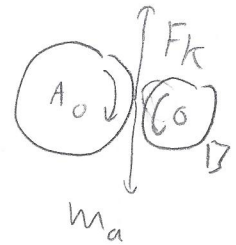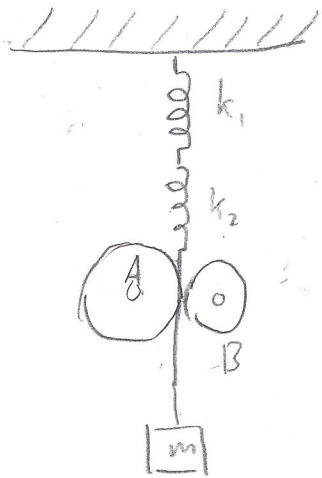20-R-VIB-D4-24

There is a 5kg mass suspended vertically from the ceiling via rope and two springs in series with spring constants 10 N/m & 25 N/m. To ensure straightness, the rope is held between two rollers of radius $r = 0.5$ & $r = 0.25$ m and mass $m = 5$ kg & $m = 2$ kg respectively. $w_n = ?$

Solution :   FBD

m_a

$$\Sigma M_A = I_A \alpha_A + I_B \alpha_B + m a \, r_A$$

$$- k y \, r_A = I_A \alpha_A + I_B \alpha_B + m a \, r_A$$

$$y = r_A \theta \qquad \frac{r_A}{r_B} = \frac{\alpha_B}{\alpha_A} \qquad \alpha_B = \alpha_A \frac{r_A}{r_B} \qquad a = \ddot{\theta} r_A$$

$$- k r_A^2 \theta = I_A \ddot{\theta} + I_B \frac{r_A}{r_B} \ddot{\theta} + m \ddot{\theta} r_A^2$$

$$\ddot{\theta} \left(I_A + I_B \frac{r_A}{r_B} + m r_A^2 \right) + k r_A^2 \theta = 0$$