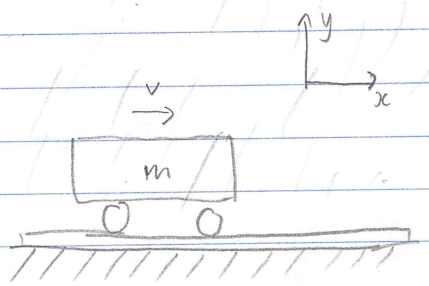
20-P-MOM-DY-36

A $m = 50\,kg$ minecart is initially traveling at $v = 4\,m/s$ collects rainwater as it travels. Rainwater collects inside the cart at a rate of $250\,g/s$. Determine the equation of velocity for the cart.



Solution: $\frac{dm}{dt} = 0.250\ kg/s$

$v = (v_r)_x + v_m$

↳ minecart initially

↳ x-component of rainwater

$v = 0 + v_m$

$$\Sigma F_x = m\frac{dv}{dt} + (v_m)\frac{dm}{dt}$$

$$0 = (50 + 0.25t)\frac{dv}{dt} + v\frac{dm}{dt}$$

$$\frac{dv}{v} = -\frac{0.25}{50+0.25t}\,dt$$

$$\int_4^v \frac{dv}{v} = \int_0^t -\frac{0.25}{50+0.25t}\,dt$$

$$\ln v\Big|_4^v = -\ln(50+0.25t)\Big|_0^t$$

$$\ln\frac{v}{4} = -\ln\left(\frac{50+0.25t}{50}\right) = \ln\left(\frac{50}{50+0.25t}\right)$$

$$\frac{v}{4} = \frac{50}{50+0.25t} \qquad v = \frac{200}{50+0.25t}\ m/s$$