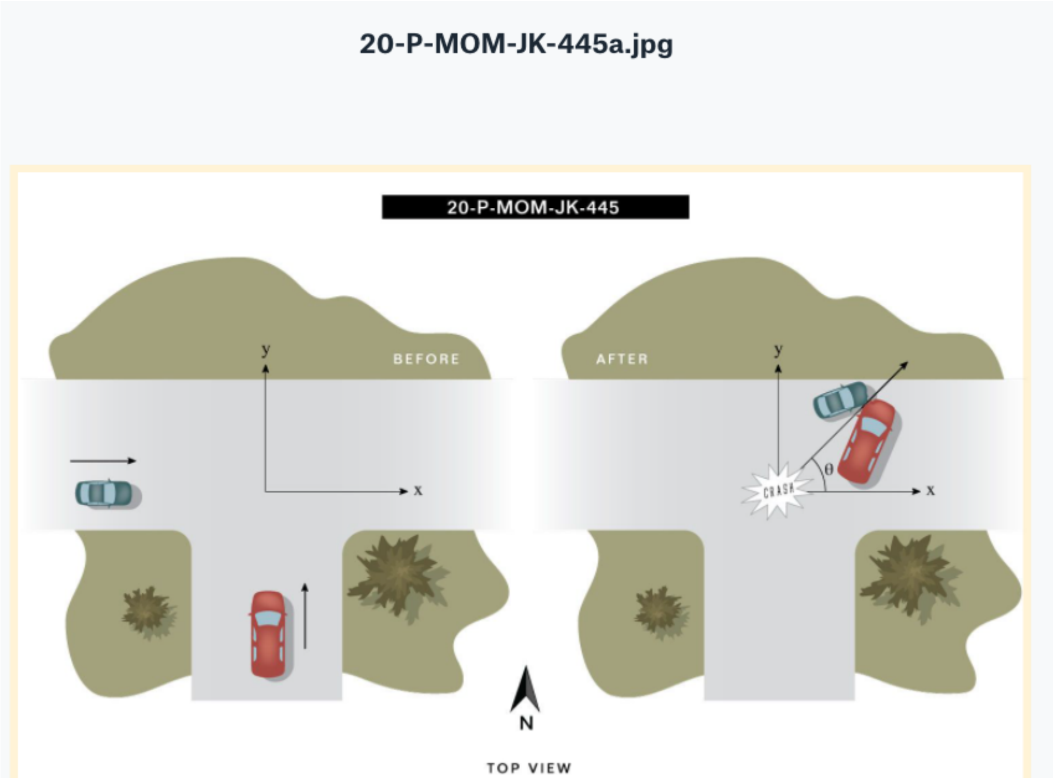
20-P-MOM-JK-445a
Collisions in two dimensions



Question: The car and the van collided in the middle of the intersection as shown. They stuck together after the collision. What was the final velocity of the car and van in metres per second immediately after the collision? Magnitude and direction.

For direction, give me the angle θ above the x-axis or North of East as shown.

The car has a mass of 999 [kg] and the van has a mass of 2345 [kg].

Before the collision, the car had been traveling at 21.2 m/s in the positive x direction (as shown) before the collision and the van had been travelling at 13.4 m/s in the positive y direction.

Assume that linear momentum was conserved.

a) What was the initial momentum of the system?

b) What was the final momentum of the system?

c) What was the final velocity of the system?

Note: on your test I will ask ONLY for the final velocity of the system. a) and b) are a hint to you.

Answers: 1a) 37900 kg m/s $\theta = 56.0°$

1b) same 379 00 kg m/s $\theta = 56.0°$

1c) 11.3 m/s at same angle , $\theta = 56.0°$

m car ranges from 1000 kg to 2000 kg
m van ranges from 2500 kg to 4000 kg

https://media.chevrolet.com/media/us/en/chevrolet/vehicles/express_cargo_van/2016.tab1.html

v car ranges from 15.1 to 19.9 m/s
v van ranges from 10.1 to 13.9 m/s

momentum before in the x direction = (m car) (v car)
momentum before in the y direction = (m van) (v van)

total momentum before, as it is a vector is
$$|\overrightarrow{momentum}| = \sqrt{(m_{car}v_{car})^2 + (m_{van}v_{van})^2}$$
magnitude of momentum = SQRT ((m car v car) ^{2} + (m van v van) ^{2})

Tangent of the angle theta = (m van v van) / (m car v car)
Angle = INV TAN ((m van v van) / (m car v car))

Final velocity, after the collision
Angle of momentum before the collision = angle of the velocity after the collision as momentum is conserved

The car and van stick together so the
velocity of the cars after the collision = total momentum before / total mass

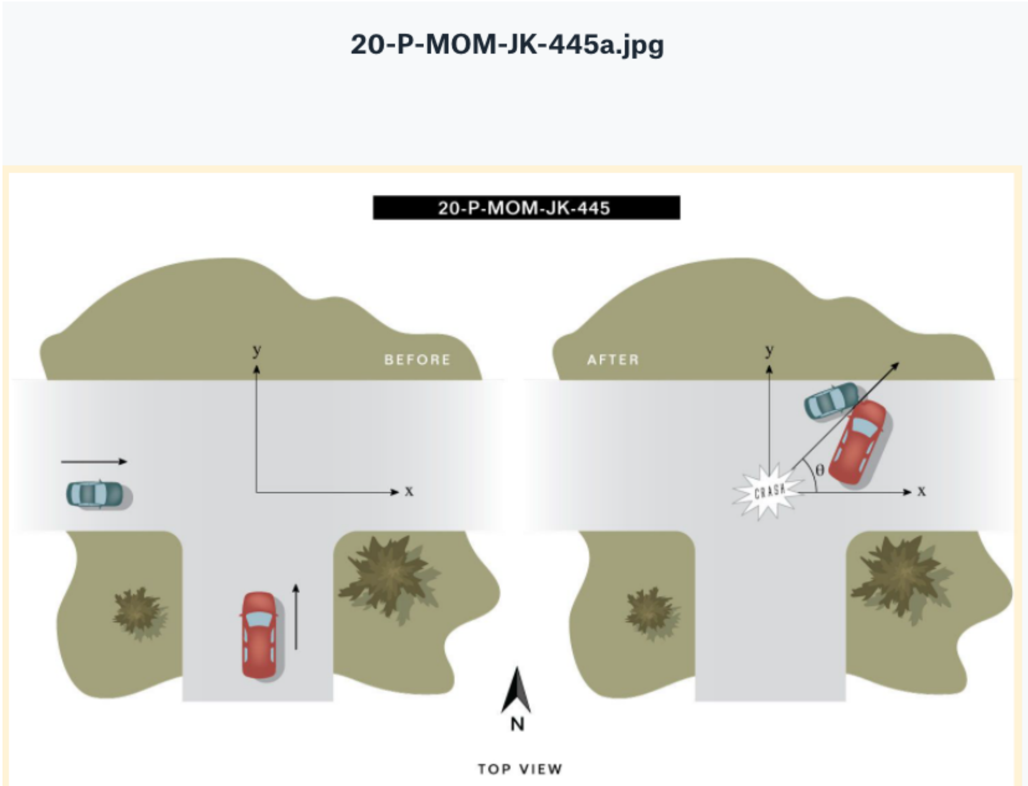
$$speed\ after = \frac{\sqrt{(m_{car}v_{car})^2 + (m_{van}v_{van})^2}}{m_{car} + m_{van}}$$

Speed after the collision = (SQRT ((m car v car) ^{2} + (m van v van) ^{2}))

(m car + m van)

If m car = 999 kg
m van = 2345 kg
v car before = 21.2 m/s in the positive x direction (as shown)
v van before = 13. 4 m/s in the positive y direction.
Momentum before = 37900 kg m/s θ = 56.0°
Momentum after = same 379 00 kg m/s θ = 56.0°
Velocity after = 11.3 m/s at same angle , θ = 56.0°

Momentum in Two Dimensions
Question Two



Same picture, but a different question. This is the type of question that frequently shows up on tests or the final exam.
The car and the van collided in the middle of the intersection as shown.
The car has a mass of 987 [kg] and the van has a mass of 2222 [kg].
After the collision, they stuck together and moved off together at an angle $\theta = 49.8°$ above the x-axis as shown.
What was the ***initial*** velocity of the van in metres per second if the car had been traveling at 50.0 [km/h] in the positive x direction (as shown) before the collision? Assume that linear momentum was conserved.
In other words, was the van speeding and thus caused the accident?
The speed limit is 50.0 km/h = 13.888889 m/s

Answers:
m car = 987 kg
m van = 2222 kg
initial momentum in the east direction = $m_{car}\, v_{car\ initial}$ = 13700 kg m/s
so total initial momentum = 21200 kg m/s = total final and v_{final} = 6.61 m/s
so initial momentum in the north direction = 16200 kg m/s

But initial momentum north = m $_{van}$ v $_{van\ initial}$ so v $_{van\ initial}$ = 7.30 m/s (north) so is was not speeding

$$tan\theta = \frac{m_{car}\ v_{car}}{m_{van}\ v_{van}}$$

$$|\overrightarrow{v_{van}}| = \frac{m_{car}\ v_{car} tan\theta}{m_{van}}$$

For the WebWork coding

m car ranges from 1501 kg to 2000 kg
m van ranges from 2501 kg to 2999 kg
v car ranges from 12.5 to 13.5 m/s (it does not speed)
Angle theta varies from 50.1 to 60.5 degrees

Van mass data from:
https://media.chevrolet.com/media/us/en/chevrolet/vehicles/express_cargo_van/2016.tab1.html

momentum before in the x direction = (m car) (v car)
momentum before in the y direction = (m van) (v van)

$$|\overrightarrow{v_{van}}| = \frac{m_{car}\ v_{car} tan\theta}{m_{van}}$$

If if if you wanted to change this question to find the speed after the collision

$$|\ v\ after\ | = \frac{(\ m\ car\ )\ (\ v\ car\ )}{(\ m\ car + m\ van)\ cos\ theta}$$

Because.

total momentum before, as it is a vector is

$\left|\overrightarrow{momentum}\right| = \sqrt{(m_{car}v_{car})^2 + (m_{van}v_{van})^2}$

magnitude of momentum = SQRT ((m car v car) ^{2} + (m van v van) ^{2})

Tangent of the angle theta = (m van v van) / (m car v car)

Angle = INV TAN ((m van v van) / (m car v car))

Final velocity, after the collision

Angle of momentum before the collision = angle of the velocity after the collision as momentum is conserved

The car and van stick together so the

velocity of the cars after the collision = total momentum before / total mass

momentum after the collision = (m car + m van) (v after) at angle theta

momentum in the x direction after the collision

= (m car + m van) (v after) cos theta

momentum in the y direction after the collision

= (m car + m van) (v after) sin theta

But

momentum before in the x direction = (m car) (v car)

momentum before in the y direction = (m van) (v van)

Momentum is conserved and as you know the momentum in the x direction before the collision,

momentum before in the x direction = (m car) (v car)

momentum in the x direction after the collision

= (m car + m van) (v after) cos theta

(m car) (v car) = (m car + m van) (v after) cos theta

$$| \text{ v after } | = \frac{(\text{ m car })(\text{ v car })}{(\text{ m car + m van}) \text{ cos theta}}$$

Yet the question was the speed of the van BEFORE the collision

momentum before in the y direction = (m van) (v van)

momentum in the y direction after the collision

= (m car + m van) (v after) sin theta

(m van) (v van) = (m car + m van) (v after) sin theta

$$v_{van} = \frac{(m_{car} + m_{van})(v_{after}) \sin \theta}{m_{van}}$$