20-R-IM-DK-11

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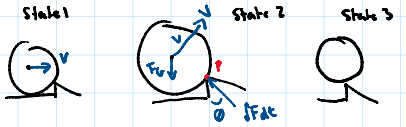
Intermediate Conservation of Momentum

Inspiration: Hibbeler pg. 542 Example

500 – 1 lb
4 – 8 cm

You are helping your mom with the groceries when you accidentally drop a can of beans. The 500 g can begins to roll on the ground and has a radius $r = 4.25$ cm. Assuming the can does not slip nor rebound, determine the minimum velocity needed for it to roll over a pebble with height $h = 0.7$ cm



Angular momentum about P is essentially conserved

$\hat{i} \times (\hat{i} + \hat{j})$ $\quad \hat{i} \times \hat{i} = 0$ Only $\hat{j}$ matters

$$(\vec{H}_P)_1 = (\vec{H}_P)_2$$

$$I_G \vec{\omega}_1 + \vec{r}_{G/P} \times m\vec{v}_G = I_G \vec{\omega}_2 + \vec{r}_{G/P} \times m\vec{v}_{G_2}$$

$$\frac{1}{2} m r^2 \omega_1 + (r - h) m v_{G_1} = \frac{1}{2} m r^2 \omega_2 + r m v_{G_2}$$

$$\frac{1}{2}(0.5)(0.0425)^2 \omega_1 + (0.0425 - 0.007)(0.5) v_{G_1} = \frac{1}{2}(0.5)(0.0425)^2 \omega_2 + 0.0425(0.5) v_{G_2}$$

No slipping, essentially pinned $v = \omega r$ $\quad \omega = \frac{v_G}{0.0425}$

$$\frac{1}{2}(0.5)(0.0425)^2 \frac{v_{G_1}}{0.0425} + (0.0425 - 0.007)(0.5) v_{G_1} = \frac{1}{2}(0.5)(0.0425)^2 \frac{v_{G_2}}{0.0425} + 0.0425(0.5) v_{G_2}$$

$$\frac{227}{8000} v_{G_1} = \frac{51}{1600} v_{G_2} \qquad v_{G_2} = \frac{227}{255} v_{G_1}$$

Taking the center of gravity at state 2 as the datum

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2} m v_{G_2}^2 + \frac{1}{2} I_G \omega_2^2 + 0 = 0 + mgh$$

$$\frac{1}{2}(0.5) v_{G_2}^2 + \frac{1}{2}(0.5)(0.0425)^2 \frac{v_{G_2}^2}{0.0425^2} = (0.5)(9.81)(0.007)$$

$$0.5\, v_{G_2}^2 = 0.034335 \qquad v_{G_2} = 0.262 \text{ m/s}$$

$$\boxed{v_{G_1} = 0.20437 \text{ m/s}}$$