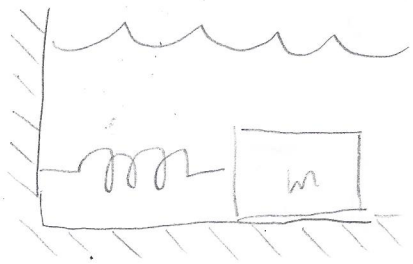
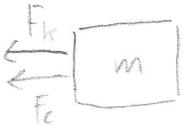
20-R-VIJ-DY-29 Intermediate

In Atlantis, the underwater city, an engineer sets up a $m = 5\,kg$ box and a spring, $k = 25\ N/m$. The water provides a damping force of $F = 2|v|$ on the box. Given that the engineer gives the box an initial displacement $x_0 = 0.02\,m$ and velocity $v = 2.5\ m/s$, determine the equation of displacement for the box.



Solution:



$$kx + 2\dot{x} = -m\ddot{x}$$

$$\ddot{x} + \frac{2\dot{x}}{m} + \frac{kx}{m} = 0$$

$$2 = c$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{5} \qquad c_c = 2m\omega_n = 10\sqrt{5}$$

$c < c_c$ underdamped $\qquad \zeta = \frac{c}{c_c} = \frac{1}{5\sqrt{5}}$

$$\omega_d = \omega_n\sqrt{1-(\zeta)^2} = 2.227$$

$$y = A\left[e^{-(\frac{c}{2m})t}\sin(\omega_d t + \phi)\right]$$

$$\dot{y} = A e^{-\frac{c}{2m}t}\left[\omega_d\cos(\omega_d t + \phi) - \frac{c}{2m}\sin(\omega_d t + \phi)\right]$$

$$0.02 = A\left[e^0\cos(0 + \phi)\right] \quad A\sin\phi = 0.02$$

$$0 = A\left[\omega_d\cos\phi - \frac{c}{2m}\sin\phi\right] \quad A \neq 0$$

$$0 = \omega_d\cos\phi - \frac{c}{2m}\sin\phi$$

$$\tan\phi = \frac{\omega_d 2m}{c} \qquad \phi = 1.481\ rad$$

$$A = 0.996 \qquad \frac{c}{2m} = \frac{1}{5}$$

$$\underline{y(t) = 0.996\,e^{-\frac{1}{5}t}\sin(2.227t + 1.481)}$$