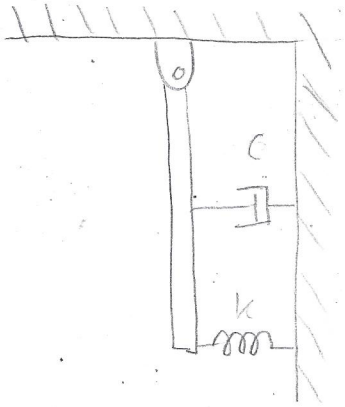
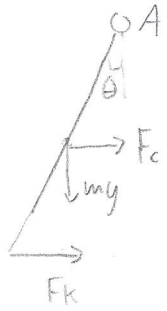
20-R-VIB-DY-30 Intermediate

A bar of length 1.5m, m = 2kg, is pinned to ceiling. A spring, k = 50 N/m, is attatched to the bottom of the bar and a damper, c = 10 Ns/m, is attached halfway down. Given a small angle displacement, find the damped frequency and the roots.



Solution: FBD



$$\Sigma M_A = -I_0\ddot{\alpha} \qquad I_0 = \frac{1}{3}ml^2$$

$$F_c\frac{l}{2}\cos\theta + F_k l\cos\theta + mg\sin\theta = -\frac{1}{3}ml^2\ddot{\theta}$$

small θ

$$cv\frac{l}{2} + kxl + mg\theta + \frac{1}{3}ml^2\ddot{\theta} = 0$$

$$v = r\dot{\theta} \qquad x = r\theta$$

$$c\frac{l^2}{4}\dot{\theta} + (kl^2 + mg)\theta + \frac{1}{3}ml^2\ddot{\theta} = 0$$

$c' = 5.625$ $\quad k' = 132.12$ $\quad m' = 1.5$

$c_c = 2m'w_n = 28.155$ $\quad w_n = \sqrt{\frac{k'}{m'}} = 9.385$ $\quad =$

$c < c_c$ underdamped $\quad \zeta = \frac{c}{c_c} = 0.2$

$$w_d = w_n\sqrt{1-\zeta^2} = 9.196$$

$$r_{1,2} = \frac{-c'}{m'} \pm i\frac{\sqrt{4m'k' - c^2}}{2m'}$$

$$= -3.75 \pm i\,9.196$$