20-R-IM-PT-7(Solution)

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Two students are sitting on spinning chairs with their arms full extended, when they fully retract their arms. Assume that their legs are retracted too, having no effect on their rotation speed.

Student 1 weighs 60kg and holds one weight that is 10kg in each arm. Assume that, when extended, student's 1 body on the chair is a cylinder of radius 0.22m, and the weights are negligible, and their arms are thin rods with a weight of 7 kg each and a length of 0.7m. When their arms are retracted, the body can be considered as a cylinder of 0.25m radius, with the weights acting as point masses on the outer radius . The initial speed is 0.7 rad/s

Student 2 weighs 55kg and holds two weights that are 12kg each. Assume, when extended, student's 2 body on the chair is a cylinder of radius 0.19m, and the weights are negligible size, and their arms are thin rods with a weight of 5 kg each and a length of 0.65m. When their arms are retracted, the body can be considered as a cylinder of 0.21m radius, with the weights acting as point masses on the outer radius. The initial speed is 0.6 rad/s

Neglecting the frictional forces, and effects due to mass of the chairs, who completes 35 revolutions first, and in how much time does that student complete it?

Both students:

$$I_1 \omega_1 = I_2 \omega_2$$

When Extended:

$$I_1 = I_{weights} + I_{arms} + I_{body}$$
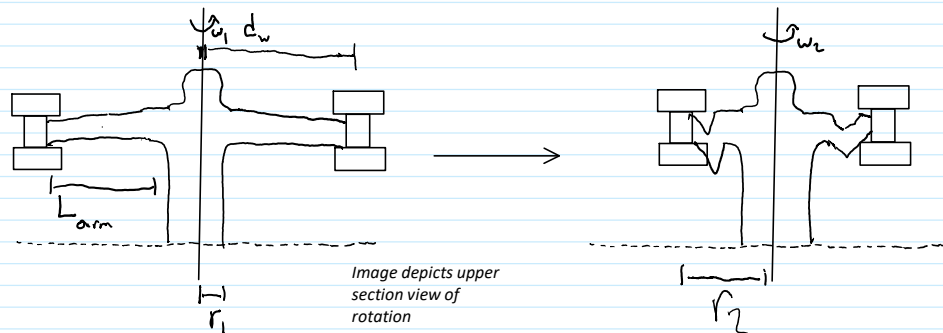
$$I_{weights} = 2 \cdot m_w \cdot d_w^2$$

$$I_{arms} = 2 \cdot \frac{1}{12} \cdot m_{arm} \cdot L_{arm}^2 + 2 \cdot m_{arm} \cdot d_{arm}^2$$

$$d_{arm} = r_1 + \frac{1}{2} L_{arm}$$

$$I_{body} = \frac{1}{2} m_{body} \, r_1^2$$

$$I_1 = 2 \cdot m_w \cdot d_w^2 + 2 \cdot \frac{1}{12} \cdot m_{arm} \cdot L_{arm}^2 + 2 \cdot m_{arm} \cdot d_{arm}^2 + \frac{1}{2} m_{body} \, r_1^2$$

When retracted:

$$I_2 = I_{2_{weight}} + I_{2_{body}} = 2 \cdot m_w \cdot d_{w_2}^2 + \frac{1}{2} (m_{body})(r_2^2)$$

Image depicts upper section view of rotation

Student 1 ~ I_1

$m_w = 10 kg$ $d_w = 0.22 m$ $m_{arm} = 7 kg$ $L_{arm} = 0.7 m$

$d_{arm} = 0.57 m$ $m_{body} = 60 kg$ $r_1 = 0.22 m$

$$I_1 = 2 \cdot m_{weights} \cdot d_{weights}^2 + 2 \cdot \frac{1}{12} \cdot m_{arm} \cdot L_{arm}^2 + 2 \cdot m_{arm} \cdot d_w^2 + \frac{1}{2} m_{body} \, r^2$$

$$I_1 = 2 \cdot 10 \cdot (0.22)^2 + 2 \left(\frac{1}{12} (7)(0.7)^2 \right) + 2(7) (0.57)^2 + \frac{1}{2}(60)(0.22)^2$$

$$= 23.5$$

I_2:

$m_w = 10 kg$ $d_{w_2} = 0.25 m$

$m_{body} = 60 kg$ $r_2 = 0.25 m$

$$I_2 = 2 \cdot m_w \cdot d_{w_2}^2 + \frac{1}{2} (m_{body}) \cdot (r_2^2)$$

$$I_2 = 2 \cdot 10 \cdot (0.25)^2 + \frac{1}{2} (60)(0.25^2)$$

$$I_2 = 3.125$$

$$I_1 \omega_1 = I_2 \omega_2$$

$$\omega_2 = \frac{I_1 \omega_1}{I_2} = \frac{23.5}{3.125} \cdot 0.7 = 5.264 \, rad/s$$

Student 2 ~ I_1

$m_w = 12 kg$ $d_w = 0.84 m$ $m_{arm} = 5 kg$ $L_{arm} = 0.67 m$

$d_{arm} = 0.515$ $m_{body} = 55 kg$ $r_1 = 0.19$

$$I_1 = 2 \cdot m_{weights} \cdot d_{weights}^2 + 2 \cdot \frac{1}{12} \cdot m_{arm} \cdot L_{arm}^2 + 2 \cdot m_{arm} \cdot d_w^2 + \frac{1}{2} m_{body} \, r^2$$

$$I_1 = 2 \cdot 12 \cdot (0.84)^2 + 2 \left(\frac{1}{12} (5)(0.67)^2 \right) + 2(5) (0.515)^2 + \frac{1}{2}(55)(0.19)^2$$

$$= 20.9$$

I_2:

$m_v = 12 kg \quad d_{v_2} = 0.21 m$

$m_{body} = 55 kg \quad r_2 = 0.21 m$

$I_2 = 2 \cdot m_v \cdot d_{v_2}^2 + \frac{1}{2}(m_{body}) \cdot (r_2^2)$

$I_2 = 2 \cdot 12 (0.21)^2 + \frac{1}{2}(55)(0.21^2)$

$I_2 = 2.2684$

$I_1 \omega_1 = I_2 \omega_2$

$\omega_2 = \frac{I_1 \omega_1}{I_2} = \frac{20.9}{2.2684} \cdot 0.6 = 5.54 \, rad/s$

$W_{student 2} > W_{student 1}$

$\frac{5.54 \, rad/s}{2\pi} = 0.8817 \, rev/s$

$t = \frac{35 \, rev}{0.8817 \, rev/s} = 39.695 = 39.7 \, s$

Student 2 is faster and completes 35 revolutions in 39.7 seconds