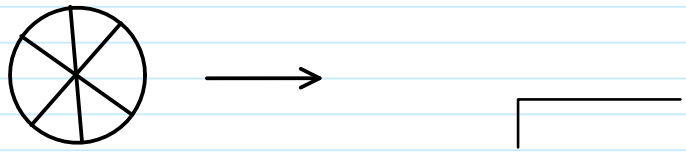
20-R-IM-PT-3

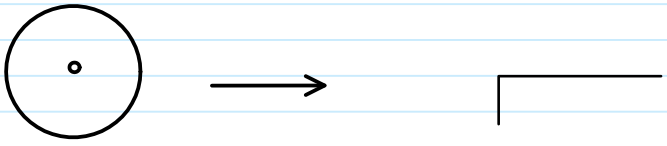
July 3, 2020 7:06 PM

Two wheels roll over identical obstructions of a height $h = 0.04\ m$ on a road.
One is a solid wooden wheel, with a mass $m_1 = 5kg$ and radius $r_1 = 0.5m$.
The other wheel has 5 spokes, a rim with mass $m_2 = 3\ kg$, and a radius of $r_2 = 0.55m$.
Each spoke has a mass of $0.25\ kg$.
Which wheel needs a higher velocity to roll over the obstacle
and by how much?

Wheel 2



Wheel 1



Both wheels:

Cons. of Angular Momentum

$(Hw)_1 = (Hw)_2$ $\qquad r_1 \simeq r$

$m(r_1 - h)(v_{b1}) + I_G \omega_1 = r\,m(v_{b2}) + I_G \omega_2 \qquad \omega_1 = \frac{v_{b1}}{r} \quad \omega_2 = \frac{v_{b2}}{r}$

$m(r_1 - h)(v_{b1}) + I_G\left(\frac{v_{b1}}{r}\right) = r\,m(v_{b2}) + I_G\left(\frac{v_{b2}}{r}\right)$

$$v_{b1} \simeq \frac{\left(r \cdot m + \frac{I_G}{r}\right)}{\left(m(r_1 - h) + \frac{I_G}{r}\right)} \cdot v_{b2} \qquad (1)$$

Conservation of Energy: (climbing over obstacle)

$T_2 + V_2 \simeq T_3 + V_3$

$\frac{1}{2}mv^2 + \frac{1}{2} I\omega^2 = mgh \qquad \omega = v/r$

$\frac{1}{2}mv^2 + \frac{1}{2} I\left(v/r\right)^2 = mgh$

$$v_2 \simeq \sqrt{\frac{2mgh}{m + \frac{I}{r^2}}} \qquad (2)$$

Wheel 1:

$m_1 = 5\,kg \quad r_1 = 0.5m \quad h = 0.04\,m$

$I_{G_1} = \frac{1}{2} m r^2 = \frac{5 \cdot (0.5^2)}{2} = 0.625 \quad \frac{I_G}{r} = 1.25$

$V_2 = \sqrt{\frac{2 \cdot 5 \cdot 0.04}{5 + \frac{0.625}{(0.5)^2}}} = 0.231\,m/s$

$V_1 = \frac{(5 \cdot 0.5 + 1.25)}{(5 \cdot (0.5 - 0.04) + 1.25)} \cdot 0.231$ $\boxed{V_1 = 0.24401\,m/s}$

Wheel 2

$m_2 = 3\,kg \quad r_1 = 0.55m \quad h = 0.04\,m \quad m_s = 0.25$

$I_{G_2} = 5 \cdot \frac{1}{3} m_s r^2 + m r^2 = 5 \cdot \frac{1}{3}(0.25)(0.55^2) + (3)(0.55)^2 = 1.0335$

$\frac{I_G}{r} = 1.879$

$V_2 = \sqrt{\frac{2 \cdot (5 \cdot 0.25 + 3) \cdot 0.04}{(5 \cdot 0.25 \cdot 3) + \frac{1.0335}{(0.55)}}} = 0.2457\,m/s$

$V_1 = \frac{((5 \cdot 0.25 + 3) \cdot 0.55 + 1.879)}{((5 \cdot 0.25 + 3)(0.55 - 0.04) + 1.879)} 0.2457 = 0.256$

$V_{w_2} - V_{w_1} = 0.256 - 0.24401 = 0.012\,m/s$

Wheel 2 needs a higher velocity by $0.012\,m/s$