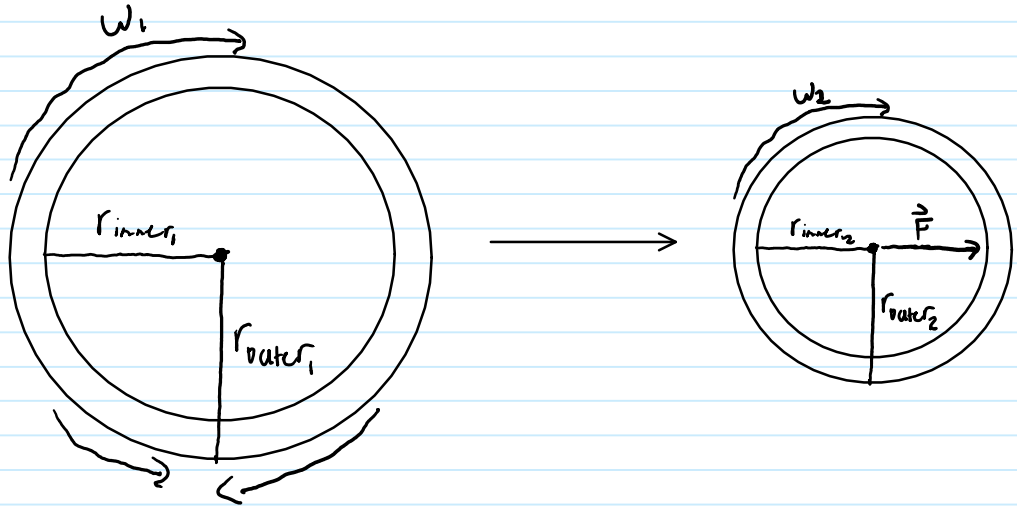
20-R-IM-PT-3

July 21, 2020 6:15 PM

A circular space station, shown below, is built with a retracting feature. If the station rotates fast enough, artificial gravity can be created from centripetal acceleration. The space station has an initial inner radius of 72 m, outer radius of 75m, a mass of 500,000kg and an initial angular speed of 0.1 radians per second. What radius should it retract to for artificial gravity? Assume that the station can be considered as a hollow circle, with a solid outer area. The angular velocity of the retracted ship should be less than 1 rad/s.



Solution

$a_c = \omega^2 r_{outer_2} = g = 9.81\ m/s^2$

$H_1 = H_2$

$r_{outer} - r_{inner} = 3m \quad r_{outer} = 3 + r_{inner}$

$I_1 \omega_1 = I_2 \omega_2$

$I_1 = \frac{1}{2} m (r_{outer_1}^2 + r_{inner_1}^2)$

$I_2 = \frac{1}{2} m (r_{outer_2}^2 + r_{inner_2}^2)$

$\frac{1}{2} m (r_{outer_1}^2 + r_{inner}^2) \cdot \omega_1 = \frac{1}{2} m (r_{outer_2}^2 + r_{inner_2}^2) \cdot \omega_2$

$(r_{outer_1}^2 + r_{inner_1}^2) \cdot \omega_1 = (r_{outer_2}^2 + r_{inner_2}^2) \cdot \omega_2$

$r_{outer_1} = 75m$
$r_{inner_1} = 72m$

$\omega_2^2 r_{outer} = 9.81 m/s^2$

$r_{outer} = \frac{9.81}{\omega_2^2} \quad r_{inner} = \frac{9.81}{\omega_2^2} - 3$

$(75^2 + 72^2) \cdot 0.1 = \left(\left(\frac{9.81}{\omega_2^2} \right)^2 + \left(\frac{9.81}{\omega_2^2} - 3 \right)^2 \right) \cdot \omega_2$

$$(75^2 + 72^2) \cdot 0.1 = \left(\left(\frac{9.81}{\omega_2^2} \right)^2 + \left(\frac{9.81}{\omega_2^2} - 3 \right)^2 \right) \cdot \omega_2$$

$$1080.9 = \left(\frac{96.2361}{\omega_2^4} + \frac{96.2361}{\omega_2^4} - 6 \cdot \frac{9.81}{\omega_2^2} + 9 \right) \cdot \omega_2$$

$$1080.9 = \frac{192.47}{\omega_2^3} - \frac{6 \cdot 9.81}{\omega_2^2} + 9 \cdot \omega_2$$

Using graphing calculator, $\omega_2 = 0.544$ rad/s

$$r_{outer} = \frac{9.81}{\omega_2^2} = \frac{9.81}{(0.544)^2} = 33.14\text{m}$$

$$r_{inner} = 33.14 - 3 = 30.14\text{m}$$

The new inner radius would be 30.14m and outer radius would be 33.14m