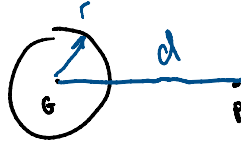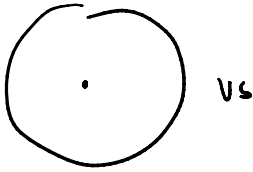20-R-KIN-DK-8

Beginner Parallel Axis Theory Homework

Inspiration: None

vs

If a disk has radius r = ? and rotates about an axis perpendicular to the plane going through point P, how many times bigger would its radius have to be to obtain the same mass moment of inertia if it were spinning about its center of mass? Point P is d = ? away from the center of mass. Assume thickness is constant and there is uniform density.

$$m = \rho V$$
$$= \rho \pi (xr)^2 h$$
$$= \rho \pi x^2 r^2 h$$

$$m = \rho V$$
$$= \rho \pi r^2 h$$

$$I = \tfrac{1}{2} m (xr)^2 \qquad I = \tfrac{1}{2} mr^2 + md^2 \qquad \text{variable}$$
$$= \tfrac{1}{2} \rho \pi x^4 r^4 h \qquad = \tfrac{1}{2} \rho \pi r^4 h + \rho \pi r^2 h \boxed{(d^2)}$$

$$\tfrac{1}{2} \rho \pi x^4 r^4 h = \tfrac{1}{2} \rho \pi r^2 h (r^2 + 32)$$

$$x^4 r^2 = r^2 + 32 \qquad x^4 r^2 - r^2 = 32 \qquad r^2 (x^4 - 1) = 32 \quad \textcolor{red}{2d^2}$$

$$x^4 = 1 + \frac{32}{r^2} \qquad x = \sqrt[4]{1 + \frac{32}{r^2}} \qquad \textcolor{red}{x = \sqrt[4]{1 + \frac{2d^2}{r^2}}}$$