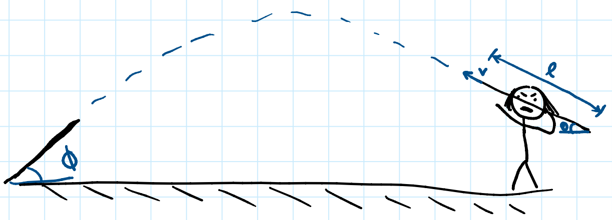
20-R-IM-DK-19

July 14, 2020 11:25 AM

Advanced Eccentric Impact

Inspiration: 16.6.2 Example 2 Mech 221 Notes

Check answer



An Olympian is practicing her javelin toss. On one throw, she tosses it a little too hard and it slips from her hand, causing the javellin to land on the smooth pavement of the track rather than the soft grass. If the 0.8 kg javelin was launched at a height of $h = 1\,m$ at an angle of $\theta = 60°$ with a velocity of $v = 28\ m/s$, determine the post-impact angular velocity of the javellin. Assume the javelin maintains a constant orientation once it returns to a height $h = 1\,m$, and that it can be treated as a slender rod. Take the javelin's length to be $\ell = 2.2\,m$ and its landing angle to be $\phi = 30°$. The coefficient of restitution is $e = 0.9$.

Use kinematics to find velocity just before impact

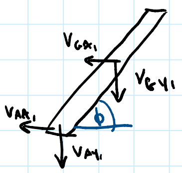
$v^2 = v_0^2 + 2a\Delta x$

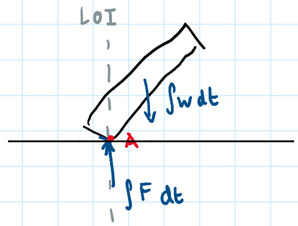
$\frac{v_y}{28} = \sin 60 \qquad v_y = 14\sqrt{3}$

$0 = (14\sqrt{3})^2 + 2(-9.81)\Delta x$

$\Delta x = 29.96942\,m \qquad$ Total height $= \Delta x + h = 30.96942$

$v^2 = 0 + 2(9.81)(30.96942) \qquad v = 24.65\ m/s \downarrow \qquad 28\cos 60 = V_G$





$\vec{V}_{G_1} = (-14\,\hat{\imath} - 24.65\,\hat{\jmath}) \qquad \vec{V}_{G_1} = \vec{V}_{A_1}$

Along LOI: $e = \dfrac{V_{Ay_2} - V_{Ground_2}}{V_{Ground} - V_{Ay_1}} \qquad 0.9 = \dfrac{V_{Ay_2} - 0}{0 - (-24.65)}$

$V_{Ay_2} = 22.185 \uparrow$

Angular momentum conserved about A:

$\vec{H}_{A_1} = \vec{H}_{A_2}$

$\vec{H}_{A_1} = I_G\vec{\omega}_1 + \vec{r}_{G/A} \times \vec{L}_1$

$= 0 + (2.2\cos 30\,\hat{\imath} + 2.2\sin 30\,\hat{\jmath}) \times (0.8)(-14\,\hat{\imath} - 24.65\,\hat{\jmath})$

$= -43.3245\,\hat{k} + 12.32\,\hat{k}$

$= -31\,\hat{k}$

$\vec{H}_{A_2} = I_G\vec{\omega}_2 + \vec{r}_{G/A} \times \vec{L}_2$

$= \frac{1}{3}(0.8)(2.2)\vec{\omega}_2 + (2.2\cos 30\,\hat{\imath} + 2.2\sin 30\,\hat{\jmath}) \times (0.8)(-V_{Gx}\,\hat{\imath} + 22.185\,\hat{\jmath})$

$= \frac{44}{75}\omega_2\,\hat{k} + 33.814481\,\hat{k} + \frac{22}{25}V_{Gx}\,\hat{k}$

Linear momentum is conserved in the x-direction

$m\vec{V}_{Gx_1} = m\vec{V}_{Gx_2} \qquad \vec{V}_{Gx_2} = -14\,\hat{\imath}\ \hat{k}$

$-31 = \frac{44}{75}\omega_2 + 33.814481 + \frac{22}{25}(14)$

$\boxed{\omega_2 = -131.479\ rad/s}$