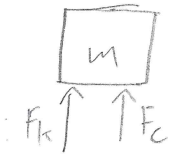
20-R-VIB-DY-44 Advanced

A mad engineer created the worlds largest jack in the box - 11 m tall. The internal mechanism consists of a spring, $k = 500 N/m$, and a damper, $c = 100 Ns/m$. The jack component has a mass of 200 kg. Given that when closed the spring is compressed 0.5 m below equilibrium, determine how long it takes for the amplitude to drop to 0.05% of the initial.

Solution: FBD



$\Sigma F_y = -ma_y$

$ky + c\dot{y} + ma_y = 0$ $\quad w_n = \sqrt{\frac{k}{m}} = \sqrt{2.5} = 1.58$

$c_c = 2mw_n = 632.46 \ Ns/m$

underdamped

$x(t) = Ae^{-w_n \zeta t} \sin(w_d t + \phi)$

$\zeta = \frac{c}{c_c} = 0.158$

$w_d = \sqrt{1-\zeta^2} w_n = 0.98$

$A = \sqrt{\frac{(v_0 + w_n \zeta x_0)^2 + (x_0 w_d)^2}{w_d^2}} = 0.516$

$\phi = \tan^{-1}\left[\frac{x_0 w_d}{v_0 + w_n \zeta x_0}\right] = 1.321$

$x(t) = 0.516 e^{-0.25t} \sin(0.98t + 1.321)$

$e^{-0.25t} \leq 0.05 \quad \therefore t \geq 11.98$

$0.98t + 1.321 = \frac{\pi}{2} + \pi n$

$t = 13.078$ ✓

$n = 4$