20-R-IM-DK-3

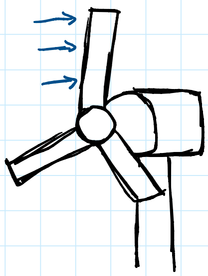
July 2, 2020 11:05 AM

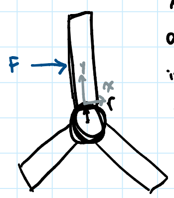
maybe advanced

Intermediate Principle of Impulse and Momentum Check

Inspiration: None

Can make easier version as another question

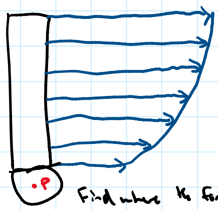




A **20 kg** wind turbine blade can be modelled as a **2 m × 0.3 m** thin plate. The edge of the blade is located a distance **$r = 0.25$ m** away from the center of the propeller. If the blade is subjected to wind that applies a force $dF = (-(y-2)^2 + 8)t$ determine the angular velocity of the blade after **$t = 2$ s** if the blade originally starts at rest.

length of blade

If I messed up, change force to only vary with time



$$F = \int dF = \int_0^2 (-(y-2)^2 + 8\,dy)t$$

$$= t\int_0^2 -y^2 + 4y + 4\,dy$$

$$= t\left[-\tfrac{1}{3}y^3 + 2y^2 + 4y\right]_0^2$$

Total Force $= \frac{40}{3}t$

Find where the force is located

$$\int_0^a dF\,y = \int_a^2 dF\,y$$

$$\cancel{t}\int_0^a -y^3 + 4y^2 + 4y\,dy = \cancel{t}\int_a^2 -y^3 + 4y^2 + 4y\,dy$$

$$-\tfrac{1}{4}a^4 + \tfrac{4}{3}a^3 + 2a^2 = \left(-\tfrac{1}{4}(2)^4 + \tfrac{4}{3}(2)^3 + 2(2)^2\right) - \left(-\tfrac{1}{4}a^4 + \tfrac{4}{3}a^3 + 2a^2\right)$$

$$-\tfrac{1}{2}a^4 + \tfrac{8}{3}a^3 + 4a^2 = 14.\overline{66}$$

Solved using Wolfram. Should I change it so they can solve easily without external help? $\longrightarrow$ $a = 1.46501$ or 6.46255

within bounds ✓ Out of bounds

Total force $F = \frac{40}{3}t$ is located at $y = 1.46501$

$$I_P\vec{\omega}_1 + \sum\int_{t_1}^{t_2}\vec{M}_A\,dt = I_A\vec{\omega}_2$$

$$0 + \int_0^2 \tfrac{40}{3}t(1.46501 + 0.25) = \left(\tfrac{1}{12}(20)(2^2 + 0.3^2) + (20)\left(\tfrac{2}{2} + 0.25\right)^2\right)\omega_2$$

$$\tfrac{40}{3}(1.71501)\left(\tfrac{1}{2}(2)^2\right) = \tfrac{571}{15}\omega_2$$

$$\boxed{\omega_2 = 1.201408056\ \text{rad/s}}$$