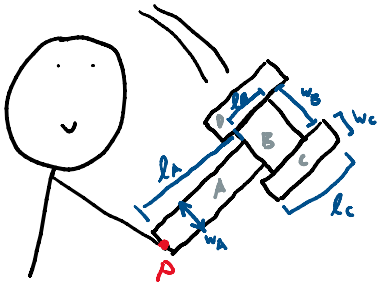
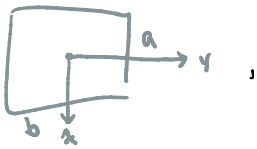
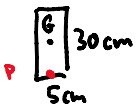
20-R-KIN-DK-2
05-26-2 Intermediate Parallel Axis Video
Inspiration: None



A kid excitedly swings his foam cutout hammer. If point P acts like a pin and the hammer rotates about that point, what is the moment of inertia of the hammer? The foam has a density of ***100*** kg/m^3 and a uniform thickness of ***0.5*** cm. Assume each cutout is a rectangular plate and the foam acts as a rigid body.
Plate A has a length ***l_A = 30 cm*** and width ***w_A = 5 cm***.
Plate B has a length ***l_B = 10 cm*** and width ***w_B = 20 cm***.
Plate C is identical to plate D, and has a length ***l_C = 14 cm*** and a width ***w_C = 7 cm***.
Plate C and D are attached to plate B such that their centers line up.



A: Moment of inertia of a plate: $I_{zz} = \frac{1}{12} m (a^2 + b^2)$

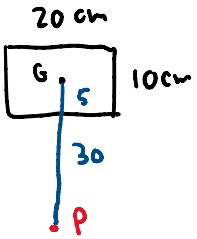


$m_A = \rho V = 100 \frac{kg}{m^3} (0.3\,m \times 0.05\,m \times 0.005\,m)$
$= 0.0075$

$I_{PA} = I_{GA} + md^2$
$= \frac{1}{12}(0.0075)(0.3^2 + 0.05^2) + (0.0075)(0.15\,m)^2$
$= 0.000226567$

Density is constant → centre of mass is in the middle

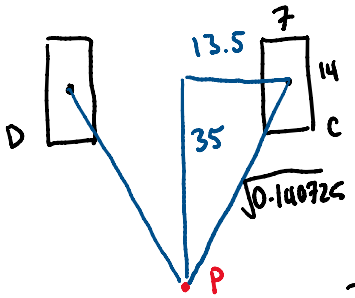
B: Plate → $I_{zz} = \frac{1}{12} m (a^2 + b^2)$



$m_B = \rho V = 100 \frac{kg}{m^3} (0.2 \times 0.1 \times 0.005)$
$= 0.01\ kg$

$I_{PB} = I_{GB} + md^2$
$= \frac{1}{12}(0.01)(0.2^2 + 0.1^2) + 0.01(0.35)^2$
$= 0.0012\bar{6}\ kg\,m^2$

C and D: Identical plates $I_{zz} = \frac{1}{12} m (a^2 + b^2)$



$m_C = m_D = \rho V = 100 \frac{kg}{m^3} (0.07 \times 0.14 \times 0.005)$
$= 0.0049$

$I_{PC} = I_{PD} = \frac{1}{12}(0.0049)(0.07^2 + 0.14^2) + 0.0049(0.140725)$
$= 0.000699456$

$I_P = I_{PA} + I_{PB} + I_{PC} + I_{PD}$
$= \boxed{0.00280\ kg\,m^2}$