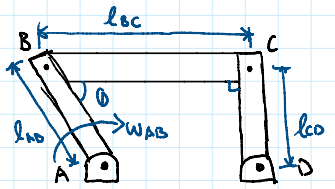
20-R-WE-DK-5

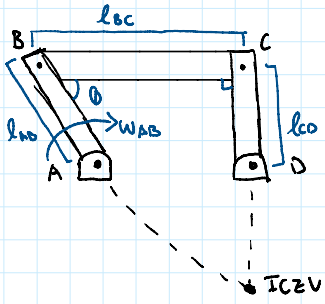
July 30, 2020 10:53 AM

20-R-WE-DK-5 Intermediate Kinetic Energy

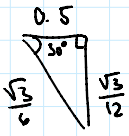
Inspiration: Example 2 - Kinetic Energy Mech 221 Notes

Students are testing a platform mechanism consisting of 3 linkages. If each linkage can be treated as a slender rod, determine the total kinetic energy of the mechanism. Each rod has a mass ***m = 5 kg*** and the lengths are given as ***l_AB = 0.4 m***, ***l_BC = 0.5 m***, and ***l_CD = 0.2m***. Rod AB forms an angle ***theta = 30 degrees*** with the horizontal. Rod AB rotates at an angular velocity of ***omega_AB = 5 rad/s***.





$\frac{1}{2}\ell_{AB} = \ell_{CD}$ for Webwork coding



$\cos 30° = \frac{0.5}{h}$ $\quad h = \frac{\sqrt{3}}{3}$

$\sin 30° = \frac{\theta}{h}$ $\quad \frac{\sqrt{3}}{3}\sin 30 = \theta = \frac{\sqrt{3}}{6}$

$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A} = 0 + -5\hat{k} \times (-0.4\cos 30\hat{i} + 0.4\sin 30\hat{j})$

$= 2\cos 30\hat{j} + 2\sin 30\hat{i}$

$\vec{V}_B = \vec{V}_{ICEV} + \vec{\omega}_{BC} \times \vec{r}_{B/IC} = 0 + \omega_{BC}\hat{k} \times (-0.5\hat{i} + \frac{\sqrt{3}}{6}\hat{j})$

$2\cos 30\hat{j} + 2\sin 30\hat{i} = -0.5\,\omega_{BC}\hat{j} - \frac{\sqrt{3}}{6}\omega_{BC}\hat{i}$

$\vec{\omega}_{BC} = -2\sqrt{3}\ \text{rad/s}\,\hat{k}$

$\vec{V}_C = \vec{V}_{ICEV} + \vec{\omega}_{BC} \times \vec{r}_{C/IC} = -2\sqrt{3}\hat{k} \times (\frac{\sqrt{3}}{6}\hat{j})$

$= 1\ \text{m/s}\,\hat{i}$

$\vec{V}_C = \vec{V}_D + \vec{\omega}_{CD} \times \vec{r}_{C/D} = 1\hat{i} = \omega_{CD}\hat{k} \times (0.2\hat{j})$ $\quad \omega_{CD} = -5\ \text{rad/s}\,\hat{k}$

Kinetic energy: $T_{tot} = T_{AB} + T_{BC} + T_{CD}$

$T_{AB} = \frac{1}{2} I_A \omega_{AB}^2 = \frac{1}{2}\left(\frac{1}{3}(5)(0.4)^2\right)(-5)^2 = \frac{10}{3}$

$T_{BC} = \frac{1}{2} I_{ICEV} \omega_{BC}^2 = \frac{1}{2}\left(\frac{1}{12}(5)(0.5)^2 + (5)\left(\left(\frac{\sqrt{3}}{6}\right)^2 + 0.25^2\right)\right)(-2\sqrt{3})^2 = \frac{35}{2}$

$T_{CD} = \frac{1}{2} I_D \omega_{CD}^2 = \frac{1}{2}\left(\frac{1}{3}(5)(0.2)^2\right)(-5)^2 = \frac{5}{6}$

$T_{tot} = 21.\overline{66}\ \text{J}$