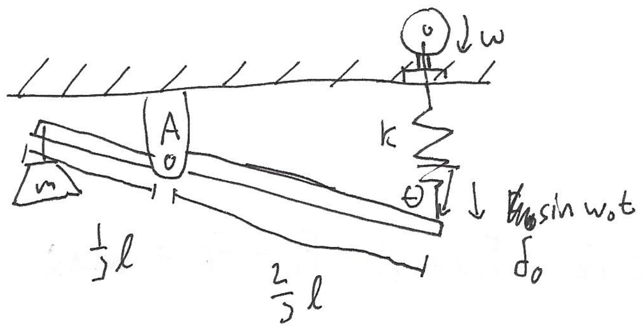
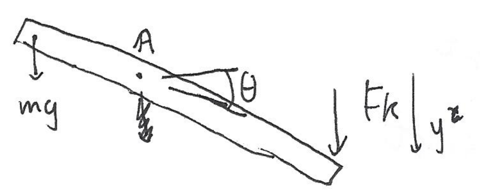
20-R-VIB-DY-19 Intermediate

A straight bar, of length l, is pinned to the ceiling at point A. On one end, there is a 10 kg weight and on the other, there is a spring with a k value of 150 N/m. A motor applies a periodic ~~force~~ displacement $\delta = 0.5 \sin 2t$ to the spring. What is the maximum velocity of the steady-state vibration?



Solution: FBD



$$\Sigma M_A :$$

$$\left(\tfrac{2}{3}l\right)k(y_2 - \delta_0 \sin w_0 t) = -ma\left(\tfrac{l}{3}\right)$$

(my & $k e_q$ cancel out)

$$2k(y_2 - \delta_0 \sin w_0 t) = -ma$$

$$2ky_2 + ma = 2k\delta_0 \sin w_0 t$$

$$y = \frac{2l}{3}\sin\theta \approx \frac{2l}{3}\theta$$

$$a = \alpha \frac{l}{3} \qquad \alpha = \ddot{\theta}$$

$$2k\left(\tfrac{2l}{3}\right)\theta + m\left(\tfrac{l}{3}\ddot{\theta}\right) = 2k\delta_0 \sin w_0 t$$

$$\ddot{\theta} + \frac{4k}{m}\theta = \frac{6k\delta_0 \sin w_0 t}{lm}$$

$$x_p = C \sin w_0 t = \theta_p$$

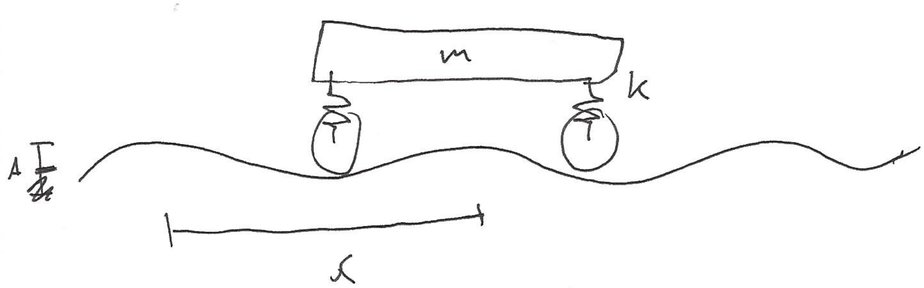
$$\ddot{x}_p = -Cw_0^2 \sin w_0 t = \ddot{\theta}_p$$

$$-Cw_0^2 \sin w_0 t + \frac{4k}{m}(C \sin w_0 t) = \frac{6k\delta_0 \sin w_0 t}{lm}$$

$$C\left(\frac{4k}{m} - w_0^2\right) = \frac{6k\delta_0}{lm}$$

20-R-VIB-DY-20 Intermediate

A 2000 kg car is driving across a bumpy road which can be described as a sinusoidal wave with an amplitude of 0.1 m and a wave length of 6 m. There are 4 springs, one for each wheel, and they all have a spring constant of 1000 N/m. Find the velocity of the car that will produce the greatest vibration.



Solution: $\delta_0 = 0.1\text{m}$ $\quad \lambda = 6\text{m}$

$k = 4(1000) = 4000$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{2} \qquad \tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{2}}$

$\omega_0 = \omega_n$ for resonance

$\lambda = 6\text{m}$ in $\frac{2\pi}{\sqrt{2}}\text{ s}$

$v = \frac{\lambda}{\tau} = 1.35 \text{ m/s}$

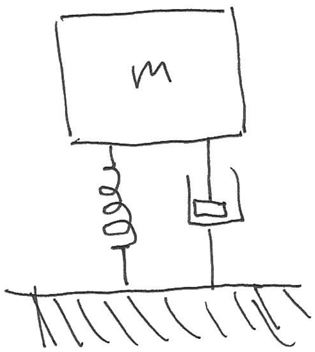
$$C = \frac{6k\delta_0}{lm\left(\frac{4k}{m} - w_0^2\right)} = \frac{6k\delta_0}{l(4k - w_0^2 m)}$$

$$\dot{\theta}_p = C w_0 \cos w_0 t = \frac{6k\delta_0 w_0}{l(4k - w_0^2 m)} \cos w_0 t$$

$$v_{max} = \dot{\theta}_p r = \frac{6k\delta_0 w_0 l}{l\,3(4k - w_0^2 m)} = \frac{2k\delta_0 w_0}{(4k - w_0^2 m)}$$

20-R-VIB-DY-21 Beginner

A simple spring, damper, and load system is configured as shown. Given that the $k = 15$ N/m and $m = 10$kg, what damping constant will make the system critically damped?



Solution: $c_c = 2m\omega_n = 2m\sqrt{\frac{k}{m}} = 24.49$ Ns/m