eBook

A simplified summary of Differentiation and Integration (with basic rules and examples)

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﴿وَقُل رَّبِّ زِدْنِي عِلْمًا﴾

Introduction

1.What is Differentiation?

Differentiation is a branch of mathematics used to study the rate of change of a function.
In simple terms, it helps us understand how a quantity changes with respect to another (like how speed changes with time).
Example:
If a car's speed is changing over time, differentiation helps us find the instantaneous speed at any moment.

2.What is Integration?

Integration is the reverse process of differentiation.

It is used to find the total value or the area under a curve.

Example:

If you know the speed of a car over a period of time, integration can help calculate the total distance traveled.

3. Importance of Calculus

1. Calculus is widely used in engineering, physics, economics, medicine, and data science.
2. It helps analyze changing systems like motion, electricity flow, or market trends.
3. It forms the foundation of technologies like robotics, AI, and autonomous systems.

📘 Key Mathematical Concepts (with Definitions)

Linear Algebra

A branch of mathematics that studies vectors, matrices, and linear transformations.

فرع من الرياضيات يدرس المتجهات والمصفوفات والتحويلات الخطية.

Trigonometry (حساب المثلثات)

The study of angles and the relationships between the sides of triangles.

يدرس الزوايا والعلاقات بين أضلاع المثلثات.

Limits(النهايات)

The value a function approaches as the input gets closer to a certain point.

القيمة التي تقترب منها الدالة عندما يقترب المتغير من نقطة معينة

Continuity (الاتصال)

A function is continuous if its graph can be drawn without lifting the pen.

الدالة متصلة إذا أمكن رسم منحناها بدون رفع القلم

Differentiability (قابلية الاشتقاق)

A function is differentiable if it has a derivative at a given point.

الدالة قابلة للاشتقاق إذا كان لها مشتقة عند نقطة معينة

Differentiation:(الاشتقاق/التفاضل)

The process of finding the derivative of a function.

عملية إيجاد المشتقة لدالة

Integration :(التكامل)

The reverse process of differentiation; used to find area under curves.

العملية العكسية للاشتقاق، وتُستخدم لحساب المساحات تحت المنحنيات

Calculus :(التفاضل والتكامل)

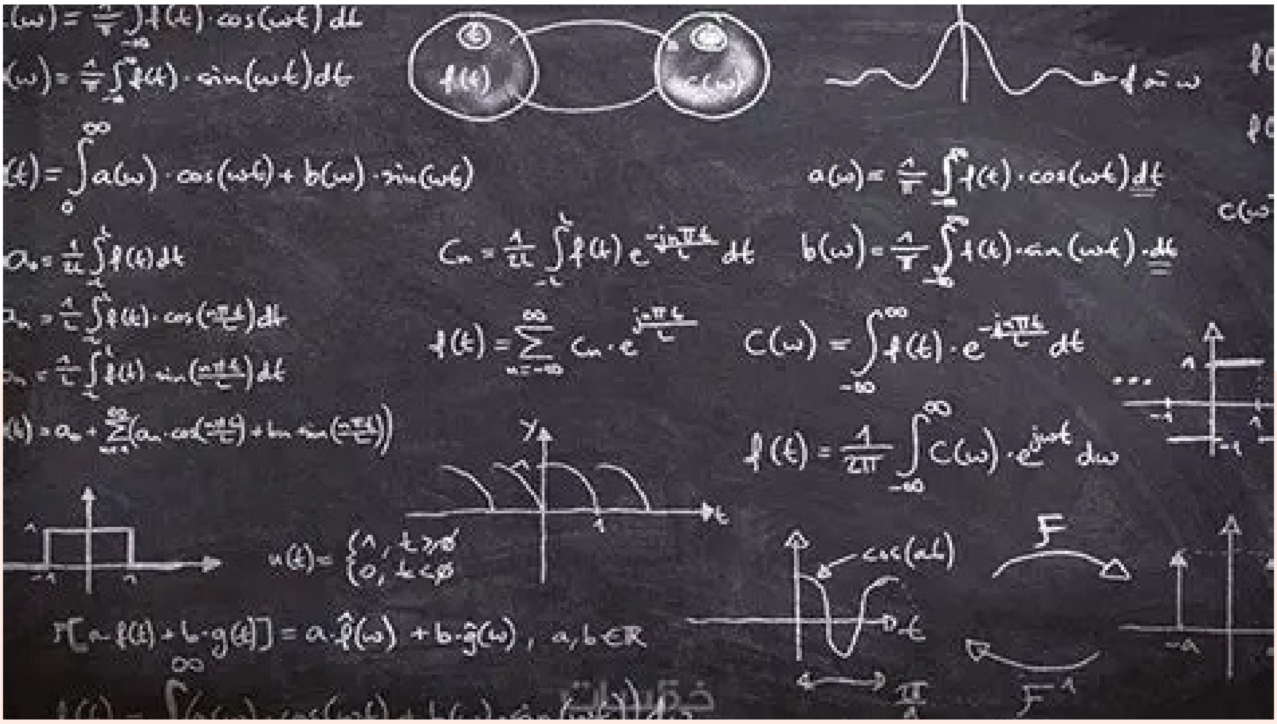
The branch of math dealing with limits, derivatives, integrals, and infinite series.

فرع الرياضيات الذي يهتم بالنهايات، المشتقات، التكاملات، والمتسلسلات اللانهائية

Functions: (الدوال)

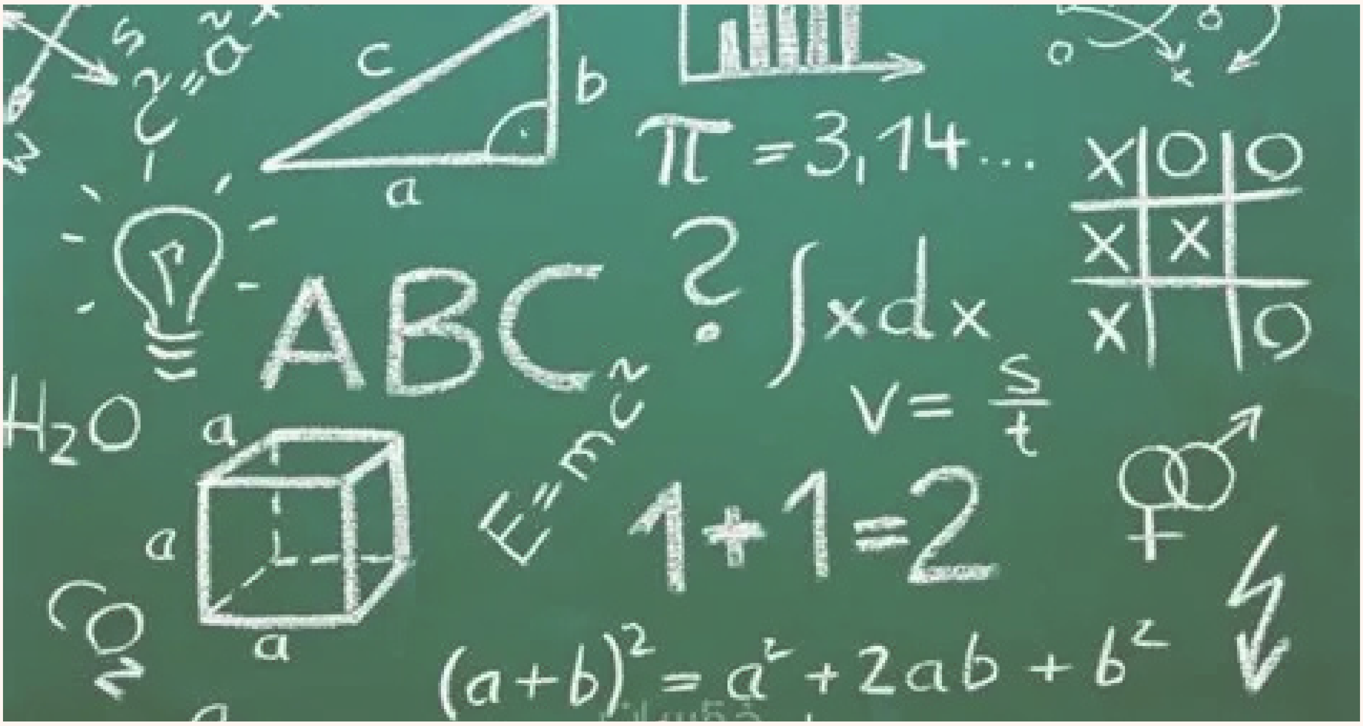
A relation between input (x) and output (f(x)) where each input has one output.

علاقة بين المدخلات والمخرجات بحيث يقابل كل مدخل مخرج واحد فقط



MODULE 2

Essential Math Formulas



This pages contains the most essential mathematical formulas and rules that every student needs.

It covers Linear Algebra, Trigonometry, Limits, Differentiation, Integration, and Continuity, along with practical examples for better understanding and quick revision.

Fonction	Fonction dérivée
a	0
ax	a
x^n	nx^{n-1}
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\ln(x)$	$\frac{1}{x}$
e^x	e^x
$\cos(x)$	$-\sin(x)$
$\sin(x)$	$\cos(x)$
$\tan(x)$	$1 + tan^2(x)$
$u^n(x)$	$n.u'(x).u^{n-1}(x)$
$\sqrt{u(x)}$	$\frac{u'(x)}{2\sqrt{u(x)}}$
$\ln(u(x))$	$\frac{u'(x)}{u(x)}$
$e^{u(x)}$	$u'(x).e^{u(x)}$
$\cos(u(x))$	$-u'(x).\sin(u(x))$
$\sin(u(x))$	$u'(x).\cos(u(x))$
$\tan(u(x))$	$u'(x).(1 + tan^2(u'(x))$
uxv	$\mathrm{u}'.\mathrm{v} + \mathrm{u}.\mathrm{v}'$
$\frac{u}{v}$	$\frac{\mathrm{u}'.\mathrm{v} - \mathrm{u}.\mathrm{v}'}{v^2}$
$u\ o\ v$	$(\mathrm{u}'\mathrm{o}\ \mathrm{v}).\mathrm{v}'$

$$\int \operatorname{sech}[x]\cdot\tanh[x]\ dx = -\operatorname{sech}[x]$$

$$\int \operatorname{cosech}[x]\cdot\coth[x]\ dx = -\coth[x]$$

$$\int \sin^{-1}[x]\ dx = x\cdot\sin^{-1}[x]+\sqrt{1-x^2}$$

$$\int \cos^{-1}[x]\ dx = x\cdot\cos^{-1}[x]-\sqrt{1-x^2}$$

$$\int \tan^{-1}[x]\ dx = x\cdot\tan^{-1}[x]-\frac{1}{2}\cdot\ln\left[1+x^2\right]$$

$$\int \cot^{-1}[x]\ dx = x\cdot\cot^{-1}[x]+\frac{1}{2}\cdot\ln\left[1+x^2\right]$$

$$\int \sec^{-1}[x]\ dx = x\cdot\sec^{-1}[x]-\cosh^{-1}[x]$$

$$\int \operatorname{cosec}^{-1}[x]\ dx = x\cdot\operatorname{cosec}^{-1}[x]+\cosh^{-1}[x]$$

$$\int \sinh^{-1}[x]\ dx = x\cdot\sinh^{-1}[x]-\sqrt{1+x^2}$$

$$\int \cosh^{-1}[x]\ dx = x\cdot\cosh^{-1}[x]-\sqrt{1+x^2}$$

$$\int \tanh^{-1}[x]\ dx = x\cdot\tanh^{-1}[x]+\frac{1}{2}\cdot\ln\left[1-x^2\right]$$

$$\int \coth^{-1}[x]\ dx = x\cdot\coth^{-1}[x]+\frac{1}{2}\cdot\ln\left[1-x^2\right]$$

$$\int \operatorname{sech}^{-1}[x]\ dx = x\cdot\operatorname{sech}^{-1}[x]+\sin^{-1}[x]$$

$$\int \operatorname{cosech}^{-1}[x]\ dx = x\cdot\operatorname{cosech}^{-1}[x]+\sinh^{-1}[x]$$

$$\int \frac{\sqrt{ax+b}}{x^2}\,dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\cdot\int\frac{dx}{x\cdot\sqrt{ax+b}}$$

$$\int \frac{1}{x^2\cdot\sqrt{ax+b}}\,dx = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\cdot\int\frac{dx}{x\cdot\sqrt{ax+b}}$$

$$\int \frac{1}{a^2+b^2}\,dx = \frac{1}{a}\cdot\tan^{-1}\left[\frac{x}{a}\right]$$

$$\int \frac{1}{\left[a^2+b^2\right]^2}\,dx = \frac{x}{2a^2\cdot\left[a^2+x^2\right]} + \frac{1}{2a^2}\cdot\tan^{-1}\left[\frac{x}{a}\right]$$

$$\int \frac{1}{a^2-b^2}\,dx = \frac{1}{2a}\cdot\ln\left|\frac{x+a}{x-a}\right|$$

$$\int \frac{1}{\left[a^2-b^2\right]^2}\,dx = \frac{x}{2a^2\cdot\left[a^2-x^2\right]} + \frac{1}{2a^2}\cdot\int\frac{dx}{a^2-x^2}$$

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \begin{cases} \sinh^{-1}\left[\frac{x}{a}\right] \\ \ln\left|x+\sqrt{a^2+x^2}\right| \end{cases}$$

$$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\cdot\sqrt{a^2+x^2} + \frac{a^2}{2}\cdot\sinh^{-1}\left[\frac{x}{a}\right]$$

$$\int x^2\cdot\sqrt{a^2+x^2}\,dx = \frac{x\cdot\left(a^2+2x^2\right)\cdot\sqrt{a^2+x^2}}{8} - \frac{a^4}{8}\cdot\sinh^{-1}\left[\frac{x}{a}\right]$$

$$\int \frac{\sqrt{a^2+x^2}}{x}\,dx = \sqrt{a^2+x^2} - a\cdot\sinh^{-1}\left[\frac{a}{x}\right]$$

$$\int \frac{\sqrt{a^2+x^2}}{x^2}\,dx = \sinh^{-1}\left[\frac{a}{x}\right] - \frac{\sqrt{a^2+x^2}}{x}$$

$$\int \frac{f'[x]}{f[x]\cdot\sqrt{a^2-f^2[x]}}\,dx = \frac{1}{a}\cdot\operatorname{sech}^{-1}\left[\frac{f[x]}{a}\right]$$

$$\int \frac{f'[x]}{f[x]\cdot\sqrt{a^2+f^2[x]}}\,dx = -\frac{1}{a}\cdot\operatorname{cosech}^{-1}\left[\frac{f[x]}{a}\right]$$

$$\int \frac{1}{\sqrt{x^2+a}}\,dx = \ln|x+\sqrt{x^2+a}|$$

$$\int \frac{x}{\sqrt{x^2+a}}\,dx = \sqrt{x^2+a}$$

$$\int e^{ax}\,dx = \frac{1}{a}\cdot e^{ax}$$

$$\int \frac{1}{\sqrt{k^2-m^2x^2}}\,dx = \frac{1}{m}\cdot\sin^{-1}\left[\frac{mx}{k}\right]$$

$$\int \frac{1}{k^2+m^2x^2}\,dx = \frac{1}{km}\cdot\tan^{-1}\left[\frac{mx}{k}\right]$$

$$\int \sqrt{x^2+m}\,dx = \frac{x}{2}\cdot\sqrt{x^2+m}+\frac{m}{2}\cdot\ln|x+\sqrt{x^2+m}|$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx = \frac{\ln\left[2\cdot\sqrt{a}\cdot\sqrt{ax^2+bx+c}+2ax+b\right]}{\sqrt{a}}$$

$\int \frac{mx+n}{\sqrt{ax^2+bx+c}}\,dx =$ ؟؟؟ متروك للطالب

$$\int \sin[ax]\cdot\cos[bx]\,dx = -\frac{1}{2}\cdot\left[\frac{\cos(a+b)x}{a+b}+\frac{\cos(a-b)x}{a-b}\right]$$

$$\int e^{ax}\cdot\cos[bx]\,dx = \frac{e^{ax}}{a^2+b^2}\cdot\left[a\cdot\cos[bx]-b\cdot\sin[bx]\right]$$

$$\int x^n\cdot\ln[x]\,dx = \frac{x^{n+1}}{(n+1)^2}\cdot\left(\left[(n+1)\cdot\ln(x)\right]-1\right)$$

$$\int [ax+b]^n\,dx = \frac{[ax+b]^{n+1}}{a\cdot[n+1]}$$

$$\int [ax+b]^{-1}\,dx = \frac{1}{a}\cdot\ln|ax+b|$$

$$\int x\cdot[ax+b]^n\,dx = \frac{[ax+b]^{n+1}}{a^2}\cdot\left[\frac{ax+b}{n+2}-\frac{b}{n+1}\right]$$

$$\int x\cdot[ax+b]^{-1}\,dx = \frac{x}{a}-\frac{b}{a^2}\cdot\ln|ax+b|$$

$$\int x\cdot[ax+b]^{-2}\,dx = \frac{1}{a^2}\cdot\left[\ln|ax+b|+\frac{b}{ax+b}\right]$$

$$\int \frac{1}{x\cdot[ax+b]}\,dx = \frac{1}{b}\cdot\ln\left|\frac{x}{ax+b}\right|$$

$$\int \left[\sqrt{ax+b}\right]^n\,dx = \frac{2}{a}\cdot\frac{\left[\sqrt{ax+b}\right]^{n+2}}{n+2}$$

$$\int \frac{\sqrt{ax+b}}{x}\,dx = 2\cdot\sqrt{ax+b}+b\cdot\int\frac{dx}{x\cdot\sqrt{ax+b}}$$

$$\int \frac{1}{x\cdot\sqrt{ax+b}}\,dx = \begin{cases}\dfrac{2}{\sqrt{(-b)}}\cdot\tan^{-1}\left[\sqrt{\dfrac{ax+b}{-b}}\right]\\[2ex] \dfrac{1}{b}\cdot\ln\left|\dfrac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right|\end{cases}$$

الدوال المثلثية لبعض الزوايا الخاصة

	0 أو 360	30	45	60	90	180	270	
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	جا
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	جتا
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	ظا

$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$	$\tan(-\theta) = \tan\theta$
$\sin(90-\theta) = \cos\theta$	$\cos(90-\theta) = \sin\theta$	$\tan(90-\theta) = \cot\theta$
$\sin(90+\theta) = \cos\theta$	$\cos(90+\theta) = -\sin\theta$	$\tan(90+\theta) = -\tan\theta$
$\sin(180-\theta) = \sin\theta$	$\cos(180-\theta) = -\cos\theta$	$\tan(180-\theta) = -\tan\theta$
$\sin(180+\theta) = -\sin\theta$	$\cos(180+\theta) = -\cos\theta$	$\tan(180+\theta) = -\tan\theta$
$\sin(360-\theta) = -\sin\theta$	$\cos(360-\theta) = \cos\theta$	$\tan(360-\theta) = -\tan\theta$

والجدول السابق صحيح لمقلوبات الدوال المثلثية أيضا.

جا sin – (مقابل÷وتر) و جتا cos – (مجاور÷وتر) و ظا tan – (مقابل÷مجاور)

المتطابقات المثلثية التى قيمتها – ١

$\sin\theta \operatorname{cosec}\theta = 1$	$\cos\theta \sec\theta = 1$	$\tan\theta \cot\theta = 1$
$\cosh^2\theta - \sinh^2\theta = 1$	$\sin^2\theta + \cos^2\theta = 1$	$\sec^2\theta - \tan^2\theta = 1$
$2\cosh^2\theta - \cosh[2\theta] = 1$	$\cosh[2\theta] - 2\sinh^2\theta = 1$	$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

قوانين مجموع زاويتين تحت تأثير دالة ما

$$\sin(a \pm b) = \sin(a)\cdot\cos(b) \pm \cos(a)\cdot\sin(b)$$

$$\cos(a \pm b) = \cos(a)\cdot\cos(b) \mp \sin(a)\cdot\sin(b)$$

$$\tan(a \pm b) = \frac{\tan(a) \pm \tan(b)}{1 \mp \tan(a)\cdot\tan(b)} \quad , \cot(a \pm b) = \frac{\cot(a)\cdot\cot(b) \mp 1}{\cot(a) \pm \cot(b)}$$

$$\sinh(a \pm b) = \sinh(a)\cdot\sinh(b) \pm \cosh(a)\cdot\cosh(b)$$

$$\cosh(a \pm b) = \cosh(a)\cdot\cosh(b) \pm \sinh(a)\cdot\sinh(b)$$

Differentiation

$(cu)' = cu' \qquad (c \text{ constant})$

$(u + v)' = u' + v'$

$(uv)' = u'v + uv'$

$\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

$\dfrac{du}{dx} = \dfrac{du}{dy} \cdot \dfrac{dy}{dx} \qquad$ (Chain rule)

$(x^n)' = nx^{n-1}$

$(e^x)' = e^x$

$(e^{ax})' = ae^{ax}$

$(a^x)' = a^x \ln a$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\tan x)' = \sec^2 x$

$(\cot x)' = -\csc^2 x$

$(\sinh x)' = \cosh x$

$(\cosh x)' = \sinh x$

$(\ln x)' = \dfrac{1}{x}$

$(\log_a x)' = \dfrac{\log_a e}{x}$

$(\arcsin x)' = \dfrac{1}{\sqrt{1 - x^2}}$

Integration

$\int uv'\,dx = uv - \int u'v\,dx$ (by parts)

$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c \qquad (n \neq -1)$

$\int \dfrac{1}{x}\,dx = \ln|x| + c$

$\int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$

$\int \sin x\,dx = -\cos x + c$

$\int \cos x\,dx = \sin x + c$

$\int \tan x\,dx = -\ln|\cos x| + c$

$\int \cot x\,dx = \ln|\sin x| + c$

$\int \sec x\,dx = \ln|\sec x + \tan x| + c$

$\int \csc x\,dx = \ln|\csc x - \cot x| + c$

$\int \dfrac{dx}{x^2 + a^2} = \dfrac{1}{a}\arctan\dfrac{x}{a} + c$

$\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \arcsin\dfrac{x}{a} + c$

$\int \dfrac{dx}{\sqrt{x^2 + a^2}} = \operatorname{arcsinh}\dfrac{x}{a} + c$

$\int \dfrac{dx}{\sqrt{x^2 - a^2}} = \operatorname{arccosh}\dfrac{x}{a} + c$

$\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$

$\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$

$\int \tan^2 x\,dx = \tan x - x + c$

$\int \cot^2 x\,dx = -\cot x - x + c$

$\int \ln x\,dx = x\ln x - x + c$

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p, p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

المصادر والمراجع | Sources & References

Free Online Courses:

Khan Academy - Mathematics

MIT OpenCourseWare - Calculus – MIT

Coursera - Mathematics for Machine Learning

Formulas & Cheat Sheets:

Paul's Online Math Notes – شرح وقوانين مع أمثلة محلولة

MathWorld (Wolfram) – موسوعة رياضيات تفاعلية

Practice & Problem Solving:

Brilliant Math – مسائل تفاعلية

Project Euler – مسائل رياضية للتفكير العميق

Limits & Continuity

Differentiation

Trigonometry

Linear Algebra