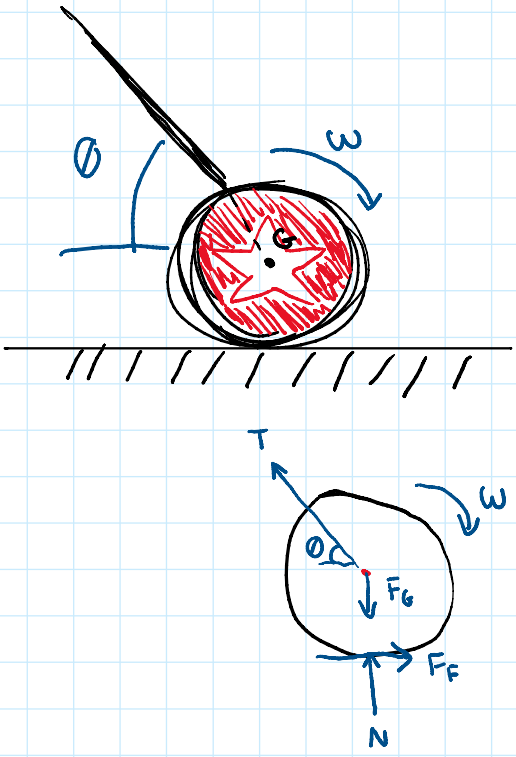
20-R-KIN-DK-31 Intermediate General Plane Motion

Inspiration: None



A mechanical engineering student has been practicing his yoyo tricks because he has too much free time. For one trick, he spins the yoyo such that it contacts the ground and moves forward, emulating someone walking their dog. If the yoyo has a radius of gyration ***k_G = 0.02 m*** and a mass of ***m = 0.2 kg***, determine the acceleration and angular acceleration of the yoyo when the tension in the string is found to be ***T = 0.4 N***. Assume the string is at its full extent and does not roll up as the yoyo rolls. Assume there is also no friction where the string slips around the yoyo's inner axle.
The coefficient of static and kinetic friction is found to be ***mu_s = 0.3*** and ***mu_k = 0.2*** respectively. The angle is ***theta = 60 degrees*** and the radius of the yoyo is ***r = 0.03 m***. Take the initial angular velocity of the yoyo to be ***omega = 6 rad/s*** an

$I_G = m k_G^2 = 0.2(0.02)^2 = 0.00008$

$\sum F_x: ma_{Gx} = F_F - T_{AB}\cos\theta$
$\sum F_y: ma_{Gy} = 0 = T_{AB}\sin\theta - F_G + N$
$\sum M_G: I_G\alpha = F_F r$

Assume rolling without slipping: $F_F \le \mu_s N$

$\vec{a}_{Gx} = \vec{\alpha}\times\vec{r} = -0.03\alpha\,\hat{i}$

$0.2(-0.03\alpha) = F_F - 0.4\cos 60$
$N = -0.4\sin 60 + (0.2)(9.81) = 1.6155898$
$0.00008\alpha = F_F(0.03)$
$0.002666\alpha = F_F$
$-0.006\alpha = 0.002666\alpha - 0.4\cos 60$
$F_F = 23.076923$ $\quad \mu_s N = (0.3)1.6155898 = 0.48467$
$F_F \le \mu_s N$ ✗
Not true so the yoyo is rolling with slipping

$\sum F_x: ma_{Gx} = \mu_k N - T_{AB}\cos\theta$
$\sum F_y: ma_{Gy} = 0 = T_{AB}\sin\theta - F_G + N$
$\sum M_G: I_G\alpha = F_F r$

$0.4\sin 60 - 0.2(9.81) + N = 0$ $\quad N = 1.61559\ N$
$0.00008\alpha = 1.61559(0.2)(0.03)$ $\quad \alpha = 121.169\ rad/s^2$
$0.2a_{Gx} = 0.2(1.61559) - 0.4\cos 60$ $\quad a_{Gx} = 0.61559\ m/s^2$