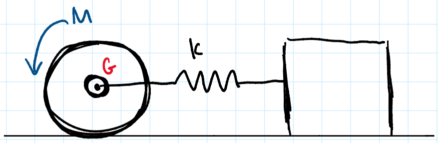
20-R-WE-DK-15

July 30, 2020 10:53 AM

20-R-WE-DK-15 Advanced Principle of Work and Energy

Inspiration: 18-24 Hibbeler

A lazy engineer is designing a robot to move things for him. He places a hub motor inside a ***2.5 kg*** disk such that a couple moment of ***M = 2.943 Nm*** is applied. If the attached package has a mass ***m = 5 kg*** and the coefficients of kinetic friction and static friction for between all objects are ***mu_k = 0.2*** and ***mu_s = 0.4*** respectively, determine the angular velocity of the disk after its center of mass has travelled a distance ***d = 0.5 m***. Assume the disk rolls without slipping and the package does not tip.
The disk has a radius ***r = 0.3 m***, the spring constant is ***k = 100 N/m*** and the spring is unstretched originally.



For coding

$\mu_k = 0.2$ $m_p = 5$ $M = 2.943\ N\cdot m$

$\mu_s = 0.4$ $m_D = 2.5$

Disk

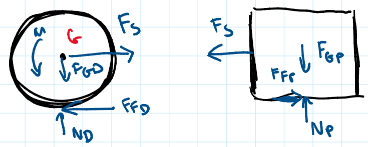
$\Sigma F_y = 0 = N_D - m_D g$ $N_D = m_D g$

$\Sigma F_x = 0 = F_{FD} - F_s$ $F_{FD} = \mu_s m_D g = F_s$ $F_s = \mu_k m_p g$

$\Sigma M_G = 0 = M - F_{FD}\cdot r$ $M = \mu_s m_D g r$ $\therefore\ m_p = \frac{\mu_s}{\mu_k} m_D$

$T_1 = 0 \quad V_1 = 0$ At state 2, the package will move at const. vel

$T_2 = \frac{1}{2} I_{GD}\omega_D^2 + \frac{1}{2} m_D V_{GD}^2 + \frac{1}{2} m_p V_{GP}^2$



The spring will stretch until it overcomes static friction then will stretch less to match kinetic friction

Const. velocity $\Rightarrow a_{GxP} = 0$

$\Sigma F_y = 0 = N_P - F_{GP}$ $N_P = F_{GP} = m_P g = (5)(9.81) = 49.05\ N$

$F_{FP} = \mu_k N_P = 0.2(49.05) = 9.81$

$\Sigma F_x = 0 = F_{FP} - F_s$ $F_{FP} = F_s$ $F_s = 9.81 = kx$ $9.81 = 100x$ $x = 0.0981$

The spring is stretched $0.0981\ m$

This means that the package moves $0.0981\ m$ less than the disk $d_P = d - x = 0.5 - 0.0981 = 0.4019\ m$

$\theta = \frac{d}{r} = \frac{0.5}{0.3} = \frac{5}{3}\ rad$

$U_M = M\theta = (2.943)(\frac{5}{3}) = 4.905\ J$

The disk is rolling without slipping thus friction on the disk does no work

$U_{FF} = U_{FFD} + U_{FFP} = 0 + F_{FP} d_P = (9.81)(0.4019) = 3.942639\ J$

$T_1 + V_1 + \Sigma U_{1\to 2\,\text{non-cons}} = T_2 + V_2 = \frac{1}{2} I_{GD}\omega_D^2 + \frac{1}{2} m_D V_{GD}^2 + \frac{1}{2} m_P V_{GP}^2 + \frac{1}{2} k x^2$

Rolling without slipping: $V = \omega r$ $V_{GD} = \omega_D(0.3)$

The package and the disk are moving at the same velocities $V_{GD} = V_{GP}$

$0 + 0 + 4.905 - 3.942639 = \frac{1}{2}\left(\frac{1}{2}(2.5)(0.3)^2\right)\omega_D^2 + \frac{1}{2}(2.5)(0.3\omega_D)^2 + \frac{1}{2}(5)(0.3\omega_D)^2 + \frac{1}{2}(100)(0.0981)^2$

$0.962361 = 0.05625\omega_D^2 + 0.1125\omega_D^2 + 0.225\omega_D^2 + 0.4811805$

$\boxed{\omega_D = 1.10546\ rad/s}$