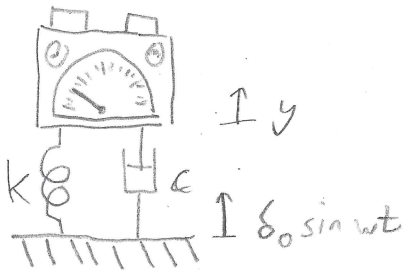
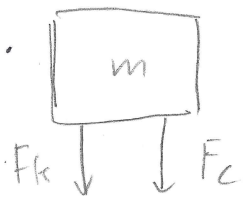
20-R-VIB-DY-41 Intermediate

A sensitive scientific instrument ($m = 20$ kg) is supported at the base by a spring, $k = 50$ N/m, and damper, $c = 100$ Ns/m. An earthquake provides a periodic vertical displacement to the ground which can be described as $\delta = 0.5 \sin 15t$. Find the equation of vibration at steady-state.



Solution:

FBD



$$\Sigma F_y = ma_y$$

$$-F_k - F_c = ma_y$$

$$-k(y - \delta_0 \sin \omega t) - c\dot{y} = m\ddot{y}$$

$$k\delta_0 \sin \omega t = m\ddot{y} + c\dot{y} + ky$$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{2.5} = 1.58$ $\quad F = k\delta_0$

$c_c = \sqrt{4mk} = \sqrt{4000} = 63.25$ $\quad D = \dfrac{\delta_0}{\sqrt{\left[1 - \left(\frac{\omega_0}{\omega_n}\right)^2\right]^2 + \left[2\frac{c}{c_c}\frac{\omega_0}{\omega_n}\right]^2}} = \dfrac{0.5}{\sqrt{(-89)^2 + (30)^2}}$

$$= 0.005324$$

$$\therefore \phi = \tan^{-1}\left[\frac{2\frac{c}{c_c}\frac{\omega_0}{\omega_n}}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2}\right] = \tan^{-1}\left[\frac{30}{-89}\right] = -0.325$$

$$y_p(t) = 0.005324 \sin(15t + 0.325)$$