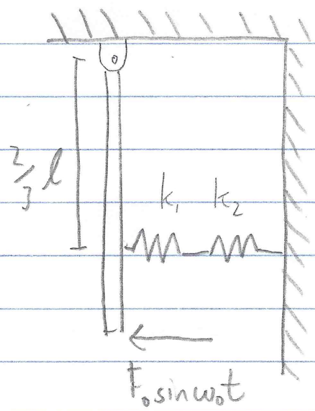
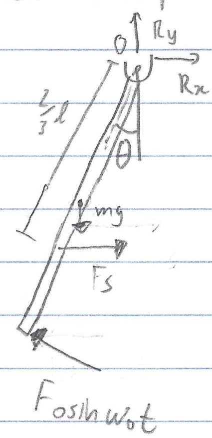
20-R-VIB-DY-17

A rod, l, is pinned to the ceiling. $\frac{2}{3}$rd of the length down, it is connected to a series of springs horizontally. The springs have a spring constant of 50 N/m & 75 N/m. Given that the rod end has a periodic force, $F = F_0 \sin \omega_0 t$, applied, find the steady-state vibration amplitude.





$$k_{tot} = \frac{k_1 k_2}{k_1 + k_2}$$

$$\Sigma M_0 = -I_0 \alpha \qquad -F_0 l \sin\omega_0 t + F_s \tfrac{2}{3} l \cos\theta + mg \tfrac{l}{2} \sin\theta = -\tfrac{1}{3} m l^2 \ddot{\theta}$$

small θ assumption $\sin\theta \approx \theta$ $\cos\theta \approx 1$

$$F_s = ks \qquad s = r\theta = \tfrac{2}{3} l \theta$$

$$\tfrac{1}{3} m l^2 \ddot{\theta} + \left[\tfrac{4}{9} k l^2 + mg \tfrac{l}{2}\right]\theta = F_0 l \sin\omega_0 t$$

$$\ddot{\theta} + \left[\frac{4k}{3m} + \frac{3g}{2l}\right]\theta = \frac{3F_0}{ml} \sin\omega_0 t$$

$\theta = C \sin\omega t$

$\ddot{\theta} = -C\omega^2 \sin\omega t$

$$-C\omega^2 \sin\omega t + \left[\frac{4k}{3m} + \frac{3g}{2l}\right] C \sin\omega t = \frac{3F_0}{ml} \sin\omega_0 t$$

$$C = \frac{3F_0}{ml\left[\left(\frac{4k}{3m} + \frac{3g}{2l}\right) - \omega^2\right]}$$