

Feb 21/20

## Possible Questions for BCENGR PHYS 3

## Chapter 13: Force and Acceleration

13.1 - 13.4

 $F=ma$ : rectilinear

8	13-24	★ ★	pulleys & masses → §12.9 needed	✓
8	13-28	★ ★	pulley, but not needing §12.9	✓
9	13-22	★	springs & pulleys, dropping mass	✓
12	13-8	★	basic kinematics	✓
12	13-12	★ ★	basic kinematics discussed	✓
13	13-28	★ ★ ★	kinematics $\vec{a} = \vec{v} \cdot \vec{v}$ , friction	✓
8	13-8	★	concepts	
11	13-12	★ ★	calculus, concept (viscous drag)	
12	13-36	★ ★ ★	friction, impendy action, variables	

13.5

Normal

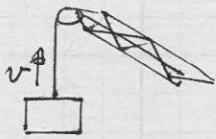
Tangential

8	13-80	★ ★ ★	$\vec{g}$ , FBD, lots of detail	✓
11	13-68	★ ★	forces along path, $\vec{g}$	✓
12	13-60	★ ★ ★	ball spinning around axis - spring	✓
12	13-80	★ ★	basic motion but disguised, $\vec{g}$	✓
8	13-69	★	basic energy and concepts; multiple techniques	✓
13	13-82	★ ★ ★	collar on guide, spring	✓
8	13-76	★ ★	spring on parabolic surface	

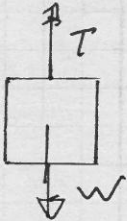
13.6

Cylindrical

8	13-104	★ ★ ★	calculus, multiple coordinates, $\vec{r}$	✓
12	13-96	★ ★ ★	calculus, multiple coordinates, $\vec{r}$	✓
8	13-102	★ ★	ball lifted on hemisphere, $\vec{r}$ , geometry	✓
13	13-102	★ ★ ★	multiple coordinate systems	✓
10	13-100	★ ★	$\vec{r}$ , $\vec{e}_r$ , $\vec{e}_\theta$ , $\vec{r}$	

Problem: Hibbeler 8<sup>th</sup> ed: 13-8

The 200 kg crate is suspended from the cable of a crane. Determine the force in the cable when  $t = 2$  s, if the crate is moving upward with  
 a) a constant velocity of 2 m/s and  
 b) a speed of  $v = (0.2t^2 + 2)$  m/s where  $t$  is in seconds



a) using  $\sum \vec{F} = m\vec{a}$   
 when  $\vec{v}$  is constant,  $\vec{a} = 0$

$$T - W = 0$$

$$\therefore T = (200 \text{ kg})(9.81 \text{ m/s}^2)$$

$$T = 1.96 \text{ kN}$$

b) when  $\vec{v} = (0.2t^2 + 2)$  m/s upward

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = 0.4t \text{ m/s}^2$$

$$\text{when } t = 2 \text{ s} \quad \vec{a} = 0.8 \text{ m/s}^2$$

$$\text{so } \sum \vec{F} = m\vec{a}$$

$$\therefore T - W = ma$$

$$T = W + ma$$

$$= (200 \text{ kg})(9.81 \text{ m/s}^2) + 200 \text{ kg}(0.8 \text{ m/s}^2)$$

$$T = 2.12 \text{ kN}$$

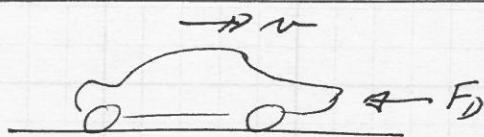
when the crate is moving at constant velocity, the tension is 1.96 kN.

when the crate is moving up at increasing speed, the tension is 2.12 kN at 2 s.

← Final Answer

Hbbeler 11<sup>th</sup> ed 13-12

1/2



A car of mass  $m$  is travelling at a slow velocity  $v_0$ . If it is subjected to the drag resistance of the wind, which is proportional to  $F_D = kv$ , determine the distance and the time the car will travel before its velocity becomes  $0.5v_0$ . Assume no other frictional forces act on the car.

Consider only the horizontal component.

The only horizontal force on the car is  $F_D$ .

$$\therefore \Sigma F = ma$$

$$-kv = ma$$

(The force is negative to the left if  $v$  is positive to the right)

$$\text{using } a = \frac{dv}{dt}$$

$$-kv = m \frac{dv}{dt}$$

$$-\frac{k}{m} dt = \frac{dv}{v}$$

$$-\frac{k}{m} \int_0^t dt = \int_{v_0}^v \frac{dv}{v}$$

$$-\frac{k}{m}(t-0) = \ln v \Big|_{v_0}^v$$

$$-\frac{k}{m}t = \ln(v/v_0)$$

when  $v = \frac{1}{2}v_0$

$$t_{\frac{1}{2}} = -\frac{m}{k} \ln \left( \frac{\frac{1}{2}v_0}{v_0} \right)$$

$$t_{\frac{1}{2}} = \frac{m}{k} \ln 2$$



Hibbeler 11<sup>th</sup> ed 13-12

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now for the distance

$$-\frac{B}{m} t = \ln(v/v_0)$$

$$e^{-\frac{B}{m} t} = v/v_0$$

$$\therefore v_0 e^{-\frac{B}{m} t} = \frac{1}{2} v_0$$

$$\int_{t_0}^t v_0 e^{-\frac{B}{m} t} dt = \int_0^s ds$$

$$v_0 \left( -\frac{m}{B} \right) e^{-\frac{B}{m} t} \Big|_{t_0}^t = s - 0$$

$$\therefore s = -\frac{m v_0}{B} (e^{-\frac{B}{m} t} - 1)$$

using  
assumption  $t_0 = 0$

so when  $t = \frac{m}{B} \ln 2$ 

$$s = -\frac{m v_0}{B} (e^{-\ln 2} - 1)$$

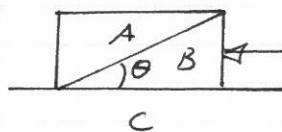
$$s = \frac{v_0 m}{B} (1 - e^{-\ln 2})$$

$$s = \frac{v_0 m}{B} \left( 1 - \frac{1}{2} \right)$$

$$\boxed{s = \frac{v_0 m}{2B}} \quad \leftarrow s$$

The car travels  $\frac{v_0 m}{2B}$  as the speed halves, and this takes  $t = \frac{m}{B} \ln 2$ .

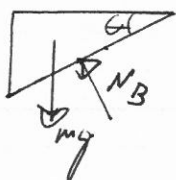
Final Answer  $\leftarrow$



Blocks A and B each have a mass  $m$ . Determine the largest horizontal force  $P$  which can be applied to B so that A will not move relative to A. All surfaces are smooth.

Without  $P$ , block A will be supported by the normal force from B and a normal from where A touches C. Eventually  $P$  will rise high enough to remove the normal from C, but will not have the normal from B raise the A block.

Consider Block A



$$\sum \vec{F} = m \vec{a}$$

$\uparrow$  component

$$-N_B \sin \theta = ma$$

$\uparrow$  component

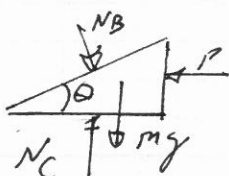
$$N_B \cos \theta - mg = 0$$

$$\therefore \boxed{a = -g \tan \theta} \quad \leftarrow a$$

and

$$\boxed{N_B = \frac{mg}{\cos \theta}} \quad \leftarrow N_B$$

Consider Block B



$$\sum \vec{F} = m \vec{a}$$

$\uparrow$  component

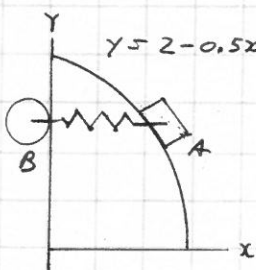
$$N_B \sin \theta - P = ma$$

combining

$$mg \tan \theta - P = -mg \tan \theta$$

$$\boxed{P = 2mg \tan \theta} \quad \leftarrow P$$

The maximum  $P$  that will not lift A is  $2mg \tan \theta$  Final Answer

Problem: Hibbeler 8<sup>th</sup> ed 13-76

The 6 kg block is confined to move along the smooth parabolic path. The attached spring restricts the motion and, due to the roller guide, always remains horizontal as the block descends. If the spring has stiffness of  $k = 10 \text{ N/m}$ , and an unstretched length of  $0.5 \text{ m}$ , determine the normal force of the path on the block at the instant  $x = 1 \text{ m}$  and the block has a speed of  $4 \text{ m/s}$ . Also, what is the rate of increase in speed of the block at this point? Neglect the mass of the roller guide and the spring.

Calculating the curvature at the point in question

$$y = 2 - 0.5x^2 \quad \text{at } x = 1 \quad y = 1.5$$

$$\frac{dy}{dx} = -x \quad \text{at } x = 1 \quad \frac{dy}{dx} = -1$$

$$\frac{d^2y}{dx^2} = -1$$

The curvature is then

$$\begin{aligned} \rho &= \frac{(1 + (\frac{dy}{dx})^2)^{\frac{3}{2}}}{|\frac{d^2y}{dx^2}|} \\ &= \frac{(1 + (-1)^2)^{\frac{3}{2}}}{|-1|} = 2\sqrt{2} \end{aligned}$$

$$\rho = 2.8284 \text{ m}$$

The direction of the surface at this point is

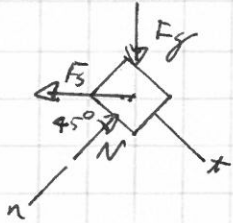
$$\theta = \arctan\left(\frac{dy}{dx}\right)$$

$$\theta = \arctan(-1)$$

$$\theta = -45^\circ$$

Problem: Hibbeler 8<sup>th</sup> ed. 13-76

Free body diagram of the mass



Force of spring

$$\begin{aligned} F_s &= k(l - l_0) \\ &= 10 \text{ N/m} (1 - 0.5) \\ &= 10 \text{ N/m} (1\text{m} - 0.5\text{m}) \end{aligned}$$

$$F_s = 5 \text{ N}$$

Newton's second Law  
 $\Sigma \vec{F} = m\vec{a}$

$x$  component:

$$mg \sin 45^\circ - F_s \cos 45^\circ = ma_x$$

$z$  component:

$$\begin{aligned} mg \cos 45^\circ + F_s \sin 45^\circ - N &= m \frac{v^2}{r} \\ (6\text{kg})(9.81\text{m/s}^2) \left(\frac{1}{\sqrt{2}}\right) + 5\text{N} \left(\frac{1}{\sqrt{2}}\right) - N &= (6\text{kg}) \frac{(2\text{m/s})^2}{2.828\text{m}} \end{aligned}$$

$$\therefore \boxed{N = 11.2 \text{ N}} \quad \text{N}$$

putting this into the  $x$  component

$$a_x = 9.81\text{m/s}^2 \frac{1}{\sqrt{2}} - \frac{5\text{N}}{6\text{kg}} \frac{1}{\sqrt{2}}$$

$$\boxed{a_x = 6.35 \text{ m/s}^2} \quad a_x$$

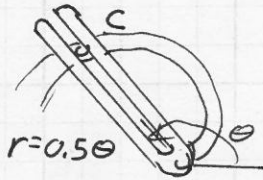
The normal force on the mass is 11.2 N and it has an increase of speed of 6.35 m/s<sup>2</sup>

Final Answer  
2



Hibbeler 10<sup>th</sup> ed 13-100

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Using a forked rod, a smooth cylinder C having a mass of 0.5 kg is forced to move along the vertical slotted line  $r = 0.5\theta$  m, where  $\theta$  is in radians. If the angular position of the arm is  $\theta = 0.5t^2$  rad, where  $t$  is in seconds, determine the force of the rod on the cylinder and the normal force of the slot on the cylinder at the instant  $t = 2.5$ . The cylinder is in contact with only one edge of the rod and slot at any instant.

computing the derivatives

$$r = 0.5\theta$$

$$\dot{r} = 0.5\dot{\theta}$$

$$\ddot{r} = 0.5\ddot{\theta}$$

$$\theta = 0.5t^2$$

$$\dot{\theta} = t$$

$$\ddot{\theta} = 1$$

so at  $t = 2.5$

$$r = 1 \text{ m} \quad \dot{r} = 1 \text{ m/s} \quad \ddot{r} = 0.5 \text{ m/s}^2$$

$$\theta = 2 \text{ rad} \quad \dot{\theta} = 2 \text{ rad/s} \quad \ddot{\theta} = 1 \text{ rad/s}^2$$

$$\text{also } dr/d\theta = 0.5 \text{ m/rad}$$

$$\begin{aligned} \text{so } \tan \psi &= \frac{r}{dr/d\theta} \\ &= \frac{1 \text{ m}}{0.5 \text{ m/rad}} \\ &= 2 \end{aligned}$$

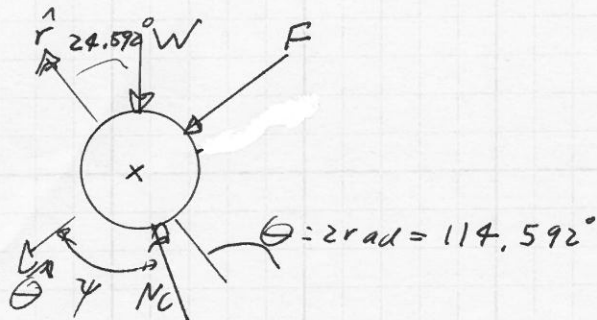
$$\psi = 63.43^\circ$$



Hibbeler 10th ed 13-100

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computing the FBD for the cylinder



I will resolve this onto  $\hat{u}_r$  and  $\hat{u}_\theta$  because I can compute  $a_r$  and  $a_\theta$  from the given data.  
 I could also use  $\hat{u}_n$  (the direction of  $N_c$ ) and  $\hat{u}_t$  but I don't have any information about  $a_n$  and  $a_t$

$$\sum \vec{F} = 0$$

$\hat{u}_r$  component

$$-W \cos(24.592^\circ) + N_c \sin \psi = m(a_r)$$

$$-(0.5 \text{ kg})(9.81 \text{ m/s}^2) \cos(24.592^\circ) + N_c \sin 63.43^\circ = 0.5 \text{ kg}(\ddot{r} - r(\dot{\theta})^2)$$

$$-4.460 \text{ N} + N_c 0.89439 = -1.75 \text{ N}$$

$$\therefore \boxed{N_c = 3.030 \text{ N}} \leftarrow N_c$$

$\hat{u}_\theta$  component

$$F + W \sin(24.592^\circ) - N_c \cos 63.43^\circ = 0.5 \text{ kg} a_\theta$$

$$F + 0.5 \text{ kg}(9.81 \text{ m/s}^2) \sin(24.592^\circ) - 3.030 \text{ N} \cos 63.43^\circ = 0.5 \text{ kg}(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$F + 2.0412 \text{ N} - 1.3553 \text{ N} = 2.5 \text{ N}$$

$$\boxed{F = 1.81 \text{ N}} \leftarrow F$$

The force of the fork on the cylinder is 1.81 N and the normal force of the slot on the cylinder is 3.03 N

Final Answer