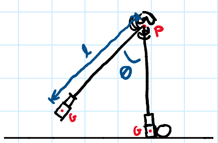
20-R-IM-DK-21

July 15, 2020 10:04 AM

Intermediate Eccentric Impact

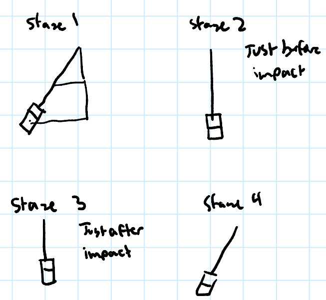
Inspiration: 19-49 Hibbeler



Double check final answer; assumed putter would rotate opposite way after impact but doesn't, leading to negative signs

You and your friends are having a great time at mini golf. You are about to take a swing at a golf ball with a putter. If the putter consists of a head with mass $m_H = 0.3$ kg and a radius of gyration $k_G = 0.05$, and a slender rod that extends from point P to G with a length $l = 0.9$ m and mass $m_r = 0.1$ kg, determine the velocity of the golf ball and the angular velocity of the putter right after impact. The coefficient of restitution is $e = 0.9$ and the golfball has mass $m_b = 0.05$ kg. The putter is released from rest when $\theta = 45°$.

Notes for artist:
These weird things around P are hands.
The length is from P to the middle of the putter head
The middle of the putter head is in line with the middle of the golfball
This is like a before and after picture; starts at an angle and ends perpendicular to the ground



Set datum to be at P

$$I_{PH} = m_H k_G^2 + m_H l^2$$
$$= (0.3)(0.05)^2 + (0.3)(0.9)^2$$
$$= 0.24375$$

$$I_{Pr} = \frac{1}{3} m_r l^2$$
$$= \frac{1}{3}(0.1)(0.9)^2$$
$$= 0.027$$

$$T_1 = 0 \qquad T_2 = \frac{1}{2} I_{PH} \omega_2^2 + \frac{1}{2} m_H V_{H_2}^2 + \frac{1}{2} I_{Pr} \omega_2^2 + \frac{1}{2} m_r V_{r_2}^2$$

Putter is pinned at P $\Rightarrow$ $V_{r_2} = \omega_2 \frac{l}{2}$ $\qquad V_H = \omega_2 l_2$

$$T_2 = \frac{1}{2}(0.24375)\omega_2^2 + \frac{1}{2}(0.3)(0.9^2)\omega_2^2 + \frac{1}{2}(0.027)\omega_2^2 + \frac{1}{2}(0.1)\left(\frac{0.9^2}{4}\right)\omega_2^2$$
$$= 0.267\omega_2^2$$

$$T_1 + V_1 = T_2 + V_2$$
$$0 - m_r g \frac{l}{2}\cos\theta - m_H g\, l\cos\theta = T_2 - m_r g \frac{l}{2} - m_H g\, l$$

$$-0.1(9.81)\frac{0.9}{2}\cos 45 - 0.3(9.81)(0.9)\cos 45 = 0.267\omega_2^2 - 0.1(9.81)\left(\frac{0.9}{2}\right) - 0.3(9.81)(0.9)$$
$$\omega_2 = 1.841148 \text{ rad/s}$$

The angular momentum of the system is conserved about point P

$$(H_P)_2 = (H_O)_3$$

$$I_{Pr}\omega_2 + \frac{l}{2} m_r V_{r_2} + I_{PH}\omega_2 + l\, m_H V_{H_2} = I_{Pb}\omega_3 + l\, m_b V_{b_3} - I_{Pr}\omega_3 - \frac{l}{2} m_r V_{r_3} - I_{PH}\omega_3 - l\, m_H V_{H_3}$$

$$0.027(1.841148) + \frac{0.9}{2}(0.1)(1.841148)\left(\frac{0.9}{2}\right) + (0.24375)(1.841148) + 0.9(0.3)(1.841148)(0.9)$$
$$= 0.983173$$

$$0.983173 = 0 + (0.9)(0.05)V_{b_3} - 0.027\omega_3 - \frac{0.9}{2}(0.1)\omega_3\left(\frac{0.9}{2}\right) - 0.24375\omega_3 - 0.9(0.3)(0.9)\omega_3$$

$$0.983173 = 0.045 V_{b_3} - 0.534\omega_3$$

$$e = \frac{V_{b_3} - V_{H_3}}{V_{H_2} - V_{b_2}} \qquad 0.9 = \frac{V_{b_3} - (-\omega_3(0.9))}{1.841148(0.9) - 0}$$

$$1.49132988 = V_{b_3} + 0.9\omega_3$$

$$0.983173 = 0.045(1.49132988 - 0.9\omega_3) - 0.534\omega_3$$

$0.916063155 = -0.5745\, \omega_3$ $\omega_3 = -1.594539$ rad/s

$\vec{\omega_3} = 1.594539$

$\boxed{V_{b_3} = 2.9264 \text{ m/s}}$