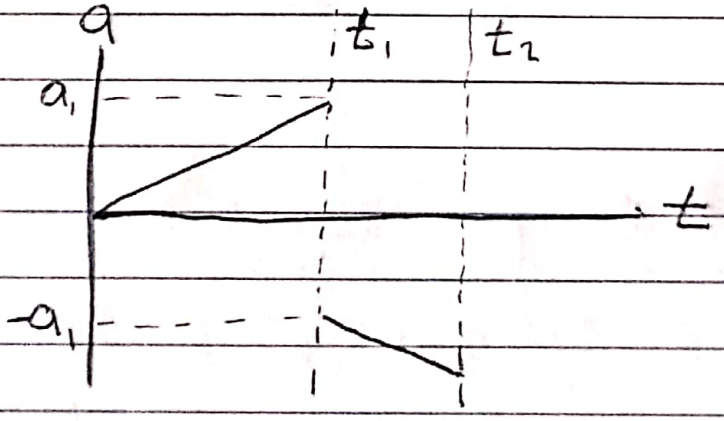
A spaceship uses a special engine that generates more thrust as it warms up. The space ship can accelerate at $\$A \cdot t \, m/s^2$ where $t = 0$ when the engine is turned on. To slow down, the spaceship turns 180° and fires its engine in the other direction. For the purposes of this question, assume this maneuver happens instantly, so that the a-t graph looks like the figure below.



find the shortest time t to travel a distance x, coming to rest at the destination.

Date

$$a = \begin{cases} 0 < t < t_1, & a = At \\ t_1 < t < t_2, & a = -At \end{cases}$$

$v = \int a\,dt \qquad 0 < t < t_1 \quad v = \frac{1}{2}At^2 + C$

$v_0 = 0, \therefore C = 0$

$v = \frac{1}{2}At^2$

$t_1 < t < t_2$

$v_+ = -\frac{1}{2}At^2 + C$

since velocity is continuous, $v_-(t_1) = v_+(t_1)$

$v_+(t_1) = \frac{1}{2}At_1^2 = -\frac{1}{2}At^2 + C$

$C = At_1^2 \qquad v_+ = -\frac{1}{2}At^2 + At_1^2$

$$v = \begin{cases} 0 \le t \le t_1, & v = \frac{1}{2}At^2 \\ t_1 < t < t_2, & v = -\frac{1}{2}At^2 + At_1^2 \end{cases}$$

$x = \int v\,dt \qquad 0 < t < t_1, \quad x = \frac{1}{6}At^3 + C$

$x_0 = C, \quad C = 0$

$t_1 < t < t_2$

$x(t) = -\frac{1}{6}At^3 + At_1^2 \cdot t + C$

since position is continuous, $x_-(t_1) = x_+(t_1)$

$x(t_1) = \frac{1}{6}At_1^3 = -\frac{1}{6}At_1^3 + At_1^3 + C$

$C = -\frac{2}{3}At_1^3$

Page

$$x = \begin{cases} 0 \le t \le t_1, \frac{1}{6}At^3 \\ t_1 \le t \le t_2, x(t) = -\frac{1}{6}At^3 + At_1^2 t - \frac{2}{3}At_1^3 \end{cases}$$

$$V(t_2) = 0 = -\frac{1}{2}At_2^2 + At_1^2$$

$$t_1^2 = \frac{1}{2}t_2^2$$

$$t_1 = \frac{1}{\sqrt{2}}t_2$$

$$x(t_2) = -\frac{1}{6}At_2^3 + At_1^2 \cdot t_2 - \frac{2}{3}At_1^3$$

$$= -\frac{1}{6}At_2^3 + A\left(\frac{1}{\sqrt{2}}t_2\right)^2 \cdot t_2 - \frac{2}{3}A\left(\frac{1}{\sqrt{2}}t_2\right)^3$$

$$= -\frac{1}{6}At_2^3 + \frac{1}{2}At_2^3 - \frac{\sqrt{2}}{6}At_2^3$$

$$x(t_2) = \frac{2-\sqrt{2}}{6}At_2^3$$

$$t_2 = \sqrt[3]{x_2 \cdot \frac{6}{2-\sqrt{2}}}$$