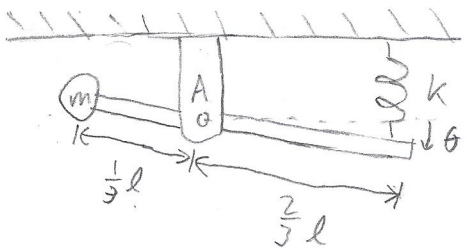
20-R-VIB-DY-5



Solution: FBD

$\Sigma M_A: I_A\alpha \quad I_A = 0$ because $m=0$ for bar

$$\left(\tfrac{2}{3}\ell\right)ky + \left(\tfrac{2}{3}\ell\right)k_{eq}y - mg\left(\tfrac{\ell}{3}\right) + ma\left(\tfrac{\ell}{3}\right)$$

cancel out

$$ky\left(\tfrac{2}{3}\ell\right) = -ma\,\tfrac{\ell}{3}$$

small angle

$$y \simeq \left(\tfrac{2}{3}\ell\right)\theta$$

$$a \simeq \left(\tfrac{1}{3}\ell\right)\ddot{\theta}$$

$$k\theta\left(\tfrac{2}{3}\ell\right)^2 + m\ddot{\theta}\left(\tfrac{\ell}{3}\right)^2 = 0$$

$$\ddot{\theta} + \frac{k\,4}{m}\theta = 0$$

$$\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{8}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{8}} = 2.22\,s$$