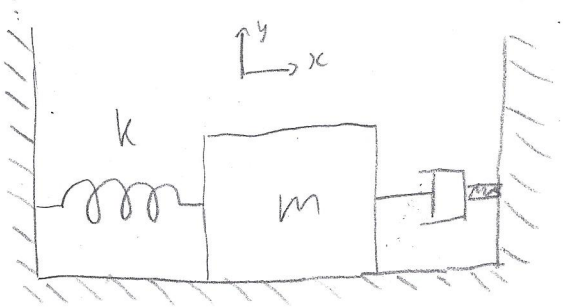
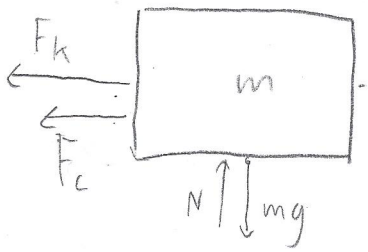
20-R-VIB-DY-25 Beginner

A box of mass $m = 5$ kg is connected to a spring, $k = 10$ N/m, and a viscous damper, $c = 10 \frac{Ns}{m}$. If the box is subject to a initial displacement $x_0 = 0$ and initial velocity $v_0 = 1 m/s$, find the equation of the solution



Solution: FBD



$\Sigma F_x = ma$

$-kx - c\dot{x} = m\ddot{x}$ $\quad \ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$

$c^2 - 4mk = 100 - 4(5)(10) = -100$

roots are complex

$\omega_n = \sqrt{2}$ $\quad \zeta = \frac{c}{2m\omega_n} = \frac{1}{\sqrt{2}}$

$r_{1,2} = -\omega_n \zeta \pm i\omega_n \sqrt{1-\zeta^2}$

$= -1 \pm i$

$x(t) = a\, e^{(-1+i)t} + b\, e^{(-1-i)t} = e^{-t}\left(a e^{it} + b e^{-it}\right)$

$= A e^{-t} \sin(-t + \phi)$

$v(t) = -A e^{-t} \sin(-t+\phi) - A e^{-t} \cos(-t+\phi)$

$0 = A \sin\phi \qquad \phi = 0$

$1 = -A \sin\phi + A\cos\phi$

$1 = -A(\sin\phi + \cos\phi)$

$\therefore A = -1$

$x(t) = -e^{-t}\sin(-t)$