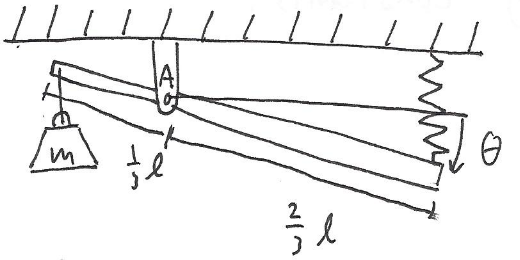
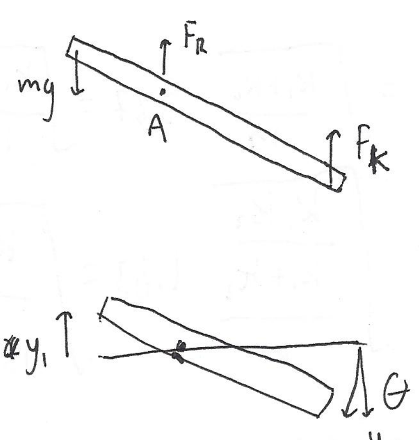
CH 22-5 Intermediate / Free Undampened Vibrations

A straight bar, 3m in length, is pinned to the ceiling at point A. On one end there is a 5kg weight and on the other end there is a spring with a k value of 10 N/m. Given that the bar is displaced a small angle θ from equilibrium, what is the natural period of vibration? Negate the bar's weight



Solution: FBD



$$\circlearrowleft \Sigma M_A: \left(\tfrac{2}{3}l\right)\underset{ky_2}{\cancel{kx}} - \left(\tfrac{2}{3}l\right)\underbrace{k_{eq} + mg\left(\tfrac{l}{3}\right)}_{\text{cancel out}} = -ma\left(\tfrac{l}{3}\right)$$

$$2ky_2\ \cancel{2kx} = -5a$$

$$y_1 = \frac{l}{3}\sin\theta$$

$$y_2 = \frac{2l}{3}\sin\theta$$

small θ assumption

$$y_1 = \frac{l}{3}\theta \qquad \dot{y}_1 = \frac{l}{3}\dot{\theta} \qquad \ddot{y}_1 = \frac{l}{3}\ddot{\theta} = a$$

$$y_2 = \frac{2l}{3}\cancel{\sin}\,\theta$$

$$2k\left(\frac{2l}{3}\theta\right) = -5\left(\frac{l}{3}\ddot{\theta}\right)$$

$$4k\theta + 5\ddot{\theta} = 0$$

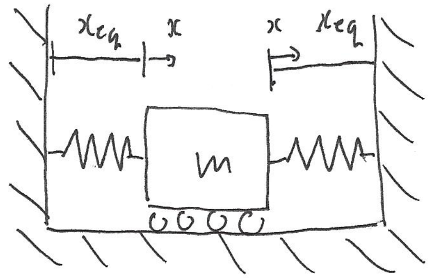
$$\ddot{\theta} + \frac{4k}{5}\theta = 0$$

$$W_n = \sqrt{\frac{4k}{5}} = \sqrt{8}$$

$$T = \frac{2\pi}{W_n} = \frac{2\pi}{\sqrt{8}} = 2.22\ s$$

20-R-VIB-6

A Solid rectangle, of mass 10kg, is attached to walls on it's right and left by springs. The springs have a k value of 50 N/m. Given an initial displacement of $x_0 = 0.15$m and an initial velocity of 1 m/s, find ~~equation~~ the natural period of vibration and the maximum amplitude



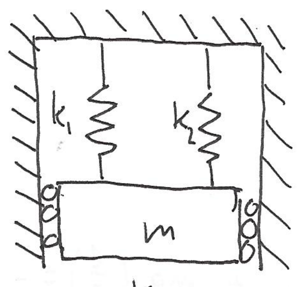
Solution: From Q1, $\omega_n = \sqrt{10}$ rad/s $\quad \tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{10}}$

$= 1.99$ s

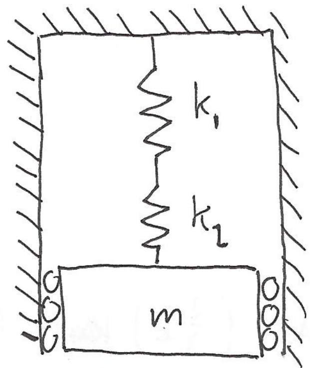
$$C = \sqrt{\left(\frac{V_0}{\omega_n}\right)^2 + x_0^2} = \sqrt{\left(\frac{1}{\sqrt{10}}\right)^2 + 0.15^2} = 0.35 \text{ m}$$

20-R-FUV-DY-7

A mechanic finds two old springs in his storage room, but the spring constants are not labeled. In order to figure out the k values, the mechanic loads the springs in parallel and series with a 15 kg weight. Given that the natural frequencies of the parallel and series setups are 3.87 Hz and 1.83 Hz respectively, What are the spring constants?



K_p



K_s

Solution: $K_p = k_1 + k_2$ $\quad \omega_n = \sqrt{\frac{k}{m}}$ $\quad (\omega_n)_p = \sqrt{\frac{k_1+k_2}{m}}$ $\quad 3.87 = \sqrt{\frac{k_1+k_2}{15}}$

$K_s = \frac{k_1 k_2}{k_1+k_2}$ $\quad (\omega_n)_s = \sqrt{\frac{\frac{k_1 k_2}{k_1+k_2}}{m}}$ $\quad 1.83 = \sqrt{\frac{k_1 k_2}{15(k_1+k_2)}}$

$224.65 = k_1 + k_2$

$50.23 = \frac{k_1 k_2}{k_1+k_2}$ $\quad k_1 + k_2 = \frac{k_1 k_2}{50.23}$ $\quad k_1 k_2 = 11284.96$ $\quad k_1 = \frac{11284.96}{k_2}$

$$50.23 = \frac{11284.96}{\frac{11284.96}{k_2} + k_2} \qquad \frac{11284.96}{k_2} + k_2 = 224.67$$

$k_1 = 75.8$ or 148.86

$k_2 = 148.86$ or 75.8

$k_2^2 - 224.67 k_2 + 11284.96 = 0$ $\quad k_2 = 148.86,\ 75.8$